Computer Algebra Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.3-Miscellaneous/52-1.3.2-Algebraic-functions

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [1025]. This is test number [52].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.41 (1019)	0.59 (6)
Mathematica	98.44 (1009)	1.56 (16)
Maple	82.34 (844)	17.66 (181)
Fricas	81.76 (838)	18.24 (187)
Giac	50.63 (519)	49.37 (506)
Mupad	44.39 (455)	55.61 (570)
Maxima	35.02 (359)	64.98 (666)
Sympy	31.41 (322)	68.59 (703)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

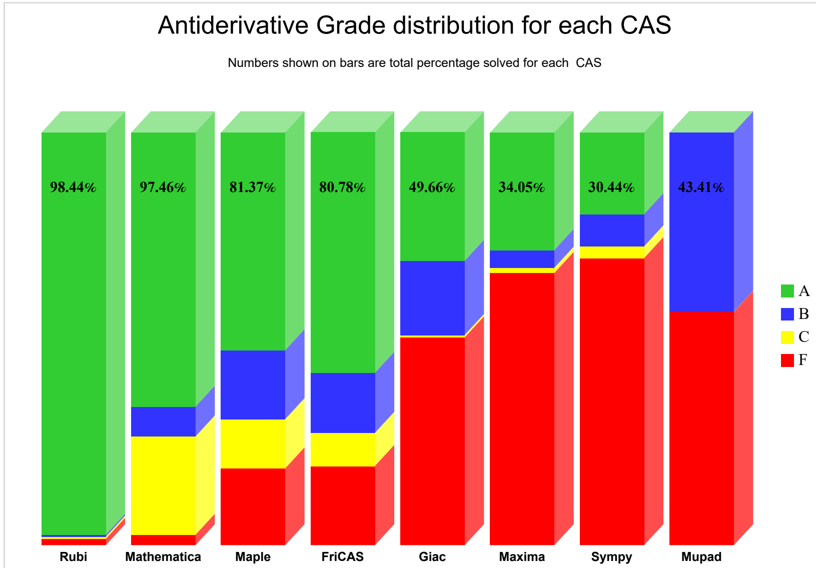
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

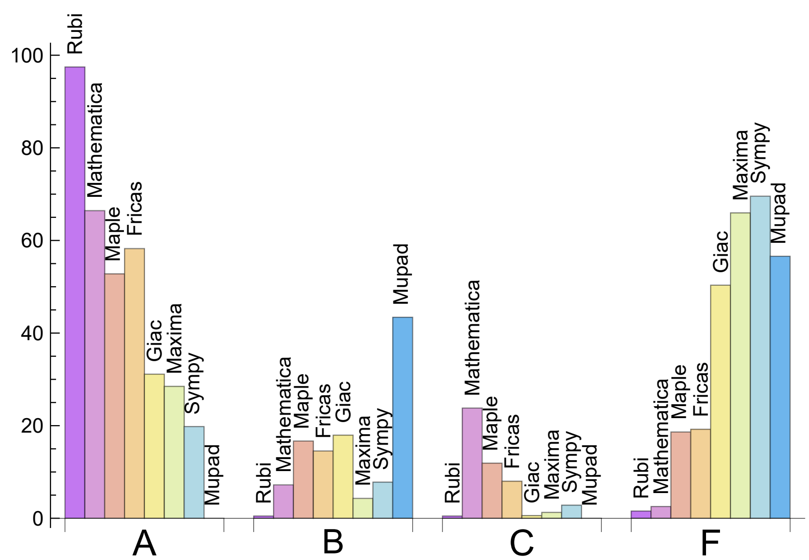
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.463	0.488	0.488	1.561
Mathematica	66.439	7.220	23.805	2.537
Fricas	58.244	14.537	8.000	19.220
Maple	52.780	16.683	11.902	18.634
Giac	31.122	17.951	0.585	50.341
Maxima	28.488	4.293	1.268	65.951
Sympy	19.805	7.805	2.829	69.561
Mupad	0.000	43.415	0.000	56.585

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	6	100.00	0.00	0.00
Mathematica	16	100.00	0.00	0.00
Maple	181	100.00	0.00	0.00
Fricas	187	52.94	40.11	6.95
Giac	506	76.09	10.08	13.83
Mupad	570	0.00	100.00	0.00
Maxima	666	93.39	0.00	6.61
Sympy	703	87.06	11.10	1.85

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.14
Maxima	0.23
Giac	0.57
Fricas	0.63
Maple	1.86
Mathematica	3.48
Sympy	3.69
Mupad	14.17

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	82.99	1.09	38.00	0.90
Rubi	143.91	1.02	89.00	1.00
Giac	220.05	2.02	59.00	1.20
Mupad	229.19	2.71	50.00	1.09
Mathematica	258.12	1.57	81.00	1.00
Fricas	263.45	2.38	81.00	1.28
Maple	381.04	2.73	74.00	1.01
Sympy	956.83	4.60	63.00	1.06

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

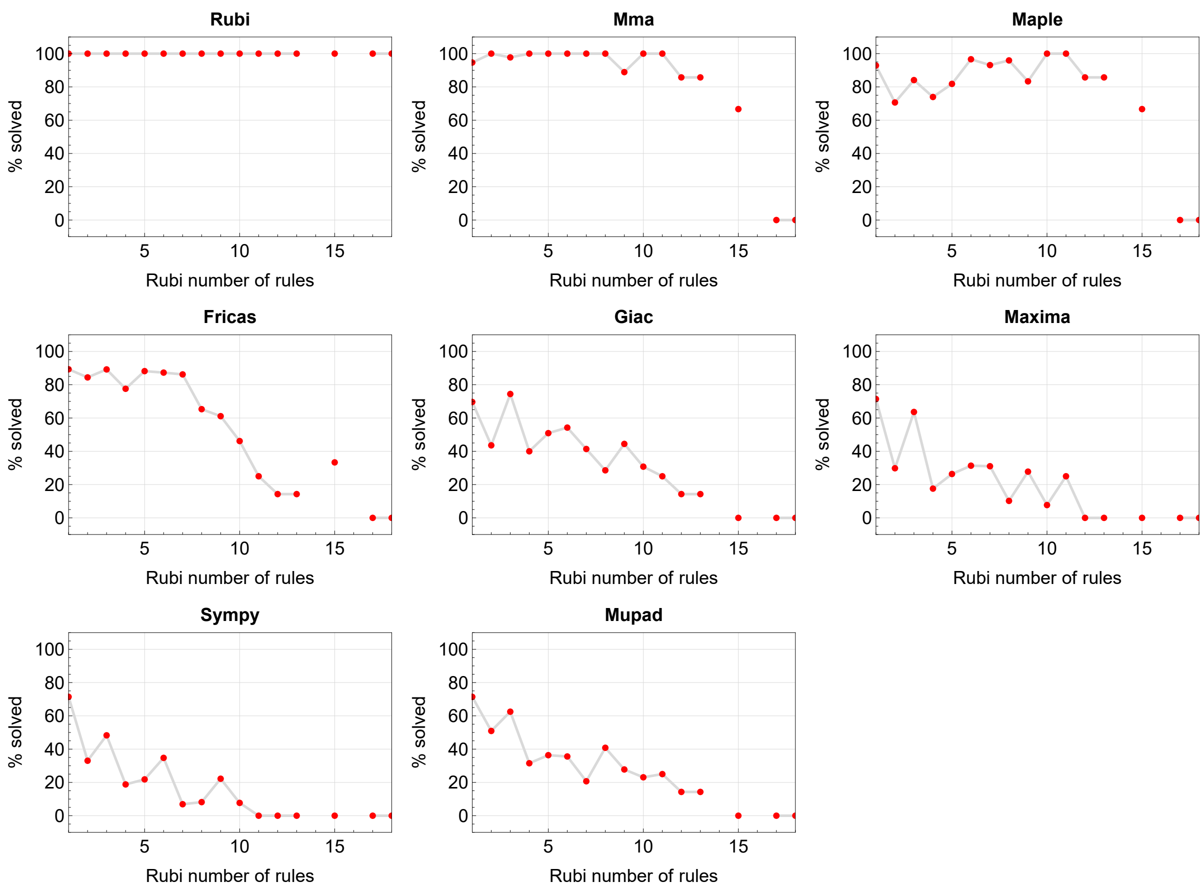


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

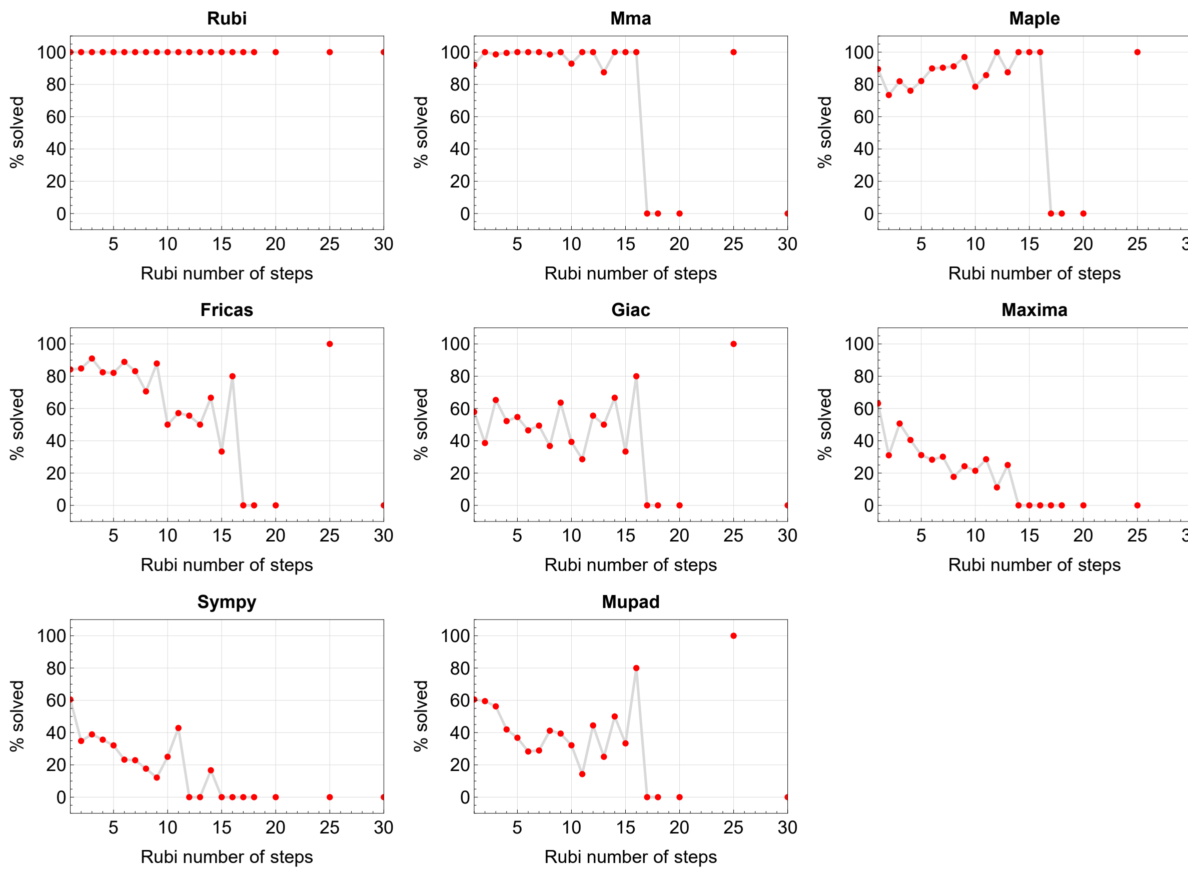


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

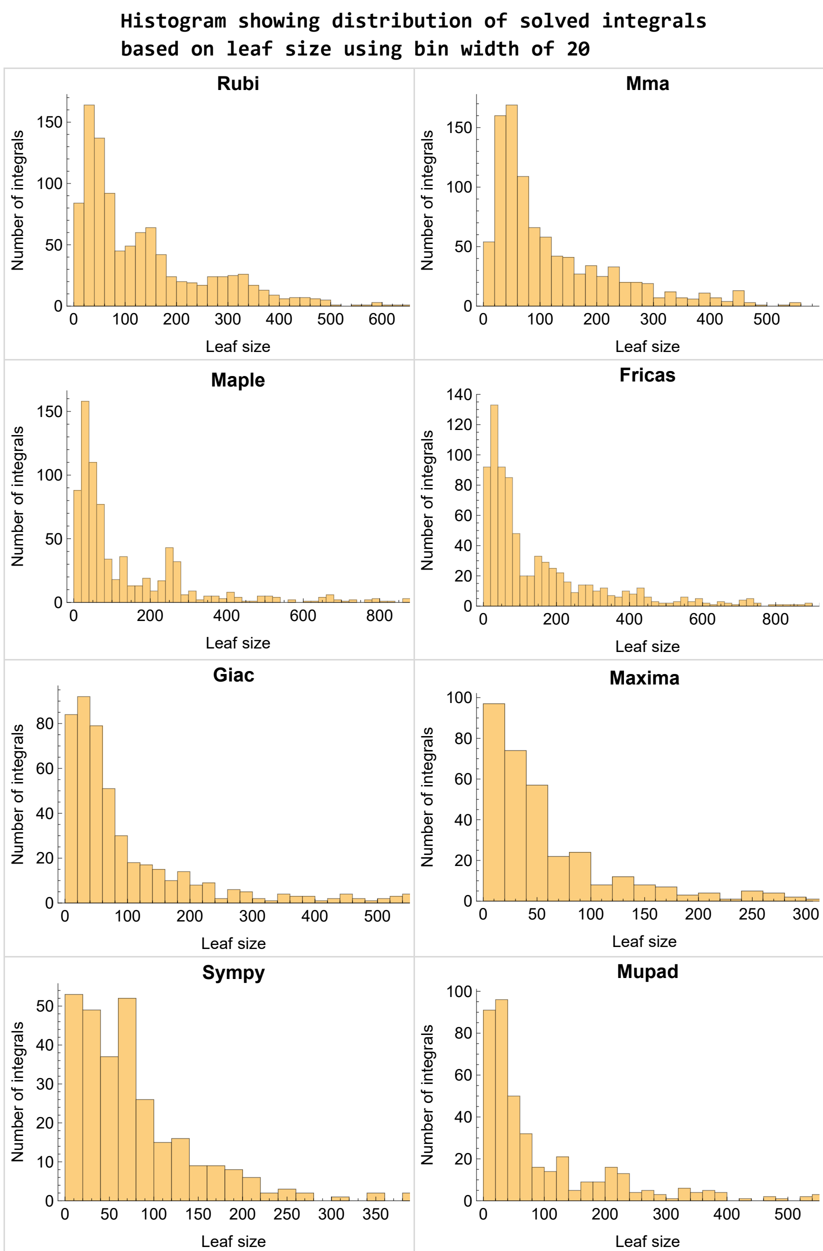


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

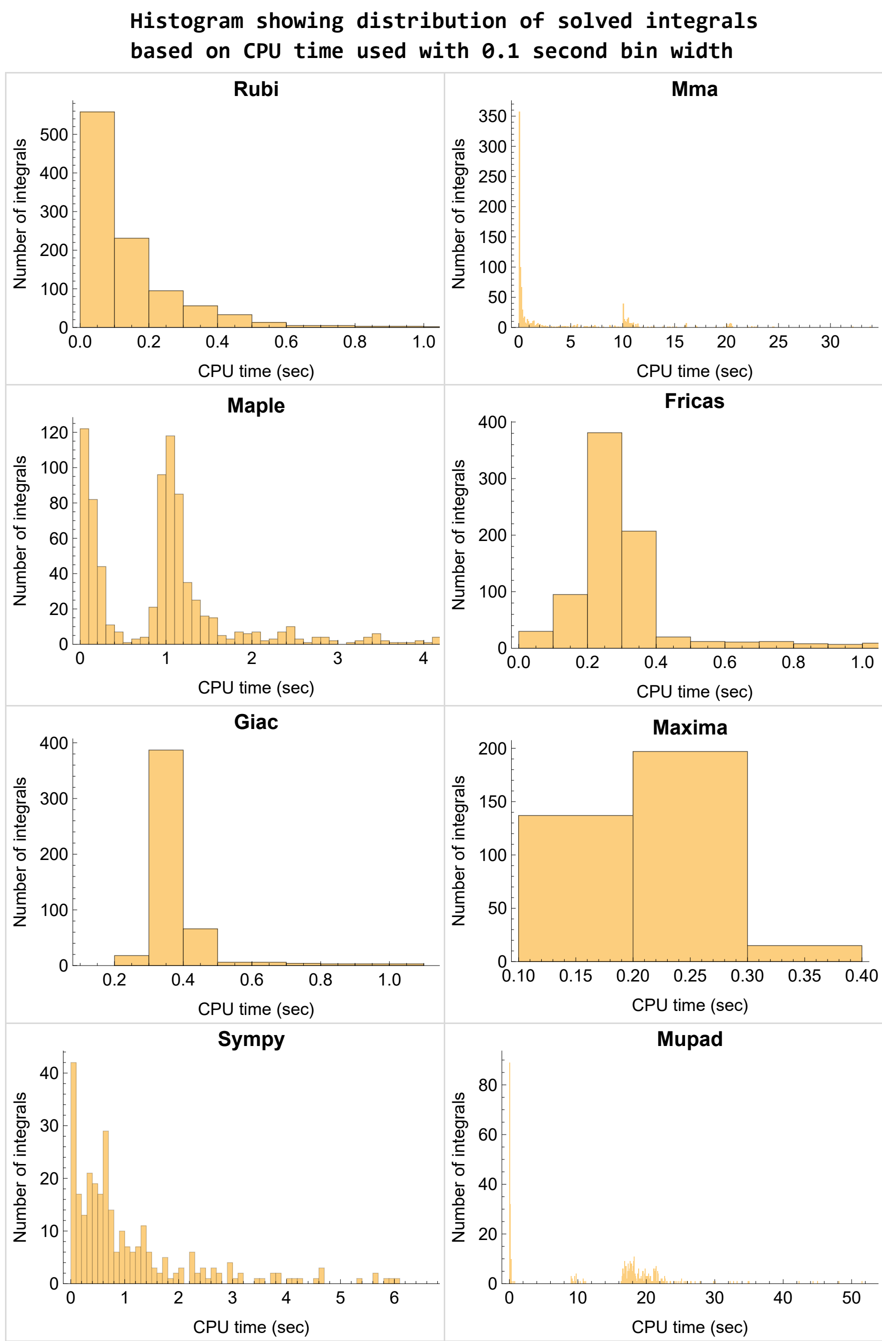


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

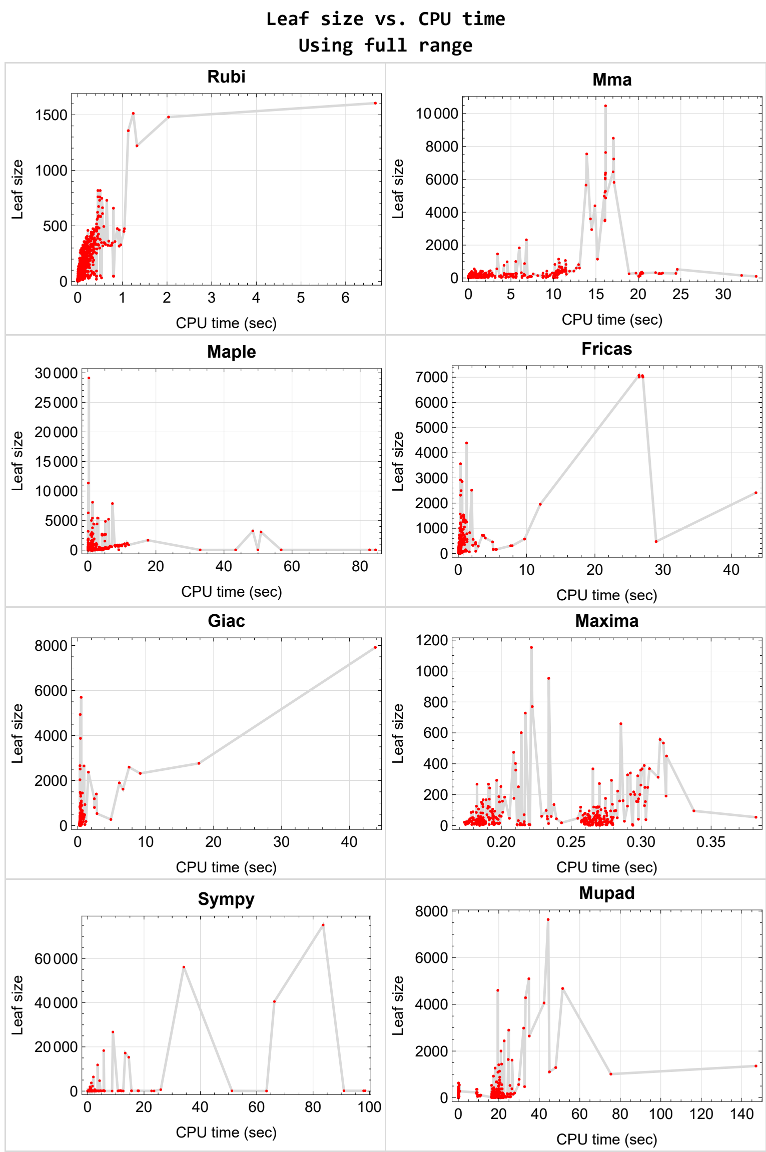


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{901, 902, 903, 904, 905, 906, 907, 908, 909, 910\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{196, 226\}$

Mathematica $\{1, 2, 3, 4, 9, 10, 11, 12, 13, 14, 15, 16, 17, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 70, 71, 72, 73, 83, 84, 85, 86, 109, 110, 111, 112, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 139, 144, 145, 146, 147, 148, 149, 150, 151, 164, 165, 166, 167, 196, 559, 561, 562, 764, 765, 766, 767, 768, 769, 770, 772, 773, 774, 775, 777, 778, 780, 796, 797, 798, 799, 800, 801, 802, 803, 804, 1009\}$

Maple $\{170, 557, 558, 559, 560, 561, 580, 774, 775, 777, 778, 780, 781, 782, 784, 785, 786, 787, 789, 790, 791, 792, 793, 794, 795, 796, 800, 803, 838, 997, 998, 1016, 1017, 1024\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
```

```
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

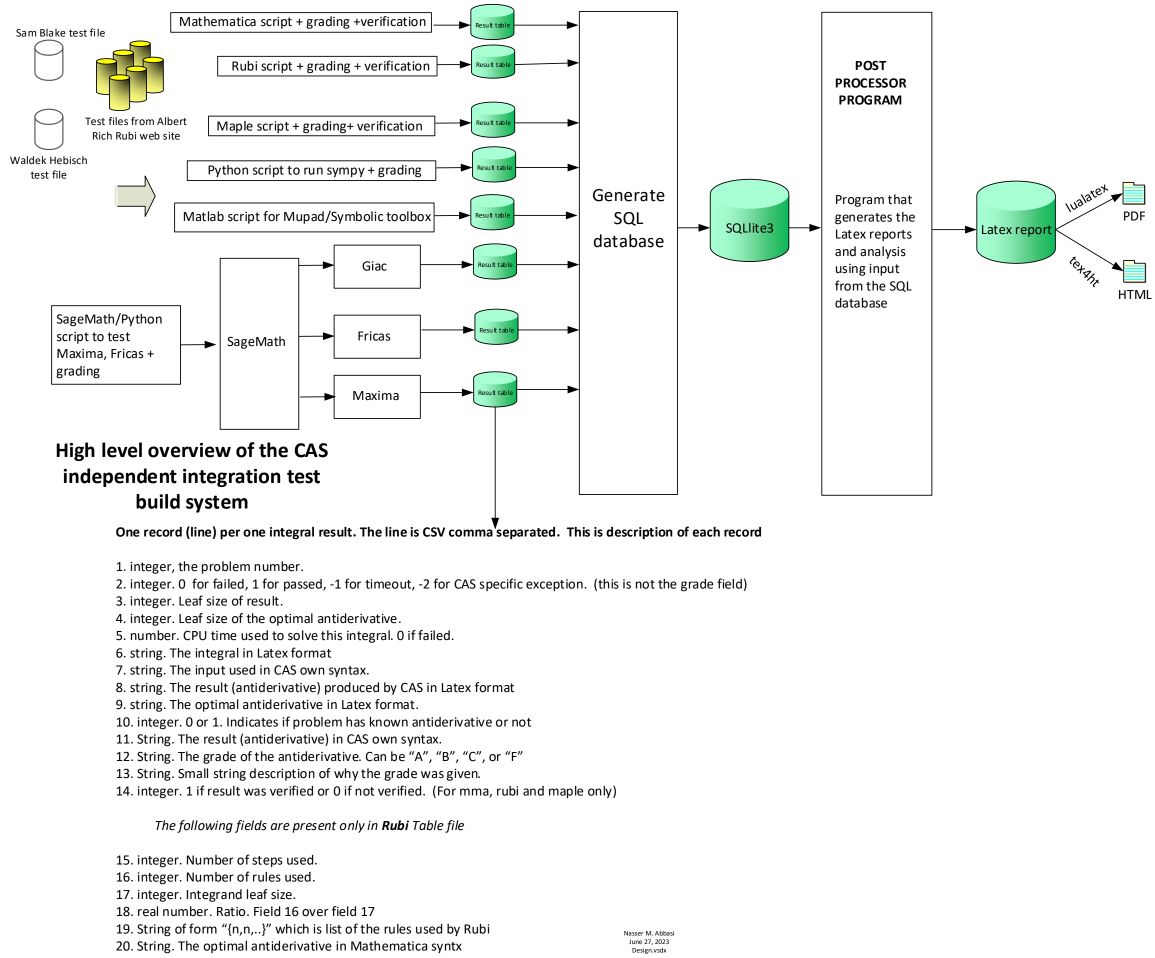
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1019, 1020, 1024 }

B grade { 413, 726, 997, 1017, 1025 }

C grade { 396, 941, 1018, 1021, 1022 }

F normal fail { 197, 616, 617, 995, 996, 1023 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 23, 24, 25, 26, 29, 30, 31, 32, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 74, 75, 76, 77, 78, 79, 80, 81, 82, 101, 102, 103, 104, 105, 106, 107, 108, 113, 114, 115, 116, 117, 118, 119, 120, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 198, 199, 200, 201, 202, 203, 204, 205, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 259, 261, 263, 264, 265, 266, 267, 268, 269, 272, 273, 276, 277, 278, 279, 280, 281, 282, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 304, 305, 307, 308, 309, 310, 311, 312, 318, 319, 320, 321, 322, 323, 324, 327, 328, 331, 332, 333, 334, 335, 336, 337, 344, 345, 346, 347, 348, 349, 352, 353, 355, 356, 357, 358, 359, 360, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 382, 384, 385, 388, 390, 393, 394, 395, 396, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 424, 427, 428, 429, 430, 432, 433, 434, 435, 436, 437, 439, 443, 444, 445, 446, 449, 452, 453, 454, 455, 456, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 528, 529, 530, 531, 534, 535, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 579, 581, 582, 583, 584, 585, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 733, 734, 735, 736, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 842, 848, 849, 850, 851, 854, 855, 856, 858, 859, 860, 862, 863, 864, 865, 866, 867, 872, 874, 876, 877, 878, 879, 880, 881, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 915, 916, 917, 920, 921, 922, 923, 924, 925, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 974, 975, 976, 977, 979, 980, 982, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 1000, 1001, 1006, 1007, 1008, 1015, 1016, 1017, 1020, 1023, 1024, 1025 }

B grade { 422, 423, 425, 426, 438, 440, 441, 447, 448, 450, 451, 457, 479, 524, 562, 681, 682, 729, 730, 731, 732, 737, 738, 739, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 839, 840, 841, 843, 844, 845, 846, 847, 852, 853, 857, 861, 868, 869, 870, 871, 873, 875, 918, 919, 926, 927, 961, 973, 978, 981, 1009, 1010, 1011, 1012, 1013, 1014 }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 109, 110, 111, 112, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 196, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 255, 256, 257, 258, 260, 262, 270, 271, 274, 275, 283, 284, 285, 286, 287, 288, 302, 303, 306, 313, 314, 315, 316, 317, 325, 326, 329, 330, 338, 339, 340, 341, 342, 343, 350, 351, 354, 361, 362, 363, 364, 365, 378, 380, 381, 383, 386, 387, 389, 391, 392, 397, 431, 442, 523, 525, 526, 527, 532, 533, 536, 537, 574, 580, 601, 603, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 882, 883, 884, 911, 912, 913, 914, 942, 984, 998, 999, 1002, 1003, 1004, 1005, 1018, 1019, 1021, 1022 }

F normal fail { 19, 20, 21, 22, 27, 28, 33, 34, 35, 40, 41, 42, 173, 195, 197, 587 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 10, 11, 12, 13, 14, 15, 16, 17, 53, 54, 55, 57, 58, 59, 66, 68, 84, 85, 86, 93, 127, 128, 129, 130, 137, 138, 144, 145, 146, 147, 148, 149, 150, 151, 154, 158, 160, 161, 162, 163, 164, 165, 166, 167, 176, 196, 200, 201, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 259, 261, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 298, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 316, 317, 318, 319, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 336, 337, 340, 344, 345, 349, 350, 351, 352, 353, 354, 355, 356, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 382, 384, 385, 388, 390, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 423, 425, 426, 427, 428, 429, 430, 431, 433, 438, 439, 440, 441, 444, 445, 446, 448, 450, 451, 455, 456, 474, 475, 498, 504, 523, 524, 525, 530, 531, 538, 539, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 678, 679, 680, 682, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 710, 711, 712, 713, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 727, 728, 729, 731, 733, 734, 735, 736, 738, 741, 742, 743, 744, 748, 749, 750, 751, 752, 756, 757, 758, 762, 763, 805, 806, 807, 808, 809, 810, 813, 814, 815, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 833, 834, 835, 836, 837, 839, 841, 842, 844, 845, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 858, 860, 864, 865, 868, 870, 872, 874, 877, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 893, 894, 895, 896, 897, 898, 899, 900, 915, 916, 917, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 943, 944, 945, 946, 947, 948, 949, 950, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 967, 969, 970, 971, 972, 975, 977, 979, 980, 981, 982, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 1004, 1005, 1006, 1007, 1009, 1010, 1019, 1020, 1023 }

B grade { 9, 52, 56, 64, 65, 67, 73, 74, 75, 83, 91, 92, 94, 95, 100, 125, 126, 135, 136, 139, 174, 175, 178, 179, 180, 182, 183, 184, 260, 262, 266, 279, 284, 299, 310, 320, 321, 322, 333, 334, 335, 338, 339, 341, 342, 343, 346, 347, 348, 357, 358, 365, 408, 413, 422, 424, 432, 442, 443, 447, 449, 452, 454, 457, 458, 459, 473, 476, 477, 478, 487, 528, 529, 534, 535, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 631, 651, 652, 681, 683, 726, 730, 732, 737, 739, 740, 745, 746, 747, 753, 759, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 811, 812, 816, 832, 840, 843, 846, 857, 859, 861, 862, 863, 867, 869, 871, 873, 875, 878, 918, 919, 920, 921, 965, 966, 974, 976, 978, 983, 984, 1015, 1025 }

C grade { 21, 22, 43, 44, 45, 46, 51, 76, 77, 82, 101, 102, 103, 104, 113, 114, 115, 116, 152, 153, 155, 156, 157, 159, 168, 169, 170, 171, 198, 199, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 226, 293, 378, 380, 381, 383, 386, 387, 389, 391, 392, 406, 407, 409, 410, 434, 435, 436, 437, 468, 469, 470, 485, 486, 526, 527, 532, 533, 536, 537, 557, 558, 559, 560, 561, 675, 676, 677, 709, 714, 754, 755, 760, 761, 796, 797, 798, 799, 800, 801, 802, 803, 804, 831, 838, 866, 892, 951, 968, 973, 997, 998, 1002, 1003, 1016, 1017, 1018, 1021, 1022, 1024 }

F normal fail { 5, 6, 7, 8, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 47, 48, 49, 50, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 87, 88, 89, 90, 96, 97, 98, 99, 105, 106, 107, 108, 109, 110, 111, 112, 117, 118, 119, 120, 121, 122, 123, 124, 131, 132, 133, 134, 140, 141, 142, 143, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 224, 225, 253, 254, 255, 256, 257, 258, 453, 460, 461, 462, 463, 464, 465, 466, 467, 471, 472, 479, 480, 481, 482, 483, 484, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 562, 563, 564, 587, 588, 589, 590, 591, 592, 593, 594, 653, 654, 655, 656, 657, 876, 911, 912, 913, 914, 942, 995, 996, 999, 1000, 1001, 1008, 1011, 1012, 1013, 1014 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 77, 103, 104, 105, 107, 109, 111, 113, 114, 117, 119, 121, 123, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 212, 213, 214, 215, 219, 220, 221, 222, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 349, 350, 351, 352, 353, 354, 355, 356, 359, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 376, 377, 379, 382, 384, 385, 390, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 454, 455, 456, 457, 460, 461, 462, 463, 464, 466, 467, 468, 469, 470, 471, 473, 474, 475, 476, 479, 480, 481, 482, 485, 486, 487, 490, 491, 492, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 514, 515, 516, 518, 519, 520, 522, 524, 525, 528, 529, 530, 531, 534, 535, 538, 539, 540, 541, 542, 543, 544, 547, 548, 549, 551, 552, 553, 555, 556, 564, 565, 567, 568, 569, 570, 571, 572, 573, 575, 576, 578, 579, 581, 582, 583, 584, 585, 586, 595, 596, 597, 599, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 646, 647, 648, 649, 656, 658, 659, 660, 661, 662, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 685, 686, 691, 692, 693, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 711, 712, 713, 714, 715, 716, 717, 718, 719, 722, 723, 724, 725, 726, 727, 728, 731, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 751, 752, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 769, 770, 805, 806, 807, 808, 809, 810, 811, 812, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 835, 836, 837, 838, 848, 849, 850, 851, 852, 853, 854, 856, 858, 860, 862, 867, 872, 874, 877, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 911, 912, 915, 916, 917, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 931, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 960, 961, 962, 963, 964, 965, 966, 967, 969, 970, 971, 972, 975, 977, 979, 980, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 1004, 1005, 1007, 1009, 1010, 1011, 1012, 1013, 1014, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025 }

B grade { 21, 22, 43, 44, 45, 46, 51, 74, 75, 76, 80, 81, 82, 101, 102, 106, 108, 110, 112, 115, 116, 118, 120, 122, 124, 174, 175, 176, 178, 179, 180, 182, 183, 184, 205, 232, 238, 310, 321, 322, 347, 348, 357, 358, 360, 367, 368, 374, 375, 388, 408, 413, 422, 447, 458, 459, 465, 477, 478, 483, 484, 526, 527, 532, 533, 536, 537, 545, 546, 550, 554, 566, 574, 580, 598, 629, 630, 631, 644, 645, 650, 651, 652, 653, 654, 655, 663, 681, 682, 683, 684, 687, 688, 689, 690, 694, 710, 720, 721, 729, 730, 732, 750, 753, 759, 832, 833, 834, 839, 840, 841, 842, 843, 844, 845, 846, 847, 859, 861, 863, 864, 865, 866, 868, 869, 870, 871, 873, 875, 878, 879, 918, 919, 935, 957, 958, 959, 968, 973, 974, 976, 978, 981, 997, 999, 1006, 1015, 1016, 1017 }

C grade { 1, 2, 9, 10, 11, 12, 13, 52, 53, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 73, 83, 84, 85, 86, 91, 92, 93, 94, 95, 100, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 152, 153, 154, 155, 156, 157, 158, 159, 168, 169, 170, 171, 378, 380, 381, 383, 387, 389, 391, 392, 397, 577, 601, 766, 767, 768, 771, 772, 773, 813, 855, 857, 998 }

F normal fail { 14, 15, 16, 17, 24, 25, 26, 31, 32, 37, 38, 39, 144, 148, 160, 161, 162, 163, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 210, 224, 225, 255, 256, 257, 386, 453, 488, 489, 493, 494, 499, 500, 505, 506, 510, 511, 513, 517, 521, 562, 563, 587, 588, 589, 590, 591, 592, 593, 594, 657, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 876, 1002, 1003 }

F(-1) timedout fail { 5, 6, 7, 8, 18, 19, 20, 23, 27, 28, 29, 30, 33, 34, 35, 36, 40, 41, 42, 47, 48, 49, 50, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 87, 88, 89, 90, 96, 97, 98, 99, 145, 146, 149, 150, 164, 165, 166, 167, 173, 211, 216, 217, 218, 223, 226, 253, 254, 472, 523, 557, 558, 560, 561, 613, 614, 615, 616, 617, 913, 914, 930, 996, 1000, 1001 }

F(-2) exception fail { 3, 4, 54, 147, 151, 172, 258, 393, 394, 395, 396, 559, 1008 }

Maxima

A grade { 174, 175, 176, 179, 180, 227, 228, 229, 230, 231, 233, 234, 235, 236, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 290, 293, 318, 319, 321, 322, 331, 332, 333, 334, 335, 336, 344, 345, 347, 348, 355, 356, 357, 358, 359, 370, 371, 372, 373, 374, 375, 376, 379, 384, 385, 396, 398, 399, 400, 419, 420, 421, 422, 423, 424, 425, 443, 444, 445, 446, 447, 448, 449, 450, 454, 455, 456, 530, 531, 538, 539, 544, 545, 546, 552, 553, 554, 565, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 579, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 646, 647, 648, 649, 654, 655, 656, 670, 676, 677, 679, 680, 681, 683, 684, 685, 686, 693, 694, 695, 696, 697, 698, 699, 700, 707, 708, 709, 711, 712, 713, 714, 715, 716, 717, 718, 719, 722, 724, 725, 733, 734, 735, 737, 738, 739, 740, 741, 742, 743, 744, 745, 747, 748, 749, 762, 763, 805, 808, 809, 810, 814, 815, 816, 817, 818, 819, 820, 822, 824, 825, 833, 835, 836, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 852, 854, 856, 857, 858, 859, 860, 861, 862, 864, 865, 866, 867, 868, 870, 872, 874, 883, 884, 915, 916, 917, 922, 923, 924, 925, 926, 927, 928, 929, 935, 936, 937, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 952, 953, 954, 959, 960, 961, 963, 964, 971, 972, 975, 977, 979, 980, 981, 982, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 1007, 1020, 1023 }

B grade { 178, 182, 183, 184, 232, 238, 320, 323, 324, 337, 346, 349, 360, 369, 564, 566, 574, 580, 612, 613, 614, 615, 616, 617, 645, 653, 675, 678, 729, 730, 731, 732, 736, 746, 750, 829, 880, 948, 957, 958, 974, 976, 978, 1015 }

C grade { 581, 582, 583, 584, 585, 586, 595, 596, 597, 598, 599, 600, 855 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 237, 239, 240, 241, 242, 243, 245, 255, 256, 257, 258, 260, 262, 270, 271, 272, 273, 274, 275, 283, 284, 285, 286, 287, 288, 289, 291, 292, 294, 295, 302, 303, 304, 305, 306, 313, 314, 315, 316, 317, 325, 326, 327, 328, 329, 330, 338, 339, 340, 341, 342, 343, 350, 351, 352, 353, 354, 361, 362, 363, 364, 365, 366, 367, 368, 377, 378, 380, 381, 382, 383, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 451, 452, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 532, 533, 534, 535, 536, 537, 540, 541, 542, 543, 547, 548, 549, 550, 551, 555, 556, 557, 558, 559, 560, 561, 562, 563, 587, 588, 589, 590, 591, 592, 593, 594, 601, 629, 630, 631, 650, 651, 652, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 682, 691, 692, 701, 702, 703, 704, 705, 706, 710, 720, 721, 723, 726, 727, 728, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 806, 807, 813, 821, 823, 826, 827, 828, 830, 831, 834, 837, 838, 850, 851, 853, 863, 869, 871, 873, 875, 876, 877, 878, 879, 881, 882, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 911, 912, 913, 914, 918, 919, 920, 921, 930, 931, 932, 933, 934, 938, 939, 940, 962, 965, 966, 967, 968, 969, 970, 973, 983, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1021, 1022, 1024, 1025 }

F(-1) timedout fail { }

F(-2) exception fail { 202, 203, 204, 205, 259, 261, 263, 264, 265, 266, 267, 268, 269, 276, 277, 278, 279, 280, 281, 282, 296, 297, 298, 299, 300, 301, 307, 308, 309, 310, 311, 312, 473, 474, 475, 687, 688, 689, 690, 811, 812, 832, 955, 956 }

Giac

A grade { 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 259, 263, 264, 265, 289, 290, 291, 292, 293, 295, 296, 297, 298, 318, 319, 344, 345, 367, 369, 371, 372, 373, 374, 375, 376, 377, 379, 382, 384, 385, 390, 398, 399, 400, 403, 409, 416, 417, 418, 452, 454, 455, 456, 458, 459, 473, 474, 475, 498, 504, 530, 531, 538, 539, 544, 545, 546, 547, 548, 550, 551, 552, 553, 554, 555, 556, 564, 565, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 602, 603, 604, 605, 607, 608, 609, 610, 618, 619, 620, 622, 623, 624, 629, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 670, 675, 676, 677, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 716, 718, 720, 721, 722, 723, 724, 725, 727, 728, 731, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 748, 749, 755, 756, 762, 763, 806, 807, 808, 809, 810, 814, 816, 817, 818, 819, 820, 821, 822, 823, 830, 833, 834, 835, 836, 837, 838, 840, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 865, 872, 874, 877, 882, 883, 891, 893, 894, 895, 896, 897, 898, 899, 900, 915, 916, 917, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 959, 960, 961, 962, 963, 964, 965, 966, 967, 970, 971, 972, 973, 974, 975, 976, 977, 979, 980, 982, 984, 1007, 1020 }

B grade { 174, 175, 176, 178, 179, 180, 183, 184, 227, 261, 267, 268, 269, 300, 301, 320, 322, 323, 324, 331, 332, 333, 346, 348, 349, 355, 356, 366, 368, 388, 401, 402, 404, 405, 406, 407, 408, 410, 411, 412, 413, 414, 415, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 457, 477, 524, 525, 526, 527, 536, 537, 566, 582, 583, 584, 585, 586, 606, 621, 625, 626, 627, 628, 630, 631, 651, 652, 653, 654, 655, 656, 678, 679, 680, 714, 715, 717, 719, 726, 729, 730, 732, 746, 747, 750, 751, 753, 754, 757, 758, 759, 760, 761, 805, 811, 812, 813, 815, 826, 827, 828, 831, 832, 839, 841, 842, 843, 844, 845, 846, 847, 863, 864, 866, 867, 868, 870, 878, 879, 880, 881, 884, 885, 918, 919, 955, 956, 957, 958, 968, 969, 978, 981, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 997, 998, 1006, 1015, 1021, 1022, 1023 }

C grade { 294, 596, 600, 824, 825, 892 }

F normal fail { 1, 9, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 51, 55, 64, 68, 73, 74, 75, 76, 77, 82, 83, 84, 85, 86, 91, 92, 93, 94, 95, 100, 125, 126, 135, 136, 139, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 255, 256, 257, 258, 260, 262, 270, 271, 272, 273, 274, 275, 283, 284, 285, 286, 287, 288, 302, 303, 304, 305, 306, 313, 314, 315, 316, 317, 325, 326, 327, 328, 329, 330, 335, 336, 337, 338, 339, 340, 341, 342, 343, 350, 351, 352, 353, 354, 359, 360, 361, 362, 363, 364, 365, 378, 380, 381, 383, 386, 387, 389, 391, 392, 393, 394, 395, 396, 397, 453, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 528, 529, 532, 533, 534, 535, 540, 541, 542, 543, 557, 558, 559, 560, 561, 562, 563, 587, 588, 589, 590, 591, 592, 593, 594, 595, 597, 599, 601, 611, 612, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 752, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 829, 869, 871, 873, 875, 876, 886, 887, 888, 889, 890, 911, 912, 913, 914, 941, 942, 995, 996, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1024, 1025 }

F(-1) timedout fail { 5, 6, 7, 8, 47, 48, 49, 50, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 87, 88, 89, 90, 96, 97, 98, 99, 105, 106, 107, 108, 117, 118, 119, 120, 131, 132, 133, 134, 140, 141, 142, 143, 476, 478, 613, 614, 615, 616, 617 }

F(-2) exception fail { 2, 3, 4, 10, 11, 12, 13, 43, 44, 45, 46, 52, 53, 54, 56, 57, 58, 59, 65, 66, 67, 101, 102, 103, 104, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 124, 127, 128, 129, 130, 137, 138, 182, 266, 276, 277, 278, 279, 280, 281, 282, 299, 307, 308, 309, 310, 311, 312, 321, 334, 347, 357, 358, 370, 549, 581, 598, 920, 921 }

Mupad

A grade { }

B grade { 14, 15, 16, 17, 43, 44, 45, 46, 47, 48, 49, 50, 51, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 92, 93, 94, 95, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 176, 178, 179, 180, 182, 183, 184, 198, 199, 200, 201, 202, 203, 204, 205, 209, 215, 221, 222, 227, 230, 232, 238, 244, 245, 247, 249, 250, 251, 252, 289, 290, 291, 292, 293, 320, 333, 346, 357, 369, 370, 379, 384, 385, 396, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 455, 456, 498, 504, 516, 518, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 536, 537, 538, 539, 544, 545, 546, 547, 548, 552, 553, 554, 555, 556, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 590, 595, 596, 597, 599, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 649, 656, 670, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 707, 708, 709, 711, 712, 722, 724, 725, 726, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 750, 751, 762, 763, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 824, 825, 826, 827, 828, 830, 831, 833, 835, 836, 837, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 854, 855, 856, 857, 858, 859, 860, 861, 862, 865, 867, 868, 870, 872, 874, 876, 877, 878, 879, 880, 881, 884, 885, 886, 887, 888, 889, 890, 900, 915, 916, 917, 918, 919, 922, 923, 924, 925, 928, 930, 931, 933, 935, 936, 937, 939, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 957, 958, 959, 960, 963, 964, 965, 966, 967, 968, 969, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 997, 1007, 1015, 1020, 1021, 1022, 1023 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 87, 88, 89, 90, 91, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 231, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 246, 248, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 371, 372, 373, 374, 375, 376, 377, 378, 380, 381, 382, 383, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 453, 454, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 519, 520, 521, 522, 523, 534, 535, 540, 541, 542, 543, 549, 550, 551, 557, 558, 559, 560, 561, 562, 563, 587, 588, 589, 591, 592, 593, 594, 598, 601, 613, 614, 625, 626, 627, 629, 630, 631, 646, 647, 648, 650, 651, 652, 653, 654, 655, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 701, 702, 703, 704, 705, 706, 710, 713, 714, 715, 716, 717, 718, 719, 720, 721, 723, 727, 728, 749, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 817, 818, 819, 820, 821, 822, 823, 829, 832, 834, 838, 850, 851, 852, 853, 863, 864, 866, 869, 871, 873, 875, 882, 883, 891, 892, 893, 894, 895, 896, 897, 898, 899, 911, 912, 913, 914, 920, 921, 926, 927, 929, 932, 934, 938, 940, 955, 956, 961, 962, 970, 995, 996, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1024, 1025 }

F(-2) exception fail { }

Sympy

A grade { 23, 24, 25, 29, 30, 31, 36, 37, 38, 152, 153, 154, 155, 156, 157, 158, 159, 168, 169, 170, 171, 206, 212, 249, 250, 251, 252, 259, 379, 388, 452, 454, 455, 456, 473, 474, 475, 524, 525, 527, 531, 537, 539, 544, 545, 547, 552, 553, 555, 565, 567, 569, 570, 571, 573, 575, 576, 602, 606, 607, 608, 609, 610, 611, 612, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 632, 633, 634, 635, 636, 639, 640, 641, 643, 646, 647, 648, 649, 658, 659, 662, 663, 664, 665, 668, 669, 670, 675, 676, 677, 678, 679, 680, 681, 682, 684, 686, 687, 689, 691, 692, 693, 694, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 806, 808, 809, 810, 811, 812, 814, 833, 835, 836, 839, 841, 842, 844, 845, 847, 848, 849, 850, 852, 854, 856, 858, 860, 868, 870, 872, 874, 878, 879, 915, 916, 917, 922, 923, 924, 928, 936, 937, 939, 943, 944, 945, 946, 948, 949, 950, 952, 953, 954, 962, 966, 970, 977, 980, 981, 984, 1007, 1015, 1020 }

B grade { 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 197, 244, 245, 398, 399, 403, 408, 412, 413, 416, 417, 418, 422, 443, 447, 469, 470, 486, 487, 498, 504, 526, 528, 529, 530, 532, 533, 534, 535, 536, 538, 546, 554, 564, 566, 603, 604, 605, 642, 683, 685, 695, 714, 762, 763, 829, 837, 877, 933, 955, 956, 957, 958, 959, 963, 964, 967, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 1023 }

C grade { 26, 32, 39, 207, 208, 209, 213, 214, 215, 221, 222, 261, 380, 468, 588, 589, 590, 729, 731, 840, 843, 846, 862, 941, 942, 951, 965, 971, 972 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 27, 28, 33, 34, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 160, 161, 162, 163, 164, 165, 166, 167, 172, 173, 192, 196, 198, 199, 200, 201, 202, 203, 204, 205, 210, 211, 216, 217, 218, 219, 220, 223, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 247, 248, 253, 254, 255, 256, 257, 258, 260, 262, 289, 290, 291, 294, 295, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 401, 402, 404, 405, 406, 407, 409, 410, 411, 414, 415, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 444, 445, 446, 448, 449, 450, 451, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 471, 476, 477, 478, 479, 480, 481, 482, 483, 484, 488, 489, 490, 491, 492, 493, 494, 496, 497, 499, 500, 502, 503, 505, 506, 509, 516, 523, 542, 543, 548, 549, 550, 551, 556, 557, 558, 559, 560, 561, 562, 563, 568, 572, 574, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 629, 630, 637, 638, 644, 645, 650, 651, 653, 654, 655, 656, 657, 660, 661, 666, 667, 671, 672, 673, 674, 688, 690, 726, 727, 728, 730, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 756, 757, 758, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 807, 813, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 830, 831, 832, 834, 838, 851, 853, 855, 857, 859, 861, 863, 864, 865, 866, 867, 869, 871, 873, 875, 876, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 911, 912, 913, 914, 918, 919, 920, 921, 925, 926, 927, 929, 930, 931, 932, 934, 935, 938, 940, 947, 960, 961, 968, 969, 973, 974, 975, 976, 978, 979, 982, 983, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1021, 1022, 1024, 1025 }

F(-1) timedout fail { 186, 187, 188, 189, 190, 191, 193, 194, 195, 224, 225, 227, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 292, 293, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 400, 485, 512, 513, 518, 522, 540, 541, 613, 614, 615, 616, 617, 631, 652, 755 }

F(-2) exception fail { 472, 495, 501, 507, 508, 510, 511, 514, 515, 517, 519, 520, 521 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	148	139	0	74	0	0	0
N.S.	1	1.00	1.02	0.96	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.110	20.141	3.109	0.000	0.113	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	148	143	0	79	0	0	0
N.S.	1	1.00	0.92	0.89	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.128	10.098	2.898	0.000	0.121	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	146	143	0	0	0	0	0
N.S.	1	1.00	0.90	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.125	20.202	2.870	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	150	139	0	0	0	0	0
N.S.	1	1.00	0.96	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	10.069	2.885	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	280	280	164	0	0	0	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.212	10.170	0.000	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	288	288	166	0	0	0	0	0	0
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.241	10.159	0.000	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	167	0	0	0	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	10.116	0.000	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	293	293	167	0	0	0	0	0	0
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	10.119	0.000	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	169	495	0	349	0	0	0
N.S.	1	1.00	0.68	1.99	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.192	10.173	1.375	0.000	0.162	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	136	132	0	57	0	0	0
N.S.	1	1.00	0.93	0.90	0.00	0.39	0.00	0.00	0.00
time (sec)	N/A	0.142	20.191	2.095	0.000	0.146	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	136	143	0	66	0	0	0
N.S.	1	1.00	0.83	0.87	0.00	0.40	0.00	0.00	0.00
time (sec)	N/A	0.123	10.100	2.127	0.000	0.122	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	134	132	0	211	0	0	0
N.S.	1	1.00	0.80	0.79	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.126	20.213	2.120	0.000	0.138	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	138	139	0	213	0	0	0
N.S.	1	1.00	0.88	0.89	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.112	10.070	2.074	0.000	0.124	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	128	123	0	0	0	0	164
N.S.	1	1.00	0.39	0.37	0.00	0.00	0.00	0.00	0.50
time (sec)	N/A	0.439	20.091	1.250	0.000	0.000	0.000	0.000	9.151

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	128	133	0	0	0	0	180
N.S.	1	1.00	0.34	0.35	0.00	0.00	0.00	0.00	0.47
time (sec)	N/A	0.494	20.086	1.067	0.000	0.000	0.000	0.000	9.149

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	126	124	0	0	0	0	164
N.S.	1	1.00	0.34	0.33	0.00	0.00	0.00	0.00	0.44
time (sec)	N/A	0.377	20.065	1.245	0.000	0.000	0.000	0.000	0.030

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	130	133	0	0	0	0	179
N.S.	1	1.00	0.38	0.39	0.00	0.00	0.00	0.00	0.53
time (sec)	N/A	0.385	20.087	1.182	0.000	0.000	0.000	0.000	0.027

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	311	0	0	0	0	0	0
N.S.	1	1.00	2.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	2.073	0.000	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	186	186	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	0	3064	0	712	0	0	0
N.S.	1	1.00	0.00	20.84	0.00	4.84	0.00	0.00	0.00
time (sec)	N/A	0.035	0.000	50.888	0.000	3.740	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	0	3247	0	720	0	0	0
N.S.	1	1.00	0.00	20.42	0.00	4.53	0.00	0.00	0.00
time (sec)	N/A	0.044	0.000	48.469	0.000	3.519	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	387	387	163	0	0	0	212	0	0
N.S.	1	1.00	0.42	0.00	0.00	0.00	0.55	0.00	0.00
time (sec)	N/A	0.190	6.644	0.000	0.000	0.000	2.638	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	242	142	0	0	0	160	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.66	0.00	0.00
time (sec)	N/A	0.165	6.486	0.000	0.000	0.000	1.754	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	192	192	111	0	0	0	114	0	0
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.59	0.00	0.00
time (sec)	N/A	0.102	6.308	0.000	0.000	0.000	1.383	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	75	0	0	0	82	0	0
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.53	0.00	0.00
time (sec)	N/A	0.065	5.627	0.000	0.000	0.000	1.208	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	435	435	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	818	818	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.503	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	310	310	392	0	0	0	206	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.66	0.00	0.00
time (sec)	N/A	0.120	10.305	0.000	0.000	0.000	2.269	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	255	255	287	0	0	0	155	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.61	0.00	0.00
time (sec)	N/A	0.092	10.066	0.000	0.000	0.000	1.908	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	201	0	0	0	110	0	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.75	0.00	0.00
time (sec)	N/A	0.071	9.639	0.000	0.000	0.000	1.304	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	163	0	0	0	78	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.63	0.00	0.00
time (sec)	N/A	0.044	9.204	0.000	0.000	0.000	0.998	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	333	333	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	761	761	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.453	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1513	1513	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.244	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	306	306	166	0	0	0	204	0	0
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.67	0.00	0.00
time (sec)	N/A	0.121	10.103	0.000	0.000	0.000	2.331	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	145	0	0	0	153	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.82	0.00	0.00
time (sec)	N/A	0.124	10.085	0.000	0.000	0.000	1.716	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	95	0	0	0	109	0	0
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.77	0.00	0.00
time (sec)	N/A	0.067	10.034	0.000	0.000	0.000	1.324	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	78	0	0	0	78	0	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.64	0.00	0.00
time (sec)	N/A	0.043	9.335	0.000	0.000	0.000	1.018	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	332	332	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	760	760	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.495	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1357	1357	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.133	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	39	108	0	75	0	0	70
N.S.	1	1.00	1.05	2.92	0.00	2.03	0.00	0.00	1.89
time (sec)	N/A	0.075	1.481	4.065	0.000	0.425	0.000	0.000	9.785

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	111	0	76	0	0	74
N.S.	1	1.00	1.02	2.78	0.00	1.90	0.00	0.00	1.85
time (sec)	N/A	0.084	1.474	4.126	0.000	0.391	0.000	0.000	9.764

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	39	108	0	238	0	0	62
N.S.	1	1.00	1.03	2.84	0.00	6.26	0.00	0.00	1.63
time (sec)	N/A	0.072	1.484	4.408	0.000	0.399	0.000	0.000	9.533

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	41	112	0	241	0	0	63
N.S.	1	1.00	1.05	2.87	0.00	6.18	0.00	0.00	1.62
time (sec)	N/A	0.069	1.446	4.633	0.000	0.398	0.000	0.000	9.532

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	106
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.68
time (sec)	N/A	0.116	5.392	0.000	0.000	0.000	0.000	0.000	11.152

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	67	0	0	0	0	0	107
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.65
time (sec)	N/A	0.131	5.393	0.000	0.000	0.000	0.000	0.000	10.966

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	68	0	0	0	0	0	102
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.55
time (sec)	N/A	0.120	5.379	0.000	0.000	0.000	0.000	0.000	10.034

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	68	0	0	0	0	0	103
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.56
time (sec)	N/A	0.118	5.392	0.000	0.000	0.000	0.000	0.000	9.762

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	51	889	0	300	0	0	95
N.S.	1	1.00	1.04	18.14	0.00	6.12	0.00	0.00	1.94
time (sec)	N/A	0.073	1.340	0.892	0.000	0.428	0.000	0.000	10.309

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	336	262	0	157	0	0	0
N.S.	1	1.00	2.13	1.66	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.142	20.428	4.340	0.000	0.163	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	335	257	0	160	0	0	0
N.S.	1	1.00	1.94	1.49	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.155	20.418	5.645	0.000	0.161	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	333	266	0	0	0	0	0
N.S.	1	1.00	1.89	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	20.398	5.348	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	338	253	0	429	0	0	0
N.S.	1	1.00	2.00	1.50	0.00	2.54	0.00	0.00	0.00
time (sec)	N/A	0.152	20.358	4.912	0.000	0.181	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	340	264	0	992	0	0	0
N.S.	1	1.00	2.14	1.66	0.00	6.24	0.00	0.00	0.00
time (sec)	N/A	0.164	20.367	1.965	0.000	0.245	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	340	261	0	1005	0	0	0
N.S.	1	1.00	1.94	1.49	0.00	5.74	0.00	0.00	0.00
time (sec)	N/A	0.179	20.424	1.961	0.000	0.225	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	338	270	0	999	0	0	0
N.S.	1	1.00	1.90	1.52	0.00	5.61	0.00	0.00	0.00
time (sec)	N/A	0.157	20.418	1.865	0.000	0.237	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	342	255	0	1002	0	0	0
N.S.	1	1.00	2.01	1.50	0.00	5.89	0.00	0.00	0.00
time (sec)	N/A	0.174	20.387	1.928	0.000	0.224	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	316	316	348	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.298	11.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	324	324	399	0	0	0	0	0	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	10.942	0.000	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	333	333	400	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	10.363	0.000	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	329	329	401	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.309	10.848	0.000	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	380	900	0	390	0	0	0
N.S.	1	1.00	1.43	3.40	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.234	11.072	0.960	0.000	0.319	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	207	258	0	82	0	0	0
N.S.	1	1.00	1.43	1.78	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.147	10.323	3.692	0.000	0.130	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	209	253	0	83	0	0	0
N.S.	1	1.00	1.31	1.58	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.165	10.346	3.716	0.000	0.132	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	207	262	0	245	0	0	0
N.S.	1	1.00	1.27	1.61	0.00	1.50	0.00	0.00	0.00
time (sec)	N/A	0.158	10.313	3.579	0.000	0.121	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	209	249	0	248	0	0	0
N.S.	1	1.00	1.34	1.60	0.00	1.59	0.00	0.00	0.00
time (sec)	N/A	0.156	10.274	3.492	0.000	0.124	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	275	275	336	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	11.002	0.000	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	283	283	388	0	0	0	0	0	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.273	10.788	0.000	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	292	292	389	0	0	0	0	0	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.270	10.775	0.000	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	288	288	389	0	0	0	0	0	0
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	10.704	0.000	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	372	892	0	350	0	0	0
N.S.	1	1.00	1.51	3.63	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.197	10.949	0.959	0.000	0.168	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	31	44	0	44	0	0	205
N.S.	1	1.00	1.35	1.91	0.00	1.91	0.00	0.00	8.91
time (sec)	N/A	0.041	0.964	1.319	0.000	0.278	0.000	0.000	0.124

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	33	49	0	47	0	0	221
N.S.	1	1.00	1.22	1.81	0.00	1.74	0.00	0.00	8.19
time (sec)	N/A	0.042	0.939	1.366	0.000	0.289	0.000	0.000	0.105

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	21	75	0	40	0	0	205
N.S.	1	1.00	0.84	3.00	0.00	1.60	0.00	0.00	8.20
time (sec)	N/A	0.037	0.951	1.473	0.000	0.288	0.000	0.000	9.072

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	79	0	38	0	0	221
N.S.	1	1.00	0.92	3.16	0.00	1.52	0.00	0.00	8.84
time (sec)	N/A	0.042	0.864	1.617	0.000	0.284	0.000	0.000	9.324

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	48	0	0	0	0	0	65
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	1.30
time (sec)	N/A	0.085	2.339	0.000	0.000	0.000	0.000	0.000	9.652

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	52	50	0	0	0	0	0	67
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	1.29
time (sec)	N/A	0.087	2.289	0.000	0.000	0.000	0.000	0.000	9.586

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	592	0	0	74
N.S.	1	1.00	0.96	0.00	0.00	11.17	0.00	0.00	1.40
time (sec)	N/A	0.095	2.311	0.000	0.000	0.903	0.000	0.000	10.750

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	641	0	0	78
N.S.	1	1.00	0.96	0.00	0.00	12.09	0.00	0.00	1.47
time (sec)	N/A	0.092	2.257	0.000	0.000	0.904	0.000	0.000	10.702

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	503	0	294	0	0	67
N.S.	1	1.00	0.96	10.93	0.00	6.39	0.00	0.00	1.46
time (sec)	N/A	0.071	1.241	1.516	0.000	0.405	0.000	0.000	9.705

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	273	246	0	61	0	0	327
N.S.	1	1.00	1.96	1.77	0.00	0.44	0.00	0.00	2.35
time (sec)	N/A	0.107	20.395	0.949	0.000	0.132	0.000	0.000	0.131

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	271	246	0	66	0	0	359
N.S.	1	1.00	1.77	1.61	0.00	0.43	0.00	0.00	2.35
time (sec)	N/A	0.107	20.229	0.960	0.000	0.092	0.000	0.000	9.264

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	269	246	0	55	0	0	327
N.S.	1	1.00	1.72	1.58	0.00	0.35	0.00	0.00	2.10
time (sec)	N/A	0.108	20.224	0.938	0.000	0.118	0.000	0.000	0.067

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	275	246	0	57	0	0	359
N.S.	1	1.00	1.83	1.64	0.00	0.38	0.00	0.00	2.39
time (sec)	N/A	0.129	20.250	0.932	0.000	0.121	0.000	0.000	9.059

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	438	0	0	0	0	0	0
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	11.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	304	304	447	0	0	0	0	0	0
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	11.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	313	313	448	0	0	0	0	0	0
N.S.	1	1.00	1.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	10.955	0.000	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	310	310	441	0	0	0	0	0	0
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.239	11.047	0.000	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	384	521	0	396	0	0	0
N.S.	1	1.00	1.74	2.36	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.206	10.862	1.010	0.000	0.257	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	193	240	0	51	0	0	207
N.S.	1	1.00	1.50	1.86	0.00	0.40	0.00	0.00	1.60
time (sec)	N/A	0.100	10.281	1.036	0.000	0.122	0.000	0.000	0.031

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	195	240	0	54	0	0	224
N.S.	1	1.00	1.34	1.66	0.00	0.37	0.00	0.00	1.54
time (sec)	N/A	0.111	10.214	0.934	0.000	0.113	0.000	0.000	9.058

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	193	240	0	47	0	0	208
N.S.	1	1.00	1.30	1.62	0.00	0.32	0.00	0.00	1.41
time (sec)	N/A	0.103	10.136	1.001	0.000	0.110	0.000	0.000	0.026

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	195	240	0	45	0	0	223
N.S.	1	1.00	1.39	1.71	0.00	0.32	0.00	0.00	1.59
time (sec)	N/A	0.114	10.229	0.947	0.000	0.118	0.000	0.000	0.034

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	260	260	369	0	0	0	0	0	0
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.196	10.652	0.000	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	371	0	0	0	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	10.578	0.000	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	277	277	372	0	0	0	0	0	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	10.470	0.000	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	273	273	372	0	0	0	0	0	0
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	10.650	0.000	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	295	509	0	356	0	0	0
N.S.	1	1.00	1.46	2.52	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	0.190	10.553	0.977	0.000	0.150	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	49	130	0	205	0	0	0
N.S.	1	1.00	1.17	3.10	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	0.081	1.833	3.439	0.000	0.340	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	49	133	0	207	0	0	0
N.S.	1	1.00	1.07	2.89	0.00	4.50	0.00	0.00	0.00
time (sec)	N/A	0.082	1.801	3.220	0.000	0.385	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	47	129	0	50	0	0	0
N.S.	1	1.00	1.07	2.93	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.075	1.812	3.428	0.000	0.339	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	51	134	0	59	0	0	0
N.S.	1	1.00	1.16	3.05	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.069	1.815	3.280	0.000	0.292	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	84	0	0	1240	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	17.97	0.00	0.00	0.00
time (sec)	N/A	0.148	7.404	0.000	0.000	1.068	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	84	0	0	1294	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	18.23	0.00	0.00	0.00
time (sec)	N/A	0.140	7.399	0.000	0.000	1.046	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	85	0	0	1245	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	17.29	0.00	0.00	0.00
time (sec)	N/A	0.130	7.339	0.000	0.000	1.056	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	87	0	0	1303	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	18.10	0.00	0.00	0.00
time (sec)	N/A	0.118	7.289	0.000	0.000	1.093	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	663	0	0	1273	0	0	0
N.S.	1	1.00	9.08	0.00	0.00	17.44	0.00	0.00	0.00
time (sec)	N/A	0.146	10.974	0.000	0.000	0.720	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	648	0	0	1330	0	0	0
N.S.	1	1.00	8.64	0.00	0.00	17.73	0.00	0.00	0.00
time (sec)	N/A	0.157	10.858	0.000	0.000	0.718	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	649	0	0	1278	0	0	0
N.S.	1	1.00	8.54	0.00	0.00	16.82	0.00	0.00	0.00
time (sec)	N/A	0.140	10.903	0.000	0.000	0.680	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	666	0	0	1339	0	0	0
N.S.	1	1.00	8.76	0.00	0.00	17.62	0.00	0.00	0.00
time (sec)	N/A	0.129	10.878	0.000	0.000	0.728	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	49	131	0	50	0	0	0
N.S.	1	1.00	1.17	3.12	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.070	2.018	2.469	0.000	0.282	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	49	135	0	59	0	0	0
N.S.	1	1.00	1.07	2.93	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.070	1.795	2.469	0.000	0.297	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	47	131	0	204	0	0	0
N.S.	1	1.00	1.07	2.98	0.00	4.64	0.00	0.00	0.00
time (sec)	N/A	0.063	1.748	2.528	0.000	0.290	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	51	135	0	206	0	0	0
N.S.	1	1.00	1.16	3.07	0.00	4.68	0.00	0.00	0.00
time (sec)	N/A	0.062	1.795	2.437	0.000	0.336	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	84	0	0	1236	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	17.91	0.00	0.00	0.00
time (sec)	N/A	0.127	7.316	0.000	0.000	1.045	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	84	0	0	1288	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	18.14	0.00	0.00	0.00
time (sec)	N/A	0.127	7.177	0.000	0.000	1.034	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	85	0	0	1239	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	17.21	0.00	0.00	0.00
time (sec)	N/A	0.125	7.238	0.000	0.000	1.039	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	87	0	0	1299	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	18.04	0.00	0.00	0.00
time (sec)	N/A	0.108	7.183	0.000	0.000	1.071	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	667	0	0	1270	0	0	0
N.S.	1	1.00	9.14	0.00	0.00	17.40	0.00	0.00	0.00
time (sec)	N/A	0.127	10.755	0.000	0.000	0.738	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	649	0	0	1324	0	0	0
N.S.	1	1.00	8.65	0.00	0.00	17.65	0.00	0.00	0.00
time (sec)	N/A	0.133	10.818	0.000	0.000	0.719	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	650	0	0	1273	0	0	0
N.S.	1	1.00	8.55	0.00	0.00	16.75	0.00	0.00	0.00
time (sec)	N/A	0.127	10.845	0.000	0.000	0.731	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	670	0	0	1335	0	0	0
N.S.	1	1.00	8.82	0.00	0.00	17.57	0.00	0.00	0.00
time (sec)	N/A	0.122	10.829	0.000	0.000	0.717	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	269	245	0	55	0	0	0
N.S.	1	1.00	1.86	1.69	0.00	0.38	0.00	0.00	0.00
time (sec)	N/A	0.135	20.393	2.427	0.000	0.106	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	267	245	0	210	0	0	0
N.S.	1	1.00	1.84	1.69	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.151	20.357	2.444	0.000	0.114	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	291	260	0	716	0	0	0
N.S.	1	1.00	1.68	1.50	0.00	4.14	0.00	0.00	0.00
time (sec)	N/A	0.177	20.465	1.587	0.000	0.159	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	291	264	0	735	0	0	0
N.S.	1	1.00	1.56	1.41	0.00	3.93	0.00	0.00	0.00
time (sec)	N/A	0.187	20.519	1.472	0.000	0.183	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	289	262	0	723	0	0	0
N.S.	1	1.00	1.52	1.38	0.00	3.81	0.00	0.00	0.00
time (sec)	N/A	0.180	20.511	1.416	0.000	0.175	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	293	258	0	724	0	0	0
N.S.	1	1.00	1.60	1.41	0.00	3.96	0.00	0.00	0.00
time (sec)	N/A	0.180	20.456	1.519	0.000	0.166	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	332	332	456	0	0	7008	0	0	0
N.S.	1	1.00	1.37	0.00	0.00	21.11	0.00	0.00	0.00
time (sec)	N/A	0.406	11.493	0.000	0.000	27.024	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	336	336	466	0	0	7063	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	21.02	0.00	0.00	0.00
time (sec)	N/A	0.401	11.257	0.000	0.000	26.954	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	345	345	467	0	0	7009	0	0	0
N.S.	1	1.00	1.35	0.00	0.00	20.32	0.00	0.00	0.00
time (sec)	N/A	0.386	11.322	0.000	0.000	26.477	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	345	345	459	0	0	7078	0	0	0
N.S.	1	1.00	1.33	0.00	0.00	20.52	0.00	0.00	0.00
time (sec)	N/A	0.372	11.444	0.000	0.000	26.451	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	209	255	0	58	0	0	0
N.S.	1	1.00	1.54	1.88	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.168	10.393	2.446	0.000	0.112	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	232	257	0	69	0	0	0
N.S.	1	1.00	1.53	1.69	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.156	10.536	2.317	0.000	0.110	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	230	255	0	213	0	0	0
N.S.	1	1.00	1.40	1.55	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.157	11.528	2.401	0.000	0.123	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	211	253	0	217	0	0	0
N.S.	1	1.00	1.35	1.62	0.00	1.39	0.00	0.00	0.00
time (sec)	N/A	0.142	10.379	2.373	0.000	0.109	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	225	253	0	214	0	0	0
N.S.	1	1.00	1.53	1.72	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.168	10.493	2.451	0.000	0.107	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	278	278	445	0	0	1288	0	0	0
N.S.	1	1.00	1.60	0.00	0.00	4.63	0.00	0.00	0.00
time (sec)	N/A	0.312	11.284	0.000	0.000	0.889	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	286	286	454	0	0	1354	0	0	0
N.S.	1	1.00	1.59	0.00	0.00	4.73	0.00	0.00	0.00
time (sec)	N/A	0.342	11.082	0.000	0.000	0.819	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	282	282	455	0	0	1295	0	0	0
N.S.	1	1.00	1.61	0.00	0.00	4.59	0.00	0.00	0.00
time (sec)	N/A	0.322	11.019	0.000	0.000	0.902	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	278	278	448	0	0	1348	0	0	0
N.S.	1	1.00	1.61	0.00	0.00	4.85	0.00	0.00	0.00
time (sec)	N/A	0.303	11.239	0.000	0.000	0.895	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	214	275	0	0	0	0	0
N.S.	1	1.00	0.68	0.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.928	10.492	1.380	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	235	264	0	0	0	0	0
N.S.	1	1.00	0.71	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.964	10.567	1.397	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	233	273	0	0	0	0	0
N.S.	1	1.00	0.72	0.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.763	10.540	1.309	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	233	266	0	0	0	0	0
N.S.	1	1.00	0.73	0.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.730	10.544	1.288	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	213	275	0	0	0	0	0
N.S.	1	1.00	0.59	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.835	10.517	1.233	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	235	268	0	0	0	0	0
N.S.	1	1.00	0.68	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.776	10.565	1.281	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	233	277	0	0	0	0	0
N.S.	1	1.00	0.68	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.608	10.536	1.092	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	233	266	0	0	0	0	0
N.S.	1	1.00	0.64	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.695	10.570	1.212	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	39	90	0	33	56	0	334
N.S.	1	1.00	0.31	0.72	0.00	0.26	0.45	0.00	2.67
time (sec)	N/A	0.037	10.036	1.445	0.000	0.121	2.268	0.000	17.624

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	40	101	0	36	99	0	373
N.S.	1	1.00	0.29	0.73	0.00	0.26	0.71	0.00	2.68
time (sec)	N/A	0.035	10.040	1.352	0.000	0.116	3.743	0.000	18.953

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	58	132	0	52	94	0	334
N.S.	1	1.00	0.41	0.93	0.00	0.37	0.66	0.00	2.35
time (sec)	N/A	0.040	10.035	1.840	0.000	0.113	3.808	0.000	18.892

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	61	94	0	61	61	0	376
N.S.	1	1.00	0.45	0.69	0.00	0.45	0.45	0.00	2.76
time (sec)	N/A	0.035	10.036	1.830	0.000	0.106	2.434	0.000	21.455

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	41	90	0	33	56	0	334
N.S.	1	1.00	0.32	0.71	0.00	0.26	0.44	0.00	2.63
time (sec)	N/A	0.034	10.030	1.067	0.000	0.092	2.268	0.000	19.813

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	42	101	0	36	99	0	373
N.S.	1	1.00	0.30	0.72	0.00	0.26	0.70	0.00	2.65
time (sec)	N/A	0.037	10.026	1.105	0.000	0.102	3.738	0.000	20.839

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	60	132	0	51	94	0	334
N.S.	1	1.00	0.42	0.92	0.00	0.35	0.65	0.00	2.32
time (sec)	N/A	0.033	10.032	1.102	0.000	0.099	3.825	0.000	20.251

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	63	94	0	60	61	0	376
N.S.	1	1.00	0.46	0.68	0.00	0.43	0.44	0.00	2.72
time (sec)	N/A	0.038	10.030	1.080	0.000	0.106	2.406	0.000	21.677

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	194	240	0	0	0	0	207
N.S.	1	1.00	0.58	0.72	0.00	0.00	0.00	0.00	0.62
time (sec)	N/A	0.458	10.228	0.958	0.000	0.000	0.000	0.000	0.269

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	195	240	0	0	0	0	224
N.S.	1	1.00	0.52	0.64	0.00	0.00	0.00	0.00	0.59
time (sec)	N/A	0.504	10.217	0.924	0.000	0.000	0.000	0.000	0.206

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	193	240	0	0	0	0	208
N.S.	1	1.00	0.52	0.64	0.00	0.00	0.00	0.00	0.56
time (sec)	N/A	0.402	10.202	0.984	0.000	0.000	0.000	0.000	19.663

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	196	240	0	0	0	0	223
N.S.	1	1.00	0.57	0.70	0.00	0.00	0.00	0.00	0.65
time (sec)	N/A	0.393	10.221	0.979	0.000	0.000	0.000	0.000	0.094

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	450	450	211	274	0	0	0	0	356
N.S.	1	1.00	0.47	0.61	0.00	0.00	0.00	0.00	0.79
time (sec)	N/A	1.036	10.446	1.024	0.000	0.000	0.000	0.000	0.147

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	233	265	0	0	0	0	387
N.S.	1	1.00	0.49	0.56	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	1.046	10.582	1.023	0.000	0.000	0.000	0.000	20.107

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	475	475	231	274	0	0	0	0	355
N.S.	1	1.00	0.49	0.58	0.00	0.00	0.00	0.00	0.75
time (sec)	N/A	0.886	10.527	1.099	0.000	0.000	0.000	0.000	0.108

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	213	265	0	0	0	0	388
N.S.	1	1.00	0.46	0.57	0.00	0.00	0.00	0.00	0.84
time (sec)	N/A	0.929	10.443	0.944	0.000	0.000	0.000	0.000	0.112

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	34	53	0	30	42	0	207
N.S.	1	1.00	0.28	0.44	0.00	0.25	0.35	0.00	1.72
time (sec)	N/A	0.038	10.039	0.984	0.000	0.109	1.265	0.000	20.504

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	34	57	0	33	65	0	223
N.S.	1	1.00	0.25	0.43	0.00	0.25	0.49	0.00	1.66
time (sec)	N/A	0.044	10.445	0.930	0.000	0.095	1.412	0.000	20.350

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	137	137	52	93	0	26	60	0	207
N.S.	1	1.00	0.38	0.68	0.00	0.19	0.44	0.00	1.51
time (sec)	N/A	0.041	10.032	0.973	0.000	0.128	1.355	0.000	0.059

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	56	56	0	35	46	0	223
N.S.	1	1.00	0.43	0.43	0.00	0.27	0.35	0.00	1.70
time (sec)	N/A	0.037	10.037	0.979	0.000	0.096	1.400	0.000	19.569

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	159	0	0	0	0	0	0
N.S.	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	1.494	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	234	234	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.152	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	133	415	253	490	6397	835	495
N.S.	1	1.00	0.83	2.59	1.58	3.06	39.98	5.22	3.09
time (sec)	N/A	0.075	0.156	0.903	0.200	0.288	2.031	0.314	19.818

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	104	283	184	348	3704	577	363
N.S.	1	1.00	0.83	2.25	1.46	2.76	29.40	4.58	2.88
time (sec)	N/A	0.054	0.096	0.890	0.202	0.336	1.304	0.402	19.170

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	167	122	222	1906	361	247
N.S.	1	1.00	1.00	1.78	1.30	2.36	20.28	3.84	2.63
time (sec)	N/A	0.034	0.094	0.878	0.187	0.277	0.814	0.428	19.414

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	94	0	0	0	675	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	6.82	0.00	0.00
time (sec)	N/A	0.041	0.087	0.000	0.000	0.000	2.560	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	252	1565	601	1565	26746	2660	1410
N.S.	1	1.00	0.86	5.32	2.04	5.32	90.97	9.05	4.80
time (sec)	N/A	0.142	0.252	1.029	0.214	0.285	8.952	0.332	19.988

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	211	893	474	1216	18328	2034	1136
N.S.	1	1.00	0.85	3.60	1.91	4.90	73.90	8.20	4.58
time (sec)	N/A	0.110	0.194	1.000	0.209	0.303	5.693	0.331	20.362

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	172	700	359	893	11851	1477	878
N.S.	1	1.00	0.85	3.45	1.77	4.40	58.38	7.28	4.33
time (sec)	N/A	0.080	0.162	0.954	0.210	0.277	3.529	0.366	20.460

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	209	209	188	0	0	0	4690	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	22.44	0.00	0.00
time (sec)	N/A	0.092	0.168	0.000	0.000	0.000	4.222	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	402	3780	1153	3564	75191	0	2896
N.S.	1	1.00	0.88	8.24	2.51	7.76	163.81	0.00	6.31
time (sec)	N/A	0.228	0.380	1.348	0.222	0.327	83.592	0.000	24.923

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	396	345	2972	953	2919	56151	4934	2436
N.S.	1	1.00	0.87	7.51	2.41	7.37	141.80	12.46	6.15
time (sec)	N/A	0.191	0.308	1.162	0.234	0.324	34.118	0.372	22.664

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	290	2280	770	2313	40536	3874	2001
N.S.	1	1.00	0.86	6.77	2.28	6.86	120.28	11.50	5.94
time (sec)	N/A	0.162	0.292	1.140	0.222	0.331	66.278	0.403	21.092

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	358	358	332	0	0	0	17189	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	48.01	0.00	0.00
time (sec)	N/A	0.144	0.302	0.000	0.000	0.000	13.326	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	324	324	284	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.677	0.558	0.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	332	332	292	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.622	0.596	0.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	293	293	239	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	0.266	0.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	253	213	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	0.202	0.000	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	288	288	237	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.231	0.225	0.000	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	263	263	222	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	0.147	0.000	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	300	300	244	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.492	0.295	0.000	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	326	326	273	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	0.337	0.000	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	253	213	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.419	0.335	0.000	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	211	211	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.331	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1480	1480	877	1126	0	0	0	0	0
N.S.	1	1.00	0.59	0.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.035	6.600	2.473	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	F	F	F	F	B	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	135	0	0	0	0	0	631	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	4.67	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	25.939	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	23	46	0	19	0	0	273
N.S.	1	1.00	1.44	2.88	0.00	1.19	0.00	0.00	17.06
time (sec)	N/A	0.043	1.121	1.616	0.000	0.282	0.000	0.000	0.239

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	23	49	0	28	0	0	292
N.S.	1	1.00	1.15	2.45	0.00	1.40	0.00	0.00	14.60
time (sec)	N/A	0.048	1.113	1.564	0.000	0.269	0.000	0.000	19.534

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	21	26	0	25	0	0	276
N.S.	1	1.00	1.17	1.44	0.00	1.39	0.00	0.00	15.33
time (sec)	N/A	0.043	1.091	1.389	0.000	0.278	0.000	0.000	0.117

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	25	30	0	28	0	0	289
N.S.	1	1.00	1.39	1.67	0.00	1.56	0.00	0.00	16.06
time (sec)	N/A	0.044	1.088	1.328	0.000	0.296	0.000	0.000	0.117

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	37	4397	0	181	0	0	632
N.S.	1	1.00	1.23	146.57	0.00	6.03	0.00	0.00	21.07
time (sec)	N/A	0.057	1.894	1.807	0.000	0.321	0.000	0.000	19.706

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	1908	0	191	0	0	677
N.S.	1	1.00	0.97	50.21	0.00	5.03	0.00	0.00	17.82
time (sec)	N/A	0.066	1.975	1.841	0.000	0.310	0.000	0.000	18.721

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	35	4437	0	187	0	0	629
N.S.	1	1.00	0.97	123.25	0.00	5.19	0.00	0.00	17.47
time (sec)	N/A	0.052	1.919	1.686	0.000	0.315	0.000	0.000	0.120

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	39	1888	0	185	0	0	680
N.S.	1	1.00	1.22	59.00	0.00	5.78	0.00	0.00	21.25
time (sec)	N/A	0.055	1.918	1.770	0.000	0.296	0.000	0.000	19.053

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	186	261	0	189	175	0	0
N.S.	1	1.00	0.52	0.74	0.00	0.53	0.49	0.00	0.00
time (sec)	N/A	0.149	10.113	1.729	0.000	0.162	2.009	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	146	234	0	165	138	0	0
N.S.	1	1.00	0.45	0.72	0.00	0.51	0.42	0.00	0.00
time (sec)	N/A	0.115	8.157	1.500	0.000	0.162	1.806	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	109	119	0	94	88	0	0
N.S.	1	1.00	0.69	0.75	0.00	0.59	0.56	0.00	0.00
time (sec)	N/A	0.059	5.290	1.231	0.000	0.129	1.518	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	89	85	0	41	37	0	37
N.S.	1	1.00	0.85	0.81	0.00	0.39	0.35	0.00	0.35
time (sec)	N/A	0.012	0.087	1.009	0.000	0.078	0.461	0.000	18.066

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	730	730	421	404	0	0	0	0	0
N.S.	1	1.00	0.58	0.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.490	9.226	2.336	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1221	1221	394	402	0	0	0	0	0
N.S.	1	1.00	0.32	0.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.326	11.459	1.023	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	157	218	0	152	141	0	0
N.S.	1	1.00	0.53	0.74	0.00	0.52	0.48	0.00	0.00
time (sec)	N/A	0.103	10.111	1.599	0.000	0.137	1.698	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	133	197	0	137	105	0	0
N.S.	1	1.00	0.51	0.75	0.00	0.52	0.40	0.00	0.00
time (sec)	N/A	0.081	10.085	1.058	0.000	0.139	1.420	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	79	96	0	72	61	0	0
N.S.	1	1.00	0.65	0.79	0.00	0.60	0.50	0.00	0.00
time (sec)	N/A	0.043	10.043	1.036	0.000	0.117	1.027	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	74	70	0	29	36	0	37
N.S.	1	1.00	0.84	0.80	0.00	0.33	0.41	0.00	0.42
time (sec)	N/A	0.007	0.038	0.968	0.000	0.083	0.420	0.000	17.623

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	200	169	0	0	0	0	0
N.S.	1	1.00	0.49	0.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.212	0.213	1.036	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	610	610	448	421	0	0	0	0	0
N.S.	1	1.00	0.73	0.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.555	0.811	1.008	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	659	659	614	483	0	0	0	0	0
N.S.	1	1.00	0.93	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.800	11.428	1.129	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	126	245	0	160	0	0	0
N.S.	1	1.00	0.42	0.82	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.088	10.075	1.282	0.000	0.106	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	108	233	0	143	0	0	0
N.S.	1	1.00	0.40	0.86	0.00	0.53	0.00	0.00	0.00
time (sec)	N/A	0.081	10.068	1.109	0.000	0.118	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	59	115	0	76	61	0	57
N.S.	1	1.00	0.52	1.01	0.00	0.67	0.54	0.00	0.50
time (sec)	N/A	0.033	10.040	1.003	0.000	0.098	2.773	0.000	18.042

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	55	94	0	64	36	0	37
N.S.	1	1.00	0.51	0.87	0.00	0.59	0.33	0.00	0.34
time (sec)	N/A	0.012	4.719	0.919	0.000	0.082	0.471	0.000	18.109

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	818	818	455	496	0	0	0	0	0
N.S.	1	1.00	0.56	0.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.452	10.646	0.972	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	349	349	274	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.591	0.427	0.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	349	349	274	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.533	0.283	0.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1605	1605	455	1153	0	0	0	0	0
N.S.	1	1.00	0.28	0.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	6.665	11.255	1.363	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	132	200	119	233	0	355	234
N.S.	1	1.00	0.82	1.24	0.74	1.45	0.00	2.20	1.45
time (sec)	N/A	0.061	0.089	0.041	0.257	0.294	0.000	0.340	18.204

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	63	60	136	74	0	72	0
N.S.	1	1.00	0.44	0.42	0.95	0.52	0.00	0.50	0.00
time (sec)	N/A	0.048	0.835	0.169	0.238	0.296	0.000	0.342	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	63	60	47	74	0	72	0
N.S.	1	1.00	0.44	0.42	0.33	0.52	0.00	0.50	0.00
time (sec)	N/A	0.038	0.084	0.952	0.255	0.317	0.000	0.305	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	63	60	98	74	0	48	50
N.S.	1	1.00	0.95	0.91	1.48	1.12	0.00	0.73	0.76
time (sec)	N/A	0.048	0.729	0.127	0.232	0.304	0.000	0.470	17.832

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	63	60	47	74	0	72	0
N.S.	1	1.00	0.44	0.42	0.33	0.52	0.00	0.50	0.00
time (sec)	N/A	0.039	0.078	0.730	0.206	0.272	0.000	0.300	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	29	26	60	73	0	25	40
N.S.	1	1.00	0.91	0.81	1.88	2.28	0.00	0.78	1.25
time (sec)	N/A	0.015	0.009	0.136	0.229	0.264	0.000	0.339	18.466

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	61	58	43	72	0	46	0
N.S.	1	1.00	0.45	0.43	0.32	0.53	0.00	0.34	0.00
time (sec)	N/A	0.019	0.076	0.483	0.233	0.327	0.000	0.292	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	234	59	171	73	0	73	0
N.S.	1	1.00	1.68	0.42	1.23	0.53	0.00	0.53	0.00
time (sec)	N/A	0.040	0.525	0.126	0.191	0.382	0.000	0.305	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	62	60	48	72	0	69	0
N.S.	1	1.00	0.46	0.45	0.36	0.54	0.00	0.51	0.00
time (sec)	N/A	0.032	0.101	0.657	0.192	0.246	0.000	0.309	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	237	61	176	76	0	91	0
N.S.	1	1.00	1.69	0.44	1.26	0.54	0.00	0.65	0.00
time (sec)	N/A	0.039	0.697	0.138	0.209	0.284	0.000	0.327	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	142	155	0	433	0	177	0
N.S.	1	1.00	0.56	0.61	0.00	1.71	0.00	0.70	0.00
time (sec)	N/A	0.089	0.104	2.045	0.000	0.310	0.000	0.463	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	29	26	70	87	0	28	62
N.S.	1	1.00	0.91	0.81	2.19	2.72	0.00	0.88	1.94
time (sec)	N/A	0.016	0.012	1.564	0.216	0.278	0.000	0.309	18.395

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	123	138	0	402	0	153	0
N.S.	1	1.00	0.59	0.67	0.00	1.94	0.00	0.74	0.00
time (sec)	N/A	0.041	0.066	1.544	0.000	0.297	0.000	0.326	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	111	221	0	391	0	185	0
N.S.	1	1.00	0.58	1.15	0.00	2.04	0.00	0.96	0.00
time (sec)	N/A	0.070	0.073	1.498	0.000	0.275	0.000	0.494	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	121	141	0	396	0	185	0
N.S.	1	1.00	0.58	0.68	0.00	1.90	0.00	0.89	0.00
time (sec)	N/A	0.061	0.047	2.262	0.000	0.304	0.000	0.326	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	118	223	0	411	0	204	0
N.S.	1	1.00	0.58	1.10	0.00	2.03	0.00	1.01	0.00
time (sec)	N/A	0.078	0.055	2.306	0.000	0.283	0.000	0.313	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	73	60	0	141	0	71	0
N.S.	1	1.00	0.95	0.78	0.00	1.83	0.00	0.92	0.00
time (sec)	N/A	0.029	0.026	0.070	0.000	0.287	0.000	0.336	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	19	19	42	28	19
N.S.	1	1.00	1.00	0.95	0.90	0.90	2.00	1.33	0.90
time (sec)	N/A	0.012	0.004	0.086	0.193	0.256	0.255	0.334	19.465

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	0	19	46	28	19
N.S.	1	1.00	1.00	0.95	0.00	0.90	2.19	1.33	0.90
time (sec)	N/A	0.006	0.005	0.048	0.000	0.248	0.297	0.333	18.460

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	59	64	80	138	0	59	0
N.S.	1	1.00	0.83	0.90	1.13	1.94	0.00	0.83	0.00
time (sec)	N/A	0.034	0.015	0.064	0.266	0.268	0.000	0.320	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	32	37	46	32	0	81	54
N.S.	1	1.00	0.67	0.77	0.96	0.67	0.00	1.69	1.12
time (sec)	N/A	0.023	0.010	0.069	0.197	0.253	0.000	0.343	19.487

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	78	81	121	175	0	103	0
N.S.	1	1.00	0.75	0.78	1.16	1.68	0.00	0.99	0.00
time (sec)	N/A	0.045	0.024	0.091	0.269	0.268	0.000	0.445	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	63	58	85	75	144	137	109
N.S.	1	1.00	0.46	0.42	0.62	0.54	1.04	0.99	0.79
time (sec)	N/A	0.072	0.056	0.046	0.193	0.296	11.192	0.311	18.884

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	52	47	64	63	116	109	88
N.S.	1	1.00	0.51	0.46	0.63	0.62	1.14	1.07	0.86
time (sec)	N/A	0.057	0.038	0.030	0.199	0.285	5.864	0.299	18.945

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	41	36	43	51	87	81	67
N.S.	1	1.00	0.62	0.55	0.65	0.77	1.32	1.23	1.02
time (sec)	N/A	0.045	0.036	0.032	0.194	0.267	3.011	0.325	18.251

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	31	26	25	37	58	17	28
N.S.	1	1.00	0.86	0.72	0.69	1.03	1.61	0.47	0.78
time (sec)	N/A	0.012	0.006	0.061	0.189	0.299	1.427	0.313	18.868

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	96	0	118	0	0	190	0
N.S.	1	1.00	0.82	0.00	1.01	0.00	0.00	1.62	0.00
time (sec)	N/A	0.051	0.084	0.000	0.269	0.000	0.000	0.305	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	107	0	138	0	0	212	0
N.S.	1	1.00	0.80	0.00	1.04	0.00	0.00	1.59	0.00
time (sec)	N/A	0.047	0.129	0.000	0.279	0.000	0.000	0.333	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	68	0	0	0	0	0	0
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	0.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	52	0	0	0	0	0	0
N.S.	1	1.00	0.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.009	0.000	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	55	0	0	0	0	0	0
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.011	0.000	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	57	0	0	0	0	0	0
N.S.	1	1.00	0.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	10.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	75	68	0	80	114	61	0
N.S.	1	1.00	1.06	0.96	0.00	1.13	1.61	0.86	0.00
time (sec)	N/A	0.016	0.071	0.992	0.000	0.314	0.900	0.413	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	59	114	0	44	0	0	0
N.S.	1	1.00	1.23	2.38	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.028	0.046	1.531	0.000	0.115	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	55	28	0	43	71	61	0
N.S.	1	1.00	1.72	0.88	0.00	1.34	2.22	1.91	0.00
time (sec)	N/A	0.009	0.012	1.032	0.000	0.336	0.872	0.351	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	43	0	9	0	0	0
N.S.	1	1.00	1.50	3.58	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.008	0.013	0.498	0.000	0.113	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	198	241	0	541	0	226	0
N.S.	1	1.00	0.81	0.99	0.00	2.22	0.00	0.93	0.00
time (sec)	N/A	0.297	0.437	0.226	0.000	0.373	0.000	0.382	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	161	189	0	407	0	169	0
N.S.	1	1.00	1.00	1.17	0.00	2.53	0.00	1.05	0.00
time (sec)	N/A	0.162	0.087	0.123	0.000	0.343	0.000	0.385	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	132	154	0	313	0	138	0
N.S.	1	1.00	1.28	1.50	0.00	3.04	0.00	1.34	0.00
time (sec)	N/A	0.074	0.049	0.104	0.000	0.342	0.000	0.395	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	147	179	0	865	0	0	0
N.S.	1	1.00	1.31	1.60	0.00	7.72	0.00	0.00	0.00
time (sec)	N/A	0.129	0.041	0.102	0.000	0.479	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	137	162	0	333	0	242	0
N.S.	1	1.00	1.08	1.28	0.00	2.62	0.00	1.91	0.00
time (sec)	N/A	0.108	0.057	0.150	0.000	0.482	0.000	0.393	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	174	201	0	427	0	549	0
N.S.	1	1.00	0.84	0.97	0.00	2.05	0.00	2.64	0.00
time (sec)	N/A	0.187	0.087	0.159	0.000	0.883	0.000	0.449	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	222	257	0	561	0	1001	0
N.S.	1	1.00	0.70	0.81	0.00	1.76	0.00	3.15	0.00
time (sec)	N/A	0.316	1.486	0.192	0.000	2.168	0.000	0.412	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	255	490	0	273	0	0	0
N.S.	1	1.00	0.71	1.37	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.254	1.321	5.207	0.000	0.122	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	208	350	0	182	0	0	0
N.S.	1	1.00	0.78	1.32	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.154	1.223	4.720	0.000	0.106	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	86	184	0	150	0	0	0
N.S.	1	1.00	0.44	0.95	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.072	1.065	1.151	0.000	0.107	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	111	192	0	132	0	0	0
N.S.	1	1.00	0.46	0.80	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.136	1.179	4.665	0.000	0.108	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	238	378	0	199	0	0	0
N.S.	1	1.00	0.74	1.18	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.223	1.442	5.432	0.000	0.113	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	302	539	0	289	0	0	0
N.S.	1	1.00	0.71	1.27	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.349	2.786	6.593	0.000	0.114	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	222	313	0	553	0	0	0
N.S.	1	1.00	0.79	1.11	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.353	4.541	0.224	0.000	1.287	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	177	252	0	417	0	0	0
N.S.	1	1.00	0.89	1.27	0.00	2.10	0.00	0.00	0.00
time (sec)	N/A	0.230	2.846	0.184	0.000	0.681	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	136	231	0	328	0	0	0
N.S.	1	1.00	0.96	1.64	0.00	2.33	0.00	0.00	0.00
time (sec)	N/A	0.112	1.894	0.161	0.000	0.462	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	187	401	0	1049	0	0	0
N.S.	1	1.00	1.24	2.66	0.00	6.95	0.00	0.00	0.00
time (sec)	N/A	0.195	2.299	0.139	0.000	0.898	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	146	239	0	350	0	0	0
N.S.	1	1.00	0.88	1.45	0.00	2.12	0.00	0.00	0.00
time (sec)	N/A	0.140	3.804	0.209	0.000	0.908	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	186	270	0	435	0	0	0
N.S.	1	1.00	0.73	1.05	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.240	4.303	0.240	0.000	2.537	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	245	333	0	573	0	0	0
N.S.	1	1.00	0.67	0.91	0.00	1.57	0.00	0.00	0.00
time (sec)	N/A	0.388	4.762	0.268	0.000	9.703	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	266	775	0	275	0	0	0
N.S.	1	1.00	0.68	1.98	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.376	5.606	9.881	0.000	0.122	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	235	734	0	229	0	0	0
N.S.	1	1.00	0.76	2.37	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.222	4.955	8.941	0.000	0.124	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	206	527	0	173	0	0	0
N.S.	1	1.00	0.79	2.01	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.136	1.210	3.336	0.000	0.108	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	228	670	0	170	0	0	0
N.S.	1	1.00	0.74	2.18	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.225	5.247	8.050	0.000	0.116	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	253	790	0	214	0	0	0
N.S.	1	1.00	0.66	2.06	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.334	5.625	8.284	0.000	0.117	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	480	480	322	908	0	293	0	0	0
N.S.	1	1.00	0.67	1.89	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.477	6.229	10.053	0.000	0.122	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	95	52	0	55	0	18	55
N.S.	1	1.00	1.86	1.02	0.00	1.08	0.00	0.35	1.08
time (sec)	N/A	0.029	0.127	0.410	0.000	0.306	0.000	0.311	17.176

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	119	78	76	77	0	30	88
N.S.	1	1.00	1.65	1.08	1.06	1.07	0.00	0.42	1.22
time (sec)	N/A	0.039	0.205	0.442	0.269	0.325	0.000	0.318	0.222

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	95	68	0	55	0	22	56
N.S.	1	1.00	1.79	1.28	0.00	1.04	0.00	0.42	1.06
time (sec)	N/A	0.028	0.206	0.830	0.000	0.361	0.000	0.366	16.809

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	95	80	0	65	0	57	101
N.S.	1	1.00	0.84	0.71	0.00	0.58	0.00	0.50	0.89
time (sec)	N/A	0.070	0.469	0.869	0.000	0.341	0.000	0.318	16.905

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	114	121	82	0	47	134
N.S.	1	1.00	1.00	1.08	1.14	0.77	0.00	0.44	1.26
time (sec)	N/A	0.063	0.649	0.777	0.270	0.342	0.000	0.322	0.232

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	49	42	0	32	0	40	0
N.S.	1	1.00	0.94	0.81	0.00	0.62	0.00	0.77	0.00
time (sec)	N/A	0.029	0.031	0.873	0.000	0.303	0.000	0.376	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	65	60	0	42	0	61	0
N.S.	1	1.00	0.96	0.88	0.00	0.62	0.00	0.90	0.00
time (sec)	N/A	0.065	0.043	0.898	0.000	0.342	0.000	0.353	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	224	243	0	545	0	235	0
N.S.	1	1.00	0.80	0.86	0.00	1.94	0.00	0.84	0.00
time (sec)	N/A	0.293	1.458	0.161	0.000	0.393	0.000	0.362	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	161	191	0	413	0	172	0
N.S.	1	1.00	0.95	1.13	0.00	2.44	0.00	1.02	0.00
time (sec)	N/A	0.158	0.886	0.127	0.000	0.352	0.000	0.366	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	136	154	0	313	0	142	0
N.S.	1	1.00	1.28	1.45	0.00	2.95	0.00	1.34	0.00
time (sec)	N/A	0.074	0.377	0.110	0.000	0.322	0.000	0.402	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	147	179	0	881	0	0	0
N.S.	1	1.00	1.31	1.60	0.00	7.87	0.00	0.00	0.00
time (sec)	N/A	0.118	0.429	0.092	0.000	0.524	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	143	162	0	333	0	238	0
N.S.	1	1.00	1.10	1.25	0.00	2.56	0.00	1.83	0.00
time (sec)	N/A	0.106	0.606	0.145	0.000	0.540	0.000	0.347	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	173	200	0	443	0	536	0
N.S.	1	1.00	0.79	0.92	0.00	2.03	0.00	2.46	0.00
time (sec)	N/A	0.171	1.164	0.155	0.000	0.883	0.000	0.393	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	258	492	0	276	0	0	0
N.S.	1	1.00	0.64	1.22	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.278	2.517	5.142	0.000	0.123	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	212	351	0	185	0	0	0
N.S.	1	1.00	0.68	1.12	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.157	2.018	3.931	0.000	0.120	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	86	127	0	130	0	0	0
N.S.	1	1.00	0.34	0.50	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.079	1.038	1.146	0.000	0.110	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	111	297	0	156	0	0	0
N.S.	1	1.00	0.38	1.03	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.134	2.082	4.657	0.000	0.112	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	372	238	377	0	206	0	0	0
N.S.	1	0.99	0.63	1.01	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.221	2.415	6.213	0.000	0.109	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	353	247	318	0	781	0	0	0
N.S.	1	1.00	0.70	0.90	0.00	2.21	0.00	0.00	0.00
time (sec)	N/A	0.363	4.435	0.240	0.000	1.251	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	179	257	0	585	0	0	0
N.S.	1	1.00	0.89	1.27	0.00	2.90	0.00	0.00	0.00
time (sec)	N/A	0.229	2.612	0.205	0.000	0.702	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	138	234	0	443	0	0	0
N.S.	1	1.00	0.95	1.60	0.00	3.03	0.00	0.00	0.00
time (sec)	N/A	0.110	1.860	0.181	0.000	0.540	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	189	401	0	1293	0	0	0
N.S.	1	1.00	1.24	2.64	0.00	8.51	0.00	0.00	0.00
time (sec)	N/A	0.196	2.092	0.143	0.000	0.906	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	148	242	0	469	0	0	0
N.S.	1	1.00	0.87	1.42	0.00	2.76	0.00	0.00	0.00
time (sec)	N/A	0.138	2.515	0.205	0.000	1.292	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	189	274	0	613	0	0	0
N.S.	1	1.00	0.74	1.07	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.247	4.200	0.227	0.000	3.953	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	453	271	780	0	425	0	0	0
N.S.	1	1.00	0.60	1.72	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.373	5.549	9.940	0.000	0.121	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	219	643	0	305	0	0	0
N.S.	1	1.00	0.58	1.70	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.227	4.909	8.864	0.000	0.116	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	203	514	0	251	0	0	0
N.S.	1	1.00	0.62	1.57	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.148	1.317	3.379	0.000	0.107	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	223	650	0	269	0	0	0
N.S.	1	1.00	0.59	1.71	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.243	5.132	8.116	0.000	0.105	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	444	266	866	0	374	0	0	0
N.S.	1	1.00	0.60	1.95	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.363	5.653	8.749	0.000	0.104	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	217	145	237	328	423	0	218	0
N.S.	1	1.00	0.67	1.10	1.52	1.96	0.00	1.01	0.00
time (sec)	N/A	0.300	0.231	0.233	0.290	0.330	0.000	0.394	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	107	193	218	325	0	158	0
N.S.	1	1.00	0.76	1.37	1.55	2.30	0.00	1.12	0.00
time (sec)	N/A	0.184	0.141	0.117	0.295	0.315	0.000	0.416	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	85	137	126	267	0	123	120
N.S.	1	1.00	1.23	1.99	1.83	3.87	0.00	1.78	1.74
time (sec)	N/A	0.042	0.094	0.155	0.290	0.290	0.000	0.431	17.767

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	102	235	159	927	0	0	0
N.S.	1	1.00	1.06	2.45	1.66	9.66	0.00	0.00	0.00
time (sec)	N/A	0.164	0.151	0.100	0.285	0.349	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	110	189	156	433	0	281	0
N.S.	1	1.00	1.06	1.82	1.50	4.16	0.00	2.70	0.00
time (sec)	N/A	0.132	0.219	0.145	0.298	0.334	0.000	0.387	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	152	239	322	577	0	713	0
N.S.	1	1.00	0.87	1.37	1.85	3.32	0.00	4.10	0.00
time (sec)	N/A	0.237	0.277	0.167	0.298	0.421	0.000	0.426	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	216	317	557	755	0	1414	0
N.S.	1	1.00	0.82	1.20	2.10	2.85	0.00	5.34	0.00
time (sec)	N/A	0.374	0.402	0.185	0.314	0.643	0.000	0.421	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	294	665	0	238	0	0	0
N.S.	1	1.00	0.80	1.81	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.366	10.064	6.524	0.000	0.094	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	243	406	0	166	0	0	0
N.S.	1	1.00	0.86	1.44	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.223	9.204	5.922	0.000	0.109	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	98	199	0	148	0	0	0
N.S.	1	1.00	0.46	0.93	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.102	8.869	1.155	0.000	0.098	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	136	272	0	196	0	0	0
N.S.	1	1.00	0.51	1.03	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.191	9.541	6.061	0.000	0.098	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	307	569	0	292	0	0	0
N.S.	1	1.00	0.85	1.57	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.291	10.739	6.997	0.000	0.105	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	466	466	400	778	0	411	0	0	0
N.S.	1	1.00	0.86	1.67	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.429	11.323	7.868	0.000	0.128	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	150	295	368	427	0	517	0
N.S.	1	1.00	0.60	1.18	1.48	1.71	0.00	2.08	0.00
time (sec)	N/A	0.328	0.242	0.215	0.306	0.411	0.000	0.802	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	113	242	247	335	0	475	0
N.S.	1	1.00	0.66	1.41	1.44	1.95	0.00	2.76	0.00
time (sec)	N/A	0.227	0.171	0.184	0.303	0.346	0.000	0.738	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	87	187	156	269	0	381	61
N.S.	1	1.00	0.93	1.99	1.66	2.86	0.00	4.05	0.65
time (sec)	N/A	0.051	0.108	0.209	0.297	0.350	0.000	0.716	18.034

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	139	652	201	1073	0	0	0
N.S.	1	1.00	1.10	5.17	1.60	8.52	0.00	0.00	0.00
time (sec)	N/A	0.250	0.213	0.127	0.299	0.402	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	124	251	202	404	0	0	0
N.S.	1	1.00	0.90	1.82	1.46	2.93	0.00	0.00	0.00
time (sec)	N/A	0.163	0.207	0.191	0.292	0.393	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	153	274	313	557	0	0	0
N.S.	1	1.00	0.75	1.34	1.53	2.72	0.00	0.00	0.00
time (sec)	N/A	0.277	0.228	0.227	0.312	0.621	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	226	371	534	733	0	0	0
N.S.	1	1.00	0.77	1.27	1.83	2.51	0.00	0.00	0.00
time (sec)	N/A	0.433	0.565	0.263	0.316	0.884	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	312	1065	0	233	0	0	0
N.S.	1	1.00	0.77	2.63	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.441	10.628	11.572	0.000	0.108	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	256	820	0	204	0	0	0
N.S.	1	1.00	0.77	2.48	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.287	10.419	9.350	0.000	0.094	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	229	515	0	160	0	0	0
N.S.	1	1.00	0.88	1.98	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.157	10.415	3.373	0.000	0.108	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	270	873	0	251	0	0	0
N.S.	1	1.00	0.87	2.80	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.295	10.447	9.403	0.000	0.109	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	352	1037	0	284	0	0	0
N.S.	1	1.00	0.91	2.67	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.446	10.923	10.724	0.000	0.121	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	494	494	472	1170	0	393	0	0	0
N.S.	1	1.00	0.96	2.37	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.584	11.228	11.584	0.000	0.106	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	148	239	340	425	0	225	0
N.S.	1	1.00	0.66	1.06	1.51	1.89	0.00	1.00	0.00
time (sec)	N/A	0.294	0.233	0.143	0.292	0.304	0.000	0.389	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	108	195	223	333	0	166	0
N.S.	1	1.00	0.73	1.32	1.51	2.25	0.00	1.12	0.00
time (sec)	N/A	0.183	0.143	0.115	0.283	0.306	0.000	0.388	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	88	134	129	267	0	128	111
N.S.	1	1.00	1.22	1.86	1.79	3.71	0.00	1.78	1.54
time (sec)	N/A	0.037	0.099	1.023	0.302	0.293	0.000	0.370	18.016

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	101	313	155	972	0	0	0
N.S.	1	1.00	1.05	3.26	1.61	10.12	0.00	0.00	0.00
time (sec)	N/A	0.145	0.124	0.084	0.303	0.364	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	114	197	173	451	0	292	0
N.S.	1	1.00	1.06	1.82	1.60	4.18	0.00	2.70	0.00
time (sec)	N/A	0.143	0.228	0.138	0.298	0.356	0.000	0.405	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	149	236	359	593	0	778	0
N.S.	1	1.00	0.84	1.33	2.03	3.35	0.00	4.40	0.00
time (sec)	N/A	0.215	0.259	0.174	0.300	0.400	0.000	0.407	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	298	664	0	240	0	0	0
N.S.	1	1.00	0.67	1.50	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.361	8.775	8.058	0.000	0.110	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	248	409	0	168	0	0	0
N.S.	1	1.00	0.70	1.16	0.00	0.47	0.00	0.00	0.00
time (sec)	N/A	0.223	7.620	5.995	0.000	0.101	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	107	164	0	127	0	0	0
N.S.	1	1.00	0.37	0.57	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.114	7.333	1.148	0.000	0.102	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	151	345	0	239	0	0	0
N.S.	1	1.00	0.44	1.01	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.203	8.738	6.062	0.000	0.101	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	431	299	568	0	297	0	0	0
N.S.	1	0.99	0.69	1.31	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.302	9.846	7.003	0.000	0.108	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	183	303	389	675	0	663	0
N.S.	1	1.00	0.59	0.98	1.25	2.18	0.00	2.14	0.00
time (sec)	N/A	0.345	0.330	0.227	0.302	0.408	0.000	0.682	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	144	248	262	541	0	552	0
N.S.	1	1.00	0.77	1.33	1.40	2.89	0.00	2.95	0.00
time (sec)	N/A	0.268	0.232	0.198	0.300	0.359	0.000	0.610	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	114	217	161	395	0	0	61
N.S.	1	1.00	1.14	2.17	1.61	3.95	0.00	0.00	0.61
time (sec)	N/A	0.049	0.129	1.135	0.287	0.343	0.000	0.000	18.323

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	140	1015	201	1477	0	0	0
N.S.	1	1.00	1.04	7.57	1.50	11.02	0.00	0.00	0.00
time (sec)	N/A	0.231	0.415	0.129	0.295	0.471	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	148	244	247	599	0	0	0
N.S.	1	1.00	1.01	1.67	1.69	4.10	0.00	0.00	0.00
time (sec)	N/A	0.174	0.239	0.211	0.305	0.392	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	192	279	450	961	0	0	0
N.S.	1	1.00	0.91	1.32	2.12	4.53	0.00	0.00	0.00
time (sec)	N/A	0.337	0.333	0.248	0.318	0.733	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	482	482	296	879	0	436	0	0	0
N.S.	1	1.00	0.61	1.82	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.452	10.611	11.979	0.000	0.120	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	253	667	0	326	0	0	0
N.S.	1	1.00	0.62	1.63	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.295	10.409	9.204	0.000	0.115	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	240	466	0	327	0	0	0
N.S.	1	1.00	0.67	1.31	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.171	10.434	3.383	0.000	0.118	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	261	685	0	432	0	0	0
N.S.	1	1.00	0.64	1.67	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.313	10.513	11.043	0.000	0.108	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	306	1080	0	601	0	0	0
N.S.	1	1.00	0.62	2.20	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.441	10.788	10.811	0.000	0.121	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	59	48	0	169	0	113	0
N.S.	1	1.00	0.79	0.64	0.00	2.25	0.00	1.51	0.00
time (sec)	N/A	0.013	0.018	1.023	0.000	0.349	0.000	0.394	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	52	40	0	153	0	68	0
N.S.	1	1.00	1.04	0.80	0.00	3.06	0.00	1.36	0.00
time (sec)	N/A	0.012	0.010	0.957	0.000	0.367	0.000	0.373	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	34	17	0	98	0	58	0
N.S.	1	1.00	1.42	0.71	0.00	4.08	0.00	2.42	0.00
time (sec)	N/A	0.005	0.006	0.955	0.000	0.360	0.000	0.322	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	41	17	0	28	17
N.S.	1	1.00	1.00	0.78	1.78	0.74	0.00	1.22	0.74
time (sec)	N/A	0.004	0.004	0.921	0.298	0.294	0.000	0.335	18.648

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	30	25	50	25	0	0	29
N.S.	1	1.00	0.61	0.51	1.02	0.51	0.00	0.00	0.59
time (sec)	N/A	0.007	0.006	0.899	0.283	0.282	0.000	0.000	17.851

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	18	26	29	0	29	0
N.S.	1	1.00	0.62	0.49	0.70	0.78	0.00	0.78	0.00
time (sec)	N/A	0.005	0.041	2.069	0.271	0.297	0.000	0.306	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	18	26	29	0	29	0
N.S.	1	1.00	0.62	0.49	0.70	0.78	0.00	0.78	0.00
time (sec)	N/A	0.010	0.001	1.483	0.275	0.340	0.000	0.315	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	34	29	40	41	0	42	0
N.S.	1	1.00	0.48	0.41	0.56	0.58	0.00	0.59	0.00
time (sec)	N/A	0.010	0.043	1.492	0.270	0.307	0.000	0.320	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	42	37	42	256	0	48	0
N.S.	1	1.00	0.86	0.76	0.86	5.22	0.00	0.98	0.00
time (sec)	N/A	0.010	0.049	0.934	0.268	0.286	0.000	0.340	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	42	37	42	256	0	48	0
N.S.	1	1.00	0.86	0.76	0.86	5.22	0.00	0.98	0.00
time (sec)	N/A	0.014	0.003	0.938	0.279	0.320	0.000	0.324	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	30	26	32	127	0	43	0
N.S.	1	1.00	0.68	0.59	0.73	2.89	0.00	0.98	0.00
time (sec)	N/A	0.010	0.023	1.309	0.279	0.290	0.000	0.343	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	42	27	0	42	0	27	0
N.S.	1	1.00	0.95	0.61	0.00	0.95	0.00	0.61	0.00
time (sec)	N/A	0.005	0.034	0.893	0.000	0.267	0.000	0.318	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	43	22	0	31	0	0	0
N.S.	1	1.00	0.52	0.27	0.00	0.37	0.00	0.00	0.00
time (sec)	N/A	0.019	10.018	0.977	0.000	0.088	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	19	18	17	19	19
N.S.	1	1.00	1.00	0.86	0.86	0.82	0.77	0.86	0.86
time (sec)	N/A	0.002	0.005	0.893	0.272	0.302	0.123	0.339	16.433

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	27	20	0	12	36	0	0
N.S.	1	1.00	0.21	0.15	0.00	0.09	0.27	0.00	0.00
time (sec)	N/A	0.059	10.020	0.925	0.000	0.078	0.532	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	27	22	0	9	0	0	0
N.S.	1	1.00	0.50	0.41	0.00	0.17	0.00	0.00	0.00
time (sec)	N/A	0.016	10.020	0.957	0.000	0.083	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	0	76	0	30	0
N.S.	1	1.00	1.00	0.86	0.00	3.45	0.00	1.36	0.00
time (sec)	N/A	0.006	0.018	0.864	0.000	0.306	0.000	0.329	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	27	22	0	30	0	0	0
N.S.	1	1.00	0.17	0.14	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.040	10.017	1.020	0.000	0.089	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	30	0	22	18
N.S.	1	1.00	1.00	0.86	1.10	1.43	0.00	1.05	0.86
time (sec)	N/A	0.003	0.022	0.921	0.278	0.298	0.000	0.332	16.616

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	28	19	0	12	20
N.S.	1	1.00	1.00	0.80	1.12	0.76	0.00	0.48	0.80
time (sec)	N/A	0.003	0.006	0.892	0.288	0.281	0.000	0.319	16.785

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	29	22	0	0	0	0	0
N.S.	1	1.00	0.10	0.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.177	10.023	2.305	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	29	22	0	19	0	0	0
N.S.	1	1.00	0.11	0.08	0.00	0.07	0.00	0.00	0.00
time (sec)	N/A	0.057	10.018	1.105	0.000	0.088	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	32	15	0	85	14	35	0
N.S.	1	1.00	1.39	0.65	0.00	3.70	0.61	1.52	0.00
time (sec)	N/A	0.011	0.212	1.915	0.000	0.320	0.524	0.322	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	27	22	0	11	0	0	0
N.S.	1	1.00	0.23	0.19	0.00	0.09	0.00	0.00	0.00
time (sec)	N/A	0.063	10.025	1.771	0.000	0.080	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	0	68	0	31	0
N.S.	1	1.00	1.00	0.79	0.00	2.83	0.00	1.29	0.00
time (sec)	N/A	0.006	0.004	0.909	0.000	0.299	0.000	0.363	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	27	22	0	14	0	0	0
N.S.	1	1.00	0.09	0.07	0.00	0.04	0.00	0.00	0.00
time (sec)	N/A	0.175	10.013	2.239	0.000	0.094	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	27	22	0	37	0	0	0
N.S.	1	1.00	0.10	0.08	0.00	0.13	0.00	0.00	0.00
time (sec)	N/A	0.065	10.015	1.194	0.000	0.094	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	36	0	0	0	0	0
N.S.	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.010	0.030	1.086	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	40	35	0	0	0	0	0
N.S.	1	1.00	0.83	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.011	0.028	0.957	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	44	37	0	0	0	0	0
N.S.	1	1.00	0.85	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.011	0.030	1.072	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	80	33	30	18	0	0	0	43
N.S.	1	2.35	0.97	0.88	0.53	0.00	0.00	0.00	1.26
time (sec)	N/A	0.026	0.054	0.956	0.243	0.000	0.000	0.000	16.697

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	106	191	0	274	0	0	0
N.S.	1	1.00	0.93	1.68	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.039	2.793	1.036	0.000	0.136	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	20	29	20	16
N.S.	1	1.00	1.00	1.06	1.06	1.25	1.81	1.25	1.00
time (sec)	N/A	0.004	0.004	0.064	0.199	0.280	0.399	0.309	17.502

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	32	32	112	32	26
N.S.	1	1.00	1.00	1.04	1.23	1.23	4.31	1.23	1.00
time (sec)	N/A	0.008	0.006	0.714	0.212	0.306	11.021	0.339	16.726

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	37	44	44	0	44	36
N.S.	1	1.00	1.00	1.03	1.22	1.22	0.00	1.22	1.00
time (sec)	N/A	0.010	0.009	49.960	0.239	0.268	0.000	0.324	16.584

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	140	90	0	94	0	390	179
N.S.	1	1.00	0.95	0.61	0.00	0.64	0.00	2.65	1.22
time (sec)	N/A	0.094	0.470	0.070	0.000	0.287	0.000	0.356	16.565

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	101	66	0	70	0	206	129
N.S.	1	1.00	1.06	0.69	0.00	0.74	0.00	2.17	1.36
time (sec)	N/A	0.052	0.357	0.058	0.000	0.276	0.000	0.351	16.944

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	35	40	0	29	136	75	79
N.S.	1	1.00	0.74	0.85	0.00	0.62	2.89	1.60	1.68
time (sec)	N/A	0.031	0.215	0.018	0.000	0.275	0.364	0.342	16.852

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	160	73	0	318	0	1015	2983
N.S.	1	1.00	1.65	0.75	0.00	3.28	0.00	10.46	30.75
time (sec)	N/A	0.065	0.504	0.025	0.000	0.317	0.000	0.509	32.295

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	187	88	0	399	0	1192	2642
N.S.	1	1.00	1.82	0.85	0.00	3.87	0.00	11.57	25.65
time (sec)	N/A	0.069	0.816	0.034	0.000	0.307	0.000	2.443	35.038

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	188	604	0	196	0	797	1358
N.S.	1	1.00	0.82	2.65	0.00	0.86	0.00	3.50	5.96
time (sec)	N/A	0.255	0.526	0.042	0.000	0.290	0.000	0.356	146.996

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	133	431	0	149	0	445	1012
N.S.	1	1.00	0.81	2.61	0.00	0.90	0.00	2.70	6.13
time (sec)	N/A	0.142	0.318	0.035	0.000	0.317	0.000	0.341	75.361

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	114	82	184	0	103	388	189	110
N.S.	1	1.81	1.30	2.92	0.00	1.63	6.16	3.00	1.75
time (sec)	N/A	0.068	0.205	0.035	0.000	0.326	0.418	0.310	0.242

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	137	258	0	290	0	194	524
N.S.	1	1.00	1.03	1.94	0.00	2.18	0.00	1.46	3.94
time (sec)	N/A	0.153	0.276	0.039	0.000	0.354	0.000	0.422	25.540

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	128	274	0	367	0	311	7637
N.S.	1	1.00	0.91	1.94	0.00	2.60	0.00	2.21	54.16
time (sec)	N/A	0.151	0.522	0.038	0.000	0.307	0.000	0.772	44.379

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	138	294	0	208	0	1447	529
N.S.	1	1.00	0.37	0.78	0.00	0.55	0.00	3.86	1.41
time (sec)	N/A	0.252	0.751	0.098	0.000	0.271	0.000	0.695	16.436

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	93	222	0	159	942	866	385
N.S.	1	1.00	0.36	0.85	0.00	0.61	3.61	3.32	1.48
time (sec)	N/A	0.166	0.586	0.096	0.000	0.276	0.754	0.354	16.830

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	151	55	146	0	106	384	427	252
N.S.	1	2.36	0.86	2.28	0.00	1.66	6.00	6.67	3.94
time (sec)	N/A	0.068	0.483	0.064	0.000	0.286	0.727	0.346	16.640

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	244	181	0	516	0	2649	4060
N.S.	1	1.00	1.55	1.15	0.00	3.29	0.00	16.87	25.86
time (sec)	N/A	0.144	0.939	0.025	0.000	0.323	0.000	0.925	42.361

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	223	187	252	0	675	0	2594	4681
N.S.	1	1.38	1.15	1.56	0.00	4.17	0.00	16.01	28.90
time (sec)	N/A	0.189	10.455	0.041	0.000	0.309	0.000	7.563	51.541

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	0	13	63	13	21
N.S.	1	1.00	1.00	0.67	0.00	0.62	3.00	0.62	1.00
time (sec)	N/A	0.004	0.007	0.033	0.000	0.264	0.163	0.325	16.606

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	0	13	63	13	21
N.S.	1	1.00	1.00	0.67	0.00	0.62	3.00	0.62	1.00
time (sec)	N/A	0.005	0.067	0.029	0.000	0.264	0.174	0.296	16.345

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	16	0	15	51	15	15
N.S.	1	1.00	1.00	0.70	0.00	0.65	2.22	0.65	0.65
time (sec)	N/A	0.017	0.113	0.018	0.000	0.281	0.191	0.317	0.044

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	44	33	31	32	0	77	45
N.S.	1	1.00	1.16	0.87	0.82	0.84	0.00	2.03	1.18
time (sec)	N/A	0.082	0.283	0.913	0.275	0.278	0.000	0.341	16.506

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	67	59	34	51	0	76	563
N.S.	1	1.00	1.40	1.23	0.71	1.06	0.00	1.58	11.73
time (sec)	N/A	0.064	0.250	0.923	0.275	0.288	0.000	0.344	29.813

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	25	24	15	23	0	51	33
N.S.	1	1.00	1.32	1.26	0.79	1.21	0.00	2.68	1.74
time (sec)	N/A	0.039	0.159	0.905	0.269	0.290	0.000	0.341	16.820

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	45	58	17	40	44	48	206
N.S.	1	1.00	2.37	3.05	0.89	2.11	2.32	2.53	10.84
time (sec)	N/A	0.018	0.136	0.968	0.278	0.317	0.904	0.358	21.699

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	72	51	41	41	0	130	122
N.S.	1	1.00	2.25	1.59	1.28	1.28	0.00	4.06	3.81
time (sec)	N/A	0.061	0.146	0.941	0.271	0.342	0.000	0.395	17.759

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	49	50	24	44	0	149	120
N.S.	1	1.00	1.88	1.92	0.92	1.69	0.00	5.73	4.62
time (sec)	N/A	0.053	0.138	0.922	0.264	0.326	0.000	0.313	17.690

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	147	58	54	44	0	235	189
N.S.	1	1.00	4.32	1.71	1.59	1.29	0.00	6.91	5.56
time (sec)	N/A	0.062	0.274	0.921	0.382	0.305	0.000	0.402	19.567

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	1832	90	0	122	0	451	179
N.S.	1	1.00	12.46	0.61	0.00	0.83	0.00	3.07	1.22
time (sec)	N/A	0.088	5.983	0.076	0.000	0.314	0.000	0.387	17.613

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	113	66	0	92	0	255	129
N.S.	1	1.00	1.19	0.69	0.00	0.97	0.00	2.68	1.36
time (sec)	N/A	0.075	0.866	0.067	0.000	0.312	0.000	0.372	16.540

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	71	40	0	50	0	107	79
N.S.	1	1.00	1.51	0.85	0.00	1.06	0.00	2.28	1.68
time (sec)	N/A	0.040	0.514	0.017	0.000	0.317	0.000	0.337	16.554

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	121	73	0	158	0	1093	213
N.S.	1	1.00	1.25	0.75	0.00	1.63	0.00	11.27	2.20
time (sec)	N/A	0.050	0.649	0.020	0.000	0.330	0.000	0.535	18.156

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	135	88	0	182	0	1402	1637
N.S.	1	1.00	1.31	0.85	0.00	1.77	0.00	13.61	15.89
time (sec)	N/A	0.065	10.169	0.038	0.000	0.318	0.000	2.749	24.654

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	75	120	0	243	0	1895	1610
N.S.	1	1.00	0.44	0.70	0.00	1.42	0.00	11.08	9.42
time (sec)	N/A	0.083	10.091	0.036	0.000	0.369	0.000	6.129	26.553

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	182	517	0	479	0	511	1107
N.S.	1	1.00	0.93	2.65	0.00	2.46	0.00	2.62	5.68
time (sec)	N/A	0.250	1.202	0.041	0.000	0.331	0.000	0.795	45.026

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	175	187	0	372	0	272	129
N.S.	1	1.00	1.23	1.32	0.00	2.62	0.00	1.92	0.91
time (sec)	N/A	0.162	0.827	0.037	0.000	0.322	0.000	0.835	0.247

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	254	266	0	346	0	359	5098
N.S.	1	1.00	1.88	1.97	0.00	2.56	0.00	2.66	37.76
time (sec)	N/A	0.141	1.191	0.046	0.000	0.348	0.000	0.918	34.870

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	239	269	0	317	0	438	4285
N.S.	1	1.00	1.73	1.95	0.00	2.30	0.00	3.17	31.05
time (sec)	N/A	0.085	1.144	0.040	0.000	0.381	0.000	0.968	33.209

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	109	313	0	126	0	532	787
N.S.	1	1.00	0.89	2.54	0.00	1.02	0.00	4.33	6.40
time (sec)	N/A	0.142	0.814	0.041	0.000	0.318	0.000	2.835	30.043

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	153	457	0	182	0	802	1290
N.S.	1	1.00	0.88	2.63	0.00	1.05	0.00	4.61	7.41
time (sec)	N/A	0.167	10.182	0.040	0.000	0.342	0.000	2.460	48.119

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	2321	246	0	225	0	932	429
N.S.	1	1.00	8.38	0.89	0.00	0.81	0.00	3.36	1.55
time (sec)	N/A	0.229	6.819	0.101	0.000	0.320	0.000	1.061	17.552

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	197	172	0	167	0	480	268
N.S.	1	1.00	1.21	1.06	0.00	1.02	0.00	2.94	1.64
time (sec)	N/A	0.164	1.076	0.062	0.000	0.313	0.000	1.021	16.856

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	1004	148	0	321	0	2374	762
N.S.	1	1.00	6.48	0.95	0.00	2.07	0.00	15.32	4.92
time (sec)	N/A	0.147	5.569	0.024	0.000	0.322	0.000	1.576	20.353

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	223	690	237	0	260	0	2318	559
N.S.	1	1.42	4.39	1.51	0.00	1.66	0.00	14.76	3.56
time (sec)	N/A	0.158	10.678	0.036	0.000	0.316	0.000	9.204	21.489

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	275	182	300	0	297	0	2766	287
N.S.	1	1.68	1.11	1.83	0.00	1.81	0.00	16.87	1.75
time (sec)	N/A	0.134	10.213	0.047	0.000	0.334	0.000	17.839	19.715

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	45	63	23	44	48	54	209
N.S.	1	1.00	1.45	2.03	0.74	1.42	1.55	1.74	6.74
time (sec)	N/A	0.039	0.008	0.937	0.270	0.295	1.035	0.297	21.797

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	44	33	31	32	0	77	42
N.S.	1	1.00	1.16	0.87	0.82	0.84	0.00	2.03	1.11
time (sec)	N/A	0.231	0.013	0.946	0.257	0.321	0.000	0.319	17.648

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	68	59	34	51	0	76	381
N.S.	1	1.00	1.42	1.23	0.71	1.06	0.00	1.58	7.94
time (sec)	N/A	0.173	0.017	0.953	0.266	0.321	0.000	0.298	26.488

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	25	26	17	25	0	54	25
N.S.	1	1.00	1.19	1.24	0.81	1.19	0.00	2.57	1.19
time (sec)	N/A	0.080	0.004	0.949	0.272	0.315	0.000	0.287	17.276

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	46	59	20	41	48	49	205
N.S.	1	1.00	2.09	2.68	0.91	1.86	2.18	2.23	9.32
time (sec)	N/A	0.037	0.005	0.945	0.263	0.328	1.061	0.284	17.973

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	72	51	41	41	0	130	122
N.S.	1	1.00	2.25	1.59	1.28	1.28	0.00	4.06	3.81
time (sec)	N/A	0.141	0.008	0.959	0.263	0.293	0.000	0.337	18.142

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	49	50	24	44	0	149	118
N.S.	1	1.00	1.88	1.92	0.92	1.69	0.00	5.73	4.54
time (sec)	N/A	0.157	0.010	1.019	0.263	0.327	0.000	0.346	17.875

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	148	57	51	43	0	233	186
N.S.	1	1.00	4.48	1.73	1.55	1.30	0.00	7.06	5.64
time (sec)	N/A	0.159	0.028	1.000	0.262	0.316	0.000	0.421	19.134

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	68	48	0	36	0	123	93
N.S.	1	1.00	2.43	1.71	0.00	1.29	0.00	4.39	3.32
time (sec)	N/A	0.241	0.164	0.952	0.000	0.326	0.000	0.369	19.041

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	46	62	0	37	223	41	200
N.S.	1	1.00	1.39	1.88	0.00	1.12	6.76	1.24	6.06
time (sec)	N/A	0.100	0.089	0.919	0.000	0.288	11.609	0.296	27.821

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	86	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	0.396	0.000	0.000	0.000	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	192	341	293	161	459	171	0
N.S.	1	1.00	1.10	1.95	1.67	0.92	2.62	0.98	0.00
time (sec)	N/A	0.090	0.763	0.932	0.197	0.287	0.604	0.372	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	113	124	107	114	185	107	210
N.S.	1	1.00	0.83	0.91	0.79	0.84	1.36	0.79	1.54
time (sec)	N/A	0.069	0.559	0.876	0.184	0.365	0.529	0.369	18.794

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	80	75	54	74	54	68	136
N.S.	1	1.00	1.18	1.10	0.79	1.09	0.79	1.00	2.00
time (sec)	N/A	0.023	0.137	0.983	0.191	0.292	1.069	0.361	17.640

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	337	651	0	187	0	299	0
N.S.	1	1.00	2.88	5.56	0.00	1.60	0.00	2.56	0.00
time (sec)	N/A	0.067	0.858	0.064	0.000	0.357	0.000	0.501	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	228	3182	0	284	0	146	0
N.S.	1	1.00	1.51	21.07	0.00	1.88	0.00	0.97	0.00
time (sec)	N/A	0.079	0.700	0.084	0.000	0.377	0.000	0.458	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	278	11352	0	536	0	182	0
N.S.	1	1.00	1.44	58.82	0.00	2.78	0.00	0.94	0.00
time (sec)	N/A	0.087	1.906	0.133	0.000	0.551	0.000	1.222	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	225	225	212	0	0	416	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	1.85	0.00	0.00	0.00
time (sec)	N/A	0.123	1.407	0.000	0.000	0.388	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	183	170	0	0	337	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	1.84	0.00	0.00	0.00
time (sec)	N/A	0.109	1.236	0.000	0.000	0.366	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	142	0	0	301	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.087	0.871	0.000	0.000	0.369	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	141	0	0	298	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	2.03	0.00	0.00	0.00
time (sec)	N/A	0.077	0.852	0.000	0.000	0.381	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	169	0	0	487	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	3.08	0.00	0.00	0.00
time (sec)	N/A	0.102	1.309	0.000	0.000	0.465	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	199	199	209	0	0	812	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	4.08	0.00	0.00	0.00
time (sec)	N/A	0.135	1.371	0.000	0.000	0.657	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	41	0	0	26	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.011	0.039	0.000	0.000	0.305	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	69	0	0	59	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.039	0.396	0.000	0.000	0.330	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	35	60	0	34	415	0	0
N.S.	1	1.00	0.78	1.33	0.00	0.76	9.22	0.00	0.00
time (sec)	N/A	0.006	0.057	0.051	0.000	0.338	0.708	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	55	0	28	197	0	0
N.S.	1	1.00	0.76	1.34	0.00	0.68	4.80	0.00	0.00
time (sec)	N/A	0.005	0.060	0.052	0.000	0.325	0.657	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	64	0	30	197	0	0
N.S.	1	1.00	0.76	1.56	0.00	0.73	4.80	0.00	0.00
time (sec)	N/A	0.005	0.048	0.050	0.000	0.327	0.671	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	72	0	0	70	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.021	3.997	0.000	0.000	0.398	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	134	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.133	0.969	0.000	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	260	809	0	345	1363	397	0
N.S.	1	1.00	0.86	2.67	0.00	1.14	4.50	1.31	0.00
time (sec)	N/A	0.260	1.271	1.261	0.000	0.334	1.136	0.349	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	176	300	0	219	490	236	0
N.S.	1	1.00	0.74	1.27	0.00	0.92	2.07	1.00	0.00
time (sec)	N/A	0.163	0.918	1.152	0.000	0.282	0.707	0.354	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	180	123	0	123	197	119	0
N.S.	1	1.00	1.53	1.04	0.00	1.04	1.67	1.01	0.00
time (sec)	N/A	0.046	0.348	1.246	0.000	0.268	0.297	0.303	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	266	1263	0	371	0	0	0
N.S.	1	1.00	1.24	5.87	0.00	1.73	0.00	0.00	0.00
time (sec)	N/A	0.138	0.746	0.082	0.000	2.519	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	427	6303	0	826	0	1618	0
N.S.	1	1.00	1.61	23.70	0.00	3.11	0.00	6.08	0.00
time (sec)	N/A	0.152	1.448	0.106	0.000	1.667	0.000	6.663	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	300	29133	0	1954	0	0	0
N.S.	1	1.00	0.91	88.28	0.00	5.92	0.00	0.00	0.00
time (sec)	N/A	0.201	10.506	0.292	0.000	12.004	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	370	370	985	0	0	923	0	0	0
N.S.	1	1.00	2.66	0.00	0.00	2.49	0.00	0.00	0.00
time (sec)	N/A	0.392	4.555	0.000	0.000	0.447	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	302	302	443	0	0	657	0	0	0
N.S.	1	1.00	1.47	0.00	0.00	2.18	0.00	0.00	0.00
time (sec)	N/A	0.274	2.570	0.000	0.000	0.441	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	233	233	327	0	0	692	0	0	0
N.S.	1	1.00	1.40	0.00	0.00	2.97	0.00	0.00	0.00
time (sec)	N/A	0.210	1.361	0.000	0.000	0.432	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	244	244	341	0	0	716	0	0	0
N.S.	1	1.00	1.40	0.00	0.00	2.93	0.00	0.00	0.00
time (sec)	N/A	0.196	1.511	0.000	0.000	0.432	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	269	269	395	0	0	1456	0	0	0
N.S.	1	1.00	1.47	0.00	0.00	5.41	0.00	0.00	0.00
time (sec)	N/A	0.273	2.408	0.000	0.000	0.785	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	335	335	557	0	0	2514	0	0	0
N.S.	1	1.00	1.66	0.00	0.00	7.50	0.00	0.00	0.00
time (sec)	N/A	0.356	3.340	0.000	0.000	1.961	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	138	216	0	158	0	0	0
N.S.	1	1.00	0.84	1.32	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.082	0.395	0.965	0.000	0.280	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	92	167	0	78	15302	0	0
N.S.	1	1.00	0.85	1.55	0.00	0.72	141.69	0.00	0.00
time (sec)	N/A	0.044	0.223	0.036	0.000	0.300	14.579	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	43	120	0	32	2236	0	0
N.S.	1	1.00	0.83	2.31	0.00	0.62	43.00	0.00	0.00
time (sec)	N/A	0.015	0.127	0.026	0.000	0.280	1.711	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.159	0.000	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	0.166	0.000	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	150	0	0	159	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.071	0.409	0.000	0.000	0.282	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	100	0	0	79	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.043	0.226	0.000	0.000	0.275	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	50	0	0	33	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.015	0.141	0.000	0.000	0.273	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	0.163	0.000	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	157	0	0	201	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.079	0.341	0.000	0.000	0.286	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	111	0	0	110	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.069	0.223	0.000	0.000	0.283	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	48	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.052	0.182	0.000	0.000	0.280	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	0	15	311	15	15
N.S.	1	1.00	1.00	0.94	0.00	0.88	18.29	0.88	0.88
time (sec)	N/A	0.038	0.023	0.854	0.000	0.276	1.509	0.310	16.967

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.075	0.000	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	201	201	173	0	0	204	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.084	0.413	0.000	0.000	0.279	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	123	0	0	113	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.068	0.224	0.000	0.000	0.279	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	51	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.052	0.193	0.000	0.000	0.284	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	18	41	18	18
N.S.	1	1.00	1.00	0.95	0.00	0.90	2.05	0.90	0.90
time (sec)	N/A	0.039	0.025	0.950	0.000	0.270	0.953	0.323	17.024

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.096	0.000	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	365	365	280	0	0	654	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.294	4.201	0.000	0.000	0.311	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	239	239	186	0	0	239	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.160	1.496	0.000	0.000	0.311	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	89	0	0	80	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.055	0.997	0.000	0.000	0.317	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.169	1.212	0.000	0.000	0.000	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.165	1.334	0.000	0.000	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	89	0	0	80	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.079	0.009	0.000	0.000	0.288	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	0.007	0.000	0.000	0.000	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	228	0	0	377	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.254	6.541	0.000	0.000	0.298	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	171	135	0	0	122	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.194	2.242	0.000	0.000	0.335	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	36	0	0	41	0	0	41
N.S.	1	1.00	0.88	0.00	0.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.146	0.252	0.000	0.000	0.273	0.000	0.000	17.344

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.181	3.108	0.000	0.000	0.000	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	36	0	0	41	0	0	39
N.S.	1	1.00	0.88	0.00	0.00	1.00	0.00	0.00	0.95
time (sec)	N/A	0.240	0.008	0.000	0.000	0.274	0.000	0.000	16.689

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	327	327	175	0	0	231	0	0	0
N.S.	1	1.00	0.54	0.00	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.356	0.165	0.000	0.000	0.309	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	76	0	0	117	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.304	0.034	0.000	0.000	0.290	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	177	152	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	0.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	76	0	0	117	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.421	0.021	0.000	0.000	0.282	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	772	272	0	0	0	0	0
N.S.	1	1.00	4.04	1.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.357	4.191	4.541	0.000	0.000	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	191	74	0	141	70	266	50
N.S.	1	1.00	2.36	0.91	0.00	1.74	0.86	3.28	0.62
time (sec)	N/A	0.047	0.087	0.159	0.000	0.332	0.318	0.403	0.111

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	233	66	0	146	63	282	28
N.S.	1	1.00	3.19	0.90	0.00	2.00	0.86	3.86	0.38
time (sec)	N/A	0.030	0.097	0.148	0.000	0.367	0.334	0.405	17.812

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	87	70	0	153	70	67	54
N.S.	1	1.00	2.29	1.84	0.00	4.03	1.84	1.76	1.42
time (sec)	N/A	0.044	0.047	0.080	0.000	0.325	0.382	0.334	0.221

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	87	72	0	155	63	63	67
N.S.	1	1.00	2.29	1.89	0.00	4.08	1.66	1.66	1.76
time (sec)	N/A	0.042	0.041	0.081	0.000	0.323	0.407	0.329	17.550

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	78	0	144	109	0	196
N.S.	1	1.00	1.00	2.05	0.00	3.79	2.87	0.00	5.16
time (sec)	N/A	0.066	0.279	0.334	0.000	0.356	15.540	0.000	17.515

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	72	0	144	99	0	139
N.S.	1	1.00	1.00	1.89	0.00	3.79	2.61	0.00	3.66
time (sec)	N/A	0.065	0.259	0.328	0.000	0.371	15.593	0.000	20.079

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	42	36	155	78	39	42
N.S.	1	1.00	1.00	1.00	0.86	3.69	1.86	0.93	1.00
time (sec)	N/A	0.048	0.016	0.075	0.271	0.329	0.351	0.421	0.113

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	46	42	67	168	75	42	199
N.S.	1	1.00	1.05	0.95	1.52	3.82	1.70	0.95	4.52
time (sec)	N/A	0.049	0.018	0.064	0.262	0.294	0.391	0.398	17.483

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	85	74	0	208	90	0	278
N.S.	1	1.00	2.12	1.85	0.00	5.20	2.25	0.00	6.95
time (sec)	N/A	0.084	0.034	0.222	0.000	0.310	0.651	0.000	17.466

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	85	77	0	213	80	0	30
N.S.	1	1.00	2.12	1.92	0.00	5.32	2.00	0.00	0.75
time (sec)	N/A	0.081	0.035	0.222	0.000	0.296	0.680	0.000	0.155

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	61	42	78	0	146	122	0	0
N.S.	1	1.45	1.00	1.86	0.00	3.48	2.90	0.00	0.00
time (sec)	N/A	0.140	1.017	1.106	0.000	0.287	98.471	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	61	42	74	0	146	112	0	0
N.S.	1	1.45	1.00	1.76	0.00	3.48	2.67	0.00	0.00
time (sec)	N/A	0.138	0.247	1.091	0.000	0.316	97.943	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	86	74	0	153	73	71	233
N.S.	1	1.00	2.15	1.85	0.00	3.82	1.82	1.78	5.82
time (sec)	N/A	0.058	0.033	0.079	0.000	0.300	0.611	0.355	16.921

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	86	74	0	155	66	67	67
N.S.	1	1.00	2.15	1.85	0.00	3.88	1.65	1.68	1.68
time (sec)	N/A	0.059	0.032	0.093	0.000	0.303	0.615	0.340	17.371

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	42	36	155	78	39	42
N.S.	1	1.00	1.00	1.00	0.86	3.69	1.86	0.93	1.00
time (sec)	N/A	0.043	0.015	0.064	0.266	0.312	0.454	0.363	17.469

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	46	42	67	168	75	42	923
N.S.	1	1.00	1.05	0.95	1.52	3.82	1.70	0.95	20.98
time (sec)	N/A	0.045	0.016	0.067	0.268	0.306	0.485	0.362	17.355

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	78	0	146	0	0	0
N.S.	1	1.00	1.00	1.86	0.00	3.48	0.00	0.00	0.00
time (sec)	N/A	0.151	2.304	2.569	0.000	0.314	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	74	0	146	0	0	0
N.S.	1	1.00	1.00	1.76	0.00	3.48	0.00	0.00	0.00
time (sec)	N/A	0.143	0.415	2.591	0.000	0.312	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	84	0	165	0	0	0
N.S.	1	1.00	1.00	2.00	0.00	3.93	0.00	0.00	0.00
time (sec)	N/A	0.160	0.470	3.539	0.000	0.324	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	78	0	165	0	0	0
N.S.	1	1.00	1.00	1.86	0.00	3.93	0.00	0.00	0.00
time (sec)	N/A	0.163	0.570	3.455	0.000	0.313	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	108	1812	125	233	138	155	167
N.S.	1	1.00	0.81	13.52	0.93	1.74	1.03	1.16	1.25
time (sec)	N/A	0.250	0.163	0.135	0.191	0.292	1.913	0.341	17.478

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	1699	62	161	85	72	123
N.S.	1	1.00	0.96	24.62	0.90	2.33	1.23	1.04	1.78
time (sec)	N/A	0.150	0.055	0.075	0.194	0.305	1.586	0.329	17.310

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	26	1628	21	105	46	22	45
N.S.	1	1.00	1.13	70.78	0.91	4.57	2.00	0.96	1.96
time (sec)	N/A	0.060	0.024	0.070	0.183	0.290	1.206	0.365	18.109

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	69	1697	0	316	139	94	1270
N.S.	1	1.00	0.78	19.28	0.00	3.59	1.58	1.07	14.43
time (sec)	N/A	0.171	0.077	0.078	0.000	0.360	2.932	0.333	18.189

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	139	1910	0	530	0	210	4602
N.S.	1	1.00	0.92	12.65	0.00	3.51	0.00	1.39	30.48
time (sec)	N/A	0.264	0.219	0.105	0.000	0.737	0.000	0.334	19.544

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	99	1780	0	1168	0	0	0
N.S.	1	1.00	0.67	12.11	0.00	7.95	0.00	0.00	0.00
time (sec)	N/A	0.159	0.258	0.083	0.000	0.395	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	63	1718	0	510	0	107	0
N.S.	1	1.00	0.61	16.68	0.00	4.95	0.00	1.04	0.00
time (sec)	N/A	0.042	0.209	0.076	0.000	0.315	0.000	0.329	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	105	1811	0	581	0	211	0
N.S.	1	1.00	0.66	11.32	0.00	3.63	0.00	1.32	0.00
time (sec)	N/A	0.159	0.360	0.086	0.000	0.374	0.000	0.344	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	108	332	125	191	146	156	200
N.S.	1	1.00	0.77	2.37	0.89	1.36	1.04	1.11	1.43
time (sec)	N/A	0.208	0.163	0.390	0.182	0.300	2.082	0.317	17.830

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	66	246	62	118	92	72	119
N.S.	1	1.00	0.90	3.37	0.85	1.62	1.26	0.99	1.63
time (sec)	N/A	0.159	0.054	0.163	0.184	0.282	1.632	0.331	18.044

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	29	160	22	61	49	23	60
N.S.	1	1.00	1.12	6.15	0.85	2.35	1.88	0.88	2.31
time (sec)	N/A	0.082	0.027	0.147	0.182	0.303	1.226	0.318	17.253

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	69	223	0	232	150	94	156
N.S.	1	1.00	0.74	2.40	0.00	2.49	1.61	1.01	1.68
time (sec)	N/A	0.164	0.076	1.111	0.000	0.301	2.973	0.383	17.758

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	139	405	0	445	0	211	248
N.S.	1	1.00	0.90	2.63	0.00	2.89	0.00	1.37	1.61
time (sec)	N/A	0.227	0.210	0.970	0.000	0.367	0.000	0.320	18.896

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	311	311	296	994	0	0	0	0	0
N.S.	1	1.00	0.95	3.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	10.432	0.992	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	304	304	252	665	0	0	0	0	0
N.S.	1	1.00	0.83	2.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	10.205	0.881	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	300	300	425	665	0	0	0	0	0
N.S.	1	1.00	1.42	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	10.461	0.863	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	319	319	496	1200	0	0	0	0	0
N.S.	1	1.00	1.55	3.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	10.523	0.834	0.000	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	324	324	604	1049	0	0	0	0	0
N.S.	1	1.00	1.86	3.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	13.067	0.914	0.000	0.000	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	320	0	0	0	0	0	0
N.S.	1	1.00	2.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	0.550	0.000	0.000	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	156	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.133	0.407	0.000	0.000	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	B	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	32	0	61	25	49	26	25
N.S.	1	1.00	1.19	0.00	2.26	0.93	1.81	0.96	0.93
time (sec)	N/A	0.076	0.059	0.000	0.199	0.263	6.000	0.311	16.922

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.004	0.014	0.945	0.276	0.267	0.087	0.322	0.047

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	21	21	26	22	9
N.S.	1	1.00	1.00	0.77	1.62	1.62	2.00	1.69	0.69
time (sec)	N/A	0.006	0.028	0.954	0.276	0.266	0.122	0.325	16.488

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	20	19	19	22	20	19
N.S.	1	1.00	0.93	0.74	0.70	0.70	0.81	0.74	0.70
time (sec)	N/A	0.008	0.019	0.957	0.182	0.255	0.089	0.333	16.555

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	33	25	24	24	0	24	24
N.S.	1	1.00	1.03	0.78	0.75	0.75	0.00	0.75	0.75
time (sec)	N/A	0.010	0.005	0.954	0.183	0.245	0.000	0.325	0.031

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	22	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.88	0.76	0.76
time (sec)	N/A	0.008	0.018	0.947	0.187	0.273	0.092	0.347	0.030

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	15	14	14	15	15	14
N.S.	1	1.00	0.90	0.75	0.70	0.70	0.75	0.75	0.70
time (sec)	N/A	0.007	0.013	0.951	0.179	0.276	0.062	0.336	0.082

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	46	45	47	68	45	73
N.S.	1	1.00	1.00	0.74	0.73	0.76	1.10	0.73	1.18
time (sec)	N/A	0.025	0.010	0.941	0.274	0.269	0.196	0.369	0.051

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	72	50	49	49	0	49	49
N.S.	1	1.00	0.99	0.68	0.67	0.67	0.00	0.67	0.67
time (sec)	N/A	0.021	0.036	1.035	0.185	0.251	0.000	0.320	0.036

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	117	83	82	76	121	82	82
N.S.	1	1.00	0.90	0.64	0.63	0.58	0.93	0.63	0.63
time (sec)	N/A	0.030	0.013	0.988	0.185	0.252	1.125	0.326	0.193

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	126	124	272	638	0	139	223
N.S.	1	1.00	0.63	0.62	1.36	3.19	0.00	0.70	1.12
time (sec)	N/A	0.281	0.011	1.033	0.270	0.983	0.000	0.431	0.095

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.003	0.002	1.009	0.275	0.281	0.106	0.346	0.143

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	17	15	15
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.89	0.79	0.79
time (sec)	N/A	0.009	0.014	0.942	0.197	0.250	0.066	0.329	17.757

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	62	66	83	74	0	83	42
N.S.	1	1.00	0.57	0.61	0.77	0.69	0.00	0.77	0.39
time (sec)	N/A	0.053	0.078	0.950	0.275	0.267	0.000	0.350	0.088

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	83	61	60	62	0	60	78
N.S.	1	1.00	1.09	0.80	0.79	0.82	0.00	0.79	1.03
time (sec)	N/A	0.024	0.046	1.026	0.274	0.272	0.000	0.341	0.071

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	110	76	75	71	0	75	75
N.S.	1	1.00	0.92	0.64	0.63	0.60	0.00	0.63	0.63
time (sec)	N/A	0.033	0.028	1.004	0.194	0.258	0.000	0.351	0.173

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	201	201	127	132	293	547	0	140	208
N.S.	1	1.00	0.63	0.66	1.46	2.72	0.00	0.70	1.03
time (sec)	N/A	0.175	0.007	1.032	0.279	0.940	0.000	0.439	17.040

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	35	36	15	44	0	0	32
N.S.	1	1.00	0.97	1.00	0.42	1.22	0.00	0.00	0.89
time (sec)	N/A	0.025	1.170	0.691	0.214	0.262	0.000	0.000	16.998

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	5	35	0	76	23
N.S.	1	1.00	1.00	0.93	0.17	1.21	0.00	2.62	0.79
time (sec)	N/A	0.022	8.770	0.974	0.213	0.285	0.000	0.352	16.946

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	27	5	35	0	56	23
N.S.	1	1.00	0.93	0.93	0.17	1.21	0.00	1.93	0.79
time (sec)	N/A	0.015	5.582	0.947	0.215	0.284	0.000	0.351	17.306

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	25	5	33	0	51	21
N.S.	1	1.00	0.96	1.00	0.20	1.32	0.00	2.04	0.84
time (sec)	N/A	0.009	4.391	0.962	0.211	0.266	0.000	0.339	17.040

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	24	5	32	0	42	20
N.S.	1	1.00	1.00	1.00	0.21	1.33	0.00	1.75	0.83
time (sec)	N/A	0.022	1.511	0.987	0.220	0.250	0.000	0.337	16.883

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	27	5	35	0	60	23
N.S.	1	1.00	0.90	0.93	0.17	1.21	0.00	2.07	0.79
time (sec)	N/A	0.021	3.142	1.041	0.214	0.251	0.000	0.377	16.739

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	112	0	0	0	117	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.85	0.00	0.00
time (sec)	N/A	0.078	0.153	0.000	0.000	0.000	11.738	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	73	0	0	0	73	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.92	0.00	0.00
time (sec)	N/A	0.026	0.102	0.000	0.000	0.000	2.666	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	50	0	0	0	34	0	51
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.85	0.00	1.28
time (sec)	N/A	0.008	0.043	0.000	0.000	0.000	0.800	0.000	16.892

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.136	0.000	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	0.116	0.000	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	99	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	0.146	0.000	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	185	185	155	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	0.204	0.000	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	8	44	0	0	29
N.S.	1	1.00	1.00	1.06	0.24	1.33	0.00	0.00	0.88
time (sec)	N/A	0.022	0.073	1.061	0.216	0.267	0.000	0.000	16.812

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	32	31	5	41	0	15	25
N.S.	1	1.00	1.03	1.00	0.16	1.32	0.00	0.48	0.81
time (sec)	N/A	0.024	0.041	1.038	0.220	0.237	0.000	0.381	16.672

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	30	7	41	0	0	27
N.S.	1	1.00	1.00	1.07	0.25	1.46	0.00	0.00	0.96
time (sec)	N/A	0.017	0.045	1.043	0.220	0.264	0.000	0.000	17.076

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	30	4	51	0	0	0
N.S.	1	1.00	1.00	1.07	0.14	1.82	0.00	0.00	0.00
time (sec)	N/A	0.013	0.056	0.935	0.217	0.277	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	7	41	0	0	22
N.S.	1	1.00	1.00	1.08	0.27	1.58	0.00	0.00	0.85
time (sec)	N/A	0.023	0.048	0.980	0.211	0.254	0.000	0.000	16.821

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	5	44	0	5	25
N.S.	1	1.00	1.00	1.00	0.16	1.42	0.00	0.16	0.81
time (sec)	N/A	0.024	0.055	0.988	0.217	0.262	0.000	0.348	17.282

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	525	623	0	235	0	0	0
N.S.	1	1.00	1.29	1.53	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.319	24.580	2.425	0.000	0.089	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	12	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.80	0.73	0.73
time (sec)	N/A	0.006	10.023	1.046	0.187	0.243	0.084	0.332	17.831

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	72	14	13	20	31	13	19
N.S.	1	1.00	4.24	0.82	0.76	1.18	1.82	0.76	1.12
time (sec)	N/A	0.006	10.040	1.085	0.196	0.289	0.097	0.373	17.770

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	31	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	2.07	0.73	0.73
time (sec)	N/A	0.006	10.026	1.073	0.188	0.270	0.096	0.359	17.503

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	31	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	2.07	0.73	0.73
time (sec)	N/A	0.005	10.030	1.037	0.194	0.257	0.093	0.411	17.398

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	23	10
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	1.77	0.77
time (sec)	N/A	0.006	0.019	1.051	0.185	0.244	0.066	0.366	17.099

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	45	34	33	32	39	33	32
N.S.	1	1.00	1.02	0.77	0.75	0.73	0.89	0.75	0.73
time (sec)	N/A	0.015	0.023	1.015	0.182	0.247	0.077	0.417	0.115

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	16	15	15	17	16	15
N.S.	1	1.00	0.90	0.76	0.71	0.71	0.81	0.76	0.71
time (sec)	N/A	0.012	0.003	0.932	0.195	0.250	0.061	0.426	17.303

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	34	27	26	25	27	26	25
N.S.	1	1.00	1.03	0.82	0.79	0.76	0.82	0.79	0.76
time (sec)	N/A	0.015	0.020	0.941	0.194	0.257	0.071	0.345	17.247

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	36	35	30	29	42	38	29
N.S.	1	1.00	1.09	1.06	0.91	0.88	1.27	1.15	0.88
time (sec)	N/A	0.015	0.030	0.982	0.186	0.285	0.097	0.340	0.051

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	17	10	12	10	10	14	0	10
N.S.	1	1.70	1.00	1.20	1.00	1.00	1.40	0.00	1.00
time (sec)	N/A	0.038	0.025	1.103	0.185	0.255	0.836	0.000	17.025

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	10	12	86	10	24	0	39
N.S.	1	1.00	0.59	0.71	5.06	0.59	1.41	0.00	2.29
time (sec)	N/A	0.029	0.007	1.197	0.201	0.241	12.624	0.000	17.096

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	38	97	0	0	0	0
N.S.	1	1.00	0.92	1.03	2.62	0.00	0.00	0.00	0.00
time (sec)	N/A	0.066	3.895	56.810	0.282	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	36	95	0	0	0	0
N.S.	1	1.00	0.91	1.03	2.71	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	10.182	43.423	0.263	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	31	35	92	0	0	0	148
N.S.	1	1.00	0.91	1.03	2.71	0.00	0.00	0.00	4.35
time (sec)	N/A	0.078	6.960	32.992	0.269	0.000	0.000	0.000	27.137

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	B	F(-1)	F(-1)	F(-1)	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	0	34	38	95	0	0	0	37
N.S.	1	0.00	0.92	1.03	2.57	0.00	0.00	0.00	1.00
time (sec)	N/A	0.000	8.767	84.549	0.278	0.000	0.000	0.000	25.134

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	B	F(-1)	F(-1)	F(-1)	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	0	34	38	95	0	0	0	146
N.S.	1	0.00	0.92	1.03	2.57	0.00	0.00	0.00	3.95
time (sec)	N/A	0.000	9.145	82.797	0.281	0.000	0.000	0.000	26.024

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	105	78	151	94	177	151	167
N.S.	1	1.00	0.57	0.42	0.82	0.51	0.96	0.82	0.90
time (sec)	N/A	0.186	0.083	0.980	0.186	0.364	0.581	0.399	17.655

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	93	66	112	81	133	127	124
N.S.	1	1.00	0.67	0.48	0.81	0.59	0.96	0.92	0.90
time (sec)	N/A	0.119	0.059	1.046	0.190	0.342	0.559	0.410	0.047

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	68	54	72	67	88	131	79
N.S.	1	1.00	0.76	0.61	0.81	0.75	0.99	1.47	0.89
time (sec)	N/A	0.064	0.041	0.981	0.194	0.339	0.557	0.498	0.030

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	35	35	49	68	82	36
N.S.	1	1.00	1.00	0.85	0.85	1.20	1.66	2.00	0.88
time (sec)	N/A	0.021	0.019	0.927	0.185	0.282	0.095	0.393	0.046

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	63	51	70	118	87	59	130
N.S.	1	1.00	1.11	0.89	1.23	2.07	1.53	1.04	2.28
time (sec)	N/A	0.043	0.056	0.895	0.262	0.285	1.390	0.402	0.089

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	64	63	73	147	73	80	131
N.S.	1	1.00	1.19	1.17	1.35	2.72	1.35	1.48	2.43
time (sec)	N/A	0.046	0.100	0.967	0.262	0.276	22.673	0.325	0.122

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	77	81	113	181	134	105	80
N.S.	1	1.00	0.96	1.01	1.41	2.26	1.68	1.31	1.00
time (sec)	N/A	0.051	0.182	0.983	0.274	0.304	90.954	0.386	0.084

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	232	385	268	286	359	915	0
N.S.	1	1.00	0.71	1.18	0.82	0.88	1.10	2.81	0.00
time (sec)	N/A	0.181	0.211	0.171	0.191	0.371	0.889	0.417	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	147	184	167	184	241	549	0
N.S.	1	1.00	0.66	0.82	0.75	0.82	1.08	2.45	0.00
time (sec)	N/A	0.113	0.127	0.148	0.189	0.315	0.748	0.391	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	84	93	93	103	156	279	0
N.S.	1	1.00	0.63	0.70	0.70	0.77	1.17	2.10	0.00
time (sec)	N/A	0.076	0.068	0.142	0.195	0.314	0.669	0.388	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	55	41	43	50	70	99	44
N.S.	1	1.00	0.98	0.73	0.77	0.89	1.25	1.77	0.79
time (sec)	N/A	0.023	0.035	0.105	0.183	0.333	0.418	0.344	17.903

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	124	154	0	194	0	157	0
N.S.	1	1.00	1.07	1.33	0.00	1.67	0.00	1.35	0.00
time (sec)	N/A	0.114	0.230	0.296	0.000	0.347	0.000	0.443	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	144	166	0	1003	0	232	0
N.S.	1	1.00	1.05	1.21	0.00	7.32	0.00	1.69	0.00
time (sec)	N/A	0.127	0.387	0.271	0.000	0.351	0.000	0.413	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	217	372	0	2856	0	895	0
N.S.	1	1.00	0.97	1.66	0.00	12.75	0.00	4.00	0.00
time (sec)	N/A	0.314	1.750	0.303	0.000	0.562	0.000	0.612	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	235	288	243	228	252	341	317
N.S.	1	1.00	1.02	1.25	1.06	0.99	1.10	1.48	1.38
time (sec)	N/A	0.195	0.199	0.057	0.192	0.289	1.402	0.424	0.070

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	151	166	148	138	167	198	184
N.S.	1	1.00	1.00	1.10	0.98	0.91	1.11	1.31	1.22
time (sec)	N/A	0.114	0.113	0.052	0.186	0.282	1.194	0.446	17.676

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	85	85	81	71	104	105	89
N.S.	1	1.00	0.94	0.94	0.90	0.79	1.16	1.17	0.99
time (sec)	N/A	0.061	0.062	0.040	0.187	0.285	1.074	0.345	0.057

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	40	36	35	33	49	38	33
N.S.	1	1.00	0.98	0.88	0.85	0.80	1.20	0.93	0.80
time (sec)	N/A	0.016	0.026	0.048	0.188	0.276	0.271	0.370	17.445

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	62	69	95	125	97	88	181
N.S.	1	1.00	0.76	0.84	1.16	1.52	1.18	1.07	2.21
time (sec)	N/A	0.054	0.050	0.077	0.338	0.298	1.344	0.353	17.828

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	118	128	191	283	0	191	220
N.S.	1	1.00	0.91	0.98	1.47	2.18	0.00	1.47	1.69
time (sec)	N/A	0.124	0.174	0.079	0.318	0.339	0.000	0.352	18.515

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	198	227	367	534	0	375	1094
N.S.	1	1.00	0.97	1.11	1.80	2.62	0.00	1.84	5.36
time (sec)	N/A	0.204	0.378	0.092	0.301	0.677	0.000	0.361	19.573

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	283	277	251	392	267	324	461
N.S.	1	1.00	1.18	1.15	1.05	1.63	1.11	1.35	1.92
time (sec)	N/A	0.206	0.194	0.072	0.212	0.316	5.690	0.339	0.094

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	194	161	158	269	189	191	197
N.S.	1	1.00	1.17	0.97	0.95	1.62	1.14	1.15	1.19
time (sec)	N/A	0.131	0.136	0.067	0.201	0.305	4.543	0.423	0.063

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	108	87	90	163	126	102	98
N.S.	1	1.00	1.14	0.92	0.95	1.72	1.33	1.07	1.03
time (sec)	N/A	0.067	0.074	0.060	0.193	0.286	3.497	0.344	0.062

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	41	43	75	124	44	43
N.S.	1	1.00	0.91	0.87	0.91	1.60	2.64	0.94	0.91
time (sec)	N/A	0.025	0.040	0.062	0.186	0.293	0.561	0.380	17.233

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	107	118	176	444	167	174	125
N.S.	1	1.00	0.83	0.91	1.36	3.44	1.29	1.35	0.97
time (sec)	N/A	0.093	0.160	0.078	0.274	0.325	4.639	0.357	17.948

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	177	182	367	854	0	311	275
N.S.	1	1.00	0.88	0.90	1.82	4.23	0.00	1.54	1.36
time (sec)	N/A	0.175	0.294	0.111	0.265	0.528	0.000	0.376	0.708

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	301	303	659	1252	0	521	1441
N.S.	1	1.00	0.98	0.99	2.15	4.09	0.00	1.70	4.71
time (sec)	N/A	0.283	0.588	0.135	0.285	1.247	0.000	0.394	21.417

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	232	384	268	231	357	409	0
N.S.	1	1.00	0.72	1.19	0.83	0.71	1.10	1.26	0.00
time (sec)	N/A	0.183	0.196	0.164	0.183	0.309	0.800	0.383	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	147	184	167	140	240	238	0
N.S.	1	1.00	0.66	0.83	0.75	0.63	1.08	1.07	0.00
time (sec)	N/A	0.117	0.119	0.163	0.187	0.340	0.721	0.406	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	84	92	93	71	155	115	0
N.S.	1	1.00	0.64	0.70	0.71	0.54	1.18	0.88	0.00
time (sec)	N/A	0.066	0.066	0.118	0.192	0.319	0.629	0.364	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	42	41	42	34	68	38	44
N.S.	1	1.00	0.78	0.76	0.78	0.63	1.26	0.70	0.81
time (sec)	N/A	0.022	0.026	0.086	0.186	0.331	0.359	0.406	17.118

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	105	92	0	743	0	140	0
N.S.	1	1.00	1.08	0.95	0.00	7.66	0.00	1.44	0.00
time (sec)	N/A	0.071	0.159	0.284	0.000	0.329	0.000	0.381	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	170	259	0	2493	0	644	0
N.S.	1	1.00	1.04	1.59	0.00	15.29	0.00	3.95	0.00
time (sec)	N/A	0.149	0.506	0.290	0.000	0.406	0.000	0.618	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	249	427	0	4390	0	1303	0
N.S.	1	1.00	0.95	1.64	0.00	16.82	0.00	4.99	0.00
time (sec)	N/A	0.356	1.306	0.418	0.000	1.219	0.000	0.444	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	350	350	555	0	728	1416	0	5699	0
N.S.	1	1.00	1.59	0.00	2.08	4.05	0.00	16.28	0.00
time (sec)	N/A	0.232	0.680	0.000	0.217	0.409	0.000	0.508	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	242	284	0	402	712	0	2511	0
N.S.	1	1.00	1.17	0.00	1.66	2.94	0.00	10.38	0.00
time (sec)	N/A	0.137	0.366	0.000	0.210	0.329	0.000	0.358	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	128	0	187	294	0	806	0
N.S.	1	1.00	0.88	0.00	1.29	2.03	0.00	5.56	0.00
time (sec)	N/A	0.084	0.194	0.000	0.198	0.314	0.000	0.352	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	53	0	60	81	0	129	146
N.S.	1	1.00	0.85	0.00	0.97	1.31	0.00	2.08	2.35
time (sec)	N/A	0.034	0.090	0.000	0.200	0.293	0.000	0.568	17.516

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	136	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	0.169	0.000	0.000	0.000	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	77	70	0	164	134	0	0
N.S.	1	1.00	0.83	0.75	0.00	1.76	1.44	0.00	0.00
time (sec)	N/A	0.058	0.196	1.353	0.000	0.275	18.081	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	54	0	130	95	0	0
N.S.	1	1.00	0.87	0.77	0.00	1.86	1.36	0.00	0.00
time (sec)	N/A	0.042	0.136	1.365	0.000	0.290	12.055	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	46	40	0	103	0	0	0
N.S.	1	1.00	0.94	0.82	0.00	2.10	0.00	0.00	0.00
time (sec)	N/A	0.030	0.109	1.281	0.000	0.287	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	78	0	0	0
N.S.	1	1.00	1.00	0.83	0.00	2.60	0.00	0.00	0.00
time (sec)	N/A	0.023	0.102	1.274	0.000	0.299	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	43	0	164	66	0	0
N.S.	1	1.00	1.00	0.83	0.00	3.15	1.27	0.00	0.00
time (sec)	N/A	0.033	0.146	1.070	0.000	0.283	3.161	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	67	59	0	262	90	0	0
N.S.	1	1.00	0.89	0.79	0.00	3.49	1.20	0.00	0.00
time (sec)	N/A	0.049	0.184	1.050	0.000	0.310	4.673	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	81	78	0	169	131	0	0
N.S.	1	1.00	0.80	0.77	0.00	1.67	1.30	0.00	0.00
time (sec)	N/A	0.059	0.161	1.406	0.000	0.265	17.815	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	66	60	0	135	94	0	0
N.S.	1	1.00	0.87	0.79	0.00	1.78	1.24	0.00	0.00
time (sec)	N/A	0.044	0.135	1.407	0.000	0.268	11.992	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	50	44	0	110	0	0	0
N.S.	1	1.00	0.94	0.83	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.031	0.115	1.284	0.000	0.279	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	80	0	0	0
N.S.	1	1.00	1.00	0.84	0.00	2.50	0.00	0.00	0.00
time (sec)	N/A	0.022	0.102	1.377	0.000	0.266	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	0	175	66	0	0
N.S.	1	1.00	1.00	0.84	0.00	3.12	1.18	0.00	0.00
time (sec)	N/A	0.032	0.146	1.109	0.000	0.284	4.180	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	70	65	0	277	88	0	0
N.S.	1	1.00	0.86	0.80	0.00	3.42	1.09	0.00	0.00
time (sec)	N/A	0.048	0.207	1.163	0.000	0.293	4.693	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	56	24	21	17
N.S.	1	1.00	1.00	0.78	1.39	2.43	1.04	0.91	0.74
time (sec)	N/A	0.006	0.003	1.077	0.265	0.294	0.517	0.347	16.702

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	78	0	0	0
N.S.	1	1.00	1.00	0.83	0.00	2.60	0.00	0.00	0.00
time (sec)	N/A	0.023	0.111	1.415	0.000	0.301	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	32	0	116	0	0	0
N.S.	1	1.00	1.00	0.86	0.00	3.14	0.00	0.00	0.00
time (sec)	N/A	0.122	0.184	1.115	0.000	0.282	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	0	147	0	0	0
N.S.	1	1.00	1.00	0.89	0.00	3.34	0.00	0.00	0.00
time (sec)	N/A	0.272	0.217	1.152	0.000	0.313	0.000	0.000	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	46	0	182	0	0	0
N.S.	1	1.00	1.00	0.90	0.00	3.57	0.00	0.00	0.00
time (sec)	N/A	0.506	0.248	1.205	0.000	0.298	0.000	0.000	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	70	63	120	55	80	77	54
N.S.	1	1.00	0.92	0.83	1.58	0.72	1.05	1.01	0.71
time (sec)	N/A	0.018	0.103	1.096	0.266	0.273	51.196	0.361	17.764

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	65	58	67	50	60	67	44
N.S.	1	1.00	1.08	0.97	1.12	0.83	1.00	1.12	0.73
time (sec)	N/A	0.014	0.079	0.991	0.269	0.268	23.559	0.336	17.645

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	46	53	30	43	39	57	30
N.S.	1	1.00	1.05	1.20	0.68	0.98	0.89	1.30	0.68
time (sec)	N/A	0.010	0.063	0.176	0.264	0.269	10.705	0.337	18.111

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	13	16	12	8	37	14
N.S.	1	1.00	1.00	1.44	1.78	1.33	0.89	4.11	1.56
time (sec)	N/A	0.003	0.024	1.017	0.182	0.261	1.120	0.335	17.763

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	24	29	30	28	20	58	25
N.S.	1	1.00	1.14	1.38	1.43	1.33	0.95	2.76	1.19
time (sec)	N/A	0.007	0.029	1.007	0.186	0.279	1.723	0.353	18.035

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	32	34	38	38	34	68	28
N.S.	1	1.00	0.94	1.00	1.12	1.12	1.00	2.00	0.82
time (sec)	N/A	0.010	0.043	1.008	0.191	0.274	2.310	0.490	18.170

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	20	23	11	28	8	11	18
N.S.	1	1.00	2.22	2.56	1.22	3.11	0.89	1.22	2.00
time (sec)	N/A	0.004	0.005	0.976	0.259	0.280	1.444	0.344	18.245

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	20	12	0	28	10	13	18
N.S.	1	1.00	2.22	1.33	0.00	3.11	1.11	1.44	2.00
time (sec)	N/A	0.003	0.002	1.000	0.000	0.274	1.397	0.381	18.373

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	20	895	16	67	71	16	26
N.S.	1	1.00	1.11	49.72	0.89	3.72	3.94	0.89	1.44
time (sec)	N/A	0.051	0.019	0.094	0.183	0.277	0.735	0.317	19.438

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	29	16	21	32	19	16	10
N.S.	1	1.00	1.81	1.00	1.31	2.00	1.19	1.00	0.62
time (sec)	N/A	0.047	0.018	1.142	0.184	0.277	0.089	0.330	18.249

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	40	27	49	36	94	29	26
N.S.	1	1.00	0.91	0.61	1.11	0.82	2.14	0.66	0.59
time (sec)	N/A	0.022	0.109	1.048	0.260	0.264	0.154	0.345	0.091

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	57	49	89	53	133	138	124
N.S.	1	1.00	0.47	0.40	0.74	0.44	1.10	1.14	1.02
time (sec)	N/A	0.077	0.031	1.314	0.269	0.330	0.907	0.342	0.053

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	42	0	268	83	41	42
N.S.	1	1.00	0.98	0.84	0.00	5.36	1.66	0.82	0.84
time (sec)	N/A	0.034	0.076	1.081	0.000	0.323	2.703	0.342	18.790

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	42	0	268	0	41	96
N.S.	1	1.00	0.98	0.84	0.00	5.36	0.00	0.82	1.92
time (sec)	N/A	0.045	0.003	1.522	0.000	0.309	0.000	0.352	19.349

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	56	0	382	131	55	56
N.S.	1	1.00	0.91	0.82	0.00	5.62	1.93	0.81	0.82
time (sec)	N/A	0.030	0.092	1.169	0.000	0.326	2.925	0.349	18.383

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	68	0	382	0	52	50
N.S.	1	1.00	0.91	1.00	0.00	5.62	0.00	0.76	0.74
time (sec)	N/A	0.380	0.057	1.148	0.000	0.320	0.000	0.420	18.896

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	37	26	0	39	32	27	39
N.S.	1	1.00	1.09	0.76	0.00	1.15	0.94	0.79	1.15
time (sec)	N/A	0.021	0.057	0.980	0.000	0.557	0.243	0.333	18.750

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	50	41	0	49	48	43	27
N.S.	1	1.00	0.68	0.55	0.00	0.66	0.65	0.58	0.36
time (sec)	N/A	0.030	0.080	0.992	0.000	0.577	0.253	0.369	18.170

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	14	13	16	17	16	13
N.S.	1	1.00	0.95	0.74	0.68	0.84	0.89	0.84	0.68
time (sec)	N/A	0.003	0.011	0.038	0.179	0.284	0.083	0.324	0.030

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	52	50	67	96	100	55	162
N.S.	1	1.00	0.96	0.93	1.24	1.78	1.85	1.02	3.00
time (sec)	N/A	0.060	0.111	9.070	0.258	0.298	0.447	0.393	0.104

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	32	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	2.29	0.86	0.86
time (sec)	N/A	0.016	0.010	0.102	0.176	0.295	0.650	0.340	17.964

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	33	46	63	54	50	71
N.S.	1	1.00	0.92	0.54	0.75	1.03	0.89	0.82	1.16
time (sec)	N/A	0.031	0.060	0.253	0.259	0.288	0.683	0.343	0.198

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	32	39	30	32
N.S.	1	1.00	1.00	0.84	0.81	0.86	1.05	0.81	0.86
time (sec)	N/A	0.027	0.012	0.237	0.261	0.309	0.578	0.329	18.250

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	32	45	64	53	49	71
N.S.	1	1.00	0.92	0.52	0.74	1.05	0.87	0.80	1.16
time (sec)	N/A	0.024	0.043	0.225	0.270	0.316	0.619	0.358	17.770

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	29	22	21	21	36	22	25
N.S.	1	1.00	0.94	0.71	0.68	0.68	1.16	0.71	0.81
time (sec)	N/A	0.017	0.002	0.136	0.174	0.286	0.696	0.347	0.051

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	60	36	51	65	53	54	79
N.S.	1	1.00	0.92	0.55	0.78	1.00	0.82	0.83	1.22
time (sec)	N/A	0.027	0.044	0.239	0.263	0.289	0.649	0.472	18.275

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	42	0	51	48	45	0
N.S.	1	1.00	0.92	0.68	0.00	0.82	0.77	0.73	0.00
time (sec)	N/A	0.019	0.071	0.044	0.000	0.579	0.290	0.327	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	59	55	0	61	58	55	0
N.S.	1	1.00	0.79	0.73	0.00	0.81	0.77	0.73	0.00
time (sec)	N/A	0.033	0.080	0.048	0.000	0.559	0.326	0.401	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	65	48	0	59	51	53	0
N.S.	1	1.00	0.96	0.71	0.00	0.87	0.75	0.78	0.00
time (sec)	N/A	0.031	0.081	0.051	0.000	0.606	0.332	0.363	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	75	60	0	73	58	67	0
N.S.	1	1.00	0.94	0.75	0.00	0.91	0.72	0.84	0.00
time (sec)	N/A	0.030	0.083	0.056	0.000	0.523	0.411	0.372	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	83	67	0	101	66	88	0
N.S.	1	1.00	0.76	0.61	0.00	0.93	0.61	0.81	0.00
time (sec)	N/A	0.056	0.200	0.056	0.000	1.205	0.471	0.339	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	32	0	47	37	33	0
N.S.	1	1.00	0.91	0.68	0.00	1.00	0.79	0.70	0.00
time (sec)	N/A	0.023	0.060	0.050	0.000	0.515	0.325	0.389	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	63	52	49	48	58	60	102
N.S.	1	1.00	0.94	0.78	0.73	0.72	0.87	0.90	1.52
time (sec)	N/A	0.107	0.095	0.280	0.260	0.283	2.300	0.362	17.319

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	67	54	51	59	60	62	118
N.S.	1	1.00	0.94	0.76	0.72	0.83	0.85	0.87	1.66
time (sec)	N/A	0.084	0.066	0.285	0.266	0.289	2.277	0.354	0.034

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	32	61	46	23	44	46	43
N.S.	1	1.00	0.62	1.17	0.88	0.44	0.85	0.88	0.83
time (sec)	N/A	0.038	0.042	1.009	0.263	0.260	0.207	0.340	18.593

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	31	14	0	37	19	25	0
N.S.	1	1.00	1.55	0.70	0.00	1.85	0.95	1.25	0.00
time (sec)	N/A	0.080	0.063	1.085	0.000	0.452	0.408	0.415	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	31	39	30	29	29	36	30	53
N.S.	1	0.72	0.91	0.70	0.67	0.67	0.84	0.70	1.23
time (sec)	N/A	0.050	0.027	1.089	0.261	0.274	0.418	0.314	18.150

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	130	90	89	76	105	89	88
N.S.	1	1.00	1.12	0.78	0.77	0.66	0.91	0.77	0.76
time (sec)	N/A	0.110	0.052	0.028	0.180	0.265	1.103	0.319	0.156

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	58	54	53	35	71	82	0
N.S.	1	1.00	0.70	0.65	0.64	0.42	0.86	0.99	0.00
time (sec)	N/A	0.041	0.055	0.365	0.196	0.292	0.403	0.391	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	62	17	40	39	216	268	0
N.S.	1	1.00	0.97	0.27	0.62	0.61	3.38	4.19	0.00
time (sec)	N/A	0.037	0.055	0.418	0.198	0.283	1.396	0.386	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	86	59	58	57	65	474	0
N.S.	1	1.00	1.05	0.72	0.71	0.70	0.79	5.78	0.00
time (sec)	N/A	0.064	0.064	0.360	0.195	0.294	0.653	0.412	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	58	54	53	35	71	82	0
N.S.	1	1.00	0.70	0.65	0.64	0.42	0.86	0.99	0.00
time (sec)	N/A	0.033	0.003	0.280	0.194	0.277	0.409	0.351	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	168	121	120	76	165	7916	0
N.S.	1	1.00	0.88	0.64	0.63	0.40	0.87	41.66	0.00
time (sec)	N/A	0.284	0.168	0.779	0.191	0.286	0.651	43.804	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	186	154	153	85	202	271	0
N.S.	1	1.00	0.80	0.66	0.66	0.36	0.87	1.16	0.00
time (sec)	N/A	0.287	0.171	0.967	0.199	0.274	0.730	4.877	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	94	107	106	62	139	859	0
N.S.	1	1.00	0.59	0.67	0.66	0.39	0.87	5.37	0.00
time (sec)	N/A	0.219	0.115	0.438	0.194	0.273	0.631	0.426	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	31	16	0	35	20	25	0
N.S.	1	1.00	1.55	0.80	0.00	1.75	1.00	1.25	0.00
time (sec)	N/A	0.062	0.002	1.015	0.000	0.446	0.394	0.333	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	52	59	38	0	85	39	49	0
N.S.	1	1.18	1.34	0.86	0.00	1.93	0.89	1.11	0.00
time (sec)	N/A	0.026	0.071	0.051	0.000	0.655	0.411	0.422	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	51	50	58	45	73	61	50
N.S.	1	1.00	0.94	0.93	1.07	0.83	1.35	1.13	0.93
time (sec)	N/A	0.293	0.048	1.012	0.185	0.263	1.278	0.424	18.262

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	64	50	0	84	48	44	0
N.S.	1	1.00	0.91	0.71	0.00	1.20	0.69	0.63	0.00
time (sec)	N/A	0.028	0.093	0.045	0.000	0.843	0.333	0.388	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	17	15	15
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.89	0.79	0.79
time (sec)	N/A	0.016	0.014	0.998	0.189	0.253	0.075	0.336	18.260

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	67	50	49	49	71	50	47
N.S.	1	1.00	0.87	0.65	0.64	0.64	0.92	0.65	0.61
time (sec)	N/A	0.046	0.031	1.044	0.186	0.267	0.345	0.361	0.051

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	224	76	289	0	92	0	171	255
N.S.	1	2.80	0.95	3.61	0.00	1.15	0.00	2.14	3.19
time (sec)	N/A	0.212	0.130	1.366	0.000	0.287	0.000	0.826	0.099

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	81	130	0	87	0	81	0
N.S.	1	1.00	0.91	1.46	0.00	0.98	0.00	0.91	0.00
time (sec)	N/A	0.186	0.098	1.001	0.000	2.554	0.000	0.538	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	55	68	0	62	0	65	0
N.S.	1	1.00	0.90	1.11	0.00	1.02	0.00	1.07	0.00
time (sec)	N/A	0.368	0.115	1.010	0.000	1.614	0.000	0.634	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	18	7	27	18	26	14	14
N.S.	1	1.00	2.25	0.88	3.38	2.25	3.25	1.75	1.75
time (sec)	N/A	0.002	0.003	1.071	0.183	0.271	0.540	0.424	0.164

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	18	32	27	27	0	22	12
N.S.	1	1.00	2.25	4.00	3.38	3.38	0.00	2.75	1.50
time (sec)	N/A	0.008	0.004	0.123	0.183	0.271	0.000	0.367	0.066

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	51	27	49	28	63	22	26
N.S.	1	1.00	2.32	1.23	2.23	1.27	2.86	1.00	1.18
time (sec)	N/A	0.003	0.036	0.993	0.184	0.267	0.904	0.323	19.424

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	51	45	51	42	0	35	35
N.S.	1	1.00	2.32	2.05	2.32	1.91	0.00	1.59	1.59
time (sec)	N/A	0.004	0.001	0.109	0.187	0.308	0.000	0.341	18.839

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	66	43	20	39	0	42	138
N.S.	1	1.00	1.83	1.19	0.56	1.08	0.00	1.17	3.83
time (sec)	N/A	0.003	0.119	1.046	0.267	0.260	0.000	0.324	19.795

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	66	56	41	36	0	51	41
N.S.	1	1.00	1.83	1.56	1.14	1.00	0.00	1.42	1.14
time (sec)	N/A	0.010	0.005	0.224	0.259	0.266	0.000	0.343	0.058

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	78	60	55	46	0	47	473
N.S.	1	1.00	1.13	0.87	0.80	0.67	0.00	0.68	6.86
time (sec)	N/A	0.009	0.099	1.038	0.185	0.281	0.000	0.347	32.745

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	78	70	138	64	0	62	119
N.S.	1	1.00	1.13	1.01	2.00	0.93	0.00	0.90	1.72
time (sec)	N/A	0.016	0.001	0.119	0.187	0.296	0.000	0.339	0.050

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	42	33	13	13	0	20	13
N.S.	1	1.00	2.80	2.20	0.87	0.87	0.00	1.33	0.87
time (sec)	N/A	0.009	0.009	0.174	0.276	0.261	0.000	0.329	0.173

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	51	30	15	15	0	16	15
N.S.	1	1.00	2.83	1.67	0.83	0.83	0.00	0.89	0.83
time (sec)	N/A	0.016	0.030	0.959	0.272	0.265	0.000	0.336	17.865

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	63	85	24	24	0	41	36
N.S.	1	1.00	2.62	3.54	1.00	1.00	0.00	1.71	1.50
time (sec)	N/A	0.045	0.053	1.057	0.277	0.278	0.000	0.379	18.265

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	77	80	59	105	0	61	31
N.S.	1	1.00	1.88	1.95	1.44	2.56	0.00	1.49	0.76
time (sec)	N/A	0.050	0.019	1.129	0.267	0.251	0.000	0.378	0.214

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	52	45	37	28	0	36	37
N.S.	1	1.00	1.62	1.41	1.16	0.88	0.00	1.12	1.16
time (sec)	N/A	0.009	0.008	0.181	0.271	0.263	0.000	0.339	0.033

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	64	39	43	32	0	29	43
N.S.	1	1.00	1.68	1.03	1.13	0.84	0.00	0.76	1.13
time (sec)	N/A	0.010	0.013	0.177	0.273	0.285	0.000	0.343	0.039

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	67	61	49	38	0	36	49
N.S.	1	1.00	1.60	1.45	1.17	0.90	0.00	0.86	1.17
time (sec)	N/A	0.011	0.032	0.154	0.271	0.289	0.000	0.326	17.923

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	67	62	70	58	0	40	51
N.S.	1	1.00	1.63	1.51	1.71	1.41	0.00	0.98	1.24
time (sec)	N/A	0.015	0.064	0.136	0.190	0.271	0.000	0.339	0.051

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	97	152	118	180	0	109	90
N.S.	1	1.00	1.28	2.00	1.55	2.37	0.00	1.43	1.18
time (sec)	N/A	0.027	0.053	0.091	0.268	0.296	0.000	0.380	0.275

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	75	76	80	54	0	74	57
N.S.	1	1.00	1.53	1.55	1.63	1.10	0.00	1.51	1.16
time (sec)	N/A	0.009	0.090	0.229	0.260	0.273	0.000	0.385	18.228

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	88	84	53	46	0	114	74
N.S.	1	1.00	1.91	1.83	1.15	1.00	0.00	2.48	1.61
time (sec)	N/A	0.017	0.172	0.243	0.269	0.292	0.000	0.338	0.159

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	17	27	17	0	17	17
N.S.	1	1.00	0.95	0.85	1.35	0.85	0.00	0.85	0.85
time (sec)	N/A	0.037	0.040	1.007	0.187	0.311	0.000	0.300	0.060

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	19	17	16	17	0	17	0
N.S.	1	1.00	1.06	0.94	0.89	0.94	0.00	0.94	0.00
time (sec)	N/A	0.020	0.005	0.985	0.197	0.255	0.000	0.312	0.000

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	68	79	103	83	0	129	62
N.S.	1	1.00	1.26	1.46	1.91	1.54	0.00	2.39	1.15
time (sec)	N/A	0.039	0.175	1.018	0.271	0.315	0.000	0.383	18.001

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	19	18	0	17	0	23	15
N.S.	1	1.00	1.73	1.64	0.00	1.55	0.00	2.09	1.36
time (sec)	N/A	0.004	0.025	1.078	0.000	0.268	0.000	0.330	18.254

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	41	40	0	34	0	0	0
N.S.	1	1.00	1.41	1.38	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.009	1.756	1.101	0.000	0.290	0.000	0.000	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	111	359	0	372	0	287	0
N.S.	1	1.00	0.62	1.99	0.00	2.07	0.00	1.59	0.00
time (sec)	N/A	0.134	0.271	1.259	0.000	0.285	0.000	0.394	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	94	104	0	171	0	350	0
N.S.	1	1.00	0.55	0.60	0.00	0.99	0.00	2.03	0.00
time (sec)	N/A	0.108	0.367	0.291	0.000	0.278	0.000	0.374	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	115	129	0	223	0	452	0
N.S.	1	1.00	0.37	0.42	0.00	0.73	0.00	1.47	0.00
time (sec)	N/A	0.181	0.438	0.332	0.000	0.268	0.000	0.380	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	78	71	0	77	0	81	0
N.S.	1	1.00	1.20	1.09	0.00	1.18	0.00	1.25	0.00
time (sec)	N/A	0.021	0.187	0.166	0.000	0.302	0.000	0.340	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	69	61	0	97	0	143	0
N.S.	1	1.00	0.83	0.73	0.00	1.17	0.00	1.72	0.00
time (sec)	N/A	0.024	0.300	0.125	0.000	0.279	0.000	0.337	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	83	69	0	129	0	184	0
N.S.	1	1.00	0.82	0.68	0.00	1.28	0.00	1.82	0.00
time (sec)	N/A	0.025	0.336	0.130	0.000	0.255	0.000	0.338	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	81	370	0	187	0	197	0
N.S.	1	1.00	0.75	3.43	0.00	1.73	0.00	1.82	0.00
time (sec)	N/A	0.086	0.225	1.039	0.000	0.278	0.000	0.329	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	84	110	0	121	0	263	0
N.S.	1	1.00	0.97	1.26	0.00	1.39	0.00	3.02	0.00
time (sec)	N/A	0.048	0.334	0.341	0.000	0.283	0.000	0.335	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	109	130	0	171	0	367	0
N.S.	1	1.00	0.73	0.87	0.00	1.15	0.00	2.46	0.00
time (sec)	N/A	0.065	0.392	0.573	0.000	0.262	0.000	0.328	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	59	42	35	59	58	182	58	38
N.S.	1	1.40	1.00	0.83	1.40	1.38	4.33	1.38	0.90
time (sec)	N/A	0.131	0.081	2.983	0.233	0.276	0.161	0.308	0.185

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	59	42	35	59	58	182	58	51
N.S.	1	1.40	1.00	0.83	1.40	1.38	4.33	1.38	1.21
time (sec)	N/A	0.146	0.071	1.561	0.236	0.251	0.766	0.320	18.468

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	278	958	0	90	0	0	0
N.S.	1	1.00	2.73	9.39	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.052	22.479	2.635	0.000	0.082	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	256	936	0	69	0	0	0
N.S.	1	1.00	4.13	15.10	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.035	22.424	1.999	0.000	0.091	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	156	200	0	16	0	0	0
N.S.	1	1.00	9.18	11.76	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.011	32.153	1.365	0.000	0.071	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	261	932	0	119	0	0	0
N.S.	1	1.00	3.58	12.77	0.00	1.63	0.00	0.00	0.00
time (sec)	N/A	0.037	22.672	2.033	0.000	0.084	0.000	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	298	972	0	195	0	0	0
N.S.	1	1.00	2.73	8.92	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.053	22.849	2.067	0.000	0.081	0.000	0.000	0.000

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	278	954	0	90	0	0	0
N.S.	1	1.00	2.73	9.35	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.049	24.415	2.223	0.000	0.105	0.000	0.000	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	256	932	0	69	0	0	0
N.S.	1	1.00	4.13	15.03	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.032	18.905	1.946	0.000	0.097	0.000	0.000	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	100	200	0	16	0	0	0
N.S.	1	1.00	5.88	11.76	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.009	33.863	1.416	0.000	0.085	0.000	0.000	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	298	963	0	119	0	0	0
N.S.	1	1.00	4.08	13.19	0.00	1.63	0.00	0.00	0.00
time (sec)	N/A	0.035	19.717	1.446	0.000	0.088	0.000	0.000	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	327	1039	0	195	0	0	0
N.S.	1	1.00	3.00	9.53	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.052	22.028	1.489	0.000	0.102	0.000	0.000	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	730	730	10468	5229	0	0	0	0	0
N.S.	1	1.00	14.34	7.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.651	16.139	6.079	0.000	0.000	0.000	0.000	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	622	622	5218	4865	0	0	0	0	0
N.S.	1	1.00	8.39	7.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.444	16.070	5.159	0.000	0.000	0.000	0.000	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	822	1056	0	0	0	0	0
N.S.	1	1.00	3.62	4.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	11.488	1.042	0.000	0.000	0.000	0.000	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	674	674	5276	5024	0	0	0	0	0
N.S.	1	1.00	7.83	7.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.467	16.124	1.033	0.000	0.000	0.000	0.000	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	663	663	7543	7887	0	0	0	0	0
N.S.	1	1.00	11.38	11.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.555	13.922	7.218	0.000	0.000	0.000	0.000	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	1065	1704	0	0	0	0	0
N.S.	1	1.00	4.53	7.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.134	11.412	1.234	0.000	0.000	0.000	0.000	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	748	748	7629	8103	0	0	0	0	0
N.S.	1	1.00	10.20	10.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.539	16.144	1.373	0.000	0.000	0.000	0.000	0.000

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	452	452	6287	2655	0	0	0	0	0
N.S.	1	1.00	13.91	5.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.425	16.104	5.237	0.000	0.000	0.000	0.000	0.000

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	397	397	3470	2519	0	0	0	0	0
N.S.	1	1.00	8.74	6.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	16.056	4.196	0.000	0.000	0.000	0.000	0.000

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	540	530	0	0	0	0	0
N.S.	1	1.00	3.75	3.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.086	11.033	0.878	0.000	0.000	0.000	0.000	0.000

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	437	437	3526	2601	0	0	0	0	0
N.S.	1	1.00	8.07	5.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	16.075	0.928	0.000	0.000	0.000	0.000	0.000

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	517	517	6386	2757	0	0	0	0	0
N.S.	1	1.00	12.35	5.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	16.135	0.959	0.000	0.000	0.000	0.000	0.000

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	558	558	7235	2694	0	0	0	0	0
N.S.	1	1.00	12.97	4.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	17.097	4.905	0.000	0.000	0.000	0.000	0.000

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	466	466	4389	2551	0	0	0	0	0
N.S.	1	1.00	9.42	5.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.371	14.891	4.755	0.000	0.000	0.000	0.000	0.000

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	813	788	0	0	0	0	0
N.S.	1	1.00	4.54	4.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.121	12.964	1.037	0.000	0.000	0.000	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	474	474	3593	2616	0	0	0	0	0
N.S.	1	1.00	7.58	5.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	14.360	1.122	0.000	0.000	0.000	0.000	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	591	591	6452	2777	0	0	0	0	0
N.S.	1	1.00	10.92	4.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.466	17.030	1.012	0.000	0.000	0.000	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	585	585	8500	2733	0	0	0	0	0
N.S.	1	1.00	14.53	4.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.514	17.053	4.146	0.000	0.000	0.000	0.000	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	485	485	5647	2582	0	0	0	0	0
N.S.	1	1.00	11.64	5.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.412	13.837	4.107	0.000	0.000	0.000	0.000	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	388	388	1145	1147	0	0	0	0	0
N.S.	1	1.00	2.95	2.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	15.188	1.437	0.000	0.000	0.000	0.000	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	311	311	2941	2607	0	0	0	0	0
N.S.	1	1.00	9.46	8.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	14.505	1.277	0.000	0.000	0.000	0.000	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	582	582	5812	2780	0	0	0	0	0
N.S.	1	1.00	9.99	4.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.511	17.139	1.511	0.000	0.000	0.000	0.000	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	129	129	927	965	0	0	0	0	0
N.S.	1	1.00	7.19	7.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	10.702	1.128	0.000	0.000	0.000	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	4865	4426	0	0	0	0	0
N.S.	1	1.00	11.29	10.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.335	16.151	2.704	0.000	0.000	0.000	0.000	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	249	961	0	0	0	0	0
N.S.	1	1.00	2.31	8.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.139	10.382	0.834	0.000	0.000	0.000	0.000	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	602	2564	0	0	0	0	0
N.S.	1	1.00	1.64	6.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	12.702	0.811	0.000	0.000	0.000	0.000	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	126	126	1148	1180	0	0	0	0	0
N.S.	1	1.00	9.11	9.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	10.615	1.158	0.000	0.000	0.000	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	434	434	6019	5421	0	0	0	0	0
N.S.	1	1.00	13.87	12.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.336	16.102	2.992	0.000	0.000	0.000	0.000	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	577	577	6084	5441	0	0	0	0	0
N.S.	1	1.00	10.54	9.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.455	16.098	2.841	0.000	0.000	0.000	0.000	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	130	130	826	1180	0	0	0	0	0
N.S.	1	1.00	6.35	9.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.176	10.174	0.990	0.000	0.000	0.000	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	444	444	4974	5421	0	0	0	0	0
N.S.	1	1.00	11.20	12.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.304	15.989	1.343	0.000	0.000	0.000	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	404	76	57	81	0	151	158
N.S.	1	1.00	7.21	1.36	1.02	1.45	0.00	2.70	2.82
time (sec)	N/A	0.148	1.665	0.627	0.263	0.269	0.000	0.487	25.149

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	68	56	0	84	63	89	55
N.S.	1	1.00	1.05	0.86	0.00	1.29	0.97	1.37	0.85
time (sec)	N/A	0.117	0.222	0.213	0.000	0.315	2.633	0.387	0.044

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	101	66	58	0	78	0	80	52
N.S.	1	1.91	1.25	1.09	0.00	1.47	0.00	1.51	0.98
time (sec)	N/A	0.162	0.167	0.139	0.000	0.256	0.000	0.367	0.044

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	12	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.86	0.86	0.86
time (sec)	N/A	0.107	0.007	1.100	0.263	0.263	0.064	0.313	20.379

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	44	45	43	60	43	43
N.S.	1	1.00	0.92	0.72	0.74	0.70	0.98	0.70	0.70
time (sec)	N/A	0.157	0.063	0.086	0.178	0.264	0.648	0.329	21.011

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	103	85	85	91	116	84	84
N.S.	1	1.00	0.90	0.75	0.75	0.80	1.02	0.74	0.74
time (sec)	N/A	0.172	0.144	0.122	0.178	0.266	0.916	0.332	0.090

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	111	102	0	180	73	92	46
N.S.	1	1.00	1.91	1.76	0.00	3.10	1.26	1.59	0.79
time (sec)	N/A	0.052	0.335	0.115	0.000	0.266	0.956	0.424	21.190

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	111	102	0	180	73	92	62
N.S.	1	1.00	1.91	1.76	0.00	3.10	1.26	1.59	1.07
time (sec)	N/A	0.107	0.003	0.069	0.000	0.272	2.218	0.426	21.204

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	55	39	0	125	0	123	67
N.S.	1	1.00	1.04	0.74	0.00	2.36	0.00	2.32	1.26
time (sec)	N/A	0.031	0.510	1.271	0.000	0.263	0.000	0.336	20.636

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	25	18	17	22	27	18	23
N.S.	1	1.00	1.09	0.78	0.74	0.96	1.17	0.78	1.00
time (sec)	N/A	0.006	0.017	1.031	0.183	0.267	0.078	0.315	0.031

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	25	26	17	25	0	54	35
N.S.	1	1.00	1.09	1.13	0.74	1.09	0.00	2.35	1.52
time (sec)	N/A	0.006	0.001	1.048	0.259	0.259	0.000	0.467	21.379

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	54	58	36	37	0	39	180
N.S.	1	1.00	1.64	1.76	1.09	1.12	0.00	1.18	5.45
time (sec)	N/A	0.015	0.061	1.054	0.183	0.300	0.000	0.340	25.922

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	57	35	2	2	0	31	0
N.S.	1	1.00	1.27	0.78	0.04	0.04	0.00	0.69	0.00
time (sec)	N/A	0.152	0.804	1.064	0.264	0.276	0.000	0.324	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	57	35	2	2	0	31	0
N.S.	1	1.00	1.27	0.78	0.04	0.04	0.00	0.69	0.00
time (sec)	N/A	0.052	0.002	1.056	0.268	0.319	0.000	0.301	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	57	35	2	2	0	31	0
N.S.	1	1.00	1.27	0.78	0.04	0.04	0.00	0.69	0.00
time (sec)	N/A	0.074	0.001	1.041	0.270	0.273	0.000	0.313	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	57	20	2	2	0	12	0
N.S.	1	1.00	1.27	0.44	0.04	0.04	0.00	0.27	0.00
time (sec)	N/A	0.108	3.792	1.073	0.263	0.276	0.000	0.308	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	39	41	0	6	0	6	0
N.S.	1	1.00	0.75	0.79	0.00	0.12	0.00	0.12	0.00
time (sec)	N/A	0.156	1.604	1.064	0.000	0.277	0.000	0.336	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	39	21	6	6	0	6	0
N.S.	1	1.00	0.75	0.40	0.12	0.12	0.00	0.12	0.00
time (sec)	N/A	0.118	1.547	1.070	0.265	0.264	0.000	0.325	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	67	40	0	8	0	8	0
N.S.	1	1.00	0.99	0.59	0.00	0.12	0.00	0.12	0.00
time (sec)	N/A	0.141	1.646	0.058	0.000	0.269	0.000	0.310	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	29	27	25	0	54	19
N.S.	1	1.00	1.26	0.94	0.87	0.81	0.00	1.74	0.61
time (sec)	N/A	0.011	0.278	1.112	0.259	0.273	0.000	0.313	0.074

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	29	25	25	0	54	19
N.S.	1	1.00	0.74	0.94	0.81	0.81	0.00	1.74	0.61
time (sec)	N/A	0.260	0.247	1.187	0.216	0.265	0.000	0.338	0.037

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	29	0	25	0	68	19
N.S.	1	1.00	0.74	0.94	0.00	0.81	0.00	2.19	0.61
time (sec)	N/A	0.121	0.273	0.092	0.000	0.283	0.000	0.369	0.068

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	29	0	25	0	68	19
N.S.	1	1.00	0.74	0.94	0.00	0.81	0.00	2.19	0.61
time (sec)	N/A	0.096	0.269	0.091	0.000	0.273	0.000	0.310	0.052

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	29	0	25	0	68	19
N.S.	1	1.00	0.74	0.94	0.00	0.81	0.00	2.19	0.61
time (sec)	N/A	0.534	0.278	1.092	0.000	0.290	0.000	0.379	21.082

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	38	58	85	44	172	0	0
N.S.	1	1.00	0.88	1.35	1.98	1.02	4.00	0.00	0.00
time (sec)	N/A	0.047	0.133	1.206	0.182	0.304	1.218	0.000	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	53	45	0	44	0	57	31
N.S.	1	1.00	1.26	1.07	0.00	1.05	0.00	1.36	0.74
time (sec)	N/A	0.108	0.101	1.113	0.000	0.287	0.000	0.340	20.710

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	50	57	0	80	0	133	60
N.S.	1	1.00	0.77	0.88	0.00	1.23	0.00	2.05	0.92
time (sec)	N/A	0.111	0.123	1.117	0.000	0.262	0.000	0.332	0.387

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	179	729	0	1174	0	558	0
N.S.	1	1.00	0.92	3.74	0.00	6.02	0.00	2.86	0.00
time (sec)	N/A	0.174	0.933	1.389	0.000	0.299	0.000	0.343	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	34	52	37	34	20
N.S.	1	1.00	1.00	0.75	1.21	1.86	1.32	1.21	0.71
time (sec)	N/A	0.010	0.029	1.543	0.264	0.256	5.908	0.295	21.788

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	0	52	0	34	0
N.S.	1	1.00	1.00	0.75	0.00	1.86	0.00	1.21	0.00
time (sec)	N/A	0.044	0.001	1.335	0.000	0.258	0.000	0.307	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	17	18	18
N.S.	1	1.00	1.00	0.86	0.82	0.82	0.77	0.82	0.82
time (sec)	N/A	0.004	0.005	1.122	0.182	0.256	0.065	0.435	21.069

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	17	18	18
N.S.	1	1.00	1.00	0.86	0.82	0.82	0.77	0.82	0.82
time (sec)	N/A	0.071	0.001	1.142	0.188	0.273	0.212	0.471	0.032

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	0	11	36	11	11
N.S.	1	1.00	1.00	0.71	0.00	0.65	2.12	0.65	0.65
time (sec)	N/A	0.047	0.031	1.247	0.000	0.281	0.226	0.297	22.172

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	17	17	17	125	0	11	0	11	0
N.S.	1	1.00	1.00	7.35	0.00	0.65	0.00	0.65	0.00
time (sec)	N/A	0.034	0.029	1.148	0.000	0.312	0.000	0.303	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	23	7	6	19	7	19	6
N.S.	1	1.00	2.30	0.70	0.60	1.90	0.70	1.90	0.60
time (sec)	N/A	0.002	0.022	1.548	0.272	0.287	0.070	0.354	0.011

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	23	34	6	25	49	12	32
N.S.	1	1.00	2.30	3.40	0.60	2.50	4.90	1.20	3.20
time (sec)	N/A	0.002	0.001	1.230	0.275	0.291	0.544	0.330	0.147

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	23	7	6	19	7	19	6
N.S.	1	1.00	2.30	0.70	0.60	1.90	0.70	1.90	0.60
time (sec)	N/A	0.004	0.001	1.091	0.280	0.253	0.519	0.309	0.014

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	23	7	8	29	7	26	6
N.S.	1	1.00	1.92	0.58	0.67	2.42	0.58	2.17	0.50
time (sec)	N/A	0.005	0.050	1.211	0.265	0.259	0.265	0.327	21.282

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	44	31	8	29	39	13	30
N.S.	1	1.00	3.67	2.58	0.67	2.42	3.25	1.08	2.50
time (sec)	N/A	0.006	0.024	1.205	0.262	0.261	0.530	0.327	0.126

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	44	7	8	29	7	26	6
N.S.	1	1.00	3.67	0.58	0.67	2.42	0.58	2.17	0.50
time (sec)	N/A	0.007	0.001	1.204	0.260	0.283	0.569	0.307	21.092

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	23	5	8	29	3	24	4
N.S.	1	1.00	5.75	1.25	2.00	7.25	0.75	6.00	1.00
time (sec)	N/A	0.004	0.046	1.264	0.269	0.266	0.271	0.444	21.150

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	44	29	8	29	39	13	33
N.S.	1	1.00	11.00	7.25	2.00	7.25	9.75	3.25	8.25
time (sec)	N/A	0.005	0.024	1.205	0.265	0.280	0.540	0.428	0.090

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	44	5	8	29	3	24	4
N.S.	1	1.00	11.00	1.25	2.00	7.25	0.75	6.00	1.00
time (sec)	N/A	0.006	0.001	1.176	0.279	0.304	0.574	0.328	0.064

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.001	0.001	0.035	0.186	0.249	0.020	0.305	0.027

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.006	0.000	1.104	0.195	0.283	0.060	0.298	0.027

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	44	24	0	26	7	10	0
N.S.	1	1.00	1.63	0.89	0.00	0.96	0.26	0.37	0.00
time (sec)	N/A	0.007	0.023	1.099	0.000	0.261	0.770	0.360	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	44	30	0	26	0	10	0
N.S.	1	1.00	1.63	1.11	0.00	0.96	0.00	0.37	0.00
time (sec)	N/A	0.014	0.001	0.160	0.000	0.271	0.000	0.316	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	56	28	14	14	15	26	0
N.S.	1	1.00	2.24	1.12	0.56	0.56	0.60	1.04	0.00
time (sec)	N/A	0.004	0.014	1.108	0.183	0.275	0.656	0.311	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	56	28	0	14	0	21	0
N.S.	1	1.00	2.24	1.12	0.00	0.56	0.00	0.84	0.00
time (sec)	N/A	0.018	0.001	0.114	0.000	0.286	0.000	0.335	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	0.82
time (sec)	N/A	0.001	0.001	1.121	0.184	0.273	0.023	0.313	21.178

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	20	17	12	16	0	15	16
N.S.	1	1.00	1.82	1.55	1.09	1.45	0.00	1.36	1.45
time (sec)	N/A	0.002	0.043	1.108	0.198	0.250	0.000	0.327	21.776

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	7	7
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.78	0.78	0.78
time (sec)	N/A	0.001	0.001	1.082	0.187	0.249	0.019	0.397	21.409

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	22	22	7	23	0	13	18
N.S.	1	1.00	2.44	2.44	0.78	2.56	0.00	1.44	2.00
time (sec)	N/A	0.001	0.041	1.138	0.195	0.254	0.000	0.316	21.496

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	12	10	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.92	0.77	0.69	0.69
time (sec)	N/A	0.001	0.001	1.199	0.186	0.242	0.022	0.337	0.095

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	22	20	12	19	0	15	20
N.S.	1	1.00	1.69	1.54	0.92	1.46	0.00	1.15	1.54
time (sec)	N/A	0.001	0.036	1.198	0.194	0.240	0.000	0.334	21.238

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.001	0.001	1.165	0.191	0.238	0.024	0.335	21.306

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	24	22	7	26	0	13	22
N.S.	1	1.00	2.18	2.00	0.64	2.36	0.00	1.18	2.00
time (sec)	N/A	0.001	0.036	1.147	0.195	0.254	0.000	0.297	21.446

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	48	67	41	52	100	39	172
N.S.	1	1.00	1.37	1.91	1.17	1.49	2.86	1.11	4.91
time (sec)	N/A	0.006	0.075	1.144	0.276	0.274	0.950	0.298	24.154

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	49	86	0	96	0	69	0
N.S.	1	1.00	1.40	2.46	0.00	2.74	0.00	1.97	0.00
time (sec)	N/A	0.006	1.678	1.296	0.000	0.279	0.000	0.374	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	69	70	53	74	0	163	0
N.S.	1	1.00	1.60	1.63	1.23	1.72	0.00	3.79	0.00
time (sec)	N/A	0.013	0.168	1.158	0.269	0.255	0.000	0.383	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	59	32	53	63	0	44	37
N.S.	1	1.00	1.69	0.91	1.51	1.80	0.00	1.26	1.06
time (sec)	N/A	0.044	0.112	1.197	0.280	0.245	0.000	0.384	25.777

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	83	134	65	93	0	196	0
N.S.	1	1.00	1.63	2.63	1.27	1.82	0.00	3.84	0.00
time (sec)	N/A	0.019	0.214	1.181	0.270	0.280	0.000	0.435	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	71	99	65	82	0	87	82
N.S.	1	1.00	1.58	2.20	1.44	1.82	0.00	1.93	1.82
time (sec)	N/A	0.063	1.344	1.233	0.275	0.254	0.000	0.340	0.105

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	20	3	2	18	2	17	2
N.S.	1	1.00	10.00	1.50	1.00	9.00	1.00	8.50	1.00
time (sec)	N/A	0.001	0.001	1.148	0.294	0.253	0.062	0.346	0.007

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	32	29	0	27	0	0	0
N.S.	1	1.00	16.00	14.50	0.00	13.50	0.00	0.00	0.00
time (sec)	N/A	0.001	0.542	1.231	0.000	0.250	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	16	3	2	14	2	25	2
N.S.	1	1.00	8.00	1.50	1.00	7.00	1.00	12.50	1.00
time (sec)	N/A	0.001	0.011	1.240	0.271	0.270	0.062	0.335	0.029

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	42	29	0	81	0	0	0
N.S.	1	1.00	21.00	14.50	0.00	40.50	0.00	0.00	0.00
time (sec)	N/A	0.001	0.511	1.270	0.000	0.255	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	37	18	17	31	15	17	17
N.S.	1	1.00	1.61	0.78	0.74	1.35	0.65	0.74	0.74
time (sec)	N/A	0.002	0.035	1.269	0.271	0.241	0.079	0.329	0.033

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	50	42	0	60	0	0	0
N.S.	1	1.00	2.17	1.83	0.00	2.61	0.00	0.00	0.00
time (sec)	N/A	0.004	0.610	1.169	0.000	0.246	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	33	16	15	25	15	25	15
N.S.	1	1.00	1.57	0.76	0.71	1.19	0.71	1.19	0.71
time (sec)	N/A	0.002	0.019	1.278	0.280	0.263	0.081	0.304	0.032

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	70	47	0	120	0	0	0
N.S.	1	1.00	3.33	2.24	0.00	5.71	0.00	0.00	0.00
time (sec)	N/A	0.002	0.616	1.118	0.000	0.250	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	57	53	0	0	0	0	0	54
N.S.	1	1.16	1.08	0.00	0.00	0.00	0.00	0.00	1.10
time (sec)	N/A	0.019	0.021	0.000	0.000	0.000	0.000	0.000	23.468

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	33	21	0	30	65	30	20
N.S.	1	1.00	1.18	0.75	0.00	1.07	2.32	1.07	0.71
time (sec)	N/A	0.011	0.022	1.140	0.000	0.254	0.189	0.309	0.036

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	39	197	0	84	17	140	105
N.S.	1	1.00	1.05	5.32	0.00	2.27	0.46	3.78	2.84
time (sec)	N/A	0.037	0.040	0.298	0.000	0.243	0.070	0.328	0.153

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	68	33	0	90	27	88	57
N.S.	1	1.00	1.70	0.82	0.00	2.25	0.68	2.20	1.42
time (sec)	N/A	0.032	0.093	0.230	0.000	0.262	0.089	0.384	0.189

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	79	45	94	67	0	117	86
N.S.	1	1.00	1.46	0.83	1.74	1.24	0.00	2.17	1.59
time (sec)	N/A	0.114	0.095	1.125	0.275	0.261	0.000	0.459	21.957

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	78	45	0	67	0	117	86
N.S.	1	1.00	1.30	0.75	0.00	1.12	0.00	1.95	1.43
time (sec)	N/A	0.227	0.079	1.131	0.000	0.240	0.000	0.377	21.712

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	45	35	0	66	0	50	0
N.S.	1	1.00	0.88	0.69	0.00	1.29	0.00	0.98	0.00
time (sec)	N/A	0.083	0.113	0.099	0.000	0.245	0.000	0.314	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	45	35	54	66	0	50	0
N.S.	1	1.00	0.88	0.69	1.06	1.29	0.00	0.98	0.00
time (sec)	N/A	0.079	0.095	1.193	0.273	0.243	0.000	0.325	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	45	51	54	64	0	109	56
N.S.	1	1.00	0.88	1.00	1.06	1.25	0.00	2.14	1.10
time (sec)	N/A	0.108	0.003	1.121	0.270	0.255	0.000	0.345	24.313

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	69	51	0	64	0	109	56
N.S.	1	1.00	1.28	0.94	0.00	1.19	0.00	2.02	1.04
time (sec)	N/A	0.037	0.105	0.036	0.000	0.260	0.000	0.376	0.063

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	0	20	0	0	20
N.S.	1	1.00	0.85	0.74	0.00	0.74	0.00	0.00	0.74
time (sec)	N/A	0.021	0.013	0.085	0.000	0.247	0.000	0.000	23.576

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	12	13	0	16	0	0	138
N.S.	1	1.00	0.86	0.93	0.00	1.14	0.00	0.00	9.86
time (sec)	N/A	0.116	0.158	1.172	0.000	0.263	0.000	0.000	22.464

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	0	16	0	0	10
N.S.	1	1.00	1.00	0.92	0.00	1.33	0.00	0.00	0.83
time (sec)	N/A	0.045	0.001	1.164	0.000	0.256	0.000	0.000	0.055

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	29	31	0	30	0	0	48
N.S.	1	1.00	0.81	0.86	0.00	0.83	0.00	0.00	1.33
time (sec)	N/A	0.096	0.008	1.158	0.000	0.260	0.000	0.000	22.126

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	31	0	30	0	0	43
N.S.	1	1.00	0.88	0.94	0.00	0.91	0.00	0.00	1.30
time (sec)	N/A	0.148	0.002	1.070	0.000	0.266	0.000	0.000	22.947

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	103	0	208	0	92	0
N.S.	1	1.00	1.00	1.47	0.00	2.97	0.00	1.31	0.00
time (sec)	N/A	0.068	0.348	1.108	0.000	0.256	0.000	0.349	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	56	60	0	38	0	46	0
N.S.	1	1.00	0.67	0.72	0.00	0.46	0.00	0.55	0.00
time (sec)	N/A	0.111	0.033	1.695	0.000	0.257	0.000	0.325	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	73	44	63	0	36	0	18	0
N.S.	1	1.55	0.94	1.34	0.00	0.77	0.00	0.38	0.00
time (sec)	N/A	0.321	5.268	1.115	0.000	0.289	0.000	0.304	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	70	75	0	74	0	39	0
N.S.	1	1.00	0.57	0.61	0.00	0.60	0.00	0.32	0.00
time (sec)	N/A	0.196	0.031	1.056	0.000	0.251	0.000	0.336	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	76	62	0	117	0	86	0
N.S.	1	1.00	0.57	0.47	0.00	0.88	0.00	0.65	0.00
time (sec)	N/A	0.055	0.255	0.331	0.000	0.258	0.000	0.323	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	56	49	0	83	0	67	0
N.S.	1	1.00	0.62	0.54	0.00	0.92	0.00	0.74	0.00
time (sec)	N/A	0.035	0.137	0.138	0.000	0.255	0.000	0.323	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	53	42	0	75	0	49	0
N.S.	1	1.00	0.87	0.69	0.00	1.23	0.00	0.80	0.00
time (sec)	N/A	0.026	0.063	0.126	0.000	0.266	0.000	0.306	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	82	79	0	142	0	88	0
N.S.	1	1.00	0.75	0.72	0.00	1.30	0.00	0.81	0.00
time (sec)	N/A	0.048	0.106	0.283	0.000	0.265	0.000	0.376	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	108	109	0	205	0	170	0
N.S.	1	1.00	0.75	0.76	0.00	1.42	0.00	1.18	0.00
time (sec)	N/A	0.064	0.266	0.293	0.000	0.288	0.000	0.399	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	25	25	0	31	0	30	23
N.S.	1	1.00	0.89	0.89	0.00	1.11	0.00	1.07	0.82
time (sec)	N/A	0.090	0.084	1.066	0.000	0.250	0.000	0.300	0.069

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	14	14	14	14
N.S.	1	1.00	1.14	0.86	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.026	0.136	0.042	0.215	0.251	0.287	0.333	20.753

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	24	14	14	14
N.S.	1	1.00	1.14	0.86	1.00	1.71	1.00	1.00	1.00
time (sec)	N/A	0.028	0.194	0.039	0.225	0.252	0.306	0.329	20.188

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	17	19	13	15	15	15	15	15
N.S.	1	1.00	1.12	0.76	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.078	0.004	0.040	0.219	0.254	1.240	0.327	20.414

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	17	19	13	15	25	15	15	15
N.S.	1	1.00	1.12	0.76	0.88	1.47	0.88	0.88	0.88
time (sec)	N/A	0.087	0.004	0.037	0.232	0.250	0.529	0.308	20.177

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.013	0.008	0.012	0.203	0.232	0.182	0.324	20.252

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	12	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.00	1.17	1.17
time (sec)	N/A	0.012	0.007	0.013	0.178	0.228	0.191	0.302	20.163

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.008	0.007	0.010	0.184	0.230	0.204	0.325	20.477

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.14
time (sec)	N/A	0.008	0.008	0.010	0.177	0.243	0.238	0.323	21.677

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	25	12	14	14
N.S.	1	1.00	1.17	1.00	1.17	2.08	1.00	1.17	1.17
time (sec)	N/A	0.007	0.010	0.010	0.176	0.238	0.237	0.306	21.461

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	27	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.93	1.00	1.14	1.14
time (sec)	N/A	0.010	0.009	0.013	0.175	0.234	0.280	0.295	19.764

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	47	47	75	0	0	135	0	0	0
N.S.	1	1.00	1.60	0.00	0.00	2.87	0.00	0.00	0.00
time (sec)	N/A	0.076	0.284	0.000	0.000	1.105	0.000	0.000	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	82	0	0	146	0	0	0
N.S.	1	1.00	1.71	0.00	0.00	3.04	0.00	0.00	0.00
time (sec)	N/A	0.113	0.280	0.000	0.000	1.076	0.000	0.000	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	364	0	0	0	0	0	0
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	0.892	0.000	0.000	0.000	0.000	0.000	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	1464	0	0	0	0	0	0
N.S.	1	1.00	5.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.263	3.421	0.000	0.000	0.000	0.000	0.000	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	42	28	27	25	37	27	27
N.S.	1	1.00	1.02	0.68	0.66	0.61	0.90	0.66	0.66
time (sec)	N/A	0.033	0.032	1.026	0.264	0.366	3.136	0.316	0.034

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	20	24	20	22
N.S.	1	1.00	1.00	0.81	0.77	0.77	0.92	0.77	0.85
time (sec)	N/A	0.037	0.032	0.007	0.268	0.343	0.951	0.330	0.055

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	30	30	39	30	42
N.S.	1	1.00	1.00	0.74	0.71	0.71	0.93	0.71	1.00
time (sec)	N/A	0.124	0.031	1.109	0.265	0.347	4.037	0.337	0.026

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	48	50	0	75	0	44	17
N.S.	1	1.00	2.40	2.50	0.00	3.75	0.00	2.20	0.85
time (sec)	N/A	0.007	0.023	1.063	0.000	0.332	0.000	0.321	19.265

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	52	61	0	84	0	40	21
N.S.	1	1.00	2.60	3.05	0.00	4.20	0.00	2.00	1.05
time (sec)	N/A	0.006	0.022	1.136	0.000	0.348	0.000	0.320	19.251

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	160	247	0	1532	0	0	0
N.S.	1	1.00	1.32	2.04	0.00	12.66	0.00	0.00	0.00
time (sec)	N/A	0.136	0.229	1.104	0.000	0.766	0.000	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	186	397	0	2411	0	0	0
N.S.	1	1.00	1.03	2.19	0.00	13.32	0.00	0.00	0.00
time (sec)	N/A	0.225	0.457	1.062	0.000	43.573	0.000	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	17	16	16	22	11	16
N.S.	1	1.00	1.00	0.65	0.62	0.62	0.85	0.42	0.62
time (sec)	N/A	0.005	6.307	0.023	0.177	0.260	63.480	0.320	19.993

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	47	23	22	32	17	22	22
N.S.	1	1.00	1.81	0.88	0.85	1.23	0.65	0.85	0.85
time (sec)	N/A	0.009	0.037	1.443	0.257	0.326	0.380	0.319	19.761

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	13	10	11	11	12	11	9
N.S.	1	1.00	0.87	0.67	0.73	0.73	0.80	0.73	0.60
time (sec)	N/A	0.003	10.009	1.089	0.180	0.260	0.069	0.319	19.903

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	19	16	28	27	0	27	18
N.S.	1	1.00	0.86	0.73	1.27	1.23	0.00	1.23	0.82
time (sec)	N/A	0.005	0.057	1.026	0.179	0.293	0.000	0.322	19.847

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	41	9	9	17	0	17	0
N.S.	1	1.00	3.42	0.75	0.75	1.42	0.00	1.42	0.00
time (sec)	N/A	0.006	0.043	1.288	0.268	0.272	0.000	0.361	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	43	11	11	17	0	17	0
N.S.	1	1.00	3.58	0.92	0.92	1.42	0.00	1.42	0.00
time (sec)	N/A	0.007	0.035	1.330	0.260	0.300	0.000	0.363	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	12	11	13	13	10	13	10
N.S.	1	1.00	0.80	0.73	0.87	0.87	0.67	0.87	0.67
time (sec)	N/A	0.003	0.028	1.065	0.180	0.269	0.063	0.302	20.276

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	82	47	42	49	0	53	0
N.S.	1	1.00	1.52	0.87	0.78	0.91	0.00	0.98	0.00
time (sec)	N/A	0.034	0.104	1.859	0.266	0.301	0.000	0.300	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	47	38	0	0	0	45	27
N.S.	1	1.00	0.80	0.64	0.00	0.00	0.00	0.76	0.46
time (sec)	N/A	0.049	0.319	3.477	0.000	0.000	0.000	1.062	20.191

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	39	28	0	30	0	33	27
N.S.	1	1.00	0.66	0.47	0.00	0.51	0.00	0.56	0.46
time (sec)	N/A	0.042	0.028	1.047	0.000	0.374	0.000	0.306	19.831

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	51	38	0	40	0	51	0
N.S.	1	1.00	0.54	0.40	0.00	0.43	0.00	0.54	0.00
time (sec)	N/A	0.083	0.032	1.061	0.000	0.373	0.000	0.313	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	20	0	20	29	18	19
N.S.	1	1.00	1.00	1.11	0.00	1.11	1.61	1.00	1.06
time (sec)	N/A	0.018	0.047	1.060	0.000	0.294	0.329	0.334	0.143

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	82	47	0	62	0	60	0
N.S.	1	1.00	0.77	0.44	0.00	0.58	0.00	0.56	0.00
time (sec)	N/A	0.024	0.067	0.345	0.000	0.284	0.000	0.332	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	14	64	0	14	25
N.S.	1	1.00	1.00	0.96	0.56	2.56	0.00	0.56	1.00
time (sec)	N/A	0.004	0.032	2.705	0.179	0.344	0.000	0.325	21.394

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	33	35	49	33	35
N.S.	1	1.00	1.00	0.81	0.79	0.83	1.17	0.79	0.83
time (sec)	N/A	0.028	0.031	0.270	0.268	0.283	0.109	0.301	0.054

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	26	25	27	37	25	27
N.S.	1	1.00	1.00	0.81	0.78	0.84	1.16	0.78	0.84
time (sec)	N/A	0.028	0.022	0.257	0.274	0.282	0.101	0.334	21.127

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	49	53	0	95	0	45	0
N.S.	1	1.00	0.64	0.70	0.00	1.25	0.00	0.59	0.00
time (sec)	N/A	0.018	0.220	0.027	0.000	0.304	0.000	0.327	0.000

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	41	43	0	64	48	79	46
N.S.	1	1.00	0.89	0.93	0.00	1.39	1.04	1.72	1.00
time (sec)	N/A	0.071	0.078	0.115	0.000	0.351	1.419	0.311	20.090

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	0	14	0	13	0
N.S.	1	1.00	1.00	0.90	0.00	0.70	0.00	0.65	0.00
time (sec)	N/A	0.004	0.266	1.023	0.000	0.277	0.000	0.301	0.000

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	36	18	20	38	19	68	0	14
N.S.	1	1.03	0.51	0.57	1.09	0.54	1.94	0.00	0.40
time (sec)	N/A	0.009	0.157	1.133	0.303	0.275	0.658	0.000	19.876

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	111	0	13	16	122	0	13
N.S.	1	1.00	7.40	0.00	0.87	1.07	8.13	0.00	0.87
time (sec)	N/A	0.014	0.193	0.000	0.234	0.300	5.334	0.000	20.460

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	28	24	37	24	46	37	37
N.S.	1	1.00	0.53	0.45	0.70	0.45	0.87	0.70	0.70
time (sec)	N/A	0.015	0.014	1.089	0.180	0.283	0.496	0.322	0.047

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	27	25	25
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.87	0.81	0.81
time (sec)	N/A	0.011	0.015	0.029	0.175	0.263	0.129	0.297	0.072

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	0.82
time (sec)	N/A	0.002	0.025	1.126	0.181	0.268	0.056	0.300	21.206

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	4	4	5	4	4
N.S.	1	1.00	1.00	0.83	0.67	0.67	0.83	0.67	0.67
time (sec)	N/A	0.002	0.009	1.074	0.265	0.303	0.085	0.351	21.159

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	17	14	16	16	0	25	16
N.S.	1	1.00	0.85	0.70	0.80	0.80	0.00	1.25	0.80
time (sec)	N/A	0.004	0.014	1.076	0.265	0.254	0.000	0.310	21.207

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	26	11	12	11	11
N.S.	1	1.00	1.00	0.71	1.53	0.65	0.71	0.65	0.65
time (sec)	N/A	0.002	0.001	0.027	0.188	0.301	0.066	0.301	0.025

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	12	11	11	15	11	12
N.S.	1	1.00	1.00	0.63	0.58	0.58	0.79	0.58	0.63
time (sec)	N/A	0.003	0.002	0.056	0.180	0.279	0.377	0.316	0.025

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	42	28	27	25	37	27	27
N.S.	1	1.00	1.02	0.68	0.66	0.61	0.90	0.66	0.66
time (sec)	N/A	0.008	0.024	1.121	0.260	0.280	1.700	0.305	0.027

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	86	48	63	65	39	64	73
N.S.	1	1.00	1.28	0.72	0.94	0.97	0.58	0.96	1.09
time (sec)	N/A	0.027	0.043	1.077	0.263	0.270	0.637	0.326	21.538

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.001	0.001	0.030	0.182	0.279	0.021	0.309	0.003

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.002	0.001	0.040	0.174	0.278	0.021	0.301	0.026

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.001	0.000	1.110	0.179	0.288	0.460	0.318	0.028

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	75	49	0	192	162	123	0
N.S.	1	1.00	1.23	0.80	0.00	3.15	2.66	2.02	0.00
time (sec)	N/A	0.017	0.061	1.260	0.000	0.390	1.070	0.309	0.000

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	78	55	0	202	162	131	0
N.S.	1	1.00	1.20	0.85	0.00	3.11	2.49	2.02	0.00
time (sec)	N/A	0.016	0.087	1.176	0.000	0.281	1.133	0.320	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	21	21	26	22	9
N.S.	1	1.00	1.00	0.77	1.62	1.62	2.00	1.69	0.69
time (sec)	N/A	0.009	0.018	1.048	0.262	0.288	0.125	0.415	0.032

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	21	21	26	22	9
N.S.	1	1.00	1.00	0.77	1.62	1.62	2.00	1.69	0.69
time (sec)	N/A	0.008	0.002	1.071	0.270	0.301	0.158	0.311	0.027

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	58	77	131	202	80	233
N.S.	1	1.00	1.00	0.81	1.07	1.82	2.81	1.11	3.24
time (sec)	N/A	0.103	0.062	1.194	0.260	0.302	0.699	0.331	20.604

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	23	15	22	0	48	22
N.S.	1	1.00	0.62	0.62	0.41	0.59	0.00	1.30	0.59
time (sec)	N/A	0.018	0.012	1.148	0.189	0.251	0.000	0.330	19.204

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	41	9	9	17	0	17	0
N.S.	1	1.00	3.42	0.75	0.75	1.42	0.00	1.42	0.00
time (sec)	N/A	0.007	0.004	1.076	0.261	0.301	0.000	0.331	0.000

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	71	67	0	89	56	51	0
N.S.	1	1.00	0.75	0.71	0.00	0.94	0.59	0.54	0.00
time (sec)	N/A	0.039	0.106	0.028	0.000	1.124	0.361	0.318	0.000

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	24	23	21	184	23	16
N.S.	1	1.00	0.80	0.69	0.66	0.60	5.26	0.66	0.46
time (sec)	N/A	0.010	0.014	0.127	0.174	0.289	0.636	0.287	19.371

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	28	27	22	265	27	24
N.S.	1	1.00	0.81	0.76	0.73	0.59	7.16	0.73	0.65
time (sec)	N/A	0.010	0.014	0.087	0.183	0.285	0.625	0.310	20.101

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	40	48	0	33	32	29	39
N.S.	1	1.00	1.38	1.66	0.00	1.14	1.10	1.00	1.34
time (sec)	N/A	0.024	0.082	1.078	0.000	0.275	1.378	0.332	21.865

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	42	48	0	36	42	32	40
N.S.	1	1.00	1.68	1.92	0.00	1.44	1.68	1.28	1.60
time (sec)	N/A	0.019	0.086	1.100	0.000	0.308	1.267	0.326	22.285

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	0	15	56	15	22
N.S.	1	1.00	1.00	0.76	0.00	0.71	2.67	0.71	1.05
time (sec)	N/A	0.020	0.052	1.045	0.000	0.300	0.170	0.470	0.042

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	49	57	0	97	0	105	127
N.S.	1	1.00	0.75	0.88	0.00	1.49	0.00	1.62	1.95
time (sec)	N/A	0.042	0.109	0.212	0.000	0.269	0.000	0.323	21.672

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	50	28	0	41	0	63	64
N.S.	1	1.00	1.61	0.90	0.00	1.32	0.00	2.03	2.06
time (sec)	N/A	0.035	0.134	0.227	0.000	0.286	0.000	0.323	0.187

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	57	46	0	54	58	50	0
N.S.	1	1.00	0.70	0.56	0.00	0.66	0.71	0.61	0.00
time (sec)	N/A	0.032	0.085	1.135	0.000	0.621	0.271	0.335	0.000

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	56	55	57	155	55	95
N.S.	1	1.00	1.00	0.76	0.74	0.77	2.09	0.74	1.28
time (sec)	N/A	0.091	0.047	1.049	0.279	0.284	2.480	0.336	20.542

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	115	81	80	80	221	80	130
N.S.	1	1.00	1.00	0.70	0.70	0.70	1.92	0.70	1.13
time (sec)	N/A	0.120	0.055	1.067	0.266	0.274	2.913	0.348	0.127

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	22	29	0	20	0	4	4
N.S.	1	1.00	5.50	7.25	0.00	5.00	0.00	1.00	1.00
time (sec)	N/A	0.035	0.099	0.142	0.000	0.270	0.000	0.337	0.030

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	39	50	40	0	31	18
N.S.	1	1.00	1.00	1.77	2.27	1.82	0.00	1.41	0.82
time (sec)	N/A	0.013	0.028	0.111	0.180	0.284	0.000	0.333	21.500

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	40	37	26	0	28	20
N.S.	1	1.00	1.00	1.67	1.54	1.08	0.00	1.17	0.83
time (sec)	N/A	0.008	0.020	0.172	0.271	0.291	0.000	0.332	20.532

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	28	34	39	51	40	0	35	24
N.S.	1	1.17	1.42	1.62	2.12	1.67	0.00	1.46	1.00
time (sec)	N/A	0.010	0.050	0.109	0.191	0.308	0.000	0.360	0.031

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	39	38	38	32	38	20
N.S.	1	1.00	1.00	1.62	1.58	1.58	1.33	1.58	0.83
time (sec)	N/A	0.023	0.019	0.092	0.177	0.274	1.000	0.496	0.037

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	51	45	51	42	0	35	35
N.S.	1	1.00	2.32	2.05	2.32	1.91	0.00	1.59	1.59
time (sec)	N/A	0.006	0.035	0.096	0.179	0.278	0.000	0.313	0.040

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	52	44	35	25	0	35	23
N.S.	1	1.00	1.79	1.52	1.21	0.86	0.00	1.21	0.79
time (sec)	N/A	0.011	0.009	0.189	0.261	0.279	0.000	0.344	0.041

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	49	27	36	35	22	25	26
N.S.	1	1.00	1.48	0.82	1.09	1.06	0.67	0.76	0.79
time (sec)	N/A	0.011	0.066	1.159	0.258	0.295	0.513	0.304	21.653

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	40	7	6	16	5	25	6
N.S.	1	1.00	5.00	0.88	0.75	2.00	0.62	3.12	0.75
time (sec)	N/A	0.006	0.026	1.167	0.262	0.291	0.497	0.313	0.017

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	11	18	0	18	13
N.S.	1	1.00	0.85	0.77	0.85	1.38	0.00	1.38	1.00
time (sec)	N/A	0.015	0.014	1.084	0.182	0.261	0.000	0.302	22.393

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	24	41	0	33	0	73	17
N.S.	1	1.00	1.09	1.86	0.00	1.50	0.00	3.32	0.77
time (sec)	N/A	0.027	0.039	1.079	0.000	0.273	0.000	0.342	22.370

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	39	30	11	13	15	13	45
N.S.	1	1.00	1.62	1.25	0.46	0.54	0.62	0.54	1.88
time (sec)	N/A	0.026	0.018	1.132	0.265	0.259	0.375	0.295	22.709

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	55	23	27	62	75	59	28
N.S.	1	1.00	1.96	0.82	0.96	2.21	2.68	2.11	1.00
time (sec)	N/A	0.007	0.038	1.122	0.185	0.287	0.329	0.306	21.388

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	55	23	27	62	75	59	28
N.S.	1	1.00	1.96	0.82	0.96	2.21	2.68	2.11	1.00
time (sec)	N/A	0.008	0.001	1.086	0.186	0.295	0.707	0.472	21.322

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	55	29	27	62	75	59	28
N.S.	1	1.00	1.96	1.04	0.96	2.21	2.68	2.11	1.00
time (sec)	N/A	0.015	0.001	0.040	0.184	0.296	0.656	0.312	21.687

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	55	29	27	62	75	59	28
N.S.	1	1.00	1.96	1.04	0.96	2.21	2.68	2.11	1.00
time (sec)	N/A	0.018	0.001	0.037	0.185	0.294	0.705	0.325	22.675

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	55	29	27	62	75	59	28
N.S.	1	1.00	1.96	1.04	0.96	2.21	2.68	2.11	1.00
time (sec)	N/A	0.012	0.001	0.041	0.176	0.278	0.662	0.351	22.632

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	55	29	27	62	75	59	28
N.S.	1	1.00	1.96	1.04	0.96	2.21	2.68	2.11	1.00
time (sec)	N/A	0.009	0.001	0.038	0.178	0.304	0.686	0.362	21.136

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	56	28	36	87	97	76	33
N.S.	1	1.00	1.40	0.70	0.90	2.18	2.42	1.90	0.82
time (sec)	N/A	0.016	0.007	1.189	0.184	0.271	0.334	0.324	21.399

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	56	28	36	87	97	76	33
N.S.	1	1.00	1.40	0.70	0.90	2.18	2.42	1.90	0.82
time (sec)	N/A	0.013	0.001	1.129	0.187	0.282	0.657	0.336	20.973

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	56	28	36	87	97	76	33
N.S.	1	1.00	1.40	0.70	0.90	2.18	2.42	1.90	0.82
time (sec)	N/A	0.011	0.001	1.095	0.181	0.335	0.711	0.301	22.701

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	56	37	36	87	97	76	33
N.S.	1	1.00	1.40	0.92	0.90	2.18	2.42	1.90	0.82
time (sec)	N/A	0.014	0.001	0.045	0.181	0.284	0.691	0.301	23.073

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	0	110	0	0	68	0	0	0
N.S.	1	0.00	1.75	0.00	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.000	0.501	0.000	0.000	0.639	0.000	0.000	0.000

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	0	91	0	0	0	0	0	0
N.S.	1	0.00	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.616	0.000	0.000	0.000	0.000	0.000	0.000

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	78	319	83	217	0	434	0	218	649
N.S.	1	4.09	1.06	2.78	0.00	5.56	0.00	2.79	8.32
time (sec)	N/A	0.562	0.323	2.327	0.000	0.289	0.000	0.409	20.649

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	126	126	97	430	0	105	0	444	0
N.S.	1	1.00	0.77	3.41	0.00	0.83	0.00	3.52	0.00
time (sec)	N/A	0.132	0.109	1.669	0.000	0.280	0.000	0.376	0.000

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	96	0	0	62	0	0	0
N.S.	1	1.00	4.36	0.00	0.00	2.82	0.00	0.00	0.00
time (sec)	N/A	0.052	0.219	0.000	0.000	0.632	0.000	0.000	0.000

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.101	0.833	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	44	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.105	0.797	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	389	1528	0	0	0	0	0
N.S.	1	1.00	2.11	8.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.170	10.356	4.993	0.000	0.000	0.000	0.000	0.000

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	90	1036	0	0	0	0	0
N.S.	1	1.00	0.69	7.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	20.049	2.355	0.000	0.000	0.000	0.000	0.000

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	419	47	0	305	0	0	0
N.S.	1	1.00	7.76	0.87	0.00	5.65	0.00	0.00	0.00
time (sec)	N/A	0.169	12.408	2.702	0.000	7.879	0.000	0.000	0.000

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	416	46	0	304	0	0	0
N.S.	1	1.00	7.85	0.87	0.00	5.74	0.00	0.00	0.00
time (sec)	N/A	0.188	11.905	2.712	0.000	7.665	0.000	0.000	0.000

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	90	69	0	154	0	164	0
N.S.	1	1.00	1.07	0.82	0.00	1.83	0.00	1.95	0.00
time (sec)	N/A	0.089	0.232	1.487	0.000	0.256	0.000	0.336	0.000

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	17	23	7	22	26	15	13
N.S.	1	1.00	0.85	1.15	0.35	1.10	1.30	0.75	0.65
time (sec)	N/A	0.005	0.004	0.068	0.294	0.257	0.183	0.299	21.024

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	38	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.016	0.026	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	246	92	0	324	0	0	0
N.S.	1	1.00	2.80	1.05	0.00	3.68	0.00	0.00	0.00
time (sec)	N/A	0.214	0.941	3.960	0.000	2.158	0.000	0.000	0.000

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	213	96	0	331	0	0	0
N.S.	1	1.00	2.42	1.09	0.00	3.76	0.00	0.00	0.00
time (sec)	N/A	0.239	0.921	3.883	0.000	2.104	0.000	0.000	0.000

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	107	0	0	161	0	0	0
N.S.	1	1.00	2.33	0.00	0.00	3.50	0.00	0.00	0.00
time (sec)	N/A	0.416	5.000	0.000	0.000	5.583	0.000	0.000	0.000

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	114	0	0	161	0	0	0
N.S.	1	1.00	2.48	0.00	0.00	3.50	0.00	0.00	0.00
time (sec)	N/A	0.425	4.440	0.000	0.000	5.158	0.000	0.000	0.000

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	107	0	0	161	0	0	0
N.S.	1	1.00	2.33	0.00	0.00	3.50	0.00	0.00	0.00
time (sec)	N/A	0.806	0.007	0.000	0.000	5.510	0.000	0.000	0.000

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	114	0	0	161	0	0	0
N.S.	1	1.00	2.48	0.00	0.00	3.50	0.00	0.00	0.00
time (sec)	N/A	0.798	0.008	0.000	0.000	5.109	0.000	0.000	0.000

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	27	147	94	96	17	58	132
N.S.	1	1.00	1.42	7.74	4.95	5.05	0.89	3.05	6.95
time (sec)	N/A	0.419	0.266	1.382	0.256	0.243	8.672	0.337	22.531

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	90	90	180	1673	0	458	0	0	0
N.S.	1	1.00	2.00	18.59	0.00	5.09	0.00	0.00	0.00
time (sec)	N/A	0.042	0.356	17.656	0.000	5.029	0.000	0.000	0.000

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	425	181	692	0	289	0	0	0
N.S.	1	4.13	1.76	6.72	0.00	2.81	0.00	0.00	0.00
time (sec)	N/A	0.464	1.344	10.010	0.000	2.921	0.000	0.000	0.000

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	47	53	49	0	51	0	0	0
N.S.	1	0.96	1.08	1.00	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.101	0.171	1.869	0.000	0.288	0.000	0.000	0.000

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	383	66	0	472	0	0	0
N.S.	1	1.00	4.79	0.82	0.00	5.90	0.00	0.00	0.00
time (sec)	N/A	0.356	10.565	3.405	0.000	28.967	0.000	0.000	0.000

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.002	0.000	1.175	0.184	0.235	0.021	0.316	0.005

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	122	129	89	0	73	0	193	549
N.S.	1	2.90	3.07	2.12	0.00	1.74	0.00	4.60	13.07
time (sec)	N/A	0.152	0.074	0.393	0.000	0.262	0.000	0.374	20.258

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	149	129	89	0	73	0	193	549
N.S.	1	3.55	3.07	2.12	0.00	1.74	0.00	4.60	13.07
time (sec)	N/A	0.345	0.070	1.071	0.000	0.261	0.000	0.347	0.506

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	0	31	32	16	31	73	81	31
N.S.	1	0.00	0.91	0.94	0.47	0.91	2.15	2.38	0.91
time (sec)	N/A	0.000	0.092	1.341	0.270	0.266	17.963	0.364	19.805

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	177	177	182	1597	0	164	0	0	0
N.S.	1	1.00	1.03	9.02	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.057	5.604	1.541	0.000	0.353	0.000	0.000	0.000

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	243	100	228	0	97	0	0	0
N.S.	1	2.43	1.00	2.28	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.094	3.632	2.072	0.000	0.337	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [42] had the largest ratio of [.9474000000000000020]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.00	19	0.210
2	A	4	4	1.00	23	0.174
3	A	4	4	1.00	21	0.190
4	A	4	4	1.00	21	0.190
5	A	4	4	1.00	33	0.121
6	A	4	4	1.00	35	0.114
7	A	4	4	1.00	36	0.111
8	A	4	4	1.00	36	0.111
9	A	4	4	1.00	24	0.167
10	A	4	4	1.00	20	0.200
11	A	4	4	1.00	24	0.167
12	A	4	4	1.00	22	0.182
13	A	4	4	1.00	22	0.182
14	A	8	8	1.00	15	0.533
15	A	8	8	1.00	17	0.471
16	A	8	8	1.00	15	0.533
17	A	8	8	1.00	17	0.471
18	A	1	1	1.00	25	0.040
19	A	3	3	1.00	25	0.120
20	A	1	1	1.00	25	0.040
21	A	1	1	1.00	21	0.048
22	A	1	1	1.00	24	0.042
23	A	11	9	1.00	19	0.474
24	A	9	8	1.00	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	8	7	1.00	19	0.368
26	A	6	5	1.00	17	0.294
27	A	13	12	1.00	19	0.632
28	A	20	17	1.00	19	0.895
29	A	10	6	1.00	19	0.316
30	A	8	6	1.00	19	0.316
31	A	7	6	1.00	19	0.316
32	A	5	4	1.00	17	0.235
33	A	10	9	1.00	19	0.474
34	A	17	13	1.00	19	0.684
35	A	32	17	1.00	19	0.895
36	A	10	8	1.00	19	0.421
37	A	8	7	1.00	19	0.368
38	A	7	6	1.00	19	0.316
39	A	5	4	1.00	17	0.235
40	A	10	9	1.00	19	0.474
41	A	18	15	1.00	19	0.790
42	A	30	18	1.00	19	0.947
43	A	2	2	1.00	28	0.071
44	A	2	2	1.00	32	0.062
45	A	2	2	1.00	30	0.067
46	A	2	2	1.00	30	0.067
47	A	2	2	1.00	53	0.038
48	A	2	2	1.00	55	0.036
49	A	2	2	1.00	56	0.036
50	A	2	2	1.00	56	0.036
51	A	2	2	1.00	30	0.067
52	A	4	4	1.00	24	0.167
53	A	4	4	1.00	28	0.143
54	A	4	4	1.00	26	0.154
55	A	4	4	1.00	26	0.154
56	A	4	4	1.00	24	0.167
57	A	4	4	1.00	28	0.143
58	A	4	4	1.00	26	0.154
59	A	4	4	1.00	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	4	4	1.00	38	0.105
61	A	4	4	1.00	40	0.100
62	A	4	4	1.00	41	0.098
63	A	4	4	1.00	41	0.098
64	A	4	4	1.00	29	0.138
65	A	4	4	1.00	20	0.200
66	A	4	4	1.00	24	0.167
67	A	4	4	1.00	22	0.182
68	A	4	4	1.00	22	0.182
69	A	4	4	1.00	34	0.118
70	A	4	4	1.00	36	0.111
71	A	4	4	1.00	37	0.108
72	A	4	4	1.00	37	0.108
73	A	4	4	1.00	25	0.160
74	A	2	2	1.00	20	0.100
75	A	2	2	1.00	22	0.091
76	A	2	2	1.00	20	0.100
77	A	2	2	1.00	22	0.091
78	A	2	2	1.00	43	0.047
79	A	2	2	1.00	44	0.045
80	A	2	2	1.00	45	0.044
81	A	2	2	1.00	46	0.043
82	A	2	2	1.00	30	0.067
83	A	4	4	1.00	22	0.182
84	A	4	4	1.00	22	0.182
85	A	4	4	1.00	20	0.200
86	A	4	4	1.00	24	0.167
87	A	4	4	1.00	35	0.114
88	A	4	4	1.00	35	0.114
89	A	4	4	1.00	36	0.111
90	A	4	4	1.00	38	0.105
91	A	4	4	1.00	29	0.138
92	A	4	4	1.00	18	0.222
93	A	4	4	1.00	18	0.222
94	A	4	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	4	4	1.00	20	0.200
96	A	4	4	1.00	31	0.129
97	A	4	4	1.00	31	0.129
98	A	4	4	1.00	32	0.125
99	A	4	4	1.00	34	0.118
100	A	4	4	1.00	25	0.160
101	A	2	2	1.00	30	0.067
102	A	2	2	1.00	36	0.056
103	A	2	2	1.00	34	0.059
104	A	2	2	1.00	32	0.062
105	A	2	2	1.00	58	0.034
106	A	2	2	1.00	61	0.033
107	A	2	2	1.00	62	0.032
108	A	2	2	1.00	61	0.033
109	A	2	2	1.00	52	0.038
110	A	2	2	1.00	55	0.036
111	A	2	2	1.00	56	0.036
112	A	2	2	1.00	55	0.036
113	A	2	2	1.00	30	0.067
114	A	2	2	1.00	36	0.056
115	A	2	2	1.00	34	0.059
116	A	2	2	1.00	32	0.062
117	A	2	2	1.00	58	0.034
118	A	2	2	1.00	61	0.033
119	A	2	2	1.00	62	0.032
120	A	2	2	1.00	61	0.033
121	A	2	2	1.00	52	0.038
122	A	2	2	1.00	55	0.036
123	A	2	2	1.00	56	0.036
124	A	2	2	1.00	55	0.036
125	A	4	4	1.00	23	0.174
126	A	4	4	1.00	25	0.160
127	A	4	4	1.00	25	0.160
128	A	4	4	1.00	29	0.138
129	A	4	4	1.00	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	4	4	1.00	27	0.148
131	A	4	4	1.00	42	0.095
132	A	4	4	1.00	44	0.091
133	A	4	4	1.00	45	0.089
134	A	4	4	1.00	45	0.089
135	A	4	4	1.00	21	0.190
136	A	4	4	1.00	25	0.160
137	A	4	4	1.00	23	0.174
138	A	4	4	1.00	23	0.174
139	A	4	4	1.00	23	0.174
140	A	4	4	1.00	38	0.105
141	A	4	4	1.00	40	0.100
142	A	4	4	1.00	41	0.098
143	A	4	4	1.00	41	0.098
144	A	6	6	1.00	25	0.240
145	A	6	6	1.00	29	0.207
146	A	6	6	1.00	27	0.222
147	A	6	6	1.00	27	0.222
148	A	6	6	1.00	27	0.222
149	A	6	6	1.00	31	0.194
150	A	6	6	1.00	29	0.207
151	A	6	6	1.00	29	0.207
152	A	5	5	1.00	21	0.238
153	A	5	5	1.00	25	0.200
154	A	5	5	1.00	23	0.217
155	A	5	5	1.00	23	0.217
156	A	5	5	1.00	23	0.217
157	A	5	5	1.00	27	0.185
158	A	5	5	1.00	25	0.200
159	A	5	5	1.00	25	0.200
160	A	8	8	1.00	16	0.500
161	A	8	8	1.00	18	0.444
162	A	8	8	1.00	16	0.500
163	A	8	8	1.00	18	0.444
164	A	8	8	1.00	22	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	8	8	1.00	24	0.333
166	A	8	8	1.00	22	0.364
167	A	8	8	1.00	24	0.333
168	A	6	6	1.00	18	0.333
169	A	6	6	1.00	20	0.300
170	A	6	6	1.00	18	0.333
171	A	6	6	1.00	20	0.300
172	A	1	1	1.00	31	0.032
173	A	3	3	1.00	30	0.100
174	A	2	1	1.00	18	0.056
175	A	2	1	1.00	16	0.062
176	A	2	1	1.00	15	0.067
177	A	3	2	1.00	18	0.111
178	A	2	1	1.00	20	0.050
179	A	2	1	1.00	18	0.056
180	A	2	1	1.00	17	0.059
181	A	3	2	1.00	20	0.100
182	A	2	1	1.00	20	0.050
183	A	2	1	1.00	18	0.056
184	A	2	1	1.00	17	0.059
185	A	3	2	1.00	20	0.100
186	A	7	2	1.00	20	0.100
187	A	7	2	1.00	20	0.100
188	A	7	2	1.00	20	0.100
189	A	5	2	1.00	20	0.100
190	A	5	2	1.00	18	0.111
191	A	5	2	1.00	17	0.118
192	A	8	3	1.00	20	0.150
193	A	8	3	1.00	20	0.150
194	A	5	2	1.00	22	0.091
195	A	8	3	1.00	20	0.150
196	A	13	12	1.00	19	0.632
197	F	0	0	N/A	0.000	N/A
198	A	2	2	1.00	27	0.074
199	A	2	2	1.00	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	2	2	1.00	27	0.074
201	A	2	2	1.00	29	0.069
202	A	2	2	1.00	31	0.065
203	A	2	2	1.00	35	0.057
204	A	2	2	1.00	33	0.061
205	A	2	2	1.00	33	0.061
206	A	11	10	1.00	19	0.526
207	A	10	9	1.00	19	0.474
208	A	8	6	1.00	17	0.353
209	A	2	2	1.00	11	0.182
210	A	15	13	1.00	19	0.684
211	A	32	15	1.00	19	0.790
212	A	9	8	1.00	19	0.421
213	A	8	7	1.00	19	0.368
214	A	6	5	1.00	17	0.294
215	A	1	1	1.00	11	0.091
216	A	7	7	1.00	19	0.368
217	A	11	11	1.00	19	0.579
218	A	12	12	1.00	19	0.632
219	A	4	4	1.00	19	0.210
220	A	4	4	1.00	19	0.210
221	A	3	3	1.00	17	0.176
222	A	2	2	1.00	11	0.182
223	A	14	13	1.00	19	0.684
224	A	10	3	1.00	20	0.150
225	A	10	3	1.00	22	0.136
226	A	16	8	1.00	24	0.333
227	A	3	2	1.00	19	0.105
228	A	4	3	1.00	19	0.158
229	A	3	2	1.00	19	0.105
230	A	4	3	1.00	19	0.158
231	A	3	2	1.00	19	0.105
232	A	3	3	1.00	17	0.176
233	A	3	2	1.00	15	0.133
234	A	4	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	3	2	1.00	19	0.105
236	A	4	3	1.00	19	0.158
237	A	8	4	1.00	19	0.210
238	A	3	3	1.00	17	0.176
239	A	7	3	1.00	15	0.200
240	A	9	5	1.00	19	0.263
241	A	7	4	1.00	19	0.210
242	A	9	6	1.00	19	0.316
243	A	3	3	1.00	19	0.158
244	A	3	3	1.00	17	0.176
245	A	2	2	1.00	15	0.133
246	A	5	5	1.00	19	0.263
247	A	3	3	1.00	19	0.158
248	A	6	6	1.00	19	0.316
249	A	4	3	1.00	21	0.143
250	A	4	3	1.00	21	0.143
251	A	4	3	1.00	21	0.143
252	A	3	3	1.00	19	0.158
253	A	7	7	1.00	21	0.333
254	A	7	7	1.00	21	0.333
255	A	5	5	1.00	21	0.238
256	A	4	4	1.00	17	0.235
257	A	4	4	1.00	21	0.190
258	A	5	5	1.00	21	0.238
259	A	4	4	1.00	15	0.267
260	A	6	6	1.00	17	0.353
261	A	3	3	1.00	15	0.200
262	A	3	3	1.00	17	0.176
263	A	6	6	1.00	26	0.231
264	A	5	5	1.00	26	0.192
265	A	4	4	1.00	24	0.167
266	A	5	4	1.00	26	0.154
267	A	4	4	1.00	26	0.154
268	A	5	5	1.00	26	0.192
269	A	6	6	1.00	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	7	7	1.00	26	0.269
271	A	6	6	1.00	26	0.231
272	A	5	5	1.00	22	0.227
273	A	7	7	1.00	26	0.269
274	A	7	7	1.00	26	0.269
275	A	8	7	1.00	26	0.269
276	A	7	7	1.00	26	0.269
277	A	6	6	1.00	26	0.231
278	A	5	5	1.00	24	0.208
279	A	6	5	1.00	26	0.192
280	A	5	5	1.00	26	0.192
281	A	6	6	1.00	26	0.231
282	A	7	7	1.00	26	0.269
283	A	8	8	1.00	26	0.308
284	A	7	7	1.00	26	0.269
285	A	6	6	1.00	22	0.273
286	A	7	7	1.00	26	0.269
287	A	8	7	1.00	26	0.269
288	A	9	7	1.00	26	0.269
289	A	4	4	1.00	21	0.190
290	A	4	4	1.00	23	0.174
291	A	4	4	1.00	23	0.174
292	A	6	6	1.00	23	0.261
293	A	5	5	1.00	25	0.200
294	A	5	5	1.00	23	0.217
295	A	5	5	1.00	28	0.179
296	A	6	6	1.00	26	0.231
297	A	5	5	1.00	26	0.192
298	A	4	4	1.00	24	0.167
299	A	5	4	1.00	26	0.154
300	A	4	4	1.00	26	0.154
301	A	5	5	1.00	26	0.192
302	A	7	7	1.00	26	0.269
303	A	6	6	1.00	26	0.231
304	A	5	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
305	A	7	7	1.00	26	0.269
306	A	7	7	0.99	26	0.269
307	A	7	6	1.00	26	0.231
308	A	6	5	1.00	26	0.192
309	A	5	5	1.00	24	0.208
310	A	6	5	1.00	26	0.192
311	A	5	5	1.00	26	0.192
312	A	6	5	1.00	26	0.192
313	A	8	8	1.00	26	0.308
314	A	7	7	1.00	26	0.269
315	A	6	6	1.00	22	0.273
316	A	7	7	1.00	26	0.269
317	A	8	7	1.00	26	0.269
318	A	7	7	1.00	21	0.333
319	A	6	6	1.00	21	0.286
320	A	5	5	1.00	19	0.263
321	A	6	5	1.00	21	0.238
322	A	5	5	1.00	21	0.238
323	A	6	6	1.00	21	0.286
324	A	7	7	1.00	21	0.333
325	A	8	8	1.00	21	0.381
326	A	7	7	1.00	21	0.333
327	A	6	6	1.00	17	0.353
328	A	8	8	1.00	21	0.381
329	A	8	8	1.00	21	0.381
330	A	9	8	1.00	21	0.381
331	A	8	8	1.00	21	0.381
332	A	7	7	1.00	21	0.333
333	A	6	6	1.00	19	0.316
334	A	7	6	1.00	21	0.286
335	A	6	6	1.00	21	0.286
336	A	7	7	1.00	21	0.333
337	A	8	8	1.00	21	0.381
338	A	9	9	1.00	21	0.429
339	A	8	8	1.00	21	0.381

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	7	7	1.00	17	0.412
341	A	8	8	1.00	21	0.381
342	A	9	8	1.00	21	0.381
343	A	10	8	1.00	21	0.381
344	A	7	7	1.00	21	0.333
345	A	6	6	1.00	21	0.286
346	A	5	5	1.00	19	0.263
347	A	6	5	1.00	21	0.238
348	A	5	5	1.00	21	0.238
349	A	6	6	1.00	21	0.286
350	A	8	8	1.00	21	0.381
351	A	7	7	1.00	21	0.333
352	A	6	6	1.00	17	0.353
353	A	8	8	1.00	21	0.381
354	A	8	8	0.99	21	0.381
355	A	8	7	1.00	21	0.333
356	A	7	6	1.00	21	0.286
357	A	6	6	1.00	19	0.316
358	A	7	6	1.00	21	0.286
359	A	6	6	1.00	21	0.286
360	A	7	6	1.00	21	0.286
361	A	9	9	1.00	21	0.429
362	A	8	8	1.00	21	0.381
363	A	7	7	1.00	17	0.412
364	A	8	8	1.00	21	0.381
365	A	9	8	1.00	21	0.381
366	A	6	5	1.00	19	0.263
367	A	5	5	1.00	19	0.263
368	A	4	4	1.00	19	0.210
369	A	2	2	1.00	19	0.105
370	A	3	3	1.00	19	0.158
371	A	4	4	1.00	22	0.182
372	A	5	5	1.00	19	0.263
373	A	6	5	1.00	22	0.227
374	A	8	5	1.00	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	9	6	1.00	30	0.200
376	A	6	6	1.00	19	0.316
377	A	3	3	1.00	19	0.158
378	A	4	4	1.00	19	0.210
379	A	2	2	1.00	19	0.105
380	A	4	4	1.00	17	0.235
381	A	3	3	1.00	19	0.158
382	A	4	4	1.00	19	0.210
383	A	6	6	1.00	19	0.316
384	A	2	2	1.00	19	0.105
385	A	2	2	1.00	19	0.105
386	A	5	5	1.00	19	0.263
387	A	4	4	1.00	19	0.210
388	A	3	3	1.00	17	0.176
389	A	3	3	1.00	19	0.158
390	A	4	4	1.00	19	0.210
391	A	6	6	1.00	19	0.316
392	A	5	5	1.00	19	0.263
393	A	2	2	1.00	21	0.095
394	A	2	2	1.00	19	0.105
395	A	2	2	1.00	23	0.087
396	C	5	3	2.35	54	0.056
397	A	2	2	1.00	26	0.077
398	A	2	2	1.00	7	0.286
399	A	3	2	1.00	15	0.133
400	A	4	2	1.00	22	0.091
401	A	5	2	1.00	25	0.080
402	A	5	2	1.00	23	0.087
403	A	2	1	1.00	21	0.048
404	A	7	4	1.00	25	0.160
405	A	7	4	1.00	25	0.160
406	A	9	7	1.00	25	0.280
407	A	8	6	1.00	23	0.261
408	A	7	5	1.81	21	0.238
409	A	9	8	1.00	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	9	8	1.00	25	0.320
411	A	10	2	1.00	25	0.080
412	A	10	2	1.00	23	0.087
413	B	6	2	2.36	21	0.095
414	A	8	4	1.00	25	0.160
415	A	14	5	1.38	25	0.200
416	A	3	3	1.00	15	0.200
417	A	3	3	1.00	15	0.200
418	A	2	1	1.00	17	0.059
419	A	5	3	1.00	23	0.130
420	A	5	4	1.00	23	0.174
421	A	3	2	1.00	21	0.095
422	A	4	3	1.00	19	0.158
423	A	6	5	1.00	23	0.217
424	A	4	3	1.00	23	0.130
425	A	6	5	1.00	23	0.217
426	A	5	2	1.00	25	0.080
427	A	5	2	1.00	25	0.080
428	A	3	2	1.00	23	0.087
429	A	8	4	1.00	21	0.190
430	A	7	4	1.00	25	0.160
431	A	9	5	1.00	25	0.200
432	A	8	6	1.00	25	0.240
433	A	7	5	1.00	25	0.200
434	A	9	8	1.00	23	0.348
435	A	9	8	1.00	21	0.381
436	A	6	4	1.00	25	0.160
437	A	7	5	1.00	25	0.200
438	A	10	2	1.00	25	0.080
439	A	6	2	1.00	25	0.080
440	A	8	4	1.00	25	0.160
441	A	14	5	1.42	23	0.217
442	A	16	5	1.68	21	0.238
443	A	4	3	1.00	27	0.111
444	A	6	4	1.00	42	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
445	A	6	5	1.00	42	0.119
446	A	4	3	1.00	40	0.075
447	A	5	4	1.00	39	0.103
448	A	7	6	1.00	42	0.143
449	A	5	4	1.00	42	0.095
450	A	7	6	1.00	42	0.143
451	A	15	7	1.00	39	0.180
452	A	9	4	1.00	35	0.114
453	A	4	3	1.00	25	0.120
454	A	3	2	1.00	25	0.080
455	A	3	2	1.00	25	0.080
456	A	4	3	1.00	23	0.130
457	A	3	2	1.00	25	0.080
458	A	3	2	1.00	25	0.080
459	A	3	2	1.00	25	0.080
460	A	6	5	1.00	27	0.185
461	A	6	5	1.00	27	0.185
462	A	6	5	1.00	27	0.185
463	A	5	5	1.00	27	0.185
464	A	5	5	1.00	27	0.185
465	A	6	5	1.00	27	0.185
466	A	3	2	1.00	17	0.118
467	A	3	2	1.00	26	0.077
468	A	1	1	1.00	17	0.059
469	A	1	1	1.00	15	0.067
470	A	1	1	1.00	15	0.067
471	A	1	1	1.00	25	0.040
472	A	4	3	1.00	28	0.107
473	A	3	2	1.00	28	0.071
474	A	3	2	1.00	28	0.071
475	A	4	3	1.00	26	0.115
476	A	3	2	1.00	28	0.071
477	A	3	2	1.00	28	0.071
478	A	3	2	1.00	28	0.071
479	A	6	5	1.00	30	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
480	A	6	5	1.00	30	0.167
481	A	6	5	1.00	30	0.167
482	A	5	5	1.00	30	0.167
483	A	5	5	1.00	30	0.167
484	A	6	5	1.00	30	0.167
485	A	3	2	1.00	21	0.095
486	A	3	2	1.00	19	0.105
487	A	3	2	1.00	13	0.154
488	A	2	2	1.00	21	0.095
489	A	2	2	1.00	21	0.095
490	A	3	2	1.00	23	0.087
491	A	3	2	1.00	21	0.095
492	A	3	2	1.00	15	0.133
493	A	2	2	1.00	23	0.087
494	A	2	2	1.00	23	0.087
495	A	3	2	1.00	23	0.087
496	A	3	2	1.00	23	0.087
497	A	3	2	1.00	23	0.087
498	A	2	2	1.00	23	0.087
499	A	2	2	1.00	23	0.087
500	A	2	2	1.00	23	0.087
501	A	3	2	1.00	25	0.080
502	A	3	2	1.00	25	0.080
503	A	3	2	1.00	25	0.080
504	A	2	2	1.00	25	0.080
505	A	2	2	1.00	25	0.080
506	A	2	2	1.00	25	0.080
507	A	4	3	1.00	56	0.054
508	A	4	3	1.00	54	0.056
509	A	4	3	1.00	33	0.091
510	A	2	2	1.00	56	0.036
511	A	3	3	1.00	56	0.054
512	A	5	4	1.00	33	0.121
513	A	3	3	1.00	56	0.054
514	A	4	3	1.00	58	0.052

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
515	A	4	3	1.00	58	0.052
516	A	3	3	1.00	58	0.052
517	A	3	3	1.00	58	0.052
518	A	4	4	1.00	58	0.069
519	A	5	4	1.00	62	0.065
520	A	4	4	1.00	62	0.065
521	A	4	4	1.00	62	0.065
522	A	5	5	1.00	60	0.083
523	A	7	6	1.00	30	0.200
524	A	4	3	1.00	37	0.081
525	A	4	3	1.00	37	0.081
526	A	2	2	1.00	37	0.054
527	A	2	2	1.00	37	0.054
528	A	2	2	1.00	42	0.048
529	A	2	2	1.00	42	0.048
530	A	4	4	1.00	30	0.133
531	A	4	4	1.00	30	0.133
532	A	2	2	1.00	42	0.048
533	A	2	2	1.00	42	0.048
534	A	2	2	1.45	51	0.039
535	A	2	2	1.45	51	0.039
536	A	2	2	1.00	40	0.050
537	A	2	2	1.00	40	0.050
538	A	4	4	1.00	32	0.125
539	A	4	4	1.00	32	0.125
540	A	2	2	1.00	51	0.039
541	A	2	2	1.00	51	0.039
542	A	2	2	1.00	56	0.036
543	A	2	2	1.00	56	0.036
544	A	4	2	1.00	29	0.069
545	A	4	2	1.00	29	0.069
546	A	3	2	1.00	27	0.074
547	A	7	6	1.00	29	0.207
548	A	8	6	1.00	29	0.207
549	A	8	7	1.00	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
550	A	4	3	1.00	25	0.120
551	A	7	6	1.00	29	0.207
552	A	4	2	1.00	29	0.069
553	A	4	2	1.00	29	0.069
554	A	3	2	1.00	29	0.069
555	A	7	6	1.00	29	0.207
556	A	8	6	1.00	29	0.207
557	A	10	10	1.00	29	0.345
558	A	9	9	1.00	27	0.333
559	A	9	9	1.00	25	0.360
560	A	10	10	1.00	29	0.345
561	A	10	10	1.00	29	0.345
562	A	4	4	1.00	25	0.160
563	A	4	4	1.00	29	0.138
564	A	3	2	1.00	31	0.065
565	A	3	3	1.00	15	0.200
566	A	5	5	1.00	15	0.333
567	A	4	3	1.00	15	0.200
568	A	4	3	1.00	13	0.231
569	A	4	3	1.00	13	0.231
570	A	4	3	1.00	15	0.200
571	A	9	9	1.00	13	0.692
572	A	4	3	1.00	13	0.231
573	A	4	3	1.00	13	0.231
574	A	9	9	1.00	15	0.600
575	A	3	3	1.00	13	0.231
576	A	4	3	1.00	13	0.231
577	A	13	10	1.00	15	0.667
578	A	10	7	1.00	19	0.368
579	A	4	3	1.00	19	0.158
580	A	10	9	1.00	21	0.429
581	A	3	3	1.00	26	0.115
582	A	3	3	1.00	26	0.115
583	A	3	3	1.00	24	0.125
584	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
585	A	3	3	1.00	26	0.115
586	A	3	3	1.00	26	0.115
587	A	4	3	1.00	17	0.176
588	A	5	5	1.00	17	0.294
589	A	4	4	1.00	15	0.267
590	A	2	2	1.00	9	0.222
591	A	5	5	1.00	17	0.294
592	A	3	3	1.00	17	0.176
593	A	4	4	1.00	17	0.235
594	A	5	5	1.00	17	0.294
595	A	3	3	1.00	28	0.107
596	A	3	3	1.00	28	0.107
597	A	3	3	1.00	26	0.115
598	A	3	3	1.00	25	0.120
599	A	3	3	1.00	28	0.107
600	A	3	3	1.00	28	0.107
601	A	8	6	1.00	21	0.286
602	A	1	1	1.00	17	0.059
603	A	1	1	1.00	21	0.048
604	A	1	1	1.00	17	0.059
605	A	1	1	1.00	19	0.053
606	A	1	1	1.00	21	0.048
607	A	4	3	1.00	25	0.120
608	A	3	2	1.00	17	0.118
609	A	4	3	1.00	27	0.111
610	A	4	3	1.00	25	0.120
611	A	5	4	1.70	22	0.182
612	A	4	3	1.00	29	0.103
613	A	1	1	1.00	176	0.006
614	A	1	1	1.00	174	0.006
615	A	1	1	1.00	164	0.006
616	F	0	0	N/A	0.000	N/A
617	F	0	0	N/A	0.000	N/A
618	A	4	3	1.00	19	0.158
619	A	4	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
620	A	4	3	1.00	17	0.176
621	A	4	3	1.00	15	0.200
622	A	7	6	1.00	19	0.316
623	A	6	6	1.00	19	0.316
624	A	6	6	1.00	19	0.316
625	A	4	3	1.00	21	0.143
626	A	4	3	1.00	21	0.143
627	A	4	3	1.00	19	0.158
628	A	4	3	1.00	17	0.176
629	A	7	6	1.00	21	0.286
630	A	8	7	1.00	21	0.333
631	A	9	8	1.00	21	0.381
632	A	4	3	1.00	19	0.158
633	A	4	3	1.00	19	0.158
634	A	4	3	1.00	17	0.176
635	A	4	3	1.00	15	0.200
636	A	7	6	1.00	19	0.316
637	A	8	7	1.00	19	0.368
638	A	9	7	1.00	19	0.368
639	A	4	3	1.00	19	0.158
640	A	4	3	1.00	19	0.158
641	A	4	3	1.00	17	0.176
642	A	4	3	1.00	15	0.200
643	A	7	6	1.00	19	0.316
644	A	8	7	1.00	19	0.368
645	A	9	7	1.00	19	0.368
646	A	4	3	1.00	21	0.143
647	A	4	3	1.00	21	0.143
648	A	4	3	1.00	19	0.158
649	A	4	3	1.00	17	0.176
650	A	6	5	1.00	21	0.238
651	A	7	6	1.00	21	0.286
652	A	8	6	1.00	21	0.286
653	A	4	3	1.00	19	0.158
654	A	4	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
655	A	4	3	1.00	17	0.176
656	A	4	3	1.00	15	0.200
657	A	6	4	1.00	19	0.210
658	A	8	6	1.00	17	0.353
659	A	7	6	1.00	17	0.353
660	A	6	6	1.00	17	0.353
661	A	5	5	1.00	17	0.294
662	A	6	6	1.00	17	0.353
663	A	7	6	1.00	17	0.353
664	A	8	6	1.00	19	0.316
665	A	7	6	1.00	19	0.316
666	A	6	6	1.00	19	0.316
667	A	5	5	1.00	19	0.263
668	A	6	6	1.00	19	0.316
669	A	7	6	1.00	19	0.316
670	A	2	2	1.00	13	0.154
671	A	5	5	1.00	17	0.294
672	A	6	5	1.00	21	0.238
673	A	7	5	1.00	25	0.200
674	A	8	5	1.00	29	0.172
675	A	8	6	1.00	20	0.300
676	A	7	6	1.00	20	0.300
677	A	6	6	1.00	18	0.333
678	A	2	2	1.00	20	0.100
679	A	4	3	1.00	20	0.150
680	A	4	3	1.00	20	0.150
681	A	2	2	1.00	18	0.111
682	A	2	2	1.00	20	0.100
683	A	3	2	1.00	22	0.091
684	A	3	3	1.00	17	0.176
685	A	3	3	1.00	23	0.130
686	A	3	2	1.00	22	0.091
687	A	5	4	1.00	34	0.118
688	A	5	5	1.00	31	0.161
689	A	6	4	1.00	47	0.085

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
690	A	9	5	1.00	58	0.086
691	A	4	4	1.00	11	0.364
692	A	6	6	1.00	11	0.546
693	A	2	1	1.00	17	0.059
694	A	8	6	1.00	13	0.462
695	A	2	1	1.00	16	0.062
696	A	4	2	1.00	14	0.143
697	A	5	4	1.00	12	0.333
698	A	4	2	1.00	13	0.154
699	A	4	2	1.00	13	0.154
700	A	4	2	1.00	15	0.133
701	A	5	5	1.00	12	0.417
702	A	6	5	1.00	14	0.357
703	A	5	4	1.00	13	0.308
704	A	5	4	1.00	17	0.235
705	A	5	4	1.00	17	0.235
706	A	4	3	1.00	13	0.231
707	A	7	5	1.00	18	0.278
708	A	7	5	1.00	20	0.250
709	A	8	5	1.00	18	0.278
710	A	4	3	1.00	26	0.115
711	A	10	6	0.72	17	0.353
712	A	3	1	1.00	27	0.037
713	A	5	3	1.00	17	0.176
714	A	5	3	1.00	17	0.176
715	A	5	3	1.00	23	0.130
716	A	5	3	1.00	17	0.176
717	A	6	2	1.00	23	0.087
718	A	5	1	1.00	25	0.040
719	A	5	2	1.00	21	0.095
720	A	3	2	1.00	23	0.087
721	A	4	3	1.18	16	0.188
722	A	3	1	1.00	28	0.036
723	A	5	5	1.00	16	0.312
724	A	4	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
725	A	8	4	1.00	21	0.190
726	B	16	10	2.80	20	0.500
727	A	9	6	1.00	25	0.240
728	A	6	4	1.00	35	0.114
729	A	2	2	1.00	13	0.154
730	A	3	3	1.00	15	0.200
731	A	3	3	1.00	13	0.231
732	A	4	4	1.00	11	0.364
733	A	3	3	1.00	18	0.167
734	A	4	4	1.00	17	0.235
735	A	4	4	1.00	18	0.222
736	A	5	5	1.00	17	0.294
737	A	2	2	1.00	16	0.125
738	A	2	2	1.00	21	0.095
739	A	3	3	1.00	26	0.115
740	A	3	3	1.00	25	0.120
741	A	3	3	1.00	12	0.250
742	A	3	3	1.00	15	0.200
743	A	3	3	1.00	15	0.200
744	A	3	3	1.00	15	0.200
745	A	3	3	1.00	17	0.176
746	A	4	4	1.00	15	0.267
747	A	4	4	1.00	21	0.190
748	A	3	3	1.00	22	0.136
749	A	4	4	1.00	20	0.200
750	A	7	7	1.00	20	0.350
751	A	2	2	1.00	15	0.133
752	A	5	5	1.00	21	0.238
753	A	8	6	1.00	18	0.333
754	A	5	4	1.00	18	0.222
755	A	6	4	1.00	18	0.222
756	A	3	2	1.00	16	0.125
757	A	3	2	1.00	16	0.125
758	A	3	2	1.00	16	0.125
759	A	10	9	1.00	18	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
760	A	5	4	1.00	18	0.222
761	A	6	5	1.00	18	0.278
762	A	5	5	1.40	35	0.143
763	A	6	6	1.40	28	0.214
764	A	7	7	1.00	23	0.304
765	A	6	6	1.00	23	0.261
766	A	3	3	1.00	23	0.130
767	A	6	6	1.00	23	0.261
768	A	7	7	1.00	23	0.304
769	A	7	7	1.00	19	0.368
770	A	6	6	1.00	19	0.316
771	A	3	3	1.00	19	0.158
772	A	6	6	1.00	19	0.316
773	A	7	7	1.00	19	0.368
774	A	6	6	1.00	31	0.194
775	A	5	5	1.00	31	0.161
776	A	2	2	1.00	31	0.065
777	A	5	5	1.00	31	0.161
778	A	5	5	1.00	34	0.147
779	A	2	2	1.00	34	0.059
780	A	5	5	1.00	34	0.147
781	A	8	8	1.00	24	0.333
782	A	7	7	1.00	24	0.292
783	A	3	3	1.00	24	0.125
784	A	7	7	1.00	24	0.292
785	A	8	8	1.00	24	0.333
786	A	14	13	1.00	26	0.500
787	A	12	12	1.00	26	0.462
788	A	7	7	1.00	26	0.269
789	A	10	10	1.00	26	0.385
790	A	12	12	1.00	26	0.462
791	A	15	13	1.00	28	0.464
792	A	13	13	1.00	28	0.464
793	A	11	11	1.00	28	0.393
794	A	10	9	1.00	28	0.321

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
795	A	13	12	1.00	28	0.429
796	A	4	4	1.00	19	0.210
797	A	10	10	1.00	19	0.526
798	A	3	3	1.00	19	0.158
799	A	9	9	1.00	19	0.474
800	A	4	4	1.00	24	0.167
801	A	10	10	1.00	24	0.417
802	A	12	11	1.00	24	0.458
803	A	4	4	1.00	24	0.167
804	A	10	10	1.00	24	0.417
805	A	12	11	1.00	27	0.407
806	A	14	7	1.00	20	0.350
807	A	12	6	1.91	21	0.286
808	A	2	1	1.00	22	0.045
809	A	4	2	1.00	18	0.111
810	A	4	2	1.00	18	0.111
811	A	5	5	1.00	23	0.217
812	A	6	6	1.00	22	0.273
813	A	6	6	1.00	20	0.300
814	A	3	2	1.00	15	0.133
815	A	3	2	1.00	21	0.095
816	A	5	4	1.00	19	0.210
817	A	9	6	1.00	24	0.250
818	A	9	6	1.00	24	0.250
819	A	10	7	1.00	21	0.333
820	A	11	8	1.00	13	0.615
821	A	12	8	1.00	25	0.320
822	A	13	9	1.00	15	0.600
823	A	14	9	1.00	19	0.474
824	A	2	1	1.00	35	0.029
825	A	8	5	1.00	37	0.135
826	A	16	8	1.00	29	0.276
827	A	16	8	1.00	36	0.222
828	A	31	13	1.00	43	0.302
829	A	5	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
830	A	6	4	1.00	27	0.148
831	A	8	6	1.00	27	0.222
832	A	4	4	1.00	27	0.148
833	A	4	4	1.00	20	0.200
834	A	6	6	1.00	29	0.207
835	A	2	1	1.00	20	0.050
836	A	3	2	1.00	25	0.080
837	A	2	2	1.00	28	0.071
838	A	3	3	1.00	26	0.115
839	A	1	1	1.00	11	0.091
840	A	2	2	1.00	19	0.105
841	A	2	2	1.00	15	0.133
842	A	2	2	1.00	14	0.143
843	A	3	3	1.00	17	0.176
844	A	3	3	1.00	13	0.231
845	A	2	2	1.00	14	0.143
846	A	3	3	1.00	17	0.176
847	A	3	3	1.00	13	0.231
848	A	1	0	1.00	9	0.000
849	A	4	3	1.00	15	0.200
850	A	2	2	1.00	13	0.154
851	A	3	3	1.00	19	0.158
852	A	2	2	1.00	11	0.182
853	A	3	3	1.00	17	0.176
854	A	1	1	1.00	9	0.111
855	A	2	2	1.00	19	0.105
856	A	1	1	1.00	7	0.143
857	A	2	2	1.00	21	0.095
858	A	1	1	1.00	9	0.111
859	A	2	2	1.00	19	0.105
860	A	1	1	1.00	7	0.143
861	A	2	2	1.00	21	0.095
862	A	3	3	1.00	17	0.176
863	A	4	4	1.00	30	0.133
864	A	7	7	1.00	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
865	A	6	6	1.00	20	0.300
866	A	7	7	1.00	23	0.304
867	A	6	6	1.00	25	0.240
868	A	1	1	1.00	11	0.091
869	A	2	2	1.00	21	0.095
870	A	1	1	1.00	9	0.111
871	A	2	2	1.00	23	0.087
872	A	2	2	1.00	11	0.182
873	A	3	3	1.00	21	0.143
874	A	2	2	1.00	9	0.222
875	A	3	3	1.00	23	0.130
876	A	3	3	1.16	14	0.214
877	A	4	4	1.00	15	0.267
878	A	7	6	1.00	17	0.353
879	A	7	6	1.00	17	0.353
880	A	10	8	1.00	34	0.235
881	A	12	7	1.00	30	0.233
882	A	5	4	1.00	21	0.190
883	A	7	5	1.00	23	0.217
884	A	9	7	1.00	25	0.280
885	A	7	6	1.00	25	0.240
886	A	3	3	1.00	13	0.231
887	A	2	2	1.00	24	0.083
888	A	2	2	1.00	22	0.091
889	A	2	2	1.00	30	0.067
890	A	3	3	1.00	27	0.111
891	A	5	5	1.00	23	0.217
892	A	7	5	1.00	28	0.179
893	A	13	10	1.55	25	0.400
894	A	8	6	1.00	29	0.207
895	A	6	6	1.00	16	0.375
896	A	6	6	1.00	16	0.375
897	A	4	4	1.00	16	0.250
898	A	7	7	1.00	16	0.438
899	A	8	8	1.00	16	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
900	A	3	3	1.00	24	0.125
901	N/A	0	0	1.00	14	0.000
902	N/A	0	0	1.00	14	0.000
903	N/A	0	0	1.00	17	0.000
904	N/A	0	0	1.00	17	0.000
905	N/A	0	0	1.00	10	0.000
906	N/A	0	0	1.00	12	0.000
907	N/A	0	0	1.00	10	0.000
908	N/A	0	0	1.00	14	0.000
909	N/A	0	0	1.00	12	0.000
910	N/A	0	0	1.00	14	0.000
911	A	2	2	1.00	37	0.054
912	A	2	2	1.00	38	0.053
913	A	5	4	1.00	40	0.100
914	A	7	5	1.00	40	0.125
915	A	6	5	1.00	18	0.278
916	A	7	6	1.00	21	0.286
917	A	8	6	1.00	22	0.273
918	A	3	3	1.00	21	0.143
919	A	3	3	1.00	24	0.125
920	A	11	10	1.00	19	0.526
921	A	10	7	1.00	24	0.292
922	A	4	3	1.00	19	0.158
923	A	3	3	1.00	17	0.176
924	A	1	1	1.00	15	0.067
925	A	1	1	1.00	17	0.059
926	A	2	2	1.00	17	0.118
927	A	2	2	1.00	17	0.118
928	A	1	1	1.00	17	0.059
929	A	6	6	1.00	17	0.353
930	A	5	5	1.00	11	0.454
931	A	3	3	1.00	11	0.273
932	A	5	3	1.00	13	0.231
933	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
934	A	5	5	1.00	16	0.312
935	A	2	2	1.00	13	0.154
936	A	6	6	1.00	16	0.375
937	A	5	4	1.00	12	0.333
938	A	4	3	1.00	18	0.167
939	A	10	6	1.00	17	0.353
940	A	1	1	1.00	17	0.059
941	C	3	2	1.03	27	0.074
942	A	2	2	1.00	37	0.054
943	A	3	2	1.00	17	0.118
944	A	5	4	1.00	17	0.235
945	A	1	1	1.00	15	0.067
946	A	3	3	1.00	14	0.214
947	A	1	1	1.00	17	0.059
948	A	2	1	1.00	13	0.077
949	A	2	1	1.00	15	0.067
950	A	7	4	1.00	15	0.267
951	A	6	6	1.00	15	0.400
952	A	1	0	1.00	9	0.000
953	A	1	0	1.00	9	0.000
954	A	2	1	1.00	19	0.053
955	A	3	3	1.00	15	0.200
956	A	3	3	1.00	16	0.188
957	A	4	4	1.00	15	0.267
958	A	5	5	1.00	15	0.333
959	A	4	4	1.00	25	0.160
960	A	2	2	1.00	11	0.182
961	A	2	2	1.00	17	0.118
962	A	6	6	1.00	22	0.273
963	A	4	3	1.00	13	0.231
964	A	4	3	1.00	15	0.200
965	A	4	4	1.00	19	0.210
966	A	4	4	1.00	21	0.190
967	A	3	3	1.00	17	0.176
968	A	7	7	1.00	19	0.368

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
969	A	7	6	1.00	19	0.316
970	A	6	6	1.00	17	0.353
971	A	10	8	1.00	17	0.471
972	A	11	9	1.00	17	0.529
973	A	3	3	1.00	22	0.136
974	A	5	5	1.00	11	0.454
975	A	5	5	1.00	13	0.385
976	A	5	5	1.17	11	0.454
977	A	5	5	1.00	15	0.333
978	A	4	4	1.00	11	0.364
979	A	5	5	1.00	13	0.385
980	A	4	4	1.00	11	0.364
981	A	3	3	1.00	11	0.273
982	A	2	2	1.00	11	0.182
983	A	5	5	1.00	19	0.263
984	A	2	2	1.00	23	0.087
985	A	2	2	1.00	13	0.154
986	A	3	3	1.00	11	0.273
987	A	3	3	1.00	15	0.200
988	A	3	3	1.00	19	0.158
989	A	3	3	1.00	19	0.158
990	A	3	3	1.00	19	0.158
991	A	2	2	1.00	15	0.133
992	A	3	3	1.00	15	0.200
993	A	3	3	1.00	12	0.250
994	A	3	3	1.00	16	0.188
995	F	0	0	N/A	0.000	N/A
996	F	0	0	N/A	0.000	N/A
997	B	25	12	4.09	31	0.387
998	A	5	4	1.00	25	0.160
999	A	2	2	1.00	27	0.074
1000	A	2	2	1.00	33	0.061
1001	A	2	2	1.00	34	0.059
1002	A	7	6	1.00	51	0.118
1003	A	2	2	1.00	49	0.041

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1004	A	2	2	1.00	43	0.047
1005	A	2	2	1.00	44	0.045
1006	A	9	8	1.00	20	0.400
1007	A	3	3	1.00	15	0.200
1008	A	3	3	1.00	15	0.200
1009	A	1	1	1.00	52	0.019
1010	A	1	1	1.00	57	0.018
1011	A	2	2	1.00	59	0.034
1012	A	2	2	1.00	58	0.034
1013	A	3	3	1.00	58	0.052
1014	A	3	3	1.00	57	0.053
1015	A	3	3	1.00	66	0.045
1016	A	3	3	1.00	31	0.097
1017	B	42	15	4.13	29	0.517
1018	C	9	6	0.96	20	0.300
1019	A	2	2	1.00	46	0.043
1020	A	1	0	1.00	15	0.000
1021	C	12	9	2.90	17	0.529
1022	C	13	10	3.55	33	0.303
1023	F	0	0	N/A	0.000	N/A
1024	A	1	1	1.00	38	0.026
1025	B	2	2	2.43	30	0.067

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 145

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 2/9*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*2^(1/3)*(1+x)*E
llipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2
))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1
/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2159, 224, 2162, 209}

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} + \frac{2\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

```
[In] Int[1/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) + (2*2^(1/3
)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipti
cF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{1+x^3}}\,dx \\
&= \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} \\
&\quad + \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1+\sqrt[3]{2}x}{\sqrt{1+x^3}}\right) \\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{4i\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right), \frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\ 2^{2/3}-i\sqrt{3}\right)\sqrt{1+x^3}}$$

```
[In] Integrate[1/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((4*I)*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*
I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3) - I
*Sqrt[3])*Sqrt[1 + x^3])
```

Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{2^{\frac{2}{3}}-1},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}\left(2^{\frac{2}{3}}-1\right)}$	139
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{2^{\frac{2}{3}}-1},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}\left(2^{\frac{2}{3}}-1\right)}$	139

```
[In] int(1/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)/(2^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1
/2),(-3/2+1/2*I*3^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1
/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.51

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(5x^{3}-2^{\frac{2}{3}}(x^{5}+x^{2})+2^{\frac{1}{3}}(7x^{4}+4x)+2\right)\sqrt{x^{3}+1}}{6\left(2x^{6}+3x^{3}+1\right)}\right) + \frac{2}{3}\cdot 2^{\frac{1}{3}}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*arctan(-1/6*sqrt(3)*(5*x^3 - 2^(2/3)*(x^5 + x^2) + 2^(1/3)*(7*
x^4 + 4*x) + 2)*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1)) + 2/3*2^(1/3)*weierstras
sPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate(1/(2**(2/3)+x)/(x**3+1)**(1/2),x)
[Out] Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Maxima [F]

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Giac [F]

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}\,(x+2^{2/3})}\,dx$$

```
[In] int(1/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
[Out] int(1/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

3.2 $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/9*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)-2/9*2^(1/3)*(1-x)
*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1
/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3
^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2159, 224, 2162, 209}

$$\int \frac{1}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{2\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

```
[In] Int[1/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3]) - (2*2^(1/
3)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{1-x^3}}\,dx \\
&= -\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} \\
&\quad - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}\right) \\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

$$\int \frac{1}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\frac{4i\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \arcsin\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right), \frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\ 2^{2/3}-i\sqrt{3}\right)\sqrt{1-x^3}}$$

```
[In] Integrate[1/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] ((-4*I)*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Ell
ipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3]
+ (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3
) - I*Sqrt[3])*Sqrt[1 - x^3])
```

Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{2 i \sqrt{3}\, \sqrt{i \left(x +\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x -1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x +\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \Pi\left(\frac{\sqrt{3}\, \sqrt{i \left(x +\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{-2^{\frac{2}{3}}-\frac{1}{2}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}\, \left(-2^{\frac{2}{3}}-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)}$	143
elliptic	$\frac{2 i \sqrt{3}\, \sqrt{i \left(x +\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x -1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x +\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \Pi\left(\frac{\sqrt{3}\, \sqrt{i \left(x +\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{-2^{\frac{2}{3}}-\frac{1}{2}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}\, \left(-2^{\frac{2}{3}}-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)}$	143

```
[In] int(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-2^(
2/3)-1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2),I*3^(1/2)/(-2^(2/3)-1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3
^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.49

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{1}{9}\,\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(5\,x^{3}-2^{\frac{2}{3}}\,(x^{5}-x^{2})-2^{\frac{1}{3}}\,(7\,x^{4}-4\,x)-2\right)\sqrt{-x^{3}+1}}{6\,\left(2\,x^{6}-3\,x^{3}+1\right)}\right) - \frac{2}{3}i\cdot 2^{\frac{1}{3}}\,\mathrm{weierstrassPInverse}(0,4,x)$$

```
[In] integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*arctan(-1/6*sqrt(3)*(5*x^3 - 2^(2/3)*(x^5 - x^2) - 2^(1/3)*(7*
x^4 - 4*x) - 2)*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1)) - 2/3*I*2^(1/3)*weierst
rassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\int \frac{1}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

```
[In] integrate(1/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
[Out] -Integral(1/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \int -\frac{1}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Arg
ument
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\int \frac{1}{\sqrt{1-x^3}\,\left(x-2^{2/3}\right)}\,dx$$

```
[In] int(-1/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
[Out] -int(1/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

3.3 $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 163

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -2/9*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)-2/9*2^(1/3)*(1-x)
*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1
/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3
^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2159, 225, 2162, 212}

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{2\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

```
[In] Int[1/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3]) - (2*2^(
1/3)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{-1+x^3}}\,dx\\
&= -\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\\
&\quad - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{1-\sqrt[3]{2}x}{\sqrt{-1+x^3}}\right)\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}}\\
&\quad - \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{4i\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \arcsin\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right), \frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\ 2^{2/3}-i\sqrt{3}\right)\sqrt{-1+x^3}}$$

```
[In] Integrate[1/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
[Out] ((-4*I)*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Ell
ipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3]
+ (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3
) - I*Sqrt[3])*Sqrt[-1 + x^3])
```

Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-2^{\frac{2}{3}}+1},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}\left(-2^{\frac{2}{3}}+1\right)}$	143
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-2^{\frac{2}{3}}+1},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}\left(-2^{\frac{2}{3}}+1\right)}$	143

```
[In] int(1/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))
^(1/2),(3/2+1/2*I*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{1}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

```
[In] integrate(1/(2**(2/3)-x)/(x**3-1)**(1/2),x)
[Out] -Integral(1/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \int -\frac{1}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[2]%
%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{1}{\sqrt{x^3-1}\,(x-2^{2/3})}\,dx$$

```
[In] int(-1/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
[Out] -int(1/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

3.4 $\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 2/9*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/9*2^(1/3)*(1+x)
*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1
/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-
3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2159, 225, 2162, 212}

$$\int \frac{1}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} + \frac{2\text{arctanh}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

```
[In] Int[1/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3]) + (2*2^(1
/3)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/
4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{-1-x^3}}\,dx \\
&= \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} \\
&\quad + \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{1+\sqrt[3]{2}x}{\sqrt{-1-x^3}}\right) \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int \frac{1}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{4i\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\ \text{EllipticPi}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right), \frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\ 2^{2/3}-i\sqrt{3}\right)\sqrt{-1-x^3}}$$

```
[In] Integrate[1/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
[Out] ((4*I)*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*
I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3) - I
*Sqrt[3])*Sqrt[-1 - x^3])
```

Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{2^{\frac{2}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}\,\left(2^{\frac{2}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}$	139
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{2^{\frac{2}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}\,\left(2^{\frac{2}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}$	139

```
[In] int(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(2^(2
/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1
/2)))^(1/2))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]

$$\int \frac{1}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate(1/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Maxima [F]

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[1]%
%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}\,\left(x+2^{2/3}\right)}\,dx$$

```
[In] int(1/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
[Out] int(1/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

3.5 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 280

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/9*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3+a)^(1/2))/b^(
1/3)*3^(1/2)/a^(1/2)+2/9*2^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2159, 224, 2162, 209}

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{\ }}{\sqrt[3]{\ }}\right.\right.}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[In] Int[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a + b*x^3]])
/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
```

```
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx}{3\ 2^{2/3}\sqrt[3]{a}} + \frac{\sqrt[3]{2}\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$= \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\text{Subst}\left(\int \frac{1}{1+3ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}} + \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.59

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{2i\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}},\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right),\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{a+bx^3}}$$

```
[In] Integrate[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^
(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3
) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[a + b*x^3])
```

Maple [F]

$$\int \frac{1}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^3+a}}dx$$

```
[In] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{1}{\sqrt{a+bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxim
a")
[Out] integrate(1/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac"
)
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{1}{\sqrt{b\,x^3+a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

```
[In] int(1/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
[Out] int(1/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.6 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 288

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] -2/9*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3+a)^(1/2))/b
^(1/3)*3^(1/2)/a^(1/2)-2/9*2^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2
*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(1/3)/(-b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2159, 224, 2162, 209}

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} - \frac{2\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[In] Int[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a - b*x^3]]
)/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(
1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^
(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)
*x)^2]*Sqrt[a - b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx}{3\,2^{2/3}\sqrt[3]{a}} + \frac{\sqrt[3]{2}\int \frac{1}{\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$= -\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$-\frac{2\text{Subst}\left(\int \frac{1}{1+3ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{3\sqrt[3]{b}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

$$-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{2i\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\operatorname{EllipticPi}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}},\arcsin\left(\right.\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{a-bx^3}}$$

```
[In] Integrate[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3)
 + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[a - b*x^3])
```

Maple [F]

$$\int \frac{1}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right)\sqrt{-b\,x^{3}+a}}dx$$

```
[In] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\int \frac{1}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

[In] integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)

[Out] -Integral(1/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)

Maxima [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{1}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] -integrate(1/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{1}{\sqrt{a-b\,x^3}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

[In] int(1/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)

[Out] int(1/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)

3.7 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 297

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}} - \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

```
[Out] -2/9*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3-a)^(1/2))/b
^(1/3)*3^(1/2)/a^(1/2)-2/9*2^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^
(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(1/3)/(b*x^3-a)^(1/2)/(-a^(
1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2159, 225, 2162, 212}

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx =$$

$$-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[In] Int[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a + b*x^3
]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*
b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx}{3\,2^{2/3}\sqrt[3]{a}} + \frac{\sqrt[3]{2}\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$= -\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}} - \frac{2\text{Subst}\left(\int \frac{1}{1-3ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}} - \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.56

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2i\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}},\arcsin\left(\right.\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{-a+bx^3}}$$

```
[In] Integrate[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3)
 + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[-a + b*x^3])
```

Maple [F]

$$\int \frac{1}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right)\sqrt{b\,x^{3}-a}}dx$$

```
[In] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\int \frac{1}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

```
[In] integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
[Out] -Integral(1/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x)
```

Maxima [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int -\frac{1}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxim
a")
[Out] -integrate(1/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac"
)
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int \frac{1}{\sqrt{b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

```
[In] int(1/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
[Out] int(1/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.8 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 293

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}} + \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] 2/9*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3-a)^(1/2))/b
^(1/3)*3^(1/2)/a^(1/2)+2/9*2^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(1/3)/(-b*x^3-a)^(1/2)/(-a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2159, 225, 2162, 212}

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-}}$$

$$+\frac{2\,\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[In] Int[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a - b*x^3]
])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b
^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
```

```
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx}{3\ 2^{2/3}\sqrt[3]{a}} + \frac{\sqrt[3]{2}\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$= \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} + \frac{2\text{Subst}\left(\int \frac{1}{1-3ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{3\sqrt[3]{b}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}} + \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2i\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}},\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right),\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{-a-bx^3}}$$

```
[In] Integrate[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^
(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3
) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[-a - b*x^3]
)
```

Maple [F]

$$\int \frac{1}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}-a}}dx$$

```
[In] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{1}{\sqrt{-a-bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
[Out] Integral(1/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{1}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxi
ma")
[Out] integrate(1/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac
")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{1}{\sqrt{-b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

```
[In] int(1/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
[Out] int(1/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.9 $\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 249

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d} + \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

```
[Out] 2/9*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/c^(3/2)/d*3^(1/
2)+2/9*2^(1/3)*(c+2^(2/3)*d*x)*EllipticF((2^(2/3)*d*x+c*(1-3^(1/2)))/(2^(2/
3)*d*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^2-2^(2/3
)*c*d*x+2*2^(1/3)*d^2*x^2)/(2^(2/3)*d*x+c*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c/d
/(4*d^3*x^3+c^3)^(1/2)/(c*(c+2^(2/3)*d*x)/(2^(2/3)*d*x+c*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

= {2159, 224, 2162, 209}

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

$$= \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

$$+\frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d}$$

```
[In] Int[1/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(3*Sqrt[3]*
c^(3/2)*d) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*d*x)*Sqrt[(c^2 - 2^(
2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)],
-7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[3])*c
 + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx}{3c} + \frac{2\int \frac{1}{\sqrt{c^3+4d^3x^3}}\,dx}{3c}$$

$$= \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}} + \frac{2\text{Subst}\left(\int \frac{1}{1+3c^3x^2}\,dx, x, \frac{1+\frac{2dx}{c}}{\sqrt{c^3+4d^3x^3}}\right)}{3d}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d} + \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.68

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = -\frac{i2^{5/6}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{2^{2/3}-\frac{2\sqrt[3]{2}dx}{c}+\frac{4d^2x^2}{c^2}}\,\text{EllipticPi}\left(\frac{i\sqrt[3]{2}\sqrt{3}}{2+\sqrt[3]{-2}}, \arcsin\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right), \sqrt[3]{-1}\right)}{\left(2+\sqrt[3]{-2}\right)d\sqrt{c^3+4d^3x^3}}$$

```
[In] Integrate[1/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
[Out] ((-I)*2^(5/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[2^(2/3) -
 (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(I*2^(1/3)*Sqrt[3])/(2 + (
-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)
]/2^(1/6)], (-1)^(1/3)])/((2 + (-2)^(1/3))*d*Sqrt[c^3 + 4*d^3*x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(200) = 400$.

Time = 1.38 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.99

method	result
default	$\frac{2\left(\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}\right)\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}}\sqrt{\frac{x+\frac{2^{\frac{1}{3}}c}{2d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}+\frac{2^{\frac{1}{3}}c}{2d}}}\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\cdots\right)c}{d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\cdots}}\cdots}{d\sqrt{4d^3x^3+c^3}\left(\left(\cdots\right.\right.}$
elliptic	$\frac{2\left(\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}\right)\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}}\sqrt{\frac{x+\frac{2^{\frac{1}{3}}c}{2d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}+\frac{2^{\frac{1}{3}}c}{2d}}}\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\cdots\right)c}{d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\cdots}}\cdots}{d\sqrt{4d^3x^3+c^3}\left(\left(\cdots\right.\right.}$

```
[In] int(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(
1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I
*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x+1
/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^
(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1
/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3
+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*EllipticPi(((x-(1
/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))
*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1
/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1
/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))
*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.40

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$
$$= \left[-\frac{\sqrt{3}\sqrt{-c}d^2\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6+\sqrt{3}\left(4\,d^4x^4-10\,cd^3x^3-18\,c^2d^2x^2-8\,c^3dx-c^4\right)\sqrt{4\,d^3x^3+c^3}\sqrt{\cdot}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right.}{18\,c^2d^3}\right.$$
$$\left.-\frac{\sqrt{3}\sqrt{c}d^2\arctan\left(\frac{\sqrt{3}\sqrt{4\,d^3x^3+c^3}\left(2\,d^3x^3-6\,cd^2x^2-6\,c^2dx-c^3\right)\sqrt{c}}{3\left(8\,cd^4x^4+4\,c^2d^3x^3+2\,c^4dx+c^5\right)}\right)-6\,c\sqrt{d^3}\text{weierstrassPInverse}\left(0,-\frac{c^3}{d^3},x\right)}{9\,c^2d^3}\right]$$

```
[In] integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/18*(sqrt(3)*sqrt(-c)*d^2*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^4
 + 28*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 + sqrt(3)*(4*d^4*x^4 - 10*c*d^3*x^
3 - 18*c^2*d^2*x^2 - 8*c^3*d*x - c^4)*sqrt(4*d^3*x^3 + c^3)*sqrt(-c))/(d^6*
x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c^
5*d*x + c^6)) - 12*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d^3, x))/(c^2*d^
3), -1/9*(sqrt(3)*sqrt(c)*d^2*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2*d
^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)*sqrt(c)/(8*c*d^4*x^4 + 4*c^2*d^3*x^
3 + 2*c^4*d*x + c^5)) - 6*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d^3, x))/
(c^2*d^3)]
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

```
[In] integrate(1/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral(1/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{1}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

```
[In] integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{1}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

```
[In] integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{1}{\sqrt{c^3+4\,d^3\,x^3}\,\left(c+d\,x\right)}\,dx$$

```
[In] int(1/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
[Out] int(1/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)), x)
```

3.10 $\int \frac{1}{(1+\sqrt{3}+x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 146

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(1/4)/(x^3+1)^(1/2)/((1+x
)/(1+x+3^(1/2))^2)^(1/2)+arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/(9
+6*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{2160, 224, 2165, 209\}$

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$= \frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$
$$+\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

```
[In] Int[1/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3*(3 + 2*Sqrt[3])]
 + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2160

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3))), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{6\left(1-\sqrt{3}\right)+6x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx}{12\sqrt{3}}+\frac{\int \frac{1}{\sqrt{1+x^3}}\,dx}{2\sqrt{3}}\\
&= \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx,x,\frac{1+x}{\sqrt{1+x^3}}\right)}{\sqrt{3}}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{1+x^3}}$$

```
[In] Integrate[1/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[3])
*Sqrt[1 + x^3])
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}$	132
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}$	132

```
[In] int(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2
),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2
)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{1}{6}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\left(\sqrt{3}(x^2-4\,x-2)-6\,x-6\right)\sqrt{2\sqrt{3}-3}}{6\sqrt{x^3+1}}\right) + \frac{1}{3}\sqrt{3}\text{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2*sqrt(3) - 3)*arctan(1/6*(sqrt(3)*(x^2 - 4*x - 2) - 6*x - 6)*sqr
t(2*sqrt(3) - 3)/sqrt(x^3 + 1)) + 1/3*sqrt(3)*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{(x+1)\,(x^2-x+1)}\,\left(x+1+\sqrt{3}\right)}\,dx$$

```
[In] integrate(1/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
[Out] Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[2]%
%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

```
[In] int(1/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.11 $\int \frac{1}{(1+\sqrt{3}-x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 164

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -1/3*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2
)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(1/4)/(-x^3+1)^(1/2)/((1
-x)/(1-x+3^(1/2))^2)^(1/2)-arctan((1-x)*(3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/2))
/(9+6*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used

$= \{2160, 224, 2165, 209\}$

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$$
$$= -\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$
$$-\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

```
[In] Int[1/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]]/Sqrt[3*(3 + 2*Sqrt[3]
)]) - (Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*El
lipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(3
/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2160

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3))), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{-6\left(1-\sqrt{3}\right)+6x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx}{12\sqrt{3}}+\frac{\int \frac{1}{\sqrt{1-x^3}}\,dx}{2\sqrt{3}}\\ &= -\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}\\ &\quad-\frac{\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx,x,\frac{1-x}{\sqrt{1-x^3}}\right)}{\sqrt{3}}\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \frac{4\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{1-x^3}}$$

```
[In] Integrate[1/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (4*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[
3])*Sqrt[1 - x^3])
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{2 i \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \Pi\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{-\frac{3}{2}-\sqrt{3}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}\, \left(-\frac{3}{2}-\sqrt{3}+\frac{i \sqrt{3}}{2}\right)}$	143
elliptic	$\frac{2 i \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \Pi\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{-\frac{3}{2}-\sqrt{3}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}\, \left(-\frac{3}{2}-\sqrt{3}+\frac{i \sqrt{3}}{2}\right)}$	143

```
[In] int(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2
-3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2),I*3^(1/2)/(-3/2-3^(1/2)+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3
^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.40

$$\int \frac{1}{\left(1+\sqrt{3}-x\right) \sqrt{1-x^3}}\, dx$$
$$= -\frac{1}{6} \sqrt{2 \sqrt{3}-3} \arctan\left(\frac{\sqrt{-x^3+1}\left(\sqrt{3}(x^2+4\,x-2)+6\,x-6\right)\sqrt{2\sqrt{3}-3}}{6\,(x^3-1)}\right)$$
$$-\frac{1}{3} i\, \sqrt{3} \text{weierstrassPInverse}(0,4,x)$$

```
[In] integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2*sqrt(3) - 3)*arctan(1/6*sqrt(-x^3 + 1)*(sqrt(3)*(x^2 + 4*x - 2)
 + 6*x - 6)*sqrt(2*sqrt(3) - 3)/(x^3 - 1)) - 1/3*I*sqrt(3)*weierstrassPInve
rse(0, 4, x)
```

Sympy [F]

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\int \frac{1}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

```
[In] integrate(1/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
[Out] -Integral(1/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)), x
)
```

Maxima [F]

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int -\frac{1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

```
[In] integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[2]%
%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

```
[In] int(1/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
[Out] \text{Hanged}
```

3.12 $\int \frac{1}{(1+\sqrt{3}-x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 167

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -1/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2
)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(1/4)/(x^3-1)^(1/2)/((-1
+x)/(1-x-3^(1/2))^2)^(1/2)-arctanh((1-x)*(3+2*3^(1/2))^(1/2)/(x^3-1)^(1/2))
/(9+6*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

= {2160, 225, 2165, 212}

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$$
$$= -\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

```
[In] Int[1/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]]/Sqrt[3*(3 + 2*Sqrt[
3])]) - (Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^
(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2160

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3))), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{6\left(1-\sqrt{3}\right)-6x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx}{12\sqrt{3}} + \frac{\int \frac{1}{\sqrt{-1+x^3}}\,dx}{2\sqrt{3}}$$

$$= -\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$-\frac{\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)}{\sqrt{3}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{4\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}}, \arcsin\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right), \frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-1+x^3}}$$

```
[In] Integrate[1/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (4*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[
3])*Sqrt[-1 + x^3])
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^{3}-1}}$	132
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^{3}-1}}$	132

```
[In] int(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/
2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)
))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$$
$$=\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8+16x^7+112x^6+16x^5+112x^4-224x^3+64x^2+4\left(2x^6+18x^5+42x^4+8x^3\right.}{}\right.$$
$$+\frac{1}{3}\sqrt{3}\text{weierstrassPInverse}(0,4,x)$$

```
[In] integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*x^4 - 2
24*x^3 + 64*x^2 + 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 + sqrt(3)*(x^6 + 12*x^
5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(x^3 - 1)*sqrt(2*sqrt(3)
- 3) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x - 4) -
 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*x^2 + 6
4*x + 16)) + 1/3*sqrt(3)*weierstrassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\int \frac{1}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

```
[In] integrate(1/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
[Out] -Integral(1/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)), x
)
```

Maxima [F]

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int -\frac{1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

```
[In] integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

```
[In] int(1/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
[Out] \text{Hanged}
```

3.13 $\int \frac{1}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)
-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(1/4)/(-x^3-1)^(1/2)/((-1
-x)/(1+x-3^(1/2))^2)^(1/2)+arctanh((1+x)*(3+2*3^(1/2))^(1/2)/(-x^3-1)^(1/2)
)/(9+6*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used

= {2160, 225, 2165, 212}

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$$
$$= \frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$
$$+\frac{\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

```
[In] Int[1/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]]/Sqrt[3*(3 + 2*Sqrt[3]
)] + (Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ell
ipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(3/
4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2160

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3))), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{-6\left(1-\sqrt{3}\right)-6x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx}{12\sqrt{3}} + \frac{\int \frac{1}{\sqrt{-1-x^3}}\,dx}{2\sqrt{3}} \\
&= \frac{\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)}{\sqrt{3}} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}}, \arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right), \frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-1-x^3}}$$

```
[In] Integrate[1/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[3])
*Sqrt[-1 - x^3])
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{3}{2}+\sqrt{3}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}\,\left(\frac{3}{2}+\sqrt{3}+\frac{i\sqrt{3}}{2}\right)}$	139
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{3}{2}+\sqrt{3}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}\,\left(\frac{3}{2}+\sqrt{3}+\frac{i\sqrt{3}}{2}\right)}$	139

```
[In] int(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+
3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1
/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.36

$$\int \frac{1}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx$$
$$=\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2-4\left(2x^6-18x^5+42x^4-8x\right.}{}\right.$$
$$-\frac{1}{3}i\sqrt{3}\text{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 + 2
24*x^3 + 64*x^2 - 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 + sqrt(3)*(x^6 - 12*x^
5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(-x^3 - 1)*sqrt(2*sqrt(3)
 - 3) - 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x + 4)
+ 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 -
64*x + 16)) - 1/3*I*sqrt(3)*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

```
[In] integrate(1/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
[Out] Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

```
[In] int(1/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.14 $\int \frac{1}{(3+x)\sqrt{1+x^3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 329

$$\int \frac{1}{(3+x)\sqrt{1+x^3}} \, dx = \frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{26+15\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right), -7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{4\sqrt[4]{3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \operatorname{EllipticPi}\left(97-56\sqrt{3}, \arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right), -7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 1/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(1+
x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*26^(1/2)/(x^3+1)^(1/
2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2))/
(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2
)/(x^3+1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3^(1/2))^2)^(1/2)+2/3
*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x
+3^(1/2))^2)^(1/2)*(3/2*6^(1/2)+5/2*2^(1/2))*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(
1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2161, 224, 2167, 2138, 551, 585, 95, 210}

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{26+15\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} + \frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[In] Int[1/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(Sqrt
[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[26 + 15*Sqr
t[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 -
 Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (4*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/(
1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sqr
t[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2161

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[-q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
```

```
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{1}{\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$+ \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-2-\sqrt{3}+\left(-2+\sqrt{3}\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx, x, \frac{-1+\sqrt{3}-x}{1+\sqrt{3}+x}\right.}{\left(-2+\sqrt{3}\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$- \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-2-\sqrt{3}\right)^2-\left(-2+\sqrt{3}\right)^2x^2\right)}\,dx, x, \frac{-1\cdots}{1+\cdots}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$+ \frac{\left(4\sqrt[4]{3}\left(-2-\sqrt{3}\right)\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-2-\sqrt{3}\right)^2-\left(-2+\sqrt{3}\right)^2x\cdots\right.}\right.}{\left(-2+\sqrt{3}\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(97-56\sqrt{3};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(2\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(-2-\sqrt{3}\right)^2-\left(-2+\sqrt{3}\right)^2x\right)}\,dx, x, \frac{\left(-1-\right.}{\left(1+\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(97-56\sqrt{3};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\mathrm{Subst}\left(\int\frac{1}{-\left(-2-\sqrt{3}\right)^2+\left(-2+\sqrt{3}\right)^2-\left(\left(-2-\sqrt{3}\right)^2+\left(7-4\sqrt{3}\right)\left(-2+\sqrt{3}\right.\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{26+15\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(97-56\sqrt{3};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.39

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx$$
$$= -\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{7i+\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(7i+\sqrt{3}\right)\sqrt{1+x^3}}$$

```
[In] Integrate[1/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(7*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((7*I + Sqrt[3])*Sqrt[1 + x^3])
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.37

method	result	size
default	$\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{3}{4}+\frac{i\sqrt{3}}{4},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$	123
elliptic	$\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{3}{4}+\frac{i\sqrt{3}}{4},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$	123

```
[In] int(1/(3+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^
(1/2)/(x^3+1)^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-3/4+1/4*I
*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 + 1)/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F]

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{(x+1)\,(x^2-x+1)}\,(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(x**3+1)**(1/2),x)
[Out] Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.50

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx$$
$$=\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(1/((x^3 + 1)^(1/2)*(x + 3)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*ellipticPi(- (3^(1/2)*1i)/4 - 3/4, asin(((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
))/(2*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.15 $\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 380

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = -\frac{(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -1/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/((x^2+x+1)/(1
-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*7^(1/2)/(-x^3+1)^(1
/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)+4/13*3^(1/4)*(1-x)*EllipticPi((-1+x+3^(1/
2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2
^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2
))^2)^(1/2)-2/3*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*
(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(4+3^(1
/2))/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2161, 224, 2167, 2138, 551, 585, 95, 212}

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx$$
$$= -\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$
$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$
$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[In] Int[1/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] -1/2*((1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt
[(1 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])]
)/(Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (2*Sqrt[2 + S
qrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1
 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*(4 + Sqrt[3])
*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (4*3^(1/4)*Sqrt[2 + Sqr
t[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304
*Sqrt[3])/169, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]]
)/(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2161

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[-q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
```

```
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt{1-x^3}}\,dx}{4+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{1-x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$
$$+\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(4-\sqrt{3}+\left(4+\sqrt{3}\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx, x, \frac{-1+\sqrt{3}+x}{1+\sqrt{3}-x}\right)}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$
$$-\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(4-\sqrt{3}\right)^2-\left(4+\sqrt{3}\right)^2x^2\right)}\,dx, x, \frac{-1+\sqrt{3}}{1+\sqrt{3}}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$
$$+\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(4-\sqrt{3}\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(4-\sqrt{3}\right)^2-\left(4+\sqrt{3}\right)^2x^2\right)}\,dx\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{\left(2\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(4-\sqrt{3}\right)^2-\left(4+\sqrt{3}\right)^2x\right)}dx,x,\frac{\left(-1+\sqrt{3}\cdots\right.}{\left(1+\sqrt{3}\cdots\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{-\left(4-\sqrt{3}\right)^2+\left(4+\sqrt{3}\right)^2-\left(\left(4-\sqrt{3}\right)^2+\left(7-4\sqrt{3}\right)\left(4+\sqrt{3}\right)^2\right)x^2\cdots}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.34

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{5i+\sqrt{3}},\arcsin\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)}{\left(5i+\sqrt{3}\right)\sqrt{1-x^3}}$$

```
[In] Integrate[1/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*EllipticP
i[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]
*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])])/((5*I + Sqrt[3])*Sqrt[1 - x^3])
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.35

method	result	size
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{5}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}\left(\frac{5}{2}+\frac{i\sqrt{3}}{2}\right)}$	133
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{5}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}\left(\frac{5}{2}+\frac{i\sqrt{3}}{2}\right)}$	133

```
[In] int(1/(3+x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(5/2
+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1
/2),I*3^(1/2)/(5/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^3 + 1)/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F]

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-(x-1)(x^2+x+1)}\,(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(-x**3+1)**(1/2),x)
[Out] Integral(1/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.47

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = -\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(1/((1 - x^3)^(1/2)*(x + 3)),x)
```

```
[Out] -(((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(
1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/8 +
 3/8, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2
)/((3^(1/2)*1i)/2 - 3/2)))/(2*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) +
 x^3)^(1/2))
```

3.16 $\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 374

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -2/39*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(5/2*6^(1/
2)-7/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-
1+x)/(1-x-3^(1/2))^2)^(1/2)-1/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3^(1
/2))^2)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2
)^(1/2)*7^(1/2)/(x^3-1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)+4/13*3^(1/4)*(1
-x)*EllipticPi((-1+x+3^(1/2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I*3^(1/
2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(x^3-1)
^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2161, 225, 2167, 2138, 551, 585, 95, 212}

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx$$
$$= -\frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$
$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$
$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[In] Int[1/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -1/2*((1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt
[(1 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])]
)/(Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3]) - (2*Sqrt[62 -
 35*Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcS
in[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(13*3^(1/4)*Sqrt[
-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[
3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*S
qrt[3])/169, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/
(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2161

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[-q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
```

```
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(4-\sqrt{3}+\left(4+\sqrt{3}\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx, x, \frac{-1+\sqrt{3}+x}{1+\sqrt{3}-x}\right)}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(4-\sqrt{3}\right)^2-\left(4+\sqrt{3}\right)^2x^2\right)}\,dx, x, \frac{-1+\sqrt{3}}{1+\sqrt{3}}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(4-\sqrt{3}\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(4-\sqrt{3}\right)^2-\left(4+\sqrt{3}\right)^2x^2\right)}\,dx\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$-\frac{\left(2\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(4-\sqrt{3}\right)^2-\left(4+\sqrt{3}\right)^2x\right)}dx,x,\frac{\left(-1+\sqrt{3}\right.}{\left(1+\sqrt{3}\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{-\left(4-\sqrt{3}\right)^2+\left(4+\sqrt{3}\right)^2-\left(\left(4-\sqrt{3}\right)^2+\left(7-4\sqrt{3}\right)\left(4+\sqrt{3}\right)^2\right)x^2}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$-\frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.34

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{5i+\sqrt{3}},\arcsin\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)}{\left(5i+\sqrt{3}\right)\sqrt{-1+x^3}}$$

```
[In] Integrate[1/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*EllipticP
i[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]
*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])])/((5*I + Sqrt[3])*Sqrt[-1 + x^3])
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.33

method	result	size
default	$\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{3}{8}+\frac{i\sqrt{3}}{8},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3-1}}$	124
elliptic	$\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{3}{8}+\frac{i\sqrt{3}}{8},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3-1}}$	124

```
[In] int(1/(3+x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),3/8+1
/8*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 - 1)/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F]

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(x**3-1)**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^3 - 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^3 - 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.44

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(1/((x^3 - 1)^(1/2)*(x + 3)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/8 + 3/8, asin((-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/
2)))/(4*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.17 $\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 340

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{4\sqrt[4]{3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(
1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((-1
-x)/(1+x-3^(1/2))^2)^(1/2)+1/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^(1/
2))^2)^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)
^(1/2)*26^(1/2)/(-x^3-1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1+x
)*EllipticPi((-1-x+3^(1/2))/(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)*((x^
2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+
x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2161, 225, 2167, 2138, 551, 585, 95, 210}

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx$$
$$= \frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$
$$-\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$
$$+\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[In] Int[1/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(Sqrt
[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3]) + (2*(1 + x)*Sqrt[(1
 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sq
rt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[-((1 + x)/(1
- Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) - (4*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/
(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2161

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[-q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
```

```
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{1}{\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-2-\sqrt{3}+\left(-2+\sqrt{3}\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx, x, \frac{-1+\sqrt{3}-x}{1+\sqrt{3}+x}$$

$$(-2+\sqrt{3})\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-2-\sqrt{3}\right)^2-\left(-2+\sqrt{3}\right)^2x^2\right)}\,dx, x, \frac{-1-}{1+}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+ \frac{\left(4\sqrt[4]{3}(-2-\sqrt{3})\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-2-\sqrt{3}\right)^2-\left(-2+\sqrt{3}\right)^2x\right.}\right.}{(-2+\sqrt{3})\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(97-56\sqrt{3};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{\left(2\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(-2-\sqrt{3}\right)^2-\left(-2+\sqrt{3}\right)^2x\right)}\,dx, x, \frac{(-1-}{(1+}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(97-56\sqrt{3};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{-\left(-2-\sqrt{3}\right)^2+\left(-2+\sqrt{3}\right)^2-\left(\left(-2-\sqrt{3}\right)^2+\left(7-4\sqrt{3}\right)\left(-2+\sqrt{3}\right.\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(97-56\sqrt{3};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.38

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{7i+\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(7i+\sqrt{3}\right)\sqrt{-1-x^3}}$$

```
[In] Integrate[1/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(7*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((7*I + Sqrt[3])*Sqrt[-1 - x^3])
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.39

method	result	size
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{7}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}\left(\frac{7}{2}+\frac{i\sqrt{3}}{2}\right)}$	133
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{7}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}\left(\frac{7}{2}+\frac{i\sqrt{3}}{2}\right)}$	133

```
[In] int(1/(3+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(7/2+
1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2),I*3^(1/2)/(7/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^3 - 1)/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F]

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-(x+1)(x^2-x+1)}\,(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(-x**3-1)**(1/2),x)
[Out] Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}(x+3)}\,dx$$

```
[In] integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.53

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx$$
$$= \frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(1/((- x^3 - 1)^(1/2)*(x + 3)),x)
```

```
[Out] (((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/
2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i
)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(- (3^(1/2)*1i)/4 -
3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/
2))^(1/2))
```

3.18 $\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}-\frac{3\log\left(d(c-dx)+2^{2/3}d\sqrt[3]{-c^3+d^3x^3}\right)}{4\sqrt[3]{2}cd}$$

```
[Out] 1/8*ln((-d*x+c)*(d*x+c)^2)*2^(2/3)/c/d-3/8*ln(d*(-d*x+c)+2^(2/3)*d*(d^3*x^3
-c^3)^(1/3))*2^(2/3)/c/d+1/4*arctan(1/3*(1-2^(1/3)*(-d*x+c)/(d^3*x^3-c^3)^(
1/3))*3^(1/2))*3^(1/2)*2^(2/3)/c/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2174}

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd} - \frac{3\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd} + \frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}$$

```
[In] Int[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/
(2*2^(1/3)*c*d) + Log[(c - d*x)*(c + d*x)^2]/(4*2^(1/3)*c*d) - (3*Log[d*(c
- d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d)
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd} + \frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd} - \frac{3\log\left(d(c-dx)+2^{2/3}d\sqrt[3]{-c^3+d^3x^3}\right)}{4\sqrt[3]{2}cd}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.24

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$
$$= \frac{\sqrt[3]{-\frac{1}{2}}\left(2i\sqrt{3}\text{arctanh}\left(\frac{\sqrt[3]{2}\left(3+i\sqrt{3}\right)c+\sqrt[3]{2}\left(-3-i\sqrt{3}\right)dx+2i\sqrt{3}\sqrt[3]{-c^3+d^3x^3}}{6\sqrt[3]{-c^3+d^3x^3}}\right)+2\log\left(\sqrt{c}\sqrt{d}\left(-c+i\sqrt{3}c+dx-\right.\right.\right.}{}$$

```
[In] Integrate[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] ((-1/2)^(1/3)*((2*I)*Sqrt[3]*ArcTanh[(2^(1/3)*(3 + I*Sqrt[3])*c + 2^(1/3)*(
-3 - I*Sqrt[3])*d*x + (2*I)*Sqrt[3]*(-c^3 + d^3*x^3)^(1/3))/(6*(-c^3 + d^3*
x^3)^(1/3))] + 2*Log[Sqrt[c]*Sqrt[d]*(-c + I*Sqrt[3]*c + d*x - I*Sqrt[3]*d*
x + 2*2^(2/3)*(-c^3 + d^3*x^3)^(1/3))] - Log[-(c*d*((1 + I*Sqrt[3])*c^2 + (
1 + I*Sqrt[3])*d^2*x^2 - 2*(-2)^(2/3)*d*x*(-c^3 + d^3*x^3)^(1/3) - 4*2^(1/3
)*(-c^3 + d^3*x^3)^(2/3) + 2*c*((-1 - I*Sqrt[3])*d*x + (-2)^(2/3)*(-c^3 + d
^3*x^3)^(1/3))))]))/(4*c*d)
```

Maple [F]

$$\int \frac{1}{(dx+c)\left(d^3x^3-c^3\right)^{\frac{1}{3}}}dx$$

```
[In] int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

```
[Out] int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{1}{\sqrt[3]{(-c+dx)\,(c^2+cdx+d^2x^2)}\,(c+dx)}\,dx$$

```
[In] integrate(1/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
[Out] Integral(1/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3x^3-c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

```
[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3x^3-c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

```
[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="giac")
[Out] integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{1}{{(d^3\,x^3-c^3)}^{1/3}\,(c+d\,x)}\,dx$$

```
[In] int(1/((d^3*x^3 - c^3)^(1/3)*(c + d*x)),x)
[Out] int(1/((d^3*x^3 - c^3)^(1/3)*(c + d*x)), x)
```

3.19 $\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 186

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \frac{\arctan\left(\frac{1+\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}cd} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2cd} - \frac{\log(c+dx)}{2cd} - \frac{\log\left(-dx+\sqrt[3]{2c^3+d^3x^3}\right)}{4cd} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd}$$

```
[Out] -1/2*ln(d*x+c)/c/d-1/4*ln(-d*x+(d^3*x^3+2*c^3)^(1/3))/c/d+3/4*ln(d*(d*x+2*c
)-d*(d^3*x^3+2*c^3)^(1/3))/c/d+1/6*arctan(1/3*(1+2*d*x/(d^3*x^3+2*c^3)^(1/3
))*3^(1/2))/c/d*3^(1/2)-1/2*arctan(1/3*(1+2*(d*x+2*c)/(d^3*x^3+2*c^3)^(1/3)
)*3^(1/2))*3^(1/2)/c/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used

$= \{2175, 245, 2176\}$

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \frac{\arctan\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}cd} - \frac{\sqrt{3}\arctan\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2cd} - \frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{4cd} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd} - \frac{\log(c+dx)}{2cd}$$

```
[In] Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]*c*d) - (Sq
rt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(2*c*d
) - Log[c + d*x]/(2*c*d) - Log[-(d*x) + (2*c^3 + d^3*x^3)^(1/3)]/(4*c*d) +
(3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(4*c*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2175

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Dist[
1/(2*c), Int[1/(a + b*x^3)^(1/3), x], x] + Dist[1/(2*c), Int[(c - d*x)/((c
+ d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2*b*c^3
- a*d^3, 0]
```

Rule 2176

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt[3]{2c^3+d^3x^3}}\,dx}{2c} + \frac{\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx}{2c}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}cd} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2cd} - \frac{\log(c+dx)}{2cd} - \frac{\log\left(-dx+\sqrt[3]{2c^3+d^3x^3}\right)}{4cd} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd}$$

Mathematica [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

```
[In] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
[Out] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]
```

Maple [F]

$$\int \frac{1}{(dx+c)\left(d^3x^3+2c^3\right)^{\frac{1}{3}}}dx$$

```
[In] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
[Out] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

```
[In] integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
[Out] Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3x^3+2\,c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3x^3+2\,c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="giac")
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{1}{(2\,c^3+d^3\,x^3)^{1/3}\,(c+d\,x)}\,dx$$

```
[In] int(1/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)),x)
[Out] int(1/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)), x)
```

3.20 $\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}} \, dx = -\frac{\arctan\left(\frac{1+\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}c^2d} + \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2c^2d} - \frac{\log(c+dx)}{2c^2d} - \frac{\log\left(dx-\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}$$

```
[Out] -1/2*ln(d*x+c)/c^2/d-1/4*ln(d*x-(d^3*x^3+2*c^3)^(1/3))/c^2/d+3/4*ln(d*(d*x+
2*c)-d*(d^3*x^3+2*c^3)^(1/3))/c^2/d-1/6*arctan(1/3*(1+2*d*x/(d^3*x^3+2*c^3)
^(1/3))*3^(1/2))/c^2/d*3^(1/2)+1/2*arctan(1/3*(1+2*(d*x+2*c)/(d^3*x^3+2*c^3
)^(1/3))*3^(1/2))*3^(1/2)/c^2/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used

$= \{2179\}$

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx = -\frac{\arctan\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}c^2d} + \frac{\sqrt{3}\arctan\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2c^2d} - \frac{\log(c+dx)}{2c^2d} - \frac{\log\left(dx-\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}$$

```
[In] Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)),x]
```

```
[Out] -1/2*ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*c^2*d)
+ (Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(
2*c^2*d) - Log[c + d*x]/(2*c^2*d) - Log[d*x - (2*c^3 + d^3*x^3)^(1/3)]/(4*c
^2*d) + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(4*c^2*d)
```

Rule 2179

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(2/3)), x_Symbol] :> With[
{q = Rt[b, 3]}, Simp[(-d)*(ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/
(2*Sqrt[3]*q^2*c^2)), x] + (Simp[Sqrt[3]*d*(ArcTan[(1 + 2*q*((2*c + d*x)/(d
*(a + b*x^3)^(1/3))))/Sqrt[3]]/(2*q^2*c^2)), x] - Simp[d*(Log[c + d*x]/(2*q
^2*c^2)), x] - Simp[d*(Log[q*x - (a + b*x^3)^(1/3)]/(4*q^2*c^2)), x] + Simp
[3*d*(Log[q*(2*c + d*x) - d*(a + b*x^3)^(1/3)]/(4*q^2*c^2)), x])] /; FreeQ[
{a, b, c, d}, x] && EqQ[2*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{1+\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}c^2d} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2c^2d} - \frac{\log(c+dx)}{2c^2d} - \frac{\log\left(dx-\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}$$

Mathematica [F]

$$\int \frac{1}{(c+dx)(2c^3+d^3x^3)^{2/3}}\,dx=\int \frac{1}{(c+dx)(2c^3+d^3x^3)^{2/3}}\,dx$$

```
[In] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)),x]
[Out] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]
```

Maple [F]

$$\int \frac{1}{(dx+c)(d^3x^3+2c^3)^{\frac{2}{3}}}dx$$

```
[In] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)
[Out] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(2c^3+d^3x^3)^{2/3}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)(2c^3+d^3x^3)^{2/3}}\,dx=\int \frac{1}{(c+dx)(2c^3+d^3x^3)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(2/3),x)
[Out] Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx = \int \frac{1}{\left(d^3x^3+2\,c^3\right)^{\frac{2}{3}}(dx+c)}\,dx$$

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx = \int \frac{1}{\left(d^3x^3+2\,c^3\right)^{\frac{2}{3}}(dx+c)}\,dx$$

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="giac")
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx = \int \frac{1}{\left(2\,c^3+d^3\,x^3\right)^{2/3}\,\left(c+d\,x\right)}\,dx$$

```
[In] int(1/((2*c^3 + d^3*x^3)^(2/3)*(c + d*x)),x)
[Out] int(1/((2*c^3 + d^3*x^3)^(2/3)*(c + d*x)), x)
```

3.21 $\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx = -\frac{\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\left(2^{2/3}+x\right)}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{2^{2/3}} - \frac{\log\left(1+\sqrt[3]{2}x\right)}{2^{2/3}} - \frac{\log\left(x-\sqrt[3]{1+x^3}\right)}{2\ 2^{2/3}} + \frac{3\log\left(2+\sqrt[3]{2}x-\sqrt[3]{2}\sqrt[3]{1+x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/2*ln(1+2^(1/3)*x)*2^(1/3)-1/4*ln(x-(x^3+1)^(1/3))*2^(1/3)+3/4*ln(2+2^(1/
3)*x-2^(1/3)*(x^3+1)^(1/3))*2^(1/3)-1/6*arctan(1/3*(1+2*x/(x^3+1)^(1/3))*3^
(1/2))*2^(1/3)*3^(1/2)+1/2*arctan(1/3*(1+2*(2^(2/3)+x)/(x^3+1)^(1/3))*3^(1/
2))*3^(1/2)*2^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2179}

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx = -\frac{\arctan\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\sqrt{3}\arctan\left(\frac{\frac{2\left(x+2^{2/3}\right)}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}} - \frac{\log\left(x-\sqrt[3]{x^3+1}\right)}{2\ 2^{2/3}} + \frac{3\log\left(-\sqrt[3]{2}\sqrt[3]{x^3+1}+\sqrt[3]{2}x+2\right)}{2\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}x+1\right)}{2^{2/3}}$$

```
[In] Int[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)),x]
```

```
[Out] -(ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + (Sqrt[3]
*ArcTan[(1 + (2*(2^(2/3) + x))/(1 + x^3)^(1/3))/Sqrt[3]])/2^(2/3) - Log[1 +
 2^(1/3)*x]/2^(2/3) - Log[x - (1 + x^3)^(1/3)]/(2*2^(2/3)) + (3*Log[2 + 2^(
1/3)*x - 2^(1/3)*(1 + x^3)^(1/3)])/(2*2^(2/3))
```

Rule 2179

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(2/3)), x_Symbol] :> With[
{q = Rt[b, 3]}, Simp[(-d)*(ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/
(2*Sqrt[3]*q^2*c^2)), x] + (Simp[Sqrt[3]*d*(ArcTan[(1 + 2*q*((2*c + d*x)/(d
*(a + b*x^3)^(1/3))))/Sqrt[3]]/(2*q^2*c^2)), x] - Simp[d*(Log[c + d*x]/(2*q
^2*c^2)), x] - Simp[d*(Log[q*x - (a + b*x^3)^(1/3)]/(4*q^2*c^2)), x] + Simp
[3*d*(Log[q*(2*c + d*x) - d*(a + b*x^3)^(1/3)]/(4*q^2*c^2)), x])] /; FreeQ[
{a, b, c, d}, x] && EqQ[2*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\left(2^{2/3}+x\right)}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{2^{2/3}} - \frac{\log\left(1+\sqrt[3]{2}x\right)}{2^{2/3}} - \frac{\log\left(x-\sqrt[3]{1+x^3}\right)}{2\ 2^{2/3}} + \frac{3\log\left(2+\sqrt[3]{2}x-\sqrt[3]{2}\sqrt[3]{1+x^3}\right)}{2\ 2^{2/3}}$$

Mathematica [F]

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)(1+x^3)^{2/3}}\,dx = \int \frac{1}{\left(1+\sqrt[3]{2}x\right)(1+x^3)^{2/3}}\,dx$$

[In] Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)),x]

[Out] Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 50.89 (sec) , antiderivative size = 3064, normalized size of antiderivative = 20.84

method	result	size
trager	Expression too large to display	3064

[In] int(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x,method=_RETURNVERBOSE)

[Out] -1/6*ln(-(-15559137585059152-1604954020235328*2^(1/3)*x^4+14712078518823840*x^3-936223178470608*x^6-12498127505504256*2^(1/3)*(x^3+1)^(1/3)-4279877387294208*x^5*2^(2/3)+6471910353179844*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)^(2/3)*x^3+1203809884289286*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)^(2/3)*x^4-3218487773589102*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)^(1/3)*x^5-72607968203490*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(2/3)*x^4-23004340956706368*2^(1/3)*x-1604954020235328*2^(2/3)*x^2-1560939140169318*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(1/3)*x^5+3775614346581480*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(2/3)*x^2-3842311729647552*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(1/3)*x^3-10498622607665136*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)^(1/3)*x^4-7759251414704196*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)^(2/3)*x-3531674097632562*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)^(1/3)*x^2+2613886855325640*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(2/3)*x+840505690860402*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(1/3)*x^2-11688730639030284*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)^(1/3)*x-5504178119120758*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^4-10391133689698608*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x+6434683875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2+6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5-10107087250606332*2^(2/3)*(x^3+1)^(2/3)-9125490357912936*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)^(1/3)+3712807561447224*(x^3+1)^(2/3)*x^4-2960082830251008*(x^3+1)^(1/3)*x^5-3259019970603674 4*(x^3+1)^(2/3)*x-12827025597754368*(x^3+1)^(1/3)*x^2+1876782797064098*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^6+2321435374812274

*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^6+10197714008127436*2^(2/3)*Root
Of(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+3217341937824168*2^(2/3)*RootOf(2^(2/3)+2^(
1/3)*_Z+_Z^2)^2*x^4+2081809489180344*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2
/3)*x+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2+56654144
13224496*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+3532767618003008*RootO
f(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^3+2613886855325640*RootOf(2^(2/3)+2^
(1/3)*_Z+_Z^2)^2*(x^3+1)^(2/3)*x^3+2884227944870616*RootOf(2^(2/3)+2^(1/3)*
_Z+_Z^2)*(x^3+1)^(2/3)*x^2-8123294120973864*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)
*(x^3+1)^(1/3)*x^3-11138422684341672*2^(1/3)*(x^3+1)^(2/3)*x^2+391907464819
4292*2^(2/3)*(x^3+1)^(2/3)*x^3-1315592369000448*2^(2/3)*(x^3+1)^(1/3)*x^4-6
964190009986188*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(1/3)*x^4+3288980
922501120*2^(1/3)*(x^3+1)^(1/3)*x^3-20062783627256832*2^(2/3)*(x^3+1)^(1/3)
*x-7115580883942020*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)^(2/3)+2
161300347926748*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(1/3)*x)/(1+2^(1/
3)*x)^6)*2^(1/3)-1/6*ln(-(-15559137585059152-1604954020235328*2^(1/3)*x^4+1
4712078518823840*x^3-936223178470608*x^6-12498127505504256*2^(1/3)*(x^3+1)^
(1/3)-4279877387294208*x^5*2^(2/3)+6471910353179844*2^(1/3)*RootOf(2^(2/3)+
2^(1/3)*_Z+_Z^2)*(x^3+1)^(2/3)*x^3+1203809884289286*2^(2/3)*RootOf(2^(2/3)+
2^(1/3)*_Z+_Z^2)*(x^3+1)^(2/3)*x^4-3218487773589102*2^(2/3)*RootOf(2^(2/3)+
2^(1/3)*_Z+_Z^2)*(x^3+1)^(1/3)*x^5-72607968203490*2^(1/3)*RootOf(2^(2/3)+2^
(1/3)*_Z+_Z^2)^2*(x^3+1)^(2/3)*x^4-23004340956706368*2^(1/3)*x-160495402023
5328*2^(2/3)*x^2-1560939140169318*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2
*(x^3+1)^(1/3)*x^5+3775614346581480*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)
^2*(x^3+1)^(2/3)*x^2-3842311729647552*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*(x^3+1)^(1/3)*x^3-10498622607665136*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)*(x^3+1)^(1/3)*x^4-7759251414704196*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)*(x^3+1)^(2/3)*x-3531674097632562*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z
^2)*(x^3+1)^(1/3)*x^2+2613886855325640*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z
^2)^2*(x^3+1)^(2/3)*x+840505690860402*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*(x^3+1)^(1/3)*x^2-11688730639030284*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)*(x^3+1)^(1/3)*x-5504178119120758*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z
^2)+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^4-10391133689698608*
RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x+6434683875648336*RootOf(2^(2/3)+2^(1/3)*_
Z+_Z^2)^2*x^2+6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5-101070
87250606332*2^(2/3)*(x^3+1)^(2/3)-9125490357912936*RootOf(2^(2/3)+2^(1/3)*_
Z+_Z^2)*(x^3+1)^(1/3)+3712807561447224*(x^3+1)^(2/3)*x^4-2960082830251008*(
x^3+1)^(1/3)*x^5-3259019970603674 4*(x^3+1)^(2/3)*x-12827025597754368*(x^3+1
)^(1/3)*x^2+1876782797064098*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^6+
2321435374812274*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^6+10197714008127
436*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+3217341937824168*2^(2/3)*Ro
otOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^4+2081809489180344*RootOf(2^(2/3)+2^(1/3)
*_Z+_Z^2)^2*2^(2/3)*x+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1
/3)*x^2+5665414413224496*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+353276
7618003008*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^3+2613886855325640*R
ootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(2/3)*x^3+2884227944870616*RootOf(

2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)^(2/3)*x^2-8123294120973864*RootOf(2^(2/3)+
2^(1/3)*_Z+_Z^2)*(x^3+1)^(1/3)*x^3-11138422684341672*2^(1/3)*(x^3+1)^(2/3)*
x^2+3919074648194292*2^(2/3)*(x^3+1)^(2/3)*x^3-1315592369000448*2^(2/3)*(x^
3+1)^(1/3)*x^4-6964190009986188*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(
1/3)*x^4+3288980922501120*2^(1/3)*(x^3+1)^(1/3)*x^3-20062783627256832*2^(2/
3)*(x^3+1)^(1/3)*x-7115580883942020*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)
*(x^3+1)^(2/3)+2161300347926748*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(
1/3)*x)/(1+2^(1/3)*x)^6)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)+1/6*RootOf(2^(2/3)
+2^(1/3)*_Z+_Z^2)*ln(-(-4550781346817636-3129477143943360*2^(1/3)*x^4+13821
85738574984*x^3-1825528333966960*x^6-3372637147591320*2^(1/3)*(x^3+1)^(1/3)
-3794491037031324*x^5*2^(2/3)-1244136642528564*2^(1/3)*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)*(x^3+1)^(2/3)*x^3-1349025820696266*2^(2/3)*RootOf(2^(2/3)+2^(1/
3)*_Z+_Z^2)*(x^3+1)^(2/3)*x^4+96609493250466*2^(2/3)*RootOf(2^(2/3)+2^(1/3)
_Z+_Z^2)(x^3+1)^(1/3)*x^5-72607968203490*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_
Z+_Z^2)^2*(x^3+1)^(2/3)*x^4-8449588288647072*2^(1/3)*x-3129477143943360*2^(
2/3)*x^2-1560939140169318*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)
^(1/3)*x^5+3775614346581480*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+
1)^(2/3)*x^2-3842311729647552*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^
3+1)^(1/3)*x^3-3429757412307240*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^
3+1)^(1/3)*x^4+12987025125355476*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x
^3+1)^(2/3)*x+5212685479353366*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3
+1)^(1/3)*x^2+2613886855325640*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x
^3+1)^(2/3)*x+840505690860402*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^
3+1)^(1/3)*x^2+16011331334883780*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x
^3+1)^(1/3)*x+5504178119120758*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)+4910
160751940304*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^4+18718371646419984*RootOf(2
^(2/3)+2^(1/3)*_Z+_Z^2)*x+6434683875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^
2*x^2+6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5-29915063666643
12*2^(2/3)*(x^3+1)^(2/3)+9125490357912936*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(
x^3+1)^(1/3)+1159971856461672*(x^3+1)^(2/3)*x^4+355014436588560*(x^3+1)^(1/
3)*x^5-11843923165977072*(x^3+1)^(2/3)*x-4082666020768440*(x^3+1)^(1/3)*x^2
+1876782797064098*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^6+14321302193
15922*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^6-3132178772121420*2^(2/3)*
RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+3217341937824168*2^(2/3)*RootOf(2^(2/3)
+2^(1/3)*_Z+_Z^2)^2*x^4+2081809489180344*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*
2^(2/3)*x+4910160751940304*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2+6636
187113750264*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+3532767618003008*R
ootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^3+2613886855325640*RootOf(2^(2/3
)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(2/3)*x^3+12218229441455304*RootOf(2^(2/3)+2^(
1/3)*_Z+_Z^2)*(x^3+1)^(2/3)*x^2-7245952797616344*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)*(x^3+1)^(1/3)*x^3-6471421936049328*2^(1/3)*(x^3+1)^(2/3)*x^2+61051150
340088*2^(2/3)*(x^3+1)^(2/3)*x^3+2218840228678500*2^(2/3)*(x^3+1)^(1/3)*x^4
-6964190009986188*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(1/3)*x^4+37276
51584179880*2^(1/3)*(x^3+1)^(1/3)*x^3-6212752640299800*2^(2/3)*(x^3+1)^(1/3
)*x+7115580883942020*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)^(2/3)+

```
2161300347926748*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(1/3)*x)/(1+2^(1
/3)*x)^6)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. 2(112) = 224.

Time = 3.74 (sec) , antiderivative size = 712, normalized size of antiderivative = 4.84

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(-1/3*(13910019318573948542*sqrt(3)*(442971093109
301727414338294053996366544517259164034007595963454201 83*x^16 + 46991175387
757729726668749336126627429821975172615651174879678821 0304*x^13 - 168603219
036433260440647021325346295645242325246375460547582960409424*x^10 - 1978806
30118237657393829295422779262737333028339787658261155 8332893440*x^7 - 14400
908916871775814229187630893019686025810368722130843899 12370301872*x^4 - 2^(
2/3)*(5227107745312510761299592397765475834939487692288555281920 9999866413*
x^15 + 5906745478545485772932858207883407784932992812555213360593997994805 17
2*x^12 + 30631426122293143161988738296663042306482221769027962533919785778 1
7900*x^9 + 73310495586975778090083525715970394034579688570667302777861149 59
327080*x^6 + 77232448067562904437597705467808729717394447501735196355441861
14816064*x^3 + 29116808987839009219563485741835514155891904460151064526080 7
0501424800) + 6*2^(1/3)*(126013559962163220933147486791491205433021406856 77
058235520929344665*x^14 - 5558690630019665139246271949192126784782079889001
98502271159380897 18*x^11 - 4503989201053205993076395360278839861317936247 29
303407436233610788504*x^8 - 7218887058809482614325170526703941062383389438 4
437355390651087986658 4*x^5 - 3386681580686843734363092730678494644056913607
513785074424729217745 44*x^2) - 62367643045453979229021701235594440425380660
140976292433240780519680*x)*(x^3 + 1)^(2/3) - 13910019318573948542*sqrt(3)*
(20244151386762728582873176440916642276036913846721964342570319874272*x^17
+ 7411461370788349909689586949569535257869682161627913691415610792313 42*x^1
4 + 21798431972717754011474383961016668755370436633451991030652907183506 60*
x^11 + 21110249350284448030276350331723739969986388702750815288350190294268
08*x^8 + 69058397930221264954184667175232357867176236156498719853237207761 7
072*x^5 + 42560446719395994043503690929493089250376947849898596094387069196
992*x^2 + 2^(2/3)*(58175953016441250552894129028785848895343146706912452780
410096144857*x^16 + 60332912344022592845951244288084636749808634046721050 84
10170807919392*x^13 + 99321772442116051464080292497021614887213800679935641
7482692017634440*x^10 - 31537366861697860036872967982882082606714520389786 0
799345951918357208*x^7 - 15359897811758984549040097640804776981234391400095
2325783379529417 1024*x^4 - 7745816539945065221850650605154579995624696708 38
```

0357107002791009604 80*x) - 2*2^(1/3)*(44250337395862623641308432146105265591584981692216944246872622437586*x^15 + 93730331994553087914588193029404165015738145719370012253256237142833*x^12 + 13218541316595455203956380934358348619932882852548406311430877544538 16*x^9 + 42477057677017468895892138257252781622024317737600109081215316558582 40*x^6 + 45932454636886436349937358513416218383598381701882855001517331858550 40*x^3 + 16158837376147892971429107707869228809509709698905305411015386387388 00))*(x^3 + 1)^(1/3) + sqrt(3)*(580845856624814138058536658925035752422341023745042657144110018434133971171392378653765*x^18 + 85128502112016585963203224235079794367450370616046622522881061739848890113983919394938 44*x^15 + 46037674634299399876464933353333393651798714498861959697684952859181279514449172348801132*x^12 + 1000163483533668123579997239485409669524356118365804202948338270587665854564636112155629 12*x^9 + 91397758625366807679053421068886729440495107689602121025455736534255642370122935700628112*x^6 + 27679206471222147818932348914707271406554121216141734785863966451139338545569046396842944*x^3 - 1391001931857394 8542*2^(2/3)*(3844366680114123938578119587438413410802428820066154040455085354797*x^17 - 4931319711549190780631731959832802785947037704060043883265521247935 91*x^14 - 22636563297337505265752397883933418042722683284040783773869796554116 28*x^11 - 360329608895964304006588260615697733294277836897086795884126627540 5688*x^8 - 2375143924145462474790789297643082581023352457583644433698318090272160*x^5 - 5385278270845367592983951643087283603473362177907843098770242601 29712*x^2) + 16692023182288738 2504*2^(1/3)*(1359589204404282836627598200670804939503290969888000412994951133922 6*x^16 + 1351333848851582503771790485959913464507711993272362079564211134619 03*x^13 + 402245899028058436823068109521885840258775610614711826343657868879359*x^10 + 5472587101498793346918329998345253082977903875633568796454680365329 66*x^7 + 3636741997036409638849600121243872631062549095216406631543023021164 04*x^4 + 97123895740704644005292055222464498011501842944639406026020532340120*x) - 1800774080838794461192653903259802591850188394016866170707655609076236167687893936558400))/(491270574577547337465577862499678580919468289682240641599400002541818630173299555553387*x^18 + 10277776658535231887928963830517649364075160462302952752368573529738604577075128345830496*x^15 + 38053074604041164955598613382506657588718033800015428848687354515819408113275280820067228*x^12 + 104552977375786496056156515228686393360634250389206134816652347595105200990156089430013680*x^9 + 193789778786217108923832562100674176189881131732280694502358058831075231461508817660387440*x^6 + 17625077361521411327021636476854594053154373128257733898997391613440934994558725195570 1568*x^3 + 58729517358150193708087322484283950706773934182867349449322904141070201590185330889048000)) + 1/12*2^(1/3)*log((6048*x^16 + 6048*x^13 - 9072*x^10 - 12204*x^7 - 2808*x^4 + 2^(2/3)*(352*x^18 - 5136*x^15 - 10632*x^12 - 3224*x^9 + 3390*x^6 + 1434*x^3 - 35) + 3*(2032*x^14 + 752*x^11 - 3000*x^8 - 1576*x^5 + 172*x^2 + 2^(2/3)*(112*x^16 - 1760*x^13 - 2228*x^10 + 356*x^7 + 707*x^4 - 22*x) - 2*2^(1/3)*(352*x^15 - 728*x^12 - 1736*x^9 - 451*x^6 + 215*x^3 - 1))*(x^3 + 1)^(2/3) - 18*2^(1/3)*(112*x^17 - 192*x^14 - 820*x^11 - 586*x^8 - 21*x^5 + 49*x^2) + 3*(2096*x^15 + 1664*x^12 - 2680*x^9 - 2492*x^6 - 224*x^3 + 2^(2/3)*(112*x^17 - 176

```
0*x^14 - 2996*x^11 - 472*x^8 + 779*x^5 + 125*x^2) - 2*2^(1/3)*(336*x^16 - 6
64*x^13 - 2132*x^10 - 1107*x^7 + 55*x^4 + 29*x) + 16)*(x^3 + 1)^(1/3) + 324
*x)/(64*x^18 + 192*x^15 + 240*x^12 + 160*x^9 + 60*x^6 + 12*x^3 + 1))
```

Sympy [F]

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)(1+x^3)^{2/3}}\,dx = \int \frac{1}{\left((x+1)\left(x^2-x+1\right)\right)^{\frac{2}{3}}\cdot\left(\sqrt[3]{2}x+1\right)}\,dx$$

```
[In] integrate(1/(1+2**(1/3)*x)/(x**3+1)**(2/3),x)
[Out] Integral(1/(((x + 1)*(x**2 - x + 1))**(2/3)*(2**(1/3)*x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)(1+x^3)^{2/3}}\,dx = \int \frac{1}{(x^3+1)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x+1\right)}\,dx$$

```
[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

Giac [F]

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)(1+x^3)^{2/3}}\,dx = \int \frac{1}{(x^3+1)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x+1\right)}\,dx$$

```
[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)(1+x^3)^{2/3}}\,dx = \int \frac{1}{(x^3+1)^{2/3}\,(2^{1/3}\,x+1)}\,dx$$

```
[In] int(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)),x)
[Out] int(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

3.22 $\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\,2^{2/3}-2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1-\sqrt[3]{2}x\right)}{2^{2/3}}+\frac{\log\left(-x-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}-\frac{3\log\left(-2+\sqrt[3]{2}x+\sqrt[3]{2}\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}$$

```
[Out] 1/2*ln(1-2^(1/3)*x)*2^(1/3)+1/4*ln(-x-(-x^3+1)^(1/3))*2^(1/3)-3/4*ln(-2+2^(
1/3)*x+2^(1/3)*(-x^3+1)^(1/3))*2^(1/3)+1/6*arctan(1/3*(1-2*x/(-x^3+1)^(1/3)
)*3^(1/2))*2^(1/3)*3^(1/2)-1/2*arctan(1/3*(1+(2*2^(2/3)-2*x)/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)*2^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2179}

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\frac{2\,2^{2/3}-2x}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(-\sqrt[3]{1-x^3}-x\right)}{2\,2^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x^3}+\sqrt[3]{2}x-2\right)}{2\,2^{2/3}}+\frac{\log\left(1-\sqrt[3]{2}x\right)}{2^{2/3}}$$

```
[In] Int[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*2^(2/3) - 2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3)
) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 -
 2^(1/3)*x]/2^(2/3) + Log[-x - (1 - x^3)^(1/3)]/(2*2^(2/3)) - (3*Log[-2 + 2
^(1/3)*x + 2^(1/3)*(1 - x^3)^(1/3)])/(2*2^(2/3))
```

Rule 2179

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(2/3)), x_Symbol] :> With[
{q = Rt[b, 3]}, Simp[(-d)*(ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/
(2*Sqrt[3]*q^2*c^2)), x] + (Simp[Sqrt[3]*d*(ArcTan[(1 + 2*q*((2*c + d*x)/(d
*(a + b*x^3)^(1/3))))/Sqrt[3]]/(2*q^2*c^2)), x] - Simp[d*(Log[c + d*x]/(2*q
^2*c^2)), x] - Simp[d*(Log[q*x - (a + b*x^3)^(1/3)]/(4*q^2*c^2)), x] + Simp
[3*d*(Log[q*(2*c + d*x) - d*(a + b*x^3)^(1/3)]/(4*q^2*c^2)), x])] /; FreeQ[
{a, b, c, d}, x] && EqQ[2*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\ 2^{2/3}-2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1-\sqrt[3]{2}x\right)}{2^{2/3}} + \frac{\log\left(-x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{3\log\left(-2+\sqrt[3]{2}x+\sqrt[3]{2}\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

Mathematica [F]

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx = \int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$$

```
[In] Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)),x]
```

```
[Out] Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 48.47 (sec) , antiderivative size = 3247, normalized size of antiderivative = 20.42

method	result	size
trager	Expression too large to display	3247

```
[In] int(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x,method=_RETURNVERBOSE)

[Out] 1/6*ln((1604954020235328*2^(1/3)*x^4+14712078518823840*x^3-3842311729647552
*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(1/3)*x^3+6471910353179
844*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(2/3)*x^3+104986226076
65136*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x^4+2613886855
325640*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x-775925141
4704196*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x-8405056908
60402*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3)*x^2+35316740
97632562*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^2+9362231
78470608*x^6+12498127505504256*2^(1/3)*(-x^3+1)^(1/3)-4279877387294208*x^5*
2^(2/3)-11688730639030284*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^
(1/3)*x+72607968203490*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(
2/3)*x^4-1203809884289286*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^
(2/3)*x^4-1560939140169318*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+
1)^(1/3)*x^5-3218487773589102*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3
+1)^(1/3)*x^5-3775614346581480*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-
x^3+1)^(2/3)*x^2-2884227944870616*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^
(2/3)*x^2-8123294120973864*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)*x
^3+2613886855325640*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(2/3)*x^3+11
138422684341672*2^(1/3)*(-x^3+1)^(2/3)*x^2+3919074648194292*2^(2/3)*(-x^3+1
)^(2/3)*x^3+6964190009986188*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/
3)*x^4+1315592369000448*2^(2/3)*(-x^3+1)^(1/3)*x^4+3288980922501120*2^(1/3)
*(-x^3+1)^(1/3)*x^3+7115580883942020*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3
)*(-x^3+1)^(2/3)+2161300347926748*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1
)^(1/3)*x-20062783627256832*2^(2/3)*(-x^3+1)^(1/3)*x-23004340956706368*2^(1
/3)*x+1604954020235328*2^(2/3)*x^2-3712807561447224*(-x^3+1)^(2/3)*x^4-2960
082830251008*(-x^3+1)^(1/3)*x^5-32590199706036744*(-x^3+1)^(2/3)*x+12827025
597754368*(-x^3+1)^(1/3)*x^2+10107087250606332*2^(2/3)*(-x^3+1)^(2/3)+91254
90357912936*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)+5504178119120758
*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)-7959206999356368*RootOf(2^(2/3)+2^
(1/3)*_Z+_Z^2)*x^4-10391133689698608*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x-6434
683875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2+6150800763487380*RootOf(
2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5-1876782797064098*2^(1/3)*RootOf(2^(2/3)+2^(1
/3)*_Z+_Z^2)^2*x^6-2321435374812274*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)
*x^6+10197714008127436*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3-32173419
37824168*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^4+2081809489180344*Roo
tOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x-7959206999356368*RootOf(2^(2/3)+2^
(1/3)*_Z+_Z^2)*2^(1/3)*x^2+5665414413224496*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)
*2^(1/3)*x^5+3532767618003008*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^3
+15559137585059152)/(2^(1/3)*x-1)^6)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)+1/6*ln
((1604954020235328*2^(1/3)*x^4+14712078518823840*x^3-3842311729647552*RootO
f(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(1/3)*x^3+6471910353179844*Ro
otOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(2/3)*x^3+10498622607665136*
RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x^4+2613886855325640
```

```
*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x-775925141470419
6*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x-840505690860402*
RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3)*x^2+35316740976325
62*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^2+9362231784706
08*x^6+12498127505504256*2^(1/3)*(-x^3+1)^(1/3)-4279877387294208*x^5*2^(2/3
)-11688730639030284*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*
x+72607968203490*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x
^4-1203809884289286*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*
x^4-1560939140169318*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/
3)*x^5-3218487773589102*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1
/3)*x^5-3775614346581480*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)
^(2/3)*x^2-2884227944870616*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(2/3)*
x^2-8123294120973864*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)*x^3+261
3886855325640*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(2/3)*x^3+11138422
684341672*2^(1/3)*(-x^3+1)^(2/3)*x^2+3919074648194292*2^(2/3)*(-x^3+1)^(2/3
)*x^3+6964190009986188*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x^4
+1315592369000448*2^(2/3)*(-x^3+1)^(1/3)*x^4+3288980922501120*2^(1/3)*(-x^3
+1)^(1/3)*x^3+7115580883942020*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^
3+1)^(2/3)+2161300347926748*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3
)*x-20062783627256832*2^(2/3)*(-x^3+1)^(1/3)*x-23004340956706368*2^(1/3)*x+
1604954020235328*2^(2/3)*x^2-3712807561447224*(-x^3+1)^(2/3)*x^4-2960082830
251008*(-x^3+1)^(1/3)*x^5-32590199706036744*(-x^3+1)^(2/3)*x+12827025597754
368*(-x^3+1)^(1/3)*x^2+10107087250606332*2^(2/3)*(-x^3+1)^(2/3)+91254903579
12936*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)+5504178119120758*2^(2/
3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)-7959206999356368*RootOf(2^(2/3)+2^(1/3)*
_Z+_Z^2)*x^4-10391133689698608*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x-6434683875
648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2+6150800763487380*RootOf(2^(2/3
)+2^(1/3)*_Z+_Z^2)^2*x^5-1876782797064098*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)^2*x^6-2321435374812274*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^6+1
0197714008127436*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3-32173419378241
68*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^4+2081809489180344*RootOf(2^
(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x-7959206999356368*RootOf(2^(2/3)+2^(1/3)*
_Z+_Z^2)*2^(1/3)*x^2+5665414413224496*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/
3)*x^5+3532767618003008*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^3+15559
137585059152)/(2^(1/3)*x-1)^6)*2^(1/3)-1/6*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*
ln((3129477143943360*2^(1/3)*x^4+1382185738574984*x^3-3842311729647552*Root
Of(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(1/3)*x^3-1244136642528564*R
ootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(2/3)*x^3+3429757412307240*
RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x^4+2613886855325640
*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x+129870251253554
76*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x-840505690860402
*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3)*x^2-5212685479353
366*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^2+182552833396
6960*x^6+3372637147591320*2^(1/3)*(-x^3+1)^(1/3)-3794491037031324*x^5*2^(2/
3)+16011331334883780*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)
```

```
*x+72607968203490*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*
x^4+1349025820696266*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)
*x^4-1560939140169318*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1
/3)*x^5+96609493250466*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/
3)*x^5-3775614346581480*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^
(2/3)*x^2-1221822944145 5304*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(2/3)*
x^2-7245952797616344*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)*x^3+261
3886855325640*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(2/3)*x^3+64714219
36049328*2^(1/3)*(-x^3+1)^(2/3)*x^2+61051150340088*2^(2/3)*(-x^3+1)^(2/3)*x
^3+6964190009986188*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x^4-22
18840228678500*2^(2/3)*(-x^3+1)^(1/3)*x^4+3727651584179880*2^(1/3)*(-x^3+1)
^(1/3)*x^3-7115580883942020*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1
)^(2/3)+2161300347926748*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x
-6212752640299800*2^(2/3)*(-x^3+1)^(1/3)*x-8449588288647072*2^(1/3)*x+31294
77143943360*2^(2/3)*x^2-1159971856461672*(-x^3+1)^(2/3)*x^4+355014436588560
*(-x^3+1)^(1/3)*x^5-1184392316597 7072*(-x^3+1)^(2/3)*x+4082666020768440*(-x
^3+1)^(1/3)*x^2+2991506366664312*2^(2/3)*(-x^3+1)^(2/3)-9125490357912936*Ro
otOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)-5504178119120758*2^(2/3)*RootO
f(2^(2/3)+2^(1/3)*_Z+_Z^2)-4910160751940304*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)
*x^4+18718371646419984*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x-6434683875648336*R
ootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2+6150800763487380*RootOf(2^(2/3)+2^(1/3
)*_Z+_Z^2)^2*x^5-1876782797064098*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2
*x^6-1432130219315922*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^6-313217877
2121420*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3-3217341937824168*2^(2/3
)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^4+2081809489180344*RootOf(2^(2/3)+2^(
1/3)*_Z+_Z^2)^2*2^(2/3)*x-4910160751940304*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*
2^(1/3)*x^2+6636187113750264*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+35
32767618003008*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^3+45507813468176
36)/(2^(1/3)*x-1)^6)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(124) = 248$.

Time = 3.52 (sec) , antiderivative size = 720, normalized size of antiderivative = 4.53

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/3*(13910019318573948542*sqrt(3)*(4429710931093
01727414338294053996366544517259164034007595963454201 83*x^16 - 469911753877
577297266687493361266274298219751726156511748796788210304*x^13 - 1686032190
364332604406470213253462956452423252463754605475829604 09424*x^10 + 19788063
```

0118237657393829295422779262737333028339787658261155833289344O*x^7 - 144009
089168717758142291876308930196860258103687221308438991237030187 2*x^4 + 2^(2
/3)*(52271077453125107612995923977654758349394876922885552819209999866413*x
^15 - 590674547854548577293285820788340778493299281255213360593997994805172
*x^12 + 306314261222931431619887382966630423064822217690279625339197857781 7
900*x^9 - 733104955869757780900835257159703940345796885706673027778611495 93
27080*x^6 + 77232448067562904437597705467808729717394447501735196355441861 1
4816064*x^3 - 29116808987839009219563485741835514155891904460151064526080 70
501424800) + 6*2^(1/3)*(12601355996216322093314748679149120543302140685677 0
58235520929344665*x^14 + 55586906300196651392462719491921267847820798890019
850227115938089718*x^11 - 45039892010532059930763953602788398613179362472 93
034074362336107885 04*x^8 + 72188870588094826143251705267039410623833894384 4
373553906510879866584*x^5 - 33866815806868437343630927306784946440569136075
137850744247292177454 4*x^2) + 62367643045453979229021701235594440425380660 1
409762924332407805196 80*x)*(-x^3 + 1)^(2/3) + 13910019318573948542*sqrt(3)*
(2024415138676272858287317644091664227603691384672196434257031987427 2*x^17
- 741146137078834990968958694956953525786968216162791369141561079231342*x^1
4 + 217984319727177540114743839610166687553704366334519910306529071835066 0*
x^11 - 21110249350284448030276350331723739969986388702750815288350190294268
08*x^8 + 69058397930221264954184667175232357867176236156498719853237207761 7
072*x^5 - 425604467193959940435036909294930892503769478498985960943870691 96
992*x^2 - 2^(2/3)*(58175953016441250552894129028785848895343146706912452780
410096144857*x^16 - 60332912344022592845951244288084636749808634046721050 84
10170807919392*x^13 + 99321772442116051464080292497021614887213800679935641
7482692017634440*x^10 + 31537366861697860036872967982882082606714520389786 0
799345951918357208*x^7 - 15359897811758984549040097640804776981234391400095
232578337952941710 24*x^4 + 77458165399450652218506506051545799956246967083 8
035710700279100960480*x) - 2*2^(1/3)*(44250337395862623641308432146105265 59
15849816922169442468726224375 86*x^15 - 93730331994553087914588193029404165 0
15738145719370012253256237142833*x^12 + 13218541316595455203956380934358348
619932882852548406311430877544538 16*x^9 - 42477057677017468895892138257252 7
81622024317737600109081215316558582 40*x^6 + 45932454636886436349937358513 41
621838359838170188285500151733185855040*x^3 - 161588373761478929714291077 07
869228809509709698905305411015386387388 00))*(-x^3 + 1)^(1/3) + sqrt(3)*(580
84585662481413805853665892503575242234102374504265714411001843413397117139 2
378653765*x^18 - 8512850211201658596320322423507979436745037061604662252288
106173984889011398391939493844*x^15 + 46037674634299399876464933353333393 65
179871449886195969768495285918127951444917234880113 2*x^12 - 100016348353366
812357999723948540966952435611836580420294833827058766585456463611215562912
*x^9 + 913977586253668076790534210688867294404951076896021210254557365342 55
64237012293570062811 2*x^6 - 276792064712221478189323489147072714065541212 16
14173478586396645113933854556904639684294 4*x^3 + 13910019318573948542*2^(2/
3)*(3844366680114123938578119587438413410802428820066154040455085354797*x^1
7 + 4931319711549190780631731959832802785947037704060043883265521247935 91*x
^14 - 2263656329733750526575239788393341804272268328404078377386979655411 62

```
8*x^11 + 360329608895964304006588260615697733294277836897086795884126627540
5688*x^8 - 237514392414546247479078929764308258102335245758364443369831809 0
272160*x^5 + 538527827084536759298395164308728360347336217790784309877024 26
0129712*x^2) + 1669202318228873825 04*2^(1/3)*(135958920440428283662759820 06
708049395032909698880004129949511339226*x^16 - 135133384885158250377179048 5
959913464507711993272362079564211134619 03*x^13 + 402245899028058436823068 10
952188584025877561061471182634365786887 9359*x^10 - 547258710149879334691832
999834525308297790387563356879645468036532966*x^7 + 363674199703640963884 96
001212438726310625490952164066315430230211 6404*x^4 - 971238957407046440052 9
205522246449801150184294463940602602053234 0120*x) - 1800774080838794461192 6
539032598025918501883940168661707076556090762361676878939365584 00))/(491270
57457754733746557786249967858091946828968224064159940000254181863017329955 5
553387*x^18 - 1027777665853523188792896383051764936407516046230295275236857
35297386045770751283458304 96*x^15 + 3805307460404116495559861338250665758 87
18033800015428848687354515819408113275280820067228*x^12 - 10455297737578649
6056156515228686393360634250389206134816652347595105200990156089430013680*x
^9 + 193789778786217108923832562100674176189881131732280694502358058831075 2
314615088176603874 40*x^6 - 176250773615214113270216364768545940531543731282
5773389899739161344093499455872519557015 68*x^3 + 58729517358150193708087322
484283950706773934182867349449322904141070201590185330889048000)) - 1/12*2^
(1/3)*log((6048*x^16 - 6048*x^13 - 9072*x^10 + 12204*x^7 - 2808*x^4 + 2^(2/
3)*(352*x^18 + 5136*x^15 - 10632*x^12 + 3224*x^9 + 3390*x^6 - 1434*x^3 - 35
) + 3*(2032*x^14 - 752*x^11 - 3000*x^8 + 1576*x^5 + 172*x^2 + 2^(2/3)*(112*
x^16 + 1760*x^13 - 2228*x^10 - 356*x^7 + 707*x^4 + 22*x) + 2*2^(1/3)*(352*x
^15 + 728*x^12 - 1736*x^9 + 451*x^6 + 215*x^3 + 1))*(-x^3 + 1)^(2/3) + 18*2
^(1/3)*(112*x^17 + 192*x^14 - 820*x^11 + 586*x^8 - 21*x^5 - 49*x^2) - 3*(20
96*x^15 - 1664*x^12 - 2680*x^9 + 2492*x^6 - 224*x^3 + 2^(2/3)*(112*x^17 + 1
760*x^14 - 2996*x^11 + 472*x^8 + 779*x^5 - 125*x^2) + 2*2^(1/3)*(336*x^16 +
 664*x^13 - 2132*x^10 + 1107*x^7 + 55*x^4 - 29*x) - 16)*(-x^3 + 1)^(1/3) -
324*x)/(64*x^18 - 192*x^15 + 240*x^12 - 160*x^9 + 60*x^6 - 12*x^3 + 1))
```

Sympy [F]

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx=-\int \frac{1}{\sqrt[3]{2}x\left(1-x^3\right)^{\frac{2}{3}}-\left(1-x^3\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(1-2**(1/3)*x)/(-x**3+1)**(2/3),x)
[Out] -Integral(1/(2**(1/3)*x*(1 - x**3)**(2/3) - (1 - x**3)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx = \int -\frac{1}{(-x^3+1)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)}\,dx$$

```
[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="maxima")
[Out] -integrate(1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)
```

Giac [F]

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx = \int -\frac{1}{(-x^3+1)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)}\,dx$$

```
[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="giac")
[Out] integrate(-1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx = -\int \frac{1}{(1-x^3)^{2/3}\,(2^{1/3}\,x-1)}\,dx$$

```
[In] int(-1/((1 - x^3)^(2/3)*(2^(1/3)*x - 1)),x)
[Out] -int(1/((1 - x^3)^(2/3)*(2^(1/3)*x - 1)), x)
```

3.23 $\int (c+dx)^4 \sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 387

$$\begin{aligned}\int (c+dx)^4 \sqrt[3]{a+bx^3}\,dx &= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}\\ &+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)\\ &-\frac{4ac^3d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{a^2d^4\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}}\\ &+\frac{ac^4x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}\\ &+\frac{acd^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}\\ &-\frac{2ac^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}}+\frac{a^2d^4\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}}\end{aligned}$$

```
[Out] 3/2*a*c^2*d^2*(b*x^3+a)^(1/3)/b+1/18*a*d^4*x^2*(b*x^3+a)^(1/3)/b+1/30*(b*x^
3+a)^(1/3)*(5*d^4*x^5+24*c*d^3*x^4+45*c^2*d^2*x^3+40*c^3*d*x^2+15*c^4*x)+1/
2*a*c^4*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^
(2/3)+1/5*a*c*d^3*x^4*(1+b*x^3/a)^(2/3)*hypergeom([2/3, 4/3],[7/3],-b*x^3/a
)/(b*x^3+a)^(2/3)-2/3*a*c^3*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)+1/18*a^
2*d^4*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)-4/9*a*c^3*d*arctan(1/3*(1+2*b^(
1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)+1/27*a^2*d^4*arctan(1/3*(1
+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1867, 1907, 252, 251, 337, 267, 372, 371, 327}

$$\int (c+dx)^4\sqrt[3]{a+bx^3}\,dx = \frac{a^2d^4\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}}+\frac{a^2d^4\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}} - \frac{4ac^3d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}-\frac{2ac^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}} + \frac{ac^4x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2(a+bx^3)^{2/3}}+\frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b} + \frac{1}{30}\sqrt[3]{a+bx^3}(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5)+\frac{acd^3x^4\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\right.}{5(a+bx^3)^{2/3}}$$

```
[In] Int[(c + d*x)^4*(a + b*x^3)^(1/3),x]
```

```
[Out] (3*a*c^2*d^2*(a + b*x^3)^(1/3))/(2*b) + (a*d^4*x^2*(a + b*x^3)^(1/3))/(18*b
) + ((a + b*x^3)^(1/3)*(15*c^4*x + 40*c^3*d*x^2 + 45*c^2*d^2*x^3 + 24*c*d^3
*x^4 + 5*d^4*x^5))/30 - (4*a*c^3*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1
/3))/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)) + (a^2*d^4*ArcTan[(1 + (2*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(5/3)) + (a*c^4*x*(1 + (b*x^3)/a)^(2
/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) +
 (a*c*d^3*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x
^3)/a)])/(5*(a + b*x^3)^(2/3)) - (2*a*c^3*d*Log[b^(1/3)*x - (a + b*x^3)^(1/
3)])/(3*b^(2/3)) + (a^2*d^4*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(18*b^(5/3)
)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right) \\
&\quad + a\int \frac{\frac{c^4}{2}+\frac{4}{3}c^3dx+\frac{3}{2}c^2d^2x^2+\frac{4}{5}cd^3x^3+\frac{d^4x^4}{6}}{\left(a+bx^3\right)^{2/3}}\,dx \\
&= \frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)+a\int\left(\frac{c^4}{2\left(a+bx^3\right)^{2/3}}\right. \\
&\quad \left. +\frac{4c^3dx}{3\left(a+bx^3\right)^{2/3}}+\frac{3c^2d^2x^2}{2\left(a+bx^3\right)^{2/3}}+\frac{4cd^3x^3}{5\left(a+bx^3\right)^{2/3}}+\frac{d^4x^4}{6\left(a+bx^3\right)^{2/3}}\right)dx \\
&= \frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right) \\
&\quad +\frac{1}{2}\left(ac^4\right)\int\frac{1}{\left(a+bx^3\right)^{2/3}}\,dx \\
&\quad +\frac{1}{3}\left(4ac^3d\right)\int\frac{x}{\left(a+bx^3\right)^{2/3}}\,dx+\frac{1}{2}\left(3ac^2d^2\right)\int\frac{x^2}{\left(a+bx^3\right)^{2/3}}\,dx \\
&\quad +\frac{1}{5}\left(4acd^3\right)\int\frac{x^3}{\left(a+bx^3\right)^{2/3}}\,dx+\frac{1}{6}\left(ad^4\right)\int\frac{x^4}{\left(a+bx^3\right)^{2/3}}\,dx \\
&= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b} \\
&\quad +\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right) \\
&\quad -\frac{4ac^3d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}-\frac{2ac^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}} \\
&\quad -\frac{\left(a^2d^4\right)\int\frac{x}{\left(a+bx^3\right)^{2/3}}\,dx}{9b}+\frac{\left(ac^4\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int\frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{2\left(a+bx^3\right)^{2/3}} \\
&\quad +\frac{\left(4acd^3\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int\frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{5\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b} + \frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b} \\
&\quad + \frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x + 40c^3dx^2 + 45c^2d^2x^3 + 24cd^3x^4 + 5d^4x^5\right) \\
&\quad - \frac{4ac^3d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}} + \frac{a^2d^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}} \\
&\quad + \frac{ac^4x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}} + \frac{acd^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}} \\
&\quad - \frac{2ac^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}} + \frac{a^2d^4\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.64 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.42

$$
\int (c+dx)^4\sqrt[3]{a+bx^3}\,dx
$$

$$
= \frac{\sqrt[3]{a+bx^3}\left(6bc^4x\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d(12bc^3-ad^3)\,x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3}\ldots\right.\right.}{6b\sqrt[3]{1+\ldots}}
$$

```
[In] Integrate[(c + d*x)^4*(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(6*b*c^4*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a
)] + d*(12*b*c^3 - a*d^3)*x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a
)] + d^2*((9*c^2 + d^2*x^2)*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + 6*b*c*d*x^4
*Hypergeometric2F1[-1/3, 4/3, 7/3, -((b*x^3)/a)])))/(6*b*(1 + (b*x^3)/a)^(1
/3))
```

Maple [F]

$$
\int (dx+c)^4\left(b\,x^3+a\right)^{\frac{1}{3}}dx
$$

```
[In] int((d*x+c)^4*(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^4*(b*x^3+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int (c+dx)^4\sqrt[3]{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.55

$$\int (c+dx)^4\sqrt[3]{a+bx^3}\,dx = \frac{\sqrt[3]{a}c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{4\sqrt[3]{a}c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{4\sqrt[3]{a}cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}d^4x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + 6c^2d^2\left(\begin{cases}\frac{\sqrt[3]{a}x^3}{3} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{4}{3}}}{4b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((d*x+c)**4*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**4*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 4*a**(1/3)*c**3*d*x**2*gamma(2/3)*hyper((-1/3, 2/3),
(5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 4*a**(1/3)*c*d**3*x**4*g
amma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3
)) + a**(1/3)*d**4*x**5*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*gamma(8/3)) + 6*c**2*d**2*Piecewise((a**(1/3)*x**3/3, Eq(b,
 0)), ((a + b*x**3)**(4/3)/(4*b), True))
```

Maxima [F]

$$\int (c+dx)^4 \sqrt[3]{a+bx^3}\, dx = \int (bx^3+a)^{\frac{1}{3}}(dx+c)^4\, dx$$

```
[In] integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^4, x)
```

Giac [F]

$$\int (c+dx)^4 \sqrt[3]{a+bx^3}\, dx = \int (bx^3+a)^{\frac{1}{3}}(dx+c)^4\, dx$$

```
[In] integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \sqrt[3]{a+bx^3}\, dx = \int (b\,x^3+a)^{1/3}\,(c+d\,x)^4\, dx$$

```
[In] int((a + b*x^3)^(1/3)*(c + d*x)^4,x)
[Out] int((a + b*x^3)^(1/3)*(c + d*x)^4, x)
```

3.24 $\int (c+dx)^3 \sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 242

$$\begin{aligned}\int (c+dx)^3 \sqrt[3]{a+bx^3}\,dx = {} & \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b} \\ & +\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right) \\ & -\frac{ac^2d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} \\ & +\frac{a(5bc^3-ad^3)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{10b\,(a+bx^3)^{2/3}} \\ & -\frac{ac^2d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\end{aligned}$$

```
[Out] 3/4*a*c*d^2*(b*x^3+a)^(1/3)/b+1/10*a*d^3*x*(b*x^3+a)^(1/3)/b+1/20*(b*x^3+a)
^(1/3)*(4*d^3*x^4+15*c*d^2*x^3+20*c^2*d*x^2+10*c^3*x)+1/10*a*(-a*d^3+5*b*c^
3)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/
3)-1/2*a*c^2*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-1/3*a*c^2*d*arctan(1/3
*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1867, 1902, 1900, 267, 1907, 252, 251, 337}

$$\int (c+dx)^3\sqrt[3]{a+bx^3}\,dx$$

$$= -\frac{ac^2d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{ac^2d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$
$$+\frac{ax\left(\frac{bx^3}{a}+1\right)^{2/3}(5bc^3-ad^3)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{10b\,(a+bx^3)^{2/3}}$$
$$+\frac{1}{20}\sqrt[3]{a+bx^3}(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4)+\frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}$$

```
[In] Int[(c + d*x)^3*(a + b*x^3)^(1/3),x]
```

```
[Out] (3*a*c*d^2*(a + b*x^3)^(1/3))/(4*b) + (a*d^3*x*(a + b*x^3)^(1/3))/(10*b) +
((a + b*x^3)^(1/3)*(10*c^3*x + 20*c^2*d*x^2 + 15*c*d^2*x^3 + 4*d^3*x^4))/20
 - (a*c^2*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]
*b^(2/3)) + (a*(5*b*c^3 - a*d^3)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[
1/3, 2/3, 4/3, -((b*x^3)/a)])/(10*b*(a + b*x^3)^(2/3)) - (a*c^2*d*Log[b^(1/
3)*x - (a + b*x^3)^(1/3)])/(2*b^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)\\ &\quad+a\int\frac{\frac{c^3}{2}+c^2dx+\frac{3}{4}cd^2x^2+\frac{d^3x^3}{5}}{\left(a+bx^3\right)^{2/3}}\,dx\end{aligned}$$

$$= \frac{ad^3x\sqrt[3]{a+bx^3}}{10b} + \frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right) + \frac{a\int \frac{\frac{1}{5}(5bc^3-ad^3)+2bc^2dx+\frac{3}{2}bcd^2x^2}{(a+bx^3)^{2/3}}\,dx}{2b}$$

$$= \frac{ad^3x\sqrt[3]{a+bx^3}}{10b} + \frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right) + \frac{a\int \frac{\frac{1}{5}(5bc^3-ad^3)+2bc^2dx}{(a+bx^3)^{2/3}}\,dx}{2b} + \frac{1}{4}\left(3acd^2\right)\int \frac{x^2}{(a+bx^3)^{2/3}}\,dx$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b} + \frac{ad^3x\sqrt[3]{a+bx^3}}{10b} + \frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right) + \frac{a\int \left(\frac{5bc^3-ad^3}{5(a+bx^3)^{2/3}} + \frac{2bc^2dx}{(a+bx^3)^{2/3}}\right)dx}{2b}$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b} + \frac{ad^3x\sqrt[3]{a+bx^3}}{10b} + \frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right) + \left(ac^2d\right)\int \frac{x}{(a+bx^3)^{2/3}}\,dx + \frac{\left(a(5bc^3-ad^3)\right)\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{10b}$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b} + \frac{ad^3x\sqrt[3]{a+bx^3}}{10b} + \frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right) - \frac{ac^2d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} - \frac{ac^2d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}} + \frac{\left(a(5bc^3-ad^3)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{10b\,(a+bx^3)^{2/3}}$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b} + \frac{ad^3x\sqrt[3]{a+bx^3}}{10b} + \frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right) - \frac{ac^2d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{a(5bc^3-ad^3)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{10b\,(a+bx^3)^{2/3}} - \frac{ac^2d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

Mathematica [A] (verified)

Time = 6.49 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.59

$$\int (c+dx)^3\sqrt[3]{a+bx^3}\,dx$$
$$=\frac{\sqrt[3]{a+bx^3}\left(4bc^3x\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d\left(6bc^2x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right.\right.}{4b\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(c + d*x)^3*(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(4*b*c^3*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a
)] + d*(6*b*c^2*x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a)] + d*(3*
c*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + b*d*x^4*Hypergeometric2F1[-1/3, 4/3,
7/3, -((b*x^3)/a)]))))/(4*b*(1 + (b*x^3)/a)^(1/3))
```

Maple [F]

$$\int \left(dx+c\right)^{3} \left(b\,x^{3}+a\right)^{\frac{1}{3}}dx$$

```
[In] int((d*x+c)^3*(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^3*(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int (c+dx)^3\sqrt[3]{a+bx^3}\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^3\,dx$$

```
[In] integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(b*x^3 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.66

$$\int (c+dx)^3\sqrt[3]{a+bx^3}\,dx = \frac{\sqrt[3]{a}c^3x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt[3]{a}c^2dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt[3]{a}d^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + 3cd^2\left(\begin{cases}\frac{\sqrt[3]{a}x^3}{3} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{4}{3}}}{4b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((d*x+c)**3*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**3*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + a**(1/3)*c**2*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5
/3,), b*x**3*exp_polar(I*pi)/a)/gamma(5/3) + a**(1/3)*d**3*x**4*gamma(4/3)*
hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 3*c*d
**2*Piecewise((a**(1/3)*x**3/3, Eq(b, 0)), ((a + b*x**3)**(4/3)/(4*b), True
))
```

Maxima [F]

$$\int (c+dx)^3\sqrt[3]{a+bx^3}\,dx = \int (bx^3+a)^{\frac{1}{3}}(dx+c)^3\,dx$$

```
[In] integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^3, x)
```

Giac [F]

$$\int (c+dx)^3\sqrt[3]{a+bx^3}\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^3\,dx$$

```
[In] integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\sqrt[3]{a+bx^3}\,dx = \int \left(b\,x^3+a\right)^{1/3}(c+d\,x)^3\,dx$$

```
[In] int((a + b*x^3)^(1/3)*(c + d*x)^3,x)
[Out] int((a + b*x^3)^(1/3)*(c + d*x)^3, x)
```

3.25 $\int (c+dx)^2 \sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 192

$$\int (c+dx)^2 \sqrt[3]{a+bx^3}\,dx = \frac{ad^2\sqrt[3]{a+bx^3}}{4b} + \frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right) - \frac{2acd\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}} + \frac{ac^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}} - \frac{acd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}}$$

```
[Out] 1/4*a*d^2*(b*x^3+a)^(1/3)/b+1/12*(b*x^3+a)^(1/3)*(3*d^2*x^3+8*c*d*x^2+6*c^2
*x)+1/2*a*c^2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x
^3+a)^(2/3)-1/3*a*c*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-2/9*a*c*d*arcta
n(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1867, 1900, 267, 1907, 252, 251, 337}

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx = -\frac{2acd\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}-\frac{acd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{ac^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}+\frac{ad^2\sqrt[3]{a+bx^3}}{4b}$$

```
[In] Int[(c + d*x)^2*(a + b*x^3)^(1/3),x]
```

```
[Out] (a*d^2*(a + b*x^3)^(1/3))/(4*b) + ((a + b*x^3)^(1/3)*(6*c^2*x + 8*c*d*x^2 +
 3*d^2*x^3))/12 - (2*a*c*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqr
t[3]])/(3*Sqrt[3]*b^(2/3)) + (a*c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2
F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) - (a*c*d*Log[b^(1/3)
*x - (a + b*x^3)^(1/3)])/(3*b^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+a\int \frac{\frac{c^2}{2}+\frac{2cdx}{3}+\frac{d^2x^2}{4}}{\left(a+bx^3\right)^{2/3}}\,dx\\
&= \frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+a\int \frac{\frac{c^2}{2}+\frac{2cdx}{3}}{\left(a+bx^3\right)^{2/3}}\,dx+\frac{1}{4}\left(ad^2\right)\int \frac{x^2}{\left(a+bx^3\right)^{2/3}}\,dx\\
&= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)\\
&\quad +a\int \left(\frac{c^2}{2\left(a+bx^3\right)^{2/3}}+\frac{2cdx}{3\left(a+bx^3\right)^{2/3}}\right)dx\\
&= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)\\
&\quad +\frac{1}{2}\left(ac^2\right)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx+\frac{1}{3}(2acd)\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx
\end{aligned}
$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b} + \frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right) - \frac{2acd\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}} - \frac{acd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}} + \frac{\left(ac^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int\frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}dx}{2\left(a+bx^3\right)^{2/3}}$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b} + \frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right) - \frac{2acd\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}} + \frac{ac^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}} - \frac{acd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}}$$

Mathematica [A] (verified)

Time = 6.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.58

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx$$

$$= \frac{\sqrt[3]{a+bx^3}\left(4bc^2x\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d\left(d(a+bx^3)\sqrt[3]{1+\frac{bx^3}{a}}+4bcx^2\,\text{Hypergeometri}\right.\right.}{4b\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(c + d*x)^2*(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(4*b*c^2*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a
)] + d*(d*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + 4*b*c*x^2*Hypergeometric2F1[-
1/3, 2/3, 5/3, -((b*x^3)/a)])))/(4*b*(1 + (b*x^3)/a)^(1/3))
```

Maple [F]

$$\int (dx+c)^2\left(b\,x^3+a\right)^{\frac{1}{3}}dx$$

```
[In] int((d*x+c)^2*(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^2*(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^2\,dx$$

```
[In] integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*(b*x^3 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.59

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx=\frac{\sqrt[3]{a}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2\sqrt[3]{a}cdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+d^2\left(\begin{cases}\frac{\sqrt[3]{a}x^3}{3} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{4}{3}}}{4b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((d*x+c)**2*(b*x**3+a)**(1/3),x)
[Out] a**(1/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(1/3)*c*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + d**2*Piecewise((a**(1/3)*x*
*3/3, Eq(b, 0)), ((a + b*x**3)**(4/3)/(4*b), True))
```

Maxima [F]

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^2\,dx$$

```
[In] integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^2, x)
```

Giac [F]

$$\int (c+dx)^2 \sqrt[3]{a+bx^3}\, dx = \int (bx^3+a)^{\frac{1}{3}}(dx+c)^2\, dx$$

```
[In] integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sqrt[3]{a+bx^3}\, dx = \int \left(b\,x^3+a\right)^{1/3} (c+d\,x)^2\, dx$$

```
[In] int((a + b*x^3)^(1/3)*(c + d*x)^2,x)
[Out] int((a + b*x^3)^(1/3)*(c + d*x)^2, x)
```

3.26 $\int (c+dx)\sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 155

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx=\frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3}-\frac{ad\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}-\frac{ad\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}}$$

```
[Out] 1/6*(2*d*x^2+3*c*x)*(b*x^3+a)^(1/3)+1/2*a*c*x*(1+b*x^3/a)^(2/3)*hypergeom([
1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)-1/6*a*d*ln(b^(1/3)*x-(b*x^3+a)^(1
/3))/b^(2/3)-1/9*a*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^
(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used

$= \{1867, 1907, 252, 251, 337\}$

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = -\frac{ad\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}} - \frac{ad\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}} + \frac{1}{6}\sqrt[3]{a+bx^3}\left(3cx+2dx^2\right) + \frac{acx\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}$$

```
[In] Int[(c + d*x)*(a + b*x^3)^(1/3),x]
```

```
[Out] ((3*c*x + 2*d*x^2)*(a + b*x^3)^(1/3))/6 - (a*d*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)) + (a*c*x*(1 + (b*x^3)/a)^(2/
3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) -
(a*d*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(6*b^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
```

```
&& IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3}+a\int\frac{\frac{c}{2}+\frac{dx}{3}}{\left(a+bx^3\right)^{2/3}}\,dx\\
&= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3}+a\int\left(\frac{c}{2\left(a+bx^3\right)^{2/3}}+\frac{dx}{3\left(a+bx^3\right)^{2/3}}\right)dx\\
&= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3}+\frac{1}{2}(ac)\int\frac{1}{\left(a+bx^3\right)^{2/3}}\,dx+\frac{1}{3}(ad)\int\frac{x}{\left(a+bx^3\right)^{2/3}}\,dx\\
&= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3}-\frac{ad\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}\\
&\quad -\frac{ad\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}}+\frac{\left(ac\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int\frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{2\left(a+bx^3\right)^{2/3}}\\
&= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3}-\frac{ad\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}\\
&\quad +\frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}-\frac{ad\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.63 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.48

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \frac{x\sqrt[3]{a+bx^3}\left(2c\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(c + d*x)*(a + b*x^3)^(1/3),x]
[Out] (x*(a + b*x^3)^(1/3)*(2*c*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)] +
 d*x*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a)]))/(2*(1 + (b*x^3)/a)^(
1/3))
```

Maple [F]

$$\int (dx+c)\left(b\,x^3+a\right)^{\frac{1}{3}}dx$$

```
[In] int((d*x+c)*(b*x^3+a)^(1/3),x)
[Out] int((d*x+c)*(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)\,dx$$

```
[In] integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(1/3)*(d*x + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.53

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \frac{\sqrt[3]{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt[3]{a}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((d*x+c)*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(1/3)*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3))
```

Maxima [F]

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \int (bx^3+a)^{\frac{1}{3}}(dx+c)\,dx$$

```
[In] integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c), x)
```

Giac [F]

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \int (bx^3+a)^{\frac{1}{3}}(dx+c)\,dx$$

```
[In] integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \int \left(b\,x^3+a\right)^{1/3}\,\left(c+d\,x\right)\,dx$$

```
[In] int((a + b*x^3)^(1/3)*(c + d*x),x)
```

```
[Out] int((a + b*x^3)^(1/3)*(c + d*x), x)
```

3.27 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 435

$$
\begin{aligned}
\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx &= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{x\sqrt[3]{a+bx^3}\,\mathrm{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}} \\
&+ \frac{\sqrt[3]{b}c\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} - \frac{\sqrt[3]{bc^3-ad^3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} \\
&+ \frac{\sqrt[3]{bc^3-ad^3}\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} \\
&+ \frac{\sqrt[3]{bc^3-ad^3}\log\left(c^3+d^3x^3\right)}{3d^2} + \frac{\sqrt[3]{b}c\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d^2} \\
&- \frac{\sqrt[3]{bc^3-ad^3}\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{2d^2} \\
&- \frac{\sqrt[3]{bc^3-ad^3}\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2d^2}
\end{aligned}
$$

```
[Out] (b*x^3+a)^(1/3)/d+x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d^3*x
^3/c^3)/c/(1+b*x^3/a)^(1/3)+1/3*(-a*d^3+b*c^3)^(1/3)*ln(d^3*x^3+c^3)/d^2+1/
2*b^(1/3)*c*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d^2-1/2*(-a*d^3+b*c^3)^(1/3)*ln((
-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/d^2-1/2*(-a*d^3+b*c^3)^(1/3)*ln((-
```

```
a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/d^2+1/3*b^(1/3)*c*arctan(1/3*(1+2*b^(
1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)-1/3*(-a*d^3+b*c^3)^(1/3)*arcta
n(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)+1
/3*(-a*d^3+b*c^3)^(1/3)*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1
/3))*3^(1/2))/d^2*3^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2181, 441, 440, 495, 337, 503, 455, 52, 60, 631, 210, 31}

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}} - \frac{\sqrt[3]{bc^3-ad^3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt[3]{bc^3-ad^3}\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt[3]{b}c\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt[3]{bc^3-ad^3}\log\left(c^3+d^3x^3\right)}{3d^2} - \frac{\sqrt[3]{bc^3-ad^3}\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2d^2} - \frac{\sqrt[3]{bc^3-ad^3}\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2d^2} + \frac{\sqrt[3]{b}c\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d^2} + \frac{\sqrt[3]{a+bx^3}}{d}$$

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x),x]
```

```
[Out] (a + b*x^3)^(1/3)/d + (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b
*x^3)/a), -((d^3*x^3)/c^3)])/(c*(1 + (b*x^3)/a)^(1/3)) + (b^(1/3)*c*ArcTan[
(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) - ((b*c^3 - a
```

```
*d^3)^(1/3)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/
Sqrt[3]])/(Sqrt[3]*d^2) + ((b*c^3 - a*d^3)^(1/3)*ArcTan[(1 - (2*d*(a + b*x^
3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) + ((b*c^3 - a*d^3)
^(1/3)*Log[c^3 + d^3*x^3])/(3*d^2) + (b^(1/3)*c*Log[b^(1/3)*x - (a + b*x^3)
^(1/3)])/(2*d^2) - ((b*c^3 - a*d^3)^(1/3)*Log[((b*c^3 - a*d^3)^(1/3)*x)/c -
 (a + b*x^3)^(1/3)])/(2*d^2) - ((b*c^3 - a*d^3)^(1/3)*Log[(b*c^3 - a*d^3)^(
1/3) + d*(a + b*x^3)^(1/3)])/(2*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\text{integral} = \int \left(\frac{c^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3} - \frac{cdx\sqrt[3]{a+bx^3}}{c^3+d^3x^3} + \frac{d^2x^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3}\right)dx$$

$$= c^2\int \frac{\sqrt[3]{a+bx^3}}{c^3+d^3x^3}\,dx - (cd)\int \frac{x\sqrt[3]{a+bx^3}}{c^3+d^3x^3}\,dx + d^2\int \frac{x^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3}\,dx$$

$$= -\frac{(bc)\int \frac{x}{(a+bx^3)^{2/3}}\,dx}{d^2} + \left(c\left(-a+\frac{bc^3}{d^3}\right)d\right)\int \frac{x}{(a+bx^3)^{2/3}(c^3+d^3x^3)}\,dx$$

$$+ \frac{1}{3}d^2\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c^3+d^3x}\,dx, x, x^3\right) + \frac{\left(c^2\sqrt[3]{a+bx^3}\right)\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{c^3+d^3x^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}} + \frac{\sqrt[3]{b}c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}$$

$$- \frac{\sqrt[3]{bc^3-ad^3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt[3]{bc^3-ad^3}\log\left(c^3+d^3x^3\right)}{6d^2}$$

$$+ \frac{\sqrt[3]{b}c\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d^2} - \frac{\sqrt[3]{bc^3-ad^3}\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{2d^2}$$

$$+ \frac{1}{3}\left(\left(a-\frac{bc^3}{d^3}\right)d^2\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c^3+d^3x)}\,dx, x, x^3\right)$$

$$=\frac{\sqrt[3]{a+bx^3}}{d}+\frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}+\frac{\sqrt[3]{b}c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}$$

$$-\frac{\sqrt[3]{bc^3-ad^3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{\sqrt[3]{bc^3-ad^3}\log\left(c^3+d^3x^3\right)}{3d^2}$$

$$+\frac{\sqrt[3]{b}c\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d^2}-\frac{\sqrt[3]{bc^3-ad^3}\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{2d^2}$$

$$-\frac{\sqrt[3]{bc^3-ad^3}\text{Subst}\left(\int\frac{1}{\frac{\sqrt[3]{bc^3-ad^3}}{d}+x}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2d^2}$$

$$-\frac{\left(bc^3-ad^3\right)^{2/3}\text{Subst}\left(\int\frac{1}{\frac{\left(bc^3-ad^3\right)^{2/3}}{d^2}-\frac{\sqrt[3]{bc^3-ad^3}x}{d}+x^2}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2d^3}$$

$$=\frac{\sqrt[3]{a+bx^3}}{d}+\frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}+\frac{\sqrt[3]{b}c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}$$

$$-\frac{\sqrt[3]{bc^3-ad^3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{\sqrt[3]{bc^3-ad^3}\log\left(c^3+d^3x^3\right)}{3d^2}$$

$$+\frac{\sqrt[3]{b}c\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d^2}-\frac{\sqrt[3]{bc^3-ad^3}\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{2d^2}$$

$$-\frac{\sqrt[3]{bc^3-ad^3}\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2d^2}$$

$$-\frac{\sqrt[3]{bc^3-ad^3}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}\right)}{d^2}$$

$$
= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}
+ \frac{\sqrt[3]{b}c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} - \frac{\sqrt[3]{bc^3-ad^3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}
+ \frac{\sqrt[3]{bc^3-ad^3}\tan^{-1}\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt[3]{bc^3-ad^3}\log\left(c^3+d^3x^3\right)}{3d^2}
+ \frac{\sqrt[3]{b}c\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d^2} - \frac{\sqrt[3]{bc^3-ad^3}\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{2d^2}
- \frac{\sqrt[3]{bc^3-ad^3}\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2d^2}
$$

Mathematica [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x),x]
[Out] Integrate[(a + b*x^3)^(1/3)/(c + d*x), x]
```

Maple [F]

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{dx+c}\,dx$$

```
[In] int((b*x^3+a)^(1/3)/(d*x+c),x)
[Out] int((b*x^3+a)^(1/3)/(d*x+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

```
[In] integrate((b*x**3+a)**(1/3)/(d*x+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx+c}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx+c}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{c+d\,x} \,d x$$

```
[In] int((a + b*x^3)^(1/3)/(c + d*x),x)
[Out] int((a + b*x^3)^(1/3)/(c + d*x), x)
```

3.28 $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 818

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = -\frac{c^2\sqrt[3]{a+bx^3}}{d\left(c^3+d^3x^3\right)}-\frac{dx^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3}+\frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}-\frac{d^3x^4\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5\sqrt[3]{1+\frac{bx^3}{a}}}-\frac{\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{2ad\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{2/3}}+\frac{\left(3bc^3-2ad^3\right)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}cd^2\left(bc^3-ad^3\right)^{2/3}}-\frac{bc^2\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2\left(bc^3-ad^3\right)^{2/3}}-\frac{bc^2\log\left(c^3+d^3x^3\right)}{6d^2\left(bc^3-ad^3\right)^{2/3}}-\frac{ad\log\left(c^3+d^3x^3\right)}{9c\left(bc^3-ad^3\right)^{2/3}}-\frac{\left(3bc^3-2ad^3\right)\log\left(c^3+d^3x^3\right)}{18cd^2\left(bc^3-ad^3\right)^{2/3}}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d^2}+\frac{ad\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\left(bc^3-ad^3\right)^{2/3}}+\frac{\left(3bc^3-2ad^3\right)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{6cd^2\left(bc^3-ad^3\right)^{2/3}}+\frac{bc^2\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2d^2\left(bc^3-ad^3\right)^{2/3}}$$

```
[Out] -c^2*(b*x^3+a)^(1/3)/d/(d^3*x^3+c^3)-d*x^2*(b*x^3+a)^(1/3)/(d^3*x^3+c^3)+x*
(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,2,4/3,-b*x^3/a,-d^3*x^3/c^3)/c^2/(1+b*x^3
/a)^(1/3)-1/2*d^3*x^4*(b*x^3+a)^(1/3)*AppellF1(4/3,-1/3,2,7/3,-b*x^3/a,-d^3
```

```
*x^3/c^3)/c^5/(1+b*x^3/a)^(1/3)-1/6*b*c^2*ln(d^3*x^3+c^3)/d^2/(-a*d^3+b*c^3
)^(2/3)-1/9*a*d*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)^(2/3)-1/18*(-2*a*d^3+3*b*c
^3)*ln(d^3*x^3+c^3)/c/d^2/(-a*d^3+b*c^3)^(2/3)-1/2*b^(1/3)*ln(b^(1/3)*x-(b*
x^3+a)^(1/3))/d^2+1/3*a*d*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-
a*d^3+b*c^3)^(2/3)+1/6*(-2*a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^
3+a)^(1/3))/c/d^2/(-a*d^3+b*c^3)^(2/3)+1/2*b*c^2*ln((-a*d^3+b*c^3)^(1/3)+d*
(b*x^3+a)^(1/3))/d^2/(-a*d^3+b*c^3)^(2/3)-1/3*b^(1/3)*arctan(1/3*(1+2*b^(1/
3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)+2/9*a*d*arctan(1/3*(1+2*(-a*d^3+
b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/c/(-a*d^3+b*c^3)^(2/3)*3^(1/2)+1
/9*(-2*a*d^3+3*b*c^3)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1
/3))*3^(1/2))/c/d^2/(-a*d^3+b*c^3)^(2/3)*3^(1/2)-1/3*b*c^2*arctan(1/3*(1-2*
d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/d^2/(-a*d^3+b*c^3)^(2/3)*3
^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.895, Rules

used = {2181, 441, 440, 480, 12, 503, 455, 43, 60, 631, 210, 31, 525, 524, 478, 598, 337}

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = -\frac{d^3\sqrt[3]{bx^3+a}\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x^4}{2c^5\sqrt[3]{\frac{bx^3}{a}+1}} - \frac{d\sqrt[3]{bx^3+a}x^2}{c^3+d^3x^3} + \frac{\sqrt[3]{bx^3+a}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x}{c^2\sqrt[3]{\frac{bx^3}{a}+1}} - \frac{\sqrt[3]{b}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{2ad\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{2/3}} + \frac{(3bc^3-2ad^3)\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{3\sqrt{3}cd^2\left(bc^3-ad^3\right)^{2/3}} - \frac{bc^2\arctan\left(\frac{1-\frac{2d\sqrt[3]{bx^3+a}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2\left(bc^3-ad^3\right)^{2/3}} - \frac{ad\log\left(c^3+d^3x^3\right)}{9c\left(bc^3-ad^3\right)^{2/3}} - \frac{(3bc^3-2ad^3)\log\left(c^3+d^3x^3\right)}{18cd^2\left(bc^3-ad^3\right)^{2/3}} - \frac{bc^2\log\left(c^3+d^3x^3\right)}{6d^2\left(bc^3-ad^3\right)^{2/3}} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{bx^3+a}\right)}{2d^2} + \frac{ad\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)}{3c\left(bc^3-ad^3\right)^{2/3}} + \frac{(3bc^3-2ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)}{6cd^2\left(bc^3-ad^3\right)^{2/3}} + \frac{bc^2\log\left(\sqrt[3]{bx^3+a}d+\sqrt[3]{bc^3-ad^3}\right)}{2d^2\left(bc^3-ad^3\right)^{2/3}} - \frac{c^2\sqrt[3]{bx^3+a}}{d\left(c^3+d^3x^3\right)}$$

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x)^2,x]
```

```
[Out] -((c^2*(a + b*x^3)^(1/3))/(d*(c^3 + d^3*x^3))) - (d*x^2*(a + b*x^3)^(1/3))/
(c^3 + d^3*x^3) + (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 2, 4/3, -((b*x^3
)/a), -((d^3*x^3)/c^3)])/(c^2*(1 + (b*x^3)/a)^(1/3)) - (d^3*x^4*(a + b*x^3)
^(1/3)*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(2*c^5*
(1 + (b*x^3)/a)^(1/3)) - (b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/
```

```
3))/Sqrt[3]])/(Sqrt[3]*d^2) + (2*a*d*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x
)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*(b*c^3 - a*d^3)^(2/3)) + ((
3*b*c^3 - 2*a*d^3)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(
1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*d^2*(b*c^3 - a*d^3)^(2/3)) - (b*c^2*ArcTan[(1
 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2*(b
*c^3 - a*d^3)^(2/3)) - (b*c^2*Log[c^3 + d^3*x^3])/(6*d^2*(b*c^3 - a*d^3)^(2
/3)) - (a*d*Log[c^3 + d^3*x^3])/(9*c*(b*c^3 - a*d^3)^(2/3)) - ((3*b*c^3 - 2
*a*d^3)*Log[c^3 + d^3*x^3])/(18*c*d^2*(b*c^3 - a*d^3)^(2/3)) - (b^(1/3)*Log
[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*d^2) + (a*d*Log[((b*c^3 - a*d^3)^(1/3)*
x)/c - (a + b*x^3)^(1/3)])/(3*c*(b*c^3 - a*d^3)^(2/3)) + ((3*b*c^3 - 2*a*d^
3)*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(6*c*d^2*(b*c^3 -
a*d^3)^(2/3)) + (b*c^2*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(2
*d^2*(b*c^3 - a*d^3)^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\text{integral} = \int \left(\frac{c^4 \sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2} - \frac{2c^3dx\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2} + \frac{3c^2d^2x^2\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2} - \frac{2cd^3x^3\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2} + \frac{d^4x^4\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2} \right) dx$$

$$= c^4 \int \frac{\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2} dx - (2c^3d) \int \frac{x\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2} dx + (3c^2d^2) \int \frac{x^2\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2} dx - (2cd^3) \int \frac{x^3\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2} dx + d^4 \int \frac{x^4\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2} dx$$

$$= -\frac{dx^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3} + \frac{1}{3}d \int \frac{x(2a+3bx^3)}{(a+bx^3)^{2/3}(c^3+d^3x^3)} dx - \frac{1}{3}(2d) \int \frac{ax}{(a+bx^3)^{2/3}(c^3+d^3x^3)} dx + (c^2d^2)\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{(c^3+d^3x)^2} dx, x, x^3 \right) + \frac{\left(c^4\sqrt[3]{a+bx^3}\right) \int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{(c^3+d^3x^3)^2} dx}{\sqrt[3]{1+\frac{bx^3}{a}}} - \frac{\left(2cd^3\sqrt[3]{a+bx^3}\right) \int \frac{x^3\sqrt[3]{1+\frac{bx^3}{a}}}{(c^3+d^3x^3)^2} dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{c^2\sqrt[3]{a+bx^3}}{d(c^3+d^3x^3)} - \frac{dx^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3} + \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3}; -\frac{1}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{d^3x^3}{c^3}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}} - \frac{d^3x^4\sqrt[3]{a+bx^3}F_1\left(\frac{4}{3}; -\frac{1}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{d^3x^3}{c^3}\right)}{2c^5\sqrt[3]{1+\frac{bx^3}{a}}} + \frac{(bc^2)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c^3+d^3x)} dx, x, x^3 \right)}{3d} + \frac{1}{3}d \int \left(\frac{3bx}{d^3(a+bx^3)^{2/3}} + \frac{(-3bc^3+2ad^3)x}{d^3(a+bx^3)^{2/3}(c^3+d^3x^3)} \right) dx - \frac{1}{3}(2ad) \int \frac{x}{(a+bx^3)^{2/3}(c^3+d^3x^3)} dx$$

$$= -\frac{c^2\sqrt[3]{a+bx^3}}{d\left(c^3+d^3x^3\right)} - \frac{dx^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3} + \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$- \frac{d^3x^4\sqrt[3]{a+bx^3}F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$+ \frac{2ad\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{2/3}} - \frac{bc^2\log\left(c^3+d^3x^3\right)}{6d^2\left(bc^3-ad^3\right)^{2/3}}$$

$$- \frac{ad\log\left(c^3+d^3x^3\right)}{9c\left(bc^3-ad^3\right)^{2/3}} + \frac{ad\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\left(bc^3-ad^3\right)^{2/3}} + \frac{b\int\frac{x}{(a+bx^3)^{2/3}}\,dx}{d^2}$$

$$+\frac{1}{3}\left(\left(2a-\frac{3bc^3}{d^3}\right)d\right)\int\frac{x}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)}\,dx+\frac{\left(bc^2\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt[3]{bc^3-ad^3}}{d}+x}\,dx,x,\sqrt[3]{a+b\right.}{2d^2\left(bc^3-ad^3\right)^{2/3}}$$

$$= -\frac{c^2\sqrt[3]{a+bx^3}}{d\left(c^3+d^3x^3\right)} - \frac{dx^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3} + \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$-\frac{d^3x^4\sqrt[3]{a+bx^3}F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5\sqrt[3]{1+\frac{bx^3}{a}}} - \frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}$$

$$+\frac{2ad\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{2/3}} - \frac{\left(2a-\frac{3bc^3}{d^3}\right)d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{2/3}}$$

$$-\frac{bc^2\log\left(c^3+d^3x^3\right)}{6d^2\left(bc^3-ad^3\right)^{2/3}} - \frac{ad\log\left(c^3+d^3x^3\right)}{9c\left(bc^3-ad^3\right)^{2/3}} + \frac{\left(2a-\frac{3bc^3}{d^3}\right)d\log\left(c^3+d^3x^3\right)}{18c\left(bc^3-ad^3\right)^{2/3}}$$

$$-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d^2} + \frac{ad\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\left(bc^3-ad^3\right)^{2/3}}$$

$$-\frac{\left(2a-\frac{3bc^3}{d^3}\right)d\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{6c\left(bc^3-ad^3\right)^{2/3}}$$

$$+\frac{bc^2\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2d^2\left(bc^3-ad^3\right)^{2/3}}$$

$$+\frac{\left(bc^2\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}\right)}{d^2\left(bc^3-ad^3\right)^{2/3}}$$

$$= -\frac{c^2\sqrt[3]{a+bx^3}}{d(c^3+d^3x^3)} - \frac{dx^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3} + \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$- \frac{d^3x^4\sqrt[3]{a+bx^3}F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5\sqrt[3]{1+\frac{bx^3}{a}}} - \frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}$$

$$+ \frac{2ad\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c(bc^3-ad^3)^{2/3}} - \frac{\left(2a-\frac{3bc^3}{d^3}\right)d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c(bc^3-ad^3)^{2/3}}$$

$$- \frac{bc^2\tan^{-1}\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2(bc^3-ad^3)^{2/3}} - \frac{bc^2\log(c^3+d^3x^3)}{6d^2(bc^3-ad^3)^{2/3}}$$

$$- \frac{ad\log(c^3+d^3x^3)}{9c(bc^3-ad^3)^{2/3}} + \frac{\left(2a-\frac{3bc^3}{d^3}\right)d\log(c^3+d^3x^3)}{18c(bc^3-ad^3)^{2/3}}$$

$$- \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d^2} + \frac{ad\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c(bc^3-ad^3)^{2/3}}$$

$$- \frac{\left(2a-\frac{3bc^3}{d^3}\right)d\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{6c(bc^3-ad^3)^{2/3}}$$

$$+ \frac{bc^2\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2d^2(bc^3-ad^3)^{2/3}}$$

Mathematica [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x)^2,x]
[Out] Integrate[(a + b*x^3)^(1/3)/(c + d*x)^2, x]
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(dx+c\right)^{2}}dx$$

```
[In] int((b*x^3+a)^(1/3)/(d*x+c)^2,x)
[Out] int((b*x^3+a)^(1/3)/(d*x+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{\left(c+dx\right)^{2}}\,dx$$

```
[In] integrate((b*x**3+a)**(1/3)/(d*x+c)**2,x)
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = \int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{\left(dx+c\right)^{2}}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx+c)^2}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{(c+d\,x)^2}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(c + d*x)^2,x)
[Out] int((a + b*x^3)^(1/3)/(c + d*x)^2, x)
```

3.29 $\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 310

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \frac{3c^2d^2(a+bx^3)^{2/3}}{b} + \frac{4cd^3x(a+bx^3)^{2/3}}{3b} + \frac{c^4\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{4acd^3\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} + \frac{2c^3dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} + \frac{d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{5\sqrt[3]{a+bx^3}} - \frac{c^4\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}} + \frac{2acd^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3b^{4/3}}$$

```
[Out] 3*c^2*d^2*(b*x^3+a)^(2/3)/b+4/3*c*d^3*x*(b*x^3+a)^(2/3)/b+2*c^3*d*x^2*(1+b*
x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)^(1/3)+1/5*d^4*x
^5*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 5/3],[8/3],-b*x^3/a)/(b*x^3+a)^(1/3)-1
/2*c^4*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+2/3*a*c*d^3*ln(-b^(1/3)*x+(b*
x^3+a)^(1/3))/b^(4/3)+1/3*c^4*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^
(1/2))/b^(1/3)*3^(1/2)-4/9*a*c*d^3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3
))*3^(1/2))/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1907, 245, 372, 371, 267, 327}

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = -\frac{4acd^3\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{c^4\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{2acd^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3b^{4/3}}-\frac{c^4\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}} + \frac{2c^3dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} + \frac{3c^2d^2(a+bx^3)^{2/3}}{b}+\frac{4cd^3x(a+bx^3)^{2/3}}{3b} + \frac{d^4x^5\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{5\sqrt[3]{a+bx^3}}$$

```
[In] Int[(c + d*x)^4/(a + b*x^3)^(1/3),x]
```

```
[Out] (3*c^2*d^2*(a + b*x^3)^(2/3))/b + (4*c*d^3*x*(a + b*x^3)^(2/3))/(3*b) + (c^
4*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3))
- (4*a*c*d^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt
[3]*b^(4/3)) + (2*c^3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/
3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) + (d^4*x^5*(1 + (b*x^3)/a)^(1/3)*H
ypergeometric2F1[1/3, 5/3, 8/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(1/3)) - (c^4
*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3)) + (2*a*c*d^3*Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(4/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c^4}{\sqrt[3]{a+bx^3}}+\frac{4c^3dx}{\sqrt[3]{a+bx^3}}+\frac{6c^2d^2x^2}{\sqrt[3]{a+bx^3}}+\frac{4cd^3x^3}{\sqrt[3]{a+bx^3}}+\frac{d^4x^4}{\sqrt[3]{a+bx^3}}\right) dx \\
&= c^4\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx+\left(4c^3d\right)\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx+\left(6c^2d^2\right)\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx \\
&\quad +\left(4cd^3\right)\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx+d^4\int \frac{x^4}{\sqrt[3]{a+bx^3}}\,dx
\end{aligned}$$

$$= \frac{3c^2d^2(a+bx^3)^{2/3}}{b} + \frac{4cd^3x(a+bx^3)^{2/3}}{3b} + \frac{c^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{c^4\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}} - \frac{(4acd^3)\int\frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3b} + \frac{\left(4c^3d\sqrt[3]{1+\frac{bx^3}{a}}\right)\int\frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{\sqrt[3]{a+bx^3}} + \frac{\left(d^4\sqrt[3]{1+\frac{bx^3}{a}}\right)\int\frac{x^4}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{\sqrt[3]{a+bx^3}}$$

$$= \frac{3c^2d^2(a+bx^3)^{2/3}}{b} + \frac{4cd^3x(a+bx^3)^{2/3}}{3b} + \frac{c^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{4acd^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} + \frac{2c^3dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} + \frac{d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{8}{3};-\frac{bx^3}{a}\right)}{5\sqrt[3]{a+bx^3}} - \frac{c^4\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}} + \frac{2acd^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3b^{4/3}}$$

Mathematica [A] (verified)

Time = 10.31 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.26

$$\int\frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{180b^{4/3}c^3dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)+18b^{4/3}d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\right.}{}$$

```
[In] Integrate[(c + d*x)^4/(a + b*x^3)^(1/3),x]
```

```
[Out] (180*b^(4/3)*c^3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/
3, -((b*x^3)/a)] + 18*b^(4/3)*d^4*x^5*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2
F1[1/3, 5/3, 8/3, -((b*x^3)/a)] + 5*c*(54*a*b^(1/3)*c*d^2 + 24*a*b^(1/3)*d^
3*x + 54*b^(4/3)*c*d^2*x^3 + 24*b^(4/3)*d^3*x^4 + 2*Sqrt[3]*(3*b*c^3 - 4*a*
```

```
d^3)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]
] + 2*(-3*b*c^3 + 4*a*d^3)*(a + b*x^3)^(1/3)*Log[1 - (b^(1/3)*x)/(a + b*x^3
)^(1/3)] + 3*b*c^3*(a + b*x^3)^(1/3)*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3
) + (b^(1/3)*x)/(a + b*x^3)^(1/3)] - 4*a*d^3*(a + b*x^3)^(1/3)*Log[1 + (b^(
2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(90*b^(4/3)*
(a + b*x^3)^(1/3))
```

Maple [F]

$$\int \frac{(dx+c)^4}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

```
[In] int((d*x+c)^4/(b*x^3+a)^(1/3),x)
[Out] int((d*x+c)^4/(b*x^3+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.66

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = 6c^2d^2\left(\begin{cases}\frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise}\end{cases}\right) + \frac{c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{4c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)} + \frac{4cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)} + \frac{d^4x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate((d*x+c)**4/(b*x**3+a)**(1/3),x)
```

```
[Out] 6*c**2*d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(
2*b), True)) + c**4*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar
(I*pi)/a)/(3*a**(1/3)*gamma(4/3)) + 4*c**3*d*x**2*gamma(2/3)*hyper((1/3, 2/
3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3)) + 4*c*d**3*x*
*4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/
3)*gamma(7/3)) + d**4*x**5*gamma(5/3)*hyper((1/3, 5/3), (8/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(1/3)*gamma(8/3))
```

Maxima [F]

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^4}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^4 + integra
te((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x)/(b*x^3 + a)^(1/3), x
)
```

Giac [F]

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^4}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(c+d\,x)^4}{(b\,x^3+a)^{1/3}}\,dx$$

```
[In] int((c + d*x)^4/(a + b*x^3)^(1/3),x)
```

```
[Out] int((c + d*x)^4/(a + b*x^3)^(1/3), x)
```

3.30 $\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 255

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}} \, dx = \frac{3cd^2(a+bx^3)^{2/3}}{2b} + \frac{d^3x(a+bx^3)^{2/3}}{3b} + \frac{c^3 \arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{ad^3 \arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} + \frac{3c^2dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}} - \frac{c^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}} + \frac{ad^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

```
[Out] 3/2*c*d^2*(b*x^3+a)^(2/3)/b+1/3*d^3*x*(b*x^3+a)^(2/3)/b+3/2*c^2*d*x^2*(1+b*
x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)^(1/3)-1/2*c^3*l
n(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+1/6*a*d^3*ln(-b^(1/3)*x+(b*x^3+a)^(1/
3))/b^(4/3)+1/3*c^3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(
1/3)*3^(1/2)-1/9*a*d^3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/
b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1907, 245, 372, 371, 267, 327}

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = -\frac{ad^3\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{c^3\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{ad^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6b^{4/3}}-\frac{c^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{3c^2dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}+\frac{3cd^2(a+bx^3)^{2/3}}{2b}+\frac{d^3x(a+bx^3)^{2/3}}{3b}$$

```
[In] Int[(c + d*x)^3/(a + b*x^3)^(1/3),x]
```

```
[Out] (3*c*d^2*(a + b*x^3)^(2/3))/(2*b) + (d^3*x*(a + b*x^3)^(2/3))/(3*b) + (c^3*
ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) -
(a*d^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^
(4/3)) + (3*c^2*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3
, -((b*x^3)/a)])/(2*(a + b*x^3)^(1/3)) - (c^3*Log[-(b^(1/3)*x) + (a + b*x^3
)^(1/3)])/(2*b^(1/3)) + (a*d^3*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6*b^
(4/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c^3}{\sqrt[3]{a+bx^3}}+\frac{3c^2dx}{\sqrt[3]{a+bx^3}}+\frac{3cd^2x^2}{\sqrt[3]{a+bx^3}}+\frac{d^3x^3}{\sqrt[3]{a+bx^3}}\right)dx\\
&= c^3\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx+\left(3c^2d\right)\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx+\left(3cd^2\right)\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx+d^3\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx\\
&= \frac{3cd^2\left(a+bx^3\right)^{2/3}}{2b}+\frac{d^3x\left(a+bx^3\right)^{2/3}}{3b}+\frac{c^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}\\
&\quad-\frac{c^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}-\frac{\left(ad^3\right)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3b}\\
&\quad+\frac{\left(3c^2d\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{\sqrt[3]{a+bx^3}}
\end{aligned}$$

$$= \frac{3cd^2(a+bx^3)^{2/3}}{2b} + \frac{d^3x(a+bx^3)^{2/3}}{3b} + \frac{c^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{ad^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} + \frac{3c^2dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}} - \frac{c^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}} + \frac{ad^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

Mathematica [A] (verified)

Time = 10.07 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{1}{18}\left(\frac{27c^2dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} + \frac{27\sqrt[3]{b}cd^2(a+bx^3)^{2/3} + 6\sqrt[3]{b}d^3x(a+bx^3)^{2/3} + 2\sqrt{3}(3bc^3-ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right) + (-6bc^3+2a\cdots}{b\cdots}\right.$$

```
[In] Integrate[(c + d*x)^3/(a + b*x^3)^(1/3),x]
```

```
[Out] ((27*c^2*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*
x^3)/a)])/(a + b*x^3)^(1/3) + (27*b^(1/3)*c*d^2*(a + b*x^3)^(2/3) + 6*b^(1/
3)*d^3*x*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(3*b*c^3 - a*d^3)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] + (-6*b*c^3 + 2*a*d^3)*Log[1 - (b^(1/3)
*x)/(a + b*x^3)^(1/3)] + 3*b*c^3*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) +
(b^(1/3)*x)/(a + b*x^3)^(1/3)] - a*d^3*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2
/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/b^(4/3))/18
```

Maple [F]

$$\int \frac{(dx+c)^3}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

[In] int((d*x+c)^3/(b*x^3+a)^(1/3),x)

[Out] int((d*x+c)^3/(b*x^3+a)^(1/3),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

[In] integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="fricas")

[Out] Timed out

Sympy [A] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = 3cd^2\left(\begin{cases}\frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise}\end{cases}\right) + \frac{c^3x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{c^2dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)} + \frac{d^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

[In] integrate((d*x+c)**3/(b*x**3+a)**(1/3),x)

[Out] 3*c*d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(2*b), True)) + c**3*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(4/3)) + c**2*d*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(a**(1/3)*gamma(5/3)) + d**3*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))

Maxima [F]

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^3}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^3 + integra
te((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x)/(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^3}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(c+d\,x)^3}{(b\,x^3+a)^{1/3}}\,dx$$

```
[In] int((c + d*x)^3/(a + b*x^3)^(1/3),x)
```

```
[Out] int((c + d*x)^3/(a + b*x^3)^(1/3), x)
```

3.31 $\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}} dx$

Optimal result

Integrand size = 19, antiderivative size = 147

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}} dx = \frac{d^2(a+bx^3)^{2/3}}{2b} + \frac{c^2 \arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{cdx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} - \frac{c^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

```
[Out] 1/2*d^2*(b*x^3+a)^(2/3)/b+c*d*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5
/3],-b*x^3/a)/(b*x^3+a)^(1/3)-1/2*c^2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3
)+1/3*c^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used

$= \{1900, 267, 1907, 245, 372, 371\}$

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{c^2\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{c^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}} + \frac{cdx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} + \frac{d^2(a+bx^3)^{2/3}}{2b}$$

```
[In] Int[(c + d*x)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (d^2*(a + b*x^3)^(2/3))/(2*b) + (c^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (c*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hyperge
ometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) - (c^2*Log[-(b^(
1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
```

```
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d^2 \int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx + \int \frac{c^2+2cdx}{\sqrt[3]{a+bx^3}}\,dx \\
&= \frac{d^2 (a+bx^3)^{2/3}}{2b} + \int \left(\frac{c^2}{\sqrt[3]{a+bx^3}} + \frac{2cdx}{\sqrt[3]{a+bx^3}}\right) dx \\
&= \frac{d^2 (a+bx^3)^{2/3}}{2b} + c^2 \int \frac{1}{\sqrt[3]{a+bx^3}}\,dx + (2cd) \int \frac{x}{\sqrt[3]{a+bx^3}}\,dx \\
&= \frac{d^2 (a+bx^3)^{2/3}}{2b} + \frac{c^2 \tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} \\
&\quad - \frac{c^2 \log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}} + \frac{\left(2cd\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{\sqrt[3]{a+bx^3}} \\
&= \frac{d^2 (a+bx^3)^{2/3}}{2b} + \frac{c^2 \tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} \\
&\quad + \frac{cdx^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} - \frac{c^2 \log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 9.64 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{d^2(a+bx^3)^{2/3}}{2b} + \frac{c^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{cdx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} - \frac{c^2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{c^2\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{b}}$$

```
[In] Integrate[(c + d*x)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (d^2*(a + b*x^3)^(2/3))/(2*b) + (c^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (c*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hyperge
ometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) - (c^2*Log[1 - (
b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(1/3)) + (c^2*Log[1 + (b^(2/3)*x^2)/(a
+ b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*b^(1/3))
```

Maple [F]

$$\int \frac{(dx+c)^2}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

```
[In] int((d*x+c)^2/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^2/(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)/(b*x^3 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.75

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = d^2\left(\begin{cases} \frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0 \\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise} \end{cases}\right) + \frac{c^2x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((d*x+c)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(2*b), T
rue)) + c**2*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*a**(1/3)*gamma(4/3)) + 2*c*d*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3))
```

Maxima [F]

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^2 + integra
te((d^2*x^2 + 2*c*d*x)/(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{{\left(c+d\,x\right)}^2}{{\left(b\,x^3+a\right)}^{1/3}} \,d x$$

```
[In] int((c + d*x)^2/(a + b*x^3)^(1/3),x)
[Out] int((c + d*x)^2/(a + b*x^3)^(1/3), x)
```

3.32 $\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 124

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \frac{c\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}} - \frac{c\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

```
[Out] 1/2*d*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)^
(1/3)-1/2*c*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+1/3*c*arctan(1/3*(1+2*b^
(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used

$= \{1907, 245, 372, 371\}$

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \frac{c\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{c\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}} + \frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}$$

```
[In] Int[(c + d*x)/(a + b*x^3)^(1/3),x]
```

```
[Out] (c*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3))
 + (d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/
a)])/(2*(a + b*x^3)^(1/3)) - (c*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b
^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{\sqrt[3]{a+bx^3}}+\frac{dx}{\sqrt[3]{a+bx^3}}\right)dx \\
&= c\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx + d\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx \\
&= \frac{c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{c\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}} + \frac{\left(d\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{\sqrt[3]{a+bx^3}} \\
&= \frac{c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}} - \frac{c\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 9.20 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.31

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \frac{1}{6}\left(\frac{3dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} + \frac{c\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)\right)}{\sqrt[3]{b}}\right)$$

```
[In] Integrate[(c + d*x)/(a + b*x^3)^(1/3),x]
```

```
[Out] ((3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/
a)])/(a + b*x^3)^(1/3) + (c*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3
)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(
2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/b^(1/3))/6
```

Maple [F]

$$\int \frac{dx+c}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}dx$$

```
[In] int((d*x+c)/(b*x^3+a)^(1/3),x)
[Out] int((d*x+c)/(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] integral((d*x + c)/(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((d*x+c)/(b*x**3+a)**(1/3),x)
[Out] c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1
/3)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(1/3)*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c + d*integra
te(x/(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((d*x + c)/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{c+d\,x}{(b\,x^3+a)^{1/3}}\,dx$$

```
[In] int((c + d*x)/(a + b*x^3)^(1/3),x)
[Out] int((c + d*x)/(a + b*x^3)^(1/3), x)
```

3.33 $\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 333

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = -\frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^2\sqrt[3]{a+bx^3}} + \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}} - \frac{\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}} + \frac{\log\left(c^3+d^3x^3\right)}{3\sqrt[3]{bc^3-ad^3}} - \frac{\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}} - \frac{\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}$$

```
[Out] -1/2*d*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1/3,1,5/3,-b*x^3/a,-d^3*x^3/c^3)/
c^2/(b*x^3+a)^(1/3)+1/3*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(1/3)-1/2*ln((-a*d^3
+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(1/3)-1/2*ln((-a*d^3+b*c^
3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(1/3)+1/3*arctan(1/3*(1+2*(-a*d^
3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(1/3)*3^(1/2)-1
/3*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(-a*d^3
+b*c^3)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {2181, 384, 525, 524, 455, 58, 631, 210, 31}

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = -\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^2\sqrt[3]{a+bx^3}} + \frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}} - \frac{\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}} + \frac{\log\left(c^3+d^3x^3\right)}{3\sqrt[3]{bc^3-ad^3}} - \frac{\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}} - \frac{\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}$$

```
[In] Int[1/((c + d*x)*(a + b*x^3)^(1/3)),x]
```

```
[Out] -1/2*(d*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a),
-((d^3*x^3)/c^3)])/(c^2*(a + b*x^3)^(1/3)) + ArcTan[(1 + (2*(b*c^3 - a*d^3)
^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*(b*c^3 - a*d^3)^(1/3)) -
 ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]]/(Sqrt[
3]*(b*c^3 - a*d^3)^(1/3)) + Log[c^3 + d^3*x^3]/(3*(b*c^3 - a*d^3)^(1/3)) -
Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)]/(2*(b*c^3 - a*d^3)^(1/
3)) - Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)]/(2*(b*c^3 - a*d^3)^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
```

```
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\text{integral} = \int \left(\frac{c^2}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)} - \frac{cdx}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)} + \frac{d^2x^2}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)}\right) dx$$

$$= c^2 \int \frac{1}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)}\, dx - (cd) \int \frac{x}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)}\, dx + d^2 \int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)}\, dx$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}} + \frac{\log\left(c^3+d^3x^3\right)}{6\sqrt[3]{bc^3-ad^3}} - \frac{\log\left(\frac{\sqrt[3]{bc^3-ad^3}\,x}{c} - \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}} + \frac{1}{3} d^2 \text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c^3+d^3x)}\, dx, x, x^3\right) - \frac{\left(cd\sqrt[3]{1+\frac{bx^3}{a}}\right) \int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}\,(c^3+d^3x^3)}\, dx}{\sqrt[3]{a+bx^3}}$$

$$= -\frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\, F_1\left(\frac{2}{3}; \frac{1}{3}, 1; \frac{5}{3}; -\frac{bx^3}{a}, -\frac{d^3x^3}{c^3}\right)}{2c^2\sqrt[3]{a+bx^3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}} + \frac{\log\left(c^3+d^3x^3\right)}{3\sqrt[3]{bc^3-ad^3}} - \frac{\log\left(\frac{\sqrt[3]{bc^3-ad^3}\,x}{c} - \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\left(bc^3-ad^3\right)^{2/3}}{d^2} - \frac{\sqrt[3]{bc^3-ad^3}\,x}{d} + x^2}\, dx, x, \sqrt[3]{a+bx^3}\right)}{2d} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc^3-ad^3}}{d} + x}\, dx, x, \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}$$

$$= -\frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^2\sqrt[3]{a+bx^3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}} + \frac{\log\left(c^3+d^3x^3\right)}{3\sqrt[3]{bc^3-ad^3}} - \frac{\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}} - \frac{\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}} + \frac{\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}\right)}{\sqrt[3]{bc^3-ad^3}}$$

$$= -\frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^2\sqrt[3]{a+bx^3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}} + \frac{\log\left(c^3+d^3x^3\right)}{3\sqrt[3]{bc^3-ad^3}} - \frac{\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}} - \frac{\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}$$

Mathematica [F]

$$\int\frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \int\frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$$

```
[In] Integrate[1/((c + d*x)*(a + b*x^3)^(1/3)),x]
[Out] Integrate[1/((c + d*x)*(a + b*x^3)^(1/3)), x]
```

Maple [F]

$$\int \frac{1}{(dx+c)\,(b\,x^{3}+a)^{\frac{1}{3}}}dx$$

```
[In] int(1/(d*x+c)/(b*x^3+a)^(1/3),x)
[Out] int(1/(d*x+c)/(b*x^3+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx)}\,dx$$

```
[In] integrate(1/(d*x+c)/(b*x**3+a)**(1/3),x)
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx+c)}\,dx$$

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx+c)}\,dx$$

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(c+d\,x)}\,dx$$

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x)),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x)), x)
```

3.34 $\int \frac{1}{(c+dx)^2 \sqrt[3]{a+bx^3}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 761

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \frac{c^2d^2(a+bx^3)^{2/3}}{(bc^3-ad^3)(c^3+d^3x^3)} - \frac{cd^3x(a+bx^3)^{2/3}}{(bc^3-ad^3)(c^3+d^3x^3)} - \frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},2,\frac{5}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\sqrt[3]{a+bx^3}} + \frac{d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{5c^6\sqrt[3]{a+bx^3}} + \frac{2ad^3\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{4/3}} + \frac{(3bc^3-2ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{4/3}} - \frac{bc^2\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{4/3}} + \frac{bc^2\log(c^3+d^3x^3)}{6(bc^3-ad^3)^{4/3}} + \frac{ad^3\log(c^3+d^3x^3)}{9c(bc^3-ad^3)^{4/3}} + \frac{(3bc^3-2ad^3)\log(c^3+d^3x^3)}{18c(bc^3-ad^3)^{4/3}} - \frac{ad^3\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c(bc^3-ad^3)^{4/3}} - \frac{(3bc^3-2ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{6c(bc^3-ad^3)^{4/3}} - \frac{bc^2\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2(bc^3-ad^3)^{4/3}}$$

```
[Out] c^2*d^2*(b*x^3+a)^(2/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)-c*d^3*x*(b*x^3+a)^(2/3
)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)-d*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1/3,2,5
/3,-b*x^3/a,-d^3*x^3/c^3)/c^3/(b*x^3+a)^(1/3)+1/5*d^4*x^5*(1+b*x^3/a)^(1/3)
*AppellF1(5/3,1/3,2,8/3,-b*x^3/a,-d^3*x^3/c^3)/c^6/(b*x^3+a)^(1/3)+1/6*b*c^
```

```
2*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(4/3)+1/9*a*d^3*ln(d^3*x^3+c^3)/c/(-a*d^3+
b*c^3)^(4/3)+1/18*(-2*a*d^3+3*b*c^3)*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)^(4/3)
-1/3*a*d^3*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-a*d^3+b*c^3)^(4
/3)-1/6*(-2*a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(
-a*d^3+b*c^3)^(4/3)-1/2*b*c^2*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-
a*d^3+b*c^3)^(4/3)+2/9*a*d^3*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^
3+a)^(1/3))*3^(1/2))/c/(-a*d^3+b*c^3)^(4/3)*3^(1/2)+1/9*(-2*a*d^3+3*b*c^3)*
arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/c/(-a*d^
3+b*c^3)^(4/3)*3^(1/2)-1/3*b*c^2*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+
b*c^3)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules

used = {2181, 390, 384, 525, 524, 455, 44, 58, 631, 210, 31, 482, 12}

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = -\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},2,\frac{5}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\sqrt[3]{a+bx^3}} + \frac{d^4x^5\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{5c^6\sqrt[3]{a+bx^3}} + \frac{2ad^3\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{4/3}} + \frac{\left(3bc^3-2ad^3\right)\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{4/3}} - \frac{bc^2\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\left(bc^3-ad^3\right)^{4/3}} - \frac{cd^3x\left(a+bx^3\right)^{2/3}}{\left(c^3+d^3x^3\right)\left(bc^3-ad^3\right)} + \frac{ad^3\log\left(c^3+d^3x^3\right)}{9c\left(bc^3-ad^3\right)^{4/3}} + \frac{\left(3bc^3-2ad^3\right)\log\left(c^3+d^3x^3\right)}{18c\left(bc^3-ad^3\right)^{4/3}} - \frac{ad^3\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{3c\left(bc^3-ad^3\right)^{4/3}} - \frac{\left(3bc^3-2ad^3\right)\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{6c\left(bc^3-ad^3\right)^{4/3}} + \frac{bc^2\log\left(c^3+d^3x^3\right)}{6\left(bc^3-ad^3\right)^{4/3}} - \frac{bc^2\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{4/3}} + \frac{c^2d^2\left(a+bx^3\right)^{2/3}}{\left(c^3+d^3x^3\right)\left(bc^3-ad^3\right)}$$

```
[In] Int[1/((c + d*x)^2*(a + b*x^3)^(1/3)),x]
```

```
[Out] (c^2*d^2*(a + b*x^3)^(2/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) - (c*d^3*x*(a
 + b*x^3)^(2/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) - (d*x^2*(1 + (b*x^3)/a)
^(1/3)*AppellF1[2/3, 1/3, 2, 5/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(c^3*(a
+ b*x^3)^(1/3)) + (d^4*x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 2, 8/3,
```

```
-((b*x^3)/a), -((d^3*x^3)/c^3)])/(5*c^6*(a + b*x^3)^(1/3)) + (2*a*d^3*ArcT
an[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqr
t[3]*c*(b*c^3 - a*d^3)^(4/3)) + ((3*b*c^3 - 2*a*d^3)*ArcTan[(1 + (2*(b*c^3
- a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*(b*c^3 - a*
d^3)^(4/3)) - (b*c^2*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1
/3))/Sqrt[3]])/(Sqrt[3]*(b*c^3 - a*d^3)^(4/3)) + (b*c^2*Log[c^3 + d^3*x^3])
/(6*(b*c^3 - a*d^3)^(4/3)) + (a*d^3*Log[c^3 + d^3*x^3])/(9*c*(b*c^3 - a*d^3
)^(4/3)) + ((3*b*c^3 - 2*a*d^3)*Log[c^3 + d^3*x^3])/(18*c*(b*c^3 - a*d^3)^(
4/3)) - (a*d^3*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(3*c*(
b*c^3 - a*d^3)^(4/3)) - ((3*b*c^3 - 2*a*d^3)*Log[((b*c^3 - a*d^3)^(1/3)*x)/
c - (a + b*x^3)^(1/3)])/(6*c*(b*c^3 - a*d^3)^(4/3)) - (b*c^2*Log[(b*c^3 - a
*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(2*(b*c^3 - a*d^3)^(4/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c^4}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2} - \frac{2c^3dx}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2} + \frac{3c^2d^2x^2}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2} - \frac{2cd^3x^3}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2} + \frac{d^4x^4}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2}\right) dx \\
&= c^4 \int \frac{1}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2}\,dx - (2c^3d) \int \frac{x}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2}\,dx \\
&\quad + (3c^2d^2) \int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2}\,dx \\
&\quad - (2cd^3) \int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2}\,dx + d^4 \int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2}\,dx \\
&= -\frac{cd^3x(a+bx^3)^{2/3}}{(bc^3-ad^3)\,(c^3+d^3x^3)} + (c^2d^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c^3+d^3x)^2}\,dx, x, x^3\right) \\
&\quad + \frac{(2cd^3)\int \frac{a}{\sqrt[3]{a+bx^3}(c^3+d^3x^3)}\,dx}{3\,(bc^3-ad^3)} + \frac{(c(3bc^3-2ad^3))\int \frac{1}{\sqrt[3]{a+bx^3}(c^3+d^3x^3)}\,dx}{3\,(bc^3-ad^3)} \\
&\quad - \frac{\left(2c^3d\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}(c^3+d^3x^3)^2}\,dx}{\sqrt[3]{a+bx^3}} + \frac{\left(d^4\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x^4}{\sqrt[3]{1+\frac{bx^3}{a}}(c^3+d^3x^3)^2}\,dx}{\sqrt[3]{a+bx^3}}
\end{aligned}$$

$$= \frac{c^2d^2(a+bx^3)^{2/3}}{(bc^3-ad^3)(c^3+d^3x^3)} - \frac{cd^3x(a+bx^3)^{2/3}}{(bc^3-ad^3)(c^3+d^3x^3)}$$

$$- \frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{3},2;\frac{5}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\sqrt[3]{a+bx^3}}$$

$$+ \frac{d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{5}{3};\frac{1}{3},2;\frac{8}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{5c^6\sqrt[3]{a+bx^3}}$$

$$+ \frac{(3bc^3-2ad^3)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c(bc^3-ad^3)^{4/3}} + \frac{(3bc^3-2ad^3)\log(c^3+d^3x^3)}{18c(bc^3-ad^3)^{4/3}}$$

$$- \frac{(3bc^3-2ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{6c(bc^3-ad^3)^{4/3}}$$

$$+ \frac{(bc^2d^2)\,\mathrm{Subst}\left(\int\frac{1}{\sqrt[3]{a+bx}(c^3+d^3x)}\,dx,x,x^3\right)}{3(bc^3-ad^3)} + \frac{(2acd^3)\int\frac{1}{\sqrt[3]{a+bx^3}(c^3+d^3x^3)}\,dx}{3(bc^3-ad^3)}$$

$$= \frac{c^2 d^2 (a+bx^3)^{2/3}}{(bc^3-ad^3)(c^3+d^3x^3)} - \frac{cd^3 x (a+bx^3)^{2/3}}{(bc^3-ad^3)(c^3+d^3x^3)}$$
$$- \frac{dx^2 \sqrt[3]{1+\frac{bx^3}{a}} F_1\left(\frac{2}{3}; \frac{1}{3}, 2; \frac{5}{3}; -\frac{bx^3}{a}, -\frac{d^3x^3}{c^3}\right)}{c^3 \sqrt[3]{a+bx^3}}$$
$$+ \frac{d^4 x^5 \sqrt[3]{1+\frac{bx^3}{a}} F_1\left(\frac{5}{3}; \frac{1}{3}, 2; \frac{8}{3}; -\frac{bx^3}{a}, -\frac{d^3x^3}{c^3}\right)}{5c^6 \sqrt[3]{a+bx^3}} + \frac{2ad^3 \tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{4/3}}$$
$$+ \frac{(3bc^3-2ad^3)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{4/3}} + \frac{bc^2 \log(c^3+d^3x^3)}{6\,(bc^3-ad^3)^{4/3}}$$
$$+ \frac{ad^3 \log(c^3+d^3x^3)}{9c\,(bc^3-ad^3)^{4/3}} + \frac{(3bc^3-2ad^3)\log(c^3+d^3x^3)}{18c\,(bc^3-ad^3)^{4/3}}$$
$$- \frac{ad^3 \log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c} - \sqrt[3]{a+bx^3}\right)}{3c\,(bc^3-ad^3)^{4/3}}$$
$$- \frac{(3bc^3-2ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c} - \sqrt[3]{a+bx^3}\right)}{6c\,(bc^3-ad^3)^{4/3}}$$
$$- \frac{(bc^2)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc^3-ad^3}}{d}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2\,(bc^3-ad^3)^{4/3}}$$
$$+ \frac{(bc^2)\,\text{Subst}\left(\int \frac{1}{\frac{(bc^3-ad^3)^{2/3}}{d^2} - \frac{\sqrt[3]{bc^3-ad^3}x}{d} + x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d\,(bc^3-ad^3)}$$

$$= \frac{c^2d^2(a+bx^3)^{2/3}}{(bc^3-ad^3)\left(c^3+d^3x^3\right)} - \frac{cd^3x(a+bx^3)^{2/3}}{(bc^3-ad^3)\left(c^3+d^3x^3\right)}$$

$$- \frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{3},2;\frac{5}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\sqrt[3]{a+bx^3}}$$

$$+ \frac{d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{5}{3};\frac{1}{3},2;\frac{8}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{5c^6\sqrt[3]{a+bx^3}} + \frac{2ad^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{4/3}}$$

$$+ \frac{(3bc^3-2ad^3)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{4/3}} + \frac{bc^2\log\left(c^3+d^3x^3\right)}{6\left(bc^3-ad^3\right)^{4/3}}$$

$$+ \frac{ad^3\log\left(c^3+d^3x^3\right)}{9c\left(bc^3-ad^3\right)^{4/3}} + \frac{(3bc^3-2ad^3)\log\left(c^3+d^3x^3\right)}{18c\left(bc^3-ad^3\right)^{4/3}}$$

$$- \frac{ad^3\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\left(bc^3-ad^3\right)^{4/3}}$$

$$- \frac{(3bc^3-2ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{6c\left(bc^3-ad^3\right)^{4/3}}$$

$$- \frac{bc^2\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{4/3}}$$

$$+ \frac{(bc^2)\,\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}\right)}{\left(bc^3-ad^3\right)^{4/3}}$$

$$= \frac{c^2d^2(a+bx^3)^{2/3}}{(bc^3-ad^3)(c^3+d^3x^3)} - \frac{cd^3x(a+bx^3)^{2/3}}{(bc^3-ad^3)(c^3+d^3x^3)} - \frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{3},2;\frac{5}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\sqrt[3]{a+bx^3}} + \frac{d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{5}{3};\frac{1}{3},2;\frac{8}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{5c^6\sqrt[3]{a+bx^3}} + \frac{2ad^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c(bc^3-ad^3)^{4/3}} + \frac{(3bc^3-2ad^3)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c(bc^3-ad^3)^{4/3}} - \frac{bc^2\tan^{-1}\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}(bc^3-ad^3)^{4/3}} + \frac{bc^2\log(c^3+d^3x^3)}{6(bc^3-ad^3)^{4/3}} + \frac{ad^3\log(c^3+d^3x^3)}{9c(bc^3-ad^3)^{4/3}} + \frac{(3bc^3-2ad^3)\log(c^3+d^3x^3)}{18c(bc^3-ad^3)^{4/3}} - \frac{ad^3\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c(bc^3-ad^3)^{4/3}} - \frac{(3bc^3-2ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{6c(bc^3-ad^3)^{4/3}} - \frac{bc^2\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2(bc^3-ad^3)^{4/3}}$$

Mathematica [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx$$

```
[In] Integrate[1/((c + d*x)^2*(a + b*x^3)^(1/3)),x]
[Out] Integrate[1/((c + d*x)^2*(a + b*x^3)^(1/3)), x]
```

Maple [F]

$$\int \frac{1}{\left(dx+c\right)^{2} \left(b\,x^{3}+a\right)^{\frac{1}{3}}}dx$$

```
[In] int(1/(d*x+c)^2/(b*x^3+a)^(1/3),x)
[Out] int(1/(d*x+c)^2/(b*x^3+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx)^2}\,dx$$

```
[In] integrate(1/(d*x+c)**2/(b*x**3+a)**(1/3),x)
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx+c)^2}\,dx$$

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx+c)^2}\,dx$$

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(c+d\,x)^2}\,dx$$

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x)^2),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x)^2), x)
```

3.35 $\int \frac{1}{(c+dx)^3 \sqrt[3]{a+bx^3}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 1513

$$\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx = \frac{3c^4d^2(a+bx^3)^{2/3}}{2(bc^3-ad^3)(c^3+d^3x^3)^2} - \frac{3c^3d^3x(a+bx^3)^{2/3}}{2(bc^3-ad^3)(c^3+d^3x^3)^2} + \frac{4bc^4d^2(a+bx^3)^{2/3}}{3(bc^3-ad^3)^2(c^3+d^3x^3)} - \frac{cd^2(bc^3-3ad^3)(a+bx^3)^{2/3}}{3(bc^3-ad^3)^2(c^3+d^3x^3)} + \frac{d^3(3bc^3-7ad^3)x(a+bx^3)^{2/3}}{18(bc^3-ad^3)^2(c^3+d^3x^3)} - \frac{d^3(9bc^3-5ad^3)x(a+bx^3)^{2/3}}{18(bc^3-ad^3)^2(c^3+d^3x^3)} - \frac{7d^3(3bc^3+ad^3)x(a+bx^3)^{2/3}}{18(bc^3-ad^3)^2(c^3+d^3x^3)} - \frac{3dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},3,\frac{5}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^4\sqrt[3]{a+bx^3}} + \frac{6d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},3,\frac{8}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{5c^7\sqrt[3]{a+bx^3}} + \frac{2a^2d^6\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^2(bc^3-ad^3)^{7/3}} + \frac{7ad^3(3bc^3-ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^2(bc^3-ad^3)^{7/3}} + \frac{(9b^2c^6-12abc^3d^3+5a^2d^6)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^2(bc^3-ad^3)^{7/3}} - \frac{4b^2c^4\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{3\sqrt{3}(bc^3-ad^3)^{7/3}} + \frac{bc(bc^3-3ad^3)\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{3\sqrt{3}(bc^3-ad^3)^{7/3}} + \frac{2b^2c^4\log(c^3+d^3x^3)}{9(bc^3-ad^3)^{7/3}} + \frac{a^2d^6\log(c^3+d^3x^3)}{27c^2(bc^3-ad^3)^{7/3}} - \frac{bc(bc^3-3ad^3)\log(c^3+d^3x^3)}{18(bc^3-ad^3)^{7/3}}$$

```
[Out] 3/2*c^4*d^2*(b*x^3+a)^(2/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2-3/2*c^3*d^3*x*(b
*x^3+a)^(2/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2+4/3*b*c^4*d^2*(b*x^3+a)^(2/3)/
(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-1/3*c*d^2*(-3*a*d^3+b*c^3)*(b*x^3+a)^(2/3)/(
-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+1/18*d^3*(-7*a*d^3+3*b*c^3)*x*(b*x^3+a)^(2/3)
/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-1/18*d^3*(-5*a*d^3+9*b*c^3)*x*(b*x^3+a)^(2/
3)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-7/18*d^3*(a*d^3+3*b*c^3)*x*(b*x^3+a)^(2/3
)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-3/2*d*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1
/3,3,5/3,-b*x^3/a,-d^3*x^3/c^3)/c^4/(b*x^3+a)^(1/3)+6/5*d^4*x^5*(1+b*x^3/a)
^(1/3)*AppellF1(5/3,1/3,3,8/3,-b*x^3/a,-d^3*x^3/c^3)/c^7/(b*x^3+a)^(1/3)+2/
9*b^2*c^4*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(7/3)+1/27*a^2*d^6*ln(d^3*x^3+c^3)
/c^2/(-a*d^3+b*c^3)^(7/3)-1/18*b*c*(-3*a*d^3+b*c^3)*ln(d^3*x^3+c^3)/(-a*d^3
+b*c^3)^(7/3)+7/54*a*d^3*(-a*d^3+3*b*c^3)*ln(d^3*x^3+c^3)/c^2/(-a*d^3+b*c^3
)^(7/3)+1/54*(5*a^2*d^6-12*a*b*c^3*d^3+9*b^2*c^6)*ln(d^3*x^3+c^3)/c^2/(-a*d
^3+b*c^3)^(7/3)-1/9*a^2*d^6*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c^
2/(-a*d^3+b*c^3)^(7/3)-7/18*a*d^3*(-a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*
x/c-(b*x^3+a)^(1/3))/c^2/(-a*d^3+b*c^3)^(7/3)-1/18*(5*a^2*d^6-12*a*b*c^3*d^
3+9*b^2*c^6)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c^2/(-a*d^3+b*c^3
)^(7/3)-2/3*b^2*c^4*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^
3)^(7/3)+1/6*b*c*(-3*a*d^3+b*c^3)*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3)
)/(-a*d^3+b*c^3)^(7/3)+2/27*a^2*d^6*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/
c/(b*x^3+a)^(1/3))*3^(1/2))/c^2/(-a*d^3+b*c^3)^(7/3)*3^(1/2)+7/27*a*d^3*(-a
*d^3+3*b*c^3)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(
1/2))/c^2/(-a*d^3+b*c^3)^(7/3)*3^(1/2)+1/27*(5*a^2*d^6-12*a*b*c^3*d^3+9*b^2
*c^6)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/c^
2/(-a*d^3+b*c^3)^(7/3)*3^(1/2)-4/9*b^2*c^4*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3
)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(7/3)*3^(1/2)+1/9*b*c*(-3*a
*d^3+b*c^3)*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2)
)/(-a*d^3+b*c^3)^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 1513, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.895$, Rules

used = {2181, 425, 541, 12, 384, 525, 524, 455, 44, 58, 631, 210, 31, 482, 457, 79, 481}

$$\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx = \frac{2a^2\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)d^6}{9\sqrt{3}c^2\left(bc^3-ad^3\right)^{7/3}} + \frac{a^2\log\left(c^3+d^3x^3\right)d^6}{27c^2\left(bc^3-ad^3\right)^{7/3}} - \frac{a^2\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)d^6}{9c^2\left(bc^3-ad^3\right)^{7/3}} + \frac{6x^5\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},3,\frac{8}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)d^4}{5c^7\sqrt[3]{bx^3+a}} + \frac{7a(3bc^3-ad^3)\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)d^3}{9\sqrt{3}c^2\left(bc^3-ad^3\right)^{7/3}} + \frac{7a(3bc^3-ad^3)\log\left(c^3+d^3x^3\right)d^3}{54c^2\left(bc^3-ad^3\right)^{7/3}} - \frac{7a(3bc^3-ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)d^3}{18c^2\left(bc^3-ad^3\right)^{7/3}} - \frac{7(3bc^3+ad^3)x(bx^3+a)^{2/3}d^3}{18\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} + \frac{(3bc^3-7ad^3)x(bx^3+a)^{2/3}d^3}{18\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} - \frac{(9bc^3-5ad^3)x(bx^3+a)^{2/3}d^3}{18\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} - \frac{3c^3x(bx^3+a)^{2/3}d^3}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2} + \frac{4bc^4(bx^3+a)^{2/3}d^2}{3\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} - \frac{c(bc^3-3ad^3)(bx^3+a)^{2/3}d^2}{3\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} + \frac{3c^4(bx^3+a)^{2/3}d^2}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2} - \frac{3x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},3,\frac{5}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)d}{2c^4\sqrt[3]{bx^3+a}} + \frac{(9b^2c^6-12abd^3c^3+5a^2d^6)\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{9\sqrt{3}c^2\left(bc^3-ad^3\right)^{7/3}} - \frac{4b^2c^4\arctan\left(\frac{1-\frac{2d\sqrt[3]{bx^3+a}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\left(bc^3-ad^3\right)^{7/3}}$$

```
[In] Int[1/((c + d*x)^3*(a + b*x^3)^(1/3)),x]
```

```
[Out] (3*c^4*d^2*(a + b*x^3)^(2/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) - (3*c^
3*d^3*x*(a + b*x^3)^(2/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) + (4*b*c^4
*d^2*(a + b*x^3)^(2/3))/(3*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) - (c*d^2*(b*c
^3 - 3*a*d^3)*(a + b*x^3)^(2/3))/(3*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) + (d
^3*(3*b*c^3 - 7*a*d^3)*x*(a + b*x^3)^(2/3))/(18*(b*c^3 - a*d^3)^2*(c^3 + d^
3*x^3)) - (d^3*(9*b*c^3 - 5*a*d^3)*x*(a + b*x^3)^(2/3))/(18*(b*c^3 - a*d^3)
^2*(c^3 + d^3*x^3)) - (7*d^3*(3*b*c^3 + a*d^3)*x*(a + b*x^3)^(2/3))/(18*(b*
c^3 - a*d^3)^2*(c^3 + d^3*x^3)) - (3*d*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2
/3, 1/3, 3, 5/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(2*c^4*(a + b*x^3)^(1/3))
 + (6*d^4*x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 3, 8/3, -((b*x^3)/a)
, -((d^3*x^3)/c^3)])/(5*c^7*(a + b*x^3)^(1/3)) + (2*a^2*d^6*ArcTan[(1 + (2*
(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^2*(b
*c^3 - a*d^3)^(7/3)) + (7*a*d^3*(3*b*c^3 - a*d^3)*ArcTan[(1 + (2*(b*c^3 - a
*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^2*(b*c^3 - a*d
^3)^(7/3)) + ((9*b^2*c^6 - 12*a*b*c^3*d^3 + 5*a^2*d^6)*ArcTan[(1 + (2*(b*c^
3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^2*(b*c^3
 - a*d^3)^(7/3)) - (4*b^2*c^4*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a
*d^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*(b*c^3 - a*d^3)^(7/3)) + (b*c*(b*c^3 - 3*
a*d^3)*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])
/(3*Sqrt[3]*(b*c^3 - a*d^3)^(7/3)) + (2*b^2*c^4*Log[c^3 + d^3*x^3])/(9*(b*c
^3 - a*d^3)^(7/3)) + (a^2*d^6*Log[c^3 + d^3*x^3])/(27*c^2*(b*c^3 - a*d^3)^(
7/3)) - (b*c*(b*c^3 - 3*a*d^3)*Log[c^3 + d^3*x^3])/(18*(b*c^3 - a*d^3)^(7/3
)) + (7*a*d^3*(3*b*c^3 - a*d^3)*Log[c^3 + d^3*x^3])/(54*c^2*(b*c^3 - a*d^3)
^(7/3)) + ((9*b^2*c^6 - 12*a*b*c^3*d^3 + 5*a^2*d^6)*Log[c^3 + d^3*x^3])/(54
*c^2*(b*c^3 - a*d^3)^(7/3)) - (a^2*d^6*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a
 + b*x^3)^(1/3)])/(9*c^2*(b*c^3 - a*d^3)^(7/3)) - (7*a*d^3*(3*b*c^3 - a*d^3
)*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(18*c^2*(b*c^3 - a*
d^3)^(7/3)) - ((9*b^2*c^6 - 12*a*b*c^3*d^3 + 5*a^2*d^6)*Log[((b*c^3 - a*d^3
)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(18*c^2*(b*c^3 - a*d^3)^(7/3)) - (2*b^2*
c^4*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(3*(b*c^3 - a*d^3)^(7
/3)) + (b*c*(b*c^3 - 3*a*d^3)*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/
3)])/(6*(b*c^3 - a*d^3)^(7/3))
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 44</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\begin{aligned}\text{integral} = \int \Bigg(& \frac{c^6}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^3} - \frac{3c^5dx}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^3} + \frac{6c^4d^2x^2}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^3} \\ & - \frac{7c^3d^3x^3}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^3} + \frac{6c^2d^4x^4}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^3} - \frac{3cd^5x^5}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^3} \\ & + \frac{d^6x^6}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^3} \Bigg) dx\end{aligned}$$

$$= c^6 \int \frac{1}{\sqrt[3]{a+bx^3}\left(c^3+d^3x^3\right)^3}\,dx - \left(3c^5d\right)\int \frac{x}{\sqrt[3]{a+bx^3}\left(c^3+d^3x^3\right)^3}\,dx$$

$$+ \left(6c^4d^2\right)\int \frac{x^2}{\sqrt[3]{a+bx^3}\left(c^3+d^3x^3\right)^3}\,dx$$

$$- \left(7c^3d^3\right)\int \frac{x^3}{\sqrt[3]{a+bx^3}\left(c^3+d^3x^3\right)^3}\,dx + \left(6c^2d^4\right)\int \frac{x^4}{\sqrt[3]{a+bx^3}\left(c^3+d^3x^3\right)^3}\,dx$$

$$- \left(3cd^5\right)\int \frac{x^5}{\sqrt[3]{a+bx^3}\left(c^3+d^3x^3\right)^3}\,dx + d^6\int \frac{x^6}{\sqrt[3]{a+bx^3}\left(c^3+d^3x^3\right)^3}\,dx$$

$$= -\frac{3c^3d^3x\left(a+bx^3\right)^{2/3}}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2} + \left(2c^4d^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\left(c^3+d^3x\right)^3}\,dx, x, x^3\right)$$

$$- \left(cd^5\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}\left(c^3+d^3x\right)^3}\,dx, x, x^3\right)$$

$$+ \frac{c^3\int \frac{6bc^3-5ad^3-3bd^3x^3}{\sqrt[3]{a+bx^3}\left(c^3+d^3x^3\right)^2}\,dx}{6\left(bc^3-ad^3\right)} + \frac{d^3\int \frac{ac^3+3\left(bc^3-2ad^3\right)x^3}{\sqrt[3]{a+bx^3}\left(c^3+d^3x^3\right)^2}\,dx}{6\left(bc^3-ad^3\right)}$$

$$+ \frac{\left(7c^3d^3\right)\int \frac{a-3bx^3}{\sqrt[3]{a+bx^3}\left(c^3+d^3x^3\right)^2}\,dx}{6\left(bc^3-ad^3\right)} - \frac{\left(3c^5d\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}\left(c^3+d^3x^3\right)^3}\,dx}{\sqrt[3]{a+bx^3}}$$

$$+ \frac{\left(6c^2d^4\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x^4}{\sqrt[3]{1+\frac{bx^3}{a}}\left(c^3+d^3x^3\right)^3}\,dx}{\sqrt[3]{a+bx^3}}$$

$$= \frac{3c^4d^2(a+bx^3)^{2/3}}{2(bc^3-ad^3)(c^3+d^3x^3)^2} - \frac{3c^3d^3x(a+bx^3)^{2/3}}{2(bc^3-ad^3)(c^3+d^3x^3)^2}$$
$$+ \frac{d^3(3bc^3-7ad^3)x(a+bx^3)^{2/3}}{18(bc^3-ad^3)^2(c^3+d^3x^3)} - \frac{d^3(9bc^3-5ad^3)x(a+bx^3)^{2/3}}{18(bc^3-ad^3)^2(c^3+d^3x^3)}$$
$$- \frac{7d^3(3bc^3+ad^3)x(a+bx^3)^{2/3}}{18(bc^3-ad^3)^2(c^3+d^3x^3)} - \frac{3dx^2\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{3},3;\frac{5}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^4\sqrt[3]{a+bx^3}}$$
$$+ \frac{6d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{5}{3};\frac{1}{3},3;\frac{8}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{5c^7\sqrt[3]{a+bx^3}} + \frac{\int \frac{2(9b^2c^6-12abc^3d^3+5a^2d^6)}{\sqrt[3]{a+bx^3}(c^3+d^3x^3)}\,dx}{18(bc^3-ad^3)^2}$$
$$+ \frac{(7d^3)\int \frac{2a(3bc^3-ad^3)}{\sqrt[3]{a+bx^3}(c^3+d^3x^3)}\,dx}{18(bc^3-ad^3)^2} + \frac{d^3\int \frac{4a^2c^3d^3}{\sqrt[3]{a+bx^3}(c^3+d^3x^3)}\,dx}{18c^3(bc^3-ad^3)^2}$$
$$+ \frac{(4bc^4d^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c^3+d^3x)^2}\,dx,x,x^3\right)}{3(bc^3-ad^3)}$$
$$- \frac{(cd^2(bc^3-3ad^3))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c^3+d^3x)^2}\,dx,x,x^3\right)}{3(bc^3-ad^3)}$$

$$
\begin{aligned}
&= \frac{3c^4d^2(a+bx^3)^{2/3}}{2(bc^3-ad^3)(c^3+d^3x^3)^2} - \frac{3c^3d^3x(a+bx^3)^{2/3}}{2(bc^3-ad^3)(c^3+d^3x^3)^2} \\
&\quad + \frac{4bc^4d^2(a+bx^3)^{2/3}}{3(bc^3-ad^3)^2(c^3+d^3x^3)} - \frac{cd^2(bc^3-3ad^3)(a+bx^3)^{2/3}}{3(bc^3-ad^3)^2(c^3+d^3x^3)} \\
&\quad + \frac{d^3(3bc^3-7ad^3)x(a+bx^3)^{2/3}}{18(bc^3-ad^3)^2(c^3+d^3x^3)} - \frac{d^3(9bc^3-5ad^3)x(a+bx^3)^{2/3}}{18(bc^3-ad^3)^2(c^3+d^3x^3)} \\
&\quad - \frac{7d^3(3bc^3+ad^3)x(a+bx^3)^{2/3}}{18(bc^3-ad^3)^2(c^3+d^3x^3)} - \frac{3dx^2\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{3},3;\frac{5}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^4\sqrt[3]{a+bx^3}} \\
&\quad + \frac{6d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{5}{3};\frac{1}{3},3;\frac{8}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{5c^7\sqrt[3]{a+bx^3}} \\
&\quad + \frac{(4b^2c^4d^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c^3+d^3x)}\,dx,x,x^3\right)}{9(bc^3-ad^3)^2} + \frac{(2a^2d^6)\int \frac{1}{\sqrt[3]{a+bx^3}(c^3+d^3x^3)}\,dx}{9(bc^3-ad^3)^2} \\
&\quad - \frac{(bcd^2(bc^3-3ad^3))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c^3+d^3x)}\,dx,x,x^3\right)}{9(bc^3-ad^3)^2} \\
&\quad + \frac{(7ad^3(3bc^3-ad^3))\int \frac{1}{\sqrt[3]{a+bx^3}(c^3+d^3x^3)}\,dx}{9(bc^3-ad^3)^2} \\
&\quad + \frac{(9b^2c^6-12abc^3d^3+5a^2d^6)\int \frac{1}{\sqrt[3]{a+bx^3}(c^3+d^3x^3)}\,dx}{9(bc^3-ad^3)^2}
\end{aligned}
$$

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Mathematica [F]

$$\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx$$

```
[In] Integrate[1/((c + d*x)^3*(a + b*x^3)^(1/3)),x]
[Out] Integrate[1/((c + d*x)^3*(a + b*x^3)^(1/3)), x]
```

Maple [F]

$$\int \frac{1}{\left(dx+c\right)^{3} \left(b\,x^{3}+a\right)^{\frac{1}{3}}}dx$$

[In] int(1/(d*x+c)^3/(b*x^3+a)^(1/3),x)

[Out] int(1/(d*x+c)^3/(b*x^3+a)^(1/3),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx)^3}\,dx$$

[In] integrate(1/(d*x+c)**3/(b*x**3+a)**(1/3),x)

[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x)**3), x)

Maxima [F]

$$\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx+c)^3}\,dx$$

[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^3), x)

Giac [F]

$$\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx+c)^3}\,dx$$

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx=\int \frac{1}{(b\,x^3+a)^{1/3}\,(c+d\,x)^3}\,dx$$

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x)^3),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x)^3), x)
```

3.36 $\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}} dx$

Optimal result

Integrand size = 19, antiderivative size = 306

$$\begin{aligned}\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}} dx =& \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b}+\frac{d^4x^2\sqrt[3]{a+bx^3}}{3b}-\frac{4c^3d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}\\&+\frac{2ad^4\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}}+\frac{c^4x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\\&+\frac{cd^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\\&-\frac{2c^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+\frac{ad^4\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}}\end{aligned}$$

```
[Out] 6*c^2*d^2*(b*x^3+a)^(1/3)/b+1/3*d^4*x^2*(b*x^3+a)^(1/3)/b+c^4*x*(1+b*x^3/a)
^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)+c*d^3*x^4*(1+b*
x^3/a)^(2/3)*hypergeom([2/3, 4/3],[7/3],-b*x^3/a)/(b*x^3+a)^(2/3)-2*c^3*d*l
n(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)+1/3*a*d^4*ln(b^(1/3)*x-(b*x^3+a)^(1/3)
)/b^(5/3)-4/3*c^3*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(
2/3)*3^(1/2)+2/9*a*d^4*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/
b^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1907, 252, 251, 337, 267, 372, 371, 327}

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = -\frac{4c^3d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{2ad^4\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}} - \frac{2c^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+\frac{ad^4\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}} + \frac{c^4x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}+\frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{cd^3x^4\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}+\frac{d^4x^2\sqrt[3]{a+bx^3}}{3b}$$

```
[In] Int[(c + d*x)^4/(a + b*x^3)^(2/3),x]
```

```
[Out] (6*c^2*d^2*(a + b*x^3)^(1/3))/b + (d^4*x^2*(a + b*x^3)^(1/3))/(3*b) - (4*c^
3*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)
) + (2*a*d^4*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt
[3]*b^(5/3)) + (c^4*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3
, -((b*x^3)/a)])/(a + b*x^3)^(2/3) + (c*d^3*x^4*(1 + (b*x^3)/a)^(2/3)*Hyper
geometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (2*c^3*d*Log
[b^(1/3)*x - (a + b*x^3)^(1/3)])/b^(2/3) + (a*d^4*Log[b^(1/3)*x - (a + b*x^
3)^(1/3)])/(3*b^(5/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c^4}{(a+bx^3)^{2/3}}+\frac{4c^3dx}{(a+bx^3)^{2/3}}+\frac{6c^2d^2x^2}{(a+bx^3)^{2/3}}+\frac{4cd^3x^3}{(a+bx^3)^{2/3}}+\frac{d^4x^4}{(a+bx^3)^{2/3}}\right)\,dx\\
&= c^4\int \frac{1}{(a+bx^3)^{2/3}}\,dx+\left(4c^3d\right)\int \frac{x}{(a+bx^3)^{2/3}}\,dx\\
&\quad +\left(6c^2d^2\right)\int \frac{x^2}{(a+bx^3)^{2/3}}\,dx+\left(4cd^3\right)\int \frac{x^3}{(a+bx^3)^{2/3}}\,dx+d^4\int \frac{x^4}{(a+bx^3)^{2/3}}\,dx
\end{aligned}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} - \frac{4c^3d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} - \frac{2c^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}} - \frac{(2ad^4)\int\frac{x}{(a+bx^3)^{2/3}}\,dx}{3b} + \frac{\left(c^4\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int\frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{(a+bx^3)^{2/3}} + \frac{\left(4cd^3\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int\frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{(a+bx^3)^{2/3}}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} - \frac{4c^3d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{2ad^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}} + \frac{c^4x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} + \frac{cd^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} - \frac{2c^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}} + \frac{ad^4\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}}$$

Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.54

$$\int\frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = \frac{3bc^4x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d\left((6bc^3-ad^3)x^2\,\text{Hypergeom}\right.}{}$$

```
[In] Integrate[(c + d*x)^4/(a + b*x^3)^(2/3),x]
```

```
[Out] (3*b*c^4*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)
/a)] + d*((6*b*c^3 - a*d^3)*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a +
 b*x^3)] + d*((18*c^2 + d^2*x^2)*(a + b*x^3) + 3*b*c*d*x^4*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])))/(3*b*(a + b*x^3)^(2
/3))
```

Maple [F]

$$\int \frac{(dx+c)^4}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

```
[In] int((d*x+c)^4/(b*x^3+a)^(2/3),x)
[Out] int((d*x+c)^4/(b*x^3+a)^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = \text{Timed out}$$

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = 6c^2d^2\left(\begin{cases} \frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b=0 \\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise} \end{cases}\right) + \frac{c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{4c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)} + \frac{4cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)} + \frac{d^4x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate((d*x+c)**4/(b*x**3+a)**(2/3),x)
[Out] 6*c**2*d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b
, True)) + c**4*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*a**(2/3)*gamma(4/3)) + 4*c**3*d*x**2*gamma(2/3)*hyper((2/3, 2/3),
(5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3)) + 4*c*d**3*x**4*g
amma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*g
amma(7/3)) + d**4*x**5*gamma(5/3)*hyper((2/3, 5/3), (8/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*a**(2/3)*gamma(8/3))
```

Maxima [F]

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^4}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] integrate((d*x + c)^4/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^4}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate((d*x + c)^4/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = \int \frac{(c+d\,x)^4}{(b\,x^3+a)^{2/3}}\,dx$$

```
[In] int((c + d*x)^4/(a + b*x^3)^(2/3),x)
[Out] int((c + d*x)^4/(a + b*x^3)^(2/3), x)
```

3.37 $\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 187

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \frac{3cd^2\sqrt[3]{a+bx^3}}{b}+\frac{d^3x\sqrt[3]{a+bx^3}}{2b}-\frac{\sqrt{3}c^2d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{(2bc^3-ad^3)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}}-\frac{3c^2d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

```
[Out] 3*c*d^2*(b*x^3+a)^(1/3)/b+1/2*d^3*x*(b*x^3+a)^(1/3)/b+1/2*(-a*d^3+2*b*c^3)*
x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/3)-
3/2*c^2*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-c^2*d*arctan(1/3*(1+2*b^(1/
3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used

$= \{1902, 1900, 267, 1907, 252, 251, 337\}$

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = -\frac{\sqrt{3}c^2 d \arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{b^{2/3}} - \frac{3c^2 d \log\left(\sqrt[3]{b}x - \sqrt[3]{a+bx^3}\right)}{2b^{2/3}} + \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc^3-ad^3)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}} + \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b}$$

```
[In] Int[(c + d*x)^3/(a + b*x^3)^(2/3),x]
```

```
[Out] (3*c*d^2*(a + b*x^3)^(1/3))/b + (d^3*x*(a + b*x^3)^(1/3))/(2*b) - (Sqrt[3]*
c^2*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/b^(2/3) + ((2*
b*c^3 - a*d^3)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((
b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3)) - (3*c^2*d*Log[b^(1/3)*x - (a + b*x^3)^
(1/3)])/(2*b^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^3x\sqrt[3]{a+bx^3}}{2b}+\frac{\int \frac{2bc^3-ad^3+6bc^2dx+6bcd^2x^2}{(a+bx^3)^{2/3}}\,dx}{2b}\\
&= \frac{d^3x\sqrt[3]{a+bx^3}}{2b}+\frac{\int \frac{2bc^3-ad^3+6bc^2dx}{(a+bx^3)^{2/3}}\,dx}{2b}+\left(3cd^2\right)\int \frac{x^2}{(a+bx^3)^{2/3}}\,dx\\
&= \frac{3cd^2\sqrt[3]{a+bx^3}}{b}+\frac{d^3x\sqrt[3]{a+bx^3}}{2b}+\frac{\int \left(\frac{2bc^3\left(1-\frac{ad^3}{2bc^3}\right)}{(a+bx^3)^{2/3}}+\frac{6bc^2dx}{(a+bx^3)^{2/3}}\right)dx}{2b}\\
&= \frac{3cd^2\sqrt[3]{a+bx^3}}{b}+\frac{d^3x\sqrt[3]{a+bx^3}}{2b}+\left(3c^2d\right)\int \frac{x}{(a+bx^3)^{2/3}}\,dx+\frac{\left(2bc^3-ad^3\right)\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{2b}\\
&= \frac{3cd^2\sqrt[3]{a+bx^3}}{b}+\frac{d^3x\sqrt[3]{a+bx^3}}{2b}-\frac{\sqrt{3}c^2d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{b^{2/3}}\\
&\quad -\frac{3c^2d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}+\frac{\left(\left(2bc^3-ad^3\right)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{2b\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} - \frac{\sqrt{3}c^2d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{(2bc^3-ad^3)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}} - \frac{3c^2d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.78

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \frac{4bc^3x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d\left(6bc^2x^2\,\text{Hypergeometric2F1}\right.}{4b\,(}$$

```
[In] Integrate[(c + d*x)^3/(a + b*x^3)^(2/3),x]
[Out] (4*b*c^3*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)
/a)] + d*(6*b*c^2*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a + b*x^3)] +
 d*(12*c*(a + b*x^3) + b*d*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3,
 4/3, 7/3, -((b*x^3)/a)])))/(4*b*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{(dx+c)^3}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

```
[In] int((d*x+c)^3/(b*x^3+a)^(2/3),x)
[Out] int((d*x+c)^3/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^3}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)/(b*x^3 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = 3cd^2\left(\begin{cases} \frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b=0 \\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise} \end{cases}\right) + \frac{c^3x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{2}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{c^2dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)} + \frac{d^3x^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((d*x+c)**3/(b*x**3+a)**(2/3),x)
```

```
[Out] 3*c*d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b, T
rue)) + c**3*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*a**(2/3)*gamma(4/3)) + c**2*d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,
), b*x**3*exp_polar(I*pi)/a)/(a**(2/3)*gamma(5/3)) + d**3*x**4*gamma(4/3)*h
yper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^3}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^3/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^3}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx=\int \frac{{\left(c+d\,x\right)}^3}{{\left(b\,x^3+a\right)}^{2/3}} \,d x$$

```
[In] int((c + d*x)^3/(a + b*x^3)^(2/3),x)
[Out] int((c + d*x)^3/(a + b*x^3)^(2/3), x)
```

3.38 $\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 141

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \frac{d^2\sqrt[3]{a+bx^3}}{b} - \frac{2cd\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} - \frac{cd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}$$

```
[Out] d^2*(b*x^3+a)^(1/3)/b+c^2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b
*x^3/a)/(b*x^3+a)^(2/3)-c*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-2/3*c*d*a
rctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1900, 267, 1907, 252, 251, 337}

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = -\frac{2cd\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} - \frac{cd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}} + \frac{c^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} + \frac{d^2\sqrt[3]{a+bx^3}}{b}$$

```
[In] Int[(c + d*x)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (d^2*(a + b*x^3)^(1/3))/b - (2*c*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1
/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeomet
ric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (c*d*Log[b^(1/3)*x
 - (a + b*x^3)^(1/3)])/b^(2/3)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\text{integral} = d^2 \int \frac{x^2}{(a+bx^3)^{2/3}}\,dx + \int \frac{c^2+2cdx}{(a+bx^3)^{2/3}}\,dx$$

$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} + \int \left(\frac{c^2}{(a+bx^3)^{2/3}} + \frac{2cdx}{(a+bx^3)^{2/3}}\right)dx$$

$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} + c^2 \int \frac{1}{(a+bx^3)^{2/3}}\,dx + (2cd)\int \frac{x}{(a+bx^3)^{2/3}}\,dx$$

$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} - \frac{2cd\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} - \frac{cd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}} + \frac{\left(c^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{(a+bx^3)^{2/3}}$$

$$= \frac{d^2\sqrt[3]{a+bx^3}}{b} - \frac{2cd\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} - \frac{cd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}$$

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \frac{bc^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d\left(d(a+bx^3)+bcx^2\,\text{Hypergeom}\right.}{b\,(a+bx^3)^{2/3}}$$

```
[In] Integrate[(c + d*x)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (b*c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a
)] + d*(d*(a + b*x^3) + b*c*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a +
 b*x^3)]))/(b*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{(dx+c)^2}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

```
[In] int((d*x+c)^2/(b*x^3+a)^(2/3),x)
[Out] int((d*x+c)^2/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)/(b*x^3 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.77

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = d^2\left(\begin{cases} \frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b=0 \\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise} \end{cases}\right) + \frac{c^2x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix} \frac{1}{3},\frac{2}{3} \\ \frac{4}{3} \end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix} \frac{2}{3},\frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((d*x+c)**2/(b*x**3+a)**(2/3),x)
[Out] d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b, True)
) + c**2*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*a**(2/3)*gamma(4/3)) + 2*c*d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3))
```

Maxima [F]

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] integrate((d*x + c)^2/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate((d*x + c)^2/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(c+d\,x)^2}{(b\,x^3+a)^{2/3}}\,dx$$

```
[In] int((c + d*x)^2/(a + b*x^3)^(2/3),x)
[Out] int((c + d*x)^2/(a + b*x^3)^(2/3), x)
```

3.39 $\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = -\frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} - \frac{d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

```
[Out] c*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)-
1/2*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-1/3*d*arctan(1/3*(1+2*b^(1/3)*x
/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1907, 252, 251, 337}

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = -\frac{d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} - \frac{d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}} + \frac{cx\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}$$

```
[In] Int[(c + d*x)/(a + b*x^3)^(2/3),x]
```

```
[Out] -((d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3
))) + (c*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)
/a)])/(a + b*x^3)^(2/3) - (d*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*b^(2/3)
)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{c}{(a+bx^3)^{2/3}}+\frac{dx}{(a+bx^3)^{2/3}}\right) dx\\
&= c\int \frac{1}{(a+bx^3)^{2/3}}\,dx+d\int \frac{x}{(a+bx^3)^{2/3}}\,dx\\
&= -\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}+\frac{\left(c\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{(a+bx^3)^{2/3}}
\end{aligned}
$$

$$= -\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} - \frac{d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

Mathematica [A] (verified)

Time = 9.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \frac{x\left(2c\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\right.}{2\,(a+bx^3)^{2/3}}$$

```
[In] Integrate[(c + d*x)/(a + b*x^3)^(2/3),x]
```

```
[Out] (x*(2*c*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)
] + d*x*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a + b*x^3)]))/(2*(a + b*x^3
)^(2/3))
```

Maple [F]

$$\int \frac{dx+c}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

```
[In] int((d*x+c)/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x+c)/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((d*x+c)/(b*x**3+a)**(2/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2
/3)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(2/3)*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \int \frac{c+d\,x}{(b\,x^3+a)^{2/3}}\,dx$$

```
[In] int((c + d*x)/(a + b*x^3)^(2/3),x)
[Out] int((c + d*x)/(a + b*x^3)^(2/3), x)
```

3.40 $\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 332

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\left(a+bx^3\right)^{2/3}} + \frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\left(bc^3-ad^3\right)^{2/3}} - \frac{d\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\left(bc^3-ad^3\right)^{2/3}} - \frac{d\log\left(c^3+d^3x^3\right)}{3\left(bc^3-ad^3\right)^{2/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{2/3}} + \frac{d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d^3*x^3/c^3)/c/(b*x^3+
a)^(2/3)-1/3*d*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(2/3)+1/2*d*ln((-a*d^3+b*c^3)
^(1/3)*x/c-(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(2/3)+1/2*d*ln((-a*d^3+b*c^3)^(1
/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(2/3)+1/3*d*arctan(1/3*(1+2*(-a*d^3+b
*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(2/3)*3^(1/2)-1/3*
d*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(-a*d^3+
b*c^3)^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {2181, 441, 440, 503, 455, 60, 631, 210, 31}

$$\int \frac{1}{(c+dx)(a+bx^3)^{2/3}}\,dx = \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\,(a+bx^3)^{2/3}} + \frac{d\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{2/3}} - \frac{d\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{2/3}} - \frac{d\log\left(c^3+d^3x^3\right)}{3\,(bc^3-ad^3)^{2/3}} + \frac{d\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2\,(bc^3-ad^3)^{2/3}} + \frac{d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\,(bc^3-ad^3)^{2/3}}$$

```
[In] Int[1/((c + d*x)*(a + b*x^3)^(2/3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d^3*x^
3)/c^3)])/(c*(a + b*x^3)^(2/3)) + (d*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x
)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*(b*c^3 - a*d^3)^(2/3)) - (d*Arc
Tan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*
(b*c^3 - a*d^3)^(2/3)) - (d*Log[c^3 + d^3*x^3])/(3*(b*c^3 - a*d^3)^(2/3)) +
 (d*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(2*(b*c^3 - a*d^3
)^(2/3)) + (d*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(2*(b*c^3 -
 a*d^3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\text{integral} = \int \left(\frac{c^2}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)} - \frac{cdx}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)} + \frac{d^2x^2}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)} \right) dx$$

$$= c^2 \int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)}\,dx - (cd) \int \frac{x}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)}\,dx + d^2 \int \frac{x^2}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)}\,dx$$

$$= \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\left(bc^3-ad^3\right)^{2/3}} - \frac{d\log\left(c^3+d^3x^3\right)}{6\left(bc^3-ad^3\right)^{2/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc^3-ad^3}\,x}{c} - \sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{2/3}} + \frac{1}{3}d^2\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}\left(c^3+d^3x\right)}\,dx, x, x^3\right) + \frac{\left(c^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c^3+d^3x^3\right)}\,dx}{\left(a+bx^3\right)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\left(a+bx^3\right)^{2/3}} + \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\left(bc^3-ad^3\right)^{2/3}} - \frac{d\log\left(c^3+d^3x^3\right)}{3\left(bc^3-ad^3\right)^{2/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc^3-ad^3}\,x}{c} - \sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{2/3}} + \frac{d\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc^3-ad^3}}{d}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\left(bc^3-ad^3\right)^{2/3}}{d^2} - \frac{\sqrt[3]{bc^3-ad^3}\,x}{d} + x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\left(a+bx^3\right)^{2/3}} + \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\left(bc^3-ad^3\right)^{2/3}} - \frac{d\log\left(c^3+d^3x^3\right)}{3\left(bc^3-ad^3\right)^{2/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{2/3}} + \frac{d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{2/3}} + \frac{d\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}\right)}{\left(bc^3-ad^3\right)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\left(a+bx^3\right)^{2/3}} + \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\left(bc^3-ad^3\right)^{2/3}} - \frac{d\tan^{-1}\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\left(bc^3-ad^3\right)^{2/3}} - \frac{d\log\left(c^3+d^3x^3\right)}{3\left(bc^3-ad^3\right)^{2/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{2/3}} + \frac{d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{2/3}}$$

Mathematica [F]

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx = \int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx$$

```
[In] Integrate[1/((c + d*x)*(a + b*x^3)^(2/3)),x]
[Out] Integrate[1/((c + d*x)*(a + b*x^3)^(2/3)), x]
```

Maple [F]

$$\int \frac{1}{(dx+c)\,(b\,x^{3}+a)^{\frac{2}{3}}}dx$$

```
[In] int(1/(d*x+c)/(b*x^3+a)^(2/3),x)
[Out] int(1/(d*x+c)/(b*x^3+a)^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\,(a+bx^3)^{2/3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)\,(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(a+bx^3)^{\frac{2}{3}}\,(c+dx)}\,dx$$

```
[In] integrate(1/(d*x+c)/(b*x**3+a)**(2/3),x)
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\,(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx+c)}\,dx$$

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx+c)}\,dx$$

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(c+d\,x)}\,dx$$

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x)),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x)), x)
```

3.41 $\int \frac{1}{(c+dx)^2 \left(a+bx^3\right)^{2/3}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 760

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \frac{c^2d^2\sqrt[3]{a+bx^3}}{(bc^3-ad^3)(c^3+d^3x^3)} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{(bc^3-ad^3)(c^3+d^3x^3)} + \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2(a+bx^3)^{2/3}} - \frac{d^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5(a+bx^3)^{2/3}} + \frac{2ad^4\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c(bc^3-ad^3)^{5/3}} + \frac{2d(3bc^3-ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c(bc^3-ad^3)^{5/3}} - \frac{2bc^2d\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}(bc^3-ad^3)^{5/3}} - \frac{bc^2d\log(c^3+d^3x^3)}{3(bc^3-ad^3)^{5/3}} - \frac{ad^4\log(c^3+d^3x^3)}{9c(bc^3-ad^3)^{5/3}} - \frac{d(3bc^3-ad^3)\log(c^3+d^3x^3)}{9c(bc^3-ad^3)^{5/3}} + \frac{ad^4\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c(bc^3-ad^3)^{5/3}} + \frac{d(3bc^3-ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c(bc^3-ad^3)^{5/3}} + \frac{bc^2d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{(bc^3-ad^3)^{5/3}}$$

```
[Out] c^2*d^2*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)+d^4*x^2*(b*x^3+a)^(1/3
)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)+x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,2,4/3,-
b*x^3/a,-d^3*x^3/c^3)/c^2/(b*x^3+a)^(2/3)-1/2*d^3*x^4*(1+b*x^3/a)^(2/3)*App
ellF1(4/3,2/3,2,7/3,-b*x^3/a,-d^3*x^3/c^3)/c^5/(b*x^3+a)^(2/3)-1/3*b*c^2*d*
ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(5/3)-1/9*a*d^4*ln(d^3*x^3+c^3)/c/(-a*d^3+b*
c^3)^(5/3)-1/9*d*(-a*d^3+3*b*c^3)*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)^(5/3)+1/
3*a*d^4*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-a*d^3+b*c^3)^(5/3)
+1/3*d*(-a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-a*
d^3+b*c^3)^(5/3)+b*c^2*d*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3
+b*c^3)^(5/3)+2/9*a*d^4*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^
(1/3))*3^(1/2))/c/(-a*d^3+b*c^3)^(5/3)*3^(1/2)+2/9*d*(-a*d^3+3*b*c^3)*arcta
n(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/c/(-a*d^3+b*c
```

```
^3)^(5/3)*3^(1/2)-2/3*b*c^2*d*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c
^3)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {2181, 441, 440, 483, 12, 503, 455, 44, 60, 631, 210, 31, 525, 524, 482}

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}} \, dx = -\frac{d^3 x^4 \left(\frac{bx^3}{a}+1\right)^{2/3} \text{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5 (a+bx^3)^{2/3}} + \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} \text{AppellF1}\left(\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2 (a+bx^3)^{2/3}} + \frac{2d(3bc^3-ad^3)\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{5/3}} + \frac{2ad^4\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{5/3}} - \frac{2bc^2 d\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{5/3}} - \frac{d(3bc^3-ad^3)\log\left(c^3+d^3x^3\right)}{9c\,(bc^3-ad^3)^{5/3}} + \frac{d(3bc^3-ad^3)\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{3c\,(bc^3-ad^3)^{5/3}} - \frac{ad^4\log\left(c^3+d^3x^3\right)}{9c\,(bc^3-ad^3)^{5/3}} + \frac{ad^4\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{3c\,(bc^3-ad^3)^{5/3}} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)(bc^3-ad^3)} - \frac{bc^2 d\log\left(c^3+d^3x^3\right)}{3\,(bc^3-ad^3)^{5/3}} + \frac{bc^2 d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{(bc^3-ad^3)^{5/3}} + \frac{c^2d^2\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)(bc^3-ad^3)}$$

```
[In] Int[1/((c + d*x)^2*(a + b*x^3)^(2/3)),x]
```

```
[Out] (c^2*d^2*(a + b*x^3)^(1/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) + (d^4*x^2*(a
 + b*x^3)^(1/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) + (x*(1 + (b*x^3)/a)^(2/
```

```
3)*AppellF1[1/3, 2/3, 2, 4/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(c^2*(a + b*
x^3)^(2/3)) - (d^3*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 2, 7/3, -((
b*x^3)/a), -((d^3*x^3)/c^3)])/(2*c^5*(a + b*x^3)^(2/3)) + (2*a*d^4*ArcTan[(
1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]
*c*(b*c^3 - a*d^3)^(5/3)) + (2*d*(3*b*c^3 - a*d^3)*ArcTan[(1 + (2*(b*c^3 -
a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*(b*c^3 - a*d^
3)^(5/3)) - (2*b*c^2*d*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c^3 - a*d^3)^(5/3)) - (b*c^2*d*Log[c^3 + d^3*x
^3])/(3*(b*c^3 - a*d^3)^(5/3)) - (a*d^4*Log[c^3 + d^3*x^3])/(9*c*(b*c^3 - a
*d^3)^(5/3)) - (d*(3*b*c^3 - a*d^3)*Log[c^3 + d^3*x^3])/(9*c*(b*c^3 - a*d^3
)^(5/3)) + (a*d^4*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(3*
c*(b*c^3 - a*d^3)^(5/3)) + (d*(3*b*c^3 - a*d^3)*Log[((b*c^3 - a*d^3)^(1/3)*
x)/c - (a + b*x^3)^(1/3)])/(3*c*(b*c^3 - a*d^3)^(5/3)) + (b*c^2*d*Log[(b*c^
3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(b*c^3 - a*d^3)^(5/3)
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 44</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

<u>Rule 60</u>

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
```

```
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\text{integral} = \int \left(\frac{c^4}{(a+bx^3)^{2/3}(c^3+d^3x^3)^2} - \frac{2c^3dx}{(a+bx^3)^{2/3}(c^3+d^3x^3)^2} + \frac{3c^2d^2x^2}{(a+bx^3)^{2/3}(c^3+d^3x^3)^2} - \frac{2cd^3x^3}{(a+bx^3)^{2/3}(c^3+d^3x^3)^2} + \frac{d^4x^4}{(a+bx^3)^{2/3}(c^3+d^3x^3)^2} \right) dx$$

$$
\begin{aligned}
&= c^4 \int \frac{1}{(a+bx^3)^{2/3}(c^3+d^3x^3)^2}\,dx \\
&\quad -(2c^3d)\int \frac{x}{(a+bx^3)^{2/3}(c^3+d^3x^3)^2}\,dx+(3c^2d^2)\int \frac{x^2}{(a+bx^3)^{2/3}(c^3+d^3x^3)^2}\,dx \\
&\quad -(2cd^3)\int \frac{x^3}{(a+bx^3)^{2/3}(c^3+d^3x^3)^2}\,dx+d^4\int \frac{x^4}{(a+bx^3)^{2/3}(c^3+d^3x^3)^2}\,dx \\
&= \frac{d^4x^2\sqrt[3]{a+bx^3}}{(bc^3-ad^3)(c^3+d^3x^3)} \\
&\quad +(c^2d^2)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c^3+d^3x)^2}\,dx,x,x^3\right)-\frac{(2d)\int \frac{(3bc^3-ad^3)x}{(a+bx^3)^{2/3}(c^3+d^3x^3)}\,dx}{3(bc^3-ad^3)} \\
&\quad -\frac{d^4\int \frac{2ax}{(a+bx^3)^{2/3}(c^3+d^3x^3)}\,dx}{3(bc^3-ad^3)}+\frac{\left(c^4\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}(c^3+d^3x^3)^2}\,dx}{(a+bx^3)^{2/3}} \\
&\quad -\frac{\left(2cd^3\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{2/3}(c^3+d^3x^3)^2}\,dx}{(a+bx^3)^{2/3}} \\
&= \frac{c^2d^2\sqrt[3]{a+bx^3}}{(bc^3-ad^3)(c^3+d^3x^3)}+\frac{d^4x^2\sqrt[3]{a+bx^3}}{(bc^3-ad^3)(c^3+d^3x^3)} \\
&\quad +\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2(a+bx^3)^{2/3}} \\
&\quad -\frac{d^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5(a+bx^3)^{2/3}} \\
&\quad +\frac{(2bc^2d^2)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c^3+d^3x)}\,dx,x,x^3\right)}{3(bc^3-ad^3)} \\
&\quad -\frac{(2ad^4)\int \frac{x}{(a+bx^3)^{2/3}(c^3+d^3x^3)}\,dx}{3(bc^3-ad^3)}-\frac{(2d(3bc^3-ad^3))\int \frac{x}{(a+bx^3)^{2/3}(c^3+d^3x^3)}\,dx}{3(bc^3-ad^3)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{c^2 d^2 \sqrt[3]{a+bx^3}}{(bc^3-ad^3)\,(c^3+d^3x^3)} + \frac{d^4 x^2 \sqrt[3]{a+bx^3}}{(bc^3-ad^3)\,(c^3+d^3x^3)} \\
&+ \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2\,(a+bx^3)^{2/3}} \\
&- \frac{d^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5\,(a+bx^3)^{2/3}} \\
&+ \frac{2ad^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{5/3}} + \frac{2d(3bc^3-ad^3)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{5/3}} \\
&- \frac{bc^2d\log\left(c^3+d^3x^3\right)}{3\,(bc^3-ad^3)^{5/3}} - \frac{ad^4\log\left(c^3+d^3x^3\right)}{9c\,(bc^3-ad^3)^{5/3}} \\
&- \frac{d(3bc^3-ad^3)\log\left(c^3+d^3x^3\right)}{9c\,(bc^3-ad^3)^{5/3}} + \frac{ad^4\log\left(\frac{\sqrt[3]{bc^3-ad^3}\,x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\,(bc^3-ad^3)^{5/3}} \\
&+ \frac{d(3bc^3-ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}\,x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\,(bc^3-ad^3)^{5/3}} \\
&+ \frac{(bc^2d)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc^3-ad^3}}{d}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{(bc^3-ad^3)^{5/3}} \\
&+ \frac{(bc^2)\,\text{Subst}\left(\int \frac{1}{\frac{(bc^3-ad^3)^{2/3}}{d^2}-\frac{\sqrt[3]{bc^3-ad^3}\,x}{d}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{(bc^3-ad^3)^{4/3}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{c^2 d^2 \sqrt[3]{a+bx^3}}{(bc^3-ad^3)(c^3+d^3x^3)} + \frac{d^4 x^2 \sqrt[3]{a+bx^3}}{(bc^3-ad^3)(c^3+d^3x^3)} \\
&+ \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2(a+bx^3)^{2/3}} \\
&- \frac{d^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5(a+bx^3)^{2/3}} \\
&+ \frac{2ad^4 \tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c(bc^3-ad^3)^{5/3}} + \frac{2d(3bc^3-ad^3)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c(bc^3-ad^3)^{5/3}} \\
&- \frac{bc^2d\log(c^3+d^3x^3)}{3(bc^3-ad^3)^{5/3}} - \frac{ad^4\log(c^3+d^3x^3)}{9c(bc^3-ad^3)^{5/3}} \\
&- \frac{d(3bc^3-ad^3)\log(c^3+d^3x^3)}{9c(bc^3-ad^3)^{5/3}} + \frac{ad^4\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c(bc^3-ad^3)^{5/3}} \\
&+ \frac{d(3bc^3-ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c(bc^3-ad^3)^{5/3}} \\
&+ \frac{bc^2d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{(bc^3-ad^3)^{5/3}} \\
&+ \frac{(2bc^2d)\,\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}\right)}{(bc^3-ad^3)^{5/3}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{c^2d^2\sqrt[3]{a+bx^3}}{(bc^3-ad^3)\,(c^3+d^3x^3)} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{(bc^3-ad^3)\,(c^3+d^3x^3)} \\
&+ \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2\,(a+bx^3)^{2/3}} \\
&- \frac{d^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5\,(a+bx^3)^{2/3}} \\
&+ \frac{2ad^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{5/3}} + \frac{2d(3bc^3-ad^3)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{5/3}} \\
&- \frac{2bc^2d\tan^{-1}\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{5/3}} - \frac{bc^2d\log\left(c^3+d^3x^3\right)}{3\,(bc^3-ad^3)^{5/3}} - \frac{ad^4\log\left(c^3+d^3x^3\right)}{9c\,(bc^3-ad^3)^{5/3}} \\
&- \frac{d(3bc^3-ad^3)\log\left(c^3+d^3x^3\right)}{9c\,(bc^3-ad^3)^{5/3}} + \frac{ad^4\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\,(bc^3-ad^3)^{5/3}} \\
&+ \frac{d(3bc^3-ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\,(bc^3-ad^3)^{5/3}} \\
&+ \frac{bc^2d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{(bc^3-ad^3)^{5/3}}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{1}{(c+dx)^2\,(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(c+dx)^2\,(a+bx^3)^{2/3}}\,dx
$$

```
[In] Integrate[1/((c + d*x)^2*(a + b*x^3)^(2/3)),x]
[Out] Integrate[1/((c + d*x)^2*(a + b*x^3)^(2/3)), x]
```

Maple [F]

$$\int \frac{1}{(dx+c)^2 (b\,x^3+a)^{\frac{2}{3}}}dx$$

```
[In] int(1/(d*x+c)^2/(b*x^3+a)^(2/3),x)
[Out] int(1/(d*x+c)^2/(b*x^3+a)^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(a+bx^3)^{\frac{2}{3}} (c+dx)^2}\,dx$$

```
[In] integrate(1/(d*x+c)**2/(b*x**3+a)**(2/3),x)
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx+c)^2}\,dx$$

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx+c)^2}\,dx$$

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(c+d\,x)^2}\,dx$$

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x)^2),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x)^2), x)
```

3.42 $\int \frac{1}{(c+dx)^3 \left(a+bx^3\right)^{2/3}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 1357

$$\int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}}\,dx = \frac{3c^4d^2\sqrt[3]{a+bx^3}}{2(bc^3-ad^3)(c^3+d^3x^3)^2} + \frac{3c^2d^4x^2\sqrt[3]{a+bx^3}}{2(bc^3-ad^3)(c^3+d^3x^3)^2} + \frac{5bc^4d^2\sqrt[3]{a+bx^3}}{3(bc^3-ad^3)^2(c^3+d^3x^3)} - \frac{cd^2(bc^3-6ad^3)\sqrt[3]{a+bx^3}}{6(bc^3-ad^3)^2(c^3+d^3x^3)} + \frac{d^4(9bc^3-4ad^3)x^2\sqrt[3]{a+bx^3}}{6c(bc^3-ad^3)^2(c^3+d^3x^3)} + \frac{d^4(3bc^3+2ad^3)x^2\sqrt[3]{a+bx^3}}{3c(bc^3-ad^3)^2(c^3+d^3x^3)} + \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3(a+bx^3)^{2/3}} - \frac{7d^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},3,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{4c^6(a+bx^3)^{2/3}} + \frac{d^6x^7\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{7}{3},\frac{2}{3},3,\frac{10}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{7c^9(a+bx^3)^{2/3}} + \frac{2ad^4(6bc^3-ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^2(bc^3-ad^3)^{8/3}} + \frac{d(9b^2c^6-6abc^3d^3+2a^2d^6)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^2(bc^3-ad^3)^{8/3}} - \frac{10b^2c^4d\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{3\sqrt{3}(bc^3-ad^3)^{8/3}} + \frac{bcd(bc^3-6ad^3)\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{3\sqrt{3}(bc^3-ad^3)^{8/3}} - \frac{5b^2c^4d\log(c^3+d^3x^3)}{9(bc^3-ad^3)^{8/3}} + \frac{bcd(bc^3-6ad^3)\log(c^3+d^3x^3)}{18(bc^3-ad^3)^{8/3}} - \frac{ad^4(6bc^3-ad^3)\log(c^3+d^3x^3)}{9c^2(bc^3-ad^3)^{8/3}} - \frac{d(9b^2c^6-6abc^3d^3+2a^2d^6)\log(c^3+d^3x^3)}{18c^2(bc^3-ad^3)^{8/3}} + \frac{ad^4(6bc^3-ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{\cdots}$$

```
[Out] 3/2*c^4*d^2*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2+3/2*c^2*d^4*x^2*
(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2+5/3*b*c^4*d^2*(b*x^3+a)^(1/3
)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-1/6*c*d^2*(-6*a*d^3+b*c^3)*(b*x^3+a)^(1/3)
/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+1/6*d^4*(-4*a*d^3+9*b*c^3)*x^2*(b*x^3+a)^(1
/3)/c/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+1/3*d^4*(2*a*d^3+3*b*c^3)*x^2*(b*x^3+a
)^(1/3)/c/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2
/3,3,4/3,-b*x^3/a,-d^3*x^3/c^3)/c^3/(b*x^3+a)^(2/3)-7/4*d^3*x^4*(1+b*x^3/a)
^(2/3)*AppellF1(4/3,2/3,3,7/3,-b*x^3/a,-d^3*x^3/c^3)/c^6/(b*x^3+a)^(2/3)+1/
7*d^6*x^7*(1+b*x^3/a)^(2/3)*AppellF1(7/3,2/3,3,10/3,-b*x^3/a,-d^3*x^3/c^3)/
c^9/(b*x^3+a)^(2/3)-5/9*b^2*c^4*d*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(8/3)+1/18
*b*c*d*(-6*a*d^3+b*c^3)*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(8/3)-1/9*a*d^4*(-a*
d^3+6*b*c^3)*ln(d^3*x^3+c^3)/c^2/(-a*d^3+b*c^3)^(8/3)-1/18*d*(2*a^2*d^6-6*a
*b*c^3*d^3+9*b^2*c^6)*ln(d^3*x^3+c^3)/c^2/(-a*d^3+b*c^3)^(8/3)+1/3*a*d^4*(-
a*d^3+6*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c^2/(-a*d^3+b*c
^3)^(8/3)+1/6*d*(2*a^2*d^6-6*a*b*c^3*d^3+9*b^2*c^6)*ln((-a*d^3+b*c^3)^(1/3)
*x/c-(b*x^3+a)^(1/3))/c^2/(-a*d^3+b*c^3)^(8/3)+5/3*b^2*c^4*d*ln((-a*d^3+b*c
^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(8/3)-1/6*b*c*d*(-6*a*d^3+b*c^3
)*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(8/3)+2/9*a*d^4
*(-a*d^3+6*b*c^3)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))
*3^(1/2))/c^2/(-a*d^3+b*c^3)^(8/3)*3^(1/2)+1/9*d*(2*a^2*d^6-6*a*b*c^3*d^3+9
*b^2*c^6)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2)
)/c^2/(-a*d^3+b*c^3)^(8/3)*3^(1/2)-10/9*b^2*c^4*d*arctan(1/3*(1-2*d*(b*x^3+
a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(8/3)*3^(1/2)+1/9*b*
c*d*(-6*a*d^3+b*c^3)*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3)
)*3^(1/2))/(-a*d^3+b*c^3)^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 1357, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.947$, Rules

used = {2181, 441, 440, 483, 593, 12, 503, 455, 44, 60, 631, 210, 31, 525, 524, 482, 457, 79}

$$\int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}}\,dx = \frac{d^6\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{7}{3},\frac{2}{3},3,\frac{10}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x^7}{7c^9(bx^3+a)^{2/3}}$$

$$-\frac{7d^3\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},3,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x^4}{4c^6(bx^3+a)^{2/3}}$$

$$+\frac{d^4(3bc^3+2ad^3)\sqrt[3]{bx^3+a}x^2}{3c(bc^3-ad^3)^2(c^3+d^3x^3)}+\frac{d^4(9bc^3-4ad^3)\sqrt[3]{bx^3+a}x^2}{6c(bc^3-ad^3)^2(c^3+d^3x^3)}$$

$$+\frac{3c^2d^4\sqrt[3]{bx^3+a}x^2}{2(bc^3-ad^3)(c^3+d^3x^3)^2}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x}{c^3(bx^3+a)^{2/3}}$$

$$+\frac{2ad^4(6bc^3-ad^3)\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^2(bc^3-ad^3)^{8/3}}$$

$$+\frac{d(9b^2c^6-6abd^3c^3+2a^2d^6)\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^2(bc^3-ad^3)^{8/3}}$$

$$-\frac{10b^2c^4d\arctan\left(\frac{1-\frac{2d\sqrt[3]{bx^3+a}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{3\sqrt{3}(bc^3-ad^3)^{8/3}}+\frac{bcd(bc^3-6ad^3)\arctan\left(\frac{1-\frac{2d\sqrt[3]{bx^3+a}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{3\sqrt{3}(bc^3-ad^3)^{8/3}}$$

$$-\frac{ad^4(6bc^3-ad^3)\log(c^3+d^3x^3)}{9c^2(bc^3-ad^3)^{8/3}}-\frac{d(9b^2c^6-6abd^3c^3+2a^2d^6)\log(c^3+d^3x^3)}{18c^2(bc^3-ad^3)^{8/3}}$$

$$-\frac{5b^2c^4d\log(c^3+d^3x^3)}{9(bc^3-ad^3)^{8/3}}+\frac{bcd(bc^3-6ad^3)\log(c^3+d^3x^3)}{18(bc^3-ad^3)^{8/3}}$$

$$+\frac{ad^4(6bc^3-ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)}{3c^2(bc^3-ad^3)^{8/3}}$$

$$+\frac{d(9b^2c^6-6abd^3c^3+2a^2d^6)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)}{6c^2(bc^3-ad^3)^{8/3}}$$

$$+\frac{5b^2c^4d\log\left(\sqrt[3]{bx^3+a}d+\sqrt[3]{bc^3-ad^3}\right)}{3(bc^3-ad^3)^{8/3}}-\frac{bcd(bc^3-6ad^3)\log\left(\sqrt[3]{bx^3+a}d+\sqrt[3]{bc^3-ad^3}\right)}{6(bc^3-ad^3)^{8/3}}$$

$$+\frac{5bc^4d^2\sqrt[3]{bx^3+a}}{3(bc^3-ad^3)^2(c^3+d^3x^3)}-\frac{cd^2(bc^3-6ad^3)\sqrt[3]{bx^3+a}}{6(bc^3-ad^3)^2(c^3+d^3x^3)}+\frac{3c^4d^2\sqrt[3]{bx^3+a}}{2(bc^3-ad^3)(c^3+d^3x^3)^2}$$

```
[In] Int[1/((c + d*x)^3*(a + b*x^3)^(2/3)),x]
```

```
[Out] (3*c^4*d^2*(a + b*x^3)^(1/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) + (3*c^
2*d^4*x^2*(a + b*x^3)^(1/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) + (5*b*c
^4*d^2*(a + b*x^3)^(1/3))/(3*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) - (c*d^2*(b
*c^3 - 6*a*d^3)*(a + b*x^3)^(1/3))/(6*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) +
(d^4*(9*b*c^3 - 4*a*d^3)*x^2*(a + b*x^3)^(1/3))/(6*c*(b*c^3 - a*d^3)^2*(c^3
 + d^3*x^3)) + (d^4*(3*b*c^3 + 2*a*d^3)*x^2*(a + b*x^3)^(1/3))/(3*c*(b*c^3
- a*d^3)^2*(c^3 + d^3*x^3)) + (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 3
, 4/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(c^3*(a + b*x^3)^(2/3)) - (7*d^3*x^
4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 3, 7/3, -((b*x^3)/a), -((d^3*x^3
)/c^3)])/(4*c^6*(a + b*x^3)^(2/3)) + (d^6*x^7*(1 + (b*x^3)/a)^(2/3)*AppellF
1[7/3, 2/3, 3, 10/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(7*c^9*(a + b*x^3)^(2
/3)) + (2*a*d^4*(6*b*c^3 - a*d^3)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(
c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^2*(b*c^3 - a*d^3)^(8/3)) + (d*
(9*b^2*c^6 - 6*a*b*c^3*d^3 + 2*a^2*d^6)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3
)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^2*(b*c^3 - a*d^3)^(8/3))
 - (10*b^2*c^4*d*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))
/Sqrt[3]])/(3*Sqrt[3]*(b*c^3 - a*d^3)^(8/3)) + (b*c*d*(b*c^3 - 6*a*d^3)*Arc
Tan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3
]*(b*c^3 - a*d^3)^(8/3)) - (5*b^2*c^4*d*Log[c^3 + d^3*x^3])/(9*(b*c^3 - a*d
^3)^(8/3)) + (b*c*d*(b*c^3 - 6*a*d^3)*Log[c^3 + d^3*x^3])/(18*(b*c^3 - a*d^
3)^(8/3)) - (a*d^4*(6*b*c^3 - a*d^3)*Log[c^3 + d^3*x^3])/(9*c^2*(b*c^3 - a*
d^3)^(8/3)) - (d*(9*b^2*c^6 - 6*a*b*c^3*d^3 + 2*a^2*d^6)*Log[c^3 + d^3*x^3]
)/(18*c^2*(b*c^3 - a*d^3)^(8/3)) + (a*d^4*(6*b*c^3 - a*d^3)*Log[((b*c^3 - a
*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(3*c^2*(b*c^3 - a*d^3)^(8/3)) + (d*(
9*b^2*c^6 - 6*a*b*c^3*d^3 + 2*a^2*d^6)*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a
 + b*x^3)^(1/3)])/(6*c^2*(b*c^3 - a*d^3)^(8/3)) + (5*b^2*c^4*d*Log[(b*c^3 -
 a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(3*(b*c^3 - a*d^3)^(8/3)) - (b*c*d*(b
*c^3 - 6*a*d^3)*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(6*(b*c^3
 - a*d^3)^(8/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\text{integral} = \int \left(\frac{c^6}{(a+bx^3)^{2/3}(c^3+d^3x^3)^3} - \frac{3c^5dx}{(a+bx^3)^{2/3}(c^3+d^3x^3)^3} + \frac{6c^4d^2x^2}{(a+bx^3)^{2/3}(c^3+d^3x^3)^3} - \frac{7c^3d^3x^3}{(a+bx^3)^{2/3}(c^3+d^3x^3)^3} + \frac{6c^2d^4x^4}{(a+bx^3)^{2/3}(c^3+d^3x^3)^3} - \frac{3cd^5x^5}{(a+bx^3)^{2/3}(c^3+d^3x^3)^3} + \frac{d^6x^6}{(a+bx^3)^{2/3}(c^3+d^3x^3)^3}\right) dx$$

$$= c^6 \int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)^3}\,dx$$
$$-\left(3c^5d\right)\int \frac{x}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)^3}\,dx+\left(6c^4d^2\right)\int \frac{x^2}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)^3}\,dx$$
$$-\left(7c^3d^3\right)\int \frac{x^3}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)^3}\,dx+\left(6c^2d^4\right)\int \frac{x^4}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)^3}\,dx-\left(3cd^5\right)\int \frac{}{(a-}$$

$$= \frac{3c^2d^4x^2\sqrt[3]{a+bx^3}}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2}+\left(2c^4d^2\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}\left(c^3+d^3x\right)^3}\,dx,x,x^3\right)$$
$$-\left(cd^5\right)\operatorname{Subst}\left(\int \frac{x}{(a+bx)^{2/3}\left(c^3+d^3x\right)^3}\,dx,x,x^3\right)$$
$$-\frac{\left(c^2d\right)\int \frac{x\left(2\left(3bc^3-2ad^3\right)-3bd^3x^3\right)}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)^2}\,dx}{2\left(bc^3-ad^3\right)}-\frac{\left(c^2d^4\right)\int \frac{x\left(2a-3bx^3\right)}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)^2}\,dx}{bc^3-ad^3}$$
$$+\frac{\left(c^6\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c^3+d^3x^3\right)^3}\,dx}{\left(a+bx^3\right)^{2/3}}$$
$$-\frac{\left(7c^3d^3\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c^3+d^3x^3\right)^3}\,dx}{\left(a+bx^3\right)^{2/3}}$$
$$+\frac{\left(d^6\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{x^6}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c^3+d^3x^3\right)^3}\,dx}{\left(a+bx^3\right)^{2/3}}$$

$$= \frac{3c^4 d^2 \sqrt[3]{a+bx^3}}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2} + \frac{3c^2 d^4 x^2 \sqrt[3]{a+bx^3}}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2}$$
$$+ \frac{d^4\left(9bc^3-4ad^3\right)x^2\sqrt[3]{a+bx^3}}{6c\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} + \frac{d^4\left(3bc^3+2ad^3\right)x^2\sqrt[3]{a+bx^3}}{3c\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)}$$
$$+ \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\left(a+bx^3\right)^{2/3}}$$
$$- \frac{7d^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{4}{3};\frac{2}{3},3;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{4c^6\left(a+bx^3\right)^{2/3}}$$
$$+ \frac{d^6x^7\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{7}{3};\frac{2}{3},3;\frac{10}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{7c^9\left(a+bx^3\right)^{2/3}} - \frac{d\int \frac{2\left(9b^2c^6-6abc^3d^3+2a^2d^6\right)x}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)}\,dx}{6c\left(bc^3-ad^3\right)^2}$$
$$- \frac{d^4\int \frac{2a\left(6bc^3-ad^3\right)x}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)}\,dx}{3c\left(bc^3-ad^3\right)^2} + \frac{\left(5bc^4d^2\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}\left(c^3+d^3x\right)^2}\,dx,x,x^3\right)}{3\left(bc^3-ad^3\right)}$$
$$- \frac{\left(cd^2\left(bc^3-6ad^3\right)\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}\left(c^3+d^3x\right)^2}\,dx,x,x^3\right)}{6\left(bc^3-ad^3\right)}$$

$$
\begin{aligned}
&= \frac{3c^4d^2\sqrt[3]{a+bx^3}}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2}+\frac{3c^2d^4x^2\sqrt[3]{a+bx^3}}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2}+\frac{5bc^4d^2\sqrt[3]{a+bx^3}}{3\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} \\
&\quad -\frac{cd^2(bc^3-6ad^3)\sqrt[3]{a+bx^3}}{6\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)}+\frac{d^4(9bc^3-4ad^3)\,x^2\sqrt[3]{a+bx^3}}{6c\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} \\
&\quad +\frac{d^4(3bc^3+2ad^3)\,x^2\sqrt[3]{a+bx^3}}{3c\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)}+\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\left(a+bx^3\right)^{2/3}} \\
&\quad -\frac{7d^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},3;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{4c^6\left(a+bx^3\right)^{2/3}} \\
&\quad +\frac{d^6x^7\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{7}{3};\frac{2}{3},3;\frac{10}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{7c^9\left(a+bx^3\right)^{2/3}} \\
&\quad +\frac{(10b^2c^4d^2)\,\mathrm{Subst}\left(\int\frac{1}{(a+bx)^{2/3}(c^3+d^3x)}\,dx,x,x^3\right)}{9\left(bc^3-ad^3\right)^2} \\
&\quad -\frac{(bcd^2(bc^3-6ad^3))\,\mathrm{Subst}\left(\int\frac{1}{(a+bx)^{2/3}(c^3+d^3x)}\,dx,x,x^3\right)}{9\left(bc^3-ad^3\right)^2} \\
&\quad -\frac{(2ad^4(6bc^3-ad^3))\int\frac{x}{(a+bx^3)^{2/3}(c^3+d^3x^3)}\,dx}{3c\left(bc^3-ad^3\right)^2} \\
&\quad -\frac{(d(9b^2c^6-6abc^3d^3+2a^2d^6))\int\frac{x}{(a+bx^3)^{2/3}(c^3+d^3x^3)}\,dx}{3c\left(bc^3-ad^3\right)^2}
\end{aligned}
$$

$= \text{Too large to display}$

Mathematica [F]

$$
\int \frac{1}{(c+dx)^3\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{1}{(c+dx)^3\left(a+bx^3\right)^{2/3}}\,dx
$$

```
[In] Integrate[1/((c + d*x)^3*(a + b*x^3)^(2/3)),x]
[Out] Integrate[1/((c + d*x)^3*(a + b*x^3)^(2/3)), x]
```

Maple [F]

$$\int \frac{1}{\left(dx+c\right)^{3} \left(b\,x^{3}+a\right)^{\frac{2}{3}}}d x$$

```
[In] int(1/(d*x+c)^3/(b*x^3+a)^(2/3),x)
[Out] int(1/(d*x+c)^3/(b*x^3+a)^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{\left(a+bx^{3}\right)^{\frac{2}{3}} \left(c+dx\right)^{3}}\,dx$$

```
[In] integrate(1/(d*x+c)**3/(b*x**3+a)**(2/3),x)
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{\left(bx^{3}+a\right)^{\frac{2}{3}} \left(dx+c\right)^{3}}\,dx$$

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^3), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx+c)^3}\,dx$$

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(c+d\,x)^3}\,dx$$

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x)^3),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x)^3), x)
```

3.43 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 37

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*2^(2/3)*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2162, 209}

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

```
[In] Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \left(2\ 2^{2/3}\right) \text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1+\sqrt[3]{2}x}{\sqrt{1+x^3}}\right)$$
$$= \frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{1+x^3}}{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}\right)}{\sqrt{3}}$$

```
[In] Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
[Out] (-2*2^(2/3)*ArcTan[Sqrt[1 + x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.92

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)\ln\left(\frac{12\sqrt{x^3+1}\,x+3\,\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)2^{\frac{2}{3}}x^2-\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)x^3+6\sqrt{x^3+1}\,2^{\frac{2}{3}}+6\,\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)2^{\frac{1}{3}}x}{\left(2^{\frac{1}{3}}x+2\right)^3}\right)}{3}$
default	$-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\,2^{\frac{2}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+}{-\frac{3}{2}+}}}{\sqrt{x^3+1}\left(2\right.}$
elliptic	$-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\,2^{\frac{2}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+}{-\frac{3}{2}+}}}{\sqrt{x^3+1}\left(2\right.}$

```
[In] int((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*RootOf(_Z^2+6*2^(1/3))*ln((12*(x^3+1)^(1/2)*x+3*RootOf(_Z^2+6*2^(1/3))
*2^(2/3)*x^2-RootOf(_Z^2+6*2^(1/3))*x^3+6*(x^3+1)^(1/2)*2^(2/3)+6*RootOf(_Z
^2+6*2^(1/3))*2^(1/3)*x+2*RootOf(_Z^2+6*2^(1/3)))/(2^(1/3)*x+2)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(27) = 54$.

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.03

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{1}{3}\sqrt{6}2^{\frac{1}{6}}\arctan\left(-\frac{\sqrt{6}2^{\frac{1}{6}}\left(2\,x^5+2\,x^2-2^{\frac{2}{3}}(7\,x^4+4\,x)-2^{\frac{1}{3}}(5\,x^3+2)\right)\sqrt{x^3+1}}{12\,(2\,x^6+3\,x^3+1)}\right)$$

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(6)*2^(1/6)*arctan(-1/12*sqrt(6)*2^(1/6)*(2*x^5 + 2*x^2 - 2^(2/3)*(
7*x^4 + 4*x) - 2^(1/3)*(5*x^3 + 2))*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1))
```

Sympy [F]

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = -\int\left(-\frac{2^{\frac{2}{3}}}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\right)dx-\int\frac{2x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx$$

```
[In] integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
[Out] -Integral(-2**(2/3)/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - Inte
gral(2*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)
```

Maxima [F]

$$\int \frac{2^{2/3} - 2x}{(2^{2/3} + x)\sqrt{1 + x^3}}\,dx = \int -\frac{2\,x - 2^{\frac{2}{3}}}{\sqrt{x^3 + 1}\left(x + 2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((2*x - 2^(2/3))/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2^{2/3} - 2x}{(2^{2/3} + x)\sqrt{1 + x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad A
rgumen
```

Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.89

$$\int \frac{2^{2/3} - 2x}{(2^{2/3} + x)\sqrt{1 + x^3}}\,dx = \frac{2^{2/3}\,\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}\,1\mathrm{i} + \sqrt{x^3+1} + 2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)\,\left(\sqrt{3}\,1\mathrm{i} - \sqrt{x^3+1} + 2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)^3}{\left(x+2^{2/3}\right)^6}\right)\,1\mathrm{i}}{3}$$

```
[In] int(-(2*x - 2^(2/3))/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
[Out] (2^(2/3)*3^(1/2)*log(((3^(1/2)*1i + (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)
*(3^(1/2)*1i - (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)^3)/(x + 2^(2/3))^6)*
1i)/3
```

3.44 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 40

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

```
[Out] -2/3*2^(2/3)*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2162, 209}

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

```
[In] Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/Sqrt[3]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\left(\left(2\ 2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)\right)$$
$$= -\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{3}\left(-1+\sqrt[3]{2}x\right)}\right)}{\sqrt{3}}$$

```
[In] Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
[Out] (-2*2^(2/3)*ArcTan[Sqrt[1 - x^3]/(Sqrt[3]*(-1 + 2^(1/3)*x))])/Sqrt[3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.78

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)\ln\left(\frac{12\sqrt{-x^3+1}\,x+3\,\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)2^{\frac{2}{3}}x^2+\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)x^3-6\sqrt{-x^3+1}\,2^{\frac{2}{3}}-6\,\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)}{\left(2^{\frac{1}{3}}x-2\right)^3}}{3}$
default	$\frac{4i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$
elliptic	$\frac{4i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$

```
[In] int((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*RootOf(_Z^2+6*2^(1/3))*ln((12*(-x^3+1)^(1/2)*x+3*RootOf(_Z^2+6*2^(1/3)
)*2^(2/3)*x^2+RootOf(_Z^2+6*2^(1/3))*x^3-6*(-x^3+1)^(1/2)*2^(2/3)-6*RootOf(
_Z^2+6*2^(1/3))*2^(1/3)*x+2*RootOf(_Z^2+6*2^(1/3)))/(2^(1/3)*x-2)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\frac{1}{3}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{\sqrt{6}2^{\frac{1}{6}}\left(2\,x^5-2\,x^2+2^{\frac{2}{3}}(7\,x^4-4\,x)-2^{\frac{1}{3}}(5\,x^3-2)\right)\sqrt{-x^3+1}}{12\,(2\,x^6-3\,x^3+1)}\right)$$

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(6)*2^(1/6)*arctan(1/12*sqrt(6)*2^(1/6)*(2*x^5 - 2*x^2 + 2^(2/3)*(
7*x^4 - 4*x) - 2^(1/3)*(5*x^3 - 2))*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1))
```

Sympy [F]

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\int \frac{2^{\frac{2}{3}}}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx - \int \frac{2x}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

```
[In] integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integ
ral(2*x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{2\,x+2^{\frac{2}{3}}}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x + 2^(2/3))/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Arg
ument
```

Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.85

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \frac{2^{2/3}\sqrt{3}\ln\left(\frac{\left(\sqrt{1-x^3}-\sqrt{3}\,1\mathrm{i}+2^{1/3}\sqrt{3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\,1\mathrm{i}+\sqrt{1-x^3}-2^{1/3}\sqrt{3}\,x\,1\mathrm{i}\right)^3}{\left(x-2^{2/3}\right)^6}\right)1\mathrm{i}}{3}$$

```
[In] int(-(2*x + 2^(2/3))/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log((((1 - x^3)^(1/2) - 3^(1/2)*1i + 2^(1/3)*3^(1/2)*x*1i)
*(3^(1/2)*1i + (1 - x^3)^(1/2) - 2^(1/3)*3^(1/2)*x*1i)^3)/(x - 2^(2/3))^6)*
1i)/3
```

3.45 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=-\frac{2\ 2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{\sqrt{3}}$$

```
[Out] -2/3*2^(2/3)*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2162, 212}

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=-\frac{2\ 2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{\sqrt{3}}$$

```
[In] Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\left(\left(2\ 2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{1-\sqrt[3]{2}x}{\sqrt{-1+x^3}}\right)\right)$$
$$= -\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{-1+x^3}}{\sqrt{3}\left(-1+\sqrt[3]{2}x\right)}\right)}{\sqrt{3}}$$

```
[In] Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
[Out] (2*2^(2/3)*ArcTanh[Sqrt[-1 + x^3]/(Sqrt[3]*(-1 + 2^(1/3)*x))])/Sqrt[3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.84

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)\ln\left(\frac{12\sqrt{x^3-1}\,x-3\,2^{\frac{2}{3}}x^2\,\mathrm{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)-\mathrm{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)x^3-6\sqrt{x^3-1}\,2^{\frac{2}{3}}+6\,\mathrm{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)2^{\frac{1}{3}}\cdots}{\left(2^{\frac{1}{3}}x-2\right)^3}\cdots\right.}{3}$
default	$-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{6\,2^{\frac{2}{3}}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\sqrt{x^3-1}}$
elliptic	$-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{6\,2^{\frac{2}{3}}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\sqrt{x^3-1}}$

```
[In] int((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*RootOf(_Z^2-6*2^(1/3))*ln((12*(x^3-1)^(1/2)*x-3*2^(2/3)*x^2*RootOf(_Z^
2-6*2^(1/3))-RootOf(_Z^2-6*2^(1/3))*x^3-6*(x^3-1)^(1/2)*2^(2/3)+6*RootOf(_Z
^2-6*2^(1/3))*2^(1/3)*x-2*RootOf(_Z^2-6*2^(1/3)))/(2^(1/3)*x-2)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(28) = 56$.

Time = 0.40 (sec) , antiderivative size = 238, normalized size of antiderivative = 6.26

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\log\left(\frac{x^{18}+1440\,x^{15}+17400\,x^{12}-21056\,x^9-10368\,x^6+15360\,x^3+2\cdots}{\cdots}\right.$$

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*log((x^18 + 1440*x^15 + 17400*x^12 - 21056*x^9 - 10368*
x^6 + 15360*x^3 + 2*sqrt(6)*2^(1/6)*(126*x^14 + 2664*x^11 - 4608*x^5 + 2304
*x^2 + 2^(2/3)*(x^16 + 310*x^13 + 2332*x^10 - 2656*x^7 - 256*x^4 + 512*x) +
 2^(1/3)*(17*x^15 + 1058*x^12 + 2528*x^9 - 5408*x^6 + 2560*x^3 - 512))*sqrt
(x^3 - 1) + 24*2^(2/3)*(x^17 + 121*x^14 + 478*x^11 - 1144*x^8 + 608*x^5 - 6
4*x^2) + 48*2^(1/3)*(5*x^16 + 176*x^13 + 83*x^10 - 680*x^7 + 544*x^4 - 128*
x) - 2048)/(x^18 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 6144*x^3 + 40
96))
```

Sympy [F]

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\int \frac{2^{\frac{2}{3}}}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx - \int \frac{2x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

```
[In] integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
[Out] -Integral(2**(2/3)/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integ
ral(2*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \int -\frac{2\,x+2^{\frac{2}{3}}}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((2*x + 2^(2/3))/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Arg
ument
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.63

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{2^{2/3}\sqrt{3}\,\ln\left(\frac{\left(\sqrt{x^3-1}-\sqrt{3}+2^{1/3}\sqrt{3}\,x\right)^3\left(\sqrt{3}+\sqrt{x^3-1}-2^{1/3}\sqrt{3}\,x\right)}{\left(x-2^{2/3}\right)^6}\right)}{3}$$

```
[In] int(-(2*x + 2^(2/3))/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
[Out] (2^(2/3)*3^(1/2)*log((((x^3 - 1)^(1/2) - 3^(1/2) + 2^(1/3)*3^(1/2)*x)^3*(3^
(1/2) + (x^3 - 1)^(1/2) - 2^(1/3)*3^(1/2)*x))/(x - 2^(2/3))^6))/3
```

3.46 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 39

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*2^(2/3)*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2162, 212}

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{\sqrt{3}}$$

```
[In] Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \left(2\ 2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{1+\sqrt[3]{2}x}{\sqrt{-1-x^3}}\right)$$
$$= \frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{-1-x^3}}{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}\right)}{\sqrt{3}}$$

```
[In] Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
[Out] (2*2^(2/3)*ArcTanh[Sqrt[-1 - x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.63 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.87

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)\ln\left(\frac{3\,2^{\frac{2}{3}}x^2\,\mathrm{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)+12\sqrt{-x^3-1}\,x-\mathrm{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)x^3+6\,\mathrm{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)2^{\frac{1}{3}}x+6\sqrt{-x^3-1}\,2}{\left(2^{\frac{1}{3}}x+2\right)^3}\right.}{3}$
default	$\frac{4i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{}}{}$
elliptic	$\frac{4i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{}}{}$

```
[In] int((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*RootOf(_Z^2-6*2^(1/3))*ln((3*2^(2/3)*x^2*RootOf(_Z^2-6*2^(1/3))+12*(-x^
3-1)^(1/2)*x-RootOf(_Z^2-6*2^(1/3))*x^3+6*RootOf(_Z^2-6*2^(1/3))*2^(1/3)*x+
6*(-x^3-1)^(1/2)*2^(2/3)+2*RootOf(_Z^2-6*2^(1/3)))/(2^(1/3)*x+2)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(29) = 58$.

Time = 0.40 (sec) , antiderivative size = 241, normalized size of antiderivative = 6.18

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\log\left(\frac{x^{18}-1440\,x^{15}+17400\,x^{12}+21056\,x^9-10368\,x^6-15360\,x^3-2}{}\right.$$

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*log((x^18 - 1440*x^15 + 17400*x^12 + 21056*x^9 - 10368*
x^6 - 15360*x^3 - 2*sqrt(6)*2^(1/6)*(126*x^14 - 2664*x^11 + 4608*x^5 + 2304
*x^2 + 2^(2/3)*(x^16 - 310*x^13 + 2332*x^10 + 2656*x^7 - 256*x^4 - 512*x) -
 2^(1/3)*(17*x^15 - 1058*x^12 + 2528*x^9 + 5408*x^6 + 2560*x^3 + 512))*sqrt
(-x^3 - 1) - 24*2^(2/3)*(x^17 - 121*x^14 + 478*x^11 + 1144*x^8 + 608*x^5 +
64*x^2) + 48*2^(1/3)*(5*x^16 - 176*x^13 + 83*x^10 + 680*x^7 + 544*x^4 + 128
*x) - 2048)/(x^18 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4
096))
```

Sympy [F]

$$\int \frac{2^{2/3} - 2x}{(2^{2/3} + x)\sqrt{-1 - x^3}}\,dx = -\int \left(-\frac{2^{\frac{2}{3}}}{x\sqrt{-x^3 - 1} + 2^{\frac{2}{3}}\sqrt{-x^3 - 1}}\right)dx - \int \frac{2x}{x\sqrt{-x^3 - 1} + 2^{\frac{2}{3}}\sqrt{-x^3 - 1}}\,dx$$

```
[In] integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
[Out] -Integral(-2**(2/3)/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x) - In
tegral(2*x/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{2^{2/3} - 2x}{(2^{2/3} + x)\sqrt{-1 - x^3}}\,dx = \int -\frac{2\,x - 2^{\frac{2}{3}}}{\sqrt{-x^3 - 1}\left(x + 2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((2*x - 2^(2/3))/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2^{2/3} - 2x}{(2^{2/3} + x)\sqrt{-1 - x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad A
rgumen
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\frac{2^{2/3}\,\sqrt{3}\,\ln\left(\frac{{\left(\sqrt{3}+\sqrt{-x^3-1}+2^{1/3}\,\sqrt{3}\,x\right)}^3\,\left(\sqrt{3}-\sqrt{-x^3-1}+2^{1/3}\,\sqrt{3}\,x\right)}{{\left(x+2^{2/3}\right)}^6}\right)}{3}$$

```
[In] int(-(2*x - 2^(2/3))/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log(((3^(1/2) + (- x^3 - 1)^(1/2) + 2^(1/3)*3^(1/2)*x)^3*(
3^(1/2) - (- x^3 - 1)^(1/2) + 2^(1/3)*3^(1/2)*x))/(x + 2^(2/3))^6))/3
```

3.47 $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 53, antiderivative size = 63

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] 2/3*2^(2/3)*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3+a)^(1
/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2162, 209}

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a +
 b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a +
b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(2\ 2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+3ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{a+bx^3}}{\sqrt{3}\left(\sqrt{a}+\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sq
rt[a + b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[Sqrt[a + b*x^3]/(Sqrt[3]*(Sqrt[a] + 2^(1/3)*a^(1/6)*b^(1
/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Maple [F]

$$\int \frac{2^{\frac{2}{3}} a^{\frac{1}{3}} - 2 b^{\frac{1}{3}} x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3} + a}}dx$$

```
[In] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/
2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/
2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{2^{2/3} \sqrt[3]{a} - 2\sqrt[3]{b}x}{\left(2^{2/3} \sqrt[3]{a} + \sqrt[3]{b}x\right) \sqrt{a + bx^3}} \, dx = \text{Timed out}$$

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{2^{2/3} \sqrt[3]{a} - 2\sqrt[3]{b}x}{\left(2^{2/3} \sqrt[3]{a} + \sqrt[3]{b}x\right) \sqrt{a + bx^3}} \, dx = - \int \left(-\frac{2^{\frac{2}{3}} \sqrt[3]{a}}{2^{\frac{2}{3}} \sqrt[3]{a}\sqrt{a + bx^3} + \sqrt[3]{b}x\sqrt{a + bx^3}}\right) dx - \int \frac{2\sqrt[3]{b}x}{2^{\frac{2}{3}} \sqrt[3]{a}\sqrt{a + bx^3} + \sqrt[3]{b}x\sqrt{a + bx^3}} dx$$

```
[In] integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(
b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)
*x*sqrt(a + b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2/3)*a**(1/3)*sqrt(a
+ b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int -\frac{2\,b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^
(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 11.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.68

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2^{2/3}\sqrt{3}\ln\left(\frac{\left(\sqrt{3}\sqrt{a}\,1\mathrm{i}-\sqrt{b\,x^3+a}+2^{1/3}\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)^3\left(\sqrt{3}\sqrt{a}\,1\mathrm{i}+\sqrt{b\,x^3+a}+2^{1/3}\sqrt{3}\,a^{1/6}\cdots\right.}{\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)^6}\right.}{3\,a^{1/6}\,b^{1/3}}$$

```
[In] int((2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b
^(1/3)*x)),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log(((3^(1/2)*a^(1/2)*1i - (a + b*x^3)^(1/2) + 2^(1/3)*3^(
1/2)*a^(1/6)*b^(1/3)*x*1i)^3*(3^(1/2)*a^(1/2)*1i + (a + b*x^3)^(1/2) + 2^(1
/3)*3^(1/2)*a^(1/6)*b^(1/3)*x*1i))/(2^(2/3)*a^(1/3) + b^(1/3)*x)^6)*1i)/(3*
a^(1/6)*b^(1/3))
```

3.48 $\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 55, antiderivative size = 65

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] -2/3*2^(2/3)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3+a)^
(1/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2162, 209}

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a -
 b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a -
 b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(2\ 2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int\frac{1}{1+3ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int\frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{a-bx^3}}{\sqrt{3}\left(\sqrt{a}-\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[a - b*x^3]),x]
[Out] (2*2^(2/3)*ArcTan[Sqrt[a - b*x^3]/(Sqrt[3]*(Sqrt[a] - 2^(1/3)*a^(1/6)*b^(1/
3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Maple [F]

$$\int \frac{2^{\frac{2}{3}} a^{\frac{1}{3}}+2 b^{\frac{1}{3}} x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}dx$$

```
[In] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1
/2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1
/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3
+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\int \frac{2^{\frac{2}{3}}\sqrt[3]{a}}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx - \int \frac{2\sqrt[3]{b}x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

```
[In] integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(
-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)
*x*sqrt(a - b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*sqrt(a
 - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{2\,b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3
+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2
^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3
+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 10.97 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{2^{2/3}\sqrt{3}\,\ln\left(\frac{\left(\sqrt{a-b\,x^3}-\sqrt{3}\,\sqrt{a}\,1\mathrm{i}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\,\sqrt{a}\,1\mathrm{i}+\sqrt{a-b\,x^3}-2^{1/3}\,\sqrt{3}\,a^{1/6}\right.}{\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)^6}\right.}{3\,a^{1/6}\,b^{1/3}}$$

```
[In] int((2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b
^(1/3)*x)),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log((((a - b*x^3)^(1/2) - 3^(1/2)*a^(1/2)*1i + 2^(1/3)*3^(
1/2)*a^(1/6)*b^(1/3)*x*1i)*(3^(1/2)*a^(1/2)*1i + (a - b*x^3)^(1/2) - 2^(1/3
)*3^(1/2)*a^(1/6)*b^(1/3)*x*1i)^3)/(2^(2/3)*a^(1/3) - b^(1/3)*x)^6)*1i)/(3*
a^(1/6)*b^(1/3))
```

3.49 $\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 56, antiderivative size = 66

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{2\ 2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] -2/3*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3-a)^
(1/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2162, 212}

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{2\ 2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a
+ b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 + b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(2\ 2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-3ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 5.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{-a+bx^3}}{\sqrt{3}\left(\sqrt{a}-\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[-a + b*x^3]),x]
[Out] (-2*2^(2/3)*ArcTanh[Sqrt[-a + b*x^3]/(Sqrt[3]*(Sqrt[a] - 2^(1/3)*a^(1/6)*b^
(1/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Maple [F]

$$\int \frac{2^{\frac{2}{3}} a^{\frac{1}{3}} + 2b^{\frac{1}{3}} x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}} - b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3} - a}}dx$$

```
[In] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/
2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/
2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{2^{2/3} \sqrt[3]{a} + 2\sqrt[3]{b}x}{\left(2^{2/3} \sqrt[3]{a} - \sqrt[3]{b}x\right) \sqrt{-a + bx^{3}}}\,dx = \text{Timed out}$$

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{2^{2/3} \sqrt[3]{a} + 2\sqrt[3]{b}x}{\left(2^{2/3} \sqrt[3]{a} - \sqrt[3]{b}x\right) \sqrt{-a + bx^{3}}}\,dx = -\int \frac{2^{\frac{2}{3}} \sqrt[3]{a}}{-2^{\frac{2}{3}} \sqrt[3]{a}\sqrt{-a + bx^{3}} + \sqrt[3]{b}x\sqrt{-a + bx^{3}}}\,dx - \int \frac{2\sqrt[3]{b}x}{-2^{\frac{2}{3}} \sqrt[3]{a}\sqrt{-a + bx^{3}} + \sqrt[3]{b}x\sqrt{-a + bx^{3}}}\,dx$$

```
[In] integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(
b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3
)*x*sqrt(-a + b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*sqrt
(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{2\,b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^
(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 10.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.55

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{\sqrt{3}\,4^{1/3}\,\ln\left(\frac{\left(\sqrt{b\,x^3-a}+\sqrt{3}\,\sqrt{a}-2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)\,\left(\sqrt{b\,x^3-a}-\sqrt{3}\,\sqrt{a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\right.}{{\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}^6}\right.}{3\,a^{1/6}\,b^{1/3}}$$

```
[In] int((2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b
^(1/3)*x)),x)
```

```
[Out] (3^(1/2)*4^(1/3)*log((((b*x^3 - a)^(1/2) + 3^(1/2)*a^(1/2) - 2^(1/3)*3^(1/2
)*a^(1/6)*b^(1/3)*x)*((b*x^3 - a)^(1/2) - 3^(1/2)*a^(1/2) + 2^(1/3)*3^(1/2)
*a^(1/6)*b^(1/3)*x)^3)/(2^(2/3)*a^(1/3) - b^(1/3)*x)^6))/(3*a^(1/6)*b^(1/3)
)
```

3.50 $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 56, antiderivative size = 66

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{2\,2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] 2/3*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3-a)^
(1/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2162, 212}

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{2\,2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a
 - b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 - b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(2\ 2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-3ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{-a-bx^3}}{\sqrt{3}\left(\sqrt{a}+\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sq
rt[-a - b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[Sqrt[-a - b*x^3]/(Sqrt[3]*(Sqrt[a] + 2^(1/3)*a^(1/6)*b^(
1/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Maple [F]

$$\int \frac{2^{\frac{2}{3}} a^{\frac{1}{3}} - 2 b^{\frac{1}{3}} x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right) \sqrt{-b\, x^{3} - a}}dx$$

```
[In] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1
/2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1
/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{2^{2/3} \sqrt[3]{a} - 2\sqrt[3]{b}x}{\left(2^{2/3} \sqrt[3]{a} + \sqrt[3]{b}x\right) \sqrt{-a - bx^3}} \, dx = \text{Timed out}$$

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3
-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\begin{aligned}\int \frac{2^{2/3} \sqrt[3]{a} - 2\sqrt[3]{b}x}{\left(2^{2/3} \sqrt[3]{a} + \sqrt[3]{b}x\right) \sqrt{-a - bx^3}} \, dx = &- \int \left(-\frac{2^{\frac{2}{3}} \sqrt[3]{a}}{2^{\frac{2}{3}} \sqrt[3]{a}\sqrt{-a - bx^3} + \sqrt[3]{b}x\sqrt{-a - bx^3}}\right) dx \\ &- \int \frac{2\sqrt[3]{b}x}{2^{\frac{2}{3}} \sqrt[3]{a}\sqrt{-a - bx^3} + \sqrt[3]{b}x\sqrt{-a - bx^3}} \, dx\end{aligned}$$

```
[In] integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(
-b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3
)*x*sqrt(-a - b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2/3)*a**(1/3)*sqrt(
-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int -\frac{2\,b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3
-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2
^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3
-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.56

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{\sqrt{3}\,4^{1/3}\,\ln\left(\frac{\left(\sqrt{-b\,x^3-a}+\sqrt{3}\,\sqrt{a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)^3\,\left(\sqrt{3}\,\sqrt{a}-\sqrt{-b\,x^3-a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\right.}{\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)^6}\right.}{3\,a^{1/6}\,b^{1/3}}$$

```
[In] int((2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) +
 b^(1/3)*x)),x)
```

```
[Out] (3^(1/2)*4^(1/3)*log((((- a - b*x^3)^(1/2) + 3^(1/2)*a^(1/2) + 2^(1/3)*3^(1
/2)*a^(1/6)*b^(1/3)*x)^3*(3^(1/2)*a^(1/2) - (- a - b*x^3)^(1/2) + 2^(1/3)*3
^(1/2)*a^(1/6)*b^(1/3)*x))/(2^(2/3)*a^(1/3) + b^(1/3)*x)^6))/(3*a^(1/6)*b^(
1/3))
```

3.51 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 49

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}d}$$

```
[Out] 2/3*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/d*3^(1/2)/c^(1/
2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2162, 209}

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}d}$$

```
[In] Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(Sqrt[3]*Sq
rt[c]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{(2c)\text{Subst}\left(\int \frac{1}{1+3c^3x^2}\,dx, x, \frac{1+\frac{2dx}{c}}{\sqrt{c^3+4d^3x^3}}\right)}{d}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}d}$$

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{c^3+4d^3x^3}}{\sqrt{3}\sqrt{c}(c+2dx)}\right)}{\sqrt{3}\sqrt{c}d}$$

```
[In] Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (-2*ArcTan[Sqrt[c^3 + 4*d^3*x^3]/(Sqrt[3]*Sqrt[c]*(c + 2*d*x))])/(Sqrt[3]*S
qrt[c]*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.89 (sec) , antiderivative size = 889, normalized size of antiderivative = 18.14

method	result	size
default	Expression too large to display	889
elliptic	Expression too large to display	889

```
[In] int((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1
/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*
3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x+1/
2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(
1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/
2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3+
c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(
```

```
1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(
1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2
^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/
2))+6*c/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/
2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)
-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)
*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*
c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*
I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d
^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*EllipticPi(
((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^
(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)+1/
4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(
1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*
c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^
(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(39) = 78$.

Time = 0.43 (sec) , antiderivative size = 300, normalized size of antiderivative = 6.12

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \left[\frac{\sqrt{3}\sqrt{-\frac{1}{c}}\log\left(\frac{2d^6x^6-36cd^5x^5-18c^2d^4x^4+28c^3d^3x^3+18c^4d^2x^2-c^6-\sqrt{3}(4cd^4x^4-10c^2d^3x^3-18c^3d^2x^2-8c^4dx-c^5)\sqrt{4d^3x^3+c^3}\sqrt{-}}{d^6x^6+6cd^5x^5+15c^2d^4x^4+20c^3d^3x^3+15c^4d^2x^2+6c^5dx+c^6}\right.}{6d}, -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{4d^3x^3+c^3}(2d^3x^3-6cd^2x^2-6c^2dx-c^3)}{3(8d^4x^4+4cd^3x^3+2c^3dx+c^4)\sqrt{c}}\right)}{3\sqrt{c}d}\right]$$

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(3)*sqrt(-1/c)*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^4 + 28
*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*c*d^4*x^4 - 10*c^2*d^3*x^3
 - 18*c^3*d^2*x^2 - 8*c^4*d*x - c^5)*sqrt(4*d^3*x^3 + c^3)*sqrt(-1/c))/(d^6
*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c
^5*d*x + c^6))/d, -1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2*
d^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)/((8*d^4*x^4 + 4*c*d^3*x^3 + 2*c^3*
d*x + c^4)*sqrt(c)))/(sqrt(c)*d)]
```

Sympy [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = -\int \left(-\frac{c}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\right)dx - \int \frac{2dx}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\,dx$$

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
[Out] -Integral(-c/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x**3)), x
) - Integral(2*d*x/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x**
3)), x)
```

Maxima [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int -\frac{2\,dx-c}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
[Out] -integrate((2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int -\frac{2\,dx-c}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
[Out] integrate(-(2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 10.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.94

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{\sqrt{3}\,\ln\left(\frac{\left(-\sqrt{c^3+4\,d^3\,x^3}+\sqrt{3}\,c^{3/2}\,1\mathrm{i}+\sqrt{3}\,\sqrt{c}\,d\,x\,2\mathrm{i}\right)^3\left(\sqrt{c^3+4\,d^3\,x^3}+\sqrt{3}\,c^{3/2}\,1\mathrm{i}+\sqrt{3}\,\sqrt{c}\,d\,x\,2\mathrm{i}\right)}{(c+d\,x)^6}\right)\,1\mathrm{i}}{3\,\sqrt{c}\,d}$$

```
[In] int((c - 2*d*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] (3^(1/2)*log(((3^(1/2)*c^(3/2)*1i - (c^3 + 4*d^3*x^3)^(1/2) + 3^(1/2)*c^(1/
2)*d*x*2i)^3*((c^3 + 4*d^3*x^3)^(1/2) + 3^(1/2)*c^(3/2)*1i + 3^(1/2)*c^(1/2
)*d*x*2i))/(c + d*x)^6)*1i)/(3*c^(1/2)*d)
```

3.52 $\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{2\left(2-3\ 2^{2/3}\right)\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 2/9*(2-3*2^(2/3))*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*(
3+2*2^(1/3))*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/
2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1
/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2164, 224, 2162, 209}

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} + \frac{2\left(2-3\ 2^{2/3}\right)\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

```
[In] Int[(2 + 3*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*(2 - 3*2^(2/3))*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt
[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\left(-3+\sqrt[3]{2}\right)\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx+\frac{1}{3}\left(3+2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx \\
&= \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} \\
&\quad +\frac{1}{3}\left(2\left(2-3\ 2^{2/3}\right)\right)\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx,x,\frac{1+\sqrt[3]{2}x}{\sqrt{1+x^3}}\right) \\
&= \frac{2\left(2-3\ 2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} \\
&\quad +\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.43 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.13

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4i\sqrt{3}-\right.\right.\right.}{}$$

```
[In] Integrate[(2 + 3*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 4*Sqrt[3]*(-3 + 2^(1/3
))*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I
+ (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(
1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqrt[3
])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(125) = 250$.

Time = 4.34 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.66

method	result
default	$\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(2-3\,2^{\frac{2}{3}}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{}}{\sqrt{x^3+1}\,(}$
elliptic	$\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(2-3\,2^{\frac{2}{3}}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{}}{\sqrt{x^3+1}\,(}$

```
[In] int((3*x+2)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(2-3*2^(2/3))*(3/2-1/2*I*3^(1/2
))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(
1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/
2)/(2^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(
1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{1}{9}\sqrt{3}\sqrt{-12\cdot 2^{\frac{2}{3}}+18\cdot 2^{\frac{1}{3}}+4}\arctan\left(\frac{\sqrt{3}\left(18\,x^5-42\,x^4-10\,x^3+18\,x^2+2^{\frac{2}{3}}(2\right.}{}\right.$$
$$+\frac{2}{3}\left(2\cdot 2^{\frac{1}{3}}+3\right)\text{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*sqrt(-12*2^(2/3) + 18*2^(1/3) + 4)*arctan(1/300*sqrt(3)*(18*x^5
 - 42*x^4 - 10*x^3 + 18*x^2 + 2^(2/3)*(2*x^5 - 63*x^4 - 15*x^3 + 2*x^2 - 36
*x - 6) + 2^(1/3)*(6*x^5 - 14*x^4 - 45*x^3 + 6*x^2 - 8*x - 18) - 24*x - 4)*
sqrt(x^3 + 1)*sqrt(-12*2^(2/3) + 18*2^(1/3) + 4)/(2*x^6 + 3*x^3 + 1)) + 2/3
*(2*2^(1/3) + 3)*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{3x+2}{\sqrt{(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((2+3*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
[Out] Integral((3*x + 2)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Maxima [F]

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{3x+2}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((3*x + 2)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad A
rgumen
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{3x+2}{\sqrt{x^3+1}\,(x+2^{2/3})}\,dx$$

```
[In] int((3*x + 2)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
[Out] int((3*x + 2)/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

3.53 $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 173

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{2\left(2+3\ 2^{2/3}\right)\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/9*(2+3*2^(2/3))*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)+2/9
*(3-2*2^(1/3))*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(
1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)
^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2164, 224, 2162, 209}

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{2\left(2+3\ 2^{2/3}\right)\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

```
[In] Int[(2 + 3*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(2 + 3*2^(2/3))*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqr
t[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int\frac{1}{\sqrt{1-x^3}}\,dx\right)+\frac{1}{3}\left(3+\sqrt[3]{2}\right)\int\frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx\\
&= \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}\\
&\quad-\frac{1}{3}\left(2\left(2+3\ 2^{2/3}\right)\right)\text{Subst}\left(\int\frac{1}{1+3x^2}\,dx,x,\frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)\\
&= -\frac{2\left(2+3\ 2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}\\
&\quad+\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.42 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.94

$$\int\frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx=\frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3i\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3i\sqrt[3]{2}\right.\right.\right.}{}$$

```
[In] Integrate[(2 + 3*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqrt[3]*(3 +
 2^(1/3))*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[
3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt
[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) +
 Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Maple [A] (verified)

Time = 5.64 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.49

method	result
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}+1}}-\frac{2i\left(-2-3\,2^{\frac{2}{3}}\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\right.}}{}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}+1}}-\frac{2i\left(-2-3\,2^{\frac{2}{3}}\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\right.}}{}$

```
[In] int((3*x+2)/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1
/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2
*I*3^(1/2)))^(1/2))-2/3*I*(-2-3*2^(2/3))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*
3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-2^(2/3)-1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-2^(2/3)-1/2+1/2*
I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.92

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx =$$

$$-\frac{1}{9}\sqrt{3}\sqrt{12\cdot 2^{\frac{2}{3}}+18\cdot 2^{\frac{1}{3}}+4}\arctan\left(\frac{\sqrt{3}\left(18\,x^5-42\,x^4-10\,x^3-18\,x^2+2^{\frac{2}{3}}\left(2\,x^5+63\,x^4+15\,x^3-2\,x^2-\right.\right.}{}\right.$$

$$-\frac{2}{3}\left(2i\cdot 2^{\frac{1}{3}}-3i\right)\text{weierstrassPInverse}(0,4,x)$$

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*sqrt(12*2^(2/3) + 18*2^(1/3) + 4)*arctan(1/348*sqrt(3)*(18*x^5
 - 42*x^4 - 10*x^3 - 18*x^2 + 2^(2/3)*(2*x^5 + 63*x^4 + 15*x^3 - 2*x^2 - 36
*x - 6) - 2^(1/3)*(6*x^5 - 14*x^4 + 45*x^3 - 6*x^2 + 8*x - 18) + 24*x + 4)*
sqrt(-x^3 + 1)*sqrt(12*2^(2/3) + 18*2^(1/3) + 4)/(2*x^6 - 3*x^3 + 1)) - 2/3
*(2*I*2^(1/3) - 3*I)*weierstrassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\int \frac{3x}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx - \int \frac{2}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

```
[In] integrate((2+3*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
[Out] -Integral(3*x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integral(2
/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \int -\frac{3\,x+2}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((3*x + 2)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Arg
ument
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \int -\frac{3\,x+2}{\sqrt{1-x^3}\,\left(x-2^{2/3}\right)}\,dx$$

```
[In] int(-(3*x + 2)/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
[Out] int(-(3*x + 2)/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

3.54 $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 176

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{2\left(2+3\ 2^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -2/9*(2+3*2^(2/3))*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)+2/9
*(3-2*2^(1/3))*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(
1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^
(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2164, 225, 2162, 212}

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{2\left(2+3\ 2^{2/3}\right)\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

```
[In] Int[(2 + 3*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*(2 + 3*2^(2/3))*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*S
qrt[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
-7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx\right)+\frac{1}{3}\left(3+\sqrt[3]{2}\right)\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$$

$$=\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$-\frac{1}{3}\left(2\left(2+3\ 2^{2/3}\right)\right)\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx,x,\frac{1-\sqrt[3]{2}x}{\sqrt{-1+x^3}}\right)$$

$$=-\frac{2\left(2+3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}}$$

$$+\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.40 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.89

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=\frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3i\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3i\sqrt[3]{}\right.\right.\right.}{}$$

```
[In] Integrate[(2 + 3*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqrt[3]*(3 +
 2^(1/3))*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[
```

```
3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt
[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) +
 Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.51

method	result
default	$-\frac{6\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-2-3\,2^{\frac{2}{3}}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+}{\frac{3}{2}}}}{\sqrt{}}$
elliptic	$-\frac{6\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-2-3\,2^{\frac{2}{3}}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+}{\frac{3}{2}}}}{\sqrt{}}$

```
[In] int((3*x+2)/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -6*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-2-3*2^(2/3))*(-3/2-1/2*I*3^(1
/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3
^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1
/2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3
^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{3x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx - \int \frac{2}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

```
[In] integrate((2+3*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(3*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integral(2
/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \int -\frac{3x+2}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((3*x + 2)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad A
rgumen
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \int -\frac{3x+2}{\sqrt{x^3-1}\,(x-2^{2/3})}\,dx$$

```
[In] int(-(3*x + 2)/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] int(-(3*x + 2)/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

3.55 $\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 169

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{2\left(2-3\ 2^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 2/9*(2-3*2^(2/3))*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/9
*(3+2*2^(1/3))*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(
1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)
^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2164, 225, 2162, 212}

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\right.}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} + \frac{2\left(2-3\ 2^{2/3}\right)\text{arctanh}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

```
[In] Int[(2 + 3*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(2 - 3*2^(2/3))*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sq
rt[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -
7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x
^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\left(-3+\sqrt[3]{2}\right)\int\frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx+\frac{1}{3}\left(3+2\sqrt[3]{2}\right)\int\frac{1}{\sqrt{-1-x^3}}\,dx\\ &= \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\\ &\quad+\frac{1}{3}\left(2\left(2-3\ 2^{2/3}\right)\right)\text{Subst}\left(\int\frac{1}{1-3x^2}\,dx,x,\frac{1+\sqrt[3]{2}x}{\sqrt{-1-x^3}}\right)\\ &= \frac{2\left(2-3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}}\\ &\quad+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.36 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.00

$$\int\frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4i\right.\right.\right.}{}$$

```
[In] Integrate[(2 + 3*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 4*Sqrt[3]*(-3 + 2^(1/3
))*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I
```

```
+ (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(
1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqrt[3
])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [A] (verified)

Time = 4.91 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.50

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}-1}}-\frac{2i\left(2-3\,2^{\frac{2}{3}}\right)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{}\right.}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}-1}}-\frac{2i\left(2-3\,2^{\frac{2}{3}}\right)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{}\right.}}{}$

```
[In] int((3*x+2)/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1
/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*
I*3^(1/2)))^(1/2))-2/3*I*(2-3*2^(2/3))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(
1/2))^(1/2)/(-x^3-1)^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/
2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I*3^(
1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.54

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{1}{18}\sqrt{3}\sqrt{-12\cdot 2^{\frac{2}{3}}+18\cdot 2^{\frac{1}{3}}+4}\log\left(\frac{25\,x^{18}-36000\,x^{15}+435000\,x^{12}+526400}{}\right.$$
$$-\frac{2}{3}\left(2i\cdot 2^{\frac{1}{3}}+3i\right)\text{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*sqrt(-12*2^(2/3) + 18*2^(1/3) + 4)*log((25*x^18 - 36000*x^15 +
 435000*x^12 + 526400*x^9 - 259200*x^6 - 384000*x^3 + 2*sqrt(3)*(6*x^16 - 3
4*x^15 + 1134*x^14 - 1860*x^13 + 2116*x^12 - 23976*x^11 + 13992*x^10 - 5056
```

```
*x^9 + 15936*x^7 - 10816*x^6 + 41472*x^5 - 1536*x^4 - 5120*x^3 + 20736*x^2
+ 3*2^(2/3)*(3*x^16 - 17*x^15 + 42*x^14 - 930*x^13 + 1058*x^12 - 888*x^11 +
 6996*x^10 - 2528*x^9 + 7968*x^7 - 5408*x^6 + 1536*x^5 - 768*x^4 - 2560*x^3
 + 768*x^2 - 1536*x - 512) + 2^(1/3)*(2*x^16 - 153*x^15 + 378*x^14 - 620*x^
13 + 9522*x^12 - 7992*x^11 + 4664*x^10 - 22752*x^9 + 5312*x^7 - 48672*x^6 +
 13824*x^5 - 512*x^4 - 23040*x^3 + 6912*x^2 - 1024*x - 4608) - 3072*x - 102
4)*sqrt(-x^3 - 1)*sqrt(-12*2^(2/3) + 18*2^(1/3) + 4) - 600*2^(2/3)*(x^17 -
121*x^14 + 478*x^11 + 1144*x^8 + 608*x^5 + 64*x^2) + 1200*2^(1/3)*(5*x^16 -
 176*x^13 + 83*x^10 + 680*x^7 + 544*x^4 + 128*x) - 51200)/(x^18 + 24*x^15 +
 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4096)) - 2/3*(2*I*2^(1/3) + 3*
I)*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{3x+2}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((2+3*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
[Out] Integral((3*x + 2)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Maxima [F]

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{3\,x+2}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Giac [F]

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{3\,x+2}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{3\,x+2}{\sqrt{-x^3-1}\,(x+2^{2/3})}\,dx$$

```
[In] int((3*x + 2)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
[Out] int((3*x + 2)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

3.56 $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{2\left(e-2^{2/3}f\right)\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 2/9*(e-2^(2/3)*f)*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*(
2^(1/3)*e+f)*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/
2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1
/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2164, 224, 2162, 209}

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} + \frac{2\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)\left(e-2^{2/3}f\right)}{3\sqrt{3}}$$

```
[In] Int[(e + f*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*(e - 2^(2/3)*f)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt
[3]) + (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{6}\left(\sqrt[3]{2}e-2f\right)\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx+\frac{1}{3}\left(\sqrt[3]{2}e+f\right)\int \frac{1}{\sqrt{1+x^3}}\,dx \\
&= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} \\
&\quad +\frac{1}{3}\left(2\left(e-2^{2/3}f\right)\right)\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx,x,\frac{1+\sqrt[3]{2}x}{\sqrt{1+x^3}}\right) \\
&= \frac{2\left(e-2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} \\
&\quad +\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.37 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.14

$$
\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(f\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4i\sqrt{3}\right.\right.\right.}{}
$$

```
[In] Integrate[(e + f*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(f*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 2*Sqrt[3]*(2^(1/3)*e -
 2*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/
(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqr
t[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(126) = 252.

Time = 1.96 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.66

method	result
default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-2^{\frac{2}{3}}f\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+1}\,(}$
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-2^{\frac{2}{3}}f\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+1}\,(}$

```
[In] int((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(e-2^(2/3)*f)*(3/2-1/2*I*3^(1
/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3
^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(
1/2)/(2^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3
^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 992, normalized size of antiderivative = 6.24

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*sqrt(3)*sqrt(2*2^(2/3)*e*f - 2*2^(1/3)*f^2 - e^2)*log(-((e^3 - 4*f^3)
*x^18 - 1440*(e^3 - 4*f^3)*x^15 + 17400*(e^3 - 4*f^3)*x^12 + 21056*(e^3 - 4
*f^3)*x^9 - 10368*(e^3 - 4*f^3)*x^6 - 15360*(e^3 - 4*f^3)*x^3 - 2048*e^3 +
8192*f^3 - 4*sqrt(3)*(2*e*f*x^16 - 17*e^2*x^15 + 252*f^2*x^14 - 620*e*f*x^1
3 + 1058*e^2*x^12 - 5328*f^2*x^11 + 4664*e*f*x^10 - 2528*e^2*x^9 + 5312*e*f
*x^7 - 5408*e^2*x^6 + 9216*f^2*x^5 - 512*e*f*x^4 - 2560*e^2*x^3 + 4608*f^2*
x^2 - 1024*e*f*x - 512*e^2 + 2^(2/3)*(2*f^2*x^16 - 17*e*f*x^15 + 63*e^2*x^1
4 - 620*f^2*x^13 + 1058*e*f*x^12 - 1332*e^2*x^11 + 4664*f^2*x^10 - 2528*e*f
*x^9 + 5312*f^2*x^7 - 5408*e*f*x^6 + 2304*e^2*x^5 - 512*f^2*x^4 - 2560*e*f*
```

```
x^3 + 1152*e^2*x^2 - 1024*f^2*x - 512*e*f) + 2^(1/3)*(e^2*x^16 - 34*f^2*x^1
5 + 126*e*f*x^14 - 310*e^2*x^13 + 2116*f^2*x^12 - 2664*e*f*x^11 + 2332*e^2*
x^10 - 5056*f^2*x^9 + 2656*e^2*x^7 - 10816*f^2*x^6 + 4608*e*f*x^5 - 256*e^2
*x^4 - 5120*f^2*x^3 + 2304*e*f*x^2 - 512*e^2*x - 1024*f^2))*sqrt(x^3 + 1)*s
qrt(2*2^(2/3)*e*f - 2*2^(1/3)*f^2 - e^2) - 24*2^(2/3)*((e^3 - 4*f^3)*x^17 -
 121*(e^3 - 4*f^3)*x^14 + 478*(e^3 - 4*f^3)*x^11 + 1144*(e^3 - 4*f^3)*x^8 +
 608*(e^3 - 4*f^3)*x^5 + 64*(e^3 - 4*f^3)*x^2) + 48*2^(1/3)*(5*(e^3 - 4*f^3
)*x^16 - 176*(e^3 - 4*f^3)*x^13 + 83*(e^3 - 4*f^3)*x^10 + 680*(e^3 - 4*f^3)
*x^7 + 544*(e^3 - 4*f^3)*x^4 + 128*(e^3 - 4*f^3)*x))/(x^18 + 24*x^15 + 240*
x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4096)) + 2/3*(2^(1/3)*e + f)*weiers
trassPInverse(0, -4, x), 1/9*sqrt(3)*sqrt(-2*2^(2/3)*e*f + 2*2^(1/3)*f^2 +
e^2)*arctan(-1/6*sqrt(3)*(4*f^2*x^5 - 14*e*f*x^4 - 5*e^2*x^3 + 4*f^2*x^2 -
8*e*f*x - 2*e^2 + 2^(2/3)*(e^2*x^5 - 14*f^2*x^4 - 5*e*f*x^3 + e^2*x^2 - 8*f
^2*x - 2*e*f) + 2^(1/3)*(2*e*f*x^5 - 7*e^2*x^4 - 10*f^2*x^3 + 2*e*f*x^2 - 4
*e^2*x - 4*f^2))*sqrt(x^3 + 1)*sqrt(-2*2^(2/3)*e*f + 2*2^(1/3)*f^2 + e^2)/(
2*(e^3 - 4*f^3)*x^6 + 3*(e^3 - 4*f^3)*x^3 + e^3 - 4*f^3)) + 2/3*(2^(1/3)*e
+ f)*weierstrassPInverse(0, -4, x)]
```

Sympy [F]

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=\int \frac{e+fx}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((f*x+e)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
[Out] Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad A
rgumen
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{e+f\,x}{\sqrt{x^3+1}\,(x+2^{2/3})}\,dx$$

```
[In] int((e + f*x)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] int((e + f*x)/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

3.57 $\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 175

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{2\left(e+2^{2/3}f\right)\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/9*(e+2^(2/3)*f)*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)-2/9
*(2^(1/3)*e-f)*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(
1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)
^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2164, 224, 2162, 209}

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{2\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)\left(e+2^{2/3}f\right)}{3\sqrt{3}}$$

```
[In] Int[(e + f*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(e + 2^(2/3)*f)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqr
t[3]) - (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{3}\left(-\sqrt[3]{2}e+f\right)\int\frac{1}{\sqrt{1-x^3}}\,dx\right)+\frac{1}{6}\left(\sqrt[3]{2}e+2f\right)\int\frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx\\
&= -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}\\
&\quad-\frac{1}{3}\left(2\left(e+2^{2/3}f\right)\right)\text{Subst}\left(\int\frac{1}{1+3x^2}\,dx,x,\frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)\\
&= -\frac{2\left(e+2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}\\
&\quad-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.42 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.94

$$\int\frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx=\frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-if\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3i\sqrt[3]{2}\right.\right.\right.}{}$$

```
[In] Integrate[(e + f*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-I)*f*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
```

```
(2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqrt[3]*(2^(
1/3)*e + 2*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*S
qrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/
3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.49

method	result
default	$\frac{2if\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2i\left(-e-2^{\frac{2}{3}}f\right)\sqrt{3}\sqrt{i\left(x+\frac{1}{2}\right.}}{}$
elliptic	$\frac{2if\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2i\left(-e-2^{\frac{2}{3}}f\right)\sqrt{3}\sqrt{i\left(x+\frac{1}{2}\right.}}{}$

```
[In] int((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-e-2^(2/3)*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-2^(2/3)-1/2+1/2*I*3^(1/2))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-2^(2/3)-1/2+
1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 1005, normalized size of antiderivative = 5.74

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*sqrt(3)*sqrt(-2*2^(2/3)*e*f - 2*2^(1/3)*f^2 - e^2)*log(((e^3 + 4*f^3)
*x^18 + 1440*(e^3 + 4*f^3)*x^15 + 17400*(e^3 + 4*f^3)*x^12 - 21056*(e^3 + 4
*f^3)*x^9 - 10368*(e^3 + 4*f^3)*x^6 + 15360*(e^3 + 4*f^3)*x^3 - 2048*e^3 -
```

```
8192*f^3 - 4*sqrt(3)*(2*e*f*x^16 - 17*e^2*x^15 - 252*f^2*x^14 + 620*e*f*x^1
3 - 1058*e^2*x^12 - 5328*f^2*x^11 + 4664*e*f*x^10 - 2528*e^2*x^9 - 5312*e*f
*x^7 + 5408*e^2*x^6 + 9216*f^2*x^5 - 512*e*f*x^4 - 2560*e^2*x^3 - 4608*f^2*
x^2 + 1024*e*f*x + 512*e^2 - 2^(2/3)*(2*f^2*x^16 - 17*e*f*x^15 + 63*e^2*x^1
4 + 620*f^2*x^13 - 1058*e*f*x^12 + 1332*e^2*x^11 + 4664*f^2*x^10 - 2528*e*f
*x^9 - 5312*f^2*x^7 + 5408*e*f*x^6 - 2304*e^2*x^5 - 512*f^2*x^4 - 2560*e*f*
x^3 + 1152*e^2*x^2 + 1024*f^2*x + 512*e*f) - 2^(1/3)*(e^2*x^16 + 34*f^2*x^1
5 - 126*e*f*x^14 + 310*e^2*x^13 + 2116*f^2*x^12 - 2664*e*f*x^11 + 2332*e^2*
x^10 + 5056*f^2*x^9 - 2656*e^2*x^7 - 10816*f^2*x^6 + 4608*e*f*x^5 - 256*e^2
*x^4 + 5120*f^2*x^3 - 2304*e*f*x^2 + 512*e^2*x - 1024*f^2))*sqrt(-x^3 + 1)*
sqrt(-2*2^(2/3)*e*f - 2*2^(1/3)*f^2 - e^2) + 24*2^(2/3)*((e^3 + 4*f^3)*x^17
 + 121*(e^3 + 4*f^3)*x^14 + 478*(e^3 + 4*f^3)*x^11 - 1144*(e^3 + 4*f^3)*x^8
 + 608*(e^3 + 4*f^3)*x^5 - 64*(e^3 + 4*f^3)*x^2) + 48*2^(1/3)*(5*(e^3 + 4*f
^3)*x^16 + 176*(e^3 + 4*f^3)*x^13 + 83*(e^3 + 4*f^3)*x^10 - 680*(e^3 + 4*f^
3)*x^7 + 544*(e^3 + 4*f^3)*x^4 - 128*(e^3 + 4*f^3)*x))/(x^18 - 24*x^15 + 24
0*x^12 - 1280*x^9 + 3840*x^6 - 6144*x^3 + 4096)) - 2/3*(I*2^(1/3)*e - I*f)*
weierstrassPInverse(0, 4, x), -1/9*sqrt(3)*sqrt(2*2^(2/3)*e*f + 2*2^(1/3)*f
^2 + e^2)*arctan(1/6*sqrt(3)*(4*f^2*x^5 - 14*e*f*x^4 - 5*e^2*x^3 - 4*f^2*x^
2 + 8*e*f*x + 2*e^2 + 2^(2/3)*(e^2*x^5 + 14*f^2*x^4 + 5*e*f*x^3 - e^2*x^2 -
 8*f^2*x - 2*e*f) - 2^(1/3)*(2*e*f*x^5 - 7*e^2*x^4 + 10*f^2*x^3 - 2*e*f*x^2
 + 4*e^2*x - 4*f^2))*sqrt(-x^3 + 1)*sqrt(2*2^(2/3)*e*f + 2*2^(1/3)*f^2 + e^
2)/(2*(e^3 + 4*f^3)*x^6 - 3*(e^3 + 4*f^3)*x^3 + e^3 + 4*f^3)) - 2/3*(I*2^(1
/3)*e - I*f)*weierstrassPInverse(0, 4, x)]
```

Sympy [F]

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\int \frac{e}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx - \int \frac{fx}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

```
[In] integrate((f*x+e)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
[Out] -Integral(e/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integral(f*x
/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{fx+e}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((f*x + e)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Arg
ument
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{e+f\,x}{\sqrt{1-x^3}\,(x-2^{2/3})}\,dx$$

```
[In] int(-(e + f*x)/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] int(-(e + f*x)/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

3.58 $\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 178

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{2\left(e+2^{2/3}f\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -2/9*(e+2^(2/3)*f)*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)-2/9
*(2^(1/3)*e-f)*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(
1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^
(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2164, 225, 2162, 212}

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{2\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)\left(e+2^{2/3}f\right)}{3\sqrt{3}}$$

```
[In] Int[(e + f*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*(e + 2^(2/3)*f)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*S
qrt[3]) - (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
-7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{3}\left(-\sqrt[3]{2}e+f\right)\int\frac{1}{\sqrt{-1+x^3}}\,dx\right)+\frac{1}{6}\left(\sqrt[3]{2}e+2f\right)\int\frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx$$

$$=-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}-\frac{1}{3}\left(2\left(e+2^{2/3}f\right)\right)\text{Subst}\left(\int\frac{1}{1-3x^2}\,dx,x,\frac{1-\sqrt[3]{2}x}{\sqrt{-1+x^3}}\right)$$

$$=-\frac{2\left(e+2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.42 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.90

$$\int\frac{e+fx}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx=\frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-if\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3i\right.\right.\right.}{}$$

```
[In] Integrate[(e + f*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-I)*f*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
```

```
(2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqrt[3]*(2^(
1/3)*e + 2*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*S
qrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/
3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.52

method	result
default	$-\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-e-2^{\frac{2}{3}}f\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$
elliptic	$-\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-e-2^{\frac{2}{3}}f\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$

```
[In] int((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-e-2^(2/3)*f)*(-3/2-1/2*I*3^
(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I
*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^
(1/2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I
*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 999, normalized size of antiderivative = 5.61

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*sqrt(3)*sqrt(2*2^(2/3)*e*f + 2*2^(1/3)*f^2 + e^2)*log(((e^3 + 4*f^3)*
x^18 + 1440*(e^3 + 4*f^3)*x^15 + 17400*(e^3 + 4*f^3)*x^12 - 21056*(e^3 + 4*
f^3)*x^9 - 10368*(e^3 + 4*f^3)*x^6 + 15360*(e^3 + 4*f^3)*x^3 - 2048*e^3 - 8
192*f^3 - 4*sqrt(3)*(2*e*f*x^16 - 17*e^2*x^15 - 252*f^2*x^14 + 620*e*f*x^13
 - 1058*e^2*x^12 - 5328*f^2*x^11 + 4664*e*f*x^10 - 2528*e^2*x^9 - 5312*e*f*
```

```
x^7 + 5408*e^2*x^6 + 9216*f^2*x^5 - 512*e*f*x^4 - 2560*e^2*x^3 - 4608*f^2*x
^2 + 1024*e*f*x + 512*e^2 - 2^(2/3)*(2*f^2*x^16 - 17*e*f*x^15 + 63*e^2*x^14
 + 620*f^2*x^13 - 1058*e*f*x^12 + 1332*e^2*x^11 + 4664*f^2*x^10 - 2528*e*f*
x^9 - 5312*f^2*x^7 + 5408*e*f*x^6 - 2304*e^2*x^5 - 512*f^2*x^4 - 2560*e*f*x
^3 + 1152*e^2*x^2 + 1024*f^2*x + 512*e*f) - 2^(1/3)*(e^2*x^16 + 34*f^2*x^15
 - 126*e*f*x^14 + 310*e^2*x^13 + 2116*f^2*x^12 - 2664*e*f*x^11 + 2332*e^2*x
^10 + 5056*f^2*x^9 - 2656*e^2*x^7 - 10816*f^2*x^6 + 4608*e*f*x^5 - 256*e^2*
x^4 + 5120*f^2*x^3 - 2304*e*f*x^2 + 512*e^2*x - 1024*f^2))*sqrt(x^3 - 1)*sq
rt(2*2^(2/3)*e*f + 2*2^(1/3)*f^2 + e^2) + 24*2^(2/3)*((e^3 + 4*f^3)*x^17 +
121*(e^3 + 4*f^3)*x^14 + 478*(e^3 + 4*f^3)*x^11 - 1144*(e^3 + 4*f^3)*x^8 +
608*(e^3 + 4*f^3)*x^5 - 64*(e^3 + 4*f^3)*x^2) + 48*2^(1/3)*(5*(e^3 + 4*f^3)
*x^16 + 176*(e^3 + 4*f^3)*x^13 + 83*(e^3 + 4*f^3)*x^10 - 680*(e^3 + 4*f^3)*
x^7 + 544*(e^3 + 4*f^3)*x^4 - 128*(e^3 + 4*f^3)*x))/(x^18 - 24*x^15 + 240*x
^12 - 1280*x^9 + 3840*x^6 - 6144*x^3 + 4096)) + 2/3*(2^(1/3)*e - f)*weierst
rassPInverse(0, 4, x), -1/9*sqrt(3)*sqrt(-2*2^(2/3)*e*f - 2*2^(1/3)*f^2 - e
^2)*arctan(1/6*sqrt(3)*(4*f^2*x^5 - 14*e*f*x^4 - 5*e^2*x^3 - 4*f^2*x^2 + 8*
e*f*x + 2*e^2 + 2^(2/3)*(e^2*x^5 + 14*f^2*x^4 + 5*e*f*x^3 - e^2*x^2 - 8*f^2
*x - 2*e*f) - 2^(1/3)*(2*e*f*x^5 - 7*e^2*x^4 + 10*f^2*x^3 - 2*e*f*x^2 + 4*e
^2*x - 4*f^2))*sqrt(x^3 - 1)*sqrt(-2*2^(2/3)*e*f - 2*2^(1/3)*f^2 - e^2)/(2*
(e^3 + 4*f^3)*x^6 - 3*(e^3 + 4*f^3)*x^3 + e^3 + 4*f^3)) + 2/3*(2^(1/3)*e -
f)*weierstrassPInverse(0, 4, x)]
```

Sympy [F]

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{e}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx - \int \frac{fx}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

```
[In] integrate((f*x+e)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
[Out] -Integral(e/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integral(f*x
/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \int -\frac{fx+e}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((f*x + e)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Arg
ument
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \int -\frac{e+f\,x}{\sqrt{x^3-1}\,\left(x-2^{2/3}\right)}\,dx$$

```
[In] int(-(e + f*x)/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] int(-(e + f*x)/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

3.59 $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 170

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{2\left(e-2^{2/3}f\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 2/9*(e-2^(2/3)*f)*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/9
*(2^(1/3)*e+f)*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(
1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)
^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2164, 225, 2162, 212}

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}\left(\sqrt[3]{2}e+f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\right.}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-1}}$$

$$+\frac{2\text{arctanh}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)\left(e-2^{2/3}f\right)}{3\sqrt{3}}$$

```
[In] Int[(e + f*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(e - 2^(2/3)*f)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sq
rt[3]) + (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqrt[(1 - x + x^2)/(1
 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -
7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x
^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}\left(\sqrt[3]{2}e-2f\right)\int\frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx+\frac{1}{3}\left(\sqrt[3]{2}e+f\right)\int\frac{1}{\sqrt{-1-x^3}}\,dx$$

$$=\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+\frac{1}{3}\left(2\left(e-2^{2/3}f\right)\right)\text{Subst}\left(\int\frac{1}{1-3x^2}\,dx,x,\frac{1+\sqrt[3]{2}x}{\sqrt{-1-x^3}}\right)$$

$$=\frac{2\left(e-2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.39 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.01

$$\int\frac{e+fx}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx=\frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(f\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4i\right.\right.\right.}{}$$

```
[In] Integrate[(e + f*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(f*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 2*Sqrt[3]*(2^(1/3)*e -
 2*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/
```

```
(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqr
t[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.50

method	result
default	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}} - \frac{2 i \left(e-2^{\frac{2}{3}} f\right) \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\right.}}{}$
elliptic	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}} - \frac{2 i \left(e-2^{\frac{2}{3}} f\right) \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\right.}}{}$

```
[In] int((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-2^(2/3)*f)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))
*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3
^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I
*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 1002, normalized size of antiderivative = 5.89

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*sqrt(3)*sqrt(-2*2^(2/3)*e*f + 2*2^(1/3)*f^2 + e^2)*log(-((e^3 - 4*f^3
)*x^18 - 1440*(e^3 - 4*f^3)*x^15 + 17400*(e^3 - 4*f^3)*x^12 + 21056*(e^3 -
4*f^3)*x^9 - 10368*(e^3 - 4*f^3)*x^6 - 15360*(e^3 - 4*f^3)*x^3 - 2048*e^3 +
 8192*f^3 - 4*sqrt(3)*(2*e*f*x^16 - 17*e^2*x^15 + 252*f^2*x^14 - 620*e*f*x^
13 + 1058*e^2*x^12 - 5328*f^2*x^11 + 4664*e*f*x^10 - 2528*e^2*x^9 + 5312*e*
```

```
f*x^7 - 5408*e^2*x^6 + 9216*f^2*x^5 - 512*e*f*x^4 - 2560*e^2*x^3 + 4608*f^2
*x^2 - 1024*e*f*x - 512*e^2 + 2^(2/3)*(2*f^2*x^16 - 17*e*f*x^15 + 63*e^2*x^
14 - 620*f^2*x^13 + 1058*e*f*x^12 - 1332*e^2*x^11 + 4664*f^2*x^10 - 2528*e*
f*x^9 + 5312*f^2*x^7 - 5408*e*f*x^6 + 2304*e^2*x^5 - 512*f^2*x^4 - 2560*e*f
*x^3 + 1152*e^2*x^2 - 1024*f^2*x - 512*e*f) + 2^(1/3)*(e^2*x^16 - 34*f^2*x^
15 + 126*e*f*x^14 - 310*e^2*x^13 + 2116*f^2*x^12 - 2664*e*f*x^11 + 2332*e^2
*x^10 - 5056*f^2*x^9 + 2656*e^2*x^7 - 10816*f^2*x^6 + 4608*e*f*x^5 - 256*e^
2*x^4 - 5120*f^2*x^3 + 2304*e*f*x^2 - 512*e^2*x - 1024*f^2))*sqrt(-x^3 - 1)
*sqrt(-2*2^(2/3)*e*f + 2*2^(1/3)*f^2 + e^2) - 24*2^(2/3)*((e^3 - 4*f^3)*x^1
7 - 121*(e^3 - 4*f^3)*x^14 + 478*(e^3 - 4*f^3)*x^11 + 1144*(e^3 - 4*f^3)*x^
8 + 608*(e^3 - 4*f^3)*x^5 + 64*(e^3 - 4*f^3)*x^2) + 48*2^(1/3)*(5*(e^3 - 4*
f^3)*x^16 - 176*(e^3 - 4*f^3)*x^13 + 83*(e^3 - 4*f^3)*x^10 + 680*(e^3 - 4*f
^3)*x^7 + 544*(e^3 - 4*f^3)*x^4 + 128*(e^3 - 4*f^3)*x))/(x^18 + 24*x^15 + 2
40*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4096)) - 2/3*(I*2^(1/3)*e + I*f)
*weierstrassPInverse(0, -4, x), 1/9*sqrt(3)*sqrt(2*2^(2/3)*e*f - 2*2^(1/3)*
f^2 - e^2)*arctan(-1/6*sqrt(3)*(4*f^2*x^5 - 14*e*f*x^4 - 5*e^2*x^3 + 4*f^2*
x^2 - 8*e*f*x - 2*e^2 + 2^(2/3)*(e^2*x^5 - 14*f^2*x^4 - 5*e*f*x^3 + e^2*x^2
 - 8*f^2*x - 2*e*f) + 2^(1/3)*(2*e*f*x^5 - 7*e^2*x^4 - 10*f^2*x^3 + 2*e*f*x
^2 - 4*e^2*x - 4*f^2))*sqrt(-x^3 - 1)*sqrt(2*2^(2/3)*e*f - 2*2^(1/3)*f^2 -
e^2)/(2*(e^3 - 4*f^3)*x^6 + 3*(e^3 - 4*f^3)*x^3 + e^3 - 4*f^3)) - 2/3*(I*2^
(1/3)*e + I*f)*weierstrassPInverse(0, -4, x)]
```

Sympy [F]

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{e+fx}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((f*x+e)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
[Out] Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad A
rgumen
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{e+f\,x}{\sqrt{-x^3-1}\,(x+2^{2/3})}\,dx$$

```
[In] int((e + f*x)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] int((e + f*x)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

3.60 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 316

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}\sqrt[3]{b}e+\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/9*(b^(1/3)*e-2^(2/3)*a^(1/3)*f)*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x
)*3^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)+2/9*(2^(1/3)*b^(1/3)*e+a
^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2
)*3^(3/4)/a^(1/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2164, 224, 2162, 209}

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx = \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{2}\sqrt[3]{b}e\right)\text{Elliptic}}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{}}$$

$$+\frac{2\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$

```
[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e - 2^(2/3)*a^(1/3)*f)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/
3)*b^(1/3)*x))/Sqrt[a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 +
Sqrt[3]]*(2^(1/3)*b^(1/3)*e + a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}} - \frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\sqrt[3]{a} - 2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a} + \sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx + \frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}} + \frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}} + \frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x}\right)|-7-\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{\left(2\left(\sqrt[3]{b}e - 2^{2/3}\sqrt[3]{a}f\right)\right)\text{Subst}\left(\int \frac{1}{1+3ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{3b^{2/3}}$$

$$= \frac{2\left(\sqrt[3]{b}e - 2^{2/3}\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a} + \sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$

$$+ \frac{2\sqrt{2+\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}} + \frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.04 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{3f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\operatorname{ar}\cdots\right.}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}\cdots\right.}{\cdots}$$

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-3*f*((-1)^(1/3
)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1
 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3
)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] + ((-1)^(1/3)*(1 + (-1)^(1/3))*(-(b^
(1/3)*e) + 2^(2/3)*a^(1/3)*f)*Sqrt[3 - (3*b^(1/3)*x)/a^(1/3) + (3*b^(2/3)*x
^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^
(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/((
-1)^(1/3) + 2^(2/3))))/(3*b^(2/3)*Sqrt[a + b*x^3])
```

Maple [F]

$$\int \frac{fx+e}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}+a}}dx$$

```
[In] int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{e+fx}{\sqrt{a+bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
[Out] Integral((e + f*x)/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{fx+e}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm=
"maxima")
[Out] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm=
"giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{e+f\,x}{\sqrt{b\,x^3+a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

```
[In] int((e + f*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
[Out] int((e + f*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.61 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 324

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] -2/9*(b^(1/3)*e+2^(2/3)*a^(1/3)*f)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*
x)*3^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)-2/9*(2^(1/3)*b^(1/3)*e
-a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*(
(a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(3/4)/a^(1/3)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x
)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2164, 224, 2162, 209}

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} - \frac{2\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)\left(2^{2/3}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$

```
[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1
/3)*b^(1/3)*x))/Sqrt[a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 +
 Sqrt[3]]*(2^(1/3)*b^(1/3)*e - a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/
3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)
*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x
^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{3}\left(-\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{a-bx^3}}\,dx\right)+\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int\frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} - \frac{\left(2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\right)\text{Subst}\left(\int\frac{1}{1+3ax^2}\,dx,x,\frac{1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{3b^{2/3}}$$

$$= -\frac{2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.94 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.23

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}\cdots}{(1\cdots}}\right.\right.\right.}{\left(\sqrt[3]{-}\cdots\right.}$$

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) + 2
^(2/3))*f*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)
^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/
3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1
)^(1/3)*(1 + (-1)^(1/3))*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3) - (-
1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3
) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), A
rcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (
-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^
(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Maple [F]

$$\int \frac{fx+e}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}+a}}dx$$

```
[In] int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\int \frac{e}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx-\int \frac{fx}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(e/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x) - Integral(f*x/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*s
qrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{fx+e}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm
="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int \frac{e+f\,x}{\sqrt{a-b\,x^3}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

```
[In] int((e + f*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

```
[Out] int((e + f*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.62 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 333

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

```
[Out] -2/9*(b^(1/3)*e+2^(2/3)*a^(1/3)*f)*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)
*x)*3^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)-2/9*(2^(1/3)*b^(1/3)*e
-a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/
(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^
(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x
)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {2164, 225, 2162, 212}

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx =$$

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)\left(2^{2/3}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$

```
[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(
1/3)*b^(1/3)*x))/Sqrt[-a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) - (2*Sqrt[2
 - Sqrt[3]]*(2^(1/3)*b^(1/3)*e - a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(
2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(
1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a
 + b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{3}\left(-\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{-a+bx^3}}\,dx\right)+\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int\frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}-\frac{\left(2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\right)\text{Subst}\left(\int\frac{1}{1-3ax^2}\,dx,x,\frac{1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{3b^{2/3}}$$

$$= -\frac{2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.36 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.20

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{\cdots}}{(1\cdots}}\right.\right.\right.}{\left(\sqrt[3]{-\cdots}\right.}$$

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) + 2
^(2/3))*f*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)
^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/
3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1
)^(1/3)*(1 + (-1)^(1/3))*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3) - (-
1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3
) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), A
rcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (
-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^
(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Maple [F]

$$\int \frac{fx+e}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}-a}}dx$$

```
[In] int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm=
"fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\int \frac{e}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx - \int \frac{fx}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
[Out] -Integral(e/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x) - Integral(f*x/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*
x*sqrt(-a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm=
"maxima")
[Out] -integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{e+f\,x}{\sqrt{b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

```
[In] int((e + f*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

```
[Out] int((e + f*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.63 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 329

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}\sqrt[3]{b}e+\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] 2/9*(b^(1/3)*e-2^(2/3)*a^(1/3)*f)*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*
x)*3^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)+2/9*(2^(1/3)*b^(1/3)*e
+a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/
2))*3^(3/4)/a^(1/3)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {2164, 225, 2162, 212}

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{2}\sqrt[3]{b}e\right)\text{Ellip}}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}}}$$

$$+\frac{2\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$

```
[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e - 2^(2/3)*a^(1/3)*f)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1
/3)*b^(1/3)*x))/Sqrt[-a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) + (2*Sqrt[2
- Sqrt[3]]*(2^(1/3)*b^(1/3)*e + a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a
- b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int\frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx+\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{-a-bx^3}}\,dx$$

$$=\frac{2\sqrt{2-\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{\left(2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\right)\text{Subst}\left(\int\frac{1}{1-3ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{3b^{2/3}}$$

$$=\frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.85 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.22

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+\cdots}{(1+\cdots}}\right.\right.\right.}{(\sqrt[3]{-1}\cdots}$$

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) + 2
^(2/3))*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^
(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3)
 + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1)^
(1/3)*(1 + (-1)^(1/3))*(-(b^(1/3)*e) + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3) + (
-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/
3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]],
(-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)
^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F]

$$\int \frac{fx+e}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}-a}}dx$$

```
[In] int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{e+fx}{\sqrt{-a-bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{e+f\,x}{\sqrt{-b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

```
[In] int((e + f*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
[Out] int((e + f*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.64 $\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 265

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{2(de-cf)\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d^2} + \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(2de+cf)\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

```
[Out] 2/9*(-c*f+d*e)*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/c^(3
/2)/d^2*3^(1/2)+1/9*2^(1/3)*(c*f+2*d*e)*(c+2^(2/3)*d*x)*EllipticF((2^(2/3)*
d*x+c*(1-3^(1/2)))/(2^(2/3)*d*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((c^2-2^(2/3)*c*d*x+2*2^(1/3)*d^2*x^2)/(2^(2/3)*d*x+c*(1+3^(1/
2)))^2)^(1/2)*3^(3/4)/c/d^2/(4*d^3*x^3+c^3)^(1/2)/(c*(c+2^(2/3)*d*x)/(2^(2/
3)*d*x+c*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used

$= \{2164, 224, 2162, 209\}$

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

$$= \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}(cf+2de)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

$$+ \frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)(de-cf)}{3\sqrt{3}c^{3/2}d^2}$$

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2*(d*e - c*f)*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])
/(3*Sqrt[3]*c^(3/2)*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c + 2^
(2/3)*d*x)*Sqrt[(c^2 - 2^(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c
+ 2^(2/3)*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sq
rt[3])*c + 2^(2/3)*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d^2*Sqrt[(c*(c + 2^
(2/3)*d*x))/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx}{3cd} + \frac{(2de+cf)\int \frac{1}{\sqrt{c^3+4d^3x^3}}\,dx}{3cd}$$

$$= \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(2de+cf)\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

$$+ \frac{(2(de-cf))\text{Subst}\left(\int \frac{1}{1+3c^3x^2}\,dx, x, \frac{1+\frac{2dx}{c}}{\sqrt{c^3+4d^3x^3}}\right)}{3d^2}$$

$$= \frac{2(de-cf)\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d^2}$$

$$+ \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(2de+cf)\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.07 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.43

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

$$= \frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(-f\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.\right.}{\left(2+\sqrt[3]{\cdots}\right.}$$

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(-(f*Sqrt[((-2)^(1/
3)*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3)*(2 + (-2)^(1/3))
*c - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*(-1
)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]) + ((-1)^(1/3)*2^(
2/3)*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((
1 + (-1)^(1/3))*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*Ell
ipticPi[(I*2^(1/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-
1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)])/Sqrt[3]))/((2 +
(-2)^(1/3))*d^2*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*S
qrt[c^3 + 4*d^3*x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(216) = 432$.

Time = 0.96 (sec) , antiderivative size = 900, normalized size of antiderivative = 3.40

method	result	size
default	Expression too large to display	900
elliptic	Expression too large to display	900

```
[In] int((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2
^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4
*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x
+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d)
)^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x
^3+c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*
2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)
)^(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2
)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^
(1/2))+2*(-c*f+d*e)/d^2*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/
3)+1/4*I*3^(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)
/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3
))*c/d))^(1/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/
d+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/
4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/
d))^(1/2)/(4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/
d)*EllipticPi(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/
4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((
1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*
```

```
c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(
1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/
4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.47

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \left[\frac{\sqrt{3}(d^3e-cd^2f)\sqrt{-c}\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6-\sqrt{3}(4\,d^4x^4-10\,cd^3x^3-18\,c^2d^2x^2-8\,c^3dx-c^4)\sqrt{4\,d}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right.}{18\,c^2d^4}\right.$$

$$-\frac{\sqrt{3}(d^3e-cd^2f)\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{4\,d^3x^3+c^3}\,(2\,d^3x^3-6\,cd^2x^2-6\,c^2dx-c^3)\sqrt{c}}{3\,(8\,cd^4x^4+4\,c^2d^3x^3+2\,c^4dx+c^5)}\right)-3\sqrt{d^3}(2\,cde+c^2f)\mathrm{weierstrassPInve}}{9\,c^2d^4}$$

```
[In] integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*(sqrt(3)*(d^3*e - c*d^2*f)*sqrt(-c)*log((2*d^6*x^6 - 36*c*d^5*x^5 - 1
8*c^2*d^4*x^4 + 28*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*d^4*x^4
- 10*c*d^3*x^3 - 18*c^2*d^2*x^2 - 8*c^3*d*x - c^4)*sqrt(4*d^3*x^3 + c^3)*sq
rt(-c))/(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d
^2*x^2 + 6*c^5*d*x + c^6)) + 6*sqrt(d^3)*(2*c*d*e + c^2*f)*weierstrassPInve
rse(0, -c^3/d^3, x))/(c^2*d^4), -1/9*(sqrt(3)*(d^3*e - c*d^2*f)*sqrt(c)*arc
tan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2*d^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x
- c^3)*sqrt(c)/(8*c*d^4*x^4 + 4*c^2*d^3*x^3 + 2*c^4*d*x + c^5)) - 3*sqrt(d^
3)*(2*c*d*e + c^2*f)*weierstrassPInverse(0, -c^3/d^3, x))/(c^2*d^4)]
```

Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{e+fx}{(c+dx)\,\sqrt{c^3+4d^3x^3}}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral((e + f*x)/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{fx+e}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{fx+e}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
[Out] integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{e+f\,x}{\sqrt{c^3+4\,d^3\,x^3}\,(c+d\,x)}\,dx$$

```
[In] int((e + f*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
[Out] int((e + f*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)), x)
```

3.65 $\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 145

$$\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -2/9*2^(2/3)*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*(1+x)*
EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/
2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(
1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2164, 224, 2162, 209}

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

```
[In] Int[x/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) +
(2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\int \frac{1}{\sqrt{1+x^3}}\,dx - \frac{1}{3}\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx \\
&= \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} \\
&\quad - \frac{1}{3}\left(2\ 2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1+\sqrt[3]{2}x}{\sqrt{1+x^3}}\right) \\
&= -\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} \\
&\quad + \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.32 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.43

$$
\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{i2^{2/3}\ldots}{\ldots}\right)}{\sqrt{1+x^3}}
$$

```
[In] Integrate[x/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[1 + x^3]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(114) = 228.

Time = 3.69 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.78

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} - \frac{2\,2^{\frac{2}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\cdots}{-\frac{3}{2}+\cdots}}\cdots}{\sqrt{x^3+1}\,(2^{\frac{2}{3}}\cdots}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} - \frac{2\,2^{\frac{2}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\cdots}{-\frac{3}{2}+\cdots}}\cdots}{\sqrt{x^3+1}\,(2^{\frac{2}{3}}\cdots}$

```
[In] int(x/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*2^(2/3)*(3/2-1/2*I*3^(1/2))*((x
+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2^
(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/
(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.57

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = -\frac{1}{9}\sqrt{3}2^{\frac{2}{3}}\arctan\left(-\frac{\sqrt{3}2^{\frac{2}{3}}\left(2x^5+2x^2-2^{\frac{2}{3}}(7x^4+4x)-2^{\frac{1}{3}}(5x^3+2)\right)\sqrt{x^3+1}}{12(2x^6+3x^3+1)}\right) + \frac{2}{3}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*2^(2/3)*arctan(-1/12*sqrt(3)*2^(2/3)*(2*x^5 + 2*x^2 - 2^(2/3)*
(7*x^4 + 4*x) - 2^(1/3)*(5*x^3 + 2))*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1)) + 2
/3*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate(x/(2**(2/3)+x)/(x**3+1)**(1/2),x)
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Maxima [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad A
rgumen
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\,(x+2^{2/3})}\,dx$$

```
[In] int(x/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
[Out] int(x/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

3.66 $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 160

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/9*2^(2/3)*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)+2/9*(1-x)
*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1
/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3
^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2164, 224, 2162, 209}

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

```
[In] Int[x/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3]) +
(2*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}\,dx\right)+\frac{1}{3}\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx\\
&= \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}\\
&\quad -\frac{1}{3}\left(2\ 2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx,x,\frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)\\
&= -\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}\\
&\quad +\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.35 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.31

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\cdots\right.}{\sqrt{1-x^3}}$$

```
[In] Integrate[x/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[1 - x^3]
```

Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.58

method	result
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$

```
[In] int(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))+2/3*I*2^(2/3)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/
2))^(1/2)/(-x^3+1)^(1/2)/(-2^(2/3)-1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-2^(2/3)-1/2+1/2*I*3^(
1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{1}{9}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2\,x^5-2\,x^2+2^{\frac{2}{3}}(7\,x^4-4\,x)-2^{\frac{1}{3}}(5\,x^3-2)\right)\sqrt{-x^3+1}}{12\,(2\,x^6-3\,x^3+1)}\right) + \frac{2}{3}i\,\text{weierstrassPInverse}(0,4,x)$$

```
[In] integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] -1/9*sqrt(3)*2^(2/3)*arctan(1/12*sqrt(3)*2^(2/3)*(2*x^5 - 2*x^2 + 2^(2/3)*(
7*x^4 - 4*x) - 2^(1/3)*(5*x^3 - 2))*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1)) + 2
/3*I*weierstrassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\int \frac{x}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

```
[In] integrate(x/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
[Out] -Integral(x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{x}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Arg
ument
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\int \frac{x}{\sqrt{1-x^3}\,(x-2^{2/3})}\,dx$$

```
[In] int(-x/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
[Out] -int(x/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

3.67 $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 163

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{2\ 2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -2/9*2^(2/3)*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)+2/9*(1-x)
*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1
/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3
^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2164, 225, 2162, 212}

$$\int \frac{x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{2\;2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

```
[In] Int[x/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3])
+ (2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{3}\int \frac{1}{\sqrt{-1+x^3}}\,dx\right)+\frac{1}{3}\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx\\
&= \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\\
&\quad -\frac{1}{3}\left(2\ 2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx,x,\frac{1-\sqrt[3]{2}x}{\sqrt{-1+x^3}}\right)\\
&= -\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}}\\
&\quad +\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.31 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.27

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i2\ldots}{\ldots}\right)}{\sqrt{-1+x^3}}$$

```
[In] Integrate[x/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[-1 + x^3]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(127) = 254.

Time = 3.58 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.61

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\,2^{\frac{2}{3}}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3-}}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\,2^{\frac{2}{3}}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3-}}$

```
[In] int(x/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*2^(2/3)*(-3/2-1/2*I*3^(1/2))*((
x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/(-2
^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))
/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.50

$$\int \frac{x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \frac{1}{18}\sqrt{3}2^{\frac{2}{3}}\log\left(\frac{x^{18}+1440\,x^{15}+17400\,x^{12}-21056\,x^{9}-10368\,x^{6}+15360\,x^{3}+}{}\right.$$
$$-\frac{2}{3}\,\text{weierstrassPInverse}(0,4,x)$$

```
[In] integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*2^(2/3)*log((x^18 + 1440*x^15 + 17400*x^12 - 21056*x^9 - 10368
*x^6 + 15360*x^3 + 2*sqrt(3)*2^(2/3)*(126*x^14 + 2664*x^11 - 4608*x^5 + 230
4*x^2 + 2^(2/3)*(x^16 + 310*x^13 + 2332*x^10 - 2656*x^7 - 256*x^4 + 512*x)
+ 2^(1/3)*(17*x^15 + 1058*x^12 + 2528*x^9 - 5408*x^6 + 2560*x^3 - 512))*sqr
t(x^3 - 1) + 24*2^(2/3)*(x^17 + 121*x^14 + 478*x^11 - 1144*x^8 + 608*x^5 -
64*x^2) + 48*2^(1/3)*(5*x^16 + 176*x^13 + 83*x^10 - 680*x^7 + 544*x^4 - 128
*x) - 2048)/(x^18 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 6144*x^3 + 4
096)) - 2/3*weierstrassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\int \frac{x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

```
[In] integrate(x/(2**(2/3)-x)/(x**3-1)**(1/2),x)
[Out] -Integral(x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \int -\frac{x}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

```
[In] integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad A
rgumen
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\int \frac{x}{\sqrt{x^3-1}\,\left(x-2^{2/3}\right)}\,dx$$

```
[In] int(-x/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
[Out] -int(x/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

3.68 $\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = -\frac{2\,2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] -2/9*2^(2/3)*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/9*(1+x
)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(
1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x
-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2164, 225, 2162, 212}

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} - \frac{2\,2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

```
[In] Int[x/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3])
+ (2*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\int \frac{1}{\sqrt{-1-x^3}}\,dx - \frac{1}{3}\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{1}{3}\left(2\ 2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{1+\sqrt[3]{2}x}{\sqrt{-1-x^3}}\right)$$

$$= -\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.34

$$\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \cdots\right)}{\sqrt{-1-x^3}}$$

```
[In] Integrate[x/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[-1 - x^3]
```

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.60

method	result
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$

```
[In] int(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))+2/3*I*2^(2/3)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2)
)^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2)
)^(1/2)/(-x^3-1)^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(
I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2)
),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.59

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{1}{18}\sqrt{3}2^{\frac{2}{3}}\log\left(\frac{x^{18}-1440\,x^{15}+17400\,x^{12}+21056\,x^9-10368\,x^6-15360\,x^3+}{}\right.$$
$$-\frac{2}{3}i\,\text{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*2^(2/3)*log((x^18 - 1440*x^15 + 17400*x^12 + 21056*x^9 - 10368
*x^6 - 15360*x^3 + 2*sqrt(3)*2^(2/3)*(126*x^14 - 2664*x^11 + 4608*x^5 + 230
4*x^2 + 2^(2/3)*(x^16 - 310*x^13 + 2332*x^10 + 2656*x^7 - 256*x^4 - 512*x)
- 2^(1/3)*(17*x^15 - 1058*x^12 + 2528*x^9 + 5408*x^6 + 2560*x^3 + 512))*sqr
t(-x^3 - 1) - 24*2^(2/3)*(x^17 - 121*x^14 + 478*x^11 + 1144*x^8 + 608*x^5 +
 64*x^2) + 48*2^(1/3)*(5*x^16 - 176*x^13 + 83*x^10 + 680*x^7 + 544*x^4 + 12
8*x) - 2048)/(x^18 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 +
4096)) - 2/3*I*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

[In] integrate(x/(2**(2/3)+x)/(-x**3-1)**(1/2),x)

[Out] Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)

Maxima [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

[In] integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")

[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)

Giac [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

[In] integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}\,(x+2^{2/3})}\,dx$$

[In] int(x/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)

[Out] int(x/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)

3.69 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 275

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/9*2^(2/3)*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3+a)^(
1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^
(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)
+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2164, 224, 2162, 209}

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x\ldots}{\sqrt[3]{b}x\ldots}\right.\right.}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

```
[In] Int[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a +
 b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(
1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}} - \frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{\left(2\ 2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+3ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{3b^{2/3}}$$

$$= -\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

$$+ \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.00 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.22

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{3\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,\text{EllipticF}\left(\text{ar}\right.}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}\right.}{}$$

```
[In] Integrate[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-3*((-1)^(1/3)*
a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
 (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*
x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-1)^(2/3)*b^
(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))
*a^(1/3)*Sqrt[3 - (3*b^(1/3)*x)/a^(1/3) + (3*b^(2/3)*x^2)/a^(2/3)]*Elliptic
Pi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^
(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3)))
)/(3*b^(2/3)*Sqrt[a + b*x^3])
```

Maple [F]

$$\int \frac{x}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}+a}}dx$$

```
[In] int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x}{\sqrt{a+bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
[Out] Integral(x/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxim
a")
[Out] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac"
)
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x}{\sqrt{b\,x^3+a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

```
[In] int(x/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
[Out] int(x/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.70 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 283

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] -2/9*2^(2/3)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3+a)^
(1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2
*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^
(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2164, 224, 2162, 209}

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{(1-\sqrt{3}\cdots}{(1+\sqrt{3}\cdots}\right.\right.}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$-\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

```
[In] Int[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a -
 b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(
1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sq
rt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*S
qrt[a - b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{1}{\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}} + \frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$- \frac{\left(2\ 2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+3ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{3b^{2/3}}$$

$$= -\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

$$+ \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.79 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.37

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}-(\cdots}{(1+\sqrt[3]{\cdots}}}\right.\right.\right.}{\left(\sqrt[3]{-1}+2^{2\cdots}\right.}$$

[In] Integrate[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]

[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) + 2^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])

Maple [F]

$$\int \frac{x}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}+a}}dx$$

[In] int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)

[Out] int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\int \frac{x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

```
[In] integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x}{\sqrt{a-b\,x^3}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

```
[In] int(x/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
[Out] int(x/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.71 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 292

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{2\,2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

```
[Out] -2/9*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3-a)^
(1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^
(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^
(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2164, 225, 2162, 212}

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\left(\frac{(1+}{(1-}\right.\right.\right.}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$-\frac{2\,2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

```
[In] Int[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 + b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2)]*Sqrt[-a + b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}} + \frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}} - \frac{\left(2\,2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-3ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{3b^{2/3}}$$

$$= -\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.78 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.33

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}-(}{(1+\sqrt[3]{}}}\right.\right.\right.}{\left(\sqrt[3]{-1}+2^{2/}\right.}$$

[In] Integrate[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]

[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) + 2^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])

Maple [F]

$$\int \frac{x}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}-a}}dx$$

[In] int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)

[Out] int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\int \frac{x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

```
[In] integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
[Out] -Integral(x/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxim
a")
[Out] -integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac"
)
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x}{\sqrt{b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

```
[In] int(x/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
[Out] int(x/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.72 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 288

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] -2/9*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3-a)
^(1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2
)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2164, 225, 2162, 212}

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}\cdots}{\sqrt[3]{b}\cdots}\right.\right.}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} - \frac{2\,2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

```
[In] Int[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 - b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2)]*Sqrt[-a - b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
```

```
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}} - \frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$- \frac{\left(2\ 2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-3ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{3b^{2/3}}$$

$$= -\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

$$+ \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.70 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.35

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,\mathrm{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+(-1\cdots}{\left(1+\sqrt[3]{-\cdots}\right.}}\right.\right.\right.}{\left(\sqrt[3]{-1}+2^{2/3}\cdots\right.}$$

```
[In] Integrate[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) + 2
^(2/3))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2
/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1)^(1
/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/
((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^
(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3)
+ (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3])
)/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F]

$$\int \frac{x}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}-a}}dx$$

```
[In] int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fric
as")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x}{\sqrt{-a-bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
[Out] Integral(x/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxi
ma")
[Out] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac
")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x}{\sqrt{-b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

```
[In] int(x/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
[Out] int(x/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.73 $\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 246

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}} \, dx = -\frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}d^2} + \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

```
[Out] -2/9*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/d^2*3^(1/2)/c^
(1/2)+1/9*2^(1/3)*(c+2^(2/3)*d*x)*EllipticF((2^(2/3)*d*x+c*(1-3^(1/2)))/(2^
(2/3)*d*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^2-2^(
2/3)*c*d*x+2*2^(1/3)*d^2*x^2)/(2^(2/3)*d*x+c*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
d^2/(4*d^3*x^3+c^3)^(1/2)/(c*(c+2^(2/3)*d*x)/(2^(2/3)*d*x+c*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used

$= \{2164, 224, 2162, 209\}$

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

$$= \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$
$$-\frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}d^2}$$

```
[In] Int[x/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(3*Sqrt[3]
*Sqrt[c]*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*d*x)*Sqrt[(c^2 - 2^
(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)],
 -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^2*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[3])*
c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
```

```
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt{c^3+4d^3x^3}}\,dx}{3d} - \frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx}{3d}$$

$$= \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}} - \frac{(2c)\text{Subst}\left(\int \frac{1}{1+3c^3x^2}\,dx, x, \frac{1+\frac{2dx}{c}}{\sqrt{c^3+4d^3x^3}}\right)}{3d^2}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}d^2} + \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.95 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.51

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

$$= \frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(-\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)\text{EllipticF}\left(\arcsin\left(\right.\right.\right.}{\left(2+\sqrt[3]{-2}\right.}$$

```
[In] Integrate[x/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(-(Sqrt[((-2)^(1/3)
*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3)*(2 + (-2)^(1/3))*c
 - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^
(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]) + ((-1)^(1/3)*2^(2/
3)*(1 + (-1)^(1/3))*c*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))
*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(I*2^(1
/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/(
(1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)])/Sqrt[3]))/((2 + (-2)^(1/3))*d^2
*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3 + 4*d^3
*x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 891 vs. $2(197) = 394$.

Time = 0.96 (sec) , antiderivative size = 892, normalized size of antiderivative = 3.63

method	result	size
default	Expression too large to display	892
elliptic	Expression too large to display	892

```
[In] int(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(
1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I
*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x+1
/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^
(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1
/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3
+c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^
(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^
(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*
2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1
/2))-2*c/d^2*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1
/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1
/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/
3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1
/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(
4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*Elliptic
Pi(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)
*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)
+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/(((1/4*
```

```
2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3
))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)
*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.42

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \left[-\frac{\sqrt{3}\sqrt{-c}d^2\log\left(\frac{2d^6x^6-36cd^5x^5-18c^2d^4x^4+28c^3d^3x^3+18c^4d^2x^2-c^6-\sqrt{3}(4d^4x^4-10cd^3x^3-18c^2d^2x^2-8c^3dx-c^4)\sqrt{4d^3x^3+c^3}\sqrt{-}}{d^6x^6+6cd^5x^5+15c^2d^4x^4+20c^3d^3x^3+15c^4d^2x^2+6c^5dx+c^6}\right.}{18cd^4}\right.$$

```
[In] integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/18*(sqrt(3)*sqrt(-c)*d^2*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^4
 + 28*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*d^4*x^4 - 10*c*d^3*x^
3 - 18*c^2*d^2*x^2 - 8*c^3*d*x - c^4)*sqrt(4*d^3*x^3 + c^3)*sqrt(-c))/(d^6*
x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c^
5*d*x + c^6)) - 6*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d^3, x))/(c*d^4),
 1/9*(sqrt(3)*sqrt(c)*d^2*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2*d^3*x
^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)*sqrt(c)/(8*c*d^4*x^4 + 4*c^2*d^3*x^3 +
2*c^4*d*x + c^5)) + 3*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d^3, x))/(c*d
^4)]
```

Sympy [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

```
[In] integrate(x/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral(x/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{x}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

```
[In] integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{x}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

```
[In] integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{x}{\sqrt{c^3+4\,d^3\,x^3}\,\left(c+d\,x\right)}\,dx$$

```
[In] int(x/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
[Out] int(x/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)), x)
```

3.74 $\int \frac{1+x}{(2-x)\sqrt{1+x^3}} dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}} dx = \frac{2}{3}\text{arctanh}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)$$

```
[Out] 2/3*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2163, 212}

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}} dx = \frac{2}{3}\text{arctanh}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

```
[In] Int[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1+x)^2}{\sqrt{1+x^3}}\right) \\
&= \frac{2}{3}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$
\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2}{3}\text{arctanh}\left(\frac{\frac{1}{3}+\frac{2x}{3}+\frac{x^2}{3}}{\sqrt{1+x^3}}\right)
$$

```
[In] Integrate[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]
[Out] (2*ArcTanh[(1/3 + (2*x)/3 + x^2/3)/Sqrt[1 + x^3]])/3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(17) = 34.

Time = 1.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

method	result
trager	$-\frac{\ln\left(\frac{-x^3+6\sqrt{x^3+1}\,x-12x^2+6\sqrt{x^3+1}+6x-10}{(x-2)^3}\right)}{3}$
default	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+1}}$
elliptic	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+1}}$

```
[In] int((x+1)/(2-x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*ln((-x^3+6*(x^3+1)^(1/2)*x-12*x^2+6*(x^3+1)^(1/2)+6*x-10)/(x-2)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(17) = 34.

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{1}{3}\log\left(\frac{x^3+12\,x^2+6\,\sqrt{x^3+1}(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)$$

```
[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
12*x - 8))
```

Sympy [F]

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = -\int \frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx - \int \frac{1}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

```
[In] integrate((1+x)/(2-x)/(x**3+1)**(1/2),x)
[Out] -Integral(x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x) - Integral(1/(x*sqrt(
x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

Maxima [F]

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

```
[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Giac [F]

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

```
[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 8.91

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(-(x + 1)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(((x
 + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1
/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.75 $\int \frac{1-x}{(2+x)\sqrt{1-x^3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}} \, dx = -\frac{2}{3}\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

```
[Out] -2/3*arctanh(1/3*(1-x)^2/(-x^3+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2163, 212}

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}} \, dx = -\frac{2}{3}\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

```
[In] Int[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(2\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1-x)^2}{\sqrt{1-x^3}}\right)\right)\\ &= -\frac{2}{3}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = -\frac{2}{3}\text{arctanh}\left(\frac{\frac{1}{3}-\frac{2x}{3}+\frac{x^2}{3}}{\sqrt{1-x^3}}\right)$$

```
[In] Integrate[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]
[Out] (-2*ArcTanh[(1/3 - (2*x)/3 + x^2/3)/Sqrt[1 - x^3]])/3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(21) = 42.

Time = 1.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

method	result
trager	$\frac{\ln\left(-\frac{-x^3+6\sqrt{-x^3+1}\,x+12x^2-6\sqrt{-x^3+1}+6x+10}{(x+2)^3}\right)}{3}$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\cdots}{}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\cdots}{}$

```
[In] int((1-x)/(x+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(-(-x^3+6*(-x^3+1)^(1/2)*x+12*x^2-6*(-x^3+1)^(1/2)+6*x+10)/(x+2)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(19) = 38$.

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = \frac{1}{3}\log\left(-\frac{x^3-12\,x^2-6\,\sqrt{-x^3+1}(x-1)-6\,x-10}{x^3+6\,x^2+12\,x+8}\right)$$

```
[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*log(-(x^3 - 12*x^2 - 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(x^3 + 6*x^2
+ 12*x + 8))
```

Sympy [F]

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = -\int \frac{x}{x\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx - \int \left(-\frac{1}{x\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

```
[In] integrate((1-x)/(2+x)/(-x**3+1)**(1/2),x)
[Out] -Integral(x/(x*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-1/(x*sqrt
(1 - x**3) + 2*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = \int -\frac{x-1}{\sqrt{-x^3+1}(x+2)}\,dx$$

```
[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Giac [F]

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = \int -\frac{x-1}{\sqrt{-x^3+1}(x+2)}\,dx$$

```
[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 221, normalized size of antiderivative = 8.19

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx$$

$$= \frac{(3+\sqrt{3}\,1\mathrm{i})\ \sqrt{x^3-1}\ \sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\ \sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\ \Bigg(\mathrm{F}\Bigg(\mathrm{asin}\Bigg(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\Bigg)\Bigg|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\Bigg)-\Pi\Bigg(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\Bigg(\cdots}{\sqrt{1-x^3}\ \sqrt{x^3+\Big(-\Big(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)\ \Big(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)-1\Big)\ x+\Big(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)\ \Big(\frac{1}{2}+\cdots}}$$

```
[In] int(-(x - 1)/((1 - x^3)^(1/2)*(x + 2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*(ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i
)/2 - 3/2)))*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((1 - x^3)^(1/2)*(((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.76 $\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 25

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{2}{3}\arctan\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)$$

```
[Out] -2/3*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2163, 209}

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{2}{3}\arctan\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

```
[In] Int[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
[Out] (-2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \frac{(1-x)^2}{\sqrt{-1+x^3}}\right)\right)$$
$$= -\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{2}{3}\arctan\left(\frac{3\sqrt{-1+x^3}}{(-1+x)^2}\right)$$

```
[In] Integrate[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
[Out] (2*ArcTan[(3*Sqrt[-1 + x^3])/(-1 + x)^2])/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.47 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.00

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^3-12\operatorname{RootOf}\left(\_Z^2+1\right)x^2+6\sqrt{x^3-1}\,x-6\operatorname{RootOf}\left(\_Z^2+1\right)x-6\sqrt{x^3-1}-10\operatorname{RootOf}\left(\_Z^2+1\right)}{(x+2)^3}\right.}{3}$
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\sqrt{x^3\cdots}}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\sqrt{x^3\cdots}}$

```
[In] int((1-x)/(x+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*RootOf(_Z^2+1)*ln(-(RootOf(_Z^2+1)*x^3-12*RootOf(_Z^2+1)*x^2+6*(x^3-1)^
(1/2)*x-6*RootOf(_Z^2+1)*x-6*(x^3-1)^(1/2)-10*RootOf(_Z^2+1))/(x+2)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(17) = 34$.

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{1}{3}\arctan\left(\frac{(x^3-12\,x^2-6\,x-10)\sqrt{x^3-1}}{6\,(x^4-x^3-x+1)}\right)$$

```
[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] -1/3*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x + 1)
)
```

Sympy [F]

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = -\int \frac{x}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx - \int \left(-\frac{1}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

```
[In] integrate((1-x)/(2+x)/(x**3-1)**(1/2),x)
[Out] -Integral(x/(x*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-1/(x*sqrt
(x**3 - 1) + 2*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = \int -\frac{x-1}{\sqrt{x^3-1}(x+2)}\,dx$$

```
[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Giac [F]

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = \int -\frac{x-1}{\sqrt{x^3-1}(x+2)}\,dx$$

```
[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 205, normalized size of antiderivative = 8.20

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx$$

$$= \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}}{2}}}\right.\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(-(x - 1)/((x^3 - 1)^(1/2)*(x + 2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticF(asin
((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2
)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*(-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1
/2)
```

3.77 $\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 25

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{2}{3}\arctan\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right)$$

```
[Out] 2/3*arctan(1/3*(1+x)^2/(-x^3-1)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2163, 209}

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{2}{3}\arctan\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)$$

```
[In] Int[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]
[Out] (2*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \frac{(1+x)^2}{\sqrt{-1-x^3}}\right)\\ &= \frac{2}{3}\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = -\frac{2}{3}\arctan\left(\frac{3\sqrt{-1-x^3}}{(1+x)^2}\right)$$

```
[In] Integrate[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]
[Out] (-2*ArcTan[(3*Sqrt[-1 - x^3])/(1 + x)^2])/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2+1\right)x^3-12\,\mathrm{RootOf}\left(\_Z^2+1\right)x^2+6\sqrt{-x^3-1}\,x+6\,\mathrm{RootOf}\left(\_Z^2+1\right)x+6\sqrt{-x^3-1}-10\,\mathrm{RootOf}\left(\_Z^2 \cdots\right.}{(x-2)^3}\right.}{3}$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x\cdots}{\frac{3}{2}+\cdots}}\cdots}{\cdots}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x\cdots}{\frac{3}{2}+\cdots}}\cdots}{\cdots}$

```
[In] int((x+1)/(2-x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*RootOf(_Z^2+1)*ln((-RootOf(_Z^2+1)*x^3-12*RootOf(_Z^2+1)*x^2+6*(-x^3-1)
^(1/2)*x+6*RootOf(_Z^2+1)*x+6*(-x^3-1)^(1/2)-10*RootOf(_Z^2+1))/(x-2)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = -\frac{1}{3}\arctan\left(\frac{(x^3+12\,x^2-6\,x+10)\sqrt{-x^3-1}}{6\,(x^4+x^3+x+1)}\right)$$

```
[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] -1/3*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 + x^3 + x + 1
))
```

Sympy [F]

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = -\int \frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx - \int \frac{1}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

```
[In] integrate((1+x)/(2-x)/(-x**3-1)**(1/2),x)
[Out] -Integral(x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x) - Integral(1/(x*sqr
t(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{x+1}{\sqrt{-x^3-1}(x-2)}\,dx$$

```
[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Giac [F]

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{x+1}{\sqrt{-x^3-1}(x-2)}\,dx$$

[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate(-(x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 221, normalized size of antiderivative = 8.84

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\right.\right.}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\right.}}$$

[In] int(-(x + 1)/((- x^3 - 1)^(1/2)*(x - 2)),x)

[Out] -((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)),
-((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6
 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/
2)/((3^(1/2)*1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/
2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((- x^3 - 1)^(1/2)*(x^3
- x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))

3.78 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 50

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] 2/3*arctanh(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(b*x^3+a)^(1/2))/a^(1/6)/b^(1
/3)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2163, 212}

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(3*a^(1/6)
*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9-ax^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{2\text{arctanh}\left(\frac{3\sqrt[6]{a}\sqrt{a+bx^3}}{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x
]
```

```
[Out] (2*ArcTanh[(3*a^(1/6)*Sqrt[a + b*x^3])/(a^(1/3) + b^(1/3)*x)^2])/(3*a^(1/6)
*b^(1/3))
```

Maple [F]

$$\int \frac{a^{\frac{1}{3}}+b^{\frac{1}{3}}x}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}+a}}dx$$

```
[In] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx - \int \frac{\sqrt[3]{b}x}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

```
[In] integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x
)
```

```
[Out] -Integral(a**(1/3)/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x*
*3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*s
qrt(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))),
 x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)\,{\left(\sqrt{b\,x^3+a}-\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x\right)}^3}{x^3\,{\left(b^{1/3}\,x-2\,a^{1/3}\right)}^3}\right)}{3\,a^{1/6}\,b^{1/3}}$$

```
[In] int(-(b^(1/3)*x + a^(1/3))/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
```

```
[Out] log((((a + b*x^3)^(1/2) + a^(1/2))*((a + b*x^3)^(1/2) - a^(1/2) + 2*a^(1/6)
*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x - 2*a^(1/3))^3))/(3*a^(1/6)*b^(1/3))
```

3.79 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 52

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\text{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] -2/3*arctanh(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(-b*x^3+a)^(1/2))/a^(1/6)/b^
(1/3)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2163, 212}

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\text{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/(3*a^(1/6
)*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9-ax^2}\,dx, x, \frac{\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{3\sqrt[6]{a}\sqrt{a-bx^3}}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x
]
```

```
[Out] (-2*ArcTanh[(3*a^(1/6)*Sqrt[a - b*x^3])/(a^(1/3) - b^(1/3)*x)^2])/(3*a^(1/6
)*b^(1/3))
```

Maple [F]

$$\int \frac{a^{\frac{1}{3}}-b^{\frac{1}{3}} x}{\left(2 a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}dx$$

```
[In] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
[Out] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, alg
orithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\int \left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\right)dx - \int \frac{\sqrt[3]{b}x}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

```
[In] integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),
x)
[Out] -Integral(-a**(1/3)/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sq
rt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, alg
orithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3)))
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{a-b\,x^3}-\sqrt{a}\right)\,\left(\sqrt{a-b\,x^3}+\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x\right)^3}{x^3\,\left(b^{1/3}\,x+2\,a^{1/3}\right)^3}\right)}{3\,a^{1/6}\,b^{1/3}}$$

```
[In] int(-(b^(1/3)*x - a^(1/3))/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
```

```
[Out] log((((a - b*x^3)^(1/2) - a^(1/2))*((a - b*x^3)^(1/2) + a^(1/2) + 2*a^(1/6)
*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x + 2*a^(1/3))^3))/(3*a^(1/6)*b^(1/3))
```

3.80 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 53

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] -2/3*arctan(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(b*x^3-a)^(1/2))/a^(1/6)/b^(1
/3)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2163, 209}

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/(3*a^(1/6
)*b^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int\frac{1}{9+ax^2}\,dx, x, \frac{\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}\\ &= -\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{2\arctan\left(\frac{3\sqrt[6]{a}\sqrt{-a+bx^3}}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),
x]
[Out] (2*ArcTan[(3*a^(1/6)*Sqrt[-a + b*x^3])/(a^(1/3) - b^(1/3)*x)^2])/(3*a^(1/6)
*b^(1/3))
```

Maple [F]

$$\int \frac{a^{\frac{1}{3}}-b^{\frac{1}{3}}x}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}-a}}dx$$

[In] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)

[Out] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(38) = 76.

Time = 0.90 (sec) , antiderivative size = 592, normalized size of antiderivative = 11.17

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$
$$=\left[\frac{1}{6}\,a^{\frac{1}{3}}\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}\log\left(\frac{b^6x^{18}-7800\,ab^5x^{15}+535272\,a^2b^4x^{12}-5147264\,a^3b^3x^9+10516992\,a^4b^2x^6-592281\right.\right.$$

[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algo rithm="fricas")

[Out] [1/6*a^(1/3)*sqrt(-1/(a*b^(2/3)))*log((b^6*x^18 - 7800*a*b^5*x^15 + 535272*a^2*b^4*x^12 - 5147264*a^3*b^3*x^9 + 10516992*a^4*b^2*x^6 - 5922816*a^5*b*x^3 + 557056*a^6 + 144*(7*b^5*x^16 - 1169*a*b^4*x^13 + 20266*a^2*b^3*x^10 - 66976*a^3*b^2*x^7 + 58112*a^4*b*x^4 - 10240*a^5*x)*a^(2/3)*b^(1/3) - 72*(b^5*x^17 - 581*a*b^4*x^14 + 19108*a^2*b^3*x^11 - 106336*a^3*b^2*x^8 + 137984*a^4*b*x^5 - 50176*a^5*x^2)*a^(1/3)*b^(2/3) - 12*sqrt(b*x^3 - a)*((b^5*x^16 - 1568*a*b^4*x^13 + 72520*a^2*b^3*x^10 - 498304*a^3*b^2*x^7 + 625664*a^4*b*x^4 - 139264*a^5*x)*a^(2/3)*b^(2/3) + 6*(41*a*b^5*x^14 - 4268*a^2*b^4*x^11 + 52896*a^3*b^3*x^8 - 116480*a^4*b^2*x^5 + 48128*a^5*b*x^2)*a^(1/3) - (25*a*b^5*x^15 - 7202*a^2*b^4*x^12 + 167392*a^3*b^3*x^9 - 647296*a^4*b^2*x^6 + 468992*a^5*b*x^3 - 40960*a^6)*b^(1/3))*sqrt(-1/(a*b^(2/3))))/(b^6*x^18 + 48*a*b^5*x^15 + 960*a^2*b^4*x^12 + 10240*a^3*b^3*x^9 + 61440*a^4*b^2*x^6 + 196608*a^5*b*x^3 + 262144*a^6)), 1/3*a^(1/3)*sqrt(1/(a*b^(2/3)))*arctan(1/6*sqrt(b*x^3 - a)*((11*b*x^4 + 16*a*x)*a^(2/3)*b^(2/3) - (b^2*x^5 - 28*a*b*x^2)*a^(1/3) + (17*a*b*x^3 + 10*a^2)*b^(1/3))*sqrt(1/(a*b^(2/3)))/(b^2*x^6 - 2*a*b*x^3 + a^2))]

Sympy [F]

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\int \left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\right)dx - \int \frac{\sqrt[3]{b}x}{2\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

```
[In] integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x
)
```

```
[Out] -Integral(-a**(1/3)/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x
*sqrt(-a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))),
 x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 10.75 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{\ln\left(\frac{\left(\sqrt{b\,x^3-a}+\sqrt{a}\,1\mathrm{i}\right)\,{\left(\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x+\sqrt{b\,x^3-a}\,1\mathrm{i}\right)}^3}{x^3\,{\left(b^{1/3}\,x+2\,a^{1/3}\right)}^3}\right)\,1\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

```
[In] int(-(b^(1/3)*x - a^(1/3))/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
```

```
[Out] (log((((b*x^3 - a)^(1/2) + a^(1/2)*1i)*((b*x^3 - a)^(1/2)*1i + a^(1/2) + 2*
a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x + 2*a^(1/3))^3))*1i)/(3*a^(1/6)*b^(1/
3))
```

3.81 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 53

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] 2/3*arctan(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(-b*x^3-a)^(1/2))/a^(1/6)/b^(1
/3)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2163, 209}

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/(3*a^(1/6)
*b^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9+ax^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2\arctan\left(\frac{3\sqrt[6]{a}\sqrt{-a-bx^3}}{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),
x]

[Out] (-2*ArcTan[(3*a^(1/6)*Sqrt[-a - b*x^3])/(a^(1/3) + b^(1/3)*x)^2])/(3*a^(1/6
)*b^(1/3))
```

Maple [F]

$$\int \frac{a^{\frac{1}{3}}+b^{\frac{1}{3}} x}{\left(2 a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}-a}}dx$$

```
[In] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(37) = 74$.

Time = 0.90 (sec) , antiderivative size = 641, normalized size of antiderivative = 12.09

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$
$$= \left[\frac{1}{6}\,a^{\frac{1}{3}}\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}\log\left(\frac{b^6x^{18}+7800\,ab^5x^{15}+535272\,a^2b^4x^{12}+5147264\,a^3b^3x^9+10516992\,a^4b^2x^6+5922816}{}\right.\right.$$
$$-\frac{1}{3}\,a^{\frac{1}{3}}\sqrt{\frac{1}{ab^{\frac{2}{3}}}}\arctan\left(\frac{\left((11\,bx^4-16\,ax)\sqrt{-bx^3-a}a^{\frac{2}{3}}b^{\frac{2}{3}}+(b^2x^5+28\,abx^2)\sqrt{-bx^3-a}a^{\frac{1}{3}}-(17\,abx^3-10\right.}{6\,(b^2x^6+2\,abx^3+a^2)}\right.$$

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/6*a^(1/3)*sqrt(-1/(a*b^(2/3)))*log((b^6*x^18 + 7800*a*b^5*x^15 + 535272*
a^2*b^4*x^12 + 5147264*a^3*b^3*x^9 + 10516992*a^4*b^2*x^6 + 5922816*a^5*b*x
^3 + 557056*a^6 + 144*(7*b^5*x^16 + 1169*a*b^4*x^13 + 20266*a^2*b^3*x^10 +
66976*a^3*b^2*x^7 + 58112*a^4*b*x^4 + 10240*a^5*x)*a^(2/3)*b^(1/3) + 72*(b^
5*x^17 + 581*a*b^4*x^14 + 19108*a^2*b^3*x^11 + 106336*a^3*b^2*x^8 + 137984*
a^4*b*x^5 + 50176*a^5*x^2)*a^(1/3)*b^(2/3) + 12*((b^5*x^16 + 1568*a*b^4*x^1
3 + 72520*a^2*b^3*x^10 + 498304*a^3*b^2*x^7 + 625664*a^4*b*x^4 + 139264*a^5
*x)*sqrt(-b*x^3 - a)*a^(2/3)*b^(2/3) + 6*(41*a*b^5*x^14 + 4268*a^2*b^4*x^11
 + 52896*a^3*b^3*x^8 + 116480*a^4*b^2*x^5 + 48128*a^5*b*x^2)*sqrt(-b*x^3 -
a)*a^(1/3) + (25*a*b^5*x^15 + 7202*a^2*b^4*x^12 + 167392*a^3*b^3*x^9 + 6472
96*a^4*b^2*x^6 + 468992*a^5*b*x^3 + 40960*a^6)*sqrt(-b*x^3 - a)*b^(1/3))*sq
rt(-1/(a*b^(2/3))))/(b^6*x^18 - 48*a*b^5*x^15 + 960*a^2*b^4*x^12 - 10240*a^
3*b^3*x^9 + 61440*a^4*b^2*x^6 - 196608*a^5*b*x^3 + 262144*a^6)), -1/3*a^(1/
3)*sqrt(1/(a*b^(2/3)))*arctan(1/6*((11*b*x^4 - 16*a*x)*sqrt(-b*x^3 - a)*a^(
2/3)*b^(2/3) + (b^2*x^5 + 28*a*b*x^2)*sqrt(-b*x^3 - a)*a^(1/3) - (17*a*b*x^
3 - 10*a^2)*sqrt(-b*x^3 - a)*b^(1/3))*sqrt(1/(a*b^(2/3)))/(b^2*x^6 + 2*a*b*
x^3 + a^2))]
```

Sympy [F]

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx - \int \frac{\sqrt[3]{b}x}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx$$

```
[In] integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),
x)
```

```
[Out] -Integral(a**(1/3)/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*
x**3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*
x*sqrt(-a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, alg
orithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3)))
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{-b\,x^3-a}-\sqrt{a}\,1\mathrm{i}\right)\,\left(2\,a^{1/6}\,b^{1/3}\,x-\sqrt{a}+\sqrt{-b\,x^3-a}\,1\mathrm{i}\right)^3}{x^3\,\left(b^{1/3}\,x-2\,a^{1/3}\right)^3}\right)\,1\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

```
[In] int(-(b^(1/3)*x + a^(1/3))/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x)
[Out] (log((((- a - b*x^3)^(1/2) - a^(1/2)*1i)*((- a - b*x^3)^(1/2)*1i - a^(1/2)
+ 2*a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x - 2*a^(1/3))^3))*1i)/(3*a^(1/6)*b
^(1/3))
```

3.82 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 46

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d}$$

```
[Out] -2/3*arctanh(1/3*(-2*d*x+c)^2/c^(1/2)/(-8*d^3*x^3+c^3)^(1/2))/d/c^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2163, 212}

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d}$$

```
[In] Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
[Out] (-2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(3*Sqrt[c]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\frac{(2c)\text{Subst}\left(\int \frac{1}{9-c^3x^2}\,dx, x, \frac{\left(1-\frac{2dx}{c}\right)^2}{\sqrt{c^3-8d^3x^3}}\right)}{d}$$
$$= -\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d}$$

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{3\sqrt{c}\sqrt{c^3-8d^3x^3}}{(c-2dx)^2}\right)}{3\sqrt{c}d}$$

```
[In] Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
[Out] (-2*ArcTanh[(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])/(c - 2*d*x)^2])/(3*Sqrt[c]*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 503, normalized size of antiderivative = 10.93

method	result
default	$-\frac{4\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}F\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}, \sqrt{\frac{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\cdots}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\cdots}{2d}}}\cdots\right)}{\sqrt{-8d^3x^3+c^3}}$
elliptic	$-\frac{4\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}F\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}, \sqrt{\frac{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\cdots}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\cdots}{2d}}}\cdots\right)}{\sqrt{-8d^3x^3+c^3}}$

```
[In] int((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(
1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-
1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1
/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x-1/
2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),((1/2*c/d-1/2*(-1/2-1/
2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))+4*(1/2*(-1
/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1
/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3
^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2-1/
2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticPi(((x-1/2*c/d)/(1/
2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),2/3*(1/2*c/d-1/2*(-1/2-1/2*I*3^(
1/2))*c/d)/c*d,((1/2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1
/2*I*3^(1/2))*c/d))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(38) = 76.

Time = 0.41 (sec) , antiderivative size = 294, normalized size of antiderivative = 6.39

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \left[\frac{\log\left(\frac{8\,d^6x^6-240\,cd^5x^5+408\,c^2d^4x^4+88\,c^3d^3x^3+156\,c^4d^2x^2+12\,c^5dx+17\,c^6-3\,(8\,d^4x^4-52\,cd^3x^3+12\,c^2d^2x^2-4\,c^3dx+5\,c^4)\sqrt{-8\,d^3x^3+c^3}\sqrt{c}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right)}{6\,\sqrt{c}d}, -\frac{\sqrt{-c}\arctan\left(\frac{(4\,d^3x^3-24\,cd^2x^2-6\,c^2dx-5\,c^3)\sqrt{-8\,d^3x^3+c^3}\sqrt{-c}}{3\,(16\,cd^4x^4-8\,c^2d^3x^3-2\,c^4dx+c^5)}\right)}{3\,cd}\right]$$

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log((8*d^6*x^6 - 240*c*d^5*x^5 + 408*c^2*d^4*x^4 + 88*c^3*d^3*x^3 + 15
6*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 - 3*(8*d^4*x^4 - 52*c*d^3*x^3 + 12*c^2*
d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*d^3*x^3 + c^3)*sqrt(c))/(d^6*x^6 + 6*c
*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c^5*d*x + c
^6))/(sqrt(c)*d), -1/3*sqrt(-c)*arctan(1/3*(4*d^3*x^3 - 24*c*d^2*x^2 - 6*c^
2*d*x - 5*c^3)*sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4 - 8*c^2*d^3*x^
3 - 2*c^4*d*x + c^5))/(c*d)]
```

Sympy [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = -\int \left(-\frac{c}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}}\right)dx - \int \frac{2dx}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}}\,dx$$

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
[Out] -Integral(-c/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x**3)), x
) - Integral(2*d*x/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x**
3)), x)
```

Maxima [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int -\frac{2\,dx-c}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
[Out] -integrate((2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int -\frac{2\,dx-c}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
[Out] integrate(-(2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{c^3-8\,d^3\,x^3}-c^{3/2}\right)\left(\sqrt{c^3-8\,d^3\,x^3}+c^{3/2}+4\,\sqrt{c}\,d\,x\right)^3}{x^3\,(c+d\,x)^3}\right)}{3\,\sqrt{c}\,d}$$

```
[In] int((c - 2*d*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
[Out] log((((c^3 - 8*d^3*x^3)^(1/2) - c^(3/2))*((c^3 - 8*d^3*x^3)^(1/2) + c^(3/2)
 + 4*c^(1/2)*d*x)^3)/(x^3*(c + d*x)^3))/(3*c^(1/2)*d)
```

3.83 $\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2}{9}(e+2f)\text{arctanh}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right) + \frac{2\sqrt{2+\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 2/9*(e+2*f)*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))+2/9*(e-f)*(1+x)*EllipticF((1
+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+
1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

= {2164, 224, 2163, 212}

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx$$
$$= \frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(e-f)\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$
$$+\frac{2}{9}\text{arctanh}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)(e+2f)$$

```
[In] Int[(e + f*x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*(e + 2*f)*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 + (2*Sqrt[2 + Sqrt[3]]
*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 +
x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
```

```
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}(e-f)\int \frac{1}{\sqrt{1+x^3}}\,dx + \frac{1}{6}(e+2f)\int \frac{2+2x}{(2-x)\sqrt{1+x^3}}\,dx \\
&= \frac{2\sqrt{2+\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} \\
&\quad + \frac{1}{3}(2(e+2f))\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1+x)^2}{\sqrt{1+x^3}}\right) \\
&= \frac{2}{9}(e+2f)\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right) \\
&\quad + \frac{2\sqrt{2+\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.40 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.96

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(1+x)}{-3i+\sqrt{3}}}\left(-3if\sqrt{i+\sqrt{3}-2ix}\left(-i-\sqrt{3}+\left(-i+\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{}}\right.\right.}{\left(3i+\sqrt{3}\right)\sqrt{-i+\sqrt{}}}$$

```
[In] Integrate[(e + f*x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-3*I)*f*Sqrt[I + Sqrt[
3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 2*Sq
rt[3]*(e + 2*f)*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(
2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3]
 + (2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(114) = 228$.

Time = 0.95 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.77

method	result
default	$-\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2(e+2f)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{3\sqrt{x^3\cdots}}$
elliptic	$-\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2(-e-2f)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{3\sqrt{x\cdots}}$

```
[In] int((f*x+e)/(2-x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*f*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e+2*f)*(3/2-1/2*I*3^(1/2)
)*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1
/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2
)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2
*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.44

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \frac{1}{9}\,(e+2\,f)\log\left(\frac{x^3+12\,x^2+6\,\sqrt{x^3+1}(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right) + \frac{2}{3}\,(e-f)\mathrm{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(e + 2*f)*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3
- 6*x^2 + 12*x - 8)) + 2/3*(e - f)*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = -\int \frac{e}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx - \int \frac{fx}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

```
[In] integrate((f*x+e)/(2-x)/(x**3+1)**(1/2),x)
[Out] -Integral(e/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x) - Integral(f*x/(x*sqr
t(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{fx+e}{\sqrt{x^3+1}(x-2)}\,dx$$

```
[In] integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((f*x + e)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Giac [F]

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{fx+e}{\sqrt{x^3+1}(x-2)}\,dx$$

```
[In] integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(f*x + e)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.35

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(e+2f)\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2f\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(-(e + f*x)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*(e + 2*f)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2
- x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2, a
sin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1
/2)*1i)/2 - 3/2)))/(3*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/
2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*f*((3^
(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2))^(1/2)
```

3.84 $\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 153

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = -\frac{2}{9}(e-2f)\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/9*(e-2*f)*arctanh(1/3*(1-x)^2/(-x^3+1)^(1/2))-2/9*(e+f)*(1-x)*EllipticF(
(1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+
x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

$= \{2164, 224, 2163, 212\}$

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx$$
$$= -\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(e+f)\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$
$$-\frac{2}{9}\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)(e-2f)$$

```
[In] Int[(e + f*x)/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(e - 2*f)*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3]
]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[
(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 -
 x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
```

```
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}(e-2f)\int \frac{2-2x}{(2+x)\sqrt{1-x^3}}\,dx + \frac{1}{3}(e+f)\int \frac{1}{\sqrt{1-x^3}}\,dx$$
$$= -\frac{2\sqrt{2+\sqrt{3}}(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$
$$-\frac{1}{3}(2(e-2f))\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1-x)^2}{\sqrt{1-x^3}}\right)$$
$$= -\frac{2}{9}(e-2f)\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$
$$-\frac{2\sqrt{2+\sqrt{3}}(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.23 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.77

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\left(3f\sqrt{i+\sqrt{3}+2ix}(-1+i\sqrt{3}+x+i\sqrt{3}x)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right), \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right) - \right.}{\left(3i+\sqrt{3}\right)\sqrt{-i+\sqrt{3}-}}$$

```
[In] Integrate[(e + f*x)/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(3*f*Sqrt[I + Sqrt[3] + (2
*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[-I + Sqrt[3
] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - 2*Sqrt[3]*
(e - 2*f)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt
[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] - (2*
I)*x]*Sqrt[1 - x^3])
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.61

method	result
default	$-\frac{2if\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}-\frac{2i(e-2f)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\right.}}{}$
elliptic	$-\frac{2if\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}-\frac{2i(e-2f)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\right.}}{}$

```
[In] int((f*x+e)/(x+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-2*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^
(1/2))^(1/2)/(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2
)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = -\frac{1}{9}\,(e-2\,f)\log\left(-\frac{x^3-12\,x^2+6\,\sqrt{-x^3+1}(x-1)-6\,x-10}{x^3+6\,x^2+12\,x+8}\right) - \frac{2}{3}\,(i\,e+i\,f)\text{weierstrassPInverse}(0,4,x)$$

```
[In] integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(e - 2*f)*log(-(x^3 - 12*x^2 + 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(x
^3 + 6*x^2 + 12*x + 8)) - 2/3*(I*e + I*f)*weierstrassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx=\int \frac{e+fx}{\sqrt{-(x-1)\left(x^2+x+1\right)}\,(x+2)}\,dx$$

```
[In] integrate((f*x+e)/(2+x)/(-x**3+1)**(1/2),x)
[Out] Integral((e + f*x)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3+1}(x+2)}\,dx$$

```
[In] integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Giac [F]

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3+1}(x+2)}\,dx$$

```
[In] integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.35

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx=$$
$$-\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$-\frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(e-2\,f)\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\right|}{3\,\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int((e + f*x)/((1 - x^3)^(1/2)*(x + 2)),x)

[Out] - (2*f*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/
((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) -
x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2*((
3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)
*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(e - 2*f)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1
i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
+ 1) + x^3)^(1/2))
```

3.85 $\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 156

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{2}{9}(e-2f)\arctan\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -2/9*(e-2*f)*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))-2/9*(e+f)*(1-x)*EllipticF((1
-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+
1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

$= \{2164, 225, 2163, 209\}$

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx$$
$$= -\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(e+f)\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$
$$-\frac{2}{9}\arctan\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)(e-2f)$$

```
[In] Int[(e + f*x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*(e - 2*f)*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3]
]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[
(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1
 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
```

```
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}(e-2f)\int \frac{2-2x}{(2+x)\sqrt{-1+x^3}}\,dx + \frac{1}{3}(e+f)\int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{1}{3}(2(e-2f))\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \frac{(1-x)^2}{\sqrt{-1+x^3}}\right)$$

$$= -\frac{2}{9}(e-2f)\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.22 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.72

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\left(3f\sqrt{i+\sqrt{3}+2ix}(-1+i\sqrt{3}+x+i\sqrt{3}x)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-\right.}{\left(3i+\sqrt{3}\right)\sqrt{-i+\sqrt{3}-}}$$

```
[In] Integrate[(e + f*x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(3*f*Sqrt[I + Sqrt[3] + (2
*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[-I + Sqrt[3
] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - 2*Sqrt[3]*
(e - 2*f)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt
[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] - (2*
I)*x]*Sqrt[-1 + x^3])
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.58

method	result
default	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2(e-2f)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{3\sqrt{}}$
elliptic	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2(e-2f)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{3\sqrt{}}$

```
[In] int((f*x+e)/(x+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e-2*f)*(-3/2-1/2*I*3^(1/2))
*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/
2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*
EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2+1/6*I*3^(1/2),((3/2+1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.35

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{1}{9}\,(e-2\,f)\arctan\left(\frac{(x^3-12\,x^2-6\,x-10)\sqrt{x^3-1}}{6\,(x^4-x^3-x+1)}\right) + \frac{2}{3}\,(e+f)\text{weierstrassPInverse}(0,4,x)$$

```
[In] integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(e - 2*f)*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^
3 - x + 1)) + 2/3*(e + f)*weierstrassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx=\int \frac{e+fx}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+2)}\,dx$$

```
[In] integrate((f*x+e)/(2+x)/(x**3-1)**(1/2),x)
[Out] Integral((e + f*x)/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3-1}(x+2)}\,dx$$

```
[In] integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Giac [F]

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3-1}(x+2)}\,dx$$

```
[In] integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.10

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx$$
$$=-\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$-\frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(e-2\,f\right)\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{3\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int((e + f*x)/((x^3 - 1)^(1/2)*(x + 2)),x)
```

```
[Out] - (2*f*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
- 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/
2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)*(e - 2*f)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2) + 1) + x^3)^(1/2))
```

3.86 $\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 150

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{2}{9}(e+2f)\arctan\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right) + \frac{2\sqrt{2-\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 2/9*(e+2*f)*arctan(1/3*(1+x)^2/(-x^3-1)^(1/2))+2/9*(e-f)*(1+x)*EllipticF((1
+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+
1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{2164, 225, 2163, 209\}$

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}(e-f)\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} + \frac{2}{9}\arctan\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)(e+2f)$$

```
[In] Int[(e + f*x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(e + 2*f)*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 + (2*Sqrt[2 - Sqrt[3]]
*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1
+ x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
```

```
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}(e-f)\int \frac{1}{\sqrt{-1-x^3}}\,dx + \frac{1}{6}(e+2f)\int \frac{2+2x}{(2-x)\sqrt{-1-x^3}}\,dx$$
$$= \frac{2\sqrt{2-\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{1}{3}(2(e+2f))\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \frac{(1+x)^2}{\sqrt{-1-x^3}}\right)$$
$$= \frac{2}{9}(e+2f)\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right) + \frac{2\sqrt{2-\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.25 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.83

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(1+x)}{-3i+\sqrt{3}}}\left(-3if\sqrt{i+\sqrt{3}-2ix}\left(-i-\sqrt{3}+\left(-i+\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right), \frac{2\sqrt{3}}{-3i+\sqrt{\ldots}}\right.\right.}{\left(3i+\sqrt{3}\right)\sqrt{-i+\sqrt{\ldots}}}$$

```
[In] Integrate[(e + f*x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-3*I)*f*Sqrt[I + Sqrt[
3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 2*Sq
rt[3]*(e + 2*f)*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(
2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3]
 + (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.64

method	result
default	$\frac{2 i f \sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}+\frac{2 i(e+2f)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\cdots}}{\cdots}$
elliptic	$\frac{2 i f \sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}-\frac{2 i(-e-2f)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\cdots\right.}}{\cdots}$

```
[In] int((f*x+e)/(2-x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1
/2*I*3^(1/2)))^(1/2))+2/3*I*(e+2*f)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2
))^(1/2)/(-x^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/
(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.38

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = -\frac{1}{9}\,(e+2\,f)\arctan\left(\frac{(x^3+12\,x^2-6\,x+10)\sqrt{-x^3-1}}{6\,(x^4+x^3+x+1)}\right) - \frac{2}{3}\,(i\,e-i\,f)\mathrm{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(e + 2*f)*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 + x
^3 + x + 1)) - 2/3*(I*e - I*f)*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx$$
$$= -\int \frac{e}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx - \int \frac{fx}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

```
[In] integrate((f*x+e)/(2-x)/(-x**3-1)**(1/2),x)
[Out] -Integral(e/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x) - Integral(f*x/(x*s
qrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{fx+e}{\sqrt{-x^3-1}(x-2)}\,dx$$

```
[In] integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((f*x + e)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Giac [F]

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{fx+e}{\sqrt{-x^3-1}(x-2)}\,dx$$

```
[In] integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(f*x + e)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.39

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx =$$
$$-\frac{2f\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(e+2f)\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+}{-\frac{3}{2}+}\right.}{3\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(-(e + f*x)/((- x^3 - 1)^(1/2)*(x - 2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(
1/2)*1i)/2 - 3/2))^(1/2)*(e + 2*f)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(
((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2
)*1i)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)
/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)
*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2))^(1/2)) - (2*f*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x +
(3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*e
llipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3
/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2))^(1/2))
```

3.87 $\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 297

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt[3]{b}e+2\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt{a}b^{2/3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/9*(b^(1/3)*e+2*a^(1/3)*f)*arctanh(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(b*x^
3+a)^(1/2))/b^(2/3)/a^(1/2)+2/9*(b^(1/3)*e-a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*E
llipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*
3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(2/3)/(b*x^
3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2164, 224, 2163, 212}

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\text{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{9\sqrt{a}b^{2/3}}$$

```
[In] Int[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e + 2*a^(1/3)*f)*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqr
t[a + b*x^3])]])/(9*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*e - a^(
1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{6}\left(-\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int\frac{2\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx\right) - \frac{1}{3}\left(-\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\left(2\left(\sqrt[3]{b}e+2\sqrt[3]{a}f\right)\right)\text{Subst}\left(\int\frac{1}{9-ax^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{a+bx^3}}\right)}{3b^{2/3}}$$

$$= \frac{2\left(\sqrt[3]{b}e+2\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt{a}b^{2/3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.12 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.47

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$
$$= \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(\left(-3-i\sqrt{3}\right)\sqrt[3]{a}+\left(3-i\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{}$$

```
[In] Integrate[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*((-3 - I*Sqrt
[3])*a^(1/3) + (3 - I*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3])*a^(1/3) - (I
 + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF[ArcSin[Sqrt[((
-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 +
 I*Sqrt[3])/2])/2 + I*(b^(1/3)*e + 2*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) + (I +
 Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/
3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[
Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]
], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/
3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [F]

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^3+a}}dx$$

```
[In] int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\int \frac{e}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx-\int \frac{fx}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

```
[In] integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
[Out] -Integral(e/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x
) - Integral(f*x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3
)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int -\frac{fx+e}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxim
a")
[Out] -integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int -\frac{e+f\,x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{b\,x^3+a}}\,dx$$

```
[In] int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
```

```
[Out] int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)), x)
```

3.88 $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 304

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt{a}b^{2/3}}$$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e+\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] -2/9*(b^(1/3)*e-2*a^(1/3)*f)*arctanh(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(-b*
x^3+a)^(1/2))/b^(2/3)/a^(1/2)-2/9*(b^(1/3)*e+a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)
*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(2/3)/
(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2164, 224, 2163, 212}

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{b}e\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} - \frac{2\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)}{9\sqrt{a}b^{2/3}}$$

```
[In] Int[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e - 2*a^(1/3)*f)*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sq
rt[a - b*x^3])])/(9*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*e + a^
(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx+\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}\,dx$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$-\frac{\left(2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\right)\text{Subst}\left(\int\frac{1}{9-ax^2}\,dx,x,\frac{\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{a-bx^3}}\right)}{3b^{2/3}}$$

$$= -\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt{a}b^{2/3}}$$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.01 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.47

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{1}{2}if\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\left(\left(-3i+\sqrt{3}\right)\sqrt[3]{a}-\left(3i+\sqrt{3}\right)\sqrt[3]{b}x\right)\text{EllipticF}\left(\arcsin\right.\right.}{}$$

[In] Integrate[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]

[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-1/2*I)*f*Sqrt[((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*((-3*I + Sqrt[3])*a^(1/3) - (3*I + Sqrt[3])*b^(1/3)*x)*EllipticF[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2] - I*(b^(1/3)*e - 2*a^(1/3)*f)*Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])

Maple [F]

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}+a}}dx$$

[In] int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)

[Out] int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fric
as")
[Out] Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

```
[In] integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
[Out] Integral((e + f*x)/((2*a**(1/3) + b**(1/3)*x)*sqrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{fx+e}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxi
ma")
[Out] integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac
")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{e+f\,x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{a-b\,x^3}}\,dx$$

```
[In] int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
[Out] int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)), x)
```

3.89 $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 313

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{9\sqrt{a}b^{2/3}}$$

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e+\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

```
[Out] -2/9*(b^(1/3)*e-2*a^(1/3)*f)*arctan(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(b*x^
3-a)^(1/2))/b^(2/3)/a^(1/2)-2/9*(b^(1/3)*e+a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)*E
llipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),
2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/(b
*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2164, 225, 2163, 209}

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{b}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}} - \frac{2\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)}{9\sqrt{a}b^{2/3}}$$

```
[In] Int[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e - 2*a^(1/3)*f)*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqr
t[-a + b*x^3])])/(9*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(b^(1/3)*e + a^
(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])
*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]]
)/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - S
qrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx+\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a+bx^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}} - \frac{\left(2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\right)\text{Subst}\left(\int \frac{1}{9+ax^2}\,dx,x,\frac{\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a+bx^3}}\right)}{3b^{2/3}}$$

$$= -\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{9\sqrt{a}b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.95 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.43

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$
$$= \frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{1}{2}if\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\left(\left(-3i+\sqrt{3}\right)\sqrt[3]{a}-\left(3i+\sqrt{3}\right)\sqrt[3]{b}x\right)\text{EllipticF}\left(\arcsin\right.\right.}{}$$

```
[In] Integrate[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-1/2*I)*f*Sqrt[
((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3
))]*((-3*I + Sqrt[3])*a^(1/3) - (3*I + Sqrt[3])*b^(1/3)*x)*EllipticF[ArcSin
[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1
/3))]], (1 + I*Sqrt[3])/2] - I*(b^(1/3)*e - 2*a^(1/3)*f)*Sqrt[((-I)*(2*a^(1
/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt
[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + S
qrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a
^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3]
)
```

Maple [F]

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^3-a}}dx$$

```
[In] int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

```
[In] integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
[Out] Integral((e + f*x)/((2*a**(1/3) + b**(1/3)*x)*sqrt(-a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int \frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxim
a")
[Out] integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac"
)
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{e+f\,x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{b\,x^3-a}}\,dx$$

```
[In] int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
[Out] int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)), x)
```

3.90 $\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 310

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{2\left(\sqrt[3]{b}e+2\sqrt[3]{a}f\right)\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt{a}b^{2/3}}$$
$$+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] 2/9*(b^(1/3)*e+2*a^(1/3)*f)*arctan(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(-b*x^
3-a)^(1/2))/b^(2/3)/a^(1/2)+2/9*(b^(1/3)*e-a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*E
llipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*
I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/(-b*x
^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2164, 225, 2163, 209}

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} + \frac{2\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{9\sqrt{a}b^{2/3}}$$

```
[In] Int[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e + 2*a^(1/3)*f)*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt
[-a - b*x^3])])/(9*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(b^(1/3)*e - a^(
1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])
/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{6}\left(-\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int\frac{2\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx\right)$$
$$-\frac{1}{3}\left(-\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{-a-bx^3}}\,dx$$
$$=\frac{2\sqrt{2-\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$+\frac{\left(2\left(\sqrt[3]{b}e+2\sqrt[3]{a}f\right)\right)\text{Subst}\left(\int\frac{1}{9+ax^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a-bx^3}}\right)}{3b^{2/3}}$$
$$=\frac{2\left(\sqrt[3]{b}e+2\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt{a}b^{2/3}}$$
$$+\frac{2\sqrt{2-\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.05 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.42

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(\left(-3-i\sqrt{3}\right)\sqrt[3]{a}+\left(3-i\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.\right.}{}$$

```
[In] Integrate[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*((-3 - I*Sqrt
[3])*a^(1/3) + (3 - I*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3])*a^(1/3) - (I
 + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF[ArcSin[Sqrt[((
-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 +
 I*Sqrt[3])/2])/2 + I*(b^(1/3)*e + 2*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) + (I +
 Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/
3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[
Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]
], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/
3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F]

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}-a}}dx$$

```
[In] int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fric
as")
[Out] Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-\int \frac{e}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx-\int \frac{fx}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx$$

```
[In] integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
[Out] -Integral(e/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)),
 x) - Integral(f*x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*
x**3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int -\frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxi
ma")
[Out] -integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int -\frac{e+f\,x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{-b\,x^3-a}}\,dx$$

```
[In] int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x)
```

```
[Out] int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)), x)
```

3.91 $\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 221

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = -\frac{2(de-cf)\operatorname{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9c^{3/2}d^2} - \frac{\sqrt{2+\sqrt{3}}(2de+cf)(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

```
[Out] -2/9*(-c*f+d*e)*arctanh(1/3*(-2*d*x+c)^2/c^(1/2)/(-8*d^3*x^3+c^3)^(1/2))/c^
(3/2)/d^2-1/9*(c*f+2*d*e)*(-2*d*x+c)*EllipticF((-2*d*x+c*(1-3^(1/2)))/(-2*d
*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((4*d^2*x^2+2*c*
d*x+c^2)/(-2*d*x+c*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c/d^2/(-8*d^3*x^3+c^3)^(1/
2)/(c*(-2*d*x+c)/(-2*d*x+c*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2164, 224, 2163, 212}

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = -\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}(cf+2de)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}} - \frac{2\text{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)(de-cf)}{9c^{3/2}d^2}$$

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (-2*(d*e - c*f)*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(
9*c^(3/2)*d^2) - (Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c - 2*d*x)*Sqrt[(c^2 + 2
*c*d*x + 4*d^2*x^2)/((1 + Sqrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*c - 2*d*x)/((1 + Sqrt[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*
d^2*Sqrt[(c*(c - 2*d*x))/((1 + Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3*x^3]
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx}{3cd} + \frac{(2de+cf)\int \frac{1}{\sqrt{c^3-8d^3x^3}}\,dx}{3cd} \\
&= -\frac{\sqrt{2+\sqrt{3}}(2de+cf)(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}} \\
&\quad - \frac{(2(de-cf))\text{Subst}\left(\int \frac{1}{9-c^3x^2}\,dx, x, \frac{\left(1-\frac{2dx}{c}\right)^2}{\sqrt{c^3-8d^3x^3}}\right)}{3d^2} \\
&= -\frac{2(de-cf)\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9c^{3/2}d^2} \\
&\quad - \frac{\sqrt{2+\sqrt{3}}(2de+cf)(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.86 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.74

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx =$$

$$-\frac{i\sqrt{\frac{c-2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(f\sqrt{\frac{\left(-i+\sqrt{3}\right)c+2\left(i+\sqrt{3}\right)dx}{\left(-3i+\sqrt{3}\right)c}}\left(\left(-3i+\sqrt{3}\right)c-2\left(3i+\sqrt{3}\right)dx\right)\text{EllipticF}\left(\arcsin\left(\sqrt{2}\sqrt{\frac{ic+idx}{3ic-}}\right.\right.\right.}{2\left(-2+\right.}$$

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] ((-1/2*I)*Sqrt[(c - 2*d*x)/((1 + (-1)^(1/3))*c)]*(f*Sqrt[((-I + Sqrt[3])*c
+ 2*(I + Sqrt[3])*d*x)/((-3*I + Sqrt[3])*c)]*((-3*I + Sqrt[3])*c - 2*(3*I +
 Sqrt[3])*d*x)*EllipticF[ArcSin[Sqrt[2]*Sqrt[(I*c + I*d*x + Sqrt[3]*d*x)/((
3*I)*c - Sqrt[3]*c)]], (1 + I*Sqrt[3])/2] + 4*Sqrt[2]*(d*e - c*f)*Sqrt[(I*c
 + I*d*x + Sqrt[3]*d*x)/((3*I)*c - Sqrt[3]*c)]*Sqrt[(c^2 + 2*c*d*x + 4*d^2*
x^2)/c^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[2]*Sqrt[(I*c
+ I*d*x + Sqrt[3]*d*x)/((3*I)*c - Sqrt[3]*c)]], (1 + I*Sqrt[3])/2]))/((-2 +
 (-1)^(1/3))*d^2*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3
 - 8*d^3*x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(192) = 384$.

Time = 1.01 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.36

method	result
default	$\frac{2f\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}F\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}},\sqrt{\frac{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\right.\cdots}{d\sqrt{-8d^3x^3+c^3}}$
elliptic	$\frac{2f\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}F\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}},\sqrt{\frac{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\right.\cdots}{d\sqrt{-8d^3x^3+c^3}}$

```
[In] int((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f/d*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*
3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2
*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/
d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x
-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),((1/2*c/d-1/2*(-1/2
-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))+4/3*(-c
*f+d*e)/d*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/
2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d
-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/
2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)/c*Ellipti
cPi(((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),2/3*(1/2*c/d
-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/c*d,((1/2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/
(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.79

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$$
$$= \left[-\frac{3\sqrt{2}\sqrt{-d^3}(2\,cde+c^2f)\text{weierstrassPInverse}\left(0,\frac{c^3}{2\,d^3},x\right)+(d^3e-cd^2f)\sqrt{c}\log\left(\frac{8\,d^6x^6-240\,cd^5x^5+408\,c^2d^4x^4}{}\right.}{18\,c^2d^4}\right.$$
$$-\frac{3\sqrt{2}\sqrt{-d^3}(2\,cde+c^2f)\text{weierstrassPInverse}\left(0,\frac{c^3}{2\,d^3},x\right)+2\,(d^3e-cd^2f)\sqrt{-c}\arctan\left(\frac{(4\,d^3x^3-24\,cd^2x^2-6\,c}{3\,(16\,cd^4x^4-8}\right.}{18\,c^2d^4}$$

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/18*(3*sqrt(2)*sqrt(-d^3)*(2*c*d*e + c^2*f)*weierstrassPInverse(0, 1/2*c
^3/d^3, x) + (d^3*e - c*d^2*f)*sqrt(c)*log((8*d^6*x^6 - 240*c*d^5*x^5 + 408
*c^2*d^4*x^4 + 88*c^3*d^3*x^3 + 156*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 + 3*(
8*d^4*x^4 - 52*c*d^3*x^3 + 12*c^2*d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*d^3*
x^3 + c^3)*sqrt(c))/(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^
3 + 15*c^4*d^2*x^2 + 6*c^5*d*x + c^6)))/(c^2*d^4), -1/18*(3*sqrt(2)*sqrt(-d
^3)*(2*c*d*e + c^2*f)*weierstrassPInverse(0, 1/2*c^3/d^3, x) + 2*(d^3*e - c
*d^2*f)*sqrt(-c)*arctan(1/3*(4*d^3*x^3 - 24*c*d^2*x^2 - 6*c^2*d*x - 5*c^3)*
sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4 - 8*c^2*d^3*x^3 - 2*c^4*d*x +
 c^5)))/(c^2*d^4)]
```

Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{e+fx}{\sqrt{-\left(-c+2dx\right)\left(c^2+2cdx+4d^2x^2\right)}\left(c+dx\right)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(-c + 2*d*x)*(c**2 + 2*c*d*x + 4*d**2*x**2))*(c +
 d*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx=\int \frac{fx+e}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx=\int \frac{fx+e}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
[Out] integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx=\int \frac{e+f\,x}{\sqrt{c^3-8\,d^3\,x^3}\,\left(c+d\,x\right)}\,dx$$

```
[In] int((e + f*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
[Out] int((e + f*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)), x)
```

3.92 $\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 129

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{4}{9}\operatorname{arctanh}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 4/9*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))-2/9*(1+x)*EllipticF((1+x-3^(1/2))/(1
+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2
))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2164, 224, 2163, 212}

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{4}{9}\operatorname{arctanh}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right) - \frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[In] Int[x/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (4*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
x)^2]*Sqrt[1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{3}\int \frac{1}{\sqrt{1+x^3}}\,dx\right)+\frac{1}{3}\int \frac{2+2x}{(2-x)\sqrt{1+x^3}}\,dx\\ &= -\frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\\ &\quad+\frac{4}{3}\text{Subst}\left(\int \frac{1}{9-x^2}\,dx,x,\frac{(1+x)^2}{\sqrt{1+x^3}}\right)\end{aligned}$$

$$=\frac{4}{9}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.50

$$\int\frac{x}{(2-x)\sqrt{1+x^3}}\,dx$$

$$=\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}},\arcsin\left(\right.\right.}{-2+\sqrt[3]{-1}}\right.}{\sqrt{1+x^3}}$$

```
[In] Integrate[x/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[1 + x^3]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(104) = 208.

Time = 1.04 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.86

method	result
default	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+i}{-\frac{3}{2}+i}}}{3\sqrt{x^3+}}$
elliptic	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+i}{-\frac{3}{2}+i}}}{3\sqrt{x^3+}}$

```
[In] int(x/(2-x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+4/3*(3/2-1/2*I*3^(1/2))*((x+1)/(
3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2
)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Elliptic
Pi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2}{9}\log\left(\frac{x^3+12\,x^2+6\,\sqrt{x^3+1}(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right) - \frac{2}{3}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
12*x - 8)) - 2/3*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = -\int \frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

```
[In] integrate(x/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

Maxima [F]

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{x}{\sqrt{x^3+1}(x-2)}\,dx$$

```
[In] integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(x^3 + 1)*(x - 2)), x)
```

Giac [F]

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{x}{\sqrt{x^3+1}(x-2)}\,dx$$

```
[In] integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-x/(sqrt(x^3 + 1)*(x - 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.60

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asi}\right.\right.}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(-x/((x^3 + 1)^(1/2)*(x - 2)),x)
[Out] -((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 2*ellipticPi((3^(
1/2)*1i)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*
1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
)^(1/2))
```

3.93 $\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 145

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \frac{4}{9}\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] 4/9*arctanh(1/3*(1-x)^2/(-x^3+1)^(1/2))-2/9*(1-x)*EllipticF((1-x-3^(1/2))/(
1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/
2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2164, 224, 2163, 212}

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \frac{4}{9}\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[In] Int[x/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (4*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(
1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] -
x)^2]*Sqrt[1 - x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule steps

$$\text{integral} = \frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}\,dx - \frac{1}{3}\int \frac{2-2x}{(2+x)\sqrt{1-x^3}}\,dx$$

$$= -\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{4}{3}\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1-x)^2}{\sqrt{1-x^3}}\right)$$

$$=\frac{4}{9}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.34

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx$$

$$=\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}},\arcsin\left(\sqrt{\right.\right.}{-2+\sqrt[3]{-1}}\right.}{\sqrt{1-x^3}}$$

```
[In] Integrate[x/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[1 - x^3]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.66

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{4i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{4i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$

```
[In] int(x/(x+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))+4/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/
2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \frac{2}{9}\log\left(-\frac{x^3-12\,x^2+6\,\sqrt{-x^3+1}(x-1)-6\,x-10}{x^3+6\,x^2+12\,x+8}\right) - \frac{2}{3}i\,\mathrm{weierstrassPInverse}(0,4,x)$$

```
[In] integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] 2/9*log(-(x^3 - 12*x^2 + 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(x^3 + 6*x^2
+ 12*x + 8)) - 2/3*I*weierstrassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-(x-1)\,(x^2+x+1)}\,(x+2)}\,dx$$

```
[In] integrate(x/(2+x)/(-x**3+1)**(1/2),x)
[Out] Integral(x/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Maxima [F]

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}(x+2)}\,dx$$

```
[In] integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Giac [F]

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}(x+2)}\,dx$$

```
[In] integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.54

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\right.\right.}{3\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(x/((1 - x^3)^(1/2)*(x + 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin((-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
- 2*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(1 - x^3)^(1/
2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.94 $\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 148

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{4}{9}\arctan\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] 4/9*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))-2/9*(1-x)*EllipticF((1-x+3^(1/2))/(1-
x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2)
)^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2164, 225, 2163, 209}

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{4}{9}\arctan\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right) - \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[In] Int[x/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (4*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3]
- x)^2)]*Sqrt[-1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\int \frac{1}{\sqrt{-1+x^3}}\,dx - \frac{1}{3}\int \frac{2-2x}{(2+x)\sqrt{-1+x^3}}\,dx \\ &= -\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} \\ &\quad + \frac{4}{3}\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \frac{(1-x)^2}{\sqrt{-1+x^3}}\right)\end{aligned}$$

$$=\frac{4}{9}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.30

$$\int\frac{x}{(2+x)\sqrt{-1+x^3}}\,dx$$

$$=\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}},\arcsin\left(\right.\right.}{-2+\sqrt[3]{-1}}\right.}{\sqrt{-1+x^3}}$$

```
[In] Integrate[x/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[-1 + x^3]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(118) = 236.

Time = 1.00 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.62

method	result
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}}{\frac{3}{2}+}}}{3\sqrt{x^3}}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}}{\frac{3}{2}+}}}{3\sqrt{x^3}}$

```
[In] int(x/(x+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4/3*(-3/2-1/2*I*3^(1/2))*((x-1)/(-
3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)
*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi
(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2+1/6*I*3^(1/2),((3/2+1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.32

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{2}{9}\arctan\left(\frac{(x^3-12\,x^2-6\,x-10)\sqrt{x^3-1}}{6\,(x^4-x^3-x+1)}\right) + \frac{2}{3}\,\text{weierstrassPInverse}(0,4,x)$$

```
[In] integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] 2/9*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x + 1))
 + 2/3*weierstrassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{(x-1)\,(x^2+x+1)}\,(x+2)}\,dx$$

```
[In] integrate(x/(2+x)/(x**3-1)**(1/2),x)
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Maxima [F]

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}(x+2)}\,dx$$

```
[In] integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 2)), x)
```

Giac [F]

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}(x+2)}\,dx$$

```
[In] integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.41

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}}{6}\right.\right.}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\right.}}$$

```
[In] int(x/((x^3 - 1)^(1/2)*(x + 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 2*ellipticPi((
3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1
/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) +
 x^3)^(1/2))
```

3.95 $\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{4}{9}\arctan\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 4/9*arctan(1/3*(1+x)^2/(-x^3-1)^(1/2))-2/9*(1+x)*EllipticF((1+x+3^(1/2))/(1
+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2
))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2164, 225, 2163, 209}

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{4}{9}\arctan\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right) - \frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[In] Int[x/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (4*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3]
+ x)^2)]*Sqrt[-1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{3}\int \frac{1}{\sqrt{-1-x^3}}\,dx\right)+\frac{1}{3}\int \frac{2+2x}{(2-x)\sqrt{-1-x^3}}\,dx\\ &= -\frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\\ &\quad+\frac{4}{3}\text{Subst}\left(\int \frac{1}{9+x^2}\,dx,x,\frac{(1+x)^2}{\sqrt{-1-x^3}}\right)\end{aligned}$$

$$=\frac{4}{9}\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right)-\frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.39

$$\int\frac{x}{(2-x)\sqrt{-1-x^3}}\,dx$$

$$=\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}},\arcsin\left(\sqrt{\ldots}\right.\right.}{-2+\sqrt[3]{-1}}\right.}{\sqrt{-1-x^3}}$$

```
[In] Integrate[x/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[-1 - x^3]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(114) = 228.

Time = 0.95 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.71

method	result
default	$\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{4i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x\ldots}{\frac{3}{2}+\ldots}}}{}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{4i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x\ldots}{\frac{3}{2}+\ldots}}}{}$

```
[In] int(x/(2-x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))+4/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(
(x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
-x^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I
*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.32

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = -\frac{2}{9}\arctan\left(\frac{(x^3+12\,x^2-6\,x+10)\sqrt{-x^3-1}}{6\,(x^4+x^3+x+1)}\right) + \frac{2}{3}i\,\text{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/9*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 + x^3 + x + 1
)) + 2/3*I*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = -\int \frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

```
[In] integrate(x/(2-x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{x}{\sqrt{-x^3-1}(x-2)}\,dx$$

```
[In] integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Giac [F]

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{x}{\sqrt{-x^3-1}(x-2)}\,dx$$

```
[In] integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-x/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.59

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\right.\right.}{3\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}\right.}}$$

```
[In] int(-x/((- x^3 - 1)^(1/2)*(x - 2)),x)
[Out] -((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
 x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin(((x + 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 2
*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(- x^3 - 1)^(1/2)
*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.96 $\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 260

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{4\text{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 4/9*arctanh(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(b*x^3+a)^(1/2))/a^(1/6)/b^(2
/3)-2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*
3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2164, 224, 2163, 212}

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{4\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[In] Int[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (4*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}}+\frac{\int \frac{2\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}}\\
&= -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\\
&\quad+\frac{\left(4\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9-ax^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{a+bx^3}}\right)}{3b^{2/3}}\\
&= \frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}\\
&\quad-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.65 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.42

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,\mathrm{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+(-1)}{\left(1+\sqrt[3]{-}\right.}}\right.\right.\right.}{\left(-2+\sqrt[3]{-1}\right)b^2}$$

```
[In] Integrate[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3)
)*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^
(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^
(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1/3)*
(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(
1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Ellip
ticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3
)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/3)
)*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]
*Sqrt[a + b*x^3])
```

Maple [F]

$$\int \frac{x}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}+a}}dx$$

```
[In] int(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\int \frac{x}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

```
[In] integrate(x/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
[Out] -Integral(x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x
)
```

Maxima [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int -\frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\int \frac{x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{b\,x^3+a}}\,dx$$

```
[In] int(-x/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
[Out] -int(x/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)), x)
```

3.97 $\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 268

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{4\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] 4/9*arctanh(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(-b*x^3+a)^(1/2))/a^(1/6)/b^(
2/3)-2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b
^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^
(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2164, 224, 2163, 212}

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{4\text{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[In] Int[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (4*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}} - \frac{\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} + \frac{\left(4\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9-ax^2}\,dx, x, \frac{\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{a-bx^3}}\right)}{3b^{2/3}}$$

$$= \frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.58 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.38

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}-(\cdots}{(1+\sqrt[3]{\cdots}}}\right.\right.\right.}{\left(-2+\sqrt[3]{-1}\right)b\cdots}$$

```
[In] Integrate[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3)
)*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b
^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1/3
)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Ell
ipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/
3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)
)]*Sqrt[a - b*x^3])
```

Maple [F]

$$\int \frac{x}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^3+a}}dx$$

```
[In] int(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

```
[In] integrate(x/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
[Out] Integral(x/((2*a**(1/3) + b**(1/3)*x)*sqrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{a-b\,x^3}}\,dx$$

```
[In] int(x/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
[Out] int(x/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)), x)
```

3.98 $\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 277

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{4\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

```
[Out] 4/9*arctan(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(b*x^3-a)^(1/2))/a^(1/6)/b^(2/
3)-2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(
1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2)
)*3^(3/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2164, 225, 2163, 209}

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{4\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

```
[In] Int[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (4*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(
1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}} - \frac{\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}} + \frac{\left(4\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9+ax^2}\,dx, x, \frac{\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a+bx^3}}\right)}{3b^{2/3}}$$

$$= \frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.47 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.34

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}-(\cdots}{(1+\sqrt[3]{\cdots}}}\right.\right.\right.}{\left(-2+\sqrt[3]{-1}\right)b^2\cdots}$$

```
[In] Integrate[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3)
)*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b
^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1/3
)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Ell
ipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/
3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)
)]*Sqrt[-a + b*x^3])
```

Maple [F]

$$\int \frac{x}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^3-a}}dx$$

```
[In] int(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

```
[In] integrate(x/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
[Out] Integral(x/((2*a**(1/3) + b**(1/3)*x)*sqrt(-a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int \frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{b\,x^3-a}}\,dx$$

```
[In] int(x/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
[Out] int(x/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)), x)
```

3.99 $\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 273

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{4\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] 4/9*arctan(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(-b*x^3-a)^(1/2))/a^(1/6)/b^(2
/3)-2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1
/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*
3^(3/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2164, 225, 2163, 209}

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{4\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[In] Int[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (4*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}} + \frac{\int \frac{2\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} + \frac{\left(4\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9+ax^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a-bx^3}}\right)}{3b^{2/3}}$$

$$= \frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.65 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.36

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+(-1}{\left(1+\sqrt[3]{-}\right.}}\right.\right.\right.}{\left(-2+\sqrt[3]{-1}\right)b^{2/3}}$$

```
[In] Integrate[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3)
)*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^
(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^
(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1/3)*
(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(
1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Ellip
ticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3
)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/3)
)*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]
*Sqrt[-a - b*x^3])
```

Maple [F]

$$\int \frac{x}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}-a}}dx$$

```
[In] int(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\int \frac{x}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx$$

```
[In] integrate(x/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
[Out] -Integral(x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)),
 x)
```

Maxima [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int -\frac{x}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-\int \frac{x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{-b\,x^3-a}}\,dx$$

```
[In] int(-x/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x)
[Out] -int(x/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)), x)
```

3.100 $\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 202

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}d^2} - \frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

```
[Out] 2/9*arctanh(1/3*(-2*d*x+c)^2/c^(1/2)/(-8*d^3*x^3+c^3)^(1/2))/d^2/c^(1/2)-1/
9*(-2*d*x+c)*EllipticF((-2*d*x+c*(1-3^(1/2)))/(-2*d*x+c*(1+3^(1/2))),I*3^(1
/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((4*d^2*x^2+2*c*d*x+c^2)/(-2*d*x+c*(1+3^
(1/2)))^2)^(1/2)*3^(3/4)/d^2/(-8*d^3*x^3+c^3)^(1/2)/(c*(-2*d*x+c)/(-2*d*x+c
*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used

$= \{2164, 224, 2163, 212\}$

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$$
$$= \frac{2\text{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}d^2} - \frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

```
[In] Int[x/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(9*Sqrt[c]*d^2
) - (Sqrt[2 + Sqrt[3]]*(c - 2*d*x)*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/((1 + S
qrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c - 2*d*x)/((1 + Sqrt
[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^2*Sqrt[(c*(c - 2*d*x))/((1
+ Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt{c^3-8d^3x^3}}\,dx}{3d} - \frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx}{3d}$$

$$= -\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}} + \frac{(2c)\text{Subst}\left(\int \frac{1}{9-c^3x^2}\,dx, x, \frac{\left(1-\frac{2dx}{c}\right)^2}{\sqrt{c^3-8d^3x^3}}\right)}{3d^2}$$

$$= \frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}d^2} - \frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.55 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.46

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$$

$$= \frac{\sqrt{\frac{c-2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}c+2dx\right)\sqrt{\frac{\sqrt[3]{-1}\left(c+2\sqrt[3]{-1}dx\right)}{\left(1+\sqrt[3]{-1}\right)c}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\right),\sqrt[3]{}\right.\right.}{\left(-2+\sqrt[3]{-1}\right)d^2\sqrt{\frac{c-2(-1)^{2/}}{\left(1+\sqrt[3]{-}\right.}}}$$

```
[In] Integrate[x/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (Sqrt[(c - 2*d*x)/((1 + (-1)^(1/3))*c)]*((-2 + (-1)^(1/3))*((-1)^(1/3)*c +
2*d*x)*Sqrt[((-1)^(1/3)*(c + 2*(-1)^(1/3)*d*x))/((1 + (-1)^(1/3))*c)]*Ellip
ticF[ArcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]], (-1)^(1/3)]
 + (2*(-1)^(1/3)*(1 + (-1)^(1/3))*c*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^
(1/3))*c)]*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/c^2]*EllipticPi[(2*Sqrt[3])/(3*
I + Sqrt[3]), ArcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]], (-
1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/3))*d^2*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1
 + (-1)^(1/3))*c)]*Sqrt[c^3 - 8*d^3*x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(173) = 346$.

Time = 0.98 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.52

method	result
default	$\frac{2\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}F\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}},\sqrt{\frac{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\right)}{d\sqrt{-8d^3x^3+c^3}}$
elliptic	$\frac{2\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}F\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}},\sqrt{\frac{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\right)}{d\sqrt{-8d^3x^3+c^3}}$

```
[In] int(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^
(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(
-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-
1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x-1
/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),((1/2*c/d-1/2*(-1/2-1
/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))-4/3/d*(1/
2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*
c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/
2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1
/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticPi(((x-1/2*c/d
)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),2/3*(1/2*c/d-1/2*(-1/2-1/2*
I*3^(1/2))*c/d)/c*d,((1/2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-
1/2+1/2*I*3^(1/2))*c/d))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.76

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$$
$$= \left[\frac{\sqrt{c}d^2 \log \left(\frac{8\,d^6x^6-240\,cd^5x^5+408\,c^2d^4x^4+88\,c^3d^3x^3+156\,c^4d^2x^2+12\,c^5dx+17\,c^6+3\,(8\,d^4x^4-52\,cd^3x^3+12\,c^2d^2x^2-4\,c^3dx+5\,c^4)\sqrt{-8\,d^3x}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6} \right.}{18\,cd^4} \right.$$

```
[In] integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*(sqrt(c)*d^2*log((8*d^6*x^6 - 240*c*d^5*x^5 + 408*c^2*d^4*x^4 + 88*c^
3*d^3*x^3 + 156*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 + 3*(8*d^4*x^4 - 52*c*d^3
*x^3 + 12*c^2*d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*d^3*x^3 + c^3)*sqrt(c))/
(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 +
 6*c^5*d*x + c^6)) - 3*sqrt(2)*sqrt(-d^3)*c*weierstrassPInverse(0, 1/2*c^3/
d^3, x))/(c*d^4), 1/18*(2*sqrt(-c)*d^2*arctan(1/3*(4*d^3*x^3 - 24*c*d^2*x^2
 - 6*c^2*d*x - 5*c^3)*sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4 - 8*c^2
*d^3*x^3 - 2*c^4*d*x + c^5)) - 3*sqrt(2)*sqrt(-d^3)*c*weierstrassPInverse(0
, 1/2*c^3/d^3, x))/(c*d^4)]
```

Sympy [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{x}{\sqrt{-(-c+2dx)\,(c^2+2cdx+4d^2x^2)}\,(c+dx)}\,dx$$

```
[In] integrate(x/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(-c + 2*d*x)*(c**2 + 2*c*d*x + 4*d**2*x**2))*(c + d*x)),
x)
```

Maxima [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{x}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

```
[In] integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{x}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

```
[In] integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{x}{\sqrt{c^3-8\,d^3\,x^3}\,\left(c+d\,x\right)}\,dx$$

```
[In] int(x/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
[Out] int(x/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)), x)
```

3.101 $\int \frac{1+\sqrt{3}+x}{(1-\sqrt{3}+x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 42

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

```
[Out] -2*arctanh((1+x)*(-3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2165, 212}

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\left(2\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right) \\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}} \end{aligned}$$

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -2\sqrt{1+\frac{2}{\sqrt{3}}}\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{1+x^3}}{1-x+x^2}\right)$$

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
[Out] -2*Sqrt[1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1 - x
+ x^2)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.44 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.10

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\ln\left(-\frac{6\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)x^2+4\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\sqrt{3}\,x^2-4\sqrt{3}\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)x}{\left(x\sqrt{3}+x-2\right)^2}\right.}{6}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3}}$

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*RootOf(_Z^2-24*3^(1/2)-36)*ln(-(6*RootOf(_Z^2-24*3^(1/2)-36)*x^2+4*Roo
tOf(_Z^2-24*3^(1/2)-36)*3^(1/2)*x^2-4*3^(1/2)*RootOf(_Z^2-24*3^(1/2)-36)*x+
48*(x^3+1)^(1/2)*3^(1/2)+4*RootOf(_Z^2-24*3^(1/2)-36)*3^(1/2)+72*(x^3+1)^(1
/2)+12*RootOf(_Z^2-24*3^(1/2)-36))/(x*3^(1/2)+x-2)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(32) = 64.

Time = 0.34 (sec) , antiderivative size = 205, normalized size of antiderivative = 4.88

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$=\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^8-16\,x^7+112\,x^6-16\,x^5+112\,x^4+224\,x^3+64\,x^2-4\left(2\,x^6-18\,x^5+42\,x^4-\cdots\right.}{}\right.$$

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*
x^4 + 224*x^3 + 64*x^2 - 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 - sqrt(3)*(x^6
- 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(x^3 + 1)*sqrt(2*s
qrt(3) + 3) + 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x
 + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*
x^2 - 64*x + 16))
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1+\sqrt{3}}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
[Out] Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1))
, x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[-1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2
]%%%}
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

```
[In] int((x + 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.102 $\int \frac{1+\sqrt{3}-x}{(1-\sqrt{3}-x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 46

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

```
[Out] 2*arctanh((1-x)*(-3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2165, 212}

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[In] Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[-3 + 2*Sqrt[
3]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = 2\sqrt{1+\frac{2}{\sqrt{3}}}\,\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{1-x^3}}{1+x+x^2}\right)$$

```
[In] Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
[Out] 2*Sqrt[1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[1 - x^3])/(1 + x +
 x^2)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.89

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\ln\left(\frac{6\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)x^2+4\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\sqrt{3}\,x^2+4\sqrt{3}\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)x-4}{\left(x\sqrt{3}+x+2\right)^2}\right.}{6}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{4i\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{4i\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{}}{}$

```
[In] int((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*RootOf(_Z^2-24*3^(1/2)-36)*ln((6*RootOf(_Z^2-24*3^(1/2)-36)*x^2+4*Root
Of(_Z^2-24*3^(1/2)-36)*3^(1/2)*x^2+4*3^(1/2)*RootOf(_Z^2-24*3^(1/2)-36)*x-4
8*(-x^3+1)^(1/2)*3^(1/2)+4*RootOf(_Z^2-24*3^(1/2)-36)*3^(1/2)-72*(-x^3+1)^(
1/2)+12*RootOf(_Z^2-24*3^(1/2)-36))/(x*3^(1/2)+x+2)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(35) = 70$.

Time = 0.39 (sec) , antiderivative size = 207, normalized size of antiderivative = 4.50

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$$
$$=\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^8+16x^7+112x^6+16x^5+112x^4-224x^3+64x^2+4\left(2x^6+18x^5+42x^4+8\right.}{}\right.$$

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*
x^4 - 224*x^3 + 64*x^2 + 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 - sqrt(3)*(x^6
+ 12*x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(-x^3 + 1)*sqrt(2*
sqrt(3) + 3) - 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*
x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64
*x^2 + 64*x + 16))
```

Sympy [F]

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int \frac{x-\sqrt{3}-1}{\sqrt{-\left(x-1\right)\left(x^2+x+1\right)}\left(x-1+\sqrt{3}\right)}\,dx$$

```
[In] integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(-x**3+1)**(1/2),x)
[Out] Integral((x - sqrt(3) - 1)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3))
), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int \frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}\left(x+\sqrt{3}-1\right)}\,dx$$

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[1,1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2]%
%%} Er
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

```
[In] int(-(3^(1/2) - x + 1)/((1 - x^3)^(1/2)*(x + 3^(1/2) - 1)),x)
[Out] \text{Hanged}
```

3.103 $\int \frac{1+\sqrt{3}-x}{(1-\sqrt{3}-x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 44

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

```
[Out] 2*arctan((1-x)*(-3+2*3^(1/2))^(1/2)/(x^3-1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2165, 209}

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[In] Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[-3 + 2*Sqrt[
3]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -2\sqrt{1+\frac{2}{\sqrt{3}}}\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{-1+x^3}}{1+x+x^2}\right)$$

```
[In] Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
[Out] -2*Sqrt[1 + 2/Sqrt[3]]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1 + x
+ x^2)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.93

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\ln\left(\frac{6\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)x^2+4\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\sqrt{3}\,x^2+4\sqrt{3}\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)x-48\sqrt{x^3}\cdots}{\left(x\sqrt{3}+x+2\right)^2}\cdots\right)}{6}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\cdots}{\frac{3}{2}+\cdots}}\cdots}{\sqrt{x^3\cdots}}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\cdots}{\frac{3}{2}+\cdots}}\cdots}{\sqrt{x^3\cdots}}$

```
[In] int((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*RootOf(_Z^2+24*3^(1/2)+36)*ln((6*RootOf(_Z^2+24*3^(1/2)+36)*x^2+4*RootO
f(_Z^2+24*3^(1/2)+36)*3^(1/2)*x^2+4*3^(1/2)*RootOf(_Z^2+24*3^(1/2)+36)*x-48
*(x^3-1)^(1/2)*3^(1/2)+4*RootOf(_Z^2+24*3^(1/2)+36)*3^(1/2)-72*(x^3-1)^(1/2
)+12*RootOf(_Z^2+24*3^(1/2)+36))/(x*3^(1/2)+x+2)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}+3}\arctan\left(\frac{\left(\sqrt{3}(x^2+4x-2)-6x+6\right)\sqrt{2\sqrt{3}+3}}{6\sqrt{x^3-1}}\right)$$

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/6*(sqrt(3)*(x^2 + 4*x - 2) - 6*x +
 6)*sqrt(2*sqrt(3) + 3)/sqrt(x^3 - 1))
```

Sympy [F]

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int \frac{x-\sqrt{3}-1}{\sqrt{(x-1)\,(x^2+x+1)}\,\left(x-1+\sqrt{3}\right)}\,dx$$

```
[In] integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) - 1)/(sqrt((x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3)))
, x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int \frac{x-\sqrt{3}-1}{\sqrt{x^3-1}\left(x+\sqrt{3}-1\right)}\,dx$$

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(x + sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[1,1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2]%
%%} Er
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

```
[In] int(-(3^(1/2) - x + 1)/((x^3 - 1)^(1/2)*(x + 3^(1/2) - 1)),x)
```

```
[Out] \text{Hanged}
```

3.104 $\int \frac{1+\sqrt{3}+x}{(1-\sqrt{3}+x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 44

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

```
[Out] -2*arctan((1+x)*(-3+2*3^(1/2))^(1/2)/(-x^3-1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2165, 209}

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right)$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = 2\sqrt{1+\frac{2}{\sqrt{3}}}\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{-1-x^3}}{1-x+x^2}\right)$$

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
[Out] 2*Sqrt[1 + 2/Sqrt[3]]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[-1 - x^3])/(1 - x +
 x^2)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.05

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\ln\left(-\frac{6\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)x^2+4\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\sqrt{3}\,x^2-4\sqrt{3}\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)x+4}{\left(x\sqrt{3}+x-2\right)^2}\right.}{6}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{4i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{3}{2}}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{4i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{3}{2}}}{}$

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*RootOf(_Z^2+24*3^(1/2)+36)*ln(-(6*RootOf(_Z^2+24*3^(1/2)+36)*x^2+4*Root
Of(_Z^2+24*3^(1/2)+36)*3^(1/2)*x^2-4*3^(1/2)*RootOf(_Z^2+24*3^(1/2)+36)*x+4
*RootOf(_Z^2+24*3^(1/2)+36)*3^(1/2)+48*(-x^3-1)^(1/2)*3^(1/2)+12*RootOf(_Z^
2+24*3^(1/2)+36)+72*(-x^3-1)^(1/2))/(x*3^(1/2)+x-2)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}+3}\arctan\left(\frac{\sqrt{-x^3-1}\left(\sqrt{3}(x^2-4x-2)+6x+6\right)\sqrt{2\sqrt{3}+3}}{6\left(x^3+1\right)}\right)$$

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/6*sqrt(-x^3 - 1)*(sqrt(3)*(x^2 - 4
*x - 2) + 6*x + 6)*sqrt(2*sqrt(3) + 3)/(x^3 + 1))
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{x+1+\sqrt{3}}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-x**3-1)**(1/2),x)
[Out] Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)
), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[-1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2
]%%%}
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\text{Hanged}$$

```
[In] int((x + 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(x - 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.105 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 58, antiderivative size = 69

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] -2*arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2)
)/a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2165, 212}

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int\frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 7.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
))/(a^(1/6)*Sqrt[a + b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [F]

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1+\sqrt{3})}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^{3}+a}}dx$$

[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)

[Out] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 1.07 (sec) , antiderivative size = 1240, normalized size of antiderivative = 17.97

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^{3}}}\,dx = \text{Too large to display}$$

[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 - 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*b^(1/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x - sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b

```
^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 - 299*a*b^6
*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024
*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 32*sqrt(
3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^
12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8))/(b
^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a
^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4
096*a^8)), sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*arctan(1/2*
sqrt(1/3)*(a^(1/3)*b*x^2 + 2*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*(sqrt(3)
*a - a)*b^(1/3))*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))/sqrt(b*x^3 + a))]
```

Sympy [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-3**(1/
2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(-sqr
t(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}+1)}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}-1)\right)}\,dx$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x -
 a^(1/3)*(sqrt(3) - 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Hanged}$$

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(
1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

3.106 $\int \frac{(1+\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 61, antiderivative size = 71

$$\int \frac{(1+\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] 2*arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2)
)/a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2165, 212}

$$\int \frac{(1+\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral} = \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 7.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
)/(a^(1/6)*Sqrt[a - b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [F]

$$\int \frac{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1+\sqrt{3}\right)}{\left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{-b\,x^{3}+a}}dx$$

```
[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x
^3+a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x
^3+a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(53) = 106$.

Time = 1.05 (sec) , antiderivative size = 1294, normalized size of antiderivative = 18.23

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^{3}}}\,dx = \text{Too large to display}$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 + 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*
a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^
13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x
- sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x
^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x
))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 +
 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^6
*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b^
5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 9318
4*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((3*b^7*x^22 +
2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x
^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x - 2*sqrt(3)*(b^7*
x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4
*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*sqrt(-b*x^3
 + a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^
5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824
*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14
 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x
^2))*sqrt(-b*x^3 + a)*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 4464
0*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^
```

6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3 + a)*b^(1/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3))) - 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*a^(1/3)*b*x^2 - 2*sqrt(-b*x^3 + a)*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*sqrt(-b*x^3 + a)*(sqrt(3)*a - a)*b^(1/3))*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))/(b*x^3 - a))]

Sympy [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{-\sqrt{3}\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a-bx^3}\left(-\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)

[Out] Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)

Maxima [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}+1)}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}-1)\right)}\,dx$$

[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Hanged}$$

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(
1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

3.107 $\int \frac{(1+\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 62, antiderivative size = 72

$$\int \frac{(1+\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] 2*arctan(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))/
a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2165, 209}

$$\int \frac{(1+\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 7.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2))
/(a^(1/6)*Sqrt[-a + b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [F]

$$\int \frac{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}(1+\sqrt{3})}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^{3}-a}}dx$$

[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)

[Out] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 1.06 (sec) , antiderivative size = 1245, normalized size of antiderivative = 17.29

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 + 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*b^(1/3))*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x - sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5

```
*b^2*x^8 - 328704*a^6*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b
^6*x^20 + 4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 1050
24*a^5*b^2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sqr
t(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*
x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8))/
(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984
*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 +
 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*arctan(-1
/2*sqrt(1/3)*(a^(1/3)*b*x^2 - 2*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*(sqrt
(3)*a - a)*b^(1/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))/sqrt(b*x^3 - a))]
```

Sympy [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{-\sqrt{3}\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a+bx^3}\left(-\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-3**(
1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-a
**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x +
 a^(1/3)*(sqrt(3) - 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Hanged}$$

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^(
1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

3.108 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 61, antiderivative size = 72

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] -2*arctan(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2)
)/a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2165, 209}

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 7.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
)/(a^(1/6)*Sqrt[-a - b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [F]

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1+\sqrt{3}\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{-b\,x^{3}-a}}dx$$

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3
-a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3
-a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(52) = 104$.

Time = 1.09 (sec) , antiderivative size = 1303, normalized size of antiderivative = 18.10

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^{3}}}\,dx = \text{Too large to display}$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 - 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672
*a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x
^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x
 - sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*
x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*
x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17
- 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^
6*b*x^5 - 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b
^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 931
84*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((3*b^7*x^22 -
 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*
x^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x - 2*sqrt(3)*(b^7
*x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^
4*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*sqrt(-b*x^
3 - a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b
^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 1382
4*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^1
4 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*
x^2))*sqrt(-b*x^3 - a)*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 446
40*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x
```

```
^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^1
8 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^
6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3 - a)*b^(1/3))*sqrt(-(2
*sqrt(3) + 3)/(a*b^(2/3))) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18
+ 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2
*x^6 + 19712*a^7*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6
*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151
552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*a^(1/3)*sqrt((2*s
qrt(3) + 3)/(a*b^(2/3)))*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*a^(1/3)*b*x
^2 + 2*sqrt(-b*x^3 - a)*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*sqrt(-b*x^3 -
 a)*(sqrt(3)*a - a)*b^(1/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))/(b*x^3 + a))
]
```

Sympy [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-3**(1/
2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(-sq
rt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
 - a^(1/3)*(sqrt(3) - 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Hanged}$$

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/((- a - b*x^3)^(1/2)*(b^(1/3)*x - a
^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

3.109 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 52, antiderivative size = 73

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] -2*arctanh((1+(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))/
(b/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2165, 212}

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b
*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.97 (sec) , antiderivative size = 663, normalized size of antiderivative = 9.08

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$$

$$= \frac{x\left(12\left(-3+\sqrt{3}\right)\sqrt[3]{\frac{b}{a}}x\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{-10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3+\frac{3bx^3}{a}}\,\text{AppellF1}\right.}{}$$

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[a + b*x^3]),x]
```

```
[Out] (x*(12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sq
rt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), (b*x^3)/(-10*a + 6
*Sqrt[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), (b*x
^3)/(-10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(-5 + 3*Sqrt[3])*a -
 b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), (b*x^3
)/(-10*a + 6*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[4/3, 1/2,
 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*A
ppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))])))
)/(a*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2
, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF
1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3
*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt
[3]*a))]))))/(24*(-5 + 3*Sqrt[3])*Sqrt[a + b*x^3])
```

Maple [F]

$$\int \frac{1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}}{\left(1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}\right) \sqrt{b\,x^{3}+a}}dx$$

[In] int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x)

[Out] int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.72 (sec) , antiderivative size = 1273, normalized size of antiderivative = 17.44

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \text{Too large to display}$$

[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 + 4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 - 145152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^7*b*x^2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 - 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4 + 7168*a^8*x))*(b/a)^(2/3) - 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 + a)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 + 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a

```
^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8) + 32*(9*a*b^7*x
^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 + 5472*a^4*b^4*x^13 + 43776*a^5*
b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 + 4608*a^8*x - sqrt(3)*(5*a*
b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 - 4928*a^4*b^4*x^13 - 28688
*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 - 2560*a^8*x))*(b/a)^(1
/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 1
21984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*
x^3 + 4096*a^8)), sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*arctan(1/2
*sqrt(1/3)*(b*x^2 + 2*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*(sqrt(3)*a - a)
*(b/a)^(1/3))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 + a))]
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}}{\sqrt{a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3+a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(a + b*x**3)*(x*(b/a)**(1/3) -
 sqrt(3) + 1)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{bx^3+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) - s
qrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{3}+x\,\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{b\,x^3+a}\,\left(x\,\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1\right)}\,dx$$

```
[In] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((a + b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1/
2) + 1)),x)
```

```
[Out] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((a + b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1/
2) + 1)), x)
```

3.110 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 55, antiderivative size = 75

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] 2*arctanh((1-(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))/
(b/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2165, 212}

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b
*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^
3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral}=\frac{2\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx,x,\frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.86 (sec) , antiderivative size = 648, normalized size of antiderivative = 8.64

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$$

$$= \frac{x\left(-12(-3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1-\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3-\frac{3bx^3}{a}}\operatorname{AppellF1}\left(\right.\right.}{}$$

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[a - b*x^3]),x]
```

```
[Out] (x*(-12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[
3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
- 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)] - b*x^3*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*Sqrt[1
- (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(
10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^
3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^
3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))))/(a*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(
8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a -
6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[a - b*x^3])
```

Maple [F]

$$\int \frac{1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}}{\left(1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}\right) \sqrt{-b\,x^{3}+a}}dx$$

```
[In] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x)
[Out] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(57) = 114$.

Time = 0.72 (sec) , antiderivative size = 1330, normalized size of antiderivative = 17.73

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="fricas")
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504
*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
+ 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^
16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a
^7*b*x^4 - 7168*a^8*x))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7*x^21 + 5
010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*
x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x
^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a
^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3
+ a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^1
4 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*
b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 49
28*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*
sqrt(-b*x^3 + a))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1
077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3
*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 - 2*sqrt(3)*(
a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 267
20*a^5*b^3*x^11 - 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b
/a)^(2/3) - 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^
```

```
15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*
b*x^3 - 512*a^8) + 32*(9*a*b^7*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16
- 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*
x^4 - 4608*a^8*x - sqrt(3)*(5*a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*
x^16 + 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a
^7*b*x^4 + 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b
^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 1
51552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt(-(2*sqrt(
3) + 3)*(b/a)^(1/3)/b)*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*b*x^2 - 2*sq
rt(-b*x^3 + a)*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*sqrt(-b*x^3 + a)*(sqrt
(3)*a - a)*(b/a)^(1/3))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)/(b*x^3 - a))]
```

Sympy [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1}{\sqrt{a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}\right)}\,dx$$

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(-b*x**3+
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(a - b*x**3)*(x*(b/a)**(1/3) -
 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{-bx^3+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1\right)}\,dx$$

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) +
sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \int -\frac{\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{a-b\,x^3}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1\right)}\,dx$$

```
[In] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)),x)
```

```
[Out] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)), x)
```

3.111 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 56, antiderivative size = 76

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] 2*arctan((1-(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))/(b
/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2165, 209}

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a +
b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x^
3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral}=\frac{2\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx,x,\frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.90 (sec) , antiderivative size = 649, normalized size of antiderivative = 8.54

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$$

$$= \frac{x\left(-12\left(-3+\sqrt{3}\right)\sqrt[3]{\frac{b}{a}}x\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3-\frac{3bx^3}{a}}\,\text{AppellF1}\left(\right.\right.}{}$$

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[-a + b*x^3]),x]
```

```
[Out] (x*(-12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[
3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
- 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)] - b*x^3*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*Sqrt[1
- (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(
10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^
3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^
3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))))/(a*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(
8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a -
6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[-a + b*x^3])
```

Maple [F]

$$\int \frac{1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}}{\left(1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}\right) \sqrt{b\,x^{3}-a}}dx$$

```
[In] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x)
[Out] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.68 (sec) , antiderivative size = 1278, normalized size of antiderivative = 16.82

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\text{Too large to display}$$

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="fricas")
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 -
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
 4*sqrt(1/3)*(486*a*b^7*x^20 + 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 + 14
5152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 + 414720*a^6*b^2*x^5 - 82944*a^7*b*x
^2 + (3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a^4*b^4
*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 + 24576*
a^8*x - 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 + 197
92*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
 - 7168*a^8*x))*(b/a)^(2/3) - 6*sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17
+ 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2
*x^5 + 8192*a^7*b*x^2) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b
^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 860
16*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 2486
4*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^
6 - 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 - a)*sqrt(-(2*sqrt
(3) + 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b
^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 3
28704*a^7*b*x^5 + 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20
+ 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 - 105024*a^6*b
^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b/a)^(2/3) - 32*sqrt(3)*(35*a*b
^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520
```

```
*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 32*(9*a*b^7
*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 - 5472*a^4*b^4*x^13 + 43776*a^
5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 - 4608*a^8*x - sqrt(3)*(5*
a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 + 4928*a^4*b^4*x^13 - 286
88*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 + 2560*a^8*x))*(b/a)^
(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 +
 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*
b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*arctan(-
1/2*sqrt(1/3)*(b*x^2 - 2*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*(sqrt(3)*a -
 a)*(b/a)^(1/3))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 - a))]
```

Sympy [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1}{\sqrt{-a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}\right)}\,dx$$

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3-a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3)
- 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{bx^3-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1\right)}\,dx$$

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) + s
qrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \int -\frac{\sqrt{3}-x\,\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{b\,x^3-a}\,\left(\sqrt{3}+x\,\left(\frac{b}{a}\right)^{1/3}-1\right)}\,dx$$

```
[In] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((b*x^3 - a)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)),x)
```

```
[Out] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((b*x^3 - a)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)), x)
```

3.112 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 55, antiderivative size = 76

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] -2*arctan((1+(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2))/
(b/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2165, 209}

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a -
b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral}=-\frac{2\text{Subst}\left(\int\frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx,x,\frac{1+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.88 (sec) , antiderivative size = 666, normalized size of antiderivative = 8.76

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$$

$$= \frac{x\left(12\left(-3+\sqrt{3}\right)\sqrt[3]{\frac{b}{a}}x\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{-10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3+\frac{3bx^3}{a}}\,\text{AppellF1}\right.}{}$$

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[-a - b*x^3]),x]
```

```
[Out] (x*(12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sq
rt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), (b*x^3)/(-10*a + 6
*Sqrt[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), (b*x
^3)/(-10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(-5 + 3*Sqrt[3])*a -
 b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), (b*x^3
)/(-10*a + 6*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -
((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[4/3, 1/2,
 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*A
ppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))])))
)/(a*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2
, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF
1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3
*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt
[3]*a))])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[-a - b*x^3])
```

Maple [F]

$$\int \frac{1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}}{\left(1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}\right) \sqrt{-b\,x^{3}-a}}dx$$

```
[In] int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x)
[Out] int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(58) = 116$.

Time = 0.73 (sec) , antiderivative size = 1339, normalized size of antiderivative = 17.62

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="fricas")
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 +
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
 4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 9350
4*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
 - 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x
^16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*
a^7*b*x^4 + 7168*a^8*x))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7*x^21 -
5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3
*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*
x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*
a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3
 - a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^
14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7
*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4
928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))
*sqrt(-b*x^3 - a))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 -
 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b
^3*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 - 2*sqrt(3)
*(a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 2
6720*a^5*b^3*x^11 + 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*
(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*
```

```
x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^
7*b*x^3 + 512*a^8) + 32*(9*a*b^7*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^1
6 + 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*
b*x^4 + 4608*a^8*x - sqrt(3)*(5*a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^
5*x^16 - 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200
*a^7*b*x^4 - 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2
*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 +
 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*sqrt((2*sqrt(
3) + 3)*(b/a)^(1/3)/b)*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*b*x^2 + 2*sqr
t(-b*x^3 - a)*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*sqrt(-b*x^3 - a)*(sqrt(
3)*a - a)*(b/a)^(1/3))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)/(b*x^3 + a))]
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}}{\sqrt{-a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(-b*x**3-
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3)
- sqrt(3) + 1)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{-bx^3-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) -
sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{\sqrt{3}+x\,\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{-b\,x^3-a}\,\left(x\,\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1\right)}\,dx$$

```
[In] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((- a - b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(
1/2) + 1)),x)
```

```
[Out] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((- a - b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(
1/2) + 1)), x)
```

3.113 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 42

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] -2*arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2165, 209}

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[3 + 2*Sqrt[3]
]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(2\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -2\sqrt{-1+\frac{2}{\sqrt{3}}}\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{1+x^3}}{1-x+x^2}\right)$$

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
[Out] -2*Sqrt[-1 + 2/Sqrt[3]]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1 - x +
 x^2)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.12

method	result
trager	$\frac{\text{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\ln\left(\frac{6\,\text{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)x^2-4\,\text{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\sqrt{3}\,x^2+4\sqrt{3}\,\text{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)x+48\sqrt{}}{\left(x\sqrt{3}-x+2\right)^2}\right.}{6}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+}}$

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*RootOf(_Z^2-36+24*3^(1/2))*ln((6*RootOf(_Z^2-36+24*3^(1/2))*x^2-4*RootO
f(_Z^2-36+24*3^(1/2))*3^(1/2)*x^2+4*3^(1/2)*RootOf(_Z^2-36+24*3^(1/2))*x+48
*(x^3+1)^(1/2)*3^(1/2)-4*RootOf(_Z^2-36+24*3^(1/2))*3^(1/2)-72*(x^3+1)^(1/2
)+12*RootOf(_Z^2-36+24*3^(1/2)))/(x*3^(1/2)-x+2)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{3}\,\sqrt{3}\sqrt{2\,\sqrt{3}-3}\arctan\left(\frac{\left(\sqrt{3}(x^2-4\,x-2)-6\,x-6\right)\sqrt{2\,\sqrt{3}-3}}{6\,\sqrt{x^3+1}}\right)$$

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*(sqrt(3)*(x^2 - 4*x - 2) - 6*x -
 6)*sqrt(2*sqrt(3) - 3)/sqrt(x^3 + 1))
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3)))
, x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%
%%} Er
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

```
[In] int((x - 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

3.114 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 46

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] 2*arctan((1-x)*(3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2165, 209}

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[In] Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[3 + 2*Sqrt[3]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
```

```
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = 2\sqrt{-1+\frac{2}{\sqrt{3}}}\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{1-x^3}}{1+x+x^2}\right)$$

```
[In] Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
[Out] 2*Sqrt[-1 + 2/Sqrt[3]]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 - x^3])/(1 + x +
x^2)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.47 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.93

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\ln\left(\frac{6\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)x^2-4\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\sqrt{3}\,x^2-4\sqrt{3}\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)x+48}{\left(x\sqrt{3}-x-2\right)^2}\right.}{6}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{4i\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{-}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{4i\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{-}}{}$

```
[In] int((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*RootOf(_Z^2-36+24*3^(1/2))*ln((6*RootOf(_Z^2-36+24*3^(1/2))*x^2-4*Root
Of(_Z^2-36+24*3^(1/2))*3^(1/2)*x^2-4*3^(1/2)*RootOf(_Z^2-36+24*3^(1/2))*x+4
8*(-x^3+1)^(1/2)*3^(1/2)-4*RootOf(_Z^2-36+24*3^(1/2))*3^(1/2)-72*(-x^3+1)^(
1/2)+12*RootOf(_Z^2-36+24*3^(1/2)))/(x*3^(1/2)-x-2)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \frac{1}{3}\,\sqrt{3}\sqrt{2\,\sqrt{3}-3}\arctan\left(\frac{\sqrt{-x^3+1}\left(\sqrt{3}(x^2+4\,x-2)+6\,x-6\right)\sqrt{2\,\sqrt{3}-3}}{6\,(x^3-1)}\right)$$

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*sqrt(-x^3 + 1)*(sqrt(3)*(x^2 + 4
*x - 2) + 6*x - 6)*sqrt(2*sqrt(3) - 3)/(x^3 - 1))
```

Sympy [F]

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int \frac{x-1+\sqrt{3}}{\sqrt{-\left(x-1\right)\left(x^2+x+1\right)}\left(x-\sqrt{3}-1\right)}\,dx$$

```
[In] integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((x - 1 + sqrt(3))/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1)
), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%
%%} Er
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

```
[In] int(-(x + 3^(1/2) - 1)/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

```
[Out] \text{Hanged}
```

3.115 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 44

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] 2*arctanh((1-x)*(3+2*3^(1/2))^(1/2)/(x^3-1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2165, 212}

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[In] Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[3 + 2*Sqrt[3
]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -2\sqrt{-1+\frac{2}{\sqrt{3}}}\,\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{-1+x^3}}{1+x+x^2}\right)$$

```
[In] Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
[Out] -2*Sqrt[-1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1 + x
 + x^2)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.53 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.98

method	result
trager	$\frac{\text{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\ln\left(\frac{6\,\text{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)x^2-4\,\text{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\sqrt{3}\,x^2-4\sqrt{3}\,\text{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)x+48\sqrt{x}}{\left(x\sqrt{3}-x-2\right)^2}\right.}{6}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+}{\frac{3}{2}+}}}{\sqrt{x}}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+}{\frac{3}{2}+}}}{\sqrt{x}}$

```
[In] int((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*RootOf(_Z^2-24*3^(1/2)+36)*ln((6*RootOf(_Z^2-24*3^(1/2)+36)*x^2-4*RootO
f(_Z^2-24*3^(1/2)+36)*3^(1/2)*x^2-4*3^(1/2)*RootOf(_Z^2-24*3^(1/2)+36)*x+48
*(x^3-1)^(1/2)*3^(1/2)-4*RootOf(_Z^2-24*3^(1/2)+36)*3^(1/2)-72*(x^3-1)^(1/2
)+12*RootOf(_Z^2-24*3^(1/2)+36))/(x*3^(1/2)-x-2)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(33) = 66$.

Time = 0.29 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.64

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$$
$$=\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8+16x^7+112x^6+16x^5+112x^4-224x^3+64x^2-4\left(2x^6+18x^5+42x^4+8\right.}{}\right.$$

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*
x^4 - 224*x^3 + 64*x^2 - 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 + sqrt(3)*(x^6
+ 12*x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(x^3 - 1)*sqrt(2*s
qrt(3) - 3) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x
 - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*
x^2 + 64*x + 16))
```

Sympy [F]

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=\int \frac{x-1+\sqrt{3}}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x-\sqrt{3}-1\right)}\,dx$$

```
[In] integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x - 1 + sqrt(3))/(sqrt((x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1))
, x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%
%%} Er
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=\text{Hanged}$$

```
[In] int(-(x + 3^(1/2) - 1)/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
[Out] \text{Hanged}
```

3.116 $\int \frac{1-\sqrt{3}+x}{(1+\sqrt{3}+x)\sqrt{-1-x^3}} dx$

Optimal result

Integrand size = 32, antiderivative size = 44

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}} \, dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] -2*arctanh((1+x)*(3+2*3^(1/2))^(1/2)/(-x^3-1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2165, 212}

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}} \, dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[3 + 2*Sqrt[
3]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(2\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = 2\sqrt{-1+\frac{2}{\sqrt{3}}}\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{-1-x^3}}{1-x+x^2}\right)$$

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
[Out] 2*Sqrt[-1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-1 - x^3])/(1 - x
+ x^2)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.07

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\ln\left(\frac{6\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)x^2-4\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\sqrt{3}\,x^2+4\sqrt{3}\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)x+4}{\left(x\sqrt{3}-x+2\right)^2}\right.}{6}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{4i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{3}{2}}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{4i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{3}{2}}}{}$

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*RootOf(_Z^2-24*3^(1/2)+36)*ln((6*RootOf(_Z^2-24*3^(1/2)+36)*x^2-4*Root
Of(_Z^2-24*3^(1/2)+36)*3^(1/2)*x^2+4*3^(1/2)*RootOf(_Z^2-24*3^(1/2)+36)*x+4
8*(-x^3-1)^(1/2)*3^(1/2)-4*RootOf(_Z^2-24*3^(1/2)+36)*3^(1/2)-72*(-x^3-1)^(
1/2)+12*RootOf(_Z^2-24*3^(1/2)+36))/(x*3^(1/2)-x+2)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(34) = 68$.

Time = 0.34 (sec) , antiderivative size = 206, normalized size of antiderivative = 4.68

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$$
$$=\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2+4\left(2x^6-18x^5+42x^4-8\right.}{}\right.$$

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*
x^4 + 224*x^3 + 64*x^2 + 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 + sqrt(3)*(x^6
- 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(-x^3 - 1)*sqrt(2*
sqrt(3) - 3) - 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*
x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64
*x^2 - 64*x + 16))
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
[Out] Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))
), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%
%%} Er
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

```
[In] int((x - 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.117 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 58, antiderivative size = 69

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] -2*arctan(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))/
a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2165, 209}

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[a + b*x^3]),x]

[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x
^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 7.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{2\arctan\left(\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3]]*
a^(1/6)*Sqrt[a + b*x^3])])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [F]

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1-\sqrt{3})}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1+\sqrt{3}\right)\right)\sqrt{b\,x^{3}+a}}dx$$

```
[In] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+
a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+
a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 1.05 (sec) , antiderivative size = 1236, normalized size of antiderivative = 17.91

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 - 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672
*a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a
^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7
- 314880*a^6*b*x^4 - 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 1
6860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^
2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 -
 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^
3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724
*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 +
 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2
*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 2
33856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2
920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*
x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*b^(1/3))*sqrt(-(2*s
qrt(3) - 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x
^16 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^
6*b*x^4 + 4608*a^7*x + sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*
x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a
^6*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 +
 13320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5
```

```
*b^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 - 299*a*b
^6*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 1050
24*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sqr
t(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*
x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8))/
(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984
*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 +
 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*arctan(-1
/2*sqrt(1/3)*(a^(1/3)*b*x^2 - 2*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*(sqrt
(3)*a + a)*b^(1/3))*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))/sqrt(b*x^3 + a))]
```

Sympy [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}\left(\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1/
2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(a**
(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)\right)}\,dx$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x +
 a^(1/3)*(sqrt(3) + 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Hanged}$$

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/((a + b*x^3)^(1/2)*(b^(1/3)*x + a^(
1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

3.118 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 61, antiderivative size = 71

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] 2*arctan(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))/
a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2165, 209}

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^
3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral} = \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 7.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{2\arctan\left(\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*ArcTan[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3]]
*a^(1/6)*Sqrt[a - b*x^3])])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [F]

$$\int \frac{-b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1-\sqrt{3})}{\left(-b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1+\sqrt{3}\right)\right) \sqrt{-b\,x^{3}+a}} dx$$

```
[In] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x
^3+a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x
^3+a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(53) = 106$.

Time = 1.03 (sec) , antiderivative size = 1288, normalized size of antiderivative = 18.14

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 + 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672
*a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x
^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x
 + sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*
x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*
x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17
+ 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^
6*b*x^5 + 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b
^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 931
84*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((3*b^7*x^22 +
 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*
x^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x + 2*sqrt(3)*(b^7
*x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^
4*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*sqrt(-b*x^
3 + a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b
^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 1382
4*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^1
4 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*
x^2))*sqrt(-b*x^3 + a)*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 446
40*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x
```

```
^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^1
8 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^
6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3 + a)*b^(1/3))*sqrt(-(2
*sqrt(3) - 3)/(a*b^(2/3))) + 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18
+ 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2
*x^6 + 19712*a^7*b*x^3 - 512*a^8))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6
*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151
552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*a^(1/3)*sqrt((2*s
qrt(3) - 3)/(a*b^(2/3)))*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*a^(1/3)*b*x
^2 + 2*sqrt(-b*x^3 + a)*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*sqrt(-b*x^3 +
 a)*(sqrt(3)*a + a)*b^(1/3))*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))/(b*x^3 - a))
]
```

Sympy [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{-\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a-bx^3}\left(-\sqrt{3}\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**(
1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-sq
rt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)\right)}\,dx$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x
- a^(1/3)*(sqrt(3) + 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Hanged}$$

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/((a - b*x^3)^(1/2)*(b^(1/3)*x - a^(
1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

3.119 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 62, antiderivative size = 72

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] 2*arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))/
a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2165, 212}

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*
x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral} = \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int\frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 7.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

$$\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3]
]*a^(1/6)*Sqrt[-a + b*x^3])])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [F]

$$\int \frac{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1-\sqrt{3})}{\left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1+\sqrt{3}\right)\right)\sqrt{b\,x^{3}-a}}dx$$

[In] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x)

[Out] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 1.04 (sec) , antiderivative size = 1239, normalized size of antiderivative = 17.21

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 + 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*b^(1/3))*sqrt((2*sqrt(3) - 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x + sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b

```
^2*x^8 - 328704*a^6*b*x^5 + 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 + 299*a*b^6
*x^20 + 4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024
*a^5*b^2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 32*sqrt(
3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^
12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8))/(b
^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a
^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4
096*a^8)), sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*arctan(1/2*
sqrt(1/3)*(a^(1/3)*b*x^2 + 2*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*(sqrt(3)
*a + a)*b^(1/3))*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))/sqrt(b*x^3 - a))]
```

Sympy [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{-\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**(
1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-s
qrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)\right)}\,dx$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x -
 a^(1/3)*(sqrt(3) + 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Hanged}$$

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/((b*x^3 - a)^(1/2)*(b^(1/3)*x - a^(
1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

3.120 $\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{-a-b x^3}} \, dx$

Optimal result

Integrand size = 61, antiderivative size = 72

$$\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{-a-b x^3}} \, dx=-\frac{2 \text{arctanh}\left(\frac{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt{-a-b x^3}}\right)}{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

```
[Out] -2*arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2)
)/a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2165, 212}

$$\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{-a-b x^3}} \, dx=-\frac{2 \text{arctanh}\left(\frac{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt{-a-b x^3}}\right)}{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b
*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 7.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{2\text{arctanh}\left(\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*ArcTanh[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3]]
*a^(1/6)*Sqrt[-a - b*x^3])])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [F]

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1-\sqrt{3}\right)}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1+\sqrt{3}\right)\right)\sqrt{-b\,x^{3}-a}}dx$$

```
[In] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3
-a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3
-a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(52) = 104$.

Time = 1.07 (sec) , antiderivative size = 1299, normalized size of antiderivative = 18.04

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^{3}}}\,dx = \text{Too large to display}$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 - 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*
a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^
13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x
+ sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x
^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*x
))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 -
 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^6
*b*x^5 - 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b^
5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 9318
4*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((3*b^7*x^22 -
2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x
^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x + 2*sqrt(3)*(b^7*
x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4
*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*sqrt(-b*x^3
 - a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^
5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824
*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14
 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x
^2))*sqrt(-b*x^3 - a)*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 4464
0*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^
```

```
6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18
 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6
*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3 - a)*b^(1/3))*sqrt((2*s
qrt(3) - 3)/(a*b^(2/3))) - 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 +
2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x
^6 + 19712*a^7*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x
^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 15155
2*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt(-(2*s
qrt(3) - 3)/(a*b^(2/3)))*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*a^(1/3)*b*
x^2 - 2*sqrt(-b*x^3 - a)*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*sqrt(-b*x^3
- a)*(sqrt(3)*a + a)*b^(1/3))*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))/(b*x^3 + a
))]
```

Sympy [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}\left(\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1/
2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(a*
*(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}+1)\right)}\,dx$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
+ a^(1/3)*(sqrt(3) + 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Hanged}$$

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/((- a - b*x^3)^(1/2)*(b^(1/3)*x + a
^(1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

3.121 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 52, antiderivative size = 73

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] -2*arctan((1+(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))/(b
/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2165, 209}

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b
*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x^3
]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral}=-\frac{2\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx,x,\frac{1+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.76 (sec) , antiderivative size = 667, normalized size of antiderivative = 9.14

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$$

$$= \frac{x\left(12(3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3+\frac{3bx^3}{a}}\operatorname{AppellF1}\left(\right.\right.}{}$$

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[a + b*x^3]),x]
```

```
[Out] (x*(12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1
, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 8*(b/a)^(2/3)*x^2*S
qrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*a +
 6*Sqrt[3]*a))] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -(
(b*x^3)/(10*a + 6*Sqrt[3]*a))] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/
3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - b*x^3*(2*(5 + 3*Sqrt[3]
)*a + b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -
((b*x^3)/(10*a + 6*Sqrt[3]*a))]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1,
4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(AppellF1[4/3
, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt
[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a
))]))))/(a*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3,
 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(App
ellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5
 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*
Sqrt[3]*a))])))))/(24*(5 + 3*Sqrt[3])*Sqrt[a + b*x^3])
```

Maple [F]

$$\int \frac{1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}}{\left(1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}\right) \sqrt{b\,x^{3}+a}}dx$$

```
[In] int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x)
[Out] int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.74 (sec) , antiderivative size = 1270, normalized size of antiderivative = 17.40

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=\text{Too large to display}$$

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="fricas")
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 +
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
 4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 - 14
5152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^7*b*x
^2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^4*b^4
*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 - 24576*
a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 - 197
92*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
 + 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17
+ 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2
*x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b
^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 860
16*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 2486
4*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^
6 - 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 + a)*sqrt(-(2*sqrt
(3) - 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b
^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 3
28704*a^7*b*x^5 - 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20
+ 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 + 105024*a^6*b
^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b/a)^(2/3) - 32*sqrt(3)*(35*a*b
^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520
```

```
*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8) + 32*(9*a*b^7
*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 + 5472*a^4*b^4*x^13 + 43776*a^
5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 + 4608*a^8*x + sqrt(3)*(5*
a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 - 4928*a^4*b^4*x^13 - 286
88*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 - 2560*a^8*x))*(b/a)^
(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 +
 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*
b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*arctan(-
1/2*sqrt(1/3)*(b*x^2 - 2*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*(sqrt(3)*a +
 a)*(b/a)^(1/3))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 + a))]
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\sqrt{a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}\right)}\,dx$$

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3+a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(a + b*x**3)*(x*(b/a)**(1/3) +
 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{bx^3+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) + s
qrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1}{\sqrt{b\,x^3+a}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

```
[In] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((a + b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1/
3) + 1)),x)
```

```
[Out] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((a + b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1/
3) + 1)), x)
```

3.122 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 55, antiderivative size = 75

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] 2*arctan((1-(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))/(b
/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2165, 209}

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b
*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^3]
])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral}=\frac{2\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx,x,\frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.82 (sec) , antiderivative size = 649, normalized size of antiderivative = 8.65

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$$

$$= \frac{x\left(-12(3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3-\frac{3bx^3}{a}}\,\text{AppellF1}\left(1,\right.\right.}{}$$

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[a - b*x^3]),x]
```

```
[Out] (x*(-12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[3
 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]
*a)] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a +
6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*
x^3)/(10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(5 + 3*Sqrt[3])*a - b*x^3)*Sqrt[1 - (
b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a
)]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
 + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(
10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a
, (b*x^3)/(10*a + 6*Sqrt[3]*a)]))))/(a*(2*(5 + 3*Sqrt[3])*a - b*x^3)*(8*(5
+ 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt
[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*
Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)
/(10*a + 6*Sqrt[3]*a)])))))/(24*(5 + 3*Sqrt[3])*Sqrt[a - b*x^3])
```

Maple [F]

$$\int \frac{1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}}{\left(1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}\right) \sqrt{-b\,x^{3}+a}}dx$$

```
[In] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x)
[Out] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(57) = 114$.

Time = 0.72 (sec) , antiderivative size = 1324, normalized size of antiderivative = 17.65

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="fricas")
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 -
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
 4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 9350
4*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
 + 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x
^16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*
a^7*b*x^4 - 7168*a^8*x))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7*x^21 +
5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3
*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*
x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*
a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3
 + a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^
14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7
*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4
928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))
*sqrt(-b*x^3 + a))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 +
 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b
^3*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 + 2*sqrt(3)
*(a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 2
6720*a^5*b^3*x^11 - 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*
(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*
```

```
x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^
7*b*x^3 - 512*a^8) + 32*(9*a*b^7*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^1
6 - 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*
b*x^4 - 4608*a^8*x + sqrt(3)*(5*a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^
5*x^16 + 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200
*a^7*b*x^4 + 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2
*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 +
 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*sqrt((2*sqrt(
3) - 3)*(b/a)^(1/3)/b)*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*b*x^2 + 2*sqr
t(-b*x^3 + a)*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*sqrt(-b*x^3 + a)*(sqrt(
3)*a + a)*(b/a)^(1/3))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)/(b*x^3 - a))]
```

Sympy [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}}{\sqrt{a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1\right)}\,dx$$

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(-b*x**3+
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(a - b*x**3)*(x*(b/a)**(1/3) -
 sqrt(3) - 1)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{-bx^3+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1\right)}\,dx$$

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) -
sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \int -\frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{a-b\,x^3}\,\left(\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

```
[In] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((a - b*x^3)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)),x)
```

```
[Out] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((a - b*x^3)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)), x)
```

3.123 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 56, antiderivative size = 76

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] 2*arctanh((1-(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))/(b
/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2165, 212}

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a +
b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x^
3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\text{integral}=\frac{2\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx,x,\frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.85 (sec) , antiderivative size = 650, normalized size of antiderivative = 8.55

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$$

$$= \frac{x\left(-12(3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1-\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3-\frac{3bx^3}{a}}\operatorname{AppellF1}\left(1,\right.\right.}{}$$

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[-a + b*x^3]),x]
```

```
[Out] (x*(-12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[3
 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]
*a)] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a +
6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*
x^3)/(10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(5 + 3*Sqrt[3])*a - b*x^3)*Sqrt[1 - (
b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a
)]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a
 + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(
10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a
, (b*x^3)/(10*a + 6*Sqrt[3]*a)]))))/(a*(2*(5 + 3*Sqrt[3])*a - b*x^3)*(8*(5
+ 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt
[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*
Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)
/(10*a + 6*Sqrt[3]*a)])))))/(24*(5 + 3*Sqrt[3])*Sqrt[-a + b*x^3])
```

Maple [F]

$$\int \frac{1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}}{\left(1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}\right) \sqrt{b\,x^{3}-a}}dx$$

```
[In] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x)
[Out] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.73 (sec) , antiderivative size = 1273, normalized size of antiderivative = 16.75

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\text{Too large to display}$$

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="fricas")
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
4*sqrt(1/3)*(486*a*b^7*x^20 + 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 + 145
152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 + 414720*a^6*b^2*x^5 - 82944*a^7*b*x^
2 + (3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a^4*b^4*
x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 + 24576*a
^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 + 1979
2*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
- 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 +
 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*
x^5 + 8192*a^7*b*x^2) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^
5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 8601
6*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864
*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6
 - 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 - a)*sqrt((2*sqrt(3
) - 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b^5
*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 328
704*a^7*b*x^5 + 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20 +
4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 - 105024*a^6*b^2
*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7
*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a
```

```
^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 32*(9*a*b^7*x
^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 - 5472*a^4*b^4*x^13 + 43776*a^5*
b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 - 4608*a^8*x + sqrt(3)*(5*a*
b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 + 4928*a^4*b^4*x^13 - 28688
*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 + 2560*a^8*x))*(b/a)^(1
/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 1
21984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*
x^3 + 4096*a^8)), sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*arctan(1/2
*sqrt(1/3)*(b*x^2 + 2*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*(sqrt(3)*a + a)
*(b/a)^(1/3))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 - a))]
```

Sympy [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}}{\sqrt{-a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1\right)}\,dx$$

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3-a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3)
- sqrt(3) - 1)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{bx^3-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1\right)}\,dx$$

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) - s
qrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \int -\frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{b\,x^3-a}\,\left(\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

```
[In] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((b*x^3 - a)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)),x)
```

```
[Out] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((b*x^3 - a)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)), x)
```

3.124 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 55, antiderivative size = 76

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] -2*arctanh((1+(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2))/
(b/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2165, 212}

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.83 (sec) , antiderivative size = 670, normalized size of antiderivative = 8.82

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$$

$$= \frac{x\left(12(3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3+\frac{3bx^3}{a}}\operatorname{AppellF1}\left(\right.\right.}{}$$

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[-a - b*x^3]),x]
```

```
[Out] (x*(12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1
, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 8*(b/a)^(2/3)*x^2*S
qrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*a +
 6*Sqrt[3]*a))] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -(
(b*x^3)/(10*a + 6*Sqrt[3]*a))] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/
3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - b*x^3*(2*(5 + 3*Sqrt[3]
)*a + b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -
((b*x^3)/(10*a + 6*Sqrt[3]*a))]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1,
4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(AppellF1[4/3
, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt
[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a
))]))))/(a*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3,
 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(App
ellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5
 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*
Sqrt[3]*a))])))))/(24*(5 + 3*Sqrt[3])*Sqrt[-a - b*x^3])
```

Maple [F]

$$\int \frac{1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}}{\left(1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}\right) \sqrt{-b\,x^{3}-a}}dx$$

```
[In] int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x)
[Out] int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(58) = 116$.

Time = 0.72 (sec) , antiderivative size = 1335, normalized size of antiderivative = 17.57

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="fricas")
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504
*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
- 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^
16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a
^7*b*x^4 + 7168*a^8*x))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7*x^21 - 5
010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*
x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x
^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a
^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3
- a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^1
4 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*
b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 49
28*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*
sqrt(-b*x^3 - a))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1
077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3
*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 + 2*sqrt(3)*(
a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 267
20*a^5*b^3*x^11 + 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b
/a)^(2/3) - 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^
```

```
15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*
b*x^3 + 512*a^8) + 32*(9*a*b^7*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16
+ 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*
x^4 + 4608*a^8*x + sqrt(3)*(5*a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*
x^16 - 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a
^7*b*x^4 - 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b
^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 1
51552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt(-(2*sqrt(
3) - 3)*(b/a)^(1/3)/b)*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*b*x^2 - 2*sq
rt(-b*x^3 - a)*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*sqrt(-b*x^3 - a)*(sqrt
(3)*a + a)*(b/a)^(1/3))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)/(b*x^3 + a))]
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\sqrt{-a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}\right)}\,dx$$

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(-b*x**3-
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3)
+ 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{-bx^3-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) +
sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1}{\sqrt{-b\,x^3-a}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

```
[In] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((- a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) + 1)),x)
```

```
[Out] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((- a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) + 1)), x)
```

3.125 $\int \frac{1+x}{(1+\sqrt{3}+x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}} + \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x
)/(1+x+3^(1/2))^2)^(1/2)-arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/(3
+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used

= {2166, 224, 2165, 209}

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[In] Int[(1 + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3 + 2*Sqrt[3]])
+ (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*
Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
```

```
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = \frac{1}{12}\int \frac{(1+\sqrt{3})\left(-22+\left(1+\sqrt{3}\right)^3\right)+6x}{(1+\sqrt{3}+x)\sqrt{1+x^3}}\,dx+\frac{1}{2}\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$-\operatorname{Subst}\left(\int \frac{1}{1+(3+2\sqrt{3})\,x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}+\frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.39 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.86

$$\int \frac{1+x}{(1+\sqrt{3}+x)\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{6}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(\sqrt{-i+\sqrt{3}+2ix}\left((-2-i)-\sqrt{3}+\left((1+2i)+i\sqrt{3}\right)x\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2}{3i}\right.\right.}{(3i+(1+2i)\sqrt{3})\sqrt{i+\sqrt{3}}}$$

```
[In] Integrate[(1 + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]*
((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqr
t[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1
 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2
*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] -
(2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(119) = 238$.

Time = 2.43 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.69

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi}{\sqrt{x^3+1}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi}{\sqrt{x^3+1}}$

```
[In] int((x+1)/(1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.38

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\left(\sqrt{3}(x^2-4x-2)-6x-6\right)\sqrt{2\sqrt{3}-3}}{6\sqrt{x^3+1}}\right) + \text{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*(sqrt(3)*(x^2 - 4*x - 2) - 6*x -
 6)*sqrt(2*sqrt(3) - 3)/sqrt(x^3 + 1)) + weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

```
[In] integrate((1+x)/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Giac [F]

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

```
[In] int((x + 1)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.126 $\int \frac{1+x}{(1-\sqrt{3}+x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}} + \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x
)/(1+x+3^(1/2))^2)^(1/2)-arctanh((1+x)*(-3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/
(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used

$= \{2166, 224, 2165, 212\}$

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \frac{\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[In] Int[(1 + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[-3 + 2*Sqrt[3]
]) + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ell
ipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
```

```
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = \frac{1}{12}\int \frac{(1-\sqrt{3})\left(-22+(1-\sqrt{3})^3\right)+6x}{(1-\sqrt{3}+x)\sqrt{1+x^3}}\,dx + \frac{1}{2}\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$- \text{Subst}\left(\int \frac{1}{1+(3-2\sqrt{3})\,x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}} + \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.36 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.84

$$\int \frac{1+x}{(1-\sqrt{3}+x)\sqrt{1+x^3}}\,dx =$$

$$-\frac{2\sqrt{6}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(\sqrt{-i+\sqrt{3}+2ix}((1+2i)-i\sqrt{3}+((-2-i)+\sqrt{3})\,x)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\right.\right.}{(-3+(2+i)\sqrt{3})\sqrt{i+\sqrt{}}}$$

```
[In] Integrate[(1 + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (2*I
)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/
(-3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4)
)], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + Sqrt[3]
 - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(119) = 238.

Time = 2.44 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.69

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3}}$

```
[In] int((x+1)/(1+x-3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.45

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$=\frac{1}{12}\sqrt{3}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2-4\left(2x^6-18x^5+42x^4-\right.}{}\right.$$
$$+\operatorname{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112
*x^4 + 224*x^3 + 64*x^2 - 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 - sqrt(3)*(x^6
 - 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(x^3 + 1)*sqrt(2*
sqrt(3) + 3) + 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*
x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64
*x^2 - 64*x + 16)) + weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x)/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)), x)
```

Maxima [F]

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Giac [F]

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

```
[In] int((x + 1)/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.127 $\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 173

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\left(e-f-\sqrt{3}f\right)\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2+\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(e-f*(1-3^(1
/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(1/4)/(
x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+arctan((1+x)*(3+2*3^(1/2))^(1/2)
/(x^3+1)^(1/2))*(e-f-f*3^(1/2))/(9+6*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2166, 224, 2165, 209}

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} + \frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)\left(e-\sqrt{3}f-f\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

```
[In] Int[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((e - f - Sqrt[3]*f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/S
qrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 + Sqrt[3]]*(e - (1 - Sqrt[3])*f)*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
```

```
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = \frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{2\sqrt{3}} + \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)}$$

$$= \frac{\sqrt{2+\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(12\left(e-\left(1+\sqrt{3}\right)f\right)\right)\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)}$$

$$= \frac{\left(e-f-\sqrt{3}f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2+\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.47 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.68

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3f\sqrt{-i+\sqrt{3}+2ix}\left((-2-i)-\sqrt{3}+\left((1+2i)+i\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\right.\right.}{\left(3i+(1+2i)\sqrt{\right.}}$$

[In] Integrate[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]

[Out] (2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*f*Sqrt[-I + Sqrt[3] + (2*I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*(-(Sqrt[3]*e) + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.50

method	result
default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-f-f\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-f-f\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$

[In] int((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*f*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e-f-f*3^(1/2))*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 716, normalized size of antiderivative = 4.14

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \left[\frac{1}{3}\left(\sqrt{3}(e-f)+3\,f\right)\text{weierstrassPInverse}(0,-4,x)\right.$$

$$+\frac{1}{12}\sqrt{3\,e^2+6\,ef-2\,\sqrt{3}(e^2+ef+f^2)}\log\left(-\frac{(e^2-2\,ef-2\,f^2)x^8-16\,(e^2-2\,ef-2\,f^2)x^7+112\,(e^2}{}\right.$$

$$-4,x)$$

$$-\frac{1}{6}\sqrt{-3\,e^2-6\,ef+2\,\sqrt{3}(e^2+ef+f^2)}\arctan\left(\frac{(3\,fx^2-6\,(e+f)x+\sqrt{3}((e-f)x^2-2\,(2\,e+f)x-}{6\,((e^2-2\,ef-2}\right.$$

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt(3)*(e - f) + 3*f)*weierstrassPInverse(0, -4, x) + 1/12*sqrt(3*e^
2 + 6*e*f - 2*sqrt(3)*(e^2 + e*f + f^2))*log(-((e^2 - 2*e*f - 2*f^2)*x^8 -
16*(e^2 - 2*e*f - 2*f^2)*x^7 + 112*(e^2 - 2*e*f - 2*f^2)*x^6 - 16*(e^2 - 2*
e*f - 2*f^2)*x^5 + 112*(e^2 - 2*e*f - 2*f^2)*x^4 + 224*(e^2 - 2*e*f - 2*f^2
)*x^3 + 64*(e^2 - 2*e*f - 2*f^2)*x^2 - 4*((2*e + f)*x^6 - 18*(e + f)*x^5 +
6*(7*e + 2*f)*x^4 - 8*(e + 5*f)*x^3 - 36*f*x^2 + 24*(e - f)*x + sqrt(3)*((e
 + f)*x^6 - 6*(2*e + f)*x^5 + 6*(3*e + 4*f)*x^4 - 8*(2*e - f)*x^3 - 12*(e -
 f)*x^2 + 24*f*x - 8*e + 16*f) + 8*e - 32*f)*sqrt(x^3 + 1)*sqrt(3*e^2 + 6*e
*f - 2*sqrt(3)*(e^2 + e*f + f^2)) + 112*e^2 - 224*e*f - 224*f^2 + 128*(e^2
- 2*e*f - 2*f^2)*x - 16*sqrt(3)*((e^2 - 2*e*f - 2*f^2)*x^7 - 2*(e^2 - 2*e*f
 - 2*f^2)*x^6 + 6*(e^2 - 2*e*f - 2*f^2)*x^5 + 5*(e^2 - 2*e*f - 2*f^2)*x^4 +
 2*(e^2 - 2*e*f - 2*f^2)*x^3 + 6*(e^2 - 2*e*f - 2*f^2)*x^2 + 4*e^2 - 8*e*f
- 8*f^2 + 4*(e^2 - 2*e*f - 2*f^2)*x))/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x
^4 + 32*x^3 + 64*x^2 - 64*x + 16)), 1/3*(sqrt(3)*(e - f) + 3*f)*weierstrass
PInverse(0, -4, x) - 1/6*sqrt(-3*e^2 - 6*e*f + 2*sqrt(3)*(e^2 + e*f + f^2))
*arctan(1/6*(3*f*x^2 - 6*(e + f)*x + sqrt(3)*((e - f)*x^2 - 2*(2*e + f)*x -
 2*e - 4*f) - 6*e)*sqrt(x^3 + 1)*sqrt(-3*e^2 - 6*e*f + 2*sqrt(3)*(e^2 + e*f
 + f^2))/((e^2 - 2*e*f - 2*f^2)*x^3 + e^2 - 2*e*f - 2*f^2))]
```

Sympy [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=\int \frac{e+fx}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

```
[In] integrate((f*x+e)/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
[Out] Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=\text{Hanged}$$

```
[In] int((e + f*x)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.128 $\int \frac{e+fx}{(1+\sqrt{3}-x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 187

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{\left(e+f+\sqrt{3}f\right)\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2+\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -1/3*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(e+f*(1-3^(
1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(1/4)/
(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-arctan((1-x)*(3+2*3^(1/2))^(1/
2)/(-x^3+1)^(1/2))*(e+f+f*3^(1/2))/(9+6*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2166, 224, 2165, 209}

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)\left(e+\sqrt{3}f+f\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

```
[In] Int[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((e + f + Sqrt[3]*f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])
/Sqrt[3*(3 + 2*Sqrt[3])]) - (Sqrt[2 + Sqrt[3]]*(e + (1 - Sqrt[3])*f)*(1 - x
)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 - x)/(1 + Sqrt[3] -
 x)^2]*Sqrt[1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
```

```
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = -\frac{\left(-e-\left(1+\sqrt{3}\right)f\right)\int \frac{\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(28-\left(1+\sqrt{3}\right)^3\right)} - \frac{\left(-6e+\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)f\right)\int \frac{1}{\sqrt{1-x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(28-\left(1+\sqrt{3}\right)^3\right)}$$

$$= -\frac{\sqrt{2+\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{\left(12\left(-e-\left(1+\sqrt{3}\right)f\right)\right)\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)}{\left(1+\sqrt{3}\right)\left(28-\left(1+\sqrt{3}\right)^3\right)}$$

$$= -\frac{\left(e+f+\sqrt{3}f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2+\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.52 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.56

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3if\sqrt{-i+\sqrt{3}-2ix}\left(-i\left((2+i)+\sqrt{3}\right)+\left((2-i)+\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}}}{\sqrt{2}\sqrt[4]{}}\right.\right.\right.}{(3i+(1+2i)}$$

```
[In] Integrate[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*f*Sqrt[-I + Sqrt
[3] - (2*I)*x]*((-I)*((2 + I) + Sqrt[3]) + ((2 - I) + Sqrt[3])*x)*EllipticF
[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I +
Sqrt[3])] + 2*(Sqrt[3]*e + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqr
t[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqr
t[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))
/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.41

method	result
default	$\frac{2if\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2i\left(-e-f-f\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x\right.}}{}$
elliptic	$\frac{2if\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2i\left(-e-f-f\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x\right.}}{}$

```
[In] int((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-e-f-f*3^(1/2))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(
1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2-3^(1/2)+1/2*I*3^(1/2))*EllipticPi
```

```
(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2-3^(1/2
)+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 735, normalized size of antiderivative = 3.93

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \left[-\frac{1}{3}\left(\sqrt{3}(i\,e+i\,f)-3i\,f\right)\text{weierstrassPInverse}(0,4,x)\right.$$
$$+\frac{1}{12}\sqrt{3\,e^2-6\,ef-2\,\sqrt{3}(e^2-ef+f^2)}\log\left(-\frac{(e^2+2\,ef-2\,f^2)x^8+16\,(e^2+2\,ef-2\,f^2)x^7+112\,(e^2\right.$$
$$-\frac{1}{3}\left(\sqrt{3}(i\,e+i\,f)-3i\,f\right)\text{weierstrassPInverse}(0,4,x)$$
$$+\frac{1}{6}\sqrt{-3\,e^2+6\,ef+2\,\sqrt{3}(e^2-ef+f^2)}\arctan\left(\frac{(3\,fx^2-6\,(e-f)x-\sqrt{3}((e+f)x^2+2\,(2\,e-f)x-}{6\,((e^2+2\,ef-2}\right.$$

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(sqrt(3)*(I*e + I*f) - 3*I*f)*weierstrassPInverse(0, 4, x) + 1/12*sqr
t(3*e^2 - 6*e*f - 2*sqrt(3)*(e^2 - e*f + f^2))*log(-((e^2 + 2*e*f - 2*f^2)*
x^8 + 16*(e^2 + 2*e*f - 2*f^2)*x^7 + 112*(e^2 + 2*e*f - 2*f^2)*x^6 + 16*(e^
2 + 2*e*f - 2*f^2)*x^5 + 112*(e^2 + 2*e*f - 2*f^2)*x^4 - 224*(e^2 + 2*e*f -
 2*f^2)*x^3 + 64*(e^2 + 2*e*f - 2*f^2)*x^2 + 4*((2*e - f)*x^6 + 18*(e - f)*
x^5 + 6*(7*e - 2*f)*x^4 + 8*(e - 5*f)*x^3 + 36*f*x^2 - 24*(e + f)*x + sqrt(
3)*((e - f)*x^6 + 6*(2*e - f)*x^5 + 6*(3*e - 4*f)*x^4 + 8*(2*e + f)*x^3 - 1
2*(e + f)*x^2 + 24*f*x - 8*e - 16*f) + 8*e + 32*f)*sqrt(-x^3 + 1)*sqrt(3*e^
2 - 6*e*f - 2*sqrt(3)*(e^2 - e*f + f^2)) + 112*e^2 + 224*e*f - 224*f^2 - 12
8*(e^2 + 2*e*f - 2*f^2)*x + 16*sqrt(3)*((e^2 + 2*e*f - 2*f^2)*x^7 + 2*(e^2
+ 2*e*f - 2*f^2)*x^6 + 6*(e^2 + 2*e*f - 2*f^2)*x^5 - 5*(e^2 + 2*e*f - 2*f^2
)*x^4 + 2*(e^2 + 2*e*f - 2*f^2)*x^3 - 6*(e^2 + 2*e*f - 2*f^2)*x^2 - 4*e^2 -
 8*e*f + 8*f^2 + 4*(e^2 + 2*e*f - 2*f^2)*x))/(x^8 - 8*x^7 + 16*x^6 + 16*x^5
 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16)), -1/3*(sqrt(3)*(I*e + I*f) - 3*I*
f)*weierstrassPInverse(0, 4, x) + 1/6*sqrt(-3*e^2 + 6*e*f + 2*sqrt(3)*(e^2
- e*f + f^2))*arctan(1/6*(3*f*x^2 - 6*(e - f)*x - sqrt(3)*((e + f)*x^2 + 2*
(2*e - f)*x - 2*e + 4*f) + 6*e)*sqrt(-x^3 + 1)*sqrt(-3*e^2 + 6*e*f + 2*sqrt
(3)*(e^2 - e*f + f^2))/((e^2 + 2*e*f - 2*f^2)*x^3 - e^2 - 2*e*f + 2*f^2))]
```

Sympy [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\int \frac{e}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx - \int \frac{fx}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

```
[In] integrate((f*x+e)/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)), x
) - Integral(f*x/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3
)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int -\frac{fx+e}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

```
[In] int((e + f*x)/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
[Out] \text{Hanged}
```

3.129 $\int \frac{e+fx}{(1+\sqrt{3}-x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 190

$$\int \frac{e+fx}{(1+\sqrt{3}-x)\sqrt{-1+x^3}}\,dx = -\frac{(e+f+\sqrt{3}f)\,\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2-\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -1/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(e+f*(1-3^(
1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(1/4)/
(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)-arctanh((1-x)*(3+2*3^(1/2))^(1
/2)/(x^3-1)^(1/2))*(e+f+f*3^(1/2))/(9+6*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2166, 225, 2165, 212}

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)\left(e+\sqrt{3}f+f\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

```
[In] Int[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((e + f + Sqrt[3]*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]
])/Sqrt[3*(3 + 2*Sqrt[3])]) - (Sqrt[2 - Sqrt[3]]*(e + (1 - Sqrt[3])*f)*(1 -
 x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
 x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[
3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
```

```
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = -\frac{\left(-e-\left(1+\sqrt{3}\right)f\right)\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)-6x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)} - \frac{\left(6e+\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)f\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)}$$

$$= -\frac{\sqrt{2-\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(12\left(-e-\left(1+\sqrt{3}\right)f\right)\right)\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)}$$

$$= -\frac{\left(e+f+\sqrt{3}f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2-\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.51 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.52

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3if\sqrt{-i+\sqrt{3}-2ix}\left(-i\left((2+i)+\sqrt{3}\right)+\left((2-i)+\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{}}}{\sqrt{2}}\right.\right.\right.}{(3i+(1+2i}$$

```
[In] Integrate[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*f*Sqrt[-I + Sqrt
[3] - (2*I)*x]*((-I)*((2 + I) + Sqrt[3]) + ((2 - I) + Sqrt[3])*x)*EllipticF
[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I +
Sqrt[3])] + 2*(Sqrt[3]*e + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqr
t[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqr
t[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))
/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.38

method	result
default	$-\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\left(-e-f-f\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$
elliptic	$-\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\left(-e-f-f\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$

```
[In] int((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(-e-f-f*3^(1/2))*(-3/2-1/2*
I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1
/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3
-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1
/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 723, normalized size of antiderivative = 3.81

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \left[\frac{1}{3}\left(\sqrt{3}(e+f)-3\,f\right)\text{weierstrassPInverse}(0,4,x)\right.$$

$$+\frac{1}{12}\sqrt{-3\,e^2+6\,ef+2\,\sqrt{3}(e^2-ef+f^2)}\log\left(-\frac{(e^2+2\,ef-2\,f^2)x^8+16\,(e^2+2\,ef-2\,f^2)x^7+112\,(e^2}{}\right.$$

$$+\frac{1}{6}\sqrt{3\,e^2-6\,ef-2\,\sqrt{3}(e^2-ef+f^2)}\arctan\left(\frac{(3\,fx^2-6\,(e-f)x-\sqrt{3}((e+f)x^2+2\,(2\,e-f)x-2\,e}{6\,((e^2+2\,ef-2\,f^2)x}\right.$$

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt(3)*(e + f) - 3*f)*weierstrassPInverse(0, 4, x) + 1/12*sqrt(-3*e^
2 + 6*e*f + 2*sqrt(3)*(e^2 - e*f + f^2))*log(-((e^2 + 2*e*f - 2*f^2)*x^8 +
16*(e^2 + 2*e*f - 2*f^2)*x^7 + 112*(e^2 + 2*e*f - 2*f^2)*x^6 + 16*(e^2 + 2*
e*f - 2*f^2)*x^5 + 112*(e^2 + 2*e*f - 2*f^2)*x^4 - 224*(e^2 + 2*e*f - 2*f^2
)*x^3 + 64*(e^2 + 2*e*f - 2*f^2)*x^2 + 4*((2*e - f)*x^6 + 18*(e - f)*x^5 +
6*(7*e - 2*f)*x^4 + 8*(e - 5*f)*x^3 + 36*f*x^2 - 24*(e + f)*x + sqrt(3)*((e
 - f)*x^6 + 6*(2*e - f)*x^5 + 6*(3*e - 4*f)*x^4 + 8*(2*e + f)*x^3 - 12*(e +
 f)*x^2 + 24*f*x - 8*e - 16*f) + 8*e + 32*f)*sqrt(x^3 - 1)*sqrt(-3*e^2 + 6*
e*f + 2*sqrt(3)*(e^2 - e*f + f^2)) + 112*e^2 + 224*e*f - 224*f^2 - 128*(e^2
 + 2*e*f - 2*f^2)*x + 16*sqrt(3)*((e^2 + 2*e*f - 2*f^2)*x^7 + 2*(e^2 + 2*e*
f - 2*f^2)*x^6 + 6*(e^2 + 2*e*f - 2*f^2)*x^5 - 5*(e^2 + 2*e*f - 2*f^2)*x^4
+ 2*(e^2 + 2*e*f - 2*f^2)*x^3 - 6*(e^2 + 2*e*f - 2*f^2)*x^2 - 4*e^2 - 8*e*f
 + 8*f^2 + 4*(e^2 + 2*e*f - 2*f^2)*x))/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*
x^4 - 32*x^3 + 64*x^2 + 64*x + 16)), 1/3*(sqrt(3)*(e + f) - 3*f)*weierstras
sPInverse(0, 4, x) + 1/6*sqrt(3*e^2 - 6*e*f - 2*sqrt(3)*(e^2 - e*f + f^2))*
arctan(1/6*(3*f*x^2 - 6*(e - f)*x - sqrt(3)*((e + f)*x^2 + 2*(2*e - f)*x -
2*e + 4*f) + 6*e)*sqrt(x^3 - 1)*sqrt(3*e^2 - 6*e*f - 2*sqrt(3)*(e^2 - e*f +
 f^2))/((e^2 + 2*e*f - 2*f^2)*x^3 - e^2 - 2*e*f + 2*f^2))]
```

Sympy [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\int \frac{e}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx - \int \frac{fx}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

```
[In] integrate((f*x+e)/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)), x
) - Integral(f*x/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1
)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int -\frac{fx+e}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

```
[In] int((e + f*x)/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
[Out] \text{Hanged}
```

3.130 $\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2-\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(e-f*(1-3^(1
/2)))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(1/4)/(
-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)+arctanh((1+x)*(3+2*3^(1/2))^(1
/2)/(-x^3-1)^(1/2))*(e-f*(1+3^(1/2)))/(9+6*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2166, 225, 2165, 212}

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} + \frac{\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)\left(e-\left(1+\sqrt{3}\right)f\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

```
[In] Int[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((e - (1 + Sqrt[3])*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]
])/Sqrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 - Sqrt[3]]*(e - (1 - Sqrt[3])*f)*(1 +
x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] +
x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 + x)/(1 - Sqrt[3
] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
```

```
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = \frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int\frac{1}{\sqrt{-1-x^3}}\,dx}{2\sqrt{3}}+\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int\frac{\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)-6x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx}{12\sqrt{3}}$$

$$=\frac{\sqrt{2-\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\text{Subst}\left(\int\frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx,x,\frac{1+x}{\sqrt{-1-x^3}}\right)}{\sqrt{3}}$$

$$=\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2-\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.46 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.60

$$\int\frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$$

$$=\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3f\sqrt{-i+\sqrt{3}+2ix}\left((-2-i)-\sqrt{3}+\left((1+2i)+i\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)\right.\right.}{(3i+(1+2i)\sqrt{}}$$

```
[In] Integrate[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*f*Sqrt[-I + Sqrt[3] + (2*
I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqr
t[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] +
 2*(-(Sqrt[3]*e) + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x
+ x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sq
rt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I +
 (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.41

method	result
default	$-\frac{2 i f \sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}-\frac{2 i\left(e-f-f\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}\right.}}{}$
elliptic	$-\frac{2 i f \sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}-\frac{2 i\left(e-f-f\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}\right.}}{}$

```
[In] int((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-f-f*3^(1/2))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+3^(1/2)+1/2
*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 724, normalized size of antiderivative = 3.96

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \left[-\frac{1}{3}\left(\sqrt{3}(i\,e-i\,f)+3i\,f\right)\text{weierstrassPInverse}(0,-4,x)\right.$$
$$+\frac{1}{12}\sqrt{-3\,e^2-6\,ef+2\,\sqrt{3}(e^2+ef+f^2)}\log\left(-\frac{(e^2-2\,ef-2\,f^2)x^8-16\,(e^2-2\,ef-2\,f^2)x^7+112\,(e\right.$$
$$-\frac{1}{3}\left(\sqrt{3}(i\,e-i\,f)+3i\,f\right)\text{weierstrassPInverse}(0,-4,x)$$
$$-\frac{1}{6}\sqrt{3\,e^2+6\,ef-2\,\sqrt{3}(e^2+ef+f^2)}\arctan\left(\frac{(3\,fx^2-6\,(e+f)x+\sqrt{3}((e-f)x^2-2\,(2\,e+f)x-2\,e}{6\,((e^2-2\,ef-2\,f^2}\right.$$

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(sqrt(3)*(I*e - I*f) + 3*I*f)*weierstrassPInverse(0, -4, x) + 1/12*sq
rt(-3*e^2 - 6*e*f + 2*sqrt(3)*(e^2 + e*f + f^2))*log(-((e^2 - 2*e*f - 2*f^2
)*x^8 - 16*(e^2 - 2*e*f - 2*f^2)*x^7 + 112*(e^2 - 2*e*f - 2*f^2)*x^6 - 16*(
e^2 - 2*e*f - 2*f^2)*x^5 + 112*(e^2 - 2*e*f - 2*f^2)*x^4 + 224*(e^2 - 2*e*f
 - 2*f^2)*x^3 + 64*(e^2 - 2*e*f - 2*f^2)*x^2 - 4*((2*e + f)*x^6 - 18*(e + f
)*x^5 + 6*(7*e + 2*f)*x^4 - 8*(e + 5*f)*x^3 - 36*f*x^2 + 24*(e - f)*x + sqr
t(3)*((e + f)*x^6 - 6*(2*e + f)*x^5 + 6*(3*e + 4*f)*x^4 - 8*(2*e - f)*x^3 -
 12*(e - f)*x^2 + 24*f*x - 8*e + 16*f) + 8*e - 32*f)*sqrt(-x^3 - 1)*sqrt(-3
*e^2 - 6*e*f + 2*sqrt(3)*(e^2 + e*f + f^2)) + 112*e^2 - 224*e*f - 224*f^2 +
 128*(e^2 - 2*e*f - 2*f^2)*x - 16*sqrt(3)*((e^2 - 2*e*f - 2*f^2)*x^7 - 2*(e
^2 - 2*e*f - 2*f^2)*x^6 + 6*(e^2 - 2*e*f - 2*f^2)*x^5 + 5*(e^2 - 2*e*f - 2*
f^2)*x^4 + 2*(e^2 - 2*e*f - 2*f^2)*x^3 + 6*(e^2 - 2*e*f - 2*f^2)*x^2 + 4*e^
2 - 8*e*f - 8*f^2 + 4*(e^2 - 2*e*f - 2*f^2)*x))/(x^8 + 8*x^7 + 16*x^6 - 16*
x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16)), -1/3*(sqrt(3)*(I*e - I*f) + 3
*I*f)*weierstrassPInverse(0, -4, x) - 1/6*sqrt(3*e^2 + 6*e*f - 2*sqrt(3)*(e
^2 + e*f + f^2))*arctan(1/6*(3*f*x^2 - 6*(e + f)*x + sqrt(3)*((e - f)*x^2 -
 2*(2*e + f)*x - 2*e - 4*f) - 6*e)*sqrt(-x^3 - 1)*sqrt(3*e^2 + 6*e*f - 2*sq
rt(3)*(e^2 + e*f + f^2))/((e^2 - 2*e*f - 2*f^2)*x^3 + e^2 - 2*e*f - 2*f^2))
]
```

Sympy [F]

$$\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{e+fx}{\sqrt{-(x+1)(x^2-x+1)}\,(x+1+\sqrt{3})}\,dx$$

```
[In] integrate((f*x+e)/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
[Out] Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3-1}(x+\sqrt{3}+1)}\,dx$$

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

```
[In] int((e + f*x)/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.131 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 332

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}} - \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))*
(b^(1/3)*e-a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)-1/3*
(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*e-a^(1/3)*f*(1+3^(1/2)))*(1/2*6^
(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(1/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*
(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2166, 224, 2165, 212}

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx =$$

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

```
[In] Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] -(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1
/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt
[a]*b^(2/3))) - (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e - (1 + Sqrt[3])*a^(1/3)*f)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*
a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3ab\right)+6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx}{12\sqrt{3}a^{4/3}b^{4/3}} + \frac{\left(-6ab^{4/3}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3ab\right)f\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{b}\left(-28ab+\left(1-\sqrt{3}\right)^3ab\right)}$$

$$= -\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\text{Subst}\left(\int\frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt{3}b^{2/3}}$$

$$= -\frac{\left(\sqrt[3]{b}e-(1-\sqrt{3})\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}}$$

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e-(1+\sqrt{3})\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\ldots}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\ldots}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.49 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.37

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}if\left(\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3i-(1+2i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\mathrm{Elli}\ldots\right.}{\ldots}$$

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x
]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((I/2)*f*((-3 +
(2 + I)*Sqrt[3])*a^(1/3) + (3*I - (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I +
 Sqrt[3])*a^(1/3) - (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*El
lipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqr
t[3])*a^(1/3))]], (1 + I*Sqrt[3])/2] + I*(b^(1/3)*e + (-1 + Sqrt[3])*a^(1/3
)*f)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1
/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*S
qrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt
[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2
 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [F]

$$\int \frac{fx+e}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^{3}+a}}dx$$

```
[In] int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
[Out] int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 27.02 (sec) , antiderivative size = 7008, normalized size of antiderivative = 21.11

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Too large to display}$$

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algori
thm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{e+fx}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate((f*x+e)/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
[Out] Integral((e + f*x)/(sqrt(a + b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/
3)*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{fx+e}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algori
thm="maxima")
[Out] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),
x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algori
thm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{e+f\,x}{\sqrt{b\,x^3+a}\,\left(b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}-1\right)\right)}\,dx$$

```
[In] int((e + f*x)/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
[Out] int((e + f*x)/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))), x)
```

3.132 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 336

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{\left(\sqrt[3]{b}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}} + \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e+\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))*
(b^(1/3)*e+a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)+1/3*
(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+
a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*e+a^(1/3)*f*(1+3^(1/2)))*(1/2*
6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(1/3)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1
/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2166, 224, 2165, 212}

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$
$$=\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$
$$+\frac{\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

```
[In] Int[(e + f*x)/((((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] ((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6
)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt[a
]*b^(2/3)) + (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e + (1 + Sqrt[3])*a^(1/3)*f)*(a^(1
/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(
1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) -
b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = -\frac{\left(\sqrt[3]{b}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3ab\right)+6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx}{12\sqrt{3}a^{4/3}b^{4/3}} - \frac{\left(-6ab^{4/3}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3ab\right)f\right)\int\frac{1}{\sqrt{a-bx^3}}\,dx}{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{b}\left(28ab-\left(1-\sqrt{3}\right)^3ab\right)}$$

$$= \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e+\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$+\frac{\left(\sqrt[3]{b}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\text{Subst}\left(\int\frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx,x,\frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt{3}b^{2/3}}$$

$$= \frac{\left(\sqrt[3]{b}e + \left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}}$$

$$+ \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e + \left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.26 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.39

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\text{Ellipt}\right.}{}$$

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x
]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*(I*(-3 + (2
+ I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3
])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF
[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3
])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 - I*(b^(1/3)*e - (-1 + Sqrt[3])*a^(1/3
)*f)*Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*
a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[
(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 -
 I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/
((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Maple [F]

$$\int \frac{fx+e}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{-b\,x^{3}+a}}dx$$

```
[In] int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
[Out] int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 26.95 (sec) , antiderivative size = 7063, normalized size of antiderivative = 21.02

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algo
rithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\begin{aligned}\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx &= -\int \frac{e}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx \\ &\quad -\int \frac{fx}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx\end{aligned}$$

```
[In] integrate((f*x+e)/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
[Out] -Integral(e/(-a**(1/3)*sqrt(a - b*x**3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3)
 + b**(1/3)*x*sqrt(a - b*x**3)), x) - Integral(f*x/(-a**(1/3)*sqrt(a - b*x*
*3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{fx+e}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1)))
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{e+f\,x}{\sqrt{a-b\,x^3}\,\left(b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}-1\right)\right)}\,dx$$

```
[In] int(-(e + f*x)/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] int(-(e + f*x)/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))), x)
```

3.133 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 345

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{\left(\sqrt[3]{b}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}} + \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e+\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

```
[Out] 1/3*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3
)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*e+a^(1/3)*f*(1+3^(1/2)))*(
(a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)
^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(1/4)/a^(1/3)/b^(2/3)/(b*x^3-a)^(1/2)/(-
a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+arcta
n(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))*(b^(1/3
)*e+a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2166, 225, 2165, 209}

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

$$= \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$+\frac{\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

```
[In] Int[(e + f*x)/((((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] ((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)
*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt[a
]*b^(2/3)) + (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e + (1 + Sqrt[3])*a^(1/3)*f)*(a^(1
/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*a^(
1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3)
- b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt[3]{b}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3ab\right)-6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx}{12\sqrt{3}a^{4/3}b^{4/3}} - \frac{\left(6ab^{4/3}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3ab\right)f\right)\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{b}\left(-28ab+\left(1-\sqrt{3}\right)^3ab\right)}$$

$$= \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e+\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}} + \frac{\left(\sqrt[3]{b}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt{3}b^{2/3}}$$

$$= \frac{\left(\sqrt[3]{b}e + \left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}}$$

$$+ \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e + \left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\cdots}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.32 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.35

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\text{Ellipt}\cdots\right.}{\cdots}$$

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),
x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*(I*(-3 + (2
+ I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3
])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF
[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3
])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 - I*(b^(1/3)*e - (-1 + Sqrt[3])*a^(1/3
)*f)*Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*
a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[
(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 -
 I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/
((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Maple [F]

$$\int \frac{fx+e}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^{3}-a}}dx$$

[In] int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)

[Out] int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 26.48 (sec) , antiderivative size = 7009, normalized size of antiderivative = 20.32

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] Too large to include

Sympy [F]

$$\begin{aligned}\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx \\ = -\int \frac{e}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx \\ -\int \frac{fx}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx\end{aligned}$$

[In] integrate((f*x+e)/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)

[Out] -Integral(e/(-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)), x) - Integral(f*x/(-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)

Maxima [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),
 x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Hanged}$$

```
[In] int(-(e + f*x)/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

3.134 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 345

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$

$$= -\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}}$$

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] -1/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)
*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*e-a^(1/3)*f*(1+3^(1/2)))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(
1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(1/4)/a^(1/3)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-arctan(
a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2))*(b^(1/3)
*e-a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2166, 225, 2165, 209}

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx =$$

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$-\frac{\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

```
[In] Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] -(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/
6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt
[a]*b^(2/3))) - (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e - (1 + Sqrt[3])*a^(1/3)*f)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*
a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = -\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3ab\right)-6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx}{12\sqrt{3}a^{4/3}b^{4/3}} + \frac{\left(6ab^{4/3}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3ab\right)f\right)\int\frac{1}{\sqrt{-a-bx^3}}\,dx}{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{b}\left(28ab-\left(1-\sqrt{3}\right)^3ab\right)}$$

$$= -\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\ldots\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} - \frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\text{Subst}\left(\int\frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}b^{2/3}}$$

$$= -\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.44 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.33

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}if\left(\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3i-(1+2i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\text{Elli}\right.}{}$$

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),
x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((I/2)*f*((-3 +
(2 + I)*Sqrt[3])*a^(1/3) + (3*I - (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I +
 Sqrt[3])*a^(1/3) - (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*El
lipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqr
t[3])*a^(1/3))]], (1 + I*Sqrt[3])/2] + I*(b^(1/3)*e + (-1 + Sqrt[3])*a^(1/3
)*f)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1
/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*S
qrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt
[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2
 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F]

$$\int \frac{fx+e}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{-b\,x^{3}-a}}dx$$

```
[In] int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
[Out] int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 26.45 (sec) , antiderivative size = 7078, normalized size of antiderivative = 20.52

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algor
ithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{e+fx}{\sqrt{-a-bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate((f*x+e)/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
[Out] Integral((e + f*x)/(sqrt(-a - b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1
/3)*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algor
ithm="maxima")
[Out] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),
 x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Hanged}$$

```
[In] int((e + f*x)/((- a - b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

3.135 $\int \frac{x}{(1+\sqrt{3}+x)\sqrt{1+x^3}} dx$

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}} + \frac{\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right), -7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -1/3*arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))*2^(1/2)*3^(1/4)+1/3*(1
+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((x^2-x+1)
/(1+x+3^(1/2))^2)^(1/2)*3^(1/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2166, 224, 2165, 209}

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

```
[In] Int[x/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/3^(3/4)) +
(Sqrt[2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)
/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = \frac{\left(-1-\sqrt{3}\right)\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)}+\frac{\left(-22+\left(1+\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{-28+\left(1+\sqrt{3}\right)^3}$$

$$=\frac{\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}-\frac{\left(12\left(-1-\sqrt{3}\right)\right)\operatorname{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx,x,\frac{1+x}{\sqrt{1+x^3}}\right)}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)}$$

$$=-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}}+\frac{\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.54

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$

$$=\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\left(1+\sqrt{3}\right)\sqrt{1-x+x^2}\operatorname{EllipticPi}\left(\frac{2}{3+(\ldots}\right.}{3+(\ldots}\right.}{\sqrt{1+x^3}}$$

```
[In] Integrate[x/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
((2*I)*(1 + Sqrt[3])*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 +
 I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)
])/(3 + (2 + I)*Sqrt[3])))/Sqrt[1 + x^3]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(108) = 216$.

Time = 2.45 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.88

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(-1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{x-}}{3}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(-1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{x-}}{3}$

```
[In] int(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(-1-3^(1/2))*(3/2-1/2*I*3^(1/
2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1
/2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^
(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.43

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{1}{3}\left(\sqrt{3}-3\right)\text{weierstrassPInverse}(0,-4,x)-\frac{1}{6}\cdot 3^{\frac{1}{4}}\sqrt{2}\arctan\left(-\frac{3^{\frac{1}{4}}\sqrt{2}\left(3\,x^2-\sqrt{3}(x^2+2\,x+4)-6\,x\right)}{12\,\sqrt{x^3+1}}\right)$$

```
[In] integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(sqrt(3) - 3)*weierstrassPInverse(0, -4, x) - 1/6*3^(1/4)*sqrt(2)*arct
an(-1/12*3^(1/4)*sqrt(2)*(3*x^2 - sqrt(3)*(x^2 + 2*x + 4) - 6*x)/sqrt(x^3 +
 1))
```

Sympy [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

```
[In] integrate(x/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Giac [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

```
[In] int(x/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.136 $\int \frac{x}{(1+\sqrt{3}-x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 152

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}} + \frac{\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -1/3*arctan((1-x)*(3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/2))*2^(1/2)*3^(1/4)+1/3*(
1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((x^2+x+1
)/(1-x+3^(1/2))^2)^(1/2)*3^(1/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used

$= \{2166, 224, 2165, 209\}$

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$$
$$= \frac{\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$
$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}}$$

```
[In] Int[x/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/3^(3/4)) +
(Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(
1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 - x)
/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
```

```
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx}{6\left(3-\sqrt{3}\right)} - \frac{\left(22-\left(1+\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{1-x^3}}\,dx}{28-\left(1+\sqrt{3}\right)^3}$$

$$= \frac{\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)}{3-\sqrt{3}}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}} + \frac{\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.53

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$$

$$= \frac{2i\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{i\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(3i+(1+2i)\sqrt{3}+\left(3+(2+i)\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + 2(1+\sqrt{3})\sqrt{\right.}$$

$$\left(3+(2+i)\sqrt{3}\right)\sqrt{1-x^3}$$

```
[In] Integrate[x/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((I*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]*(3*I + (1 + 2*I)*Sqrt[3] + (3 + (2 + I)*Sqrt[3])*x)*Ellip
```

```
ticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + 2*(1 + Sqrt[3])*Sqrt[1 + x + x^2]*Ell
ipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((3 + (2 + I)*Sqrt[3])*Sqrt[1 - x^3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(124) = 248.

Time = 2.32 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.69

method	result
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}-\frac{2i\left(-1-\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-i\right.}}{}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}-\frac{2i\left(-1-\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-i\right.}}{}$

```
[In] int(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))-2/3*I*(-1-3^(1/2))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*
3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2-3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2-3^(1/2)+1/2*
I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.45

$$\begin{aligned}&\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^{3}}}\,dx\\&=-\frac{1}{3}\left(i\sqrt{3}-3i\right)\mathrm{weierstrassPInverse}(0,4,x)+\frac{1}{6}\\&\quad\cdot 3^{\frac{1}{4}}\sqrt{2}\arctan\left(-\frac{3^{\frac{1}{4}}\sqrt{2}\sqrt{-x^{3}+1}\left(3\,x^{2}-\sqrt{3}(x^{2}-2\,x+4)+6\,x\right)}{12\left(x^{3}-1\right)}\right)\end{aligned}$$

```
[In] integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(I*sqrt(3) - 3*I)*weierstrassPInverse(0, 4, x) + 1/6*3^(1/4)*sqrt(2)*a
rctan(-1/12*3^(1/4)*sqrt(2)*sqrt(-x^3 + 1)*(3*x^2 - sqrt(3)*(x^2 - 2*x + 4)
 + 6*x)/(x^3 - 1))
```

Sympy [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\int \frac{x}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

```
[In] integrate(x/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)), x
)
```

Maxima [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int -\frac{x}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

```
[In] integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Giac [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int -\frac{x}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

```
[In] integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

```
[In] int(x/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
[Out] \text{Hanged}
```

3.137 $\int \frac{x}{(1+\sqrt{3}-x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 164

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -1/3*arctanh((1-x)*(3+2*3^(1/2))^(1/2)/(x^3-1)^(1/2))*2^(1/2)*3^(1/4)+2/3*(
1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/3*6^(1/2)-1/2*
2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1
-x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used

$= \{2166, 225, 2165, 212\}$

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$
$$- \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{3^{3/4}}$$

```
[In] Int[x/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/3^(3/4))
+ (2*Sqrt[7/6 - 2/Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^
(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
```

```
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = -\frac{(-1-\sqrt{3})\int \frac{(1+\sqrt{3})\left(-22+(1+\sqrt{3})^3\right)-6x}{(1+\sqrt{3}-x)\sqrt{-1+x^3}}\,dx}{(1+\sqrt{3})\left(-28+(1+\sqrt{3})^3\right)} - \frac{\left(-22+(1+\sqrt{3})^3\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx}{-28+(1+\sqrt{3})^3}$$

$$= \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1-x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}\sqrt{-1+x^3}}$$

$$- \frac{(12(-1-\sqrt{3}))\,\text{Subst}\left(\int \frac{1}{1-(3+2\sqrt{3})x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)}{(1+\sqrt{3})\left(-28+(1+\sqrt{3})^3\right)}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1-x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.53 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.40

$$\int \frac{x}{(1+\sqrt{3}-x)\sqrt{-1+x^3}}\,dx$$

$$= \frac{2i\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{i\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(3i+(1+2i)\sqrt{3}+\left(3+(2+i)\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + 2(1+\sqrt{3})\sqrt{}\right.}{(3+(2+i)\sqrt{3})\sqrt{-1+x^3}}$$

```
[In] Integrate[x/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((I*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]*(3*I + (1 + 2*I)*Sqrt[3] + (3 + (2 + I)*Sqrt[3])*x)*Ellip
```

```
ticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + 2*(1 + Sqrt[3])*Sqrt[1 + x + x^2]*Ell
ipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-1 + x^3])
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.55

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\left(-1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-}{\frac{3}{2}-i}}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\left(-1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-}{\frac{3}{2}-i}}$

```
[In] int(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(-1-3^(1/2))*(-3/2-1/2*I*3^(1
/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3
^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1
/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3
^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.30

$$\int \frac{x}{(1+\sqrt{3}-x)\sqrt{-1+x^3}}\,dx = \frac{1}{3}\left(\sqrt{3}-3\right)\text{weierstrassPInverse}(0,4,x)+\frac{1}{12}\cdot 3^{\frac{1}{4}}\sqrt{2}\log\left(\frac{x^8+16x^7+112x^6+16x^5+112x^4-224x^3+2\cdot 3^{\frac{1}{4}}\sqrt{2}\left(x^6+18x^5+12x^4+40x^3-36x^2-\right.}{x^{}}\right.$$

```
[In] integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(3) - 3)*weierstrassPInverse(0, 4, x) + 1/12*3^(1/4)*sqrt(2)*log((
x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*x^4 - 224*x^3 + 2*3^(1/4)*sqrt(2)*(x^
6 + 18*x^5 + 12*x^4 + 40*x^3 - 36*x^2 + sqrt(3)*(x^6 + 6*x^5 + 24*x^4 - 8*x
^3 + 12*x^2 - 24*x + 16) + 24*x - 32)*sqrt(x^3 - 1) + 64*x^2 + 16*sqrt(3)*(
```

```
x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x - 4) - 128*x + 112)/(x^8
- 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16))
```

Sympy [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\int \frac{x}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

```
[In] integrate(x/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
[Out] -Integral(x/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)), x
)
```

Maxima [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int -\frac{x}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

```
[In] integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

```
[In] int(x/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
[Out] \text{Hanged}
```

3.138 $\int \frac{x}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 156

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] -1/3*arctanh((1+x)*(3+2*3^(1/2))^(1/2)/(-x^3-1)^(1/2))*2^(1/2)*3^(1/4)+2/3*
(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/3*6^(1/2)-1/2
*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/
(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used

$= \{2166, 225, 2165, 212\}$

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$
$$-\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{3^{3/4}}$$

```
[In] Int[x/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/3^(3/4))
+ (2*Sqrt[7/6 - 2/Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^
(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
```

```
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)-6x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx}{6\left(3-\sqrt{3}\right)} + \frac{\left(22-\left(1+\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx}{28-\left(1+\sqrt{3}\right)^3}$$

$$= \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{2\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)}{3-\sqrt{3}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.38 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.35

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{2i\left(1+\sqrt{3}\right)\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2}{3+(\ldots}\right.}{3+(\ldots}\right.}{\sqrt{-1-x^3}}$$

```
[In] Integrate[x/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
```

```
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
((2*I)*(1 + Sqrt[3])*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 +
 I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)
])/(3 + (2 + I)*Sqrt[3])))/Sqrt[-1 - x^3]
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.62

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}} - \frac{2i\left(-1-\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{}\right.}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}} - \frac{2i\left(-1-\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{}\right.}}{}$

```
[In] int(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))-2/3*I*(-1-3^(1/2))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^
(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1
/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+3^(1/2)+1/2*I*3^
(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.39

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = -\frac{1}{3}\left(-i\sqrt{3}+3i\right)\text{weierstrassPInverse}(0,-4,x)+\frac{1}{12}$$
$$\cdot 3^{\frac{1}{4}}\sqrt{2}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+2\cdot 3^{\frac{1}{4}}\sqrt{2}\left(x^6-18x^5+12x^4-40x^3-36x^2\right.}{}\right.$$

```
[In] integrate(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(-I*sqrt(3) + 3*I)*weierstrassPInverse(0, -4, x) + 1/12*3^(1/4)*sqrt(2
)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 + 224*x^3 + 2*3^(1/4)*sqrt
```

```
(2)*(x^6 - 18*x^5 + 12*x^4 - 40*x^3 - 36*x^2 + sqrt(3)*(x^6 - 6*x^5 + 24*x^
4 + 8*x^3 + 12*x^2 + 24*x + 16) - 24*x - 32)*sqrt(-x^3 - 1) + 64*x^2 - 16*s
qrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x + 4) + 128*x + 11
2)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16))
```

Sympy [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{x}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

```
[In] integrate(x/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
[Out] Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{x}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

```
[In] int(x/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.139 $\int \frac{x}{(1-\sqrt{3}+x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -1/3*arctanh((1+x)*(-3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))*2^(1/2)*3^(1/4)+2/3*
(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/3*6^(1/2)+1/2
*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1
+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used

= {2166, 224, 2165, 212}

$$\int \frac{x}{(1-\sqrt{3}+x)\sqrt{1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$
$$-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

```
[In] Int[x/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/3^(3/4))
+ (2*Sqrt[7/6 + 2/Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*
EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
```

```
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{(1-\sqrt{3})\left(-22+(1-\sqrt{3})^3\right)+6x}{(1-\sqrt{3}+x)\sqrt{1+x^3}}\,dx}{6(3+\sqrt{3})} + \frac{\left(-22+(1-\sqrt{3})^3\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{-28+(1-\sqrt{3})^3}$$

$$= \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}(1+x)\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}\sqrt{1+x^3}} - \frac{2\text{Subst}\left(\int \frac{1}{1+(3-2\sqrt{3})x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)}{3+\sqrt{3}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}(1+x)\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.49 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.53

$$\int \frac{x}{(1-\sqrt{3}+x)\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(\frac{\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(3-(2+i)\sqrt{3}+\left(-3i+(1+2i)\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} - 2(-1+\sqrt{3})\right.}{(-3i+(1+2i)\sqrt{3})\sqrt{1+x^3}}$$

```
[In] Integrate[x/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((Sqrt[((-1)^(1/3) - (-1)^(2/3)*x)/(1 + (
-1)^(1/3))]*(3 - (2 + I)*Sqrt[3] + (-3*I + (1 + 2*I)*Sqrt[3])*x)*EllipticF[
ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (
-1)^(2/3)*x)/(1 + (-1)^(1/3))] - 2*(-1 + Sqrt[3])*Sqrt[1 - x + x^2]*Ellipti
cPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[1 + x^3]
)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(116) = 232$.

Time = 2.45 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.72

method	result
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\sqrt{3}-1\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{3\cdots}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{3\cdots}$

```
[In] int(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(3^(1/2)-1)*(3/2-1/2*I*3^(1/2
))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(
1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/
2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^
(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.46

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{3}\left(\sqrt{3}+3\right)\text{weierstrassPInverse}(0,-4,x)+\frac{1}{12}$$
$$\cdot 3^{\frac{1}{4}}\sqrt{2}\log\left(\frac{x^8-16\,x^7+112\,x^6-16\,x^5+112\,x^4+224\,x^3+2\cdot 3^{\frac{1}{4}}\sqrt{2}\left(x^6-18\,x^5+12\,x^4-40\,x^3-36\,x^2\cdots\right.}{x\cdots}\right.$$

```
[In] integrate(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(3) + 3)*weierstrassPInverse(0, -4, x) + 1/12*3^(1/4)*sqrt(2)*log(
(x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 + 224*x^3 + 2*3^(1/4)*sqrt(2)*(x
^6 - 18*x^5 + 12*x^4 - 40*x^3 - 36*x^2 - sqrt(3)*(x^6 - 6*x^5 + 24*x^4 + 8*
x^3 + 12*x^2 + 24*x + 16) - 24*x - 32)*sqrt(x^3 + 1) + 64*x^2 + 16*sqrt(3)*
(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x + 4) + 128*x + 112)/(x^8
 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16))
```

Sympy [F]

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate(x/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)), x)
```

Maxima [F]

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Giac [F]

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

```
[In] int(x/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.140 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 278

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/3*arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/
2))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1
/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(
1/3*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2166, 224, 2165, 212}

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$
$$=\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqr
t[a + b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (2*Sqrt[7/6 + 2/Sqrt[3]]*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3ab\right)+6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx}{6\left(3+\sqrt{3}\right)ab^{4/3}} + \frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$= \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\left(3+\sqrt{3}\right)b^{2/3}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.28 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.60

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}i\left(\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3i-(1+2i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\text{Ellip}\right.}{}$$

```
[In] Integrate[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((I/2)*((-3 + (2
 + I)*Sqrt[3])*a^(1/3) + (3*I - (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + S
qrt[3])*a^(1/3) - (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Elli
pticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[
3])*a^(1/3))]], (1 + I*Sqrt[3])/2] + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-2*I)*
a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^
(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1
 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((
-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(
2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt
[a + b*x^3])
```

Maple [F]

$$\int \frac{x}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^{3}+a}}dx$$

```
[In] int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
[Out] int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.89 (sec) , antiderivative size = 1288, normalized size of antiderivative = 4.63

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="f
ricas")
[Out] [1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(sqrt(3)/a)*log((b^8*x^24 - 1840*a*b^7*x
^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 21401
60*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 - 2*sq
rt(2)*(26*a*b^7*x^21 - 4180*a^2*b^6*x^18 + 39552*a^3*b^5*x^15 + 10432*a^4*b
^4*x^12 + 271744*a^5*b^3*x^9 + 699648*a^6*b^2*x^6 + 284672*a^7*b*x^3 + 8192
*a^8 - (b^7*x^22 - 1160*a*b^6*x^19 + 23232*a^2*b^5*x^16 - 53920*a^3*b^4*x^1
3 - 148288*a^4*b^3*x^10 - 586752*a^5*b^2*x^7 - 496640*a^6*b*x^4 - 38912*a^7
*x - sqrt(3)*(b^7*x^22 - 632*a*b^6*x^19 + 14736*a^2*b^5*x^16 - 8416*a^3*b^4
*x^13 + 105920*a^4*b^3*x^10 + 334848*a^5*b^2*x^7 + 286720*a^6*b*x^4 + 22528
*a^7*x))*a^(2/3)*b^(1/3) - 12*(17*a*b^6*x^20 - 1014*a^2*b^5*x^17 + 2748*a^3
*b^4*x^14 - 9632*a^4*b^3*x^11 - 36096*a^5*b^2*x^8 - 53376*a^6*b*x^5 - 11008
*a^7*x^2 - 2*sqrt(3)*(5*a*b^6*x^20 - 285*a^2*b^5*x^17 + 1038*a^3*b^4*x^14 +
 784*a^4*b^3*x^11 + 11424*a^5*b^2*x^8 + 15168*a^6*b*x^5 + 3200*a^7*x^2))*a^
(1/3)*b^(2/3) - 2*sqrt(3)*(7*a*b^7*x^21 - 1250*a^2*b^6*x^18 + 9984*a^3*b^5*
x^15 - 19456*a^4*b^4*x^12 - 82624*a^5*b^3*x^9 - 193920*a^6*b^2*x^6 - 84992*
a^7*b*x^3 - 2048*a^8))*sqrt(b*x^3 + a)*sqrt(sqrt(3)/a) + 32*(9*b^7*x^22 - 8
46*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10
+ 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x - sqrt(3)*(5*b^7*x^22 -
505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10
 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(
3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 11
1360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 -
 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^
14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*
```

```
x^2))*a^(1/3)*b^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 254
4*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6
+ 19712*a^7*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18
 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a
^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)) + 4*b^(7/6)*(sqrt(3) + 3)*weierst
rassPInverse(0, -4*a/b, x))/b^2, 1/6*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(-sqrt(3)
/a)*arctan(1/12*(2*sqrt(2)*(sqrt(3)*x - 3*x)*a^(2/3)*b^(1/3)*sqrt(-sqrt(3)/
a) + sqrt(2)*(sqrt(3)*x^2 + 3*x^2)*a^(1/3)*b^(2/3)*sqrt(-sqrt(3)/a) + 4*sqr
t(3)*sqrt(2)*a*sqrt(-sqrt(3)/a))/sqrt(b*x^3 + a)) + 2*b^(7/6)*(sqrt(3) + 3)
*weierstrassPInverse(0, -4*a/b, x))/b^2]
```

Sympy [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate(x/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
[Out] Integral(x/(sqrt(a + b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)),
x)
```

Maxima [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="m
axima")
[Out] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="g
iac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Hanged}$$

```
[In] int(x/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
[Out] \text{Hanged}
```

3.141 $\int \frac{x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 286

$$\int \frac{x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] -1/3*arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1
/2))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+2/3*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^
(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I
)*(1/3*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2166, 224, 2165, 212}

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$
$$=\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$
$$-\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqr
t[a - b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (2*Sqrt[7/6 + 2/Sqrt[3]]*(a^(1/
3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)^2]*Sqrt[a - b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3 ab\right)+6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx}{6\left(3+\sqrt{3}\right)ab^{4/3}} - \frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{a-bx^3}}\,dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$= \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$- \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\left(3+\sqrt{3}\right)b^{2/3}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.08 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.59

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\text{Ellipt}\right.}{}$$

```
[In] Integrate[x/((((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((I*(-3 + (2 +
I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3])
*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF[A
rcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])
*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-I)*(2
*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 +
 (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3
)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt
[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[a - b*x^3])
```

Maple [F]

$$\int \frac{x}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{-b\,x^{3}+a}}dx$$

```
[In] int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
[Out] int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.82 (sec) , antiderivative size = 1354, normalized size of antiderivative = 4.73

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm=
"fricas")
[Out] [1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(sqrt(3)/a)*log((b^8*x^24 + 1840*a*b^7*x
^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 21401
60*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 + 32*(
9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 + 43776
*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x - sqrt(3)*
(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13 - 2868
8*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x))*a^(2/3)
*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 + 19200*a^3
*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^6*b*x^5 + 6
1440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b^5*x^17 - 1
520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 93184*a^6*b*x^
5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 2*sqrt(2)*((b^7*x^22 + 1160*a*b^6*x^1
9 + 23232*a^2*b^5*x^16 + 53920*a^3*b^4*x^13 - 148288*a^4*b^3*x^10 + 586752*
a^5*b^2*x^7 - 496640*a^6*b*x^4 + 38912*a^7*x - sqrt(3)*(b^7*x^22 + 632*a*b^
6*x^19 + 14736*a^2*b^5*x^16 + 8416*a^3*b^4*x^13 + 105920*a^4*b^3*x^10 - 334
848*a^5*b^2*x^7 + 286720*a^6*b*x^4 - 22528*a^7*x))*sqrt(-b*x^3 + a)*a^(2/3)
*b^(1/3) + 12*(17*a*b^6*x^20 + 1014*a^2*b^5*x^17 + 2748*a^3*b^4*x^14 + 9632
*a^4*b^3*x^11 - 36096*a^5*b^2*x^8 + 53376*a^6*b*x^5 - 11008*a^7*x^2 - 2*sqr
t(3)*(5*a*b^6*x^20 + 285*a^2*b^5*x^17 + 1038*a^3*b^4*x^14 - 784*a^4*b^3*x^1
1 + 11424*a^5*b^2*x^8 - 15168*a^6*b*x^5 + 3200*a^7*x^2))*sqrt(-b*x^3 + a)*a
^(1/3)*b^(2/3) + 2*(13*a*b^7*x^21 + 2090*a^2*b^6*x^18 + 19776*a^3*b^5*x^15
- 5216*a^4*b^4*x^12 + 135872*a^5*b^3*x^9 - 349824*a^6*b^2*x^6 + 142336*a^7*
b*x^3 - 4096*a^8 - sqrt(3)*(7*a*b^7*x^21 + 1250*a^2*b^6*x^18 + 9984*a^3*b^5
*x^15 + 19456*a^4*b^4*x^12 - 82624*a^5*b^3*x^9 + 193920*a^6*b^2*x^6 - 84992
```

```
*a^7*b*x^3 + 2048*a^8))*sqrt(-b*x^3 + a))*sqrt(sqrt(3)/a) - 32*sqrt(3)*(35*
a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39
520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8))/(b^8*x^24
 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*
x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8
)) + 4*sqrt(-b)*b^(2/3)*(sqrt(3) + 3)*weierstrassPInverse(0, 4*a/b, x))/b^2
, -1/6*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(-sqrt(3)/a)*arctan(-1/12*(2*sqrt(2)*sq
rt(-b*x^3 + a)*(sqrt(3)*x - 3*x)*a^(2/3)*b^(1/3)*sqrt(-sqrt(3)/a) - sqrt(2)
*sqrt(-b*x^3 + a)*(sqrt(3)*x^2 + 3*x^2)*a^(1/3)*b^(2/3)*sqrt(-sqrt(3)/a) -
4*sqrt(3)*sqrt(2)*sqrt(-b*x^3 + a)*a*sqrt(-sqrt(3)/a))/(b*x^3 - a)) - 2*sqr
t(-b)*b^(2/3)*(sqrt(3) + 3)*weierstrassPInverse(0, 4*a/b, x))/b^2]
```

Sympy [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$
$$=-\int \frac{x}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

```
[In] integrate(x/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
[Out] -Integral(x/(-a**(1/3)*sqrt(a - b*x**3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3)
 + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm=
"maxima")
[Out] -integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm=
"giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Hanged}$$

```
[In] int(-x/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
[Out] \text{Hanged}
```

3.142 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 282

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

```
[Out] -1/3*arctan(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2
))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+1/3*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*
2^(1/2)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)*3^(1/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*
x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {2166, 225, 2165, 209}

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt
[-a + b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2]*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqr
t[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[-((a^(1
/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a
+ b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3 ab\right)-6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx}{6\left(3+\sqrt{3}\right)ab^{4/3}} - \frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$= \frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$- \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\left(3+\sqrt{3}\right)b^{2/3}}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.02 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.61

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\text{Elliptic}\right.}{}$$

```
[In] Integrate[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((I*(-3 + (2 +
I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3])
*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF[A
rcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])
*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-I)*(2
*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 +
 (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3
)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt
[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[-a + b*x^3])
```

Maple [F]

$$\int \frac{x}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^{3}-a}}dx$$

[In] int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)

[Out] int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 1295, normalized size of antiderivative = 4.59

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] [1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(-sqrt(3)/a)*log((b^8*x^24 + 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x - sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^6*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 2*sqrt(b*x^3 - a)*(sqrt(2)*(b^7*x^22 + 1160*a*b^6*x^19 + 23232*a^2*b^5*x^16 + 53920*a^3*b^4*x^13 - 148288*a^4*b^3*x^10 + 586752*a^5*b^2*x^7 - 496640*a^6*b*x^4 + 38912*a^7*x - sqrt(3)*(b^7*x^22 + 632*a*b^6*x^19 + 14736*a^2*b^5*x^16 + 8416*a^3*b^4*x^13 + 105920*a^4*b^3*x^10 - 334848*a^5*b^2*x^7 + 286720*a^6*b*x^4 - 22528*a^7*x))*a^(2/3)*b^(1/3)*sqrt(-sqrt(3)/a) + 12*sqrt(2)*(17*a*b^6*x^20 + 1014*a^2*b^5*x^17 + 2748*a^3*b^4*x^14 + 9632*a^4*b^3*x^11 - 36096*a^5*b^2*x^8 + 53376*a^6*b*x^5 - 11008*a^7*x^2 - 2*sqrt(3)*(5*a*b^6*x^20 + 285*a^2*b^5*x^17 + 1038*a^3*b^4*x^14 - 784*a^4*b^3*x^11 + 11424*a^5*b^2*x^8 - 15168*a^6*b*x^5 + 3200*a^7*x^2))*a^(1/3)*b^(2/3)*sqrt(-sqrt(3)/a) + 2*sqrt(2)*(13*a*b^7*x^21 + 2090*a^2*b^6*x^18 + 19776*a^3*b^5*x^15 - 5216*a^4*b^4*x^12 + 135872*a^5*b^

```
3*x^9 - 349824*a^6*b^2*x^6 + 142336*a^7*b*x^3 - 4096*a^8 - sqrt(3)*(7*a*b^7
*x^21 + 1250*a^2*b^6*x^18 + 9984*a^3*b^5*x^15 + 19456*a^4*b^4*x^12 - 82624*
a^5*b^3*x^9 + 193920*a^6*b^2*x^6 - 84992*a^7*b*x^3 + 2048*a^8))*sqrt(-sqrt(
3)/a)))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15
+ 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7
*b*x^3 + 4096*a^8)) - 4*b^(7/6)*(sqrt(3) + 3)*weierstrassPInverse(0, 4*a/b,
 x))/b^2, -1/6*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(sqrt(3)/a)*arctan(-1/12*sqrt(2
)*(2*(sqrt(3)*x - 3*x)*a^(2/3)*b^(1/3) - (sqrt(3)*x^2 + 3*x^2)*a^(1/3)*b^(2
/3) - 4*sqrt(3)*a)*sqrt(sqrt(3)/a)/sqrt(b*x^3 - a)) + 2*b^(7/6)*(sqrt(3) +
3)*weierstrassPInverse(0, 4*a/b, x))/b^2]
```

Sympy [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$
$$=-\int \frac{x}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

```
[In] integrate(x/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x**
3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="
maxima")
```

```
[Out] -integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="
giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Hanged}$$

```
[In] int(-x/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
[Out] \text{Hanged}
```

3.143 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 278

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] -1/3*arctan(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/
2))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+1/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2
^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/
2)))^2)^(1/2)*3^(1/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x
)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {2166, 225, 2165, 209}

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$
$$=\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt
[-a - b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2]*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[-((a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a
- b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3 ab\right)-6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx}{6\left(3+\sqrt{3}\right)ab^{4/3}}+\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$= \frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$-\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\left(3+\sqrt{3}\right)b^{2/3}}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.24 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.61

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}i\left(\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3i-(1+2i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\mathrm{Elli}\right.}{}$$

```
[In] Integrate[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((I/2)*((-3 + (2
 + I)*Sqrt[3])*a^(1/3) + (3*I - (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + S
qrt[3])*a^(1/3) - (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Elli
pticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[
3])*a^(1/3))]], (1 + I*Sqrt[3])/2] + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-2*I)*
a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^
(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1
 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((
-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(
2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt
[-a - b*x^3])
```

Maple [F]

$$\int \frac{x}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{-b\,x^{3}-a}}dx$$

[In] int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)

[Out] int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 1348, normalized size of antiderivative = 4.85

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Too large to display}$$

[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] [1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(-sqrt(3)/a)*log((b^8*x^24 - 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 - 4*sqrt(2)*(13*a*b^7*x^21 - 2090*a^2*b^6*x^18 + 19776*a^3*b^5*x^15 + 5216*a^4*b^4*x^12 + 135872*a^5*b^3*x^9 + 349824*a^6*b^2*x^6 + 142336*a^7*b*x^3 + 4096*a^8 - sqrt(3)*(7*a*b^7*x^21 - 1250*a^2*b^6*x^18 + 9984*a^3*b^5*x^15 - 19456*a^4*b^4*x^12 - 82624*a^5*b^3*x^9 - 193920*a^6*b^2*x^6 - 84992*a^7*b*x^3 - 2048*a^8))*sqrt(-b*x^3 - a)*sqrt(-sqrt(3)/a) + 2*(144*b^7*x^22 - 13536*a*b^6*x^19 + 73872*a^2*b^5*x^16 + 87552*a^3*b^4*x^13 + 700416*a^4*b^3*x^10 + 1575936*a^5*b^2*x^7 + 949248*a^6*b*x^4 + 73728*a^7*x + sqrt(2)*(b^7*x^22 - 1160*a*b^6*x^19 + 23232*a^2*b^5*x^16 - 53920*a^3*b^4*x^13 - 148288*a^4*b^3*x^10 - 586752*a^5*b^2*x^7 - 496640*a^6*b*x^4 - 38912*a^7*x - sqrt(3)*(b^7*x^22 - 632*a*b^6*x^19 + 14736*a^2*b^5*x^16 - 8416*a^3*b^4*x^13 + 105920*a^4*b^3*x^10 + 334848*a^5*b^2*x^7 + 286720*a^6*b*x^4 + 22528*a^7*x))*sqrt(-b*x^3 - a)*sqrt(-sqrt(3)/a) - 16*sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 - 3*sqrt(2)*(17*a*b^6*x^20 - 1014*a^2*b^5*x^17 + 2748*a^3*b^4*x^14 - 9632*a^4*b^3*x^11 - 36096*a^5*b^2*x^8 - 53376*a^6*b*x^5 - 11008*a^7*x^2 - 2*sqrt(3)*(5*a*b^6*x^20 - 285*a^2*b^5*x^17 + 1038*a^3*b^4*x^14 + 784*a^4*b^3*x^11 + 11424*a^5*b^2*x^8 + 15168*a^6*b*x^5 + 3200*a^7*x^2))*sqrt(-b*x^3 - a)*sqrt(-sqrt(3)/a) - 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*

```
a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/
3)*b^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x
^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7
*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^
3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6
- 40960*a^7*b*x^3 + 4096*a^8)) - 4*sqrt(-b)*b^(2/3)*(sqrt(3) + 3)*weierstra
ssPInverse(0, -4*a/b, x))/b^2, 1/6*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(sqrt(3)/a)
*arctan(1/12*sqrt(2)*(2*sqrt(-b*x^3 - a)*(sqrt(3)*x - 3*x)*a^(2/3)*b^(1/3)
+ sqrt(-b*x^3 - a)*(sqrt(3)*x^2 + 3*x^2)*a^(1/3)*b^(2/3) + 4*sqrt(3)*sqrt(-
b*x^3 - a)*a)*sqrt(sqrt(3)/a)/(b*x^3 + a)) - 2*sqrt(-b)*b^(2/3)*(sqrt(3) +
3)*weierstrassPInverse(0, -4*a/b, x))/b^2]
```

Sympy [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x}{\sqrt{-a-bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

```
[In] integrate(x/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
[Out] Integral(x/(sqrt(-a - b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)),
 x)
```

Maxima [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="
maxima")
[Out] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="
giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Hanged}$$

```
[In] int(x/((- a - b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
[Out] \text{Hanged}
```

3.144 $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 317

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = -\frac{\left(c-\left(1+\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -(1+x)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)
/d^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*(c-d*(1+3^(1/2)))*((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(x^3+1)^(1/2)/
((1+x)/(1+x+3^(1/2))^2)^(1/2)-4*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2))/(1+
x+3^(1/2)),(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(1/2*6^(1
/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c-d*(1-3^(1/2)))/(x^3+1
)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2167, 2138, 551, 585, 95, 211}

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1-\sqrt{3}\right)d\right)} - \frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\arctan\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

```
[In] Int[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(((c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ar
cTan[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]
*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sq
rt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])) + (4*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*E
llipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, ArcSin[(1 - Sqrt
[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c - (1 - Sqrt[3])*d)*Sqrt[(
1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

integral

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-c+\left(1-\sqrt{3}\right)d+\left(-c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx, x, \frac{-1+\sqrt{3}-}{1+\sqrt{3}+}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(-c+d+\sqrt{3}d)\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-c+\left(1-\sqrt{3}\right)d\right)^2-\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$+\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(-c+\left(1-\sqrt{3}\right)d)\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-c+\left(1-\sqrt{3}\right)d\right.\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$=\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$-\frac{\left(2\sqrt[4]{3}\sqrt{2-\sqrt{3}}(-c+d+\sqrt{3}d)\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(-c+\left(1-\sqrt{3}\right)d\right)^2-\left(\right.\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$=\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(-c+d+\sqrt{3}d)\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{}{-\left(-c+\left(1-\sqrt{3}\right)d\right)^2+\left(-c+\left(1+\sqrt{3}\right)d\right)^2-}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$=-\frac{\left(c-d-\sqrt{3}d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.49 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.68

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i\left(c-\left(1+\sqrt{3}\right)d\right)\sqrt{1-x+x^2}\,\text{EllipticPi}\ldots}{\ldots}\right)}{d\sqrt{1+x^3}}$$

```
[In] Integrate[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*(c - (1 + Sqrt[3])*d)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-
1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]
)/(c + (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.87

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}+\frac{2\left(d\sqrt{3}-c+d\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\ldots}{d^2\sqrt{x^3+\ldots}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}+\frac{2\left(d\sqrt{3}-c+d\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\ldots}{d^2\sqrt{x^3+\ldots}}$

```
[In] int((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(d*3^(1/2)-c+d)/d^2*(3/2-1/2*
I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3+1)^(1/2)/(-1+c/d)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*
I*3^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 + 1)*(x + sqrt(3) + 1)/(d*x^4 + c*x^3 + d*x + c), x)
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{x+1+\sqrt{3}}{\sqrt{(x+1)\,(x^2-x+1)}\,(c+dx)}\,dx$$

```
[In] integrate((1+x+3**(1/2))/(d*x+c)/(x**3+1)**(1/2),x)
[Out] Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

```
[In] int((x + 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

3.145 $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 329

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\frac{\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2},\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -(1-x)*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/d^(1/2)/(c
+d)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*(c+d+d*3^(1/2))*((x^2+x+1)/(1-
x+3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(-x^3+1)^(1/2)/
((1-x)/(1-x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1-x)*EllipticPi((-1+x+3^(1/2))/(1-
x+3^(1/2)),(c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c+d-d*3^(1/2))/(-x^3+1)^(1/
2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2167, 2138, 551, 585, 95, 214}

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2},\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c-\sqrt{3}d+d\right)} - \frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\text{arctanh}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

```
[In] Int[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcT
anh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt
[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])) - (4*3
^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*El
lipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, ArcSin[(1 - Sqrt[3] -
 x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c + d - Sqrt[3]*d)*Sqrt[(1 - x)/
(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

integral

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\operatorname{Subst}\left(\int \frac{1}{\left(c+\left(1-\sqrt{3}\right)d+\left(c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx, x, \frac{-1+\sqrt{3}+x}{1+\sqrt{3}-x}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(c+\left(1-\sqrt{3}\right)d\right)^2-\left(c+\cdots\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(c+\left(1-\sqrt{3}\right)d\right)^2-\left(\cdots\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$=-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{\left(2\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(c+\left(1-\sqrt{3}\right)d\right)^2-\left(c+\cdots\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$=-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{\cdots}{-\left(c+\left(1-\sqrt{3}\right)d\right)^2+\left(c+\left(1+\sqrt{3}\right)d\right)^2-\left(\left(c\cdots\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$=-\frac{\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.57 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$$

$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c+\left(3+\sqrt{3}\right)d\right)\cdots}{\cdots}\right)}{3d\sqrt{1-x^3}}$$

```
[In] Integrate[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (3 + Sqrt[3])*d)*Sqrt[1 + x + x^2
]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^3
])
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.80

method	result
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^3+1}}-\frac{2i\left(c+d+d\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\cdots\right.}}{\cdots}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^3+1}}-\frac{2i\left(c+d+d\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\cdots\right.}}{\cdots}$

```
[In] int((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))-2/3*I*(c+d+d*3^(1/2))/d^2*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*
```

```
3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3
^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \text{Timed out}$$

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\int \left(-\frac{\sqrt{3}}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\right)dx - \int \frac{x}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\,dx - \int \left(-\frac{1}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\right)dx$$

```
[In] integrate((1-x+3**(1/2))/(d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(-sqrt(3)/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(x
/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(-1/(c*sqrt(1 - x**3
) + d*x*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

```
[In] int((3^(1/2) - x + 1)/((1 - x^3)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

3.146 $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 325

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = -\frac{\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2},\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -(1-x)*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/d^(1/2)/(c
+d)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*(c+d+d*3^(1/2))*((x^2+x+1)/(1-
x+3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(x^3-1)^(1/2)/(
(1-x)/(1-x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1-x)*EllipticPi((-1+x+3^(1/2))/(1-x
+3^(1/2)),(c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1
/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c+d-d*3^(1/2))/(x^3-1)^(1/2)
/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2167, 2138, 551, 585, 95, 214}

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2},\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c-\sqrt{3}d+d\right)} - \frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\text{arctanh}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

```
[In] Int[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcT
anh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt
[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])) - (4*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*E
llipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, ArcSin[(1 - Sqrt[3]
- x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c + d - Sqrt[3]*d)*Sqrt[(1 - x)
/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

integral

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(c+\left(1-\sqrt{3}\right)d+\left(c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx, x, \frac{-1+\sqrt{3}+x}{1+\sqrt{3}-x}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(c+d-\sqrt{3}d)\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(c+\left(1-\sqrt{3}\right)d\right)^2-\left(c+\left(1\right.\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(c+d+\sqrt{3}d)\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(c+\left(1-\sqrt{3}\right)d\right)^2-\left(c+\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$-\frac{\left(2\sqrt[4]{3}\sqrt{2-\sqrt{3}}(c+d+\sqrt{3}d)\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(c+\left(1-\sqrt{3}\right)d\right)^2-\left(c+\left(\right.\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(c+d+\sqrt{3}d)\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{}{-\left(c+\left(1-\sqrt{3}\right)d\right)^2+\left(c+\left(1+\sqrt{3}\right)d\right)^2-\left(\left(c+\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.72

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c+\left(3+\sqrt{3}\right.\right.}{}\right.}{3d\sqrt{-1+x^3}}$$

```
[In] Integrate[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (3 + Sqrt[3])*d)*Sqrt[1 + x + x^2
]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x^
3])
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.84

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}+\frac{2\left(c+d+d\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}+\frac{2\left(c+d+d\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$

```
[In] int((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(c+d+d*3^(1/2))/d^2*(-3/2-1/2
*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-
1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3-1)^(1/2)/(1+c/d)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I
*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \text{Timed out}$$

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = -\int \left(-\frac{\sqrt{3}}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right) dx - \int \frac{x}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx - \int \left(-\frac{1}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right) dx$$

```
[In] integrate((1-x+3**(1/2))/(d*x+c)/(x**3-1)**(1/2),x)
[Out] -Integral(-sqrt(3)/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(x
/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(-1/(c*sqrt(x**3 - 1
) + d*x*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

```
[In] int((3^(1/2) - x + 1)/((x^3 - 1)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

3.147 $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 321

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = -\frac{\left(c-\left(1+\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] -(1+x)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)
/d^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*(c-d*(1+3^(1/2)))*((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(-x^3-1)^(1/2)
/((1+x)/(1+x+3^(1/2))^2)^(1/2)-4*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2))/(1
+x+3^(1/2)),(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c-d*(1-3^(1/2)))/(-x^3
-1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2167, 2138, 551, 585, 95, 211}

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$$
$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1-\sqrt{3}\right)d\right)}$$
$$-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\arctan\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

```
[In] Int[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] -(((c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ar
cTan[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]
*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sq
rt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])) + (4
*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*
EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, ArcSin[(1 - Sqr
t[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c - (1 - Sqrt[3])*d)*Sqrt[
(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule steps

integral

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-c+\left(1-\sqrt{3}\right)d+\left(-c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx, x, \frac{-1+\sqrt{3}-x}{1+\sqrt{3}+x}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= -\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(-c+d+\sqrt{3}d)\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-c+\left(1-\sqrt{3}\right)d\right)\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(-c+\left(1-\sqrt{3}\right)d)\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-c+\left(1-\sqrt{3}\right.\right.\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$-\frac{\left(2\sqrt[4]{3}\sqrt{2-\sqrt{3}}(-c+d+\sqrt{3}d)\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(-c+\left(1-\sqrt{3}\right)d\right)^2-\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(-c+d+\sqrt{3}d)\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{}{-\left(-c+\left(1-\sqrt{3}\right)d\right)^2+\left(-c+\left(1+\sqrt{3}\right)d\right)^2}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= -\frac{\left(c-d-\sqrt{3}d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.73

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c-\left(3+\sqrt{3}\right)d\right.\cdots}{\cdots}\right.\cdots}{3d\sqrt{-1-x^3}}$$

```
[In] Integrate[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c - (3 + Sqrt[3])*d)*Sqrt[1 - x + x^2
]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1/3)*d)))/(3*d*Sqrt[-1 - x^3
])
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.83

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^3-1}}-\frac{2i\left(d\sqrt{3}-c+d\right)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\cdots\right.}}{\cdots}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^3-1}}-\frac{2i\left(d\sqrt{3}-c+d\right)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\cdots\right.}}{\cdots}$

```
[In] int((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*I*(d*3^(1/2)-c+d)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^
```

```
(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/
2)+c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x+1+\sqrt{3}}{\sqrt{-(x+1)(x^2-x+1)}\,(c+dx)}\,dx$$

```
[In] integrate((1+x+3**(1/2))/(d*x+c)/(-x**3-1)**(1/2),x)
[Out] Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

```
[In] int((x + 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

3.148 $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 358

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = -\frac{\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\mathrm{arctanh}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{7+4\sqrt{3}+\frac{\left(1+\sqrt{3}+x\right)^2}{\left(1-\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\mathrm{EllipticPi}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -(1+x)*arctanh(2*(c^2+c*d+d^2)^(1/2)*((-1-x)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6^
(1/2)+1/2*2^(1/2))/(c-d)^(1/2)/d^(1/2)/(7+4*3^(1/2)+(1+x+3^(1/2))^2/(1+x-3^
(1/2))^2)^(1/2))*(c-d*(1-3^(1/2)))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)/(c-d)^
(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(x^3+1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1
/2)+4*3^(1/4)*(1+x)*EllipticPi((-1-x-3^(1/2))/(1+x-3^(1/2)),(c-d*(1-3^(1/2)
))^2/(c-d*(1+3^(1/2)))^2,2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1
)/(1+x-3^(1/2))^2)^(1/2)/(-d*3^(1/2)+c-d)/(x^3+1)^(1/2)/((-1-x)/(1+x-3^(1/2
))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2168, 2138, 551, 585, 95, 214}

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\sqrt{3}d-d\right)} - \frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\text{arctanh}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

```
[In] Int[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(((c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ar
cTanh[(2*Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1 + x)/(1 - Sqrt[3
] + x)^2)])/(Sqrt[c - d]*Sqrt[d]*Sqrt[7 + 4*Sqrt[3] + (1 + Sqrt[3] + x)^2/(
1 - Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1
 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[1 + x^3])) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*
(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticPi[(c - (1 - Sqrt[3
])*d)^2/(c - (1 + Sqrt[3])*d)^2, ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)
], -7 + 4*Sqrt[3]])/((c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2
)]*Sqrt[1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2168

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2])), Subst[Int[1/(((1
 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*S
qrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; Fr
eeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sqr
t[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

integral =

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\left(-c+\left(1+\sqrt{3}\right)d+\left(-c+\left(1-\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}}\,dx,x,\frac{1+\sqrt{}}{-1+\sqrt{}}\right)}{\sqrt{\frac{-1-x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(-c+d+\sqrt{3}d)\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}\left(\left(-c+\left(1+\sqrt{3}\right)d\right)^2-\right.}\right.}{\sqrt{\frac{-1-x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$+\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(-c+\left(1-\sqrt{3}\right)d)\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{x}{\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}\left(\left(-c+\left(1+\sqrt{3}\right)d\right.\right.}\right.}{\sqrt{\frac{-1-x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$+\frac{\left(2\sqrt[4]{3}\sqrt{2+\sqrt{3}}(-c+\left(1-\sqrt{3}\right)d)\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7+4\sqrt{3}+x}\left(\left(-c+\left(1+\sqrt{3}\right)d\right)^2\right.}\right.}{\sqrt{\frac{-1-x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$+\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(-c+\left(1-\sqrt{3}\right)d)\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{}{\left(-c+\left(1-\sqrt{3}\right)d\right)^2-\left(-c+\left(1+\sqrt{3}\right)d\right)^2}\right.}{\sqrt{\frac{-1-x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{7+4\sqrt{3}+\frac{\left(1+\sqrt{3}+x\right)^2}{\left(1-\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.52 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.59

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\mathrm{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i\left(c+\left(-1+\sqrt{3}\right)d\right)\sqrt{1-x+x^2}\,\mathrm{EllipticI}}{}\right.}{d\sqrt{1+x^3}}$$

```
[In] Integrate[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*(c + (-1 + Sqrt[3])*d)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (
-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)
])/(c + (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.77

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}-\frac{2\left(d\sqrt{3}+c-d\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{d^2\sqrt{x^3+}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}-\frac{2\left(d\sqrt{3}+c-d\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{d^2\sqrt{x^3+}}$

```
[In] int((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(d*3^(1/2)+c-d)/d^2*(3/2-1/2*
I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3+1)^(1/2)/(-1+c/d)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*
I*3^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 + 1)*(x - sqrt(3) + 1)/(d*x^4 + c*x^3 + d*x + c), x)
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{(x+1)\,(x^2-x+1)}\,(c+dx)}\,dx$$

```
[In] integrate((1+x-3**(1/2))/(d*x+c)/(x**3+1)**(1/2),x)
[Out] Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

```
[In] int((x - 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

3.149 $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 346

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}} \, dx = -\frac{\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\arctan\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2},\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -(1-x)*arctan((c^2-c*d+d^2)^(1/2)*((-1+x)/(1-x-3^(1/2))^2)^(1/2)/d^(1/2)/(c
+d)^(1/2)/((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2))*(c+d-d*3^(1/2))*((x^2+x+1)/(1-
x-3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(-x^3+1)^(1/2)/
((-1+x)/(1-x-3^(1/2))^2)^(1/2)-4*3^(1/4)*(1-x)*EllipticPi((-1+x-3^(1/2))/(1
-x-3^(1/2)),(c+d-d*3^(1/2))^2/(c+d+d*3^(1/2))^2,2*I-I*3^(1/2))*(1/2*6^(1/2)
-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)/(c+d+d*3^(1/2))/(-x^3+1)^(1
/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2168, 2138, 551, 585, 95, 211}

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$$
$$= \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2},\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$
$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\arctan\left(\frac{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

```
[In] Int[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*ArcT
an[(Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)])/(Sqrt[d]*Sq
rt[c + d]*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*S
qrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])) +
 (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^
2]*EllipticPi[(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, ArcSin[(1 + Sqrt
[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((c + d + Sqrt[3]*d)*Sqrt[-((
1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2168

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2])), Subst[Int[1/(((1
 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*S
qrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; Fr
eeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sqr
t[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

integral =

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\left(c+\left(1+\sqrt{3}\right)d+\left(c+\left(1-\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}}\,dx,x,\frac{1+\sqrt{3}-x}{-1+\sqrt{3}+x}}{\sqrt{\frac{-1+x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$=\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{x}{\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}\left(\left(c+\left(1+\sqrt{3}\right)d\right)^2-\left(c+\right.\right.}\right.}{\sqrt{\frac{-1+x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}\left(\left(c+\left(1+\sqrt{3}\right)d\right)^2-\left(\right.\right.}\right.}{\sqrt{\frac{-1+x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$=\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$+\frac{\left(2\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7+4\sqrt{3}+x}\left(\left(c+\left(1+\sqrt{3}\right)d\right)^2-\left(c+\right.\right.}\right.}{\sqrt{\frac{-1+x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$=\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$+\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{}{\left(c+\left(1-\sqrt{3}\right)d\right)^2-\left(c+\left(1+\sqrt{3}\right)d\right)^2-\left(\left(7+\right.\right.}\right.}{\sqrt{\frac{-1+x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$=-\frac{\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.56 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.68

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$$

$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c+\left(-3+\sqrt{3}\right)\right.}{}\right.}{3d\sqrt{1-x^3}}$$

```
[In] Integrate[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (-3 + Sqrt[3])*d)*Sqrt[1 + x + x^
2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^
3])
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.77

method	result
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^3+1}}+\frac{2i\left(d\sqrt{3}-c-d\right)\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\right.}}{}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^3+1}}+\frac{2i\left(d\sqrt{3}-c-d\right)\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\right.}}{}$

```
[In] int((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))+2/3*I*(d*3^(1/2)-c-d)/d^2*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*
```

```
3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3
^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \text{Timed out}$$

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\int \frac{\sqrt{3}}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\,dx - \int \frac{x}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\,dx - \int \left(-\frac{1}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\right)dx$$

```
[In] integrate((1-x-3**(1/2))/(d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(sqrt(3)/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(x/
(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(-1/(c*sqrt(1 - x**3)
 + d*x*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx=\int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx=\text{Hanged}$$

```
[In] int(-(x + 3^(1/2) - 1)/((1 - x^3)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

3.150 $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 342

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = -\frac{\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\arctan\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2},\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -(1-x)*arctan((c^2-c*d+d^2)^(1/2)*((-1+x)/(1-x-3^(1/2))^2)^(1/2)/d^(1/2)/(c
+d)^(1/2)/((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2))*(c+d-d*3^(1/2))*((x^2+x+1)/(1-
x-3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(x^3-1)^(1/2)/(
(-1+x)/(1-x-3^(1/2))^2)^(1/2)-4*3^(1/4)*(1-x)*EllipticPi((-1+x-3^(1/2))/(1-
x-3^(1/2)),(c+d-d*3^(1/2))^2/(c+d+d*3^(1/2))^2,2*I-I*3^(1/2))*(1/2*6^(1/2)-
1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)/(c+d+d*3^(1/2))/(x^3-1)^(1/2
)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2168, 2138, 551, 585, 95, 211}

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2},\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)} - \frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\arctan\left(\frac{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

```
[In] Int[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*ArcT
an[(Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)])/(Sqrt[d]*Sq
rt[c + d]*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*S
qrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]))
+ (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)
^2]*EllipticPi[(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, ArcSin[(1 + Sqr
t[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((c + d + Sqrt[3]*d)*Sqrt[-(
(1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2168

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2])), Subst[Int[1/(((1
 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*S
qrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; Fr
eeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sqr
t[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

integral =

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\left(c+\left(1+\sqrt{3}\right)d+\left(c+\left(1-\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}}\,dx,x,\frac{1+\sqrt{3}-}{-1+\sqrt{3}+}\right.}{\sqrt{\frac{-1+x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(c+d-\sqrt{3}d)\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}\left(\left(c+\left(1+\sqrt{3}\right)d\right)^2-\left(c+\left(1\right.\right.\right.}\right.}{\sqrt{\frac{-1+x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$- \frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(c+d+\sqrt{3}d)\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}\left(\left(c+\left(1+\sqrt{3}\right)d\right)^2-\left(c+\right.\right.}\right.}{\sqrt{\frac{-1+x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$+ \frac{\left(2\sqrt[4]{3}\sqrt{2+\sqrt{3}}(c+d-\sqrt{3}d)\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{7+4\sqrt{3}+x}\left(\left(c+\left(1+\sqrt{3}\right)d\right)^2-\left(c+\left(\right.\right.\right.}\right.}{\sqrt{\frac{-1+x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$+ \frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(c+d-\sqrt{3}d)\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(c+\left(1-\sqrt{3}\right)d\right)^2-\left(c+\left(1+\sqrt{3}\right)d\right)^2-\left(\left(7+4\right.\right.}\right.}{\sqrt{\frac{-1+x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$+ \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.68

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c+\left(-3+\sqrt{}\right.\right.}{}\right.}{3d\sqrt{-1+x^3}}$$

```
[In] Integrate[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (-3 + Sqrt[3])*d)*Sqrt[1 + x + x^
2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x
^3])
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.81

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}-\frac{2\left(d\sqrt{3}-c-d\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{}}{}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}-\frac{2\left(d\sqrt{3}-c-d\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{}}{}$

```
[In] int((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*(d*3^(1/2)-c-d)/d^2*(-3/2-1/2
*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-
1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3-1)^(1/2)/(1+c/d)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I
*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \text{Timed out}$$

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = -\int \frac{\sqrt{3}}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx - \int \frac{x}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx - \int \left(-\frac{1}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx$$

```
[In] integrate((1-x-3**(1/2))/(d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(sqrt(3)/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(x/
(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(-1/(c*sqrt(x**3 - 1)
 + d*x*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

```
[In] int(-(x + 3^(1/2) - 1)/((x^3 - 1)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

3.151 $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 362

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = -\frac{\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{arctanh}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{7+4\sqrt{3}+\frac{\left(1+\sqrt{3}+x\right)^2}{\left(1-\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] -(1+x)*arctanh(2*(c^2+c*d+d^2)^(1/2)*((-1-x)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6^
(1/2)+1/2*2^(1/2))/(c-d)^(1/2)/d^(1/2)/(7+4*3^(1/2)+(1+x+3^(1/2))^2/(1+x-3^
(1/2))^2)^(1/2))*(c-d*(1-3^(1/2)))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)/(c-d)^
(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(
1/2)+4*3^(1/4)*(1+x)*EllipticPi((-1-x-3^(1/2))/(1+x-3^(1/2)),(c-d*(1-3^(1/2
)))^2/(c-d*(1+3^(1/2)))^2,2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+
1)/(1+x-3^(1/2))^2)^(1/2)/(-d*3^(1/2)+c-d)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1
/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2168, 2138, 551, 585, 95, 214}

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\sqrt{3}d-d\right)} - \frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\text{arctanh}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

```
[In] Int[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] -(((c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ar
cTanh[(2*Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1 + x)/(1 - Sqrt[3
] + x)^2)])/(Sqrt[c - d]*Sqrt[d]*Sqrt[7 + 4*Sqrt[3] + (1 + Sqrt[3] + x)^2/(
1 - Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1
 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticPi[(c - (1 - Sqrt[
3])*d)^2/(c - (1 + Sqrt[3])*d)^2, ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x
)], -7 + 4*Sqrt[3]])/((c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^
2)]*Sqrt[-1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2168

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2])), Subst[Int[1/(((1
 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*S
qrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; Fr
eeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sqr
t[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

integral =

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\left(-c+\left(1+\sqrt{3}\right)d+\left(-c+\left(1-\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}}\,dx,x,\frac{1+\sqrt{3}-}{-1+\sqrt{3}}}{\sqrt{\frac{-1-x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(-c+d+\sqrt{3}d)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}\left(\left(-c+\left(1+\sqrt{3}\right)d\right)\right.}\right.}{\sqrt{\frac{-1-x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(-c+(1-\sqrt{3})d)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{x}{\sqrt{1-x^2}\sqrt{7+4\sqrt{3}+x^2}\left(\left(-c+\left(1+\sqrt{3}\right)\right.\right.}\right.}{\sqrt{\frac{-1-x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{(c-d-\sqrt{3}d)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+\frac{\left(2\sqrt[4]{3}\sqrt{2+\sqrt{3}}(-c+(1-\sqrt{3})d)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7+4\sqrt{3}+x}\left(\left(-c+\left(1+\sqrt{3}\right)d\right.\right.}\right.}{\sqrt{\frac{-1-x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{(c-d-\sqrt{3}d)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(-c+(1-\sqrt{3})d)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{}{\left(-c+\left(1-\sqrt{3}\right)d\right)^2-\left(-c+\left(1+\sqrt{3}\right)d\right.}\right.}{\sqrt{\frac{-1-x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= -\frac{(c-(1-\sqrt{3})d)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{7+4\sqrt{3}+\frac{\left(1+\sqrt{3}+x\right)^2}{\left(1-\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{(c-d-\sqrt{3}d)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.57 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.64

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c-\left(-3+\sqrt{3}\right)\ldots\right.}{\ldots}\right.}{3d\sqrt{-1-x^3}}$$

```
[In] Integrate[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c - (-3 + Sqrt[3])*d)*Sqrt[1 - x + x^
2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1/3)*d)))/(3*d*Sqrt[-1 - x^
3])
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.73

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^3-1}}+\frac{2i\left(d\sqrt{3}+c-d\right)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\ldots\right.}}{\ldots}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^3-1}}+\frac{2i\left(d\sqrt{3}+c-d\right)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\ldots\right.}}{\ldots}$

```
[In] int((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))+2/3*I*(d*3^(1/2)+c-d)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^
```

```
(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/
2)+c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-(x+1)(x^2-x+1)}\,(c+dx)}\,dx$$

```
[In] integrate((1+x-3**(1/2))/(d*x+c)/(-x**3-1)**(1/2),x)
[Out] Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

```
[In] int((x - 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

3.152 $\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -2/3*arctanh((x^3+1)^(1/2))*(1+3^(1/2))+2/3*(1+x)*EllipticF((1+x-3^(1/2))/(
1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/
2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1846, 272, 65, 213, 224}

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \frac{2}{3}\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{x^3+1}\right)$$

```
[In] Int[(1 + Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\text{integral} = \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{1+x^3}}\,dx + \int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\left(1+\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+x}}\,dx,x,x^3\right)$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\left(2\left(1+\sqrt{3}\right)\right)\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^3}\right)$$

$$= -\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.31

$$\int\frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}\left(1+\sqrt{3}\right)\text{arctanh}\left(\sqrt{1+x^3}\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)$$

```
[In] Integrate[(1 + Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + x*Hypergeometric2F1[1/3, 1/2,
 4/3, -x^3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.72

method	result
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}}{3\sqrt{\pi}}+x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+\frac{\sqrt{3}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}\right.}{3\sqrt{\pi}}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)\left(1+\sqrt{3}\right)}{3}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)\left(1+\sqrt{3}\right)}{3}$

```
[In] int((1+x+3^(1/2))/x/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^(
1/2))+x*hypergeom([1/3,1/2],[4/3],-x^3)+1/3*3^(1/2)/Pi^(1/2)*(-2*Pi^(1/2)*l
n(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.26

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx=\frac{1}{3}\left(\sqrt{3}+1\right)\log\left(\frac{x^3-2\sqrt{x^3+1}+2}{x^3}\right)+2\,\mathrm{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(3) + 1)*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3) + 2*weierstrassPInve
rse(0, -4, x)
```

Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.45

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx=\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\,x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2\sqrt{3}\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{2\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

```
[In] integrate((1+x+3**(1/2))/x/(x**3+1)**(1/2),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
 - 2*sqrt(3)*asinh(x**(-3/2))/3 - 2*asinh(x**(-3/2))/3
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

```
[In] integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

```
[In] integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 17.62 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.67

$$\begin{aligned}\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx &= -\frac{2\sqrt{3}\,\mathrm{atanh}\left(\sqrt{x^3+1}\right)}{3} \\ &+ \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} \\ &- \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\end{aligned}$$

```
[In] int((x + 3^(1/2) + 1)/(x*(x^3 + 1)^(1/2)),x)
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2))^(1/2) - (2*3^(1/2)*atanh((x^3 + 1)^(1/2)))/3 - (2*((3^
```

```
(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
)/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.153 $\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1-x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/3*arctanh((-x^3+1)^(1/2))*(1+3^(1/2))+2/3*(1-x)*EllipticF((1-x-3^(1/2))/
(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1
/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1846, 272, 65, 212, 224}

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{2}{3}\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1-x^3}\right)$$

```
[In] Int[(1 + Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/
(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x
)^2]*Sqrt[1 - x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\text{integral} = \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}\,dx - \int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{1}{3}\left(1+\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-xx}}\,dx,x,x^3\right)$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{1}{3}\left(2\left(1+\sqrt{3}\right)\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{1-x^3}\right)$$

$$= -\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.29

$$\int\frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}\left(1+\sqrt{3}\right)\text{arctanh}\left(\sqrt{1-x^3}\right) - x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)$$

```
[In] Integrate[(1 + Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 - x*Hypergeometric2F1[1/3, 1/2,
 4/3, x^3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.73

method	result
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}}{3\sqrt{\pi}} - x{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right) + \frac{\sqrt{3}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x\right.}{3\sqrt{\pi}}$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2\,\mathrm{arctanh}\left(\sqrt{-x^3+1}\right)\left(1+\sqrt{\right.}{3}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2\,\mathrm{arctanh}\left(\sqrt{-x^3+1}\right)\left(1+\sqrt{\right.}{3}$

```
[In] int((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(-x^3+1)^(1/2))+(-2*ln(2)+3*ln(x)+I*Pi
)*Pi^(1/2))-x*hypergeom([1/3,1/2],[4/3],x^3)+1/3*3^(1/2)/Pi^(1/2)*(-2*Pi^(1
/2)*ln(1/2+1/2*(-x^3+1)^(1/2))+(-2*ln(2)+3*ln(x)+I*Pi)*Pi^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.26

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \frac{1}{3}\left(\sqrt{3}+1\right)\log\left(-\frac{x^3+2\sqrt{-x^3+1}-2}{x^3}\right) + 2i\,\mathrm{weierstrassPInverse}(0,4,x)$$

```
[In] integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(3) + 1)*log(-(x^3 + 2*sqrt(-x^3 + 1) - 2)/x^3) + 2*I*weierstrassP
Inverse(0, 4, x)
```

Sympy [A] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \begin{cases} -\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|} > 1 \\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases} + \sqrt{3}\left(\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|} > 1 \\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((1-x+3**(1/2))/x/(-x**3+1)**(1/2),x)
[Out] -x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/
3)) + Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin(x**(-3/
2))/3, True)) + sqrt(3)*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1),
 (2*I*asin(x**(-3/2))/3, True))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

```
[In] integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

```
[In] integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 18.95 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.68

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{1-x^3}-1\right)^3\left(\sqrt{1-x^3}+1\right)}{x^6}\right)}{3} + \frac{\sqrt{x^3-1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\cdots}{-\frac{3}{2}+\cdots}}\cdots}{\sqrt{x^3\cdots}}\right)}{\sqrt{1-x^3}}$$

```
[In] int((3^(1/2) - x + 1)/(x*(1 - x^3)^(1/2)),x)
```

```
[Out] (3^(1/2)*log((((1 - x^3)^(1/2) - 1)^3*((1 - x^3)^(1/2) + 1))/x^6))/3 + ((x^
3 - 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(
x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/
2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2)))/(1 - x^3)^(1/2)
```

3.154 $\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 142

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{-1+x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] 2/3*arctan((x^3-1)^(1/2))*(1+3^(1/2))+2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-
x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2)
)^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1846, 272, 65, 209, 225}

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} + \frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{x^3-1}\right)$$

```
[In] Int[(1 + Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
x)^2)]*Sqrt[-1 + x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\text{integral} = \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{-1+x^3}}\,dx - \int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$+\frac{1}{3}\left(1+\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1+x}x}\,dx,x,x^3\right)$$

$$= \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$+\frac{1}{3}\left(2\left(1+\sqrt{3}\right)\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^3}\right)$$

$$= \frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1+x^3}\right)$$

$$+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.41

$$\int\frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{-1+x^3}\right) - \frac{x\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)}{\sqrt{-1+x^3}}$$

```
[In] Integrate[(1 + Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 - (x*Sqrt[1 - x^3]*Hypergeometri
c2F1[1/3, 1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}+\frac{2\arctan\left(\sqrt{x^{3}-1}\right)\left(1+\sqrt{3}\right)}{3}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}+\frac{2\arctan\left(\sqrt{x^{3}-1}\right)\left(1+\sqrt{3}\right)}{3}$
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^{3}-1)}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{3}+1}}{2}\right)+(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}\right)}{3\sqrt{\pi}\,\sqrt{\operatorname{signum}(x^{3}-1)}}-\frac{\sqrt{-\operatorname{signum}(x^{3}-1)}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^{3}\right)}{\sqrt{\operatorname{signum}(x^{3}-1)}}+\frac{\sqrt{3}\,\sqrt{-\operatorname{sign}}}{}$

[In] int((1-x+3^(1/2))/x/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2/3*arctan((x^3-1)^(1/2))*(1+3^(1/2))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.37

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{1}{3}\sqrt{2\sqrt{3}+4}\arctan\left(-\frac{\left(x^3-\sqrt{3}(x^3-2)-2\right)\sqrt{2\sqrt{3}+4}}{4\sqrt{x^3-1}}\right) - 2\,\mathrm{weierstrassPInverse}(0,4,x)$$

[In] integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="fricas")

[Out] 1/3*sqrt(2*sqrt(3) + 4)*arctan(-1/4*(x^3 - sqrt(3)*(x^3 - 2) - 2)*sqrt(2*sqrt(3) + 4)/sqrt(x^3 - 1)) - 2*weierstrassPInverse(0, 4, x)

Sympy [A] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.66

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} + \begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases} + \sqrt{3}\left(\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((1-x+3**(1/2))/x/(x**3-1)**(1/2),x)
[Out] I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + Piecewise((
2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/3, True)) + s
qrt(3)*Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3
/2))/3, True))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

```
[In] integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

```
[In] integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 18.89 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.35

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{x^3-1}\right)}{3} + \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int((3^(1/2) - x + 1)/(x*(x^3 - 1)^(1/2)),x)
```

```
[Out] (2*3^(1/2)*atan((x^3 - 1)^(1/2)))/3 + (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(
1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*elli
pticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2
)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) -
 x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((
3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(
1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

3.155 $\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{-1-x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 2/3*arctan((-x^3-1)^(1/2))*(1+3^(1/2))+2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1
+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2
))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1846, 272, 65, 210, 225}

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} + \frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{-x^3-1}\right)$$

```
[In] Int[(1 + Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] +
x)^2)]*Sqrt[-1 - x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\text{integral} = \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{-1-x^3}}\,dx + \int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{1}{3}\left(1+\sqrt{3}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{-1-xx}}\,dx,x,x^3\right)$$

$$= \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{1}{3}\left(2\left(1+\sqrt{3}\right)\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,\sqrt{-1-x^3}\right)$$

$$= \frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1-x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{-1-x^3}\right) + \frac{x\sqrt{1+x^3}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)}{\sqrt{-1-x^3}}$$

```
[In] Integrate[(1 + Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (x*Sqrt[1 + x^3]*Hypergeometri
c2F1[1/3, 1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

method	result
meijerg	$-\frac{i\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}\right)}{3\sqrt{\pi}} - ix\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right) - \frac{i\sqrt{3}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\right.}{3\sqrt{\pi}}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}} + \frac{2\arctan\left(\sqrt{-x^3-1}\right)\left(1+\sqrt{3}\right)}{3}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}} + \frac{2\arctan\left(\sqrt{-x^3-1}\right)\left(1+\sqrt{3}\right)}{3}$

```
[In] int((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*P
i^(1/2))-I*x*hypergeom([1/3,1/2],[4/3],-x^3)-1/3*I*3^(1/2)/Pi^(1/2)*(-2*Pi^
(1/2)*ln(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{1}{3}\sqrt{2\sqrt{3}+4}\arctan\left(-\frac{\left(x^3-\sqrt{3}(x^3+2)+2\right)\sqrt{-x^3-1}\sqrt{2\sqrt{3}+4}}{4\,(x^3+1)}\right) - 2i\,\text{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2*sqrt(3) + 4)*arctan(-1/4*(x^3 - sqrt(3)*(x^3 + 2) + 2)*sqrt(-x^3
 - 1)*sqrt(2*sqrt(3) + 4)/(x^3 + 1)) - 2*I*weierstrassPInverse(0, -4, x)
```

Sympy [A] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = -\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2\sqrt{3}i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

```
[In] integrate((1+x+3**(1/2))/x/(-x**3-1)**(1/2),x)
[Out] -I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/
3)) + 2*I*asinh(x**(-3/2))/3 + 2*sqrt(3)*I*asinh(x**(-3/2))/3
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

```
[In] integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

```
[In] integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 21.45 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.76

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{-x^3-1}-\mathrm{i}\right)\left(\sqrt{-x^3-1}+1\mathrm{i}\right)^3}{x^6}\right)\,1\mathrm{i}}{3} + \frac{\sqrt{x^3+1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{x^3+\left(-\right.}}\right.}{\sqrt{-x^3-1}}$$

```
[In] int((x + 3^(1/2) + 1)/(x*(- x^3 - 1)^(1/2)),x)
```

```
[Out] (3^(1/2)*log((((- x^3 - 1)^(1/2) - 1i)*((- x^3 - 1)^(1/2) + 1i)^3)/x^6)*1i)
/3 + ((x^3 + 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2
)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3
^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x +
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)/2 + 3/
2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2))^(1/2)))/(- x^3 - 1)^(1/2)
```

3.156 $\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}\left(1-\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -2/3*arctanh((x^3+1)^(1/2))*(1-3^(1/2))+2/3*(1+x)*EllipticF((1+x-3^(1/2))/(
1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/
2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1846, 272, 65, 213, 224}

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \frac{2}{3}\left(1-\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{x^3+1}\right)$$

```
[In] Int[(1 - Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\text{integral} = \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{1+x^3}}\,dx + \int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\left(1-\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+x}}\,dx,x,x^3\right)$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\left(2\left(1-\sqrt{3}\right)\right)\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^3}\right)$$

$$= -\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.32

$$\int\frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}\left(1-\sqrt{3}\right)\text{arctanh}\left(\sqrt{1+x^3}\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)$$

```
[In] Integrate[(1 - Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + x*Hypergeometric2F1[1/3, 1/2,
 4/3, -x^3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

method	result
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}}{3\sqrt{\pi}}+x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)-\frac{\sqrt{3}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}\right.}{3\sqrt{\pi}}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\sqrt{3}-1\right)\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\operatorname{arctanh}\left(\sqrt{x^3+1}\right)\left(1-\sqrt{3}\right)}{3}$

```
[In] int((1+x-3^(1/2))/x/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^(
1/2))+x*hypergeom([1/3,1/2],[4/3],-x^3)-1/3*3^(1/2)/Pi^(1/2)*(-2*Pi^(1/2)*l
n(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.26

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx=\frac{1}{3}\left(\sqrt{3}-1\right)\log\left(\frac{x^3+2\sqrt{x^3+1}+2}{x^3}\right)+2\,\text{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(3) - 1)*log((x^3 + 2*sqrt(x^3 + 1) + 2)/x^3) + 2*weierstrassPInve
rse(0, -4, x)
```

Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.44

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx=\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\,x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2\sqrt{3}\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

```
[In] integrate((1+x-3**(1/2))/x/(x**3+1)**(1/2),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
 - 2*asinh(x**(-3/2))/3 + 2*sqrt(3)*asinh(x**(-3/2))/3
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

```
[In] integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

```
[In] integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 19.81 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.63

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \frac{2\sqrt{3}\,\mathrm{atanh}\left(\sqrt{x^3+1}\right)}{3} + \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int((x - 3^(1/2) + 1)/(x*(x^3 + 1)^(1/2)),x)
[Out] (2*3^(1/2)*atanh((x^3 + 1)^(1/2)))/3 + (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(
1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellip
ticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^
```

```
(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
)/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.157 $\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 141

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}\left(1-\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1-x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/3*arctanh((-x^3+1)^(1/2))*(1-3^(1/2))+2/3*(1-x)*EllipticF((1-x-3^(1/2))/
(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1
/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1846, 272, 65, 212, 224}

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{2}{3}\left(1-\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1-x^3}\right)$$

```
[In] Int[(1 - Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/
(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x
)^2]*Sqrt[1 - x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\text{integral} = \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}\,dx - \int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}
+\frac{1}{3}\left(1-\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}x}\,dx,x,x^3\right)$$

$$= \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}
-\frac{1}{3}\left(2\left(1-\sqrt{3}\right)\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{1-x^3}\right)$$

$$= -\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)
+\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.30

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}\left(1-\sqrt{3}\right)\text{arctanh}\left(\sqrt{1-x^3}\right)-x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)$$

```
[In] Integrate[(1 - Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 - x*Hypergeometric2F1[1/3, 1/2,
 4/3, x^3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

method	result
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}}{3\sqrt{\pi}} - x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right) - \frac{\sqrt{3}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x\right.}{3\sqrt{\pi}}$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} + \frac{2\left(\sqrt{3}-1\right)\operatorname{arctanh}\left(\sqrt{-x^3+1}\right.}{3}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2\operatorname{arctanh}\left(\sqrt{-x^3+1}\right)\left(1-\sqrt{\right.}}{3}$

```
[In] int((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(-x^3+1)^(1/2))+(-2*ln(2)+3*ln(x)+I*Pi
)*Pi^(1/2))-x*hypergeom([1/3,1/2],[4/3],x^3)-1/3*3^(1/2)/Pi^(1/2)*(-2*Pi^(1
/2)*ln(1/2+1/2*(-x^3+1)^(1/2))+(-2*ln(2)+3*ln(x)+I*Pi)*Pi^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.26

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \frac{1}{3}\left(\sqrt{3}-1\right)\log\left(-\frac{x^3-2\sqrt{-x^3+1}-2}{x^3}\right)+2i\,\mathrm{weierstrassPInverse}(0,4,x)$$

```
[In] integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*(sqrt(3) - 1)*log(-(x^3 - 2*sqrt(-x^3 + 1) - 2)/x^3) + 2*I*weierstrassP
Inverse(0, 4, x)
```

Sympy [A] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.70

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \sqrt{3}\left(\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right) + \begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}$$

```
[In] integrate((1-x-3**(1/2))/x/(-x**3+1)**(1/2),x)
```

```
[Out] -x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/
3)) - sqrt(3)*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin
(x**(-3/2))/3, True)) + Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1),
 (2*I*asin(x**(-3/2))/3, True))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

```
[In] integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

```
[In] integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 20.84 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.65

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \frac{\sqrt{3}\ln\left(\frac{\left(\sqrt{1-x^3}-1\right)\left(\sqrt{1-x^3}+1\right)^3}{x^6}\right)}{3} + \frac{\sqrt{x^3-1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}\cdots}{-\frac{3}{2}+\cdots}}\cdots}{\sqrt{x\cdots}}\cdots\right)}{\sqrt{1-x^3}}$$

```
[In] int(-(x + 3^(1/2) - 1)/(x*(1 - x^3)^(1/2)),x)
```

```
[Out] (3^(1/2)*log((((1 - x^3)^(1/2) - 1)*((1 - x^3)^(1/2) + 1)^3)/x^6))/3 + ((x^
3 - 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1
```

```
/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(
x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/
2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2)))/(1 - x^3)^(1/2)
```

3.158 $\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 144

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{-1+x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] 2/3*arctan((x^3-1)^(1/2))*(1-3^(1/2))+2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-
x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2)
)^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1846, 272, 65, 209, 225}

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} + \frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{x^3-1}\right)$$

```
[In] Int[(1 - Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
x)^2)]*Sqrt[-1 + x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\text{integral} = \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{-1+x^3}}\,dx - \int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$+\frac{1}{3}\left(1-\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1+xx}}\,dx,x,x^3\right)$$
$$= \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$+\frac{1}{3}\left(2\left(1-\sqrt{3}\right)\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^3}\right)$$
$$= \frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1+x^3}\right)$$
$$+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

$$\int\frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{-1+x^3}\right) - \frac{x\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)}{\sqrt{-1+x^3}}$$

```
[In] Integrate[(1 - Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 - (x*Sqrt[1 - x^3]*Hypergeometri
c2F1[1/3, 1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\left(\sqrt{3}-1\right)\arctan\left(\sqrt{x^3-1}\right)}{3}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\arctan\left(\sqrt{x^3-1}\right)\left(1-\sqrt{3}\right)}{3}$
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}\right)}{3\sqrt{\pi}\,\sqrt{\operatorname{signum}(x^3-1)}}-\frac{\sqrt{-\operatorname{signum}(x^3-1)}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}-\frac{\sqrt{3}\,\sqrt{-\operatorname{sign}}}{}$

```
[In] int((1-x-3^(1/2))/x/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(3^(1/2)-1)*arctan((x^3-1)^(1
/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.35

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = -\frac{1}{3}\sqrt{-2\sqrt{3}+4}\arctan\left(\frac{\left(x^3+\sqrt{3}(x^3-2)-2\right)\sqrt{-2\sqrt{3}+4}}{4\sqrt{x^3-1}}\right) - 2\,\text{weierstrassPInverse}(0,4,x)$$

```
[In] integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(-2*sqrt(3) + 4)*arctan(1/4*(x^3 + sqrt(3)*(x^3 - 2) - 2)*sqrt(-2*
sqrt(3) + 4)/sqrt(x^3 - 1)) - 2*weierstrassPInverse(0, 4, x)
```

Sympy [A] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.65

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} - \sqrt{3}\left(\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right) + \begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}$$

```
[In] integrate((1-x-3**(1/2))/x/(x**3-1)**(1/2),x)
[Out] I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - sqrt(3)*Pie
cewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/3, Tr
ue)) + Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3
/2))/3, True))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

```
[In] integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

```
[In] integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 20.25 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.32

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = -\frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{x^3-1}\right)}{3} + \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(-(x + 3^(1/2) - 1)/(x*(x^3 - 1)^(1/2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*3^(1/2)*atan((x^3 - 1)^(1/2)))/3 - (2*((
3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(
1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

3.159 $\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\begin{aligned}\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx &= \frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{-1-x^3}\right)\\ &\quad+\frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\end{aligned}$$

```
[Out] 2/3*arctan((-x^3-1)^(1/2))*(1-3^(1/2))+2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1
+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2
))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1846, 272, 65, 210, 225}

$$\begin{aligned}\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx &= \frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}\\ &\quad+\frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{-x^3-1}\right)\end{aligned}$$

```
[In] Int[(1 - Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] +
x)^2)]*Sqrt[-1 - x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\text{integral} = \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{-1-x^3}}\,dx + \int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$
= \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}
+\frac{1}{3}\left(1-\sqrt{3}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1-xx}}\,dx,x,x^3\right)
$$

$$
= \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}
-\frac{1}{3}\left(2\left(1-\sqrt{3}\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,\sqrt{-1-x^3}\right)
$$

$$
= \frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1-x^3}\right)
+\frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

$$
\int\frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{-1-x^3}\right)
+\frac{x\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)}{\sqrt{-1-x^3}}
$$

```
[In] Integrate[(1 - Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (x*Sqrt[1 + x^3]*Hypergeometri
c2F1[1/3, 1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.68

method	result
meijerg	$-\frac{i\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+\left(-2\ln(2)+3\ln(x)\right)\sqrt{\pi}\right)}{3\sqrt{\pi}}-ix\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+\frac{i\sqrt{3}\,\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+\left(-2\ln(2)+3\right.\right.}{3\sqrt{\pi}}$
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2\left(\sqrt{3}-1\right)\arctan\left(\sqrt{-x^3-1}\right)}{3}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2\arctan\left(\sqrt{-x^3-1}\right)\left(1-\sqrt{3}\right)}{3}$

```
[In] int((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*P
i^(1/2))-I*x*hypergeom([1/3,1/2],[4/3],-x^3)+1/3*I*3^(1/2)/Pi^(1/2)*(-2*Pi^
(1/2)*ln(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.43

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$$
$$=-\frac{1}{3}\,\sqrt{-2\,\sqrt{3}+4}\arctan\left(\frac{\left(x^3+\sqrt{3}(x^3+2)+2\right)\sqrt{-x^3-1}\sqrt{-2\,\sqrt{3}+4}}{4\,(x^3+1)}\right)$$
$$-2i\,\mathrm{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(-2*sqrt(3) + 4)*arctan(1/4*(x^3 + sqrt(3)*(x^3 + 2) + 2)*sqrt(-x^
3 - 1)*sqrt(-2*sqrt(3) + 4)/(x^3 + 1)) - 2*I*weierstrassPInverse(0, -4, x)
```

Sympy [A] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.44

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = -\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2\sqrt{3}i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} + \frac{2i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

```
[In] integrate((1+x-3**(1/2))/x/(-x**3-1)**(1/2),x)
[Out] -I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/
3)) - 2*sqrt(3)*I*asinh(x**(-3/2))/3 + 2*I*asinh(x**(-3/2))/3
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

```
[In] integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

```
[In] integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 21.68 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.72

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{-x^3-1}-\mathrm{i}\right)^3\left(\sqrt{-x^3-1}+1\mathrm{i}\right)}{x^6}\right)\,1\mathrm{i}}{3} + \frac{\sqrt{x^3+1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{x^3+\left(-\left(\right.\right.}}\right.}{\sqrt{-x^3-1}}$$

```
[In] int((x - 3^(1/2) + 1)/(x*(- x^3 - 1)^(1/2)),x)
```

```
[Out] (3^(1/2)*log((((- x^3 - 1)^(1/2) - 1i)^3*((- x^3 - 1)^(1/2) + 1i))/x^6)*1i)
/3 + ((x^3 + 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2
)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3
^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x +
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)/2 + 3/
2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2))^(1/2)))/(- x^3 - 1)^(1/2)
```

3.160 $\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 332

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = -\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{12\sqrt[4]{3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -3/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*26^(1/2)/(x^3+1)^(1
/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-12*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2)
)/(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1
/2)/(x^3+1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3^(1/2))^2)^(1/2)-2
/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2)*(7*2^(1/2)+4*6^(1/2))*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+
x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2169, 224, 2167, 2138, 551, 585, 95, 210}

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} + \frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[In] Int[x/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-3*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(S
qrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*Sqrt[2*(97 +
56*Sqrt[3])]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcS
in[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (12*3^(1/4)*(1 + x)*Sqrt[(1 - x
+ x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], ArcSin[(1 - Sqrt[3]
 + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/
(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\text{integral} = -\frac{3\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-2-\sqrt{3}+\left(-2+\sqrt{3}\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx, x, \frac{-1+\sqrt{3}-}{1+\sqrt{3}+}\right.}{\left(-2+\sqrt{3}\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-2-\sqrt{3}\right)^2-\left(-2+\sqrt{3}\right)^2x^2\right)}\,dx, x, \frac{-}{1}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{\left(12\sqrt[4]{3}\left(-2-\sqrt{3}\right)\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-2-\sqrt{3}\right)^2-\left(-2+\sqrt{3}\right)^2\right.}\right.}{\left(-2+\sqrt{3}\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$
$$+\frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(97-56\sqrt{3};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$
$$+\frac{\left(6\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(-2-\sqrt{3}\right)^2-\left(-2+\sqrt{3}\right)^2x\right)}\,dx,x,\frac{\left(-1+\cdots\right.}{\left(1+\cdots\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$
$$+\frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(97-56\sqrt{3};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$
$$+\frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{-\left(-2-\sqrt{3}\right)^2+\left(-2+\sqrt{3}\right)^2-\left(\left(-2-\sqrt{3}\right)^2+\left(7-4\sqrt{3}\right)\left(-2+\cdots\right.\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$
$$-\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$
$$+\frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(97-56\sqrt{3};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.58

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{3+\sqrt[3]{-1}},\text{arcs}\ldots\right.}{3+\sqrt[3]{-\ldots}}\right.}{\sqrt{1+x^3}}$$

[In] Integrate[x/((3 + x)*Sqrt[1 + x^3]),x]

[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
((3*I)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/(3 + (-1)^(1/3)), ArcSin[Sq
rt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 + (-1)^(1/3))))/S
qrt[1 + x^3]

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.72

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\ldots}{\sqrt{x^3+1}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\ldots}{\sqrt{x^3+1}}$

[In] int(x/(3+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-3/4+1/4*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))

Fricas [F]

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 + 1)*x/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F]

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\,(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(x**3+1)**(1/2),x)
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.62

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx$$
$$= \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(2\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\mathrm{asin}\right.\right.}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(x/((x^3 + 1)^(1/2)*(x + 3)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*(2*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 3*ellipticPi(- (3^
(1/2)*1i)/4 - 3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)
*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(2*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2
))^(1/2))
```

3.161 $\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 377

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = \frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] 3/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/((x^2+x+1)/(1-
x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*7^(1/2)/(-x^3+1)^(1/
2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-12/13*3^(1/4)*(1-x)*EllipticPi((-1+x+3^(1/
2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2
^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2
))^2)^(1/2)-2/39*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)
*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*(5*2^(1/2)+2*6^(1/2))*3^(3/4)/(-x^3+1)^(
1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2169, 224, 2167, 2138, 551, 585, 95, 212}

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx$$
$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$
$$+\frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$
$$+\frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[In] Int[x/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (3*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1
 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(
2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (2*Sqrt[2*(37
+ 20*Sqrt[3])]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[Ar
cSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(13*3^(1/4)*Sqr
t[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) + (12*3^(1/4)*Sqrt[2 + Sqrt[3
]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*Sq
rt[3])/169, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(
13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\text{integral} = -\frac{3\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{1-x^3}}\,dx}{4+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(4-\sqrt{3}+\left(4+\sqrt{3}\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx, x, \frac{-1+\sqrt{3}+x}{1+\sqrt{3}-x}\right)}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(4-\sqrt{3}\right)^2-\left(4+\sqrt{3}\right)^2x^2\right)}\,dx, x, \frac{-1+\sqrt{}}{1+\sqrt{}}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(4-\sqrt{3}\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(4-\sqrt{3}\right)^2-\left(4+\sqrt{3}\right)^2x^2\right)}\,d\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$
$$+\frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$
$$+\frac{\left(6\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(4-\sqrt{3}\right)^2-\left(4+\sqrt{3}\right)^2x\right)}\,dx,x,\frac{\left(-1+\sqrt{3}\right.}{\left(1+\sqrt{3}\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$
$$+\frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$
$$+\frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{-\left(4-\sqrt{3}\right)^2+\left(4+\sqrt{3}\right)^2-\left(\left(4-\sqrt{3}\right)^2+\left(7-4\sqrt{3}\right)\left(4+\sqrt{3}\right)^2\right)x\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= \frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$
$$-\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$
$$+\frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.52

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1+x+x^2}\operatorname{EllipticPi}\left(\frac{2\sqrt{3}}{5i+\sqrt{3}},\arcsin\left(\sqrt{\right.\right.}{-3+\sqrt[3]{-1}}\right.}{\sqrt{1-x^3}}$$

```
[In] Integrate[x/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((3
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-3 + (-1)^(1/3))))/Sqrt
[1 - x^3]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.64

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$

```
[In] int(x/(3+x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*
((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(-x^3+1)^(1/2)/(5/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*
```

```
3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(5/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*
I*3^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^3 + 1)*x/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F]

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-(x-1)(x^2+x+1)}\,(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(-x**3+1)**(1/2),x)
[Out] Integral(x/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.59

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(4\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(\right.\right.}{4\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int(x/((1 - x^3)^(1/2)*(x + 3)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(4*ellipticF(asin((-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
- 3*ellipticPi((3^(1/2)*1i)/8 + 3/8, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(4*(1 - x^3)^(1/
2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.162 $\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 373

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((x
^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(4+3^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1
-x-3^(1/2))^2)^(1/2)+3/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3^(1/2))^2)
^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)
*7^(1/2)/(x^3-1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-12/13*3^(1/4)*(1-x)*El
lipticPi((-1+x+3^(1/2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I*3^(1/2)+2*I
)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(x^3-1)^(1/2)
/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2169, 225, 2167, 2138, 551, 585, 95, 212}

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{2\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} + \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} + \frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[In] Int[x/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (3*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1
 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(
2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3]) - (2*Sqrt[2]*(1
 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3]
 - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*(4 + Sqrt[3])*Sqrt[-((1
 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (12*3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*Sqrt
[3])/169, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(13
*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\text{integral} = -\frac{3\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(4-\sqrt{3}+\left(4+\sqrt{3}\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx, x, \frac{-1+\sqrt{3}+x}{1+\sqrt{3}-x}\right)}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(4-\sqrt{3}\right)^2-\left(4+\sqrt{3}\right)^2x^2\right)}\,dx, x, \frac{-1+\cdots}{1+\cdots}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(4-\sqrt{3}\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(4-\sqrt{3}\right)^2-\left(4+\sqrt{3}\right)^2x^2\right)}\,d\cdots\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$+\frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$+\frac{\left(6\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(4-\sqrt{3}\right)^2-\left(4+\sqrt{3}\right)^2x\right)}\,dx,x,\frac{\left(-1+\sqrt{3}\cdots\right.}{\left(1+\sqrt{3}\cdots\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$+\frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$+\frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{-\left(4-\sqrt{3}\right)^2+\left(4+\sqrt{3}\right)^2-\left(\left(4-\sqrt{3}\right)^2+\left(7-4\sqrt{3}\right)\left(4+\sqrt{3}\right)^2\right)x\cdots}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= \frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$-\frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$+\frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.52

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{5i+\sqrt{3}},\arcsin\left(\sqrt{\ldots}\right.\right.}{-3+\sqrt[3]{-1}}\right.}{\sqrt{-1+x^3}}$$

```
[In] Integrate[x/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((3
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-3 + (-1)^(1/3))))/Sqrt
[-1 + x^3]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.64

method	result
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{3\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\ldots}{\frac{3}{2}+\ldots}}}{2\sqrt{x^3-\ldots}}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{3\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\ldots}{\frac{3}{2}+\ldots}}}{2\sqrt{x^3-\ldots}}$

```
[In] int(x/(3+x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-3/2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-
3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)
*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi
(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),3/8+1/8*I*3^(1/2),((3/2+1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 - 1)*x/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F]

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(x**3-1)**(1/2),x)
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 19.66 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.56

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(4\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(\frac{3}{8}+\frac{\sqrt{3}\,1\ldots}{8}\right.\right.}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\ldots}{2}\right.}}$$

```
[In] int(x/((x^3 - 1)^(1/2)*(x + 3)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(4*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 3*ellipticPi((
3^(1/2)*1i)/8 + 3/8, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1
/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(4*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) +
 x^3)^(1/2))
```

3.163 $\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 341

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = -\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{2\sqrt{14+8\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{12\sqrt[4]{3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] -3/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*26^(1/2)/(-x^3-1)^(
1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-12*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2
))/(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x+3^(1/2))^2)^(
1/2)/(-x^3-1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3^(1/2))^2)^(1/2)
-2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/
(1+x-3^(1/2))^2)^(1/2)*(2*2^(1/2)+6^(1/2))*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(
1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2169, 225, 2167, 2138, 551, 585, 95, 210}

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx$$
$$= -\frac{2\sqrt{14+8\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$
$$+\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$
$$-\frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[In] Int[x/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-3*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(S
qrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3]) - (2*Sqrt[14 + 8*
Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 +
x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) + (12*3^(1/4)*(1 + x)*Sqrt[(1 - x
+ x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], ArcSin[(1 - Sqrt[3]
 + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/
(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\text{integral} = -\frac{3\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}}$$

$$= -\frac{2\sqrt{14+8\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-2-\sqrt{3}+\left(-2+\sqrt{3}\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx, x, \frac{-1+\sqrt{3}-}{1+\sqrt{3}+}\right.}{\left(-2+\sqrt{3}\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= -\frac{2\sqrt{14+8\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-2-\sqrt{3}\right)^2-\left(-2+\sqrt{3}\right)^2x^2\right)}\,dx, x, \frac{-}{1}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{\left(12\sqrt[4]{3}\left(-2-\sqrt{3}\right)\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-2-\sqrt{3}\right)^2-\left(-2+\sqrt{3}\right)^2\right.}\right.}{\left(-2+\sqrt{3}\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$=-\frac{2\sqrt{14+8\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+\frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(97-56\sqrt{3};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+\frac{\left(6\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(-2-\sqrt{3}\right)^2-\left(-2+\sqrt{3}\right)^2x\right)}\,dx,x,\frac{\left(-1+\right.}{(1+}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$=-\frac{2\sqrt{14+8\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+\frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(97-56\sqrt{3};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+\frac{\left(12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{-\left(-2-\sqrt{3}\right)^2+\left(-2+\sqrt{3}\right)^2-\left(\left(-2-\sqrt{3}\right)^2+\left(7-4\sqrt{3}\right)\left(-2+\sqrt{}\right.\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$=-\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$-\frac{2\sqrt{14+8\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+\frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(97-56\sqrt{3};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.57

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{3+\sqrt[3]{-1}},\text{arcs}\ldots\right)}{3+\sqrt[3]{-1}}\ldots\right)}{\sqrt{-1-x^3}}$$

[In] Integrate[x/((3 + x)*Sqrt[-1 - x^3]),x]

[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) + ((3*I)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/(3 + (-1)^(1/3)), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 + (-1)^(1/3))))/Sqrt[-1 - x^3]

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.70

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ldots}{\ldots}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ldots}{\ldots}$

[In] int(x/(3+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(7/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))

```
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(7/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^
(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^3 - 1)*x/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F]

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-(x+1)(x^2-x+1)}(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(-x**3-1)**(1/2),x)
[Out] Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}(x+3)}\,dx$$

```
[In] integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.65

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx$$
$$= \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(2\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(-\frac{3}{4}-\right.\right.}{2\,\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}\right.}}$$

```
[In] int(x/((- x^3 - 1)^(1/2)*(x + 3)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(2*ellipticF(asin(((x + 1)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 3*
ellipticPi(- (3^(1/2)*1i)/4 - 3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(2*(- x^3 - 1)^(1/2
)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.164 $\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 450

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{2+\sqrt{3}}(e-f-\sqrt{3}f)\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,(c-d-\sqrt{3}d)\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{(c^2-2cd-2d^2)\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] (-c*f+d*e)*(1+x)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(
c-d)^(1/2)/d^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/
2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(
1+x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1+x)*EllipticPi((-1-x+3^(1/2))/
(1+x+3^(1/2)),(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(1/2*6
^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c^2-2*c*d-2*d^2)/(x^
3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+2/3*(1+x)*EllipticF((1+x-3^(1/2))/
(1+x+3^(1/2)),I*3^(1/2)+2*I)*(e-f-f*3^(1/2))*(1/2*6^(1/2)+1/2*2^(1/2))*((x^
2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(-d*3^(1/2)+c-d)/(x^3+1)^(1/2)/((1+x)
/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2169, 224, 2167, 2138, 551, 585, 95, 211}

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\,\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\,(c^2-2cd-2d^2)} + \frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\sqrt{3}f-f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\sqrt{3}d-d\right)} + \frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\arctan\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)(de-cf)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[c
^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]*Sqrt[d]*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d
 + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2 + Sqrt
[3]]*(e - f - Sqrt[3]*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*El
lipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1
/4)*(c - d - Sqrt[3]*d)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) -
(4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sq
rt[3] + x)^2]*EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, A
rcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c^2 - 2*c*d
- 2*d^2)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int\frac{1}{\sqrt{1+x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d}-\frac{(de-cf)\int\frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\left(-c+\left(1-\sqrt{3}\right)d+\left(-c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{}}}\right.}{\left(c-\left(1+\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$+\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(-c+d+\sqrt{3}d\right)(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-c+\right.\right.}\right.}{\left(c-\left(1+\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(-c+\left(1-\sqrt{3}\right)d\right)(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-\right.\right.}\right.}{\left(c-\left(1+\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\left(c^2-2cd-2d^2\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$+\frac{\left(2\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(-c+d+\sqrt{3}d\right)(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(-c+\left(\right.\right.\right.}\right.}{\left(c-\left(1+\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\left(c^2-2cd-2d^2\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$+\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(-c+d+\sqrt{3}d\right)(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\frac{}{-\left(-c+\left(1-\sqrt{3}\right)d\right)^2+\left(-c+\right.}\right.}{\left(c-\left(1+\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right.}}}$$

$$= \frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\left(c^2-2cd-2d^2\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.45 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.47

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{i(-de+cf)\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{i \ldots}{c+\sqrt[3]{\ldots}}\ldots\right)}{c+\ldots}\right)}{d\sqrt{1+x^3}}$$

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((f*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
+ (I*(-(d*e) + c*f)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1
/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c
+ (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.61

method	result
default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}+\frac{2(-cf+ed)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{}}{d^2\sqrt{x^3+}}$
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}-\frac{2(cf-ed)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{}}{d^2\sqrt{x^3+}}$

```
[In] int((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f/d*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(-c*f+d*e)/d^2*(3/2-1/2*I*3
^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*
I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1
)^(1/2)/(-1+c/d)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3
^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{e+fx}{\sqrt{(x+1)(x^2-x+1)}\,(c+dx)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3+1}(dx+c)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3+1}(dx+c)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.79

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(c\,f-d\,e\right)\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{c}{d}-1};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d^2\,\left(\frac{c}{d}-1\right)\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int((e + f*x)/((x^3 + 1)^(1/2)*(c + d*x)),x)
[Out] (2*f*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d*(x^3 - x*
(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)
*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*(c*f - d*e)*((x + 1)/((3^(1/2)*
```

```
1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(c/d - 1), asin(((x + 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(
c/d - 1)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.165 $\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 474

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx = -\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2},\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\left(c^2+2cd-2d^2\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -(-c*f+d*e)*(1-x)*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)
/d^(1/2)/(c+d)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(
1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(-x^3+1)^(1/2)/((1-x
)/(1-x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1-x)*EllipticPi((-1+x+3^(1/2
))/(1-x+3^(1/2)),(c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1/2*6^
(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c^2+2*c*d-2*d^2)/(-x^
3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-2/3*(1-x)*EllipticF((1-x-3^(1/2))/
(1-x+3^(1/2)),I*3^(1/2)+2*I)*(e+f+f*3^(1/2))*(1/2*6^(1/2)+1/2*2^(1/2))*((x^
2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(c+d+d*3^(1/2))/(-x^3+1)^(1/2)/((1-x)
/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2169, 224, 2167, 2138, 551, 585, 95, 214}

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx = -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\,\text{EllipticPi}\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2},\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\,(c^2+2cd-2d^2)} - \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)} - \frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{arctanh}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)(de-cf)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt[c + d]*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 -
c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])) - (2*Sqrt[2 +
Sqrt[3]]*(e + f + Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2
]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*(c + d + Sqrt[3]*d)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]
) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, A
rcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c^2 + 2*c*d
- 2*d^2)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(e+f+\sqrt{3}f\right)\int \frac{1}{\sqrt{1-x^3}}\,dx}{c+d+\sqrt{3}d} + \frac{(de-cf)\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx}{c+d+\sqrt{3}d}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$+\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(c+\left(1-\sqrt{3}\right)d+\left(c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}}}\right.}{\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(c+\left(1-\sqrt{3}\right)d\right)^2-\left(c+\left(1+\right.\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$+\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(c+d-\sqrt{3}d\right)(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(c+\left(1\right.\right.\right.}\right.}{\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{4\sqrt[4]{3}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\left(c^2+2cd-2d^2\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{\left(2\sqrt[4]{3}\sqrt{2-\sqrt{3}}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(c+\left(1-\sqrt{3}\right)d\right)^2-\left(c+\left(1+\sqrt{3}\right)\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{4\sqrt[4]{3}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\left(c^2+2cd-2d^2\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int\frac{1}{-\left(c+\left(1-\sqrt{3}\right)d\right)^2+\left(c+\left(1+\sqrt{3}\right)d\right)^2-\left(\left(c+\left(1-\sqrt{3}\right.\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$
$$-\frac{2\sqrt{2+\sqrt{3}}(e+f+\sqrt{3}f)\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,(c+d+\sqrt{3}d)\,\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$
$$-\frac{4\sqrt[4]{3}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\,(c^2+2cd-2d^2)\,\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.58 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.49

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{3f\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)(-de+cf)\sqrt{1}}{}\right.}{3d\sqrt{1-x^3}}$$

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*f*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
 ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[1 + x + x^2]*Elli
pticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/(-c + (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^3])
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.56

method	result
default	$-\frac{2 i f \sqrt{3}\, \sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, \sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^{3}+1}} - \frac{2i(-cf+ed)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2} \cdots\right.}}{\cdots}$
elliptic	$-\frac{2 i f \sqrt{3}\, \sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, \sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^{3}+1}} + \frac{2i(cf-ed)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}- \cdots\right.}}{\cdots}$

```
[In] int((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*f/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2
*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-
3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-c*f+d*e)/d^2*3^(1/2)*(I*(x+1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^
(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(
1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx = \int \frac{e+fx}{\sqrt{-(x-1)(x^2+x+1)}\,(c+dx)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x - 1)*(x**2 + x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3+1}(dx+c)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3+1}(dx+c)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 20.11 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.82

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx=$$

$$-\frac{2 f \left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{d\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

$$+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,(c\,f-d\,e)\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{c}{d}+1};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|\right.}{d^2\sqrt{1-x^3}\left(\frac{c}{d}+1\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right.}}$$

```
[In] int((e + f*x)/((1 - x^3)^(1/2)*(c + d*x)),x)
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)*(c*f - d*e)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3
^(1/2)*1i)/2 + 3/2)/(c/d + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(1 - x^3)^(1/2)*(c
/d + 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2*f*((3^(1/2)*1i)/2 +
```

```
3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(
1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d*(1 - x^3)^(1/
2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.166 $\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 475

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx = -\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$-\frac{2\sqrt{2-\sqrt{3}}(e+f+\sqrt{3}f)\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2},\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{(c^2+2cd-2d^2)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(e+f+f*3^(1
/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(c
+d+d*3^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)-(-c*f+d*e)*(1-x)
*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1
/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/d^
(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(x^3-1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)
^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1-x)*EllipticPi((-1+x+3^(1/2))/(1-x+3^(1/2)),(
c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))
*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c^2+2*c*d-2*d^2)/(x^3-1)^(1/2)/((1-x)/(
1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2169, 225, 2167, 2138, 551, 585, 95, 214}

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx = -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\,\text{EllipticPi}\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2},\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\,(c^2+2cd-2d^2)} - \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)} - \frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{arctanh}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)(de-cf)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt[c + d]*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 -
c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])) - (2*Sqrt[2 -
 Sqrt[3]]*(e + f + Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^
2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/
(3^(1/4)*(c + d + Sqrt[3]*d)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
 x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2
)/(1 + Sqrt[3] - x)^2]*EllipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)
^2, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c^2 + 2
*c*d - 2*d^2)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(e+f+\sqrt{3}f\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx}{c+d+\sqrt{3}d} + \frac{(de-cf)\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx}{c+d+\sqrt{3}d}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$+\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(c+\left(1-\sqrt{3}\right)d+\left(c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+\cdots}}\right.}{\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$-\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(c+\left(1-\sqrt{3}\right)d\right)^2-\left(c+\left(1+\sqrt{\cdots}\right.\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$+\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(c+d-\sqrt{3}d\right)(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(c+\left(1-\cdots\right.\right.\right.}\right.}{\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}(e+f+\sqrt{3}f)(1-x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}(c+d+\sqrt{3}d)\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}\sqrt{-1+x^3}} - \frac{4\sqrt[4]{3}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}\Pi\left(\frac{(c+d+\sqrt{3}d)^2}{(c+d-\sqrt{3}d)^2};\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}(c^2+2cd-2d^2)\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}\sqrt{-1+x^3}} - \frac{\left(2\sqrt[4]{3}\sqrt{2-\sqrt{3}}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(c+\left(1-\sqrt{3}\right)d\right)^2-\left(c+\left(1+\sqrt{3}\right.\right.\right.}\right.}{\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}(e+f+\sqrt{3}f)(1-x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}(c+d+\sqrt{3}d)\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}\sqrt{-1+x^3}} - \frac{4\sqrt[4]{3}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}\Pi\left(\frac{(c+d+\sqrt{3}d)^2}{(c+d-\sqrt{3}d)^2};\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}(c^2+2cd-2d^2)\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}\sqrt{-1+x^3}} - \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}\right)\text{Subst}\left(\int\frac{1}{-\left(c+\left(1-\sqrt{3}\right)d\right)^2+\left(c+\left(1+\sqrt{3}\right)d\right)^2-\left(\left(c+\left(1-\sqrt{}\right.\right.\right.}\right.}{\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$-\frac{2\sqrt{2-\sqrt{3}}(e+f+\sqrt{3}f)\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,(c+d+\sqrt{3}d)\,\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$-\frac{4\sqrt[4]{3}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\Pi\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\,(c^2+2cd-2d^2)\,\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.53 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.49

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{3f\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)(-de+cf)\sqrt{1+x}\cdots}{\cdots}\right)}{3d\sqrt{-1+x^3}}$$

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*f*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
 ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[1 + x + x^2]*Elli
pticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/(-c + (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x^3])
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.58

method	result
default	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}+\frac{2(-cf+ed)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\cdots}{\frac{3}{2}-\cdots}}}{d^2\cdots}$
elliptic	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}-\frac{2(cf-ed)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\cdots}{\frac{3}{2}-\cdots}}}{d^2\cdots}$

```
[In] int((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f/d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-c*f+d*e)/d^2*(-3/2-1/2*I*3
^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*
I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)
^(1/2)/(1+c/d)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(
1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx = \int \frac{e+fx}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(c+dx)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x - 1)*(x**2 + x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3-1}(dx+c)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3-1}(dx+c)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.75

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx$$
$$=-\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$+\frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(c\,f-d\,e\right)\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{c}{d}+1};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d^2\,\left(\frac{c}{d}+1\right)\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int((e + f*x)/((x^3 - 1)^(1/2)*(c + d*x)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(c*f -
d*e)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3
/2)/(c/d + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i
)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(c/d + 1)*(((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
 1) + x^3)^(1/2)) - (2*f*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2
```

```
)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/
2 - 3/2)))/(d*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)
*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.167 $\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 463

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx = \frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{(c^2-2cd-2d^2)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(e-f-f*3^(1/
2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-d
*3^(1/2)+c-d)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)+(-c*f+d*e)*(1+x
)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1
/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c
-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(-x^3-1)^(1/2)/((1+x)/(1+x+3^(1/2))^2
)^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1+x)*EllipticPi((-1-x+3^(1/2))/(1+x+3^(1/2)),
(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(
1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c^2-2*c*d-2*d^2)/(-x^3-1)^(1/2)/((
1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2169, 225, 2167, 2138, 551, 585, 95, 211}

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx = -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\,\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\,(c^2-2cd-2d^2)} + \frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\sqrt{3}f-f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\sqrt{3}d-d\right)} + \frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\arctan\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)(de-cf)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[c
^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]*Sqrt[d]*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d
 + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3]) + (2*Sqrt[2 - Sqr
t[3]]*(e - f - Sqrt[3]*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*E
llipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(
1/4)*(c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3
]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1
 + Sqrt[3] + x)^2]*EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)
^2, ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c^2 - 2
*c*d - 2*d^2)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d} - \frac{(de-cf)\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$- \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-c+\left(1-\sqrt{3}\right)d+\left(-c+\left(1+\sqrt{3}\right)d\right)x\right)\sqrt{1-x^2}\sqrt{7-}}\right.}{\left(c-\left(1+\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+ \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(-c+d+\sqrt{3}d\right)(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(-c\right.\right.}\right.}{\left(c-\left(1+\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$- \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(-c+\left(1-\sqrt{3}\right)d\right)(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}\left(\left(\right.\right.}\right.}{\left(c-\left(1+\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$- \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{(c^2-2cd-2d^2)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+ \frac{\left(2\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(-c+d+\sqrt{3}d\right)(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x}\sqrt{7-4\sqrt{3}+x}\left(\left(-c+\left(1\right.\right.\right.}\right.}{\left(c-\left(1+\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$- \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{(c^2-2cd-2d^2)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+ \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(-c+d+\sqrt{3}d\right)(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{}{-\left(-c+\left(1-\sqrt{3}\right)d\right)^2+\left(-c+\left(\right.\right.}\right.}{\left(c-\left(1+\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}$$

$$= \frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$
$$+ \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$
$$- \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\left(c^2-2cd-2d^2\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.44 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.46

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i(-de+cf)\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{}{c+}\right.}{c-}\right.}{d\sqrt{-1-x^3}}$$

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((f*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
+ (I*(-(d*e) + c*f)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1
/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c
+ (-1)^(1/3)*d)))/(d*Sqrt[-1 - x^3])
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.57

method	result
default	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 d \sqrt{-x^{3}-1}}-\frac{2 i \left(-c f+e d\right) \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\right.}}{}$
elliptic	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 d \sqrt{-x^{3}-1}}+\frac{2 i \left(c f-e d\right) \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{}}{2}\right.}}{}$

```
[In] int((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*f/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/
2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-c*f+d*e)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/3*3^
(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+
c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx=\text{Timed out}$$

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx=\int \frac{e+fx}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\,\left(c+dx\right)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3-1}(dx+c)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3-1}(dx+c)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.84

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx$$
$$=\frac{2\,f\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(c\,f-d\,e)\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{c}{d}-1};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\ldots\right.}{d^2\sqrt{-x^3-1}\left(\frac{c}{d}-1\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\ldots}}$$

```
[In] int((e + f*x)/((- x^3 - 1)^(1/2)*(c + d*x)),x)
[Out] (2*f*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2
)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/(d*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*((3
^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1
```

```
i)/2 - 3/2))^(1/2)*(c*f - d*e)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^
(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*
1i)/2 + 3/2)/(c/d - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(- x^3 - 1)^(1/2)*(c/d - 1
)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.168 $\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 120

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}e\,\text{arctanh}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}f(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -2/3*e*arctanh((x^3+1)^(1/2))+2/3*f*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1
/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(
1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1846, 272, 65, 213, 12, 224}

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = \frac{2\sqrt{2+\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \frac{2}{3}e\,\text{arctanh}\left(\sqrt{x^3+1}\right)$$

```
[In] Int[(e + f*x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*f*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[
3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\text{integral} = e\int \frac{1}{x\sqrt{1+x^3}}\,dx + \int \frac{f}{\sqrt{1+x^3}}\,dx$$

$$= \frac{1}{3}e\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right) + f\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}f(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$+ \frac{1}{3}(2e)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)$$

$$= -\frac{2}{3}e\tanh^{-1}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}f(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.28

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}e\text{arctanh}\left(\sqrt{1+x^3}\right) + fx\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)$$

```
[In] Integrate[(e + f*x)/(x*Sqrt[1 + x^3]),x]
[Out] (-2*e*ArcTanh[Sqrt[1 + x^3]])/3 + f*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.98 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

method	result	size
meijerg	$f x {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+\frac{e\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+\left(-2\ln(2)+3\ln(x)\right)\sqrt{\pi}\right)}{3\sqrt{\pi}}$	53
default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2e\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}$	129
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2e\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}$	129

```
[In] int((f*x+e)/x/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*x*hypergeom([1/3,1/2],[4/3],-x^3)+1/3*e/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*
(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.25

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx=\frac{1}{3}\,e\log\left(\frac{x^3-2\sqrt{x^3+1}+2}{x^3}\right)+2\,f\mathrm{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*e*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3) + 2*f*weierstrassPInverse(0, -4,
 x)
```

Sympy [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.35

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx=-\frac{2e\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{fx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((f*x+e)/x/(x**3+1)**(1/2),x)
```

```
[Out] -2*e*asinh(x**(-3/2))/3 + f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp
_polar(I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3+1}x}\,dx$$

```
[In] integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*x), x)
```

Giac [F]

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3+1}x}\,dx$$

```
[In] integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 20.50 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.72

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx$$
$$= \frac{(3+\sqrt{3}\,1i)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(f\,F\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}}{}}\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

```
[In] int((e + f*x)/(x*(x^3 + 1)^(1/2)),x)
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*(f*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/
2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - e*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(
((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*
1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x
+ 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1
/2)
```

3.169 $\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 134

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}e\operatorname{arctanh}\left(\sqrt{1-x^3}\right) - \frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/3*e*arctanh((-x^3+1)^(1/2))-2/3*f*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(
1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^
(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1846, 272, 65, 212, 12, 224}

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{2}{3}e\operatorname{arctanh}\left(\sqrt{1-x^3}\right)$$

```
[In] Int[(e + f*x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 - x^3]])/3 - (2*Sqrt[2 + Sqrt[3]]*f*(1 - x)*Sqrt[(1 +
x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[
3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= e\int \frac{1}{x\sqrt{1-x^3}}\,dx + \int \frac{f}{\sqrt{1-x^3}}\,dx \\
&= \frac{1}{3}e\,\text{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx, x, x^3\right) + f\int \frac{1}{\sqrt{1-x^3}}\,dx \\
&= -\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} \\
&\quad - \frac{1}{3}(2e)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^3}\right) \\
&= -\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right) - \frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.25

$$
\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}e\,\text{arctanh}\left(\sqrt{1-x^3}\right) + fx\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)
$$

```
[In] Integrate[(e + f*x)/(x*Sqrt[1 - x^3]),x]
[Out] (-2*e*ArcTanh[Sqrt[1 - x^3]])/3 + f*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.93 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.43

method	result
meijerg	$f x {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+\frac{e\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}\right)}{3\sqrt{\pi}}$
default	$-\frac{2if\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2e\,\text{arctanh}\left(\sqrt{-x^3+1}\right)}{3}$
elliptic	$-\frac{2if\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2e\,\text{arctanh}\left(\sqrt{-x^3+1}\right)}{3}$

```
[In] int((f*x+e)/x/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] f*x*hypergeom([1/3,1/2],[4/3],x^3)+1/3*e/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(
-x^3+1)^(1/2))+(-2*ln(2)+3*ln(x)+I*Pi)*Pi^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.25

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx=\frac{1}{3}\,e\log\left(-\frac{x^3+2\,\sqrt{-x^3+1}-2}{x^3}\right)-2i\,f\text{weierstrassPInverse}(0,4,x)$$

```
[In] integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*e*log(-(x^3 + 2*sqrt(-x^3 + 1) - 2)/x^3) - 2*I*f*weierstrassPInverse(0,
 4, x)
```

Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.49

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx=e\left(\begin{cases}-\frac{2\,\text{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ \frac{2i\,\text{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)+\frac{fx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((f*x+e)/x/(-x**3+1)**(1/2),x)
[Out] e*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin(x**(-3/2))/
3, True)) + f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi)
)/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3+1}x}\,dx$$

[In] integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*x), x)

Giac [F]

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3+1}x}\,dx$$

[In] integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*x), x)

Mupad [B] (verification not implemented)

Time = 20.35 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.66

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = -\frac{\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{-\frac{x-}{\frac{3}{2}+}}\right.\right.\right.}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

[In] int((e + f*x)/(x*(1 - x^3)^(1/2)),x)

```
[Out] -((x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(f*ellipticF(as
in((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1
/2)*1i)/2 - 3/2)) + e*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))
*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3^(1/2) - 3i)*1i)/((1 - x^3)^(1/2
)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.170 $\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 137

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}e\arctan\left(\sqrt{-1+x^3}\right) - \frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] 2/3*e*arctan((x^3-1)^(1/2))-2/3*f*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2
)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/
2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1846, 272, 65, 209, 12, 225}

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}e\arctan\left(\sqrt{x^3-1}\right) - \frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[In] Int[(e + f*x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 + x^3]])/3 - (2*Sqrt[2 - Sqrt[3]]*f*(1 - x)*Sqrt[(1 + x
 + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3
] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqr
t[-1 + x^3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= e\int \frac{1}{x\sqrt{-1+x^3}}\,dx + \int \frac{f}{\sqrt{-1+x^3}}\,dx\\
&= \frac{1}{3}e\text{Subst}\left(\int \frac{1}{\sqrt{-1+xx}}\,dx, x, x^3\right) + f\int \frac{1}{\sqrt{-1+x^3}}\,dx\\
&= -\frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\\
&\quad + \frac{1}{3}(2e)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^3}\right)\\
&= \frac{2}{3}e\tan^{-1}\left(\sqrt{-1+x^3}\right) - \frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.38

$$
\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}e\arctan\left(\sqrt{-1+x^3}\right) + \frac{fx\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)}{\sqrt{-1+x^3}}
$$

```
[In] Integrate[(e + f*x)/(x*Sqrt[-1 + x^3]),x]
[Out] (2*e*ArcTan[Sqrt[-1 + x^3]])/3 + (f*x*Sqrt[1 - x^3]*Hypergeometric2F1[1/3,
1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.97 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68

method	result	size
meijerg	$\frac{f\sqrt{-\operatorname{signum}(x^3-1)}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}+\frac{e\sqrt{-\operatorname{signum}(x^3-1)}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}\right)}{3\sqrt{\pi}\,\sqrt{\operatorname{signum}(x^3-1)}}$	93
default	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2e\arctan\left(\sqrt{x^3-1}\right)}{3}$	129
elliptic	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2e\arctan\left(\sqrt{x^3-1}\right)}{3}$	129

```
[In] int((f*x+e)/x/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/2],[4/3],x^
3)+1/3*e/Pi^(1/2)/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*(-2*Pi^(1/2)*l
n(1/2+1/2*(-x^3+1)^(1/2))+(-2*ln(2)+3*ln(x)+I*Pi)*Pi^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.19

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx=\frac{1}{3}\,e\arctan\left(\frac{x^3-2}{2\,\sqrt{x^3-1}}\right)+2\,f\mathrm{weierstrassPInverse}(0,4,x)$$

```
[In] integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*e*arctan(1/2*(x^3 - 2)/sqrt(x^3 - 1)) + 2*f*weierstrassPInverse(0, 4, x
)
```

Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.44

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx=e\left(\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|}>1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)-\frac{ifx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((f*x+e)/x/(x**3-1)**(1/2),x)
```

```
[Out] e*Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/
3, True)) - I*f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3-1}x}\,dx$$

```
[In] integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*x), x)
```

Giac [F]

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3-1}x}\,dx$$

```
[In] integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.51

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = -\frac{\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\sqrt{}\right.}}$$

```
[In] int((e + f*x)/(x*(x^3 - 1)^(1/2)),x)
```

```
[Out] -((-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)
*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(f*ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
) + e*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*(-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(3^(1/2) - 3i)*1i)/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x
^3)^(1/2)
```

3.171 $\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 131

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}e\arctan\left(\sqrt{-1-x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}f(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 2/3*e*arctan((-x^3-1)^(1/2))+2/3*f*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/
2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1
/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1846, 272, 65, 210, 12, 225}

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{2-\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} + \frac{2}{3}e\arctan\left(\sqrt{-x^3-1}\right)$$

```
[In] Int[(e + f*x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*f*(1 + x)*Sqrt[(1 - x
 + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3
] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqr
t[-1 - x^3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= e\int \frac{1}{x\sqrt{-1-x^3}}\,dx + \int \frac{f}{\sqrt{-1-x^3}}\,dx \\
&= \frac{1}{3}e\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}x}\,dx, x, x^3\right) + f\int \frac{1}{\sqrt{-1-x^3}}\,dx \\
&= \frac{2\sqrt{2-\sqrt{3}}f(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} \\
&\quad - \frac{1}{3}(2e)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{-1-x^3}\right) \\
&= \frac{2}{3}e\tan^{-1}\left(\sqrt{-1-x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}f(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}e\arctan\left(\sqrt{-1-x^3}\right) + \frac{fx\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)}{\sqrt{-1-x^3}}$$

```
[In] Integrate[(e + f*x)/(x*Sqrt[-1 - x^3]),x]
[Out] (2*e*ArcTan[Sqrt[-1 - x^3]])/3 + (f*x*Sqrt[1 + x^3]*Hypergeometric2F1[1/3,
1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.98 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

method	result	s
meijerg	$-ifx\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)-\frac{ie\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}\right)}{3\sqrt{\pi}}$	5
default	$-\frac{2 i f \sqrt{3}\, \sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, \sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2 e \arctan\left(\sqrt{-x^3-1}\right)}{3}$	1
elliptic	$-\frac{2 i f \sqrt{3}\, \sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, \sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2 e \arctan\left(\sqrt{-x^3-1}\right)}{3}$	1

```
[In] int((f*x+e)/x/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*f*x*hypergeom([1/3,1/2],[4/3],-x^3)-1/3*I*e/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2
+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.27

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx=\frac{1}{3}\,e\arctan\left(\frac{(x^3+2)\sqrt{-x^3-1}}{2\,(x^3+1)}\right)-2i\,f\text{weierstrassPInverse}(0,-4,x)$$

```
[In] integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*e*arctan(1/2*(x^3 + 2)*sqrt(-x^3 - 1)/(x^3 + 1)) - 2*I*f*weierstrassPIn
verse(0, -4, x)
```

Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.35

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx=\frac{2ie\,\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{ifx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\,x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((f*x+e)/x/(-x**3-1)**(1/2),x)
```

```
[Out] 2*I*e*asinh(x**(-3/2))/3 - I*f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*
exp_polar(I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3-1}x}\,dx$$

[In] integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="maxima")

[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*x), x)

Giac [F]

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3-1}x}\,dx$$

[In] integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*x), x)

Mupad [B] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.70

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\right.\right.}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\right.}}$$

[In] int((e + f*x)/(x*(- x^3 - 1)^(1/2)),x)

[Out] ((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*(f*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)),
-((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - e*ellipticPi((3^(1/2)*1i
)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
3/2)/((3^(1/2)*1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^
(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((- x^3 - 1)^(1/2)*(x
^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))

3.172 $\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{d} - \frac{\log(c+dx)}{d} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}$$

```
[Out] -ln(d*x+c)/d+3/2*ln(d*(d*x+2*c)-d*(d^3*x^3+2*c^3)^(1/3))/d-arctan(1/3*(1+2*
(d*x+2*c)/(d^3*x^3+2*c^3)^(1/3))*3^(1/2))*3^(1/2)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2176}

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d} - \frac{\log(c+dx)}{d}$$

```
[In] Int[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/d
) - Log[c + d*x]/d + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(2*
d)
```

Rule 2176

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{d} - \frac{\log(c+dx)}{d} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}$$

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.67

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{2c^3+d^3x^3}}{4c+2dx+\sqrt[3]{2c^3+d^3x^3}}\right)}{d} + \frac{\log\left(-2c-dx+\sqrt[3]{2c^3+d^3x^3}\right)}{d} - \frac{\log\left(4c^2+4cdx+d^2x^2+(2c+dx)\sqrt[3]{2c^3+d^3x^3}+(2c^3+d^3x^3)^{2/3}\right)}{2d}$$

```
[In] Integrate[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(2*c^3 + d^3*x^3)^(1/3))/(4*c + 2*d*x + (2*c^3 + d
^3*x^3)^(1/3))])/d + Log[-2*c - d*x + (2*c^3 + d^3*x^3)^(1/3)]/d - Log[4*c^
2 + 4*c*d*x + d^2*x^2 + (2*c + d*x)*(2*c^3 + d^3*x^3)^(1/3) + (2*c^3 + d^3*
x^3)^(2/3)]/(2*d)
```

Maple [F]

$$\int \frac{-dx+c}{(dx+c)\,(d^3x^3+2c^3)^{\frac{1}{3}}}dx$$

[In] int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)

[Out] int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)

Fricas [F(-2)]

Exception generated.

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (residue poly has multiple non-linear factors)

Sympy [F]

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = -\int \left(-\frac{c}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\right)dx - \int \frac{dx}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\,dx$$

[In] integrate((-d*x+c)/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)

[Out] -Integral(-c/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**3*x**3)**(1/3)), x) - Integral(d*x/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**3*x**3)**(1/3)), x)

Maxima [F]

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int -\frac{dx-c}{(d^3x^3+2\,c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")

[Out] -integrate((d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)

Giac [F]

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int -\frac{dx-c}{(d^3x^3+2\,c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

```
[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="giac")
[Out] integrate(-(d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{c-d\,x}{(2\,c^3+d^3\,x^3)^{1/3}\,(c+d\,x)}\,dx$$

```
[In] int((c - d*x)/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)),x)
[Out] int((c - d*x)/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)), x)
```

3.173 $\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 234

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \frac{f\arctan\left(\frac{1+\frac{2dx}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt{3}(de-cf)\arctan\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd^2} + \frac{(de-cf)\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd^2} - \frac{f\log\left(-dx+\sqrt[3]{-c^3+d^3x^3}\right)}{2d^2} - \frac{3(de-cf)\log\left(d(c-dx)+2^{2/3}d\sqrt[3]{-c^3+d^3x^3}\right)}{4\sqrt[3]{2}cd^2}$$

```
[Out] 1/8*(-c*f+d*e)*ln((-d*x+c)*(d*x+c)^2)*2^(2/3)/c/d^2-1/2*f*ln(-d*x+(d^3*x^3-
c^3)^(1/3))/d^2-3/8*(-c*f+d*e)*ln(d*(-d*x+c)+2^(2/3)*d*(d^3*x^3-c^3)^(1/3))
*2^(2/3)/c/d^2+1/3*f*arctan(1/3*(1+2*d*x/(d^3*x^3-c^3)^(1/3))*3^(1/2))/d^2*
3^(1/2)+1/4*(-c*f+d*e)*arctan(1/3*(1-2^(1/3)*(-d*x+c)/(d^3*x^3-c^3)^(1/3))*
3^(1/2))*3^(1/2)*2^(2/3)/c/d^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2177, 245, 2174}

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)(de-cf)}{2\sqrt[3]{2}cd^2} + \frac{f\arctan\left(\frac{\frac{2dx}{\sqrt[3]{d^3x^3-c^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2} - \frac{3(de-cf)\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd^2} - \frac{f\log\left(\sqrt[3]{d^3x^3-c^3}-dx\right)}{2d^2} + \frac{(de-cf)\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd^2}$$

```
[In] Int[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] (f*ArcTan[(1 + (2*d*x)/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) + (S
qrt[3]*(d*e - c*f)*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/
Sqrt[3]])/(2*2^(1/3)*c*d^2) + ((d*e - c*f)*Log[(c - d*x)*(c + d*x)^2])/(4*2
^(1/3)*c*d^2) - (f*Log[-(d*x) + (-c^3 + d^3*x^3)^(1/3)])/(2*d^2) - (3*(d*e
- c*f)*Log[d*(c - d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d^
2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rubi steps

$$\text{integral} = \frac{f \int \frac{1}{\sqrt[3]{-c^3+d^3x^3}} dx}{d} + \frac{(de-cf) \int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}} dx}{d}$$

$$= \frac{f \tan^{-1}\left(\frac{1+\frac{2dx}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt{3}(de-cf)\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd^2}$$
$$+ \frac{(de-cf)\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd^2} - \frac{f\log\left(-dx+\sqrt[3]{-c^3+d^3x^3}\right)}{2d^2}$$
$$- \frac{3(de-cf)\log\left(d(c-dx)+2^{2/3}d\sqrt[3]{-c^3+d^3x^3}\right)}{4\sqrt[3]{2}cd^2}$$

Mathematica [F]

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}} dx = \int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}} dx$$

```
[In] Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
[Out] Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]
```

Maple [F]

$$\int \frac{fx+e}{(dx+c)\left(d^3x^3-c^3\right)^{\frac{1}{3}}} dx$$

```
[In] int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
[Out] int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{e+fx}{\sqrt[3]{(-c+dx)\,(c^2+cdx+d^2x^2)}\,(c+dx)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
[Out] Integral((e + f*x)/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x
)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{fx+e}{(d^3x^3-c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")
[Out] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{fx+e}{(d^3x^3-c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

```
[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="giac")
[Out] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx=\int \frac{e+f\,x}{\left(d^3\,x^3-c^3\right)^{1/3}\,\left(c+d\,x\right)}\,dx$$

```
[In] int((e + f*x)/((d^3*x^3 - c^3)^(1/3)*(c + d*x)),x)
[Out] int((e + f*x)/((d^3*x^3 - c^3)^(1/3)*(c + d*x)), x)
```

3.174 $\int x^2(a+bx)^n (c+dx^3) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 160

$$\int x^2(a+bx)^n (c+dx^3) \, dx = \frac{a^2(b^3c-a^3d)(a+bx)^{1+n}}{b^6(1+n)} - \frac{a(2b^3c-5a^3d)(a+bx)^{2+n}}{b^6(2+n)} + \frac{(b^3c-10a^3d)(a+bx)^{3+n}}{b^6(3+n)} + \frac{10a^2d(a+bx)^{4+n}}{b^6(4+n)} - \frac{5ad(a+bx)^{5+n}}{b^6(5+n)} + \frac{d(a+bx)^{6+n}}{b^6(6+n)}$$

```
[Out] a^2*(-a^3*d+b^3*c)*(b*x+a)^(1+n)/b^6/(1+n)-a*(-5*a^3*d+2*b^3*c)*(b*x+a)^(2+
n)/b^6/(2+n)+(-10*a^3*d+b^3*c)*(b*x+a)^(3+n)/b^6/(3+n)+10*a^2*d*(b*x+a)^(4+
n)/b^6/(4+n)-5*a*d*(b*x+a)^(5+n)/b^6/(5+n)+d*(b*x+a)^(6+n)/b^6/(6+n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1634}

$$\int x^2(a+bx)^n (c+dx^3) \, dx = -\frac{a(2b^3c-5a^3d)(a+bx)^{n+2}}{b^6(n+2)} + \frac{(b^3c-10a^3d)(a+bx)^{n+3}}{b^6(n+3)} + \frac{10a^2d(a+bx)^{n+4}}{b^6(n+4)} + \frac{a^2(b^3c-a^3d)(a+bx)^{n+1}}{b^6(n+1)} - \frac{5ad(a+bx)^{n+5}}{b^6(n+5)} + \frac{d(a+bx)^{n+6}}{b^6(n+6)}$$

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] (a^2*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*(2*b^3*c - 5*a^3
*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + ((b^3*c - 10*a^3*d)*(a + b*x)^(3 + n
))/(b^6*(3 + n)) + (10*a^2*d*(a + b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d*(a +
 b*x)^(5 + n))/(b^6*(5 + n)) + (d*(a + b*x)^(6 + n))/(b^6*(6 + n))
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a^2b^3c - a^5d)(a+bx)^n}{b^5} + \frac{a(-2b^3c + 5a^3d)(a+bx)^{1+n}}{b^5} + \frac{(b^3c - 10a^3d)(a+bx)^{2+n}}{b^5} + \frac{10a^2d(a+bx)^{3+n}}{b^5} - \frac{5ad(a+bx)^{4+n}}{b^5} + \frac{d(a+bx)^{5+n}}{b^5}\right) dx$$

$$= \frac{a^2(b^3c - a^3d)(a+bx)^{1+n}}{b^6(1+n)} - \frac{a(2b^3c - 5a^3d)(a+bx)^{2+n}}{b^6(2+n)} + \frac{(b^3c - 10a^3d)(a+bx)^{3+n}}{b^6(3+n)} + \frac{10a^2d(a+bx)^{4+n}}{b^6(4+n)} - \frac{5ad(a+bx)^{5+n}}{b^6(5+n)} + \frac{d(a+bx)^{6+n}}{b^6(6+n)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.83

$$\int x^2(a+bx)^n (c+dx^3) \, dx = \frac{(a+bx)^{1+n} \left(\frac{a^2b^3c - a^5d}{1+n} + \frac{a(-2b^3c+5a^3d)(a+bx)}{2+n} + \frac{(b^3c-10a^3d)(a+bx)^2}{3+n} + \frac{10a^2d(a+bx)^3}{4+n} - \frac{5ad(a+bx)^4}{5+n} + \frac{d(a+bx)^5}{6+n}\right)}{b^6}$$

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*b^3*c - a^5*d)/(1 + n) + (a*(-2*b^3*c + 5*a^3*d)*(
a + b*x))/(2 + n) + ((b^3*c - 10*a^3*d)*(a + b*x)^2)/(3 + n) + (10*a^2*d*(a
 + b*x)^3)/(4 + n) - (5*a*d*(a + b*x)^4)/(5 + n) + (d*(a + b*x)^5)/(6 + n))
)/b^6
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(160) = 320.

Time = 0.90 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.59

method	result
norman	$\frac{d\,x^{6} e^{n \ln(bx+a)}}{6+n}+\frac{(b^{3} c\,n^{3}+15 b^{3} c\,n^{2}+20 a^{3} d n+74 b^{3} c n+120 b^{3} c) x^{3} e^{n \ln(bx+a)}}{b^{3}(n^{4}+18 n^{3}+119 n^{2}+342 n+360)}+\frac{a d n\,x^{5} e^{n \ln(bx+a)}}{b(n^{2}+11 n+30)}-\frac{2 a^{3}(-b^{3} c\,n^{3}-15 b^{3} c\,n^{2}}{b^{6}(n^{6}+21 n^{5}+175}$
gosper	$-\frac{(bx+a)^{1+n}(-b^{5} d\,n^{5} x^{5}-15 b^{5} d\,n^{4} x^{5}+5 a\,b^{4} d\,n^{4} x^{4}-85 b^{5} d\,n^{3} x^{5}+50 a\,b^{4} d\,n^{3} x^{4}-b^{5} c\,n^{5} x^{2}-225 b^{5} d\,n^{2} x^{5}-20 a^{2} b^{3} d\,n^{3} x^{3}+175 a\,b}{}$
risch	$-\frac{(-b^{6} d\,n^{5} x^{6}-a\,b^{5} d\,n^{5} x^{5}-15 b^{6} d\,n^{4} x^{6}-10 a\,b^{5} d\,n^{4} x^{5}-85 b^{6} d\,n^{3} x^{6}+5 a^{2} b^{4} d\,n^{4} x^{4}-35 a\,b^{5} d\,n^{3} x^{5}-b^{6} c\,n^{5} x^{3}-225 b^{6} d\,n^{2} x^{6}+30 a^{2}}{}$
parallelrisch	$\frac{121 x^{3}(bx+a)^{n} a\,b^{6} c\,n^{3}+16 x^{2}(bx+a)^{n} a^{2} b^{5} c\,n^{4}-30 x^{4}(bx+a)^{n} a^{3} b^{4} d n+60 x^{3}(bx+a)^{n} a^{4} b^{3} d\,n^{2}+372 x^{3}(bx+a)^{n} a\,b^{6} c\,n^{2}+89 x^{2}(bx}{}$

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] d/(6+n)*x^6*exp(n*ln(b*x+a))+(b^3*c*n^3+15*b^3*c*n^2+20*a^3*d*n+74*b^3*c*n+
120*b^3*c)/b^3/(n^4+18*n^3+119*n^2+342*n+360)*x^3*exp(n*ln(b*x+a))+a*d*n/b/
(n^2+11*n+30)*x^5*exp(n*ln(b*x+a))-2*a^3*(-b^3*c*n^3-15*b^3*c*n^2-74*b^3*c*
n+60*a^3*d-120*b^3*c)/b^6/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)*
exp(n*ln(b*x+a))+2/b^5*n*a^2*(-b^3*c*n^3-15*b^3*c*n^2-74*b^3*c*n+60*a^3*d-1
20*b^3*c)/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)*x*exp(n*ln(b*x+a
))-5*n*d*a^2/b^2/(n^3+15*n^2+74*n+120)*x^4*exp(n*ln(b*x+a))-(-b^3*c*n^3-15*
b^3*c*n^2-74*b^3*c*n+60*a^3*d-120*b^3*c)*a/b^4*n/(n^5+20*n^4+155*n^3+580*n^
2+1044*n+720)*x^2*exp(n*ln(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(160) = 320.

Time = 0.29 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.06

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx$$
$$=\frac{(2\,a^3b^3cn^3+30\,a^3b^3cn^2+148\,a^3b^3cn+240\,a^3b^3c-120\,a^6d+(b^6dn^5+15\,b^6dn^4+85\,b^6dn^3+225\,b^6dn^2+}{}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] (2*a^3*b^3*c*n^3 + 30*a^3*b^3*c*n^2 + 148*a^3*b^3*c*n + 240*a^3*b^3*c - 120
*a^6*d + (b^6*d*n^5 + 15*b^6*d*n^4 + 85*b^6*d*n^3 + 225*b^6*d*n^2 + 274*b^6
*d*n + 120*b^6*d)*x^6 + (a*b^5*d*n^5 + 10*a*b^5*d*n^4 + 35*a*b^5*d*n^3 + 50
*a*b^5*d*n^2 + 24*a*b^5*d*n)*x^5 - 5*(a^2*b^4*d*n^4 + 6*a^2*b^4*d*n^3 + 11*
a^2*b^4*d*n^2 + 6*a^2*b^4*d*n)*x^4 + (b^6*c*n^5 + 18*b^6*c*n^4 + 240*b^6*c
+ (121*b^6*c + 20*a^3*b^3*d)*n^3 + 12*(31*b^6*c + 5*a^3*b^3*d)*n^2 + 4*(127
*b^6*c + 10*a^3*b^3*d)*n)*x^3 + (a*b^5*c*n^5 + 16*a*b^5*c*n^4 + 89*a*b^5*c*
```

```
n^3 + 2*(97*a*b^5*c - 30*a^4*b^2*d)*n^2 + 60*(2*a*b^5*c - a^4*b^2*d)*n)*x^2
 - 2*(a^2*b^4*c*n^4 + 15*a^2*b^4*c*n^3 + 74*a^2*b^4*c*n^2 + 60*(2*a^2*b^4*c
 - a^5*b*d)*n)*x)*(b*x + a)^n/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6
*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6397 vs. $2(144) = 288$.

Time = 2.03 (sec) , antiderivative size = 6397, normalized size of antiderivative = 39.98

$$\int x^2(a+bx)^n \left(c+dx^3\right) dx = \text{Too large to display}$$

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x**3/3 + d*x**6/6), Eq(b, 0)), (60*a**5*d*log(a/b + x)/(
60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 +
300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5*d/(60*a**5*b**6 + 300*a**4*b**
7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11
*x**5) + 300*a**4*b*d*x*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*
a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 6
25*a**4*b*d*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a*
*2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*d*x**2*log
(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b
**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*d*x**2/(60*a*
*5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a
*b**10*x**4 + 60*b**11*x**5) - 2*a**2*b**3*c/(60*a**5*b**6 + 300*a**4*b**7*
x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x
**5) + 600*a**2*b**3*d*x**3*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x +
600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5)
 + 900*a**2*b**3*d*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**
2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 10*a*b**4*c*x/
(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 +
 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d*x**4*log(a/b + x)/(60*a**
5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*
b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d*x**4/(60*a**5*b**6 + 300*a**4*b*
*7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**1
1*x**5) - 20*b**5*c*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x*
*2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 60*b**5*d*x**
5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a
**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-60*a**5*d*
log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9
*x**3 + 12*b**10*x**4) - 125*a**5*d/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**
2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**4*b*d*x*log(a/b + x)
/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b
```

10*x4) - 440*a**4*b*d*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x
2 + 48*a*b9*x**3 + 12*b**10*x**4) - 360*a**3*b**2*d*x**2*log(a/b + x)/(
12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**
10*x**4) - 540*a**3*b**2*d*x**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b*
*8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - a**2*b**3*c/(12*a**4*b**6 + 48*
a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**
2*b**3*d*x**3*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x*
*2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*d*x**3/(12*a**4*b**6 +
48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 4*a
*b**4*c*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x*
*3 + 12*b**10*x**4) - 60*a*b**4*d*x**4*log(a/b + x)/(12*a**4*b**6 + 48*a**3
*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 6*b**5*c*x*
*2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12
*b**10*x**4) + 12*b**5*d*x**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8
*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4), Eq(n, -5)), (60*a**5*d*log(a/b + x
)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 110*a**5*
d/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**4*
b*d*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*
x**3) + 270*a**4*b*d*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b
9*x3) + 180*a**3*b**2*d*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x
+ 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*d*x**2/(6*a**3*b**6 + 18*a
2*b7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 2*a**2*b**3*c/(6*a**3*b**6 + 1
8*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*d*x**3*log(a/b
+ x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 6*a*b
4*c*x/(6*a3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*
a*b**4*d*x**4/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3)
- 6*b**5*c*x**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x*
*3) + 3*b**5*d*x**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9
*x**3), Eq(n, -4)), (-60*a**5*d*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6
*b**8*x**2) - 90*a**5*d/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**
4*b*d*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b
*d*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 60*a**3*b**2*d*x**2*log(a/
b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 6*a**2*b**3*c*log(a/b +
x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 9*a**2*b**3*c/(6*a**2*b**6 +
12*a*b**7*x + 6*b**8*x**2) + 20*a**2*b**3*d*x**3/(6*a**2*b**6 + 12*a*b**7*
x + 6*b**8*x**2) + 12*a*b**4*c*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x +
6*b**8*x**2) + 12*a*b**4*c*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 5*
a*b**4*d*x**4/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 6*b**5*c*x**2*log
(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 2*b**5*d*x**5/(6*a**2
*b**6 + 12*a*b**7*x + 6*b**8*x**2), Eq(n, -3)), (60*a**5*d*log(a/b + x)/(12
*a*b**6 + 12*b**7*x) + 60*a**5*d/(12*a*b**6 + 12*b**7*x) + 60*a**4*b*d*x*lo
g(a/b + x)/(12*a*b**6 + 12*b**7*x) - 30*a**3*b**2*d*x**2/(12*a*b**6 + 12*b*
*7*x) - 24*a**2*b**3*c*log(a/b + x)/(12*a*b**6 + 12*b**7*x) - 24*a**2*b**3*
c/(12*a*b**6 + 12*b**7*x) + 10*a**2*b**3*d*x**3/(12*a*b**6 + 12*b**7*x) - 2
4*a*b**4*c*x*log(a/b + x)/(12*a*b**6 + 12*b**7*x) - 5*a*b**4*d*x**4/(12*a*b

6 + 12*b7*x) + 12*b**5*c*x**2/(12*a*b**6 + 12*b**7*x) + 3*b**5*d*x**5/(
12*a*b**6 + 12*b**7*x), Eq(n, -2)), (-a**5*d*log(a/b + x)/b**6 + a**4*d*x/b
5 - a3*d*x**2/(2*b**4) + a**2*c*log(a/b + x)/b**3 + a**2*d*x**3/(3*b**3
) - a*c*x/b**2 - a*d*x**4/(4*b**2) + c*x**2/(2*b) + d*x**5/(5*b), Eq(n, -1)
), (-120*a**6*d*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*a**5*b*d*n*x*(
a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*d*n**2*x**2*(a + b*x)*
*n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n*
*2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*d*n*x**2*(a + b*x)**n/(b**6*n**
6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b*
*6*n + 720*b**6) + 2*a**3*b**3*c*n**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**
5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6
) + 30*a**3*b**3*c*n**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n
4 + 735*b6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 148*a**3*b
3*c*n*(a + b*x)n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n
3 + 1624*b6*n**2 + 1764*b**6*n + 720*b**6) + 240*a**3*b**3*c*(a + b*x)*
*n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n*
*2 + 1764*b**6*n + 720*b**6) + 20*a**3*b**3*d*n**3*x**3*(a + b*x)**n/(b**6*
n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764
*b**6*n + 720*b**6) + 60*a**3*b**3*d*n**2*x**3*(a + b*x)**n/(b**6*n**6 + 21
*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) + 40*a**3*b**3*d*n*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 +
175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) -
2*a**2*b**4*c*n**4*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**
4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4
*c*n**3*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6
*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 148*a**2*b**4*c*n**2*x*(
a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) - 240*a**2*b**4*c*n*x*(a + b*x)**n/(b
6*n6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 +
1764*b**6*n + 720*b**6) - 5*a**2*b**4*d*n**4*x**4*(a + b*x)**n/(b**6*n**6 +
21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*
n + 720*b**6) - 30*a**2*b**4*d*n**3*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*
n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b
6) - 55*a2*b**4*d*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30
*a**2*b**4*d*n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4
+ 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b**5*c*n**5*
x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
+ 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 16*a*b**5*c*n**4*x**2*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 89*a*b**5*c*n**3*x**2*(a + b*x)**n/(b**6*
n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764
*b**6*n + 720*b**6) + 194*a*b**5*c*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b

```
**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 7
20*b**6) + 120*a*b**5*c*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175
*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b
**5*d*n**5*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 10*a*b**5*d*n**4*x
**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
+ 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 35*a*b**5*d*n**3*x**5*(a + b*x
)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*
n**2 + 1764*b**6*n + 720*b**6) + 50*a*b**5*d*n**2*x**5*(a + b*x)**n/(b**6*n
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 24*a*b**5*d*n*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n
**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b*
*6) + b**6*c*n**5*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n*
*4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 18*b**6*c*n
**4*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*
n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 121*b**6*c*n**3*x**3*(a +
 b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b
**6*n**2 + 1764*b**6*n + 720*b**6) + 372*b**6*c*n**2*x**3*(a + b*x)**n/(b**
6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 17
64*b**6*n + 720*b**6) + 508*b**6*c*n*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6
*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*
b**6) + 240*b**6*c*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n
**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*d*n**
5*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n*
*3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 15*b**6*d*n**4*x**6*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 85*b**6*d*n**3*x**6*(a + b*x)**n/(b**6*n*
*6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b
**6*n + 720*b**6) + 225*b**6*d*n**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*
n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b
**6) + 274*b**6*d*n*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*
n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*b**6*
d*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n*
*3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.58

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx$$
$$=\frac{((n^2+3\,n+2)b^3x^3+(n^2+n)ab^2x^2-2\,a^2bnx+2\,a^3)(bx+a)^nc}{(n^3+6\,n^2+11\,n+6)b^3}+\frac{((n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^6x^6+(n^5+10\,n^4+35\,n^3+50\,n^2+24\,n)ab^5x^5-5\,(n^4+}{(n^6+21\,n^5+175\,n^4+73}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2
 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5
 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*
b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*d
/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. 2(160) = 320.

Time = 0.31 (sec) , antiderivative size = 835, normalized size of antiderivative = 5.22

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx$$
$$=\frac{(bx+a)^nb^6dn^5x^6+(bx+a)^nab^5dn^5x^5+15\,(bx+a)^nb^6dn^4x^6+10\,(bx+a)^nab^5dn^4x^5+85\,(bx+a)^nb^6dn}{}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^6*d*n^5*x^6 + (b*x + a)^n*a*b^5*d*n^5*x^5 + 15*(b*x + a)^n*b
^6*d*n^4*x^6 + 10*(b*x + a)^n*a*b^5*d*n^4*x^5 + 85*(b*x + a)^n*b^6*d*n^3*x^
6 + (b*x + a)^n*b^6*c*n^5*x^3 - 5*(b*x + a)^n*a^2*b^4*d*n^4*x^4 + 35*(b*x +
 a)^n*a*b^5*d*n^3*x^5 + 225*(b*x + a)^n*b^6*d*n^2*x^6 + (b*x + a)^n*a*b^5*c
*n^5*x^2 + 18*(b*x + a)^n*b^6*c*n^4*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n^3*x^4
+ 50*(b*x + a)^n*a*b^5*d*n^2*x^5 + 274*(b*x + a)^n*b^6*d*n*x^6 + 16*(b*x +
a)^n*a*b^5*c*n^4*x^2 + 121*(b*x + a)^n*b^6*c*n^3*x^3 + 20*(b*x + a)^n*a^3*b
^3*d*n^3*x^3 - 55*(b*x + a)^n*a^2*b^4*d*n^2*x^4 + 24*(b*x + a)^n*a*b^5*d*n*
x^5 + 120*(b*x + a)^n*b^6*d*x^6 - 2*(b*x + a)^n*a^2*b^4*c*n^4*x + 89*(b*x +
 a)^n*a*b^5*c*n^3*x^2 + 372*(b*x + a)^n*b^6*c*n^2*x^3 + 60*(b*x + a)^n*a^3*
b^3*d*n^2*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n*x^4 - 30*(b*x + a)^n*a^2*b^4*c*n
^3*x + 194*(b*x + a)^n*a*b^5*c*n^2*x^2 - 60*(b*x + a)^n*a^4*b^2*d*n^2*x^2 +
 508*(b*x + a)^n*b^6*c*n*x^3 + 40*(b*x + a)^n*a^3*b^3*d*n*x^3 + 2*(b*x + a)
^n*a^3*b^3*c*n^3 - 148*(b*x + a)^n*a^2*b^4*c*n^2*x + 120*(b*x + a)^n*a*b^5*
```

```
c*n*x^2 - 60*(b*x + a)^n*a^4*b^2*d*n*x^2 + 240*(b*x + a)^n*b^6*c*x^3 + 30*(
b*x + a)^n*a^3*b^3*c*n^2 - 240*(b*x + a)^n*a^2*b^4*c*n*x + 120*(b*x + a)^n*
a^5*b*d*n*x + 148*(b*x + a)^n*a^3*b^3*c*n + 240*(b*x + a)^n*a^3*b^3*c - 120
*(b*x + a)^n*a^6*d)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 + 162
4*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

Mupad [B] (verification not implemented)

Time = 19.82 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.09

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx = (a+b\,x)^n\left(\frac{d\,x^6\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}{n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720} + \frac{2\,a^3\,(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3)}{b^6\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)} + \frac{x^3\,(n^2+3\,n+2)\,(20\,d\,a^3\,n+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3)}{b^3\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)} - \frac{2\,a^2\,n\,x\,(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3)}{b^5\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)} + \frac{a\,d\,n\,x^5\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{b\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)} + \frac{a\,n\,x^2\,(n+1)\,(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3)}{b^4\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)} - \frac{5\,a^2\,d\,n\,x^4\,(n^3+6\,n^2+11\,n+6)}{b^2\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}\right)$$

```
[In] int(x^2*(c + d*x^3)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^6*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(1764*
n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) + (2*a^3*(120*b^3*c
- 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n^3 + 74*b^3*c*n))/(b^6*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (x^3*(3*n + n^2 + 2)*(120*b^
3*c + 15*b^3*c*n^2 + b^3*c*n^3 + 20*a^3*d*n + 74*b^3*c*n))/(b^3*(1764*n + 1
624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (2*a^2*n*x*(120*b^3*c
- 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n^3 + 74*b^3*c*n))/(b^5*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*d*n*x^5*(50*n + 35*n^2 +
10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^
6 + 720)) + (a*n*x^2*(n + 1)*(120*b^3*c - 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n
^3 + 74*b^3*c*n))/(b^4*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^
6 + 720)) - (5*a^2*d*n*x^4*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)))
```

3.175 $\int x(a+bx)^n \left(c+dx^3\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int x(a+bx)^n \left(c+dx^3\right) dx = -\frac{a(b^3c-a^3d)(a+bx)^{1+n}}{b^5(1+n)} + \frac{(b^3c-4a^3d)(a+bx)^{2+n}}{b^5(2+n)} + \frac{6a^2d(a+bx)^{3+n}}{b^5(3+n)} - \frac{4ad(a+bx)^{4+n}}{b^5(4+n)} + \frac{d(a+bx)^{5+n}}{b^5(5+n)}$$

```
[Out] -a*(-a^3*d+b^3*c)*(b*x+a)^(1+n)/b^5/(1+n)+(-4*a^3*d+b^3*c)*(b*x+a)^(2+n)/b^
5/(2+n)+6*a^2*d*(b*x+a)^(3+n)/b^5/(3+n)-4*a*d*(b*x+a)^(4+n)/b^5/(4+n)+d*(b*
x+a)^(5+n)/b^5/(5+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1634}

$$\int x(a+bx)^n \left(c+dx^3\right) dx = -\frac{a(b^3c-a^3d)(a+bx)^{n+1}}{b^5(n+1)} + \frac{(b^3c-4a^3d)(a+bx)^{n+2}}{b^5(n+2)} + \frac{6a^2d(a+bx)^{n+3}}{b^5(n+3)} - \frac{4ad(a+bx)^{n+4}}{b^5(n+4)} + \frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

```
[In] Int[x*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] -((a*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^5*(1 + n))) + ((b^3*c - 4*a^3*d)
*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (6*a^2*d*(a + b*x)^(3 + n))/(b^5*(3 + n
)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)^(5 + n))/(b^5*(
5 + n))
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a(-b^3c+a^3d)\,(a+bx)^n}{b^4} + \frac{(b^3c-4a^3d)\,(a+bx)^{1+n}}{b^4} + \frac{6a^2d(a+bx)^{2+n}}{b^4} - \frac{4ad(a+bx)^{3+n}}{b^4} + \frac{d(a+bx)^{4+n}}{b^4}\right) dx$$

$$= -\frac{a(b^3c-a^3d)\,(a+bx)^{1+n}}{b^5(1+n)} + \frac{(b^3c-4a^3d)\,(a+bx)^{2+n}}{b^5(2+n)} + \frac{6a^2d(a+bx)^{3+n}}{b^5(3+n)} - \frac{4ad(a+bx)^{4+n}}{b^5(4+n)} + \frac{d(a+bx)^{5+n}}{b^5(5+n)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int x(a+bx)^n\left(c+dx^3\right)\,dx = \frac{(a+bx)^{1+n}\left(\frac{a(-b^3c+a^3d)}{1+n} + \frac{(b^3c-4a^3d)(a+bx)}{2+n} + \frac{6a^2d(a+bx)^2}{3+n} - \frac{4ad(a+bx)^3}{4+n} + \frac{d(a+bx)^4}{5+n}\right)}{b^5}$$

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3),x]
[Out] ((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d))/(1 + n) + ((b^3*c - 4*a^3*d)*(a
+ b*x))/(2 + n) + (6*a^2*d*(a + b*x)^2)/(3 + n) - (4*a*d*(a + b*x)^3)/(4 +
n) + (d*(a + b*x)^4)/(5 + n)))/b^5
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(126) = 252.

Time = 0.89 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.25

method	result
gosper	$\frac{(bx+a)^{1+n}\left(b^4 d\,n^4 x^4+10b^4 d\,n^3 x^4-4a\,b^3 d\,n^3 x^3+35b^4 d\,n^2 x^4-24a\,b^3 d\,n^2 x^3+b^4 c\,n^4 x+50b^4 dn\,x^4+12a^2 b^2 d\,n^2 x^2-44a\,b^3 dn\,x^3\right.}{b}$
norman	$\frac{d\,x^5 \mathrm{e}^{n\ln(bx+a)}}{5+n}+\frac{a^2\left(-b^3 c\,n^3-12b^3 c\,n^2-47b^3 cn+24a^3 d-60b^3 c\right)\mathrm{e}^{n\ln(bx+a)}}{b^5\left(n^5+15n^4+85n^3+225n^2+274n+120\right)}+\frac{\left(b^3 c\,n^3+12b^3 c\,n^2+12a^3 dn+47b^3 cn+60b^3 c\right)}{b^3\left(n^4+14n^3+71n^2+154n+120\right.}$
risch	$\frac{\left(b^5 d\,n^4 x^5+a\,b^4 d\,n^4 x^4+10b^5 d\,n^3 x^5+6a\,b^4 d\,n^3 x^4+35b^5 d\,n^2 x^5-4a^2 b^3 d\,n^3 x^3+11a\,b^4 d\,n^2 x^4+b^5 c\,n^4 x^2+50b^5 dn\,x^5-12a^2 b^3 d\,n^2\right.}{}$
parallelrisch	$\frac{x^5(bx+a)^n b^5 d\,n^4+10x^5(bx+a)^n b^5 d\,n^3+35x^5(bx+a)^n b^5 d\,n^2+50x^5(bx+a)^n b^5 dn+x^2(bx+a)^n b^5 c\,n^4+13x^2(bx+a)^n b^5 c\,n^3+59}{}$

```
[In] int(x*(b*x+a)^n*(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(b*x+a)^(1+n)/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)*(b^4*d*n^4*x^4+10
*b^4*d*n^3*x^4-4*a*b^3*d*n^3*x^3+35*b^4*d*n^2*x^4-24*a*b^3*d*n^2*x^3+b^4*c*
n^4*x+50*b^4*d*n*x^4+12*a^2*b^2*d*n^2*x^2-44*a*b^3*d*n*x^3+13*b^4*c*n^3*x+2
4*b^4*d*x^4+36*a^2*b^2*d*n*x^2-a*b^3*c*n^3-24*a*b^3*d*x^3+59*b^4*c*n^2*x-24
*a^3*b*d*n*x+24*a^2*b^2*d*x^2-12*a*b^3*c*n^2+107*b^4*c*n*x-24*a^3*b*d*x-47*
a*b^3*c*n+60*b^4*c*x+24*a^4*d-60*a*b^3*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(126) = 252$.

Time = 0.34 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.76

$$\int x(a+bx)^n \left(c+dx^3\right)\,dx =$$

$$-\frac{\left(a^2b^3cn^3+12\,a^2b^3cn^2+47\,a^2b^3cn+60\,a^2b^3c-24\,a^5d-\left(b^5dn^4+10\,b^5dn^3+35\,b^5dn^2+50\,b^5dn+24\,b\right.\right.}{}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -(a^2*b^3*c*n^3 + 12*a^2*b^3*c*n^2 + 47*a^2*b^3*c*n + 60*a^2*b^3*c - 24*a^5
*d - (b^5*d*n^4 + 10*b^5*d*n^3 + 35*b^5*d*n^2 + 50*b^5*d*n + 24*b^5*d)*x^5
- (a*b^4*d*n^4 + 6*a*b^4*d*n^3 + 11*a*b^4*d*n^2 + 6*a*b^4*d*n)*x^4 + 4*(a^2
*b^3*d*n^3 + 3*a^2*b^3*d*n^2 + 2*a^2*b^3*d*n)*x^3 - (b^5*c*n^4 + 13*b^5*c*n
^3 + 60*b^5*c + (59*b^5*c + 12*a^3*b^2*d)*n^2 + (107*b^5*c + 12*a^3*b^2*d)*
n)*x^2 - (a*b^4*c*n^4 + 12*a*b^4*c*n^3 + 47*a*b^4*c*n^2 + 12*(5*a*b^4*c - 2
*a^4*b*d)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^
2 + 274*b^5*n + 120*b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3704 vs. $2(112) = 224$.

Time = 1.30 (sec) , antiderivative size = 3704, normalized size of antiderivative = 29.40

$$\int x(a+bx)^n\left(c+dx^3\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x**2/2 + d*x**5/5), Eq(b, 0)), (12*a**4*d*log(a/b + x)/(
12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**
9*x**4) + 25*a**4*d/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48
*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d*x*log(a/b + x)/(12*a**4*b**5 + 4
8*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 88*a**
3*b*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
 + 12*b**9*x**4) + 72*a**2*b**2*d*x**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3
*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 108*a**2*b**
2*d*x**2/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**
3 + 12*b**9*x**4) - a*b**3*c/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*
x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d*x**3*log(a/b + x)/(12*a
**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x*
*4) + 48*a*b**3*d*x**3/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 +
 48*a*b**8*x**3 + 12*b**9*x**4) - 4*b**4*c*x/(12*a**4*b**5 + 48*a**3*b**6*x
 + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 12*b**4*d*x**4*log(
a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**
3 + 12*b**9*x**4), Eq(n, -5)), (-24*a**4*d*log(a/b + x)/(6*a**3*b**5 + 18*a
**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 44*a**4*d/(6*a**3*b**5 + 18*a*
*2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a**3*b*d*x*log(a/b + x)/(6*a
**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 108*a**3*b*d*x/
(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a**2*b**
2*d*x**2*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b*
*8*x**3) - 72*a**2*b**2*d*x**2/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x*
*2 + 6*b**8*x**3) - a*b**3*c/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2
 + 6*b**8*x**3) - 24*a*b**3*d*x**3*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6
*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 3*b**4*c*x/(6*a**3*b**5 + 18*a**2*b**6
*x + 18*a*b**7*x**2 + 6*b**8*x**3) + 6*b**4*d*x**4/(6*a**3*b**5 + 18*a**2*b
**6*x + 18*a*b**7*x**2 + 6*b**8*x**3), Eq(n, -4)), (12*a**4*d*log(a/b + x)/
(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 18*a**4*d/(2*a**2*b**5 + 4*a*b**
6*x + 2*b**7*x**2) + 24*a**3*b*d*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x +
 2*b**7*x**2) + 24*a**3*b*d*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12
*a**2*b**2*d*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - a
*b**3*c/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - 4*a*b**3*d*x**3/(2*a**2*
b**5 + 4*a*b**6*x + 2*b**7*x**2) - 2*b**4*c*x/(2*a**2*b**5 + 4*a*b**6*x + 2
*b**7*x**2) + b**4*d*x**4/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2), Eq(n, -
```

3)), (-12*a**4*d*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 12*a**4*d/(3*a*b**5 + 3*b**6*x) - 12*a**3*b*d*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + 6*a**2*b**2*d*x**2/(3*a*b**5 + 3*b**6*x) + 3*a*b**3*c*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + 3*a*b**3*c/(3*a*b**5 + 3*b**6*x) - 2*a*b**3*d*x**3/(3*a*b**5 + 3*b**6*x) + 3*b**4*c*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + b**4*d*x**4/(3*a*b**5 + 3*b**6*x), Eq(n, -2)), (a**4*d*log(a/b + x)/b**5 - a**3*d*x/b**4 + a**2*d*x**2/(2*b**3) - a*c*log(a/b + x)/b**2 - a*d*x**3/(3*b**2) + c*x/b + d*x**4/(4*b), Eq(n, -1)), (24*a**5*d*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a**4*b*d*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - a**2*b**3*c*n**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 12*a**2*b**3*c*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 47*a**2*b**3*c*n*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 60*a**2*b**3*c*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*a**2*b**3*d*n**3*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 12*a**2*b**3*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 8*a**2*b**3*d*n*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*c*n**4*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a*b**4*c*n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 47*a*b**4*c*n**2*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 60*a*b**4*c*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*d*n**4*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d*n**3*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 11*a*b**4*d*n**2*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d*n*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*c*n**4*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 13*b**5*c*n**3*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 59*b**5*c*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 107*b**5*c*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 60*b**5*c*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*d*n**4*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**

```
5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 10*b**5*d*
n**3*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*
n**2 + 274*b**5*n + 120*b**5) + 35*b**5*d*n**2*x**5*(a + b*x)**n/(b**5*n**5
 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 5
0*b**5*d*n*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225
*b**5*n**2 + 274*b**5*n + 120*b**5) + 24*b**5*d*x**5*(a + b*x)**n/(b**5*n**
5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5), T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.46

$$\int x(a+bx)^n\left(c+dx^3\right)\,dx=\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^nc}{(n^2+3\,n+2)b^2}+\frac{((n^4+10\,n^3+35\,n^2+50\,n+24)b^5x^5+(n^4+6\,n^3+11\,n^2+6\,n)ab^4x^4-4\,(n^3+3\,n^2+2\,n)a^2b^3x^3+12}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^5}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c/((n^2 + 3*n + 2)*b^2) + ((n
^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*
b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24
*a^4*b*n*x + 24*a^5)*(b*x + a)^n*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*
n + 120)*b^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(126) = 252.

Time = 0.40 (sec) , antiderivative size = 577, normalized size of antiderivative = 4.58

$$\int x(a+bx)^n\left(c+dx^3\right)\,dx=\frac{(bx+a)^nb^5dn^4x^5+(bx+a)^nab^4dn^4x^4+10\,(bx+a)^nb^5dn^3x^5+6\,(bx+a)^nab^4dn^3x^4+35\,(bx+a)^nb^5dn^2x}{}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*d*n^4*x^5 + (b*x + a)^n*a*b^4*d*n^4*x^4 + 10*(b*x + a)^n*b
^5*d*n^3*x^5 + 6*(b*x + a)^n*a*b^4*d*n^3*x^4 + 35*(b*x + a)^n*b^5*d*n^2*x^5
 + (b*x + a)^n*b^5*c*n^4*x^2 - 4*(b*x + a)^n*a^2*b^3*d*n^3*x^3 + 11*(b*x +
a)^n*a*b^4*d*n^2*x^4 + 50*(b*x + a)^n*b^5*d*n*x^5 + (b*x + a)^n*a*b^4*c*n^4
*x + 13*(b*x + a)^n*b^5*c*n^3*x^2 - 12*(b*x + a)^n*a^2*b^3*d*n^2*x^3 + 6*(b
```

```
*x + a)^n*a*b^4*d*n*x^4 + 24*(b*x + a)^n*b^5*d*x^5 + 12*(b*x + a)^n*a*b^4*c
*n^3*x + 59*(b*x + a)^n*b^5*c*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n^2*x^2 -
8*(b*x + a)^n*a^2*b^3*d*n*x^3 - (b*x + a)^n*a^2*b^3*c*n^3 + 47*(b*x + a)^n*
a*b^4*c*n^2*x + 107*(b*x + a)^n*b^5*c*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n*x^
2 - 12*(b*x + a)^n*a^2*b^3*c*n^2 + 60*(b*x + a)^n*a*b^4*c*n*x - 24*(b*x + a
)^n*a^4*b*d*n*x + 60*(b*x + a)^n*b^5*c*x^2 - 47*(b*x + a)^n*a^2*b^3*c*n - 6
0*(b*x + a)^n*a^2*b^3*c + 24*(b*x + a)^n*a^5*d)/(b^5*n^5 + 15*b^5*n^4 + 85*
b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Mupad [B] (verification not implemented)

Time = 19.17 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.88

$$\int x(a+bx)^n\,(c+dx^3)\,dx = (a+b\,x)^n\left(\frac{d\,x^5\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120} - \frac{a^2\,(-24\,d\,a^3+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b^3)}{b^5\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)} + \frac{x^2\,(n+1)\,(12\,d\,a^3\,n+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b^3)}{b^3\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)} + \frac{a\,n\,x\,(-24\,d\,a^3+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b^3)}{b^4\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)} + \frac{a\,d\,n\,x^4\,(n^3+6\,n^2+11\,n+6)}{b\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)} - \frac{4\,a^2\,d\,n\,x^3\,(n^2+3\,n+2)}{b^2\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}\right)$$

```
[In] int(x*(c + d*x^3)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^5*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*n + 225*n^2 +
 85*n^3 + 15*n^4 + n^5 + 120) - (a^2*(60*b^3*c - 24*a^3*d + 12*b^3*c*n^2 +
b^3*c*n^3 + 47*b^3*c*n))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 12
0)) + (x^2*(n + 1)*(60*b^3*c + 12*b^3*c*n^2 + b^3*c*n^3 + 12*a^3*d*n + 47*b
^3*c*n))/(b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*n*x*(60
*b^3*c - 24*a^3*d + 12*b^3*c*n^2 + b^3*c*n^3 + 47*b^3*c*n))/(b^4*(274*n + 2
25*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d*n*x^4*(11*n + 6*n^2 + n^3 + 6
))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (4*a^2*d*n*x^3*(3*
n + n^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

3.176 $\int (a+bx)^n (c+dx^3) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int (a+bx)^n (c+dx^3) \, dx = \frac{(b^3c-a^3d)(a+bx)^{1+n}}{b^4(1+n)} + \frac{3a^2d(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad(a+bx)^{3+n}}{b^4(3+n)} + \frac{d(a+bx)^{4+n}}{b^4(4+n)}$$

```
[Out] (-a^3*d+b^3*c)*(b*x+a)^(1+n)/b^4/(1+n)+3*a^2*d*(b*x+a)^(2+n)/b^4/(2+n)-3*a*
d*(b*x+a)^(3+n)/b^4/(3+n)+d*(b*x+a)^(4+n)/b^4/(4+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1864}

$$\int (a+bx)^n (c+dx^3) \, dx = \frac{(b^3c-a^3d)(a+bx)^{n+1}}{b^4(n+1)} + \frac{3a^2d(a+bx)^{n+2}}{b^4(n+2)} - \frac{3ad(a+bx)^{n+3}}{b^4(n+3)} + \frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

```
[In] Int[(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2 +
 n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*x)
^(4 + n))/(b^4*(4 + n))
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
```

```
, 0] || EqQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{(b^3c - a^3d)(a + bx)^n}{b^3} + \frac{3a^2d(a + bx)^{1+n}}{b^3} - \frac{3ad(a + bx)^{2+n}}{b^3} + \frac{d(a + bx)^{3+n}}{b^3} \right) dx \\
&= \frac{(b^3c - a^3d)(a + bx)^{1+n}}{b^4(1 + n)} + \frac{3a^2d(a + bx)^{2+n}}{b^4(2 + n)} - \frac{3ad(a + bx)^{3+n}}{b^4(3 + n)} + \frac{d(a + bx)^{4+n}}{b^4(4 + n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$
\int (a + bx)^n (c + dx^3) \, dx = \frac{(b^3c - a^3d)(a + bx)^{1+n}}{b^4(1 + n)} + \frac{3a^2d(a + bx)^{2+n}}{b^4(2 + n)} - \frac{3ad(a + bx)^{3+n}}{b^4(3 + n)} + \frac{d(a + bx)^{4+n}}{b^4(4 + n)}
$$

```
[In] Integrate[(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2 +
 n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*x)
^(4 + n))/(b^4*(4 + n))
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.78

method	result
gosper	$-\frac{(bx+a)^{1+n}(-b^3dn^3x^3-6b^3dn^2x^3+3ab^2dn^2x^2-11b^3dnx^3+9ab^2dnx^2-b^3cn^3-6x^3db^3-6a^2bdnx+6adx^2b^2-9b^3cn^2-6}{b^4(n^4+10n^3+35n^2+50n+24)}$
risch	$-\frac{(-b^4dn^3x^4-ab^3dn^3x^3-6b^4dn^2x^4-3ab^3dn^2x^3-11b^4dnx^4+3a^2b^2dn^2x^2-2ab^3dnx^3-b^4cn^3x-6dx^4b^4+3a^2b^2dnx^2-a}{(3+n)(4+n)(2+n)(1+n)b^4}$
norman	$\frac{dx^4e^{n\ln(bx+a)}}{4+n}+\frac{(b^3cn^3+9b^3cn^2+6a^3dn+26b^3cn+24b^3c)x\,e^{n\ln(bx+a)}}{b^3(n^4+10n^3+35n^2+50n+24)}+\frac{ndax^3e^{n\ln(bx+a)}}{b(n^2+7n+12)}-\frac{a(-b^3cn^3-9b^3cn^2-26b}{b^4(n^4+10n^3}$
parallelrisch	$\frac{x^4(bx+a)^nab^4dn^3+6x^4(bx+a)^nab^4dn^2+x^3(bx+a)^na^2b^3dn^3+11x^4(bx+a)^nab^4dn+3x^3(bx+a)^na^2b^3dn^2+6x^4(bx+a)^nab^4}{}$

```
[In] int((b*x+a)^n*(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(b*x+a)^(1+n)/(n^4+10*n^3+35*n^2+50*n+24)*(-b^3*d*n^3*x^3-6*b^3*d*n^
2*x^3+3*a*b^2*d*n^2*x^2-11*b^3*d*n*x^3+9*a*b^2*d*n*x^2-b^3*c*n^3-6*b^3*d*x^
3-6*a^2*b*d*n*x+6*a*b^2*d*x^2-9*b^3*c*n^2-6*a^2*b*d*x-26*b^3*c*n+6*a^3*d-24
*b^3*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(94) = 188$.

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.36

$$\int (a+bx)^n (c+dx^3)\,dx = \frac{(ab^3cn^3 + 9\,ab^3cn^2 + 26\,ab^3cn + 24\,ab^3c - 6\,a^4d + (b^4dn^3 + 6\,b^4dn^2 + 11\,b^4dn + 6\,b^4d)x^4 + (ab^3dn^3 + 3\,ab^3}{b^4n^4 + 10\,b^4n^3 + 35\,b}$$

```
[In] integrate((b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] (a*b^3*c*n^3 + 9*a*b^3*c*n^2 + 26*a*b^3*c*n + 24*a*b^3*c - 6*a^4*d + (b^4*d
*n^3 + 6*b^4*d*n^2 + 11*b^4*d*n + 6*b^4*d)*x^4 + (a*b^3*d*n^3 + 3*a*b^3*d*n
^2 + 2*a*b^3*d*n)*x^3 - 3*(a^2*b^2*d*n^2 + a^2*b^2*d*n)*x^2 + (b^4*c*n^3 +
9*b^4*c*n^2 + 24*b^4*c + 2*(13*b^4*c + 3*a^3*b*d)*n)*x)*(b*x + a)^n/(b^4*n^
4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1906 vs. $2(83) = 166$.

Time = 0.81 (sec) , antiderivative size = 1906, normalized size of antiderivative = 20.28

$$\int (a+bx)^n (c+dx^3)\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x + d*x**4/4), Eq(b, 0)), (6*a**3*d*log(a/b + x)/(6*a**3
*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3*d/(6*a**3*
b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*x*log(a
/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*
a**2*b*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
18*a*b**2*d*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**
2 + 6*b**7*x**3) + 18*a*b**2*d*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) - 2*b**3*c/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6
*x**2 + 6*b**7*x**3) + 6*b**3*d*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(n, -4)), (-6*a**3*d*log(a/b + x)/(
2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d/(2*a**2*b**4 + 4*a*b**5*
x + 2*b**6*x**2) - 12*a**2*b*d*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2
*b**6*x**2) - 12*a**2*b*d*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*
b**2*d*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - b**3*c/
(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*x**3/(2*a**2*b**4 + 4*a
*b**5*x + 2*b**6*x**2), Eq(n, -3)), (6*a**3*d*log(a/b + x)/(2*a*b**4 + 2*b*
*5*x) + 6*a**3*d/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*x*log(a/b + x)/(2*a*b**
```

```
4 + 2*b**5*x) - 3*a*b**2*d*x**2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c/(2*a*b**4
+ 2*b**5*x) + b**3*d*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)), (-a**3*d*log(a
/b + x)/b**4 + a**2*d*x/b**3 - a*d*x**2/(2*b**2) + c*log(a/b + x)/b + d*x**
3/(3*b), Eq(n, -1)), (-6*a**4*d*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35
*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*d*n*x*(a + b*x)**n/(b**4*n**4
+ 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*d*n**2*x
**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*
b**4) - 3*a**2*b**2*d*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b*
*4*n**2 + 50*b**4*n + 24*b**4) + a*b**3*c*n**3*(a + b*x)**n/(b**4*n**4 + 10
*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*a*b**3*c*n**2*(a + b*x
)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 26*a
*b**3*c*n*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n
 + 24*b**4) + 24*a*b**3*c*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4) + a*b**3*d*n**3*x**3*(a + b*x)**n/(b**4*n**4 +
10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*d*n**2*x**3*(
a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
 + 2*a*b**3*d*n*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2
+ 50*b**4*n + 24*b**4) + b**4*c*n**3*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*b**4*c*n**2*x*(a + b*x)**n/(b*
*4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 26*b**4*c*n*
x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b*
*4) + 24*b**4*c*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 5
0*b**4*n + 24*b**4) + b**4*d*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*d*n**2*x**4*(a + b*x)**n/
(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*d
*n*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n +
 24*b**4) + 6*b**4*d*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

$$\int (a+bx)^n\left(c+dx^3\right)\,dx=\frac{(bx+a)^{n+1}c}{b(n+1)}+\frac{((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4)(bx+a)^nd}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}$$

```
[In] integrate((b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c/(b*(n + 1)) + ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3
+ 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(
b*x + a)^n*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(94) = 188.

Time = 0.43 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.84

$$\int (a+bx)^n \left(c+dx^3\right) dx = \frac{(bx+a)^n b^4 dn^3 x^4 + (bx+a)^n ab^3 dn^3 x^3 + 6\,(bx+a)^n b^4 dn^2 x^4 + 3\,(bx+a)^n ab^3 dn^2 x^3 + 11\,(bx+a)^n b^4 dnx^4}{}$$

```
[In] integrate((b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*d*n^3*x^4 + (b*x + a)^n*a*b^3*d*n^3*x^3 + 6*(b*x + a)^n*b^
4*d*n^2*x^4 + 3*(b*x + a)^n*a*b^3*d*n^2*x^3 + 11*(b*x + a)^n*b^4*d*n*x^4 +
(b*x + a)^n*b^4*c*n^3*x - 3*(b*x + a)^n*a^2*b^2*d*n^2*x^2 + 2*(b*x + a)^n*a
*b^3*d*n*x^3 + 6*(b*x + a)^n*b^4*d*x^4 + (b*x + a)^n*a*b^3*c*n^3 + 9*(b*x +
 a)^n*b^4*c*n^2*x - 3*(b*x + a)^n*a^2*b^2*d*n*x^2 + 9*(b*x + a)^n*a*b^3*c*n
^2 + 26*(b*x + a)^n*b^4*c*n*x + 6*(b*x + a)^n*a^3*b*d*n*x + 26*(b*x + a)^n*
a*b^3*c*n + 24*(b*x + a)^n*b^4*c*x + 24*(b*x + a)^n*a*b^3*c - 6*(b*x + a)^n
*a^4*d)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

Mupad [B] (verification not implemented)

Time = 19.41 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.63

$$\int (a+bx)^n \left(c+dx^3\right) dx = (a+b\,x)^n \left(\frac{x\,(6\,d\,a^3\,b\,n + c\,b^4\,n^3 + 9\,c\,b^4\,n^2 + 26\,c\,b^4\,n + 24\,c\,b^4)}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)} + \frac{a\,(-6\,d\,a^3 + c\,b^3\,n^3 + 9\,c\,b^3\,n^2 + 26\,c\,b^3\,n + 24\,c\,b^3)}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)} + \frac{d\,x^4\,(n^3+6\,n^2+11\,n+6)}{n^4+10\,n^3+35\,n^2+50\,n+24} - \frac{3\,a^2\,d\,n\,x^2\,(n+1)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)} + \frac{a\,d\,n\,x^3\,(n^2+3\,n+2)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)$$

```
[In] int((c + d*x^3)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((x*(24*b^4*c + 9*b^4*c*n^2 + b^4*c*n^3 + 26*b^4*c*n + 6*a^3*b*
d*n))/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*(24*b^3*c - 6*a^3*d +
9*b^3*c*n^2 + b^3*c*n^3 + 26*b^3*c*n))/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 +
 24)) + (d*x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4 + 24
) - (3*a^2*d*n*x^2*(n + 1))/(b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*
d*n*x^3*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

3.177 $\int \frac{(a+bx)^n(c+dx^3)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 99

$$\int \frac{(a+bx)^n(c+dx^3)}{x}\,dx = \frac{a^2d(a+bx)^{1+n}}{b^3(1+n)} - \frac{2ad(a+bx)^{2+n}}{b^3(2+n)} + \frac{d(a+bx)^{3+n}}{b^3(3+n)} - \frac{c(a+bx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)}{a(1+n)}$$

```
[Out] a^2*d*(b*x+a)^(1+n)/b^3/(1+n)-2*a*d*(b*x+a)^(2+n)/b^3/(2+n)+d*(b*x+a)^(3+n)
/b^3/(3+n)-c*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1634, 67}

$$\int \frac{(a+bx)^n(c+dx^3)}{x}\,dx = \frac{a^2d(a+bx)^{n+1}}{b^3(n+1)} - \frac{2ad(a+bx)^{n+2}}{b^3(n+2)} + \frac{d(a+bx)^{n+3}}{b^3(n+3)} - \frac{c(a+bx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{bx}{a}+1\right)}{a(n+1)}$$

```
[In] Int[((a + b*x)^n*(c + d*x^3))/x,x]
```

```
[Out] (a^2*d*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 + n))/(b^3*(2
 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n)) - (c*(a + b*x)^(1 + n)*Hyperge
ometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a^2d(a+bx)^n}{b^2}+\frac{c(a+bx)^n}{x}-\frac{2ad(a+bx)^{1+n}}{b^2}+\frac{d(a+bx)^{2+n}}{b^2}\right)dx\\ &= \frac{a^2d(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}+c\int\frac{(a+bx)^n}{x}\,dx\\ &= \frac{a^2d(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}-\frac{c(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int\frac{(a+bx)^n\,(c+dx^3)}{x}\,dx$$
$$=\frac{(a+bx)^{1+n}\left(ad(2a^2-2ab(1+n)x+b^2(2+3n+n^2)\,x^2)-b^3c(6+5n+n^2)\,\text{Hypergeometric2F1}\left(1,1+n\right.\right.}{ab^3(1+n)(2+n)(3+n)}$$

```
[In] Integrate[((a + b*x)^n*(c + d*x^3))/x,x]
```

```
[Out] ((a + b*x)^(1 + n)*(a*d*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2)
 - b^3*c*(6 + 5*n + n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a]))/
(a*b^3*(1 + n)*(2 + n)*(3 + n))
```

Maple [F]

$$\int \frac{(bx+a)^n (x^3d+c)}{x}dx$$

[In] int((b*x+a)^n*(d*x^3+c)/x,x)

[Out] int((b*x+a)^n*(d*x^3+c)/x,x)

Fricas [F]

$$\int \frac{(a+bx)^n (c+dx^3)}{x} dx = \int \frac{(dx^3+c)(bx+a)^n}{x} dx$$

[In] integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="fricas")

[Out] integral((d*x^3 + c)*(b*x + a)^n/x, x)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(83) = 166$.

Time = 2.56 (sec) , antiderivative size = 675, normalized size of antiderivative = 6.82

$$\int \frac{(a+bx)^n (c+dx^3)}{x} dx$$
$$= d \left(\begin{cases} \frac{a^n x^3}{3} \\ \frac{2a^2 \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{3a^2}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{2b^2x^2 \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} \\ -\frac{2a^2 \log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} - \frac{2a^2}{ab^3+b^4x} - \frac{2abx \log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} + \frac{b^2x^2}{ab^3+b^4x} \\ \frac{a^2 \log\left(\frac{a}{b}+x\right)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b} \\ \frac{2a^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} - \frac{2a^2bnx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{ab^2n^2x^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{ab^2nx^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{b^3n^2x^3(a+b}{b^3n^3+6b^3n^2+11} \end{cases} \right.$$
$$- \frac{b^{n+1}cn\left(\frac{a}{b}+x\right)^{n+1} \Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma(n+2)}$$
$$- \frac{b^{n+1}c\left(\frac{a}{b}+x\right)^{n+1} \Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma(n+2)}$$

[In] integrate((b*x+a)**n*(d*x**3+c)/x,x)

[Out] d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x/(2

```
*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2*a**2*b
**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b + x)/(a*b**3
+ b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b**4*
x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b + x)/b**3 - a*x
/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n*
*2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n*
*2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**
3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b**3*n**3 + 6*b*
*3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)**n/(b**3*n**3 + 6*
b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x)**n/(b**3*n**3 + 6
*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)**n/(b**3*n**3 + 6*
b**3*n**2 + 11*b**3*n + 6*b**3), True)) - b**(n + 1)*c*n*(a/b + x)**(n + 1)
*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)) - b**(n + 1)*c
*(a/b + x)**(n + 1)*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n +
 2))
```

Maxima [F]

$$\int \frac{(a+bx)^n (c+dx^3)}{x} \, dx = \int \frac{(dx^3+c)(bx+a)^n}{x} \, dx$$

```
[In] integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)*(b*x + a)^n/x, x)
```

Giac [F]

$$\int \frac{(a+bx)^n (c+dx^3)}{x} \, dx = \int \frac{(dx^3+c)(bx+a)^n}{x} \, dx$$

```
[In] integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="giac")
[Out] integrate((d*x^3 + c)*(b*x + a)^n/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^n\,(c+dx^3)}{x}\,dx = \int \frac{(d\,x^3+c)\,\left(a+b\,x\right)^n}{x}\,dx$$

```
[In] int(((c + d*x^3)*(a + b*x)^n)/x,x)
[Out] int(((c + d*x^3)*(a + b*x)^n)/x, x)
```

3.178 $\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 294

$$\begin{aligned}\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx &= \frac{a^2(b^3c-a^3d)^2\,(a+bx)^{1+n}}{b^9(1+n)}\\ &\quad-\frac{2a(b^3c-4a^3d)\,(b^3c-a^3d)\,(a+bx)^{2+n}}{b^9(2+n)}\\ &\quad+\frac{(b^6c^2-20a^3b^3cd+28a^6d^2)\,(a+bx)^{3+n}}{b^9(3+n)}\\ &\quad+\frac{4a^2d(5b^3c-14a^3d)\,(a+bx)^{4+n}}{b^9(4+n)}\\ &\quad-\frac{10ad(b^3c-7a^3d)\,(a+bx)^{5+n}}{b^9(5+n)}\\ &\quad+\frac{2d(b^3c-28a^3d)\,(a+bx)^{6+n}}{b^9(6+n)}+\frac{28a^2d^2(a+bx)^{7+n}}{b^9(7+n)}\\ &\quad-\frac{8ad^2(a+bx)^{8+n}}{b^9(8+n)}+\frac{d^2(a+bx)^{9+n}}{b^9(9+n)}\end{aligned}$$

```
[Out] a^2*(-a^3*d+b^3*c)^2*(b*x+a)^(1+n)/b^9/(1+n)-2*a*(-4*a^3*d+b^3*c)*(-a^3*d+b
^3*c)*(b*x+a)^(2+n)/b^9/(2+n)+(28*a^6*d^2-20*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^(
3+n)/b^9/(3+n)+4*a^2*d*(-14*a^3*d+5*b^3*c)*(b*x+a)^(4+n)/b^9/(4+n)-10*a*d*(
-7*a^3*d+b^3*c)*(b*x+a)^(5+n)/b^9/(5+n)+2*d*(-28*a^3*d+b^3*c)*(b*x+a)^(6+n)
/b^9/(6+n)+28*a^2*d^2*(b*x+a)^(7+n)/b^9/(7+n)-8*a*d^2*(b*x+a)^(8+n)/b^9/(8+
n)+d^2*(b*x+a)^(9+n)/b^9/(9+n)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1634}

$$\int x^2(a+bx)^n \left(c+dx^3\right)^2 dx = -\frac{2a(b^3c-4a^3d)(b^3c-a^3d)(a+bx)^{n+2}}{b^9(n+2)} - \frac{10ad(b^3c-7a^3d)(a+bx)^{n+5}}{b^9(n+5)} + \frac{2d(b^3c-28a^3d)(a+bx)^{n+6}}{b^9(n+6)} + \frac{28a^2d^2(a+bx)^{n+7}}{b^9(n+7)} + \frac{(28a^6d^2-20a^3b^3cd+b^6c^2)(a+bx)^{n+3}}{b^9(n+3)} + \frac{a^2(b^3c-a^3d)^2(a+bx)^{n+1}}{b^9(n+1)} + \frac{4a^2d(5b^3c-14a^3d)(a+bx)^{n+4}}{b^9(n+4)} - \frac{8ad^2(a+bx)^{n+8}}{b^9(n+8)} + \frac{d^2(a+bx)^{n+9}}{b^9(n+9)}$$

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] (a^2*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^3*c - 4*a
^3*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^6*c^2 - 20*a^3
*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) + (4*a^2*d*(5*b^3*c
 - 14*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4 + n)) - (10*a*d*(b^3*c - 7*a^3*d)*(
a + b*x)^(5 + n))/(b^9*(5 + n)) + (2*d*(b^3*c - 28*a^3*d)*(a + b*x)^(6 + n)
)/(b^9*(6 + n)) + (28*a^2*d^2*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^2*(
a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^2*(a + b*x)^(9 + n))/(b^9*(9 + n))
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(ab^3c-a^4d)^2(a+bx)^n}{b^8} - \frac{2(ab^6c^2-5a^4b^3cd+4a^7d^2)(a+bx)^{1+n}}{b^8} + \frac{(b^6c^2-20a^3b^3cd+28a^6d^2)(a+bx)^{2+n}}{b^8} - \frac{4a^2d(-5b^3c+14a^3d)(a+bx)^{3+n}}{b^8} + \frac{10ad(-b^3c+7a^3d)(a+bx)^{4+n}}{b^8} + \frac{2d(b^3c-28a^3d)(a+bx)^{5+n}}{b^8} + \frac{28a^2d^2(a+bx)^{6+n}}{b^8} - \frac{8ad^2(a+bx)^{7+n}}{b^8} + \frac{d^2(a+bx)^{8+n}}{b^8} \right) dx$$

$$= \frac{a^2(b^3c-a^3d)^2(a+bx)^{1+n}}{b^9(1+n)} - \frac{2a(b^3c-4a^3d)(b^3c-a^3d)(a+bx)^{2+n}}{b^9(2+n)} + \frac{(b^6c^2-20a^3b^3cd+28a^6d^2)(a+bx)^{3+n}}{b^9(3+n)} + \frac{4a^2d(5b^3c-14a^3d)(a+bx)^{4+n}}{b^9(4+n)} - \frac{10ad(b^3c-7a^3d)(a+bx)^{5+n}}{b^9(5+n)} + \frac{2d(b^3c-28a^3d)(a+bx)^{6+n}}{b^9(6+n)} + \frac{28a^2d^2(a+bx)^{7+n}}{b^9(7+n)} - \frac{8ad^2(a+bx)^{8+n}}{b^9(8+n)} + \frac{d^2(a+bx)^{9+n}}{b^9(9+n)}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.86

$$\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx = \frac{(a+bx)^{1+n}\left(\frac{(ab^3c-a^4d)^2}{1+n} - \frac{2a(b^3c-4a^3d)(b^3c-a^3d)(a+bx)}{2+n} + \frac{(b^6c^2-20a^3b^3cd+28a^6d^2)(a+bx)^2}{3+n} + \frac{4a^2d(5b^3c-14a^3d)(a+bx)^3}{4+n} + 1\right.}{b^9}$$

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*b^3*c - a^4*d)^2/(1 + n) - (2*a*(b^3*c - 4*a^3*d)*(b
^3*c - a^3*d)*(a + b*x))/(2 + n) + ((b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)
*(a + b*x)^2)/(3 + n) + (4*a^2*d*(5*b^3*c - 14*a^3*d)*(a + b*x)^3)/(4 + n)
+ (10*a*d*(-(b^3*c) + 7*a^3*d)*(a + b*x)^4)/(5 + n) + (2*d*(b^3*c - 28*a^3*
d)*(a + b*x)^5)/(6 + n) + (28*a^2*d^2*(a + b*x)^6)/(7 + n) - (8*a*d^2*(a +
b*x)^7)/(8 + n) + (d^2*(a + b*x)^8)/(9 + n)))/b^9
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. 2(294) = 588.

Time = 1.03 (sec) , antiderivative size = 1565, normalized size of antiderivative = 5.32

method	result	size
gosper	Expression too large to display	1565
risch	Expression too large to display	1799
parallelrisch	Expression too large to display	2710

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^9*(b*x+a)^(1+n)/(n^9+45*n^8+870*n^7+9450*n^6+63273*n^5+269325*n^4+72368
0*n^3+1172700*n^2+1026576*n+362880)*(b^8*d^2*n^8*x^8+36*b^8*d^2*n^7*x^8-8*a
*b^7*d^2*n^7*x^7+546*b^8*d^2*n^6*x^8-224*a*b^7*d^2*n^6*x^7+2*b^8*c*d*n^8*x^
5+4536*b^8*d^2*n^5*x^8+56*a^2*b^6*d^2*n^6*x^6-2576*a*b^7*d^2*n^5*x^7+78*b^8
*c*d*n^7*x^5+22449*b^8*d^2*n^4*x^8+1176*a^2*b^6*d^2*n^5*x^6-10*a*b^7*c*d*n^
7*x^4-15680*a*b^7*d^2*n^4*x^7+1272*b^8*c*d*n^6*x^5+67284*b^8*d^2*n^3*x^8-33
6*a^3*b^5*d^2*n^5*x^5+9800*a^2*b^6*d^2*n^4*x^6-340*a*b^7*c*d*n^6*x^4-54152*
a*b^7*d^2*n^3*x^7+b^8*c^2*n^8*x^2+11268*b^8*c*d*n^5*x^5+118124*b^8*d^2*n^2*
x^8-5040*a^3*b^5*d^2*n^4*x^5+40*a^2*b^6*c*d*n^6*x^3+41160*a^2*b^6*d^2*n^3*x
^6-4660*a*b^7*c*d*n^5*x^4-105056*a*b^7*d^2*n^2*x^7+42*b^8*c^2*n^7*x^2+58938
*b^8*c*d*n^4*x^5+109584*b^8*d^2*n*x^8+1680*a^4*b^4*d^2*n^4*x^4-28560*a^3*b^
5*d^2*n^3*x^5+1200*a^2*b^6*c*d*n^5*x^3+90944*a^2*b^6*d^2*n^2*x^6-2*a*b^7*c^
2*n^7*x-33040*a*b^7*c*d*n^4*x^4-104544*a*b^7*d^2*n*x^7+744*b^8*c^2*n^6*x^2+
185022*b^8*c*d*n^3*x^5+40320*b^8*d^2*x^8+16800*a^4*b^4*d^2*n^3*x^4-120*a^3*
b^5*c*d*n^5*x^2-75600*a^3*b^5*d^2*n^2*x^5+13840*a^2*b^6*c*d*n^4*x^3+98784*a
^2*b^6*d^2*n*x^6-80*a*b^7*c^2*n^6*x-129490*a*b^7*c*d*n^3*x^4-40320*a*b^7*d^
2*x^7+7218*b^8*c^2*n^5*x^2+337228*b^8*c*d*n^2*x^5-6720*a^5*b^3*d^2*n^3*x^3+
58800*a^4*b^4*d^2*n^2*x^4-3240*a^3*b^5*c*d*n^4*x^2-92064*a^3*b^5*d^2*n*x^5+
2*a^2*b^6*c^2*n^6+76800*a^2*b^6*c*d*n^3*x^3+40320*a^2*b^6*d^2*x^6-1328*a*b^
7*c^2*n^5*x-277660*a*b^7*c*d*n^2*x^4+41619*b^8*c^2*n^4*x^2+322032*b^8*c*d*n
*x^5-40320*a^5*b^3*d^2*n^2*x^3+240*a^4*b^4*c*d*n^4*x+84000*a^4*b^4*d^2*n*x^
4-31800*a^3*b^5*c*d*n^3*x^2-40320*a^3*b^5*d^2*x^5+78*a^2*b^6*c^2*n^5+210760
*a^2*b^6*c*d*n^2*x^3-11780*a*b^7*c^2*n^4*x-297840*a*b^7*c*d*n*x^4+144468*b^
8*c^2*n^3*x^2+120960*b^8*c*d*x^5+20160*a^6*b^2*d^2*n^2*x^2-73920*a^5*b^3*d^
2*n*x^3+6000*a^4*b^4*c*d*n^3*x+40320*a^4*b^4*d^2*x^4-135000*a^3*b^5*c*d*n^2
*x^2+1250*a^2*b^6*c^2*n^4+267600*a^2*b^6*c*d*n*x^3-59678*a*b^7*c^2*n^3*x-12
0960*a*b^7*c*d*x^4+290276*b^8*c^2*n^2*x^2+60480*a^6*b^2*d^2*n*x^2-240*a^5*b
^3*c*d*n^3-40320*a^5*b^3*d^2*x^3+51600*a^4*b^4*c*d*n^2*x-227280*a^3*b^5*c*d
*n*x^2+10530*a^2*b^6*c^2*n^3+120960*a^2*b^6*c*d*x^3-169580*a*b^7*c^2*n^2*x+
301872*b^8*c^2*n*x^2-40320*a^7*b*d^2*n*x+40320*a^6*b^2*d^2*x^2-5760*a^5*b^3
*c*d*n^2+166800*a^4*b^4*c*d*n*x-120960*a^3*b^5*c*d*x^2+49148*a^2*b^6*c^2*n^
2-241392*a*b^7*c^2*n*x+120960*b^8*c^2*x^2-40320*a^7*b*d^2*x-45840*a^5*b^3*c
*d*n+120960*a^4*b^4*c*d*x+120432*a^2*b^6*c^2*n-120960*a*b^7*c^2*x+40320*a^8
```

```
*d^2-120960*a^5*b^3*c*d+120960*a^2*b^6*c^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1565 vs. $2(294) = 588$.

Time = 0.29 (sec) , antiderivative size = 1565, normalized size of antiderivative = 5.32

$$\int x^2(a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^6*c^2*n^6 + 78*a^3*b^6*c^2*n^5 + 1250*a^3*b^6*c^2*n^4 + 120960*a^3
*b^6*c^2 - 120960*a^6*b^3*c*d + 40320*a^9*d^2 + (b^9*d^2*n^8 + 36*b^9*d^2*n
^7 + 546*b^9*d^2*n^6 + 4536*b^9*d^2*n^5 + 22449*b^9*d^2*n^4 + 67284*b^9*d^2
*n^3 + 118124*b^9*d^2*n^2 + 109584*b^9*d^2*n + 40320*b^9*d^2)*x^9 + (a*b^8*
d^2*n^8 + 28*a*b^8*d^2*n^7 + 322*a*b^8*d^2*n^6 + 1960*a*b^8*d^2*n^5 + 6769*
a*b^8*d^2*n^4 + 13132*a*b^8*d^2*n^3 + 13068*a*b^8*d^2*n^2 + 5040*a*b^8*d^2*
n)*x^8 - 8*(a^2*b^7*d^2*n^7 + 21*a^2*b^7*d^2*n^6 + 175*a^2*b^7*d^2*n^5 + 73
5*a^2*b^7*d^2*n^4 + 1624*a^2*b^7*d^2*n^3 + 1764*a^2*b^7*d^2*n^2 + 720*a^2*b
^7*d^2*n)*x^7 + 2*(b^9*c*d*n^8 + 39*b^9*c*d*n^7 + 60480*b^9*c*d + 4*(159*b^
9*c*d + 7*a^3*b^6*d^2)*n^6 + 6*(939*b^9*c*d + 70*a^3*b^6*d^2)*n^5 + (29469*
b^9*c*d + 2380*a^3*b^6*d^2)*n^4 + 9*(10279*b^9*c*d + 700*a^3*b^6*d^2)*n^3 +
 2*(84307*b^9*c*d + 3836*a^3*b^6*d^2)*n^2 + 24*(6709*b^9*c*d + 140*a^3*b^6*
d^2)*n)*x^6 + 2*(a*b^8*c*d*n^8 + 34*a*b^8*c*d*n^7 + 466*a*b^8*c*d*n^6 + 56*
(59*a*b^8*c*d - 3*a^4*b^5*d^2)*n^5 + (12949*a*b^8*c*d - 1680*a^4*b^5*d^2)*n
^4 + 2*(13883*a*b^8*c*d - 2940*a^4*b^5*d^2)*n^3 + 24*(1241*a*b^8*c*d - 350*
a^4*b^5*d^2)*n^2 + 4032*(3*a*b^8*c*d - a^4*b^5*d^2)*n)*x^5 - 10*(a^2*b^7*c*
d*n^7 + 30*a^2*b^7*c*d*n^6 + 346*a^2*b^7*c*d*n^5 + 24*(80*a^2*b^7*c*d - 7*a
^5*b^4*d^2)*n^4 + (5269*a^2*b^7*c*d - 1008*a^5*b^4*d^2)*n^3 + 6*(1115*a^2*b
^7*c*d - 308*a^5*b^4*d^2)*n^2 + 1008*(3*a^2*b^7*c*d - a^5*b^4*d^2)*n)*x^4 +
 30*(351*a^3*b^6*c^2 - 8*a^6*b^3*c*d)*n^3 + (b^9*c^2*n^8 + 42*b^9*c^2*n^7 +
 120960*b^9*c^2 + 8*(93*b^9*c^2 + 5*a^3*b^6*c*d)*n^6 + 18*(401*b^9*c^2 + 60
*a^3*b^6*c*d)*n^5 + (41619*b^9*c^2 + 10600*a^3*b^6*c*d)*n^4 + 12*(12039*b^9
*c^2 + 3750*a^3*b^6*c*d - 560*a^6*b^3*d^2)*n^3 + 4*(72569*b^9*c^2 + 18940*a
^3*b^6*c*d - 5040*a^6*b^3*d^2)*n^2 + 48*(6289*b^9*c^2 + 840*a^3*b^6*c*d - 2
80*a^6*b^3*d^2)*n)*x^3 + 4*(12287*a^3*b^6*c^2 - 1440*a^6*b^3*c*d)*n^2 + (a*
b^8*c^2*n^8 + 40*a*b^8*c^2*n^7 + 664*a*b^8*c^2*n^6 + 10*(589*a*b^8*c^2 - 12
*a^4*b^5*c*d)*n^5 + (29839*a*b^8*c^2 - 3000*a^4*b^5*c*d)*n^4 + 10*(8479*a*b
^8*c^2 - 2580*a^4*b^5*c*d)*n^3 + 24*(5029*a*b^8*c^2 - 3475*a^4*b^5*c*d + 84
0*a^7*b^2*d^2)*n^2 + 20160*(3*a*b^8*c^2 - 3*a^4*b^5*c*d + a^7*b^2*d^2)*n)*x
^2 + 48*(2509*a^3*b^6*c^2 - 955*a^6*b^3*c*d)*n - 2*(a^2*b^7*c^2*n^7 + 39*a^
2*b^7*c^2*n^6 + 625*a^2*b^7*c^2*n^5 + 15*(351*a^2*b^7*c^2 - 8*a^5*b^4*c*d)*
n^4 + 2*(12287*a^2*b^7*c^2 - 1440*a^5*b^4*c*d)*n^3 + 24*(2509*a^2*b^7*c^2 -
 955*a^5*b^4*c*d)*n^2 + 20160*(3*a^2*b^7*c^2 - 3*a^5*b^4*c*d + a^8*b*d^2)*n
```

```
)*x)*(b*x + a)^n/(b^9*n^9 + 45*b^9*n^8 + 870*b^9*n^7 + 9450*b^9*n^6 + 63273
*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 1172700*b^9*n^2 + 1026576*b^9*
n + 362880*b^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26746 vs. 2(275) = 550.

Time = 8.95 (sec) , antiderivative size = 26746, normalized size of antiderivative = 90.97

$$\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x**3/3 + c*d*x**6/3 + d**2*x**9/9), Eq(b, 0)), (840*a
**8*d**2*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11
*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x*
*5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 2283*a**
8*d**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a
**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2
*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 6720*a**7*b*d**2*x*log(
a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040
*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a*
*2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 17424*a**7*b*d**2*x/(
840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**1
2*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x
**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 23520*a**6*b**2*d**2*x**2*log(a
/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*
a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**
2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 57624*a**6*b**2*d**2*x
**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5
*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b*
*15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 10*a**5*b**3*c*d/(840*a**8
*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 +
 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 67
20*a*b**16*x**7 + 840*b**17*x**8) + 47040*a**5*b**3*d**2*x**3*log(a/b + x)/
(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**
12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*
x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 107408*a**5*b**3*d**2*x**3/(84
0*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*
x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**
6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 80*a**4*b**4*c*d*x/(840*a**8*b**9
 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 5880
0*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*
b**16*x**7 + 840*b**17*x**8) + 58800*a**4*b**4*d**2*x**4*log(a/b + x)/(840*
```

a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x*
*3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6
+ 6720*a*b**16*x**7 + 840*b**17*x**8) + 122500*a**4*b**4*d**2*x**4/(840*a**
8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3
+ 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6
720*a*b**16*x**7 + 840*b**17*x**8) - 280*a**3*b**5*c*d*x**2/(840*a**8*b**9
+ 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800
*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b
16*x7 + 840*b**17*x**8) + 47040*a**3*b**5*d**2*x**5*log(a/b + x)/(840*a
8*b9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**
3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 +
6720*a*b**16*x**7 + 840*b**17*x**8) + 86240*a**3*b**5*d**2*x**5/(840*a**8*
b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 +
58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 672
0*a*b**16*x**7 + 840*b**17*x**8) - 5*a**2*b**6*c**2/(840*a**8*b**9 + 6720*a
7*b10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b*
*13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**
7 + 840*b**17*x**8) - 560*a**2*b**6*c*d*x**3/(840*a**8*b**9 + 6720*a**7*b**
10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**
4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840
*b**17*x**8) + 23520*a**2*b**6*d**2*x**6*log(a/b + x)/(840*a**8*b**9 + 6720
*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*
b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x
7 + 840*b17*x**8) + 35280*a**2*b**6*d**2*x**6/(840*a**8*b**9 + 6720*a**
7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**1
3*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7
+ 840*b**17*x**8) - 40*a*b**7*c**2*x/(840*a**8*b**9 + 6720*a**7*b**10*x + 2
3520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 4704
0*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x
8) - 700*a*b7*c*d*x**4/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*
b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**
14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 672
0*a*b**7*d**2*x**7*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*
a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**
3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8)
+ 6720*a*b**7*d**2*x**7/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**
11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*
x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 140*b*
*8*c**2*x**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 4
7040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 2352
0*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 560*b**8*c*d*x**5
/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b*
*12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15
*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 840*b**8*d**2*x**8*log(a/b +
x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*

b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**
15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8), Eq(n, -9)), (-840*a**8*d**2*
log(a/b + x)/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 367
5*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**
15*x**6 + 105*b**16*x**7) - 2178*a**8*d**2/(105*a**7*b**9 + 735*a**6*b**10*
x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 22
05*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 5880*a**7*b*d**2*
x*log(a/b + x)/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3
675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b
15*x6 + 105*b**16*x**7) - 14406*a**7*b*d**2*x/(105*a**7*b**9 + 735*a**6
*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x*
*4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 17640*a**6
*b**2*d**2*x**2*log(a/b + x)/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*
b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*
x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 40278*a**6*b**2*d**2*x**2/(105*
a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3
+ 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**1
6*x**7) - 5*a**5*b**3*c*d/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**
11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**
5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 29400*a**5*b**3*d**2*x**3*log(a/b
+ x)/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b
12*x3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6
+ 105*b**16*x**7) - 61250*a**5*b**3*d**2*x**3/(105*a**7*b**9 + 735*a**6*b**
10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 +
2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 35*a**4*b**4*c
*d*x/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b
12*x3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6
+ 105*b**16*x**7) - 29400*a**4*b**4*d**2*x**4*log(a/b + x)/(105*a**7*b**9 +
735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3
*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 5
3900*a**4*b**4*d**2*x**4/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**1
1*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5
+ 735*a*b**15*x**6 + 105*b**16*x**7) - 105*a**3*b**5*c*d*x**2/(105*a**7*b*
*9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*
a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7)
- 17640*a**3*b**5*d**2*x**5*log(a/b + x)/(105*a**7*b**9 + 735*a**6*b**10*x
+ 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 220
5*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 26460*a**3*b**5*d*
*2*x**5/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**
4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x*
*6 + 105*b**16*x**7) - a**2*b**6*c**2/(105*a**7*b**9 + 735*a**6*b**10*x + 2
205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a*
*2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 175*a**2*b**6*c*d*x**3
/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12
*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 10

5*b**16*x**7) - 5880*a**2*b**6*d**2*x**6*log(a/b + x)/(105*a**7*b**9 + 735*
a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**1
3*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 5880*a
2*b6*d**2*x**6/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2
 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735
*a*b**15*x**6 + 105*b**16*x**7) - 7*a*b**7*c**2*x/(105*a**7*b**9 + 735*a**6
*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x*
*4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 175*a*b**7
*c*d*x**4/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a
4*b12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*
x**6 + 105*b**16*x**7) - 840*a*b**7*d**2*x**7*log(a/b + x)/(105*a**7*b**9 +
 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3
*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 2
1*b**8*c**2*x**2/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 +
 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a
*b**15*x**6 + 105*b**16*x**7) - 105*b**8*c*d*x**5/(105*a**7*b**9 + 735*a**6
*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x*
*4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) + 105*b**8*d
2*x8/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a*
*4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x
6 + 105*b16*x**7), Eq(n, -8)), (1680*a**8*d**2*log(a/b + x)/(60*a**6*b*
*9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a*
*2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 4116*a**8*d**2/(60*a**6
*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900
*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 10080*a**7*b*d**2*x*
log(a/b + x)/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*
a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) +
 23016*a**7*b*d**2*x/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2
 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15
*x**6) + 25200*a**6*b**2*d**2*x**2*log(a/b + x)/(60*a**6*b**9 + 360*a**5*b*
*10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 +
360*a*b**14*x**5 + 60*b**15*x**6) + 52500*a**6*b**2*d**2*x**2/(60*a**6*b**9
 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2
*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) - 20*a**5*b**3*c*d/(60*a**6
*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900
*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 33600*a**5*b**3*d**2
*x**3*log(a/b + x)/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 +
 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x
6) + 61600*a5*b**3*d**2*x**3/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**
4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**
5 + 60*b**15*x**6) - 120*a**4*b**4*c*d*x/(60*a**6*b**9 + 360*a**5*b**10*x +
 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b
14*x5 + 60*b**15*x**6) + 25200*a**4*b**4*d**2*x**4*log(a/b + x)/(60*a**
6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 90
0*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 37800*a**4*b**4*d**

2*x**4/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b
12*x3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) - 300*a
3*b5*c*d*x**2/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 +
1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x*
*6) + 10080*a**3*b**5*d**2*x**5*log(a/b + x)/(60*a**6*b**9 + 360*a**5*b**10
*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360
*a*b**14*x**5 + 60*b**15*x**6) + 10080*a**3*b**5*d**2*x**5/(60*a**6*b**9 +
360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b*
*13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) - a**2*b**6*c**2/(60*a**6*b**9
 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2
*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) - 400*a**2*b**6*c*d*x**3/(6
0*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3
 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 1680*a**2*b**6
*d**2*x**6*log(a/b + x)/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**11*x
2 + 1200*a3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 60*b*
*15*x**6) - 6*a*b**7*c**2*x/(60*a**6*b**9 + 360*a**5*b**10*x + 900*a**4*b**
11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*x**5 + 6
0*b**15*x**6) - 300*a*b**7*c*d*x**4/(60*a**6*b**9 + 360*a**5*b**10*x + 900*
a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360*a*b**14*
x**5 + 60*b**15*x**6) - 240*a*b**7*d**2*x**7/(60*a**6*b**9 + 360*a**5*b**10
*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4 + 360
*a*b**14*x**5 + 60*b**15*x**6) - 15*b**8*c**2*x**2/(60*a**6*b**9 + 360*a**5
*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**13*x**4
 + 360*a*b**14*x**5 + 60*b**15*x**6) - 120*b**8*c*d*x**5/(60*a**6*b**9 + 36
0*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**2*b**1
3*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6) + 30*b**8*d**2*x**8/(60*a**6*b**
9 + 360*a**5*b**10*x + 900*a**4*b**11*x**2 + 1200*a**3*b**12*x**3 + 900*a**
2*b**13*x**4 + 360*a*b**14*x**5 + 60*b**15*x**6), Eq(n, -7)), (-1680*a**8*d
2*log(a/b + x)/(30*a5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 3
00*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 3836*a**8*d**2/(30
*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 +
 150*a*b**13*x**4 + 30*b**14*x**5) - 8400*a**7*b*d**2*x*log(a/b + x)/(30*a*
*5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 15
0*a*b**13*x**4 + 30*b**14*x**5) - 17500*a**7*b*d**2*x/(30*a**5*b**9 + 150*a
4*b10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4
+ 30*b**14*x**5) - 16800*a**6*b**2*d**2*x**2*log(a/b + x)/(30*a**5*b**9 + 1
50*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x
4 + 30*b14*x**5) - 30800*a**6*b**2*d**2*x**2/(30*a**5*b**9 + 150*a**4*b
10*x + 300*a3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*
b**14*x**5) + 60*a**5*b**3*c*d*log(a/b + x)/(30*a**5*b**9 + 150*a**4*b**10*
x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14
*x**5) + 137*a**5*b**3*c*d/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**1
1*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 16800*a*
*5*b**3*d**2*x**3*log(a/b + x)/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*
b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 2520

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0*a**5*b**3*d**2*x**3/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**
2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) + 300*a**4*b**4
*c*d*x*log(a/b + x)/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2
+ 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) + 625*a**4*b**4*c
*d*x/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**1
2*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 8400*a**4*b**4*d**2*x**4*log(a
/b + x)/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b
**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 8400*a**4*b**4*d**2*x**4/(3
0*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3
+ 150*a*b**13*x**4 + 30*b**14*x**5) + 600*a**3*b**5*c*d*x**2*log(a/b + x)/(
30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3
 + 150*a*b**13*x**4 + 30*b**14*x**5) + 1100*a**3*b**5*c*d*x**2/(30*a**5*b**
9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b*
*13*x**4 + 30*b**14*x**5) - 1680*a**3*b**5*d**2*x**5*log(a/b + x)/(30*a**5*
b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a
*b**13*x**4 + 30*b**14*x**5) - a**2*b**6*c**2/(30*a**5*b**9 + 150*a**4*b**1
0*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**
14*x**5) + 600*a**2*b**6*c*d*x**3*log(a/b + x)/(30*a**5*b**9 + 150*a**4*b**
10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b*
*14*x**5) + 900*a**2*b**6*c*d*x**3/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a
**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) +
280*a**2*b**6*d**2*x**6/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x
**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) - 5*a*b**7*c*
*2*x/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**1
2*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) + 300*a*b**7*c*d*x**4*log(a/b +
x)/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*
x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) + 300*a*b**7*c*d*x**4/(30*a**5*b**
9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b*
*13*x**4 + 30*b**14*x**5) - 40*a*b**7*d**2*x**7/(30*a**5*b**9 + 150*a**4*b*
*10*x + 300*a**3*b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b
**14*x**5) - 10*b**8*c**2*x**2/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*
b**11*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) + 60*b
**8*c*d*x**5*log(a/b + x)/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11
*x**2 + 300*a**2*b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5) + 10*b**8*d
**2*x**8/(30*a**5*b**9 + 150*a**4*b**10*x + 300*a**3*b**11*x**2 + 300*a**2*
b**12*x**3 + 150*a*b**13*x**4 + 30*b**14*x**5), Eq(n, -6)), (840*a**8*d**2*
log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b*
*12*x**3 + 12*b**13*x**4) + 1750*a**8*d**2/(12*a**4*b**9 + 48*a**3*b**10*x
+ 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 3360*a**7*b*d**2*
x*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*
b**12*x**3 + 12*b**13*x**4) + 6160*a**7*b*d**2*x/(12*a**4*b**9 + 48*a**3*b*
*10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 5040*a**6*b
**2*d**2*x**2*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*
x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 7560*a**6*b**2*d**2*x**2/(12*a**4
*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x
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4) - 120*a5*b**3*c*d*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 250*a**5*b**3*c*d/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 3360*a**5*b**3*d**2*x**3*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 3360*a**5*b**3*d**2*x**3/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 480*a**4*b**4*c*d*x*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 880*a**4*b**4*c*d*x/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 840*a**4*b**4*d**2*x**4*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 720*a**3*b**5*c*d*x**2*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 1080*a**3*b**5*c*d*x**2/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 168*a**3*b**5*d**2*x**5/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - a**2*b**6*c**2/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 480*a**2*b**6*c*d*x**3*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 480*a**2*b**6*c*d*x**3/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 28*a**2*b**6*d**2*x**6/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 4*a*b**7*c**2*x/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 120*a*b**7*c*d*x**4*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 8*a*b**7*d**2*x**7/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 6*b**8*c**2*x**2/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 24*b**8*c*d*x**5/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 3*b**8*d**2*x**8/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4), Eq(n, -5)), (-840*a**8*d**2*log(a/b + x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 1540*a**8*d**2/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 2520*a**7*b*d**2*x*log(a/b + x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 3780*a**7*b*d**2*x/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 2520*a**6*b**2*d**2*x**2*log(a/b + x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 2520*a**6*b**2*d**2*x**2/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) + 300*a**5*b**3*c*d*log(a/b + x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) + 550*a**5*b**3*c*d/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 840*a**5*b**3*d**2*x**3*log(a/b + x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) + 900*a**4*b**4*c*d*x*log(a/b + x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) + 1350*a**4*b**4*c*d*x/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3)

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+ 210*a**4*b**4*d**2*x**4/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2
 + 15*b**12*x**3) + 900*a**3*b**5*c*d*x**2*log(a/b + x)/(15*a**3*b**9 + 45*
a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) + 900*a**3*b**5*c*d*x**2/(1
5*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 42*a**3*
b**5*d**2*x**5/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12
*x**3) - 5*a**2*b**6*c**2/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2
 + 15*b**12*x**3) + 300*a**2*b**6*c*d*x**3*log(a/b + x)/(15*a**3*b**9 + 45*
a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) + 14*a**2*b**6*d**2*x**6/(1
5*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 15*a*b**
7*c**2*x/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3)
 - 75*a*b**7*c*d*x**4/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 1
5*b**12*x**3) - 6*a*b**7*d**2*x**7/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b
**11*x**2 + 15*b**12*x**3) - 15*b**8*c**2*x**2/(15*a**3*b**9 + 45*a**2*b**1
0*x + 45*a*b**11*x**2 + 15*b**12*x**3) + 15*b**8*c*d*x**5/(15*a**3*b**9 + 4
5*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) + 3*b**8*d**2*x**8/(15*a*
*3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3), Eq(n, -4)), (
840*a**8*d**2*log(a/b + x)/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) +
1260*a**8*d**2/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) + 1680*a**7*b*
d**2*x*log(a/b + x)/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) + 1680*a*
*7*b*d**2*x/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) + 840*a**6*b**2*d
**2*x**2*log(a/b + x)/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) - 600*a
**5*b**3*c*d*log(a/b + x)/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) - 9
00*a**5*b**3*c*d/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) - 280*a**5*b
**3*d**2*x**3/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) - 1200*a**4*b**
4*c*d*x*log(a/b + x)/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) - 1200*a
**4*b**4*c*d*x/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) + 70*a**4*b**4
*d**2*x**4/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) - 600*a**3*b**5*c*
d*x**2*log(a/b + x)/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) - 28*a**3
*b**5*d**2*x**5/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) + 30*a**2*b**
6*c**2*log(a/b + x)/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) + 45*a**2
*b**6*c**2/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) + 200*a**2*b**6*c*
d*x**3/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) + 14*a**2*b**6*d**2*x*
*6/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) + 60*a*b**7*c**2*x*log(a/b
 + x)/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) + 60*a*b**7*c**2*x/(30*
a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) - 50*a*b**7*c*d*x**4/(30*a**2*b**
9 + 60*a*b**10*x + 30*b**11*x**2) - 8*a*b**7*d**2*x**7/(30*a**2*b**9 + 60*a
*b**10*x + 30*b**11*x**2) + 30*b**8*c**2*x**2*log(a/b + x)/(30*a**2*b**9 +
60*a*b**10*x + 30*b**11*x**2) + 20*b**8*c*d*x**5/(30*a**2*b**9 + 60*a*b**10
*x + 30*b**11*x**2) + 5*b**8*d**2*x**8/(30*a**2*b**9 + 60*a*b**10*x + 30*b*
*11*x**2), Eq(n, -3)), (-1680*a**8*d**2*log(a/b + x)/(210*a*b**9 + 210*b**1
0*x) - 1680*a**8*d**2/(210*a*b**9 + 210*b**10*x) - 1680*a**7*b*d**2*x*log(a
/b + x)/(210*a*b**9 + 210*b**10*x) + 840*a**6*b**2*d**2*x**2/(210*a*b**9 +
210*b**10*x) + 2100*a**5*b**3*c*d*log(a/b + x)/(210*a*b**9 + 210*b**10*x) +
 2100*a**5*b**3*c*d/(210*a*b**9 + 210*b**10*x) - 280*a**5*b**3*d**2*x**3/(2
10*a*b**9 + 210*b**10*x) + 2100*a**4*b**4*c*d*x*log(a/b + x)/(210*a*b**9 +
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210*b**10*x) + 140*a**4*b**4*d**2*x**4/(210*a*b**9 + 210*b**10*x) - 1050*a*
*3*b**5*c*d*x**2/(210*a*b**9 + 210*b**10*x) - 84*a**3*b**5*d**2*x**5/(210*a
*b**9 + 210*b**10*x) - 420*a**2*b**6*c**2*log(a/b + x)/(210*a*b**9 + 210*b*
*10*x) - 420*a**2*b**6*c**2/(210*a*b**9 + 210*b**10*x) + 350*a**2*b**6*c*d*
x**3/(210*a*b**9 + 210*b**10*x) + 56*a**2*b**6*d**2*x**6/(210*a*b**9 + 210*
b**10*x) - 420*a*b**7*c**2*x*log(a/b + x)/(210*a*b**9 + 210*b**10*x) - 175*
a*b**7*c*d*x**4/(210*a*b**9 + 210*b**10*x) - 40*a*b**7*d**2*x**7/(210*a*b**
9 + 210*b**10*x) + 210*b**8*c**2*x**2/(210*a*b**9 + 210*b**10*x) + 105*b**8
*c*d*x**5/(210*a*b**9 + 210*b**10*x) + 30*b**8*d**2*x**8/(210*a*b**9 + 210*
b**10*x), Eq(n, -2)), (a**8*d**2*log(a/b + x)/b**9 - a**7*d**2*x/b**8 + a**
6*d**2*x**2/(2*b**7) - 2*a**5*c*d*log(a/b + x)/b**6 - a**5*d**2*x**3/(3*b**
6) + 2*a**4*c*d*x/b**5 + a**4*d**2*x**4/(4*b**5) - a**3*c*d*x**2/b**4 - a**
3*d**2*x**5/(5*b**4) + a**2*c**2*log(a/b + x)/b**3 + 2*a**2*c*d*x**3/(3*b**
3) + a**2*d**2*x**6/(6*b**3) - a*c**2*x/b**2 - a*c*d*x**4/(2*b**2) - a*d**2
*x**7/(7*b**2) + c**2*x**2/(2*b) + 2*c*d*x**5/(5*b) + d**2*x**8/(8*b), Eq(n
, -1)), (40320*a**9*d**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*
n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n*
*3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 40320*a**8*b*d**2*
n*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6
 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**
2 + 1026576*b**9*n + 362880*b**9) + 20160*a**7*b**2*d**2*n**2*x**2*(a + b*x
)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**
9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*
b**9*n + 362880*b**9) + 20160*a**7*b**2*d**2*n*x**2*(a + b*x)**n/(b**9*n**9
 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325
*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880
*b**9) - 240*a**6*b**3*c*d*n**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 87
0*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*
b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 5760*a**6*b
3*c*d*n2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*
b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700
*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 45840*a**6*b**3*c*d*n*(a + b*x
)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**
9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*
b**9*n + 362880*b**9) - 120960*a**6*b**3*c*d*(a + b*x)**n/(b**9*n**9 + 45*b
9*n8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n
4 + 723680*b9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9)
- 6720*a**6*b**3*d**2*n**3*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 87
0*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*
b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 20160*a**6*
b**3*d**2*n**2*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7
+ 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 +
1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 13440*a**6*b**3*d**2*n*
x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**
6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n*

*2 + 1026576*b**9*n + 362880*b**9) + 240*a**5*b**4*c*d*n**4*x*(a + b*x)**n/
(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**
5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*
n + 362880*b**9) + 5760*a**5*b**4*c*d*n**3*x*(a + b*x)**n/(b**9*n**9 + 45*b
9*n8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n
4 + 723680*b9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9)
+ 45840*a**5*b**4*c*d*n**2*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b
9*n7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**
9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 120960*a**5*b*
*4*c*d*n*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b*
*9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b
9*n2 + 1026576*b**9*n + 362880*b**9) + 1680*a**5*b**4*d**2*n**4*x**4*(a
 + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 632
73*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 10
26576*b**9*n + 362880*b**9) + 10080*a**5*b**4*d**2*n**3*x**4*(a + b*x)**n/(
b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5
 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n
 + 362880*b**9) + 18480*a**5*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**9*n**9 +
45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b*
*9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b*
*9) + 10080*a**5*b**4*d**2*n*x**4*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 +
870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 72368
0*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 120*a**4*
b**5*c*d*n**5*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 +
 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1
172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 3000*a**4*b**5*c*d*n**4*
x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**
6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n*
*2 + 1026576*b**9*n + 362880*b**9) - 25800*a**4*b**5*c*d*n**3*x**2*(a + b*x
)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**
9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*
b**9*n + 362880*b**9) - 83400*a**4*b**5*c*d*n**2*x**2*(a + b*x)**n/(b**9*n*
*9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 2693
25*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 3628
80*b**9) - 60480*a**4*b**5*c*d*n*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**
8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 7
23680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 336*a
4*b5*d**2*n**5*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n
7 + 9450*b9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**
3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 3360*a**4*b**5*d**2
*n**4*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b*
*9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b
9*n2 + 1026576*b**9*n + 362880*b**9) - 11760*a**4*b**5*d**2*n**3*x**5*(
a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63
273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1

026576*b**9*n + 362880*b**9) - 16800*a**4*b**5*d**2*n**2*x**5*(a + b*x)**n/
(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**
5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*
n + 362880*b**9) - 8064*a**4*b**5*d**2*n*x**5*(a + b*x)**n/(b**9*n**9 + 45*
b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*
n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9)
 + 2*a**3*b**6*c**2*n**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*
n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n*
*3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 78*a**3*b**6*c**2*
n**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**
6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n*
*2 + 1026576*b**9*n + 362880*b**9) + 1250*a**3*b**6*c**2*n**4*(a + b*x)**n/
(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**
5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*
n + 362880*b**9) + 10530*a**3*b**6*c**2*n**3*(a + b*x)**n/(b**9*n**9 + 45*b
9*n8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n
4 + 723680*b9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9)
 + 49148*a**3*b**6*c**2*n**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b*
*9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9
*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 120432*a**3*b**
6*c**2*n*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9
*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**
9*n**2 + 1026576*b**9*n + 362880*b**9) + 120960*a**3*b**6*c**2*(a + b*x)**n
/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n*
*5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9
*n + 362880*b**9) + 40*a**3*b**6*c*d*n**6*x**3*(a + b*x)**n/(b**9*n**9 + 45
*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9
*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9
) + 1080*a**3*b**6*c*d*n**5*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 8
70*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680
*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 10600*a**3
*b**6*c*d*n**4*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7
 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 +
1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 45000*a**3*b**6*c*d*n**
3*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n
6 + 63273*b9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*
n**2 + 1026576*b**9*n + 362880*b**9) + 75760*a**3*b**6*c*d*n**2*x**3*(a + b
*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b
9*n5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 102657
6*b**9*n + 362880*b**9) + 40320*a**3*b**6*c*d*n*x**3*(a + b*x)**n/(b**9*n**
9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 26932
5*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 36288
0*b**9) + 56*a**3*b**6*d**2*n**6*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**
8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 7
23680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 840*a

3*b6*d**2*n**5*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n
7 + 9450*b9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**
3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 4760*a**3*b**6*d**2
*n**4*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b*
*9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b
9*n2 + 1026576*b**9*n + 362880*b**9) + 12600*a**3*b**6*d**2*n**3*x**6*(
a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63
273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1
026576*b**9*n + 362880*b**9) + 15344*a**3*b**6*d**2*n**2*x**6*(a + b*x)**n/
(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**
5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*
n + 362880*b**9) + 6720*a**3*b**6*d**2*n*x**6*(a + b*x)**n/(b**9*n**9 + 45*
b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*
n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9)
 - 2*a**2*b**7*c**2*n**7*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**
9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*
n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 78*a**2*b**7*c**
2*n**6*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9
*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**
9*n**2 + 1026576*b**9*n + 362880*b**9) - 1250*a**2*b**7*c**2*n**5*x*(a + b*
x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b*
*9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576
*b**9*n + 362880*b**9) - 10530*a**2*b**7*c**2*n**4*x*(a + b*x)**n/(b**9*n**
9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 26932
5*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 36288
0*b**9) - 49148*a**2*b**7*c**2*n**3*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**
8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 7
23680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 12043
2*a**2*b**7*c**2*n**2*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n
7 + 9450*b9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**
3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 120960*a**2*b**7*c*
*2*n*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n
6 + 63273*b9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*
n**2 + 1026576*b**9*n + 362880*b**9) - 10*a**2*b**7*c*d*n**7*x**4*(a + b*x)
n/(b9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9
*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b
9*n + 362880*b9) - 300*a**2*b**7*c*d*n**6*x**4*(a + b*x)**n/(b**9*n**9
 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*
b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*
b**9) - 3460*a**2*b**7*c*d*n**5*x**4*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8
 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 72
3680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 19200*
a**2*b**7*c*d*n**4*x**4*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n
7 + 9450*b9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**
3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 52690*a**2*b**7*c*d

*n**3*x**4*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b*
*9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b
9*n2 + 1026576*b**9*n + 362880*b**9) - 66900*a**2*b**7*c*d*n**2*x**4*(a
 + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 632
73*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 10
26576*b**9*n + 362880*b**9) - 30240*a**2*b**7*c*d*n*x**4*(a + b*x)**n/(b**9
*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 2
69325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 3
62880*b**9) - 8*a**2*b**7*d**2*n**7*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**9*
n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4
 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 16
8*a**2*b**7*d**2*n**6*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**
9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*
n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 1400*a**2*b**7*d
2*n5*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450
*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 117270
0*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 5880*a**2*b**7*d**2*n**4*x**7
*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 +
63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 +
 1026576*b**9*n + 362880*b**9) - 12992*a**2*b**7*d**2*n**3*x**7*(a + b*x)**
n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n
5 + 269325*b9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**
9*n + 362880*b**9) - 14112*a**2*b**7*d**2*n**2*x**7*(a + b*x)**n/(b**9*n**9
 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325
*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880
*b**9) - 5760*a**2*b**7*d**2*n*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8
 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723
680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + a*b**8*
c**2*n**8*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 945
0*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 11727
00*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 40*a*b**8*c**2*n**7*x**2*(a
 + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 6327
3*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 102
6576*b**9*n + 362880*b**9) + 664*a*b**8*c**2*n**6*x**2*(a + b*x)**n/(b**9*n
9 + 45*b9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269
325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362
880*b**9) + 5890*a*b**8*c**2*n**5*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n*
*8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 +
723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 2983
9*a*b**8*c**2*n**4*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n
7 + 9450*b9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**
3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 84790*a*b**8*c**2*n
3*x2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9
*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**
9*n**2 + 1026576*b**9*n + 362880*b**9) + 120696*a*b**8*c**2*n**2*x**2*(a +

b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*
b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 10265
76*b**9*n + 362880*b**9) + 60480*a*b**8*c**2*n*x**2*(a + b*x)**n/(b**9*n**9
 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325
*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880
*b**9) + 2*a*b**8*c*d*n**8*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 87
0*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*
b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 68*a*b**8*c
*d*n**7*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*
b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700
*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 932*a*b**8*c*d*n**6*x**5*(a +
b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*
b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 10265
76*b**9*n + 362880*b**9) + 6608*a*b**8*c*d*n**5*x**5*(a + b*x)**n/(b**9*n**
9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 26932
5*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 36288
0*b**9) + 25898*a*b**8*c*d*n**4*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8
 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 72
3680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 55532*
a*b**8*c*d*n**3*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7
 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 +
 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 59568*a*b**8*c*d*n**2*
x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**
6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n*
*2 + 1026576*b**9*n + 362880*b**9) + 24192*a*b**8*c*d*n*x**5*(a + b*x)**n/(
b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5
 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n
 + 362880*b**9) + a*b**8*d**2*n**8*x**8*(a + b*x)**n/(b**9*n**9 + 45*b**9*n
8 + 870*b9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 +
 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 28*
a*b**8*d**2*n**7*x**8*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**
7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3
+ 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 322*a*b**8*d**2*n**6*
x**8*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**
6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n*
*2 + 1026576*b**9*n + 362880*b**9) + 1960*a*b**8*d**2*n**5*x**8*(a + b*x)**
n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n
5 + 269325*b9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**
9*n + 362880*b**9) + 6769*a*b**8*d**2*n**4*x**8*(a + b*x)**n/(b**9*n**9 + 4
5*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**
9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**
9) + 13132*a*b**8*d**2*n**3*x**8*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 8
70*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680
*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 13068*a*b*
*8*d**2*n**2*x**8*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 +

9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 11
72700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 5040*a*b**8*d**2*n*x**8*(
a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63
273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1
026576*b**9*n + 362880*b**9) + b**9*c**2*n**8*x**3*(a + b*x)**n/(b**9*n**9
+ 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*
b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*
b**9) + 42*b**9*c**2*n**7*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870
*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b
9*n3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 744*b**9*c**
2*n**6*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b
9*n6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*
b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 7218*b**9*c**2*n**5*x**3*(a + b
*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b
9*n5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 102657
6*b**9*n + 362880*b**9) + 41619*b**9*c**2*n**4*x**3*(a + b*x)**n/(b**9*n**9
 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325
*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880
*b**9) + 144468*b**9*c**2*n**3*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8
+ 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723
680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 290276*
b**9*c**2*n**2*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7
+ 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 +
1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 301872*b**9*c**2*n*x**3
*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 +
63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 +
 1026576*b**9*n + 362880*b**9) + 120960*b**9*c**2*x**3*(a + b*x)**n/(b**9*n
9 + 45*b9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269
325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362
880*b**9) + 2*b**9*c*d*n**8*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 8
70*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680
*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 78*b**9*c*
d*n**7*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b
9*n6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*
b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 1272*b**9*c*d*n**6*x**6*(a + b*
x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b*
*9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576
*b**9*n + 362880*b**9) + 11268*b**9*c*d*n**5*x**6*(a + b*x)**n/(b**9*n**9 +
 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b
9*n4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b
9) + 58938*b9*c*d*n**4*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 87
0*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*
b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 185022*b**9
*c*d*n**3*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 945
0*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 11727

```
00*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 337228*b**9*c*d*n**2*x**6*(a
 + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 632
73*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 10
26576*b**9*n + 362880*b**9) + 322032*b**9*c*d*n*x**6*(a + b*x)**n/(b**9*n**
9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 26932
5*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 36288
0*b**9) + 120960*b**9*c*d*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870
*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b
**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + b**9*d**2*n*
*8*x**9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*
n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9
*n**2 + 1026576*b**9*n + 362880*b**9) + 36*b**9*d**2*n**7*x**9*(a + b*x)**n
/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n*
*5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9
*n + 362880*b**9) + 546*b**9*d**2*n**6*x**9*(a + b*x)**n/(b**9*n**9 + 45*b*
*9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n*
*4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) +
 4536*b**9*d**2*n**5*x**9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9
*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n
**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 22449*b**9*d**2*n
**4*x**9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9
*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**
9*n**2 + 1026576*b**9*n + 362880*b**9) + 67284*b**9*d**2*n**3*x**9*(a + b*x
)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**
9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*
b**9*n + 362880*b**9) + 118124*b**9*d**2*n**2*x**9*(a + b*x)**n/(b**9*n**9
+ 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*
b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*
b**9) + 109584*b**9*d**2*n*x**9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 87
0*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*
b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 40320*b**9*
d**2*x**9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**
9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b*
*9*n**2 + 1026576*b**9*n + 362880*b**9), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(294) = 588$.

Time = 0.21 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.04

$$\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx$$
$$=\frac{((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3)(bx+a)^nc^2}{(n^3+6n^2+11n+6)b^3}$$
$$+\frac{2\left((n^5+15n^4+85n^3+225n^2+274n+120)b^6x^6+(n^5+10n^4+35n^3+50n^2+24n)ab^5x^5-5(n^4\right.}{(n^6+21n^5+175n^4+7}$$
$$+\frac{((n^8+36n^7+546n^6+4536n^5+22449n^4+67284n^3+118124n^2+109584n+40320)b^9x^9+(n^8+}{}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^2/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 2*((n^5 + 15*n^4 + 85*n^3 + 225
*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5
*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*
a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)
^n*c*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6) +
 ((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 +
 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4
+ 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5 + 7
35*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 + 85*n
^4 + 225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^3 +
50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*x^4
- 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 - 4032
0*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*d^2/((n^9 + 45*n^8 + 870*n^7 + 9450*n^
6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 362880)
*b^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2660 vs. 2(294) = 588.

Time = 0.33 (sec) , antiderivative size = 2660, normalized size of antiderivative = 9.05

$$\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^9*d^2*n^8*x^9 + (b*x + a)^n*a*b^8*d^2*n^8*x^8 + 36*(b*x + a)
^n*b^9*d^2*n^7*x^9 + 28*(b*x + a)^n*a*b^8*d^2*n^7*x^8 + 546*(b*x + a)^n*b^9
*d^2*n^6*x^9 + 2*(b*x + a)^n*b^9*c*d*n^8*x^6 - 8*(b*x + a)^n*a^2*b^7*d^2*n^
7*x^7 + 322*(b*x + a)^n*a*b^8*d^2*n^6*x^8 + 4536*(b*x + a)^n*b^9*d^2*n^5*x^
```

9 + 2*(b*x + a)^n*a*b^8*c*d*n^8*x^5 + 78*(b*x + a)^n*b^9*c*d*n^7*x^6 - 168*
(b*x + a)^n*a^2*b^7*d^2*n^6*x^7 + 1960*(b*x + a)^n*a*b^8*d^2*n^5*x^8 + 2244
9*(b*x + a)^n*b^9*d^2*n^4*x^9 + 68*(b*x + a)^n*a*b^8*c*d*n^7*x^5 + 1272*(b*
x + a)^n*b^9*c*d*n^6*x^6 + 56*(b*x + a)^n*a^3*b^6*d^2*n^6*x^6 - 1400*(b*x +
a)^n*a^2*b^7*d^2*n^5*x^7 + 6769*(b*x + a)^n*a*b^8*d^2*n^4*x^8 + 67284*(b*x
+ a)^n*b^9*d^2*n^3*x^9 + (b*x + a)^n*b^9*c^2*n^8*x^3 - 10*(b*x + a)^n*a^2*
b^7*c*d*n^7*x^4 + 932*(b*x + a)^n*a*b^8*c*d*n^6*x^5 + 11268*(b*x + a)^n*b^9
*c*d*n^5*x^6 + 840*(b*x + a)^n*a^3*b^6*d^2*n^5*x^6 - 5880*(b*x + a)^n*a^2*b
^7*d^2*n^4*x^7 + 13132*(b*x + a)^n*a*b^8*d^2*n^3*x^8 + 118124*(b*x + a)^n*b
^9*d^2*n^2*x^9 + (b*x + a)^n*a*b^8*c^2*n^8*x^2 + 42*(b*x + a)^n*b^9*c^2*n^7
x^3 - 300(b*x + a)^n*a^2*b^7*c*d*n^6*x^4 + 6608*(b*x + a)^n*a*b^8*c*d*n^5
x^5 - 336(b*x + a)^n*a^4*b^5*d^2*n^5*x^5 + 58938*(b*x + a)^n*b^9*c*d*n^4*
x^6 + 4760*(b*x + a)^n*a^3*b^6*d^2*n^4*x^6 - 12992*(b*x + a)^n*a^2*b^7*d^2*
n^3*x^7 + 13068*(b*x + a)^n*a*b^8*d^2*n^2*x^8 + 109584*(b*x + a)^n*b^9*d^2*
n*x^9 + 40*(b*x + a)^n*a*b^8*c^2*n^7*x^2 + 744*(b*x + a)^n*b^9*c^2*n^6*x^3
+ 40*(b*x + a)^n*a^3*b^6*c*d*n^6*x^3 - 3460*(b*x + a)^n*a^2*b^7*c*d*n^5*x^4
+ 25898*(b*x + a)^n*a*b^8*c*d*n^4*x^5 - 3360*(b*x + a)^n*a^4*b^5*d^2*n^4*x
^5 + 185022*(b*x + a)^n*b^9*c*d*n^3*x^6 + 12600*(b*x + a)^n*a^3*b^6*d^2*n^3
x^6 - 14112(b*x + a)^n*a^2*b^7*d^2*n^2*x^7 + 5040*(b*x + a)^n*a*b^8*d^2*n
x^8 + 40320(b*x + a)^n*b^9*d^2*x^9 - 2*(b*x + a)^n*a^2*b^7*c^2*n^7*x + 66
4*(b*x + a)^n*a*b^8*c^2*n^6*x^2 + 7218*(b*x + a)^n*b^9*c^2*n^5*x^3 + 1080*(
b*x + a)^n*a^3*b^6*c*d*n^5*x^3 - 19200*(b*x + a)^n*a^2*b^7*c*d*n^4*x^4 + 16
80*(b*x + a)^n*a^5*b^4*d^2*n^4*x^4 + 55532*(b*x + a)^n*a*b^8*c*d*n^3*x^5 -
11760*(b*x + a)^n*a^4*b^5*d^2*n^3*x^5 + 337228*(b*x + a)^n*b^9*c*d*n^2*x^6
+ 15344*(b*x + a)^n*a^3*b^6*d^2*n^2*x^6 - 5760*(b*x + a)^n*a^2*b^7*d^2*n*x^
7 - 78*(b*x + a)^n*a^2*b^7*c^2*n^6*x + 5890*(b*x + a)^n*a*b^8*c^2*n^5*x^2 -
120*(b*x + a)^n*a^4*b^5*c*d*n^5*x^2 + 41619*(b*x + a)^n*b^9*c^2*n^4*x^3 +
10600*(b*x + a)^n*a^3*b^6*c*d*n^4*x^3 - 52690*(b*x + a)^n*a^2*b^7*c*d*n^3*x
^4 + 10080*(b*x + a)^n*a^5*b^4*d^2*n^3*x^4 + 59568*(b*x + a)^n*a*b^8*c*d*n^
2*x^5 - 16800*(b*x + a)^n*a^4*b^5*d^2*n^2*x^5 + 322032*(b*x + a)^n*b^9*c*d*
n*x^6 + 6720*(b*x + a)^n*a^3*b^6*d^2*n*x^6 + 2*(b*x + a)^n*a^3*b^6*c^2*n^6
- 1250*(b*x + a)^n*a^2*b^7*c^2*n^5*x + 29839*(b*x + a)^n*a*b^8*c^2*n^4*x^2
- 3000*(b*x + a)^n*a^4*b^5*c*d*n^4*x^2 + 144468*(b*x + a)^n*b^9*c^2*n^3*x^3
+ 45000*(b*x + a)^n*a^3*b^6*c*d*n^3*x^3 - 6720*(b*x + a)^n*a^6*b^3*d^2*n^3
x^3 - 66900(b*x + a)^n*a^2*b^7*c*d*n^2*x^4 + 18480*(b*x + a)^n*a^5*b^4*d^
2*n^2*x^4 + 24192*(b*x + a)^n*a*b^8*c*d*n*x^5 - 8064*(b*x + a)^n*a^4*b^5*d^
2*n*x^5 + 120960*(b*x + a)^n*b^9*c*d*x^6 + 78*(b*x + a)^n*a^3*b^6*c^2*n^5 -
10530*(b*x + a)^n*a^2*b^7*c^2*n^4*x + 240*(b*x + a)^n*a^5*b^4*c*d*n^4*x +
84790*(b*x + a)^n*a*b^8*c^2*n^3*x^2 - 25800*(b*x + a)^n*a^4*b^5*c*d*n^3*x^2
+ 290276*(b*x + a)^n*b^9*c^2*n^2*x^3 + 75760*(b*x + a)^n*a^3*b^6*c*d*n^2*x
^3 - 20160*(b*x + a)^n*a^6*b^3*d^2*n^2*x^3 - 30240*(b*x + a)^n*a^2*b^7*c*d*
n*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^2*n*x^4 + 1250*(b*x + a)^n*a^3*b^6*c^2*
n^4 - 49148*(b*x + a)^n*a^2*b^7*c^2*n^3*x + 5760*(b*x + a)^n*a^5*b^4*c*d*n^
3*x + 120696*(b*x + a)^n*a*b^8*c^2*n^2*x^2 - 83400*(b*x + a)^n*a^4*b^5*c*d*
n^2*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^2*n^2*x^2 + 301872*(b*x + a)^n*b^9*c^

```
2*n*x^3 + 40320*(b*x + a)^n*a^3*b^6*c*d*n*x^3 - 13440*(b*x + a)^n*a^6*b^3*d
^2*n*x^3 + 10530*(b*x + a)^n*a^3*b^6*c^2*n^3 - 240*(b*x + a)^n*a^6*b^3*c*d*
n^3 - 120432*(b*x + a)^n*a^2*b^7*c^2*n^2*x + 45840*(b*x + a)^n*a^5*b^4*c*d*
n^2*x + 60480*(b*x + a)^n*a*b^8*c^2*n*x^2 - 60480*(b*x + a)^n*a^4*b^5*c*d*n
*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^2*n*x^2 + 120960*(b*x + a)^n*b^9*c^2*x^3
 + 49148*(b*x + a)^n*a^3*b^6*c^2*n^2 - 5760*(b*x + a)^n*a^6*b^3*c*d*n^2 - 1
20960*(b*x + a)^n*a^2*b^7*c^2*n*x + 120960*(b*x + a)^n*a^5*b^4*c*d*n*x - 40
320*(b*x + a)^n*a^8*b*d^2*n*x + 120432*(b*x + a)^n*a^3*b^6*c^2*n - 45840*(b
*x + a)^n*a^6*b^3*c*d*n + 120960*(b*x + a)^n*a^3*b^6*c^2 - 120960*(b*x + a)
^n*a^6*b^3*c*d + 40320*(b*x + a)^n*a^9*d^2)/(b^9*n^9 + 45*b^9*n^8 + 870*b^9
*n^7 + 9450*b^9*n^6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 117
2700*b^9*n^2 + 1026576*b^9*n + 362880*b^9)
```

Mupad [B] (verification not implemented)

Time = 19.99 (sec) , antiderivative size = 1410, normalized size of antiderivative = 4.80

$$\int x^2(a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

```
[In] int(x^2*(c + d*x^3)^2*(a + b*x)^n,x)
```

```
[Out] (d^2*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*
n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(1026576*n + 1172700*n^2 + 723680*n^
3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880) +
(2*a^3*(a + b*x)^n*(20160*a^6*d^2 + 60480*b^6*c^2 + 60216*b^6*c^2*n + 24574
*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b^6*c^2*n^5 + b^6*c^
2*n^6 - 60480*a^3*b^3*c*d - 22920*a^3*b^3*c*d*n - 2880*a^3*b^3*c*d*n^2 - 12
0*a^3*b^3*c*d*n^3))/(b^9*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4
 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (x^3*(a + b*x
)^n*(3*n + n^2 + 2)*(60480*b^6*c^2 - 6720*a^6*d^2*n + 60216*b^6*c^2*n + 245
74*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b^6*c^2*n^5 + b^6*
c^2*n^6 + 20160*a^3*b^3*c*d*n + 7640*a^3*b^3*c*d*n^2 + 960*a^3*b^3*c*d*n^3
+ 40*a^3*b^3*c*d*n^4))/(b^6*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*
n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (2*d*x^6*(
a + b*x)^n*(504*b^3*c + 24*b^3*c*n^2 + b^3*c*n^3 + 28*a^3*d*n + 191*b^3*c*n
)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^3*(1026576*n + 117270
0*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 +
 n^9 + 362880)) - (2*a^2*n*x*(a + b*x)^n*(20160*a^6*d^2 + 60480*b^6*c^2 + 6
0216*b^6*c^2*n + 24574*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 3
9*b^6*c^2*n^5 + b^6*c^2*n^6 - 60480*a^3*b^3*c*d - 22920*a^3*b^3*c*d*n - 288
0*a^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^8*(1026576*n + 1172700*n^2 + 7
23680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 36
2880)) + (a*n*x^2*(n + 1)*(a + b*x)^n*(20160*a^6*d^2 + 60480*b^6*c^2 + 6021
6*b^6*c^2*n + 24574*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b
```

```
^6*c^2*n^5 + b^6*c^2*n^6 - 60480*a^3*b^3*c*d - 22920*a^3*b^3*c*d*n - 2880*a
^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^7*(1026576*n + 1172700*n^2 + 7236
80*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 36288
0)) + (a*d^2*n*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 +
 322*n^5 + 28*n^6 + n^7 + 5040))/(b*(1026576*n + 1172700*n^2 + 723680*n^3 +
 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) - (8
*a^2*d^2*n*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5
+ n^6 + 720))/(b^2*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 632
73*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) - (10*a^2*d*n*x^4*(a
+ b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(504*b^3*c - 168*a^3*d + 24*b^3*c*n^2 + b
^3*c*n^3 + 191*b^3*c*n))/(b^5*(1026576*n + 1172700*n^2 + 723680*n^3 + 26932
5*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (2*a*d*n
*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(504*b^3*c - 168*a^3*d
 + 24*b^3*c*n^2 + b^3*c*n^3 + 191*b^3*c*n))/(b^4*(1026576*n + 1172700*n^2 +
 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 +
362880))
```

3.179 $\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 248

$$\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx = -\frac{a(b^3c-a^3d)^2 (a+bx)^{1+n}}{b^8(1+n)} + \frac{(b^3c-7a^3d)(b^3c-a^3d)(a+bx)^{2+n}}{b^8(2+n)} + \frac{3a^2d(4b^3c-7a^3d)(a+bx)^{3+n}}{b^8(3+n)} - \frac{ad(8b^3c-35a^3d)(a+bx)^{4+n}}{b^8(4+n)} + \frac{d(2b^3c-35a^3d)(a+bx)^{5+n}}{b^8(5+n)} + \frac{21a^2d^2(a+bx)^{6+n}}{b^8(6+n)} - \frac{7ad^2(a+bx)^{7+n}}{b^8(7+n)} + \frac{d^2(a+bx)^{8+n}}{b^8(8+n)}$$

```
[Out] -a*(-a^3*d+b^3*c)^2*(b*x+a)^(1+n)/b^8/(1+n)+(-7*a^3*d+b^3*c)*(-a^3*d+b^3*c)
*(b*x+a)^(2+n)/b^8/(2+n)+3*a^2*d*(-7*a^3*d+4*b^3*c)*(b*x+a)^(3+n)/b^8/(3+n)
-a*d*(-35*a^3*d+8*b^3*c)*(b*x+a)^(4+n)/b^8/(4+n)+d*(-35*a^3*d+2*b^3*c)*(b*x
+a)^(5+n)/b^8/(5+n)+21*a^2*d^2*(b*x+a)^(6+n)/b^8/(6+n)-7*a*d^2*(b*x+a)^(7+n
)/b^8/(7+n)+d^2*(b*x+a)^(8+n)/b^8/(8+n)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1634}

$$\int x(a+bx)^n\left(c+dx^3\right)^2\,dx=-\frac{a(b^3c-a^3d)^2(a+bx)^{n+1}}{b^8(n+1)}+\frac{(b^3c-7a^3d)(b^3c-a^3d)(a+bx)^{n+2}}{b^8(n+2)}-\frac{ad(8b^3c-35a^3d)(a+bx)^{n+4}}{b^8(n+4)}+\frac{d(2b^3c-35a^3d)(a+bx)^{n+5}}{b^8(n+5)}+\frac{21a^2d^2(a+bx)^{n+6}}{b^8(n+6)}+\frac{3a^2d(4b^3c-7a^3d)(a+bx)^{n+3}}{b^8(n+3)}-\frac{7ad^2(a+bx)^{n+7}}{b^8(n+7)}+\frac{d^2(a+bx)^{n+8}}{b^8(n+8)}$$

```
[In] Int[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] -((a*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^3*c - 7*a^3*
d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) + (3*a^2*d*(4*b^3*c - 7
*a^3*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) - (a*d*(8*b^3*c - 35*a^3*d)*(a + b
*x)^(4 + n))/(b^8*(4 + n)) + (d*(2*b^3*c - 35*a^3*d)*(a + b*x)^(5 + n))/(b^
8*(5 + n)) + (21*a^2*d^2*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^2*(a + b
*x)^(7 + n))/(b^8*(7 + n)) + (d^2*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\text{integral}=\int\left(-\frac{a(-b^3c+a^3d)^2(a+bx)^n}{b^7}+\frac{(b^3c-7a^3d)(b^3c-a^3d)(a+bx)^{1+n}}{b^7}-\frac{3a^2d(-4b^3c+7a^3d)(a+bx)^{2+n}}{b^7}+\frac{ad(-8b^3c+35a^3d)(a+bx)^{3+n}}{b^7}+\frac{d(2b^3c-35a^3d)(a+bx)^{4+n}}{b^7}+\frac{21a^2d^2(a+bx)^{5+n}}{b^7}-\frac{7ad^2(a+bx)^{6+n}}{b^7}+\frac{d^2(a+bx)^{7+n}}{b^7}\right)dx$$

$$= -\frac{a(b^3c - a^3d)^2 (a+bx)^{1+n}}{b^8(1+n)} + \frac{(b^3c - 7a^3d)(b^3c - a^3d)(a+bx)^{2+n}}{b^8(2+n)} + \frac{3a^2d(4b^3c - 7a^3d)(a+bx)^{3+n}}{b^8(3+n)} - \frac{ad(8b^3c - 35a^3d)(a+bx)^{4+n}}{b^8(4+n)} + \frac{d(2b^3c - 35a^3d)(a+bx)^{5+n}}{b^8(5+n)} + \frac{21a^2d^2(a+bx)^{6+n}}{b^8(6+n)} - \frac{7ad^2(a+bx)^{7+n}}{b^8(7+n)} + \frac{d^2(a+bx)^{8+n}}{b^8(8+n)}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.85

$$\int x(a+bx)^n \left(c+dx^3\right)^2 dx$$

$$= \frac{(a+bx)^{1+n} \left(-\frac{a(b^3c-a^3d)^2}{1+n} + \frac{(b^3c-7a^3d)(b^3c-a^3d)(a+bx)}{2+n} + \frac{3a^2d(4b^3c-7a^3d)(a+bx)^2}{3+n} + \frac{ad(-8b^3c+35a^3d)(a+bx)^3}{4+n} + \frac{d(2b^3c-\cdots}{\cdots}\right.}{b^8}$$

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-((a*(b^3*c - a^3*d)^2)/(1 + n)) + ((b^3*c - 7*a^3*d)*(
b^3*c - a^3*d)*(a + b*x))/(2 + n) + (3*a^2*d*(4*b^3*c - 7*a^3*d)*(a + b*x)^
2)/(3 + n) + (a*d*(-8*b^3*c + 35*a^3*d)*(a + b*x)^3)/(4 + n) + (d*(2*b^3*c
- 35*a^3*d)*(a + b*x)^4)/(5 + n) + (21*a^2*d^2*(a + b*x)^5)/(6 + n) - (7*a*
d^2*(a + b*x)^6)/(7 + n) + (d^2*(a + b*x)^7)/(8 + n)))/b^8
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. 2(248) = 496.

Time = 1.00 (sec) , antiderivative size = 893, normalized size of antiderivative = 3.60

method	result
norman	$\frac{d^2x^8e^{n\ln(bx+a)}}{8+n} + \frac{na(b^6c^2n^6+33b^6c^2n^5+445b^6c^2n^4-48a^3b^3cd\,n^3+3135b^6c^2n^3-1008a^3b^3cd\,n^2+12154b^6c^2n^2-7008a^3b^3c\cdots}{b^7(n^8+36n^7+546n^6+4536n^5+22449n^4+67284n^3+118124\cdots}$
gosper	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

```
[In] int(x*(b*x+a)^n*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/(8+n)*x^8*exp(n*ln(b*x+a))+1/b^7*n*a*(b^6*c^2*n^6+33*b^6*c^2*n^5+445*b^
6*c^2*n^4-48*a^3*b^3*c*d*n^3+3135*b^6*c^2*n^3-1008*a^3*b^3*c*d*n^2+12154*b^
6*c^2*n^2-7008*a^3*b^3*c*d*n+24552*b^6*c^2*n+5040*a^6*d^2-16128*a^3*b^3*c*d
```

```
+20160*b^6*c^2)/(n^8+36*n^7+546*n^6+4536*n^5+22449*n^4+67284*n^3+118124*n^2
+109584*n+40320)*x*exp(n*ln(b*x+a))+d^2*a*n/b/(n^2+15*n+56)*x^7*exp(n*ln(b*
x+a))-a^2*(b^6*c^2*n^6+33*b^6*c^2*n^5+445*b^6*c^2*n^4-48*a^3*b^3*c*d*n^3+31
35*b^6*c^2*n^3-1008*a^3*b^3*c*d*n^2+12154*b^6*c^2*n^2-7008*a^3*b^3*c*d*n+24
552*b^6*c^2*n+5040*a^6*d^2-16128*a^3*b^3*c*d+20160*b^6*c^2)/b^8/(n^8+36*n^7
+546*n^6+4536*n^5+22449*n^4+67284*n^3+118124*n^2+109584*n+40320)*exp(n*ln(b
*x+a))-(-b^6*c^2*n^6-33*b^6*c^2*n^5-24*a^3*b^3*c*d*n^4-445*b^6*c^2*n^4-504*
a^3*b^3*c*d*n^3-3135*b^6*c^2*n^3-3504*a^3*b^3*c*d*n^2-12154*b^6*c^2*n^2+252
0*a^6*d^2*n-8064*a^3*b^3*c*d*n-24552*b^6*c^2*n-20160*b^6*c^2)/b^6/(n^7+35*n
^6+511*n^5+4025*n^4+18424*n^3+48860*n^2+69264*n+40320)*x^2*exp(n*ln(b*x+a))
+2*d*(b^3*c*n^3+21*b^3*c*n^2+21*a^3*d*n+146*b^3*c*n+336*b^3*c)/b^3/(n^4+26*
n^3+251*n^2+1066*n+1680)*x^5*exp(n*ln(b*x+a))-7*n*a^2*d^2/b^2/(n^3+21*n^2+1
46*n+336)*x^6*exp(n*ln(b*x+a))-2*n*a*d*(-b^3*c*n^3-21*b^3*c*n^2-146*b^3*c*n
+105*a^3*d-336*b^3*c)/b^4/(n^5+30*n^4+355*n^3+2070*n^2+5944*n+6720)*x^4*exp
(n*ln(b*x+a))+8*(-b^3*c*n^3-21*b^3*c*n^2-146*b^3*c*n+105*a^3*d-336*b^3*c)*a
^2/b^5*d*n/(n^6+33*n^5+445*n^4+3135*n^3+12154*n^2+24552*n+20160)*x^3*exp(n*
ln(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. $2(248) = 496$.

Time = 0.30 (sec) , antiderivative size = 1216, normalized size of antiderivative = 4.90

$$\int x(a+bx)^n \left(c+dx^3\right)^2 dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b^6*c^2*n^6 + 33*a^2*b^6*c^2*n^5 + 445*a^2*b^6*c^2*n^4 + 20160*a^2*b^
6*c^2 - 16128*a^5*b^3*c*d + 5040*a^8*d^2 - (b^8*d^2*n^7 + 28*b^8*d^2*n^6 +
322*b^8*d^2*n^5 + 1960*b^8*d^2*n^4 + 6769*b^8*d^2*n^3 + 13132*b^8*d^2*n^2 +
 13068*b^8*d^2*n + 5040*b^8*d^2)*x^8 - (a*b^7*d^2*n^7 + 21*a*b^7*d^2*n^6 +
175*a*b^7*d^2*n^5 + 735*a*b^7*d^2*n^4 + 1624*a*b^7*d^2*n^3 + 1764*a*b^7*d^2
*n^2 + 720*a*b^7*d^2*n)*x^7 + 7*(a^2*b^6*d^2*n^6 + 15*a^2*b^6*d^2*n^5 + 85*
a^2*b^6*d^2*n^4 + 225*a^2*b^6*d^2*n^3 + 274*a^2*b^6*d^2*n^2 + 120*a^2*b^6*d
^2*n)*x^6 - 2*(b^8*c*d*n^7 + 31*b^8*c*d*n^6 + 8064*b^8*c*d + (391*b^8*c*d +
 21*a^3*b^5*d^2)*n^5 + (2581*b^8*c*d + 210*a^3*b^5*d^2)*n^4 + (9544*b^8*c*d
 + 735*a^3*b^5*d^2)*n^3 + 2*(9782*b^8*c*d + 525*a^3*b^5*d^2)*n^2 + 72*(282*
b^8*c*d + 7*a^3*b^5*d^2)*n)*x^5 - 2*(a*b^7*c*d*n^7 + 27*a*b^7*c*d*n^6 + 283
*a*b^7*c*d*n^5 + 21*(69*a*b^7*c*d - 5*a^4*b^4*d^2)*n^4 + 2*(1874*a*b^7*c*d
- 315*a^4*b^4*d^2)*n^3 + 3*(1524*a*b^7*c*d - 385*a^4*b^4*d^2)*n^2 + 126*(16
*a*b^7*c*d - 5*a^4*b^4*d^2)*n)*x^4 + 3*(1045*a^2*b^6*c^2 - 16*a^5*b^3*c*d)*
n^3 + 8*(a^2*b^6*c*d*n^6 + 24*a^2*b^6*c*d*n^5 + 211*a^2*b^6*c*d*n^4 + 3*(27
2*a^2*b^6*c*d - 35*a^5*b^3*d^2)*n^3 + 5*(260*a^2*b^6*c*d - 63*a^5*b^3*d^2)*
n^2 + 42*(16*a^2*b^6*c*d - 5*a^5*b^3*d^2)*n)*x^3 + 2*(6077*a^2*b^6*c^2 - 50
```

```
4*a^5*b^3*c*d)*n^2 - (b^8*c^2*n^7 + 34*b^8*c^2*n^6 + 20160*b^8*c^2 + 2*(239
*b^8*c^2 + 12*a^3*b^5*c*d)*n^5 + 4*(895*b^8*c^2 + 132*a^3*b^5*c*d)*n^4 + (1
5289*b^8*c^2 + 4008*a^3*b^5*c*d)*n^3 + 2*(18353*b^8*c^2 + 5784*a^3*b^5*c*d
- 1260*a^6*b^2*d^2)*n^2 + 72*(621*b^8*c^2 + 112*a^3*b^5*c*d - 35*a^6*b^2*d^
2)*n)*x^2 + 24*(1023*a^2*b^6*c^2 - 292*a^5*b^3*c*d)*n - (a*b^7*c^2*n^7 + 33
*a*b^7*c^2*n^6 + 445*a*b^7*c^2*n^5 + 3*(1045*a*b^7*c^2 - 16*a^4*b^4*c*d)*n^
4 + 2*(6077*a*b^7*c^2 - 504*a^4*b^4*c*d)*n^3 + 24*(1023*a*b^7*c^2 - 292*a^4
*b^4*c*d)*n^2 + 1008*(20*a*b^7*c^2 - 16*a^4*b^4*c*d + 5*a^7*b*d^2)*n)*x)*(b
*x + a)^n/(b^8*n^8 + 36*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^
4 + 67284*b^8*n^3 + 118124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18328 vs. 2(228) = 456.

Time = 5.69 (sec) , antiderivative size = 18328, normalized size of antiderivative = 73.90

$$\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x**2/2 + 2*c*d*x**5/5 + d**2*x**8/8), Eq(b, 0)), (420
*a**7*d**2*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10
*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**
5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 1089*a**7*d**2/(420*a**7*b**8 + 2
940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3
*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) +
2940*a**6*b*d**2*x*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a*
*5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b
**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 7203*a**6*b*d**2*x/(420*a
**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3
+ 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b*
*15*x**7) + 8820*a**5*b**2*d**2*x**2*log(a/b + x)/(420*a**7*b**8 + 2940*a**
6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*
x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 20139*a
**5*b**2*d**2*x**2/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2
 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2
940*a*b**14*x**6 + 420*b**15*x**7) - 8*a**4*b**3*c*d/(420*a**7*b**8 + 2940*
a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**
12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 1470
0*a**4*b**3*d**2*x**3*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820
*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**
2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 30625*a**4*b**3*d**2*x
**3/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b
**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**
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6 + 420*b**15*x**7) - 56*a**3*b**4*c*d*x/(420*a**7*b**8 + 2940*a**6*b**9*x
+ 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 88
20*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 14700*a**3*b**4*
d**2*x**4*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*
x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5
 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 26950*a**3*b**4*d**2*x**4/(420*a**
7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 +
14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**1
5*x**7) - 168*a**2*b**5*c*d*x**2/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a
5*b10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*
b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 8820*a**2*b**5*d**2*x**5
*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14
700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a
*b**14*x**6 + 420*b**15*x**7) + 13230*a**2*b**5*d**2*x**5/(420*a**7*b**8 +
2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**
3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) -
 10*a*b**6*c**2/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 +
14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940
*a*b**14*x**6 + 420*b**15*x**7) - 280*a*b**6*c*d*x**3/(420*a**7*b**8 + 2940
*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b*
*12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 294
0*a*b**6*d**2*x**6*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a*
*5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b
13*x5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 2940*a*b**6*d**2*x**6/(42
0*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x*
*3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420
*b**15*x**7) - 70*b**7*c**2*x/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5
*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**
13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) - 280*b**7*c*d*x**4/(420*a**7
*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 1
4700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15
*x**7) + 420*b**7*d**2*x**7*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x
+ 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 88
20*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7), Eq(n, -8)), (-420
*a**7*d**2*log(a/b + x)/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x*
*2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**
14*x**6) - 1029*a**7*d**2/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*
x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b
14*x6) - 2520*a**6*b*d**2*x*log(a/b + x)/(60*a**6*b**8 + 360*a**5*b**9*
x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*
a*b**13*x**5 + 60*b**14*x**6) - 5754*a**6*b*d**2*x/(60*a**6*b**8 + 360*a**5
*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4
+ 360*a*b**13*x**5 + 60*b**14*x**6) - 6300*a**5*b**2*d**2*x**2*log(a/b + x)
/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x*
*3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 13125*a**5*b

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**2*d**2*x**2/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*
a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) -
 4*a**4*b**3*c*d/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 12
00*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6
) - 8400*a**4*b**3*d**2*x**3*log(a/b + x)/(60*a**6*b**8 + 360*a**5*b**9*x +
 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b
**13*x**5 + 60*b**14*x**6) - 15400*a**4*b**3*d**2*x**3/(60*a**6*b**8 + 360*
a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x
**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 24*a**3*b**4*c*d*x/(60*a**6*b**8
+ 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b
**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 6300*a**3*b**4*d**2*x**4*lo
g(a/b + x)/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**
3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 94
50*a**3*b**4*d**2*x**4/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**
2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**1
4*x**6) - 60*a**2*b**5*c*d*x**2/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*
b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5
+ 60*b**14*x**6) - 2520*a**2*b**5*d**2*x**5*log(a/b + x)/(60*a**6*b**8 + 36
0*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12
*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 2520*a**2*b**5*d**2*x**5/(60*a*
*6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 90
0*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 2*a*b**6*c**2/(60*a
**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 9
00*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 80*a*b**6*c*d*x**3
/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x*
*3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) - 420*a*b**6*d
**2*x**6*log(a/b + x)/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2
 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14
*x**6) - 12*b**7*c**2*x/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x*
*2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**
14*x**6) - 60*b**7*c*d*x**4/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**1
0*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60
*b**14*x**6) + 60*b**7*d**2*x**7/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4
*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5
 + 60*b**14*x**6), Eq(n, -7)), (420*a**7*d**2*log(a/b + x)/(20*a**5*b**8 +
100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x
**4 + 20*b**13*x**5) + 959*a**7*d**2/(20*a**5*b**8 + 100*a**4*b**9*x + 200*
a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) +
 2100*a**6*b*d**2*x*log(a/b + x)/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3
*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 437
5*a**6*b*d**2*x/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200
*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 4200*a**5*b**2*d**2*
x**2*log(a/b + x)/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 2
00*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 7700*a**5*b**2*d**
2*x**2/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**
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11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) - 8*a**4*b**3*c*d/(20*a**5*b**8
 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**1
2*x**4 + 20*b**13*x**5) + 4200*a**4*b**3*d**2*x**3*log(a/b + x)/(20*a**5*b*
*8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b*
*12*x**4 + 20*b**13*x**5) + 6300*a**4*b**3*d**2*x**3/(20*a**5*b**8 + 100*a*
*4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 +
20*b**13*x**5) - 40*a**3*b**4*c*d*x/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a
3*b10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) +
2100*a**3*b**4*d**2*x**4*log(a/b + x)/(20*a**5*b**8 + 100*a**4*b**9*x + 200
*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5)
+ 2100*a**3*b**4*d**2*x**4/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10
*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) - 80*a**2*b
5*c*d*x2/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a*
*2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) + 420*a**2*b**5*d**2*x**5
*log(a/b + x)/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a
2*b11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) - a*b**6*c**2/(20*a**5*b
8 + 100*a4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b
12*x4 + 20*b**13*x**5) - 80*a*b**6*c*d*x**3/(20*a**5*b**8 + 100*a**4*b*
*9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b*
*13*x**5) - 70*a*b**6*d**2*x**6/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*
b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) - 5*b*
*7*c**2*x/(20*a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*
b**11*x**3 + 100*a*b**12*x**4 + 20*b**13*x**5) - 40*b**7*c*d*x**4/(20*a**5*
b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*
b**12*x**4 + 20*b**13*x**5) + 10*b**7*d**2*x**7/(20*a**5*b**8 + 100*a**4*b*
*9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 100*a*b**12*x**4 + 20*b*
*13*x**5), Eq(n, -6)), (-420*a**7*d**2*log(a/b + x)/(12*a**4*b**8 + 48*a**3
*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 875*a**7*
d**2/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3
+ 12*b**12*x**4) - 1680*a**6*b*d**2*x*log(a/b + x)/(12*a**4*b**8 + 48*a**3*
b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 3080*a**6*
b*d**2*x/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x
3 + 12*b12*x**4) - 2520*a**5*b**2*d**2*x**2*log(a/b + x)/(12*a**4*b**8
+ 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) -
3780*a**5*b**2*d**2*x**2/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**
2 + 48*a*b**11*x**3 + 12*b**12*x**4) + 24*a**4*b**3*c*d*log(a/b + x)/(12*a*
*4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*
x**4) + 50*a**4*b**3*c*d/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**
2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 1680*a**4*b**3*d**2*x**3*log(a/b + x
)/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 1
2*b**12*x**4) - 1680*a**4*b**3*d**2*x**3/(12*a**4*b**8 + 48*a**3*b**9*x + 7
2*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) + 96*a**3*b**4*c*d*x*l
og(a/b + x)/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**1
1*x**3 + 12*b**12*x**4) + 176*a**3*b**4*c*d*x/(12*a**4*b**8 + 48*a**3*b**9*
x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 420*a**3*b**4*d

```
**2*x**4*log(a/b + x)/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 +
 48*a*b**11*x**3 + 12*b**12*x**4) + 144*a**2*b**5*c*d*x**2*log(a/b + x)/(12
*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**
12*x**4) + 216*a**2*b**5*c*d*x**2/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*
b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) + 84*a**2*b**5*d**2*x**5/(12*
a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**1
2*x**4) - a*b**6*c**2/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 +
 48*a*b**11*x**3 + 12*b**12*x**4) + 96*a*b**6*c*d*x**3*log(a/b + x)/(12*a**
4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x
**4) + 96*a*b**6*c*d*x**3/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x*
*2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 14*a*b**6*d**2*x**6/(12*a**4*b**8 +
 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) - 4
*b**7*c**2*x/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**
11*x**3 + 12*b**12*x**4) + 24*b**7*c*d*x**4*log(a/b + x)/(12*a**4*b**8 + 48
*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) + 4*b*
*7*d**2*x**7/(12*a**4*b**8 + 48*a**3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**
11*x**3 + 12*b**12*x**4), Eq(n, -5)), (420*a**7*d**2*log(a/b + x)/(12*a**3*
b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) + 770*a**7*d**2/(1
2*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) + 1260*a**6
*b*d**2*x*log(a/b + x)/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 1
2*b**11*x**3) + 1890*a**6*b*d**2*x/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b*
*10*x**2 + 12*b**11*x**3) + 1260*a**5*b**2*d**2*x**2*log(a/b + x)/(12*a**3*
b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) + 1260*a**5*b**2*d
**2*x**2/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3)
- 96*a**4*b**3*c*d*log(a/b + x)/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10
*x**2 + 12*b**11*x**3) - 176*a**4*b**3*c*d/(12*a**3*b**8 + 36*a**2*b**9*x +
 36*a*b**10*x**2 + 12*b**11*x**3) + 420*a**4*b**3*d**2*x**3*log(a/b + x)/(1
2*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) - 288*a**3*
b**4*c*d*x*log(a/b + x)/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 +
12*b**11*x**3) - 432*a**3*b**4*c*d*x/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*
b**10*x**2 + 12*b**11*x**3) - 105*a**3*b**4*d**2*x**4/(12*a**3*b**8 + 36*a*
*2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) - 288*a**2*b**5*c*d*x**2*log(a
/b + x)/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) -
 288*a**2*b**5*c*d*x**2/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 +
12*b**11*x**3) + 21*a**2*b**5*d**2*x**5/(12*a**3*b**8 + 36*a**2*b**9*x + 36
*a*b**10*x**2 + 12*b**11*x**3) - 2*a*b**6*c**2/(12*a**3*b**8 + 36*a**2*b**9
*x + 36*a*b**10*x**2 + 12*b**11*x**3) - 96*a*b**6*c*d*x**3*log(a/b + x)/(12
*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3) - 7*a*b**6*d
**2*x**6/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11*x**3)
- 6*b**7*c**2*x/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2 + 12*b**11
*x**3) + 24*b**7*c*d*x**4/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b**10*x**2
+ 12*b**11*x**3) + 3*b**7*d**2*x**7/(12*a**3*b**8 + 36*a**2*b**9*x + 36*a*b
**10*x**2 + 12*b**11*x**3), Eq(n, -4)), (-420*a**7*d**2*log(a/b + x)/(20*a*
*2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 630*a**7*d**2/(20*a**2*b**8 + 40*a
*b**9*x + 20*b**10*x**2) - 840*a**6*b*d**2*x*log(a/b + x)/(20*a**2*b**8 + 4
```

0*a*b**9*x + 20*b**10*x**2) - 840*a**6*b*d**2*x/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 420*a**5*b**2*d**2*x**2*log(a/b + x)/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) + 240*a**4*b**3*c*d*log(a/b + x)/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) + 360*a**4*b**3*c*d/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) + 140*a**4*b**3*d**2*x**3/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) + 480*a**3*b**4*c*d*x*log(a/b + x)/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) + 480*a**3*b**4*c*d*x/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 35*a**3*b**4*d**2*x**4/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) + 240*a**2*b**5*c*d*x**2*log(a/b + x)/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) + 14*a**2*b**5*d**2*x**5/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 10*a*b**6*c**2/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 80*a*b**6*c*d*x**3/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 7*a*b**6*d**2*x**6/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) - 20*b**7*c**2*x/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) + 20*b**7*c*d*x**4/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2) + 4*b**7*d**2*x**7/(20*a**2*b**8 + 40*a*b**9*x + 20*b**10*x**2), Eq(n, -3)), (420*a**7*d**2*log(a/b + x)/(60*a*b**8 + 60*b**9*x) + 420*a**7*d**2/(60*a*b**8 + 60*b**9*x) + 420*a**6*b*d**2*x*log(a/b + x)/(60*a*b**8 + 60*b**9*x) - 210*a**5*b**2*d**2*x**2/(60*a*b**8 + 60*b**9*x) - 480*a**4*b**3*c*d*log(a/b + x)/(60*a*b**8 + 60*b**9*x) - 480*a**4*b**3*c*d/(60*a*b**8 + 60*b**9*x) + 70*a**4*b**3*d**2*x**3/(60*a*b**8 + 60*b**9*x) - 480*a**3*b**4*c*d*x*log(a/b + x)/(60*a*b**8 + 60*b**9*x) - 35*a**3*b**4*d**2*x**4/(60*a*b**8 + 60*b**9*x) + 240*a**2*b**5*c*d*x**2/(60*a*b**8 + 60*b**9*x) + 21*a**2*b**5*d**2*x**5/(60*a*b**8 + 60*b**9*x) + 60*a*b**6*c**2*log(a/b + x)/(60*a*b**8 + 60*b**9*x) + 60*a*b**6*c**2/(60*a*b**8 + 60*b**9*x) - 80*a*b**6*c*d*x**3/(60*a*b**8 + 60*b**9*x) - 14*a*b**6*d**2*x**6/(60*a*b**8 + 60*b**9*x) + 60*b**7*c**2*x*log(a/b + x)/(60*a*b**8 + 60*b**9*x) + 40*b**7*c*d*x**4/(60*a*b**8 + 60*b**9*x) + 10*b**7*d**2*x**7/(60*a*b**8 + 60*b**9*x), Eq(n, -2)), (-a**7*d**2*log(a/b + x)/b**8 + a**6*d**2*x/b**7 - a**5*d**2*x**2/(2*b**6) + 2*a**4*c*d*log(a/b + x)/b**5 + a**4*d**2*x**3/(3*b**5) - 2*a**3*c*d*x/b**4 - a**3*d**2*x**4/(4*b**4) + a**2*c*d*x**2/b**3 + a**2*d**2*x**5/(5*b**3) - a*c**2*log(a/b + x)/b**2 - 2*a*c*d*x**3/(3*b**2) - a*d**2*x**6/(6*b**2) + c**2*x/b + c*d*x**4/(2*b) + d**2*x**7/(7*b), Eq(n, -1)), (-5040*a**8*d**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 5040*a**7*b*d**2*n*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 2520*a**6*b**2*d**2*n**2*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 2520*a**6*b**2*d**2*n*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 48*a**5*b**3*c*d*n**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 1008*a**5*b**3*c*d*n**2*(a + b*x)**n/(b**8*n**8 + 36

*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*
n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 7008*a**5*b**3*c*d*
n*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 +
 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 403
20*b**8) + 16128*a**5*b**3*c*d*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546
*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b*
*8*n**2 + 109584*b**8*n + 40320*b**8) + 840*a**5*b**3*d**2*n**3*x**3*(a + b
*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b
8*n4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8)
 + 2520*a**5*b**3*d**2*n**2*x**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 5
46*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*
b**8*n**2 + 109584*b**8*n + 40320*b**8) + 1680*a**5*b**3*d**2*n*x**3*(a + b
*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b
8*n4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8)
 - 48*a**4*b**4*c*d*n**4*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**
8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n
2 + 109584*b8*n + 40320*b**8) - 1008*a**4*b**4*c*d*n**3*x*(a + b*x)**n/
(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**
4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 7008
*a**4*b**4*c*d*n**2*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**
6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 +
 109584*b**8*n + 40320*b**8) - 16128*a**4*b**4*c*d*n*x*(a + b*x)**n/(b**8*n
8 + 36*b8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 672
84*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 210*a**4*b*
*4*d**2*n**4*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 +
4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109
584*b**8*n + 40320*b**8) - 1260*a**4*b**4*d**2*n**3*x**4*(a + b*x)**n/(b**8
*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 6
7284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 2310*a**4
*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6
 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 +
109584*b**8*n + 40320*b**8) - 1260*a**4*b**4*d**2*n*x**4*(a + b*x)**n/(b**8
*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 6
7284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 24*a**3*b
5*c*d*n5*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 +
4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109
584*b**8*n + 40320*b**8) + 528*a**3*b**5*c*d*n**4*x**2*(a + b*x)**n/(b**8*n
8 + 36*b8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 672
84*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 4008*a**3*b
5*c*d*n3*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 +
4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109
584*b**8*n + 40320*b**8) + 11568*a**3*b**5*c*d*n**2*x**2*(a + b*x)**n/(b**8
*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 6
7284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 8064*a**3
*b**5*c*d*n*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4

536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 1095
84*b**8*n + 40320*b**8) + 42*a**3*b**5*d**2*n**5*x**5*(a + b*x)**n/(b**8*n*
*8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 6728
4*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 420*a**3*b**
5*d**2*n**4*x**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4
536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 1095
84*b**8*n + 40320*b**8) + 1470*a**3*b**5*d**2*n**3*x**5*(a + b*x)**n/(b**8*
n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67
284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 2100*a**3*
b**5*d**2*n**2*x**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6
+ 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 1
09584*b**8*n + 40320*b**8) + 1008*a**3*b**5*d**2*n*x**5*(a + b*x)**n/(b**8*
n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67
284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - a**2*b**6*
c**2*n**6*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**
8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8
*n + 40320*b**8) - 33*a**2*b**6*c**2*n**5*(a + b*x)**n/(b**8*n**8 + 36*b**8
*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3
+ 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 445*a**2*b**6*c**2*n**4*
(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 2
2449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320
*b**8) - 3135*a**2*b**6*c**2*n**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 +
546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124
*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 12154*a**2*b**6*c**2*n**2*(a + b
*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b
8*n4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8)
 - 24552*a**2*b**6*c**2*n*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8
*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n*
*2 + 109584*b**8*n + 40320*b**8) - 20160*a**2*b**6*c**2*(a + b*x)**n/(b**8*
n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67
284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 8*a**2*b**
6*c*d*n**6*x**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 45
36*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 10958
4*b**8*n + 40320*b**8) - 192*a**2*b**6*c*d*n**5*x**3*(a + b*x)**n/(b**8*n**
8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284
*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 1688*a**2*b**
6*c*d*n**4*x**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 45
36*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 10958
4*b**8*n + 40320*b**8) - 6528*a**2*b**6*c*d*n**3*x**3*(a + b*x)**n/(b**8*n*
*8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 6728
4*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 10400*a**2*b
6*c*d*n2*x**3*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 +
4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109
584*b**8*n + 40320*b**8) - 5376*a**2*b**6*c*d*n*x**3*(a + b*x)**n/(b**8*n**
8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284

*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 7*a**2*b**6*d
2*n6*x**6*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536
*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*
b**8*n + 40320*b**8) - 105*a**2*b**6*d**2*n**5*x**6*(a + b*x)**n/(b**8*n**8
 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*
b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 595*a**2*b**6*
d**2*n**4*x**6*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 453
6*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584
*b**8*n + 40320*b**8) - 1575*a**2*b**6*d**2*n**3*x**6*(a + b*x)**n/(b**8*n*
*8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 6728
4*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) - 1918*a**2*b*
*6*d**2*n**2*x**6*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 +
4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109
584*b**8*n + 40320*b**8) - 840*a**2*b**6*d**2*n*x**6*(a + b*x)**n/(b**8*n**
8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284
*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + a*b**7*c**2*n
7*x*(a + b*x)n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n*
*5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n +
 40320*b**8) + 33*a*b**7*c**2*n**6*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7
 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118
124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 445*a*b**7*c**2*n**5*x*(a + b
*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b
8*n4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8)
 + 3135*a*b**7*c**2*n**4*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**
8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n
2 + 109584*b8*n + 40320*b**8) + 12154*a*b**7*c**2*n**3*x*(a + b*x)**n/(
b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4
 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 24552
*a*b**7*c**2*n**2*x*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6
+ 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 1
09584*b**8*n + 40320*b**8) + 20160*a*b**7*c**2*n*x*(a + b*x)**n/(b**8*n**8
+ 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b
8*n3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 2*a*b**7*c*d*n*
*7*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*
n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n
 + 40320*b**8) + 54*a*b**7*c*d*n**6*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*
n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 +
 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 566*a*b**7*c*d*n**5*x**4*
(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 2
2449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320
*b**8) + 2898*a*b**7*c*d*n**4*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 +
 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 11812
4*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 7496*a*b**7*c*d*n**3*x**4*(a +
b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*
b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8

) + 9144*a*b**7*c*d*n**2*x**4*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*
b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**
8*n**2 + 109584*b**8*n + 40320*b**8) + 4032*a*b**7*c*d*n*x**4*(a + b*x)**n/
(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**
4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + a*b*
*7*d**2*n**7*x**7*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 +
4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109
584*b**8*n + 40320*b**8) + 21*a*b**7*d**2*n**6*x**7*(a + b*x)**n/(b**8*n**8
 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*
b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 175*a*b**7*d**
2*n**5*x**7*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b
8*n5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b*
*8*n + 40320*b**8) + 735*a*b**7*d**2*n**4*x**7*(a + b*x)**n/(b**8*n**8 + 36
*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*
n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 1624*a*b**7*d**2*n*
*3*x**7*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*
n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n
 + 40320*b**8) + 1764*a*b**7*d**2*n**2*x**7*(a + b*x)**n/(b**8*n**8 + 36*b*
*8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**
3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 720*a*b**7*d**2*n*x**7
*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 +
22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 4032
0*b**8) + b**8*c**2*n**7*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*
b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**
8*n**2 + 109584*b**8*n + 40320*b**8) + 34*b**8*c**2*n**6*x**2*(a + b*x)**n/
(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**
4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 478*
b**8*c**2*n**5*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6
+ 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 1
09584*b**8*n + 40320*b**8) + 3580*b**8*c**2*n**4*x**2*(a + b*x)**n/(b**8*n*
*8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 6728
4*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 15289*b**8*c
2*n3*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536
*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*
b**8*n + 40320*b**8) + 36706*b**8*c**2*n**2*x**2*(a + b*x)**n/(b**8*n**8 +
36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**
8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 44712*b**8*c**2*n
*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n*
*5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n +
 40320*b**8) + 20160*b**8*c**2*x**2*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7
+ 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 1181
24*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 2*b**8*c*d*n**7*x**5*(a + b*x)
n/(b8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8
*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) +
62*b**8*c*d*n**6*x**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**

```
6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 +
 109584*b**8*n + 40320*b**8) + 782*b**8*c*d*n**5*x**5*(a + b*x)**n/(b**8*n*
*8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 6728
4*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 5162*b**8*c*
d*n**4*x**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b
**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b*
*8*n + 40320*b**8) + 19088*b**8*c*d*n**3*x**5*(a + b*x)**n/(b**8*n**8 + 36*
b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n
**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 39128*b**8*c*d*n**2*
x**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**
5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n +
40320*b**8) + 40608*b**8*c*d*n*x**5*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7
+ 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 1181
24*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 16128*b**8*c*d*x**5*(a + b*x)*
*n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*
n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + b
**8*d**2*n**7*x**8*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 +
 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 10
9584*b**8*n + 40320*b**8) + 28*b**8*d**2*n**6*x**8*(a + b*x)**n/(b**8*n**8
+ 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b
**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 322*b**8*d**2*n
**5*x**8*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8
*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*
n + 40320*b**8) + 1960*b**8*d**2*n**4*x**8*(a + b*x)**n/(b**8*n**8 + 36*b**
8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3
 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 6769*b**8*d**2*n**3*x**
8*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 +
 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 403
20*b**8) + 13132*b**8*d**2*n**2*x**8*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7
 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118
124*b**8*n**2 + 109584*b**8*n + 40320*b**8) + 13068*b**8*d**2*n*x**8*(a + b
*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n**6 + 4536*b**8*n**5 + 22449*b
**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2 + 109584*b**8*n + 40320*b**8)
 + 5040*b**8*d**2*x**8*(a + b*x)**n/(b**8*n**8 + 36*b**8*n**7 + 546*b**8*n*
*6 + 4536*b**8*n**5 + 22449*b**8*n**4 + 67284*b**8*n**3 + 118124*b**8*n**2
+ 109584*b**8*n + 40320*b**8), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.91

$$\int x(a+bx)^n\left(c+dx^3\right)^2\,dx=\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^nc^2}{(n^2+3\,n+2)b^2}+\frac{2\,((n^4+10\,n^3+35\,n^2+50\,n+24)b^5x^5+(n^4+6\,n^3+11\,n^2+6\,n)ab^4x^4-4\,(n^3+3\,n^2+2\,n)a^2b^3x^3+1}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^5}+\frac{((n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040)b^8x^8+(n^7+21\,n^6+175\,n^5+7}{}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2)*b^2) + 2
*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n
)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2
- 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2
+ 274*n + 120)*b^5) + ((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 1313
2*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*
n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 27
4*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^3
*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*n^2
+ 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040*a^8
)*(b*x + a)^n*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n
^3 + 118124*n^2 + 109584*n + 40320)*b^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2034 vs. 2(248) = 496.

Time = 0.33 (sec) , antiderivative size = 2034, normalized size of antiderivative = 8.20

$$\int x(a+bx)^n\left(c+dx^3\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^8*d^2*n^7*x^8 + (b*x + a)^n*a*b^7*d^2*n^7*x^7 + 28*(b*x + a)
^n*b^8*d^2*n^6*x^8 + 21*(b*x + a)^n*a*b^7*d^2*n^6*x^7 + 322*(b*x + a)^n*b^8
*d^2*n^5*x^8 + 2*(b*x + a)^n*b^8*c*d*n^7*x^5 - 7*(b*x + a)^n*a^2*b^6*d^2*n^
6*x^6 + 175*(b*x + a)^n*a*b^7*d^2*n^5*x^7 + 1960*(b*x + a)^n*b^8*d^2*n^4*x^
8 + 2*(b*x + a)^n*a*b^7*c*d*n^7*x^4 + 62*(b*x + a)^n*b^8*c*d*n^6*x^5 - 105*
(b*x + a)^n*a^2*b^6*d^2*n^5*x^6 + 735*(b*x + a)^n*a*b^7*d^2*n^4*x^7 + 6769*
(b*x + a)^n*b^8*d^2*n^3*x^8 + 54*(b*x + a)^n*a*b^7*c*d*n^6*x^4 + 782*(b*x +
```

a)^n*b^8*c*d*n^5*x^5 + 42*(b*x + a)^n*a^3*b^5*d^2*n^5*x^5 - 595*(b*x + a)^
n*a^2*b^6*d^2*n^4*x^6 + 1624*(b*x + a)^n*a*b^7*d^2*n^3*x^7 + 13132*(b*x + a
)^n*b^8*d^2*n^2*x^8 + (b*x + a)^n*b^8*c^2*n^7*x^2 - 8*(b*x + a)^n*a^2*b^6*c
*d*n^6*x^3 + 566*(b*x + a)^n*a*b^7*c*d*n^5*x^4 + 5162*(b*x + a)^n*b^8*c*d*n
^4*x^5 + 420*(b*x + a)^n*a^3*b^5*d^2*n^4*x^5 - 1575*(b*x + a)^n*a^2*b^6*d^2
*n^3*x^6 + 1764*(b*x + a)^n*a*b^7*d^2*n^2*x^7 + 13068*(b*x + a)^n*b^8*d^2*n
*x^8 + (b*x + a)^n*a*b^7*c^2*n^7*x + 34*(b*x + a)^n*b^8*c^2*n^6*x^2 - 192*(
b*x + a)^n*a^2*b^6*c*d*n^5*x^3 + 2898*(b*x + a)^n*a*b^7*c*d*n^4*x^4 - 210*(
b*x + a)^n*a^4*b^4*d^2*n^4*x^4 + 19088*(b*x + a)^n*b^8*c*d*n^3*x^5 + 1470*(
b*x + a)^n*a^3*b^5*d^2*n^3*x^5 - 1918*(b*x + a)^n*a^2*b^6*d^2*n^2*x^6 + 720
*(b*x + a)^n*a*b^7*d^2*n*x^7 + 5040*(b*x + a)^n*b^8*d^2*x^8 + 33*(b*x + a)^
n*a*b^7*c^2*n^6*x + 478*(b*x + a)^n*b^8*c^2*n^5*x^2 + 24*(b*x + a)^n*a^3*b^
5*c*d*n^5*x^2 - 1688*(b*x + a)^n*a^2*b^6*c*d*n^4*x^3 + 7496*(b*x + a)^n*a*b
^7*c*d*n^3*x^4 - 1260*(b*x + a)^n*a^4*b^4*d^2*n^3*x^4 + 39128*(b*x + a)^n*b
^8*c*d*n^2*x^5 + 2100*(b*x + a)^n*a^3*b^5*d^2*n^2*x^5 - 840*(b*x + a)^n*a^2
*b^6*d^2*n*x^6 - (b*x + a)^n*a^2*b^6*c^2*n^6 + 445*(b*x + a)^n*a*b^7*c^2*n^
5*x + 3580*(b*x + a)^n*b^8*c^2*n^4*x^2 + 528*(b*x + a)^n*a^3*b^5*c*d*n^4*x^
2 - 6528*(b*x + a)^n*a^2*b^6*c*d*n^3*x^3 + 840*(b*x + a)^n*a^5*b^3*d^2*n^3*
x^3 + 9144*(b*x + a)^n*a*b^7*c*d*n^2*x^4 - 2310*(b*x + a)^n*a^4*b^4*d^2*n^2
x^4 + 40608(b*x + a)^n*b^8*c*d*n*x^5 + 1008*(b*x + a)^n*a^3*b^5*d^2*n*x^5
 - 33*(b*x + a)^n*a^2*b^6*c^2*n^5 + 3135*(b*x + a)^n*a*b^7*c^2*n^4*x - 48*(
b*x + a)^n*a^4*b^4*c*d*n^4*x + 15289*(b*x + a)^n*b^8*c^2*n^3*x^2 + 4008*(b*
x + a)^n*a^3*b^5*c*d*n^3*x^2 - 10400*(b*x + a)^n*a^2*b^6*c*d*n^2*x^3 + 2520
*(b*x + a)^n*a^5*b^3*d^2*n^2*x^3 + 4032*(b*x + a)^n*a*b^7*c*d*n*x^4 - 1260*
(b*x + a)^n*a^4*b^4*d^2*n*x^4 + 16128*(b*x + a)^n*b^8*c*d*x^5 - 445*(b*x +
a)^n*a^2*b^6*c^2*n^4 + 12154*(b*x + a)^n*a*b^7*c^2*n^3*x - 1008*(b*x + a)^n
*a^4*b^4*c*d*n^3*x + 36706*(b*x + a)^n*b^8*c^2*n^2*x^2 + 11568*(b*x + a)^n*
a^3*b^5*c*d*n^2*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^2*n^2*x^2 - 5376*(b*x + a)
^n*a^2*b^6*c*d*n*x^3 + 1680*(b*x + a)^n*a^5*b^3*d^2*n*x^3 - 3135*(b*x + a)^
n*a^2*b^6*c^2*n^3 + 48*(b*x + a)^n*a^5*b^3*c*d*n^3 + 24552*(b*x + a)^n*a*b^
7*c^2*n^2*x - 7008*(b*x + a)^n*a^4*b^4*c*d*n^2*x + 44712*(b*x + a)^n*b^8*c^
2*n*x^2 + 8064*(b*x + a)^n*a^3*b^5*c*d*n*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^2
*n*x^2 - 12154*(b*x + a)^n*a^2*b^6*c^2*n^2 + 1008*(b*x + a)^n*a^5*b^3*c*d*n
^2 + 20160*(b*x + a)^n*a*b^7*c^2*n*x - 16128*(b*x + a)^n*a^4*b^4*c*d*n*x +
5040*(b*x + a)^n*a^7*b*d^2*n*x + 20160*(b*x + a)^n*b^8*c^2*x^2 - 24552*(b*x
 + a)^n*a^2*b^6*c^2*n + 7008*(b*x + a)^n*a^5*b^3*c*d*n - 20160*(b*x + a)^n*
a^2*b^6*c^2 + 16128*(b*x + a)^n*a^5*b^3*c*d - 5040*(b*x + a)^n*a^8*d^2)/(b^
8*n^8 + 36*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8
*n^3 + 118124*b^8*n^2 + 109584*b^8*n + 40320*b^8)

Mupad [B] (verification not implemented)

Time = 20.36 (sec) , antiderivative size = 1136, normalized size of antiderivative = 4.58

$$\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx$$
$$= \frac{d^2 x^8 (a+b x)^n (n^7+28 n^6+322 n^5+1960 n^4+6769 n^3+13132 n^2+13068 n+5040)}{n^8+36 n^7+546 n^6+4536 n^5+22449 n^4+67284 n^3+118124 n^2+109584 n+40320}$$
$$- \frac{a^2 (a+b x)^n (5040 a^6 d^2-48 a^3 b^3 c d n^3-1008 a^3 b^3 c d n^2-7008 a^3 b^3 c d n-16128 a^3 b^3 c d+b^6 c^2 n^6+\dots}{b^8 (n^8+36 n^7+546 n^6+4536 n^5+22449 n^4+67284 \dots}$$
$$+ \frac{x^2 (n+1) (a+b x)^n (-2520 a^6 d^2 n+24 a^3 b^3 c d n^4+504 a^3 b^3 c d n^3+3504 a^3 b^3 c d n^2+8064 a^3 b^3 c d n \dots}{b^6 (n^8+36 n^7+546 n^6+4536 n^5+22449 n^4+\dots}$$
$$+ \frac{a n x (a+b x)^n (5040 a^6 d^2-48 a^3 b^3 c d n^3-1008 a^3 b^3 c d n^2-7008 a^3 b^3 c d n-16128 a^3 b^3 c d+b^6 c^2 n \dots}{b^7 (n^8+36 n^7+546 n^6+4536 n^5+22449 n^4+6728 \dots}$$
$$+ \frac{2 d x^5 (a+b x)^n (n^4+10 n^3+35 n^2+50 n+24) (21 d a^3 n+c b^3 n^3+21 c b^3 n^2+146 c b^3 n+336 c b^3)}{b^3 (n^8+36 n^7+546 n^6+4536 n^5+22449 n^4+67284 n^3+118124 n^2+109584 n+40320)}$$
$$+ \frac{a d^2 n x^7 (a+b x)^n (n^6+21 n^5+175 n^4+735 n^3+1624 n^2+1764 n+720)}{b (n^8+36 n^7+546 n^6+4536 n^5+22449 n^4+67284 n^3+118124 n^2+109584 n+40320)}$$
$$- \frac{7 a^2 d^2 n x^6 (a+b x)^n (n^5+15 n^4+85 n^3+225 n^2+274 n+120)}{b^2 (n^8+36 n^7+546 n^6+4536 n^5+22449 n^4+67284 n^3+118124 n^2+109584 n+40320)}$$
$$+ \frac{2 a d n x^4 (a+b x)^n (n^3+6 n^2+11 n+6) (-105 d a^3+c b^3 n^3+21 c b^3 n^2+146 c b^3 n+336 c b^3)}{b^4 (n^8+36 n^7+546 n^6+4536 n^5+22449 n^4+67284 n^3+118124 n^2+109584 n+40320)}$$
$$- \frac{8 a^2 d n x^3 (a+b x)^n (n^2+3 n+2) (-105 d a^3+c b^3 n^3+21 c b^3 n^2+146 c b^3 n+336 c b^3)}{b^5 (n^8+36 n^7+546 n^6+4536 n^5+22449 n^4+67284 n^3+118124 n^2+109584 n+40320)}$$

```
[In] int(x*(c + d*x^3)^2*(a + b*x)^n,x)
```

```
[Out] (d^2*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
 28*n^6 + n^7 + 5040))/(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 453
6*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320) - (a^2*(a + b*x)^n*(5040*a^6*d^2 +
20160*b^6*c^2 + 24552*b^6*c^2*n + 12154*b^6*c^2*n^2 + 3135*b^6*c^2*n^3 + 44
5*b^6*c^2*n^4 + 33*b^6*c^2*n^5 + b^6*c^2*n^6 - 16128*a^3*b^3*c*d - 7008*a^3
*b^3*c*d*n - 1008*a^3*b^3*c*d*n^2 - 48*a^3*b^3*c*d*n^3))/(b^8*(109584*n + 1
18124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 403
20)) + (x^2*(n + 1)*(a + b*x)^n*(20160*b^6*c^2 - 2520*a^6*d^2*n + 24552*b^6
*c^2*n + 12154*b^6*c^2*n^2 + 3135*b^6*c^2*n^3 + 445*b^6*c^2*n^4 + 33*b^6*c^
2*n^5 + b^6*c^2*n^6 + 8064*a^3*b^3*c*d*n + 3504*a^3*b^3*c*d*n^2 + 504*a^3*b
^3*c*d*n^3 + 24*a^3*b^3*c*d*n^4))/(b^6*(109584*n + 118124*n^2 + 67284*n^3 +
 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*n*x*(a + b*x)
^n*(5040*a^6*d^2 + 20160*b^6*c^2 + 24552*b^6*c^2*n + 12154*b^6*c^2*n^2 + 31
35*b^6*c^2*n^3 + 445*b^6*c^2*n^4 + 33*b^6*c^2*n^5 + b^6*c^2*n^6 - 16128*a^3
*b^3*c*d - 7008*a^3*b^3*c*d*n - 1008*a^3*b^3*c*d*n^2 - 48*a^3*b^3*c*d*n^3))
/(b^7*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 +
 36*n^7 + n^8 + 40320)) + (2*d*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^
```

4 + 24)*(336*b^3*c + 21*b^3*c*n^2 + b^3*c*n^3 + 21*a^3*d*n + 146*b^3*c*n))/
(b^3*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 +
36*n^7 + n^8 + 40320)) + (a*d^2*n*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*
n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(b*(109584*n + 118124*n^2 + 67284*n^3
+ 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (7*a^2*d^2*n*x^
6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^2*(109584
*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8
 + 40320)) + (2*a*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(336*b^3*c -
 105*a^3*d + 21*b^3*c*n^2 + b^3*c*n^3 + 146*b^3*c*n))/(b^4*(109584*n + 1181
24*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)
) - (8*a^2*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(336*b^3*c - 105*a^3*d + 21*
b^3*c*n^2 + b^3*c*n^3 + 146*b^3*c*n))/(b^5*(109584*n + 118124*n^2 + 67284*n
^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))

3.180 $\int (a+bx)^n \left(c+dx^3\right)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 203

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \frac{\left(b^3c-a^3d\right)^2 (a+bx)^{1+n}}{b^7(1+n)}+\frac{6a^2d(b^3c-a^3d)\,(a+bx)^{2+n}}{b^7(2+n)} - \frac{3ad(2b^3c-5a^3d)\,(a+bx)^{3+n}}{b^7(3+n)}+\frac{2d(b^3c-10a^3d)\,(a+bx)^{4+n}}{b^7(4+n)} + \frac{15a^2d^2(a+bx)^{5+n}}{b^7(5+n)}-\frac{6ad^2(a+bx)^{6+n}}{b^7(6+n)}+\frac{d^2(a+bx)^{7+n}}{b^7(7+n)}$$

```
[Out] (-a^3*d+b^3*c)^2*(b*x+a)^(1+n)/b^7/(1+n)+6*a^2*d*(-a^3*d+b^3*c)*(b*x+a)^(2+
n)/b^7/(2+n)-3*a*d*(-5*a^3*d+2*b^3*c)*(b*x+a)^(3+n)/b^7/(3+n)+2*d*(-10*a^3*
d+b^3*c)*(b*x+a)^(4+n)/b^7/(4+n)+15*a^2*d^2*(b*x+a)^(5+n)/b^7/(5+n)-6*a*d^2
*(b*x+a)^(6+n)/b^7/(6+n)+d^2*(b*x+a)^(7+n)/b^7/(7+n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1864}

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \frac{\left(b^3c-a^3d\right)^2 (a+bx)^{n+1}}{b^7(n+1)}-\frac{3ad(2b^3c-5a^3d)\,(a+bx)^{n+3}}{b^7(n+3)} + \frac{2d(b^3c-10a^3d)\,(a+bx)^{n+4}}{b^7(n+4)} + \frac{15a^2d^2(a+bx)^{n+5}}{b^7(n+5)}+\frac{6a^2d(b^3c-a^3d)\,(a+bx)^{n+2}}{b^7(n+2)} - \frac{6ad^2(a+bx)^{n+6}}{b^7(n+6)}+\frac{d^2(a+bx)^{n+7}}{b^7(n+7)}$$

```
[In] Int[(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) + (6*a^2*d*(b^3*c - a^3
*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) - (3*a*d*(2*b^3*c - 5*a^3*d)*(a + b*x)
^(3 + n))/(b^7*(3 + n)) + (2*d*(b^3*c - 10*a^3*d)*(a + b*x)^(4 + n))/(b^7*(
4 + n)) + (15*a^2*d^2*(a + b*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)
^(6 + n))/(b^7*(6 + n)) + (d^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \int \left(\frac{(b^3c-a^3d)^2 (a+bx)^n}{b^6} - \frac{6a^2d(-b^3c+a^3d)(a+bx)^{1+n}}{b^6} + \frac{3ad(-2b^3c+5a^3d)(a+bx)^{2+n}}{b^6} + \frac{2d(b^3c-10a^3d)(a+bx)^{3+n}}{b^6} + \frac{15a^2d^2(a+bx)^{4+n}}{b^6} - \frac{6ad^2(a+bx)^{5+n}}{b^6} + \frac{d^2(a+bx)^{6+n}}{b^6}\right) dx$$

$$= \frac{(b^3c-a^3d)^2 (a+bx)^{1+n}}{b^7(1+n)} + \frac{6a^2d(b^3c-a^3d)(a+bx)^{2+n}}{b^7(2+n)} - \frac{3ad(2b^3c-5a^3d)(a+bx)^{3+n}}{b^7(3+n)} + \frac{2d(b^3c-10a^3d)(a+bx)^{4+n}}{b^7(4+n)} + \frac{15a^2d^2(a+bx)^{5+n}}{b^7(5+n)} - \frac{6ad^2(a+bx)^{6+n}}{b^7(6+n)} + \frac{d^2(a+bx)^{7+n}}{b^7(7+n)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.85

$$\int (a+bx)^n (c+dx^3)^2 dx = \frac{(a+bx)^{1+n} \left(\frac{(b^3c-a^3d)^2}{1+n} + \frac{6a^2d(b^3c-a^3d)(a+bx)}{2+n} + \frac{3ad(-2b^3c+5a^3d)(a+bx)^2}{3+n} + \frac{2d(b^3c-10a^3d)(a+bx)^3}{4+n} + \frac{15a^2d^2(a+bx)^4}{5+n} - 6\right.}{b^7}$$

```
[In] Integrate[(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b^3*c - a^3*d)^2/(1 + n) + (6*a^2*d*(b^3*c - a^3*d)*(a
 + b*x))/(2 + n) + (3*a*d*(-2*b^3*c + 5*a^3*d)*(a + b*x)^2)/(3 + n) + (2*d*
(b^3*c - 10*a^3*d)*(a + b*x)^3)/(4 + n) + (15*a^2*d^2*(a + b*x)^4)/(5 + n)
- (6*a*d^2*(a + b*x)^5)/(6 + n) + (d^2*(a + b*x)^6)/(7 + n)))/b^7
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. 2(203) = 406.

Time = 0.95 (sec) , antiderivative size = 700, normalized size of antiderivative = 3.45

method	result
norman	$\frac{d^2x^7\mathrm{e}^{n\ln(bx+a)}}{7+n}+\frac{a\left(b^6c^2n^6+27b^6c^2n^5+295b^6c^2n^4-12a^3b^3cd\,n^3+1665b^6c^2n^3-216a^3b^3cd\,n^2+5104b^6c^2n^2-1284a^3b^3cdn+80}{b^7\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n-\right.}$
gosper	$\frac{(bx+a)^{1+n}\left(b^6d^2n^6x^6+21b^6d^2n^5x^6-6a\,b^5d^2n^5x^5+175b^6d^2n^4x^6-90a\,b^5d^2n^4x^5+2b^6cd\,n^6x^3+735b^6d^2n^3x^6+30a^2b^4d^2n^4x^4-5\right.}{}$
risch	$\frac{\left(b^7d^2n^6x^7+a\,b^6d^2n^6x^6+21b^7d^2n^5x^7+15a\,b^6d^2n^5x^6+175b^7d^2n^4x^7-6a^2b^5d^2n^5x^5+85a\,b^6d^2n^4x^6+2b^7cd\,n^6x^4+735b^7d^2n^3x^7-\right.}{}$
parallelrisch	Expression too large to display

```
[In] int((b*x+a)^n*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/(7+n)*x^7*exp(n*ln(b*x+a))+a*(b^6*c^2*n^6+27*b^6*c^2*n^5+295*b^6*c^2*n^
4-12*a^3*b^3*c*d*n^3+1665*b^6*c^2*n^3-216*a^3*b^3*c*d*n^2+5104*b^6*c^2*n^2-
1284*a^3*b^3*c*d*n+8028*b^6*c^2*n+720*a^6*d^2-2520*a^3*b^3*c*d+5040*b^6*c^2
)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)*exp(n*l
n(b*x+a))+d^2*a*n/b/(n^2+13*n+42)*x^6*exp(n*ln(b*x+a))-(-b^6*c^2*n^6-27*b^6
*c^2*n^5-12*a^3*b^3*c*d*n^4-295*b^6*c^2*n^4-216*a^3*b^3*c*d*n^3-1665*b^6*c^
2*n^3-1284*a^3*b^3*c*d*n^2-5104*b^6*c^2*n^2+720*a^6*d^2*n-2520*a^3*b^3*c*d*
n-8028*b^6*c^2*n-5040*b^6*c^2)/b^6/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13
132*n^2+13068*n+5040)*x*exp(n*ln(b*x+a))+2*(b^3*c*n^3+18*b^3*c*n^2+15*a^3*d
*n+107*b^3*c*n+210*b^3*c)*d/b^3/(n^4+22*n^3+179*n^2+638*n+840)*x^4*exp(n*ln
(b*x+a))-6*n*a^2*d^2/b^2/(n^3+18*n^2+107*n+210)*x^5*exp(n*ln(b*x+a))-2*n*a*
d*(-b^3*c*n^3-18*b^3*c*n^2-107*b^3*c*n+60*a^3*d-210*b^3*c)/b^4/(n^5+25*n^4+
245*n^3+1175*n^2+2754*n+2520)*x^3*exp(n*ln(b*x+a))+6*(-b^3*c*n^3-18*b^3*c*n
^2-107*b^3*c*n+60*a^3*d-210*b^3*c)*d*a^2/b^5*n/(n^6+27*n^5+295*n^4+1665*n^3
+5104*n^2+8028*n+5040)*x^2*exp(n*ln(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. 2(203) = 406.

Time = 0.28 (sec) , antiderivative size = 893, normalized size of antiderivative = 4.40

$$\int (a+bx)^n\left(c+dx^3\right)^2\,dx$$
$$=\frac{\left(ab^6c^2n^6+27\,ab^6c^2n^5+295\,ab^6c^2n^4+5040\,ab^6c^2-2520\,a^4b^3cd+720\,a^7d^2+\left(b^7d^2n^6+21\,b^7d^2n^5+175\,b^7\right.\right.}{}$$

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^6*c^2*n^6 + 27*a*b^6*c^2*n^5 + 295*a*b^6*c^2*n^4 + 5040*a*b^6*c^2 - 25
20*a^4*b^3*c*d + 720*a^7*d^2 + (b^7*d^2*n^6 + 21*b^7*d^2*n^5 + 175*b^7*d^2*
```

```
n^4 + 735*b^7*d^2*n^3 + 1624*b^7*d^2*n^2 + 1764*b^7*d^2*n + 720*b^7*d^2)*x^
7 + (a*b^6*d^2*n^6 + 15*a*b^6*d^2*n^5 + 85*a*b^6*d^2*n^4 + 225*a*b^6*d^2*n^
3 + 274*a*b^6*d^2*n^2 + 120*a*b^6*d^2*n)*x^6 - 6*(a^2*b^5*d^2*n^5 + 10*a^2*
b^5*d^2*n^4 + 35*a^2*b^5*d^2*n^3 + 50*a^2*b^5*d^2*n^2 + 24*a^2*b^5*d^2*n)*x
^5 + 2*(b^7*c*d*n^6 + 24*b^7*c*d*n^5 + 1260*b^7*c*d + (226*b^7*c*d + 15*a^3
*b^4*d^2)*n^4 + 6*(176*b^7*c*d + 15*a^3*b^4*d^2)*n^3 + 5*(509*b^7*c*d + 33*
a^3*b^4*d^2)*n^2 + 18*(164*b^7*c*d + 5*a^3*b^4*d^2)*n)*x^4 + 3*(555*a*b^6*c
^2 - 4*a^4*b^3*c*d)*n^3 + 2*(a*b^6*c*d*n^6 + 21*a*b^6*c*d*n^5 + 163*a*b^6*c
*d*n^4 + 3*(189*a*b^6*c*d - 20*a^4*b^3*d^2)*n^3 + 4*(211*a*b^6*c*d - 45*a^4
*b^3*d^2)*n^2 + 60*(7*a*b^6*c*d - 2*a^4*b^3*d^2)*n)*x^3 + 8*(638*a*b^6*c^2
- 27*a^4*b^3*c*d)*n^2 - 6*(a^2*b^5*c*d*n^5 + 19*a^2*b^5*c*d*n^4 + 125*a^2*b
^5*c*d*n^3 + (317*a^2*b^5*c*d - 60*a^5*b^2*d^2)*n^2 + 30*(7*a^2*b^5*c*d - 2
*a^5*b^2*d^2)*n)*x^2 + 12*(669*a*b^6*c^2 - 107*a^4*b^3*c*d)*n + (b^7*c^2*n^
6 + 27*b^7*c^2*n^5 + 5040*b^7*c^2 + (295*b^7*c^2 + 12*a^3*b^4*c*d)*n^4 + 9*
(185*b^7*c^2 + 24*a^3*b^4*c*d)*n^3 + 4*(1276*b^7*c^2 + 321*a^3*b^4*c*d)*n^2
 + 36*(223*b^7*c^2 + 70*a^3*b^4*c*d - 20*a^6*b*d^2)*n)*x)*(b*x + a)^n/(b^7*
n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^
2 + 13068*b^7*n + 5040*b^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11851 vs. $2(187) = 374$.

Time = 3.53 (sec) , antiderivative size = 11851, normalized size of antiderivative = 58.38

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x + c*d*x**4/2 + d**2*x**7/7), Eq(b, 0)), (60*a**6*d*
*2*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200
*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6)
+ 147*a**6*d**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200
*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6)
+ 360*a**5*b*d**2*x*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4
*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5
+ 60*b**13*x**6) + 822*a**5*b*d**2*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*
a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x
**5 + 60*b**13*x**6) + 900*a**4*b**2*d**2*x**2*log(a/b + x)/(60*a**6*b**7 +
 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**
11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1875*a**4*b**2*d**2*x**2/(60*
a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 9
00*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 2*a**3*b**3*c*d/(6
0*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 +
 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1200*a**3*b**3*d
```

2*x3*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 2200*a**3*b**3*d**2*x**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 12*a**2*b**4*c*d*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 900*a**2*b**4*d**2*x**4*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1350*a**2*b**4*d**2*x**4/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 30*a*b**5*c*d*x**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 360*a*b**5*d**2*x**5*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 360*a*b**5*d**2*x**5/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 10*b**6*c**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 40*b**6*c*d*x**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 60*b**6*d**2*x**6*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6), Eq(n, -7)), (-60*a**6*d**2*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 137*a**6*d**2/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 300*a**5*b*d**2*x*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 625*a**5*b*d**2*x/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 600*a**4*b**2*d**2*x**2*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 1100*a**4*b**2*d**2*x**2/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - a**3*b**3*c*d/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 600*a**3*b**3*d**2*x**3*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 900*a**3*b**3*d**2*x**3/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 5*a**2*b**4*c*d*x/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 300*a**2*b**4*d**2*x**4*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 300*a**2*b**4*d**2*x**4/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 5

0*a*b**11*x**4 + 10*b**12*x**5) - 10*a*b**5*c*d*x**2/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 60*a*b**5*d**2*x**5*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 2*b**6*c**2/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 10*b**6*c*d*x**3/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) + 10*b**6*d**2*x**6/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5), Eq(n, -6)), (60*a**6*d**2*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 125*a**6*d**2/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 240*a**5*b*d**2*x*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 440*a**5*b*d**2*x/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 360*a**4*b**2*d**2*x**2*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 540*a**4*b**2*d**2*x**2/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - 2*a**3*b**3*c*d/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 240*a**3*b**3*d**2*x**3*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 240*a**3*b**3*d**2*x**3/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - 8*a**2*b**4*c*d*x/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 60*a**2*b**4*d**2*x**4*log(a/b + x)/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - 12*a*b**5*c*d*x**2/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - 12*a*b**5*d**2*x**5/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - b**6*c**2/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) - 8*b**6*c*d*x**3/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4) + 2*b**6*d**2*x**6/(4*a**4*b**7 + 16*a**3*b**8*x + 24*a**2*b**9*x**2 + 16*a*b**10*x**3 + 4*b**11*x**4), Eq(n, -5)), (-60*a**6*d**2*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 110*a**6*d**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**5*b*d**2*x*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 270*a**5*b*d**2*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**4*b**2*d**2*x**2*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**4*b**2*d**2*x**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 6*a**3*b**3*c*d*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 11*a**3*b**3*c*d/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 60*a**3*b**3*d**2*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 18*a**2*b**4*c*d*x*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 27*a**2*b**4*c*d*x/(3*a**3*b*

*7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 15*a**2*b**4*d**2*x**4
/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 18*a*b**5*c
*d*x**2*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10
*x**3) + 18*a*b**5*c*d*x**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 +
3*b**10*x**3) - 3*a*b**5*d**2*x**5/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*
x**2 + 3*b**10*x**3) - b**6*c**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x*
*2 + 3*b**10*x**3) + 6*b**6*c*d*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**
8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + b**6*d**2*x**6/(3*a**3*b**7 + 9*a**2*
b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3), Eq(n, -4)), (60*a**6*d**2*log(a/b +
x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 90*a**6*d**2/(4*a**2*b**7 +
8*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x*log(a/b + x)/(4*a**2*b**7 + 8
*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x/(4*a**2*b**7 + 8*a*b**8*x + 4*
b**9*x**2) + 60*a**4*b**2*d**2*x**2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x
+ 4*b**9*x**2) - 24*a**3*b**3*c*d*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x +
4*b**9*x**2) - 36*a**3*b**3*c*d/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) -
20*a**3*b**3*d**2*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 48*a**2*b
4*c*d*x*log(a/b + x)/(4*a2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 48*a**2*b
4*c*d*x/(4*a2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 5*a**2*b**4*d**2*x**4/
(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 24*a*b**5*c*d*x**2*log(a/b + x)/
(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 2*a*b**5*d**2*x**5/(4*a**2*b**7
+ 8*a*b**8*x + 4*b**9*x**2) - 2*b**6*c**2/(4*a**2*b**7 + 8*a*b**8*x + 4*b**
9*x**2) + 8*b**6*c*d*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + b**6*d
2*x6/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2), Eq(n, -3)), (-60*a**6*d*
*2*log(a/b + x)/(10*a*b**7 + 10*b**8*x) - 60*a**6*d**2/(10*a*b**7 + 10*b**8
*x) - 60*a**5*b*d**2*x*log(a/b + x)/(10*a*b**7 + 10*b**8*x) + 30*a**4*b**2*
d**2*x**2/(10*a*b**7 + 10*b**8*x) + 60*a**3*b**3*c*d*log(a/b + x)/(10*a*b**
7 + 10*b**8*x) + 60*a**3*b**3*c*d/(10*a*b**7 + 10*b**8*x) - 10*a**3*b**3*d*
*2*x**3/(10*a*b**7 + 10*b**8*x) + 60*a**2*b**4*c*d*x*log(a/b + x)/(10*a*b**
7 + 10*b**8*x) + 5*a**2*b**4*d**2*x**4/(10*a*b**7 + 10*b**8*x) - 30*a*b**5*
c*d*x**2/(10*a*b**7 + 10*b**8*x) - 3*a*b**5*d**2*x**5/(10*a*b**7 + 10*b**8*
x) - 10*b**6*c**2/(10*a*b**7 + 10*b**8*x) + 10*b**6*c*d*x**3/(10*a*b**7 + 1
0*b**8*x) + 2*b**6*d**2*x**6/(10*a*b**7 + 10*b**8*x), Eq(n, -2)), (a**6*d**
2*log(a/b + x)/b**7 - a**5*d**2*x/b**6 + a**4*d**2*x**2/(2*b**5) - 2*a**3*c
*d*log(a/b + x)/b**4 - a**3*d**2*x**3/(3*b**4) + 2*a**2*c*d*x/b**3 + a**2*d
2*x4/(4*b**3) - a*c*d*x**2/b**2 - a*d**2*x**5/(5*b**2) + c**2*log(a/b +
x)/b + 2*c*d*x**3/(3*b) + d**2*x**6/(6*b), Eq(n, -1)), (720*a**7*d**2*(a +
b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*
b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 720*a**6*b*d**2*n
x(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
+ 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 360*a**5*b
2*d2*n**2*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b*
*7) + 360*a**5*b**2*d**2*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 32
2*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**
7*n + 5040*b**7) - 12*a**4*b**3*c*d*n**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*

n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 216*a**4*b**3*c*d*n**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 1284*a**4*b**3*c*d*n*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 2520*a**4*b**3*c*d*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 120*a**4*b**3*d**2*n**3*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 360*a**4*b**3*d**2*n**2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 240*a**4*b**3*d**2*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 12*a**3*b**4*c*d*n**4*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 216*a**3*b**4*c*d*n**3*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1284*a**3*b**4*c*d*n**2*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2520*a**3*b**4*c*d*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 30*a**3*b**4*d**2*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 180*a**3*b**4*d**2*n**3*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 330*a**3*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 180*a**3*b**4*d**2*n*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 6*a**2*b**5*c*d*n**5*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 114*a**2*b**5*c*d*n**4*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 750*a**2*b**5*c*d*n**3*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 1902*a**2*b**5*c*d*n**2*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 1260*a**2*b**5*c*d*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 6*a**2*b**5*d**2*n**5*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 60*a**2*b**

5*d**2*n**4*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1
960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7
) - 210*a**2*b**5*d**2*n**3*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 3
22*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b*
*7*n + 5040*b**7) - 300*a**2*b**5*d**2*n**2*x**5*(a + b*x)**n/(b**7*n**7 +
28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7
*n**2 + 13068*b**7*n + 5040*b**7) - 144*a**2*b**5*d**2*n*x**5*(a + b*x)**n/
(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3
 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + a*b**6*c**2*n**6*(a + b*x)
n/(b7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*
n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 27*a*b**6*c**2*n**5*(a
 + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 676
9*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 295*a*b**6*c**2
*n**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n*
*4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1665*a*
b**6*c**2*n**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 196
0*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7)
+ 5104*a*b**6*c**2*n**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n
5 + 1960*b7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 50
40*b**7) + 8028*a*b**6*c**2*n*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*
b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*
n + 5040*b**7) + 5040*a*b**6*c**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b
7*n + 5040*b7) + 2*a*b**6*c*d*n**6*x**3*(a + b*x)**n/(b**7*n**7 + 28*b*
*7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2
 + 13068*b**7*n + 5040*b**7) + 42*a*b**6*c*d*n**5*x**3*(a + b*x)**n/(b**7*n
7 + 28*b7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1313
2*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 326*a*b**6*c*d*n**4*x**3*(a + b*x
)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7
*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1134*a*b**6*c*d*n**3*
x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**
4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1688*a*b
6*c*d*n2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**
7) + 840*a*b**6*c*d*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**
7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n +
 5040*b**7) + a*b**6*d**2*n**6*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
+ 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068
*b**7*n + 5040*b**7) + 15*a*b**6*d**2*n**5*x**6*(a + b*x)**n/(b**7*n**7 + 2
8*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*
n**2 + 13068*b**7*n + 5040*b**7) + 85*a*b**6*d**2*n**4*x**6*(a + b*x)**n/(b
7*n7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 225*a*b**6*d**2*n**3*x**6*(a
 + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 676
9*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 274*a*b**6*d**2

*n**2*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b*
*7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 12
0*a*b**6*d**2*n*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) + b**7*c**2*n**6*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*
n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5
040*b**7) + 27*b**7*c**2*n**5*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 32
2*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**
7*n + 5040*b**7) + 295*b**7*c**2*n**4*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n
6 + 322*b7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1
3068*b**7*n + 5040*b**7) + 1665*b**7*c**2*n**3*x*(a + b*x)**n/(b**7*n**7 +
28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7
*n**2 + 13068*b**7*n + 5040*b**7) + 5104*b**7*c**2*n**2*x*(a + b*x)**n/(b**
7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1
3132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 8028*b**7*c**2*n*x*(a + b*x)**
n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n*
*3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 5040*b**7*c**2*x*(a + b*
x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**
7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2*b**7*c*d*n**6*x**4
*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 48*b**7*c*d*
n**5*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**
7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 452
*b**7*c*d*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
+ 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b
7) + 2112*b7*c*d*n**3*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322
*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7
*n + 5040*b**7) + 5090*b**7*c*d*n**2*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7
*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
 13068*b**7*n + 5040*b**7) + 5904*b**7*c*d*n*x**4*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 2520*b**7*c*d*x**4*(a + b*x)**n/(b**7*
n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 131
32*b**7*n**2 + 13068*b**7*n + 5040*b**7) + b**7*d**2*n**6*x**7*(a + b*x)**n
/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**
3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 21*b**7*d**2*n**5*x**7*(a
 + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 676
9*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 175*b**7*d**2*n
4*x7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7
*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 735*
b**7*d**2*n**3*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
+ 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b
7) + 1624*b7*d**2*n**2*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 32
2*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**
7*n + 5040*b**7) + 1764*b**7*d**2*n*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*

```
n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
13068*b**7*n + 5040*b**7) + 720*b**7*d**2*x**7*(a + b*x)**n/(b**7*n**7 + 28
*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n
**2 + 13068*b**7*n + 5040*b**7), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.77

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \frac{(bx+a)^{n+1}c^2}{b(n+1)} + \frac{2\left((n^3+6n^2+11n+6)b^4x^4+(n^3+3n^2+2n)ab^3x^3-3(n^2+n)a^2b^2x^2+6a^3bnx-6a^4\right)(bx+a)^n cd}{(n^4+10n^3+35n^2+50n+24)b^4} + \frac{((n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)b^7x^7+(n^6+15n^5+85n^4+225n^3+274n^2}{}$$

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^2/(b*(n + 1)) + 2*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (
n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^
4)*(b*x + a)^n*c*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4) + ((n^6 + 21*n
^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n^5 +
 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n^3 +
 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x^4 -
 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*a^6*
b*n*x + 720*a^7)*(b*x + a)^n*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769
*n^3 + 13132*n^2 + 13068*n + 5040)*b^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. 2(203) = 406.

Time = 0.37 (sec) , antiderivative size = 1477, normalized size of antiderivative = 7.28

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^7*d^2*n^6*x^7 + (b*x + a)^n*a*b^6*d^2*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^2*n^5*x^7 + 15*(b*x + a)^n*a*b^6*d^2*n^5*x^6 + 175*(b*x + a)^n*b^7
*d^2*n^4*x^7 + 2*(b*x + a)^n*b^7*c*d*n^6*x^4 - 6*(b*x + a)^n*a^2*b^5*d^2*n^
5*x^5 + 85*(b*x + a)^n*a*b^6*d^2*n^4*x^6 + 735*(b*x + a)^n*b^7*d^2*n^3*x^7
+ 2*(b*x + a)^n*a*b^6*c*d*n^6*x^3 + 48*(b*x + a)^n*b^7*c*d*n^5*x^4 - 60*(b*
```

```
x + a)^n*a^2*b^5*d^2*n^4*x^5 + 225*(b*x + a)^n*a*b^6*d^2*n^3*x^6 + 1624*(b*
x + a)^n*b^7*d^2*n^2*x^7 + 42*(b*x + a)^n*a*b^6*c*d*n^5*x^3 + 452*(b*x + a)
^n*b^7*c*d*n^4*x^4 + 30*(b*x + a)^n*a^3*b^4*d^2*n^4*x^4 - 210*(b*x + a)^n*a
^2*b^5*d^2*n^3*x^5 + 274*(b*x + a)^n*a*b^6*d^2*n^2*x^6 + 1764*(b*x + a)^n*b
^7*d^2*n*x^7 + (b*x + a)^n*b^7*c^2*n^6*x - 6*(b*x + a)^n*a^2*b^5*c*d*n^5*x^
2 + 326*(b*x + a)^n*a*b^6*c*d*n^4*x^3 + 2112*(b*x + a)^n*b^7*c*d*n^3*x^4 +
180*(b*x + a)^n*a^3*b^4*d^2*n^3*x^4 - 300*(b*x + a)^n*a^2*b^5*d^2*n^2*x^5 +
 120*(b*x + a)^n*a*b^6*d^2*n*x^6 + 720*(b*x + a)^n*b^7*d^2*x^7 + (b*x + a)^
n*a*b^6*c^2*n^6 + 27*(b*x + a)^n*b^7*c^2*n^5*x - 114*(b*x + a)^n*a^2*b^5*c*
d*n^4*x^2 + 1134*(b*x + a)^n*a*b^6*c*d*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^
2*n^3*x^3 + 5090*(b*x + a)^n*b^7*c*d*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^2*
n^2*x^4 - 144*(b*x + a)^n*a^2*b^5*d^2*n*x^5 + 27*(b*x + a)^n*a*b^6*c^2*n^5
+ 295*(b*x + a)^n*b^7*c^2*n^4*x + 12*(b*x + a)^n*a^3*b^4*c*d*n^4*x - 750*(b
*x + a)^n*a^2*b^5*c*d*n^3*x^2 + 1688*(b*x + a)^n*a*b^6*c*d*n^2*x^3 - 360*(b
*x + a)^n*a^4*b^3*d^2*n^2*x^3 + 5904*(b*x + a)^n*b^7*c*d*n*x^4 + 180*(b*x +
 a)^n*a^3*b^4*d^2*n*x^4 + 295*(b*x + a)^n*a*b^6*c^2*n^4 + 1665*(b*x + a)^n*
b^7*c^2*n^3*x + 216*(b*x + a)^n*a^3*b^4*c*d*n^3*x - 1902*(b*x + a)^n*a^2*b^
5*c*d*n^2*x^2 + 360*(b*x + a)^n*a^5*b^2*d^2*n^2*x^2 + 840*(b*x + a)^n*a*b^6
*c*d*n*x^3 - 240*(b*x + a)^n*a^4*b^3*d^2*n*x^3 + 2520*(b*x + a)^n*b^7*c*d*x
^4 + 1665*(b*x + a)^n*a*b^6*c^2*n^3 - 12*(b*x + a)^n*a^4*b^3*c*d*n^3 + 5104
*(b*x + a)^n*b^7*c^2*n^2*x + 1284*(b*x + a)^n*a^3*b^4*c*d*n^2*x - 1260*(b*x
 + a)^n*a^2*b^5*c*d*n*x^2 + 360*(b*x + a)^n*a^5*b^2*d^2*n*x^2 + 5104*(b*x +
 a)^n*a*b^6*c^2*n^2 - 216*(b*x + a)^n*a^4*b^3*c*d*n^2 + 8028*(b*x + a)^n*b^
7*c^2*n*x + 2520*(b*x + a)^n*a^3*b^4*c*d*n*x - 720*(b*x + a)^n*a^6*b*d^2*n*
x + 8028*(b*x + a)^n*a*b^6*c^2*n - 1284*(b*x + a)^n*a^4*b^3*c*d*n + 5040*(b
*x + a)^n*b^7*c^2*x + 5040*(b*x + a)^n*a*b^6*c^2 - 2520*(b*x + a)^n*a^4*b^3
*c*d + 720*(b*x + a)^n*a^7*d^2)/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*
b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

Mupad [B] (verification not implemented)

Time = 20.46 (sec) , antiderivative size = 878, normalized size of antiderivative = 4.33

$$\int (a+bx)^n\left(c+dx^3\right)^2\,dx$$
$$=\frac{a\,(a+b\,x)^n\,(720\,a^6\,d^2-12\,a^3\,b^3\,c\,d\,n^3-216\,a^3\,b^3\,c\,d\,n^2-1284\,a^3\,b^3\,c\,d\,n-2520\,a^3\,b^3\,c\,d+b^6\,c^2\,n^6+27\,b^6\,c}{b^7\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\right.}$$
$$+\frac{d^2\,x^7\,(a+b\,x)^n\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}{n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040}$$
$$+\frac{x\,(a+b\,x)^n\,(-720\,a^6\,b\,d^2\,n+12\,a^3\,b^4\,c\,d\,n^4+216\,a^3\,b^4\,c\,d\,n^3+1284\,a^3\,b^4\,c\,d\,n^2+2520\,a^3\,b^4\,c\,d\,n+b^7\,c^2\,n}{b^7\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+\right.}$$
$$+\frac{2\,d\,x^4\,(a+b\,x)^n\,(n^3+6\,n^2+11\,n+6)\,\left(15\,d\,a^3\,n+c\,b^3\,n^3+18\,c\,b^3\,n^2+107\,c\,b^3\,n+210\,c\,b^3\right)}{b^3\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$
$$+\frac{a\,d^2\,n\,x^6\,(a+b\,x)^n\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}{b\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$
$$-\frac{6\,a^2\,d^2\,n\,x^5\,(a+b\,x)^n\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{b^2\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$
$$+\frac{2\,a\,d\,n\,x^3\,(a+b\,x)^n\,(n^2+3\,n+2)\,\left(-60\,d\,a^3+c\,b^3\,n^3+18\,c\,b^3\,n^2+107\,c\,b^3\,n+210\,c\,b^3\right)}{b^4\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$
$$-\frac{6\,a^2\,d\,n\,x^2\,(n+1)\,(a+b\,x)^n\,\left(-60\,d\,a^3+c\,b^3\,n^3+18\,c\,b^3\,n^2+107\,c\,b^3\,n+210\,c\,b^3\right)}{b^5\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$

```
[In] int((c + d*x^3)^2*(a + b*x)^n,x)
```

```
[Out] (a*(a + b*x)^n*(720*a^6*d^2 + 5040*b^6*c^2 + 8028*b^6*c^2*n + 5104*b^6*c^2*
n^2 + 1665*b^6*c^2*n^3 + 295*b^6*c^2*n^4 + 27*b^6*c^2*n^5 + b^6*c^2*n^6 - 2
520*a^3*b^3*c*d - 1284*a^3*b^3*c*d*n - 216*a^3*b^3*c*d*n^2 - 12*a^3*b^3*c*d
*n^3))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 +
 n^7 + 5040)) + (d^2*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4
 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^
5 + 28*n^6 + n^7 + 5040) + (x*(a + b*x)^n*(5040*b^7*c^2 + 8028*b^7*c^2*n +
5104*b^7*c^2*n^2 + 1665*b^7*c^2*n^3 + 295*b^7*c^2*n^4 + 27*b^7*c^2*n^5 + b^
7*c^2*n^6 - 720*a^6*b*d^2*n + 2520*a^3*b^4*c*d*n + 1284*a^3*b^4*c*d*n^2 + 2
16*a^3*b^4*c*d*n^3 + 12*a^3*b^4*c*d*n^4))/(b^7*(13068*n + 13132*n^2 + 6769*
n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*d*x^4*(a + b*x)^n*(11
*n + 6*n^2 + n^3 + 6)*(210*b^3*c + 18*b^3*c*n^2 + b^3*c*n^3 + 15*a^3*d*n +
107*b^3*c*n))/(b^3*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 2
8*n^6 + n^7 + 5040)) + (a*d^2*n*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 +
 15*n^4 + n^5 + 120))/(b*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n
^5 + 28*n^6 + n^7 + 5040)) - (6*a^2*d^2*n*x^5*(a + b*x)^n*(50*n + 35*n^2 +
10*n^3 + n^4 + 24))/(b^2*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n
^5 + 28*n^6 + n^7 + 5040)) + (2*a*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(210*
b^3*c - 60*a^3*d + 18*b^3*c*n^2 + b^3*c*n^3 + 107*b^3*c*n))/(b^4*(13068*n +
```

```
 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) - (6*a^2
*d*n*x^2*(n + 1)*(a + b*x)^n*(210*b^3*c - 60*a^3*d + 18*b^3*c*n^2 + b^3*c*n
^3 + 107*b^3*c*n))/(b^5*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^
5 + 28*n^6 + n^7 + 5040))
```

3.181 $\int \frac{(a+bx)^n(c+dx^3)^2}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 209

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x}\,dx = \frac{a^2d(2b^3c-a^3d)\,(a+bx)^{1+n}}{b^6(1+n)} - \frac{ad(4b^3c-5a^3d)\,(a+bx)^{2+n}}{b^6(2+n)} + \frac{2d(b^3c-5a^3d)\,(a+bx)^{3+n}}{b^6(3+n)} + \frac{10a^2d^2(a+bx)^{4+n}}{b^6(4+n)} - \frac{5ad^2(a+bx)^{5+n}}{b^6(5+n)} + \frac{d^2(a+bx)^{6+n}}{b^6(6+n)} - \frac{c^2(a+bx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)}{a(1+n)}$$

```
[Out] a^2*d*(-a^3*d+2*b^3*c)*(b*x+a)^(1+n)/b^6/(1+n)-a*d*(-5*a^3*d+4*b^3*c)*(b*x+
a)^(2+n)/b^6/(2+n)+2*d*(-5*a^3*d+b^3*c)*(b*x+a)^(3+n)/b^6/(3+n)+10*a^2*d^2*
(b*x+a)^(4+n)/b^6/(4+n)-5*a*d^2*(b*x+a)^(5+n)/b^6/(5+n)+d^2*(b*x+a)^(6+n)/b
^6/(6+n)-c^2*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used

= {1634, 67}

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x}\,dx = -\frac{ad(4b^3c-5a^3d)(a+bx)^{n+2}}{b^6(n+2)} + \frac{2d(b^3c-5a^3d)(a+bx)^{n+3}}{b^6(n+3)} + \frac{10a^2d^2(a+bx)^{n+4}}{b^6(n+4)} + \frac{a^2d(2b^3c-a^3d)(a+bx)^{n+1}}{b^6(n+1)} - \frac{5ad^2(a+bx)^{n+5}}{b^6(n+5)} + \frac{d^2(a+bx)^{n+6}}{b^6(n+6)} - \frac{c^2(a+bx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{bx}{a}+1\right)}{a(n+1)}$$

```
[In] Int[((a + b*x)^n*(c + d*x^3)^2)/x,x]
```

```
[Out] (a^2*d*(2*b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*d*(4*b^3*c -
 5*a^3*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + (2*d*(b^3*c - 5*a^3*d)*(a + b*
x)^(3 + n))/(b^6*(3 + n)) + (10*a^2*d^2*(a + b*x)^(4 + n))/(b^6*(4 + n)) -
(5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d^2*(a + b*x)^(6 + n))/(b^6*(6
 + n)) - (c^2*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x
)/a])/(a*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{a^2d(-2b^3c+a^3d)(a+bx)^n}{b^5} + \frac{c^2(a+bx)^n}{x} + \frac{ad(-4b^3c+5a^3d)(a+bx)^{1+n}}{b^5} + \frac{2d(b^3c-5a^3d)(a+bx)^{2+n}}{b^5} + \frac{10a^2d^2(a+bx)^{3+n}}{b^5} - \frac{5ad^2(a+bx)^{4+n}}{b^5} + \frac{d^2(a+bx)^{5+n}}{b^5}\right) dx$$

$$= \frac{a^2d(2b^3c-a^3d)(a+bx)^{1+n}}{b^6(1+n)} - \frac{ad(4b^3c-5a^3d)(a+bx)^{2+n}}{b^6(2+n)} + \frac{2d(b^3c-5a^3d)(a+bx)^{3+n}}{b^6(3+n)} + \frac{10a^2d^2(a+bx)^{4+n}}{b^6(4+n)} - \frac{5ad^2(a+bx)^{5+n}}{b^6(5+n)} + \frac{d^2(a+bx)^{6+n}}{b^6(6+n)} + c^2\int \frac{(a+bx)^n}{x}\,dx$$

$$= \frac{a^2d(2b^3c-a^3d)(a+bx)^{1+n}}{b^6(1+n)} - \frac{ad(4b^3c-5a^3d)(a+bx)^{2+n}}{b^6(2+n)} + \frac{2d(b^3c-5a^3d)(a+bx)^{3+n}}{b^6(3+n)} + \frac{10a^2d^2(a+bx)^{4+n}}{b^6(4+n)} - \frac{5ad^2(a+bx)^{5+n}}{b^6(5+n)} + \frac{d^2(a+bx)^{6+n}}{b^6(6+n)} - \frac{c^2(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^n\left(c+dx^3\right)^2}{x}\,dx = (a+bx)^{1+n}\left(\frac{a^2d(2b^3c-a^3d)}{b^6(1+n)} + \frac{ad(-4b^3c+5a^3d)(a+bx)}{b^6(2+n)} + \frac{2d(b^3c-5a^3d)(a+bx)^2}{b^6(3+n)} + \frac{10a^2d^2(a+bx)^3}{b^6(4+n)} - \frac{5ad^2(a+bx)^4}{b^6(5+n)} + \frac{d^2(a+bx)^5}{b^6(6+n)} - \frac{c^2\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+bx}{a}\right)}{a+an}\right)$$

```
[In] Integrate[((a + b*x)^n*(c + d*x^3)^2)/x,x]
```

```
[Out] (a + b*x)^(1 + n)*((a^2*d*(2*b^3*c - a^3*d))/(b^6*(1 + n)) + (a*d*(-4*b^3*c
 + 5*a^3*d)*(a + b*x))/(b^6*(2 + n)) + (2*d*(b^3*c - 5*a^3*d)*(a + b*x)^2)/
(b^6*(3 + n)) + (10*a^2*d^2*(a + b*x)^3)/(b^6*(4 + n)) - (5*a*d^2*(a + b*x)
^4)/(b^6*(5 + n)) + (d^2*(a + b*x)^5)/(b^6*(6 + n)) - (c^2*Hypergeometric2F
1[1, 1 + n, 2 + n, (a + b*x)/a])/(a + a*n))
```

Maple [F]

$$\int \frac{(bx+a)^n\left(x^3d+c\right)^2}{x}dx$$

```
[In] int((b*x+a)^n*(d*x^3+c)^2/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^3+c)^2/x,x)
```

Fricas [F]

$$\int \frac{(a+bx)^n\,(c+dx^3)^2}{x}\,dx = \int \frac{(dx^3+c)^2(bx+a)^n}{x}\,dx$$

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x + a)^n/x, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4007 vs. $2(187) = 374$.

Time = 4.22 (sec) , antiderivative size = 4690, normalized size of antiderivative = 22.44

$$\int \frac{(a+bx)^n\,(c+dx^3)^2}{x}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)**n*(d*x**3+c)**2/x,x)
```

```
[Out] 2*c*d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3
+ 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**
2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*
x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2*a*
*2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b + x)/(a*b
**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b
**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b + x)/b**3 -
 a*x/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3 + 6*b**
3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**
3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**3*n**3 + 6
*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b**3*n**3 +
6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)**n/(b**3*n**3
+ 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x)**n/(b**3*n**3
 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)**n/(b**3*n**3
+ 6*b**3*n**2 + 11*b**3*n + 6*b**3), True)) + d**2*Piecewise((a**n*x**6/6,
Eq(b, 0)), (60*a**5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3
*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 137*a
**5/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x*
*3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*x*log(a/b + x)/(60*a**5
*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b
**10*x**4 + 60*b**11*x**5) + 625*a**4*b*x/(60*a**5*b**6 + 300*a**4*b**7*x +
 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5
) + 600*a**3*b**2*x**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a
**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 11
```

00*a**3*b**2*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 60
0*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**2*b**3*x**3*l
og(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2
*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 900*a**2*b**3*x**3/(60*a**
5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*
b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*x**4*log(a/b + x)/(60*a**5*b**6 +
300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**
4 + 60*b**11*x**5) + 300*a*b**4*x**4/(60*a**5*b**6 + 300*a**4*b**7*x + 600*
a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 6
0*b**5*x**5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x*
*2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-
60*a**5*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 4
8*a*b**9*x**3 + 12*b**10*x**4) - 125*a**5/(12*a**4*b**6 + 48*a**3*b**7*x +
72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**4*b*x*log(a/b
+ x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 +
12*b**10*x**4) - 440*a**4*b*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8
*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 360*a**3*b**2*x**2*log(a/b + x)/(
12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**
10*x**4) - 540*a**3*b**2*x**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8
*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*x**3*log(a/b + x)/(
12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**
10*x**4) - 240*a**2*b**3*x**3/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8
*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 60*a*b**4*x**4*log(a/b + x)/(12*a
4*b6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x
4) + 12*b5*x**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48
*a*b**9*x**3 + 12*b**10*x**4), Eq(n, -5)), (60*a**5*log(a/b + x)/(6*a**3*b*
*6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 110*a**5/(6*a**3*b**6
 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**4*b*x*log(a/b +
x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 270*a**4
*b*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a*
*3*b**2*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 +
6*b**9*x**3) + 180*a**3*b**2*x**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8
*x**2 + 6*b**9*x**3) + 60*a**2*b**3*x**3*log(a/b + x)/(6*a**3*b**6 + 18*a**
2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*a*b**4*x**4/(6*a**3*b**6 + 18
*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 3*b**5*x**5/(6*a**3*b**6 + 1
8*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3), Eq(n, -4)), (-60*a**5*log(a/
b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 90*a**5/(6*a**2*b**6 + 1
2*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b
7*x + 6*b8*x**2) - 120*a**4*b*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**
2) - 60*a**3*b**2*x**2*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**
2) + 20*a**2*b**3*x**3/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 5*a*b**4
*x**4/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 2*b**5*x**5/(6*a**2*b**6
+ 12*a*b**7*x + 6*b**8*x**2), Eq(n, -3)), (60*a**5*log(a/b + x)/(12*a*b**6
+ 12*b**7*x) + 60*a**5/(12*a*b**6 + 12*b**7*x) + 60*a**4*b*x*log(a/b + x)/(
12*a*b**6 + 12*b**7*x) - 30*a**3*b**2*x**2/(12*a*b**6 + 12*b**7*x) + 10*a**

2*b**3*x**3/(12*a*b**6 + 12*b**7*x) - 5*a*b**4*x**4/(12*a*b**6 + 12*b**7*x)
 + 3*b**5*x**5/(12*a*b**6 + 12*b**7*x), Eq(n, -2)), (-a**5*log(a/b + x)/b**
6 + a**4*x/b**5 - a**3*x**2/(2*b**4) + a**2*x**3/(3*b**3) - a*x**4/(4*b**2)
 + x**5/(5*b), Eq(n, -1)), (-120*a**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**
5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6
) + 120*a**5*b*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 +
 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*n*
*2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n
3 + 1624*b6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*n*x**2*(a + b
*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**
6*n**2 + 1764*b**6*n + 720*b**6) + 20*a**3*b**3*n**3*x**3*(a + b*x)**n/(b**
6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 17
64*b**6*n + 720*b**6) + 60*a**3*b**3*n**2*x**3*(a + b*x)**n/(b**6*n**6 + 21
*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
 720*b**6) + 40*a**3*b**3*n*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 1
75*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 5
*a**2*b**4*n**4*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4
 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4*
n**3*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6
*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 55*a**2*b**4*n**2*x**4*(
a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4*n*x**4*(a + b*x)**n/(b
6*n6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 +
1764*b**6*n + 720*b**6) + a*b**5*n**5*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**
6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720
*b**6) + 10*a*b**5*n**4*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b
6*n4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 35*a*
b**5*n**3*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735
*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 50*a*b**5*n**2*x**5
*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1
624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 24*a*b**5*n*x**5*(a + b*x)**n/(b*
*6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1
764*b**6*n + 720*b**6) + b**6*n**5*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n
5 + 175*b6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b*
*6) + 15*b**6*n**4*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n
4 + 735*b6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 85*b**6*n*
*3*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n
3 + 1624*b6*n**2 + 1764*b**6*n + 720*b**6) + 225*b**6*n**2*x**6*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 274*b**6*n*x**6*(a + b*x)**n/(b**6*n**6 +
 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*
n + 720*b**6) + 120*b**6*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*
b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6), True)
) - b**(n + 1)*c**2*n*(a/b + x)**(n + 1)*lerchphi(1 + b*x/a, 1, n + 1)*gamm
a(n + 1)/(a*gamma(n + 2)) - b**(n + 1)*c**2*(a/b + x)**(n + 1)*lerchphi(1 +

```
b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

Maxima [F]

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x} \, dx = \int \frac{(dx^3+c)^2 (bx+a)^n}{x} \, dx$$

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^2*(b*x + a)^n/x, x)
```

Giac [F]

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x} \, dx = \int \frac{(dx^3+c)^2 (bx+a)^n}{x} \, dx$$

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^2*(b*x + a)^n/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x} \, dx = \int \frac{(d\,x^3+c)^2 (a+b\,x)^n}{x} \, dx$$

```
[In] int(((c + d*x^3)^2*(a + b*x)^n)/x,x)
[Out] int(((c + d*x^3)^2*(a + b*x)^n)/x, x)
```

3.182 $\int x^2(a+bx)^n \left(c+dx^3\right)^3 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 459

$$\begin{aligned}\int x^2(a+bx)^n \left(c+dx^3\right)^3 \, dx = {} & \frac{a^2(b^3c-a^3d)^3(a+bx)^{1+n}}{b^{12}(1+n)} \\ & - \frac{a(2b^3c-11a^3d)\left(b^3c-a^3d\right)^2(a+bx)^{2+n}}{b^{12}(2+n)} \\ & + \frac{(b^3c-a^3d)\left(b^6c^2-29a^3b^3cd+55a^6d^2\right)(a+bx)^{3+n}}{b^{12}(3+n)} \\ & + \frac{3a^2d(10b^6c^2-56a^3b^3cd+55a^6d^2)(a+bx)^{4+n}}{b^{12}(4+n)} \\ & - \frac{15ad(b^6c^2-14a^3b^3cd+22a^6d^2)(a+bx)^{5+n}}{b^{12}(5+n)} \\ & + \frac{3d(b^6c^2-56a^3b^3cd+154a^6d^2)(a+bx)^{6+n}}{b^{12}(6+n)} \\ & + \frac{42a^2d^2(2b^3c-11a^3d)(a+bx)^{7+n}}{b^{12}(7+n)} \\ & - \frac{6ad^2(4b^3c-55a^3d)(a+bx)^{8+n}}{b^{12}(8+n)} \\ & + \frac{3d^2(b^3c-55a^3d)(a+bx)^{9+n}}{b^{12}(9+n)} + \frac{55a^2d^3(a+bx)^{10+n}}{b^{12}(10+n)} \\ & - \frac{11ad^3(a+bx)^{11+n}}{b^{12}(11+n)} + \frac{d^3(a+bx)^{12+n}}{b^{12}(12+n)}\end{aligned}$$

```
[Out] a^2*(-a^3*d+b^3*c)^3*(b*x+a)^(1+n)/b^12/(1+n)-a*(-11*a^3*d+2*b^3*c)*(-a^3*d
+b^3*c)^2*(b*x+a)^(2+n)/b^12/(2+n)+(-a^3*d+b^3*c)*(55*a^6*d^2-29*a^3*b^3*c*
d+b^6*c^2)*(b*x+a)^(3+n)/b^12/(3+n)+3*a^2*d*(55*a^6*d^2-56*a^3*b^3*c*d+10*b
^6*c^2)*(b*x+a)^(4+n)/b^12/(4+n)-15*a*d*(22*a^6*d^2-14*a^3*b^3*c*d+b^6*c^2)
```

```
*(b*x+a)^(5+n)/b^12/(5+n)+3*d*(154*a^6*d^2-56*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^
(6+n)/b^12/(6+n)+42*a^2*d^2*(-11*a^3*d+2*b^3*c)*(b*x+a)^(7+n)/b^12/(7+n)-6*
a*d^2*(-55*a^3*d+4*b^3*c)*(b*x+a)^(8+n)/b^12/(8+n)+3*d^2*(-55*a^3*d+b^3*c)*
(b*x+a)^(9+n)/b^12/(9+n)+55*a^2*d^3*(b*x+a)^(10+n)/b^12/(10+n)-11*a*d^3*(b*
x+a)^(11+n)/b^12/(11+n)+d^3*(b*x+a)^(12+n)/b^12/(12+n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1634}

$$\begin{aligned}\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx=&-\frac{6ad^2(4b^3c-55a^3d)\,(a+bx)^{n+8}}{b^{12}(n+8)}\\&+\frac{3d^2(b^3c-55a^3d)\,(a+bx)^{n+9}}{b^{12}(n+9)}\\&-\frac{a(2b^3c-11a^3d)\,(b^3c-a^3d)^2\,(a+bx)^{n+2}}{b^{12}(n+2)}\\&+\frac{55a^2d^3(a+bx)^{n+10}}{b^{12}(n+10)}\\&+\frac{(b^3c-a^3d)\,(55a^6d^2-29a^3b^3cd+b^6c^2)\,(a+bx)^{n+3}}{b^{12}(n+3)}\\&-\frac{15ad(22a^6d^2-14a^3b^3cd+b^6c^2)\,(a+bx)^{n+5}}{b^{12}(n+5)}\\&+\frac{3d(154a^6d^2-56a^3b^3cd+b^6c^2)\,(a+bx)^{n+6}}{b^{12}(n+6)}\\&+\frac{42a^2d^2(2b^3c-11a^3d)\,(a+bx)^{n+7}}{b^{12}(n+7)}\\&+\frac{a^2(b^3c-a^3d)^3\,(a+bx)^{n+1}}{b^{12}(n+1)}\\&+\frac{3a^2d(55a^6d^2-56a^3b^3cd+10b^6c^2)\,(a+bx)^{n+4}}{b^{12}(n+4)}\\&-\frac{11ad^3(a+bx)^{n+11}}{b^{12}(n+11)}+\frac{d^3(a+bx)^{n+12}}{b^{12}(n+12)}\end{aligned}$$

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] (a^2*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^12*(1 + n)) - (a*(2*b^3*c - 11
*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^12*(2 + n)) + ((b^3*c - a^3
*d)*(b^6*c^2 - 29*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(3 + n))/(b^12*(3 + n
)) + (3*a^2*d*(10*b^6*c^2 - 56*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(4 + n))
/(b^12*(4 + n)) - (15*a*d*(b^6*c^2 - 14*a^3*b^3*c*d + 22*a^6*d^2)*(a + b*x)
^(5 + n))/(b^12*(5 + n)) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d + 154*a^6*d^2)*(a
```

```
 + b*x)^(6 + n))/(b^12*(6 + n)) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a + b*x
)^(7 + n))/(b^12*(7 + n)) - (6*a*d^2*(4*b^3*c - 55*a^3*d)*(a + b*x)^(8 + n)
)/(b^12*(8 + n)) + (3*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^(9 + n))/(b^12*(9 +
n)) + (55*a^2*d^3*(a + b*x)^(10 + n))/(b^12*(10 + n)) - (11*a*d^3*(a + b*x)
^(11 + n))/(b^12*(11 + n)) + (d^3*(a + b*x)^(12 + n))/(b^12*(12 + n))
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{a^2(-b^3c+a^3d)^3(a+bx)^n}{b^{11}} + \frac{a(-b^3c+a^3d)^2(-2b^3c+11a^3d)(a+bx)^{1+n}}{b^{11}} + \frac{(b^3c-a^3d)(b^6c^2-29a^3b^3cd+55a^6d^2)(a+bx)^{2+n}}{b^{11}} + \frac{3a^2d(10b^6c^2-56a^3b^3cd+55a^6d^2)(a+bx)^{3+n}}{b^{11}} - \frac{15ad(b^6c^2-14a^3b^3cd+22a^6d^2)(a+bx)^{4+n}}{b^{11}} + \frac{3d(b^6c^2-56a^3b^3cd+154a^6d^2)(a+bx)^{5+n}}{b^{11}} - \frac{42a^2d^2(-2b^3c+11a^3d)(a+bx)^{6+n}}{b^{11}} + \frac{6ad^2(-4b^3c+55a^3d)(a+bx)^{7+n}}{b^{11}} + \frac{3d^2(b^3c-55a^3d)(a+bx)^{8+n}}{b^{11}} + \frac{55a^2d^3(a+bx)^{9+n}}{b^{11}} - \frac{11ad^3(a+bx)^{10+n}}{b^{11}} + \frac{d^3(a+bx)^{11+n}}{b^{11}}\right) dx$$

$$= \frac{a^2(b^3c-a^3d)^3(a+bx)^{1+n}}{b^{12}(1+n)} - \frac{a(2b^3c-11a^3d)(b^3c-a^3d)^2(a+bx)^{2+n}}{b^{12}(2+n)}$$
$$+ \frac{(b^3c-a^3d)(b^6c^2-29a^3b^3cd+55a^6d^2)(a+bx)^{3+n}}{b^{12}(3+n)}$$
$$+ \frac{3a^2d(10b^6c^2-56a^3b^3cd+55a^6d^2)(a+bx)^{4+n}}{b^{12}(4+n)}$$
$$- \frac{15ad(b^6c^2-14a^3b^3cd+22a^6d^2)(a+bx)^{5+n}}{b^{12}(5+n)}$$
$$+ \frac{3d(b^6c^2-56a^3b^3cd+154a^6d^2)(a+bx)^{6+n}}{b^{12}(6+n)} + \frac{42a^2d^2(2b^3c-11a^3d)(a+bx)^{7+n}}{b^{12}(7+n)}$$
$$- \frac{6ad^2(4b^3c-55a^3d)(a+bx)^{8+n}}{b^{12}(8+n)} + \frac{3d^2(b^3c-55a^3d)(a+bx)^{9+n}}{b^{12}(9+n)}$$
$$+ \frac{55a^2d^3(a+bx)^{10+n}}{b^{12}(10+n)} - \frac{11ad^3(a+bx)^{11+n}}{b^{12}(11+n)} + \frac{d^3(a+bx)^{12+n}}{b^{12}(12+n)}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.88

$$\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx$$
$$= \frac{(a+bx)^{1+n}\left(\frac{a^2(b^3c-a^3d)^3}{1+n} + \frac{a(b^3c-a^3d)^2(-2b^3c+11a^3d)(a+bx)}{2+n} + \frac{(b^3c-a^3d)(b^6c^2-29a^3b^3cd+55a^6d^2)(a+bx)^2}{3+n} + \frac{3a^2d(10b^6c^2-56a\ldots}{}\right.}{}$$

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*(b^3*c - a^3*d)^3)/(1 + n) + (a*(b^3*c - a^3*d)^2*
(-2*b^3*c + 11*a^3*d)*(a + b*x))/(2 + n) + ((b^3*c - a^3*d)*(b^6*c^2 - 29*a
^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^2)/(3 + n) + (3*a^2*d*(10*b^6*c^2 - 56*a
^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^3)/(4 + n) - (15*a*d*(b^6*c^2 - 14*a^3*b
^3*c*d + 22*a^6*d^2)*(a + b*x)^4)/(5 + n) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d
+ 154*a^6*d^2)*(a + b*x)^5)/(6 + n) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a +
 b*x)^6)/(7 + n) + (6*a*d^2*(-4*b^3*c + 55*a^3*d)*(a + b*x)^7)/(8 + n) + (3
*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^8)/(9 + n) + (55*a^2*d^3*(a + b*x)^9)/(10
 + n) - (11*a*d^3*(a + b*x)^10)/(11 + n) + (d^3*(a + b*x)^11)/(12 + n)))/b^
12
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3779 vs. 2(459) = 918.

Time = 1.35 (sec) , antiderivative size = 3780, normalized size of antiderivative = 8.24

method	result	size
gosper	Expression too large to display	3780
risch	Expression too large to display	4231
parallelrisch	Expression too large to display	6192

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^12*(b*x+a)^(1+n)/(n^12+78*n^11+2717*n^10+55770*n^9+749463*n^8+6926634*
n^7+44990231*n^6+206070150*n^5+657206836*n^4+1414014888*n^3+1931559552*n^2+
1486442880*n+479001600)*(-b^11*d^3*n^11*x^11-66*b^11*d^3*n^10*x^11+11*a*b^1
0*d^3*n^10*x^10-1925*b^11*d^3*n^9*x^11+605*a*b^10*d^3*n^9*x^10-3*b^11*c*d^2
*n^11*x^8-32670*b^11*d^3*n^8*x^11-110*a^2*b^9*d^3*n^9*x^9+14520*a*b^10*d^3*
n^8*x^10-207*b^11*c*d^2*n^10*x^8-357423*b^11*d^3*n^7*x^11-4950*a^2*b^9*d^3*
n^8*x^9+24*a*b^10*c*d^2*n^10*x^7+199650*a*b^10*d^3*n^7*x^10-6288*b^11*c*d^2
*n^9*x^8-2637558*b^11*d^3*n^6*x^11+990*a^3*b^8*d^3*n^8*x^8-95700*a^2*b^9*d^
3*n^7*x^9+1464*a*b^10*c*d^2*n^9*x^7+1735503*a*b^10*d^3*n^6*x^10-3*b^11*c^2*
d*n^11*x^5-110718*b^11*c*d^2*n^8*x^8-13339535*b^11*d^3*n^5*x^11+35640*a^3*b
^8*d^3*n^7*x^8-168*a^2*b^9*c*d^2*n^9*x^6-1039500*a^2*b^9*d^3*n^6*x^9+38592*
a*b^10*c*d^2*n^8*x^7+9922605*a*b^10*d^3*n^5*x^10-216*b^11*c^2*d*n^10*x^5-12
51927*b^11*c*d^2*n^7*x^8-45995730*b^11*d^3*n^4*x^11-7920*a^4*b^7*d^3*n^7*x^
7+540540*a^3*b^8*d^3*n^6*x^8-9072*a^2*b^9*c*d^2*n^8*x^6-6960030*a^2*b^9*d^3
*n^5*x^9+15*a*b^10*c^2*d*n^10*x^4+577008*a*b^10*c*d^2*n^7*x^7+37586230*a*b^
10*d^3*n^4*x^10-6855*b^11*c^2*d*n^9*x^5-9512559*b^11*c*d^2*n^6*x^8-10525807
6*b^11*d^3*n^3*x^11-221760*a^4*b^7*d^3*n^6*x^7+1008*a^3*b^8*c*d^2*n^8*x^5+4
490640*a^3*b^8*d^3*n^5*x^8-206640*a^2*b^9*c*d^2*n^7*x^6-29625750*a^2*b^9*d^
3*n^4*x^9+1005*a*b^10*c^2*d*n^9*x^4+5399352*a*b^10*c*d^2*n^6*x^7+92504500*a
*b^10*d^3*n^3*x^10-b^11*c^3*n^11*x^2-126180*b^11*c^2*d*n^8*x^5-49357662*b^1
1*c*d^2*n^5*x^8-150917976*b^11*d^3*n^2*x^11+55440*a^5*b^6*d^3*n^6*x^6-25502
40*a^4*b^7*d^3*n^5*x^7+48384*a^3*b^8*c*d^2*n^7*x^5+22224510*a^3*b^8*d^3*n^4
*x^8-60*a^2*b^9*c^2*d*n^9*x^3-2592576*a^2*b^9*c*d^2*n^6*x^6-79604800*a^2*b^
9*d^3*n^3*x^9+29250*a*b^10*c^2*d*n^8*x^4+32905656*a*b^10*c*d^2*n^5*x^7+1402
89336*a*b^10*d^3*n^2*x^10-75*b^11*c^3*n^10*x^2-1491309*b^11*c^2*d*n^7*x^5-1
73991492*b^11*c*d^2*n^4*x^8-120543840*b^11*d^3*n*x^11+1164240*a^5*b^6*d^3*n
^5*x^6-5040*a^4*b^7*c*d^2*n^7*x^4-15523200*a^4*b^7*d^3*n^4*x^7+949536*a^3*b
^8*c*d^2*n^6*x^5+66611160*a^3*b^8*d^3*n^3*x^8-3780*a^2*b^9*c^2*d*n^8*x^3-19
647432*a^2*b^9*c*d^2*n^5*x^6-128997000*a^2*b^9*d^3*n^2*x^9+2*a*b^10*c^3*n^1
0*x+484650*a*b^10*c^2*d*n^7*x^4+131616048*a*b^10*c*d^2*n^4*x^7+116915040*a*
b^10*d^3*n*x^10-2492*b^11*c^3*n^9*x^2-11832048*b^11*c^2*d*n^6*x^5-405697080
*b^11*c*d^2*n^3*x^8-39916800*b^11*d^3*x^11-332640*a^6*b^5*d^3*n^5*x^5+97020
00*a^5*b^6*d^3*n^4*x^6-216720*a^4*b^7*c*d^2*n^6*x^4-53610480*a^4*b^7*d^3*n^
```

3*x^7+180*a^3*b^8*c^2*d*n^8*x^2+9858240*a^3*b^8*c*d^2*n^5*x^5+116942760*a^3
*b^8*d^3*n^2*x^8-101880*a^2*b^9*c^2*d*n^7*x^3-92807568*a^2*b^9*c*d^2*n^4*x^
6-112923360*a^2*b^9*d^3*n*x^9+146*a*b^10*c^3*n^9*x+5033295*a*b^10*c^2*d*n^6
*x^4+339003552*a*b^10*c*d^2*n^3*x^7+39916800*a*b^10*d^3*x^10-48294*b^11*c^3
*n^8*x^2-63978405*b^11*c^2*d*n^5*x^5-590770944*b^11*c*d^2*n^2*x^8-4989600*a
^6*b^5*d^3*n^4*x^5+20160*a^5*b^6*c*d^2*n^6*x^3+40748400*a^5*b^6*d^3*n^3*x^6
-3664080*a^4*b^7*c*d^2*n^5*x^4-104005440*a^4*b^7*d^3*n^2*x^7+10800*a^3*b^8*
c^2*d*n^7*x^2+58735152*a^3*b^8*c*d^2*n^4*x^5+108488160*a^3*b^8*d^3*n*x^8-2*
a^2*b^9*c^3*n^9-1531080*a^2*b^9*c^2*d*n^6*x^3-271659360*a^2*b^9*c*d^2*n^3*x
^6-39916800*a^2*b^9*d^3*x^9+4692*a*b^10*c^3*n^8*x+33993765*a*b^10*c^2*d*n^5
*x^4+533548224*a*b^10*c*d^2*n^2*x^7-604581*b^11*c^3*n^7*x^2-234340020*b^11*
c^2*d*n^4*x^5-477740160*b^11*c*d^2*n*x^8+1663200*a^7*b^4*d^3*n^4*x^4-282744
00*a^6*b^5*d^3*n^3*x^5+786240*a^5*b^6*c*d^2*n^5*x^3+90034560*a^5*b^6*d^3*n^
2*x^6-360*a^4*b^7*c^2*d*n^7*x-30970800*a^4*b^7*c*d^2*n^4*x^4-103498560*a^4*
b^7*d^3*n*x^7+273240*a^3*b^8*c^2*d*n^6*x^2+204434496*a^3*b^8*c*d^2*n^3*x^5+
39916800*a^3*b^8*d^3*x^8-144*a^2*b^9*c^3*n^8-14008860*a^2*b^9*c^2*d*n^5*x^3
-471409344*a^2*b^9*c*d^2*n^2*x^6+87204*a*b^10*c^3*n^7*x+149923200*a*b^10*c^
2*d*n^4*x^4+457781760*a*b^10*c*d^2*n*x^7-5112891*b^11*c^3*n^6*x^2-565580388
*b^11*c^2*d*n^3*x^5-159667200*b^11*c*d^2*x^8+16632000*a^7*b^4*d^3*n^3*x^4-6
0480*a^6*b^5*c*d^2*n^5*x^2-74844000*a^6*b^5*d^3*n^2*x^5+11511360*a^5*b^6*c*
d^2*n^4*x^3+97796160*a^5*b^6*d^3*n*x^6-20880*a^4*b^7*c^2*d*n^6*x-138821760*
a^4*b^7*c*d^2*n^3*x^4-39916800*a^4*b^7*d^3*x^7+3773520*a^3*b^8*c^2*d*n^5*x^
2+403349184*a^3*b^8*c*d^2*n^2*x^5-4548*a^2*b^9*c^3*n^7-79939620*a^2*b^9*c^2
*d*n^4*x^3-434972160*a^2*b^9*c*d^2*n*x^6+1034754*a*b^10*c^3*n^6*x+422084100
*a*b^10*c^2*d*n^3*x^4+159667200*a*b^10*c*d^2*x^7-29651558*b^11*c^3*n^5*x^2-
848562336*b^11*c^2*d*n^2*x^5-6652800*a^8*b^3*d^3*n^3*x^3+58212000*a^7*b^4*d
^3*n^2*x^4-2177280*a^6*b^5*c*d^2*n^4*x^2-91143360*a^6*b^5*d^3*n*x^5+360*a^5
*b^6*c^2*d*n^6+77837760*a^5*b^6*c*d^2*n^3*x^3+39916800*a^5*b^6*d^3*x^6-5047
20*a^4*b^7*c^2*d*n^5*x-328063680*a^4*b^7*c*d^2*n^2*x^4+30706020*a^3*b^8*c^2
*d*n^4*x^2+408360960*a^3*b^8*c*d^2*n*x^5-82656*a^2*b^9*c^3*n^6-279934320*a^
2*b^9*c^2*d*n^3*x^3-159667200*a^2*b^9*c*d^2*x^6+8156274*a*b^10*c^3*n^5*x+71
7481440*a*b^10*c^2*d*n^2*x^4-117115476*b^11*c^3*n^4*x^2-703304640*b^11*c^2*
d*n*x^5-39916800*a^8*b^3*d^3*n^2*x^3+120960*a^7*b^4*c*d^2*n^4*x+83160000*a^
7*b^4*d^3*n*x^4-28002240*a^6*b^5*c*d^2*n^3*x^2-39916800*a^6*b^5*d^3*x^5+205
20*a^5*b^6*c^2*d*n^5+243936000*a^5*b^6*c*d^2*n^2*x^3-6537600*a^4*b^7*c^2*d*
n^4*x-376427520*a^4*b^7*c*d^2*n*x^4+147700800*a^3*b^8*c^2*d*n^3*x^2+1596672
00*a^3*b^8*c*d^2*x^5-952098*a^2*b^9*c^3*n^5-568599120*a^2*b^9*c^2*d*n^2*x^3
+42990568*a*b^10*c^3*n^4*x+655404480*a*b^10*c^2*d*n*x^4-305860408*b^11*c^3*
n^3*x^2-239500800*b^11*c^2*d*x^5+19958400*a^9*b^2*d^3*n^2*x^2-73180800*a^8*
b^3*d^3*n*x^3+4112640*a^7*b^4*c*d^2*n^3*x+39916800*a^7*b^4*d^3*x^4-14950656
0*a^6*b^5*c*d^2*n^2*x^2+484200*a^5*b^6*c^2*d*n^4+336510720*a^5*b^6*c*d^2*n*
x^3-48336840*a^4*b^7*c^2*d*n^3*x-159667200*a^4*b^7*c*d^2*x^4+396700560*a^3*
b^8*c^2*d*n^2*x^2-7204176*a^2*b^9*c^3*n^4-595529280*a^2*b^9*c^2*d*n*x^3+148
249816*a*b^10*c^3*n^3*x+239500800*a*b^10*c^2*d*x^4-496433664*b^11*c^3*n^2*x
^2+59875200*a^9*b^2*d^3*n*x^2-120960*a^8*b^3*c*d^2*n^3-39916800*a^8*b^3*d^3

```
*x^3+47779200*a^7*b^4*c*d^2*n^2*x-283288320*a^6*b^5*c*d^2*n*x^2+6053400*a^5
*b^6*c^2*d*n^3+159667200*a^5*b^6*c*d^2*x^3-198727920*a^4*b^7*c^2*d*n^2*x+51
5695680*a^3*b^8*c^2*d*n*x^2-35786392*a^2*b^9*c^3*n^3-239500800*a^2*b^9*c^2*
d*x^3+315221184*a*b^10*c^3*n^2*x-442258560*b^11*c^3*n*x^2-39916800*a^10*b*d
^3*n*x+39916800*a^9*b^2*d^3*x^2-3991680*a^8*b^3*c*d^2*n^2+203454720*a^7*b^4
*c*d^2*n*x-159667200*a^6*b^5*c*d^2*x^2+42283440*a^5*b^6*c^2*d*n^2-395945280
*a^4*b^7*c^2*d*n*x+239500800*a^3*b^8*c^2*d*x^2-112463424*a^2*b^9*c^3*n^2+36
2424960*a*b^10*c^3*n*x-159667200*b^11*c^3*x^2-39916800*a^10*b*d^3*x-4378752
0*a^8*b^3*c*d^2*n+159667200*a^7*b^4*c*d^2*x+156444480*a^5*b^6*c^2*d*n-23950
0800*a^4*b^7*c^2*d*x-202757760*a^2*b^9*c^3*n+159667200*a*b^10*c^3*x+3991680
0*a^11*d^3-159667200*a^8*b^3*c*d^2+239500800*a^5*b^6*c^2*d-159667200*a^2*b^
9*c^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3564 vs. 2(459) = 918.

Time = 0.33 (sec) , antiderivative size = 3564, normalized size of antiderivative = 7.76

$$\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^9*c^3*n^9 + 144*a^3*b^9*c^3*n^8 + 4548*a^3*b^9*c^3*n^7 + 159667200
*a^3*b^9*c^3 - 239500800*a^6*b^6*c^2*d + 159667200*a^9*b^3*c*d^2 - 39916800
*a^12*d^3 + (b^12*d^3*n^11 + 66*b^12*d^3*n^10 + 1925*b^12*d^3*n^9 + 32670*b
^12*d^3*n^8 + 357423*b^12*d^3*n^7 + 2637558*b^12*d^3*n^6 + 13339535*b^12*d^
3*n^5 + 45995730*b^12*d^3*n^4 + 105258076*b^12*d^3*n^3 + 150917976*b^12*d^3
*n^2 + 120543840*b^12*d^3*n + 39916800*b^12*d^3)*x^12 + (a*b^11*d^3*n^11 +
55*a*b^11*d^3*n^10 + 1320*a*b^11*d^3*n^9 + 18150*a*b^11*d^3*n^8 + 157773*a*
b^11*d^3*n^7 + 902055*a*b^11*d^3*n^6 + 3416930*a*b^11*d^3*n^5 + 8409500*a*b
^11*d^3*n^4 + 12753576*a*b^11*d^3*n^3 + 10628640*a*b^11*d^3*n^2 + 3628800*a
*b^11*d^3*n)*x^11 - 11*(a^2*b^10*d^3*n^10 + 45*a^2*b^10*d^3*n^9 + 870*a^2*b
^10*d^3*n^8 + 9450*a^2*b^10*d^3*n^7 + 63273*a^2*b^10*d^3*n^6 + 269325*a^2*b
^10*d^3*n^5 + 723680*a^2*b^10*d^3*n^4 + 1172700*a^2*b^10*d^3*n^3 + 1026576*
a^2*b^10*d^3*n^2 + 362880*a^2*b^10*d^3*n)*x^10 + (3*b^12*c*d^2*n^11 + 207*b
^12*c*d^2*n^10 + 159667200*b^12*c*d^2 + 2*(3144*b^12*c*d^2 + 55*a^3*b^9*d^3
)*n^9 + 18*(6151*b^12*c*d^2 + 220*a^3*b^9*d^3)*n^8 + 3*(417309*b^12*c*d^2 +
 20020*a^3*b^9*d^3)*n^7 + 567*(16777*b^12*c*d^2 + 880*a^3*b^9*d^3)*n^6 + 6*
(8226277*b^12*c*d^2 + 411565*a^3*b^9*d^3)*n^5 + 36*(4833097*b^12*c*d^2 + 20
5590*a^3*b^9*d^3)*n^4 + 40*(10142427*b^12*c*d^2 + 324841*a^3*b^9*d^3)*n^3 +
 288*(2051288*b^12*c*d^2 + 41855*a^3*b^9*d^3)*n^2 + 5760*(82941*b^12*c*d^2
+ 770*a^3*b^9*d^3)*n)*x^9 + 3*(a*b^11*c*d^2*n^11 + 61*a*b^11*c*d^2*n^10 + 1
608*a*b^11*c*d^2*n^9 + 6*(4007*a*b^11*c*d^2 - 55*a^4*b^8*d^3)*n^8 + 21*(107
13*a*b^11*c*d^2 - 440*a^4*b^8*d^3)*n^7 + 21*(65289*a*b^11*c*d^2 - 5060*a^4*
```

b^8*d^3)*n^6 + 2*(2742001*a*b^11*c*d^2 - 323400*a^4*b^8*d^3)*n^5 + 2*(70625
74*a*b^11*c*d^2 - 1116885*a^4*b^8*d^3)*n^4 + 264*(84209*a*b^11*c*d^2 - 1641
5*a^4*b^8*d^3)*n^3 + 360*(52984*a*b^11*c*d^2 - 11979*a^4*b^8*d^3)*n^2 + 166
3200*(4*a*b^11*c*d^2 - a^4*b^8*d^3)*n)*x^8 - 24*(a^2*b^10*c*d^2*n^10 + 54*a
^2*b^10*c*d^2*n^9 + 1230*a^2*b^10*c*d^2*n^8 + 6*(2572*a^2*b^10*c*d^2 - 55*a
^5*b^7*d^3)*n^7 + 21*(5569*a^2*b^10*c*d^2 - 330*a^5*b^7*d^3)*n^6 + 42*(1315
3*a^2*b^10*c*d^2 - 1375*a^5*b^7*d^3)*n^5 + 10*(161702*a^2*b^10*c*d^2 - 2425
5*a^5*b^7*d^3)*n^4 + 24*(116917*a^2*b^10*c*d^2 - 22330*a^5*b^7*d^3)*n^3 + 3
60*(7192*a^2*b^10*c*d^2 - 1617*a^5*b^7*d^3)*n^2 + 237600*(4*a^2*b^10*c*d^2
- a^5*b^7*d^3)*n)*x^7 + 72*(1148*a^3*b^9*c^3 - 5*a^6*b^6*c^2*d)*n^6 + 3*(b^
12*c^2*d*n^11 + 72*b^12*c^2*d*n^10 + 79833600*b^12*c^2*d + (2285*b^12*c^2*d
 + 56*a^3*b^9*c*d^2)*n^9 + 12*(3505*b^12*c^2*d + 224*a^3*b^9*c*d^2)*n^8 + 3
*(165701*b^12*c^2*d + 17584*a^3*b^9*c*d^2)*n^7 + 48*(82167*b^12*c^2*d + 114
10*a^3*b^9*c*d^2 - 385*a^6*b^6*d^3)*n^6 + (21326135*b^12*c^2*d + 3263064*a^
3*b^9*c*d^2 - 277200*a^6*b^6*d^3)*n^5 + 12*(6509445*b^12*c^2*d + 946456*a^3
*b^9*c*d^2 - 130900*a^6*b^6*d^3)*n^4 + 4*(47131699*b^12*c^2*d + 5602072*a^3
*b^9*c*d^2 - 1039500*a^6*b^6*d^3)*n^3 + 96*(2946397*b^12*c^2*d + 236320*a^3
*b^9*c*d^2 - 52745*a^6*b^6*d^3)*n^2 + 2880*(81401*b^12*c^2*d + 3080*a^3*b^9
*c*d^2 - 770*a^6*b^6*d^3)*n)*x^6 + 6*(158683*a^3*b^9*c^3 - 3420*a^6*b^6*c^2
*d)*n^5 + 3*(a*b^11*c^2*d*n^11 + 67*a*b^11*c^2*d*n^10 + 1950*a*b^11*c^2*d*n
^9 + 6*(5385*a*b^11*c^2*d - 56*a^4*b^8*c*d^2)*n^8 + 3*(111851*a*b^11*c^2*d
- 4816*a^4*b^8*c*d^2)*n^7 + 3*(755417*a*b^11*c^2*d - 81424*a^4*b^8*c*d^2)*n
^6 + 560*(17848*a*b^11*c^2*d - 3687*a^4*b^8*c*d^2 + 198*a^7*b^5*d^3)*n^5 +
4*(7034735*a*b^11*c^2*d - 2313696*a^4*b^8*c*d^2 + 277200*a^7*b^5*d^3)*n^4 +
 96*(498251*a*b^11*c^2*d - 227822*a^4*b^8*c*d^2 + 40425*a^7*b^5*d^3)*n^3 +
576*(75857*a*b^11*c^2*d - 43568*a^4*b^8*c*d^2 + 9625*a^7*b^5*d^3)*n^2 + 266
1120*(6*a*b^11*c^2*d - 4*a^4*b^8*c*d^2 + a^7*b^5*d^3)*n)*x^5 + 72*(100058*a
^3*b^9*c^3 - 6725*a^6*b^6*c^2*d)*n^4 - 15*(a^2*b^10*c^2*d*n^10 + 63*a^2*b^1
0*c^2*d*n^9 + 1698*a^2*b^10*c^2*d*n^8 + 6*(4253*a^2*b^10*c^2*d - 56*a^5*b^7
*c*d^2)*n^7 + 3*(77827*a^2*b^10*c^2*d - 4368*a^5*b^7*c*d^2)*n^6 + 3*(444109
*a^2*b^10*c^2*d - 63952*a^5*b^7*c*d^2)*n^5 + 4*(1166393*a^2*b^10*c^2*d - 32
4324*a^5*b^7*c*d^2 + 27720*a^8*b^4*d^3)*n^4 + 12*(789721*a^2*b^10*c^2*d - 3
38800*a^5*b^7*c*d^2 + 55440*a^8*b^4*d^3)*n^3 + 144*(68927*a^2*b^10*c^2*d -
38948*a^5*b^7*c*d^2 + 8470*a^8*b^4*d^3)*n^2 + 665280*(6*a^2*b^10*c^2*d - 4*
a^5*b^7*c*d^2 + a^8*b^4*d^3)*n)*x^4 + 8*(4473299*a^3*b^9*c^3 - 756675*a^6*b
^6*c^2*d + 15120*a^9*b^3*c*d^2)*n^3 + (b^12*c^3*n^11 + 75*b^12*c^3*n^10 + 1
59667200*b^12*c^3 + 4*(623*b^12*c^3 + 15*a^3*b^9*c^2*d)*n^9 + 18*(2683*b^12
*c^3 + 200*a^3*b^9*c^2*d)*n^8 + 3*(201527*b^12*c^3 + 30360*a^3*b^9*c^2*d)*n
^7 + 9*(568099*b^12*c^3 + 139760*a^3*b^9*c^2*d - 2240*a^6*b^6*c*d^2)*n^6 +
2*(14825779*b^12*c^3 + 5117670*a^3*b^9*c^2*d - 362880*a^6*b^6*c*d^2)*n^5 +
12*(9759623*b^12*c^3 + 4102800*a^3*b^9*c^2*d - 777840*a^6*b^6*c*d^2)*n^4 +
8*(38232551*b^12*c^3 + 16529190*a^3*b^9*c^2*d - 6229440*a^6*b^6*c*d^2 + 831
600*a^9*b^3*d^3)*n^3 + 576*(861864*b^12*c^3 + 298435*a^3*b^9*c^2*d - 163940
*a^6*b^6*c*d^2 + 34650*a^9*b^3*d^3)*n^2 + 5760*(76781*b^12*c^3 + 13860*a^3*
b^9*c^2*d - 9240*a^6*b^6*c*d^2 + 2310*a^9*b^3*d^3)*n)*x^3 + 144*(780996*a^3

```
*b^9*c^3 - 293635*a^6*b^6*c^2*d + 27720*a^9*b^3*c*d^2)*n^2 + (a*b^11*c^3*n^
11 + 73*a*b^11*c^3*n^10 + 2346*a*b^11*c^3*n^9 + 6*(7267*a*b^11*c^3 - 30*a^4
*b^8*c^2*d)*n^8 + 3*(172459*a*b^11*c^3 - 3480*a^4*b^8*c^2*d)*n^7 + 3*(13593
79*a*b^11*c^3 - 84120*a^4*b^8*c^2*d)*n^6 + 4*(5373821*a*b^11*c^3 - 817200*a
^4*b^8*c^2*d + 15120*a^7*b^5*c*d^2)*n^5 + 4*(18531227*a*b^11*c^3 - 6042105*
a^4*b^8*c^2*d + 514080*a^7*b^5*c*d^2)*n^4 + 72*(2189036*a*b^11*c^3 - 138005
5*a^4*b^8*c^2*d + 331800*a^7*b^5*c*d^2)*n^3 + 1440*(125842*a*b^11*c^3 - 137
481*a^4*b^8*c^2*d + 70644*a^7*b^5*c*d^2 - 13860*a^10*b^2*d^3)*n^2 + 1995840
0*(4*a*b^11*c^3 - 6*a^4*b^8*c^2*d + 4*a^7*b^5*c*d^2 - a^10*b^2*d^3)*n)*x^2
+ 2880*(70402*a^3*b^9*c^3 - 54321*a^6*b^6*c^2*d + 15204*a^9*b^3*c*d^2)*n -
2*(a^2*b^10*c^3*n^10 + 72*a^2*b^10*c^3*n^9 + 2274*a^2*b^10*c^3*n^8 + 36*(11
48*a^2*b^10*c^3 - 5*a^5*b^7*c^2*d)*n^7 + 3*(158683*a^2*b^10*c^3 - 3420*a^5*
b^7*c^2*d)*n^6 + 36*(100058*a^2*b^10*c^3 - 6725*a^5*b^7*c^2*d)*n^5 + 4*(447
3299*a^2*b^10*c^3 - 756675*a^5*b^7*c^2*d + 15120*a^8*b^4*c*d^2)*n^4 + 72*(7
80996*a^2*b^10*c^3 - 293635*a^5*b^7*c^2*d + 27720*a^8*b^4*c*d^2)*n^3 + 1440
*(70402*a^2*b^10*c^3 - 54321*a^5*b^7*c^2*d + 15204*a^8*b^4*c*d^2)*n^2 + 199
58400*(4*a^2*b^10*c^3 - 6*a^5*b^7*c^2*d + 4*a^8*b^4*c*d^2 - a^11*b*d^3)*n)*
x)*(b*x + a)^n/(b^12*n^12 + 78*b^12*n^11 + 2717*b^12*n^10 + 55770*b^12*n^9
+ 749463*b^12*n^8 + 6926634*b^12*n^7 + 44990231*b^12*n^6 + 206070150*b^12*n
^5 + 657206836*b^12*n^4 + 1414014888*b^12*n^3 + 1931559552*b^12*n^2 + 14864
42880*b^12*n + 479001600*b^12)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75191 vs. $2(439) = 878$.

Time = 83.59 (sec) , antiderivative size = 75191, normalized size of antiderivative = 163.81

$$\int x^2(a+bx)^n \left(c+dx^3\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] Piecewise((a**n*(c**3*x**3/3 + c**2*d*x**6/2 + c*d**2*x**9/3 + d**3*x**12/1
2), Eq(b, 0)), (27720*a**11*d**3*log(a/b + x)/(27720*a**11*b**12 + 304920*a
**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*
a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 914
7600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 +
304920*a*b**22*x**10 + 27720*b**23*x**11) + 83711*a**11*d**3/(27720*a**11*b
**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*
x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b
**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**
2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) + 304920*a**10*b*d
**3*x*log(a/b + x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9
*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*
a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573
```

800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 2772
0*b**23*x**11) + 893101*a**10*b*d**3*x/(27720*a**11*b**12 + 304920*a**10*b*
*13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b*
*16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a*
*4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*
a*b**22*x**10 + 27720*b**23*x**11) + 1524600*a**9*b**2*d**3*x**2*log(a/b +
x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 45
73800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5
+ 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x
8 + 1524600*a2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) +
4313045*a**9*b**2*d**3*x**2/(27720*a**11*b**12 + 304920*a**10*b**13*x + 15
24600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 +
12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x
7 + 4573800*a3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x*
*10 + 27720*b**23*x**11) - 168*a**8*b**3*c*d**2/(27720*a**11*b**12 + 304920
*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 914760
0*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9
147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9
+ 304920*a*b**22*x**10 + 27720*b**23*x**11) + 4573800*a**8*b**3*d**3*x**3*l
og(a/b + x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*
x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b*
*17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**
3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23
*x**11) + 12430935*a**8*b**3*d**3*x**3/(27720*a**11*b**12 + 304920*a**10*b*
*13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b*
*16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a*
*4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*
a*b**22*x**10 + 27720*b**23*x**11) - 1848*a**7*b**4*c*d**2*x/(27720*a**11*b
12 + 304920*a10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*
x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b
18*x6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**
2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) + 9147600*a**7*b**
4*d**3*x**4*log(a/b + x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 152460
0*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 128
06640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7
+ 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10
+ 27720*b**23*x**11) + 23718420*a**7*b**4*d**3*x**4/(27720*a**11*b**12 + 30
4920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 91
47600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6
+ 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x
9 + 304920*a*b22*x**10 + 27720*b**23*x**11) - 9240*a**6*b**5*c*d**2*x**
2/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 457
3800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 +
12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x*
*8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) +

12806640*a**6*b**5*d**3*x**5*log(a/b + x)/(27720*a**11*b**12 + 304920*a**10
*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7
*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600
*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 3049
20*a*b**22*x**10 + 27720*b**23*x**11) + 31376268*a**6*b**5*d**3*x**5/(27720
*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**
8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 1280664
0*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 152
4600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) - 30*a**5*
b**6*c**2*d/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*
x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b*
*17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**
3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23
*x**11) - 27720*a**5*b**6*c*d**2*x**3/(27720*a**11*b**12 + 304920*a**10*b**
13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**
16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**
4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a
*b**22*x**10 + 27720*b**23*x**11) + 12806640*a**5*b**6*d**3*x**6*log(a/b +
x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 45
73800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5
+ 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x
8 + 1524600*a2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) +
 29241828*a**5*b**6*d**3*x**6/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1
524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4
+ 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*
x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x
10 + 27720*b23*x**11) - 330*a**4*b**7*c**2*d*x/(27720*a**11*b**12 + 304
920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 914
7600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6
+ 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x*
*9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) - 55440*a**4*b**7*c*d**2*x**
4/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 457
3800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 +
 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x*
*8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) +
9147600*a**4*b**7*d**3*x**7*log(a/b + x)/(27720*a**11*b**12 + 304920*a**10*
b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*
b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*
a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 30492
0*a*b**22*x**10 + 27720*b**23*x**11) + 19057500*a**4*b**7*d**3*x**7/(27720*
a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8
*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640
*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524
600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) - 1650*a**3
*b**8*c**2*d*x**2/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*

b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) - 77616*a**3*b**8*c*d**2*x**5/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) + 4573800*a**3*b**8*d**3*x**8*log(a/b + x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) + 8385300*a**3*b**8*d**3*x**8/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) - 56*a**2*b**9*c**3/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) - 4950*a**2*b**9*c**2*d*x**3/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) - 77616*a**2*b**9*c*d**2*x**6/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) + 1524600*a**2*b**9*d**3*x**9*log(a/b + x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) + 2286900*a**2*b**9*d**3*x**9/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) - 616*a*b**10*c**3*x/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) - 9900*a*b**10*c**2*d*x**4/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*

x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**
21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) - 55440*a*b**10*c*d**2*
x**7/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 +
4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**
5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20
*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11)
 + 304920*a*b**10*d**3*x**10*log(a/b + x)/(27720*a**11*b**12 + 304920*a**10
*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7
*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600
*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 3049
20*a*b**22*x**10 + 27720*b**23*x**11) + 304920*a*b**10*d**3*x**10/(27720*a*
*11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b
15*x3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a
5*b18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 152460
0*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) - 3080*b**11*
c**3*x**2/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x*
*2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**1
7*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*
b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x
11) - 13860*b11*c**2*d*x**5/(27720*a**11*b**12 + 304920*a**10*b**13*x +
 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**
4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**1
9*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22
*x**10 + 27720*b**23*x**11) - 27720*b**11*c*d**2*x**8/(27720*a**11*b**12 +
304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 +
9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x*
*6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21
*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) + 27720*b**11*d**3*x**11*
log(a/b + x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14
*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b
17*x5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a*
*3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**2
3*x**11), Eq(n, -12)), (-27720*a**11*d**3*log(a/b + x)/(2520*a**10*b**12 +
25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 5292
00*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 3024
00*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**
22*x**10) - 81191*a**11*d**3/(2520*a**10*b**12 + 25200*a**9*b**13*x + 11340
0*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 63504
0*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 11340
0*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 277200*a**10*b
*d**3*x*log(a/b + x)/(2520*a**10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b
14*x2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b
17*x5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b
20*x8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 784190*a**10*b*d**3*x/
(2520*a**10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a*

```
*7*b**15*x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a*
*4*b**18*x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b
**21*x**9 + 2520*b**22*x**10) - 1247400*a**9*b**2*d**3*x**2*log(a/b + x)/(2
520*a**10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a**7
*b**15*x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4
*b**18*x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**
21*x**9 + 2520*b**22*x**10) - 3390255*a**9*b**2*d**3*x**2/(2520*a**10*b**12
 + 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 5
29200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 3
02400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*
b**22*x**10) - 84*a**8*b**3*c*d**2/(2520*a**10*b**12 + 25200*a**9*b**13*x +
 113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 +
 635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 +
 113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 3326400*
a**8*b**3*d**3*x**3*log(a/b + x)/(2520*a**10*b**12 + 25200*a**9*b**13*x + 1
13400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 6
35040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 1
13400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 8624880*a*
*8*b**3*d**3*x**3/(2520*a**10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b**1
4*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b**1
7*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b**2
0*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 840*a**7*b**4*c*d**2*x/(2
520*a**10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a**7
*b**15*x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4
*b**18*x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**
21*x**9 + 2520*b**22*x**10) - 5821200*a**7*b**4*d**3*x**4*log(a/b + x)/(252
0*a**10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a**7*b
**15*x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4*b
**18*x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**21
*x**9 + 2520*b**22*x**10) - 14261940*a**7*b**4*d**3*x**4/(2520*a**10*b**12
+ 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 52
9200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 30
2400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b
**22*x**10) - 3780*a**6*b**5*c*d**2*x**2/(2520*a**10*b**12 + 25200*a**9*b**
13*x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*
x**4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*
x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 69
85440*a**6*b**5*d**3*x**5*log(a/b + x)/(2520*a**10*b**12 + 25200*a**9*b**13
*x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x*
*4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x*
*7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 1595
0088*a**6*b**5*d**3*x**5/(2520*a**10*b**12 + 25200*a**9*b**13*x + 113400*a*
*8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 635040*a*
*5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 113400*a*
*2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 6*a**5*b**6*c**2*d
```

/(2520*a**10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a
7*b15*x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a
4*b18*x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*
b**21*x**9 + 2520*b**22*x**10) - 10080*a**5*b**6*c*d**2*x**3/(2520*a**10*b*
*12 + 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3
+ 529200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6
+ 302400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 25
20*b**22*x**10) - 5821200*a**5*b**6*d**3*x**6*log(a/b + x)/(2520*a**10*b**1
2 + 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 +
529200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 +
302400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520
*b**22*x**10) - 12127500*a**5*b**6*d**3*x**6/(2520*a**10*b**12 + 25200*a**9
*b**13*x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b*
*16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b*
*19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10)
- 60*a**4*b**7*c**2*d*x/(2520*a**10*b**12 + 25200*a**9*b**13*x + 113400*a**
8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 635040*a**
5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 113400*a**
2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 17640*a**4*b**7*c*d
2*x4/(2520*a**10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 +
302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 +
529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 +
25200*a*b**21*x**9 + 2520*b**22*x**10) - 3326400*a**4*b**7*d**3*x**7*log(a/
b + x)/(2520*a**10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 30
2400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 52
9200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25
200*a*b**21*x**9 + 2520*b**22*x**10) - 6098400*a**4*b**7*d**3*x**7/(2520*a*
*10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15
*x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18
*x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**
9 + 2520*b**22*x**10) - 270*a**3*b**8*c**2*d*x**2/(2520*a**10*b**12 + 25200
*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a*
*6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a*
*3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x*
*10) - 21168*a**3*b**8*c*d**2*x**5/(2520*a**10*b**12 + 25200*a**9*b**13*x +
113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 +
635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 +
113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 1247400*
a**3*b**8*d**3*x**8*log(a/b + x)/(2520*a**10*b**12 + 25200*a**9*b**13*x + 1
13400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 6
35040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 1
13400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 1871100*a*
*3*b**8*d**3*x**8/(2520*a**10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b**1
4*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b**1
7*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b**2

0*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 7*a**2*b**9*c**3/(2520*a*
*10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15
*x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18
*x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**
9 + 2520*b**22*x**10) - 720*a**2*b**9*c**2*d*x**3/(2520*a**10*b**12 + 25200
*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a*
*6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a*
*3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x*
*10) - 17640*a**2*b**9*c*d**2*x**6/(2520*a**10*b**12 + 25200*a**9*b**13*x +
113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 +
635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 +
113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 277200*a
2*b9*d**3*x**9*log(a/b + x)/(2520*a**10*b**12 + 25200*a**9*b**13*x + 11
3400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 63
5040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 11
3400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 277200*a**2
*b**9*d**3*x**9/(2520*a**10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b**14*
x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b**17*
x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b**20*
x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 70*a*b**10*c**3*x/(2520*a**
10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15*
x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18*
x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**9
+ 2520*b**22*x**10) - 1260*a*b**10*c**2*d*x**4/(2520*a**10*b**12 + 25200*a
9*b13*x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6
*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3
*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**1
0) - 10080*a*b**10*c*d**2*x**7/(2520*a**10*b**12 + 25200*a**9*b**13*x + 113
400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 635
040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 113
400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 27720*a*b**1
0*d**3*x**10*log(a/b + x)/(2520*a**10*b**12 + 25200*a**9*b**13*x + 113400*a
8*b14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 635040*a
5*b17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 113400*a
2*b20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) - 315*b**11*c**3*x*
*2/(2520*a**10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400
*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200
*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*
a*b**21*x**9 + 2520*b**22*x**10) - 1512*b**11*c**2*d*x**5/(2520*a**10*b**12
+ 25200*a**9*b**13*x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 5
29200*a**6*b**16*x**4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 3
02400*a**3*b**19*x**7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*
b**22*x**10) - 3780*b**11*c*d**2*x**8/(2520*a**10*b**12 + 25200*a**9*b**13*
x + 113400*a**8*b**14*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**
4 + 635040*a**5*b**17*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**

7 + 113400*a**2*b**20*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10) + 2520*
b**11*d**3*x**11/(2520*a**10*b**12 + 25200*a**9*b**13*x + 113400*a**8*b**14
*x**2 + 302400*a**7*b**15*x**3 + 529200*a**6*b**16*x**4 + 635040*a**5*b**17
*x**5 + 529200*a**4*b**18*x**6 + 302400*a**3*b**19*x**7 + 113400*a**2*b**20
*x**8 + 25200*a*b**21*x**9 + 2520*b**22*x**10), Eq(n, -11)), (27720*a**11*d
3*log(a/b + x)/(504*a9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**
2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 +
 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b*
*21*x**9) + 78419*a**11*d**3/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a*
*7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*
b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x
8 + 504*b21*x**9) + 249480*a**10*b*d**3*x*log(a/b + x)/(504*a**9*b**12
+ 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504
*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a*
*2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) + 678051*a**10*b*d**3*x
/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b
15*x3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**1
8*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) + 9979
20*a**9*b**2*d**3*x**2*log(a/b + x)/(504*a**9*b**12 + 4536*a**8*b**13*x + 1
8144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 6350
4*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*
b**20*x**8 + 504*b**21*x**9) + 2587464*a**9*b**2*d**3*x**2/(504*a**9*b**12
+ 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504
*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a*
*2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) - 168*a**8*b**3*c*d**2/
(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b*
*15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18
*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) + 23284
80*a**8*b**3*d**3*x**3*log(a/b + x)/(504*a**9*b**12 + 4536*a**8*b**13*x + 1
8144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 6350
4*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*
b**20*x**8 + 504*b**21*x**9) + 5704776*a**8*b**3*d**3*x**3/(504*a**9*b**12
+ 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504
*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a*
*2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) - 1512*a**7*b**4*c*d**2
*x/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6
*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b*
*18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) + 34
92720*a**7*b**4*d**3*x**4*log(a/b + x)/(504*a**9*b**12 + 4536*a**8*b**13*x
+ 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 6
3504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536
*a*b**20*x**8 + 504*b**21*x**9) + 7975044*a**7*b**4*d**3*x**4/(504*a**9*b**
12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63
504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144
*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) - 6048*a**6*b**5*c*d

2*x2/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 4233
6*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a
3*b18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9
) + 3492720*a**6*b**5*d**3*x**5*log(a/b + x)/(504*a**9*b**12 + 4536*a**8*b*
*13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x*
*4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7
+ 4536*a*b**20*x**8 + 504*b**21*x**9) + 7276500*a**6*b**5*d**3*x**5/(504*a*
*9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**
3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 +
18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) - 3*a**5*b**6*
c**2*d/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*
a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**
3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9)
- 14112*a**5*b**6*c*d**2*x**3/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a
7*b14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4
*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*
x**8 + 504*b**21*x**9) + 2328480*a**5*b**6*d**3*x**6*log(a/b + x)/(504*a**9
*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3
+ 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 1
8144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) + 4268880*a**5*b
6*d3*x**6/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 +
42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42
336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21
*x**9) - 27*a**4*b**7*c**2*d*x/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*
a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**
4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20
*x**8 + 504*b**21*x**9) - 21168*a**4*b**7*c*d**2*x**4/(504*a**9*b**12 + 453
6*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5
*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b*
*19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) + 997920*a**4*b**7*d**3*x**7
*log(a/b + x)/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 +
42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42
336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21
*x**9) + 1496880*a**4*b**7*d**3*x**7/(504*a**9*b**12 + 4536*a**8*b**13*x +
18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 635
04*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a
*b**20*x**8 + 504*b**21*x**9) - 108*a**3*b**8*c**2*d*x**2/(504*a**9*b**12 +
4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504*
a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**
2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) - 21168*a**3*b**8*c*d**2
*x**5/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a
6*b15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3
*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) +
249480*a**3*b**8*d**3*x**8*log(a/b + x)/(504*a**9*b**12 + 4536*a**8*b**13*
x + 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 +

63504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 45
36*a*b**20*x**8 + 504*b**21*x**9) + 249480*a**3*b**8*d**3*x**8/(504*a**9*b*
*12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 6
3504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 1814
4*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) - 2*a**2*b**9*c**3/
(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b*
*15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18
*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) - 252*a
2*b9*c**2*d*x**3/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14
*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x*
*5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 50
4*b**21*x**9) - 14112*a**2*b**9*c*d**2*x**6/(504*a**9*b**12 + 4536*a**8*b**
13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**
4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 +
4536*a*b**20*x**8 + 504*b**21*x**9) + 27720*a**2*b**9*d**3*x**9*log(a/b +
x)/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6
*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b*
*18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) - 18
*a*b**10*c**3*x/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2
+ 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 +
42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**
21*x**9) - 378*a*b**10*c**2*d*x**4/(504*a**9*b**12 + 4536*a**8*b**13*x + 18
144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504
*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b
20*x8 + 504*b**21*x**9) - 6048*a*b**10*c*d**2*x**7/(504*a**9*b**12 + 45
36*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**
5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b
19*x7 + 4536*a*b**20*x**8 + 504*b**21*x**9) - 2772*a*b**10*d**3*x**10/(
504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b**
15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18*
x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9) - 72*b**
11*c**3*x**2/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b**14*x**2 +
42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 423
36*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*
x**9) - 378*b**11*c**2*d*x**5/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a
7*b14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4
*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*
x**8 + 504*b**21*x**9) - 1512*b**11*c*d**2*x**8/(504*a**9*b**12 + 4536*a**8
*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16
*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x*
*7 + 4536*a*b**20*x**8 + 504*b**21*x**9) + 252*b**11*d**3*x**11/(504*a**9*b
12 + 4536*a8*b**13*x + 18144*a**7*b**14*x**2 + 42336*a**6*b**15*x**3 +
63504*a**5*b**16*x**4 + 63504*a**4*b**17*x**5 + 42336*a**3*b**18*x**6 + 181
44*a**2*b**19*x**7 + 4536*a*b**20*x**8 + 504*b**21*x**9), Eq(n, -10)), (-13
8600*a**11*d**3*log(a/b + x)/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a*

*6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*
b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) -
376695*a**11*d**3/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a**6*b**14*x*
*2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*b**17*x**5
+ 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) - 1108800*a**
10*b*d**3*x*log(a/b + x)/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a**6*b
14*x2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*b**1
7*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) - 2874
960*a**10*b*d**3*x/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a**6*b**14*x
2 + 47040*a5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*b**17*x**5
 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) - 3880800*a*
*9*b**2*d**3*x**2*log(a/b + x)/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*
a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**
3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8)
- 9507960*a**9*b**2*d**3*x**2/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a
6*b14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3
*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) +
 2520*a**8*b**3*c*d**2*log(a/b + x)/(840*a**8*b**12 + 6720*a**7*b**13*x + 2
3520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 4704
0*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x
8) + 6849*a8*b**3*c*d**2/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a*
*6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*
b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) -
7761600*a**8*b**3*d**3*x**3*log(a/b + x)/(840*a**8*b**12 + 6720*a**7*b**13*
x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 +
 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b*
*20*x**8) - 17722320*a**8*b**3*d**3*x**3/(840*a**8*b**12 + 6720*a**7*b**13*
x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 +
 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b*
*20*x**8) + 20160*a**7*b**4*c*d**2*x*log(a/b + x)/(840*a**8*b**12 + 6720*a*
*7*b**13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**
16*x**4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7
 + 840*b**20*x**8) + 52272*a**7*b**4*c*d**2*x/(840*a**8*b**12 + 6720*a**7*b
13*x + 23520*a6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x
4 + 47040*a3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 8
40*b**20*x**8) - 9702000*a**7*b**4*d**3*x**4*log(a/b + x)/(840*a**8*b**12 +
 6720*a**7*b**13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*
a**4*b**16*x**4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b*
*19*x**7 + 840*b**20*x**8) - 20212500*a**7*b**4*d**3*x**4/(840*a**8*b**12 +
 6720*a**7*b**13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*
a**4*b**16*x**4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b*
*19*x**7 + 840*b**20*x**8) + 70560*a**6*b**5*c*d**2*x**2*log(a/b + x)/(840*
a**8*b**12 + 6720*a**7*b**13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x
3 + 58800*a4*b**16*x**4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6
 + 6720*a*b**19*x**7 + 840*b**20*x**8) + 172872*a**6*b**5*c*d**2*x**2/(840*

a**8*b**12 + 6720*a**7*b**13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x
3 + 58800*a4*b**16*x**4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6
 + 6720*a*b**19*x**7 + 840*b**20*x**8) - 7761600*a**6*b**5*d**3*x**5*log(a/
b + x)/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a**6*b**14*x**2 + 47040*
a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*b**17*x**5 + 23520*a**
2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) - 14229600*a**6*b**5*d**
3*x**5/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a**6*b**14*x**2 + 47040*
a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*b**17*x**5 + 23520*a**
2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) - 15*a**5*b**6*c**2*d/(8
40*a**8*b**12 + 6720*a**7*b**13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**1
5*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x
6 + 6720*a*b19*x**7 + 840*b**20*x**8) + 141120*a**5*b**6*c*d**2*x**3*lo
g(a/b + x)/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a**6*b**14*x**2 + 47
040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*b**17*x**5 + 23520
*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) + 322224*a**5*b**6*c
*d**2*x**3/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a**6*b**14*x**2 + 47
040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*b**17*x**5 + 23520
*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) - 3880800*a**5*b**6*
d**3*x**6*log(a/b + x)/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a**6*b**
14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*b**17*
x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) - 582120
0*a**5*b**6*d**3*x**6/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a**6*b**1
4*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*b**17*x
5 + 23520*a2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) - 120*a**
4*b**7*c**2*d*x/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a**6*b**14*x**2
 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*b**17*x**5 +
23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) + 176400*a**4*b
7*c*d2*x**4*log(a/b + x)/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a*
*6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*
b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) +
367500*a**4*b**7*c*d**2*x**4/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a*
*6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*
b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) -
1108800*a**4*b**7*d**3*x**7*log(a/b + x)/(840*a**8*b**12 + 6720*a**7*b**13*
x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 +
 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b*
*20*x**8) - 1108800*a**4*b**7*d**3*x**7/(840*a**8*b**12 + 6720*a**7*b**13*x
 + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 +
47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**
20*x**8) - 420*a**3*b**8*c**2*d*x**2/(840*a**8*b**12 + 6720*a**7*b**13*x +
23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 470
40*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*
x**8) + 141120*a**3*b**8*c*d**2*x**5*log(a/b + x)/(840*a**8*b**12 + 6720*a*
*7*b**13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**
16*x**4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7

+ 840*b**20*x**8) + 258720*a**3*b**8*c*d**2*x**5/(840*a**8*b**12 + 6720*a*
*7*b**13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**
16*x**4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7
+ 840*b**20*x**8) - 138600*a**3*b**8*d**3*x**8*log(a/b + x)/(840*a**8*b**1
2 + 6720*a**7*b**13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 588
00*a**4*b**16*x**4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a
*b**19*x**7 + 840*b**20*x**8) - 5*a**2*b**9*c**3/(840*a**8*b**12 + 6720*a**
7*b**13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**1
6*x**4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7
+ 840*b**20*x**8) - 840*a**2*b**9*c**2*d*x**3/(840*a**8*b**12 + 6720*a**7*b
13*x + 23520*a6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x
4 + 47040*a3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 8
40*b**20*x**8) + 70560*a**2*b**9*c*d**2*x**6*log(a/b + x)/(840*a**8*b**12 +
6720*a**7*b**13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*
a**4*b**16*x**4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b*
*19*x**7 + 840*b**20*x**8) + 105840*a**2*b**9*c*d**2*x**6/(840*a**8*b**12 +
6720*a**7*b**13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*
a**4*b**16*x**4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b*
*19*x**7 + 840*b**20*x**8) + 15400*a**2*b**9*d**3*x**9/(840*a**8*b**12 + 67
20*a**7*b**13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**
4*b**16*x**4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19
*x**7 + 840*b**20*x**8) - 40*a*b**10*c**3*x/(840*a**8*b**12 + 6720*a**7*b**
13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**
4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840
*b**20*x**8) - 1050*a*b**10*c**2*d*x**4/(840*a**8*b**12 + 6720*a**7*b**13*x
+ 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 +
47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**
20*x**8) + 20160*a*b**10*c*d**2*x**7*log(a/b + x)/(840*a**8*b**12 + 6720*a*
*7*b**13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**
16*x**4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7
+ 840*b**20*x**8) + 20160*a*b**10*c*d**2*x**7/(840*a**8*b**12 + 6720*a**7*
b**13*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*
x**4 + 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 +
840*b**20*x**8) - 1540*a*b**10*d**3*x**10/(840*a**8*b**12 + 6720*a**7*b**13
*x + 23520*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4
+ 47040*a**3*b**17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b
20*x8) - 140*b**11*c**3*x**2/(840*a**8*b**12 + 6720*a**7*b**13*x + 2352
0*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a
3*b17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8
) - 840*b**11*c**2*d*x**5/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a**6*
b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*b**
17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8) + 252
0*b**11*c*d**2*x**8*log(a/b + x)/(840*a**8*b**12 + 6720*a**7*b**13*x + 2352
0*a**6*b**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a
3*b17*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8

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) + 280*b**11*d**3*x**11/(840*a**8*b**12 + 6720*a**7*b**13*x + 23520*a**6*b
**14*x**2 + 47040*a**5*b**15*x**3 + 58800*a**4*b**16*x**4 + 47040*a**3*b**1
7*x**5 + 23520*a**2*b**18*x**6 + 6720*a*b**19*x**7 + 840*b**20*x**8), Eq(n,
 -9)), (138600*a**11*d**3*log(a/b + x)/(420*a**7*b**12 + 2940*a**6*b**13*x
+ 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 88
20*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) + 359370*a**11*d**
3/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b
**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**
6 + 420*b**19*x**7) + 970200*a**10*b*d**3*x*log(a/b + x)/(420*a**7*b**12 +
2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a*
*3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7)
+ 2376990*a**10*b*d**3*x/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b*
*14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*
x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) + 2910600*a**9*b**2*d**3*x**2*lo
g(a/b + x)/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 147
00*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*
b**18*x**6 + 420*b**19*x**7) + 6645870*a**9*b**2*d**3*x**2/(420*a**7*b**12
+ 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*
a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7
) - 10080*a**8*b**3*c*d**2*log(a/b + x)/(420*a**7*b**12 + 2940*a**6*b**13*x
 + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8
820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 26136*a**8*b**3
*c*d**2/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*
a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**
18*x**6 + 420*b**19*x**7) + 4851000*a**8*b**3*d**3*x**3*log(a/b + x)/(420*a
**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**
3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*
b**19*x**7) + 10106250*a**8*b**3*d**3*x**3/(420*a**7*b**12 + 2940*a**6*b**1
3*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4
+ 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 70560*a**7*b
**4*c*d**2*x*log(a/b + x)/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b
**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17
*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 172872*a**7*b**4*c*d**2*x/(42
0*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*
x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 4
20*b**19*x**7) + 4851000*a**7*b**4*d**3*x**4*log(a/b + x)/(420*a**7*b**12 +
 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a
**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7)
 + 8893500*a**7*b**4*d**3*x**4/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a
**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*
b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 211680*a**6*b**5*c*d**2*
x**2*log(a/b + x)/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**
2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 +
2940*a*b**18*x**6 + 420*b**19*x**7) - 483336*a**6*b**5*c*d**2*x**2/(420*a**
7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3
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+ 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b*
*19*x**7) + 2910600*a**6*b**5*d**3*x**5*log(a/b + x)/(420*a**7*b**12 + 2940
*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b
16*x4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) + 43
65900*a**6*b**5*d**3*x**5/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b
14*x2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17
*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 30*a**5*b**6*c**2*d/(420*a**7
*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 +
14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**
19*x**7) - 352800*a**5*b**6*c*d**2*x**3*log(a/b + x)/(420*a**7*b**12 + 2940
*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b
16*x4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 73
5000*a**5*b**6*c*d**2*x**3/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*
b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**1
7*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) + 970200*a**5*b**6*d**3*x**6*l
og(a/b + x)/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14
700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a
*b**18*x**6 + 420*b**19*x**7) + 970200*a**5*b**6*d**3*x**6/(420*a**7*b**12
+ 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*
a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7
) - 210*a**4*b**7*c**2*d*x/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*
b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**1
7*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 352800*a**4*b**7*c*d**2*x**4
*log(a/b + x)/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 +
14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940
*a*b**18*x**6 + 420*b**19*x**7) - 646800*a**4*b**7*c*d**2*x**4/(420*a**7*b*
*12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14
700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*
x**7) + 138600*a**4*b**7*d**3*x**7*log(a/b + x)/(420*a**7*b**12 + 2940*a**6
*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*
x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 630*a**
3*b**8*c**2*d*x**2/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x*
*2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 +
2940*a*b**18*x**6 + 420*b**19*x**7) - 211680*a**3*b**8*c*d**2*x**5*log(a/b
+ x)/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a*
*4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18
*x**6 + 420*b**19*x**7) - 317520*a**3*b**8*c*d**2*x**5/(420*a**7*b**12 + 29
40*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3
*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) -
17325*a**3*b**8*d**3*x**8/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b
14*x2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17
*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 4*a**2*b**9*c**3/(420*a**7*b*
*12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14
700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*
x**7) - 1050*a**2*b**9*c**2*d*x**3/(420*a**7*b**12 + 2940*a**6*b**13*x + 88

20*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a **2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 70560*a**2*b**9*c*d* *2*x**6*log(a/b + x)/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14* x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 70560*a**2*b**9*c*d**2*x**6/(420*a **7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x** 3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420* b**19*x**7) + 1925*a**2*b**9*d**3*x**9/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 88 20*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 28*a*b**10*c**3* x/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b **15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x** 6 + 420*b**19*x**7) - 1050*a*b**10*c**2*d*x**4/(420*a**7*b**12 + 2940*a**6* b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x **4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 10080*a* b**10*c*d**2*x**7*log(a/b + x)/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a **5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2* b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 385*a*b**10*d**3*x**10/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**1 5*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 84*b**11*c**3*x**2/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 882 0*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) - 630*b**11*c**2*d* x**5/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a** 4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18* x**6 + 420*b**19*x**7) + 1260*b**11*c*d**2*x**8/(420*a**7*b**12 + 2940*a**6 *b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16* x**4 + 8820*a**2*b**17*x**5 + 2940*a*b**18*x**6 + 420*b**19*x**7) + 105*b** 11*d**3*x**11/(420*a**7*b**12 + 2940*a**6*b**13*x + 8820*a**5*b**14*x**2 + 14700*a**4*b**15*x**3 + 14700*a**3*b**16*x**4 + 8820*a**2*b**17*x**5 + 2940 *a*b**18*x**6 + 420*b**19*x**7), Eq(n, -8)), (-27720*a**11*d**3*log(a/b + x)/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15 *x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 67914*a** 11*d**3/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3 *b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 166 320*a**10*b*d**3*x*log(a/b + x)/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a** 4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x** 5 + 60*b**18*x**6) - 379764*a**10*b*d**3*x/(60*a**6*b**12 + 360*a**5*b**13* x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360* a*b**17*x**5 + 60*b**18*x**6) - 415800*a**9*b**2*d**3*x**2*log(a/b + x)/(60 *a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 866250*a**9*b* *2*d**3*x**2/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200 *a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) + 5040*a**8*b**3*c*d**2*log(a/b + x)/(60*a**6*b**12 + 360*a**5*b**13*x + 90

0*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**1
7*x**5 + 60*b**18*x**6) + 12348*a**8*b**3*c*d**2/(60*a**6*b**12 + 360*a**5*
b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4
+ 360*a*b**17*x**5 + 60*b**18*x**6) - 554400*a**8*b**3*d**3*x**3*log(a/b +
x)/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**1
5*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 1016400*
a**8*b**3*d**3*x**3/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2
 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18
*x**6) + 30240*a**7*b**4*c*d**2*x*log(a/b + x)/(60*a**6*b**12 + 360*a**5*b*
*13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 +
360*a*b**17*x**5 + 60*b**18*x**6) + 69048*a**7*b**4*c*d**2*x/(60*a**6*b**12
 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2
*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 415800*a**7*b**4*d**3*x**
4*log(a/b + x)/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 12
00*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6
) - 623700*a**7*b**4*d**3*x**4/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4
*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5
 + 60*b**18*x**6) + 75600*a**6*b**5*c*d**2*x**2*log(a/b + x)/(60*a**6*b**12
 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2
*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) + 157500*a**6*b**5*c*d**2*x
2/(60*a6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**
15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 166320*
a**6*b**5*d**3*x**5*log(a/b + x)/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a*
*4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x*
*5 + 60*b**18*x**6) - 166320*a**6*b**5*d**3*x**5/(60*a**6*b**12 + 360*a**5*
b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4
+ 360*a*b**17*x**5 + 60*b**18*x**6) - 30*a**5*b**6*c**2*d/(60*a**6*b**12 +
360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b*
*16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) + 100800*a**5*b**6*c*d**2*x**3
*log(a/b + x)/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 120
0*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6)
 + 184800*a**5*b**6*c*d**2*x**3/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**
4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**
5 + 60*b**18*x**6) - 27720*a**5*b**6*d**3*x**6*log(a/b + x)/(60*a**6*b**12
+ 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*
b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 180*a**4*b**7*c**2*d*x/(60
*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3
 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) + 75600*a**4*b**
7*c*d**2*x**4*log(a/b + x)/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**
14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 6
0*b**18*x**6) + 113400*a**4*b**7*c*d**2*x**4/(60*a**6*b**12 + 360*a**5*b**1
3*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 36
0*a*b**17*x**5 + 60*b**18*x**6) + 3960*a**4*b**7*d**3*x**7/(60*a**6*b**12 +
 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b
16*x4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 450*a**3*b**8*c**2*d*x**2/(

60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) + 30240*a**3*b**8*c*d**2*x**5*log(a/b + x)/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) + 30240*a**3*b**8*c*d**2*x**5/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 495*a**3*b**8*d**3*x**8/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - a**2*b**9*c**3/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 600*a**2*b**9*c**2*d*x**3/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) + 5040*a**2*b**9*c*d**2*x**6*log(a/b + x)/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) + 110*a**2*b**9*d**3*x**9/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 6*a*b**10*c**3*x/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 450*a*b**10*c**2*d*x**4/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 720*a*b**10*c*d**2*x**7/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 33*a*b**10*d**3*x**10/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 15*b**11*c**3*x**2/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) - 180*b**11*c**2*d*x**5/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) + 90*b**11*c*d**2*x**8/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6) + 12*b**11*d**3*x**11/(60*a**6*b**12 + 360*a**5*b**13*x + 900*a**4*b**14*x**2 + 1200*a**3*b**15*x**3 + 900*a**2*b**16*x**4 + 360*a*b**17*x**5 + 60*b**18*x**6), Eq(n, -7)), (27720*a**11*d**3*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 63294*a**11*d**3/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 138600*a**10*b*d**3*x*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 288750*a**10*b*d**3*x/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 277200*a**9*b**2*d**3*x**2*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 508200*a**9*b**2*d**3*x**2/(60*a**5*b**12 + 300*a

4*b13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 10080*a**8*b**3*c*d**2*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 23016*a**8*b**3*c*d**2/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 277200*a**8*b**3*d**3*x**3*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 415800*a**8*b**3*d**3*x**3/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 50400*a**7*b**4*c*d**2*x*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 105000*a**7*b**4*c*d**2*x/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 138600*a**7*b**4*d**3*x**4*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 138600*a**7*b**4*d**3*x**4/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 100800*a**6*b**5*c*d**2*x**2*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 184800*a**6*b**5*c*d**2*x**2/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 27720*a**6*b**5*d**3*x**5*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 180*a**5*b**6*c**2*d*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 411*a**5*b**6*c**2*d/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 100800*a**5*b**6*c*d**2*x**3*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 151200*a**5*b**6*c*d**2*x**3/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 4620*a**5*b**6*d**3*x**6/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 900*a**4*b**7*c**2*d*x*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 1875*a**4*b**7*c**2*d*x/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 50400*a**4*b**7*c*d**2*x**4*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 50400*a**4*b**7*c*d**2*x**4/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 660*a**4*b**7*d**3*x**7/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 1800*a**3*b**8*c**2*d*x**2*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 3300*a**3*b**8*c**

2*d*x**2/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2
*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 10080*a**3*b**8*c*d**2*x*
*5*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 6
00*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 165*a**3*b**8*d**3
*x**8/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b*
*15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 2*a**2*b**9*c**3/(60*a**5*b*
*12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*
b**16*x**4 + 60*b**17*x**5) + 1800*a**2*b**9*c**2*d*x**3*log(a/b + x)/(60*a
5*b12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 +
300*a*b**16*x**4 + 60*b**17*x**5) + 2700*a**2*b**9*c**2*d*x**3/(60*a**5*b**
12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b
16*x4 + 60*b**17*x**5) + 1680*a**2*b**9*c*d**2*x**6/(60*a**5*b**12 + 30
0*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x*
*4 + 60*b**17*x**5) + 55*a**2*b**9*d**3*x**9/(60*a**5*b**12 + 300*a**4*b**1
3*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**
17*x**5) - 10*a*b**10*c**3*x/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b
14*x2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 900*a
*b**10*c**2*d*x**4*log(a/b + x)/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**
3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) + 90
0*a*b**10*c**2*d*x**4/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x*
*2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 240*a*b**10*
c*d**2*x**7/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a
2*b15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 22*a*b**10*d**3*x**10/
(60*a**5*b**12 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x*
*3 + 300*a*b**16*x**4 + 60*b**17*x**5) - 20*b**11*c**3*x**2/(60*a**5*b**12
+ 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**1
6*x**4 + 60*b**17*x**5) + 180*b**11*c**2*d*x**5*log(a/b + x)/(60*a**5*b**12
 + 300*a**4*b**13*x + 600*a**3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**
16*x**4 + 60*b**17*x**5) + 60*b**11*c*d**2*x**8/(60*a**5*b**12 + 300*a**4*b
13*x + 600*a3*b**14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*
b**17*x**5) + 10*b**11*d**3*x**11/(60*a**5*b**12 + 300*a**4*b**13*x + 600*a
3*b14*x**2 + 600*a**2*b**15*x**3 + 300*a*b**16*x**4 + 60*b**17*x**5), E
q(n, -6)), (-27720*a**11*d**3*log(a/b + x)/(84*a**4*b**12 + 336*a**3*b**13*
x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) - 57750*a**11*d
3/(84*a4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x
3 + 84*b16*x**4) - 110880*a**10*b*d**3*x*log(a/b + x)/(84*a**4*b**12 +
336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4)
- 203280*a**10*b*d**3*x/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*
x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) - 166320*a**9*b**2*d**3*x**2*log(a
/b + x)/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**
15*x**3 + 84*b**16*x**4) - 249480*a**9*b**2*d**3*x**2/(84*a**4*b**12 + 336*
a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) + 17
640*a**8*b**3*c*d**2*log(a/b + x)/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a
2*b14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) + 36750*a**8*b**3*c*d**2
/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3

+ 84*b**16*x**4) - 110880*a**8*b**3*d**3*x**3*log(a/b + x)/(84*a**4*b**12
+ 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4
) - 110880*a**8*b**3*d**3*x**3/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2
*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) + 70560*a**7*b**4*c*d**2*x*
log(a/b + x)/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*
a*b**15*x**3 + 84*b**16*x**4) + 129360*a**7*b**4*c*d**2*x/(84*a**4*b**12 +
336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4)
- 27720*a**7*b**4*d**3*x**4*log(a/b + x)/(84*a**4*b**12 + 336*a**3*b**13*x
+ 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) + 105840*a**6*b**
5*c*d**2*x**2*log(a/b + x)/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**
14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) + 158760*a**6*b**5*c*d**2*x**2/
(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3
+ 84*b**16*x**4) + 5544*a**6*b**5*d**3*x**5/(84*a**4*b**12 + 336*a**3*b**13
*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) - 1260*a**5*b*
*6*c**2*d*log(a/b + x)/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x
2 + 336*a*b15*x**3 + 84*b**16*x**4) - 2625*a**5*b**6*c**2*d/(84*a**4*b*
*12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*
x**4) + 70560*a**5*b**6*c*d**2*x**3*log(a/b + x)/(84*a**4*b**12 + 336*a**3*
b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) + 70560*a
5*b6*c*d**2*x**3/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x**
2 + 336*a*b**15*x**3 + 84*b**16*x**4) - 924*a**5*b**6*d**3*x**6/(84*a**4*b*
*12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*
x**4) - 5040*a**4*b**7*c**2*d*x*log(a/b + x)/(84*a**4*b**12 + 336*a**3*b**1
3*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) - 9240*a**4*b
7*c2*d*x/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*
a*b**15*x**3 + 84*b**16*x**4) + 17640*a**4*b**7*c*d**2*x**4*log(a/b + x)/(8
4*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 +
84*b**16*x**4) + 264*a**4*b**7*d**3*x**7/(84*a**4*b**12 + 336*a**3*b**13*x
+ 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) - 7560*a**3*b**8*
c**2*d*x**2*log(a/b + x)/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14
*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) - 11340*a**3*b**8*c**2*d*x**2/(84
*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 8
4*b**16*x**4) - 3528*a**3*b**8*c*d**2*x**5/(84*a**4*b**12 + 336*a**3*b**13*
x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) - 99*a**3*b**8*
d**3*x**8/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b
15*x3 + 84*b**16*x**4) - 7*a**2*b**9*c**3/(84*a**4*b**12 + 336*a**3*b**
13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) - 5040*a**2*
b**9*c**2*d*x**3*log(a/b + x)/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*
b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) - 5040*a**2*b**9*c**2*d*x**3
/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3
+ 84*b**16*x**4) + 588*a**2*b**9*c*d**2*x**6/(84*a**4*b**12 + 336*a**3*b**
13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) + 44*a**2*b*
*9*d**3*x**9/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*
a*b**15*x**3 + 84*b**16*x**4) - 28*a*b**10*c**3*x/(84*a**4*b**12 + 336*a**3
*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) - 1260*a

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*b**10*c**2*d*x**4*log(a/b + x)/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**
2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) - 168*a*b**10*c*d**2*x**7/
(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3
+ 84*b**16*x**4) - 22*a*b**10*d**3*x**10/(84*a**4*b**12 + 336*a**3*b**13*x
+ 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) - 42*b**11*c**3*x
**2/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x
**3 + 84*b**16*x**4) + 252*b**11*c**2*d*x**5/(84*a**4*b**12 + 336*a**3*b**1
3*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4) + 63*b**11*c*
d**2*x**8/(84*a**4*b**12 + 336*a**3*b**13*x + 504*a**2*b**14*x**2 + 336*a*b
**15*x**3 + 84*b**16*x**4) + 12*b**11*d**3*x**11/(84*a**4*b**12 + 336*a**3*
b**13*x + 504*a**2*b**14*x**2 + 336*a*b**15*x**3 + 84*b**16*x**4), Eq(n, -5
)), (138600*a**11*d**3*log(a/b + x)/(840*a**3*b**12 + 2520*a**2*b**13*x + 2
520*a*b**14*x**2 + 840*b**15*x**3) + 254100*a**11*d**3/(840*a**3*b**12 + 25
20*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**3) + 415800*a**10*b*d**3
*x*log(a/b + x)/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 8
40*b**15*x**3) + 623700*a**10*b*d**3*x/(840*a**3*b**12 + 2520*a**2*b**13*x
+ 2520*a*b**14*x**2 + 840*b**15*x**3) + 415800*a**9*b**2*d**3*x**2*log(a/b
+ x)/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x*
*3) + 415800*a**9*b**2*d**3*x**2/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520
*a*b**14*x**2 + 840*b**15*x**3) - 141120*a**8*b**3*c*d**2*log(a/b + x)/(840
*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**3) - 258
720*a**8*b**3*c*d**2/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**
2 + 840*b**15*x**3) + 138600*a**8*b**3*d**3*x**3*log(a/b + x)/(840*a**3*b**
12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**3) - 423360*a**7*
b**4*c*d**2*x*log(a/b + x)/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b**
14*x**2 + 840*b**15*x**3) - 635040*a**7*b**4*c*d**2*x/(840*a**3*b**12 + 252
0*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**3) - 34650*a**7*b**4*d**3
*x**4/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x
**3) - 423360*a**6*b**5*c*d**2*x**2*log(a/b + x)/(840*a**3*b**12 + 2520*a**
2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**3) - 423360*a**6*b**5*c*d**2*x
**2/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**
3) + 6930*a**6*b**5*d**3*x**5/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*
b**14*x**2 + 840*b**15*x**3) + 25200*a**5*b**6*c**2*d*log(a/b + x)/(840*a**
3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**3) + 46200*a
**5*b**6*c**2*d/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 8
40*b**15*x**3) - 141120*a**5*b**6*c*d**2*x**3*log(a/b + x)/(840*a**3*b**12
+ 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**3) - 2310*a**5*b**6*
d**3*x**6/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**
15*x**3) + 75600*a**4*b**7*c**2*d*x*log(a/b + x)/(840*a**3*b**12 + 2520*a**
2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**3) + 113400*a**4*b**7*c**2*d*x
/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**3)
+ 35280*a**4*b**7*c*d**2*x**4/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*
b**14*x**2 + 840*b**15*x**3) + 990*a**4*b**7*d**3*x**7/(840*a**3*b**12 + 25
20*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**3) + 75600*a**3*b**8*c**
2*d*x**2*log(a/b + x)/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x*
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*2 + 840*b**15*x**3) + 75600*a**3*b**8*c**2*d*x**2/(840*a**3*b**12 + 2520*a
2*b13*x + 2520*a*b**14*x**2 + 840*b**15*x**3) - 7056*a**3*b**8*c*d**2*x
5/(840*a3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**
3) - 495*a**3*b**8*d**3*x**8/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b
14*x2 + 840*b**15*x**3) - 280*a**2*b**9*c**3/(840*a**3*b**12 + 2520*a**
2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**3) + 25200*a**2*b**9*c**2*d*x*
*3*log(a/b + x)/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 8
40*b**15*x**3) + 2352*a**2*b**9*c*d**2*x**6/(840*a**3*b**12 + 2520*a**2*b**
13*x + 2520*a*b**14*x**2 + 840*b**15*x**3) + 275*a**2*b**9*d**3*x**9/(840*a
3*b12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**3) - 840*a
*b**10*c**3*x/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 840
*b**15*x**3) - 6300*a*b**10*c**2*d*x**4/(840*a**3*b**12 + 2520*a**2*b**13*x
 + 2520*a*b**14*x**2 + 840*b**15*x**3) - 1008*a*b**10*c*d**2*x**7/(840*a**3
*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**3) - 165*a*b*
*10*d**3*x**10/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 84
0*b**15*x**3) - 840*b**11*c**3*x**2/(840*a**3*b**12 + 2520*a**2*b**13*x + 2
520*a*b**14*x**2 + 840*b**15*x**3) + 1260*b**11*c**2*d*x**5/(840*a**3*b**12
 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15*x**3) + 504*b**11*c*d*
*2*x**8/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*b**14*x**2 + 840*b**15
*x**3) + 105*b**11*d**3*x**11/(840*a**3*b**12 + 2520*a**2*b**13*x + 2520*a*
b**14*x**2 + 840*b**15*x**3), Eq(n, -4)), (-138600*a**11*d**3*log(a/b + x)/
(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) - 207900*a**11*d**3/(2
520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) - 277200*a**10*b*d**3*x*
log(a/b + x)/(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) - 277200*
a**10*b*d**3*x/(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) - 13860
0*a**9*b**2*d**3*x**2*log(a/b + x)/(2520*a**2*b**12 + 5040*a*b**13*x + 2520
*b**14*x**2) + 211680*a**8*b**3*c*d**2*log(a/b + x)/(2520*a**2*b**12 + 5040
*a*b**13*x + 2520*b**14*x**2) + 317520*a**8*b**3*c*d**2/(2520*a**2*b**12 +
5040*a*b**13*x + 2520*b**14*x**2) + 46200*a**8*b**3*d**3*x**3/(2520*a**2*b*
*12 + 5040*a*b**13*x + 2520*b**14*x**2) + 423360*a**7*b**4*c*d**2*x*log(a/b
 + x)/(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) + 423360*a**7*b*
*4*c*d**2*x/(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) - 11550*a*
*7*b**4*d**3*x**4/(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) + 21
1680*a**6*b**5*c*d**2*x**2*log(a/b + x)/(2520*a**2*b**12 + 5040*a*b**13*x +
 2520*b**14*x**2) + 4620*a**6*b**5*d**3*x**5/(2520*a**2*b**12 + 5040*a*b**1
3*x + 2520*b**14*x**2) - 75600*a**5*b**6*c**2*d*log(a/b + x)/(2520*a**2*b**
12 + 5040*a*b**13*x + 2520*b**14*x**2) - 113400*a**5*b**6*c**2*d/(2520*a**2
*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) - 70560*a**5*b**6*c*d**2*x**3/(2
520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) - 2310*a**5*b**6*d**3*x*
*6/(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) - 151200*a**4*b**7*
c**2*d*x*log(a/b + x)/(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2)
- 151200*a**4*b**7*c**2*d*x/(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*
x**2) + 17640*a**4*b**7*c*d**2*x**4/(2520*a**2*b**12 + 5040*a*b**13*x + 252
0*b**14*x**2) + 1320*a**4*b**7*d**3*x**7/(2520*a**2*b**12 + 5040*a*b**13*x
+ 2520*b**14*x**2) - 75600*a**3*b**8*c**2*d*x**2*log(a/b + x)/(2520*a**2*b*

*12 + 5040*a*b**13*x + 2520*b**14*x**2) - 7056*a**3*b**8*c*d**2*x**5/(2520*
a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) - 825*a**3*b**8*d**3*x**8/(2
520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) + 2520*a**2*b**9*c**3*lo
g(a/b + x)/(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) + 3780*a**2
*b**9*c**3/(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) + 25200*a**
2*b**9*c**2*d*x**3/(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) + 3
528*a**2*b**9*c*d**2*x**6/(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x*
*2) + 550*a**2*b**9*d**3*x**9/(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**1
4*x**2) + 5040*a*b**10*c**3*x*log(a/b + x)/(2520*a**2*b**12 + 5040*a*b**13*
x + 2520*b**14*x**2) + 5040*a*b**10*c**3*x/(2520*a**2*b**12 + 5040*a*b**13*
x + 2520*b**14*x**2) - 6300*a*b**10*c**2*d*x**4/(2520*a**2*b**12 + 5040*a*b
13*x + 2520*b14*x**2) - 2016*a*b**10*c*d**2*x**7/(2520*a**2*b**12 + 504
0*a*b**13*x + 2520*b**14*x**2) - 385*a*b**10*d**3*x**10/(2520*a**2*b**12 +
5040*a*b**13*x + 2520*b**14*x**2) + 2520*b**11*c**3*x**2*log(a/b + x)/(2520
*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) + 2520*b**11*c**2*d*x**5/(2
520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) + 1260*b**11*c*d**2*x**8
/(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2) + 280*b**11*d**3*x**1
1/(2520*a**2*b**12 + 5040*a*b**13*x + 2520*b**14*x**2), Eq(n, -3)), (27720*
a**11*d**3*log(a/b + x)/(2520*a*b**12 + 2520*b**13*x) + 27720*a**11*d**3/(2
520*a*b**12 + 2520*b**13*x) + 27720*a**10*b*d**3*x*log(a/b + x)/(2520*a*b**
12 + 2520*b**13*x) - 13860*a**9*b**2*d**3*x**2/(2520*a*b**12 + 2520*b**13*x
) - 60480*a**8*b**3*c*d**2*log(a/b + x)/(2520*a*b**12 + 2520*b**13*x) - 604
80*a**8*b**3*c*d**2/(2520*a*b**12 + 2520*b**13*x) + 4620*a**8*b**3*d**3*x**
3/(2520*a*b**12 + 2520*b**13*x) - 60480*a**7*b**4*c*d**2*x*log(a/b + x)/(25
20*a*b**12 + 2520*b**13*x) - 2310*a**7*b**4*d**3*x**4/(2520*a*b**12 + 2520*
b**13*x) + 30240*a**6*b**5*c*d**2*x**2/(2520*a*b**12 + 2520*b**13*x) + 1386
*a**6*b**5*d**3*x**5/(2520*a*b**12 + 2520*b**13*x) + 37800*a**5*b**6*c**2*d
*log(a/b + x)/(2520*a*b**12 + 2520*b**13*x) + 37800*a**5*b**6*c**2*d/(2520*
a*b**12 + 2520*b**13*x) - 10080*a**5*b**6*c*d**2*x**3/(2520*a*b**12 + 2520*
b**13*x) - 924*a**5*b**6*d**3*x**6/(2520*a*b**12 + 2520*b**13*x) + 37800*a*
*4*b**7*c**2*d*x*log(a/b + x)/(2520*a*b**12 + 2520*b**13*x) + 5040*a**4*b**
7*c*d**2*x**4/(2520*a*b**12 + 2520*b**13*x) + 660*a**4*b**7*d**3*x**7/(2520
*a*b**12 + 2520*b**13*x) - 18900*a**3*b**8*c**2*d*x**2/(2520*a*b**12 + 2520
*b**13*x) - 3024*a**3*b**8*c*d**2*x**5/(2520*a*b**12 + 2520*b**13*x) - 495*
a**3*b**8*d**3*x**8/(2520*a*b**12 + 2520*b**13*x) - 5040*a**2*b**9*c**3*log
(a/b + x)/(2520*a*b**12 + 2520*b**13*x) - 5040*a**2*b**9*c**3/(2520*a*b**12
 + 2520*b**13*x) + 6300*a**2*b**9*c**2*d*x**3/(2520*a*b**12 + 2520*b**13*x)
 + 2016*a**2*b**9*c*d**2*x**6/(2520*a*b**12 + 2520*b**13*x) + 385*a**2*b**9
*d**3*x**9/(2520*a*b**12 + 2520*b**13*x) - 5040*a*b**10*c**3*x*log(a/b + x)
/(2520*a*b**12 + 2520*b**13*x) - 3150*a*b**10*c**2*d*x**4/(2520*a*b**12 + 2
520*b**13*x) - 1440*a*b**10*c*d**2*x**7/(2520*a*b**12 + 2520*b**13*x) - 308
*a*b**10*d**3*x**10/(2520*a*b**12 + 2520*b**13*x) + 2520*b**11*c**3*x**2/(2
520*a*b**12 + 2520*b**13*x) + 1890*b**11*c**2*d*x**5/(2520*a*b**12 + 2520*b
13*x) + 1080*b11*c*d**2*x**8/(2520*a*b**12 + 2520*b**13*x) + 252*b**11*
d**3*x**11/(2520*a*b**12 + 2520*b**13*x), Eq(n, -2)), (-a**11*d**3*log(a/b

+ x)/b**12 + a**10*d**3*x/b**11 - a**9*d**3*x**2/(2*b**10) + 3*a**8*c*d**2*
log(a/b + x)/b**9 + a**8*d**3*x**3/(3*b**9) - 3*a**7*c*d**2*x/b**8 - a**7*d
3*x4/(4*b**8) + 3*a**6*c*d**2*x**2/(2*b**7) + a**6*d**3*x**5/(5*b**7) -
3*a**5*c**2*d*log(a/b + x)/b**6 - a**5*c*d**2*x**3/b**6 - a**5*d**3*x**6/(
6*b**6) + 3*a**4*c**2*d*x/b**5 + 3*a**4*c*d**2*x**4/(4*b**5) + a**4*d**3*x*
*7/(7*b**5) - 3*a**3*c**2*d*x**2/(2*b**4) - 3*a**3*c*d**2*x**5/(5*b**4) - a
3*d3*x**8/(8*b**4) + a**2*c**3*log(a/b + x)/b**3 + a**2*c**2*d*x**3/b**
3 + a**2*c*d**2*x**6/(2*b**3) + a**2*d**3*x**9/(9*b**3) - a*c**3*x/b**2 - 3
*a*c**2*d*x**4/(4*b**2) - 3*a*c*d**2*x**7/(7*b**2) - a*d**3*x**10/(10*b**2)
+ c**3*x**2/(2*b) + 3*c**2*d*x**5/(5*b) + 3*c*d**2*x**8/(8*b) + d**3*x**11
/(11*b), Eq(n, -1)), (-39916800*a**12*d**3*(a + b*x)**n/(b**12*n**12 + 78*b
12*n11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926
634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**
12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*
n + 479001600*b**12) + 39916800*a**11*b*d**3*n*x*(a + b*x)**n/(b**12*n**12
+ 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8
+ 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 6572068
36*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*
b**12*n + 479001600*b**12) - 19958400*a**10*b**2*d**3*n**2*x**2*(a + b*x)**
n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749
463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12
*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**
2 + 1486442880*b**12*n + 479001600*b**12) - 19958400*a**10*b**2*d**3*n*x**2
*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**1
2*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206
070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 19315595
52*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 120960*a**9*b**3*c*
d**2*n**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 5
5770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n
6 + 206070150*b12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 +
1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 3991680*a*
*9*b**3*c*d**2*n**2*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12
*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 449902
31*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b*
*12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) +
43787520*a**9*b**3*c*d**2*n*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 27
17*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7
+ 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 14140
14888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b
12) + 159667200*a9*b**3*c*d**2*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**
11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**1
2*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4
+ 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 4790
01600*b**12) + 6652800*a**9*b**3*d**3*n**3*x**3*(a + b*x)**n/(b**12*n**12 +
78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 +

6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 65720683
6*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b
12*n + 479001600*b12) + 19958400*a**9*b**3*d**3*n**2*x**3*(a + b*x)**n/
(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 74946
3*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n
5 + 657206836*b12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2
+ 1486442880*b**12*n + 479001600*b**12) + 13305600*a**9*b**3*d**3*n*x**3*(a
 + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n
9 + 749463*b12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070
150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*
b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 120960*a**8*b**4*c*d**
2*n**4*x*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55
770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n*
*6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 +
1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 3991680*a**
8*b**4*c*d**2*n**3*x*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**1
2*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990
231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b
12*n3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) -
 43787520*a**8*b**4*c*d**2*n**2*x*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**1
1 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12
*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 +
 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 47900
1600*b**12) - 159667200*a**8*b**4*c*d**2*n*x*(a + b*x)**n/(b**12*n**12 + 78
*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 69
26634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b
12*n4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**1
2*n + 479001600*b**12) - 1663200*a**8*b**4*d**3*n**4*x**4*(a + b*x)**n/(b**
12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b*
*12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5
+ 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 14
86442880*b**12*n + 479001600*b**12) - 9979200*a**8*b**4*d**3*n**3*x**4*(a +
 b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**
9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 20607015
0*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b*
*12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 18295200*a**8*b**4*d**3*
n**2*x**4*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 5
5770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n
6 + 206070150*b12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 +
 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 9979200*a*
*8*b**4*d**3*n*x**4*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12
*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 449902
31*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b*
*12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) +
60480*a**7*b**5*c*d**2*n**5*x**2*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11

+ 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*
n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 +
1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001
600*b**12) + 2056320*a**7*b**5*c*d**2*n**4*x**2*(a + b*x)**n/(b**12*n**12 +
78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 +
6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 65720683
6*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b
12*n + 479001600*b12) + 23889600*a**7*b**5*c*d**2*n**3*x**2*(a + b*x)**
n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749
463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12
*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**
2 + 1486442880*b**12*n + 479001600*b**12) + 101727360*a**7*b**5*c*d**2*n**2
*x**2*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770
*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6
+ 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 193
1559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 79833600*a**7*
b**5*c*d**2*n*x**2*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*
n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 4499023
1*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**
12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 3
32640*a**7*b**5*d**3*n**5*x**5*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 +
2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n*
*7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 14
14014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 47900160
0*b**12) + 3326400*a**7*b**5*d**3*n**4*x**5*(a + b*x)**n/(b**12*n**12 + 78*
b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 692
6634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b*
*12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12
*n + 479001600*b**12) + 11642400*a**7*b**5*d**3*n**3*x**5*(a + b*x)**n/(b**
12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b*
*12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5
+ 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 14
86442880*b**12*n + 479001600*b**12) + 16632000*a**7*b**5*d**3*n**2*x**5*(a
+ b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n*
*9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 2060701
50*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b
12*n2 + 1486442880*b**12*n + 479001600*b**12) + 7983360*a**7*b**5*d**3*
n*x**5*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 5577
0*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6
+ 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 19
31559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 360*a**6*b**6
*c**2*d*n**6*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10
+ 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**1
2*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**
3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 20520*a

6*b6*c**2*d*n**5*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**1
2*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990
231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b
12*n3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) -
484200*a**6*b**6*c**2*d*n**4*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 +
2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**
7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 141
4014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600
*b**12) - 6053400*a**6*b**6*c**2*d*n**3*(a + b*x)**n/(b**12*n**12 + 78*b**1
2*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634
*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*
n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n +
479001600*b**12) - 42283440*a**6*b**6*c**2*d*n**2*(a + b*x)**n/(b**12*n**1
2 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**
8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 65720
6836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 148644288
0*b**12*n + 479001600*b**12) - 156444480*a**6*b**6*c**2*d*n*(a + b*x)**n/(b
12*n12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*
b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**
5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 +
1486442880*b**12*n + 479001600*b**12) - 239500800*a**6*b**6*c**2*d*(a + b*x
)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 +
749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b*
*12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*
n**2 + 1486442880*b**12*n + 479001600*b**12) - 20160*a**6*b**6*c*d**2*n**6*
x**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*
b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 +
206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931
559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 725760*a**6*b**
6*c*d**2*n**5*x**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*
n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 4499023
1*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**
12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 9
334080*a**6*b**6*c*d**2*n**4*x**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**1
1 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12
*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 +
1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 47900
1600*b**12) - 49835520*a**6*b**6*c*d**2*n**3*x**3*(a + b*x)**n/(b**12*n**12
+ 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8
+ 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206
836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880
*b**12*n + 479001600*b**12) - 94429440*a**6*b**6*c*d**2*n**2*x**3*(a + b*x)
n/(b12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 7
49463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**
12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n

2 + 1486442880*b12*n + 479001600*b**12) - 53222400*a**6*b**6*c*d**2*n*x
3*(a + b*x)n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b
12*n9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 +
206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 19315
59552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 55440*a**6*b**6*
d**3*n**6*x**6*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**1
0 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b*
*12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n
3 + 1931559552*b12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 83160
0*a**6*b**6*d**3*n**5*x**6*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 271
7*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 +
 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 141401
4888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b*
*12) - 4712400*a**6*b**6*d**3*n**4*x**6*(a + b*x)**n/(b**12*n**12 + 78*b**1
2*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634
*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*
n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n +
 479001600*b**12) - 12474000*a**6*b**6*d**3*n**3*x**6*(a + b*x)**n/(b**12*n
12 + 78*b12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*
n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 65
7206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 148644
2880*b**12*n + 479001600*b**12) - 15190560*a**6*b**6*d**3*n**2*x**6*(a + b*
x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 +
 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b
12*n5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12
*n**2 + 1486442880*b**12*n + 479001600*b**12) - 6652800*a**6*b**6*d**3*n*x*
6(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b*
*12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 2
06070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 193155
9552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 360*a**5*b**7*c**
2*d*n**7*x*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 +
55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*
n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3
+ 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 20520*a**
5*b**7*c**2*d*n**6*x*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**1
2*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990
231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b
12*n3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) +
 484200*a**5*b**7*c**2*d*n**5*x*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11
+ 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n
7 + 44990231*b12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1
414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 4790016
00*b**12) + 6053400*a**5*b**7*c**2*d*n**4*x*(a + b*x)**n/(b**12*n**12 + 78*
b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 692
6634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b*

*12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12
*n + 479001600*b**12) + 42283440*a**5*b**7*c**2*d*n**3*x*(a + b*x)**n/(b**1
2*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**
12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 +
657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 148
6442880*b**12*n + 479001600*b**12) + 156444480*a**5*b**7*c**2*d*n**2*x*(a +
b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**
9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 20607015
0*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b*
*12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 239500800*a**5*b**7*c**2
*d*n*x*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 5577
0*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6
+ 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 19
31559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 5040*a**5*b**
7*c*d**2*n**7*x**4*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*
n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 4499023
1*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**
12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 1
96560*a**5*b**7*c*d**2*n**6*x**4*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11
+ 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*
n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 +
1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001
600*b**12) + 2877840*a**5*b**7*c*d**2*n**5*x**4*(a + b*x)**n/(b**12*n**12 +
78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 +
6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 65720683
6*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b
12*n + 479001600*b12) + 19459440*a**5*b**7*c*d**2*n**4*x**4*(a + b*x)**
n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749
463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12
*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**
2 + 1486442880*b**12*n + 479001600*b**12) + 60984000*a**5*b**7*c*d**2*n**3*
x**4*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*
b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 +
206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931
559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 84127680*a**5*b
7*c*d2*n**2*x**4*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**1
2*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990
231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b
12*n3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) +
39916800*a**5*b**7*c*d**2*n*x**4*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**1
1 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12
*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 +
1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 47900
1600*b**12) + 7920*a**5*b**7*d**3*n**7*x**7*(a + b*x)**n/(b**12*n**12 + 78*
b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 692

6634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b*
*12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12
*n + 479001600*b**12) + 166320*a**5*b**7*d**3*n**6*x**7*(a + b*x)**n/(b**12
*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**1
2*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 +
657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486
442880*b**12*n + 479001600*b**12) + 1386000*a**5*b**7*d**3*n**5*x**7*(a + b
*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9
+ 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*
b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**1
2*n**2 + 1486442880*b**12*n + 479001600*b**12) + 5821200*a**5*b**7*d**3*n**
4*x**7*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 5577
0*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6
 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 19
31559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 12862080*a**5
*b**7*d**3*n**3*x**7*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**1
2*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990
231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b
12*n3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) +
 13970880*a**5*b**7*d**3*n**2*x**7*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**
11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**1
2*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4
+ 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 4790
01600*b**12) + 5702400*a**5*b**7*d**3*n*x**7*(a + b*x)**n/(b**12*n**12 + 78
*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 69
26634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b
12*n4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**1
2*n + 479001600*b**12) - 180*a**4*b**8*c**2*d*n**8*x**2*(a + b*x)**n/(b**12
*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**1
2*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 +
657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486
442880*b**12*n + 479001600*b**12) - 10440*a**4*b**8*c**2*d*n**7*x**2*(a + b
*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9
+ 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*
b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**1
2*n**2 + 1486442880*b**12*n + 479001600*b**12) - 252360*a**4*b**8*c**2*d*n*
*6*x**2*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 557
70*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**
6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1
931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 3268800*a**4
*b**8*c**2*d*n**5*x**2*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b*
*12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 449
90231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888
*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12)
 - 24168420*a**4*b**8*c**2*d*n**4*x**2*(a + b*x)**n/(b**12*n**12 + 78*b**12

*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*
b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n
4 + 1414014888*b12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n +
479001600*b**12) - 99363960*a**4*b**8*c**2*d*n**3*x**2*(a + b*x)**n/(b**12*
n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12
*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 6
57206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 14864
42880*b**12*n + 479001600*b**12) - 197972640*a**4*b**8*c**2*d*n**2*x**2*(a
+ b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n*
*9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 2060701
50*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b
12*n2 + 1486442880*b**12*n + 479001600*b**12) - 119750400*a**4*b**8*c**
2*d*n*x**2*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 +
55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*
n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3
+ 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 1008*a**4
*b**8*c*d**2*n**8*x**5*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b*
*12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 449
90231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888
*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12)
 - 43344*a**4*b**8*c*d**2*n**7*x**5*(a + b*x)**n/(b**12*n**12 + 78*b**12*n*
*11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**
12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4
 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479
001600*b**12) - 732816*a**4*b**8*c*d**2*n**6*x**5*(a + b*x)**n/(b**12*n**12
 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8
 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206
836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880
*b**12*n + 479001600*b**12) - 6194160*a**4*b**8*c*d**2*n**5*x**5*(a + b*x)*
*n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 74
9463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**1
2*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n*
*2 + 1486442880*b**12*n + 479001600*b**12) - 27764352*a**4*b**8*c*d**2*n**4
*x**5*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770
*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6
+ 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 193
1559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 65612736*a**4*
b**8*c*d**2*n**3*x**5*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**
12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 4499
0231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*
b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12)
- 75285504*a**4*b**8*c*d**2*n**2*x**5*(a + b*x)**n/(b**12*n**12 + 78*b**12*
n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b
12*n7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n*
*4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 4

79001600*b**12) - 31933440*a**4*b**8*c*d**2*n*x**5*(a + b*x)**n/(b**12*n**1
2 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**
8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 65720
6836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 148644288
0*b**12*n + 479001600*b**12) - 990*a**4*b**8*d**3*n**8*x**8*(a + b*x)**n/(b
12*n12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*
b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**
5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 +
1486442880*b**12*n + 479001600*b**12) - 27720*a**4*b**8*d**3*n**7*x**8*(a +
 b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**
9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 20607015
0*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b*
*12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 318780*a**4*b**8*d**3*n*
*6*x**8*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 557
70*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**
6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1
931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 1940400*a**4
*b**8*d**3*n**5*x**8*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**1
2*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990
231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b
12*n3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) -
 6701310*a**4*b**8*d**3*n**4*x**8*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**1
1 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12
*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 +
 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 47900
1600*b**12) - 13000680*a**4*b**8*d**3*n**3*x**8*(a + b*x)**n/(b**12*n**12 +
 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 +
 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 65720683
6*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b
12*n + 479001600*b12) - 12937320*a**4*b**8*d**3*n**2*x**8*(a + b*x)**n/
(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 74946
3*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n
5 + 657206836*b12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2
+ 1486442880*b**12*n + 479001600*b**12) - 4989600*a**4*b**8*d**3*n*x**8*(a
+ b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n*
*9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 2060701
50*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b
12*n2 + 1486442880*b**12*n + 479001600*b**12) + 2*a**3*b**9*c**3*n**9*(
a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*
n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 20607
0150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552
*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 144*a**3*b**9*c**3*n*
8(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b*
*12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 2
06070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 193155

9552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 4548*a**3*b**9*c*
*3*n**7*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 557
70*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**
6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1
931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 82656*a**3*b
9*c3*n**6*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10
 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**
12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n*
*3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 952098
*a**3*b**9*c**3*n**5*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**1
2*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990
231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b
12*n3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) +
 7204176*a**3*b**9*c**3*n**4*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2
717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7
 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414
014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*
b**12) + 35786392*a**3*b**9*c**3*n**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*
n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b
12*n7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n*
*4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 4
79001600*b**12) + 112463424*a**3*b**9*c**3*n**2*(a + b*x)**n/(b**12*n**12 +
 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 +
 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 65720683
6*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b
12*n + 479001600*b12) + 202757760*a**3*b**9*c**3*n*(a + b*x)**n/(b**12*
n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12
*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 6
57206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 14864
42880*b**12*n + 479001600*b**12) + 159667200*a**3*b**9*c**3*(a + b*x)**n/(b
12*n12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*
b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**
5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 +
1486442880*b**12*n + 479001600*b**12) + 60*a**3*b**9*c**2*d*n**9*x**3*(a +
b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9
 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150
*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**
12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 3600*a**3*b**9*c**2*d*n**
8*x**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 5577
0*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6
 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 19
31559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 91080*a**3*b*
*9*c**2*d*n**7*x**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12
*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 449902
31*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b*

*12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) +
1257840*a**3*b**9*c**2*d*n**6*x**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**
11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**1
2*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4
+ 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 4790
01600*b**12) + 10235340*a**3*b**9*c**2*d*n**5*x**3*(a + b*x)**n/(b**12*n**1
2 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**
8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 65720
6836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 148644288
0*b**12*n + 479001600*b**12) + 49233600*a**3*b**9*c**2*d*n**4*x**3*(a + b*x
)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 +
749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b*
*12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*
n**2 + 1486442880*b**12*n + 479001600*b**12) + 132233520*a**3*b**9*c**2*d*n
3*x3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55
770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n*
*6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 +
1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 171898560*a
3*b9*c**2*d*n**2*x**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717
*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 +
44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014
888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**
12) + 79833600*a**3*b**9*c**2*d*n*x**3*(a + b*x)**n/(b**12*n**12 + 78*b**12
*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*
b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n
4 + 1414014888*b12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n +
479001600*b**12) + 168*a**3*b**9*c*d**2*n**9*x**6*(a + b*x)**n/(b**12*n**12
 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8
 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206
836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880
*b**12*n + 479001600*b**12) + 8064*a**3*b**9*c*d**2*n**8*x**6*(a + b*x)**n/
(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 74946
3*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n
5 + 657206836*b12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2
+ 1486442880*b**12*n + 479001600*b**12) + 158256*a**3*b**9*c*d**2*n**7*x**6
*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**1
2*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206
070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 19315595
52*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 1643040*a**3*b**9*c
*d**2*n**6*x**6*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**
10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b
12*n6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*
n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 9789
192*a**3*b**9*c*d**2*n**5*x**6*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 +
 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n*

*7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 14
14014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 47900160
0*b**12) + 34072416*a**3*b**9*c*d**2*n**4*x**6*(a + b*x)**n/(b**12*n**12 +
78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 +
6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836
*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b*
*12*n + 479001600*b**12) + 67224864*a**3*b**9*c*d**2*n**3*x**6*(a + b*x)**n
/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 7494
63*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*
n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2
 + 1486442880*b**12*n + 479001600*b**12) + 68060160*a**3*b**9*c*d**2*n**2*x
6*(a + b*x)n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b
12*n9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 +
206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 19315
59552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 26611200*a**3*b*
*9*c*d**2*n*x**6*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n*
*10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*
b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12
*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 110
*a**3*b**9*d**3*n**9*x**9*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717
*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 +
44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014
888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**
12) + 3960*a**3*b**9*d**3*n**8*x**9*(a + b*x)**n/(b**12*n**12 + 78*b**12*n*
*11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**
12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4
 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479
001600*b**12) + 60060*a**3*b**9*d**3*n**7*x**9*(a + b*x)**n/(b**12*n**12 +
78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 +
6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836
*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b*
*12*n + 479001600*b**12) + 498960*a**3*b**9*d**3*n**6*x**9*(a + b*x)**n/(b*
*12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b
12*n8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5
 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1
486442880*b**12*n + 479001600*b**12) + 2469390*a**3*b**9*d**3*n**5*x**9*(a
+ b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n*
*9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 2060701
50*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b
12*n2 + 1486442880*b**12*n + 479001600*b**12) + 7401240*a**3*b**9*d**3*
n**4*x**9*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 5
5770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n
6 + 206070150*b12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 +
 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 12993640*a
3*b9*d**3*n**3*x**9*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b

12*n10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44
990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 141401488
8*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12
) + 12054240*a**3*b**9*d**3*n**2*x**9*(a + b*x)**n/(b**12*n**12 + 78*b**12*
n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b
12*n7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n*
*4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 4
79001600*b**12) + 4435200*a**3*b**9*d**3*n*x**9*(a + b*x)**n/(b**12*n**12 +
 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 +
 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 65720683
6*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b
12*n + 479001600*b12) - 2*a**2*b**10*c**3*n**10*x*(a + b*x)**n/(b**12*n
12 + 78*b12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*
n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 65
7206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 148644
2880*b**12*n + 479001600*b**12) - 144*a**2*b**10*c**3*n**9*x*(a + b*x)**n/(
b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463
*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n*
*5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 +
 1486442880*b**12*n + 479001600*b**12) - 4548*a**2*b**10*c**3*n**8*x*(a + b
*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9
+ 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*
b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**1
2*n**2 + 1486442880*b**12*n + 479001600*b**12) - 82656*a**2*b**10*c**3*n**7
x(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b*
*12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 2
06070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 193155
9552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 952098*a**2*b**10
*c**3*n**6*x*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10
+ 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**1
2*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**
3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 7204176
*a**2*b**10*c**3*n**5*x*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b
12*n10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44
990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 141401488
8*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12
) - 35786392*a**2*b**10*c**3*n**4*x*(a + b*x)**n/(b**12*n**12 + 78*b**12*n*
*11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**
12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4
 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479
001600*b**12) - 112463424*a**2*b**10*c**3*n**3*x*(a + b*x)**n/(b**12*n**12
+ 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8
+ 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 6572068
36*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*
b**12*n + 479001600*b**12) - 202757760*a**2*b**10*c**3*n**2*x*(a + b*x)**n/

(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 74946
3*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n
5 + 657206836*b12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2
+ 1486442880*b**12*n + 479001600*b**12) - 159667200*a**2*b**10*c**3*n*x*(a
+ b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n*
*9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 2060701
50*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b
12*n2 + 1486442880*b**12*n + 479001600*b**12) - 15*a**2*b**10*c**2*d*n*
*10*x**4*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55
770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n*
*6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 +
1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 945*a**2*b*
*10*c**2*d*n**9*x**4*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**1
2*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990
231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b
12*n3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) -
25470*a**2*b**10*c**2*d*n**8*x**4*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**
11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**1
2*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4
+ 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 4790
01600*b**12) - 382770*a**2*b**10*c**2*d*n**7*x**4*(a + b*x)**n/(b**12*n**12
+ 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8
+ 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206
836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880
*b**12*n + 479001600*b**12) - 3502215*a**2*b**10*c**2*d*n**6*x**4*(a + b*x)
n/(b12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 7
49463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**
12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n
2 + 1486442880*b12*n + 479001600*b**12) - 19984905*a**2*b**10*c**2*d*n*
*5*x**4*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 557
70*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**
6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1
931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 69983580*a**
2*b**10*c**2*d*n**4*x**4*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*
b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 4
4990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 14140148
88*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**1
2) - 142149780*a**2*b**10*c**2*d*n**3*x**4*(a + b*x)**n/(b**12*n**12 + 78*b
12*n11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926
634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**
12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*
n + 479001600*b**12) - 148882320*a**2*b**10*c**2*d*n**2*x**4*(a + b*x)**n/(
b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463
*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n*
*5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 +

1486442880*b**12*n + 479001600*b**12) - 59875200*a**2*b**10*c**2*d*n*x**4*
(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12
*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 2060
70150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 193155955
2*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 24*a**2*b**10*c*d**2
*n**10*x**7*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 +
55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12
*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3
+ 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 1296*a**
2*b**10*c*d**2*n**9*x**7*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*
b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 4
4990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 14140148
88*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**1
2) - 29520*a**2*b**10*c*d**2*n**8*x**7*(a + b*x)**n/(b**12*n**12 + 78*b**12
*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*
b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n
4 + 1414014888*b12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n +
479001600*b**12) - 370368*a**2*b**10*c*d**2*n**7*x**7*(a + b*x)**n/(b**12*n
12 + 78*b12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*
n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 65
7206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 148644
2880*b**12*n + 479001600*b**12) - 2806776*a**2*b**10*c*d**2*n**6*x**7*(a +
b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9
+ 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150
*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**
12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 13258224*a**2*b**10*c*d**
2*n**5*x**7*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 +
55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12
*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3
+ 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 38808480
*a**2*b**10*c*d**2*n**4*x**7*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2
717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7
+ 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414
014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*
b**12) - 67344192*a**2*b**10*c*d**2*n**3*x**7*(a + b*x)**n/(b**12*n**12 + 7
8*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6
926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*
b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**
12*n + 479001600*b**12) - 62138880*a**2*b**10*c*d**2*n**2*x**7*(a + b*x)**n
/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 7494
63*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*
n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2
+ 1486442880*b**12*n + 479001600*b**12) - 22809600*a**2*b**10*c*d**2*n*x**
7*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**
12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 20

6070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559
552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 11*a**2*b**10*d**3
*n**10*x**10*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10
+ 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**1
2*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**
3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 495*a**
2*b**10*d**3*n**9*x**10*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b
12*n10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44
990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 141401488
8*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12
) - 9570*a**2*b**10*d**3*n**8*x**10*(a + b*x)**n/(b**12*n**12 + 78*b**12*n*
*11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**
12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4
 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479
001600*b**12) - 103950*a**2*b**10*d**3*n**7*x**10*(a + b*x)**n/(b**12*n**12
 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8
 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206
836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880
*b**12*n + 479001600*b**12) - 696003*a**2*b**10*d**3*n**6*x**10*(a + b*x)**
n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749
463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12
*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**
2 + 1486442880*b**12*n + 479001600*b**12) - 2962575*a**2*b**10*d**3*n**5*x*
10(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b
12*n9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 +
206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 19315
59552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 7960480*a**2*b**
10*d**3*n**4*x**10*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*
n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 4499023
1*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**
12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) - 1
2899700*a**2*b**10*d**3*n**3*x**10*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**
11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**1
2*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4
+ 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 4790
01600*b**12) - 11292336*a**2*b**10*d**3*n**2*x**10*(a + b*x)**n/(b**12*n**1
2 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**
8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 65720
6836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 148644288
0*b**12*n + 479001600*b**12) - 3991680*a**2*b**10*d**3*n*x**10*(a + b*x)**n
/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 7494
63*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*
n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2
 + 1486442880*b**12*n + 479001600*b**12) + a*b**11*c**3*n**11*x**2*(a + b*x
)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 +

749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b*
*12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*
n**2 + 1486442880*b**12*n + 479001600*b**12) + 73*a*b**11*c**3*n**10*x**2*(
a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*
n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 20607
0150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552
*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 2346*a*b**11*c**3*n**
9*x**2*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 5577
0*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6
 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 19
31559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 43602*a*b**11
*c**3*n**8*x**2*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**
10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b
12*n6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*
n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 5173
77*a*b**11*c**3*n**7*x**2*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717
*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 +
44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014
888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**
12) + 4078137*a*b**11*c**3*n**6*x**2*(a + b*x)**n/(b**12*n**12 + 78*b**12*n
11 + 2717*b12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b*
*12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**
4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 47
9001600*b**12) + 21495284*a*b**11*c**3*n**5*x**2*(a + b*x)**n/(b**12*n**12
+ 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8
+ 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 6572068
36*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*
b**12*n + 479001600*b**12) + 74124908*a*b**11*c**3*n**4*x**2*(a + b*x)**n/(
b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463
*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n*
*5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 +
 1486442880*b**12*n + 479001600*b**12) + 157610592*a*b**11*c**3*n**3*x**2*(
a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*
n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 20607
0150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552
*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 181212480*a*b**11*c**
3*n**2*x**2*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 +
 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12
*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3
 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 79833600
*a*b**11*c**3*n*x**2*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**1
2*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990
231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b
12*n3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) +
 3*a*b**11*c**2*d*n**11*x**5*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2

717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7
 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414
014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*
b**12) + 201*a*b**11*c**2*d*n**10*x**5*(a + b*x)**n/(b**12*n**12 + 78*b**12
*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*
b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n
4 + 1414014888*b12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n +
479001600*b**12) + 5850*a*b**11*c**2*d*n**9*x**5*(a + b*x)**n/(b**12*n**12
+ 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8
+ 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 6572068
36*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*
b**12*n + 479001600*b**12) + 96930*a*b**11*c**2*d*n**8*x**5*(a + b*x)**n/(b
12*n12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*
b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**
5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 +
1486442880*b**12*n + 479001600*b**12) + 1006659*a*b**11*c**2*d*n**7*x**5*(a
 + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n
9 + 749463*b12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070
150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*
b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 6798753*a*b**11*c**2*d
*n**6*x**5*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 +
55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*
n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3
+ 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 29984640*
a*b**11*c**2*d*n**5*x**5*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*
b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 4
4990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 14140148
88*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**1
2) + 84416820*a*b**11*c**2*d*n**4*x**5*(a + b*x)**n/(b**12*n**12 + 78*b**12
*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*
b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n
4 + 1414014888*b12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n +
479001600*b**12) + 143496288*a*b**11*c**2*d*n**3*x**5*(a + b*x)**n/(b**12*n
12 + 78*b12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*
n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 65
7206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 148644
2880*b**12*n + 479001600*b**12) + 131080896*a*b**11*c**2*d*n**2*x**5*(a + b
*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9
+ 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*
b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**1
2*n**2 + 1486442880*b**12*n + 479001600*b**12) + 47900160*a*b**11*c**2*d*n*
x**5*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*
b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 +
 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931
559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 3*a*b**11*c*d**

2*n**11*x**8*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10
+ 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**1
2*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**
3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 183*a*b
11*c*d2*n**10*x**8*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b*
*12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 449
90231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888
*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12)
 + 4824*a*b**11*c*d**2*n**9*x**8*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11
 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*
n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 +
1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001
600*b**12) + 72126*a*b**11*c*d**2*n**8*x**8*(a + b*x)**n/(b**12*n**12 + 78*
b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 692
6634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b*
*12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12
*n + 479001600*b**12) + 674919*a*b**11*c*d**2*n**7*x**8*(a + b*x)**n/(b**12
*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**1
2*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 +
657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486
442880*b**12*n + 479001600*b**12) + 4113207*a*b**11*c*d**2*n**6*x**8*(a + b
*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9
+ 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*
b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**1
2*n**2 + 1486442880*b**12*n + 479001600*b**12) + 16452006*a*b**11*c*d**2*n*
*5*x**8*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 557
70*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**
6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1
931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 42375444*a*b
11*c*d2*n**4*x**8*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**
12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 4499
0231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*
b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12)
+ 66693528*a*b**11*c*d**2*n**3*x**8*(a + b*x)**n/(b**12*n**12 + 78*b**12*n*
*11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**
12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4
 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479
001600*b**12) + 57222720*a*b**11*c*d**2*n**2*x**8*(a + b*x)**n/(b**12*n**12
 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8
 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206
836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880
*b**12*n + 479001600*b**12) + 19958400*a*b**11*c*d**2*n*x**8*(a + b*x)**n/(
b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463
*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n*
*5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 +

1486442880*b**12*n + 479001600*b**12) + a*b**11*d**3*n**11*x**11*(a + b*x)
n/(b12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 7
49463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**
12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n
2 + 1486442880*b12*n + 479001600*b**12) + 55*a*b**11*d**3*n**10*x**11*(
a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*
n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 20607
0150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552
*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 1320*a*b**11*d**3*n**
9*x**11*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 557
70*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**
6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1
931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 18150*a*b**1
1*d**3*n**8*x**11*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n
10 + 55770*b12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231
*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**1
2*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 15
7773*a*b**11*d**3*n**7*x**11*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2
717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7
 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414
014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*
b**12) + 902055*a*b**11*d**3*n**6*x**11*(a + b*x)**n/(b**12*n**12 + 78*b**1
2*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634
*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*
n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n +
 479001600*b**12) + 3416930*a*b**11*d**3*n**5*x**11*(a + b*x)**n/(b**12*n**
12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n*
*8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 6572
06836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 14864428
80*b**12*n + 479001600*b**12) + 8409500*a*b**11*d**3*n**4*x**11*(a + b*x)**
n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749
463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12
*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**
2 + 1486442880*b**12*n + 479001600*b**12) + 12753576*a*b**11*d**3*n**3*x**1
1*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**
12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 20
6070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559
552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 10628640*a*b**11*d
3*n2*x**11*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**1
0 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b*
*12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n
3 + 1931559552*b12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 36288
00*a*b**11*d**3*n*x**11*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b
12*n10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44
990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 141401488

8*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12
) + b**12*c**3*n**11*x**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717
*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 +
44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014
888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**
12) + 75*b**12*c**3*n**10*x**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 +
 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n*
*7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 14
14014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 47900160
0*b**12) + 2492*b**12*c**3*n**9*x**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n
11 + 2717*b12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b*
*12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**
4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 47
9001600*b**12) + 48294*b**12*c**3*n**8*x**3*(a + b*x)**n/(b**12*n**12 + 78*
b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 692
6634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b*
*12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12
*n + 479001600*b**12) + 604581*b**12*c**3*n**7*x**3*(a + b*x)**n/(b**12*n**
12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n*
*8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 6572
06836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 14864428
80*b**12*n + 479001600*b**12) + 5112891*b**12*c**3*n**6*x**3*(a + b*x)**n/(
b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463
*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n*
*5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 +
 1486442880*b**12*n + 479001600*b**12) + 29651558*b**12*c**3*n**5*x**3*(a +
 b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**
9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 20607015
0*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b*
*12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 117115476*b**12*c**3*n**
4*x**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 5577
0*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6
 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 19
31559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 305860408*b**
12*c**3*n**3*x**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n
10 + 55770*b12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231
*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**1
2*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 49
6433664*b**12*c**3*n**2*x**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2
717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7
 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414
014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*
b**12) + 442258560*b**12*c**3*n*x**3*(a + b*x)**n/(b**12*n**12 + 78*b**12*n
11 + 2717*b12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b*
*12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**

4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 47
9001600*b**12) + 159667200*b**12*c**3*x**3*(a + b*x)**n/(b**12*n**12 + 78*b
12*n11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926
634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**
12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*
n + 479001600*b**12) + 3*b**12*c**2*d*n**11*x**6*(a + b*x)**n/(b**12*n**12
+ 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8
+ 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 6572068
36*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*
b**12*n + 479001600*b**12) + 216*b**12*c**2*d*n**10*x**6*(a + b*x)**n/(b**1
2*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**
12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 +
657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 148
6442880*b**12*n + 479001600*b**12) + 6855*b**12*c**2*d*n**9*x**6*(a + b*x)*
*n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 74
9463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**1
2*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n*
*2 + 1486442880*b**12*n + 479001600*b**12) + 126180*b**12*c**2*d*n**8*x**6*
(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12
*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 2060
70150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 193155955
2*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 1491309*b**12*c**2*d
*n**7*x**6*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 +
55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*
n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3
+ 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 11832048*
b**12*c**2*d*n**6*x**6*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b*
*12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 449
90231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888
*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12)
+ 63978405*b**12*c**2*d*n**5*x**6*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**
11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**1
2*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4
+ 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 4790
01600*b**12) + 234340020*b**12*c**2*d*n**4*x**6*(a + b*x)**n/(b**12*n**12 +
78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 +
6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 65720683
6*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b
12*n + 479001600*b12) + 565580388*b**12*c**2*d*n**3*x**6*(a + b*x)**n/(
b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463
*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n*
*5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 +
1486442880*b**12*n + 479001600*b**12) + 848562336*b**12*c**2*d*n**2*x**6*(
a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*
n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 20607

0150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552
*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 703304640*b**12*c**2*
d*n*x**6*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55
770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n*
*6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 +
1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 239500800*b
12*c2*d*x**6*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n*
*10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*
b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12
*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 3*b
12*c*d2*n**11*x**9*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b*
*12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 449
90231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888
*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12)
 + 207*b**12*c*d**2*n**10*x**9*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 +
 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n*
*7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 14
14014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 47900160
0*b**12) + 6288*b**12*c*d**2*n**9*x**9*(a + b*x)**n/(b**12*n**12 + 78*b**12
*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*
b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n
4 + 1414014888*b12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n +
479001600*b**12) + 110718*b**12*c*d**2*n**8*x**9*(a + b*x)**n/(b**12*n**12
+ 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8
+ 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 6572068
36*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*
b**12*n + 479001600*b**12) + 1251927*b**12*c*d**2*n**7*x**9*(a + b*x)**n/(b
12*n12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*
b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**
5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 +
1486442880*b**12*n + 479001600*b**12) + 9512559*b**12*c*d**2*n**6*x**9*(a +
 b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**
9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 20607015
0*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b*
*12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 49357662*b**12*c*d**2*n*
*5*x**9*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 557
70*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**
6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1
931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 173991492*b*
*12*c*d**2*n**4*x**9*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**1
2*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990
231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b
12*n3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) +
 405697080*b**12*c*d**2*n**3*x**9*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**1
1 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12

*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 +
1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 47900
1600*b**12) + 590770944*b**12*c*d**2*n**2*x**9*(a + b*x)**n/(b**12*n**12 +
78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 +
6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836
*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b*
*12*n + 479001600*b**12) + 477740160*b**12*c*d**2*n*x**9*(a + b*x)**n/(b**1
2*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**
12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 +
657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 148
6442880*b**12*n + 479001600*b**12) + 159667200*b**12*c*d**2*x**9*(a + b*x)*
*n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 74
9463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**1
2*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n*
*2 + 1486442880*b**12*n + 479001600*b**12) + b**12*d**3*n**11*x**12*(a + b*
x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 +
749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b
12*n5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12
*n**2 + 1486442880*b**12*n + 479001600*b**12) + 66*b**12*d**3*n**10*x**12*(
a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*
n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 20607
0150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552
*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 1925*b**12*d**3*n**9*
x**12*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770
*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6
+ 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 193
1559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 32670*b**12*d*
*3*n**8*x**12*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10
+ 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**
12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n*
*3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 357423
*b**12*d**3*n**7*x**12*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b*
*12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 449
90231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888
*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12)
+ 2637558*b**12*d**3*n**6*x**12*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11
+ 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*
n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 +
1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001
600*b**12) + 13339535*b**12*d**3*n**5*x**12*(a + b*x)**n/(b**12*n**12 + 78*
b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 692
6634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b*
*12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12
*n + 479001600*b**12) + 45995730*b**12*d**3*n**4*x**12*(a + b*x)**n/(b**12*
n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 + 749463*b**12

```
*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b**12*n**5 + 6
57206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*n**2 + 14864
42880*b**12*n + 479001600*b**12) + 105258076*b**12*d**3*n**3*x**12*(a + b*x
)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b**12*n**9 +
749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 + 206070150*b*
*12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 1931559552*b**12*
n**2 + 1486442880*b**12*n + 479001600*b**12) + 150917976*b**12*d**3*n**2*x*
*12*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10 + 55770*b
**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**12*n**6 +
206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**3 + 19315
59552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 120543840*b**12*
d**3*n*x**12*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*n**10
+ 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 44990231*b**1
2*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**12*n**
3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12) + 3991680
0*b**12*d**3*x**12*(a + b*x)**n/(b**12*n**12 + 78*b**12*n**11 + 2717*b**12*
n**10 + 55770*b**12*n**9 + 749463*b**12*n**8 + 6926634*b**12*n**7 + 4499023
1*b**12*n**6 + 206070150*b**12*n**5 + 657206836*b**12*n**4 + 1414014888*b**
12*n**3 + 1931559552*b**12*n**2 + 1486442880*b**12*n + 479001600*b**12), Tr
ue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1153 vs. 2(459) = 918.

Time = 0.22 (sec) , antiderivative size = 1153, normalized size of antiderivative = 2.51

$$\int x^2(a+bx)^n \left(c+dx^3\right)^3 dx = \text{Too large to display}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^5 + 15*n^4 + 85*n^3 + 225
*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5
*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*
a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)
^n*c^2*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
 + 3*((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n
^2 + 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*
n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5
 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 +
85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^
3 + 50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*
x^4 - 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 -
40320*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*c*d^2/((n^9 + 45*n^8 + 870*n^7 + 9
```

```
450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 3
62880)*b^9) + ((n^11 + 66*n^10 + 1925*n^9 + 32670*n^8 + 357423*n^7 + 263755
8*n^6 + 13339535*n^5 + 45995730*n^4 + 105258076*n^3 + 150917976*n^2 + 12054
3840*n + 39916800)*b^12*x^12 + (n^11 + 55*n^10 + 1320*n^9 + 18150*n^8 + 157
773*n^7 + 902055*n^6 + 3416930*n^5 + 8409500*n^4 + 12753576*n^3 + 10628640*
n^2 + 3628800*n)*a*b^11*x^11 - 11*(n^10 + 45*n^9 + 870*n^8 + 9450*n^7 + 632
73*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3 + 1026576*n^2 + 362880*n)*a^
2*b^10*x^10 + 110*(n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67284*n^
4 + 118124*n^3 + 109584*n^2 + 40320*n)*a^3*b^9*x^9 - 990*(n^8 + 28*n^7 + 32
2*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^4*b^8*x^8 +
 7920*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^5*
b^7*x^7 - 55440*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^6*b^6
*x^6 + 332640*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^7*b^5*x^5 - 1663200
*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^8*b^4*x^4 + 6652800*(n^3 + 3*n^2 + 2*n)*a^9
*b^3*x^3 - 19958400*(n^2 + n)*a^10*b^2*x^2 + 39916800*a^11*b*n*x - 39916800
*a^12)*(b*x + a)^n*d^3/((n^12 + 78*n^11 + 2717*n^10 + 55770*n^9 + 749463*n^
8 + 6926634*n^7 + 44990231*n^6 + 206070150*n^5 + 657206836*n^4 + 1414014888
*n^3 + 1931559552*n^2 + 1486442880*n + 479001600)*b^12)
```

Giac [F(-2)]

Exception generated.

$$\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Polynomial exponent overflow. Error:
Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 24.92 (sec) , antiderivative size = 2896, normalized size of antiderivative = 6.31

$$\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx=\text{Too large to display}$$

```
[In] int(x^2*(c + d*x^3)^3*(a + b*x)^n,x)
```

```
[Out] (2*a^3*(a + b*x)^n*(79833600*b^9*c^3 - 19958400*a^9*d^3 + 101378880*b^9*c^3
*n + 56231712*b^9*c^3*n^2 + 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 47
6049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 +
b^9*c^3*n^9 - 119750400*a^3*b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 78222240*a
```

^3*b^6*c^2*d*n + 21893760*a^6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*n^2 + 19
95840*a^6*b^3*c*d^2*n^2 - 3026700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3*c*d^2*n
^3 - 242100*a^3*b^6*c^2*d*n^4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b^6*c^2*d
n^6))/(b^12(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^
4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2
717*n^10 + 78*n^11 + n^12 + 479001600)) + (d^3*x^12*(a + b*x)^n*(120543840*
n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n
^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800))/(14864
42880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 +
44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11
+ n^12 + 479001600) + (x^3*(a + b*x)^n*(3*n + n^2 + 2)*(79833600*b^9*c^3 +
6652800*a^9*d^3*n + 101378880*b^9*c^3*n + 56231712*b^9*c^3*n^2 + 17893196*b
^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 476049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 +
2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 + b^9*c^3*n^9 + 39916800*a^3*b^6*c^2*d*n
- 26611200*a^6*b^3*c*d^2*n + 26074080*a^3*b^6*c^2*d*n^2 - 7297920*a^6*b^3*
c*d^2*n^2 + 7047240*a^3*b^6*c^2*d*n^3 - 665280*a^6*b^3*c*d^2*n^3 + 1008900*
a^3*b^6*c^2*d*n^4 - 20160*a^6*b^3*c*d^2*n^4 + 80700*a^3*b^6*c^2*d*n^5 + 342
0*a^3*b^6*c^2*d*n^6 + 60*a^3*b^6*c^2*d*n^7))/(b^9*(1486442880*n + 193155955
2*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 692
6634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)
) + (3*d*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)*(6
65280*b^6*c^2 - 18480*a^6*d^2*n + 434568*b^6*c^2*n + 117454*b^6*c^2*n^2 + 1
6815*b^6*c^2*n^3 + 1345*b^6*c^2*n^4 + 57*b^6*c^2*n^5 + b^6*c^2*n^6 + 73920*
a^3*b^3*c*d*n + 20272*a^3*b^3*c*d*n^2 + 1848*a^3*b^3*c*d*n^3 + 56*a^3*b^3*c
*d*n^4))/(b^6*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n
^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 +
2717*n^10 + 78*n^11 + n^12 + 479001600)) - (2*a^2*n*x*(a + b*x)^n*(79833600
*b^9*c^3 - 19958400*a^9*d^3 + 101378880*b^9*c^3*n + 56231712*b^9*c^3*n^2 +
17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 476049*b^9*c^3*n^5 + 41328*b^9
*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 + b^9*c^3*n^9 - 119750400*a^3*
b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 78222240*a^3*b^6*c^2*d*n + 21893760*a^
6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*n^2 + 1995840*a^6*b^3*c*d^2*n^2 - 30
26700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3*c*d^2*n^3 - 242100*a^3*b^6*c^2*d*n^
4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b^6*c^2*d*n^6))/(b^11*(1486442880*n +
1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231
*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 +
479001600)) + (d^2*x^9*(a + b*x)^n*(3960*b^3*c + 99*b^3*c*n^2 + 3*b^3*c*n^3
+ 110*a^3*d*n + 1086*b^3*c*n)*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n
^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(b^3*(1486442880*n + 19315
59552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 +
6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001
600)) + (a*d^3*n*x^11*(a + b*x)^n*(10628640*n + 12753576*n^2 + 8409500*n^3
+ 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n
^10 + 3628800))/(b*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206
836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n

^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) - (11*a^2*d^3*n*x^10*(a + b*x
)^n*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n
^6 + 870*n^7 + 45*n^8 + n^9 + 362880))/(b^2*(1486442880*n + 1931559552*n^2
+ 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n
^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) + (a
*n*x^2*(n + 1)*(a + b*x)^n*(79833600*b^9*c^3 - 19958400*a^9*d^3 + 101378880
*b^9*c^3*n + 56231712*b^9*c^3*n^2 + 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*
n^4 + 476049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^
3*n^8 + b^9*c^3*n^9 - 119750400*a^3*b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 78
222240*a^3*b^6*c^2*d*n + 21893760*a^6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*
n^2 + 1995840*a^6*b^3*c*d^2*n^2 - 3026700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3
*c*d^2*n^3 - 242100*a^3*b^6*c^2*d*n^4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b
^6*c^2*d*n^6))/(b^10*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 6572
06836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770
*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) + (3*a*d*n*x^5*(a + b*x)^n*
(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(110880*a^6*d^2 + 665280*b^6*c^2 + 4345
68*b^6*c^2*n + 117454*b^6*c^2*n^2 + 16815*b^6*c^2*n^3 + 1345*b^6*c^2*n^4 +
57*b^6*c^2*n^5 + b^6*c^2*n^6 - 443520*a^3*b^3*c*d - 121632*a^3*b^3*c*d*n -
11088*a^3*b^3*c*d*n^2 - 336*a^3*b^3*c*d*n^3))/(b^7*(1486442880*n + 19315595
52*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 69
26634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600
)) - (15*a^2*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(110880*a^6*d^2 +
665280*b^6*c^2 + 434568*b^6*c^2*n + 117454*b^6*c^2*n^2 + 16815*b^6*c^2*n^3
+ 1345*b^6*c^2*n^4 + 57*b^6*c^2*n^5 + b^6*c^2*n^6 - 443520*a^3*b^3*c*d - 1
21632*a^3*b^3*c*d*n - 11088*a^3*b^3*c*d*n^2 - 336*a^3*b^3*c*d*n^3))/(b^8*(1
486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n
^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n
^11 + n^12 + 479001600)) + (3*a*d^2*n*x^8*(a + b*x)^n*(1320*b^3*c - 330*a^3
*d + 33*b^3*c*n^2 + b^3*c*n^3 + 362*b^3*c*n)*(13068*n + 13132*n^2 + 6769*n^
3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040))/(b^4*(1486442880*n + 1931559
552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6
926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 47900160
0)) - (24*a^2*d^2*n*x^7*(a + b*x)^n*(1320*b^3*c - 330*a^3*d + 33*b^3*c*n^2
+ b^3*c*n^3 + 362*b^3*c*n)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5
+ n^6 + 720))/(b^5*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206
836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n
^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600))

3.183 $\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 396

$$\begin{aligned}\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx = & -\frac{a\left(b^3c-a^3d\right)^3 (a+bx)^{1+n}}{b^{11}(1+n)} \\ & +\frac{\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2 (a+bx)^{2+n}}{b^{11}(2+n)} \\ & +\frac{9a^2d\left(2b^3c-5a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{3+n}}{b^{11}(3+n)} \\ & -\frac{3ad\left(4b^6c^2-35a^3b^3cd+40a^6d^2\right)(a+bx)^{4+n}}{b^{11}(4+n)} \\ & +\frac{3d\left(b^6c^2-35a^3b^3cd+70a^6d^2\right)(a+bx)^{5+n}}{b^{11}(5+n)} \\ & +\frac{63a^2d^2\left(b^3c-4a^3d\right)(a+bx)^{6+n}}{b^{11}(6+n)} \\ & -\frac{21ad^2\left(b^3c-10a^3d\right)(a+bx)^{7+n}}{b^{11}(7+n)} \\ & +\frac{3d^2\left(b^3c-40a^3d\right)(a+bx)^{8+n}}{b^{11}(8+n)}+\frac{45a^2d^3(a+bx)^{9+n}}{b^{11}(9+n)} \\ & -\frac{10ad^3(a+bx)^{10+n}}{b^{11}(10+n)}+\frac{d^3(a+bx)^{11+n}}{b^{11}(11+n)}\end{aligned}$$

```
[Out] -a*(-a^3*d+b^3*c)^3*(b*x+a)^(1+n)/b^11/(1+n)+(-10*a^3*d+b^3*c)*(-a^3*d+b^3*
c)^2*(b*x+a)^(2+n)/b^11/(2+n)+9*a^2*d*(-5*a^3*d+2*b^3*c)*(-a^3*d+b^3*c)*(b*
x+a)^(3+n)/b^11/(3+n)-3*a*d*(40*a^6*d^2-35*a^3*b^3*c*d+4*b^6*c^2)*(b*x+a)^(
4+n)/b^11/(4+n)+3*d*(70*a^6*d^2-35*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^(5+n)/b^11/
(5+n)+63*a^2*d^2*(-4*a^3*d+b^3*c)*(b*x+a)^(6+n)/b^11/(6+n)-21*a*d^2*(-10*a^
3*d+b^3*c)*(b*x+a)^(7+n)/b^11/(7+n)+3*d^2*(-40*a^3*d+b^3*c)*(b*x+a)^(8+n)/b
```

```
^11/(8+n)+45*a^2*d^3*(b*x+a)^(9+n)/b^11/(9+n)-10*a*d^3*(b*x+a)^(10+n)/b^11/
(10+n)+d^3*(b*x+a)^(11+n)/b^11/(11+n)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1634}

$$\int x(a+bx)^n\left(c+dx^3\right)^3\,dx=-\frac{21ad^2\left(b^3c-10a^3d\right)(a+bx)^{n+7}}{b^{11}(n+7)}+\frac{3d^2\left(b^3c-40a^3d\right)(a+bx)^{n+8}}{b^{11}(n+8)}-\frac{a\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{11}(n+1)}+\frac{\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{11}(n+2)}+\frac{45a^2d^3(a+bx)^{n+9}}{b^{11}(n+9)}-\frac{3ad\left(40a^6d^2-35a^3b^3cd+4b^6c^2\right)(a+bx)^{n+4}}{b^{11}(n+4)}+\frac{3d\left(70a^6d^2-35a^3b^3cd+b^6c^2\right)(a+bx)^{n+5}}{b^{11}(n+5)}+\frac{63a^2d^2\left(b^3c-4a^3d\right)(a+bx)^{n+6}}{b^{11}(n+6)}+\frac{9a^2d\left(2b^3c-5a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+3}}{b^{11}(n+3)}-\frac{10ad^3(a+bx)^{n+10}}{b^{11}(n+10)}+\frac{d^3(a+bx)^{n+11}}{b^{11}(n+11)}$$

```
[In] Int[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] -((a*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^11*(1 + n))) + ((b^3*c - 10*a^
3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^11*(2 + n)) + (9*a^2*d*(2*b^3*
c - 5*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(3 + n))/(b^11*(3 + n)) - (3*a*d*(4*
b^6*c^2 - 35*a^3*b^3*c*d + 40*a^6*d^2)*(a + b*x)^(4 + n))/(b^11*(4 + n)) +
(3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^2)*(a + b*x)^(5 + n))/(b^11*(5 +
n)) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^(6 + n))/(b^11*(6 + n)) - (21
*a*d^2*(b^3*c - 10*a^3*d)*(a + b*x)^(7 + n))/(b^11*(7 + n)) + (3*d^2*(b^3*c
 - 40*a^3*d)*(a + b*x)^(8 + n))/(b^11*(8 + n)) + (45*a^2*d^3*(a + b*x)^(9 +
 n))/(b^11*(9 + n)) - (10*a*d^3*(a + b*x)^(10 + n))/(b^11*(10 + n)) + (d^3*
(a + b*x)^(11 + n))/(b^11*(11 + n))
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
```

```
xpon[Px, x], 2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a(-b^3c+a^3d)^3(a+bx)^n}{b^{10}} + \frac{(b^3c-10a^3d)(b^3c-a^3d)^2(a+bx)^{1+n}}{b^{10}} + \frac{9a^2d(2b^3c-5a^3d)(b^3c-a^3d)(a+bx)^{2+n}}{b^{10}} - \frac{3ad(4b^6c^2-35a^3b^3cd+40a^6d^2)(a+bx)^{3+n}}{b^{10}} + \frac{3d(b^6c^2-35a^3b^3cd+70a^6d^2)(a+bx)^{4+n}}{b^{10}} - \frac{63a^2d^2(-b^3c+4a^3d)(a+bx)^{5+n}}{b^{10}} + \frac{21ad^2(-b^3c+10a^3d)(a+bx)^{6+n}}{b^{10}} + \frac{3d^2(b^3c-40a^3d)(a+bx)^{7+n}}{b^{10}} + \frac{45a^2d^3(a+bx)^{8+n}}{b^{10}} - \frac{10ad^3(a+bx)^{9+n}}{b^{10}} + \frac{d^3(a+bx)^{10+n}}{b^{10}}\right) dx$$

$$= -\frac{a(b^3c-a^3d)^3(a+bx)^{1+n}}{b^{11}(1+n)} + \frac{(b^3c-10a^3d)(b^3c-a^3d)^2(a+bx)^{2+n}}{b^{11}(2+n)} + \frac{9a^2d(2b^3c-5a^3d)(b^3c-a^3d)(a+bx)^{3+n}}{b^{11}(3+n)} - \frac{3ad(4b^6c^2-35a^3b^3cd+40a^6d^2)(a+bx)^{4+n}}{b^{11}(4+n)} + \frac{3d(b^6c^2-35a^3b^3cd+70a^6d^2)(a+bx)^{5+n}}{b^{11}(5+n)} + \frac{63a^2d^2(b^3c-4a^3d)(a+bx)^{6+n}}{b^{11}(6+n)} - \frac{21ad^2(b^3c-10a^3d)(a+bx)^{7+n}}{b^{11}(7+n)} + \frac{3d^2(b^3c-40a^3d)(a+bx)^{8+n}}{b^{11}(8+n)} + \frac{45a^2d^3(a+bx)^{9+n}}{b^{11}(9+n)} - \frac{10ad^3(a+bx)^{10+n}}{b^{11}(10+n)} + \frac{d^3(a+bx)^{11+n}}{b^{11}(11+n)}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.87

$$\int x(a+bx)^n\left(c+dx^3\right)^3 dx$$

$$= \frac{(a+bx)^{1+n}\left(\frac{a(-b^3c+a^3d)^3}{1+n} + \frac{(b^3c-10a^3d)(b^3c-a^3d)^2(a+bx)}{2+n} + \frac{9a^2d(-b^3c+a^3d)(-2b^3c+5a^3d)(a+bx)^2}{3+n} - \frac{3ad(4b^6c^2-35a^3b^3cd+40a^6d^2)}{4+n}\right.}{}$$

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d)^3)/(1 + n) + ((b^3*c - 10*a^3*d)*
(b^3*c - a^3*d)^2*(a + b*x))/(2 + n) + (9*a^2*d*(-(b^3*c) + a^3*d)*(-2*b^3*
c + 5*a^3*d)*(a + b*x)^2)/(3 + n) - (3*a*d*(4*b^6*c^2 - 35*a^3*b^3*c*d + 40
*a^6*d^2)*(a + b*x)^3)/(4 + n) + (3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^
2)*(a + b*x)^4)/(5 + n) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^5)/(6 + n
) + (21*a*d^2*(-(b^3*c) + 10*a^3*d)*(a + b*x)^6)/(7 + n) + (3*d^2*(b^3*c -
40*a^3*d)*(a + b*x)^7)/(8 + n) + (45*a^2*d^3*(a + b*x)^8)/(9 + n) - (10*a*d
^3*(a + b*x)^9)/(10 + n) + (d^3*(a + b*x)^10)/(11 + n)))/b^11
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2971 vs. $2(396) = 792$.

Time = 1.16 (sec) , antiderivative size = 2972, normalized size of antiderivative = 7.51

method	result	size
gosper	Expression too large to display	2972
risch	Expression too large to display	3409
parallelrisch	Expression too large to display	4900

```
[In] int(x*(b*x+a)^n*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^11*(b*x+a)^(1+n)/(n^11+66*n^10+1925*n^9+32670*n^8+357423*n^7+2637558*n^
6+13339535*n^5+45995730*n^4+105258076*n^3+150917976*n^2+120543840*n+3991680
0)*(b^10*d^3*n^10*x^10+55*b^10*d^3*n^9*x^10-10*a*b^9*d^3*n^9*x^9+1320*b^10*
d^3*n^8*x^10-450*a*b^9*d^3*n^8*x^9+3*b^10*c*d^2*n^10*x^7+18150*b^10*d^3*n^7
*x^10+90*a^2*b^8*d^3*n^8*x^8-8700*a*b^9*d^3*n^7*x^9+174*b^10*c*d^2*n^9*x^7+
157773*b^10*d^3*n^6*x^10+3240*a^2*b^8*d^3*n^7*x^8-21*a*b^9*c*d^2*n^9*x^6-94
500*a*b^9*d^3*n^6*x^9+4383*b^10*c*d^2*n^8*x^7+902055*b^10*d^3*n^5*x^10-720*
a^3*b^7*d^3*n^7*x^7+49140*a^2*b^8*d^3*n^6*x^8-1071*a*b^9*c*d^2*n^8*x^6-6327
30*a*b^9*d^3*n^5*x^9+3*b^10*c^2*d*n^10*x^4+62946*b^10*c*d^2*n^7*x^7+3416930
*b^10*d^3*n^4*x^10-20160*a^3*b^7*d^3*n^6*x^7+126*a^2*b^8*c*d^2*n^8*x^5+4082
40*a^2*b^8*d^3*n^5*x^8-23184*a*b^9*c*d^2*n^7*x^6-2693250*a*b^9*d^3*n^4*x^9+
183*b^10*c^2*d*n^9*x^4+568701*b^10*c*d^2*n^6*x^7+8409500*b^10*d^3*n^3*x^10+
5040*a^4*b^6*d^3*n^6*x^6-231840*a^3*b^7*d^3*n^5*x^7+5670*a^2*b^8*c*d^2*n^7*
x^5+2020410*a^2*b^8*d^3*n^4*x^8-12*a*b^9*c^2*d*n^9*x^3-278334*a*b^9*c*d^2*n
^6*x^6-7236800*a*b^9*d^3*n^3*x^9+4860*b^10*c^2*d*n^8*x^4+3363066*b^10*c*d^2
*n^5*x^7+12753576*b^10*d^3*n^2*x^10+105840*a^4*b^6*d^3*n^5*x^6-630*a^3*b^7*
c*d^2*n^7*x^4-1411200*a^3*b^7*d^3*n^4*x^7+105084*a^2*b^8*c*d^2*n^6*x^5+6055
560*a^2*b^8*d^3*n^3*x^8-684*a*b^9*c^2*d*n^8*x^3-2032569*a*b^9*c*d^2*n^5*x^6
-11727000*a*b^9*d^3*n^2*x^9+b^10*c^3*n^10*x+73710*b^10*c^2*d*n^7*x^4+131140
77*b^10*c*d^2*n^4*x^7+10628640*b^10*d^3*n*x^10-30240*a^5*b^5*d^3*n^5*x^5+88
2000*a^4*b^6*d^3*n^4*x^6-25200*a^3*b^7*c*d^2*n^6*x^4-4873680*a^3*b^7*d^3*n^
3*x^7+36*a^2*b^8*c^2*d*n^8*x^2+1039500*a^2*b^8*c*d^2*n^5*x^5+10631160*a^2*b
^8*d^3*n^2*x^8-16704*a*b^9*c^2*d*n^7*x^3-9313479*a*b^9*c*d^2*n^4*x^6-102657
```

60*a*b^9*d^3*n*x^9+64*b^10*c^3*n^9*x+703719*b^10*c^2*d*n^6*x^4+33074574*b^1
0*c*d^2*n^3*x^7+3628800*b^10*d^3*x^10-453600*a^5*b^5*d^3*n^4*x^5+2520*a^4*b
^6*c*d^2*n^6*x^3+3704400*a^4*b^6*d^3*n^3*x^6-399420*a^3*b^7*c*d^2*n^5*x^4-9
455040*a^3*b^7*d^3*n^2*x^7+1944*a^2*b^8*c^2*d*n^7*x^2+5958414*a^2*b^8*c*d^2
*n^4*x^5+9862560*a^2*b^8*d^3*n*x^8-a*b^9*c^3*n^9-228024*a*b^9*c^2*d*n^6*x^3
-26604186*a*b^9*c*d^2*n^3*x^6-3628800*a*b^9*d^3*x^9+1797*b^10*c^3*n^8*x+439
4079*b^10*c^2*d*n^5*x^4+51177636*b^10*c*d^2*n^2*x^7+151200*a^6*b^4*d^3*n^4*
x^4-2570400*a^5*b^5*d^3*n^3*x^5+90720*a^4*b^6*c*d^2*n^5*x^3+8184960*a^4*b^6
*d^3*n^2*x^6-72*a^3*b^7*c^2*d*n^7*x-3200400*a^3*b^7*c*d^2*n^4*x^4-9408960*a
^3*b^7*d^3*n*x^7+44280*a^2*b^8*c^2*d*n^6*x^2+20130390*a^2*b^8*c*d^2*n^3*x^5
+3628800*a^2*b^8*d^3*x^8-63*a*b^9*c^3*n^8-1902780*a*b^9*c^2*d*n^5*x^3-45292
716*a*b^9*c*d^2*n^2*x^6+29076*b^10*c^3*n^7*x+18048210*b^10*c^2*d*n^4*x^4+43
332840*b^10*c*d^2*n*x^7+1512000*a^6*b^4*d^3*n^3*x^4-7560*a^5*b^5*c*d^2*n^5*
x^2-6804000*a^5*b^5*d^3*n^2*x^5+1234800*a^4*b^6*c*d^2*n^4*x^3+8890560*a^4*b
^6*d^3*n*x^6-3744*a^3*b^7*c^2*d*n^6*x-13790070*a^3*b^7*c*d^2*n^3*x^4-362880
0*a^3*b^7*d^3*x^7+551232*a^2*b^8*c^2*d*n^5*x^2+38842776*a^2*b^8*c*d^2*n^2*x
^5-1734*a*b^9*c^3*n^7-9965196*a*b^9*c^2*d*n^4*x^3-41194440*a*b^9*c*d^2*n*x^
6+299271*b^10*c^3*n^6*x+47746140*b^10*c^2*d*n^3*x^4+14968800*b^10*c*d^2*x^7
-604800*a^7*b^3*d^3*n^3*x^3+5292000*a^6*b^4*d^3*n^2*x^4-249480*a^5*b^5*c*d^
2*n^4*x^2-8285760*a^5*b^5*d^3*n*x^5+72*a^4*b^6*c^2*d*n^6+7862400*a^4*b^6*c*
d^2*n^3*x^3+3628800*a^4*b^6*d^3*x^6-81072*a^3*b^7*c^2*d*n^5*x-31701600*a^3*
b^7*c*d^2*n^2*x^4+4054644*a^2*b^8*c^2*d*n^4*x^2+38699640*a^2*b^8*c*d^2*n*x^
5-27342*a*b^9*c^3*n^6-32332056*a*b^9*c^2*d*n^3*x^3-14968800*a*b^9*c*d^2*x^6
+2039016*b^10*c^3*n^5*x+77043528*b^10*c^2*d*n^2*x^4-3628800*a^7*b^3*d^3*n^2
*x^3+15120*a^6*b^4*c*d^2*n^4*x+7560000*a^6*b^4*d^3*n*x^4-2955960*a^5*b^5*c*
d^2*n^3*x^2-3628800*a^5*b^5*d^3*x^5+3672*a^4*b^6*c^2*d*n^5+23710680*a^4*b^6
*c*d^2*n^2*x^3-940320*a^3*b^7*c^2*d*n^4*x-35705880*a^3*b^7*c*d^2*n*x^4+1773
1656*a^2*b^8*c^2*d*n^3*x^2+14968800*a^2*b^8*c*d^2*x^5-271929*a*b^9*c^3*n^5-
61656336*a*b^9*c^2*d*n^2*x^3+9261503*b^10*c^3*n^4*x+67536288*b^10*c^2*d*n*x
^4+1814400*a^8*b^2*d^3*n^2*x^2-6652800*a^7*b^3*d^3*n*x^3+468720*a^6*b^4*c*d
^2*n^3*x+3628800*a^6*b^4*d^3*x^4-14719320*a^5*b^5*c*d^2*n^2*x^2+77400*a^4*b
^6*c^2*d*n^4+31963680*a^4*b^6*c*d^2*n*x^3-6228648*a^3*b^7*c^2*d*n^3*x-14968
800*a^3*b^7*c*d^2*x^4+43801200*a^2*b^8*c^2*d*n^2*x^2-1767087*a*b^9*c^3*n^4-
61548768*a*b^9*c^2*d*n*x^3+27472724*b^10*c^3*n^3*x+23950080*b^10*c^2*d*x^4+
5443200*a^8*b^2*d^3*n*x^2-15120*a^7*b^3*c*d^2*n^3-3628800*a^7*b^3*d^3*x^3+4
974480*a^6*b^4*c*d^2*n^2*x-26974080*a^5*b^5*c*d^2*n*x^2+862920*a^4*b^6*c^2*
d*n^3+14968800*a^4*b^6*c*d^2*x^3-23006016*a^3*b^7*c^2*d*n^2*x+53565408*a^2*
b^8*c^2*d*n*x^2-7494416*a*b^9*c^3*n^3-23950080*a*b^9*c^2*d*x^3+50312628*b^1
0*c^3*n^2*x-3628800*a^9*b*d^3*n*x+3628800*a^8*b^2*d^3*x^2-453600*a^7*b^3*c*
d^2*n^2+19489680*a^6*b^4*c*d^2*n*x-14968800*a^5*b^5*c*d^2*x^2+5365728*a^4*b
^6*c^2*d*n^2-41590368*a^3*b^7*c^2*d*n*x+23950080*a^2*b^8*c^2*d*x^2-19978308
*a*b^9*c^3*n^2+50292720*b^10*c^3*n*x-3628800*a^9*b*d^3*x-4520880*a^7*b^3*c*
d^2*n+14968800*a^6*b^4*c*d^2*x+17640288*a^4*b^6*c^2*d*n-23950080*a^3*b^7*c^
2*d*x-30334320*a*b^9*c^3*n+19958400*b^10*c^3*x+3628800*a^10*d^3-14968800*a^
7*b^3*c*d^2+23950080*a^4*b^6*c^2*d-19958400*a*b^9*c^3)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2919 vs. 2(396) = 792.

Time = 0.32 (sec) , antiderivative size = 2919, normalized size of antiderivative = 7.37

$$\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*b^9*c^3*n^9 + 63*a^2*b^9*c^3*n^8 + 1734*a^2*b^9*c^3*n^7 + 19958400*a^
2*b^9*c^3 - 23950080*a^5*b^6*c^2*d + 14968800*a^8*b^3*c*d^2 - 3628800*a^11*
d^3 - (b^11*d^3*n^10 + 55*b^11*d^3*n^9 + 1320*b^11*d^3*n^8 + 18150*b^11*d^3
*n^7 + 157773*b^11*d^3*n^6 + 902055*b^11*d^3*n^5 + 3416930*b^11*d^3*n^4 + 8
409500*b^11*d^3*n^3 + 12753576*b^11*d^3*n^2 + 10628640*b^11*d^3*n + 3628800
*b^11*d^3)*x^11 - (a*b^10*d^3*n^10 + 45*a*b^10*d^3*n^9 + 870*a*b^10*d^3*n^8
 + 9450*a*b^10*d^3*n^7 + 63273*a*b^10*d^3*n^6 + 269325*a*b^10*d^3*n^5 + 723
680*a*b^10*d^3*n^4 + 1172700*a*b^10*d^3*n^3 + 1026576*a*b^10*d^3*n^2 + 3628
80*a*b^10*d^3*n)*x^10 + 10*(a^2*b^9*d^3*n^9 + 36*a^2*b^9*d^3*n^8 + 546*a^2*
b^9*d^3*n^7 + 4536*a^2*b^9*d^3*n^6 + 22449*a^2*b^9*d^3*n^5 + 67284*a^2*b^9*
d^3*n^4 + 118124*a^2*b^9*d^3*n^3 + 109584*a^2*b^9*d^3*n^2 + 40320*a^2*b^9*d
^3*n)*x^9 - 3*(b^11*c*d^2*n^10 + 58*b^11*c*d^2*n^9 + 4989600*b^11*c*d^2 + 3
*(487*b^11*c*d^2 + 10*a^3*b^8*d^3)*n^8 + 6*(3497*b^11*c*d^2 + 140*a^3*b^8*d
^3)*n^7 + 21*(9027*b^11*c*d^2 + 460*a^3*b^8*d^3)*n^6 + 294*(3813*b^11*c*d^2
 + 200*a^3*b^8*d^3)*n^5 + (4371359*b^11*c*d^2 + 203070*a^3*b^8*d^3)*n^4 + 2
*(5512429*b^11*c*d^2 + 196980*a^3*b^8*d^3)*n^3 + 36*(473867*b^11*c*d^2 + 10
890*a^3*b^8*d^3)*n^2 + 360*(40123*b^11*c*d^2 + 420*a^3*b^8*d^3)*n)*x^8 - 3*
(a*b^10*c*d^2*n^10 + 51*a*b^10*c*d^2*n^9 + 1104*a*b^10*c*d^2*n^8 + 6*(2209*
a*b^10*c*d^2 - 40*a^4*b^7*d^3)*n^7 + 21*(4609*a*b^10*c*d^2 - 240*a^4*b^7*d^
3)*n^6 + 21*(21119*a*b^10*c*d^2 - 2000*a^4*b^7*d^3)*n^5 + 2*(633433*a*b^10*
c*d^2 - 88200*a^4*b^7*d^3)*n^4 + 12*(179733*a*b^10*c*d^2 - 32480*a^4*b^7*d^
3)*n^3 + 360*(5449*a*b^10*c*d^2 - 1176*a^4*b^7*d^3)*n^2 + 21600*(33*a*b^10*
c*d^2 - 8*a^4*b^7*d^3)*n)*x^7 + 18*(1519*a^2*b^9*c^3 - 4*a^5*b^6*c^2*d)*n^6
 + 21*(a^2*b^9*c*d^2*n^9 + 45*a^2*b^9*c*d^2*n^8 + 834*a^2*b^9*c*d^2*n^7 + 3
0*(275*a^2*b^9*c*d^2 - 8*a^5*b^6*d^3)*n^6 + 3*(15763*a^2*b^9*c*d^2 - 1200*a
^5*b^6*d^3)*n^5 + 15*(10651*a^2*b^9*c*d^2 - 1360*a^5*b^6*d^3)*n^4 + 4*(7706
9*a^2*b^9*c*d^2 - 13500*a^5*b^6*d^3)*n^3 + 60*(5119*a^2*b^9*c*d^2 - 1096*a^
5*b^6*d^3)*n^2 + 3600*(33*a^2*b^9*c*d^2 - 8*a^5*b^6*d^3)*n)*x^6 + 3*(90643*
a^2*b^9*c^3 - 1224*a^5*b^6*c^2*d)*n^5 - 3*(b^11*c^2*d*n^10 + 61*b^11*c^2*d*
n^9 + 7983360*b^11*c^2*d + 6*(270*b^11*c^2*d + 7*a^3*b^8*c*d^2)*n^8 + 210*(
117*b^11*c^2*d + 8*a^3*b^8*c*d^2)*n^7 + 3*(78191*b^11*c^2*d + 8876*a^3*b^8*
c*d^2)*n^6 + 3*(488231*b^11*c^2*d + 71120*a^3*b^8*c*d^2 - 3360*a^6*b^5*d^3)
*n^5 + 2*(3008035*b^11*c^2*d + 459669*a^3*b^8*c*d^2 - 50400*a^6*b^5*d^3)*n^
4 + 20*(795769*b^11*c^2*d + 105672*a^3*b^8*c*d^2 - 17640*a^6*b^5*d^3)*n^3 +
 72*(356683*b^11*c^2*d + 33061*a^3*b^8*c*d^2 - 7000*a^6*b^5*d^3)*n^2 + 288*
```

```
(78167*b^11*c^2*d + 3465*a^3*b^8*c*d^2 - 840*a^6*b^5*d^3)*n)*x^5 + 9*(19634
3*a^2*b^9*c^3 - 8600*a^5*b^6*c^2*d)*n^4 - 3*(a*b^10*c^2*d*n^10 + 57*a*b^10*
c^2*d*n^9 + 1392*a*b^10*c^2*d*n^8 + 6*(3167*a*b^10*c^2*d - 35*a^4*b^7*c*d^2
)*n^7 + 15*(10571*a*b^10*c^2*d - 504*a^4*b^7*c*d^2)*n^6 + 3*(276811*a*b^10*
c^2*d - 34300*a^4*b^7*c*d^2)*n^5 + 2*(1347169*a*b^10*c^2*d - 327600*a^4*b^7
*c*d^2 + 25200*a^7*b^4*d^3)*n^4 + 42*(122334*a*b^10*c^2*d - 47045*a^4*b^7*c
*d^2 + 7200*a^7*b^4*d^3)*n^3 + 72*(71237*a*b^10*c^2*d - 36995*a^4*b^7*c*d^2
 + 7700*a^7*b^4*d^3)*n^2 + 7560*(264*a*b^10*c^2*d - 165*a^4*b^7*c*d^2 + 40*
a^7*b^4*d^3)*n)*x^4 + 8*(936802*a^2*b^9*c^3 - 107865*a^5*b^6*c^2*d + 1890*a
^8*b^3*c*d^2)*n^3 + 12*(a^2*b^9*c^2*d*n^9 + 54*a^2*b^9*c^2*d*n^8 + 1230*a^2
*b^9*c^2*d*n^7 + 6*(2552*a^2*b^9*c^2*d - 35*a^5*b^6*c*d^2)*n^6 + 33*(3413*a
^2*b^9*c^2*d - 210*a^5*b^6*c*d^2)*n^5 + 6*(82091*a^2*b^9*c^2*d - 13685*a^5*
b^6*c*d^2)*n^4 + 10*(121670*a^2*b^9*c^2*d - 40887*a^5*b^6*c*d^2 + 5040*a^8*
b^3*d^3)*n^3 + 24*(61997*a^2*b^9*c^2*d - 31220*a^5*b^6*c*d^2 + 6300*a^8*b^3
*d^3)*n^2 + 2520*(264*a^2*b^9*c^2*d - 165*a^5*b^6*c*d^2 + 40*a^8*b^3*d^3)*n
)*x^3 + 36*(554953*a^2*b^9*c^3 - 149048*a^5*b^6*c^2*d + 12600*a^8*b^3*c*d^2
)*n^2 - (b^11*c^3*n^10 + 64*b^11*c^3*n^9 + 19958400*b^11*c^3 + 3*(599*b^11*
c^3 + 12*a^3*b^8*c^2*d)*n^8 + 12*(2423*b^11*c^3 + 156*a^3*b^8*c^2*d)*n^7 +
3*(99757*b^11*c^3 + 13512*a^3*b^8*c^2*d)*n^6 + 24*(84959*b^11*c^3 + 19590*a
^3*b^8*c^2*d - 315*a^6*b^5*c*d^2)*n^5 + (9261503*b^11*c^3 + 3114324*a^3*b^8
*c^2*d - 234360*a^6*b^5*c*d^2)*n^4 + 4*(6868181*b^11*c^3 + 2875752*a^3*b^8*
c^2*d - 621810*a^6*b^5*c*d^2)*n^3 + 36*(1397573*b^11*c^3 + 577644*a^3*b^8*c
^2*d - 270690*a^6*b^5*c*d^2 + 50400*a^9*b^2*d^3)*n^2 + 720*(69851*b^11*c^3
+ 16632*a^3*b^8*c^2*d - 10395*a^6*b^5*c*d^2 + 2520*a^9*b^2*d^3)*n)*x^2 + 14
4*(210655*a^2*b^9*c^3 - 122502*a^5*b^6*c^2*d + 31395*a^8*b^3*c*d^2)*n - (a*
b^10*c^3*n^10 + 63*a*b^10*c^3*n^9 + 1734*a*b^10*c^3*n^8 + 18*(1519*a*b^10*c
^3 - 4*a^4*b^7*c^2*d)*n^7 + 3*(90643*a*b^10*c^3 - 1224*a^4*b^7*c^2*d)*n^6 +
 9*(196343*a*b^10*c^3 - 8600*a^4*b^7*c^2*d)*n^5 + 8*(936802*a*b^10*c^3 - 10
7865*a^4*b^7*c^2*d + 1890*a^7*b^4*c*d^2)*n^4 + 36*(554953*a*b^10*c^3 - 1490
48*a^4*b^7*c^2*d + 12600*a^7*b^4*c*d^2)*n^3 + 144*(210655*a*b^10*c^3 - 1225
02*a^4*b^7*c^2*d + 31395*a^7*b^4*c*d^2)*n^2 + 90720*(220*a*b^10*c^3 - 264*a
^4*b^7*c^2*d + 165*a^7*b^4*c*d^2 - 40*a^10*b*d^3)*n)*x)*(b*x + a)^n/(b^11*n
^11 + 66*b^11*n^10 + 1925*b^11*n^9 + 32670*b^11*n^8 + 357423*b^11*n^7 + 263
7558*b^11*n^6 + 13339535*b^11*n^5 + 45995730*b^11*n^4 + 105258076*b^11*n^3
+ 150917976*b^11*n^2 + 120543840*b^11*n + 39916800*b^11)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56151 vs. $2(374) = 748$.

Time = 34.12 (sec) , antiderivative size = 56151, normalized size of antiderivative = 141.80

$$\int x(a+bx)^n \left(c+dx^3\right)^3 dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x+a)**n*(d*x**3+c)**3,x)
```

[Out] Piecewise((a**n*(c**3*x**2/2 + 3*c**2*d*x**5/5 + 3*c*d**2*x**8/8 + d**3*x**11/11), Eq(b, 0)), (2520*a**10*d**3*log(a/b + x)/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 7381*a**10*d**3/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 25200*a**9*b*d**3*x*log(a/b + x)/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 71290*a**9*b*d**3*x/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 113400*a**8*b**2*d**3*x**2*log(a/b + x)/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 308205*a**8*b**2*d**3*x**2/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) - 21*a**7*b**3*c*d**2/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 302400*a**7*b**3*d**3*x**3*log(a/b + x)/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 784080*a**7*b**3*d**3*x**3/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) - 210*a**6*b**4*c*d**2*x/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 529200*a**6*b**4*d**3*x**4*log(a/b + x)/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 1296540*a**6*b**4*d**3*x**4/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x

```
**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x
**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) - 945
*a**5*b**5*c*d**2*x**2/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8
*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5
*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2
*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 635040*a**5*b**5*d**
3*x**5*log(a/b + x)/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b*
*13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b*
*16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b*
*19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 1450008*a**5*b**5*d**3*
x**5/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 3024
00*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 5292
00*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 2520
0*a*b**20*x**9 + 2520*b**21*x**10) - 6*a**4*b**6*c**2*d/(2520*a**10*b**11 +
 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529
200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302
400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b*
*21*x**10) - 2520*a**4*b**6*c*d**2*x**3/(2520*a**10*b**11 + 25200*a**9*b**1
2*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x
**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x
**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 529
200*a**4*b**6*d**3*x**6*log(a/b + x)/(2520*a**10*b**11 + 25200*a**9*b**12*x
 + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4
 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7
 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 110250
0*a**4*b**6*d**3*x**6/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*
b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*
b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*
b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) - 60*a**3*b**7*c**2*d*x
/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a
**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a
**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*
b**20*x**9 + 2520*b**21*x**10) - 4410*a**3*b**7*c*d**2*x**4/(2520*a**10*b**
11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 +
 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 +
 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 252
0*b**21*x**10) + 302400*a**3*b**7*d**3*x**7*log(a/b + x)/(2520*a**10*b**11
+ 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 52
9200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 30
2400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b
**21*x**10) + 554400*a**3*b**7*d**3*x**7/(2520*a**10*b**11 + 25200*a**9*b**
12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*
x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*
x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) - 27
0*a**2*b**8*c**2*d*x**2/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**
```

8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) - 5292*a**2*b**8*c*d**2*x**5/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 113400*a**2*b**8*d**3*x**8*log(a/b + x)/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 170100*a**2*b**8*d**3*x**8/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) - 28*a*b**9*c**3/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) - 720*a*b**9*c**2*d*x**3/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) - 4410*a*b**9*c*d**2*x**6/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 25200*a*b**9*d**3*x**9*log(a/b + x)/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 25200*a*b**9*d**3*x**9/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) - 280*b**10*c**3*x/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) - 1260*b**10*c**2*d*x**4/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) - 2520*b**10*c*d**2*x**7/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 2520*b**10*d**3*x**10*log(a/b + x)/(2520*a**10*b**11 + 25200*a**

9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b **15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b **18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) , Eq(n, -11)), (-25200*a**10*d**3*log(a/b + x)/(2520*a**9*b**11 + 22680*a** 8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520*a**5*b* *15*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*a**2*b** 18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 71290*a**10*d**3/(2520*a* *9*b**11 + 22680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a**6*b**14*x **3 + 317520*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b**17*x **6 + 90720*a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 22680 0*a**9*b*d**3*x*log(a/b + x)/(2520*a**9*b**11 + 22680*a**8*b**12*x + 90720* a**7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520*a**5*b**15*x**4 + 317520* a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22680*a* b**19*x**8 + 2520*b**20*x**9) - 616410*a**9*b*d**3*x/(2520*a**9*b**11 + 226 80*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520*a **5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*a* *2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 907200*a**8*b**2*d* *3*x**2*log(a/b + x)/(2520*a**9*b**11 + 22680*a**8*b**12*x + 90720*a**7*b** 13*x**2 + 211680*a**6*b**14*x**3 + 317520*a**5*b**15*x**4 + 317520*a**4*b** 16*x**5 + 211680*a**3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b**19*x* *8 + 2520*b**20*x**9) - 2352240*a**8*b**2*d**3*x**2/(2520*a**9*b**11 + 2268 0*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520*a* *5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*a** 2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 105*a**7*b**3*c*d**2 /(2520*a**9*b**11 + 22680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a** 6*b**14*x**3 + 317520*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*a** 3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 2116800*a**7*b**3*d**3*x**3*log(a/b + x)/(2520*a**9*b**11 + 22680*a**8* b**12*x + 90720*a**7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520*a**5*b**1 5*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*a**2*b**18 *x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 5186160*a**7*b**3*d**3*x**3 /(2520*a**9*b**11 + 22680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a** 6*b**14*x**3 + 317520*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*a** 3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 945*a**6*b**4*c*d**2*x/(2520*a**9*b**11 + 22680*a**8*b**12*x + 90720*a* *7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520*a**5*b**15*x**4 + 317520*a* *4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b* *19*x**8 + 2520*b**20*x**9) - 3175200*a**6*b**4*d**3*x**4*log(a/b + x)/(252 0*a**9*b**11 + 22680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a**6*b** 14*x**3 + 317520*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b** 17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 7 250040*a**6*b**4*d**3*x**4/(2520*a**9*b**11 + 22680*a**8*b**12*x + 90720*a* *7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520*a**5*b**15*x**4 + 317520*a* *4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b* *19*x**8 + 2520*b**20*x**9) - 3780*a**5*b**5*c*d**2*x**2/(2520*a**9*b**11 +

22680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a**6*b**14*x**3 + 3175
20*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 9072
0*a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 3175200*a**5*b*
*5*d**3*x**5*log(a/b + x)/(2520*a**9*b**11 + 22680*a**8*b**12*x + 90720*a**
7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520*a**5*b**15*x**4 + 317520*a**
4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b**
19*x**8 + 2520*b**20*x**9) - 6615000*a**5*b**5*d**3*x**5/(2520*a**9*b**11 +
22680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a**6*b**14*x**3 + 3175
20*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 9072
0*a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 12*a**4*b**6*c*
*2*d/(2520*a**9*b**11 + 22680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680
*a**6*b**14*x**3 + 317520*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680
*a**3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*
x**9) - 8820*a**4*b**6*c*d**2*x**3/(2520*a**9*b**11 + 22680*a**8*b**12*x +
90720*a**7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520*a**5*b**15*x**4 + 3
17520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22
680*a*b**19*x**8 + 2520*b**20*x**9) - 2116800*a**4*b**6*d**3*x**6*log(a/b +
x)/(2520*a**9*b**11 + 22680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*
a**6*b**14*x**3 + 317520*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*
a**3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x
9) - 3880800*a4*b**6*d**3*x**6/(2520*a**9*b**11 + 22680*a**8*b**12*x +
90720*a**7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520*a**5*b**15*x**4 + 3
17520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22
680*a*b**19*x**8 + 2520*b**20*x**9) - 108*a**3*b**7*c**2*d*x/(2520*a**9*b**
11 + 22680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a**6*b**14*x**3 +
317520*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 +
90720*a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 13230*a**3*
b**7*c*d**2*x**4/(2520*a**9*b**11 + 22680*a**8*b**12*x + 90720*a**7*b**13*x
2 + 211680*a6*b**14*x**3 + 317520*a**5*b**15*x**4 + 317520*a**4*b**16*x
5 + 211680*a3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b**19*x**8 +
2520*b**20*x**9) - 907200*a**3*b**7*d**3*x**7*log(a/b + x)/(2520*a**9*b**1
1 + 22680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a**6*b**14*x**3 + 3
17520*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 9
0720*a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 1360800*a**3
*b**7*d**3*x**7/(2520*a**9*b**11 + 22680*a**8*b**12*x + 90720*a**7*b**13*x*
*2 + 211680*a**6*b**14*x**3 + 317520*a**5*b**15*x**4 + 317520*a**4*b**16*x*
*5 + 211680*a**3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b**19*x**8 +
2520*b**20*x**9) - 432*a**2*b**8*c**2*d*x**2/(2520*a**9*b**11 + 22680*a**8*
b**12*x + 90720*a**7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520*a**5*b**1
5*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*a**2*b**18
*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 13230*a**2*b**8*c*d**2*x**5
/(2520*a**9*b**11 + 22680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a**
6*b**14*x**3 + 317520*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*a**
3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9
) - 226800*a**2*b**8*d**3*x**8*log(a/b + x)/(2520*a**9*b**11 + 22680*a**8*b

12*x + 90720*a7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520*a**5*b**15
*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*a**2*b**18*
x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 226800*a**2*b**8*d**3*x**8/(
2520*a**9*b**11 + 22680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a**6*
b**14*x**3 + 317520*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*
b**17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9)
- 35*a*b**9*c**3/(2520*a**9*b**11 + 22680*a**8*b**12*x + 90720*a**7*b**13*x
2 + 211680*a6*b**14*x**3 + 317520*a**5*b**15*x**4 + 317520*a**4*b**16*x
5 + 211680*a3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b**19*x**8 +
 2520*b**20*x**9) - 1008*a*b**9*c**2*d*x**3/(2520*a**9*b**11 + 22680*a**8*b
12*x + 90720*a7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520*a**5*b**15
*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*a**2*b**18*
x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 8820*a*b**9*c*d**2*x**6/(252
0*a**9*b**11 + 22680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a**6*b**
14*x**3 + 317520*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b**
17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 2
5200*a*b**9*d**3*x**9*log(a/b + x)/(2520*a**9*b**11 + 22680*a**8*b**12*x +
90720*a**7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520*a**5*b**15*x**4 + 3
17520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22
680*a*b**19*x**8 + 2520*b**20*x**9) - 315*b**10*c**3*x/(2520*a**9*b**11 + 2
2680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520
*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*
a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9) - 1512*b**10*c**2*d
*x**4/(2520*a**9*b**11 + 22680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 21168
0*a**6*b**14*x**3 + 317520*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 21168
0*a**3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20
*x**9) - 3780*b**10*c*d**2*x**7/(2520*a**9*b**11 + 22680*a**8*b**12*x + 907
20*a**7*b**13*x**2 + 211680*a**6*b**14*x**3 + 317520*a**5*b**15*x**4 + 3175
20*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 90720*a**2*b**18*x**7 + 22680
*a*b**19*x**8 + 2520*b**20*x**9) + 2520*b**10*d**3*x**10/(2520*a**9*b**11 +
 22680*a**8*b**12*x + 90720*a**7*b**13*x**2 + 211680*a**6*b**14*x**3 + 3175
20*a**5*b**15*x**4 + 317520*a**4*b**16*x**5 + 211680*a**3*b**17*x**6 + 9072
0*a**2*b**18*x**7 + 22680*a*b**19*x**8 + 2520*b**20*x**9), Eq(n, -10)), (12
600*a**10*d**3*log(a/b + x)/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6
*b**13*x**2 + 15680*a**5*b**14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b*
*16*x**5 + 7840*a**2*b**17*x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) + 342
45*a**10*d**3/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 +
15680*a**5*b**14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 784
0*a**2*b**17*x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) + 100800*a**9*b*d**
3*x*log(a/b + x)/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2
 + 15680*a**5*b**14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 +
7840*a**2*b**17*x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) + 261360*a**9*b*
d**3*x/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a
5*b14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*
b**17*x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) + 352800*a**8*b**2*d**3*x*

*2*log(a/b + x)/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2
+ 15680*a**5*b**14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7
840*a**2*b**17*x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) + 864360*a**8*b**
2*d**3*x**2/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15
680*a**5*b**14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*
a**2*b**17*x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) - 105*a**7*b**3*c*d**
2/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b
14*x3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17
*x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) + 705600*a**7*b**3*d**3*x**3*lo
g(a/b + x)/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 156
80*a**5*b**14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a
2*b17*x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) + 1611120*a**7*b**3*d*
*3*x**3/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*
a**5*b**14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2
*b**17*x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) - 840*a**6*b**4*c*d**2*x/
(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b**
14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*x
6 + 2240*a*b18*x**7 + 280*b**19*x**8) + 882000*a**6*b**4*d**3*x**4*log(
a/b + x)/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680
*a**5*b**14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**
2*b**17*x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) + 1837500*a**6*b**4*d**3
*x**4/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a*
*5*b**14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b
17*x6 + 2240*a*b**18*x**7 + 280*b**19*x**8) - 2940*a**5*b**5*c*d**2*x**
2/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b
14*x3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17
*x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) + 705600*a**5*b**5*d**3*x**5*lo
g(a/b + x)/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 156
80*a**5*b**14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a
2*b17*x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) + 1293600*a**5*b**5*d*
*3*x**5/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*
a**5*b**14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2
*b**17*x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) - 3*a**4*b**6*c**2*d/(280
*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b**14*x
3 + 19600*a4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*x**6
+ 2240*a*b**18*x**7 + 280*b**19*x**8) - 5880*a**4*b**6*c*d**2*x**3/(280*a**
8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b**14*x**3
+ 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*x**6 + 22
40*a*b**18*x**7 + 280*b**19*x**8) + 352800*a**4*b**6*d**3*x**6*log(a/b + x)
/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b*
*14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*
x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) + 529200*a**4*b**6*d**3*x**6/(28
0*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b**14*
x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*x**6
+ 2240*a*b**18*x**7 + 280*b**19*x**8) - 24*a**3*b**7*c**2*d*x/(280*a**8*b*

*11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b**14*x**3 + 19
600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*x**6 + 2240*a
*b**18*x**7 + 280*b**19*x**8) - 7350*a**3*b**7*c*d**2*x**4/(280*a**8*b**11
+ 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b**14*x**3 + 19600*
a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*x**6 + 2240*a*b**
18*x**7 + 280*b**19*x**8) + 100800*a**3*b**7*d**3*x**7*log(a/b + x)/(280*a*
*8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b**14*x**3
+ 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*x**6 + 2
240*a*b**18*x**7 + 280*b**19*x**8) + 100800*a**3*b**7*d**3*x**7/(280*a**8*b
11 + 2240*a7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b**14*x**3 + 1
9600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*x**6 + 2240*
a*b**18*x**7 + 280*b**19*x**8) - 84*a**2*b**8*c**2*d*x**2/(280*a**8*b**11 +
2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b**14*x**3 + 19600*a
4*b15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*x**6 + 2240*a*b**1
8*x**7 + 280*b**19*x**8) - 5880*a**2*b**8*c*d**2*x**5/(280*a**8*b**11 + 224
0*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b**14*x**3 + 19600*a**4*
b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*x**6 + 2240*a*b**18*x*
*7 + 280*b**19*x**8) + 12600*a**2*b**8*d**3*x**8*log(a/b + x)/(280*a**8*b**
11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b**14*x**3 + 196
00*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*x**6 + 2240*a*
b**18*x**7 + 280*b**19*x**8) - 5*a*b**9*c**3/(280*a**8*b**11 + 2240*a**7*b*
*12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b**14*x**3 + 19600*a**4*b**15*x**
4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*x**6 + 2240*a*b**18*x**7 + 280*
b**19*x**8) - 168*a*b**9*c**2*d*x**3/(280*a**8*b**11 + 2240*a**7*b**12*x +
7840*a**6*b**13*x**2 + 15680*a**5*b**14*x**3 + 19600*a**4*b**15*x**4 + 1568
0*a**3*b**16*x**5 + 7840*a**2*b**17*x**6 + 2240*a*b**18*x**7 + 280*b**19*x*
*8) - 2940*a*b**9*c*d**2*x**6/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a*
*6*b**13*x**2 + 15680*a**5*b**14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*
b**16*x**5 + 7840*a**2*b**17*x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) - 1
400*a*b**9*d**3*x**9/(280*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*
x**2 + 15680*a**5*b**14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**
5 + 7840*a**2*b**17*x**6 + 2240*a*b**18*x**7 + 280*b**19*x**8) - 40*b**10*c
3*x/(280*a8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a*
*5*b**14*x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b
17*x6 + 2240*a*b**18*x**7 + 280*b**19*x**8) - 210*b**10*c**2*d*x**4/(28
0*a**8*b**11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b**14*
x**3 + 19600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*x**6
+ 2240*a*b**18*x**7 + 280*b**19*x**8) - 840*b**10*c*d**2*x**7/(280*a**8*b*
*11 + 2240*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b**14*x**3 + 19
600*a**4*b**15*x**4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*x**6 + 2240*a
*b**18*x**7 + 280*b**19*x**8) + 140*b**10*d**3*x**10/(280*a**8*b**11 + 2240
*a**7*b**12*x + 7840*a**6*b**13*x**2 + 15680*a**5*b**14*x**3 + 19600*a**4*b
15*x4 + 15680*a**3*b**16*x**5 + 7840*a**2*b**17*x**6 + 2240*a*b**18*x**
7 + 280*b**19*x**8), Eq(n, -9)), (-50400*a**10*d**3*log(a/b + x)/(420*a**7*
b**11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 +

14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**1
8*x**7) - 130680*a**10*d**3/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5
*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**
16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) - 352800*a**9*b*d**3*x*log(a/
b + x)/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a
4*b14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**1
7*x**6 + 420*b**18*x**7) - 864360*a**9*b*d**3*x/(420*a**7*b**11 + 2940*a**6
*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*
x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) - 1058400
*a**8*b**2*d**3*x**2*log(a/b + x)/(420*a**7*b**11 + 2940*a**6*b**12*x + 882
0*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a*
*2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) - 2416680*a**8*b**2*d**
3*x**2/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a
4*b14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**1
7*x**6 + 420*b**18*x**7) + 1260*a**7*b**3*c*d**2*log(a/b + x)/(420*a**7*b**
11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 147
00*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x
7) + 3267*a7*b**3*c*d**2/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**
5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b*
*16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) - 1764000*a**7*b**3*d**3*x**
3*log(a/b + x)/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 +
 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 294
0*a*b**17*x**6 + 420*b**18*x**7) - 3675000*a**7*b**3*d**3*x**3/(420*a**7*b*
*11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14
700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*
x**7) + 8820*a**6*b**4*c*d**2*x*log(a/b + x)/(420*a**7*b**11 + 2940*a**6*b*
*12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**
4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) + 21609*a**6
*b**4*c*d**2*x/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 +
 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 294
0*a*b**17*x**6 + 420*b**18*x**7) - 1764000*a**6*b**4*d**3*x**4*log(a/b + x)
/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b*
*14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6
 + 420*b**18*x**7) - 3234000*a**6*b**4*d**3*x**4/(420*a**7*b**11 + 2940*a**
6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15
*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) + 26460*
a**5*b**5*c*d**2*x**2*log(a/b + x)/(420*a**7*b**11 + 2940*a**6*b**12*x + 88
20*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a
2*b16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) + 60417*a**5*b**5*c*d*
*2*x**2/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*
a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**
17*x**6 + 420*b**18*x**7) - 1058400*a**5*b**5*d**3*x**5*log(a/b + x)/(420*a
7*b11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**
3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*
b**18*x**7) - 1587600*a**5*b**5*d**3*x**5/(420*a**7*b**11 + 2940*a**6*b**12

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*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 +
 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) - 12*a**4*b**6*
c**2*d/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a
**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**1
7*x**6 + 420*b**18*x**7) + 44100*a**4*b**6*c*d**2*x**3*log(a/b + x)/(420*a*
*7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3
 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b
**18*x**7) + 91875*a**4*b**6*c*d**2*x**3/(420*a**7*b**11 + 2940*a**6*b**12*
x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 +
8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) - 352800*a**4*b*
*6*d**3*x**6*log(a/b + x)/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b
**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16
*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) - 352800*a**4*b**6*d**3*x**6/(4
20*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14
*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 +
420*b**18*x**7) - 84*a**3*b**7*c**2*d*x/(420*a**7*b**11 + 2940*a**6*b**12*x
 + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8
820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) + 44100*a**3*b**7
*c*d**2*x**4*log(a/b + x)/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b
**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16
*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) + 80850*a**3*b**7*c*d**2*x**4/(
420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**1
4*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 +
 420*b**18*x**7) - 50400*a**3*b**7*d**3*x**7*log(a/b + x)/(420*a**7*b**11 +
 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a
**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7)
 - 252*a**2*b**8*c**2*d*x**2/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**
5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b*
*16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) + 26460*a**2*b**8*c*d**2*x**
5*log(a/b + x)/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 +
 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 294
0*a*b**17*x**6 + 420*b**18*x**7) + 39690*a**2*b**8*c*d**2*x**5/(420*a**7*b*
*11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14
700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*
x**7) + 6300*a**2*b**8*d**3*x**8/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820
*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**
2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) - 10*a*b**9*c**3/(420*a*
*7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3
 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b
**18*x**7) - 420*a*b**9*c**2*d*x**3/(420*a**7*b**11 + 2940*a**6*b**12*x + 8
820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*
a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) + 8820*a*b**9*c*d**2*
x**6*log(a/b + x)/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**
2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 +
2940*a*b**17*x**6 + 420*b**18*x**7) + 8820*a*b**9*c*d**2*x**6/(420*a**7*b**
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11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 147
00*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x
7) - 700*a*b9*d**3*x**9/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5
*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**
16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) - 70*b**10*c**3*x/(420*a**7*b
11 + 2940*a6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 1
4700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18
*x**7) - 420*b**10*c**2*d*x**4/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a
5*b13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*
b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7) + 1260*b**10*c*d**2*x**7*l
og(a/b + x)/(420*a**7*b**11 + 2940*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14
700*a**4*b**14*x**3 + 14700*a**3*b**15*x**4 + 8820*a**2*b**16*x**5 + 2940*a
*b**17*x**6 + 420*b**18*x**7) + 140*b**10*d**3*x**10/(420*a**7*b**11 + 2940
*a**6*b**12*x + 8820*a**5*b**13*x**2 + 14700*a**4*b**14*x**3 + 14700*a**3*b
15*x4 + 8820*a**2*b**16*x**5 + 2940*a*b**17*x**6 + 420*b**18*x**7), Eq(
n, -8)), (12600*a**10*d**3*log(a/b + x)/(60*a**6*b**11 + 360*a**5*b**12*x +
 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a**2*b**15*x**4 + 360*a*b
16*x5 + 60*b**17*x**6) + 30870*a**10*d**3/(60*a**6*b**11 + 360*a**5*b**
12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a**2*b**15*x**4 + 3
60*a*b**16*x**5 + 60*b**17*x**6) + 75600*a**9*b*d**3*x*log(a/b + x)/(60*a**
6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 9
00*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) + 172620*a**9*b*d**3
*x/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**1
4*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) + 189000*a
8*b2*d**3*x**2*log(a/b + x)/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**
4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**
5 + 60*b**17*x**6) + 393750*a**8*b**2*d**3*x**2/(60*a**6*b**11 + 360*a**5*b
12*x + 900*a4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a**2*b**15*x**4 +
 360*a*b**16*x**5 + 60*b**17*x**6) - 1260*a**7*b**3*c*d**2*log(a/b + x)/(60
*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3
 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) - 3087*a**7*b**3
*c*d**2/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3
*b**14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) + 252
000*a**7*b**3*d**3*x**3*log(a/b + x)/(60*a**6*b**11 + 360*a**5*b**12*x + 90
0*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**1
6*x**5 + 60*b**17*x**6) + 462000*a**7*b**3*d**3*x**3/(60*a**6*b**11 + 360*a
5*b12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a**2*b**15*x
4 + 360*a*b16*x**5 + 60*b**17*x**6) - 7560*a**6*b**4*c*d**2*x*log(a/b +
 x)/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**
14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) - 17262*a
6*b4*c*d**2*x/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 +
 1200*a**3*b**14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x
6) + 189000*a6*b**4*d**3*x**4*log(a/b + x)/(60*a**6*b**11 + 360*a**5*b*
*12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a**2*b**15*x**4 +
360*a*b**16*x**5 + 60*b**17*x**6) + 283500*a**6*b**4*d**3*x**4/(60*a**6*b**

11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a*
*2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) - 18900*a**5*b**5*c*d**2*
x**2*log(a/b + x)/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 +
 1200*a**3*b**14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x
6) - 39375*a5*b**5*c*d**2*x**2/(60*a**6*b**11 + 360*a**5*b**12*x + 900*
a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*
x**5 + 60*b**17*x**6) + 75600*a**5*b**5*d**3*x**5*log(a/b + x)/(60*a**6*b**
11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a*
*2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) + 75600*a**5*b**5*d**3*x*
*5/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**1
4*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) - 6*a**4*b
6*c2*d/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a
3*b14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) -
25200*a**4*b**6*c*d**2*x**3*log(a/b + x)/(60*a**6*b**11 + 360*a**5*b**12*x
+ 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a**2*b**15*x**4 + 360*a*
b**16*x**5 + 60*b**17*x**6) - 46200*a**4*b**6*c*d**2*x**3/(60*a**6*b**11 +
360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a**2*b*
*15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) + 12600*a**4*b**6*d**3*x**6*lo
g(a/b + x)/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a
3*b14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) -
36*a**3*b**7*c**2*d*x/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x*
*2 + 1200*a**3*b**14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**
17*x**6) - 18900*a**3*b**7*c*d**2*x**4*log(a/b + x)/(60*a**6*b**11 + 360*a*
*5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a**2*b**15*x*
*4 + 360*a*b**16*x**5 + 60*b**17*x**6) - 28350*a**3*b**7*c*d**2*x**4/(60*a*
*6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 +
900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) - 1800*a**3*b**7*d*
*3*x**7/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3
*b**14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) - 90*
a**2*b**8*c**2*d*x**2/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x*
*2 + 1200*a**3*b**14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**
17*x**6) - 7560*a**2*b**8*c*d**2*x**5*log(a/b + x)/(60*a**6*b**11 + 360*a**
5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a**2*b**15*x**
4 + 360*a*b**16*x**5 + 60*b**17*x**6) - 7560*a**2*b**8*c*d**2*x**5/(60*a**6
*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 90
0*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) + 225*a**2*b**8*d**3*
x**8/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b*
*14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) - 2*a*b*
*9*c**3/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3
*b**14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) - 120
*a*b**9*c**2*d*x**3/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2
 + 1200*a**3*b**14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17
*x**6) - 1260*a*b**9*c*d**2*x**6*log(a/b + x)/(60*a**6*b**11 + 360*a**5*b**
12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a**2*b**15*x**4 + 3
60*a*b**16*x**5 + 60*b**17*x**6) - 50*a*b**9*d**3*x**9/(60*a**6*b**11 + 360

*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a**2*b**15
*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) - 12*b**10*c**3*x/(60*a**6*b**11
+ 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 + 900*a**2*
b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) - 90*b**10*c**2*d*x**4/(60*a
6*b11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b**14*x**3 +
900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) + 180*b**10*c*d**2
*x**7/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 1200*a**3*b
14*x3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6) + 15*b*
*10*d**3*x**10/(60*a**6*b**11 + 360*a**5*b**12*x + 900*a**4*b**13*x**2 + 12
00*a**3*b**14*x**3 + 900*a**2*b**15*x**4 + 360*a*b**16*x**5 + 60*b**17*x**6
), Eq(n, -7)), (-5040*a**10*d**3*log(a/b + x)/(20*a**5*b**11 + 100*a**4*b**
12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*a*b**15*x**4 + 20*b*
*16*x**5) - 11508*a**10*d**3/(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b
13*x2 + 200*a**2*b**14*x**3 + 100*a*b**15*x**4 + 20*b**16*x**5) - 25200
*a**9*b*d**3*x*log(a/b + x)/(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b*
*13*x**2 + 200*a**2*b**14*x**3 + 100*a*b**15*x**4 + 20*b**16*x**5) - 52500*
a**9*b*d**3*x/(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200
*a**2*b**14*x**3 + 100*a*b**15*x**4 + 20*b**16*x**5) - 50400*a**8*b**2*d**3
*x**2*log(a/b + x)/(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2
+ 200*a**2*b**14*x**3 + 100*a*b**15*x**4 + 20*b**16*x**5) - 92400*a**8*b**2
*d**3*x**2/(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a*
*2*b**14*x**3 + 100*a*b**15*x**4 + 20*b**16*x**5) + 1260*a**7*b**3*c*d**2*l
og(a/b + x)/(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a
2*b14*x**3 + 100*a*b**15*x**4 + 20*b**16*x**5) + 2877*a**7*b**3*c*d**2/
(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x*
*3 + 100*a*b**15*x**4 + 20*b**16*x**5) - 50400*a**7*b**3*d**3*x**3*log(a/b
+ x)/(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**
14*x**3 + 100*a*b**15*x**4 + 20*b**16*x**5) - 75600*a**7*b**3*d**3*x**3/(20
*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3
+ 100*a*b**15*x**4 + 20*b**16*x**5) + 6300*a**6*b**4*c*d**2*x*log(a/b + x)/
(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x*
*3 + 100*a*b**15*x**4 + 20*b**16*x**5) + 13125*a**6*b**4*c*d**2*x/(20*a**5*
b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*
a*b**15*x**4 + 20*b**16*x**5) - 25200*a**6*b**4*d**3*x**4*log(a/b + x)/(20*
a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 +
100*a*b**15*x**4 + 20*b**16*x**5) - 25200*a**6*b**4*d**3*x**4/(20*a**5*b**
11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*a*b
15*x4 + 20*b**16*x**5) + 12600*a**5*b**5*c*d**2*x**2*log(a/b + x)/(20*a
5*b11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 +
100*a*b**15*x**4 + 20*b**16*x**5) + 23100*a**5*b**5*c*d**2*x**2/(20*a**5*b*
*11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*a*
b**15*x**4 + 20*b**16*x**5) - 5040*a**5*b**5*d**3*x**5*log(a/b + x)/(20*a**
5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 10
0*a*b**15*x**4 + 20*b**16*x**5) - 12*a**4*b**6*c**2*d/(20*a**5*b**11 + 100*
a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*a*b**15*x**4

+ 20*b**16*x**5) + 12600*a**4*b**6*c*d**2*x**3*log(a/b + x)/(20*a**5*b**11
+ 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*a*b**
15*x**4 + 20*b**16*x**5) + 18900*a**4*b**6*c*d**2*x**3/(20*a**5*b**11 + 100
*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*a*b**15*x**
4 + 20*b**16*x**5) + 840*a**4*b**6*d**3*x**6/(20*a**5*b**11 + 100*a**4*b**1
2*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*a*b**15*x**4 + 20*b**
16*x**5) - 60*a**3*b**7*c**2*d*x/(20*a**5*b**11 + 100*a**4*b**12*x + 200*a*
*3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*a*b**15*x**4 + 20*b**16*x**5) + 6
300*a**3*b**7*c*d**2*x**4*log(a/b + x)/(20*a**5*b**11 + 100*a**4*b**12*x +
200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*a*b**15*x**4 + 20*b**16*x**
5) + 6300*a**3*b**7*c*d**2*x**4/(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**
3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*a*b**15*x**4 + 20*b**16*x**5) - 12
0*a**3*b**7*d**3*x**7/(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x*
*2 + 200*a**2*b**14*x**3 + 100*a*b**15*x**4 + 20*b**16*x**5) - 120*a**2*b**
8*c**2*d*x**2/(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200
*a**2*b**14*x**3 + 100*a*b**15*x**4 + 20*b**16*x**5) + 1260*a**2*b**8*c*d**
2*x**5*log(a/b + x)/(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2
+ 200*a**2*b**14*x**3 + 100*a*b**15*x**4 + 20*b**16*x**5) + 30*a**2*b**8*d
3*x8/(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2
*b**14*x**3 + 100*a*b**15*x**4 + 20*b**16*x**5) - a*b**9*c**3/(20*a**5*b**1
1 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*a*b*
*15*x**4 + 20*b**16*x**5) - 120*a*b**9*c**2*d*x**3/(20*a**5*b**11 + 100*a**
4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*a*b**15*x**4 +
20*b**16*x**5) - 210*a*b**9*c*d**2*x**6/(20*a**5*b**11 + 100*a**4*b**12*x +
200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*a*b**15*x**4 + 20*b**16*x*
*5) - 10*a*b**9*d**3*x**9/(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b**1
3*x**2 + 200*a**2*b**14*x**3 + 100*a*b**15*x**4 + 20*b**16*x**5) - 5*b**10*
c**3*x/(20*a**5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b
14*x3 + 100*a*b**15*x**4 + 20*b**16*x**5) - 60*b**10*c**2*d*x**4/(20*a*
*5*b**11 + 100*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 1
00*a*b**15*x**4 + 20*b**16*x**5) + 30*b**10*c*d**2*x**7/(20*a**5*b**11 + 10
0*a**4*b**12*x + 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*a*b**15*x*
*4 + 20*b**16*x**5) + 4*b**10*d**3*x**10/(20*a**5*b**11 + 100*a**4*b**12*x
+ 200*a**3*b**13*x**2 + 200*a**2*b**14*x**3 + 100*a*b**15*x**4 + 20*b**16*x
5), Eq(n, -6)), (2520*a10*d**3*log(a/b + x)/(12*a**4*b**11 + 48*a**3*b*
*12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) + 5250*a**10*
d**3/(12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**
3 + 12*b**15*x**4) + 10080*a**9*b*d**3*x*log(a/b + x)/(12*a**4*b**11 + 48*a
3*b12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) + 18480
*a**9*b*d**3*x/(12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a
*b**14*x**3 + 12*b**15*x**4) + 15120*a**8*b**2*d**3*x**2*log(a/b + x)/(12*a
4*b11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**
15*x**4) + 22680*a**8*b**2*d**3*x**2/(12*a**4*b**11 + 48*a**3*b**12*x + 72*
a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) - 1260*a**7*b**3*c*d**2*
log(a/b + x)/(12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b

14*x3 + 12*b**15*x**4) - 2625*a**7*b**3*c*d**2/(12*a**4*b**11 + 48*a**3
*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) + 10080*a*
*7*b**3*d**3*x**3*log(a/b + x)/(12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b
13*x2 + 48*a*b**14*x**3 + 12*b**15*x**4) + 10080*a**7*b**3*d**3*x**3/(1
2*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*
b**15*x**4) - 5040*a**6*b**4*c*d**2*x*log(a/b + x)/(12*a**4*b**11 + 48*a**3
*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) - 9240*a**
6*b**4*c*d**2*x/(12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*
a*b**14*x**3 + 12*b**15*x**4) + 2520*a**6*b**4*d**3*x**4*log(a/b + x)/(12*a
4*b11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**
15*x**4) - 7560*a**5*b**5*c*d**2*x**2*log(a/b + x)/(12*a**4*b**11 + 48*a**3
*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) - 11340*a*
*5*b**5*c*d**2*x**2/(12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 +
48*a*b**14*x**3 + 12*b**15*x**4) - 504*a**5*b**5*d**3*x**5/(12*a**4*b**11
+ 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) +
36*a**4*b**6*c**2*d*log(a/b + x)/(12*a**4*b**11 + 48*a**3*b**12*x + 72*a**
2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) + 75*a**4*b**6*c**2*d/(12*a
4*b11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**
15*x**4) - 5040*a**4*b**6*c*d**2*x**3*log(a/b + x)/(12*a**4*b**11 + 48*a**3
*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) - 5040*a**
4*b**6*c*d**2*x**3/(12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 +
48*a*b**14*x**3 + 12*b**15*x**4) + 84*a**4*b**6*d**3*x**6/(12*a**4*b**11 +
48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) + 1
44*a**3*b**7*c**2*d*x*log(a/b + x)/(12*a**4*b**11 + 48*a**3*b**12*x + 72*a*
*2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) + 264*a**3*b**7*c**2*d*x/(
12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12
*b**15*x**4) - 1260*a**3*b**7*c*d**2*x**4*log(a/b + x)/(12*a**4*b**11 + 48*
a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) - 24*a
3*b7*d**3*x**7/(12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 +
48*a*b**14*x**3 + 12*b**15*x**4) + 216*a**2*b**8*c**2*d*x**2*log(a/b + x)/(
12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12
*b**15*x**4) + 324*a**2*b**8*c**2*d*x**2/(12*a**4*b**11 + 48*a**3*b**12*x +
72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) + 252*a**2*b**8*c*d*
*2*x**5/(12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*
x**3 + 12*b**15*x**4) + 9*a**2*b**8*d**3*x**8/(12*a**4*b**11 + 48*a**3*b**1
2*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) - a*b**9*c**3/(
12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12
*b**15*x**4) + 144*a*b**9*c**2*d*x**3*log(a/b + x)/(12*a**4*b**11 + 48*a**3
*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) + 144*a*b*
*9*c**2*d*x**3/(12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a
*b**14*x**3 + 12*b**15*x**4) - 42*a*b**9*c*d**2*x**6/(12*a**4*b**11 + 48*a*
*3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) - 4*a*b*
*9*d**3*x**9/(12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b
14*x3 + 12*b**15*x**4) - 4*b**10*c**3*x/(12*a**4*b**11 + 48*a**3*b**12*
x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) + 36*b**10*c**2*d

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*x**4*log(a/b + x)/(12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 +
48*a*b**14*x**3 + 12*b**15*x**4) + 12*b**10*c*d**2*x**7/(12*a**4*b**11 + 48
*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*a*b**14*x**3 + 12*b**15*x**4) + 2*b
**10*d**3*x**10/(12*a**4*b**11 + 48*a**3*b**12*x + 72*a**2*b**13*x**2 + 48*
a*b**14*x**3 + 12*b**15*x**4), Eq(n, -5)), (-10080*a**10*d**3*log(a/b + x)/
(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x**3) - 184
80*a**10*d**3/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b**
14*x**3) - 30240*a**9*b*d**3*x*log(a/b + x)/(84*a**3*b**11 + 252*a**2*b**12
*x + 252*a*b**13*x**2 + 84*b**14*x**3) - 45360*a**9*b*d**3*x/(84*a**3*b**11
 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x**3) - 30240*a**8*b**2*d
**3*x**2*log(a/b + x)/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2
+ 84*b**14*x**3) - 30240*a**8*b**2*d**3*x**2/(84*a**3*b**11 + 252*a**2*b**1
2*x + 252*a*b**13*x**2 + 84*b**14*x**3) + 8820*a**7*b**3*c*d**2*log(a/b + x
)/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x**3) + 1
6170*a**7*b**3*c*d**2/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2
+ 84*b**14*x**3) - 10080*a**7*b**3*d**3*x**3*log(a/b + x)/(84*a**3*b**11 +
252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x**3) + 26460*a**6*b**4*c*d*
*2*x*log(a/b + x)/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84
*b**14*x**3) + 39690*a**6*b**4*c*d**2*x/(84*a**3*b**11 + 252*a**2*b**12*x +
 252*a*b**13*x**2 + 84*b**14*x**3) + 2520*a**6*b**4*d**3*x**4/(84*a**3*b**1
1 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x**3) + 26460*a**5*b**5*
c*d**2*x**2*log(a/b + x)/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x*
*2 + 84*b**14*x**3) + 26460*a**5*b**5*c*d**2*x**2/(84*a**3*b**11 + 252*a**2
*b**12*x + 252*a*b**13*x**2 + 84*b**14*x**3) - 504*a**5*b**5*d**3*x**5/(84*
a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x**3) - 1008*a*
*4*b**6*c**2*d*log(a/b + x)/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13
*x**2 + 84*b**14*x**3) - 1848*a**4*b**6*c**2*d/(84*a**3*b**11 + 252*a**2*b*
*12*x + 252*a*b**13*x**2 + 84*b**14*x**3) + 8820*a**4*b**6*c*d**2*x**3*log(
a/b + x)/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x*
*3) + 168*a**4*b**6*d**3*x**6/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**
13*x**2 + 84*b**14*x**3) - 3024*a**3*b**7*c**2*d*x*log(a/b + x)/(84*a**3*b*
*11 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x**3) - 4536*a**3*b**7
*c**2*d*x/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x
**3) - 2205*a**3*b**7*c*d**2*x**4/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a
*b**13*x**2 + 84*b**14*x**3) - 72*a**3*b**7*d**3*x**7/(84*a**3*b**11 + 252*
a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x**3) - 3024*a**2*b**8*c**2*d*x*
*2*log(a/b + x)/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b
**14*x**3) - 3024*a**2*b**8*c**2*d*x**2/(84*a**3*b**11 + 252*a**2*b**12*x +
 252*a*b**13*x**2 + 84*b**14*x**3) + 441*a**2*b**8*c*d**2*x**5/(84*a**3*b**
11 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x**3) + 36*a**2*b**8*d*
*3*x**8/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x**
3) - 14*a*b**9*c**3/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2 +
84*b**14*x**3) - 1008*a*b**9*c**2*d*x**3*log(a/b + x)/(84*a**3*b**11 + 252*
a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x**3) - 147*a*b**9*c*d**2*x**6/(
84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x**3) - 20*a
```

*b**9*d**3*x**9/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b **14*x**3) - 42*b**10*c**3*x/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**1 3*x**2 + 84*b**14*x**3) + 252*b**10*c**2*d*x**4/(84*a**3*b**11 + 252*a**2*b **12*x + 252*a*b**13*x**2 + 84*b**14*x**3) + 63*b**10*c*d**2*x**7/(84*a**3* b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x**3) + 12*b**10*d** 3*x**10/(84*a**3*b**11 + 252*a**2*b**12*x + 252*a*b**13*x**2 + 84*b**14*x** 3), Eq(n, -4)), (12600*a**10*d**3*log(a/b + x)/(280*a**2*b**11 + 560*a*b**1 2*x + 280*b**13*x**2) + 18900*a**10*d**3/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) + 25200*a**9*b*d**3*x*log(a/b + x)/(280*a**2*b**11 + 560*a* b**12*x + 280*b**13*x**2) + 25200*a**9*b*d**3*x/(280*a**2*b**11 + 560*a*b** 12*x + 280*b**13*x**2) + 12600*a**8*b**2*d**3*x**2*log(a/b + x)/(280*a**2*b **11 + 560*a*b**12*x + 280*b**13*x**2) - 17640*a**7*b**3*c*d**2*log(a/b + x)/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) - 26460*a**7*b**3*c*d** 2/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) - 4200*a**7*b**3*d**3*x **3/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) - 35280*a**6*b**4*c*d **2*x*log(a/b + x)/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) - 3528 0*a**6*b**4*c*d**2*x/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) + 10 50*a**6*b**4*d**3*x**4/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) - 17640*a**5*b**5*c*d**2*x**2*log(a/b + x)/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) - 420*a**5*b**5*d**3*x**5/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) + 5040*a**4*b**6*c**2*d*log(a/b + x)/(280*a**2*b**11 + 560 *a*b**12*x + 280*b**13*x**2) + 7560*a**4*b**6*c**2*d/(280*a**2*b**11 + 560* a*b**12*x + 280*b**13*x**2) + 5880*a**4*b**6*c*d**2*x**3/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) + 210*a**4*b**6*d**3*x**6/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) + 10080*a**3*b**7*c**2*d*x*log(a/b + x)/(2 80*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) + 10080*a**3*b**7*c**2*d*x/ (280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) - 1470*a**3*b**7*c*d**2*x **4/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) - 120*a**3*b**7*d**3* x**7/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) + 5040*a**2*b**8*c** 2*d*x**2*log(a/b + x)/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) + 5 88*a**2*b**8*c*d**2*x**5/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) + 75*a**2*b**8*d**3*x**8/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) - 140*a*b**9*c**3/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) - 1680* a*b**9*c**2*d*x**3/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) - 294* a*b**9*c*d**2*x**6/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) - 50*a *b**9*d**3*x**9/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) - 280*b** 10*c**3*x/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) + 420*b**10*c** 2*d*x**4/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) + 168*b**10*c*d* *2*x**7/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2) + 35*b**10*d**3*x **10/(280*a**2*b**11 + 560*a*b**12*x + 280*b**13*x**2), Eq(n, -3)), (-12600 *a**10*d**3*log(a/b + x)/(1260*a*b**11 + 1260*b**12*x) - 12600*a**10*d**3/(1260*a*b**11 + 1260*b**12*x) - 12600*a**9*b*d**3*x*log(a/b + x)/(1260*a*b** 11 + 1260*b**12*x) + 6300*a**8*b**2*d**3*x**2/(1260*a*b**11 + 1260*b**12*x) + 26460*a**7*b**3*c*d**2*log(a/b + x)/(1260*a*b**11 + 1260*b**12*x) + 2646 0*a**7*b**3*c*d**2/(1260*a*b**11 + 1260*b**12*x) - 2100*a**7*b**3*d**3*x**3

/(1260*a*b**11 + 1260*b**12*x) + 26460*a**6*b**4*c*d**2*x*log(a/b + x)/(126
0*a*b**11 + 1260*b**12*x) + 1050*a**6*b**4*d**3*x**4/(1260*a*b**11 + 1260*b
12*x) - 13230*a5*b**5*c*d**2*x**2/(1260*a*b**11 + 1260*b**12*x) - 630*a
5*b5*d**3*x**5/(1260*a*b**11 + 1260*b**12*x) - 15120*a**4*b**6*c**2*d*l
og(a/b + x)/(1260*a*b**11 + 1260*b**12*x) - 15120*a**4*b**6*c**2*d/(1260*a*
b**11 + 1260*b**12*x) + 4410*a**4*b**6*c*d**2*x**3/(1260*a*b**11 + 1260*b**
12*x) + 420*a**4*b**6*d**3*x**6/(1260*a*b**11 + 1260*b**12*x) - 15120*a**3*
b**7*c**2*d*x*log(a/b + x)/(1260*a*b**11 + 1260*b**12*x) - 2205*a**3*b**7*c
*d**2*x**4/(1260*a*b**11 + 1260*b**12*x) - 300*a**3*b**7*d**3*x**7/(1260*a*
b**11 + 1260*b**12*x) + 7560*a**2*b**8*c**2*d*x**2/(1260*a*b**11 + 1260*b**
12*x) + 1323*a**2*b**8*c*d**2*x**5/(1260*a*b**11 + 1260*b**12*x) + 225*a**2
*b**8*d**3*x**8/(1260*a*b**11 + 1260*b**12*x) + 1260*a*b**9*c**3*log(a/b +
x)/(1260*a*b**11 + 1260*b**12*x) + 1260*a*b**9*c**3/(1260*a*b**11 + 1260*b*
*12*x) - 2520*a*b**9*c**2*d*x**3/(1260*a*b**11 + 1260*b**12*x) - 882*a*b**9
*c*d**2*x**6/(1260*a*b**11 + 1260*b**12*x) - 175*a*b**9*d**3*x**9/(1260*a*b
11 + 1260*b12*x) + 1260*b**10*c**3*x*log(a/b + x)/(1260*a*b**11 + 1260*
b**12*x) + 1260*b**10*c**2*d*x**4/(1260*a*b**11 + 1260*b**12*x) + 630*b**10
*c*d**2*x**7/(1260*a*b**11 + 1260*b**12*x) + 140*b**10*d**3*x**10/(1260*a*b
11 + 1260*b12*x), Eq(n, -2)), (a**10*d**3*log(a/b + x)/b**11 - a**9*d**
3*x/b**10 + a**8*d**3*x**2/(2*b**9) - 3*a**7*c*d**2*log(a/b + x)/b**8 - a**
7*d**3*x**3/(3*b**8) + 3*a**6*c*d**2*x/b**7 + a**6*d**3*x**4/(4*b**7) - 3*a
5*c*d2*x**2/(2*b**6) - a**5*d**3*x**5/(5*b**6) + 3*a**4*c**2*d*log(a/b
+ x)/b**5 + a**4*c*d**2*x**3/b**5 + a**4*d**3*x**6/(6*b**5) - 3*a**3*c**2*d
*x/b**4 - 3*a**3*c*d**2*x**4/(4*b**4) - a**3*d**3*x**7/(7*b**4) + 3*a**2*c*
*2*d*x**2/(2*b**3) + 3*a**2*c*d**2*x**5/(5*b**3) + a**2*d**3*x**8/(8*b**3)
- a*c**3*log(a/b + x)/b**2 - a*c**2*d*x**3/b**2 - a*c*d**2*x**6/(2*b**2) -
a*d**3*x**9/(9*b**2) + c**3*x/b + 3*c**2*d*x**4/(4*b) + 3*c*d**2*x**7/(7*b)
 + d**3*x**10/(10*b), Eq(n, -1)), (3628800*a**11*d**3*(a + b*x)**n/(b**11*n
11 + 66*b11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n
7 + 2637558*b11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 1052
58076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**1
1) - 3628800*a**10*b*d**3*n*x*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 +
1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6
 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 15091
7976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 1814400*a**9*b**2*d
3*n2*x**2*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9
+ 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**1
1*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2
+ 120543840*b**11*n + 39916800*b**11) + 1814400*a**9*b**2*d**3*n*x**2*(a +
b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8
+ 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b
11*n4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n
 + 39916800*b**11) - 15120*a**8*b**3*c*d**2*n**3*(a + b*x)**n/(b**11*n**11
+ 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 +
 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076

*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) -
453600*a**8*b**3*c*d**2*n**2*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1
925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6
+ 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917
976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 4520880*a**8*b**3*c*
d**2*n*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670
*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5
+ 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 12054
3840*b**11*n + 39916800*b**11) - 14968800*a**8*b**3*c*d**2*(a + b*x)**n/(b*
*11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b*
*11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 +
105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800
*b**11) - 604800*a**8*b**3*d**3*n**3*x**3*(a + b*x)**n/(b**11*n**11 + 66*b*
*11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 263755
8*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*
n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 1814400
*a**8*b**3*d**3*n**2*x**3*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925
*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 1
3339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976
*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 1209600*a**8*b**3*d**3*
n*x**3*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670
*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5
+ 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 12054
3840*b**11*n + 39916800*b**11) + 15120*a**7*b**4*c*d**2*n**4*x*(a + b*x)**n
/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 35742
3*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n*
*4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 3991
6800*b**11) + 453600*a**7*b**4*c*d**2*n**3*x*(a + b*x)**n/(b**11*n**11 + 66
*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 263
7558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**
11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 4520
880*a**7*b**4*c*d**2*n**2*x*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 19
25*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 +
13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 1509179
76*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 14968800*a**7*b**4*c*
d**2*n*x*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 326
70*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**
5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120
543840*b**11*n + 39916800*b**11) + 151200*a**7*b**4*d**3*n**4*x**4*(a + b*x
)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 3
57423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**1
1*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n +
39916800*b**11) + 907200*a**7*b**4*d**3*n**3*x**4*(a + b*x)**n/(b**11*n**11
+ 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7
+ 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 10525807

6*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) +
1663200*a**7*b**4*d**3*n**2*x**4*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**1
0 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*
n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 1
50917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 907200*a**7*b**
4*d**3*n*x**4*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9
+ 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**1
1*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2
+ 120543840*b**11*n + 39916800*b**11) - 7560*a**6*b**5*c*d**2*n**5*x**2*(a
+ b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**
8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730
*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11
*n + 39916800*b**11) - 234360*a**6*b**5*c*d**2*n**4*x**2*(a + b*x)**n/(b**1
1*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**1
1*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 1
05258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b
11) - 2487240*a6*b**5*c*d**2*n**3*x**2*(a + b*x)**n/(b**11*n**11 + 66*b
11*n10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 26375
58*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11
*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 974484
0*a**6*b**5*c*d**2*n**2*x**2*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1
925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6
+ 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917
976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 7484400*a**6*b**5*c*
d**2*n*x**2*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 +
32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*
n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 +
120543840*b**11*n + 39916800*b**11) - 30240*a**6*b**5*d**3*n**5*x**5*(a + b
*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 +
357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b*
*11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n
+ 39916800*b**11) - 302400*a**6*b**5*d**3*n**4*x**5*(a + b*x)**n/(b**11*n**
11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**
7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258
076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11)
- 1058400*a**6*b**5*d**3*n**3*x**5*(a + b*x)**n/(b**11*n**11 + 66*b**11*n*
*10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**1
1*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 +
150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 1512000*a**6*
b**5*d**3*n**2*x**5*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11
*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 1333953
5*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11
*n**2 + 120543840*b**11*n + 39916800*b**11) - 725760*a**6*b**5*d**3*n*x**5*
(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*
n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995

730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b*
*11*n + 39916800*b**11) + 72*a**5*b**6*c**2*d*n**6*(a + b*x)**n/(b**11*n**1
1 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7
 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 1052580
76*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11)
+ 3672*a**5*b**6*c**2*d*n**5*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1
925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6
+ 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917
976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 77400*a**5*b**6*c**2
*d*n**4*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 3267
0*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5
 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 1205
43840*b**11*n + 39916800*b**11) + 862920*a**5*b**6*c**2*d*n**3*(a + b*x)**n
/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 35742
3*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n*
*4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 3991
6800*b**11) + 5365728*a**5*b**6*c**2*d*n**2*(a + b*x)**n/(b**11*n**11 + 66*
b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637
558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**1
1*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 17640
288*a**5*b**6*c**2*d*n*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b*
*11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 1333
9535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b*
*11*n**2 + 120543840*b**11*n + 39916800*b**11) + 23950080*a**5*b**6*c**2*d*
(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*
n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995
730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b*
*11*n + 39916800*b**11) + 2520*a**5*b**6*c*d**2*n**6*x**3*(a + b*x)**n/(b**
11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**
11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 +
105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*
b**11) + 83160*a**5*b**6*c*d**2*n**5*x**3*(a + b*x)**n/(b**11*n**11 + 66*b*
*11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 263755
8*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*
n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 985320*
a**5*b**6*c*d**2*n**4*x**3*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 192
5*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 +
13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 15091797
6*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 4906440*a**5*b**6*c*d*
*2*n**3*x**3*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 +
 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11
*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 +
 120543840*b**11*n + 39916800*b**11) + 8991360*a**5*b**6*c*d**2*n**2*x**3*(
a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n
8 + 357423*b11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 459957

30*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 4989600*a**5*b**6*c*d**2*n*x**3*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 5040*a**5*b**6*d**3*n**6*x**6*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 75600*a**5*b**6*d**3*n**5*x**6*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 428400*a**5*b**6*d**3*n**4*x**6*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 1134000*a**5*b**6*d**3*n**3*x**6*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 1380960*a**5*b**6*d**3*n**2*x**6*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 604800*a**5*b**6*d**3*n*x**6*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 72*a**4*b**7*c**2*d*n**7*x*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 3672*a**4*b**7*c**2*d*n**6*x*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 77400*a**4*b**7*c**2*d*n**5*x*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 862920*a**4*b**7*c**2*d*n**4*x*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 5365728*a**4*b**7*c**2*d*n**3*x*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 +

45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 1205438
40*b**11*n + 39916800*b**11) - 17640288*a**4*b**7*c**2*d*n**2*x*(a + b*x)**
n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 3574
23*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n
4 + 105258076*b11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 399
16800*b**11) - 23950080*a**4*b**7*c**2*d*n*x*(a + b*x)**n/(b**11*n**11 + 66
*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 263
7558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**
11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 630*
a**4*b**7*c*d**2*n**7*x**4*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 192
5*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 +
13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 15091797
6*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 22680*a**4*b**7*c*d**2
*n**6*x**4*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 3
2670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n
5 + 45995730*b11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 1
20543840*b**11*n + 39916800*b**11) - 308700*a**4*b**7*c*d**2*n**5*x**4*(a +
 b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8
 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*
b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*
n + 39916800*b**11) - 1965600*a**4*b**7*c*d**2*n**4*x**4*(a + b*x)**n/(b**1
1*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**1
1*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 1
05258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b
11) - 5927670*a4*b**7*c*d**2*n**3*x**4*(a + b*x)**n/(b**11*n**11 + 66*b
11*n10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 26375
58*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11
*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 799092
0*a**4*b**7*c*d**2*n**2*x**4*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1
925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6
+ 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917
976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 3742200*a**4*b**7*c*
d**2*n*x**4*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 +
32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*
n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 +
120543840*b**11*n + 39916800*b**11) - 720*a**4*b**7*d**3*n**7*x**7*(a + b*x
)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 3
57423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**1
1*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n +
39916800*b**11) - 15120*a**4*b**7*d**3*n**6*x**7*(a + b*x)**n/(b**11*n**11
+ 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 +
 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076
*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) -
126000*a**4*b**7*d**3*n**5*x**7*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10
+ 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n*

*6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150
917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 529200*a**4*b**7*
d**3*n**4*x**7*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9
 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**
11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2
 + 120543840*b**11*n + 39916800*b**11) - 1169280*a**4*b**7*d**3*n**3*x**7*(
a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n
8 + 357423*b11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 459957
30*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**
11*n + 39916800*b**11) - 1270080*a**4*b**7*d**3*n**2*x**7*(a + b*x)**n/(b**
11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**
11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 +
105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*
b**11) - 518400*a**4*b**7*d**3*n*x**7*(a + b*x)**n/(b**11*n**11 + 66*b**11*
n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b*
*11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3
 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 36*a**3*b**
8*c**2*d*n**8*x**2*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*
n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535
*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*
n**2 + 120543840*b**11*n + 39916800*b**11) + 1872*a**3*b**8*c**2*d*n**7*x**
2*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**1
1*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 459
95730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*
b**11*n + 39916800*b**11) + 40536*a**3*b**8*c**2*d*n**6*x**2*(a + b*x)**n/(
b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*
b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4
 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 399168
00*b**11) + 470160*a**3*b**8*c**2*d*n**5*x**2*(a + b*x)**n/(b**11*n**11 + 6
6*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 26
37558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b*
*11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 311
4324*a**3*b**8*c**2*d*n**4*x**2*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10
+ 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n*
*6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150
917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 11503008*a**3*b**
8*c**2*d*n**3*x**2*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*
n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535
*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*
n**2 + 120543840*b**11*n + 39916800*b**11) + 20795184*a**3*b**8*c**2*d*n**2
*x**2*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*
b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 +
 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543
840*b**11*n + 39916800*b**11) + 11975040*a**3*b**8*c**2*d*n*x**2*(a + b*x)*
*n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357

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423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*
n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39
916800*b**11) + 126*a**3*b**8*c*d**2*n**8*x**5*(a + b*x)**n/(b**11*n**11 +
66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2
637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b
**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 50
40*a**3*b**8*c*d**2*n**7*x**5*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 +
1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6
 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 15091
7976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 79884*a**3*b**8*c*d
**2*n**6*x**5*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9
+ 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**1
1*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2
+ 120543840*b**11*n + 39916800*b**11) + 640080*a**3*b**8*c*d**2*n**5*x**5*(
a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n
**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 459957
30*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**
11*n + 39916800*b**11) + 2758014*a**3*b**8*c*d**2*n**4*x**5*(a + b*x)**n/(b
**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b
**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4
+ 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 3991680
0*b**11) + 6340320*a**3*b**8*c*d**2*n**3*x**5*(a + b*x)**n/(b**11*n**11 + 6
6*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 26
37558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b*
*11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 714
1176*a**3*b**8*c*d**2*n**2*x**5*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10
+ 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n*
*6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150
917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 2993760*a**3*b**8
*c*d**2*n*x**5*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9
 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**
11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2
 + 120543840*b**11*n + 39916800*b**11) + 90*a**3*b**8*d**3*n**8*x**8*(a + b
*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 +
 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b*
*11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n
+ 39916800*b**11) + 2520*a**3*b**8*d**3*n**7*x**8*(a + b*x)**n/(b**11*n**11
 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7
+ 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 10525807
6*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) +
 28980*a**3*b**8*d**3*n**6*x**8*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10
+ 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n*
*6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150
917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 176400*a**3*b**8*
d**3*n**5*x**8*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9
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+ 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 609210*a**3*b**8*d**3*n**4*x**8*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 1181880*a**3*b**8*d**3*n**3*x**8*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 1176120*a**3*b**8*d**3*n**2*x**8*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 453600*a**3*b**8*d**3*n*x**8*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - a**2*b**9*c**3*n**9*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 63*a**2*b**9*c**3*n**8*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 1734*a**2*b**9*c**3*n**7*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 27342*a**2*b**9*c**3*n**6*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 271929*a**2*b**9*c**3*n**5*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 1767087*a**2*b**9*c**3*n**4*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 7494416*a**2*b**9*c**3*n**3*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 19978308*a**2*b**9*c**3*n**2*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b

11*n7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4
+ 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 3991680
0*b**11) - 30334320*a**2*b**9*c**3*n*(a + b*x)**n/(b**11*n**11 + 66*b**11*n
10 + 1925*b11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**
11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3
+ 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 19958400*a**
2*b**9*c**3*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 +
32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*
n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 +
120543840*b**11*n + 39916800*b**11) - 12*a**2*b**9*c**2*d*n**9*x**3*(a + b*
x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 +
357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**
11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n +
39916800*b**11) - 648*a**2*b**9*c**2*d*n**8*x**3*(a + b*x)**n/(b**11*n**11
+ 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7
+ 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 10525807
6*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) -
14760*a**2*b**9*c**2*d*n**7*x**3*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**1
0 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*
n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 1
50917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 183744*a**2*b**
9*c**2*d*n**6*x**3*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*
n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535
*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*
n**2 + 120543840*b**11*n + 39916800*b**11) - 1351548*a**2*b**9*c**2*d*n**5*
x**3*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b
11*n8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 +
45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 1205438
40*b**11*n + 39916800*b**11) - 5910552*a**2*b**9*c**2*d*n**4*x**3*(a + b*x)
n/(b11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 35
7423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11
*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 3
9916800*b**11) - 14600400*a**2*b**9*c**2*d*n**3*x**3*(a + b*x)**n/(b**11*n*
*11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n*
*7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 10525
8076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11
) - 17855136*a**2*b**9*c**2*d*n**2*x**3*(a + b*x)**n/(b**11*n**11 + 66*b**1
1*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*
b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n*
*3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 7983360*a
2*b9*c**2*d*n*x**3*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b*
*11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 1333
9535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b*
*11*n**2 + 120543840*b**11*n + 39916800*b**11) - 21*a**2*b**9*c*d**2*n**9*x
6*(a + b*x)n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b*

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*11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 4
5995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 12054384
0*b**11*n + 39916800*b**11) - 945*a**2*b**9*c*d**2*n**8*x**6*(a + b*x)**n/(
b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*
b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4
 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 399168
00*b**11) - 17514*a**2*b**9*c*d**2*n**7*x**6*(a + b*x)**n/(b**11*n**11 + 66
*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 263
7558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**
11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 1732
50*a**2*b**9*c*d**2*n**6*x**6*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 +
1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6
 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 15091
7976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 993069*a**2*b**9*c*
d**2*n**5*x**6*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9
 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**
11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2
 + 120543840*b**11*n + 39916800*b**11) - 3355065*a**2*b**9*c*d**2*n**4*x**6
*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11
*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 4599
5730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b
**11*n + 39916800*b**11) - 6473796*a**2*b**9*c*d**2*n**3*x**6*(a + b*x)**n/
(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423
*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**
4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916
800*b**11) - 6449940*a**2*b**9*c*d**2*n**2*x**6*(a + b*x)**n/(b**11*n**11 +
 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 +
2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*
b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 2
494800*a**2*b**9*c*d**2*n*x**6*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 +
 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**
6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 1509
17976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 10*a**2*b**9*d**3*
n**9*x**9*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32
670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n*
*5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 12
0543840*b**11*n + 39916800*b**11) - 360*a**2*b**9*d**3*n**8*x**9*(a + b*x)*
*n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357
423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*
n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39
916800*b**11) - 5460*a**2*b**9*d**3*n**7*x**9*(a + b*x)**n/(b**11*n**11 + 6
6*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 26
37558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b*
*11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 453
60*a**2*b**9*d**3*n**6*x**9*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 19
```

25*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 +
 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 1509179
76*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 224490*a**2*b**9*d**3
*n**5*x**9*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 3
2670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n
5 + 45995730*b11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 1
20543840*b**11*n + 39916800*b**11) - 672840*a**2*b**9*d**3*n**4*x**9*(a + b
*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 +
 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b*
*11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n
+ 39916800*b**11) - 1181240*a**2*b**9*d**3*n**3*x**9*(a + b*x)**n/(b**11*n*
*11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n*
*7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 10525
8076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11
) - 1095840*a**2*b**9*d**3*n**2*x**9*(a + b*x)**n/(b**11*n**11 + 66*b**11*n
10 + 1925*b11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**
11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3
+ 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) - 403200*a**2*
b**9*d**3*n*x**9*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n*
*9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b
11*n5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n*
*2 + 120543840*b**11*n + 39916800*b**11) + a*b**10*c**3*n**10*x*(a + b*x)**
n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 3574
23*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n
4 + 105258076*b11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 399
16800*b**11) + 63*a*b**10*c**3*n**9*x*(a + b*x)**n/(b**11*n**11 + 66*b**11*
n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b*
*11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3
 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 1734*a*b**1
0*c**3*n**8*x*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9
+ 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**1
1*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2
+ 120543840*b**11*n + 39916800*b**11) + 27342*a*b**10*c**3*n**7*x*(a + b*x)
n/(b11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 35
7423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11
*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 3
9916800*b**11) + 271929*a*b**10*c**3*n**6*x*(a + b*x)**n/(b**11*n**11 + 66*
b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637
558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**1
1*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 17670
87*a*b**10*c**3*n**5*x*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b*
*11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 1333
9535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b*
*11*n**2 + 120543840*b**11*n + 39916800*b**11) + 7494416*a*b**10*c**3*n**4*
x*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**1

1*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 459
95730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*
b**11*n + 39916800*b**11) + 19978308*a*b**10*c**3*n**3*x*(a + b*x)**n/(b**1
1*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**1
1*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 1
05258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b
11) + 30334320*a*b10*c**3*n**2*x*(a + b*x)**n/(b**11*n**11 + 66*b**11*n
10 + 1925*b11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**
11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3
+ 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 19958400*a*b
10*c3*n*x*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9
+ 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**1
1*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2
+ 120543840*b**11*n + 39916800*b**11) + 3*a*b**10*c**2*d*n**10*x**4*(a + b*
x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 +
357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**
11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n +
39916800*b**11) + 171*a*b**10*c**2*d*n**9*x**4*(a + b*x)**n/(b**11*n**11 +
66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 +
2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*
b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 4
176*a*b**10*c**2*d*n**8*x**4*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1
925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6
+ 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917
976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 57006*a*b**10*c**2*d
*n**7*x**4*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 3
2670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n
5 + 45995730*b11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 1
20543840*b**11*n + 39916800*b**11) + 475695*a*b**10*c**2*d*n**6*x**4*(a + b
*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 +
357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b*
*11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n
+ 39916800*b**11) + 2491299*a*b**10*c**2*d*n**5*x**4*(a + b*x)**n/(b**11*n*
*11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n*
*7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 10525
8076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11
) + 8083014*a*b**10*c**2*d*n**4*x**4*(a + b*x)**n/(b**11*n**11 + 66*b**11*n
10 + 1925*b11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**
11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3
+ 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 15414084*a*b
10*c2*d*n**3*x**4*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**
11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339
535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**
11*n**2 + 120543840*b**11*n + 39916800*b**11) + 15387192*a*b**10*c**2*d*n**
2*x**4*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670

*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5
+ 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 12054
3840*b**11*n + 39916800*b**11) + 5987520*a*b**10*c**2*d*n*x**4*(a + b*x)**n
/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 35742
3*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n*
*4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 3991
6800*b**11) + 3*a*b**10*c*d**2*n**10*x**7*(a + b*x)**n/(b**11*n**11 + 66*b*
*11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 263755
8*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*
n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 153*a*b
10*c*d2*n**9*x**7*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**
11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339
535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**
11*n**2 + 120543840*b**11*n + 39916800*b**11) + 3312*a*b**10*c*d**2*n**8*x*
7(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**
11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45
995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840
*b**11*n + 39916800*b**11) + 39762*a*b**10*c*d**2*n**7*x**7*(a + b*x)**n/(b
11*n11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b
11*n7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4
+ 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 3991680
0*b**11) + 290367*a*b**10*c*d**2*n**6*x**7*(a + b*x)**n/(b**11*n**11 + 66*b
11*n10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 26375
58*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11
*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 133049
7*a*b**10*c*d**2*n**5*x**7*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 192
5*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 +
13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 15091797
6*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 3800598*a*b**10*c*d**2
*n**4*x**7*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 3
2670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n
5 + 45995730*b11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 1
20543840*b**11*n + 39916800*b**11) + 6470388*a*b**10*c*d**2*n**3*x**7*(a +
b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8
+ 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b
11*n4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n
 + 39916800*b**11) + 5884920*a*b**10*c*d**2*n**2*x**7*(a + b*x)**n/(b**11*n
11 + 66*b11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n
7 + 2637558*b11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 1052
58076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**1
1) + 2138400*a*b**10*c*d**2*n*x**7*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**
10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11
*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 +
150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + a*b**10*d**3*n
10*x10*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 3

2670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n
5 + 45995730*b11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 1
20543840*b**11*n + 39916800*b**11) + 45*a*b**10*d**3*n**9*x**10*(a + b*x)**
n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 3574
23*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n
4 + 105258076*b11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 399
16800*b**11) + 870*a*b**10*d**3*n**8*x**10*(a + b*x)**n/(b**11*n**11 + 66*b
11*n10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 26375
58*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11
*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 9450*a
*b**10*d**3*n**7*x**10*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b*
*11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 1333
9535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b*
*11*n**2 + 120543840*b**11*n + 39916800*b**11) + 63273*a*b**10*d**3*n**6*x*
10(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b*
*11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 4
5995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 12054384
0*b**11*n + 39916800*b**11) + 269325*a*b**10*d**3*n**5*x**10*(a + b*x)**n/(
b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*
b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4
 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 399168
00*b**11) + 723680*a*b**10*d**3*n**4*x**10*(a + b*x)**n/(b**11*n**11 + 66*b
11*n10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 26375
58*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11
*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 117270
0*a*b**10*d**3*n**3*x**10*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925
*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 1
3339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976
*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 1026576*a*b**10*d**3*n*
*2*x**10*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 326
70*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**
5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120
543840*b**11*n + 39916800*b**11) + 362880*a*b**10*d**3*n*x**10*(a + b*x)**n
/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 35742
3*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n*
*4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 3991
6800*b**11) + b**11*c**3*n**10*x**2*(a + b*x)**n/(b**11*n**11 + 66*b**11*n*
*10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**1
1*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 +
 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 64*b**11*c**3
*n**9*x**2*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 3
2670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n
5 + 45995730*b11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 1
20543840*b**11*n + 39916800*b**11) + 1797*b**11*c**3*n**8*x**2*(a + b*x)**n
/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 35742

3*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n*
*4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 3991
6800*b**11) + 29076*b**11*c**3*n**7*x**2*(a + b*x)**n/(b**11*n**11 + 66*b**
11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558
*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n
3 + 150917976*b11*n**2 + 120543840*b**11*n + 39916800*b**11) + 299271*b
11*c3*n**6*x**2*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11
*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 1333953
5*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11
*n**2 + 120543840*b**11*n + 39916800*b**11) + 2039016*b**11*c**3*n**5*x**2*
(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*
n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995
730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b*
*11*n + 39916800*b**11) + 9261503*b**11*c**3*n**4*x**2*(a + b*x)**n/(b**11*
n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*
n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105
258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**
11) + 27472724*b**11*c**3*n**3*x**2*(a + b*x)**n/(b**11*n**11 + 66*b**11*n*
*10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**1
1*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 +
 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 50312628*b**1
1*c**3*n**2*x**2*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n*
*9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b
11*n5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n*
*2 + 120543840*b**11*n + 39916800*b**11) + 50292720*b**11*c**3*n*x**2*(a +
b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8
 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b
11*n4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n
 + 39916800*b**11) + 19958400*b**11*c**3*x**2*(a + b*x)**n/(b**11*n**11 + 6
6*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 26
37558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b*
*11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 3*b
11*c2*d*n**10*x**5*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b*
*11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 1333
9535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b*
*11*n**2 + 120543840*b**11*n + 39916800*b**11) + 183*b**11*c**2*d*n**9*x**5
*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11
*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 4599
5730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b
11*n + 39916800*b11) + 4860*b**11*c**2*d*n**8*x**5*(a + b*x)**n/(b**11*
n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*
n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105
258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**
11) + 73710*b**11*c**2*d*n**7*x**5*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**
10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11

*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 +
150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 703719*b**11*c
2*d*n6*x**5*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**
9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b*
*11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**
2 + 120543840*b**11*n + 39916800*b**11) + 4394079*b**11*c**2*d*n**5*x**5*(a
 + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n*
*8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 4599573
0*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**1
1*n + 39916800*b**11) + 18048210*b**11*c**2*d*n**4*x**5*(a + b*x)**n/(b**11
*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11
*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 10
5258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b*
*11) + 47746140*b**11*c**2*d*n**3*x**5*(a + b*x)**n/(b**11*n**11 + 66*b**11
*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b
11*n6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**
3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 77043528*b
11*c2*d*n**2*x**5*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**
11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339
535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**
11*n**2 + 120543840*b**11*n + 39916800*b**11) + 67536288*b**11*c**2*d*n*x**
5*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**1
1*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 459
95730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*
b**11*n + 39916800*b**11) + 23950080*b**11*c**2*d*x**5*(a + b*x)**n/(b**11*
n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*
n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105
258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**
11) + 3*b**11*c*d**2*n**10*x**8*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10
 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n*
*6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150
917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 174*b**11*c*d**2*
n**9*x**8*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32
670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n*
*5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 12
0543840*b**11*n + 39916800*b**11) + 4383*b**11*c*d**2*n**8*x**8*(a + b*x)**
n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 3574
23*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n
4 + 105258076*b11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 399
16800*b**11) + 62946*b**11*c*d**2*n**7*x**8*(a + b*x)**n/(b**11*n**11 + 66*
b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637
558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**1
1*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 56870
1*b**11*c*d**2*n**6*x**8*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*
b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13

339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*
b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 3363066*b**11*c*d**2*n**
5*x**8*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670
*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5
+ 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 12054
3840*b**11*n + 39916800*b**11) + 13114077*b**11*c*d**2*n**4*x**8*(a + b*x)*
*n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357
423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*
n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39
916800*b**11) + 33074574*b**11*c*d**2*n**3*x**8*(a + b*x)**n/(b**11*n**11 +
66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 +
2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*
b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 5
1177636*b**11*c*d**2*n**2*x**8*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 +
1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**
6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 1509
17976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 43332840*b**11*c*d
2*n*x8*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 3
2670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n
5 + 45995730*b11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 1
20543840*b**11*n + 39916800*b**11) + 14968800*b**11*c*d**2*x**8*(a + b*x)**
n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 3574
23*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n
4 + 105258076*b11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 399
16800*b**11) + b**11*d**3*n**10*x**11*(a + b*x)**n/(b**11*n**11 + 66*b**11*
n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b*
*11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3
+ 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 55*b**11*d*
*3*n**9*x**11*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9
+ 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**1
1*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2
+ 120543840*b**11*n + 39916800*b**11) + 1320*b**11*d**3*n**8*x**11*(a + b*x
)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 3
57423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**1
1*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n +
39916800*b**11) + 18150*b**11*d**3*n**7*x**11*(a + b*x)**n/(b**11*n**11 + 6
6*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 26
37558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b*
*11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 157
773*b**11*d**3*n**6*x**11*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925
*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 1
3339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976
*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 902055*b**11*d**3*n**5*
x**11*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*
b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 +

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 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543
840*b**11*n + 39916800*b**11) + 3416930*b**11*d**3*n**4*x**11*(a + b*x)**n/
(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423
*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**
4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916
800*b**11) + 8409500*b**11*d**3*n**3*x**11*(a + b*x)**n/(b**11*n**11 + 66*b
**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 26375
58*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11
*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 127535
76*b**11*d**3*n**2*x**11*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*
b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13
339535*b**11*n**5 + 45995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*
b**11*n**2 + 120543840*b**11*n + 39916800*b**11) + 10628640*b**11*d**3*n*x*
*11*(a + b*x)**n/(b**11*n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b*
*11*n**8 + 357423*b**11*n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 4
5995730*b**11*n**4 + 105258076*b**11*n**3 + 150917976*b**11*n**2 + 12054384
0*b**11*n + 39916800*b**11) + 3628800*b**11*d**3*x**11*(a + b*x)**n/(b**11*
n**11 + 66*b**11*n**10 + 1925*b**11*n**9 + 32670*b**11*n**8 + 357423*b**11*
n**7 + 2637558*b**11*n**6 + 13339535*b**11*n**5 + 45995730*b**11*n**4 + 105
258076*b**11*n**3 + 150917976*b**11*n**2 + 120543840*b**11*n + 39916800*b**
11), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. 2(396) = 792.

Time = 0.23 (sec) , antiderivative size = 953, normalized size of antiderivative = 2.41

$$\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^3/((n^2 + 3*n + 2)*b^2) + 3
*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n
)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2
- 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c^2*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^
2 + 274*n + 120)*b^5) + 3*((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 +
13132*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1
624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3
+ 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)
*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*
n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040
*a^8)*(b*x + a)^n*c*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 6
7284*n^3 + 118124*n^2 + 109584*n + 40320)*b^8) + ((n^10 + 55*n^9 + 1320*n^8
 + 18150*n^7 + 157773*n^6 + 902055*n^5 + 3416930*n^4 + 8409500*n^3 + 127535
```

```
76*n^2 + 10628640*n + 3628800)*b^11*x^11 + (n^10 + 45*n^9 + 870*n^8 + 9450*
n^7 + 63273*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3 + 1026576*n^2 + 362
880*n)*a*b^10*x^10 - 10*(n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67
284*n^4 + 118124*n^3 + 109584*n^2 + 40320*n)*a^2*b^9*x^9 + 90*(n^8 + 28*n^7
 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^3*b^8*
x^8 - 720*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*
a^4*b^7*x^7 + 5040*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^5*
b^6*x^6 - 30240*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^6*b^5*x^5 + 15120
0*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^7*b^4*x^4 - 604800*(n^3 + 3*n^2 + 2*n)*a^8
*b^3*x^3 + 1814400*(n^2 + n)*a^9*b^2*x^2 - 3628800*a^10*b*n*x + 3628800*a^1
1)*(b*x + a)^n*d^3/((n^11 + 66*n^10 + 1925*n^9 + 32670*n^8 + 357423*n^7 + 2
637558*n^6 + 13339535*n^5 + 45995730*n^4 + 105258076*n^3 + 150917976*n^2 +
120543840*n + 39916800)*b^11)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4934 vs. 2(396) = 792.

Time = 0.37 (sec) , antiderivative size = 4934, normalized size of antiderivative = 12.46

$$\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^11*d^3*n^10*x^11 + (b*x + a)^n*a*b^10*d^3*n^10*x^10 + 55*(b*
x + a)^n*b^11*d^3*n^9*x^11 + 45*(b*x + a)^n*a*b^10*d^3*n^9*x^10 + 1320*(b*x
 + a)^n*b^11*d^3*n^8*x^11 + 3*(b*x + a)^n*b^11*c*d^2*n^10*x^8 - 10*(b*x + a
)^n*a^2*b^9*d^3*n^9*x^9 + 870*(b*x + a)^n*a*b^10*d^3*n^8*x^10 + 18150*(b*x
+ a)^n*b^11*d^3*n^7*x^11 + 3*(b*x + a)^n*a*b^10*c*d^2*n^10*x^7 + 174*(b*x +
 a)^n*b^11*c*d^2*n^9*x^8 - 360*(b*x + a)^n*a^2*b^9*d^3*n^8*x^9 + 9450*(b*x
+ a)^n*a*b^10*d^3*n^7*x^10 + 157773*(b*x + a)^n*b^11*d^3*n^6*x^11 + 153*(b*
x + a)^n*a*b^10*c*d^2*n^9*x^7 + 4383*(b*x + a)^n*b^11*c*d^2*n^8*x^8 + 90*(b
*x + a)^n*a^3*b^8*d^3*n^8*x^8 - 5460*(b*x + a)^n*a^2*b^9*d^3*n^7*x^9 + 6327
3*(b*x + a)^n*a*b^10*d^3*n^6*x^10 + 902055*(b*x + a)^n*b^11*d^3*n^5*x^11 +
3*(b*x + a)^n*b^11*c^2*d*n^10*x^5 - 21*(b*x + a)^n*a^2*b^9*c*d^2*n^9*x^6 +
3312*(b*x + a)^n*a*b^10*c*d^2*n^8*x^7 + 62946*(b*x + a)^n*b^11*c*d^2*n^7*x^
8 + 2520*(b*x + a)^n*a^3*b^8*d^3*n^7*x^8 - 45360*(b*x + a)^n*a^2*b^9*d^3*n^
6*x^9 + 269325*(b*x + a)^n*a*b^10*d^3*n^5*x^10 + 3416930*(b*x + a)^n*b^11*d
^3*n^4*x^11 + 3*(b*x + a)^n*a*b^10*c^2*d*n^10*x^4 + 183*(b*x + a)^n*b^11*c^
2*d*n^9*x^5 - 945*(b*x + a)^n*a^2*b^9*c*d^2*n^8*x^6 + 39762*(b*x + a)^n*a*b
^10*c*d^2*n^7*x^7 - 720*(b*x + a)^n*a^4*b^7*d^3*n^7*x^7 + 568701*(b*x + a)^
n*b^11*c*d^2*n^6*x^8 + 28980*(b*x + a)^n*a^3*b^8*d^3*n^6*x^8 - 224490*(b*x
+ a)^n*a^2*b^9*d^3*n^5*x^9 + 723680*(b*x + a)^n*a*b^10*d^3*n^4*x^10 + 84095
00*(b*x + a)^n*b^11*d^3*n^3*x^11 + 171*(b*x + a)^n*a*b^10*c^2*d*n^9*x^4 + 4
860*(b*x + a)^n*b^11*c^2*d*n^8*x^5 + 126*(b*x + a)^n*a^3*b^8*c*d^2*n^8*x^5
```

- 17514*(b*x + a)^n*a^2*b^9*c*d^2*n^7*x^6 + 290367*(b*x + a)^n*a*b^10*c*d^2*n^6*x^7 - 15120*(b*x + a)^n*a^4*b^7*d^3*n^6*x^7 + 3363066*(b*x + a)^n*b^11*c*d^2*n^5*x^8 + 176400*(b*x + a)^n*a^3*b^8*d^3*n^5*x^8 - 672840*(b*x + a)^n*a^2*b^9*d^3*n^4*x^9 + 1172700*(b*x + a)^n*a*b^10*d^3*n^3*x^10 + 12753576*(b*x + a)^n*b^11*d^3*n^2*x^11 + (b*x + a)^n*b^11*c^3*n^10*x^2 - 12*(b*x + a)^n*a^2*b^9*c^2*d*n^9*x^3 + 4176*(b*x + a)^n*a*b^10*c^2*d*n^8*x^4 + 73710*(b*x + a)^n*b^11*c^2*d*n^7*x^5 + 5040*(b*x + a)^n*a^3*b^8*c*d^2*n^7*x^5 - 173250*(b*x + a)^n*a^2*b^9*c*d^2*n^6*x^6 + 5040*(b*x + a)^n*a^5*b^6*d^3*n^6*x^6 + 1330497*(b*x + a)^n*a*b^10*c*d^2*n^5*x^7 - 126000*(b*x + a)^n*a^4*b^7*d^3*n^5*x^7 + 13114077*(b*x + a)^n*b^11*c*d^2*n^4*x^8 + 609210*(b*x + a)^n*a^3*b^8*d^3*n^4*x^8 - 1181240*(b*x + a)^n*a^2*b^9*d^3*n^3*x^9 + 1026576*(b*x + a)^n*a*b^10*d^3*n^2*x^10 + 10628640*(b*x + a)^n*b^11*d^3*n*x^11 + (b*x + a)^n*a*b^10*c^3*n^10*x + 64*(b*x + a)^n*b^11*c^3*n^9*x^2 - 648*(b*x + a)^n*a^2*b^9*c^2*d*n^8*x^3 + 57006*(b*x + a)^n*a*b^10*c^2*d*n^7*x^4 - 630*(b*x + a)^n*a^4*b^7*c*d^2*n^7*x^4 + 703719*(b*x + a)^n*b^11*c^2*d*n^6*x^5 + 79884*(b*x + a)^n*a^3*b^8*c*d^2*n^6*x^5 - 993069*(b*x + a)^n*a^2*b^9*c*d^2*n^5*x^6 + 75600*(b*x + a)^n*a^5*b^6*d^3*n^5*x^6 + 3800598*(b*x + a)^n*a*b^10*c*d^2*n^4*x^7 - 529200*(b*x + a)^n*a^4*b^7*d^3*n^4*x^7 + 33074574*(b*x + a)^n*b^11*c*d^2*n^3*x^8 + 1181880*(b*x + a)^n*a^3*b^8*d^3*n^3*x^8 - 1095840*(b*x + a)^n*a^2*b^9*d^3*n^2*x^9 + 362880*(b*x + a)^n*a*b^10*d^3*n*x^10 + 3628800*(b*x + a)^n*b^11*d^3*x^11 + 63*(b*x + a)^n*a*b^10*c^3*n^9*x + 1797*(b*x + a)^n*b^11*c^3*n^8*x^2 + 36*(b*x + a)^n*a^3*b^8*c^2*d*n^8*x^2 - 14760*(b*x + a)^n*a^2*b^9*c^2*d*n^7*x^3 + 475695*(b*x + a)^n*a*b^10*c^2*d*n^6*x^4 - 22680*(b*x + a)^n*a^4*b^7*c*d^2*n^6*x^4 + 4394079*(b*x + a)^n*b^11*c^2*d*n^5*x^5 + 640080*(b*x + a)^n*a^3*b^8*c*d^2*n^5*x^5 - 30240*(b*x + a)^n*a^6*b^5*d^3*n^5*x^5 - 3355065*(b*x + a)^n*a^2*b^9*c*d^2*n^4*x^6 + 428400*(b*x + a)^n*a^5*b^6*d^3*n^4*x^6 + 6470388*(b*x + a)^n*a*b^10*c*d^2*n^3*x^7 - 1169280*(b*x + a)^n*a^4*b^7*d^3*n^3*x^7 + 51177636*(b*x + a)^n*b^11*c*d^2*n^2*x^8 + 1176120*(b*x + a)^n*a^3*b^8*d^3*n^2*x^8 - 403200*(b*x + a)^n*a^2*b^9*d^3*n*x^9 - (b*x + a)^n*a^2*b^9*c^3*n^9 + 1734*(b*x + a)^n*a*b^10*c^3*n^8*x + 29076*(b*x + a)^n*b^11*c^3*n^7*x^2 + 1872*(b*x + a)^n*a^3*b^8*c^2*d*n^7*x^2 - 183744*(b*x + a)^n*a^2*b^9*c^2*d*n^6*x^3 + 2520*(b*x + a)^n*a^5*b^6*c*d^2*n^6*x^3 + 2491299*(b*x + a)^n*a*b^10*c^2*d*n^5*x^4 - 308700*(b*x + a)^n*a^4*b^7*c*d^2*n^5*x^4 + 18048210*(b*x + a)^n*b^11*c^2*d*n^4*x^5 + 2758014*(b*x + a)^n*a^3*b^8*c*d^2*n^4*x^5 - 302400*(b*x + a)^n*a^6*b^5*d^3*n^4*x^5 - 6473796*(b*x + a)^n*a^2*b^9*c*d^2*n^3*x^6 + 1134000*(b*x + a)^n*a^5*b^6*d^3*n^3*x^6 + 5884920*(b*x + a)^n*a*b^10*c*d^2*n^2*x^7 - 1270080*(b*x + a)^n*a^4*b^7*d^3*n^2*x^7 + 43332840*(b*x + a)^n*b^11*c*d^2*n*x^8 + 453600*(b*x + a)^n*a^3*b^8*d^3*n*x^8 - 63*(b*x + a)^n*a^2*b^9*c^3*n^8 + 27342*(b*x + a)^n*a*b^10*c^3*n^7*x - 72*(b*x + a)^n*a^4*b^7*c^2*d*n^7*x + 299271*(b*x + a)^n*b^11*c^3*n^6*x^2 + 40536*(b*x + a)^n*a^3*b^8*c^2*d*n^6*x^2 - 1351548*(b*x + a)^n*a^2*b^9*c^2*d*n^5*x^3 + 83160*(b*x + a)^n*a^5*b^6*c*d^2*n^5*x^3 + 8083014*(b*x + a)^n*a*b^10*c^2*d*n^4*x^4 - 1965600*(b*x + a)^n*a^4*b^7*c*d^2*n^4*x^4 + 151200*(b*x + a)^n*a^7*b^4*d^3*n^4*x^4 + 47746140*(b*x + a)^n*b^11*c^2*d*n^3*x^5 + 6340320*(b*x + a)^n*a^3*b^8*c*d^2*n^3*x^5 - 1058400*(b*x

+ a)^n*a^6*b^5*d^3*n^3*x^5 - 6449940*(b*x + a)^n*a^2*b^9*c*d^2*n^2*x^6 + 13
80960*(b*x + a)^n*a^5*b^6*d^3*n^2*x^6 + 2138400*(b*x + a)^n*a*b^10*c*d^2*n*
x^7 - 518400*(b*x + a)^n*a^4*b^7*d^3*n*x^7 + 14968800*(b*x + a)^n*b^11*c*d^
2*x^8 - 1734*(b*x + a)^n*a^2*b^9*c^3*n^7 + 271929*(b*x + a)^n*a*b^10*c^3*n^
6*x - 3672*(b*x + a)^n*a^4*b^7*c^2*d*n^6*x + 2039016*(b*x + a)^n*b^11*c^3*n
^5*x^2 + 470160*(b*x + a)^n*a^3*b^8*c^2*d*n^5*x^2 - 7560*(b*x + a)^n*a^6*b^
5*c*d^2*n^5*x^2 - 5910552*(b*x + a)^n*a^2*b^9*c^2*d*n^4*x^3 + 985320*(b*x +
 a)^n*a^5*b^6*c*d^2*n^4*x^3 + 15414084*(b*x + a)^n*a*b^10*c^2*d*n^3*x^4 - 5
927670*(b*x + a)^n*a^4*b^7*c*d^2*n^3*x^4 + 907200*(b*x + a)^n*a^7*b^4*d^3*n
^3*x^4 + 77043528*(b*x + a)^n*b^11*c^2*d*n^2*x^5 + 7141176*(b*x + a)^n*a^3*
b^8*c*d^2*n^2*x^5 - 1512000*(b*x + a)^n*a^6*b^5*d^3*n^2*x^5 - 2494800*(b*x
+ a)^n*a^2*b^9*c*d^2*n*x^6 + 604800*(b*x + a)^n*a^5*b^6*d^3*n*x^6 - 27342*(
b*x + a)^n*a^2*b^9*c^3*n^6 + 72*(b*x + a)^n*a^5*b^6*c^2*d*n^6 + 1767087*(b*
x + a)^n*a*b^10*c^3*n^5*x - 77400*(b*x + a)^n*a^4*b^7*c^2*d*n^5*x + 9261503
*(b*x + a)^n*b^11*c^3*n^4*x^2 + 3114324*(b*x + a)^n*a^3*b^8*c^2*d*n^4*x^2 -
 234360*(b*x + a)^n*a^6*b^5*c*d^2*n^4*x^2 - 14600400*(b*x + a)^n*a^2*b^9*c^
2*d*n^3*x^3 + 4906440*(b*x + a)^n*a^5*b^6*c*d^2*n^3*x^3 - 604800*(b*x + a)^
n*a^8*b^3*d^3*n^3*x^3 + 15387192*(b*x + a)^n*a*b^10*c^2*d*n^2*x^4 - 7990920
*(b*x + a)^n*a^4*b^7*c*d^2*n^2*x^4 + 1663200*(b*x + a)^n*a^7*b^4*d^3*n^2*x^
4 + 67536288*(b*x + a)^n*b^11*c^2*d*n*x^5 + 2993760*(b*x + a)^n*a^3*b^8*c*d
^2*n*x^5 - 725760*(b*x + a)^n*a^6*b^5*d^3*n*x^5 - 271929*(b*x + a)^n*a^2*b^
9*c^3*n^5 + 3672*(b*x + a)^n*a^5*b^6*c^2*d*n^5 + 7494416*(b*x + a)^n*a*b^10
*c^3*n^4*x - 862920*(b*x + a)^n*a^4*b^7*c^2*d*n^4*x + 15120*(b*x + a)^n*a^7
*b^4*c*d^2*n^4*x + 27472724*(b*x + a)^n*b^11*c^3*n^3*x^2 + 11503008*(b*x +
a)^n*a^3*b^8*c^2*d*n^3*x^2 - 2487240*(b*x + a)^n*a^6*b^5*c*d^2*n^3*x^2 - 17
855136*(b*x + a)^n*a^2*b^9*c^2*d*n^2*x^3 + 8991360*(b*x + a)^n*a^5*b^6*c*d^
2*n^2*x^3 - 1814400*(b*x + a)^n*a^8*b^3*d^3*n^2*x^3 + 5987520*(b*x + a)^n*a
*b^10*c^2*d*n*x^4 - 3742200*(b*x + a)^n*a^4*b^7*c*d^2*n*x^4 + 907200*(b*x +
 a)^n*a^7*b^4*d^3*n*x^4 + 23950080*(b*x + a)^n*b^11*c^2*d*x^5 - 1767087*(b*
x + a)^n*a^2*b^9*c^3*n^4 + 77400*(b*x + a)^n*a^5*b^6*c^2*d*n^4 + 19978308*(
b*x + a)^n*a*b^10*c^3*n^3*x - 5365728*(b*x + a)^n*a^4*b^7*c^2*d*n^3*x + 453
600*(b*x + a)^n*a^7*b^4*c*d^2*n^3*x + 50312628*(b*x + a)^n*b^11*c^3*n^2*x^2
 + 20795184*(b*x + a)^n*a^3*b^8*c^2*d*n^2*x^2 - 9744840*(b*x + a)^n*a^6*b^5
*c*d^2*n^2*x^2 + 1814400*(b*x + a)^n*a^9*b^2*d^3*n^2*x^2 - 7983360*(b*x + a
)^n*a^2*b^9*c^2*d*n*x^3 + 4989600*(b*x + a)^n*a^5*b^6*c*d^2*n*x^3 - 1209600
*(b*x + a)^n*a^8*b^3*d^3*n*x^3 - 7494416*(b*x + a)^n*a^2*b^9*c^3*n^3 + 8629
20*(b*x + a)^n*a^5*b^6*c^2*d*n^3 - 15120*(b*x + a)^n*a^8*b^3*c*d^2*n^3 + 30
334320*(b*x + a)^n*a*b^10*c^3*n^2*x - 17640288*(b*x + a)^n*a^4*b^7*c^2*d*n^
2*x + 4520880*(b*x + a)^n*a^7*b^4*c*d^2*n^2*x + 50292720*(b*x + a)^n*b^11*c
^3*n*x^2 + 11975040*(b*x + a)^n*a^3*b^8*c^2*d*n*x^2 - 7484400*(b*x + a)^n*a
^6*b^5*c*d^2*n*x^2 + 1814400*(b*x + a)^n*a^9*b^2*d^3*n*x^2 - 19978308*(b*x
+ a)^n*a^2*b^9*c^3*n^2 + 5365728*(b*x + a)^n*a^5*b^6*c^2*d*n^2 - 453600*(b*
x + a)^n*a^8*b^3*c*d^2*n^2 + 19958400*(b*x + a)^n*a*b^10*c^3*n*x - 23950080
*(b*x + a)^n*a^4*b^7*c^2*d*n*x + 14968800*(b*x + a)^n*a^7*b^4*c*d^2*n*x - 3
628800*(b*x + a)^n*a^10*b*d^3*n*x + 19958400*(b*x + a)^n*b^11*c^3*x^2 - 303

```
34320*(b*x + a)^n*a^2*b^9*c^3*n + 17640288*(b*x + a)^n*a^5*b^6*c^2*d*n - 45
20880*(b*x + a)^n*a^8*b^3*c*d^2*n - 19958400*(b*x + a)^n*a^2*b^9*c^3 + 2395
0080*(b*x + a)^n*a^5*b^6*c^2*d - 14968800*(b*x + a)^n*a^8*b^3*c*d^2 + 36288
00*(b*x + a)^n*a^11*d^3)/(b^11*n^11 + 66*b^11*n^10 + 1925*b^11*n^9 + 32670*
b^11*n^8 + 357423*b^11*n^7 + 2637558*b^11*n^6 + 13339535*b^11*n^5 + 4599573
0*b^11*n^4 + 105258076*b^11*n^3 + 150917976*b^11*n^2 + 120543840*b^11*n + 3
9916800*b^11)
```

Mupad [B] (verification not implemented)

Time = 22.66 (sec) , antiderivative size = 2436, normalized size of antiderivative = 6.15

$$\int x(a+bx)^n \left(c+dx^3\right)^3 dx = \text{Too large to display}$$

```
[In] int(x*(c + d*x^3)^3*(a + b*x)^n,x)
```

```
[Out] (d^3*x^11*(a + b*x)^n*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^
4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 362880
0))/(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*
n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39
916800) - (a^2*(a + b*x)^n*(19958400*b^9*c^3 - 3628800*a^9*d^3 + 30334320*b
^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9*c^3*n^3 + 1767087*b^9*c^3*n^4
 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1734*b^9*c^3*n^7 + 63*b^9*c^3*n
^8 + b^9*c^3*n^9 - 23950080*a^3*b^6*c^2*d + 14968800*a^6*b^3*c*d^2 - 176402
88*a^3*b^6*c^2*d*n + 4520880*a^6*b^3*c*d^2*n - 5365728*a^3*b^6*c^2*d*n^2 +
453600*a^6*b^3*c*d^2*n^2 - 862920*a^3*b^6*c^2*d*n^3 + 15120*a^6*b^3*c*d^2*n
^3 - 77400*a^3*b^6*c^2*d*n^4 - 3672*a^3*b^6*c^2*d*n^5 - 72*a^3*b^6*c^2*d*n^
6))/(b^11*(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 133
39535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^1
1 + 39916800)) + (x^2*(n + 1)*(a + b*x)^n*(19958400*b^9*c^3 + 1814400*a^9*d
^3*n + 30334320*b^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9*c^3*n^3 + 17
67087*b^9*c^3*n^4 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1734*b^9*c^3*n
^7 + 63*b^9*c^3*n^8 + b^9*c^3*n^9 + 11975040*a^3*b^6*c^2*d*n - 7484400*a^6*
b^3*c*d^2*n + 8820144*a^3*b^6*c^2*d*n^2 - 2260440*a^6*b^3*c*d^2*n^2 + 26828
64*a^3*b^6*c^2*d*n^3 - 226800*a^6*b^3*c*d^2*n^3 + 431460*a^3*b^6*c^2*d*n^4
- 7560*a^6*b^3*c*d^2*n^4 + 38700*a^3*b^6*c^2*d*n^5 + 1836*a^3*b^6*c^2*d*n^6
 + 36*a^3*b^6*c^2*d*n^7))/(b^9*(120543840*n + 150917976*n^2 + 105258076*n^3
 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 192
5*n^9 + 66*n^10 + n^11 + 39916800)) + (a*n*x*(a + b*x)^n*(19958400*b^9*c^3
- 3628800*a^9*d^3 + 30334320*b^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9
*c^3*n^3 + 1767087*b^9*c^3*n^4 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1
734*b^9*c^3*n^7 + 63*b^9*c^3*n^8 + b^9*c^3*n^9 - 23950080*a^3*b^6*c^2*d + 1
4968800*a^6*b^3*c*d^2 - 17640288*a^3*b^6*c^2*d*n + 4520880*a^6*b^3*c*d^2*n
- 5365728*a^3*b^6*c^2*d*n^2 + 453600*a^6*b^3*c*d^2*n^2 - 862920*a^3*b^6*c^2
```

*d*n^3 + 15120*a^6*b^3*c*d^2*n^3 - 77400*a^3*b^6*c^2*d*n^4 - 3672*a^3*b^6*c
^2*d*n^5 - 72*a^3*b^6*c^2*d*n^6))/(b^10*(120543840*n + 150917976*n^2 + 1052
58076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*
n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) + (3*d*x^5*(a + b*x)^n*(50*n +
 35*n^2 + 10*n^3 + n^4 + 24)*(332640*b^6*c^2 - 10080*a^6*d^2*n + 245004*b^6
*c^2*n + 74524*b^6*c^2*n^2 + 11985*b^6*c^2*n^3 + 1075*b^6*c^2*n^4 + 51*b^6*
c^2*n^5 + b^6*c^2*n^6 + 41580*a^3*b^3*c*d*n + 12558*a^3*b^3*c*d*n^2 + 1260*
a^3*b^3*c*d*n^3 + 42*a^3*b^3*c*d*n^4))/(b^6*(120543840*n + 150917976*n^2 +
105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32
670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) + (3*d^2*x^8*(a + b*x)^n*(
990*b^3*c + 30*b^3*c*n^2 + b^3*c*n^3 + 30*a^3*d*n + 299*b^3*c*n)*(13068*n +
 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040))/(b^3*(12
0543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2
637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)
) + (a*d^3*n*x^10*(a + b*x)^n*(1026576*n + 1172700*n^2 + 723680*n^3 + 26932
5*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880))/(b*(120543
840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 26375
58*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) -
(10*a^2*d^3*n*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^
4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(b^2*(120543840*n + 1509179
76*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423
*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) + (3*a*d^2*n*x^7*
(a + b*x)^n*(990*b^3*c - 240*a^3*d + 30*b^3*c*n^2 + b^3*c*n^3 + 299*b^3*c*n
)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(b^4*(12054
3840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637
558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) -
 (21*a^2*d^2*n*x^6*(a + b*x)^n*(990*b^3*c - 240*a^3*d + 30*b^3*c*n^2 + b^3*
c*n^3 + 299*b^3*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^5*
(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5
+ 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 399168
00)) + (3*a*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(50400*a^6*d^2 + 3
32640*b^6*c^2 + 245004*b^6*c^2*n + 74524*b^6*c^2*n^2 + 11985*b^6*c^2*n^3 +
1075*b^6*c^2*n^4 + 51*b^6*c^2*n^5 + b^6*c^2*n^6 - 207900*a^3*b^3*c*d - 6279
0*a^3*b^3*c*d*n - 6300*a^3*b^3*c*d*n^2 - 210*a^3*b^3*c*d*n^3))/(b^7*(120543
840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 26375
58*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) -
(12*a^2*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(50400*a^6*d^2 + 332640*b^6*c^2
 + 245004*b^6*c^2*n + 74524*b^6*c^2*n^2 + 11985*b^6*c^2*n^3 + 1075*b^6*c^2*
n^4 + 51*b^6*c^2*n^5 + b^6*c^2*n^6 - 207900*a^3*b^3*c*d - 62790*a^3*b^3*c*d
*n - 6300*a^3*b^3*c*d*n^2 - 210*a^3*b^3*c*d*n^3))/(b^8*(120543840*n + 15091
7976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 3574
23*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800))

3.184 $\int (a+bx)^n \left(c+dx^3\right)^3 dx$

Optimal result

Integrand size = 17, antiderivative size = 337

$$\int (a+bx)^n \left(c+dx^3\right)^3 dx = \frac{\left(b^3c-a^3d\right)^3 (a+bx)^{1+n}}{b^{10}(1+n)} + \frac{9a^2d\left(b^3c-a^3d\right)^2 (a+bx)^{2+n}}{b^{10}(2+n)} - \frac{9ad(b^3c-4a^3d)\left(b^3c-a^3d\right)(a+bx)^{3+n}}{b^{10}(3+n)} + \frac{3d(b^6c^2-20a^3b^3cd+28a^6d^2)(a+bx)^{4+n}}{b^{10}(4+n)} + \frac{9a^2d^2(5b^3c-14a^3d)(a+bx)^{5+n}}{b^{10}(5+n)} - \frac{18ad^2(b^3c-7a^3d)(a+bx)^{6+n}}{b^{10}(6+n)} + \frac{3d^2(b^3c-28a^3d)(a+bx)^{7+n}}{b^{10}(7+n)} + \frac{36a^2d^3(a+bx)^{8+n}}{b^{10}(8+n)} - \frac{9ad^3(a+bx)^{9+n}}{b^{10}(9+n)} + \frac{d^3(a+bx)^{10+n}}{b^{10}(10+n)}$$

```
[Out] (-a^3*d+b^3*c)^3*(b*x+a)^(1+n)/b^10/(1+n)+9*a^2*d*(-a^3*d+b^3*c)^2*(b*x+a)^
(2+n)/b^10/(2+n)-9*a*d*(-4*a^3*d+b^3*c)*(-a^3*d+b^3*c)*(b*x+a)^(3+n)/b^10/(
3+n)+3*d*(28*a^6*d^2-20*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^(4+n)/b^10/(4+n)+9*a^2
*d^2*(-14*a^3*d+5*b^3*c)*(b*x+a)^(5+n)/b^10/(5+n)-18*a*d^2*(-7*a^3*d+b^3*c)
*(b*x+a)^(6+n)/b^10/(6+n)+3*d^2*(-28*a^3*d+b^3*c)*(b*x+a)^(7+n)/b^10/(7+n)+
36*a^2*d^3*(b*x+a)^(8+n)/b^10/(8+n)-9*a*d^3*(b*x+a)^(9+n)/b^10/(9+n)+d^3*(b
*x+a)^(10+n)/b^10/(10+n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1864}

$$\int (a+bx)^n \left(c+dx^3\right)^3 dx = -\frac{18ad^2(b^3c-7a^3d)(a+bx)^{n+6}}{b^{10}(n+6)} + \frac{3d^2(b^3c-28a^3d)(a+bx)^{n+7}}{b^{10}(n+7)} + \frac{(b^3c-a^3d)^3(a+bx)^{n+1}}{b^{10}(n+1)} - \frac{9ad(b^3c-4a^3d)(b^3c-a^3d)(a+bx)^{n+3}}{b^{10}(n+3)} + \frac{36a^2d^3(a+bx)^{n+8}}{b^{10}(n+8)} + \frac{3d(28a^6d^2-20a^3b^3cd+b^6c^2)(a+bx)^{n+4}}{b^{10}(n+4)} + \frac{9a^2d^2(5b^3c-14a^3d)(a+bx)^{n+5}}{b^{10}(n+5)} + \frac{9a^2d(b^3c-a^3d)^2(a+bx)^{n+2}}{b^{10}(n+2)} - \frac{9ad^3(a+bx)^{n+9}}{b^{10}(n+9)} + \frac{d^3(a+bx)^{n+10}}{b^{10}(n+10)}$$

```
[In] Int[(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^10*(1 + n)) + (9*a^2*d*(b^3*c - a^
3*d)^2*(a + b*x)^(2 + n))/(b^10*(2 + n)) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*c
- a^3*d)*(a + b*x)^(3 + n))/(b^10*(3 + n)) + (3*d*(b^6*c^2 - 20*a^3*b^3*c*d
 + 28*a^6*d^2)*(a + b*x)^(4 + n))/(b^10*(4 + n)) + (9*a^2*d^2*(5*b^3*c - 14
*a^3*d)*(a + b*x)^(5 + n))/(b^10*(5 + n)) - (18*a*d^2*(b^3*c - 7*a^3*d)*(a
+ b*x)^(6 + n))/(b^10*(6 + n)) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^(7 + n
))/(b^10*(7 + n)) + (36*a^2*d^3*(a + b*x)^(8 + n))/(b^10*(8 + n)) - (9*a*d^
3*(a + b*x)^(9 + n))/(b^10*(9 + n)) + (d^3*(a + b*x)^(10 + n))/(b^10*(10 +
n))
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \int \left(\frac{(b^3c-a^3d)^3(a+bx)^n}{b^9}+\frac{9d(ab^3c-a^4d)^2(a+bx)^{1+n}}{b^9}+\frac{9ad(b^3c-4a^3d)(-b^3c+a^3d)(a+bx)^{2+n}}{b^9}+\frac{3d(b^6c^2-20a^3b^3cd+28a^6d^2)(a+bx)^{3+n}}{b^9}-\frac{9a^2d^2(-5b^3c+14a^3d)(a+bx)^{4+n}}{b^9}+\frac{18ad^2(-b^3c+7a^3d)(a+bx)^{5+n}}{b^9}+\frac{3d^2(b^3c-28a^3d)(a+bx)^{6+n}}{b^9}+\frac{36a^2d^3(a+bx)^{7+n}}{b^9}-\frac{9ad^3(a+bx)^{8+n}}{b^9}+\frac{d^3(a+bx)^{9+n}}{b^9}\right)dx$$

$$=\frac{(b^3c-a^3d)^3(a+bx)^{1+n}}{b^{10}(1+n)}+\frac{9a^2d(b^3c-a^3d)^2(a+bx)^{2+n}}{b^{10}(2+n)}-\frac{9ad(b^3c-4a^3d)(b^3c-a^3d)(a+bx)^{3+n}}{b^{10}(3+n)}+\frac{3d(b^6c^2-20a^3b^3cd+28a^6d^2)(a+bx)^{4+n}}{b^{10}(4+n)}+\frac{9a^2d^2(5b^3c-14a^3d)(a+bx)^{5+n}}{b^{10}(5+n)}-\frac{18ad^2(b^3c-7a^3d)(a+bx)^{6+n}}{b^{10}(6+n)}+\frac{3d^2(b^3c-28a^3d)(a+bx)^{7+n}}{b^{10}(7+n)}+\frac{36a^2d^3(a+bx)^{8+n}}{b^{10}(8+n)}-\frac{9ad^3(a+bx)^{9+n}}{b^{10}(9+n)}+\frac{d^3(a+bx)^{10+n}}{b^{10}(10+n)}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.86

$$\int (a+bx)^n\left(c+dx^3\right)^3\,dx = \frac{(a+bx)^{1+n}\left(\frac{(b^3c-a^3d)^3}{1+n}+\frac{9d(ab^3c-a^4d)^2(a+bx)}{2+n}-\frac{9ad(b^3c-4a^3d)(b^3c-a^3d)(a+bx)^2}{3+n}+\frac{3d(b^6c^2-20a^3b^3cd+28a^6d^2)(a+bx)^3}{4+n}+9a\right.}{b^{10}}$$

```
[In] Integrate[(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b^3*c - a^3*d)^3/(1 + n) + (9*d*(a*b^3*c - a^4*d)^2*(a
 + b*x))/(2 + n) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^2)/(3
 + n) + (3*d*(b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^3)/(4 + n) +
 (9*a^2*d^2*(5*b^3*c - 14*a^3*d)*(a + b*x)^4)/(5 + n) + (18*a*d^2*(-(b^3*c)
 + 7*a^3*d)*(a + b*x)^5)/(6 + n) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^6)/(
7 + n) + (36*a^2*d^3*(a + b*x)^7)/(8 + n) - (9*a*d^3*(a + b*x)^8)/(9 + n) +
 (d^3*(a + b*x)^9)/(10 + n)))/b^10
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2279 vs. 2(337) = 674.

Time = 1.14 (sec) , antiderivative size = 2280, normalized size of antiderivative = 6.77

method	result	size
gosper	Expression too large to display	2280
risch	Expression too large to display	2665
parallelrisch	Expression too large to display	3960

```
[In] int((b*x+a)^n*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^10*(b*x+a)^(1+n)/(n^10+55*n^9+1320*n^8+18150*n^7+157773*n^6+902055*n^5
+3416930*n^4+8409500*n^3+12753576*n^2+10628640*n+3628800)*(-b^9*d^3*n^9*x^9
-45*b^9*d^3*n^8*x^9+9*a*b^8*d^3*n^8*x^8-870*b^9*d^3*n^7*x^9+324*a*b^8*d^3*n
^7*x^8-3*b^9*c*d^2*n^9*x^6-9450*b^9*d^3*n^6*x^9-72*a^2*b^7*d^3*n^7*x^7+4914
*a*b^8*d^3*n^6*x^8-144*b^9*c*d^2*n^8*x^6-63273*b^9*d^3*n^5*x^9-2016*a^2*b^7
*d^3*n^6*x^7+18*a*b^8*c*d^2*n^8*x^5+40824*a*b^8*d^3*n^5*x^8-2952*b^9*c*d^2*
n^7*x^6-269325*b^9*d^3*n^4*x^9+504*a^3*b^6*d^3*n^6*x^6-23184*a^2*b^7*d^3*n^
5*x^7+756*a*b^8*c*d^2*n^7*x^5+202041*a*b^8*d^3*n^4*x^8-3*b^9*c^2*d*n^9*x^3-
33786*b^9*c*d^2*n^6*x^6-723680*b^9*d^3*n^3*x^9+10584*a^3*b^6*d^3*n^5*x^6-90
*a^2*b^7*c*d^2*n^7*x^4-141120*a^2*b^7*d^3*n^4*x^7+13176*a*b^8*c*d^2*n^6*x^5
+605556*a*b^8*d^3*n^3*x^8-153*b^9*c^2*d*n^8*x^3-236817*b^9*c*d^2*n^5*x^6-11
72700*b^9*d^3*n^2*x^9-3024*a^4*b^5*d^3*n^5*x^5+88200*a^3*b^6*d^3*n^4*x^6-33
30*a^2*b^7*c*d^2*n^6*x^4-487368*a^2*b^7*d^3*n^3*x^7+9*a*b^8*c^2*d*n^8*x^2+1
23660*a*b^8*c*d^2*n^5*x^5+1063116*a*b^8*d^3*n^2*x^8-3348*b^9*c^2*d*n^7*x^3-
1048446*b^9*c*d^2*n^4*x^6-1026576*b^9*d^3*n*x^9-45360*a^4*b^5*d^3*n^4*x^5+3
60*a^3*b^6*c*d^2*n^6*x^3+370440*a^3*b^6*d^3*n^3*x^6-49230*a^2*b^7*c*d^2*n^5
*x^4-945504*a^2*b^7*d^3*n^2*x^7+432*a*b^8*c^2*d*n^7*x^2+678942*a*b^8*c*d^2*
n^4*x^5+986256*a*b^8*d^3*n*x^8-b^9*c^3*n^9-41058*b^9*c^2*d*n^6*x^3-2911668*
b^9*c*d^2*n^3*x^6-362880*b^9*d^3*x^9+15120*a^5*b^4*d^3*n^4*x^4-257040*a^4*b
^5*d^3*n^3*x^5+11880*a^3*b^6*c*d^2*n^5*x^3+818496*a^3*b^6*d^3*n^2*x^6-18*a^
2*b^7*c^2*d*n^7*x-372150*a^2*b^7*c*d^2*n^4*x^4-940896*a^2*b^7*d^3*n*x^7+874
8*a*b^8*c^2*d*n^6*x^2+2217024*a*b^8*c*d^2*n^3*x^5+362880*a*b^8*d^3*x^8-54*b
^9*c^3*n^8-309087*b^9*c^2*d*n^5*x^3-4846824*b^9*c*d^2*n^2*x^6+151200*a^5*b^
4*d^3*n^3*x^4-1080*a^4*b^5*c*d^2*n^5*x^2-680400*a^4*b^5*d^3*n^2*x^5+149400*
a^3*b^6*c*d^2*n^4*x^3+889056*a^3*b^6*d^3*n*x^6-828*a^2*b^7*c^2*d*n^6*x-1533
960*a^2*b^7*c*d^2*n^3*x^4-362880*a^2*b^7*d^3*x^7+96930*a*b^8*c^2*d*n^5*x^2+
4167864*a*b^8*c*d^2*n^2*x^5-1266*b^9*c^3*n^7-1469817*b^9*c^2*d*n^4*x^3-4332
960*b^9*c*d^2*n*x^6-60480*a^6*b^3*d^3*n^3*x^3+529200*a^5*b^4*d^3*n^2*x^4-32
400*a^4*b^5*c*d^2*n^4*x^2-828576*a^4*b^5*d^3*n*x^5+18*a^3*b^6*c^2*d*n^6+891
000*a^3*b^6*c*d^2*n^3*x^3+362880*a^3*b^6*d^3*x^6-15840*a^2*b^7*c^2*d*n^5*x-
3415320*a^2*b^7*c*d^2*n^2*x^4+636471*a*b^8*c^2*d*n^4*x^2+4073760*a*b^8*c*d^
2*n*x^5-16884*b^9*c^3*n^6-4371522*b^9*c^2*d*n^3*x^3-1555200*b^9*c*d^2*x^6-3
62880*a^6*b^3*d^3*n^2*x^3+2160*a^5*b^4*c*d^2*n^4*x+756000*a^5*b^4*d^3*n*x^4
```

```
-351000*a^4*b^5*c*d^2*n^3*x^2-362880*a^4*b^5*d^3*x^5+810*a^3*b^6*c^2*d*n^5+
2571840*a^3*b^6*c*d^2*n^2*x^3-162180*a^2*b^7*c^2*d*n^4*x-3762720*a^2*b^7*c*
d^2*n*x^4+2500038*a*b^8*c^2*d*n^3*x^2+1555200*a*b^8*c*d^2*x^5-140889*b^9*c^
3*n^5-7742412*b^9*c^2*d*n^2*x^3+181440*a^7*b^2*d^3*n^2*x^2-665280*a^6*b^3*d
^3*n*x^3+60480*a^5*b^4*c*d^2*n^3*x+362880*a^5*b^4*d^3*x^4-1620000*a^4*b^5*c
*d^2*n^2*x^2+15030*a^3*b^6*c^2*d*n^4+3373920*a^3*b^6*c*d^2*n*x^3-948582*a^2
*b^7*c^2*d*n^3*x-1555200*a^2*b^7*c*d^2*x^4+5614452*a*b^8*c^2*d*n^2*x^2-7611
66*b^9*c^3*n^4-7291080*b^9*c^2*d*n*x^3+544320*a^7*b^2*d^3*n*x^2-2160*a^6*b^
3*c*d^2*n^3-362880*a^6*b^3*d^3*x^3+581040*a^5*b^4*c*d^2*n^2*x-2855520*a^4*b
^5*c*d^2*n*x^2+147150*a^3*b^6*c^2*d*n^3+1555200*a^3*b^6*c*d^2*x^3-3102912*a
^2*b^7*c^2*d*n^2*x+6383880*a*b^8*c^2*d*n*x^2-2655764*b^9*c^3*n^3-2721600*b^
9*c^2*d*x^3-362880*a^8*b*d^3*n*x+362880*a^7*b^2*d^3*x^2-58320*a^6*b^3*c*d^2
*n^2+2077920*a^5*b^4*c*d^2*n*x-1555200*a^4*b^5*c*d^2*x^2+801432*a^3*b^6*c^2
*d*n^2-5023080*a^2*b^7*c^2*d*n*x+2721600*a*b^8*c^2*d*x^2-5753736*b^9*c^3*n^
2-362880*a^8*b*d^3*x-522720*a^6*b^3*c*d^2*n+1555200*a^5*b^4*c*d^2*x+2301480
*a^3*b^6*c^2*d*n-2721600*a^2*b^7*c^2*d*x-6999840*b^9*c^3*n+362880*a^9*d^3-1
555200*a^6*b^3*c*d^2+2721600*a^3*b^6*c^2*d-3628800*b^9*c^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2313 vs. $2(337) = 674$.

Time = 0.33 (sec) , antiderivative size = 2313, normalized size of antiderivative = 6.86

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] (a*b^9*c^3*n^9 + 54*a*b^9*c^3*n^8 + 1266*a*b^9*c^3*n^7 + 3628800*a*b^9*c^3
- 2721600*a^4*b^6*c^2*d + 1555200*a^7*b^3*c*d^2 - 362880*a^10*d^3 + (b^10*d
^3*n^9 + 45*b^10*d^3*n^8 + 870*b^10*d^3*n^7 + 9450*b^10*d^3*n^6 + 63273*b^1
0*d^3*n^5 + 269325*b^10*d^3*n^4 + 723680*b^10*d^3*n^3 + 1172700*b^10*d^3*n^
2 + 1026576*b^10*d^3*n + 362880*b^10*d^3)*x^10 + (a*b^9*d^3*n^9 + 36*a*b^9*
d^3*n^8 + 546*a*b^9*d^3*n^7 + 4536*a*b^9*d^3*n^6 + 22449*a*b^9*d^3*n^5 + 67
284*a*b^9*d^3*n^4 + 118124*a*b^9*d^3*n^3 + 109584*a*b^9*d^3*n^2 + 40320*a*b
^9*d^3*n)*x^9 - 9*(a^2*b^8*d^3*n^8 + 28*a^2*b^8*d^3*n^7 + 322*a^2*b^8*d^3*n
^6 + 1960*a^2*b^8*d^3*n^5 + 6769*a^2*b^8*d^3*n^4 + 13132*a^2*b^8*d^3*n^3 +
13068*a^2*b^8*d^3*n^2 + 5040*a^2*b^8*d^3*n)*x^8 + 3*(b^10*c*d^2*n^9 + 48*b^
10*c*d^2*n^8 + 518400*b^10*c*d^2 + 24*(41*b^10*c*d^2 + a^3*b^7*d^3)*n^7 + 6
*(1877*b^10*c*d^2 + 84*a^3*b^7*d^3)*n^6 + 21*(3759*b^10*c*d^2 + 200*a^3*b^7
*d^3)*n^5 + 42*(8321*b^10*c*d^2 + 420*a^3*b^7*d^3)*n^4 + 4*(242639*b^10*c*d
^2 + 9744*a^3*b^7*d^3)*n^3 + 72*(22439*b^10*c*d^2 + 588*a^3*b^7*d^3)*n^2 +
1440*(1003*b^10*c*d^2 + 12*a^3*b^7*d^3)*n)*x^7 + 18*(938*a*b^9*c^3 - a^4*b^
6*c^2*d)*n^6 + 3*(a*b^9*c*d^2*n^9 + 42*a*b^9*c*d^2*n^8 + 732*a*b^9*c*d^2*n^
7 + 6*(1145*a*b^9*c*d^2 - 28*a^4*b^6*d^3)*n^6 + 9*(4191*a*b^9*c*d^2 - 280*a
```

^4*b^6*d^3)*n^5 + 24*(5132*a*b^9*c*d^2 - 595*a^4*b^6*d^3)*n^4 + 4*(57887*a*b^9*c*d^2 - 9450*a^4*b^6*d^3)*n^3 + 48*(4715*a*b^9*c*d^2 - 959*a^4*b^6*d^3)*n^2 + 2880*(30*a*b^9*c*d^2 - 7*a^4*b^6*d^3)*n)*x^6 + 3*(46963*a*b^9*c^3 - 270*a^4*b^6*c^2*d)*n^5 - 18*(a^2*b^8*c*d^2*n^8 + 37*a^2*b^8*c*d^2*n^7 + 547*a^2*b^8*c*d^2*n^6 + (4135*a^2*b^8*c*d^2 - 168*a^5*b^5*d^3)*n^5 + 4*(4261*a^2*b^8*c*d^2 - 420*a^5*b^5*d^3)*n^4 + 4*(9487*a^2*b^8*c*d^2 - 1470*a^5*b^5*d^3)*n^3 + 48*(871*a^2*b^8*c*d^2 - 175*a^5*b^5*d^3)*n^2 + 576*(30*a^2*b^8*c*d^2 - 7*a^5*b^5*d^3)*n)*x^5 + 18*(42287*a*b^9*c^3 - 835*a^4*b^6*c^2*d)*n^4 + 3*(b^10*c^2*d*n^9 + 51*b^10*c^2*d*n^8 + 907200*b^10*c^2*d + 6*(186*b^10*c^2*d + 5*a^3*b^7*c*d^2)*n^7 + 6*(2281*b^10*c^2*d + 165*a^3*b^7*c*d^2)*n^6 + 3*(34343*b^10*c^2*d + 4150*a^3*b^7*c*d^2)*n^5 + 3*(163313*b^10*c^2*d + 24750*a^3*b^7*c*d^2 - 1680*a^6*b^4*d^3)*n^4 + 2*(728587*b^10*c^2*d + 107160*a^3*b^7*c*d^2 - 15120*a^6*b^4*d^3)*n^3 + 36*(71689*b^10*c^2*d + 7810*a^3*b^7*c*d^2 - 1540*a^6*b^4*d^3)*n^2 + 360*(6751*b^10*c^2*d + 360*a^3*b^7*c*d^2 - 84*a^6*b^4*d^3)*n)*x^4 + 2*(1327882*a*b^9*c^3 - 73575*a^4*b^6*c^2*d + 1080*a^7*b^3*c*d^2)*n^3 + 3*(a*b^9*c^2*d*n^9 + 48*a*b^9*c^2*d*n^8 + 972*a*b^9*c^2*d*n^7 + 30*(359*a*b^9*c^2*d - 4*a^4*b^6*c*d^2)*n^6 + 3*(23573*a*b^9*c^2*d - 1200*a^4*b^6*c*d^2)*n^5 + 6*(46297*a*b^9*c^2*d - 6500*a^4*b^6*c*d^2)*n^4 + 4*(155957*a*b^9*c^2*d - 45000*a^4*b^6*c*d^2 + 5040*a^7*b^3*d^3)*n^3 + 120*(5911*a*b^9*c^2*d - 2644*a^4*b^6*c*d^2 + 504*a^7*b^3*d^3)*n^2 + 2880*(105*a*b^9*c^2*d - 60*a^4*b^6*c*d^2 + 14*a^7*b^3*d^3)*n)*x^3 + 72*(79913*a*b^9*c^3 - 11131*a^4*b^6*c^2*d + 810*a^7*b^3*c*d^2)*n^2 - 9*(a^2*b^8*c^2*d*n^8 + 46*a^2*b^8*c^2*d*n^7 + 880*a^2*b^8*c^2*d*n^6 + 10*(901*a^2*b^8*c^2*d - 12*a^5*b^5*c*d^2)*n^5 + (52699*a^2*b^8*c^2*d - 3360*a^5*b^5*c*d^2)*n^4 + 8*(21548*a^2*b^8*c^2*d - 4035*a^5*b^5*c*d^2)*n^3 + 60*(4651*a^2*b^8*c^2*d - 1924*a^5*b^5*c*d^2 + 336*a^8*b^2*d^3)*n^2 + 1440*(105*a^2*b^8*c^2*d - 60*a^5*b^5*c*d^2 + 14*a^8*b^2*d^3)*n)*x^2 + 360*(19444*a*b^9*c^3 - 6393*a^4*b^6*c^2*d + 1452*a^7*b^3*c*d^2)*n + (b^10*c^3*n^9 + 54*b^10*c^3*n^8 + 3628800*b^10*c^3 + 6*(211*b^10*c^3 + 3*a^3*b^7*c^2*d)*n^7 + 18*(938*b^10*c^3 + 45*a^3*b^7*c^2*d)*n^6 + 3*(46963*b^10*c^3 + 5010*a^3*b^7*c^2*d)*n^5 + 18*(42287*b^10*c^3 + 8175*a^3*b^7*c^2*d - 120*a^6*b^4*c*d^2)*n^4 + 4*(663941*b^10*c^3 + 200358*a^3*b^7*c^2*d - 14580*a^6*b^4*c*d^2)*n^3 + 72*(79913*b^10*c^3 + 31965*a^3*b^7*c^2*d - 7260*a^6*b^4*c*d^2)*n^2 + 1440*(4861*b^10*c^3 + 1890*a^3*b^7*c^2*d - 1080*a^6*b^4*c*d^2 + 252*a^9*b*d^3)*n)*x)*(b*x + a)^n/(b^10*n^10 + 55*b^10*n^9 + 1320*b^10*n^8 + 18150*b^10*n^7 + 157773*b^10*n^6 + 902055*b^10*n^5 + 3416930*b^10*n^4 + 8409500*b^10*n^3 + 12753576*b^10*n^2 + 10628640*b^10*n + 3628800*b^10)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40536 vs. 2(316) = 632.

Time = 66.28 (sec) , antiderivative size = 40536, normalized size of antiderivative = 120.28

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] Piecewise((a**n*(c**3*x + 3*c**2*d*x**4/4 + 3*c*d**2*x**7/7 + d**3*x**10/10
), Eq(b, 0)), (2520*a**9*d**3*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b*
*11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*
x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x
**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 7129*a**9*d**3/(2520*a**9*b**
10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 +
317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 +
90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 22680*a**8*
b*d**3*x*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b*
*12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b*
*15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x
**8 + 2520*b**19*x**9) + 61641*a**8*b*d**3*x/(2520*a**9*b**10 + 22680*a**8*
b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**1
4*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17
*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 90720*a**7*b**2*d**3*x**2*l
og(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 +
 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 +
 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520
*b**19*x**9) + 235224*a**7*b**2*d**3*x**2/(2520*a**9*b**10 + 22680*a**8*b**
11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x
**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x*
*7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) - 30*a**6*b**3*c*d**2/(2520*a**9
*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**
3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**
6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 211680*
a**6*b**3*d**3*x**3*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90
720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317
520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 2268
0*a*b**18*x**8 + 2520*b**19*x**9) + 518616*a**6*b**3*d**3*x**3/(2520*a**9*b
**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3
+ 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6
+ 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) - 270*a**5*
b**4*c*d**2*x/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2
 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5
 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 25
```

20*b**19*x**9) + 317520*a**5*b**4*d**3*x**4*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 725004*a**5*b**4*d**3*x**4/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) - 1080*a**4*b**5*c*d**2*x**2/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 317520*a**4*b**5*d**3*x**5*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 661500*a**4*b**5*d**3*x**5/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) - 15*a**3*b**6*c**2*d/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) - 2520*a**3*b**6*c*d**2*x**3/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 211680*a**3*b**6*d**3*x**6*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 388080*a**3*b**6*d**3*x**6/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) - 135*a**2*b**7*c**2*d*x/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) - 3780*a**2*b**7*c*d**2*x**4/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 90720*a**2*b**7*d**3*x**7*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 136080*a**2*b**7*d**3*x**7/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) - 540

*a*b**8*c**2*d*x**2/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**1
2*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**1
5*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**
8 + 2520*b**19*x**9) - 3780*a*b**8*c*d**2*x**5/(2520*a**9*b**10 + 22680*a**
8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b*
*14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**
17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 22680*a*b**8*d**3*x**8*lo
g(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 +
211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 +
211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*
b**19*x**9) + 22680*a*b**8*d**3*x**8/(2520*a**9*b**10 + 22680*a**8*b**11*x
+ 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 +
 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 +
22680*a*b**18*x**8 + 2520*b**19*x**9) - 280*b**9*c**3/(2520*a**9*b**10 + 22
680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*
a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a
2*b17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) - 1260*b**9*c**2*d*x
3/(2520*a9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*
a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*
a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x
9) - 2520*b9*c*d**2*x**6/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*
a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*
a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*
b**18*x**8 + 2520*b**19*x**9) + 2520*b**9*d**3*x**9*log(a/b + x)/(2520*a**9
*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**
3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**
6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9), Eq(n, -1
0)), (-2520*a**9*d**3*log(a/b + x)/(280*a**8*b**10 + 2240*a**7*b**11*x + 78
40*a**6*b**12*x**2 + 15680*a**5*b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*
a**3*b**15*x**5 + 7840*a**2*b**16*x**6 + 2240*a*b**17*x**7 + 280*b**18*x**8
) - 6849*a**9*d**3/(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x*
*2 + 15680*a**5*b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5
+ 7840*a**2*b**16*x**6 + 2240*a*b**17*x**7 + 280*b**18*x**8) - 20160*a**8*b
*d**3*x*log(a/b + x)/(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*
x**2 + 15680*a**5*b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**
5 + 7840*a**2*b**16*x**6 + 2240*a*b**17*x**7 + 280*b**18*x**8) - 52272*a**8
*b*d**3*x/(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 1568
0*a**5*b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a*
*2*b**16*x**6 + 2240*a*b**17*x**7 + 280*b**18*x**8) - 70560*a**7*b**2*d**3*
x**2*log(a/b + x)/(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**
2 + 15680*a**5*b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 +
 7840*a**2*b**16*x**6 + 2240*a*b**17*x**7 + 280*b**18*x**8) - 172872*a**7*b
2*d3*x**2/(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 +
15680*a**5*b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 784
0*a**2*b**16*x**6 + 2240*a*b**17*x**7 + 280*b**18*x**8) - 15*a**6*b**3*c*d*

*2/(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*a**5*
b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b**1
6*x**6 + 2240*a*b**17*x**7 + 280*b**18*x**8) - 141120*a**6*b**3*d**3*x**3*l
og(a/b + x)/(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15
680*a**5*b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*
a**2*b**16*x**6 + 2240*a*b**17*x**7 + 280*b**18*x**8) - 322224*a**6*b**3*d*
*3*x**3/(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*
a**5*b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2
*b**16*x**6 + 2240*a*b**17*x**7 + 280*b**18*x**8) - 120*a**5*b**4*c*d**2*x/
(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*a**5*b**
13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b**16*x
6 + 2240*a*b17*x**7 + 280*b**18*x**8) - 176400*a**5*b**4*d**3*x**4*log(
a/b + x)/(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680
*a**5*b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**
2*b**16*x**6 + 2240*a*b**17*x**7 + 280*b**18*x**8) - 367500*a**5*b**4*d**3*
x**4/(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*a**
5*b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b*
*16*x**6 + 2240*a*b**17*x**7 + 280*b**18*x**8) - 420*a**4*b**5*c*d**2*x**2/
(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*a**5*b**
13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b**16*x
6 + 2240*a*b17*x**7 + 280*b**18*x**8) - 141120*a**4*b**5*d**3*x**5*log(
a/b + x)/(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680
*a**5*b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**
2*b**16*x**6 + 2240*a*b**17*x**7 + 280*b**18*x**8) - 258720*a**4*b**5*d**3*
x**5/(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*a**
5*b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b*
*16*x**6 + 2240*a*b**17*x**7 + 280*b**18*x**8) - 3*a**3*b**6*c**2*d/(280*a*
*8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*a**5*b**13*x**3
 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b**16*x**6 + 2
240*a*b**17*x**7 + 280*b**18*x**8) - 840*a**3*b**6*c*d**2*x**3/(280*a**8*b*
*10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*a**5*b**13*x**3 + 19
600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b**16*x**6 + 2240*a
*b**17*x**7 + 280*b**18*x**8) - 70560*a**3*b**6*d**3*x**6*log(a/b + x)/(280
*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*a**5*b**13*x
3 + 19600*a4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b**16*x**6
+ 2240*a*b**17*x**7 + 280*b**18*x**8) - 105840*a**3*b**6*d**3*x**6/(280*a**
8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*a**5*b**13*x**3
+ 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b**16*x**6 + 22
40*a*b**17*x**7 + 280*b**18*x**8) - 24*a**2*b**7*c**2*d*x/(280*a**8*b**10 +
 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*a**5*b**13*x**3 + 19600*a
4*b14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b**16*x**6 + 2240*a*b**1
7*x**7 + 280*b**18*x**8) - 1050*a**2*b**7*c*d**2*x**4/(280*a**8*b**10 + 224
0*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*a**5*b**13*x**3 + 19600*a**4*
b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b**16*x**6 + 2240*a*b**17*x*
*7 + 280*b**18*x**8) - 20160*a**2*b**7*d**3*x**7*log(a/b + x)/(280*a**8*b**

10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*a**5*b**13*x**3 + 196
00*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b**16*x**6 + 2240*a*
b**17*x**7 + 280*b**18*x**8) - 20160*a**2*b**7*d**3*x**7/(280*a**8*b**10 +
2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*a**5*b**13*x**3 + 19600*a*
*4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b**16*x**6 + 2240*a*b**17
*x**7 + 280*b**18*x**8) - 84*a*b**8*c**2*d*x**2/(280*a**8*b**10 + 2240*a**7
*b**11*x + 7840*a**6*b**12*x**2 + 15680*a**5*b**13*x**3 + 19600*a**4*b**14*
x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b**16*x**6 + 2240*a*b**17*x**7 + 2
80*b**18*x**8) - 840*a*b**8*c*d**2*x**5/(280*a**8*b**10 + 2240*a**7*b**11*x
 + 7840*a**6*b**12*x**2 + 15680*a**5*b**13*x**3 + 19600*a**4*b**14*x**4 + 1
5680*a**3*b**15*x**5 + 7840*a**2*b**16*x**6 + 2240*a*b**17*x**7 + 280*b**18
*x**8) - 2520*a*b**8*d**3*x**8*log(a/b + x)/(280*a**8*b**10 + 2240*a**7*b**
11*x + 7840*a**6*b**12*x**2 + 15680*a**5*b**13*x**3 + 19600*a**4*b**14*x**4
 + 15680*a**3*b**15*x**5 + 7840*a**2*b**16*x**6 + 2240*a*b**17*x**7 + 280*b
18*x8) - 35*b**9*c**3/(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b
12*x2 + 15680*a**5*b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**1
5*x**5 + 7840*a**2*b**16*x**6 + 2240*a*b**17*x**7 + 280*b**18*x**8) - 168*b
9*c2*d*x**3/(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2
+ 15680*a**5*b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7
840*a**2*b**16*x**6 + 2240*a*b**17*x**7 + 280*b**18*x**8) - 420*b**9*c*d**2
*x**6/(280*a**8*b**10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*a*
*5*b**13*x**3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b
16*x6 + 2240*a*b**17*x**7 + 280*b**18*x**8) + 280*b**9*d**3*x**9/(280*a
8*b10 + 2240*a**7*b**11*x + 7840*a**6*b**12*x**2 + 15680*a**5*b**13*x**
3 + 19600*a**4*b**14*x**4 + 15680*a**3*b**15*x**5 + 7840*a**2*b**16*x**6 +
2240*a*b**17*x**7 + 280*b**18*x**8), Eq(n, -9)), (5040*a**9*d**3*log(a/b +
x)/(140*a**7*b**10 + 980*a**6*b**11*x + 2940*a**5*b**12*x**2 + 4900*a**4*b*
*13*x**3 + 4900*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980*a*b**16*x**6 +
 140*b**17*x**7) + 13068*a**9*d**3/(140*a**7*b**10 + 980*a**6*b**11*x + 294
0*a**5*b**12*x**2 + 4900*a**4*b**13*x**3 + 4900*a**3*b**14*x**4 + 2940*a**2
*b**15*x**5 + 980*a*b**16*x**6 + 140*b**17*x**7) + 35280*a**8*b*d**3*x*log(
a/b + x)/(140*a**7*b**10 + 980*a**6*b**11*x + 2940*a**5*b**12*x**2 + 4900*a
4*b13*x**3 + 4900*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980*a*b**16*
x**6 + 140*b**17*x**7) + 86436*a**8*b*d**3*x/(140*a**7*b**10 + 980*a**6*b**
11*x + 2940*a**5*b**12*x**2 + 4900*a**4*b**13*x**3 + 4900*a**3*b**14*x**4 +
 2940*a**2*b**15*x**5 + 980*a*b**16*x**6 + 140*b**17*x**7) + 105840*a**7*b*
*2*d**3*x**2*log(a/b + x)/(140*a**7*b**10 + 980*a**6*b**11*x + 2940*a**5*b*
*12*x**2 + 4900*a**4*b**13*x**3 + 4900*a**3*b**14*x**4 + 2940*a**2*b**15*x*
*5 + 980*a*b**16*x**6 + 140*b**17*x**7) + 241668*a**7*b**2*d**3*x**2/(140*a
7*b10 + 980*a**6*b**11*x + 2940*a**5*b**12*x**2 + 4900*a**4*b**13*x**3
+ 4900*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980*a*b**16*x**6 + 140*b**1
7*x**7) - 60*a**6*b**3*c*d**2/(140*a**7*b**10 + 980*a**6*b**11*x + 2940*a**
5*b**12*x**2 + 4900*a**4*b**13*x**3 + 4900*a**3*b**14*x**4 + 2940*a**2*b**1
5*x**5 + 980*a*b**16*x**6 + 140*b**17*x**7) + 176400*a**6*b**3*d**3*x**3*lo
g(a/b + x)/(140*a**7*b**10 + 980*a**6*b**11*x + 2940*a**5*b**12*x**2 + 4900

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*a**4*b**13*x**3 + 4900*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980*a*b**1
6*x**6 + 140*b**17*x**7) + 367500*a**6*b**3*d**3*x**3/(140*a**7*b**10 + 980
*a**6*b**11*x + 2940*a**5*b**12*x**2 + 4900*a**4*b**13*x**3 + 4900*a**3*b**
14*x**4 + 2940*a**2*b**15*x**5 + 980*a*b**16*x**6 + 140*b**17*x**7) - 420*a
**5*b**4*c*d**2*x/(140*a**7*b**10 + 980*a**6*b**11*x + 2940*a**5*b**12*x**2
 + 4900*a**4*b**13*x**3 + 4900*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980
*a*b**16*x**6 + 140*b**17*x**7) + 176400*a**5*b**4*d**3*x**4*log(a/b + x)/(
140*a**7*b**10 + 980*a**6*b**11*x + 2940*a**5*b**12*x**2 + 4900*a**4*b**13*
x**3 + 4900*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980*a*b**16*x**6 + 140
*b**17*x**7) + 323400*a**5*b**4*d**3*x**4/(140*a**7*b**10 + 980*a**6*b**11*
x + 2940*a**5*b**12*x**2 + 4900*a**4*b**13*x**3 + 4900*a**3*b**14*x**4 + 29
40*a**2*b**15*x**5 + 980*a*b**16*x**6 + 140*b**17*x**7) - 1260*a**4*b**5*c*
d**2*x**2/(140*a**7*b**10 + 980*a**6*b**11*x + 2940*a**5*b**12*x**2 + 4900*
a**4*b**13*x**3 + 4900*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980*a*b**16
*x**6 + 140*b**17*x**7) + 105840*a**4*b**5*d**3*x**5*log(a/b + x)/(140*a**7
*b**10 + 980*a**6*b**11*x + 2940*a**5*b**12*x**2 + 4900*a**4*b**13*x**3 + 4
900*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980*a*b**16*x**6 + 140*b**17*x
**7) + 158760*a**4*b**5*d**3*x**5/(140*a**7*b**10 + 980*a**6*b**11*x + 2940
*a**5*b**12*x**2 + 4900*a**4*b**13*x**3 + 4900*a**3*b**14*x**4 + 2940*a**2*
b**15*x**5 + 980*a*b**16*x**6 + 140*b**17*x**7) - 3*a**3*b**6*c**2*d/(140*a
**7*b**10 + 980*a**6*b**11*x + 2940*a**5*b**12*x**2 + 4900*a**4*b**13*x**3
+ 4900*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980*a*b**16*x**6 + 140*b**1
7*x**7) - 2100*a**3*b**6*c*d**2*x**3/(140*a**7*b**10 + 980*a**6*b**11*x + 2
940*a**5*b**12*x**2 + 4900*a**4*b**13*x**3 + 4900*a**3*b**14*x**4 + 2940*a*
*2*b**15*x**5 + 980*a*b**16*x**6 + 140*b**17*x**7) + 35280*a**3*b**6*d**3*x
**6*log(a/b + x)/(140*a**7*b**10 + 980*a**6*b**11*x + 2940*a**5*b**12*x**2
+ 4900*a**4*b**13*x**3 + 4900*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980*
a*b**16*x**6 + 140*b**17*x**7) + 35280*a**3*b**6*d**3*x**6/(140*a**7*b**10
+ 980*a**6*b**11*x + 2940*a**5*b**12*x**2 + 4900*a**4*b**13*x**3 + 4900*a**
3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980*a*b**16*x**6 + 140*b**17*x**7) -
21*a**2*b**7*c**2*d*x/(140*a**7*b**10 + 980*a**6*b**11*x + 2940*a**5*b**12*
x**2 + 4900*a**4*b**13*x**3 + 4900*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 +
 980*a*b**16*x**6 + 140*b**17*x**7) - 2100*a**2*b**7*c*d**2*x**4/(140*a**7*
b**10 + 980*a**6*b**11*x + 2940*a**5*b**12*x**2 + 4900*a**4*b**13*x**3 + 49
00*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980*a*b**16*x**6 + 140*b**17*x*
*7) + 5040*a**2*b**7*d**3*x**7*log(a/b + x)/(140*a**7*b**10 + 980*a**6*b**1
1*x + 2940*a**5*b**12*x**2 + 4900*a**4*b**13*x**3 + 4900*a**3*b**14*x**4 +
2940*a**2*b**15*x**5 + 980*a*b**16*x**6 + 140*b**17*x**7) - 63*a*b**8*c**2*
d*x**2/(140*a**7*b**10 + 980*a**6*b**11*x + 2940*a**5*b**12*x**2 + 4900*a**
4*b**13*x**3 + 4900*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980*a*b**16*x*
*6 + 140*b**17*x**7) - 1260*a*b**8*c*d**2*x**5/(140*a**7*b**10 + 980*a**6*b
**11*x + 2940*a**5*b**12*x**2 + 4900*a**4*b**13*x**3 + 4900*a**3*b**14*x**4
 + 2940*a**2*b**15*x**5 + 980*a*b**16*x**6 + 140*b**17*x**7) - 630*a*b**8*d
**3*x**8/(140*a**7*b**10 + 980*a**6*b**11*x + 2940*a**5*b**12*x**2 + 4900*a
**4*b**13*x**3 + 4900*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980*a*b**16*
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x**6 + 140*b**17*x**7) - 20*b**9*c**3/(140*a**7*b**10 + 980*a**6*b**11*x +
2940*a**5*b**12*x**2 + 4900*a**4*b**13*x**3 + 4900*a**3*b**14*x**4 + 2940*a
2*b15*x**5 + 980*a*b**16*x**6 + 140*b**17*x**7) - 105*b**9*c**2*d*x**3/
(140*a**7*b**10 + 980*a**6*b**11*x + 2940*a**5*b**12*x**2 + 4900*a**4*b**13
*x**3 + 4900*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980*a*b**16*x**6 + 14
0*b**17*x**7) - 420*b**9*c*d**2*x**6/(140*a**7*b**10 + 980*a**6*b**11*x + 2
940*a**5*b**12*x**2 + 4900*a**4*b**13*x**3 + 4900*a**3*b**14*x**4 + 2940*a*
*2*b**15*x**5 + 980*a*b**16*x**6 + 140*b**17*x**7) + 70*b**9*d**3*x**9/(140
*a**7*b**10 + 980*a**6*b**11*x + 2940*a**5*b**12*x**2 + 4900*a**4*b**13*x**
3 + 4900*a**3*b**14*x**4 + 2940*a**2*b**15*x**5 + 980*a*b**16*x**6 + 140*b*
*17*x**7), Eq(n, -8)), (-5040*a**9*d**3*log(a/b + x)/(60*a**6*b**10 + 360*a
5*b11*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 900*a**2*b**14*x
4 + 360*a*b15*x**5 + 60*b**16*x**6) - 12348*a**9*d**3/(60*a**6*b**10 +
360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 900*a**2*b*
*14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) - 30240*a**8*b*d**3*x*log(a/b
+ x)/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*b*
*13*x**3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) - 69048*
a**8*b*d**3*x/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 120
0*a**3*b**13*x**3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6)
 - 75600*a**7*b**2*d**3*x**2*log(a/b + x)/(60*a**6*b**10 + 360*a**5*b**11*x
 + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 900*a**2*b**14*x**4 + 360*a
*b**15*x**5 + 60*b**16*x**6) - 157500*a**7*b**2*d**3*x**2/(60*a**6*b**10 +
360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 900*a**2*b*
*14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) + 180*a**6*b**3*c*d**2*log(a/b
 + x)/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*b
13*x3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) + 441*a
6*b3*c*d**2/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1
200*a**3*b**13*x**3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**
6) - 100800*a**6*b**3*d**3*x**3*log(a/b + x)/(60*a**6*b**10 + 360*a**5*b**1
1*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 900*a**2*b**14*x**4 + 36
0*a*b**15*x**5 + 60*b**16*x**6) - 184800*a**6*b**3*d**3*x**3/(60*a**6*b**10
 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 900*a**2
*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) + 1080*a**5*b**4*c*d**2*x*l
og(a/b + x)/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*
a**3*b**13*x**3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) +
 2466*a**5*b**4*c*d**2*x/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12
*x**2 + 1200*a**3*b**13*x**3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*
b**16*x**6) - 75600*a**5*b**4*d**3*x**4*log(a/b + x)/(60*a**6*b**10 + 360*a
5*b11*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 900*a**2*b**14*x
4 + 360*a*b15*x**5 + 60*b**16*x**6) - 113400*a**5*b**4*d**3*x**4/(60*a*
*6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 +
900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) + 2700*a**4*b**5*c*
d**2*x**2*log(a/b + x)/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x
2 + 1200*a3*b**13*x**3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b*
*16*x**6) + 5625*a**4*b**5*c*d**2*x**2/(60*a**6*b**10 + 360*a**5*b**11*x +

900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 900*a**2*b**14*x**4 + 360*a*b*
*15*x**5 + 60*b**16*x**6) - 30240*a**4*b**5*d**3*x**5*log(a/b + x)/(60*a**6
*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 90
0*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) - 30240*a**4*b**5*d**
3*x**5/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*
b**13*x**3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) - 3*a*
*3*b**6*c**2*d/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 12
00*a**3*b**13*x**3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6
) + 3600*a**3*b**6*c*d**2*x**3*log(a/b + x)/(60*a**6*b**10 + 360*a**5*b**11
*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 900*a**2*b**14*x**4 + 360
*a*b**15*x**5 + 60*b**16*x**6) + 6600*a**3*b**6*c*d**2*x**3/(60*a**6*b**10
+ 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 900*a**2*
b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) - 5040*a**3*b**6*d**3*x**6*l
og(a/b + x)/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*
a**3*b**13*x**3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) -
18*a**2*b**7*c**2*d*x/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x
2 + 1200*a3*b**13*x**3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b*
*16*x**6) + 2700*a**2*b**7*c*d**2*x**4*log(a/b + x)/(60*a**6*b**10 + 360*a*
*5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 900*a**2*b**14*x*
*4 + 360*a*b**15*x**5 + 60*b**16*x**6) + 4050*a**2*b**7*c*d**2*x**4/(60*a**
6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 9
00*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) + 720*a**2*b**7*d**3
*x**7/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*b
13*x3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) - 45*a*
b**8*c**2*d*x**2/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 +
1200*a**3*b**13*x**3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x*
*6) + 1080*a*b**8*c*d**2*x**5*log(a/b + x)/(60*a**6*b**10 + 360*a**5*b**11*
x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 900*a**2*b**14*x**4 + 360*
a*b**15*x**5 + 60*b**16*x**6) + 1080*a*b**8*c*d**2*x**5/(60*a**6*b**10 + 36
0*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 900*a**2*b**1
4*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) - 90*a*b**8*d**3*x**8/(60*a**6*b
10 + 360*a5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 900*
a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) - 10*b**9*c**3/(60*a**6
*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*x**3 + 90
0*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) - 60*b**9*c**2*d*x**3
/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*x**2 + 1200*a**3*b**13*
x**3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b**16*x**6) + 180*b**9*c
*d**2*x**6*log(a/b + x)/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4*b**12*
x**2 + 1200*a**3*b**13*x**3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5 + 60*b
16*x6) + 20*b**9*d**3*x**9/(60*a**6*b**10 + 360*a**5*b**11*x + 900*a**4
*b**12*x**2 + 1200*a**3*b**13*x**3 + 900*a**2*b**14*x**4 + 360*a*b**15*x**5
+ 60*b**16*x**6), Eq(n, -7)), (2520*a**9*d**3*log(a/b + x)/(20*a**5*b**10
+ 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**1
4*x**4 + 20*b**15*x**5) + 5754*a**9*d**3/(20*a**5*b**10 + 100*a**4*b**11*x
+ 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x

5) + 12600*a8*b*d**3*x*log(a/b + x)/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) + 26250*a**8*b*d**3*x/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) + 25200*a**7*b**2*d**3*x**2*log(a/b + x)/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) + 46200*a**7*b**2*d**3*x**2/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 360*a**6*b**3*c*d**2*log(a/b + x)/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 822*a**6*b**3*c*d**2/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) + 25200*a**6*b**3*d**3*x**3*log(a/b + x)/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) + 37800*a**6*b**3*d**3*x**3/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 1800*a**5*b**4*c*d**2*x*log(a/b + x)/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 3750*a**5*b**4*c*d**2*x/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) + 12600*a**5*b**4*d**3*x**4*log(a/b + x)/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) + 12600*a**5*b**4*d**3*x**4/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 3600*a**4*b**5*c*d**2*x**2*log(a/b + x)/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 6600*a**4*b**5*c*d**2*x**2/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) + 2520*a**4*b**5*d**3*x**5*log(a/b + x)/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 3*a**3*b**6*c**2*d/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 3600*a**3*b**6*c*d**2*x**3*log(a/b + x)/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 5400*a**3*b**6*c*d**2*x**3/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 420*a**3*b**6*d**3*x**6/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 15*a**2*b**7*c**2*d*x/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 1800*a**2*b**7*c*d**2*x**4*log(a/b + x)/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 1800*a**2*b**7*c*d**2*x**4/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) + 60*a**2*b**7*d**3*x**7/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 30*a*b**8

*c**2*d*x**2/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*
a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 360*a*b**8*c*d**2*x**
5*log(a/b + x)/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 20
0*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) - 15*a*b**8*d**3*x**8
/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x
3 + 100*a*b14*x**4 + 20*b**15*x**5) - 4*b**9*c**3/(20*a**5*b**10 + 100*
a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4
 + 20*b**15*x**5) - 30*b**9*c**2*d*x**3/(20*a**5*b**10 + 100*a**4*b**11*x +
 200*a**3*b**12*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x*
*5) + 60*b**9*c*d**2*x**6/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**1
2*x**2 + 200*a**2*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5) + 5*b**9*d
3*x9/(20*a**5*b**10 + 100*a**4*b**11*x + 200*a**3*b**12*x**2 + 200*a**2
*b**13*x**3 + 100*a*b**14*x**4 + 20*b**15*x**5), Eq(n, -6)), (-2520*a**9*d*
*3*log(a/b + x)/(20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2*b**12*x**2 + 80
*a*b**13*x**3 + 20*b**14*x**4) - 5250*a**9*d**3/(20*a**4*b**10 + 80*a**3*b*
*11*x + 120*a**2*b**12*x**2 + 80*a*b**13*x**3 + 20*b**14*x**4) - 10080*a**8
*b*d**3*x*log(a/b + x)/(20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2*b**12*x*
*2 + 80*a*b**13*x**3 + 20*b**14*x**4) - 18480*a**8*b*d**3*x/(20*a**4*b**10
+ 80*a**3*b**11*x + 120*a**2*b**12*x**2 + 80*a*b**13*x**3 + 20*b**14*x**4)
- 15120*a**7*b**2*d**3*x**2*log(a/b + x)/(20*a**4*b**10 + 80*a**3*b**11*x +
 120*a**2*b**12*x**2 + 80*a*b**13*x**3 + 20*b**14*x**4) - 22680*a**7*b**2*d
3*x2/(20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2*b**12*x**2 + 80*a*b**1
3*x**3 + 20*b**14*x**4) + 900*a**6*b**3*c*d**2*log(a/b + x)/(20*a**4*b**10
+ 80*a**3*b**11*x + 120*a**2*b**12*x**2 + 80*a*b**13*x**3 + 20*b**14*x**4)
+ 1875*a**6*b**3*c*d**2/(20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2*b**12*x
2 + 80*a*b13*x**3 + 20*b**14*x**4) - 10080*a**6*b**3*d**3*x**3*log(a/b
+ x)/(20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2*b**12*x**2 + 80*a*b**13*x*
*3 + 20*b**14*x**4) - 10080*a**6*b**3*d**3*x**3/(20*a**4*b**10 + 80*a**3*b*
*11*x + 120*a**2*b**12*x**2 + 80*a*b**13*x**3 + 20*b**14*x**4) + 3600*a**5*
b**4*c*d**2*x*log(a/b + x)/(20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2*b**1
2*x**2 + 80*a*b**13*x**3 + 20*b**14*x**4) + 6600*a**5*b**4*c*d**2*x/(20*a**
4*b**10 + 80*a**3*b**11*x + 120*a**2*b**12*x**2 + 80*a*b**13*x**3 + 20*b**1
4*x**4) - 2520*a**5*b**4*d**3*x**4*log(a/b + x)/(20*a**4*b**10 + 80*a**3*b*
*11*x + 120*a**2*b**12*x**2 + 80*a*b**13*x**3 + 20*b**14*x**4) + 5400*a**4*
b**5*c*d**2*x**2*log(a/b + x)/(20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2*b
12*x2 + 80*a*b**13*x**3 + 20*b**14*x**4) + 8100*a**4*b**5*c*d**2*x**2/(
20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2*b**12*x**2 + 80*a*b**13*x**3 + 2
0*b**14*x**4) + 504*a**4*b**5*d**3*x**5/(20*a**4*b**10 + 80*a**3*b**11*x +
120*a**2*b**12*x**2 + 80*a*b**13*x**3 + 20*b**14*x**4) - 15*a**3*b**6*c**2*
d/(20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2*b**12*x**2 + 80*a*b**13*x**3
+ 20*b**14*x**4) + 3600*a**3*b**6*c*d**2*x**3*log(a/b + x)/(20*a**4*b**10 +
 80*a**3*b**11*x + 120*a**2*b**12*x**2 + 80*a*b**13*x**3 + 20*b**14*x**4) +
 3600*a**3*b**6*c*d**2*x**3/(20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2*b**
12*x**2 + 80*a*b**13*x**3 + 20*b**14*x**4) - 84*a**3*b**6*d**3*x**6/(20*a**
4*b**10 + 80*a**3*b**11*x + 120*a**2*b**12*x**2 + 80*a*b**13*x**3 + 20*b**1

4*x**4) - 60*a**2*b**7*c**2*d*x/(20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2
*b**12*x**2 + 80*a*b**13*x**3 + 20*b**14*x**4) + 900*a**2*b**7*c*d**2*x**4*
log(a/b + x)/(20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2*b**12*x**2 + 80*a*
b**13*x**3 + 20*b**14*x**4) + 24*a**2*b**7*d**3*x**7/(20*a**4*b**10 + 80*a*
*3*b**11*x + 120*a**2*b**12*x**2 + 80*a*b**13*x**3 + 20*b**14*x**4) - 90*a*
b**8*c**2*d*x**2/(20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2*b**12*x**2 + 8
0*a*b**13*x**3 + 20*b**14*x**4) - 180*a*b**8*c*d**2*x**5/(20*a**4*b**10 + 8
0*a**3*b**11*x + 120*a**2*b**12*x**2 + 80*a*b**13*x**3 + 20*b**14*x**4) - 9
*a*b**8*d**3*x**8/(20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2*b**12*x**2 +
80*a*b**13*x**3 + 20*b**14*x**4) - 5*b**9*c**3/(20*a**4*b**10 + 80*a**3*b**
11*x + 120*a**2*b**12*x**2 + 80*a*b**13*x**3 + 20*b**14*x**4) - 60*b**9*c**
2*d*x**3/(20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2*b**12*x**2 + 80*a*b**1
3*x**3 + 20*b**14*x**4) + 30*b**9*c*d**2*x**6/(20*a**4*b**10 + 80*a**3*b**1
1*x + 120*a**2*b**12*x**2 + 80*a*b**13*x**3 + 20*b**14*x**4) + 4*b**9*d**3*
x**9/(20*a**4*b**10 + 80*a**3*b**11*x + 120*a**2*b**12*x**2 + 80*a*b**13*x*
*3 + 20*b**14*x**4), Eq(n, -5)), (2520*a**9*d**3*log(a/b + x)/(30*a**3*b**1
0 + 90*a**2*b**11*x + 90*a*b**12*x**2 + 30*b**13*x**3) + 4620*a**9*d**3/(30
*a**3*b**10 + 90*a**2*b**11*x + 90*a*b**12*x**2 + 30*b**13*x**3) + 7560*a**
8*b*d**3*x*log(a/b + x)/(30*a**3*b**10 + 90*a**2*b**11*x + 90*a*b**12*x**2
+ 30*b**13*x**3) + 11340*a**8*b*d**3*x/(30*a**3*b**10 + 90*a**2*b**11*x + 9
0*a*b**12*x**2 + 30*b**13*x**3) + 7560*a**7*b**2*d**3*x**2*log(a/b + x)/(30
*a**3*b**10 + 90*a**2*b**11*x + 90*a*b**12*x**2 + 30*b**13*x**3) + 7560*a**
7*b**2*d**3*x**2/(30*a**3*b**10 + 90*a**2*b**11*x + 90*a*b**12*x**2 + 30*b*
*13*x**3) - 1800*a**6*b**3*c*d**2*log(a/b + x)/(30*a**3*b**10 + 90*a**2*b**
11*x + 90*a*b**12*x**2 + 30*b**13*x**3) - 3300*a**6*b**3*c*d**2/(30*a**3*b*
*10 + 90*a**2*b**11*x + 90*a*b**12*x**2 + 30*b**13*x**3) + 2520*a**6*b**3*d
3*x3*log(a/b + x)/(30*a**3*b**10 + 90*a**2*b**11*x + 90*a*b**12*x**2 +
30*b**13*x**3) - 5400*a**5*b**4*c*d**2*x*log(a/b + x)/(30*a**3*b**10 + 90*a
2*b11*x + 90*a*b**12*x**2 + 30*b**13*x**3) - 8100*a**5*b**4*c*d**2*x/(3
0*a**3*b**10 + 90*a**2*b**11*x + 90*a*b**12*x**2 + 30*b**13*x**3) - 630*a**
5*b**4*d**3*x**4/(30*a**3*b**10 + 90*a**2*b**11*x + 90*a*b**12*x**2 + 30*b*
*13*x**3) - 5400*a**4*b**5*c*d**2*x**2*log(a/b + x)/(30*a**3*b**10 + 90*a**
2*b**11*x + 90*a*b**12*x**2 + 30*b**13*x**3) - 5400*a**4*b**5*c*d**2*x**2/(
30*a**3*b**10 + 90*a**2*b**11*x + 90*a*b**12*x**2 + 30*b**13*x**3) + 126*a*
*4*b**5*d**3*x**5/(30*a**3*b**10 + 90*a**2*b**11*x + 90*a*b**12*x**2 + 30*b
13*x3) + 90*a**3*b**6*c**2*d*log(a/b + x)/(30*a**3*b**10 + 90*a**2*b**1
1*x + 90*a*b**12*x**2 + 30*b**13*x**3) + 165*a**3*b**6*c**2*d/(30*a**3*b**1
0 + 90*a**2*b**11*x + 90*a*b**12*x**2 + 30*b**13*x**3) - 1800*a**3*b**6*c*d
2*x3*log(a/b + x)/(30*a**3*b**10 + 90*a**2*b**11*x + 90*a*b**12*x**2 +
30*b**13*x**3) - 42*a**3*b**6*d**3*x**6/(30*a**3*b**10 + 90*a**2*b**11*x +
90*a*b**12*x**2 + 30*b**13*x**3) + 270*a**2*b**7*c**2*d*x*log(a/b + x)/(30*
a**3*b**10 + 90*a**2*b**11*x + 90*a*b**12*x**2 + 30*b**13*x**3) + 405*a**2*
b**7*c**2*d*x/(30*a**3*b**10 + 90*a**2*b**11*x + 90*a*b**12*x**2 + 30*b**13
*x**3) + 450*a**2*b**7*c*d**2*x**4/(30*a**3*b**10 + 90*a**2*b**11*x + 90*a*
b**12*x**2 + 30*b**13*x**3) + 18*a**2*b**7*d**3*x**7/(30*a**3*b**10 + 90*a*

*2*b**11*x + 90*a*b**12*x**2 + 30*b**13*x**3) + 270*a*b**8*c**2*d*x**2*log(
a/b + x)/(30*a**3*b**10 + 90*a**2*b**11*x + 90*a*b**12*x**2 + 30*b**13*x**3
) + 270*a*b**8*c**2*d*x**2/(30*a**3*b**10 + 90*a**2*b**11*x + 90*a*b**12*x*
*2 + 30*b**13*x**3) - 90*a*b**8*c*d**2*x**5/(30*a**3*b**10 + 90*a**2*b**11*
x + 90*a*b**12*x**2 + 30*b**13*x**3) - 9*a*b**8*d**3*x**8/(30*a**3*b**10 +
90*a**2*b**11*x + 90*a*b**12*x**2 + 30*b**13*x**3) - 10*b**9*c**3/(30*a**3*
b**10 + 90*a**2*b**11*x + 90*a*b**12*x**2 + 30*b**13*x**3) + 90*b**9*c**2*d
*x**3*log(a/b + x)/(30*a**3*b**10 + 90*a**2*b**11*x + 90*a*b**12*x**2 + 30*
b**13*x**3) + 30*b**9*c*d**2*x**6/(30*a**3*b**10 + 90*a**2*b**11*x + 90*a*b
12*x2 + 30*b**13*x**3) + 5*b**9*d**3*x**9/(30*a**3*b**10 + 90*a**2*b**1
1*x + 90*a*b**12*x**2 + 30*b**13*x**3), Eq(n, -4)), (-5040*a**9*d**3*log(a/
b + x)/(140*a**2*b**10 + 280*a*b**11*x + 140*b**12*x**2) - 7560*a**9*d**3/(
140*a**2*b**10 + 280*a*b**11*x + 140*b**12*x**2) - 10080*a**8*b*d**3*x*log(
a/b + x)/(140*a**2*b**10 + 280*a*b**11*x + 140*b**12*x**2) - 10080*a**8*b*d
3*x/(140*a2*b**10 + 280*a*b**11*x + 140*b**12*x**2) - 5040*a**7*b**2*d*
*3*x**2*log(a/b + x)/(140*a**2*b**10 + 280*a*b**11*x + 140*b**12*x**2) + 63
00*a**6*b**3*c*d**2*log(a/b + x)/(140*a**2*b**10 + 280*a*b**11*x + 140*b**1
2*x**2) + 9450*a**6*b**3*c*d**2/(140*a**2*b**10 + 280*a*b**11*x + 140*b**12
*x**2) + 1680*a**6*b**3*d**3*x**3/(140*a**2*b**10 + 280*a*b**11*x + 140*b**
12*x**2) + 12600*a**5*b**4*c*d**2*x*log(a/b + x)/(140*a**2*b**10 + 280*a*b*
*11*x + 140*b**12*x**2) + 12600*a**5*b**4*c*d**2*x/(140*a**2*b**10 + 280*a*
b**11*x + 140*b**12*x**2) - 420*a**5*b**4*d**3*x**4/(140*a**2*b**10 + 280*a
*b**11*x + 140*b**12*x**2) + 6300*a**4*b**5*c*d**2*x**2*log(a/b + x)/(140*a
2*b10 + 280*a*b**11*x + 140*b**12*x**2) + 168*a**4*b**5*d**3*x**5/(140*
a**2*b**10 + 280*a*b**11*x + 140*b**12*x**2) - 1260*a**3*b**6*c**2*d*log(a/
b + x)/(140*a**2*b**10 + 280*a*b**11*x + 140*b**12*x**2) - 1890*a**3*b**6*c
2*d/(140*a2*b**10 + 280*a*b**11*x + 140*b**12*x**2) - 2100*a**3*b**6*c*
d**2*x**3/(140*a**2*b**10 + 280*a*b**11*x + 140*b**12*x**2) - 84*a**3*b**6*
d**3*x**6/(140*a**2*b**10 + 280*a*b**11*x + 140*b**12*x**2) - 2520*a**2*b**
7*c**2*d*x*log(a/b + x)/(140*a**2*b**10 + 280*a*b**11*x + 140*b**12*x**2) -
 2520*a**2*b**7*c**2*d*x/(140*a**2*b**10 + 280*a*b**11*x + 140*b**12*x**2)
+ 525*a**2*b**7*c*d**2*x**4/(140*a**2*b**10 + 280*a*b**11*x + 140*b**12*x**
2) + 48*a**2*b**7*d**3*x**7/(140*a**2*b**10 + 280*a*b**11*x + 140*b**12*x**
2) - 1260*a*b**8*c**2*d*x**2*log(a/b + x)/(140*a**2*b**10 + 280*a*b**11*x +
 140*b**12*x**2) - 210*a*b**8*c*d**2*x**5/(140*a**2*b**10 + 280*a*b**11*x +
 140*b**12*x**2) - 30*a*b**8*d**3*x**8/(140*a**2*b**10 + 280*a*b**11*x + 14
0*b**12*x**2) - 70*b**9*c**3/(140*a**2*b**10 + 280*a*b**11*x + 140*b**12*x*
*2) + 420*b**9*c**2*d*x**3/(140*a**2*b**10 + 280*a*b**11*x + 140*b**12*x**2
) + 105*b**9*c*d**2*x**6/(140*a**2*b**10 + 280*a*b**11*x + 140*b**12*x**2)
+ 20*b**9*d**3*x**9/(140*a**2*b**10 + 280*a*b**11*x + 140*b**12*x**2), Eq(n
, -3)), (2520*a**9*d**3*log(a/b + x)/(280*a*b**10 + 280*b**11*x) + 2520*a**
9*d**3/(280*a*b**10 + 280*b**11*x) + 2520*a**8*b*d**3*x*log(a/b + x)/(280*a
*b**10 + 280*b**11*x) - 1260*a**7*b**2*d**3*x**2/(280*a*b**10 + 280*b**11*x
) - 5040*a**6*b**3*c*d**2*log(a/b + x)/(280*a*b**10 + 280*b**11*x) - 5040*a
6*b3*c*d**2/(280*a*b**10 + 280*b**11*x) + 420*a**6*b**3*d**3*x**3/(280*

a*b**10 + 280*b**11*x) - 5040*a**5*b**4*c*d**2*x*log(a/b + x)/(280*a*b**10 + 280*b**11*x) - 210*a**5*b**4*d**3*x**4/(280*a*b**10 + 280*b**11*x) + 2520*a**4*b**5*c*d**2*x**2/(280*a*b**10 + 280*b**11*x) + 126*a**4*b**5*d**3*x**5/(280*a*b**10 + 280*b**11*x) + 2520*a**3*b**6*c**2*d*log(a/b + x)/(280*a*b**10 + 280*b**11*x) + 2520*a**3*b**6*c**2*d/(280*a*b**10 + 280*b**11*x) - 840*a**3*b**6*c*d**2*x**3/(280*a*b**10 + 280*b**11*x) - 84*a**3*b**6*d**3*x**6/(280*a*b**10 + 280*b**11*x) + 2520*a**2*b**7*c**2*d*x*log(a/b + x)/(280*a*b**10 + 280*b**11*x) + 420*a**2*b**7*c*d**2*x**4/(280*a*b**10 + 280*b**11*x) + 60*a**2*b**7*d**3*x**7/(280*a*b**10 + 280*b**11*x) - 1260*a*b**8*c**2*d*x**2/(280*a*b**10 + 280*b**11*x) - 252*a*b**8*c*d**2*x**5/(280*a*b**10 + 280*b**11*x) - 45*a*b**8*d**3*x**8/(280*a*b**10 + 280*b**11*x) - 280*b**9*c**3/(280*a*b**10 + 280*b**11*x) + 420*b**9*c**2*d*x**3/(280*a*b**10 + 280*b**11*x) + 168*b**9*c*d**2*x**6/(280*a*b**10 + 280*b**11*x) + 35*b**9*d**3*x**9/(280*a*b**10 + 280*b**11*x), Eq(n, -2)), (-a**9*d**3*log(a/b + x)/b**10 + a**8*d**3*x/b**9 - a**7*d**3*x**2/(2*b**8) + 3*a**6*c*d**2*log(a/b + x)/b**7 + a**6*d**3*x**3/(3*b**7) - 3*a**5*c*d**2*x/b**6 - a**5*d**3*x**4/(4*b**6) + 3*a**4*c*d**2*x**2/(2*b**5) + a**4*d**3*x**5/(5*b**5) - 3*a**3*c**2*d*log(a/b + x)/b**4 - a**3*c*d**2*x**3/b**4 - a**3*d**3*x**6/(6*b**4) + 3*a**2*c**2*d*x/b**3 + 3*a**2*c*d**2*x**4/(4*b**3) + a**2*d**3*x**7/(7*b**3) - 3*a*c**2*d*x**2/(2*b**2) - 3*a*c*d**2*x**5/(5*b**2) - a*d**3*x**8/(8*b**2) + c**3*log(a/b + x)/b + c**2*d*x**3/b + c*d**2*x**6/(2*b) + d**3*x**9/(9*b), Eq(n, -1)), (-362880*a**10*d**3*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 362880*a**9*b*d**3*n*x*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 181440*a**8*b**2*d**3*n**2*x**2*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 181440*a**8*b**2*d**3*n*x**2*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 2160*a**7*b**3*c*d**2*n**3*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 58320*a**7*b**3*c*d**2*n**2*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 522720*a**7*b**3*c*d**2*n*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 1555200*a**7*b**3*c*d**2*(a + b*x)**n/(b**10*n**10 +

55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 9
02055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10
*n**2 + 10628640*b**10*n + 3628800*b**10) + 60480*a**7*b**3*d**3*n**3*x**3*
(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n
7 + 157773*b10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*
b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 1814
40*a**7*b**3*d**3*n**2*x**3*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 132
0*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3
416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**
10*n + 3628800*b**10) + 120960*a**7*b**3*d**3*n*x**3*(a + b*x)**n/(b**10*n*
*10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**
6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*
b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 2160*a**6*b**4*c*d**2*n**4
x(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**1
0*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 84095
00*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 5
8320*a**6*b**4*c*d**2*n**3*x*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 13
20*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 +
3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b*
*10*n + 3628800*b**10) - 522720*a**6*b**4*c*d**2*n**2*x*(a + b*x)**n/(b**10
*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*
n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 127535
76*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 1555200*a**6*b**4*c*d**
2*n*x*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b
10*n7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 84
09500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10)
- 15120*a**6*b**4*d**3*n**4*x**4*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9
+ 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**
5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 1062864
0*b**10*n + 3628800*b**10) - 90720*a**6*b**4*d**3*n**3*x**4*(a + b*x)**n/(b
10*n10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b*
*10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12
753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 166320*a**6*b**4*d*
*3*n**2*x**4*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 +
18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n*
*4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*
b**10) - 90720*a**6*b**4*d**3*n*x**4*(a + b*x)**n/(b**10*n**10 + 55*b**10*n
9 + 1320*b10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10
*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 106
28640*b**10*n + 3628800*b**10) + 1080*a**5*b**5*c*d**2*n**5*x**2*(a + b*x)*
*n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 1577
73*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3
 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 30240*a**5*b**
5*c*d**2*n**4*x**2*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n
8 + 18150*b10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b*

*10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 36
28800*b**10) + 290520*a**5*b**5*c*d**2*n**3*x**2*(a + b*x)**n/(b**10*n**10
+ 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 +
902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**1
0*n**2 + 10628640*b**10*n + 3628800*b**10) + 1038960*a**5*b**5*c*d**2*n**2*
x**2*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b*
*10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 840
9500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) +
777600*a**5*b**5*c*d**2*n*x**2*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 +
1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5
+ 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640
*b**10*n + 3628800*b**10) + 3024*a**5*b**5*d**3*n**5*x**5*(a + b*x)**n/(b**
10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**1
0*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 1275
3576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 30240*a**5*b**5*d**3*
n**4*x**5*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 181
50*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4
+ 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**
10) + 105840*a**5*b**5*d**3*n**3*x**5*(a + b*x)**n/(b**10*n**10 + 55*b**10*
n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**1
0*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10
628640*b**10*n + 3628800*b**10) + 151200*a**5*b**5*d**3*n**2*x**5*(a + b*x)
n/(b10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157
773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**
3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 72576*a**5*b*
*5*d**3*n*x**5*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8
+ 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*
n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 362880
0*b**10) - 18*a**4*b**6*c**2*d*n**6*(a + b*x)**n/(b**10*n**10 + 55*b**10*n*
*9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*
n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 1062
8640*b**10*n + 3628800*b**10) - 810*a**4*b**6*c**2*d*n**5*(a + b*x)**n/(b**
10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**1
0*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 1275
3576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 15030*a**4*b**6*c**2*
d*n**4*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*
b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8
409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10)
- 147150*a**4*b**6*c**2*d*n**3*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 +
1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5
+ 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640
*b**10*n + 3628800*b**10) - 801432*a**4*b**6*c**2*d*n**2*(a + b*x)**n/(b**1
0*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10
*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753
576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 2301480*a**4*b**6*c**2

*d*n*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b*
*10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 840
9500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) -
2721600*a**4*b**6*c**2*d*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*
b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 341
6930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10
*n + 3628800*b**10) - 360*a**4*b**6*c*d**2*n**6*x**3*(a + b*x)**n/(b**10*n*
*10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**
6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*
b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 10800*a**4*b**6*c*d**2*n**
5*x**3*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*
b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8
409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10)
- 117000*a**4*b**6*c*d**2*n**4*x**3*(a + b*x)**n/(b**10*n**10 + 55*b**10*n
9 + 1320*b10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10
*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 106
28640*b**10*n + 3628800*b**10) - 540000*a**4*b**6*c*d**2*n**3*x**3*(a + b*x
)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 15
7773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n*
*3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 951840*a**4*
b**6*c*d**2*n**2*x**3*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**1
0*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930
*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n +
3628800*b**10) - 518400*a**4*b**6*c*d**2*n*x**3*(a + b*x)**n/(b**10*n**10
+ 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 +
902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**1
0*n**2 + 10628640*b**10*n + 3628800*b**10) - 504*a**4*b**6*d**3*n**6*x**6*(
a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n*
*7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b
10*n3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 7560*
a**4*b**6*d**3*n**5*x**6*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b
10*n8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416
930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*
n + 3628800*b**10) - 42840*a**4*b**6*d**3*n**4*x**6*(a + b*x)**n/(b**10*n**
10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6
+ 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b
10*n2 + 10628640*b**10*n + 3628800*b**10) - 113400*a**4*b**6*d**3*n**3*
x**6*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b*
*10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 840
9500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) -
138096*a**4*b**6*d**3*n**2*x**6*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9
+ 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**
5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 1062864
0*b**10*n + 3628800*b**10) - 60480*a**4*b**6*d**3*n*x**6*(a + b*x)**n/(b**1
0*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10

*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753
576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 18*a**3*b**7*c**2*d*n*
*7*x*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b*
*10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 840
9500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) +
810*a**3*b**7*c**2*d*n**6*x*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 13
20*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 +
3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b*
*10*n + 3628800*b**10) + 15030*a**3*b**7*c**2*d*n**5*x*(a + b*x)**n/(b**10*
n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n
6 + 902055*b10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 1275357
6*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 147150*a**3*b**7*c**2*d*
n**4*x*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*
b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8
409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10)
+ 801432*a**3*b**7*c**2*d*n**3*x*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9
+ 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n*
*5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 106286
40*b**10*n + 3628800*b**10) + 2301480*a**3*b**7*c**2*d*n**2*x*(a + b*x)**n/
(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*
b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 +
12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 2721600*a**3*b**7
*c**2*d*n*x*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 1
8150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**
4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b
10) + 90*a3*b**7*c*d**2*n**7*x**4*(a + b*x)**n/(b**10*n**10 + 55*b**10*
n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**1
0*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10
628640*b**10*n + 3628800*b**10) + 2970*a**3*b**7*c*d**2*n**6*x**4*(a + b*x)
n/(b10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157
773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**
3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 37350*a**3*b*
*7*c*d**2*n**5*x**4*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*
n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b
10*n4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3
628800*b**10) + 222750*a**3*b**7*c*d**2*n**4*x**4*(a + b*x)**n/(b**10*n**10
+ 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 +
902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**
10*n**2 + 10628640*b**10*n + 3628800*b**10) + 642960*a**3*b**7*c*d**2*n**3*
x**4*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b*
*10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 840
9500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) +
843480*a**3*b**7*c*d**2*n**2*x**4*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**
9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n
5 + 3416930*b10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628

640*b**10*n + 3628800*b**10) + 388800*a**3*b**7*c*d**2*n*x**4*(a + b*x)**n/
(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*
b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 +
12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 72*a**3*b**7*d**3
*n**7*x**7*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18
150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4
 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b*
*10) + 1512*a**3*b**7*d**3*n**6*x**7*(a + b*x)**n/(b**10*n**10 + 55*b**10*n
9 + 1320*b10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10
*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 106
28640*b**10*n + 3628800*b**10) + 12600*a**3*b**7*d**3*n**5*x**7*(a + b*x)**
n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 15777
3*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3
+ 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 52920*a**3*b**7
*d**3*n**4*x**7*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8
 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10
*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 36288
00*b**10) + 116928*a**3*b**7*d**3*n**3*x**7*(a + b*x)**n/(b**10*n**10 + 55*
b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 90205
5*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**
2 + 10628640*b**10*n + 3628800*b**10) + 127008*a**3*b**7*d**3*n**2*x**7*(a
+ b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7
 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**
10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 51840*a
3*b7*d**3*n*x**7*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10
*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*
b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n +
3628800*b**10) - 9*a**2*b**8*c**2*d*n**8*x**2*(a + b*x)**n/(b**10*n**10 + 5
5*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902
055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n
2 + 10628640*b10*n + 3628800*b**10) - 414*a**2*b**8*c**2*d*n**7*x**2*(a
 + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**
7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b*
*10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 7920*a
2*b8*c**2*d*n**6*x**2*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*
b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 341
6930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10
*n + 3628800*b**10) - 81090*a**2*b**8*c**2*d*n**5*x**2*(a + b*x)**n/(b**10*
n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n
6 + 902055*b10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 1275357
6*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 474291*a**2*b**8*c**2*d*
n**4*x**2*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 181
50*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4
+ 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**
10) - 1551456*a**2*b**8*c**2*d*n**3*x**2*(a + b*x)**n/(b**10*n**10 + 55*b**

10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b
10*n5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 +
10628640*b**10*n + 3628800*b**10) - 2511540*a**2*b**8*c**2*d*n**2*x**2*(a
+ b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7
+ 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**
10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 1360800
*a**2*b**8*c**2*d*n*x**2*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b
10*n8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416
930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*
n + 3628800*b**10) - 18*a**2*b**8*c*d**2*n**8*x**5*(a + b*x)**n/(b**10*n**1
0 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6
+ 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b*
*10*n**2 + 10628640*b**10*n + 3628800*b**10) - 666*a**2*b**8*c*d**2*n**7*x*
5(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**1
0*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 84095
00*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 9
846*a**2*b**8*c*d**2*n**6*x**5*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 +
1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5
+ 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*
b**10*n + 3628800*b**10) - 74430*a**2*b**8*c*d**2*n**5*x**5*(a + b*x)**n/(b
10*n10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b*
*10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12
753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 306792*a**2*b**8*c*
d**2*n**4*x**5*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8
+ 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*
n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 362880
0*b**10) - 683064*a**2*b**8*c*d**2*n**3*x**5*(a + b*x)**n/(b**10*n**10 + 55
*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 9020
55*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n*
*2 + 10628640*b**10*n + 3628800*b**10) - 752544*a**2*b**8*c*d**2*n**2*x**5*
(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n
7 + 157773*b10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*
b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 3110
40*a**2*b**8*c*d**2*n*x**5*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320
*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 34
16930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**1
0*n + 3628800*b**10) - 9*a**2*b**8*d**3*n**8*x**8*(a + b*x)**n/(b**10*n**10
+ 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 +
902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**
10*n**2 + 10628640*b**10*n + 3628800*b**10) - 252*a**2*b**8*d**3*n**7*x**8*
(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n
7 + 157773*b10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*
b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 2898
*a**2*b**8*d**3*n**6*x**8*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*
b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 341

6930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10
*n + 3628800*b**10) - 17640*a**2*b**8*d**3*n**5*x**8*(a + b*x)**n/(b**10*n*
*10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**
6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*
b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 60921*a**2*b**8*d**3*n**4*
x**8*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b*
*10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 840
9500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) -
118188*a**2*b**8*d**3*n**3*x**8*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9
+ 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**
5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 1062864
0*b**10*n + 3628800*b**10) - 117612*a**2*b**8*d**3*n**2*x**8*(a + b*x)**n/(
b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b
10*n6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 1
2753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) - 45360*a**2*b**8*d*
*3*n*x**8*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 181
50*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4
+ 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**
10) + a*b**9*c**3*n**9*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**
10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 341693
0*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n
+ 3628800*b**10) + 54*a*b**9*c**3*n**8*(a + b*x)**n/(b**10*n**10 + 55*b**10
*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**
10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 1
0628640*b**10*n + 3628800*b**10) + 1266*a*b**9*c**3*n**7*(a + b*x)**n/(b**1
0*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10
*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753
576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 16884*a*b**9*c**3*n**6
*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*
n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500
*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 140
889*a*b**9*c**3*n**5*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10
*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*
b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n +
3628800*b**10) + 761166*a*b**9*c**3*n**4*(a + b*x)**n/(b**10*n**10 + 55*b**
10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b
10*n5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 +
10628640*b**10*n + 3628800*b**10) + 2655764*a*b**9*c**3*n**3*(a + b*x)**n/
(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*
b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 +
12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 5753736*a*b**9*c*
*3*n**2*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150
*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 +
8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10
) + 6999840*a*b**9*c**3*n*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*

b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 341
6930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10
*n + 3628800*b**10) + 3628800*a*b**9*c**3*(a + b*x)**n/(b**10*n**10 + 55*b*
*10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*
b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2
+ 10628640*b**10*n + 3628800*b**10) + 3*a*b**9*c**2*d*n**9*x**3*(a + b*x)**
n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 15777
3*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3
+ 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 144*a*b**9*c**2
*d*n**8*x**3*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 +
18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n*
*4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*
b**10) + 2916*a*b**9*c**2*d*n**7*x**3*(a + b*x)**n/(b**10*n**10 + 55*b**10*
n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**1
0*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10
628640*b**10*n + 3628800*b**10) + 32310*a*b**9*c**2*d*n**6*x**3*(a + b*x)**
n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 15777
3*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3
+ 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 212157*a*b**9*c
2*d*n5*x**3*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8
 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10
*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 36288
00*b**10) + 833346*a*b**9*c**2*d*n**4*x**3*(a + b*x)**n/(b**10*n**10 + 55*b
10*n9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055
*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2
 + 10628640*b**10*n + 3628800*b**10) + 1871484*a*b**9*c**2*d*n**3*x**3*(a +
 b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7
+ 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**1
0*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 2127960*
a*b**9*c**2*d*n**2*x**3*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b*
*10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 34169
30*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n
 + 3628800*b**10) + 907200*a*b**9*c**2*d*n*x**3*(a + b*x)**n/(b**10*n**10 +
 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 9
02055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10
*n**2 + 10628640*b**10*n + 3628800*b**10) + 3*a*b**9*c*d**2*n**9*x**6*(a +
b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 +
 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10
*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 126*a*b**
9*c*d**2*n**8*x**6*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n
8 + 18150*b10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b*
*10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 36
28800*b**10) + 2196*a*b**9*c*d**2*n**7*x**6*(a + b*x)**n/(b**10*n**10 + 55*
b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 90205
5*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**

2 + 10628640*b**10*n + 3628800*b**10) + 20610*a*b**9*c*d**2*n**6*x**6*(a +
b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 +
157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10
*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 113157*a*
b**9*c*d**2*n**5*x**6*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**1
0*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930
*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n +
3628800*b**10) + 369504*a*b**9*c*d**2*n**4*x**6*(a + b*x)**n/(b**10*n**10
+ 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 +
902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**1
0*n**2 + 10628640*b**10*n + 3628800*b**10) + 694644*a*b**9*c*d**2*n**3*x**6
*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*
n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500
*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 678
960*a*b**9*c*d**2*n**2*x**6*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 132
0*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3
416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**
10*n + 3628800*b**10) + 259200*a*b**9*c*d**2*n*x**6*(a + b*x)**n/(b**10*n**
10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6
+ 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b
10*n2 + 10628640*b**10*n + 3628800*b**10) + a*b**9*d**3*n**9*x**9*(a +
b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 +
157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10
*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 36*a*b**9
*d**3*n**8*x**9*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8
+ 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10
*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 36288
00*b**10) + 546*a*b**9*d**3*n**7*x**9*(a + b*x)**n/(b**10*n**10 + 55*b**10*
n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**1
0*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10
628640*b**10*n + 3628800*b**10) + 4536*a*b**9*d**3*n**6*x**9*(a + b*x)**n/(
b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b
10*n6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 1
2753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 22449*a*b**9*d**3*
n**5*x**9*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 181
50*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4
+ 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**
10) + 67284*a*b**9*d**3*n**4*x**9*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9
+ 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n*
*5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 106286
40*b**10*n + 3628800*b**10) + 118124*a*b**9*d**3*n**3*x**9*(a + b*x)**n/(b*
*10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**
10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 127
53576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 109584*a*b**9*d**3*n
2*x9*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 1815

0*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 +
 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**1
0) + 40320*a*b**9*d**3*n*x**9*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1
320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 +
 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b
10*n + 3628800*b10) + b**10*c**3*n**9*x*(a + b*x)**n/(b**10*n**10 + 55*
b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 90205
5*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**
2 + 10628640*b**10*n + 3628800*b**10) + 54*b**10*c**3*n**8*x*(a + b*x)**n/(
b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b
10*n6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 1
2753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 1266*b**10*c**3*n*
*7*x*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b*
*10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 840
9500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) +
 16884*b**10*c**3*n**6*x*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b
10*n8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416
930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*
n + 3628800*b**10) + 140889*b**10*c**3*n**5*x*(a + b*x)**n/(b**10*n**10 + 5
5*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902
055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n
2 + 10628640*b10*n + 3628800*b**10) + 761166*b**10*c**3*n**4*x*(a + b*x
)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 15
7773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n*
*3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 2655764*b**1
0*c**3*n**3*x*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 +
 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n
4 + 8409500*b10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800
*b**10) + 5753736*b**10*c**3*n**2*x*(a + b*x)**n/(b**10*n**10 + 55*b**10*n*
*9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*
n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 1062
8640*b**10*n + 3628800*b**10) + 6999840*b**10*c**3*n*x*(a + b*x)**n/(b**10*
n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n
6 + 902055*b10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 1275357
6*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 3628800*b**10*c**3*x*(a
+ b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7
 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**
10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 3*b**10
*c**2*d*n**9*x**4*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n*
*8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**
10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 362
8800*b**10) + 153*b**10*c**2*d*n**8*x**4*(a + b*x)**n/(b**10*n**10 + 55*b**
10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b
10*n5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 +
 10628640*b**10*n + 3628800*b**10) + 3348*b**10*c**2*d*n**7*x**4*(a + b*x)*

*n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 1577
73*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3
 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 41058*b**10*c*
*2*d*n**6*x**4*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8
+ 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*
n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 362880
0*b**10) + 309087*b**10*c**2*d*n**5*x**4*(a + b*x)**n/(b**10*n**10 + 55*b**
10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b
10*n5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 +
 10628640*b**10*n + 3628800*b**10) + 1469817*b**10*c**2*d*n**4*x**4*(a + b*
x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 1
57773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n
3 + 12753576*b10*n**2 + 10628640*b**10*n + 3628800*b**10) + 4371522*b**
10*c**2*d*n**3*x**4*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*
n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b
10*n4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3
628800*b**10) + 7742412*b**10*c**2*d*n**2*x**4*(a + b*x)**n/(b**10*n**10 +
55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 90
2055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*
n**2 + 10628640*b**10*n + 3628800*b**10) + 7291080*b**10*c**2*d*n*x**4*(a +
 b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7
+ 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**1
0*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 2721600*
b**10*c**2*d*x**4*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n*
*8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**
10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 362
8800*b**10) + 3*b**10*c*d**2*n**9*x**7*(a + b*x)**n/(b**10*n**10 + 55*b**10
*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**
10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 1
0628640*b**10*n + 3628800*b**10) + 144*b**10*c*d**2*n**8*x**7*(a + b*x)**n/
(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*
b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 +
12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 2952*b**10*c*d**2
*n**7*x**7*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18
150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4
 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b*
*10) + 33786*b**10*c*d**2*n**6*x**7*(a + b*x)**n/(b**10*n**10 + 55*b**10*n*
*9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*
n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 1062
8640*b**10*n + 3628800*b**10) + 236817*b**10*c*d**2*n**5*x**7*(a + b*x)**n/
(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*
b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 +
12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 1048446*b**10*c*d
2*n4*x**7*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 +
 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n

4 + 8409500*b10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800
*b**10) + 2911668*b**10*c*d**2*n**3*x**7*(a + b*x)**n/(b**10*n**10 + 55*b**
10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b
10*n5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 +
 10628640*b**10*n + 3628800*b**10) + 4846824*b**10*c*d**2*n**2*x**7*(a + b*
x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 1
57773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n
3 + 12753576*b10*n**2 + 10628640*b**10*n + 3628800*b**10) + 4332960*b**
10*c*d**2*n*x**7*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**
8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**1
0*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628
800*b**10) + 1555200*b**10*c*d**2*x**7*(a + b*x)**n/(b**10*n**10 + 55*b**10
*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**
10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 1
0628640*b**10*n + 3628800*b**10) + b**10*d**3*n**9*x**10*(a + b*x)**n/(b**1
0*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10
*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753
576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 45*b**10*d**3*n**8*x**
10*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**1
0*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 84095
00*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 8
70*b**10*d**3*n**7*x**10*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b
10*n8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416
930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*
n + 3628800*b**10) + 9450*b**10*d**3*n**6*x**10*(a + b*x)**n/(b**10*n**10 +
 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 9
02055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10
*n**2 + 10628640*b**10*n + 3628800*b**10) + 63273*b**10*d**3*n**5*x**10*(a
+ b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7
 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**
10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 269325*
b**10*d**3*n**4*x**10*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**1
0*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930
*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n +
 3628800*b**10) + 723680*b**10*d**3*n**3*x**10*(a + b*x)**n/(b**10*n**10 +
55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 90
2055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*
n**2 + 10628640*b**10*n + 3628800*b**10) + 1172700*b**10*d**3*n**2*x**10*(a
 + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**
7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*b**10*n**4 + 8409500*b*
*10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n + 3628800*b**10) + 102657
6*b**10*d**3*n*x**10*(a + b*x)**n/(b**10*n**10 + 55*b**10*n**9 + 1320*b**10
*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b**10*n**5 + 3416930*
b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 + 10628640*b**10*n +
3628800*b**10) + 362880*b**10*d**3*x**10*(a + b*x)**n/(b**10*n**10 + 55*b**

```
10*n**9 + 1320*b**10*n**8 + 18150*b**10*n**7 + 157773*b**10*n**6 + 902055*b
**10*n**5 + 3416930*b**10*n**4 + 8409500*b**10*n**3 + 12753576*b**10*n**2 +
 10628640*b**10*n + 3628800*b**10), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(337) = 674$.

Time = 0.22 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.28

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = \frac{(bx+a)^{n+1}c^3}{b(n+1)} + \frac{3\left((n^3+6n^2+11n+6)b^4x^4+(n^3+3n^2+2n)ab^3x^3-3(n^2+n)a^2b^2x^2+6a^3bnx-6a^4\right)(bx+a)^n c^2}{(n^4+10n^3+35n^2+50n+24)b^4} + \frac{3\left((n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)b^7x^7+(n^6+15n^5+85n^4+225n^3+274n\right.}{} + \frac{\left((n^9+45n^8+870n^7+9450n^6+63273n^5+269325n^4+723680n^3+1172700n^2+1026576n+3628\right.}{}$$

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^3/(b*(n + 1)) + 3*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (
n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^
4)*(b*x + a)^n*c^2*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4) + 3*((n^6 +
21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n
^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n
^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x
^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*
a^6*b*n*x + 720*a^7)*(b*x + a)^n*c*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4
+ 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7) + ((n^9 + 45*n^8 + 870*n^7 +
9450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n +
362880)*b^10*x^10 + (n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67284*
n^4 + 118124*n^3 + 109584*n^2 + 40320*n)*a*b^9*x^9 - 9*(n^8 + 28*n^7 + 322*
n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^2*b^8*x^8 + 7
2*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^3*b^7*
x^7 - 504*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^4*b^6*x^6 +
 3024*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^5*b^5*x^5 - 15120*(n^4 + 6*
n^3 + 11*n^2 + 6*n)*a^6*b^4*x^4 + 60480*(n^3 + 3*n^2 + 2*n)*a^7*b^3*x^3 - 1
81440*(n^2 + n)*a^8*b^2*x^2 + 362880*a^9*b*n*x - 362880*a^10)*(b*x + a)^n*d
^3/((n^10 + 55*n^9 + 1320*n^8 + 18150*n^7 + 157773*n^6 + 902055*n^5 + 34169
30*n^4 + 8409500*n^3 + 12753576*n^2 + 10628640*n + 3628800)*b^10)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3874 vs. 2(337) = 674.

Time = 0.40 (sec) , antiderivative size = 3874, normalized size of antiderivative = 11.50

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^10*d^3*n^9*x^10 + (b*x + a)^n*a*b^9*d^3*n^9*x^9 + 45*(b*x +
a)^n*b^10*d^3*n^8*x^10 + 36*(b*x + a)^n*a*b^9*d^3*n^8*x^9 + 870*(b*x + a)^n
*b^10*d^3*n^7*x^10 + 3*(b*x + a)^n*b^10*c*d^2*n^9*x^7 - 9*(b*x + a)^n*a^2*b
^8*d^3*n^8*x^8 + 546*(b*x + a)^n*a*b^9*d^3*n^7*x^9 + 9450*(b*x + a)^n*b^10*
d^3*n^6*x^10 + 3*(b*x + a)^n*a*b^9*c*d^2*n^9*x^6 + 144*(b*x + a)^n*b^10*c*d
^2*n^8*x^7 - 252*(b*x + a)^n*a^2*b^8*d^3*n^7*x^8 + 4536*(b*x + a)^n*a*b^9*d
^3*n^6*x^9 + 63273*(b*x + a)^n*b^10*d^3*n^5*x^10 + 126*(b*x + a)^n*a*b^9*c*
d^2*n^8*x^6 + 2952*(b*x + a)^n*b^10*c*d^2*n^7*x^7 + 72*(b*x + a)^n*a^3*b^7*
d^3*n^7*x^7 - 2898*(b*x + a)^n*a^2*b^8*d^3*n^6*x^8 + 22449*(b*x + a)^n*a*b^
9*d^3*n^5*x^9 + 269325*(b*x + a)^n*b^10*d^3*n^4*x^10 + 3*(b*x + a)^n*b^10*c
^2*d*n^9*x^4 - 18*(b*x + a)^n*a^2*b^8*c*d^2*n^8*x^5 + 2196*(b*x + a)^n*a*b^
9*c*d^2*n^7*x^6 + 33786*(b*x + a)^n*b^10*c*d^2*n^6*x^7 + 1512*(b*x + a)^n*a
^3*b^7*d^3*n^6*x^7 - 17640*(b*x + a)^n*a^2*b^8*d^3*n^5*x^8 + 67284*(b*x + a
)^n*a*b^9*d^3*n^4*x^9 + 723680*(b*x + a)^n*b^10*d^3*n^3*x^10 + 3*(b*x + a)^
n*a*b^9*c^2*d*n^9*x^3 + 153*(b*x + a)^n*b^10*c^2*d*n^8*x^4 - 666*(b*x + a)^
n*a^2*b^8*c*d^2*n^7*x^5 + 20610*(b*x + a)^n*a*b^9*c*d^2*n^6*x^6 - 504*(b*x
+ a)^n*a^4*b^6*d^3*n^6*x^6 + 236817*(b*x + a)^n*b^10*c*d^2*n^5*x^7 + 12600*
(b*x + a)^n*a^3*b^7*d^3*n^5*x^7 - 60921*(b*x + a)^n*a^2*b^8*d^3*n^4*x^8 + 1
18124*(b*x + a)^n*a*b^9*d^3*n^3*x^9 + 1172700*(b*x + a)^n*b^10*d^3*n^2*x^10
 + 144*(b*x + a)^n*a*b^9*c^2*d*n^8*x^3 + 3348*(b*x + a)^n*b^10*c^2*d*n^7*x^
4 + 90*(b*x + a)^n*a^3*b^7*c*d^2*n^7*x^4 - 9846*(b*x + a)^n*a^2*b^8*c*d^2*n
^6*x^5 + 113157*(b*x + a)^n*a*b^9*c*d^2*n^5*x^6 - 7560*(b*x + a)^n*a^4*b^6*
d^3*n^5*x^6 + 1048446*(b*x + a)^n*b^10*c*d^2*n^4*x^7 + 52920*(b*x + a)^n*a^
3*b^7*d^3*n^4*x^7 - 118188*(b*x + a)^n*a^2*b^8*d^3*n^3*x^8 + 109584*(b*x +
a)^n*a*b^9*d^3*n^2*x^9 + 1026576*(b*x + a)^n*b^10*d^3*n*x^10 + (b*x + a)^n*
b^10*c^3*n^9*x - 9*(b*x + a)^n*a^2*b^8*c^2*d*n^8*x^2 + 2916*(b*x + a)^n*a*b
^9*c^2*d*n^7*x^3 + 41058*(b*x + a)^n*b^10*c^2*d*n^6*x^4 + 2970*(b*x + a)^n*
a^3*b^7*c*d^2*n^6*x^4 - 74430*(b*x + a)^n*a^2*b^8*c*d^2*n^5*x^5 + 3024*(b*x
 + a)^n*a^5*b^5*d^3*n^5*x^5 + 369504*(b*x + a)^n*a*b^9*c*d^2*n^4*x^6 - 4284
0*(b*x + a)^n*a^4*b^6*d^3*n^4*x^6 + 2911668*(b*x + a)^n*b^10*c*d^2*n^3*x^7
+ 116928*(b*x + a)^n*a^3*b^7*d^3*n^3*x^7 - 117612*(b*x + a)^n*a^2*b^8*d^3*n
^2*x^8 + 40320*(b*x + a)^n*a*b^9*d^3*n*x^9 + 362880*(b*x + a)^n*b^10*d^3*x^
10 + (b*x + a)^n*a*b^9*c^3*n^9 + 54*(b*x + a)^n*b^10*c^3*n^8*x - 414*(b*x +
 a)^n*a^2*b^8*c^2*d*n^7*x^2 + 32310*(b*x + a)^n*a*b^9*c^2*d*n^6*x^3 - 360*(
b*x + a)^n*a^4*b^6*c*d^2*n^6*x^3 + 309087*(b*x + a)^n*b^10*c^2*d*n^5*x^4 +
```

37350*(b*x + a)^n*a^3*b^7*c*d^2*n^5*x^4 - 306792*(b*x + a)^n*a^2*b^8*c*d^2*n^4*x^5 + 30240*(b*x + a)^n*a^5*b^5*d^3*n^4*x^5 + 694644*(b*x + a)^n*a*b^9*c*d^2*n^3*x^6 - 113400*(b*x + a)^n*a^4*b^6*d^3*n^3*x^6 + 4846824*(b*x + a)^n*b^10*c*d^2*n^2*x^7 + 127008*(b*x + a)^n*a^3*b^7*d^3*n^2*x^7 - 45360*(b*x + a)^n*a^2*b^8*d^3*n*x^8 + 54*(b*x + a)^n*a*b^9*c^3*n^8 + 1266*(b*x + a)^n*b^10*c^3*n^7*x + 18*(b*x + a)^n*a^3*b^7*c^2*d*n^7*x - 7920*(b*x + a)^n*a^2*b^8*c^2*d*n^6*x^2 + 212157*(b*x + a)^n*a*b^9*c^2*d*n^5*x^3 - 10800*(b*x + a)^n*a^4*b^6*c*d^2*n^5*x^3 + 1469817*(b*x + a)^n*b^10*c^2*d*n^4*x^4 + 222750*(b*x + a)^n*a^3*b^7*c*d^2*n^4*x^4 - 15120*(b*x + a)^n*a^6*b^4*d^3*n^4*x^4 - 683064*(b*x + a)^n*a^2*b^8*c*d^2*n^3*x^5 + 105840*(b*x + a)^n*a^5*b^5*d^3*n^3*x^5 + 678960*(b*x + a)^n*a*b^9*c*d^2*n^2*x^6 - 138096*(b*x + a)^n*a^4*b^6*d^3*n^2*x^6 + 4332960*(b*x + a)^n*b^10*c*d^2*n*x^7 + 51840*(b*x + a)^n*a^3*b^7*d^3*n*x^7 + 1266*(b*x + a)^n*a*b^9*c^3*n^7 + 16884*(b*x + a)^n*b^10*c^3*n^6*x + 810*(b*x + a)^n*a^3*b^7*c^2*d*n^6*x - 81090*(b*x + a)^n*a^2*b^8*c^2*d*n^5*x^2 + 1080*(b*x + a)^n*a^5*b^5*c*d^2*n^5*x^2 + 833346*(b*x + a)^n*a*b^9*c^2*d*n^4*x^3 - 117000*(b*x + a)^n*a^4*b^6*c*d^2*n^4*x^3 + 4371522*(b*x + a)^n*b^10*c^2*d*n^3*x^4 + 642960*(b*x + a)^n*a^3*b^7*c*d^2*n^3*x^4 - 90720*(b*x + a)^n*a^6*b^4*d^3*n^3*x^4 - 752544*(b*x + a)^n*a^2*b^8*c*d^2*n^2*x^5 + 151200*(b*x + a)^n*a^5*b^5*d^3*n^2*x^5 + 259200*(b*x + a)^n*a*b^9*c*d^2*n*x^6 - 60480*(b*x + a)^n*a^4*b^6*d^3*n*x^6 + 1555200*(b*x + a)^n*b^10*c*d^2*x^7 + 16884*(b*x + a)^n*a*b^9*c^3*n^6 - 18*(b*x + a)^n*a^4*b^6*c^2*d*n^6 + 140889*(b*x + a)^n*b^10*c^3*n^5*x + 15030*(b*x + a)^n*a^3*b^7*c^2*d*n^5*x - 474291*(b*x + a)^n*a^2*b^8*c^2*d*n^4*x^2 + 30240*(b*x + a)^n*a^5*b^5*c*d^2*n^4*x^2 + 1871484*(b*x + a)^n*a*b^9*c^2*d*n^3*x^3 - 540000*(b*x + a)^n*a^4*b^6*c*d^2*n^3*x^3 + 60480*(b*x + a)^n*a^7*b^3*d^3*n^3*x^3 + 7742412*(b*x + a)^n*b^10*c^2*d*n^2*x^4 + 843480*(b*x + a)^n*a^3*b^7*c*d^2*n^2*x^4 - 166320*(b*x + a)^n*a^6*b^4*d^3*n^2*x^4 - 311040*(b*x + a)^n*a^2*b^8*c*d^2*n*x^5 + 72576*(b*x + a)^n*a^5*b^5*d^3*n*x^5 + 140889*(b*x + a)^n*a*b^9*c^3*n^5 - 810*(b*x + a)^n*a^4*b^6*c^2*d*n^5 + 761166*(b*x + a)^n*b^10*c^3*n^4*x + 147150*(b*x + a)^n*a^3*b^7*c^2*d*n^4*x - 2160*(b*x + a)^n*a^6*b^4*c*d^2*n^4*x - 1551456*(b*x + a)^n*a^2*b^8*c^2*d*n^3*x^2 + 290520*(b*x + a)^n*a^5*b^5*c*d^2*n^3*x^2 + 2127960*(b*x + a)^n*a*b^9*c^2*d*n^2*x^3 - 951840*(b*x + a)^n*a^4*b^6*c*d^2*n^2*x^3 + 181440*(b*x + a)^n*a^7*b^3*d^3*n^2*x^3 + 7291080*(b*x + a)^n*b^10*c^2*d*n*x^4 + 388800*(b*x + a)^n*a^3*b^7*c*d^2*n*x^4 - 90720*(b*x + a)^n*a^6*b^4*d^3*n*x^4 + 761166*(b*x + a)^n*a*b^9*c^3*n^4 - 15030*(b*x + a)^n*a^4*b^6*c^2*d*n^4 + 2655764*(b*x + a)^n*b^10*c^3*n^3*x + 801432*(b*x + a)^n*a^3*b^7*c^2*d*n^3*x - 58320*(b*x + a)^n*a^6*b^4*c*d^2*n^3*x - 2511540*(b*x + a)^n*a^2*b^8*c^2*d*n^2*x^2 + 1038960*(b*x + a)^n*a^5*b^5*c*d^2*n^2*x^2 - 181440*(b*x + a)^n*a^8*b^2*d^3*n^2*x^2 + 907200*(b*x + a)^n*a*b^9*c^2*d*n*x^3 - 518400*(b*x + a)^n*a^4*b^6*c*d^2*n*x^3 + 120960*(b*x + a)^n*a^7*b^3*d^3*n*x^3 + 2721600*(b*x + a)^n*b^10*c^2*d*x^4 + 2655764*(b*x + a)^n*a*b^9*c^3*n^3 - 147150*(b*x + a)^n*a^4*b^6*c^2*d*n^3 + 2160*(b*x + a)^n*a^7*b^3*c*d^2*n^3 + 5753736*(b*x + a)^n*b^10*c^3*n^2*x + 2301480*(b*x + a)^n*a^3*b^7*c^2*d*n^2*x - 522720*(b*x + a)^n*a^6*b^4*c*d^2*n^2*x - 1360800*(b*x + a)^n*a^2*b^8*c^2*d*n*x^2 + 777600*(b*x + a)^n*a^5*b^5*c*d^2* n*x^2 - 181440*(b*x + a)^n*a^8*b^2*d^3*n*x^2 + 5753736*(b*x + a)^n*a*b^9*c^3*n^2 - 801432*(b*x + a)^n*a^4*b^6*c^2*d*n^2 + 58320*(b*x + a)^n*a^7*b^3*c*d^2*n^2 + 6999840*(b*x + a)^n*b^10*c^3*n*x + 2721600*(b*x + a)^n*a^3*b^7*c^2*d*n*x - 1555200*(b*x + a)^n*a^6*b^4*c*d^2*n*x + 362880*(b*x + a)^n*a^9*b*d^3*n*x + 6999840*(b*x + a)^n*a*b^9*c^3*n - 2301480*(b*x + a)^n*a^4*b^6*c^2*d*n + 522720*(b*x + a)^n*a^7*b^3*c*d^2*n + 3628800*(b*x + a)^n*b^10*c^3*x + 3628800*(b*x + a)^n*a*b^9*c^3 - 2721600*(b*x + a)^n*a^4*b^6*c^2*d + 1555200*(b*x + a)^n*a^7*b^3*c*d^2 - 362880*(b*x + a)^n*a^10*d^3)/(b^10*n^10 + 55*b^10*n^9 + 1320*b^10*n^8 + 18150*b^10*n^7 + 157773*b^10*n^6 + 902055*b^10*n^5 + 3416930*b^10*n^4 + 8409500*b^10*n^3 + 12753576*b^10*n^2 + 10628640*b^10*n + 3628800*b^10)

Mupad [B] (verification not implemented)

Time = 21.09 (sec) , antiderivative size = 2001, normalized size of antiderivative = 5.94

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

[In] int((c + d*x^3)^3*(a + b*x)^n,x)

[Out] ((a + b*x)^n*(3628800*a*b^9*c^3 - 362880*a^10*d^3 - 2721600*a^4*b^6*c^2*d + 1555200*a^7*b^3*c*d^2 + 5753736*a*b^9*c^3*n^2 + 2655764*a*b^9*c^3*n^3 + 761166*a*b^9*c^3*n^4 + 140889*a*b^9*c^3*n^5 + 16884*a*b^9*c^3*n^6 + 1266*a*b^9*c^3*n^7 + 54*a*b^9*c^3*n^8 + a*b^9*c^3*n^9 + 6999840*a*b^9*c^3*n - 2301480*a^4*b^6*c^2*d*n + 522720*a^7*b^3*c*d^2*n - 801432*a^4*b^6*c^2*d*n^2 + 58320*a^7*b^3*c*d^2*n^2 - 147150*a^4*b^6*c^2*d*n^3 + 2160*a^7*b^3*c*d^2*n^3 - 15030*a^4*b^6*c^2*d*n^4 - 810*a^4*b^6*c^2*d*n^5 - 18*a^4*b^6*c^2*d*n^6))/(b^10*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (x*(a + b*x)^n*(3628800*b^10*c^3 + 6999840*b^10*c^3*n + 5753736*b^10*c^3*n^2 + 2655764*b^10*c^3*n^3 + 761166*b^10*c^3*n^4 + 140889*b^10*c^3*n^5 + 16884*b^10*c^3*n^6 + 1266*b^10*c^3*n^7 + 54*b^10*c^3*n^8 + b^10*c^3*n^9 + 362880*a^9*b*d^3*n + 2721600*a^3*b^7*c^2*d*n - 1555200*a^6*b^4*c*d^2*n + 2301480*a^3*b^7*c^2*d*n^2 - 522720*a^6*b^4*c*d^2*n^2 + 801432*a^3*b^7*c^2*d*n^3 - 58320*a^6*b^4*c*d^2*n^3 + 147150*a^3*b^7*c^2*d*n^4 - 2160*a^6*b^4*c*d^2*n^4 + 15030*a^3*b^7*c^2*d*n^5 + 810*a^3*b^7*c^2*d*n^6 + 18*a^3*b^7*c^2*d*n^7))/(b^10*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (d^3*x^10*(a + b*x)^n*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880))/(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800) + (3*d^2*x^7*(a + b*x)^n*(720*b^3*c + 27*b^3*c*n^2 + b^3*c*n^3 + 24*a^3*d*n + 242*b^3*c*n)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(b^3*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930

```
*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 362
8800)) + (3*d*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(151200*b^6*c^2 - 50
40*a^6*d^2*n + 127860*b^6*c^2*n + 44524*b^6*c^2*n^2 + 8175*b^6*c^2*n^3 + 83
5*b^6*c^2*n^4 + 45*b^6*c^2*n^5 + b^6*c^2*n^6 + 21600*a^3*b^3*c*d*n + 7260*a
^3*b^3*c*d*n^2 + 810*a^3*b^3*c*d*n^3 + 30*a^3*b^3*c*d*n^4))/(b^6*(10628640*
n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18
150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (a*d^3*n*x^9*(a + b*x)^n*(
109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7
 + n^8 + 40320))/(b*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4
+ 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)
) - (9*a^2*d^3*n*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4
 + 322*n^5 + 28*n^6 + n^7 + 5040))/(b^2*(10628640*n + 12753576*n^2 + 840950
0*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n
^9 + n^10 + 3628800)) + (3*a*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(20160*a^6
*d^2 + 151200*b^6*c^2 + 127860*b^6*c^2*n + 44524*b^6*c^2*n^2 + 8175*b^6*c^2
*n^3 + 835*b^6*c^2*n^4 + 45*b^6*c^2*n^5 + b^6*c^2*n^6 - 86400*a^3*b^3*c*d -
 29040*a^3*b^3*c*d*n - 3240*a^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^7*(1
0628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*
n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) - (9*a^2*d*n*x^2*(n
+ 1)*(a + b*x)^n*(20160*a^6*d^2 + 151200*b^6*c^2 + 127860*b^6*c^2*n + 44524
*b^6*c^2*n^2 + 8175*b^6*c^2*n^3 + 835*b^6*c^2*n^4 + 45*b^6*c^2*n^5 + b^6*c^
2*n^6 - 86400*a^3*b^3*c*d - 29040*a^3*b^3*c*d*n - 3240*a^3*b^3*c*d*n^2 - 12
0*a^3*b^3*c*d*n^3))/(b^8*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930
*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 362
8800)) + (3*a*d^2*n*x^6*(a + b*x)^n*(720*b^3*c - 168*a^3*d + 27*b^3*c*n^2 +
 b^3*c*n^3 + 242*b^3*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/
(b^4*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 +
157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) - (18*a^2*d^2
*n*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(720*b^3*c - 168*a^3
*d + 27*b^3*c*n^2 + b^3*c*n^3 + 242*b^3*c*n))/(b^5*(10628640*n + 12753576*n
^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320
*n^8 + 55*n^9 + n^10 + 3628800))
```

3.185 $\int \frac{(a+bx)^n(c+dx^3)^3}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 358

$$\begin{aligned}\int \frac{(a+bx)^n\,(c+dx^3)^3}{x}\,dx &= \frac{a^2d(3b^6c^2-3a^3b^3cd+a^6d^2)\,(a+bx)^{1+n}}{b^9(1+n)} \\ &- \frac{ad(6b^6c^2-15a^3b^3cd+8a^6d^2)\,(a+bx)^{2+n}}{b^9(2+n)} \\ &+ \frac{d(3b^6c^2-30a^3b^3cd+28a^6d^2)\,(a+bx)^{3+n}}{b^9(3+n)} \\ &+ \frac{2a^2d^2(15b^3c-28a^3d)\,(a+bx)^{4+n}}{b^9(4+n)} \\ &- \frac{5ad^2(3b^3c-14a^3d)\,(a+bx)^{5+n}}{b^9(5+n)} \\ &+ \frac{d^2(3b^3c-56a^3d)\,(a+bx)^{6+n}}{b^9(6+n)} + \frac{28a^2d^3(a+bx)^{7+n}}{b^9(7+n)} \\ &- \frac{8ad^3(a+bx)^{8+n}}{b^9(8+n)} + \frac{d^3(a+bx)^{9+n}}{b^9(9+n)} \\ &- \frac{c^3(a+bx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)}{a(1+n)}\end{aligned}$$

```
[Out] a^2*d*(a^6*d^2-3*a^3*b^3*c*d+3*b^6*c^2)*(b*x+a)^(1+n)/b^9/(1+n)-a*d*(8*a^6*
d^2-15*a^3*b^3*c*d+6*b^6*c^2)*(b*x+a)^(2+n)/b^9/(2+n)+d*(28*a^6*d^2-30*a^3*
b^3*c*d+3*b^6*c^2)*(b*x+a)^(3+n)/b^9/(3+n)+2*a^2*d^2*(-28*a^3*d+15*b^3*c)*(
b*x+a)^(4+n)/b^9/(4+n)-5*a*d^2*(-14*a^3*d+3*b^3*c)*(b*x+a)^(5+n)/b^9/(5+n)+
d^2*(-56*a^3*d+3*b^3*c)*(b*x+a)^(6+n)/b^9/(6+n)+28*a^2*d^3*(b*x+a)^(7+n)/b^
9/(7+n)-8*a*d^3*(b*x+a)^(8+n)/b^9/(8+n)+d^3*(b*x+a)^(9+n)/b^9/(9+n)-c^3*(b*
x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1634, 67}

$$\int \frac{(a+bx)^n (c+dx^3)^3}{x} dx = -\frac{5ad^2(3b^3c-14a^3d)(a+bx)^{n+5}}{b^9(n+5)} + \frac{d^2(3b^3c-56a^3d)(a+bx)^{n+6}}{b^9(n+6)} + \frac{28a^2d^3(a+bx)^{n+7}}{b^9(n+7)} - \frac{ad(8a^6d^2-15a^3b^3cd+6b^6c^2)(a+bx)^{n+2}}{b^9(n+2)} + \frac{d(28a^6d^2-30a^3b^3cd+3b^6c^2)(a+bx)^{n+3}}{b^9(n+3)} + \frac{2a^2d^2(15b^3c-28a^3d)(a+bx)^{n+4}}{b^9(n+4)} + \frac{a^2d(a^6d^2-3a^3b^3cd+3b^6c^2)(a+bx)^{n+1}}{b^9(n+1)} - \frac{8ad^3(a+bx)^{n+8}}{b^9(n+8)} + \frac{d^3(a+bx)^{n+9}}{b^9(n+9)} - \frac{c^3(a+bx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{bx}{a}+1\right)}{a(n+1)}$$

```
[In] Int[((a + b*x)^n*(c + d*x^3)^3)/x,x]
```

```
[Out] (a^2*d*(3*b^6*c^2 - 3*a^3*b^3*c*d + a^6*d^2)*(a + b*x)^(1 + n))/(b^9*(1 + n
)) - (a*d*(6*b^6*c^2 - 15*a^3*b^3*c*d + 8*a^6*d^2)*(a + b*x)^(2 + n))/(b^9*
(2 + n)) + (d*(3*b^6*c^2 - 30*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/
(b^9*(3 + n)) + (2*a^2*d^2*(15*b^3*c - 28*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4
 + n)) - (5*a*d^2*(3*b^3*c - 14*a^3*d)*(a + b*x)^(5 + n))/(b^9*(5 + n)) + (
d^2*(3*b^3*c - 56*a^3*d)*(a + b*x)^(6 + n))/(b^9*(6 + n)) + (28*a^2*d^3*(a
+ b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^3*(a + b*x)^(8 + n))/(b^9*(8 + n)) +
 (d^3*(a + b*x)^(9 + n))/(b^9*(9 + n)) - (c^3*(a + b*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
```

```
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2d(3b^6c^2-3a^3b^3cd+a^6d^2)(a+bx)^n}{b^8}+\frac{c^3(a+bx)^n}{x}-\frac{ad(6b^6c^2-15a^3b^3cd+8a^6d^2)(a+bx)^{1+n}}{b^8}+\frac{d(3b^6c^2-30a^3b^3cd+28a^6d^2)(a+bx)^{2+n}}{b^8}-\frac{2a^2d^2(-15b^3c+28a^3d)(a+bx)^{3+n}}{b^8}+\frac{5ad^2(-3b^3c+14a^3d)(a+bx)^{4+n}}{b^8}+\frac{d^2(3b^3c-56a^3d)(a+bx)^{5+n}}{b^8}+\frac{28a^2d^3(a+bx)^{6+n}}{b^8}-\frac{8ad^3(a+bx)^{7+n}}{b^8}+\frac{d^3(a+bx)^{8+n}}{b^8}\right)dx$$

$$=\frac{a^2d(3b^6c^2-3a^3b^3cd+a^6d^2)(a+bx)^{1+n}}{b^9(1+n)}-\frac{ad(6b^6c^2-15a^3b^3cd+8a^6d^2)(a+bx)^{2+n}}{b^9(2+n)}+\frac{d(3b^6c^2-30a^3b^3cd+28a^6d^2)(a+bx)^{3+n}}{b^9(3+n)}+\frac{2a^2d^2(15b^3c-28a^3d)(a+bx)^{4+n}}{b^9(4+n)}-\frac{5ad^2(3b^3c-14a^3d)(a+bx)^{5+n}}{b^9(5+n)}+\frac{d^2(3b^3c-56a^3d)(a+bx)^{6+n}}{b^9(6+n)}+\frac{28a^2d^3(a+bx)^{7+n}}{b^9(7+n)}-\frac{8ad^3(a+bx)^{8+n}}{b^9(8+n)}+\frac{d^3(a+bx)^{9+n}}{b^9(9+n)}+c^3\int\frac{(a+bx)^n}{x}\,dx$$

$$=\frac{a^2d(3b^6c^2-3a^3b^3cd+a^6d^2)(a+bx)^{1+n}}{b^9(1+n)}-\frac{ad(6b^6c^2-15a^3b^3cd+8a^6d^2)(a+bx)^{2+n}}{b^9(2+n)}+\frac{d(3b^6c^2-30a^3b^3cd+28a^6d^2)(a+bx)^{3+n}}{b^9(3+n)}+\frac{2a^2d^2(15b^3c-28a^3d)(a+bx)^{4+n}}{b^9(4+n)}-\frac{5ad^2(3b^3c-14a^3d)(a+bx)^{5+n}}{b^9(5+n)}+\frac{d^2(3b^3c-56a^3d)(a+bx)^{6+n}}{b^9(6+n)}+\frac{28a^2d^3(a+bx)^{7+n}}{b^9(7+n)}-\frac{8ad^3(a+bx)^{8+n}}{b^9(8+n)}+\frac{d^3(a+bx)^{9+n}}{b^9(9+n)}-\frac{c^3(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^n (c+dx^3)^3}{x} dx = (a+bx)^{1+n} \left(\frac{a^2d(3b^6c^2-3a^3b^3cd+a^6d^2)}{b^9(1+n)} - \frac{ad(6b^6c^2-15a^3b^3cd+8a^6d^2)(a+bx)}{b^9(2+n)} + \frac{d(3b^6c^2-30a^3b^3cd+28a^6d^2)(a+bx)^2}{b^9(3+n)} + \frac{2a^2d^2(15b^3c-28a^3d)(a+bx)^3}{b^9(4+n)} + \frac{5ad^2(-3b^3c+14a^3d)(a+bx)^4}{b^9(5+n)} + \frac{d^2(3b^3c-56a^3d)(a+bx)^5}{b^9(6+n)} + \frac{28a^2d^3(a+bx)^6}{b^9(7+n)} - \frac{8ad^3(a+bx)^7}{b^9(8+n)} + \frac{d^3(a+bx)^8}{b^9(9+n)} - \frac{c^3 \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+bx}{a}\right)}{a+an} \right)$$

```
[In] Integrate[((a + b*x)^n*(c + d*x^3)^3)/x,x]
```

```
[Out] (a + b*x)^(1 + n)*((a^2*d*(3*b^6*c^2 - 3*a^3*b^3*c*d + a^6*d^2))/(b^9*(1 +
n)) - (a*d*(6*b^6*c^2 - 15*a^3*b^3*c*d + 8*a^6*d^2)*(a + b*x))/(b^9*(2 + n)
) + (d*(3*b^6*c^2 - 30*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^2)/(b^9*(3 + n))
 + (2*a^2*d^2*(15*b^3*c - 28*a^3*d)*(a + b*x)^3)/(b^9*(4 + n)) + (5*a*d^2*(
-3*b^3*c + 14*a^3*d)*(a + b*x)^4)/(b^9*(5 + n)) + (d^2*(3*b^3*c - 56*a^3*d)
*(a + b*x)^5)/(b^9*(6 + n)) + (28*a^2*d^3*(a + b*x)^6)/(b^9*(7 + n)) - (8*a
*d^3*(a + b*x)^7)/(b^9*(8 + n)) + (d^3*(a + b*x)^8)/(b^9*(9 + n)) - (c^3*Hy
pergeometric2F1[1, 1 + n, 2 + n, (a + b*x)/a])/(a + a*n))
```

Maple [F]

$$\int \frac{(bx+a)^n (x^3d+c)^3}{x} dx$$

```
[In] int((b*x+a)^n*(d*x^3+c)^3/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^3+c)^3/x,x)
```

Fricas [F]

$$\int \frac{(a+bx)^n\left(c+dx^3\right)^3}{x}\,dx=\int \frac{\left(dx^3+c\right)^3(bx+a)^n}{x}\,dx$$

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3)*(b*x + a)^n/x, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12492 vs. 2(338) = 676.

Time = 13.33 (sec) , antiderivative size = 17189, normalized size of antiderivative = 48.01

$$\int \frac{(a+bx)^n\left(c+dx^3\right)^3}{x}\,dx=\text{Too large to display}$$

```
[In] integrate((b*x+a)**n*(d*x**3+c)**3/x,x)
```

```
[Out] 3*c**2*d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b*
*3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*
x**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a
*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2
*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b + x)/(
a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3
+ b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b + x)/b**
3 - a*x/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3 + 6*
b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**3 + 6*
b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**3*n**3
+ 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b**3*n**3
 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)**n/(b**3*n*
*3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x)**n/(b**3*n
**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)**n/(b**3*n*
*3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True)) + 3*c*d**2*Piecewise((a**n*x
**6/6, Eq(b, 0)), (60*a**5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 6
00*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5)
+ 137*a**5/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*
b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*x*log(a/b + x)/(
60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 +
300*a*b**10*x**4 + 60*b**11*x**5) + 625*a**4*b*x/(60*a**5*b**6 + 300*a**4*b
**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**
11*x**5) + 600*a**3*b**2*x**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x
+ 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**
```

```
5) + 1100*a**3*b**2*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x*
*2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**2*b**3
*x**3*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 6
00*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 900*a**2*b**3*x**3/
(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 +
 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*x**4*log(a/b + x)/(60*a**5*
b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b*
*10*x**4 + 60*b**11*x**5) + 300*a*b**4*x**4/(60*a**5*b**6 + 300*a**4*b**7*x
 + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x*
*5) + 60*b**5*x**5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*
b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5), Eq(n, -
6)), (-60*a**5*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x
**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 125*a**5/(12*a**4*b**6 + 48*a**3*b*
*7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**4*b*x*l
og(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*
x**3 + 12*b**10*x**4) - 440*a**4*b*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a*
*2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 360*a**3*b**2*x**2*log(a/b
 + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 +
 12*b**10*x**4) - 540*a**3*b**2*x**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a*
*2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*x**3*log(a/b
 + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 +
 12*b**10*x**4) - 240*a**2*b**3*x**3/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a*
*2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 60*a*b**4*x**4*log(a/b + x
)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*
b**10*x**4) + 12*b**5*x**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x*
*2 + 48*a*b**9*x**3 + 12*b**10*x**4), Eq(n, -5)), (60*a**5*log(a/b + x)/(6*
a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 110*a**5/(6*a*
*3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**4*b*x*log
(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 2
70*a**4*b*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) +
 180*a**3*b**2*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*
x**2 + 6*b**9*x**3) + 180*a**3*b**2*x**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18
*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*x**3*log(a/b + x)/(6*a**3*b**6 +
 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*a*b**4*x**4/(6*a**3*b*
*6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 3*b**5*x**5/(6*a**3*b
**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3), Eq(n, -4)), (-60*a**5
*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 90*a**5/(6*a**2*b
**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*x*log(a/b + x)/(6*a**2*b**6 +
 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b
**8*x**2) - 60*a**3*b**2*x**2*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b
**8*x**2) + 20*a**2*b**3*x**3/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 5
*a*b**4*x**4/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 2*b**5*x**5/(6*a**
2*b**6 + 12*a*b**7*x + 6*b**8*x**2), Eq(n, -3)), (60*a**5*log(a/b + x)/(12*
a*b**6 + 12*b**7*x) + 60*a**5/(12*a*b**6 + 12*b**7*x) + 60*a**4*b*x*log(a/b
 + x)/(12*a*b**6 + 12*b**7*x) - 30*a**3*b**2*x**2/(12*a*b**6 + 12*b**7*x) +
```

10*a**2*b**3*x**3/(12*a*b**6 + 12*b**7*x) - 5*a*b**4*x**4/(12*a*b**6 + 12*b**7*x) + 3*b**5*x**5/(12*a*b**6 + 12*b**7*x), Eq(n, -2)), (-a**5*log(a/b + x)/b**6 + a**4*x/b**5 - a**3*x**2/(2*b**4) + a**2*x**3/(3*b**3) - a*x**4/(4*b**2) + x**5/(5*b), Eq(n, -1)), (-120*a**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*a**5*b*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 20*a**3*b**3*n**3*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 60*a**3*b**3*n**2*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 40*a**3*b**3*n*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 5*a**2*b**4*n**4*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4*n**3*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 55*a**2*b**4*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4*n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b**5*n**5*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 10*a*b**5*n**4*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 35*a*b**5*n**3*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 50*a*b**5*n**2*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 24*a*b**5*n*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*n**5*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 15*b**6*n**4*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 85*b**6*n**3*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 225*b**6*n**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 274*b**6*n*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*b**6*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6), True)) + d**3*Piecewise((a**n*x**9/9, Eq(b, 0)), (840*a**8*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**1

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2*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x
**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 2283*a**8/(840*a**8*b**9 + 6720
*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*
b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x
**7 + 840*b**17*x**8) + 6720*a**7*b*x*log(a/b + x)/(840*a**8*b**9 + 6720*a*
*7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**
13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7
 + 840*b**17*x**8) + 17424*a**7*b*x/(840*a**8*b**9 + 6720*a**7*b**10*x + 23
520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040
*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x*
*8) + 23520*a**6*b**2*x**2*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x
 + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 4
7040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**1
7*x**8) + 57624*a**6*b**2*x**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a
**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3
*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) +
 47040*a**5*b**3*x**3*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 235
20*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*
a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**
8) + 107408*a**5*b**3*x**3/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*
b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**
14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 588
00*a**4*b**4*x**4*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a
**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3
*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) +
 122500*a**4*b**4*x**4/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**1
1*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x
**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 47040*a
**3*b**5*x**5*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*
b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**
14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 862
40*a**3*b**5*x**5/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**
2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 +
 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 23520*a**2*b
**6*x**6*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11
*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x*
*5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 35280*a*
*2*b**6*x**6/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 4
7040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 2352
0*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 6720*a*b**7*x**7*
log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 4
7040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 2352
0*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 6720*a*b**7*x**7/
(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**
12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*
```

x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 840*b**8*x**8*log(a/b + x)/(84
0*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*
x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**
6 + 6720*a*b**16*x**7 + 840*b**17*x**8), Eq(n, -9)), (-840*a**8*log(a/b + x
)/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**1
2*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 1
05*b**16*x**7) - 2178*a**8/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b*
*11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x*
*5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 5880*a**7*b*x*log(a/b + x)/(105*a
7*b9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 +
3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16
*x**7) - 14406*a**7*b*x/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11
*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5
+ 735*a*b**15*x**6 + 105*b**16*x**7) - 17640*a**6*b**2*x**2*log(a/b + x)/(1
05*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x*
*3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b
16*x7) - 40278*a**6*b**2*x**2/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*
a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b
14*x5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 29400*a**5*b**3*x**3*log(a
/b + x)/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**
4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x*
*6 + 105*b**16*x**7) - 61250*a**5*b**3*x**3/(105*a**7*b**9 + 735*a**6*b**10
*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2
205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 29400*a**4*b**4*
x**4*log(a/b + x)/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2
+ 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*
a*b**15*x**6 + 105*b**16*x**7) - 53900*a**4*b**4*x**4/(105*a**7*b**9 + 735*
a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**1
3*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 17640*
a**3*b**5*x**5*log(a/b + x)/(105*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b
11*x2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x
5 + 735*a*b15*x**6 + 105*b**16*x**7) - 26460*a**3*b**5*x**5/(105*a**7*b
9 + 735*a6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675
*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7
) - 5880*a**2*b**6*x**6*log(a/b + x)/(105*a**7*b**9 + 735*a**6*b**10*x + 22
05*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205*a**
2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) - 5880*a**2*b**6*x**6/(10
5*a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**
3 + 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b*
*16*x**7) - 840*a*b**7*x**7*log(a/b + x)/(105*a**7*b**9 + 735*a**6*b**10*x
+ 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3 + 3675*a**3*b**13*x**4 + 2205
*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**16*x**7) + 105*b**8*x**8/(105*
a**7*b**9 + 735*a**6*b**10*x + 2205*a**5*b**11*x**2 + 3675*a**4*b**12*x**3
+ 3675*a**3*b**13*x**4 + 2205*a**2*b**14*x**5 + 735*a*b**15*x**6 + 105*b**1
6*x**7), Eq(n, -8)), (840*a**8*log(a/b + x)/(30*a**6*b**9 + 180*a**5*b**10*

x + 450*a**4*b**11*x**2 + 600*a**3*b**12*x**3 + 450*a**2*b**13*x**4 + 180*a
*b**14*x**5 + 30*b**15*x**6) + 2058*a**8/(30*a**6*b**9 + 180*a**5*b**10*x +
 450*a**4*b**11*x**2 + 600*a**3*b**12*x**3 + 450*a**2*b**13*x**4 + 180*a*b*
*14*x**5 + 30*b**15*x**6) + 5040*a**7*b*x*log(a/b + x)/(30*a**6*b**9 + 180*
a**5*b**10*x + 450*a**4*b**11*x**2 + 600*a**3*b**12*x**3 + 450*a**2*b**13*x
4 + 180*a*b14*x**5 + 30*b**15*x**6) + 11508*a**7*b*x/(30*a**6*b**9 + 18
0*a**5*b**10*x + 450*a**4*b**11*x**2 + 600*a**3*b**12*x**3 + 450*a**2*b**13
*x**4 + 180*a*b**14*x**5 + 30*b**15*x**6) + 12600*a**6*b**2*x**2*log(a/b +
x)/(30*a**6*b**9 + 180*a**5*b**10*x + 450*a**4*b**11*x**2 + 600*a**3*b**12*
x**3 + 450*a**2*b**13*x**4 + 180*a*b**14*x**5 + 30*b**15*x**6) + 26250*a**6
*b**2*x**2/(30*a**6*b**9 + 180*a**5*b**10*x + 450*a**4*b**11*x**2 + 600*a**
3*b**12*x**3 + 450*a**2*b**13*x**4 + 180*a*b**14*x**5 + 30*b**15*x**6) + 16
800*a**5*b**3*x**3*log(a/b + x)/(30*a**6*b**9 + 180*a**5*b**10*x + 450*a**4
*b**11*x**2 + 600*a**3*b**12*x**3 + 450*a**2*b**13*x**4 + 180*a*b**14*x**5
+ 30*b**15*x**6) + 30800*a**5*b**3*x**3/(30*a**6*b**9 + 180*a**5*b**10*x +
450*a**4*b**11*x**2 + 600*a**3*b**12*x**3 + 450*a**2*b**13*x**4 + 180*a*b**
14*x**5 + 30*b**15*x**6) + 12600*a**4*b**4*x**4*log(a/b + x)/(30*a**6*b**9
+ 180*a**5*b**10*x + 450*a**4*b**11*x**2 + 600*a**3*b**12*x**3 + 450*a**2*b
13*x4 + 180*a*b**14*x**5 + 30*b**15*x**6) + 18900*a**4*b**4*x**4/(30*a*
*6*b**9 + 180*a**5*b**10*x + 450*a**4*b**11*x**2 + 600*a**3*b**12*x**3 + 45
0*a**2*b**13*x**4 + 180*a*b**14*x**5 + 30*b**15*x**6) + 5040*a**3*b**5*x**5
*log(a/b + x)/(30*a**6*b**9 + 180*a**5*b**10*x + 450*a**4*b**11*x**2 + 600*
a**3*b**12*x**3 + 450*a**2*b**13*x**4 + 180*a*b**14*x**5 + 30*b**15*x**6) +
 5040*a**3*b**5*x**5/(30*a**6*b**9 + 180*a**5*b**10*x + 450*a**4*b**11*x**2
 + 600*a**3*b**12*x**3 + 450*a**2*b**13*x**4 + 180*a*b**14*x**5 + 30*b**15*
x**6) + 840*a**2*b**6*x**6*log(a/b + x)/(30*a**6*b**9 + 180*a**5*b**10*x +
450*a**4*b**11*x**2 + 600*a**3*b**12*x**3 + 450*a**2*b**13*x**4 + 180*a*b**
14*x**5 + 30*b**15*x**6) - 120*a*b**7*x**7/(30*a**6*b**9 + 180*a**5*b**10*x
 + 450*a**4*b**11*x**2 + 600*a**3*b**12*x**3 + 450*a**2*b**13*x**4 + 180*a*
b**14*x**5 + 30*b**15*x**6) + 15*b**8*x**8/(30*a**6*b**9 + 180*a**5*b**10*x
 + 450*a**4*b**11*x**2 + 600*a**3*b**12*x**3 + 450*a**2*b**13*x**4 + 180*a*
b**14*x**5 + 30*b**15*x**6), Eq(n, -7)), (-840*a**8*log(a/b + x)/(15*a**5*b
9 + 75*a4*b**10*x + 150*a**3*b**11*x**2 + 150*a**2*b**12*x**3 + 75*a*b*
*13*x**4 + 15*b**14*x**5) - 1918*a**8/(15*a**5*b**9 + 75*a**4*b**10*x + 150
*a**3*b**11*x**2 + 150*a**2*b**12*x**3 + 75*a*b**13*x**4 + 15*b**14*x**5) -
 4200*a**7*b*x*log(a/b + x)/(15*a**5*b**9 + 75*a**4*b**10*x + 150*a**3*b**1
1*x**2 + 150*a**2*b**12*x**3 + 75*a*b**13*x**4 + 15*b**14*x**5) - 8750*a**7
*b*x/(15*a**5*b**9 + 75*a**4*b**10*x + 150*a**3*b**11*x**2 + 150*a**2*b**12
*x**3 + 75*a*b**13*x**4 + 15*b**14*x**5) - 8400*a**6*b**2*x**2*log(a/b + x)
/(15*a**5*b**9 + 75*a**4*b**10*x + 150*a**3*b**11*x**2 + 150*a**2*b**12*x**
3 + 75*a*b**13*x**4 + 15*b**14*x**5) - 15400*a**6*b**2*x**2/(15*a**5*b**9 +
 75*a**4*b**10*x + 150*a**3*b**11*x**2 + 150*a**2*b**12*x**3 + 75*a*b**13*x
4 + 15*b14*x**5) - 8400*a**5*b**3*x**3*log(a/b + x)/(15*a**5*b**9 + 75*
a**4*b**10*x + 150*a**3*b**11*x**2 + 150*a**2*b**12*x**3 + 75*a*b**13*x**4
+ 15*b**14*x**5) - 12600*a**5*b**3*x**3/(15*a**5*b**9 + 75*a**4*b**10*x + 1

50*a**3*b**11*x**2 + 150*a**2*b**12*x**3 + 75*a*b**13*x**4 + 15*b**14*x**5)
 - 4200*a**4*b**4*x**4*log(a/b + x)/(15*a**5*b**9 + 75*a**4*b**10*x + 150*a
3*b11*x**2 + 150*a**2*b**12*x**3 + 75*a*b**13*x**4 + 15*b**14*x**5) - 4
200*a**4*b**4*x**4/(15*a**5*b**9 + 75*a**4*b**10*x + 150*a**3*b**11*x**2 +
150*a**2*b**12*x**3 + 75*a*b**13*x**4 + 15*b**14*x**5) - 840*a**3*b**5*x**5
*log(a/b + x)/(15*a**5*b**9 + 75*a**4*b**10*x + 150*a**3*b**11*x**2 + 150*a
2*b12*x**3 + 75*a*b**13*x**4 + 15*b**14*x**5) + 140*a**2*b**6*x**6/(15*
a**5*b**9 + 75*a**4*b**10*x + 150*a**3*b**11*x**2 + 150*a**2*b**12*x**3 + 7
5*a*b**13*x**4 + 15*b**14*x**5) - 20*a*b**7*x**7/(15*a**5*b**9 + 75*a**4*b*
*10*x + 150*a**3*b**11*x**2 + 150*a**2*b**12*x**3 + 75*a*b**13*x**4 + 15*b*
*14*x**5) + 5*b**8*x**8/(15*a**5*b**9 + 75*a**4*b**10*x + 150*a**3*b**11*x*
*2 + 150*a**2*b**12*x**3 + 75*a*b**13*x**4 + 15*b**14*x**5), Eq(n, -6)), (8
40*a**8*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 +
 48*a*b**12*x**3 + 12*b**13*x**4) + 1750*a**8/(12*a**4*b**9 + 48*a**3*b**10
*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 3360*a**7*b*x*
log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b*
*12*x**3 + 12*b**13*x**4) + 6160*a**7*b*x/(12*a**4*b**9 + 48*a**3*b**10*x +
 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 5040*a**6*b**2*x**
2*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*
b**12*x**3 + 12*b**13*x**4) + 7560*a**6*b**2*x**2/(12*a**4*b**9 + 48*a**3*b
10*x + 72*a2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 3360*a**5*
b**3*x**3*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2
 + 48*a*b**12*x**3 + 12*b**13*x**4) + 3360*a**5*b**3*x**3/(12*a**4*b**9 + 4
8*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 84
0*a**4*b**4*x**4*log(a/b + x)/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**
11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) - 168*a**3*b**5*x**5/(12*a**4*b*
*9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4
) + 28*a**2*b**6*x**6/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2
+ 48*a*b**12*x**3 + 12*b**13*x**4) - 8*a*b**7*x**7/(12*a**4*b**9 + 48*a**3*
b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 + 12*b**13*x**4) + 3*b**8*x*
*8/(12*a**4*b**9 + 48*a**3*b**10*x + 72*a**2*b**11*x**2 + 48*a*b**12*x**3 +
 12*b**13*x**4), Eq(n, -5)), (-840*a**8*log(a/b + x)/(15*a**3*b**9 + 45*a**
2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 1540*a**8/(15*a**3*b**9 + 45
*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 2520*a**7*b*x*log(a/b +
x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 378
0*a**7*b*x/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**
3) - 2520*a**6*b**2*x**2*log(a/b + x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*
a*b**11*x**2 + 15*b**12*x**3) - 2520*a**6*b**2*x**2/(15*a**3*b**9 + 45*a**2
*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) - 840*a**5*b**3*x**3*log(a/b +
x)/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3) + 210
*a**4*b**4*x**4/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x**2 + 15*b**1
2*x**3) - 42*a**3*b**5*x**5/(15*a**3*b**9 + 45*a**2*b**10*x + 45*a*b**11*x*
*2 + 15*b**12*x**3) + 14*a**2*b**6*x**6/(15*a**3*b**9 + 45*a**2*b**10*x + 4
5*a*b**11*x**2 + 15*b**12*x**3) - 6*a*b**7*x**7/(15*a**3*b**9 + 45*a**2*b**
10*x + 45*a*b**11*x**2 + 15*b**12*x**3) + 3*b**8*x**8/(15*a**3*b**9 + 45*a*

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*2*b**10*x + 45*a*b**11*x**2 + 15*b**12*x**3), Eq(n, -4)), (840*a**8*log(a/
b + x)/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) + 1260*a**8/(30*a**2*b
**9 + 60*a*b**10*x + 30*b**11*x**2) + 1680*a**7*b*x*log(a/b + x)/(30*a**2*b
**9 + 60*a*b**10*x + 30*b**11*x**2) + 1680*a**7*b*x/(30*a**2*b**9 + 60*a*b*
*10*x + 30*b**11*x**2) + 840*a**6*b**2*x**2*log(a/b + x)/(30*a**2*b**9 + 60
*a*b**10*x + 30*b**11*x**2) - 280*a**5*b**3*x**3/(30*a**2*b**9 + 60*a*b**10
*x + 30*b**11*x**2) + 70*a**4*b**4*x**4/(30*a**2*b**9 + 60*a*b**10*x + 30*b
**11*x**2) - 28*a**3*b**5*x**5/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2
) + 14*a**2*b**6*x**6/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) - 8*a*b
**7*x**7/(30*a**2*b**9 + 60*a*b**10*x + 30*b**11*x**2) + 5*b**8*x**8/(30*a*
*2*b**9 + 60*a*b**10*x + 30*b**11*x**2), Eq(n, -3)), (-840*a**8*log(a/b + x
)/(105*a*b**9 + 105*b**10*x) - 840*a**8/(105*a*b**9 + 105*b**10*x) - 840*a*
*7*b*x*log(a/b + x)/(105*a*b**9 + 105*b**10*x) + 420*a**6*b**2*x**2/(105*a*
b**9 + 105*b**10*x) - 140*a**5*b**3*x**3/(105*a*b**9 + 105*b**10*x) + 70*a*
*4*b**4*x**4/(105*a*b**9 + 105*b**10*x) - 42*a**3*b**5*x**5/(105*a*b**9 + 1
05*b**10*x) + 28*a**2*b**6*x**6/(105*a*b**9 + 105*b**10*x) - 20*a*b**7*x**7
/(105*a*b**9 + 105*b**10*x) + 15*b**8*x**8/(105*a*b**9 + 105*b**10*x), Eq(n
, -2)), (a**8*log(a/b + x)/b**9 - a**7*x/b**8 + a**6*x**2/(2*b**7) - a**5*x
**3/(3*b**6) + a**4*x**4/(4*b**5) - a**3*x**5/(5*b**4) + a**2*x**6/(6*b**3)
 - a*x**7/(7*b**2) + x**8/(8*b), Eq(n, -1)), (40320*a**9*(a + b*x)**n/(b**9
*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 2
69325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 3
62880*b**9) - 40320*a**8*b*n*x*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870
*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b
**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 20160*a**7*b
**2*n**2*x**2*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450
*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 117270
0*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 20160*a**7*b**2*n*x**2*(a + b
*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b
**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 102657
6*b**9*n + 362880*b**9) - 6720*a**6*b**3*n**3*x**3*(a + b*x)**n/(b**9*n**9
 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*
b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*
b**9) - 20160*a**6*b**3*n**2*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 +
870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 72368
0*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 13440*a**
6*b**3*n*x**3*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450
*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 117270
0*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 1680*a**5*b**4*n**4*x**4*(a +
 b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273
*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026
576*b**9*n + 362880*b**9) + 10080*a**5*b**4*n**3*x**4*(a + b*x)**n/(b**9*n*
*9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 2693
25*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 3628
80*b**9) + 18480*a**5*b**4*n**2*x**4*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8
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+ 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 72
3680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 10080*
a**5*b**4*n*x**4*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9
450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 117
2700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 336*a**4*b**5*n**5*x**5*(a
+ b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 632
73*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 10
26576*b**9*n + 362880*b**9) - 3360*a**4*b**5*n**4*x**5*(a + b*x)**n/(b**9*n
9 + 45*b9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269
325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362
880*b**9) - 11760*a**4*b**5*n**3*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**
8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 7
23680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 16800
*a**4*b**5*n**2*x**5*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7
+ 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 +
1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 8064*a**4*b**5*n*x**5*
(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 6
3273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 +
1026576*b**9*n + 362880*b**9) + 56*a**3*b**6*n**6*x**6*(a + b*x)**n/(b**9*n
9 + 45*b9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269
325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362
880*b**9) + 840*a**3*b**6*n**5*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8
+ 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723
680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 4760*a*
*3*b**6*n**4*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 +
9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 11
72700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 12600*a**3*b**6*n**3*x**6
*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 +
63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 +
1026576*b**9*n + 362880*b**9) + 15344*a**3*b**6*n**2*x**6*(a + b*x)**n/(b*
*9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 +
269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n +
362880*b**9) + 6720*a**3*b**6*n*x**6*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**
8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 7
23680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 8*a**
2*b**7*n**7*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9
450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 117
2700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 168*a**2*b**7*n**6*x**7*(a
+ b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 632
73*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 10
26576*b**9*n + 362880*b**9) - 1400*a**2*b**7*n**5*x**7*(a + b*x)**n/(b**9*n
9 + 45*b9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269
325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362
880*b**9) - 5880*a**2*b**7*n**4*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8
+ 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 72

3680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 12992*
a**2*b**7*n**3*x**7*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7
+ 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 +
1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) - 14112*a**2*b**7*n**2*x*
7(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6
+ 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2
 + 1026576*b**9*n + 362880*b**9) - 5760*a**2*b**7*n*x**7*(a + b*x)**n/(b**9
*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 2
69325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 3
62880*b**9) + a*b**8*n**8*x**8*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870
*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b
9*n3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 28*a*b**8*n*
*7*x**8*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*
n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9
*n**2 + 1026576*b**9*n + 362880*b**9) + 322*a*b**8*n**6*x**8*(a + b*x)**n/(
b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5
 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n
 + 362880*b**9) + 1960*a*b**8*n**5*x**8*(a + b*x)**n/(b**9*n**9 + 45*b**9*n
8 + 870*b9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 +
 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 676
9*a*b**8*n**4*x**8*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 +
 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1
172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 13132*a*b**8*n**3*x**8*(
a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63
273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1
026576*b**9*n + 362880*b**9) + 13068*a*b**8*n**2*x**8*(a + b*x)**n/(b**9*n*
*9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 2693
25*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 3628
80*b**9) + 5040*a*b**8*n*x**8*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*
b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b*
*9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + b**9*n**8*x**
9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 +
 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2
+ 1026576*b**9*n + 362880*b**9) + 36*b**9*n**7*x**9*(a + b*x)**n/(b**9*n**9
 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325
*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880
*b**9) + 546*b**9*n**6*x**9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b*
*9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9
*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 4536*b**9*n**5*
x**9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**
6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n*
*2 + 1026576*b**9*n + 362880*b**9) + 22449*b**9*n**4*x**9*(a + b*x)**n/(b**
9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 +
269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n +
362880*b**9) + 67284*b**9*n**3*x**9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8

```
+ 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723
680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 118124*
b**9*n**2*x**9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 945
0*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 11727
00*b**9*n**2 + 1026576*b**9*n + 362880*b**9) + 109584*b**9*n*x**9*(a + b*x)
**n/(b**9*n**9 + 45*b**9*n**8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9
*n**5 + 269325*b**9*n**4 + 723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b
**9*n + 362880*b**9) + 40320*b**9*x**9*(a + b*x)**n/(b**9*n**9 + 45*b**9*n*
*8 + 870*b**9*n**7 + 9450*b**9*n**6 + 63273*b**9*n**5 + 269325*b**9*n**4 +
723680*b**9*n**3 + 1172700*b**9*n**2 + 1026576*b**9*n + 362880*b**9), True)
) - b**(n + 1)*c**3*n*(a/b + x)**(n + 1)*lerchphi(1 + b*x/a, 1, n + 1)*gamm
a(n + 1)/(a*gamma(n + 2)) - b**(n + 1)*c**3*(a/b + x)**(n + 1)*lerchphi(1 +
 b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

Maxima [F]

$$\int \frac{(a+bx)^n (c+dx^3)^3}{x} \, dx = \int \frac{(dx^3+c)^3 (bx+a)^n}{x} \, dx$$

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^3*(b*x + a)^n/x, x)
```

Giac [F]

$$\int \frac{(a+bx)^n (c+dx^3)^3}{x} \, dx = \int \frac{(dx^3+c)^3 (bx+a)^n}{x} \, dx$$

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^3*(b*x + a)^n/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^n (c+dx^3)^3}{x} \, dx = \int \frac{(d\,x^3+c)^3\,(a+b\,x)^n}{x} \, dx$$

```
[In] int(((c + d*x^3)^3*(a + b*x)^n)/x,x)
[Out] int(((c + d*x^3)^3*(a + b*x)^n)/x, x)
```

3.186 $\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 324

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} + \frac{a(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{5/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{a(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{5/3}\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)} + \frac{a(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{5/3}\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)(1+n)}$$

```
[Out] e^2*(f*x+e)^(1+n)/b/f^3/(1+n)-2*e*(f*x+e)^(2+n)/b/f^3/(2+n)+(f*x+e)^(3+n)/b
/f^3/(3+n)+1/3*a*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^
(1/3)*e-a^(1/3)*f))/b^(5/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*a*(f*x+e)^(1+n)
*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f))
/b^(5/3)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f)/(1+n)+1/3*a*(f*x+e)^(1+n)*hyperge
om([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f))/b^(5/3)
/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f)/(1+n)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6857, 70}

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \frac{a(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)} + \frac{a(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)} + \frac{a(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)} + \frac{e^2(e+fx)^{n+1}}{bf^3(n+1)} - \frac{2e(e+fx)^{n+2}}{bf^3(n+2)} + \frac{(e+fx)^{n+3}}{bf^3(n+3)}$$

```
[In] Int[(x^5*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] (e^2*(e + f*x)^(1 + n))/(b*f^3*(1 + n)) - (2*e*(e + f*x)^(2 + n))/(b*f^3*(2
 + n)) + (e + f*x)^(3 + n)/(b*f^3*(3 + n)) + (a*(e + f*x)^(1 + n)*Hypergeom
etric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*
b^(5/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (a*(e + f*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)
*f)])/(3*b^(5/3)*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)*(1 + n)) + (a*(e + f*x)
^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e
- (-1)^(2/3)*a^(1/3)*f)])/(3*b^(5/3)*(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)*(1
+ n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^2(e+fx)^n}{bf^2} - \frac{2e(e+fx)^{1+n}}{bf^2} + \frac{(e+fx)^{2+n}}{bf^2} - \frac{ax^2(e+fx)^n}{b(a+bx^3)}\right) dx$$

$$= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} - \frac{a\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx}{b}$$

$$= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} - \frac{a\int \left(\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right) dx}{b}$$

$$= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} - \frac{a\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{5/3}} - \frac{a\int \frac{(e+fx)^n}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{5/3}} - \frac{a\int \frac{(e+fx)^n}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{5/3}}$$

$$= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} + \frac{a(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{5/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{a(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{5/3}\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)} + \frac{a(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{5/3}\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)(1+n)}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.88

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}\left(\frac{3b^{2/3}e^2}{f^3(1+n)} - \frac{6b^{2/3}e(e+fx)}{f^3(2+n)} + \frac{3b^{2/3}(e+fx)^2}{f^3(3+n)} + \frac{a\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{a\,\text{Hypergeometric}\ldots}{\ldots}\right.}{3b^{5/3}}$$

[In] Integrate[(x^5*(e + f*x)^n)/(a + b*x^3),x]

[Out] ((e + f*x)^(1 + n)*((3*b^(2/3)*e^2)/(f^3*(1 + n)) - (6*b^(2/3)*e*(e + f*x))/(f^3*(2 + n)) + (3*b^(2/3)*(e + f*x)^2)/(f^3*(3 + n)) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/((b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/((b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)*(1 + n)) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/((b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)*(1 + n))))/(3*b^(5/3))

Maple [F]

$$\int \frac{x^{5} (f x +e)^{n}}{b \,x^{3}+a}dx$$

[In] int(x^5*(f*x+e)^n/(b*x^3+a),x)

[Out] int(x^5*(f*x+e)^n/(b*x^3+a),x)

Fricas [F]

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx=\int \frac{(fx+e)^n x^5}{bx^3+a}\,dx$$

[In] integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")

[Out] integral((f*x + e)^n*x^5/(b*x^3 + a), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx=\text{Timed out}$$

[In] integrate(x**5*(f*x+e)**n/(b*x**3+a),x)

[Out] Timed out

Maxima [F]

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^5}{bx^3+a}\,dx$$

```
[In] integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^5/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^5}{bx^3+a}\,dx$$

```
[In] integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^5/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \int \frac{x^5\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

```
[In] int((x^5*(e + f*x)^n)/(a + b*x^3),x)
[Out] int((x^5*(e + f*x)^n)/(a + b*x^3), x)
```

3.187 $\int \frac{x^4(e+fx)^n}{a+bx^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 332

$$\begin{aligned}
&\int \frac{x^4(e+fx)^n}{a+bx^3} \, dx \\
&= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)} + \frac{(e+fx)^{2+n}}{bf^2(2+n)} \\
&\quad - \frac{a^{2/3}(e+fx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} \\
&\quad + \frac{\sqrt[3]{-1}a^{2/3}(e+fx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} \\
&\quad + \frac{(-1)^{2/3}a^{2/3}(e+fx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b^{4/3}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}
\end{aligned}$$

```
[Out] -e*(f*x+e)^(1+n)/b/f^2/(1+n)+(f*x+e)^(2+n)/b/f^2/(2+n)-1/3*a^(2/3)*(f*x+e)^
(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1/3)*f))/b^(4/
3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(-1)^(1/3)*a^(2/3)*(f*x+e)^(1+n)*hyperge
om([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*
f))/b^(4/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(-1)^(2/3)*a^(2/3)*(
f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e)/((-1)^(1/3
)*b^(1/3)*e+a^(1/3)*f))/b^(4/3)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f)/(1+n)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6857, 70}

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx$$
$$= -\frac{a^{2/3}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}$$
$$+\frac{\sqrt[3]{-1}a^{2/3}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}$$
$$+\frac{(-1)^{2/3}a^{2/3}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}$$
$$-\frac{e(e+fx)^{n+1}}{bf^2(n+1)}+\frac{(e+fx)^{n+2}}{bf^2(n+2)}$$

```
[In] Int[(x^4*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] -((e*(e + f*x)^(1 + n))/(b*f^2*(1 + n))) + (e + f*x)^(2 + n)/(b*f^2*(2 + n)
) - (a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*
(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b^(4/3)*(b^(1/3)*e - a^(1/3)*f)*(1
+ n)) + ((-1)^(1/3)*a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(
3*b^(4/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*a^(2/3)
*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(
e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b^(4/3)*((-1)^(1/3)*b^(1/
3)*e + a^(1/3)*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

`[n, 0]`

Rubi steps

$$\text{integral} = \int \left(-\frac{e(e+fx)^n}{bf} + \frac{(e+fx)^{1+n}}{bf} - \frac{ax(e+fx)^n}{b(a+bx^3)}\right) dx$$

$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)} + \frac{(e+fx)^{2+n}}{bf^2(2+n)} - \frac{a\int \frac{x(e+fx)^n}{a+bx^3}\,dx}{b}$$

$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)} + \frac{(e+fx)^{2+n}}{bf^2(2+n)} - \frac{a\int \left(-\frac{(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx}{b}$$

$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)} + \frac{(e+fx)^{2+n}}{bf^2(2+n)} + \frac{a^{2/3}\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}a^{2/3}\right)\int \frac{(e+fx)^n}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b^{4/3}} + \frac{\left((-1)^{2/3}a^{2/3}\right)\int \frac{(e+fx)^n}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3b^{4/3}}$$

$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)} + \frac{(e+fx)^{2+n}}{bf^2(2+n)} - \frac{a^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{\sqrt[3]{-1}a^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{(-1)^{2/3}a^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b^{4/3}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.88

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}\left(-\frac{3\sqrt[3]{b}e}{f^2(1+n)}+\frac{3\sqrt[3]{b}(e+fx)}{f^2(2+n)}-\frac{a^{2/3}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}+\frac{\sqrt[3]{-1}a^{2/3}\,\text{Hypergeometric2F}\cdots}{\left((-1)^{2/3}\sqrt[3]{\cdots}\right.}\cdots\right.}{3b^{4/3}}$$

```
[In] Integrate[(x^4*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*((-3*b^(1/3)*e)/(f^2*(1 + n)) + (3*b^(1/3)*(e + f*x))/(f
^2*(2 + n)) - (a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x
))/(b^(1/3)*e - a^(1/3)*f)])/((b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(1/3
)*a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))
/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(
1 + n)) + ((-1)^(2/3)*a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3
)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(((-1)^(1/3)*b^(1
/3)*e + a^(1/3)*f)*(1 + n))))/(3*b^(4/3))
```

Maple [F]

$$\int \frac{x^4(fx+e)^n}{b\,x^3+a}dx$$

```
[In] int(x^4*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x^4*(f*x+e)^n/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^4}{bx^3+a}\,dx$$

```
[In] integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^4/(b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(f*x+e)**n/(b*x**3+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^4}{bx^3+a}\,dx$$

```
[In] integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^4/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^4}{bx^3+a}\,dx$$

```
[In] integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^4/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \int \frac{x^4\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

```
[In] int((x^4*(e + f*x)^n)/(a + b*x^3),x)
[Out] int((x^4*(e + f*x)^n)/(a + b*x^3), x)
```

3.188 $\int \frac{x^3(e+fx)^n}{a+bx^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 293

$$\int \frac{x^3(e+fx)^n}{a+bx^3} dx = \frac{(e+fx)^{1+n}}{bf(1+n)} + \frac{\sqrt[3]{a}(e+fx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{\sqrt[3]{a}(e+fx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{\sqrt[3]{a}(e+fx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}$$

```
[Out] (f*x+e)^(1+n)/b/f/(1+n)+1/3*a^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],
b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1/3)*f))/b/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*a^
(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-
1)^(2/3)*b^(1/3)*e-a^(1/3)*f))/b/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3
*a^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e)/
((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/b/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f)/(1+n)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6857, 70}

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx$$
$$=\frac{\sqrt[3]{a}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}$$
$$+\frac{\sqrt[3]{a}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}$$
$$-\frac{\sqrt[3]{a}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}+\frac{(e+fx)^{n+1}}{bf(n+1)}$$

```
[In] Int[(x^3*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] (e + f*x)^(1 + n)/(b*f*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b*(b^
(1/3)*e - a^(1/3)*f)*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F
1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a
^(1/3)*f)])/(3*b*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - (a^(1/3)*(e
+ f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e +
f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b*((-1)^(1/3)*b^(1/3)*e + a^(
1/3)*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(e+fx)^n}{b} - \frac{a(e+fx)^n}{b(a+bx^3)}\right) dx$$

$$= \frac{(e+fx)^{1+n}}{bf(1+n)} - \frac{a\int \frac{(e+fx)^n}{a+bx^3}\,dx}{b}$$

$$= \frac{(e+fx)^{1+n}}{bf(1+n)} - \frac{a\int \left(-\frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx}{b}$$

$$= \frac{(e+fx)^{1+n}}{bf(1+n)} + \frac{\sqrt[3]{a}\int \frac{(e+fx)^n}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{(e+fx)^n}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{(e+fx)^n}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b}$$

$$= \frac{(e+fx)^{1+n}}{bf(1+n)} + \frac{\sqrt[3]{a}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{\sqrt[3]{a}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{\sqrt[3]{a}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.82

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}\left(\frac{3}{f} + \frac{\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f} + \frac{\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right.}{3b(1+n)}$$

```
[In] Integrate[(x^3*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(3/f + (a^(1/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1
/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f) + (a^(1/3)
*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2
/3)*b^(1/3)*e - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) - (a^(1/3)*
Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/
3)*b^(1/3)*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*b*(1 +
n))
```

Maple [F]

$$\int \frac{x^{3} \left(f x +e \right)^{n}}{b \,x^{3}+a}d x$$

```
[In] int(x^3*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x^3*(f*x+e)^n/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^3}{bx^3+a}\,dx$$

```
[In] integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^3/(b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^3}{bx^3+a}\,dx$$

```
[In] integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^3/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^3}{bx^3+a}\,dx$$

```
[In] integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^3/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \int \frac{x^3\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

```
[In] int((x^3*(e + f*x)^n)/(a + b*x^3),x)
```

```
[Out] int((x^3*(e + f*x)^n)/(a + b*x^3), x)
```

3.189 $\int \frac{x^2(e+fx)^n}{a+bx^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 253

$$\int \frac{x^2(e+fx)^n}{a+bx^3} dx = -\frac{(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{2/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{2/3}\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)} - \frac{(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{2/3}\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)(1+n)}$$

```
[Out] -1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1
/3)*f))/b^(2/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*(f*x+e)^(1+n)*hypergeom([1,
 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f))/b^(2/3)/(b^(1
/3)*e+(-1)^(1/3)*a^(1/3)*f)/(1+n)-1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n
],b^(1/3)*(f*x+e)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f))/b^(2/3)/(b^(1/3)*e-(-1)
^(2/3)*a^(1/3)*f)/(1+n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6857, 70}

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = -\frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)} - \frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)} - \frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)}$$

```
[In] Int[(x^2*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] -1/3*((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*
x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(2/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) - (
(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b
^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e + (-1)^(1/3)*a^(1/
3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(
1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e
- (-1)^(2/3)*a^(1/3)*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)} \right) dx$$

$$= \frac{\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}} + \frac{\int \frac{(e+fx)^n}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}} + \frac{\int \frac{(e+fx)^n}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{2/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{2/3}\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)} - \frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{2/3}\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)(1+n)}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.84

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f} - \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f} - \text{Hyperg}\right.}{3b^{2/3}(1+n)}$$

```
[In] Integrate[(x^2*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x)
)/(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f)) - Hypergeometric2F1[1,
1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)]/(b^(1
/3)*e + (-1)^(1/3)*a^(1/3)*f) - Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)
*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)]/(b^(1/3)*e - (-1)^(2/3)*a^(
1/3)*f)))/(3*b^(2/3)*(1 + n))
```

Maple [F]

$$\int \frac{x^2 (fx+e)^n}{b\,x^3+a}dx$$

```
[In] int(x^2*(f*x+e)^n/(b*x^3+a),x)
[Out] int(x^2*(f*x+e)^n/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^2}{bx^3+a}\,dx$$

```
[In] integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x^2/(b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(f*x+e)**n/(b*x**3+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^2}{bx^3+a}\,dx$$

```
[In] integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^2/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^2}{bx^3+a}\,dx$$

```
[In] integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^2/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \int \frac{x^2\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

```
[In] int((x^2*(e + f*x)^n)/(a + b*x^3),x)
[Out] int((x^2*(e + f*x)^n)/(a + b*x^3), x)
```

3.190 $\int \frac{x(e+fx)^n}{a+bx^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 288

$$\int \frac{x(e+fx)^n}{a+bx^3} \, dx = \frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{\sqrt[3]{-1}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{(-1)^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}$$

```
[Out] 1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1/
3)*f))/a^(1/3)/b^(1/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*(-1)^(1/3)*(f*x+e)^(
1+n)*hypergeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/3)*b^(1/3
)*e-a^(1/3)*f))/a^(1/3)/b^(1/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*
(-1)^(2/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e
)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/a^(1/3)/b^(1/3)/((-1)^(1/3)*b^(1/3)*e+a
^(1/3)*f)/(1+n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6857, 70}

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx$$
$$= \frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}$$
$$-\frac{\sqrt[3]{-1}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}$$
$$-\frac{(-1)^{2/3}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}$$

```
[In] Int[(x*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(
b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)
) - ((-1)^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^
(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^
(1/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - ((-1)^(2/3)*(e + f*x)^(
1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((
-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*((-1)^(1/3)*b^(1/3)*e
 + a^(1/3)*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx$$

$$= -\frac{\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{(e+fx)^n}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{(-1)^{2/3}\int \frac{(e+fx)^n}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$= \frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{\sqrt[3]{-1}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{(-1)^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.82

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}\left(\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f} - \frac{\sqrt[3]{-1}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f} - \right.}{3\sqrt[3]{a}\sqrt[3]{b}(1+n)}$$

```
[In] Integrate[(x*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/
(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f) - ((-1)^(1/3)*Hypergeometr
ic2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e
 - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) - ((-1)^(2/3)*Hypergeome
tric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)
*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*a^(1/3)*b^(1/3)*(
1 + n))
```

Maple [F]

$$\int \frac{x(fx+e)^n}{b\,x^3+a}dx$$

[In] int(x*(f*x+e)^n/(b*x^3+a),x)

[Out] int(x*(f*x+e)^n/(b*x^3+a),x)

Fricas [F]

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x}{bx^3+a}\,dx$$

[In] integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")

[Out] integral((f*x + e)^n*x/(b*x^3 + a), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

[In] integrate(x*(f*x+e)**n/(b*x**3+a),x)

[Out] Timed out

Maxima [F]

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x}{bx^3+a}\,dx$$

[In] integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")

[Out] integrate((f*x + e)^n*x/(b*x^3 + a), x)

Giac [F]

$$\int \frac{x(e+fx)^n}{a+bx^3} dx = \int \frac{(fx+e)^n x}{bx^3+a} dx$$

```
[In] integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(e+fx)^n}{a+bx^3} dx = \int \frac{x (e+f x)^n}{b x^3+a} dx$$

```
[In] int((x*(e + f*x)^n)/(a + b*x^3),x)
[Out] int((x*(e + f*x)^n)/(a + b*x^3), x)
```

3.191 $\int \frac{(e+fx)^n}{a+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 263

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = -\frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}$$

```
[Out] -1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1
/3)*f))/a^(2/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*(f*x+e)^(1+n)*hypergeom([1,
 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f))/a^
(2/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(f*x+e)^(1+n)*hypergeom([1
, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/a
^(2/3)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f)/(1+n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6857, 70}

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = -\frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)} - \frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)} + \frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}$$

```
[In] Int[(e + f*x)^n/(a + b*x^3),x]
```

```
[Out] -1/3*((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*
x))/(b^(1/3)*e - a^(1/3)*f)])/(a^(2/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) - (
(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e
 + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(2/3)*((-1)^(2/3)*b^(1/3
)*e - a^(1/3)*f)*(1 + n)) + ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/
(3*a^(2/3)*((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx$$

$$= -\frac{\int \frac{(e+fx)^n}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{(e+fx)^n}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{(e+fx)^n}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.84

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f} - \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f} + \frac{\text{Hyperg}}{}\right.}{3a^{2/3}(1+n)}$$

```
[In] Integrate[(e + f*x)^n/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x)
)/(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f)) - Hypergeometric2F1[1,
1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3
)*f)]/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) + Hypergeometric2F1[1, 1 + n, 2 +
n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)]/((-1)
^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*a^(2/3)*(1 + n))
```

Maple [F]

$$\int \frac{(fx+e)^n}{b\,x^3+a}dx$$

```
[In] int((f*x+e)^n/(b*x^3+a),x)
[Out] int((f*x+e)^n/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n}{bx^3+a}\,dx$$

```
[In] integrate((f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n/(b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**n/(b*x**3+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n}{bx^3+a}\,dx$$

```
[In] integrate((f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx=\int \frac{(fx+e)^n}{bx^3+a}\,dx$$

```
[In] integrate((f*x+e)^n/(b*x^3+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx=\int \frac{(e+f\,x)^n}{b\,x^3+a}\,dx$$

```
[In] int((e + f*x)^n/(a + b*x^3),x)
[Out] int((e + f*x)^n/(a + b*x^3), x)
```

3.192 $\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 300

$$\begin{aligned}\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx &= \frac{\sqrt[3]{b}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}\\
&+\frac{\sqrt[3]{b}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3a\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)}\\
&+\frac{\sqrt[3]{b}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3a\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)(1+n)}\\
&-\frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{fx}{e}\right)}{ae(1+n)}\end{aligned}$$

```
[Out] 1/3*b^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)
*e-a^(1/3)*f))/a/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*b^(1/3)*(f*x+e)^(1+n)*hype
rgeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f))/a/(b
^(1/3)*e+(-1)^(1/3)*a^(1/3)*f)/(1+n)+1/3*b^(1/3)*(f*x+e)^(1+n)*hypergeom([1
, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f))/a/(b^(1/3)*e
-(-1)^(2/3)*a^(1/3)*f)/(1+n)-(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],1+f*x/e
)/a/e/(1+n)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6857, 67, 70}

$$\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx$$
$$= \frac{\sqrt[3]{b}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}$$
$$+\frac{\sqrt[3]{b}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}$$
$$+\frac{\sqrt[3]{b}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)}$$
$$-\frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{fx}{e}+1\right)}{ae(n+1)}$$

```
[In] Int[(e + f*x)^n/(x*(a + b*x^3)),x]
```

```
[Out] (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e +
 f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (b
^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f
*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*a*(b^(1/3)*e + (-1)^(1/3)*a^(1
/3)*f)*(1 + n)) + (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
+ n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*a*(b^(1/3)
*e - (-1)^(2/3)*a^(1/3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{(e+fx)^n}{ax}-\frac{bx^2(e+fx)^n}{a\left(a+bx^3\right)}\right) dx \\
&= \frac{\int \frac{(e+fx)^n}{x}\,dx}{a}-\frac{b\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx}{a} \\
&= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)} \\
&\quad -\frac{b\int \left(\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{(e+fx)^n}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{(e+fx)^n}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right) dx}{a} \\
&= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)}-\frac{\sqrt[3]{b}\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a} \\
&\quad -\frac{\sqrt[3]{b}\int \frac{(e+fx)^n}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}-\frac{\sqrt[3]{b}\int \frac{(e+fx)^n}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a} \\
&= \frac{\sqrt[3]{b}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} \\
&\quad +\frac{\sqrt[3]{b}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3a\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)} \\
&\quad +\frac{\sqrt[3]{b}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3a\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)(1+n)} \\
&\quad -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.81

$$\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx = \frac{(e+fx)^{1+n}\left(\frac{\sqrt[3]{b}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f}+\frac{\sqrt[3]{b}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}+\right.}{3a(1+n)}$$

```
[In] Integrate[(e + f*x)^n/(x*(a + b*x^3)),x]
```

```
[Out] ((e + f*x)^(1 + n)*((b^(1/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e
 + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f) + (b^(1/3)*Hyper
geometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a
^(1/3)*f)])/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f) + (b^(1/3)*Hypergeometric2F1
[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/
(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f) - (3*Hypergeometric2F1[1, 1 + n, 2 + n,
1 + (f*x)/e])/e))/(3*a*(1 + n))
```

Maple [F]

$$\int \frac{(fx+e)^n}{x(b\,x^3+a)}dx$$

```
[In] int((f*x+e)^n/x/(b*x^3+a),x)
```

```
[Out] int((f*x+e)^n/x/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx = \int \frac{(fx+e)^n}{(bx^3+a)x}\,dx$$

```
[In] integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*x^4 + a*x), x)
```

Sympy [F]

$$\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx = \int \frac{(e+fx)^n}{x(a+bx^3)}\,dx$$

```
[In] integrate((f*x+e)**n/x/(b*x**3+a),x)
[Out] Integral((e + f*x)**n/(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx = \int \frac{(fx+e)^n}{(bx^3+a)x}\,dx$$

```
[In] integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x), x)
```

Giac [F]

$$\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx = \int \frac{(fx+e)^n}{(bx^3+a)x}\,dx$$

```
[In] integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx = \int \frac{(e+f\,x)^n}{x\,(b\,x^3+a)}\,dx$$

```
[In] int((e + f*x)^n/(x*(a + b*x^3)),x)
[Out] int((e + f*x)^n/(x*(a + b*x^3)), x)
```

3.193 $\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 326

$$\begin{aligned}\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx &= -\frac{b^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}\\
&\quad+\frac{\sqrt[3]{-1}b^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}\\
&\quad+\frac{(-1)^{2/3}b^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{4/3}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}\\
&\quad+\frac{f(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{fx}{e}\right)}{ae^2(1+n)}\end{aligned}$$

```
[Out] -1/3*b^(2/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3
)*e-a^(1/3)*f))/a^(4/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(-1)^(1/3)*b^(2/3)*
(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/
3)*b^(1/3)*e-a^(1/3)*f))/a^(4/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3
*(-1)^(2/3)*b^(2/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/
3)*(f*x+e)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/a^(4/3)/((-1)^(1/3)*b^(1/3)*e+
a^(1/3)*f)/(1+n)+f*(f*x+e)^(1+n)*hypergeom([2, 1+n],[2+n],1+f*x/e)/a/e^2/(1
+n)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6857, 67, 70}

$$\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx$$
$$= -\frac{b^{2/3}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}$$
$$+\frac{\sqrt[3]{-1}b^{2/3}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}$$
$$+\frac{(-1)^{2/3}b^{2/3}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}$$
$$+\frac{f(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{fx}{e}+1\right)}{ae^2(n+1)}$$

```
[In] Int[(e + f*x)^n/(x^2*(a + b*x^3)),x]

[Out] -1/3*(b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)
*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(a^(4/3)*(b^(1/3)*e - a^(1/3)*f)*(1 +
 n)) + ((-1)^(1/3)*b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
+ n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3
*a^(4/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*b^(2/3)*
(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e
 + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(4/3)*((-1)^(1/3)*b^(1/3
)*e + a^(1/3)*f)*(1 + n)) + (f*(e + f*x)^(1 + n)*Hypergeometric2F1[2, 1 + n
, 2 + n, 1 + (f*x)/e])/(a*e^2*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
```

```
+ 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(e+fx)^n}{ax^2} - \frac{bx(e+fx)^n}{a\,(a+bx^3)}\right) dx$$

$$= \frac{\int \frac{(e+fx)^n}{x^2}\,dx}{a} - \frac{b\int \frac{x(e+fx)^n}{a+bx^3}\,dx}{a}$$

$$= \frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)} - \frac{b\int \left(-\frac{(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx}{a}$$

$$= \frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)} + \frac{b^{2/3}\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}} - \frac{\left(\sqrt[3]{-1}b^{2/3}\right)\int \frac{(e+fx)^n}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{4/3}} + \frac{\left((-1)^{2/3}b^{2/3}\right)\int \frac{(e+fx)^n}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{4/3}}$$

$$= -\frac{b^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{\sqrt[3]{-1}b^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{(-1)^{2/3}b^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{4/3}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)} + \frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.84

$$\int \frac{(e+fx)^n}{x^2\left(a+bx^3\right)}\,dx$$
$$= \frac{(e+fx)^{1+n}\left(-\frac{b^{2/3}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f}+\frac{\sqrt[3]{-1}b^{2/3}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+\cdots}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{\cdots}}\right.}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\cdots}{3a^{4/3}(1\cdots}$$

```
[In] Integrate[(e + f*x)^n/(x^2*(a + b*x^3)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*
(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f)) + ((-1)^(1/3)
*b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/
((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) + (
(-1)^(2/3)*b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(
e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1
/3)*f) + (3*a^(1/3)*f*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/e^2)
)/(3*a^(4/3)*(1 + n))
```

Maple [F]

$$\int \frac{(fx+e)^n}{x^2\left(b\,x^3+a\right)}dx$$

```
[In] int((f*x+e)^n/x^2/(b*x^3+a),x)
```

```
[Out] int((f*x+e)^n/x^2/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{(e+fx)^n}{x^2\left(a+bx^3\right)}\,dx=\int \frac{(fx+e)^n}{(bx^3+a)x^2}\,dx$$

```
[In] integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*x^5 + a*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x^2\,(a+bx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**n/x**2/(b*x**3+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^n}{x^2\,(a+bx^3)}\,dx = \int \frac{(fx+e)^n}{(bx^3+a)x^2}\,dx$$

```
[In] integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x^2), x)
```

Giac [F]

$$\int \frac{(e+fx)^n}{x^2\,(a+bx^3)}\,dx = \int \frac{(fx+e)^n}{(bx^3+a)x^2}\,dx$$

```
[In] integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x^2\,(a+bx^3)}\,dx = \int \frac{(e+f\,x)^n}{x^2\,(b\,x^3+a)}\,dx$$

```
[In] int((e + f*x)^n/(x^2*(a + b*x^3)),x)
[Out] int((e + f*x)^n/(x^2*(a + b*x^3)), x)
```

3.194 $\int \frac{x^2(c+dx)^{1+n}}{a+bx^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 253

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3} dx = -\frac{(c+dx)^{2+n} \text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)(2+n)} - \frac{(c+dx)^{2+n} \text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d\right)(2+n)} - \frac{(c+dx)^{2+n} \text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d\right)(2+n)}$$

```
[Out] -1/3*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(1
/3)*d))/b^(2/3)/(b^(1/3)*c-a^(1/3)*d)/(2+n)-1/3*(d*x+c)^(2+n)*hypergeom([1,
 2+n],[3+n],b^(1/3)*(d*x+c)/(b^(1/3)*c+(-1)^(1/3)*a^(1/3)*d))/b^(2/3)/(b^(1
/3)*c+(-1)^(1/3)*a^(1/3)*d)/(2+n)-1/3*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n
],b^(1/3)*(d*x+c)/(b^(1/3)*c-(-1)^(2/3)*a^(1/3)*d))/b^(2/3)/(b^(1/3)*c-(-1)
^(2/3)*a^(1/3)*d)/(2+n)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6857, 70}

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = -\frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)} - \frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c\right)} - \frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d\right)}$$

```
[In] Int[(x^2*(c + d*x)^(1 + n))/(a + b*x^3),x]
```

```
[Out] -1/3*((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*
x))/(b^(1/3)*c - a^(1/3)*d)])/(b^(2/3)*(b^(1/3)*c - a^(1/3)*d)*(2 + n)) - (
(c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x))/(b
^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c + (-1)^(1/3)*a^(1/
3)*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(
1/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c
- (-1)^(2/3)*a^(1/3)*d)*(2 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(c+dx)^{1+n}}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(c+dx)^{1+n}}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(c+dx)^{1+n}}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)} \right) dx$$

$$= \frac{\int \frac{(c+dx)^{1+n}}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}} + \frac{\int \frac{(c+dx)^{1+n}}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}} + \frac{\int \frac{(c+dx)^{1+n}}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$= -\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)(2+n)} - \frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d\right)(2+n)} - \frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d\right)(2+n)}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.84

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \frac{(c+dx)^{2+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d} - \frac{\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d} - \text{Hype}\right.}{3b^{2/3}(2+n)}$$

```
[In] Integrate[(x^2*(c + d*x)^(1 + n))/(a + b*x^3),x]
```

```
[Out] ((c + d*x)^(2 + n)*(-(Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x)
)/(b^(1/3)*c - a^(1/3)*d)]/(b^(1/3)*c - a^(1/3)*d)) - Hypergeometric2F1[1,
2 + n, 3 + n, (b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]/(b^(1
/3)*c + (-1)^(1/3)*a^(1/3)*d) - Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)
*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)]/(b^(1/3)*c - (-1)^(2/3)*a^(
1/3)*d)))/(3*b^(2/3)*(2 + n))
```

Maple [F]

$$\int \frac{x^{2} \left(d x +c \right)^{1+n}}{b \,x^{3}+a}d x$$

```
[In] int(x^2*(d*x+c)^(1+n)/(b*x^3+a),x)
[Out] int(x^2*(d*x+c)^(1+n)/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \int \frac{(dx+c)^{n+1}x^2}{bx^3+a}\,dx$$

```
[In] integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="fricas")
[Out] integral((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(d*x+c)**(1+n)/(b*x**3+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \int \frac{(dx+c)^{n+1}x^2}{bx^3+a}\,dx$$

```
[In] integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \int \frac{(dx+c)^{n+1}x^2}{bx^3+a}\,dx$$

```
[In] integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="giac")
[Out] integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \int \frac{x^2\,(c+d\,x)^{n+1}}{b\,x^3+a}\,dx$$

```
[In] int((x^2*(c + d*x)^(n + 1))/(a + b*x^3),x)
[Out] int((x^2*(c + d*x)^(n + 1))/(a + b*x^3), x)
```

3.195 $\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 211

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(1+m)} + \frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(1+m)} + \frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{(-1)^{2/3}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(1+m)}$$

```
[Out] 1/3*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-b^(1/3)*x/a^(1/3),-f*x/e)/a/(1
+m)/((1+f*x/e)^n)+1/3*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,(-1)^(1/3)*b^
(1/3)*x/a^(1/3),-f*x/e)/a/(1+m)/((1+f*x/e)^n)+1/3*x^(1+m)*(f*x+e)^n*AppellF
1(1+m,-n,1,2+m,-f*x/e,-(-1)^(2/3)*b^(1/3)*x/a^(1/3))/a/(1+m)/((1+f*x/e)^n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used

= {6857, 140, 138}

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)} + \frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)} + \frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},-\frac{(-1)^{2/3}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}$$

```
[In] Int[(x^m*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] (x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((b^(1/3)
*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x)/e)^n) + (x^(1 + m)*(e + f*x)^n*Appel
lF1[1 + m, -n, 1, 2 + m, -((f*x)/e), ((-1)^(1/3)*b^(1/3)*x)/a^(1/3)])/(3*a*
(1 + m)*(1 + (f*x)/e)^n) + (x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2
+ m, -((f*x)/e), -(((-1)^(2/3)*b^(1/3)*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x
)/e)^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx$$

$$= -\frac{\int \frac{x^m(e+fx)^n}{-\sqrt[3]{a}-\sqrt[3]{b}x} dx}{3a^{2/3}} - \frac{\int \frac{x^m(e+fx)^n}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x} dx}{3a^{2/3}} - \frac{\int \frac{x^m(e+fx)^n}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x} dx}{3a^{2/3}}$$

$$= -\frac{\left((e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{-\sqrt[3]{a}-\sqrt[3]{b}x} dx}{3a^{2/3}} - \frac{\left((e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x} dx}{3a^{2/3}} - \frac{\left((e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x} dx}{3a^{2/3}}$$

$$= \frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(1+m)} + \frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(1+m)} + \frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{(-1)^{2/3}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(1+m)}$$

Mathematica [F]

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \int \frac{x^m(e+fx)^n}{a+bx^3}\,dx$$

```
[In] Integrate[(x^m*(e + f*x)^n)/(a + b*x^3),x]
[Out] Integrate[(x^m*(e + f*x)^n)/(a + b*x^3), x]
```

Maple [F]

$$\int \frac{x^m (fx+e)^n}{b\,x^3+a}dx$$

[In] int(x^m*(f*x+e)^n/(b*x^3+a),x)

[Out] int(x^m*(f*x+e)^n/(b*x^3+a),x)

Fricas [F]

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^m}{bx^3+a}\,dx$$

[In] integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")

[Out] integral((f*x + e)^n*x^m/(b*x^3 + a), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

[In] integrate(x**m*(f*x+e)**n/(b*x**3+a),x)

[Out] Timed out

Maxima [F]

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^m}{bx^3+a}\,dx$$

[In] integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")

[Out] integrate((f*x + e)^n*x^m/(b*x^3 + a), x)

Giac [F]

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^m}{bx^3+a}\,dx$$

```
[In] integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^m/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \int \frac{x^m\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

```
[In] int((x^m*(e + f*x)^n)/(a + b*x^3),x)
```

```
[Out] int((x^m*(e + f*x)^n)/(a + b*x^3), x)
```

3.196 $\int \frac{\sqrt{c+dx^3}}{a+bx} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 1480

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx = \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$-\frac{\sqrt[6]{c}\sqrt{b\sqrt[3]{c}-a\sqrt[3]{d}}\sqrt{b^2c^{2/3}+ab\sqrt[3]{c}\sqrt[3]{d}+a^2d^{2/3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}\left(1-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}\sqrt{b^2c^{2/}}}{\sqrt[4]{3}\sqrt{b}\sqrt[6]{c}\sqrt{}}\right.}{b^{5/2}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}a\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{b^2\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}a\left(\left(1-\sqrt{3}\right)b\sqrt[3]{c}+a\sqrt[3]{d}\right)\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{\sqrt[4]{3}b^3\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{2\sqrt{2+\sqrt{3}}(b^3c-a^3d)\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-\right.}{\sqrt[4]{3}b^3\left(\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{c}(b^3c-a^3d)\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}\left(1-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)^2}{\left(\left(1-\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)^2},\arcsi\right.}{b^2\left(2b^2c^{2/3}+2ab\sqrt[3]{c}\sqrt[3]{d}-a^2d^{2/3}\right)\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 2/3*(d*x^3+c)^(1/2)/b-2*a*d^(1/3)*(d*x^3+c)^(1/2)/b^2/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2)))+3^(1/4)*a*c^(1/3)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)
```

```
*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2
*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^2/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*
x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/3*a*d^(1/3)*(c^(1/3)+d^(1/3)*
x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*(a*d^(1/3)+b*c^(1/3)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2)
)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)*3^(3/4)/b^3/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-2/3*(-a^3*d+b^3*c)*(c^(1/3)+d^(1/3)*x)*Elli
pticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^3/(-a*d^(1/3)+b*c^(1/3
)*(1+3^(1/2)))/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))^2)^(1/2)-c^(1/6)*(c^(1/3)+d^(1/3)*x)*arctanh(1/3*(b^2*c^(2
/3)+a*b*c^(1/3)*d^(1/3)+a^2*d^(2/3))^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(1-(d^
(1/3)*x+c^(1/3)*(1-3^(1/2)))^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(
3/4)/c^(1/6)/b^(1/2)/(b*c^(1/3)-a*d^(1/3))^(1/2)/(7-4*3^(1/2)+(d^(1/3)*x+c^
(1/3)*(1-3^(1/2)))^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2))*(b*c^(1/3)-a
*d^(1/3))^(1/2)*(b^2*c^(2/3)+a*b*c^(1/3)*d^(1/3)+a^2*d^(2/3))^(1/2)*(c^(2/3
)*(1-d^(1/3)*x/c^(1/3)+d^(2/3)*x^2/c^(2/3))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
^2)^(1/2)/b^(5/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))^2)^(1/2)-4*3^(1/4)*c^(1/3)*(-a^3*d+b^3*c)*(c^(1/3)+d^(1
/3)*x)*EllipticPi((-d^(1/3)*x-c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2))),(-a*d^(1/3)+b*c^(1/3)*(1+3^(1/2)))^2/(-a*d^(1/3)+b*c^(1/3)*(1-3^(1/
2)))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*(c^(2/3)*(1-d^(1/3)*x/c^(1/
3)+d^(2/3)*x^2/c^(2/3))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^2/(2*b^2
*c^(2/3)+2*a*b*c^(1/3)*d^(1/3)-a^2*d^(2/3))/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/
3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 2.03 (sec) , antiderivative size = 1480, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules

used = {2173, 267, 2161, 224, 2167, 2138, 551, 585, 95, 214, 1892, 1891}

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$$

$$= \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)a}{b^2\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}a+\left(1-\sqrt{3}\right)b\sqrt[3]{c}\right)\sqrt[3]{d}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+(1-\cdots}{\sqrt[3]{d}x+(1+\cdots}\right.\right.}{\sqrt[4]{3}b^3\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}$$

$$-\frac{2\sqrt[3]{d}\sqrt{dx^3+c}a}{b^2\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)}$$

$$-\frac{\sqrt[6]{c}\sqrt{b\sqrt[3]{c}-a\sqrt[3]{d}}\sqrt{d^{2/3}a^2+b\sqrt[3]{c}\sqrt[3]{d}a+b^2c^{2/3}}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}\sqrt{d^{2/3}a\cdots}}{\sqrt[4]{3}\sqrt{b}\sqrt[6]{c}\sqrt{\cdots}}\right.}{b^{5/2}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}$$

$$-\frac{2\sqrt{2+\sqrt{3}}(b^3c-a^3d)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-\cdots\right.}{\sqrt[4]{3}b^3\left(\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}$$

$$+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{c}(b^3c-a^3d)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)^2}{\left(\left(1-\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)^2},\arcsin\cdots\right.}{b^2\left(-d^{2/3}a^2+2b\sqrt[3]{c}\sqrt[3]{d}a+2b^2c^{2/3}\right)\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}$$

$$+\frac{2\sqrt{dx^3+c}}{3b}$$

```
[In] Int[Sqrt[c + d*x^3]/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b) - (2*a*d^(1/3)*Sqrt[c + d*x^3])/(b^2*((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)) - (c^(1/6)*Sqrt[b*c^(1/3) - a*d^(1/3)]*Sqrt[b^2*c^
(2/3) + a*b*c^(1/3)*d^(1/3) + a^2*d^(2/3)]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2
/3)*(1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*ArcTanh[(Sqrt[2 - Sqrt[3]]*Sqrt[b^2*c^(2/3) + a*b*c^(1/3
)*d^(1/3) + a^2*d^(2/3)]*Sqrt[1 - ((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)^2/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2])/(3^(1/4)*Sqrt[b]*c^(1/6)*Sqrt[b*c^(1/3
) - a*d^(1/3)]*Sqrt[7 - 4*Sqrt[3] + ((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)^2/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2])])/(b^(5/2)*Sqrt[(c^(1/3)*(c^(1/3) +
d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a*c^(1/3)*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^2*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2 + Sqrt[
3]]*a*((1 - Sqrt[3])*b*c^(1/3) + a*d^(1/3))*d^(1/3)*(c^(1/3) + d^(1/3)*x)*S
qrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^3*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 - (2*Sqrt[2 + Sqrt[3]]*(b^3*c - a^3*d)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^3*((1 + Sqrt[3])*b*c^(1/3) - a*
d^(1/3))*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*Sqrt[c + d*x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*c^(1/3)*(b^3*c -
a^3*d)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)*(1 - (d^(1/3)*x)/c^(1/3) + (d^(2
/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticPi[((1 +
Sqrt[3])*b*c^(1/3) - a*d^(1/3))^2/((1 - Sqrt[3])*b*c^(1/3) - a*d^(1/3))^2,
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(b^2*(2*b^2*c^(2/3) + 2*a*b*c^(1/3)*d^(1/3) - a^2*d^
(2/3))*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
```

```
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2161

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[-q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2173

```
Int[Sqrt[(a_) + (b_.)*(x_)^3]/((c_) + (d_.)*(x_)), x_Symbol] :> Dist[b/d, I
nt[x^2/Sqrt[a + b*x^3], x], x] + (-Dist[(b*c^3 - a*d^3)/d^3, Int[1/((c + d*
x)*Sqrt[a + b*x^3]), x], x] + Dist[b*(c/d^3), Int[(c - d*x)/Sqrt[a + b*x^3]
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[b*c^3 - a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{(ad)\int \frac{a-bx}{\sqrt{c+dx^3}}\,dx}{b^3} + \frac{d\int \frac{x^2}{\sqrt{c+dx^3}}\,dx}{b} - \left(-c+\frac{a^3d}{b^3}\right)\int \frac{1}{(a+bx)\sqrt{c+dx^3}}\,dx$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{\left(ad^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{b^2} + \frac{\left(a\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)d\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{b^3} + \frac{\left(b\left(c-\frac{a^3 d}{b^3}\right)\right)\int \frac{1+\sqrt{3}+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{(a+bx)\sqrt{c+dx^3}}\,dx}{b+\sqrt{3}b-\frac{a\sqrt[3]{d}}{\sqrt[3]{c}}} - \frac{\left(\sqrt[3]{d}\left(c-\frac{a^3 d}{b^3}\right)\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}}$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}a\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{b^2\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{2\sqrt{2+\sqrt{3}}a\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)d^{2/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)\right.}{\sqrt[4]{3}b^3\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(c-\frac{a^3 d}{b^3}\right)\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}b\left(c-\frac{a^3 d}{b^3}\right)\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{\frac{1-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}}{\left(1+\sqrt{3}+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\left(\left(1-\sqrt{3}\right)b-\frac{a\sqrt[3]{d}}{\sqrt[3]{c}}+\left(\left(1+\sqrt{3}\right)b-\right.\right.}\right.}{\left(b+\sqrt{3}b-\frac{a\sqrt[3]{d}}{\sqrt[3]{c}}\right)\sqrt{\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(1+\sqrt{3}+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}}\sqrt{c+dx^3}}$$

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Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.60 (sec) , antiderivative size = 877, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$$

$$= \frac{2\left(c+dx^3+\frac{3\sqrt{2}a\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{-1}\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{d}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{c}}}\sqrt{\frac{i\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{3i+\sqrt{3}}}\left(\left(-1+(-1)^{2/3}\right)E\left(\arcsin\left(\sqrt{-\frac{(-1)^{2/3}\cdots}{\cdots}}\right.\right.\right.}{b\sqrt{\frac{\sqrt[3]{c}+(-1)^{2/3}\cdots}{\left(1+\sqrt[3]{-1}\right)\cdots}}}\right.}{\cdots}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(a + b*x),x]
```

```
[Out] (2*(c + d*x^3 + (3*Sqrt[2]*a*c^(1/3)*d^(1/3)*((-1)^(1/3)*c^(1/3) - d^(1/3)*
x)*Sqrt[((-1)^(1/3)*c^(1/3) - (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/
3))]*Sqrt[(I*(1 + (d^(1/3)*x)/c^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))
*EllipticE[ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(2/3)*c^(1/3) + d^(1/3)*x))/((1
+ (-1)^(1/3))*c^(1/3)))]], (-1)^(1/3)/(-1 + (-1)^(1/3))] + EllipticF[ArcSin
[Sqrt[-(((-1)^(2/3)*((-1)^(2/3)*c^(1/3) + d^(1/3)*x))/((1 + (-1)^(1/3))*c^(
1/3)))]], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(b*Sqrt[(c^(1/3) + (-1)^(2/3)*d^(
1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]) - (3*a^2*d^(2/3)*((-1)^(1/3)*c^(1/3) -
 d^(1/3)*x)*Sqrt[(c^(1/3) + d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]*Sqrt[((-
1)^(1/3)*c^(1/3) - (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]*Ellipt
icF[ArcSin[Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))
]], (-1)^(1/3)])/(b^2*Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3
))*c^(1/3))]) - ((3*I)*b*c^(4/3)*Sqrt[(c^(1/3) + d^(1/3)*x)/((1 + (-1)^(1/3
))*c^(1/3))]*Sqrt[1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(2/3)]*Elliptic
Pi[(I*Sqrt[3]*b*c^(1/3))/((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)), ArcSin[Sqrt[(c
^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]], (-1)^(1/3)])/(
(-1)^(1/3)*b*c^(1/3) + a*d^(1/3)) + ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*a^
3*c^(1/3)*d*Sqrt[(c^(1/3) + d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]*Sqrt[1 -
 (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(2/3)]*EllipticPi[(I*Sqrt[3]*b*c^(1/
3))/((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)), ArcSin[Sqrt[(c^(1/3) + (-1)^(2/3)*d
^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]], (-1)^(1/3)])/(b^2*((-1)^(1/3)*b*c^(
1/3) + a*d^(1/3)))))/(3*b*Sqrt[c + d*x^3])
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 1126, normalized size of antiderivative = 0.76

method	result	size
default	Expression too large to display	1126
elliptic	Expression too large to display	1126
risch	Expression too large to display	1137

```
[In] int((d*x^3+c)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b-2/3*I*a^2/b^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))+2/3*I*a/b^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*(
(x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*
(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
)+2/3*I*(a^3*d-b^3*c)/b^4*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*
(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)/(-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)+a/b)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)+a/b),(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx = \int \frac{\sqrt{dx^3+c}}{bx+a}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="fricas")
[Out] integral(sqrt(d*x^3 + c)/(b*x + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx = \int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/(b*x+a),x)
[Out] Integral(sqrt(c + d*x**3)/(a + b*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx = \int \frac{\sqrt{dx^3+c}}{bx+a}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/(b*x + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx = \int \frac{\sqrt{dx^3+c}}{bx+a}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx = \int \frac{\sqrt{d\,x^3+c}}{a+b\,x}\,dx$$

```
[In] int((c + d*x^3)^(1/2)/(a + b*x),x)
[Out] int((c + d*x^3)^(1/2)/(a + b*x), x)
```

3.197 $\int \frac{(d^3+e^3x^3)^p}{d+ex} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{(d^3+e^3x^3)^p}{d+ex} \, dx = \frac{(d^3+e^3x^3)^p \left(1+\frac{2(d+ex)}{\left(-3+i\sqrt{3}\right)d}\right)^{-p} \left(1-\frac{2(d+ex)}{\left(3+i\sqrt{3}\right)d}\right)^{-p} \operatorname{AppellF1}\left(p,-p,-p,1+p,-\frac{2(d+ex)}{\left(-3+i\sqrt{3}\right)d},\frac{2(d+ex)}{\left(3+i\sqrt{3}\right)d}\right)}{ep}$$

```
[Out] (e^3*x^3+d^3)^p*AppellF1(p,-p,-p,p+1,-2*(e*x+d)/d/(-3+I*3^(1/2)),2*(e*x+d)/
d/(3+I*3^(1/2)))/e/p/((1+2*(e*x+d)/d/(-3+I*3^(1/2)))^p)/((1-2*(e*x+d)/d/(3+
I*3^(1/2)))^p)
```

Rubi [F]

$$\int \frac{(d^3+e^3x^3)^p}{d+ex} \, dx = \int \frac{(d^3+e^3x^3)^p}{d+ex} \, dx$$

```
[In] Int[(d^3 + e^3*x^3)^p/(d + e*x),x]
```

```
[Out] Defer[Int][(d^3 + e^3*x^3)^p/(d + e*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{(d^3+e^3x^3)^p}{d+ex} \, dx$$

Mathematica [F]

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx = \int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx$$

```
[In] Integrate[(d^3 + e^3*x^3)^p/(d + e*x),x]
[Out] Integrate[(d^3 + e^3*x^3)^p/(d + e*x), x]
```

Maple [F]

$$\int \frac{(e^3x^3+d^3)^p}{ex+d}dx$$

```
[In] int((e^3*x^3+d^3)^p/(e*x+d),x)
[Out] int((e^3*x^3+d^3)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx = \int \frac{(e^3x^3+d^3)^p}{ex+d}\,dx$$

```
[In] integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="fricas")
[Out] integral((e^3*x^3 + d^3)^p/(e*x + d), x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(102) = 204$.

Time = 25.94 (sec) , antiderivative size = 631, normalized size of antiderivative = 4.67

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx$$
$$= \frac{0^p \log\left(1+\frac{e^3x^3}{d^3}\right)\Gamma\left(-\frac{2}{3}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{5}{3}\right)}{4\pi^2 e}$$
$$+ \frac{0^p e^{\frac{i\pi}{3}}\log\left(1-\frac{exe^{\frac{i\pi}{3}}}{d}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}{6\pi^2 e\Gamma\left(\frac{5}{3}\right)}$$
$$+ \frac{0^p e^{\frac{2i\pi}{3}}\log\left(1-\frac{exe^{\frac{i\pi}{3}}}{d}\right)\Gamma^3\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)}{12\pi^2 e\Gamma\left(\frac{4}{3}\right)} - \frac{0^p \log\left(1-\frac{exe^{i\pi}}{d}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}{6\pi^2 e\Gamma\left(\frac{5}{3}\right)}$$
$$+ \frac{0^p \log\left(1-\frac{exe^{i\pi}}{d}\right)\Gamma^3\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)}{12\pi^2 e\Gamma\left(\frac{4}{3}\right)} + \frac{0^p e^{-\frac{2i\pi}{3}}\log\left(1-\frac{exe^{\frac{5i\pi}{3}}}{d}\right)\Gamma^3\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)}{12\pi^2 e\Gamma\left(\frac{4}{3}\right)}$$
$$+ \frac{0^p e^{-\frac{i\pi}{3}}\log\left(1-\frac{exe^{\frac{5i\pi}{3}}}{d}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}{6\pi^2 e\Gamma\left(\frac{5}{3}\right)}$$
$$- \frac{d^2e^{3p-3}px^{3p-2}\Gamma\left(-\frac{2}{3}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{5}{3}\right)\Gamma(p)\Gamma\left(\frac{2}{3}-p\right){}_2F_1\left(\begin{matrix}1-p,\frac{2}{3}-p\\ \frac{5}{3}-p\end{matrix}\middle|\frac{d^3e^{i\pi}}{e^3x^3}\right)}{4\pi^2\Gamma\left(\frac{5}{3}-p\right)\Gamma(p+1)}$$
$$- \frac{de^{3p-2}px^{3p-1}\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)\Gamma(p)\Gamma\left(\frac{1}{3}-p\right){}_2F_1\left(\begin{matrix}1-p,\frac{1}{3}-p\\ \frac{4}{3}-p\end{matrix}\middle|\frac{d^3e^{i\pi}}{e^3x^3}\right)}{4\pi^2\Gamma\left(\frac{4}{3}-p\right)\Gamma(p+1)}$$
$$- \frac{d^{3p}e^2x^3\Gamma^2\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix}2,1,1-p\\ 2,2\end{matrix}\middle|\frac{e^3x^3e^{i\pi}}{d^3}\right)}{4\pi^2d^3\Gamma(-p)\Gamma(p+1)}$$

```
[In] integrate((e**3*x**3+d**3)**p/(e*x+d),x)
```

```
[Out] 0**p*log(1 + e**3*x**3/d**3)*gamma(-2/3)*gamma(-1/3)*gamma(4/3)*gamma(5/3)/
(4*pi**2*e) + 0**p*exp(I*pi/3)*log(1 - e*x*exp_polar(I*pi/3)/d)*gamma(-1/3)
*gamma(1/3)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3)) + 0**p*exp(2*I*
pi/3)*log(1 - e*x*exp_polar(I*pi/3)/d)*gamma(1/3)**3*gamma(2/3)**2/(12*pi**
2*e*gamma(4/3)) - 0**p*log(1 - e*x*exp_polar(I*pi)/d)*gamma(-1/3)*gamma(1/3
)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3)) + 0**p*log(1 - e*x*exp_po
lar(I*pi)/d)*gamma(1/3)**3*gamma(2/3)**2/(12*pi**2*e*gamma(4/3)) + 0**p*exp
(-2*I*pi/3)*log(1 - e*x*exp_polar(5*I*pi/3)/d)*gamma(1/3)**3*gamma(2/3)**2/
(12*pi**2*e*gamma(4/3)) + 0**p*exp(-I*pi/3)*log(1 - e*x*exp_polar(5*I*pi/3)
/d)*gamma(-1/3)*gamma(1/3)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3))
- d**2*e**(3*p - 3)*p*x**(3*p - 2)*gamma(-2/3)*gamma(-1/3)*gamma(4/3)*gamma
```

```
(5/3)*gamma(p)*gamma(2/3 - p)*hyper((1 - p, 2/3 - p), (5/3 - p,), d**3*exp_
polar(I*pi)/(e**3*x**3))/(4*pi**2*gamma(5/3 - p)*gamma(p + 1)) - d*e**(3*p
- 2)*p*x**(3*p - 1)*gamma(-1/3)*gamma(1/3)*gamma(2/3)*gamma(4/3)*gamma(p)*g
amma(1/3 - p)*hyper((1 - p, 1/3 - p), (4/3 - p,), d**3*exp_polar(I*pi)/(e**
3*x**3))/(4*pi**2*gamma(4/3 - p)*gamma(p + 1)) - d**(3*p)*e**2*x**3*gamma(1
/3)**2*gamma(2/3)**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e**
3*x**3*exp_polar(I*pi)/d**3)/(4*pi**2*d**3*gamma(-p)*gamma(p + 1))
```

Maxima [F]

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx = \int \frac{(e^3x^3+d^3)^p}{ex+d}\,dx$$

```
[In] integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((e^3*x^3 + d^3)^p/(e*x + d), x)
```

Giac [F]

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx = \int \frac{(e^3x^3+d^3)^p}{ex+d}\,dx$$

```
[In] integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((e^3*x^3 + d^3)^p/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx = \int \frac{(d^3+e^3\,x^3)^p}{d+e\,x}\,dx$$

```
[In] int((d^3 + e^3*x^3)^p/(d + e*x),x)
[Out] int((d^3 + e^3*x^3)^p/(d + e*x), x)
```

3.198 $\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 16

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = 2\arctan\left(\frac{1+x}{\sqrt{1+x^3}}\right)$$

```
[Out] 2*arctan((1+x)/(x^3+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2171, 209}

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = 2\arctan\left(\frac{x+1}{\sqrt{x^3+1}}\right)$$

```
[In] Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] 2*ArcTan[(1 + x)/Sqrt[1 + x^3]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2171

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Dist[-g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
```

```
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\\ &= 2\tan^{-1}\left(\frac{1+x}{\sqrt{1+x^3}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = 2\arctan\left(\frac{\sqrt{1+x^3}}{1-x+x^2}\right)$$

```
[In] Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
[Out] 2*ArcTan[Sqrt[1 + x^3]/(1 - x + x^2)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.88

method	result	size
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^2-2\operatorname{RootOf}\left(\_Z^2+1\right)x+2\sqrt{x^3+1}}{x^2+2}\right)$	46
default	Expression too large to display	1640
elliptic	Expression too large to display	1845

```
[In] int((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] RootOf(_Z^2+1)*ln((RootOf(_Z^2+1)*x^2-2*RootOf(_Z^2+1)*x+2*(x^3+1)^(1/2))/(
x^2+2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = -\arctan\left(\frac{x^2-2\,x}{2\,\sqrt{x^3+1}}\right)$$

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] -arctan(1/2*(x^2 - 2*x)/sqrt(x^3 + 1))
```

Sympy [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = -\int \frac{2x}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx - \int \frac{x^2}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx - \int \left(-\frac{2}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\right)dx$$

```
[In] integrate((-x**2-2*x+2)/(x**2+2)/(x**3+1)**(1/2),x)
[Out] -Integral(2*x/(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(x**2/
(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(-2/(x**2*sqrt(x**3
+ 1) + 2*sqrt(x**3 + 1)), x)
```

Maxima [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{x^3+1}(x^2+2)}\,dx$$

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

Giac [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx=\int -\frac{x^2+2\,x-2}{\sqrt{x^3+1}(x^2+2)}\,dx$$

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 273, normalized size of antiderivative = 17.06

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{1+\sqrt{2}\,1\mathrm{i}};\mathrm{asin}\left(\sqrt{\frac{}{\frac{3}{2}}}\right.\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\right.}}$$

```
[In] int(-(2*x + x^2 - 2)/((x^2 + 2)*(x^3 + 1)^(1/2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i + 1),
 asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^
(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3^(1/2)*
1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))
, -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(x^3 - x*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i
)/2 + 1/2))^(1/2)
```

3.199 $\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 20

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = -2\arctan\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

```
[Out] -2*arctan((1-x)/(-x^3+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2171, 209}

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = -2\arctan\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

```
[In] Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] -2*ArcTan[(1 - x)/Sqrt[1 - x^3]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2171

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Dist[-g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
```

```
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)\right)\\ &= -2\tan^{-1}\left(\frac{1-x}{\sqrt{1-x^3}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = -2\arctan\left(\frac{\sqrt{1-x^3}}{1+x+x^2}\right)$$

```
[In] Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
[Out] -2*ArcTan[Sqrt[1 - x^3]/(1 + x + x^2)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.45

method	result
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^2+2\operatorname{RootOf}\left(\_Z^2+1\right)x-2\sqrt{-x^3+1}}{x^2+2}\right)$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2i\sqrt{3}\sqrt{i\sqrt{3}x+\frac{i\sqrt{3}}{2}+\frac{3}{2}}\cdots}{\cdots}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\sqrt{3}x+\frac{i\sqrt{3}}{2}+\frac{3}{2}}\sqrt{\frac{x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\sqrt{3}x-\frac{i\sqrt{3}}{2}+\frac{3}{2}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2\sqrt{2}\sqrt{3}\sqrt{i\sqrt{3}x+\frac{i\sqrt{3}}{2}\cdots}}{\cdots}$

```
[In] int((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] RootOf(_Z^2+1)*ln(-(RootOf(_Z^2+1)*x^2+2*RootOf(_Z^2+1)*x-2*(-x^3+1)^(1/2))
/(x^2+2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = -\arctan\left(\frac{\sqrt{-x^3+1}(x^2+2\,x)}{2\,(x^3-1)}\right)$$

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] -arctan(1/2*sqrt(-x^3 + 1)*(x^2 + 2*x)/(x^3 - 1))
```

Sympy [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = -\int\left(-\frac{2x}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx - \int\frac{x^2}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx - \int\left(-\frac{2}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

```
[In] integrate((-x**2+2*x+2)/(x**2+2)/(-x**3+1)**(1/2),x)
[Out] -Integral(-2*x/(x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(x**2
/(x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-2/(x**2*sqrt(1 -
x**3) + 2*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = \int -\frac{x^2-2\,x-2}{\sqrt{-x^3+1}(x^2+2)}\,dx$$

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

Giac [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = \int -\frac{x^2-2\,x-2}{\sqrt{-x^3+1}(x^2+2)}\,dx$$

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 19.53 (sec) , antiderivative size = 292, normalized size of antiderivative = 14.60

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\cdots\right)\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\cdots\right)}}$$

```
[In] int((2*x - x^2 + 2)/((x^2 + 2)*(1 - x^3)^(1/2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2
)/(2^(1/2)*1i + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/
2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-(x - 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + e
llipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin((-(x - 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(
(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.200 $\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 18

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = -2\operatorname{arctanh}\left(\frac{1-x}{\sqrt{-1+x^3}}\right)$$

```
[Out] -2*arctanh((1-x)/(x^3-1)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2171, 212}

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = -2\operatorname{arctanh}\left(\frac{1-x}{\sqrt{x^3-1}}\right)$$

```
[In] Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] -2*ArcTanh[(1 - x)/Sqrt[-1 + x^3]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2171

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Dist[-g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
```

```
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\right)$$
$$= -2\tanh^{-1}\left(\frac{1-x}{\sqrt{-1+x^3}}\right)$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = 2\text{arctanh}\left(\frac{\sqrt{-1+x^3}}{1+x+x^2}\right)$$

```
[In] Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
[Out] 2*ArcTanh[Sqrt[-1 + x^3]/(1 + x + x^2)]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

method	result	size
trager	$\ln\left(\frac{x^2+2\sqrt{x^3-1}+2x}{x^2+2}\right)$	26
default	Expression too large to display	1656
elliptic	Expression too large to display	1865

```
[In] int((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln((x^2+2*(x^3-1)^(1/2)+2*x)/(x^2+2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = \log\left(\frac{x^2+2\,x+2\,\sqrt{x^3-1}}{x^2+2}\right)$$

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] log((x^2 + 2*x + 2*sqrt(x^3 - 1))/(x^2 + 2))
```

Sympy [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = -\int\left(-\frac{2x}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx - \int\frac{x^2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx - \int\left(-\frac{2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

```
[In] integrate((-x**2+2*x+2)/(x**2+2)/(x**3-1)**(1/2),x)
[Out] -Integral(-2*x/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(x**2
/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-2/(x**2*sqrt(x**3
 - 1) + 2*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = \int -\frac{x^2-2\,x-2}{\sqrt{x^3-1}(x^2+2)}\,dx$$

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

Giac [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = \int -\frac{x^2-2\,x-2}{\sqrt{x^3-1}(x^2+2)}\,dx$$

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 276, normalized size of antiderivative = 15.33

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = -\frac{(3+\sqrt{3}\,1\mathrm{i})\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{1+\sqrt{2}\,1\mathrm{i}};\mathrm{a}\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(\right.}}$$

```
[In] int((2*x - x^2 + 2)/((x^2 + 2)*(x^3 - 1)^(1/2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i +
1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3^(
1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2)
```

3.201 $\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 18

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = 2\text{arctanh}\left(\frac{1+x}{\sqrt{-1-x^3}}\right)$$

```
[Out] 2*arctanh((1+x)/(-x^3-1)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2171, 212}

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = 2\text{arctanh}\left(\frac{x+1}{\sqrt{-x^3-1}}\right)$$

```
[In] Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] 2*ArcTanh[(1 + x)/Sqrt[-1 - x^3]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2171

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Dist[-g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
```

```
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\\ &= 2\tanh^{-1}\left(\frac{1+x}{\sqrt{-1-x^3}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = -2\text{arctanh}\left(\frac{\sqrt{-1-x^3}}{1-x+x^2}\right)$$

```
[In] Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
[Out] -2*ArcTanh[Sqrt[-1 - x^3]/(1 - x + x^2)]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

method	result
trager	$-\ln\left(\frac{x^2+2\sqrt{-x^3-1}-2x}{x^2+2}\right)$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2\sqrt{2}\sqrt{3}\sqrt{i\sqrt{3}\,x-\frac{i\sqrt{3}}{2}+\frac{3}{2}}\sqrt{\frac{}{\frac{3}{2}+}}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\sqrt{3}\,x-\frac{i\sqrt{3}}{2}+\frac{3}{2}}\sqrt{\frac{x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\sqrt{3}\,x+\frac{i\sqrt{3}}{2}+\frac{3}{2}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2\sqrt{2}\sqrt{3}\sqrt{i\sqrt{3}\,x-\frac{i\sqrt{3}}{2}+\frac{3}{2}}}{}$

```
[In] int((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -ln((x^2+2*(-x^3-1)^(1/2)-2*x)/(x^2+2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = \log\left(-\frac{x^2-2\,x-2\,\sqrt{-x^3-1}}{x^2+2}\right)$$

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] log(-(x^2 - 2*x - 2*sqrt(-x^3 - 1))/(x^2 + 2))
```

Sympy [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = -\int \frac{2x}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx - \int \frac{x^2}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx - \int \left(-\frac{2}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\right)dx$$

```
[In] integrate((-x**2-2*x+2)/(x**2+2)/(-x**3-1)**(1/2),x)
[Out] -Integral(2*x/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(x**
2/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/(x**2*sqrt(-
x**3 - 1) + 2*sqrt(-x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{-x^3-1}(x^2+2)}\,dx$$

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

Giac [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{-x^3-1}(x^2+2)}\,dx$$

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 289, normalized size of antiderivative = 16.06

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{1+\sqrt{2}\,1\mathrm{i}};\,a\right.\right.}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x\,-}}$$

```
[In] int(-(2*x + x^2 - 2)/((x^2 + 2)*(- x^3 - 1)^(1/2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(
2^(1/2)*1i + 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1
i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipt
icPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin(((x + 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/((- x^3
 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) -
((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.202 $\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 30

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{1+d}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{1+d}}$$

```
[Out] 2*arctan((1+x)*(1+d)^(1/2)/(x^3+1)^(1/2))/(1+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2170, 210}

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{d+1}}$$

```
[In] Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
[Out] (2*ArcTan[(Sqrt[1 + d]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[1 + d]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
```

```
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(4\text{Subst}\left(\int \frac{1}{-2-(2+2d)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{1+d}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{1+d}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{1+d}\sqrt{1+x^3}}{1-x+x^2}\right)}{\sqrt{1+d}}$$

```
[In] Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
[Out] (2*ArcTan[(Sqrt[1 + d]*Sqrt[1 + x^3])/(1 - x + x^2)])/Sqrt[1 + d]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.81 (sec) , antiderivative size = 4397, normalized size of antiderivative = 146.57

method	result	size
default	Expression too large to display	4397
elliptic	Expression too large to display	4602

```
[In] int((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3/2/(d^2-4*d-8)^(1/2)*(1/(3/2-1/
2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(
-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*
I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-
1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))
```

,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2-1/2*I/(d^2-4*d-8)^(
1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3
^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1
/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*Elliptic
Pi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^
2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2*3^(1
/2)+3/2*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*
I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3
^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*Ellip
ticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*
(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d-1/2
I(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(
1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*
(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi
(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-
4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)+
6/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(
1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2)
)*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(
-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^
(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-
1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2))*d+I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1
/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2)
)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*
I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*El
lipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1
/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^
(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*
I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3
^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*Ellip
ticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*
(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+1/2*I
/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1
/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))
3^(1/2))^(1/2)(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-
3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(
1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1
+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(
1/2))*d^2*3^(1/2)+12/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*
I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/

(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2
*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2
d-1/2(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/
2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3
/2-1/2*I*3^(1/2)))^(1/2))-4*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/
(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2
))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/
(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2
)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(
1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+3/2/(d^2-4*d-8)^(1/2)*(1/(3/2-1/
2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(
-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*
I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-
1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))
,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2-1/2*I*(1/(3/2-1/2*I
*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/
2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3
^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2
*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((
-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)+3/2*(1/(3/2-1/2*
I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3
/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*
3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/
2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),(
(-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d+2*I/(d^2-4*d-8)^(1/2)*(
1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2)
)*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(
-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3
^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x
+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-
8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-6/(d
^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-
3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2
+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2
))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/
2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2
))*d+4*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^
(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*
3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+
1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-

```
4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1
/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(
1/2)))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/
2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2
*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d
-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2)
)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2
)))^(1/2))+I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2
-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/
2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/
2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*
EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d
+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*
3^(1/2)-12/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2))
)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2)
)+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^
2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^
(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3
^(1/2)))^(1/2))-2*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I
*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(
-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*
I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*
d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2
+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/
2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 181, normalized size of antiderivative = 6.03

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx = \left[-\frac{\sqrt{-d-1}\log\left(-\frac{2\,(3\,d+4)x^3-x^4-(d^2+2\,d+4)x^2-d^2+4\,\sqrt{x^3+1}\left((d+2)x-x^2+d\right)\sqrt{-d-1}-2\,(d^2+2\,d)x+4\,d+4}{2\,dx^3+x^4+(d^2+2\,d+4)x^2+d^2+2\,(d^2+2\,d)x+4\,d+4}\right)}{2\,(d+1)}, -\frac{\arctan\left(-\frac{\sqrt{x^3+1}\left((d+2)x-x^2+d\right)\sqrt{d+1}}{2\,\left((d+1)x^3+d+1\right)}\right)}{\sqrt{d+1}}\right]$$

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-d - 1)*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - d^2
+ 4*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(-d - 1) - 2*(d^2 + 2*d)*x + 4*
d + 4)/(2*d*x^3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d +
 4))/(d + 1), -arctan(-1/2*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(d + 1)/
((d + 1)*x^3 + d + 1))/sqrt(d + 1)]
```

Sympy [F]

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx$$
$$= -\int \frac{2x}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx$$
$$-\int \frac{x^2}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx$$
$$-\int \left(-\frac{2}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\right)\,dx$$

```
[In] integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2*x/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3 + 1) + x**2*sqrt(x**3 + 1)
+ 2*sqrt(x**3 + 1)), x) - Integral(x**2/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3 +
 1) + x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(-2/(d*x*sqrt(x
**3 + 1) + d*sqrt(x**3 + 1) + x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(d+2)>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{x^3+1}(dx+x^2+d+2)}\,dx$$

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(d*x + x^2 + d + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 19.71 (sec) , antiderivative size = 632, normalized size of antiderivative = 21.07

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx = \text{Too large to display}$$

```
[In] int(-(2*x + x^2 - 2)/((x^3 + 1)^(1/2)*(d + d*x + x^2 + 2)),x)
[Out] - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*(
(3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)
/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((
3^(1/2)*1i)/2 + 3/2)/((d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1), asin(((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
)*(d - (d - 2)*(d/2 - (d^2 - 4*d - 8)^(1/2)/2) + 4))/((x^3 - x*(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*((d^2 - 4*d - 8)^(1/2)/2 - d/2 +
1)) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2
 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x
+ 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(d/
2 + (d^2 - 4*d - 8)^(1/2)/2 - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d - (d - 2)*(d/2 + (d
^2 - 4*d - 8)^(1/2)/2) + 4))/((x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i
)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)*(d^2
 - 4*d - 8)^(1/2)*(d/2 + (d^2 - 4*d - 8)^(1/2)/2 - 1))
```

3.203 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 38

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

```
[Out] -2*arctan((1-x)*(1-d)^(1/2)/(-x^3+1)^(1/2))/(1-d)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2170, 210}

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

```
[In] Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
[Out] (-2*ArcTan[(Sqrt[1 - d]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[1 - d]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
```

```
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = 4\text{Subst}\left(\int \frac{1}{-2-(2-2d)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{-1+d}\sqrt{1-x^3}}{1+x+x^2}\right)}{\sqrt{-1+d}}$$

```
[In] Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
[Out] (-2*ArcTanh[(Sqrt[-1 + d]*Sqrt[1 - x^3])/(1 + x + x^2)])/Sqrt[-1 + d]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.84 (sec) , antiderivative size = 1908, normalized size of antiderivative = 50.21

method	result	size
default	Expression too large to display	1908
elliptic	Expression too large to display	1919

```
[In] int((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))+1/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^
(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-
I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2
*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),(I
*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*
```

3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*
(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1
/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),
(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d+4/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(
I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1
/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(
d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d-2/3*I*3^(1/2)*(
I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1
/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(
d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-8/3*I/(d^2+4*d-8)
^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2)
)*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-
x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(
1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-1
/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3
/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/
2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)
/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^
(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(
-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(
1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1
/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/
2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*
3^(1/2)))^(1/2))*d-4/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/
2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3
^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+
1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^
(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/
2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3
^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+
1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^
(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+8/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/
2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/
2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*
I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d

```
-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 5.03

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = \left[\frac{\log\left(-\frac{2\,(3\,d-4)x^3-x^4-(d^2-2\,d+4)x^2-4\,\sqrt{-x^3+1}\,((d-2)x-x^2-d)\,\sqrt{d-1}-d^2+2\,(d^2-2\,d)x-4\,d+4}{2\,dx^3+x^4+(d^2-2\,d+4)x^2+d^2-2\,(d^2-2\,d)x-4\,d+4}\right)}{2\,\sqrt{d-1}}, -\frac{\sqrt{-d+1}\arctan\left(-\frac{\sqrt{-x^3+1}\,((d-2)x-x^2-d)\,\sqrt{-d+1}}{2\,((d-1)x^3-d+1)}\right)}{d-1}\right]$$

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - 4*sqrt(-x^3 + 1)*(
(d - 2)*x - x^2 - d)*sqrt(d - 1) - d^2 + 2*(d^2 - 2*d)*x - 4*d + 4)/(2*d*x^
3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d + 4))/sqrt(d -
1), -sqrt(-d + 1)*arctan(-1/2*sqrt(-x^3 + 1)*((d - 2)*x - x^2 - d)*sqrt(-d
+ 1)/((d - 1)*x^3 - d + 1))/(d - 1)]
```

Sympy [F]

$$\begin{aligned}\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx &= -\int\left(-\frac{2x}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx \\ &\quad -\int\frac{x^2}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx \\ &\quad -\int\left(-\frac{2}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx\end{aligned}$$

```
[In] integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(d*x*sqrt(1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3)
 + 2*sqrt(1 - x**3)), x) - Integral(x**2/(d*x*sqrt(1 - x**3) - d*sqrt(1 - x
**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-2/(d*x*sqrt(
1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(2-d)>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = \int -\frac{x^2-2\,x-2}{\sqrt{-x^3+1}(dx+x^2-d+2)}\,dx$$

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(d*x + x^2 - d + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 18.72 (sec) , antiderivative size = 677, normalized size of antiderivative = 17.82

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = \text{Too large to display}$$

```
[In] int((2*x - x^2 + 2)/((1 - x^3)^(1/2)*(d*x - d + x^2 + 2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/
2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*((
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) + (2*((3^(1
/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*
(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(
d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d + (d + 2)*(d/2 -
 (4*d + d^2 - 8)^(1/2)/2) - 4))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((
```

```
3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1
) + x^3)^(1/2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8)^(1/2)) -
 (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/
2 + 3/2)/(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d + (d +
 2)*(d/2 + (4*d + d^2 - 8)^(1/2)/2) - 4))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2) + 1) + x^3)^(1/2)*(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8
)^(1/2))
```

3.204 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 36

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{1-d}}$$

```
[Out] -2*arctanh((1-x)*(1-d)^(1/2)/(x^3-1)^(1/2))/(1-d)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2170, 213}

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{1-d}}$$

```
[In] Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
[Out] (-2*ArcTanh[(Sqrt[1 - d]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[1 - d]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
```

```
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = 4\text{Subst}\left(\int \frac{1}{-2-(-2+2d)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{1-d}}$$

Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{-1+d}\sqrt{-1+x^3}}{1+x+x^2}\right)}{\sqrt{-1+d}}$$

```
[In] Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
[Out] (2*ArcTan[(Sqrt[-1 + d]*Sqrt[-1 + x^3])/(1 + x + x^2)])/Sqrt[-1 + d]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.69 (sec) , antiderivative size = 4437, normalized size of antiderivative = 123.25

method	result	size
default	Expression too large to display	4437
elliptic	Expression too large to display	4646

```
[In] int((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+3/2/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/
2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(
3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3
^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I
*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+
```

1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d^2+4*I/(d^2+4*d-8)^(1/2)*(1/(-3
/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+
1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/
2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-
1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(
(3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3/2*(1/(-3/2-1/2*I*
3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-
1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/
2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3
-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(
1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*
I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d-4*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2
*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3
/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^
(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*
3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+6/(d^2+4*d-8)^(1/2)*(1/(-
3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x
+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1
/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^
(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2
-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),
((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d-1/2*I/(d^2+4*d-8)^(1/2)*
(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/
2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(
3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1
/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/
(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1
/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d^2*3^(1/2)-3*(1/(-3/
2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1
/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2
*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1
/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1
/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((
3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/2*I*(1/(-3/2-1/2*I*3^(1/2)
)*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3
^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1
/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/
2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^
(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/
2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-12/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2
*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3
/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^

```
(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*
3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/
2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(
3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3
^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I
*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-3/2/(d^2+4*d-8)^(1/2)*
(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/
2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(
3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1
/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/
(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1
/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d^2+1/2*I/(d^2+4*d-8)
^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*
I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(
((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*
d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d^2*3^(1/2)-3/
2*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(
1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1
/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^
(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1
)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^
(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d-I*(1/(-3/2-1/2*I*
3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-
1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/
2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3
-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(
1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*
I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-6/(d^2+4*d-8)^(1/2)*(1/(-3/2
-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/
2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*
I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/
2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3
/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d-2*I/(d^2+4*d-8)^(1/2)*(1/(-
3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x
+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1
/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^
(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2
-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),
```

```
((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-3*(1/(-3/2-1/2*I
*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2
-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1
/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^
3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^
(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2
*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2
-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2
*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2
*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d
-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+
1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/
2*I*3^(1/2)))^(1/2))*3^(1/2)+12/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x
-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1
/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/
(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/
(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/
2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))
/(3/2-1/2*I*3^(1/2)))^(1/2))-1/2*I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*
3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2
-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1
/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d
^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3
^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2))*d*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 187, normalized size of antiderivative = 5.19

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = \left[-\frac{\sqrt{-d+1}\log\left(-\frac{2\,(3\,d-4)x^3-x^4-(d^2-2\,d+4)x^2-d^2+4\,\sqrt{x^3-1}\left((d-2)x-x^2-d\right)\sqrt{-d+1}+2\,(d^2-2\,d)x-4\,d+4}{2\,dx^3+x^4+(d^2-2\,d+4)x^2+d^2-2\,(d^2-2\,d)x-4\,d+4}\right)}{2\,(d-1)},\ -\frac{\arctan\left(-\frac{\sqrt{x^3-1}\left((d-2)x-x^2-d\right)\sqrt{d-1}}{2\,((d-1)x^3-d+1)}\right)}{\sqrt{d-1}}\right]$$

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-d + 1)*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - d^2
+ 4*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(-d + 1) + 2*(d^2 - 2*d)*x - 4*
```

```
d + 4)/(2*d*x^3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d +
 4))/(d - 1), -arctan(-1/2*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(d - 1)/
((d - 1)*x^3 - d + 1))/sqrt(d - 1)]
```

Sympy [F]

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = -\int \left(-\frac{2x}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx - \int \frac{x^2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx - \int \left(-\frac{2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

```
[In] integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(x**3-1)**(1/2),x)
[Out] -Integral(-2*x/(d*x*sqrt(x**3 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt(x**3 - 1)
 + 2*sqrt(x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(x**3 - 1) - d*sqrt(x**3
- 1) + x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-2/(d*x*sqrt(
x**3 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(2-d)>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = \int -\frac{x^2-2\,x-2}{\sqrt{x^3-1}(dx+x^2-d+2)}\,dx$$

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(d*x + x^2 - d + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 629, normalized size of antiderivative = 17.47

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = \text{Too large to display}$$

```
[In] int((2*x - x^2 + 2)/((x^3 - 1)^(1/2)*(d*x - d + x^2 + 2)),x)
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) + x^3)^(1/2) + (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1
i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi
(((3^(1/2)*1i)/2 + 3/2)/(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2))*(d + (d + 2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2) - 4))/((((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8)
^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2
)/(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d + (d + 2)*(d/
2 + (4*d + d^2 - 8)^(1/2)/2) - 4))/((((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2
)*(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8)^(1/2))
```

3.205 $\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 32

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{1+d}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{1+d}}$$

```
[Out] 2*arctanh((1+x)*(1+d)^(1/2)/(-x^3-1)^(1/2))/(1+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2170, 213}

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{d+1}}$$

```
[In] Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
[Out] (2*ArcTanh[(Sqrt[1 + d]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[1 + d]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
```

```
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(4\text{Subst}\left(\int \frac{1}{-2-(-2-2d)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{1+d}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{1+d}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{1+d}\sqrt{-1-x^3}}{1-x+x^2}\right)}{\sqrt{1+d}}$$

```
[In] Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
[Out] (-2*ArcTanh[(Sqrt[1 + d]*Sqrt[-1 - x^3])/(1 - x + x^2)])/Sqrt[1 + d]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.77 (sec) , antiderivative size = 1888, normalized size of antiderivative = 59.00

method	result	size
default	Expression too large to display	1888
elliptic	Expression too large to display	1897

```
[In] int((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))+1/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1
/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^
(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/
2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/
2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)
```

+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/
2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(
d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/
(3/2+1/2*I*3^(1/2)))^(1/2))*d-4/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-
1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1
/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2
)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)
),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+2/3*I*3^(1/2)*(I*3^(1/2)*x-1/2*I
3^(1/2)+3/2)^(1/2)(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(
-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2
d-1/2(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*
3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-8/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1
/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2
)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*
3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)
^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-1/3*I/(d^2-4*d-8)^(1/2)*3^(1
/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1
/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/
(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(
x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*
(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)
*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/
2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^
2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+4/3*I/(d^2-4*d-8)^
(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*
x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3
-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^
(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+
1/2*d+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+2/3*I
3^(1/2)(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(
1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)
(I(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d
+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+8/3*I/(d^2-4
*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(
1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)
/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(
1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^
(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 5.78

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = \left[\frac{\log\left(-\frac{2\,(3\,d+4)x^3-x^4-(d^2+2\,d+4)x^2-4\,\sqrt{-x^3-1}((d+2)x-x^2+d)\sqrt{d+1}-d^2-2\,(d^2+2\,d)x+4\,d+4}{2\,dx^3+x^4+(d^2+2\,d+4)x^2+d^2+2\,(d^2+2\,d)x+4\,d+4}\right)}{2\,\sqrt{d+1}}, -\frac{\sqrt{-d-1}\arctan\left(-\frac{\sqrt{-x^3-1}((d+2)x-x^2+d)\sqrt{-d-1}}{2\,((d+1)x^3+d+1)}\right)}{d+1}\right]$$

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - 4*sqrt(-x^3 - 1)*(
(d + 2)*x - x^2 + d)*sqrt(d + 1) - d^2 - 2*(d^2 + 2*d)*x + 4*d + 4)/(2*d*x^
3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d + 4))/sqrt(d +
1), -sqrt(-d - 1)*arctan(-1/2*sqrt(-x^3 - 1)*((d + 2)*x - x^2 + d)*sqrt(-d
- 1)/((d + 1)*x^3 + d + 1))/(d + 1)]
```

Sympy [F]

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = -\int \frac{2x}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx - \int \frac{x^2}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx - \int \left(-\frac{2}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\right)dx$$

```
[In] integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(2*x/(d*x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 -
1) + 2*sqrt(-x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(-x**3 - 1) + d*sqrt(-
x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/(d*
x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3
 - 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(d+2)>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{-x^3-1}(dx+x^2+d+2)}\,dx$$

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(d*x + x^2 + d + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 19.05 (sec) , antiderivative size = 680, normalized size of antiderivative = 21.25

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = \text{Too large to display}$$

```
[In] int(-(2*x + x^2 - 2)/((- x^3 - 1)^(1/2)*(d + d*x + x^2 + 2)),x)
```

```
[Out] - (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2
)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*((3^(
1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/((
d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d - (d - 2)*(d/2 - (
d^2 - 4*d - 8)^(1/2)/2) + 4))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2
```

```
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*((d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1)) -
 (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)
*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*1i)/2
 + 3/2)/(d/2 + (d^2 - 4*d - 8)^(1/2)/2 - 1), asin(((x + 1)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d - (d - 2
)*(d/2 + (d^2 - 4*d - 8)^(1/2)/2) + 4))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*(d/2 + (d^2 - 4*d - 8)^(1/2)
/2 - 1))
```

3.206 $\int (d+ex)^3\sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 355

$$\begin{aligned}\int (d+ex)^3\sqrt{a+cx^4}\,dx &= \frac{3}{4}d^2ex^2\sqrt{a+cx^4}+\frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4}+\frac{e^3(a+cx^4)^{3/2}}{6c}\\&\quad+\frac{3ad^2e\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}-\frac{6a^{5/4}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}\\&\quad+\frac{a^{3/4}d\left(5\sqrt{c}d^2+9\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}\end{aligned}$$

```
[Out] 1/6*e^3*(c*x^4+a)^(3/2)/c+3/4*a*d^2*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/
c^(1/2)+3/4*d^2*e*x^2*(c*x^4+a)^(1/2)+1/15*d*x*(9*e^2*x^2+5*d^2)*(c*x^4+a)^
(1/2)+6/5*a*d*e^2*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-6/5*a^(5/
4)*d*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/
a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1
/2)+1/15*a^(3/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(
c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))
*(9*e^2*a^(1/2)+5*d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^
2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1899, 1191, 1212, 226, 1210, 1262, 655, 201, 223, 212}

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx$$

$$= \frac{a^{3/4}d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(9\sqrt{a}e^2+5\sqrt{c}d^2)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}} - \frac{6a^{5/4}de^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}} + \frac{3ad^2e\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}} + \frac{1}{15}dx\sqrt{a+cx^4}(5d^2+9e^2x^2) + \frac{3}{4}d^2ex^2\sqrt{a+cx^4} + \frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}(\sqrt{a}+\sqrt{c}x^2)} + \frac{e^3(a+cx^4)^{3/2}}{6c}$$

```
[In] Int[(d + e*x)^3*Sqrt[a + c*x^4],x]
```

```
[Out] (3*d^2*e*x^2*Sqrt[a + c*x^4])/4 + (6*a*d*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt[c]*
(Sqrt[a] + Sqrt[c]*x^2)) + (d*x*(5*d^2 + 9*e^2*x^2)*Sqrt[a + c*x^4])/15 + (
e^3*(a + c*x^4)^(3/2))/(6*c) + (3*a*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*
x^4]])/(4*Sqrt[c]) - (6*a^(5/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)*d*(5*Sqrt[c]*d^2 + 9*Sqrt[a]*e^2)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
```

```
x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\left(d^3+3de^2x^2\right)\sqrt{a+cx^4}+x\left(3d^2e+e^3x^2\right)\sqrt{a+cx^4}\right)\,dx \\
&= \int \left(d^3+3de^2x^2\right)\sqrt{a+cx^4}\,dx+\int x\left(3d^2e+e^3x^2\right)\sqrt{a+cx^4}\,dx \\
&= \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4}+\frac{1}{15}\int \frac{10ad^3+18ade^2x^2}{\sqrt{a+cx^4}}\,dx \\
&\quad +\frac{1}{2}\text{Subst}\left(\int \left(3d^2e+e^3x\right)\sqrt{a+cx^2}\,dx,x,x^2\right) \\
&= \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4}+\frac{e^3\left(a+cx^4\right)^{3/2}}{6c} \\
&\quad +\frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \sqrt{a+cx^2}\,dx,x,x^2\right)-\frac{\left(6a^{3/2}de^2\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{5\sqrt{c}} \\
&\quad +\frac{1}{15}\left(2ad\left(5d^2+\frac{9\sqrt{a}e^2}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx \\
&= \frac{3}{4}d^2ex^2\sqrt{a+cx^4}+\frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} \\
&\quad +\frac{e^3\left(a+cx^4\right)^{3/2}}{6c}-\frac{6a^{5/4}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}} \\
&\quad +\frac{a^{3/4}d\left(5\sqrt{c}d^2+9\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}} \\
&\quad +\frac{1}{4}\left(3ad^2e\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx,x,x^2\right) \\
&= \frac{3}{4}d^2ex^2\sqrt{a+cx^4}+\frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} \\
&\quad +\frac{e^3\left(a+cx^4\right)^{3/2}}{6c}-\frac{6a^{5/4}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}} \\
&\quad +\frac{a^{3/4}d\left(5\sqrt{c}d^2+9\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}} \\
&\quad +\frac{1}{4}\left(3ad^2e\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{a+cx^4}}\right)
\end{aligned}$$

$$= \frac{3}{4}d^2ex^2\sqrt{a+cx^4} + \frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4}$$
$$+ \frac{e^3(a+cx^4)^{3/2}}{6c} + \frac{3ad^2e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}} - \frac{6a^{5/4}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}}{\sqrt[4]{a}}\right.\right.}{5c^{3/4}\sqrt{a+cx^4}}$$
$$+ \frac{a^{3/4}d\left(5\sqrt{c}d^2+9\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.52

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx$$
$$= \frac{\sqrt{a+cx^4}\left(2ae^3\sqrt{1+\frac{cx^4}{a}} + 9cd^2ex^2\sqrt{1+\frac{cx^4}{a}} + 2ce^3x^4\sqrt{1+\frac{cx^4}{a}} + 9\sqrt{a}\sqrt{c}d^2e\,\mathrm{arcsinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) + 12cd^3x\,\mathrm{Hy}\right.}{12c\sqrt{1+\frac{cx^4}{a}}}$$

```
[In] Integrate[(d + e*x)^3*Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[a + c*x^4]*(2*a*e^3*Sqrt[1 + (c*x^4)/a] + 9*c*d^2*e*x^2*Sqrt[1 + (c*x
^4)/a] + 2*c*e^3*x^4*Sqrt[1 + (c*x^4)/a] + 9*Sqrt[a]*Sqrt[c]*d^2*e*ArcSinh[
(Sqrt[c]*x^2)/Sqrt[a]] + 12*c*d^3*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*
x^4)/a)] + 12*c*d*e^2*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^4)/a)]))
/(12*c*Sqrt[1 + (c*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.74

method	result
risch	$\frac{\left(10 e^{3} x^{4} c+36 d\,e^{2} x^{3} c+45 c\,d^{2} e\,x^{2}+20 d^{3} c x+10 a\,e^{3}\right) \sqrt{c\,x^{4}+a}}{60 c}+\frac{d a \left(\frac{20 d^{2} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}+\frac{36 i e^{2} \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1 \cdots}}{\cdots}\right)}{30 \cdots}$
default	$d^{3} \left(\frac{x \sqrt{c\,x^{4}+a}}{3}+\frac{2 a \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{3 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}\right)+\frac{e^{3} \left(c\,x^{4}+a\right)^{\frac{3}{2}}}{6 c}+3 d\,e^{2} \left(\frac{x^{3} \sqrt{c\,x^{4}+a}}{5}+\frac{2 i a^{\frac{3}{2}} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{\cdots}}{\cdots}\right.$
elliptic	$\frac{e^{3} x^{4} \sqrt{c\,x^{4}+a}}{6}+\frac{3 d\,e^{2} x^{3} \sqrt{c\,x^{4}+a}}{5}+\frac{3 d^{2} e\,x^{2} \sqrt{c\,x^{4}+a}}{4}+\frac{d^{3} x \sqrt{c\,x^{4}+a}}{3}+\frac{e^{3} a \sqrt{c\,x^{4}+a}}{6 c}+\frac{2 a\,d^{3} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\cdots}\right.}{3 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}$

```
[In] int((e*x+d)^3*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(10*c*e^3*x^4+36*c*d*e^2*x^3+45*c*d^2*e*x^2+20*c*d^3*x+10*a*e^3)/c*(c*
x^4+a)^(1/2)+1/30*d*a*(20*d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I
/a^(1/2)*c^(1/2))^(1/2),I)+36*I*e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/
a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/
c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^
(1/2))^(1/2),I))+45/2*e*d*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.16 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.53

$$\int (d+ex)^3 \sqrt{a+cx^4}\, dx$$
$$= \frac{144\, a \sqrt{c} d e^2 x \left(-\frac{a}{c}\right)^{\frac{3}{4}} E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right) \mid -1\right) + 45\, a \sqrt{c} d^2 e x \log\left(-2\, c x^4 - 2\, \sqrt{c x^4 + a} \sqrt{c} x^2 - a\right) + 16\, (5\, c d^3 \cdots}{\cdots}$$

```
[In] integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/120*(144*a*sqrt(c)*d*e^2*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x)
, -1) + 45*a*sqrt(c)*d^2*e*x*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 -
 a) + 16*(5*c*d^3 - 9*a*d*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a
/c)^(1/4)/x), -1) + 2*(10*c*e^3*x^5 + 36*c*d*e^2*x^4 + 45*c*d^2*e*x^3 + 20*
c*d^3*x^2 + 10*a*e^3*x + 72*a*d*e^2)*sqrt(c*x^4 + a))/(c*x)
```

Sympy [A] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.49

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx = \frac{\sqrt{a}d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{3\sqrt{a}d^2ex^2\sqrt{1+\frac{cx^4}{a}}}{4} + \frac{3\sqrt{a}de^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{3ad^2e\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{c}} + e^3\left(\begin{cases} \frac{\sqrt{a}x^4}{4} & \text{for } c=0 \\ \frac{(a+cx^4)^{\frac{3}{2}}}{6c} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((e*x+d)**3*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d**3*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)
/a)/(4*gamma(5/4)) + 3*sqrt(a)*d**2*e*x**2*sqrt(1 + c*x**4/a)/4 + 3*sqrt(a)
*d*e**2*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a
)/(4*gamma(7/4)) + 3*a*d**2*e*asinh(sqrt(c)*x**2/sqrt(a))/(4*sqrt(c)) + e**
3*Piecewise((sqrt(a)*x**4/4, Eq(c, 0)), ((a + c*x**4)**(3/2)/(6*c), True))
```

Maxima [F]

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}(ex+d)^3\,dx$$

```
[In] integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^3, x)
```

Giac [F]

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}(ex+d)^3\,dx$$

```
[In] integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx=\int \sqrt{c\,x^4+a}\,{\left(d+e\,x\right)}^3\,dx$$

```
[In] int((a + c*x^4)^(1/2)*(d + e*x)^3,x)
[Out] int((a + c*x^4)^(1/2)*(d + e*x)^3, x)
```

3.207 $\int (d+ex)^2\sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 326

$$\begin{aligned}\int (d+ex)^2\sqrt{a+cx^4}\,dx &= \frac{1}{2}dex^2\sqrt{a+cx^4}+\frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4}\\&+\frac{ade\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}-\frac{2a^{5/4}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}\\&+\frac{a^{3/4}\left(5\sqrt{c}d^2+3\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}\end{aligned}$$

```
[Out] 1/2*a*d*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/2*d*e*x^2*(c*x^4+a
)^(1/2)+1/15*x*(3*e^2*x^2+5*d^2)*(c*x^4+a)^(1/2)+2/5*a*e^2*x*(c*x^4+a)^(1/2
)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-2/5*a^(5/4)*e^2*(cos(2*arctan(c^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(
1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2
*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/15*a^(3/4)*(cos(2*arctan(c^(1/
4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*ar
ctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*e^2*a^(1/2)+5*d^2*c^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^
(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1899, 281, 201, 223, 212, 1191, 1212, 226, 1210}

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx$$
$$= \frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}e^2+5\sqrt{c}d^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}} - \frac{2a^{5/4}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}} + \frac{ade\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} + \frac{1}{15}x\sqrt{a+cx^4}\left(5d^2+3e^2x^2\right) + \frac{1}{2}dex^2\sqrt{a+cx^4} + \frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[In] Int[(d + e*x)^2*Sqrt[a + c*x^4],x]
```

```
[Out] (d*e*x^2*Sqrt[a + c*x^4])/2 + (2*a*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt[c]*(Sqrt[
a] + Sqrt[c]*x^2)) + (x*(5*d^2 + 3*e^2*x^2)*Sqrt[a + c*x^4])/15 + (a*d*e*Ar
cTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) - (2*a^(5/4)*e^2*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)*(5*S
qrt[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(
3/4)*Sqrt[a + c*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\text{integral} = \int \left(2dex\sqrt{a+cx^4} + \left(d^2+e^2x^2\right)\sqrt{a+cx^4}\right)\,dx$$

$$= (2de)\int x\sqrt{a+cx^4}\,dx + \int \left(d^2+e^2x^2\right)\sqrt{a+cx^4}\,dx$$

$$= \frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4}+\frac{1}{15}\int \frac{10ad^2+6ae^2x^2}{\sqrt{a+cx^4}}\,dx+(de)\text{Subst}\left(\int \sqrt{a+cx^2}\,dx,x,x^2\right)$$

$$= \frac{1}{2}dex^2\sqrt{a+cx^4}+\frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4}$$
$$+\frac{1}{2}(ade)\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx,x,x^2\right)-\frac{\left(2a^{3/2}e^2\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{5\sqrt{c}}$$
$$+\frac{1}{15}\left(2a\left(5d^2+\frac{3\sqrt{a}e^2}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{1}{2}dex^2\sqrt{a+cx^4}+\frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4}$$
$$-\frac{2a^{5/4}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}$$
$$+\frac{a^{3/4}\left(5\sqrt{c}d^2+3\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}$$
$$+\frac{1}{2}(ade)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{a+cx^4}}\right)$$

$$= \frac{1}{2}dex^2\sqrt{a+cx^4}+\frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+\frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4}+\frac{ade\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$
$$-\frac{2a^{5/4}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}$$
$$+\frac{a^{3/4}\left(5\sqrt{c}d^2+3\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.45

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx$$
$$=\frac{\sqrt{a+cx^4}\left(6\sqrt{c}d^2x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^4}{a}\right)+e\left(3d\left(\sqrt{c}x^2\sqrt{1+\frac{cx^4}{a}}+\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)\right)\right.\right.}{6\sqrt{c}\sqrt{1+\frac{cx^4}{a}}}$$

```
[In] Integrate[(d + e*x)^2*Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[a + c*x^4]*(6*Sqrt[c]*d^2*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^
4)/a)] + e*(3*d*(Sqrt[c]*x^2*Sqrt[1 + (c*x^4)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]
*x^2)/Sqrt[a]]) + 2*Sqrt[c]*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^
4)/a)])))/(6*Sqrt[c]*Sqrt[1 + (c*x^4)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x(6e^2x^2+15edx+10d^2)\sqrt{cx^4+a}}{30}+\frac{a\left(\frac{10d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4+a}}+\frac{6ie^2\sqrt{a}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4+a}\sqrt{c}}\right.}{15}$
default	$d^2\left(\frac{x\sqrt{cx^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4+a}}\right)+e^2\left(\frac{x^3\sqrt{cx^4+a}}{5}+\frac{2ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\right.\right.}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4+a}}\right.$
elliptic	$\frac{e^2x^3\sqrt{cx^4+a}}{5}+\frac{dex^2\sqrt{cx^4+a}}{2}+\frac{d^2x\sqrt{cx^4+a}}{3}+\frac{2ad^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4+a}}+\frac{eda\ln\left(2x^2\sqrt{c}+2\sqrt{cx^4+a}\right)}{2\sqrt{c}}$

```
[In] int((e*x+d)^2*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*x*(6*e^2*x^2+15*d*e*x+10*d^2)*(c*x^4+a)^(1/2)+1/15*a*(10*d^2/(I/a^(1/2
)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^
(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+6*I*e^2*a^(1
/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))
^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+15/2*e*d*ln(x^2*c^(1/2)
+(c*x^4+a)^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.51

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx = \frac{24\,a\sqrt{c}e^2x\left(-\frac{a}{c}\right)^{\frac{3}{4}}\,E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+15\,a\sqrt{c}dex\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+8\,(5\,cd^2-3}{60\,cx}$$

[In] integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] 1/60*(24*a*sqrt(c)*e^2*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1
) + 15*a*sqrt(c)*d*e*x*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) +
8*(5*c*d^2 - 3*a*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)
/x), -1) + 2*(6*c*e^2*x^4 + 15*c*d*e*x^3 + 10*c*d^2*x^2 + 12*a*e^2)*sqrt(c*
x^4 + a))/(c*x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.42

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx = \frac{\sqrt{a}d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}dex^2\sqrt{1+\frac{cx^4}{a}}}{2}+\frac{\sqrt{a}e^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ade\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}}$$

[In] integrate((e*x+d)**2*(c*x**4+a)**(1/2),x)

[Out] sqrt(a)*d**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)
/a)/(4*gamma(5/4)) + sqrt(a)*d*e*x**2*sqrt(1 + c*x**4/a)/2 + sqrt(a)*e**2*x
3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x4*exp_polar(I*pi)/a)/(4*gamm
a(7/4)) + a*d*e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c))

Maxima [F]

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}(ex+d)^2\,dx$$

```
[In] integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^2, x)
```

Giac [F]

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}(ex+d)^2\,dx$$

```
[In] integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx = \int \sqrt{c\,x^4+a}\,(d+e\,x)^2\,dx$$

```
[In] int((a + c*x^4)^(1/2)*(d + e*x)^2,x)
[Out] int((a + c*x^4)^(1/2)*(d + e*x)^2, x)
```

3.208 $\int (d+ex)\sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 158

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{a\text{earctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}+\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}$$

```
[Out] 1/4*a*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/3*d*x*(c*x^4+a)^(1/2
)+1/4*e*x^2*(c*x^4+a)^(1/2)+1/3*a^(3/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1899, 201, 226, 281, 223, 212}

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{a\text{earctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}+\frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}$$

```
[In] Int[(d + e*x)*Sqrt[a + c*x^4],x]
```

```
[Out] (d*x*Sqrt[a + c*x^4])/3 + (e*x^2*Sqrt[a + c*x^4])/4 + (a*e*ArcTanh[(Sqrt[c]
*x^2)/Sqrt[a + c*x^4]])/(4*Sqrt[c]) + (a^(3/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(3*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(d\sqrt{a+cx^4}+ex\sqrt{a+cx^4}\right)\,dx \\
&= d\int \sqrt{a+cx^4}\,dx+e\int x\sqrt{a+cx^4}\,dx \\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{3}(2ad)\int \frac{1}{\sqrt{a+cx^4}}\,dx+\frac{1}{2}e\text{Subst}\left(\int \sqrt{a+cx^2}\,dx,x,x^2\right) \\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4} \\
&\quad+\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}} \\
&\quad+\frac{1}{4}(ae)\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx,x,x^2\right) \\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4} \\
&\quad+\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}} \\
&\quad+\frac{1}{4}(ae)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{a+cx^4}}\right) \\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}} \\
&\quad+\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.69

$$
\int (d+ex)\sqrt{a+cx^4}\,dx = \frac{\sqrt{a+cx^4}\left(\sqrt{c}ex^2\sqrt{1+\frac{cx^4}{a}}+\sqrt{a}\text{earcsinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)+4\sqrt{c}dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^4}{a}\right)\right)}{4\sqrt{c}\sqrt{1+\frac{cx^4}{a}}}
$$

```
[In] Integrate[(d + e*x)*Sqrt[a + c*x^4],x]
```

[Out] (Sqrt[a + c*x^4]*(Sqrt[c]*e*x^2*Sqrt[1 + (c*x^4)/a] + Sqrt[a]*e*ArcSinh[(Sqrt[c]*x^2)/Sqrt[a]] + 4*Sqrt[c]*d*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^4)/a)]))/(4*Sqrt[c]*Sqrt[1 + (c*x^4)/a])

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{x(3ex+4d)\sqrt{c\,x^4+a}}{12}+\frac{2ad\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{ae\ln\left(x^2\sqrt{c}+\sqrt{c\,x^4+a}\right)}{4\sqrt{c}}$	119
default	$d\left(\frac{x\sqrt{c\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)+e\left(\frac{x^2\sqrt{c\,x^4+a}}{4}+\frac{a\ln\left(x^2\sqrt{c}+\sqrt{c\,x^4+a}\right)}{4\sqrt{c}}\right)$	129
elliptic	$\frac{e\,x^2\sqrt{c\,x^4+a}}{4}+\frac{dx\sqrt{c\,x^4+a}}{3}+\frac{2ad\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{ae\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+a}\right)}{4\sqrt{c}}$	130

[In] int((e*x+d)*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/12*x*(3*e*x+4*d)*(c*x^4+a)^(1/2)+2/3*a*d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/4*a*e*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.59

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \frac{16\,c^{\frac{3}{2}}d\left(-\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)+3\,a\sqrt{c}e\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+2\,\sqrt{cx^4+a}(3\,cex^2}{24\,c}$$

[In] integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] 1/24*(16*c^(3/2)*d*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + 3*a*sqrt(c)*e*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) + 2*sqrt(c*x^4 + a)*(3*c*e*x^2 + 4*c*d*x))/c

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.56

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \frac{\sqrt{a}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}ex^2\sqrt{1+\frac{cx^4}{a}}}{4}+\frac{ae\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{c}}$$

```
[In] integrate((e*x+d)*(c*x**4+a)**(1/2),x)
[Out] sqrt(a)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a)
/(4*gamma(5/4)) + sqrt(a)*e*x**2*sqrt(1 + c*x**4/a)/4 + a*e*asinh(sqrt(c)*x
**2/sqrt(a))/(4*sqrt(c))
```

Maxima [F]

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}(ex+d)\,dx$$

```
[In] integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d), x)
```

Giac [F]

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}(ex+d)\,dx$$

```
[In] integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \int \sqrt{c\,x^4+a}\,(d+e\,x)\,dx$$

```
[In] int((a + c*x^4)^(1/2)*(d + e*x),x)
[Out] int((a + c*x^4)^(1/2)*(d + e*x), x)
```

3.209 $\int \sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 105

$$\int \sqrt{a+cx^4}\,dx=\frac{1}{3}x\sqrt{a+cx^4}+\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}$$

```
[Out] 1/3*x*(c*x^4+a)^(1/2)+1/3*a^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4)
)),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(
1/2)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {201, 226}

$$\int \sqrt{a+cx^4}\,dx=\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}$$

```
[In] Int[Sqrt[a + c*x^4],x]
```

```
[Out] (x*Sqrt[a + c*x^4])/3 + (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(3
*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}x\sqrt{a+cx^4}+\frac{1}{3}(2a)\int\frac{1}{\sqrt{a+cx^4}}\,dx\\ &= \frac{1}{3}x\sqrt{a+cx^4}+\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int\sqrt{a+cx^4}\,dx=\frac{x(a+cx^4)-\frac{2ia\sqrt{1+\frac{cx^4}{a}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{3\sqrt{a+cx^4}}$$

```
[In] Integrate[Sqrt[a + c*x^4],x]
```

```
[Out] (x*(a + c*x^4) - ((2*I)*a*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*S
qrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(3*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x\sqrt{c\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	85
risch	$\frac{x\sqrt{c\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	85
elliptic	$\frac{x\sqrt{c\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	85

```
[In] int((c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(c*x^4+a)^(1/2)+2/3*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a
^(1/2)*c^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.39

$$\int \sqrt{a+cx^4}\,dx=\frac{2}{3}\,\sqrt{c}\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+\frac{1}{3}\,\sqrt{cx^4+a}x$$

```
[In] integrate((c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c)*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + 1/3*sqrt(
c*x^4 + a)*x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int \sqrt{a+cx^4}\,dx=\frac{\sqrt{a}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((c*x**4+a)**(1/2),x)
[Out] sqrt(a)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a)/(
4*gamma(5/4))
```

Maxima [F]

$$\int \sqrt{a+cx^4}\,dx=\int \sqrt{cx^4+a}\,dx$$

```
[In] integrate((c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \sqrt{a+cx^4}\,dx=\int \sqrt{cx^4+a}\,dx$$

```
[In] integrate((c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 18.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int \sqrt{a+cx^4}\,dx=\frac{x\sqrt{c\,x^4+a}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\sqrt{\frac{c\,x^4}{a}+1}}$$

```
[In] int((a + c*x^4)^(1/2),x)
[Out] (x*(a + c*x^4)^(1/2)*hypergeom([-1/2, 1/4], 5/4, -(c*x^4)/a))/((c*x^4)/a +
1)^(1/2)
```

3.210 $\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 730

$$\begin{aligned}\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx &= \frac{\sqrt{a+cx^4}}{2e} - \frac{\sqrt{c}dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt{-cd^4-ae^4}\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2e^3} \\ &+ \frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2e^3} - \frac{\sqrt{cd^4+ae^4}\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2e^3} \\ &+ \frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{a+cx^4}} \\ &- \frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2e^4\sqrt{a+cx^4}} \\ &+ \frac{\sqrt[4]{c}d(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} \\ &- \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}\end{aligned}$$

```
[Out] 1/2*d^2*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))*c^(1/2)/e^3-1/2*arctan(x*(-a*e
^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))*(-a*e^4-c*d^4)^(1/2)/e^3-1/2*arctanh((
c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))*(a*e^4+c*d^4)^(1/2)/e
^3+1/2*(c*x^4+a)^(1/2)/e-d*x*c^(1/2)*(c*x^4+a)^(1/2)/e^2/(a^(1/2)+x^2*c^(1/
2))+a^(1/4)*c^(1/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/e^2/(c*
```

```
x^4+a)^(1/2)+1/2*c^(1/4)*d*(a*e^4+c*d^4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^
2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2
))^2)^(1/2)/a^(1/4)/e^4/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/4*(a*e^
4+c*d^4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/
a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*
c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*
(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(
1/4)/d/e^4/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/2*a^(1/4)*c^(1/4)*d*
(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))
*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/
2))*(e^2+d^2*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/e^4
/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {1743, 1223, 1212, 226, 1210, 1231, 1721, 1262, 749, 858, 223, 212, 739}

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = -\frac{\sqrt{-ae^4-cd^4}\arctan\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2e^3} - \frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2e^4\sqrt{a+cx^4}} + \frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^4+cd^4\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)} - \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de^4\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)} + \frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{a+cx^4}} + \frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2e^3} - \frac{\sqrt{ae^4+cd^4}\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2e^3} - \frac{\sqrt{c}dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{\sqrt{a+cx^4}}{2e}$$

```
[In] Int[Sqrt[a + c*x^4]/(d + e*x),x]
```

```
[Out] Sqrt[a + c*x^4]/(2*e) - (Sqrt[c]*d*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[a] + Sqrt[
c]*x^2)) - (Sqrt[-(c*d^4) - a*e^4]*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*S
qrt[a + c*x^4])])/(2*e^3) + (Sqrt[c]*d^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x
```

```
^4]])/(2*e^3) - (Sqrt[c*d^4 + a*e^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^
4 + a*e^4]*Sqrt[a + c*x^4])])/(2*e^3) + (a^(1/4)*c^(1/4)*d*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4]) - (a^(1/4)*c^(1/4)*d*((Sqrt[c
]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*e^4*Sqrt[a
 + c*x^4]) + (c^(1/4)*d*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(2*a^(1/4)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c
]*d^2 - Sqrt[a]*e^2)*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*S
qrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^
(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1223

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[-(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
```

```
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1743

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[x*((a + c*x^4)^p/(d^
2 - e^2*x^2)), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int \frac{\sqrt{a+cx^4}}{d^2-e^2x^2}\,dx - e\int \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2}\,dx \\
&= \left(d\left(a+\frac{cd^4}{e^4}\right)\right)\int \frac{1}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx \\
&\quad - \frac{d\int \frac{cd^2+ce^2x^2}{\sqrt{a+cx^4}}\,dx}{e^4} - \frac{1}{2}e\,\text{Subst}\left(\int \frac{\sqrt{a+cx^2}}{d^2-e^2x}\,dx, x, x^2\right) \\
&= \frac{\sqrt{a+cx^4}}{2e} + \frac{\left(\sqrt{a}\sqrt{c}d\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{e^2} + \frac{\text{Subst}\left(\int \frac{-ae^2-cd^2x}{(d^2-e^2x)\sqrt{a+cx^2}}\,dx, x, x^2\right)}{2e} \\
&\quad + \frac{\left(\sqrt{c}d\left(a+\frac{cd^4}{e^4}\right)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}d^2+\sqrt{a}e^2} + \frac{\left(\sqrt{a}d\left(a+\frac{cd^4}{e^4}\right)e^2\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{\sqrt{c}d^2+\sqrt{a}e^2} \\
&\quad - \frac{\left(\sqrt{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{e^4}
\end{aligned}$$

$$= \frac{\sqrt{a+cx^4}}{2e} - \frac{\sqrt{c}dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt{-cd^4-ae^4}\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2e^3}$$
$$+ \frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{a+cx^4}}$$
$$+ \frac{\sqrt[4]{c}d\left(a+\frac{cd^4}{e^4}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$
$$- \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\sqrt{a+cx^4}}$$
$$- \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$
$$+ \frac{\left(cd^2\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+cx^2}}\,dx,x,x^2\right)}{2e^3} - \frac{\left(cd^4+ae^4\right)\text{Subst}\left(\int\frac{1}{\left(d^2-e^2x\right)\sqrt{a+cx^2}}\,dx,x,x^2\right)}{2e^3}$$

$$= \frac{\sqrt{a+cx^4}}{2e} - \frac{\sqrt{c}dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt{-cd^4-ae^4}\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2e^3}$$
$$+ \frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{a+cx^4}}$$
$$+ \frac{\sqrt[4]{c}d\left(a+\frac{cd^4}{e^4}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$
$$- \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\sqrt{a+cx^4}}$$
$$- \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$
$$+ \frac{\left(cd^2\right)\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{a+cx^4}}\right)}{2e^3}$$
$$+ \frac{\left(cd^4+ae^4\right)\text{Subst}\left(\int\frac{1}{cd^4+ae^4-x^2}\,dx,x,\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}}\right)}{2e^3}$$

$$= \frac{\sqrt{a+cx^4}}{2e} - \frac{\sqrt{c}dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt{-cd^4-ae^4}\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2e^3} + \frac{\sqrt{c}d^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2e^3} - \frac{\sqrt{cd^4+ae^4}\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2e^3} + \frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{a+cx^4}} + \frac{\sqrt[4]{c}d\left(a+\frac{cd^4}{e^4}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} - \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\sqrt{a+cx^4}} - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.23 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$$

$$= \frac{-2\sqrt{a}c^{3/4}d^2e^2\sqrt{1+\frac{cx^4}{a}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right) + 2c^{3/4}d^2\left(i\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{1+\frac{cx^4}{a}}\,\text{EllipticF}\left(i\text{arcsi}\right.}{}$$

```
[In] Integrate[Sqrt[a + c*x^4]/(d + e*x),x]
```

```
[Out] (-2*Sqrt[a]*c^(3/4)*d^2*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I
*Sqrt[c])/Sqrt[a]]*x], -1] + 2*c^(3/4)*d^2*(I*Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sq
rt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + S
qrt[(I*Sqrt[c])/Sqrt[a]]*(-2*(-1)^(1/4)*a^(1/4)*(c*d^4 + a*e^4)*Sqrt[1 + (c
*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/
4)*x)/a^(1/4)], -1] + c^(1/4)*d*e*(e^2*(a + c*x^4) - 2*Sqrt[-(c*d^4) - a*e^
4]*Sqrt[a + c*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/S
qrt[-(c*d^4) - a*e^4]] - Sqrt[c]*d^2*Sqrt[a + c*x^4]*Log[-(Sqrt[c]*x^2) + S
qrt[a + c*x^4]])))/(2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^(1/4)*d*e^4*Sqrt[a + c*x^
4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.34 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.55

method	result
default	$\frac{\sqrt{c\,x^4+a}}{2e}-\frac{d^3c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{\sqrt{c}\,d^2\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+a}\right)}{2e^3}-\frac{i\sqrt{c}\,d\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(\right.\right.}{e^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x}}$
elliptic	$\frac{\sqrt{c\,x^4+a}}{2e}-\frac{d^3c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{\sqrt{c}\,d^2\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+a}\right)}{2e^3}-\frac{i\sqrt{c}\,d\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(\right.\right.}{e^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x}}$
risch	$\frac{\sqrt{c\,x^4+a}}{2e}-\frac{\left(-e^4a-d^4c\right)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}+2a}{2\sqrt{\frac{c\,d^4}{e^4}+a}\,\sqrt{c\,x^4+a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}+a}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4+a}}\right)}{e^4}+\frac{cd\left(\frac{d^2\sqrt{1-\frac{i\sqrt{c}\,x}{\sqrt{a}}}}{}\right.}{e}$

[In] int((c*x^4+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/2*(c*x^4+a)^(1/2)/e-d^3*c/e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*c^(1/2)*d^2/e^3*ln(2*x^2*c^(1/2)+2*(c*x^4+a)^(1/2))-I*c^(1/2)/e^2*d*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+(a*e^4+c*d^4)/e^5*(-1/2/(c/e^4*d^4+a)^(1/2)*arctanh(1/2*(2*c*x^2/e^2*d^2+2*a)/(c/e^4*d^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)*e/d*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)*e^2/d^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))

Fricas [F]

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{cx^4+a}}{ex+d}\,dx$$

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + a)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$$

```
[In] integrate((c*x**4+a)**(1/2)/(e*x+d),x)
[Out] Integral(sqrt(a + c*x**4)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{cx^4+a}}{ex+d}\,dx$$

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + a)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{cx^4+a}}{ex+d}\,dx$$

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + a)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx=\int \frac{\sqrt{c\,x^4+a}}{d+e\,x}\,dx$$

```
[In] int((a + c*x^4)^(1/2)/(d + e*x),x)
[Out] int((a + c*x^4)^(1/2)/(d + e*x), x)
```

3.211 $\int \frac{\sqrt{a+cx^4}}{(d+ex)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 1221

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx$$

$$= \frac{2\sqrt{c}x\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)} + \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2} + \frac{\sqrt{-cd^4-ae^4}\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2de^3}$$

$$- \frac{(cd^4-ae^4)\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2de^3\sqrt{-cd^4-ae^4}} - \frac{\sqrt{c}d\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{e^3} + \frac{cd^3\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{e^3\sqrt{cd^4+ae^4}}$$

$$- \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{a+cx^4}}$$

$$+ \frac{3\sqrt[4]{a}\sqrt[4]{c}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4e^4\sqrt{a+cx^4}}$$

$$- \frac{\sqrt[4]{c}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\sqrt{a+cx^4}}$$

$$+ \frac{\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}e^4\sqrt{a+cx^4}}$$

$$- \frac{\sqrt[4]{c}(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$

$$+ \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d^2e^4\sqrt{a+cx^4}}$$

$$+ \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d^2e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$

```
[Out] -d*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))*c^(1/2)/e^3-1/2*(-a*e^4+c*d^4)*arct
an(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/d/e^3/(-a*e^4-c*d^4)^(1/2)+1
/2*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))*(-a*e^4-c*d^4)^(1/2)/
d/e^3+c*d^3*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/
e^3/(a*e^4+c*d^4)^(1/2)-d*(c*x^4+a)^(1/2)/e/(-e^2*x^2+d^2)+x*(c*x^4+a)^(1/2
)/(-e^2*x^2+d^2)+2*x*c^(1/2)*(c*x^4+a)^(1/2)/e^2/(a^(1/2)+x^2*c^(1/2))-2*a^
(1/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/
4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(
1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/e^2/(c*x^4+a)^(
```

```
1/2)-1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(
-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(
1/2))^2)^(1/2)/a^(1/4)/e^4/(c*x^4+a)^(1/2)+1/4*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(
1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2),1/
2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))^2*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a
^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d^2/e^4/(c*x^4+a)^(1/2)-1/2*c^
(1/4)*(a*e^4+c*d^4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/e
^4/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)+1/4*(a*e^4+c*d^4)*(cos(2*arcta
n(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(
sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^
(1/2)/c^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))
*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d^2/e^4/(e^2*a^(
1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)+1/4*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/
4)*x/a^(1/4))),1/2*2^(1/2))*(e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))
*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/e^4/(c*x^4+a)^(1/2)+3/4*
a^(1/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*c^(1/2))*(e^2+d^2*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/e^4/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 1221, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules

used = {2184, 1241, 1212, 226, 1210, 1231, 1721, 1262, 747, 858, 223, 212, 739, 1350, 1223}

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \frac{c\,\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{cx^4+a}}\right)d^3}{e^3\sqrt{cd^4+ae^4}} - \frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{cx^4+a}}\right)d}{e^3} - \frac{\sqrt{cx^4+a}\,d}{e\left(d^2-e^2x^2\right)} - \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{cx^4+a}} + \frac{3\sqrt[4]{a}\sqrt[4]{c}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4e^4\sqrt{cx^4+a}} - \frac{\sqrt[4]{c}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\sqrt{cx^4+a}} + \frac{\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}e^4\sqrt{cx^4+a}} - \frac{\sqrt[4]{c}\left(cd^4+ae^4\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{cx^4+a}} + \frac{2\sqrt{c}x\sqrt{cx^4+a}}{e^2\left(\sqrt{c}x^2+\sqrt{a}\right)} + \frac{x\sqrt{cx^4+a}}{d^2-e^2x^2} - \frac{\left(cd^4-ae^4\right)\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{cx^4+a}}\right)}{2e^3\sqrt{-cd^4-ae^4}d} + \frac{\sqrt{-cd^4-ae^4}\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{cx^4+a}}\right)}{2e^3d} + \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)^2\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}e^4\sqrt{cx^4+a}d^2} + \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{cx^4+a}d^2}$$

```
[In] Int[Sqrt[a + c*x^4]/(d + e*x)^2,x]
```

```
[Out] (2*Sqrt[c]*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[a] + Sqrt[c]*x^2)) - (d*Sqrt[a + c
*x^4])/(e*(d^2 - e^2*x^2)) + (x*Sqrt[a + c*x^4])/(d^2 - e^2*x^2) + (Sqrt[-(
c*d^4) - a*e^4]*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(
2*d*e^3) - ((c*d^4 - a*e^4)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a +
 c*x^4])])/(2*d*e^3*Sqrt[-(c*d^4) - a*e^4]) - (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x
^2)/Sqrt[a + c*x^4]])/e^3 + (c*d^3*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4
+ a*e^4]*Sqrt[a + c*x^4])])/(e^3*Sqrt[c*d^4 + a*e^4]) - (2*a^(1/4)*c^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4]) + (3*a^(1/4)*
```

```
c^(1/4)*((Sqrt[c]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(4*e^4*Sqrt[a + c*x^4]) - (c^(1/4)*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*e^4*Sqrt[a + c*x^4]) + (c^(1/4)*(
Sqrt[c]*d^2 + Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/
4)*e^4*Sqrt[a + c*x^4]) - (c^(1/4)*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(2*a^(1/4)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
 + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt
[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/
4)*d^2*e^4*Sqrt[a + c*x^4]) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(c*d^4 + a*e^4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d^2*e^4*(Sqrt[c]*d^2 + Sqrt[a]*
e^2)*Sqrt[a + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
```

```
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1223

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[-(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1241

```
Int[Sqrt[(a_) + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Symbol] :> Simp[x*
(Sqrt[a + c*x^4]/(2*d*(d + e*x^2))), x] + (Dist[c/(2*d*e^2), Int[(d - e*x^2
)/Sqrt[a + c*x^4], x], x] - Dist[(c*d^2 - a*e^2)/(2*d*e^2), Int[1/((d + e*x
```

```
^2)*Sqrt[a + c*x^4]), x], x]) /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^
2, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1350

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 2184

```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^nn)^p, (c/(c^2 - d^2*x^(2*n)) - d*(x^n/(
c^2 - d^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !
IntegerQ[p] && ILtQ[q, 0] && IGtQ[Log[2, nn/n], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d^2\sqrt{a+cx^4}}{(d^2-e^2x^2)^2} - \frac{2dex\sqrt{a+cx^4}}{(d^2-e^2x^2)^2} + \frac{e^2x^2\sqrt{a+cx^4}}{(-d^2+e^2x^2)^2}\right) dx \\
&= d^2 \int \frac{\sqrt{a+cx^4}}{(d^2-e^2x^2)^2}\, dx - (2de) \int \frac{x\sqrt{a+cx^4}}{(d^2-e^2x^2)^2}\, dx + e^2 \int \frac{x^2\sqrt{a+cx^4}}{(-d^2+e^2x^2)^2}\, dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{x\sqrt{a+cx^4}}{2\left(d^2-e^2x^2\right)} + \frac{1}{2}\left(a-\frac{cd^4}{e^4}\right)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx + \frac{c\int \frac{d^2+e^2x^2}{\sqrt{a+cx^4}}\,dx}{2e^4} \\
&\quad - (de)\text{Subst}\left(\int \frac{\sqrt{a+cx^2}}{\left(d^2-e^2x\right)^2}\,dx, x, x^2\right) + e^2\int \left(\frac{d^2\sqrt{a+cx^4}}{e^2\left(-d^2+e^2x^2\right)^2} \right. \\
&\qquad \left. + \frac{\sqrt{a+cx^4}}{e^2\left(-d^2+e^2x^2\right)}\right)dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)} + \frac{x\sqrt{a+cx^4}}{2\left(d^2-e^2x^2\right)} + d^2\int \frac{\sqrt{a+cx^4}}{\left(-d^2+e^2x^2\right)^2}\,dx \\
&\quad + \frac{1}{2}\left(\sqrt{a}\left(\sqrt{a}-\frac{\sqrt{c}d^2}{e^2}\right)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx - \frac{\left(\sqrt{a}\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2e^2} \\
&\quad + \frac{(cd)\text{Subst}\left(\int \frac{x}{\left(d^2-e^2x\right)\sqrt{a+cx^2}}\,dx, x, x^2\right)}{e} - \frac{\left(\sqrt{c}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2e^4} \\
&\quad + \frac{\left(c\left(d^2+\frac{\sqrt{a}e^2}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2e^4} + \int \frac{\sqrt{a+cx^4}}{-d^2+e^2x^2}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\sqrt{c}x\sqrt{a+cx^4}}{2e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)} + \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2} - \frac{\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{4de^3\sqrt{-cd^4-ae^4}} \\
&\quad - \frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2e^2\sqrt{a+cx^4}} \\
&\quad - \frac{\sqrt[4]{c}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}e^4\sqrt{a+cx^4}} \\
&\quad + \frac{\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}e^4\sqrt{a+cx^4}} \\
&\quad + \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{c}d^2e^4\sqrt{a+cx^4}} \\
&\quad + \frac{1}{2}\left(-a+\frac{cd^4}{e^4}\right)\int \frac{1}{\left(-d^2+e^2x^2\right)\sqrt{a+cx^4}}\,dx + \left(a+\frac{cd^4}{e^4}\right)\int \frac{1}{\left(-d^2+e^2x^2\right)\sqrt{a+cx^4}}\,dx - \frac{\int \frac{-cd^2-c}{\sqrt{a+c}}}{e^4}
\end{aligned}
$$

$$= \frac{\sqrt{c}x\sqrt{a+cx^4}}{2e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)} + \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2} - \frac{\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{4de^3\sqrt{-cd^4-ae^4}}$$

$$- \frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2e^2\sqrt{a+cx^4}}$$

$$- \frac{\sqrt[4]{c}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}e^4\sqrt{a+cx^4}}$$

$$+ \frac{\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}e^4\sqrt{a+cx^4}}$$

$$+ \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{c}d^2e^4\sqrt{a+cx^4}}$$

$$- \frac{1}{2}\left(\sqrt{a}\left(\sqrt{a}-\frac{\sqrt{c}d^2}{e^2}\right)\right)\int\frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(-d^2+e^2x^2\right)\sqrt{a+cx^4}}\,dx - \frac{(cd)\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{a+cx^4}}\right)}{e^3} - \frac{(cd^3}{}$$

$$
\begin{aligned}
&= \frac{2\sqrt{c}x\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)} + \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2} \\
&+ \frac{\sqrt{-cd^4-ae^4}\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2de^3} - \frac{(cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2de^3\sqrt{-cd^4-ae^4}} \\
&- \frac{\sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{e^3} + \frac{cd^3\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{e^3\sqrt{cd^4+ae^4}} \\
&- \frac{2\sqrt[4]{a}\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{a+cx^4}} \\
&- \frac{\sqrt[4]{c}(\sqrt{c}d^2-\sqrt{a}e^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\sqrt{a+cx^4}} \\
&- \frac{\sqrt[4]{c}\left(a+\frac{cd^4}{e^4}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} \\
&+ \frac{\sqrt[4]{c}(\sqrt{c}d^2+\sqrt{a}e^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}e^4\sqrt{a+cx^4}} \\
&+ \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\Pi\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d^2e^4\sqrt{a+cx^4}} \\
&+ \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\Pi\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d^2e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.46 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.32

$$
\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx
$$

$$
= \frac{-\frac{e^3(a+cx^4)}{d+ex} - \frac{2cd^3e\sqrt{a+cx^4}\arctan\left(\frac{\sqrt{c}\left(d^2-e^2x^2\right)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)}{\sqrt{-cd^4-ae^4}} - 2ia\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}e^2\sqrt{1+\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right) - \frac{2\sqrt{}}{}}{}
$$

```
[In] Integrate[Sqrt[a + c*x^4]/(d + e*x)^2,x]
```

```
[Out] (-((e^3*(a + c*x^4))/(d + e*x)) - (2*c*d^3*e*Sqrt[a + c*x^4]*ArcTan[(Sqrt[c
]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]])/Sqrt[-(c*
```

d^4) - a*e^4] - (2*I)*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (2*Sqrt[c]*(I*Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]] + 2*(-1)^(1/4)*a^(1/4)*c^(3/4)*d^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1] + Sqrt[c]*d*e*Sqrt[a + c*x^4]*Log[-(Sqrt[c]*x^2) + Sqrt[a + c*x^4]])/(e^4*Sqrt[a + c*x^4])

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.33

method	result
default	$-\frac{\sqrt{c\,x^4+a}}{e(ex+d)}+\frac{2c\,d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{d\sqrt{c}\,\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+a}\right)}{e^3}+\frac{2i\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}}{e^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}$
elliptic	$-\frac{\sqrt{c\,x^4+a}}{e(ex+d)}+\frac{2c\,d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{d\sqrt{c}\,\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+a}\right)}{e^3}+\frac{2i\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}}{e^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}$

[In] int((c*x^4+a)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] -1/e*(c*x^4+a)^(1/2)/(e*x+d)+2*c*d^2/e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-d*c^(1/2)/e^3*ln(2*x^2*c^(1/2)+2*(c*x^4+a)^(1/2))+2*I*c^(1/2)/e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))-2*d^3*c/e^5*(-1/2/(c/e^4*d^4+a)^(1/2)*arctanh(1/2*(2*c*x^2/e^2*d^2+2*a)/(c/e^4*d^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)*e/d*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)*e^2/d^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx$$

```
[In] integrate((c*x**4+a)**(1/2)/(e*x+d)**2,x)
[Out] Integral(sqrt(a + c*x**4)/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^4+a}}{(ex+d)^2}\,dx$$

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + a)/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^4+a}}{(ex+d)^2}\,dx$$

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + a)/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx=\int \frac{\sqrt{c\,x^4+a}}{{\left(d+e\,x\right)}^2}\,dx$$

```
[In] int((a + c*x^4)^(1/2)/(d + e*x)^2,x)
[Out] int((a + c*x^4)^(1/2)/(d + e*x)^2, x)
```

3.212 $\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 295

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \frac{e^3\sqrt{a+cx^4}}{2c}+\frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{3d^2e\,\mathrm{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} - \frac{3\sqrt[4]{a}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{d\left(\sqrt{c}d^2+3\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] 3/2*d^2*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/2*e^3*(c*x^4+a)^(1
/2)/c+3*d*e^2*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-3*a^(1/4)*d*e
^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4
)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^
(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/
2*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*e^2*a^(1/2)
+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/
2)/a^(1/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1899, 1212, 226, 1210, 1262, 655, 223, 212}

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}} - \frac{3\sqrt[4]{a}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{3d^2e\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} + \frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^3\sqrt{a+cx^4}}{2c}$$

```
[In] Int[(d + e*x)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (e^3*Sqrt[a + c*x^4])/(2*c) + (3*d*e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a]
 + Sqrt[c]*x^2)) + (3*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt
[c]) - (3*a^(1/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sq
rt[a + c*x^4]) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a
^(1/4)], 1/2])/(2*a^(1/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d^3+3de^2x^2}{\sqrt{a+cx^4}}+\frac{x(3d^2e+e^3x^2)}{\sqrt{a+cx^4}}\right)dx\\
&= \int \frac{d^3+3de^2x^2}{\sqrt{a+cx^4}}\,dx+\int \frac{x(3d^2e+e^3x^2)}{\sqrt{a+cx^4}}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{3d^2e+e^3x}{\sqrt{a+cx^2}}\,dx,x,x^2\right)-\frac{\left(3\sqrt{a}de^2\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}\\
&\quad+\left(d\left(d^2+\frac{3\sqrt{a}e^2}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx
\end{aligned}$$

$$= \frac{e^3\sqrt{a+cx^4}}{2c} + \frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{3\sqrt[4]{a}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{d\left(\sqrt{c}d^2+3\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}} + \frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+cx^2}}\,dx,x,x^2\right)$$

$$= \frac{e^3\sqrt{a+cx^4}}{2c} + \frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{3\sqrt[4]{a}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{d\left(\sqrt{c}d^2+3\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}} + \frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{a+cx^4}}\right)$$

$$= \frac{e^3\sqrt{a+cx^4}}{2c} + \frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{3d^2e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} - \frac{3\sqrt[4]{a}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{d\left(\sqrt{c}d^2+3\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \frac{e^3\sqrt{a+cx^4}}{2c} + \frac{3d^2e\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} + \frac{d^3x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}} + \frac{de^2x^3\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}}$$

```
[In] Integrate[(d + e*x)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (e^3*Sqrt[a + c*x^4])/(2*c) + (3*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4
]])/(2*Sqrt[c]) + (d^3*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/
4, -((c*x^4)/a)])/Sqrt[a + c*x^4] + (d*e^2*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/Sqrt[a + c*x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.74

method	result
default	$\frac{d^3\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}+\frac{e^3\sqrt{c\,x^4+a}}{2c}+\frac{3id\,e^2\sqrt{a}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}\sqrt{c}}+$
risch	$\frac{e^3\sqrt{c\,x^4+a}}{2c}+d\left(\frac{d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}+\frac{3ie^2\sqrt{a}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}\sqrt{c}}\right.$
elliptic	$\frac{e^3\sqrt{c\,x^4+a}}{2c}+\frac{d^3\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}+\frac{3d^2e\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+a}\right)}{2\sqrt{c}}+\frac{3id\,e^2\sqrt{a}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(F\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+}}$

```
[In] int((e*x+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d^3/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
+1/2*e^3*(c*x^4+a)^(1/2)/c+3*I*d*e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I
/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)
/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c
^(1/2))^(1/2),I))+3/2*d^2*e*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{12\,a\sqrt{c}de^2x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1) + 3\,a\sqrt{c}d^2ex\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+4\,(cd^3-3}{4\,acx}$$

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(12*a*sqrt(c)*d*e^2*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -
1) + 3*a*sqrt(c)*d^2*e*x*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a)
+ 4*(c*d^3 - 3*a*d*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/
4)/x), -1) + 2*(a*e^3*x + 6*a*d*e^2)*sqrt(c*x^4 + a))/(a*c*x)
```

Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.48

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = e^3\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for}\ c=0\\ \frac{\sqrt{a+cx^4}}{2c} & \text{otherwise}\end{cases}\right)+\frac{3d^2e\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}} + \frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{3de^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((e*x+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] e**3*Piecewise((x**4/(4*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**4)/(2*c), True)
) + 3*d**2*e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)) + d**3*x*gamma(1/4)*hy
per((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) +
3*d*e**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a
)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex+d)^3}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^3/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex+d)^3}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)^3/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(d+e\,x)^3}{\sqrt{c\,x^4+a}}\,dx$$

```
[In] int((d + e*x)^3/(a + c*x^4)^(1/2),x)
[Out] int((d + e*x)^3/(a + c*x^4)^(1/2), x)
```

3.213 $\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 263

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{d e\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}} - \frac{\sqrt[4]{a}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] d*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+e^2*x*(c*x^4+a)^(1/2)/c^(1
/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^
2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4
)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e^2+d^2*c^(1/2)/a^(1/2))*
((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1899, 281, 223, 212, 1212, 226, 1210}

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}} - \frac{\sqrt[4]{a}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{de\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}} + \frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[In] Int[(d + e*x)^2/Sqrt[a + c*x^4],x]
```

```
[Out] (e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (d*e*ArcTanh[(S
qrt[c]*x^2)/Sqrt[a + c*x^4]])/Sqrt[c] - (a^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*((Sqrt[c]*d^2)/Sqrt[
a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4
])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{2dex}{\sqrt{a+cx^4}}+\frac{d^2+e^2x^2}{\sqrt{a+cx^4}}\right) dx \\
&= (2de)\int \frac{x}{\sqrt{a+cx^4}}\,dx+\int \frac{d^2+e^2x^2}{\sqrt{a+cx^4}}\,dx \\
&= (de)\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx,x,x^2\right)-\frac{\left(\sqrt{a}e^2\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}+\left(d^2+\frac{\sqrt{a}e^2}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx \\
&= \frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} \\
&\quad +\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}} \\
&\quad +(de)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{a+cx^4}}\right)
\end{aligned}
$$

$$
= \frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{de\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}}
- \frac{\sqrt[4]{a}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}
+ \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.51

$$
\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \frac{d e \operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}} + \frac{d^2x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}}
+ \frac{e^2x^3\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)}{3\sqrt{a+cx^4}}
$$

```
[In] Integrate[(d + e*x)^2/Sqrt[a + c*x^4],x]
```

```
[Out] (d*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/Sqrt[c] + (d^2*x*Sqrt[1 + (c*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/Sqrt[a + c*x^4] + (e
^2*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/
(3*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.75

method	result
default	$\frac{d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}} + \frac{ie^2\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}} + \frac{ed\ln\left(x^2\sqrt{c}+\sqrt{c\,x}\right.}{\sqrt{c}}$
elliptic	$\frac{d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}} + \frac{ed\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+a}\right)}{\sqrt{c}} + \frac{ie^2\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$

```
[In] int((e*x+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
+I*e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/
2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+e*d*ln(x^2*c
^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \frac{2\,a\sqrt{c}e^2x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1) + a\sqrt{c}dex\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+2\,(cd^2-ae^2)\sqrt{}}{2\,acx}$$

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*sqrt(c)*e^2*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1)
+ a*sqrt(c)*d*e*x*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) + 2*(c*
d^2 - a*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1)
+ 2*sqrt(c*x^4 + a)*a*e^2)/(a*c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.40

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \frac{de\,\mathrm{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}} + \frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{e^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((e*x+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] d*e*asinh(sqrt(c)*x**2/sqrt(a))/sqrt(c) + d**2*x*gamma(1/4)*hyper((1/4, 1/2
), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e**2*x**3*gam
ma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamm
a(7/4))
```

Maxima [F]

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^2/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)^2/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(d+e\,x)^2}{\sqrt{c\,x^4+a}}\,dx$$

```
[In] int((d + e*x)^2/(a + c*x^4)^(1/2),x)
[Out] int((d + e*x)^2/(a + c*x^4)^(1/2), x)
```

3.214 $\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx=\frac{e\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

```
[Out] 1/2*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/2*d*(cos(2*arctan(c^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^
(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1899, 226, 281, 223, 212}

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx=\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{e\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

```
[In] Int[(d + e*x)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) + (d*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d}{\sqrt{a+cx^4}}+\frac{ex}{\sqrt{a+cx^4}}\right) dx \\
&= d\int \frac{1}{\sqrt{a+cx^4}}\,dx+e\int \frac{x}{\sqrt{a+cx^4}}\,dx \\
&= \frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{2}e\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx,x,x^2\right) \\
&= \frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{2}e\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{a+cx^4}}\right)
\end{aligned}
$$

$$= \frac{e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} + \frac{d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx = \frac{e\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} + \frac{dx\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}}$$

```
[In] Integrate[(d + e*x)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) + (d*x*Sqrt[1 + (c*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/Sqrt[a + c*x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{e\ln\left(x^2\sqrt{c}+\sqrt{c\,x^4+a}\right)}{2\sqrt{c}}$	96
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{e\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+a}\right)}{2\sqrt{c}}$	99

```
[In] int((e*x+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1
/2*e*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.60

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx = \frac{4\,c^{\frac{3}{2}}d\left(-\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+a\sqrt{c}e\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)}{4\,ac}$$

```
[In] integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] 1/4*(4*c^(3/2)*d*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + a*sq
rt(c)*e*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a))/(a*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx = \frac{e\,\mathrm{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}}+\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((e*x+d)/(c*x**4+a)**(1/2),x)
[Out] e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)) + d*x*gamma(1/4)*hyper((1/4, 1/2)
, (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx = \int \frac{ex+d}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x + d)/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx=\int \frac{ex+d}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x + d)/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx=\int \frac{d+e\,x}{\sqrt{c\,x^4+a}}\,dx$$

```
[In] int((d + e*x)/(a + c*x^4)^(1/2),x)
[Out] int((d + e*x)/(a + c*x^4)^(1/2), x)
```

3.215 $\int \frac{1}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 88

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx=\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

```
[Out] 1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c
^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(c*x^4+a)
^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {226}

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx=\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

```
[In] Int[1/Sqrt[a + c*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]
)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = -\frac{i\sqrt{1+\frac{cx^4}{a}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

```
[In] Integrate[1/Sqrt[a + c*x^4],x]
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x],
 -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	70
elliptic	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	70

```
[In] int(1/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = -\frac{\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}} F\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right) \mid -1\right)}{c}$$

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] -sqrt(a)*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)), -1)/c
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate(1/(c*x**4+a)**(1/2),x)
[Out] x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 17.62 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = \frac{x\sqrt{\frac{c\,x^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\sqrt{c\,x^4+a}}$$

```
[In] int(1/(a + c*x^4)^(1/2),x)
[Out] (x*((c*x^4)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(c*x^4)/a))/(a + c*x^4
)^(1/2)
```

3.216 $\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 405

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \frac{e\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} - \frac{e\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}} + \frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$

```
[Out] 1/2*e*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-c*d^4)^(1/
2)-1/2*e*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/(a*
e^4+c*d^4)^(1/2)+1/2*c^(1/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/c
os(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),
1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2
)/a^(1/4)/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/4*(cos(2*arctan(c^(1/
4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c
^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^
4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d/(e^2*a^(1/2)+d^2*c^(1
/2))/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1739, 1231, 226, 1721, 1262, 739, 212}

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx$$
$$= \frac{e\arctan\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-ae^4-cd^4}} + \frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}$$
$$- \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}$$
$$- \frac{e\,\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

```
[In] Int[1/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] (e*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*Sqrt[-(c*d^
4) - a*e^4]) - (e*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a +
 c*x^4])])/(2*Sqrt[c*d^4 + a*e^4]) + (c^(1/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sq
rt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[
c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*(Sqr
t[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d\int \frac{1}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx - e\int \frac{x}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx\\ &= -\left(\frac{1}{2}e\text{Subst}\left(\int \frac{1}{(d^2-e^2x)\sqrt{a+cx^2}}\,dx, x, x^2\right)\right)\\ &\quad+\frac{\left(\sqrt{c}d\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}d^2+\sqrt{a}e^2}+\frac{\left(\sqrt{a}de^2\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{\sqrt{c}d^2+\sqrt{a}e^2}\end{aligned}$$

$$= \frac{e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} + \frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$
$$- \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$
$$+ \frac{1}{2}e\text{Subst}\left(\int\frac{1}{cd^4+ae^4-x^2}\,dx,x,\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}}\right)$$
$$= \frac{e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} - \frac{e\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}}$$
$$+ \frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$
$$- \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.49

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx$$
$$= \frac{\sqrt{1+\frac{cx^4}{a}}\left(-2\sqrt[4]{-1}\sqrt[4]{a}\sqrt{1+\frac{cd^4}{ae^4}}e\,\text{EllipticPi}\left(\frac{i\sqrt{a}e^2}{\sqrt{c}d^2},\arcsin\left(\frac{(-1)^{3/4}\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)+\sqrt[4]{c}d\log\left(\frac{-d^2+\cdots}{cd^2x^2+ae^2\left(1+\sqrt{\cdots}\right.}\right.\right.}{2\sqrt[4]{c}d\sqrt{1+\frac{cd^4}{ae^4}}e\sqrt{a+cx^4}}$$

```
[In] Integrate[1/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] (Sqrt[1 + (c*x^4)/a]*(-2*(-1)^(1/4)*a^(1/4)*Sqrt[1 + (c*d^4)/(a*e^4)]*e*Ell
ipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4
)], -1] + c^(1/4)*d*Log[(-d^2 + e^2*x^2)/(c*d^2*x^2 + a*e^2*(1 + Sqrt[1 + (
c*d^4)/(a*e^4)]*Sqrt[1 + (c*x^4)/a]))]))/(2*c^(1/4)*d*Sqrt[1 + (c*d^4)/(a*e
^4)]*e*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.42

method	result	size
default	$\frac{-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}+2a}{2\sqrt{\frac{c\,d^4}{e^4}+a}\,\sqrt{c\,x^4+a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}+a}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4+a}}}{e}$	169
elliptic	$\frac{-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}+2a}{2\sqrt{\frac{c\,d^4}{e^4}+a}\,\sqrt{c\,x^4+a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}+a}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4+a}}}{e}$	169

```
[In] int(1/(e*x+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/e*(-1/2/(c/e^4*d^4+a)^(1/2)*arctanh(1/2*(2*c*x^2/e^2*d^2+2*a)/(c/e^4*d^4+
a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)*e/d*(1-I/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x
*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)*e^2/d^2,(-I/a^(1/2)*c^(1/2))^
(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\,(d+ex)}\,dx$$

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)}\,dx$$

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)}\,dx$$

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+a}\,(d+e\,x)}\,dx$$

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x)),x)
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x)), x)
```

3.217 $\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 610

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = -\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)(d+ex)}+\frac{\sqrt{c}e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{cd^3e\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{(-cd^4-ae^4)^{3/2}} - \frac{cd^3e\,\mathrm{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{(cd^4+ae^4)^{3/2}} - \frac{\sqrt[4]{a}\sqrt[4]{c}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{(cd^4+ae^4)\sqrt{a+cx^4}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} - \frac{c^{3/4}d^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)(cd^4+ae^4)\sqrt{a+cx^4}}$$

```
[Out] -c*d^3*e*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-c*d^4)^
(3/2)-c*d^3*e*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2)
)/(a*e^4+c*d^4)^(3/2)-e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)+e^2*x*c^(1/
2)*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*c^(1/4)*e^2*
(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))
*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/
2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)
+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1
/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(e^2*a^(
```

```
1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/2*c^(3/4)*d^2*(cos(2*arctan(c^(1/4)*x/a
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(
c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2)
,1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(
a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a*e^4+c*d^4)/(e^2*a^(1/2)+d^2*c^(1/2
))/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {1741, 1756, 12, 1262, 739, 212, 1729, 1210, 1723, 226, 1721}

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx =$$
$$-\frac{c^{3/4}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\left(ae^4+cd^4\right)}$$
$$-\frac{\sqrt[4]{a}\sqrt[4]{c}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+cx^4}\left(ae^4+cd^4\right)}$$
$$+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}$$
$$-\frac{cd^3e\arctan\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{\left(-ae^4-cd^4\right)^{3/2}}-\frac{cd^3e\,\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\left(ae^4+cd^4\right)^{3/2}}$$
$$-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}+\frac{\sqrt{c}e^2x\sqrt{a+cx^4}}{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^4+cd^4\right)}$$

```
[In] Int[1/((d + e*x)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] -((e^3*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d + e*x))) + (Sqrt[c]*e^2*x*Sqrt[
a + c*x^4])/((c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)) - (c*d^3*e*ArcTan[(Sq
rt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(-(c*d^4) - a*e^4)^(3/2) -
(c*d^3*e*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])]
)/(c*d^4 + a*e^4)^(3/2) - (a^(1/4)*c^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], 1/2])/((c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/
4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4
]) - (c^(3/4)*d^2*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^
```

```
2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a
^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
```

```
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1723

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1741

```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[e
^3*(d + e*x)^(q + 1)*(Sqrt[a + c*x^4]/((q + 1)*(c*d^4 + a*e^4))), x] + Dist
[c/((q + 1)*(c*d^4 + a*e^4)), Int[((d + e*x)^(q + 1)/Sqrt[a + c*x^4])*Simp[
d^3*(q + 1) - d^2*e*(q + 1)*x + d*e^2*(q + 1)*x^2 - e^3*(q + 3)*x^3, x], x]
, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^4 + a*e^4, 0] && ILtQ[q, -1]
```

Rule 1756

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rubi steps

$$\text{integral} = -\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)(d+ex)} - \frac{c\int \frac{-d^3+d^2ex-de^2x^2-e^3x^3}{(d+ex)\sqrt{a+cx^4}}\,dx}{cd^4+ae^4}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)(d+ex)} - \frac{c\int \frac{2d^3ex}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{cd^4+ae^4} - \frac{c\int \frac{-d^4-2d^2e^2x^2+e^4x^4}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{cd^4+ae^4}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)(d+ex)} + \frac{\int \frac{cd^4e^2+\sqrt{a}\sqrt{c}d^2e^4+(2cd^2e^4-e^4(cd^2+\sqrt{a}\sqrt{c}e^2))x^2}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{e^2(cd^4+ae^4)} - \frac{(2cd^3e)\int \frac{x}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{cd^4+ae^4} - \frac{\left(\sqrt{a}\sqrt{c}e^2\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{cd^4+ae^4}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)(d+ex)} + \frac{\sqrt{c}e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}\sqrt[4]{c}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{(cd^4+ae^4)\sqrt{a+cx^4}} + \frac{\sqrt{c}\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}d^2+\sqrt{a}e^2} - \frac{(cd^3e)\,\mathrm{Subst}\left(\int \frac{1}{(d^2-e^2x)\sqrt{a+cx^2}}\,dx, x, x^2\right)}{cd^4+ae^4} + \frac{\left(2\sqrt{a}cd^4e^2\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)(cd^4+ae^4)}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)(d+ex)} + \frac{\sqrt{c}e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{cd^3e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{(-cd^4-ae^4)^{3/2}} - \frac{\sqrt[4]{a}\sqrt[4]{c}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{(cd^4+ae^4)\sqrt{a+cx^4}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} - \frac{c^{3/4}d^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)(cd^4+ae^4)\sqrt{a+cx^4}} + \frac{(cd^3e)\,\mathrm{Subst}\left(\int \frac{1}{cd^4+ae^4-x^2}\,dx, x, \frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}}\right)}{cd^4+ae^4}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)(d+ex)} + \frac{\sqrt{c}e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$- \frac{cd^3e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{(-cd^4-ae^4)^{3/2}} - \frac{cd^3e\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{(cd^4+ae^4)^{3/2}}$$
$$- \frac{\sqrt[4]{a}\sqrt[4]{c}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{(cd^4+ae^4)\sqrt{a+cx^4}}$$
$$+ \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$
$$- \frac{c^{3/4}d^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)(cd^4+ae^4)\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.73

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx =$$
$$-\frac{\sqrt{a}\sqrt{c}e^2\sqrt{-cd^4-ae^4}(d+ex)\sqrt{1+\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\Big|-1\right)+i\sqrt{c}\left(\sqrt{c}d^2+i\sqrt{a}e^2\right)\sqrt{-cd^4-a}}{}$$

```
[In] Integrate[1/((d + e*x)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] -((Sqrt[a]*Sqrt[c]*e^2*Sqrt[-(c*d^4) - a*e^4]*(d + e*x)*Sqrt[1 + (c*x^4)/a]
*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*Sqrt[c]*(Sqrt[c]
*d^2 + I*Sqrt[a]*e^2)*Sqrt[-(c*d^4) - a*e^4]*(d + e*x)*Sqrt[1 + (c*x^4)/a]*
EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - Sqrt[(I*Sqrt[c])/Sq
rt[a]]*(e^3*Sqrt[-(c*d^4) - a*e^4]*(a + c*x^4) - 2*c*d^3*e*(d + e*x)*Sqrt[a
 + c*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d
^4) - a*e^4]] + 2*(-1)^(1/4)*a^(1/4)*c^(3/4)*d^2*Sqrt[-(c*d^4) - a*e^4]*(d
+ e*x)*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin
[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*(-(c*d^4
) - a*e^4)^(3/2)*(d + e*x)*Sqrt[a + c*x^4]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.69

method	result
default	$-\frac{e^{3}\sqrt{c\,x^{4}+a}}{\left(e^{4}a+d^{4}c\right)\left(ex+d\right)}-\frac{c\,d^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\left(e^{4}a+d^{4}c\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{i e^{2}\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\right.\right.}{\left(e^{4}a+d^{4}c\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$
elliptic	$-\frac{e^{3}\sqrt{c\,x^{4}+a}}{\left(e^{4}a+d^{4}c\right)\left(ex+d\right)}-\frac{c\,d^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\left(e^{4}a+d^{4}c\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{i e^{2}\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\right.\right.}{\left(e^{4}a+d^{4}c\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$

```
[In] int(1/(e*x+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)-c*d^2/(a*e^4+c*d^4)/(I/a^(1/2)*c
^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/
2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I*e^2*c^(1/2)/(
a*e^4+c*d^4)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)
*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+2*d^3*c/(a*e^4
+c*d^4)/e*(-1/2/(c/e^4*d^4+a)^(1/2)*arctanh(1/2*(2*c*x^2/e^2*d^2+2*a)/(c/e^
4*d^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)*e/d*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellipt
icPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)*e^2/d^2,(-I/a^(1/2)*c^(
1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\,(d+ex)^2}\,dx$$

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)^2}\,dx$$

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^2), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)^2}\,dx$$

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+a}\,(d+e\,x)^2}\,dx$$

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x)^2),x)
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x)^2), x)
```

3.218 $\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 659

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = -\frac{e^3\sqrt{a+cx^4}}{2(cd^4+ae^4)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{(cd^4+ae^4)^2(d+ex)} + \frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)^2(\sqrt{a}+\sqrt{c}x^2)} + \frac{3cd^2e(cd^4-ae^4)\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2(-cd^4-ae^4)^{5/2}} - \frac{3cd^2e(cd^4-ae^4)\operatorname{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2(cd^4+ae^4)^{5/2}} - \frac{3\sqrt[4]{a}c^{5/4}d^3e^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{(cd^4+ae^4)^2\sqrt{a+cx^4}} + \frac{c^{3/4}d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(cd^4+ae^4)\sqrt{a+cx^4}} - \frac{3c^{3/4}d(\sqrt{c}d^2-\sqrt{a}e^2)^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticPi}\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}(cd^4+ae^4)^2\sqrt{a+cx^4}}$$

```
[Out] 3/2*c*d^2*e*(-a*e^4+c*d^4)*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2
))/(-a*e^4-c*d^4)^(5/2)-3/2*c*d^2*e*(-a*e^4+c*d^4)*arctanh((c*d^2*x^2+a*e^2
)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/(a*e^4+c*d^4)^(5/2)-1/2*e^3*(c*x^4+a
)^(1/2)/(a*e^4+c*d^4)/(e*x+d)^2-3*c*d^3*e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)^2
/(e*x+d)+3*c^(3/2)*d^3*e^2*x*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)^2/(a^(1/2)+x^2*c
^(1/2))-3*a^(1/4)*c^(5/4)*d^3*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4)
)),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(
```

```
1/2)/(a*e^4+c*d^4)^2/(c*x^4+a)^(1/2)+1/2*c^(3/4)*d*(cos(2*arctan(c^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(
c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+
x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)-3/4*c^(3/4)*d*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2
/d^2/e^2/a^(1/2)/c^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))^2*(a^(1/2)
+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a*e^4+c*d^
4)^2/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {1741, 1753, 1756, 12, 1262, 739, 212, 1729, 1210, 1723, 226, 1721}

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx$$
$$=\frac{c^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\,(ae^4+cd^4)}$$
$$-\frac{3\sqrt[4]{a}c^{5/4}d^3e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+cx^4}\,(ae^4+cd^4)^2}$$
$$-\frac{3c^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)^2\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{a+cx^4}\,(ae^4+cd^4)^2}$$
$$+\frac{3cd^2e(cd^4-ae^4)\arctan\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2\left(-ae^4-cd^4\right)^{5/2}}-\frac{3cd^2e(cd^4-ae^4)\,\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\left(ae^4+cd^4\right)^{5/2}}$$
$$+\frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{\left(\sqrt{a}+\sqrt{c}x^2\right)(ae^4+cd^4)^2}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\,(ae^4+cd^4)}-\frac{3cd^3e^3\sqrt{a+cx^4}}{(d+ex)\,(ae^4+cd^4)^2}$$

```
[In] Int[1/((d + e*x)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] -1/2*(e^3*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d + e*x)^2) - (3*c*d^3*e^3*Sqr
t[a + c*x^4])/((c*d^4 + a*e^4)^2*(d + e*x)) + (3*c^(3/2)*d^3*e^2*x*Sqrt[a +
 c*x^4])/((c*d^4 + a*e^4)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (3*c*d^2*e*(c*d^4 -
a*e^4)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*(-(c*d^
4) - a*e^4)^(5/2)) - (3*c*d^2*e*(c*d^4 - a*e^4)*ArcTanh[(a*e^2 + c*d^2*x^2)
/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(2*(c*d^4 + a*e^4)^(5/2)) - (3*a^(
1/4)*c^(5/4)*d^3*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/((c*d^4 + a*e^
```

```
4)^2*Sqrt[a + c*x^4]) + (c^(3/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)
/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(2*a^(1/4)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (3*c^(3/4)*d*(Sqrt[c]*d^2 - S
qrt[a]*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2)
, 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(c*d^4 + a*e^4)^2*Sqrt[a
+ c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1723

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1741

```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[e
^3*(d + e*x)^(q + 1)*(Sqrt[a + c*x^4]/((q + 1)*(c*d^4 + a*e^4))), x] + Dist
[c/((q + 1)*(c*d^4 + a*e^4)), Int[((d + e*x)^(q + 1)/Sqrt[a + c*x^4])*Simp[
d^3*(q + 1) - d^2*e*(q + 1)*x + d*e^2*(q + 1)*x^2 - e^3*(q + 3)*x^3, x], x]
, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^4 + a*e^4, 0] && ILtQ[q, -1]
```

Rule 1753

```
Int[((Px_)*((d_) + (e_.)*(x_))^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :
> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D =
Coeff[Px, x, 3]}, Simp[(-(d^3*D - C*d^2*e + B*d*e^2 - A*e^3))*(d + e*x)^(q
+ 1)*(Sqrt[a + c*x^4]/((q + 1)*(c*d^4 + a*e^4))), x] + Dist[1/((q + 1)*(c*d
^4 + a*e^4)), Int[((d + e*x)^(q + 1)/Sqrt[a + c*x^4])*Simp[(q + 1)*(a*e*(d^
2*D - C*d*e + B*e^2) + A*d*(c*d^2)) - (e*(q + 1)*(A*c*d^2 + a*e*(d*D - C*e)
) - B*d*(c*d^2*(q + 1)))*x + (q + 1)*(D*e*(a*e^2) + c*d*(C*d^2 - e*(B*d - A
*e)))*x^2 + c*(q + 3)*(d^3*D - C*d^2*e + B*d*e^2 - A*e^3)*x^3, x], x], x]]
/; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*
```

```
d^4 + a*e^4, 0] && LtQ[q, -1]
```

Rule 1756

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rubi steps

$$\text{integral} = -\frac{e^3\sqrt{a+cx^4}}{2(cd^4+ae^4)(d+ex)^2} - \frac{c\int \frac{-2d^3+2d^2ex-2de^2x^2}{(d+ex)^2\sqrt{a+cx^4}}\,dx}{2(cd^4+ae^4)}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2(cd^4+ae^4)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{(cd^4+ae^4)^2(d+ex)} + \frac{c\int \frac{2d^2(cd^4-2ae^4)-2de(2cd^4-ae^4)x+6cd^4e^2x^2+6cd^3e^3x^3}{(d+ex)\sqrt{a+cx^4}}\,dx}{2(cd^4+ae^4)^2}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2(cd^4+ae^4)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{(cd^4+ae^4)^2(d+ex)} + \frac{c\int \frac{(-2d^2e(cd^4-2ae^4)-2d^2e(2cd^4-ae^4))x}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{2(cd^4+ae^4)^2} + \frac{c\int \frac{2d^3(cd^4-2ae^4)+(6cd^5e^2+2de^2(2cd^4-ae^4))x^2-6cd^3e^4x^4}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{2(cd^4+ae^4)^2}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2(cd^4+ae^4)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{(cd^4+ae^4)^2(d+ex)} - \frac{\int \frac{-6\sqrt{a}c^{3/2}d^5e^4-2cd^3e^2(cd^4-2ae^4)+(6cd^3e^4(cd^2+\sqrt{a}\sqrt{c}e^2)-ce^2(6cd^5e^2+2de^2(2cd^4-ae^4)))x^2}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{2e^2(cd^4+ae^4)^2}$$
$$- \frac{\left(3\sqrt{a}c^{3/2}d^3e^2\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{(cd^4+ae^4)^2} - \frac{\left(3cd^2e(cd^4-ae^4)\right)\int \frac{x}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{(cd^4+ae^4)^2}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2(cd^4+ae^4)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{(cd^4+ae^4)^2(d+ex)} + \frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)^2(\sqrt{a}+\sqrt{c}x^2)}$$

$$- \frac{3\sqrt[4]{a}c^{5/4}d^3e^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{(cd^4+ae^4)^2\sqrt{a+cx^4}}$$

$$+ \frac{\left(3\sqrt{a}cd^3e^2(\sqrt{c}d^2-\sqrt{a}e^2)\right)\int\frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{(cd^4+ae^4)^2}$$

$$- \frac{(3cd^2e(cd^4-ae^4))\,\text{Subst}\left(\int\frac{1}{(d^2-e^2x)\sqrt{a+cx^2}}\,dx,x,x^2\right)}{2(cd^4+ae^4)^2} + \frac{(cd)\int\frac{1}{\sqrt{a+cx^4}}\,dx}{cd^4+ae^4}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2(cd^4+ae^4)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{(cd^4+ae^4)^2(d+ex)}$$

$$+ \frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)^2(\sqrt{a}+\sqrt{c}x^2)} + \frac{3cd^2e(cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2(-cd^4-ae^4)^{5/2}}$$

$$- \frac{3\sqrt[4]{a}c^{5/4}d^3e^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{(cd^4+ae^4)^2\sqrt{a+cx^4}}$$

$$+ \frac{c^{3/4}d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}(cd^4+ae^4)\sqrt{a+cx^4}}$$

$$- \frac{3c^{3/4}d(\sqrt{c}d^2-\sqrt{a}e^2)^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\Pi\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}(cd^4+ae^4)^2\sqrt{a+cx^4}}$$

$$+ \frac{(3cd^2e(cd^4-ae^4))\,\text{Subst}\left(\int\frac{1}{cd^4+ae^4-x^2}\,dx,x,\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}}\right)}{2(cd^4+ae^4)^2}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2(cd^4+ae^4)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{(cd^4+ae^4)^2(d+ex)} + \frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)^2(\sqrt{a}+\sqrt{c}x^2)} + \frac{3cd^2e(cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2(-cd^4-ae^4)^{5/2}} - \frac{3cd^2e(cd^4-ae^4)\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2(cd^4+ae^4)^{5/2}} - \frac{3\sqrt[4]{a}c^{5/4}d^3e^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{(cd^4+ae^4)^2\sqrt{a+cx^4}} + \frac{c^{3/4}d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}(cd^4+ae^4)\sqrt{a+cx^4}} - \frac{3c^{3/4}d(\sqrt{c}d^2-\sqrt{a}e^2)^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\Pi\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}(cd^4+ae^4)^2\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.43 (sec) , antiderivative size = 614, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx =$$

$$-\frac{\frac{cd^4e^3\sqrt{a+cx^4}}{(d+ex)^2} + \frac{ae^7\sqrt{a+cx^4}}{(d+ex)^2} + \frac{6cd^3e^3\sqrt{a+cx^4}}{d+ex} - \frac{6c^2d^6e\arctan\left(\frac{\sqrt{c}(d^2-e^2x^2)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)}{\sqrt{-cd^4-ae^4}} + \frac{6acd^2e^5\arctan\left(\frac{\sqrt{c}(d^2-e^2x^2)+e^2\sqrt{a+c}}{\sqrt{-cd^4-ae^4}}\right)}{\sqrt{-cd^4-ae^4}}}{}$$

```
[In] Integrate[1/((d + e*x)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] -1/2*((c*d^4*e^3*Sqrt[a + c*x^4])/(d + e*x)^2 + (a*e^7*Sqrt[a + c*x^4])/(d
+ e*x)^2 + (6*c*d^3*e^3*Sqrt[a + c*x^4])/(d + e*x) - (6*c^2*d^6*e*ArcTan[(S
qrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]])/Sqrt
[-(c*d^4) - a*e^4] + (6*a*c*d^2*e^5*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*S
qrt[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]])/Sqrt[-(c*d^4) - a*e^4] + ((6*I)*a*
Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d^3*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh
[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[a + c*x^4] + ((2*I)*c*d*(-2*c*d^4
- (3*I)*Sqrt[a]*Sqrt[c]*d^2*e^2 + a*e^4)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*Ar
cSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[a
+ c*x^4]) + (6*(-1)^(1/4)*a^(1/4)*c^(7/4)*d^5*Sqrt[1 + (c*x^4)/a]*EllipticP
i[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1
```

```
])/Sqrt[a + c*x^4] - (6*(-1)^(1/4)*a^(5/4)*c^(3/4)*d*e^4*Sqrt[1 + (c*x^4)/a
]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a
^(1/4)], -1])/Sqrt[a + c*x^4])/(c*d^4 + a*e^4)^2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.73

method	result
default	$-\frac{e^3\sqrt{c\,x^4+a}}{2(e^4a+d^4c)(ex+d)^2}-\frac{3c\,d^3e^3\sqrt{c\,x^4+a}}{(e^4a+d^4c)^2(ex+d)}+\frac{dc(e^4a-2d^4c)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{(e^4a+d^4c)^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}+\frac{3ic^{\frac{3}{2}}d^3e^2\sqrt{a}\sqrt{1-\frac{i\sqrt{c}}{\sqrt{}}}$
elliptic	$-\frac{e^3\sqrt{c\,x^4+a}}{2(e^4a+d^4c)(ex+d)^2}-\frac{3c\,d^3e^3\sqrt{c\,x^4+a}}{(e^4a+d^4c)^2(ex+d)}+\frac{dc(e^4a-2d^4c)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{(e^4a+d^4c)^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}+\frac{3ic^{\frac{3}{2}}d^3e^2\sqrt{a}\sqrt{1-\frac{i\sqrt{c}}{\sqrt{}}}$

```
[In] int(1/(e*x+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)^2-3*c*d^3*e^3*(c*x^4+a)^(1/2
)/(a*e^4+c*d^4)^2/(e*x+d)+d*c*(a*e^4-2*c*d^4)/(a*e^4+c*d^4)^2/(I/a^(1/2)*c^
(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3*I*c^(3/2)*d^3*
e^2/(a*e^4+c*d^4)^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/
a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))-3*c*d^2
*(a*e^4-c*d^4)/(a*e^4+c*d^4)^2/e*(-1/2/(c/e^4*d^4+a)^(1/2)*arctanh(1/2*(2*c
*x^2/e^2*d^2+2*a)/(c/e^4*d^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2)
)^(1/2)*e/d*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)
/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)*
e^2/d^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\,(d+ex)^3}\,dx$$

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)^3}\,dx$$

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^3), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)^3}\,dx$$

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx=\int \frac{1}{\sqrt{c\,x^4+a}\,{\left(d+e\,x\right)}^3}\,dx$$

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x)^3),x)
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x)^3), x)
```

3.219 $\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 298

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = -\frac{3de^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{2ac\sqrt{a+cx^4}} + \frac{3de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} + \frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] 1/2*(-a*e^3+c*x*(3*d*e^2*x^2+3*d^2*e*x+d^3))/a/c/(c*x^4+a)^(1/2)-3/2*d*e^2*
x*(c*x^4+a)^(1/2)/a/c^(1/2)/(a^(1/2)+x^2*c^(1/2))+3/2*d*e^2*(cos(2*arctan(c
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(
2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/
(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/c^(3/4)/(c*x^4+a)^(1/2)+1/4*d*(cos(2
*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellip
ticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-3*e^2*a^(1/2)+d^2*c^(1
/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(5/4
)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1868, 1212, 226, 1210}

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \frac{d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(\sqrt{c}d^2-3\sqrt{a}e^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}} + \frac{3de^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{2ac\sqrt{a+cx^4}} - \frac{3de^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[In] Int[(d + e*x)^3/(a + c*x^4)^(3/2),x]
```

```
[Out] (-3*d*e^2*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a*e^3
 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(2*a*c*Sqrt[a + c*x^4]) + (3*d*e^2*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4])
 + (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])
/(4*a^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{2ac\sqrt{a + cx^4}} - \frac{\int \frac{-d^3+3de^2x^2}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{2ac\sqrt{a + cx^4}} + \frac{(3de^2)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}\sqrt{c}} + \frac{\left(d\left(d^2 - \frac{3\sqrt{a}e^2}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= -\frac{3de^2x\sqrt{a + cx^4}}{2a\sqrt{c}\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{2ac\sqrt{a + cx^4}} + \frac{3de^2\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a + cx^4}} + \frac{d\left(d^2 - \frac{3\sqrt{a}e^2}{\sqrt{c}}\right)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a + cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.42

$$\int \frac{(d + ex)^3}{(a + cx^4)^{3/2}}\,dx = \frac{-ae^3 + cd^3x + 3cd^2ex^2 + cd^3x\sqrt{1 + \frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{cx^4}{a}\right) + 2cde^2x^3\sqrt{\ldots}}{2ac\sqrt{a + cx^4}}$$

```
[In] Integrate[(d + e*x)^3/(a + c*x^4)^(3/2),x]
```

```
[Out] (-(a*e^3) + c*d^3*x + 3*c*d^2*e*x^2 + c*d^3*x*Sqrt[1 + (c*x^4)/a]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + 2*c*d*e^2*x^3*Sqrt[1 + (c*x^4)/a]*H
ypergeometric2F1[3/4, 3/2, 7/4, -((c*x^4)/a)])/(2*a*c*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.82

method	result
elliptic	$-\frac{2c\left(-\frac{3d\,e^2x^3}{4ac}-\frac{3d^2e\,x^2}{4ca}-\frac{d^3x}{4ac}+\frac{e^3}{4c^2}\right)}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{d^3\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{3id\,e^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right.\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{}}$
default	$d^3\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)-\frac{e^3}{2c\sqrt{c\,x^4+a}}+3d\,e^2\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}-\frac{i\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{}}{}\right.$

```
[In] int((e*x+d)^3/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(-3/4/a*d*e^2/c*x^3-3/4*d^2*e/c/a*x^2-1/4*d^3/a/c*x+1/4*e^3/c^2)/((x^4
+a/c)*c)^(1/2)+1/2*d^3/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2
)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1
/2)*c^(1/2))^(1/2),I)-3/2*I/a^(1/2)*d*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^
(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^
(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1
/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.54

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx=\frac{3\left(cde^2x^4+ade^2\right)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-\left(\left(cd^3+3\,cde^2\right)x^4+ad^3+3\,a\right.}{2\left(ac^2x\right.}$$

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*(c*d*e^2*x^4 + a*d*e^2)*sqrt(a)*(-c/a)^(3/4)*elliptic_e(arcsin(x*(-c
/a)^(1/4)), -1) - ((c*d^3 + 3*c*d*e^2)*x^4 + a*d^3 + 3*a*d*e^2)*sqrt(a)*(-c
/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)), -1) + (3*c*d*e^2*x^3 + 3*c*d^2
*e*x^2 + c*d^3*x - a*e^3)*sqrt(c*x^4 + a))/(a*c^2*x^4 + a^2*c)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(d+ex)^3}{(a+cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)**3/(c*x**4+a)**(3/2),x)
[Out] Integral((d + e*x)**3/(a + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(ex+d)^3}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^3/(c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(ex+d)^3}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x + d)^3/(c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(d+e\,x)^3}{(c\,x^4+a)^{3/2}}\,dx$$

```
[In] int((d + e*x)^3/(a + c*x^4)^(3/2),x)
[Out] int((d + e*x)^3/(a + c*x^4)^(3/2), x)
```

3.220 $\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 270

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} - \frac{e^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} + \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] 1/2*x*(e*x+d)^2/a/(c*x^4+a)^(1/2)-1/2*e^2*x*(c*x^4+a)^(1/2)/a/c^(1/2)/(a^(1
/2)+x^2*c^(1/2))+1/2*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^
(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3
/4)/c^(3/4)/(c*x^4+a)^(1/2)+1/4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4)))
,1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(
a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(5/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1869, 1212, 226, 1210}

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}} + \frac{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} + \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} - \frac{e^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[In] Int[(d + e*x)^2/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(d + e*x)^2)/(2*a*Sqrt[a + c*x^4]) - (e^2*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c
]*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4]) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
```

```
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rubi steps

$$\text{integral} = \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} - \frac{\int \frac{-d^2+e^2x^2}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} + \frac{e^2\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}\sqrt{c}} + \frac{\left(d^2-\frac{\sqrt{a}e^2}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} - \frac{e^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} + \frac{\left(d^2-\frac{\sqrt{a}e^2}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.40

$$\int \frac{(d+ex)^2}{\left(a+cx^4\right)^{3/2}}\,dx = \frac{x\left(3d(d+2ex)+3d^2\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+2e^2x^2\sqrt{1+\frac{cx^4}{a}}\,\text{H}\right.}{6a\sqrt{a+cx^4}}$$

```
[In] Integrate[(d + e*x)^2/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(3*d*(d + 2*e*x) + 3*d^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2,
 5/4, -((c*x^4)/a)] + 2*e^2*x^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[3/4,
3/2, 7/4, -((c*x^4)/a)]))/(6*a*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.86

method	result
elliptic	$-\frac{2c\left(-\frac{e^2x^3}{4ac}-\frac{ed\,x^2}{2ca}-\frac{d^2x}{4ac}\right)}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{ie^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{}}{\sqrt{}}}\right.\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$
default	$d^2\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)+e^2\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}-\frac{i\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right.\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right.$

```
[In] int((e*x+d)^2/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(-1/4/a*e^2/c*x^3-1/2/c*e*d/a*x^2-1/4/a*d^2/c*x)/((x^4+a/c)*c)^(1/2)+1
/2/a*d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(
1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/
2),I)-1/2*I/a^(1/2)*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x
*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^2}{\left(a+cx^4\right)^{3/2}}\,dx=\frac{(ce^2x^4+ae^2)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-\left((cd^2+ce^2)x^4+ad^2+ae^2\right)\sqrt{a}\left(-\frac{c}{a}\right.}{2\left(ac^2x^4+a^2c\right)}$$

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((c*e^2*x^4 + a*e^2)*sqrt(a)*(-c/a)^(3/4)*elliptic_e(arcsin(x*(-c/a)^(1
/4)), -1) - ((c*d^2 + c*e^2)*x^4 + a*d^2 + a*e^2)*sqrt(a)*(-c/a)^(3/4)*elli
ptic_f(arcsin(x*(-c/a)^(1/4)), -1) + (c*e^2*x^3 + 2*c*d*e*x^2 + c*d^2*x)*sq
rt(c*x^4 + a))/(a*c^2*x^4 + a^2*c)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{(d+ex)^2}{(a+cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)**2/(c*x**4+a)**(3/2),x)
[Out] Integral((d + e*x)**2/(a + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{(ex+d)^2}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((e*x + d)^2/(c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{(ex+d)^2}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x + d)^2/(c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{(d+e\,x)^2}{(c\,x^4+a)^{3/2}}\,dx$$

```
[In] int((d + e*x)^2/(a + c*x^4)^(3/2),x)
[Out] int((d + e*x)^2/(a + c*x^4)^(3/2), x)
```

3.221 $\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 114

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx = \frac{x(d+ex)}{2a\sqrt{a+cx^4}} + \frac{d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}$$

```
[Out] 1/2*x*(e*x+d)/a/(c*x^4+a)^(1/2)+1/4*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^
2)^(1/2)/a^(5/4)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1869, 12, 226}

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx = \frac{d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}} + \frac{x(d+ex)}{2a\sqrt{a+cx^4}}$$

```
[In] Int[(d + e*x)/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(d + e*x))/(2*a*Sqrt[a + c*x^4]) + (d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
1/2])/(4*a^(5/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{x(d+ex)}{2a\sqrt{a+cx^4}} + \frac{\int \frac{d}{\sqrt{a+cx^4}}\,dx}{2a} \\ &= \frac{x(d+ex)}{2a\sqrt{a+cx^4}} + \frac{d\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a} \\ &= \frac{x(d+ex)}{2a\sqrt{a+cx^4}} + \frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}} \end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

$$\int \frac{d+ex}{\left(a+cx^4\right)^{3/2}}\,dx = \frac{x\left(d+ex+d\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)\right)}{2a\sqrt{a+cx^4}}$$

```
[In] Integrate[(d + e*x)/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(d + e*x + d*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x
^4)/a)]))/(2*a*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

method	result	size
default	$d\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)+\frac{e\,x^2}{2\sqrt{c\,x^4+a}\,a}$	115
elliptic	$-\frac{2c\left(-\frac{e\,x^2}{4ca}-\frac{dx}{4ac}\right)}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	115

```
[In] int((e*x+d)/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] d*(1/2*x/a/((x^4+a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)
*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellipti
cF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+1/2*e/(c*x^4+a)^(1/2)*x^2/a
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int \frac{d+ex}{\left(a+cx^4\right)^{3/2}}\,dx = -\frac{\left(cdx^4+ad\right)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-\sqrt{cx^4+a}\left(cex^2+cdx\right)}{2\left(ac^2x^4+a^2c\right)}$$

```
[In] integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*d*x^4 + a*d)*sqrt(a)*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)
), -1) - sqrt(c*x^4 + a)*(c*e*x^2 + c*d*x))/(a*c^2*x^4 + a^2*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.77 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{d+ex}{\left(a+cx^4\right)^{3/2}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{cx^4}{a}}}$$

```
[In] integrate((e*x+d)/(c*x**4+a)**(3/2),x)
```

```
[Out] d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3
/2)*gamma(5/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + c*x**4/a))
```

Maxima [F]

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx = \int \frac{ex+d}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((e*x + d)/(c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx = \int \frac{ex+d}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((e*x + d)/(c*x^4 + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 18.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx = \frac{e\,x^2}{2\,a\,\sqrt{c\,x^4+a}} + \frac{d\,x\,\left(\frac{c\,x^4}{a}+1\right)^{3/2}\,{}_2F_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{(c\,x^4+a)^{3/2}}$$

```
[In] int((d + e*x)/(a + c*x^4)^(3/2),x)
[Out] (e*x^2)/(2*a*(a + c*x^4)^(1/2)) + (d*x*((c*x^4)/a + 1)^(3/2)*hypergeom([1/4
, 3/2], 5/4, -(c*x^4)/a))/(a + c*x^4)^(3/2)
```

3.222 $\int \frac{1}{(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 108

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx=\frac{x}{2a\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}$$

```
[Out] 1/2*x/a/(c*x^4+a)^(1/2)+1/4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a
^(5/4)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {205, 226}

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx=\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{x}{2a\sqrt{a+cx^4}}$$

```
[In] Int[(a + c*x^4)^(-3/2),x]
```

```
[Out] x/(2*a*Sqrt[a + c*x^4]) + ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/
4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{x}{2a\sqrt{a+cx^4}} + \frac{\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= \frac{x}{2a\sqrt{a+cx^4}} + \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.72 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int \frac{1}{\left(a+cx^4\right)^{3/2}}\,dx = \frac{x + x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)}{2a\sqrt{a+cx^4}}$$

```
[In] Integrate[(a + c*x^4)^(-3/2),x]
[Out] (x + x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/
(2*a*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	94
elliptic	$\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	94

```
[In] int(1/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/a/((x^4+a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \frac{1}{\left(a+cx^{4}\right)^{3/2}}\,dx=-\frac{(cx^{4}+a)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\mid-1\right)-\sqrt{cx^{4}+a}cx}{2\left(ac^{2}x^{4}+a^{2}c\right)}$$

```
[In] integrate(1/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*x^4 + a)*sqrt(a)*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)), -
1) - sqrt(c*x^4 + a)*c*x)/(a*c^2*x^4 + a^2*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.33

$$\int \frac{1}{\left(a+cx^{4}\right)^{3/2}}\,dx=\frac{x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate(1/(c*x**4+a)**(3/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3/2
)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + a)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 18.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.34

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = \frac{x\left(\frac{c\,x^4}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{(c\,x^4+a)^{3/2}}$$

```
[In] int(1/(a + c*x^4)^(3/2),x)
[Out] (x*((c*x^4)/a + 1)^(3/2)*hypergeom([1/4, 3/2], 5/4, -(c*x^4)/a))/(a + c*x^4
)^(3/2)
```

3.223 $\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 818

$$\begin{aligned}\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx &= \frac{e(ae^2-cd^2x^2)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}}+\frac{cdx(d^2+e^2x^2)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}}\\
&-\frac{\sqrt{c}de^2x\sqrt{a+cx^4}}{2a\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{e^5\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2\left(-cd^4-ae^4\right)^{3/2}}-\frac{e^5\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2\left(cd^4+ae^4\right)^{3/2}}\\
&+\frac{\sqrt[4]{c}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\left(cd^4+ae^4\right)\sqrt{a+cx^4}}\\
&+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\left(cd^4+ae^4\right)\sqrt{a+cx^4}}\\
&+\frac{\sqrt[4]{c}de^4\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\sqrt{a+cx^4}}\\
&-\frac{e^4\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\sqrt{a+cx^4}}\end{aligned}$$

```
[Out] -1/2*e^5*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-c*d^4)^
(3/2)-1/2*e^5*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2)
)/(a*e^4+c*d^4)^(3/2)+1/2*e*(-c*d^2*x^2+a*e^2)/a/(a*e^4+c*d^4)/(c*x^4+a)^(1
/2)+1/2*c*d*x*(e^2*x^2+d^2)/a/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)-1/2*d*e^2*x*c^(
1/2)*(c*x^4+a)^(1/2)/a/(a*e^4+c*d^4)/(a^(1/2)+x^2*c^(1/2))+1/2*c^(1/4)*d*e^
2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)
))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(
```

```
1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(a*e^4+c*d^4)/(c*x^
4+a)^(1/2)+1/4*c^(1/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^
(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)
+x^2*c^(1/2))^2)^(1/2)/a^(5/4)/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)+1/2*c^(1/4)*d*
e^4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c
^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a*e^4+c*d^4)/(e^
2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/4*e^4*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(
1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2),1/
2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(
1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d/(a*e^4+c*d^4)/(e^2*a^(1/2)+d^2
*c^(1/2))/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {1743, 1236, 1193, 1212, 226, 1210, 1231, 1721, 1262, 755, 12, 739, 212}

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{cx^4+a}}\right)e^5}{2\left(-cd^4-ae^4\right)^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{cx^4+a}}\right)e^5}{2\left(cd^4+ae^4\right)^{3/2}}$$
$$+\frac{\sqrt[4]{c}d\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e^4}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$
$$-\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e^4}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$
$$+\frac{\sqrt[4]{c}d\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^2}{2a^{3/4}\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$
$$-\frac{\sqrt{c}dx\sqrt{cx^4+a}ae^2}{2a\left(cd^4+ae^4\right)\left(\sqrt{c}x^2+\sqrt{a}\right)}+\frac{\left(ae^2-cd^2x^2\right)e}{2a\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$
$$+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$
$$+\frac{cdx\left(d^2+e^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$

```
[In] Int[1/((d + e*x)*(a + c*x^4)^(3/2)),x]
```

```
[Out] (e*(a*e^2 - c*d^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c*d*x*(d^2
 + e^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (Sqrt[c]*d*e^2*x*Sqrt[
a + c*x^4])/(2*a*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)) - (e^5*ArcTan[(Sq
rt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*(-(c*d^4) - a*e^4)^(3/2)
) - (e^5*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])]
)/(2*(c*d^4 + a*e^4)^(3/2)) + (c^(1/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(2*a^(3/4)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*d*(Sqrt[c]
*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*(c*d
^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*d*e^4*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a +
c*x^4]) - (e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^
2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1
/4)*c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
```

```
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1236

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/(
c*d^2 + a*e^2), Int[(c*d - c*e*x^2)*(a + c*x^4)^p, x], x] + Dist[e^2/(c*d^2
 + a*e^2), Int[(a + c*x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1743

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[x*((a + c*x^4)^p/(d^
2 - e^2*x^2)), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d\int \frac{1}{(d^2-e^2x^2)\,(a+cx^4)^{3/2}}\,dx - e\int \frac{x}{(d^2-e^2x^2)\,(a+cx^4)^{3/2}}\,dx \\
&= -\left(\frac{1}{2}e\text{Subst}\left(\int \frac{1}{(d^2-e^2x)\,(a+cx^2)^{3/2}}\,dx, x, x^2\right)\right) \\
&\quad + \frac{d\int \frac{cd^2+ce^2x^2}{(a+cx^4)^{3/2}}\,dx}{cd^4+ae^4} + \frac{(de^4)\int \frac{1}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{cd^4+ae^4} \\
&= \frac{e(ae^2-cd^2x^2)}{2a\,(cd^4+ae^4)\sqrt{a+cx^4}} + \frac{cdx(d^2+e^2x^2)}{2a\,(cd^4+ae^4)\sqrt{a+cx^4}} \\
&\quad - \frac{d\int \frac{-cd^2+ce^2x^2}{\sqrt{a+cx^4}}\,dx}{2a\,(cd^4+ae^4)} - \frac{e\text{Subst}\left(\int \frac{ae^4}{(d^2-e^2x)\sqrt{a+cx^2}}\,dx, x, x^2\right)}{2a\,(cd^4+ae^4)} \\
&\quad + \frac{\left(\sqrt{c}de^4\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)(cd^4+ae^4)} + \frac{\left(\sqrt{a}de^6\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)(cd^4+ae^4)}
\end{aligned}
$$

$$= \frac{e(ae^2-cd^2x^2)}{2a(cd^4+ae^4)\sqrt{a+cx^4}} + \frac{cdx(d^2+e^2x^2)}{2a(cd^4+ae^4)\sqrt{a+cx^4}} - \frac{e^5\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2(-cd^4-ae^4)^{3/2}}$$
$$+ \frac{\sqrt[4]{c}de^4(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}(\sqrt{c}d^2+\sqrt{a}e^2)(cd^4+ae^4)\sqrt{a+cx^4}}$$
$$+ \frac{d\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)e^6(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\Pi\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}(\sqrt{c}d^2+\sqrt{a}e^2)(cd^4+ae^4)\sqrt{a+cx^4}}$$
$$+ \frac{(\sqrt{c}de^2)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}(cd^4+ae^4)} - \frac{e^5\text{Subst}\left(\int\frac{1}{(d^2-e^2x)\sqrt{a+cx^2}}\,dx,x,x^2\right)}{2(cd^4+ae^4)}$$
$$+ \frac{\left(\sqrt{c}d(\sqrt{c}d^2-\sqrt{a}e^2)\right)\int\frac{1}{\sqrt{a+cx^4}}\,dx}{2a(cd^4+ae^4)}$$

$$= \frac{e(ae^2-cd^2x^2)}{2a(cd^4+ae^4)\sqrt{a+cx^4}} + \frac{cdx(d^2+e^2x^2)}{2a(cd^4+ae^4)\sqrt{a+cx^4}}$$
$$- \frac{\sqrt{c}de^2x\sqrt{a+cx^4}}{2a(cd^4+ae^4)(\sqrt{a}+\sqrt{c}x^2)} - \frac{e^5\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2(-cd^4-ae^4)^{3/2}}$$
$$+ \frac{\sqrt[4]{c}de^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}(cd^4+ae^4)\sqrt{a+cx^4}}$$
$$+ \frac{\sqrt[4]{c}d(\sqrt{c}d^2-\sqrt{a}e^2)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}(cd^4+ae^4)\sqrt{a+cx^4}}$$
$$+ \frac{\sqrt[4]{c}de^4(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}(\sqrt{c}d^2+\sqrt{a}e^2)(cd^4+ae^4)\sqrt{a+cx^4}}$$
$$+ \frac{d\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)e^6(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\Pi\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}(\sqrt{c}d^2+\sqrt{a}e^2)(cd^4+ae^4)\sqrt{a+cx^4}}$$
$$+ \frac{e^5\text{Subst}\left(\int\frac{1}{cd^4+ae^4-x^2}\,dx,x,\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}}\right)}{2(cd^4+ae^4)}$$

$$= \frac{e(ae^2-cd^2x^2)}{2a(cd^4+ae^4)\sqrt{a+cx^4}} + \frac{cdx(d^2+e^2x^2)}{2a(cd^4+ae^4)\sqrt{a+cx^4}} - \frac{\sqrt{c}de^2x\sqrt{a+cx^4}}{2a(cd^4+ae^4)(\sqrt{a}+\sqrt{c}x^2)} - \frac{e^5\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2(-cd^4-ae^4)^{3/2}} - \frac{e^5\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2(cd^4+ae^4)^{3/2}} + \frac{\sqrt[4]{c}de^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}(cd^4+ae^4)\sqrt{a+cx^4}} + \frac{\sqrt[4]{c}d(\sqrt{c}d^2-\sqrt{a}e^2)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}(cd^4+ae^4)\sqrt{a+cx^4}} + \frac{\sqrt[4]{c}de^4(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}(\sqrt{c}d^2+\sqrt{a}e^2)(cd^4+ae^4)\sqrt{a+cx^4}} + \frac{d\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)e^6(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\Pi\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}(\sqrt{c}d^2+\sqrt{a}e^2)(cd^4+ae^4)\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.65 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.56

$$\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx =$$

$$-\frac{-\sqrt{a}c^{3/4}d^2e^2\sqrt{-cd^4-ae^4}\sqrt{1+\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\Big|-1\right)+c^{3/4}d^2\left(-i\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{-cd^4-ae^4}\sqrt{}}{}$$

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)^(3/2)),x]
```

```
[Out] -1/2*(-(Sqrt[a]*c^(3/4)*d^2*e^2*Sqrt[-(c*d^4) - a*e^4]*Sqrt[1 + (c*x^4)/a]*
EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]) + c^(3/4)*d^2*((-I)*
Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[-(c*d^4) - a*e^4]*Sqrt[1 + (c*x^4)/a]*Ellip
ticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[(I*Sqrt[c])/Sqrt[a]
]*(c^(1/4)*d*(Sqrt[-(c*d^4) - a*e^4]*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)
) + 2*a*e^5*Sqrt[a + c*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a +
c*x^4])/Sqrt[-(c*d^4) - a*e^4]]) - 2*(-1)^(1/4)*a^(5/4)*e^4*Sqrt[-(c*d^4) -
 a*e^4]*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSi
n[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1]))/(a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^(1/
4)*d*(-(c*d^4) - a*e^4)^(3/2)*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 496, normalized size of antiderivative = 0.61

method	result
default	$-\frac{2c\left(-\frac{d\,e^{2}x^{3}}{4a\left(e^{4}a+d^{4}c\right)}+\frac{d^{2}e\,x^{2}}{4a\left(e^{4}a+d^{4}c\right)}-\frac{d^{3}x}{4a\left(e^{4}a+d^{4}c\right)}-\frac{e^{3}}{4\left(e^{4}a+d^{4}c\right)c}\right)}{\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{d^{3}c\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\left(e^{4}a+d^{4}c\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}-\frac{i\sqrt{c}\,d\,e^{2}\sqrt{1\cdots}}{\cdots}$
elliptic	$-\frac{2c\left(-\frac{d\,e^{2}x^{3}}{4a\left(e^{4}a+d^{4}c\right)}+\frac{d^{2}e\,x^{2}}{4a\left(e^{4}a+d^{4}c\right)}-\frac{d^{3}x}{4a\left(e^{4}a+d^{4}c\right)}-\frac{e^{3}}{4\left(e^{4}a+d^{4}c\right)c}\right)}{\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{d^{3}c\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\left(e^{4}a+d^{4}c\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}-\frac{i\sqrt{c}\,d\,e^{2}\sqrt{1\cdots}}{\cdots}$

```
[In] int(1/(e*x+d)/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(-1/4/a*d*e^2/(a*e^4+c*d^4)*x^3+1/4/a*d^2*e/(a*e^4+c*d^4)*x^2-1/4*d^3/
a/(a*e^4+c*d^4)*x-1/4*e^3/(a*e^4+c*d^4)/c)/((x^4+a/c)*c)^(1/2)+1/2*d^3/a*c/
(a*e^4+c*d^4)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2)
)^(1/2),I)-1/2*I*c^(1/2)*d/a^(1/2)*e^2/(a*e^4+c*d^4)/(I/a^(1/2)*c^(1/2))^(1
/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+
a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c
^(1/2))^(1/2),I))+e^3/(a*e^4+c*d^4)*(-1/2/(c/e^4*d^4+a)^(1/2)*arctanh(1/2*(
2*c*x^2/e^2*d^2+2*a)/(c/e^4*d^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1
/2))^(1/2)*e/d*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/
2)*e^2/d^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx = \int \frac{1}{\left(a+cx^4\right)^{\frac{3}{2}}(d+ex)}\,dx$$

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**(3/2),x)
[Out] Integral(1/((a + c*x**4)**(3/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx = \int \frac{1}{\left(cx^4+a\right)^{\frac{3}{2}}(ex+d)}\,dx$$

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx = \int \frac{1}{\left(cx^4+a\right)^{\frac{3}{2}}(ex+d)}\,dx$$

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx = \int \frac{1}{\left(c\,x^4+a\right)^{3/2}\,(d+e\,x)}\,dx$$

```
[In] int(1/((a + c*x^4)^(3/2)*(d + e*x)),x)
[Out] int(1/((a + c*x^4)^(3/2)*(d + e*x)), x)
```

3.224 $\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 349

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = -\frac{(c+dx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)(1+n)} - \frac{(c+dx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d\right)(1+n)} - \frac{(c+dx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)(1+n)} - \frac{(c+dx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt[4]{-a}d\right)(1+n)}$$

```
[Out] -1/4*(d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)
^(1/4)*d))/b^(3/4)/(b^(1/4)*c-(-a)^(1/4)*d)/(1+n)-1/4*(d*x+c)^(1+n)*hyperge
om([1, 1+n],[2+n],b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b^(3/4)/(b^(1/4
)*c+(-a)^(1/4)*d)/(1+n)-1/4*(d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/4)*
(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(1+n)/(b^(1/4)*c-d*(-(-a
)^(1/2))^(1/2))-1/4*(d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/4)*(d*x+c)/
(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(1+n)/(b^(1/4)*c+d*(-(-a)^(1/2))
^(1/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6857, 845, 70}

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = -\frac{(c+dx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)} - \frac{(c+dx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+1)\left(\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c\right)} - \frac{(c+dx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)} - \frac{(c+dx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{-a}d+\sqrt[4]{b}c\right)}$$

```
[In] Int[(x^3*(c + d*x)^n)/(a + b*x^4),x]
```

```
[Out] -1/4*((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*
x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(b^(3/4)*(b^(1/4)*c - Sqrt[-Sqrt[-a]]
*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/
4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c + Sqr
t[-Sqrt[-a]]*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*
c - (-a)^(1/4)*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4
)*c + (-a)^(1/4)*d)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
```

]

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{x(c+dx)^n}{2\left(-\sqrt{-a}\sqrt{b}+bx^2\right)}+\frac{x(c+dx)^n}{2\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx \\
&= \frac{1}{2}\int \frac{x(c+dx)^n}{-\sqrt{-a}\sqrt{b}+bx^2}\,dx+\frac{1}{2}\int \frac{x(c+dx)^n}{\sqrt{-a}\sqrt{b}+bx^2}\,dx \\
&= \frac{1}{2}\int \left(-\frac{(c+dx)^n}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}+\frac{(c+dx)^n}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}\right)dx \\
&\quad +\frac{1}{2}\int \left(-\frac{(c+dx)^n}{2b^{3/4}\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}+\frac{(c+dx)^n}{2b^{3/4}\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}\right)dx \\
&= -\frac{\int \frac{(c+dx)^n}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x}\,dx}{4b^{3/4}}-\frac{\int \frac{(c+dx)^n}{\sqrt[4]{-a}-\sqrt[4]{b}x}\,dx}{4b^{3/4}}+\frac{\int \frac{(c+dx)^n}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x}\,dx}{4b^{3/4}}+\frac{\int \frac{(c+dx)^n}{\sqrt[4]{-a}+\sqrt[4]{b}x}\,dx}{4b^{3/4}} \\
&= -\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)(1+n)} \\
&\quad -\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d\right)(1+n)} \\
&\quad -\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)(1+n)} \\
&\quad -\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt[4]{-a}d\right)(1+n)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.79

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx$$
$$= \frac{(c+dx)^{1+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}-\text{Hyperge}\ldots\right)}{4b^{3/4}(1+n)}$$

[In] Integrate[(x^3*(c + d*x)^n)/(a + b*x^4),x]

[Out] ((c + d*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)]/(b^(1/4)*c - (-a)^(1/4)*d)) - Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)]/(b^(1/4)*c - I*(-a)^(1/4)*d) - Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]/(b^(1/4)*c + I*(-a)^(1/4)*d) - Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)]/(b^(1/4)*c + (-a)^(1/4)*d)))/(4*b^(3/4)*(1 + n))

Maple [F]

$$\int \frac{x^3(dx+c)^n}{b\,x^4+a}dx$$

[In] int(x^3*(d*x+c)^n/(b*x^4+a),x)

[Out] int(x^3*(d*x+c)^n/(b*x^4+a),x)

Fricas [F]

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \int \frac{(dx+c)^n x^3}{bx^4+a}\,dx$$

[In] integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="fricas")

[Out] integral((d*x + c)^n*x^3/(b*x^4 + a), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(d*x+c)**n/(b*x**4+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \int \frac{(dx+c)^n x^3}{bx^4+a}\,dx$$

```
[In] integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^n*x^3/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \int \frac{(dx+c)^n x^3}{bx^4+a}\,dx$$

```
[In] integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="giac")
[Out] integrate((d*x + c)^n*x^3/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \int \frac{x^3\,(c+d\,x)^n}{b\,x^4+a}\,dx$$

```
[In] int((x^3*(c + d*x)^n)/(a + b*x^4),x)
[Out] int((x^3*(c + d*x)^n)/(a + b*x^4), x)
```

3.225 $\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 349

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = -\frac{(c+dx)^{2+n}\,\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)(2+n)} - \frac{(c+dx)^{2+n}\,\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d\right)(2+n)} - \frac{(c+dx)^{2+n}\,\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)(2+n)} - \frac{(c+dx)^{2+n}\,\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt[4]{-a}d\right)(2+n)}$$

```
[Out] -1/4*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)
^(1/4)*d))/b^(3/4)/(b^(1/4)*c-(-a)^(1/4)*d)/(2+n)-1/4*(d*x+c)^(2+n)*hyperge
om([1, 2+n],[3+n],b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b^(3/4)/(b^(1/4
)*c+(-a)^(1/4)*d)/(2+n)-1/4*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/4)*
(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(2+n)/(b^(1/4)*c-d*(-(-a
)^(1/2))^(1/2))-1/4*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/4)*(d*x+c)/
(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(2+n)/(b^(1/4)*c+d*(-(-a)^(1/2))
^(1/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6857, 845, 70}

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = -\frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)} - \frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+2)\left(\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c\right)} - \frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)} - \frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{-a}d+\sqrt[4]{b}c\right)}$$

```
[In] Int[(x^3*(c + d*x)^(1 + n))/(a + b*x^4),x]
```

```
[Out] -1/4*((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*
x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(b^(3/4)*(b^(1/4)*c - Sqrt[-Sqrt[-a]]
*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/
4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c + Sqr
t[-Sqrt[-a]]*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3
 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*
c - (-a)^(1/4)*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n,
 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4
)*c + (-a)^(1/4)*d)*(2 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
```

```
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x(c+dx)^{1+n}}{2\left(-\sqrt{-a}\sqrt{b}+bx^2\right)}+\frac{x(c+dx)^{1+n}}{2\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx \\
&= \frac{1}{2}\int \frac{x(c+dx)^{1+n}}{-\sqrt{-a}\sqrt{b}+bx^2}\,dx+\frac{1}{2}\int \frac{x(c+dx)^{1+n}}{\sqrt{-a}\sqrt{b}+bx^2}\,dx \\
&= \frac{1}{2}\int \left(-\frac{(c+dx)^{1+n}}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x\right)}+\frac{(c+dx)^{1+n}}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x\right)}\right)dx \\
&\quad +\frac{1}{2}\int \left(-\frac{(c+dx)^{1+n}}{2b^{3/4}\left(\sqrt[4]{-a}-\sqrt[4]{b}x\right)}+\frac{(c+dx)^{1+n}}{2b^{3/4}\left(\sqrt[4]{-a}+\sqrt[4]{b}x\right)}\right)dx \\
&= -\frac{\int \frac{(c+dx)^{1+n}}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}x}\,dx}{4b^{3/4}}-\frac{\int \frac{(c+dx)^{1+n}}{\sqrt[4]{-a}-\sqrt[4]{b}x}\,dx}{4b^{3/4}}+\frac{\int \frac{(c+dx)^{1+n}}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}x}\,dx}{4b^{3/4}}+\frac{\int \frac{(c+dx)^{1+n}}{\sqrt[4]{-a}+\sqrt[4]{b}x}\,dx}{4b^{3/4}} \\
&= -\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)(2+n)} \\
&\quad -\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d\right)(2+n)} \\
&\quad -\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)(2+n)} \\
&\quad -\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt[4]{-a}d\right)(2+n)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.79

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \frac{(c+dx)^{2+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}-\frac{\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}-\frac{\text{Hyperg}}{}\right.}{4b^{3/4}(2+n)}$$

```
[In] Integrate[(x^3*(c + d*x)^(1 + n))/(a + b*x^4),x]
```

```
[Out] ((c + d*x)^(2 + n)*(-(Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x)
)/(b^(1/4)*c - (-a)^(1/4)*d)]/(b^(1/4)*c - (-a)^(1/4)*d)) - Hypergeometric2
F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)]/(b^(1
/4)*c - I*(-a)^(1/4)*d) - Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c +
d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]/(b^(1/4)*c + I*(-a)^(1/4)*d) - Hypergeo
metric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)]/
(b^(1/4)*c + (-a)^(1/4)*d)))/(4*b^(3/4)*(2 + n))
```

Maple [F]

$$\int \frac{x^3(dx+c)^{1+n}}{b\,x^4+a}dx$$

```
[In] int(x^3*(d*x+c)^(1+n)/(b*x^4+a),x)
```

```
[Out] int(x^3*(d*x+c)^(1+n)/(b*x^4+a),x)
```

Fricas [F]

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \int \frac{(dx+c)^{n+1}x^3}{bx^4+a}\,dx$$

```
[In] integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(d*x+c)**(1+n)/(b*x**4+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \int \frac{(dx+c)^{n+1}x^3}{bx^4+a}\,dx$$

```
[In] integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \int \frac{(dx+c)^{n+1}x^3}{bx^4+a}\,dx$$

```
[In] integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="giac")
[Out] integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \int \frac{x^3\,(c+d\,x)^{n+1}}{b\,x^4+a}\,dx$$

```
[In] int((x^3*(c + d*x)^(n + 1))/(a + b*x^4),x)
[Out] int((x^3*(c + d*x)^(n + 1))/(a + b*x^4), x)
```

3.226 $\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1605

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx$$
$$= -\frac{e^2\arctan\left(\frac{\sqrt{2}\sqrt{-bd^4+4bcd^2e-2bc^2e^2-2ae^4-bd\sqrt{d^2-4ce}(d^2-2ce)}x}{e\left(d+\sqrt{d^2-4ce}\right)\sqrt{a+bx^4}}\right)}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4-4cd^2e+2c^2e^2+d^3\sqrt{d^2-4ce}-2cde\sqrt{d^2-4ce}\right)}}$$
$$+\frac{e^2\arctan\left(\frac{\sqrt{2}\sqrt{-bd^4+4bcd^2e-2bc^2e^2-2ae^4+bd\sqrt{d^2-4ce}(d^2-2ce)}x}{e\left(d-\sqrt{d^2-4ce}\right)\sqrt{a+bx^4}}\right)}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4-4cd^2e+2c^2e^2-d^3\sqrt{d^2-4ce}+2cde\sqrt{d^2-4ce}\right)}}$$
$$-\frac{e^2\text{arctanh}\left(\frac{4ae^2+b\left(d-\sqrt{d^2-4ce}\right)^2x^2}{2\sqrt{2}\sqrt{bd^4-4bcd^2e+2bc^2e^2+2ae^4-bd\sqrt{d^2-4ce}(d^2-2ce)}\sqrt{a+bx^4}}\right)}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{bd^4-4bcd^2e+2bc^2e^2+2ae^4-bd\sqrt{d^2-4ce}\,(d^2-2ce)}}$$
$$+\frac{e^2\text{arctanh}\left(\frac{4ae^2+b\left(d+\sqrt{d^2-4ce}\right)^2x^2}{2\sqrt{2}\sqrt{bd^4-4bcd^2e+2bc^2e^2+2ae^4+bd\sqrt{d^2-4ce}(d^2-2ce)}\sqrt{a+bx^4}}\right)}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{bd^4-4bcd^2e+2bc^2e^2+2ae^4+bd\sqrt{d^2-4ce}\,(d^2-2ce)}}$$
$$+\frac{\sqrt[4]{b}e\left(d-\sqrt{d^2-4ce}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{d^2-4ce}\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce-d\sqrt{d^2-4ce}\right)\right)\sqrt{a+bx^4}}$$
$$-\frac{\sqrt[4]{b}e\left(d+\sqrt{d^2-4ce}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{d^2-4ce}\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce+d\sqrt{d^2-4ce}\right)\right)\sqrt{a+bx^4}}$$
$$+\frac{e\left(2\sqrt{a}e^2-\sqrt{b}\left(d^2-2ce-d\sqrt{d^2-4ce}\right)\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce-d\sqrt{d^2\right.\right.}}{4\sqrt{a}\sqrt{b}e^2\left(d-\sqrt{d^2-4ce}\right.}\right.}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}\right)\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce-d\sqrt{d^2-4ce}\right)\right)\sqrt{a}}$$
$$-\frac{e\left(2\sqrt{a}e^2-\sqrt{b}\left(d^2-2ce+d\sqrt{d^2-4ce}\right)\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce+d\sqrt{d^2\right.\right.}}{4\sqrt{a}\sqrt{b}e^2\left(d+\sqrt{d^2-4ce}\right.}\right.}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}\right)\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce+d\sqrt{d^2-4ce}\right)\right)\sqrt{a}}$$

```
[Out] 1/2*b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/
4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(
1/2)+x^2*b^(1/2))*(d-(-4*c*e+d^2)^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2
)^(1/2)/a^(1/4)/(-4*c*e+d^2)^(1/2)/(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e-d*(-4*
```

```
c*e+d^2)^(1/2)))/(b*x^4+a)^(1/2)+1/2*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a
^(1/4))),1/4*(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e-d*(-4*c*e+d^2)^(1/2)))^2/e^2
/a^(1/2)/b^(1/2)/(d-(-4*c*e+d^2)^(1/2))^2,1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2)
)*(2*e^2*a^(1/2)-b^(1/2)*(d^2-2*c*e-d*(-4*c*e+d^2)^(1/2)))*((b*x^4+a)/(a^(1
/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/(d-(-4*c*e+d^2)^(1/2))/(-4*c*e+d^
2)^(1/2)/(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e-d*(-4*c*e+d^2)^(1/2)))/(b*x^4+a)
^(1/2)-1/2*b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2
))*(a^(1/2)+x^2*b^(1/2))*(d+(-4*c*e+d^2)^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(
1/2))^2)^(1/2)/a^(1/4)/(-4*c*e+d^2)^(1/2)/(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e
+d*(-4*c*e+d^2)^(1/2)))/(b*x^4+a)^(1/2)-1/2*e*(cos(2*arctan(b^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1
/4)*x/a^(1/4))),1/4*(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e+d*(-4*c*e+d^2)^(1/2))
)^2/e^2/a^(1/2)/b^(1/2)/(d+(-4*c*e+d^2)^(1/2))^2,1/2*2^(1/2))*(a^(1/2)+x^2*
b^(1/2))*(2*e^2*a^(1/2)-b^(1/2)*(d^2-2*c*e+d*(-4*c*e+d^2)^(1/2)))*((b*x^4+a
)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/(-4*c*e+d^2)^(1/2)/(d+(-4*
c*e+d^2)^(1/2))/(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e+d*(-4*c*e+d^2)^(1/2)))/(b
*x^4+a)^(1/2)-1/2*e^2*arctanh(1/4*(4*a*e^2+b*x^2*(d-(-4*c*e+d^2)^(1/2))^2)*
2^(1/2)/(b*x^4+a)^(1/2)/(b*d^4-4*b*c*d^2*e+2*b*c^2*e^2+2*a*e^4-b*d*(-2*c*e+
d^2)*(-4*c*e+d^2)^(1/2))^(1/2))*2^(1/2)/(-4*c*e+d^2)^(1/2)/(b*d^4-4*b*c*d^2
*e+2*b*c^2*e^2+2*a*e^4-b*d*(-2*c*e+d^2)*(-4*c*e+d^2)^(1/2))^(1/2)+1/2*e^2*a
rctanh(1/4*(4*a*e^2+b*x^2*(d+(-4*c*e+d^2)^(1/2))^2)*2^(1/2)/(b*x^4+a)^(1/2)
/(b*d^4-4*b*c*d^2*e+2*b*c^2*e^2+2*a*e^4+b*d*(-2*c*e+d^2)*(-4*c*e+d^2)^(1/2)
)^(1/2))*2^(1/2)/(-4*c*e+d^2)^(1/2)/(b*d^4-4*b*c*d^2*e+2*b*c^2*e^2+2*a*e^4+
b*d*(-2*c*e+d^2)*(-4*c*e+d^2)^(1/2))^(1/2)-1/2*e^2*arctan(x*2^(1/2)*(-b*d^4
+4*b*c*d^2*e-2*b*c^2*e^2-2*a*e^4-b*d*(-2*c*e+d^2)*(-4*c*e+d^2)^(1/2))^(1/2)
/e/(d+(-4*c*e+d^2)^(1/2))/(b*x^4+a)^(1/2))*2^(1/2)/(-4*c*e+d^2)^(1/2)/(-2*a
*e^4-b*(d^4-4*c*d^2*e+2*c^2*e^2+d^3*(-4*c*e+d^2)^(1/2)-2*c*d*e*(-4*c*e+d^2)
^(1/2)))^(1/2)+1/2*e^2*arctan(x*2^(1/2)*(-b*d^4+4*b*c*d^2*e-2*b*c^2*e^2-2*a
*e^4+b*d*(-2*c*e+d^2)*(-4*c*e+d^2)^(1/2))^(1/2)/e/(d-(-4*c*e+d^2)^(1/2))/(b
*x^4+a)^(1/2))*2^(1/2)/(-4*c*e+d^2)^(1/2)/(-2*a*e^4-b*(d^4-4*c*d^2*e+2*c^2*
e^2-d^3*(-4*c*e+d^2)^(1/2)+2*c*d*e*(-4*c*e+d^2)^(1/2)))^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 6.67 (sec) , antiderivative size = 1605, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{6860, 1739, 1231, 226, 1721, 1262, 739, 212\}$

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx$$

$$= -\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-bd^4+4bced^2-b\sqrt{d^2-4ce}(d^2-2ce)d-2ae^4-2bc^2e^2}x}{e\left(d+\sqrt{d^2-4ce}\right)\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4+\sqrt{d^2-4ce}d^3-4ced^2-2ce\sqrt{d^2-4ce}d+2c^2e^2\right)}}$$
$$+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-bd^4+4bced^2+b\sqrt{d^2-4ce}(d^2-2ce)d-2ae^4-2bc^2e^2}x}{e\left(d-\sqrt{d^2-4ce}\right)\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4-\sqrt{d^2-4ce}d^3-4ced^2+2ce\sqrt{d^2-4ce}d+2c^2e^2\right)}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{4ae^2+b\left(d-\sqrt{d^2-4ce}\right)^2x^2}{2\sqrt{2}\sqrt{bd^4-4bced^2-b\sqrt{d^2-4ce}(d^2-2ce)d+2ae^4+2bc^2e^2}\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{bd^4-4bced^2-b\sqrt{d^2-4ce}\,(d^2-2ce)\,d+2ae^4+2bc^2e^2}}$$
$$+\frac{\operatorname{arctanh}\left(\frac{4ae^2+b\left(d+\sqrt{d^2-4ce}\right)^2x^2}{2\sqrt{2}\sqrt{bd^4-4bced^2+b\sqrt{d^2-4ce}(d^2-2ce)d+2ae^4+2bc^2e^2}\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{bd^4-4bced^2+b\sqrt{d^2-4ce}\,(d^2-2ce)\,d+2ae^4+2bc^2e^2}}$$
$$+\frac{\sqrt[4]{b}\left(d-\sqrt{d^2-4ce}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}\sqrt{d^2-4ce}\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-\sqrt{d^2-4ce}d-2ce\right)\right)\sqrt{bx^4+a}}$$
$$-\frac{\sqrt[4]{b}\left(d+\sqrt{d^2-4ce}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}\sqrt{d^2-4ce}\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2+\sqrt{d^2-4ce}d-2ce\right)\right)\sqrt{bx^4+a}}$$
$$+\frac{\left(2\sqrt{a}e^2-\sqrt{b}\left(d^2-\sqrt{d^2-4ce}d-2ce\right)\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-\sqrt{d^2-4ce}d-\right.\right.}{4\sqrt{a}\sqrt{b}e^2\left(d-\sqrt{d^2-4ce}\right)}\right.}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}\right)\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-\sqrt{d^2-4ce}d-2ce\right)\right)\sqrt{bx}}$$
$$-\frac{\left(2\sqrt{a}e^2-\sqrt{b}\left(d^2+\sqrt{d^2-4ce}d-2ce\right)\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2+\sqrt{d^2-4ce}d-\right.\right.}{4\sqrt{a}\sqrt{b}e^2\left(d+\sqrt{d^2-4ce}\right)}\right.}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}\right)\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2+\sqrt{d^2-4ce}d-2ce\right)\right)\sqrt{bx}}$$

```
[In] Int[1/((c + d*x + e*x^2)*Sqrt[a + b*x^4]),x]
```

```
[Out] -((e^2*ArcTan[(Sqrt[2]*Sqrt[-(b*d^4) + 4*b*c*d^2*e - 2*b*c^2*e^2 - 2*a*e^4
- b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*x)/(e*(d + Sqrt[d^2 - 4*c*e])*Sqrt[a
 + b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[-2*a*e^4 - b*(d^4 - 4*c*d^2*e
+ 2*c^2*e^2 + d^3*Sqrt[d^2 - 4*c*e] - 2*c*d*e*Sqrt[d^2 - 4*c*e])])) + (e^2*
```

```
ArcTan[(Sqrt[2]*Sqrt[-(b*d^4) + 4*b*c*d^2*e - 2*b*c^2*e^2 - 2*a*e^4 + b*d*S
qrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*x)/(e*(d - Sqrt[d^2 - 4*c*e])*Sqrt[a + b*x^
4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[-2*a*e^4 - b*(d^4 - 4*c*d^2*e + 2*c^2
*e^2 - d^3*Sqrt[d^2 - 4*c*e] + 2*c*d*e*Sqrt[d^2 - 4*c*e])]) - (e^2*ArcTanh[
(4*a*e^2 + b*(d - Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d
^2*e + 2*b*c^2*e^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a
+ b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e
^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]) + (e^2*ArcTanh[(4*a*e^
2 + b*(d + Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d^2*e +
2*b*c^2*e^2 + 2*a*e^4 + b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a + b*x^4
])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e^2 + 2*
a*e^4 + b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]) + (b^(1/4)*e*(d - Sqrt[d^2 -
4*c*e])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*Sqrt[d^2 - 4*c*e
]*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e - d*Sqrt[d^2 - 4*c*e]))*Sqrt[a + b*
x^4]) - (b^(1/4)*e*(d + Sqrt[d^2 - 4*c*e])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
+ b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/(2*a^(1/4)*Sqrt[d^2 - 4*c*e]*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e
+ d*Sqrt[d^2 - 4*c*e]))*Sqrt[a + b*x^4]) + (e*(2*Sqrt[a]*e^2 - Sqrt[b]*(d^2
 - 2*c*e - d*Sqrt[d^2 - 4*c*e]))*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e
- d*Sqrt[d^2 - 4*c*e]))^2/(4*Sqrt[a]*Sqrt[b]*e^2*(d - Sqrt[d^2 - 4*c*e])^2)
, 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[d^2 - 4*c*e]
*(d - Sqrt[d^2 - 4*c*e])*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e - d*Sqrt[d^2
 - 4*c*e]))*Sqrt[a + b*x^4]) - (e*(2*Sqrt[a]*e^2 - Sqrt[b]*(d^2 - 2*c*e + d
*Sqrt[d^2 - 4*c*e]))*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sq
rt[b]*x^2)^2]*EllipticPi[(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e + d*Sqrt[d^2
 - 4*c*e]))^2/(4*Sqrt[a]*Sqrt[b]*e^2*(d + Sqrt[d^2 - 4*c*e])^2), 2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[d^2 - 4*c*e]*(d + Sqrt[d
^2 - 4*c*e])*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e + d*Sqrt[d^2 - 4*c*e]))*
Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
```

```
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{2e}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)\sqrt{a+bx^4}} - \frac{2e}{\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}+2ex\right)\sqrt{a+bx^4}} \right) dx$$

$$= \frac{(2e)\int \frac{1}{\left(d-\sqrt{d^2-4ce}+2ex\right)\sqrt{a+bx^4}}\,dx}{\sqrt{d^2-4ce}} - \frac{(2e)\int \frac{1}{\left(d+\sqrt{d^2-4ce}+2ex\right)\sqrt{a+bx^4}}\,dx}{\sqrt{d^2-4ce}}$$

$$= -\frac{(4e^2)\int \frac{x}{\left(\left(d-\sqrt{d^2-4ce}\right)^2-4e^2x^2\right)\sqrt{a+bx^4}}\,dx}{\sqrt{d^2-4ce}} + \frac{(4e^2)\int \frac{x}{\left(\left(d+\sqrt{d^2-4ce}\right)^2-4e^2x^2\right)\sqrt{a+bx^4}}\,dx}{\sqrt{d^2-4ce}}$$
$$- \left(2e\left(1-\frac{d}{\sqrt{d^2-4ce}}\right)\right)\int \frac{1}{\left(\left(d-\sqrt{d^2-4ce}\right)^2-4e^2x^2\right)\sqrt{a+bx^4}}\,dx$$
$$- \left(2e\left(1+\frac{d}{\sqrt{d^2-4ce}}\right)\right)\int \frac{1}{\left(\left(d+\sqrt{d^2-4ce}\right)^2-4e^2x^2\right)\sqrt{a+bx^4}}\,dx$$

$$= -\frac{(2e^2)\,\mathrm{Subst}\left(\int \frac{1}{\left(\left(d-\sqrt{d^2-4ce}\right)^2-4e^2x\right)\sqrt{a+bx^2}}\,dx, x, x^2\right)}{\sqrt{d^2-4ce}}$$
$$+ \frac{(2e^2)\,\mathrm{Subst}\left(\int \frac{1}{\left(\left(d+\sqrt{d^2-4ce}\right)^2-4e^2x\right)\sqrt{a+bx^2}}\,dx, x, x^2\right)}{\sqrt{d^2-4ce}}$$
$$- \frac{\left(\sqrt{b}e\left(1-\frac{d}{\sqrt{d^2-4ce}}\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce-d\sqrt{d^2-4ce}\right)}$$
$$- \frac{\left(4\sqrt{a}e^3\left(1-\frac{d}{\sqrt{d^2-4ce}}\right)\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\left(\left(d-\sqrt{d^2-4ce}\right)^2-4e^2x^2\right)\sqrt{a+bx^4}}\,dx}{2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce-d\sqrt{d^2-4ce}\right)}$$
$$- \frac{\left(\sqrt{b}e\left(1+\frac{d}{\sqrt{d^2-4ce}}\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce+d\sqrt{d^2-4ce}\right)}$$
$$- \frac{\left(4\sqrt{a}e^3\left(1+\frac{d}{\sqrt{d^2-4ce}}\right)\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\left(\left(d+\sqrt{d^2-4ce}\right)^2-4e^2x^2\right)\sqrt{a+bx^4}}\,dx}{2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce+d\sqrt{d^2-4ce}\right)}$$

$$= -\frac{e^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-bd^4+4bcd^2e-2bc^2e^2-2ae^4-bd\sqrt{d^2-4ce}(d^2-2ce)}x}{e\left(d+\sqrt{d^2-4ce}\right)\sqrt{a+bx^4}}\right)}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4-4cd^2e+2c^2e^2+d^3\sqrt{d^2-4ce}-2cde\sqrt{d^2-4ce}\right)}} + \frac{e^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-bd^4+4bcd^2e-2bc^2e^2-2ae^4+bd\sqrt{d^2-4ce}(d^2-2ce)}x}{e\left(d-\sqrt{d^2-4ce}\right)\sqrt{a+bx^4}}\right)}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4-4cd^2e+2c^2e^2-d^3\sqrt{d^2-4ce}+2cde\sqrt{d^2-4ce}\right)}} - \frac{\sqrt[4]{b}e\left(1-\frac{d}{\sqrt{d^2-4ce}}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce-d\sqrt{d^2-4ce}\right)\right)\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}e\left(1+\frac{d}{\sqrt{d^2-4ce}}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce+d\sqrt{d^2-4ce}\right)\right)\sqrt{a+bx^4}} - \frac{\sqrt[4]{a}e\left(1-\frac{d}{\sqrt{d^2-4ce}}\right)\left(4e^2-\frac{\sqrt{b}\left(d-\sqrt{d^2-4ce}\right)^2}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\Pi\left(\frac{\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce-d\sqrt{d\right.\right.}{4\sqrt{a}\sqrt{b}e^2\left(d-\sqrt{d^2-4\right.}\right.}{4\sqrt[4]{b}\left(d-\sqrt{d^2-4ce}\right)^2\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce-d\sqrt{d^2-4ce}\right)\right)\sqrt{a+}} - \frac{\sqrt[4]{a}e\left(1+\frac{d}{\sqrt{d^2-4ce}}\right)\left(4e^2-\frac{\sqrt{b}\left(d+\sqrt{d^2-4ce}\right)^2}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\Pi\left(\frac{\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce+d\sqrt{d\right.\right.}{4\sqrt{a}\sqrt{b}e^2\left(d+\sqrt{d^2-4\right.}\right.}{4\sqrt[4]{b}\left(d+\sqrt{d^2-4ce}\right)^2\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-2ce+d\sqrt{d^2-4ce}\right)\right)\sqrt{a+}} + \frac{(2e^2)\,\text{Subst}\left(\int\frac{1}{16ae^4+b\left(d-\sqrt{d^2-4ce}\right)^4-x^2}dx,x,\frac{-4ae^2-b\left(d-\sqrt{d^2-4ce}\right)^2x^2}{\sqrt{a+bx^4}}\right)}{\sqrt{d^2-4ce}} - \frac{(2e^2)\,\text{Subst}\left(\int\frac{1}{16ae^4+b\left(d+\sqrt{d^2-4ce}\right)^4-x^2}dx,x,\frac{-4ae^2-b\left(d+\sqrt{d^2-4ce}\right)^2x^2}{\sqrt{a+bx^4}}\right)}{\sqrt{d^2-4ce}}$$

= Too large to display

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 11.25 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.28

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = -\frac{i\sqrt{1+\frac{bx^4}{a}}\left(\left(-d^2+\sqrt{d^4-4cd^2e}\right)\text{EllipticPi}\left(\frac{2i\sqrt{a}e^2}{\sqrt{b}\left(d^2-2ce+\sqrt{d^4-4cd^2e}\right)},i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right),-1\right)+\left(d^2+\sqrt{\right.\right.}{2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}c\sqrt{d^4-4cd^2e}\sqrt{a+bx}}$$

$$+\sqrt{b}d\text{RootSum}\Bigg[a^2e^2-2a\sqrt{b}d^2\#1+4a\sqrt{b}ce\#1+4bc^2\#1^2-2ae^2\#1^2+2\sqrt{b}d^2\#1^3-4\sqrt{b}ce\#1^3+e^2\#1^4\&,\frac{\log\left(-\sqrt{b}x^2+\sqrt{a+bx^4}-\#1\right)\#1}{-a\sqrt{b}d^2+2a\sqrt{b}ce+4bc^2\#1-2ae^2\#1+3\sqrt{b}d^2\#1^2-6\sqrt{b}ce\#1^2+2e^2\#1^3}\&\Bigg]$$

```
[In] Integrate[1/((c + d*x + e*x^2)*Sqrt[a + b*x^4]),x]
```

```
[Out] ((-1/2*I)*Sqrt[1 + (b*x^4)/a]*((-d^2 + Sqrt[d^4 - 4*c*d^2*e])*EllipticPi[((
2*I)*Sqrt[a]*e^2)/(Sqrt[b]*(d^2 - 2*c*e + Sqrt[d^4 - 4*c*d^2*e])), I*ArcSin
h[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] + (d^2 + Sqrt[d^4 - 4*c*d^2*e])*Ellipti
cPi[((-2*I)*Sqrt[a]*e^2)/(Sqrt[b]*(-d^2 + 2*c*e + Sqrt[d^4 - 4*c*d^2*e])),
I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*c*
Sqrt[d^4 - 4*c*d^2*e]*Sqrt[a + b*x^4]) + Sqrt[b]*d*RootSum[a^2*e^2 - 2*a*Sq
rt[b]*d^2*#1 + 4*a*Sqrt[b]*c*e*#1 + 4*b*c^2*#1^2 - 2*a*e^2*#1^2 + 2*Sqrt[b]
*d^2*#1^3 - 4*Sqrt[b]*c*e*#1^3 + e^2*#1^4 & , (Log[-(Sqrt[b]*x^2) + Sqrt[a
+ b*x^4] - #1]*#1)/(-(a*Sqrt[b]*d^2) + 2*a*Sqrt[b]*c*e + 4*b*c^2*#1 - 2*a*e
^2*#1 + 3*Sqrt[b]*d^2*#1^2 - 6*Sqrt[b]*c*e*#1^2 + 2*e^2*#1^3) & ]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 1153, normalized size of antiderivative = 0.72

method	result	size
default	Expression too large to display	1153
elliptic	Expression too large to display	1153

```
[In] int(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-4*c*e+d^2)^(1/2)/(1/2*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b
/e^3*c*d^2+b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)*arctanh(1/2/(1/2
*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c*d*(-4*c*e
```

```
+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d^2-1/2/(1/2*b/e^4
*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c*d*(-4*c*e+d^2)^
(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d*(-4*c*e+d^2)^(1/2)-1/(
1/2*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c*d*(-4*
c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e*c+1/(1/2*b/e^4*d^
4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c*d*(-4*c*e+d^2)^(1/
2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*a)-2/(-4*c*e+d^2)^(1/2)/(I/a^(1/2)*b^
(1/2))^(1/2)*e/(-d+(-4*c*e+d^2)^(1/2))*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2)
)^(1/2),-4*I*a^(1/2)/b^(1/2)*e^2/(-d+(-4*c*e+d^2)^(1/2))^2,(-I/a^(1/2)*b^(1
/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))+1/2/(-4*c*e+d^2)^(1/2)/(1/2*b/e^4*d^4
+1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2
)+b/e^2*c^2+a)^(1/2)*arctanh(1/2/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^2)^
(1/2)-2*b/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+
a)^(1/2)*b*x^2/e^2*d^2+1/2/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-
2*b/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/
2)*b*x^2/e^2*d*(-4*c*e+d^2)^(1/2)-1/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^
2)^(1/2)-2*b/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x
^4+a)^(1/2)*b*x^2/e*c+1/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b
/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*
a)-2/(-4*c*e+d^2)^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)/(d+(-4*c*e+d^2)^(1/2))*e*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(
1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),-4*I*a^(1/2)/b^(1/2)/(d+(-4*c*e
+d^2)^(1/2))^2*e^2,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{a+bx^4}\,(c+dx+ex^2)}\,dx$$

```
[In] integrate(1/(e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x**4)*(c + d*x + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+a}(ex^2+dx+c)}\,dx$$

```
[In] integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^4 + a)*(e*x^2 + d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+a}(ex^2+dx+c)}\,dx$$

```
[In] integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^4 + a)*(e*x^2 + d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{b\,x^4+a}\,(e\,x^2+d\,x+c)}\,dx$$

```
[In] int(1/((a + b*x^4)^(1/2)*(c + d*x + e*x^2)),x)
[Out] int(1/((a + b*x^4)^(1/2)*(c + d*x + e*x^2)), x)
```

3.227 $\int x^m \left(c(a+bx^2)^2\right)^{3/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 161

$$\int x^m \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{a^3cx^{1+m}\sqrt{c\,(a+bx^2)^2}}{(1+m)\,(a+bx^2)} + \frac{3a^2bcx^{3+m}\sqrt{c\,(a+bx^2)^2}}{(3+m)\,(a+bx^2)} + \frac{3ab^2cx^{5+m}\sqrt{c\,(a+bx^2)^2}}{(5+m)\,(a+bx^2)} + \frac{b^3cx^{7+m}\sqrt{c\,(a+bx^2)^2}}{(7+m)\,(a+bx^2)}$$

```
[Out] a^3*c*x^(1+m)*(c*(b*x^2+a)^2)^(1/2)/(1+m)/(b*x^2+a)+3*a^2*b*c*x^(3+m)*(c*(b
*x^2+a)^2)^(1/2)/(3+m)/(b*x^2+a)+3*a*b^2*c*x^(5+m)*(c*(b*x^2+a)^2)^(1/2)/(5
+m)/(b*x^2+a)+b^3*c*x^(7+m)*(c*(b*x^2+a)^2)^(1/2)/(7+m)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1973, 276}

$$\int x^m \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{a^3cx^{m+1}\sqrt{c\,(a+bx^2)^2}}{(m+1)\,(a+bx^2)} + \frac{3a^2bcx^{m+3}\sqrt{c\,(a+bx^2)^2}}{(m+3)\,(a+bx^2)} + \frac{b^3cx^{m+7}\sqrt{c\,(a+bx^2)^2}}{(m+7)\,(a+bx^2)} + \frac{3ab^2cx^{m+5}\sqrt{c\,(a+bx^2)^2}}{(m+5)\,(a+bx^2)}$$

```
[In] Int[x^m*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^(1 + m)*Sqrt[c*(a + b*x^2)^2])/((1 + m)*(a + b*x^2)) + (3*a^2*b*c*
x^(3 + m)*Sqrt[c*(a + b*x^2)^2])/((3 + m)*(a + b*x^2)) + (3*a*b^2*c*x^(5 +
```

```
m)*Sqrt[c*(a + b*x^2)^2])/((5 + m)*(a + b*x^2)) + (b^3*c*x^(7 + m)*Sqrt[c*(
a + b*x^2)^2])/((7 + m)*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^2}\right)\int x^m\left(1+\frac{bx^2}{a}\right)^3\,dx}{1+\frac{bx^2}{a}}\\
&= \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^2}\right)\int\left(x^m+\frac{3bx^{2+m}}{a}+\frac{3b^2x^{4+m}}{a^2}+\frac{b^3x^{6+m}}{a^3}\right)dx}{1+\frac{bx^2}{a}}\\
&= \frac{a^3cx^{1+m}\sqrt{c\left(a+bx^2\right)^2}}{(1+m)\left(a+bx^2\right)}+\frac{3a^2bcx^{3+m}\sqrt{c\left(a+bx^2\right)^2}}{(3+m)\left(a+bx^2\right)}\\
&\quad+\frac{3ab^2cx^{5+m}\sqrt{c\left(a+bx^2\right)^2}}{(5+m)\left(a+bx^2\right)}+\frac{b^3cx^{7+m}\sqrt{c\left(a+bx^2\right)^2}}{(7+m)\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int x^m\left(c\left(a+bx^2\right)^2\right)^{3/2}dx=\frac{x^{1+m}\left(c(a+bx^2)^2\right)^{3/2}\left(a^3(105+71m+15m^2+m^3)+3a^2b(35+47m+13m^2+m^3)\,x^2+\ldots\right.}{(1+m)(3+m)(5+m)(7+m)\,(\ldots}$$

```
[In] Integrate[x^m*(c*(a + b*x^2)^2)^(3/2),x]
[Out] (x^(1 + m)*(c*(a + b*x^2)^2)^(3/2)*(a^3*(105 + 71*m + 15*m^2 + m^3) + 3*a^2
*b*(35 + 47*m + 13*m^2 + m^3)*x^2 + 3*a*b^2*(21 + 31*m + 11*m^2 + m^3)*x^4
+ b^3*(15 + 23*m + 9*m^2 + m^3)*x^6))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*(a +
 b*x^2)^3)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.24

method	result
gosper	$\frac{x^{1+m}\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}\left(b^{3}m^{3}x^{6}+9b^{3}m^{2}x^{6}+3a\,b^{2}m^{3}x^{4}+23m\,x^{6}b^{3}+33a\,b^{2}m^{2}x^{4}+15b^{3}x^{6}+3a^{2}b\,m^{3}x^{2}+93m\,x^{4}b^{2}a+39a^{2}b\,m^{2}x^{2}+63b^{2}a\right.}{(1+m)(3+m)(5+m)(7+m)\left(b\,x^{2}+a\right)^{3}}$
risch	$\frac{c\sqrt{c\left(b\,x^{2}+a\right)^{2}}\,\left(b^{3}m^{3}x^{6}+9b^{3}m^{2}x^{6}+3a\,b^{2}m^{3}x^{4}+23m\,x^{6}b^{3}+33a\,b^{2}m^{2}x^{4}+15b^{3}x^{6}+3a^{2}b\,m^{3}x^{2}+93m\,x^{4}b^{2}a+39a^{2}b\,m^{2}x^{2}+63b^{2}x^{4}a+a^{3}\right.}{\left(b\,x^{2}+a\right)(7+m)(5+m)(3+m)(1+m)}$

```
[In] int(x^m*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1+m)/(1+m)/(3+m)/(5+m)/(7+m)/(b*x^2+a)^3*(c*(b*x^2+a)^2)^(3/2)*(b^3*m^3*
x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^2*m^2*x^4+15*b^3*x^6+
3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*x^2+63*a*b^2*x^4+a^3*m^3+141*a^
2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2+71*a^3*m+105*a^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.45

$$\int x^m\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{\left(\left(b^3cm^3+9\,b^3cm^2+23\,b^3cm+15\,b^3c\right)x^7+3\left(ab^2cm^3+11\,ab^2cm^2+31\,ab^2cm+21\,ab^2c\right)x^5\right.}{am^4+16\,am^3+86\,am^2+(bm}$$

```
[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*c*m^3 + 9*b^3*c*m^2 + 23*b^3*c*m + 15*b^3*c)*x^7 + 3*(a*b^2*c*m^3 + 1
1*a*b^2*c*m^2 + 31*a*b^2*c*m + 21*a*b^2*c)*x^5 + 3*(a^2*b*c*m^3 + 13*a^2*b*
c*m^2 + 47*a^2*b*c*m + 35*a^2*b*c)*x^3 + (a^3*c*m^3 + 15*a^3*c*m^2 + 71*a^3
*c*m + 105*a^3*c)*x)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*x^m/(a*m^4 + 16*
a*m^3 + 86*a*m^2 + (b*m^4 + 16*b*m^3 + 86*b*m^2 + 176*b*m + 105*b)*x^2 + 17
6*a*m + 105*a)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \left(c(a+bx^2)^2\right)^{3/2} dx = \text{Timed out}$$

```
[In] integrate(x**m*(c*(b*x**2+a)**2)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.74

$$\int x^m \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{\left((m^3+9m^2+23m+15)b^3c^{\frac{3}{2}}x^7+3(m^3+11m^2+31m+21)ab^2c^{\frac{3}{2}}x^5+3(m^3+13m^2+\cdots\right.}{m^4+16m^3+86m^2+176m+\cdots}$$

```
[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*c^(3/2)*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a
*b^2*c^(3/2)*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*c^(3/2)*x^3 + (m^3 +
15*m^2 + 71*m + 105)*a^3*c^(3/2)*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 10
5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(153) = 306.

Time = 0.34 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.20

$$\int x^m \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{(b^3m^3x^7x^m\text{sgn}(bx^2+a)+9b^3m^2x^7x^m\text{sgn}(bx^2+a)+3ab^2m^3x^5x^m\text{sgn}(bx^2+a)+23b^3mx\cdots}{\cdots}$$

```
[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
[Out] (b^3*m^3*x^7*x^m*sgn(b*x^2 + a) + 9*b^3*m^2*x^7*x^m*sgn(b*x^2 + a) + 3*a*b^
2*m^3*x^5*x^m*sgn(b*x^2 + a) + 23*b^3*m*x^7*x^m*sgn(b*x^2 + a) + 33*a*b^2*m
^2*x^5*x^m*sgn(b*x^2 + a) + 15*b^3*x^7*x^m*sgn(b*x^2 + a) + 3*a^2*b*m^3*x^3
*x^m*sgn(b*x^2 + a) + 93*a*b^2*m*x^5*x^m*sgn(b*x^2 + a) + 39*a^2*b*m^2*x^3*
x^m*sgn(b*x^2 + a) + 63*a*b^2*x^5*x^m*sgn(b*x^2 + a) + a^3*m^3*x*x^m*sgn(b*
x^2 + a) + 141*a^2*b*m*x^3*x^m*sgn(b*x^2 + a) + 15*a^3*m^2*x*x^m*sgn(b*x^2
+ a) + 105*a^2*b*x^3*x^m*sgn(b*x^2 + a) + 71*a^3*m*x*x^m*sgn(b*x^2 + a) + 1
05*a^3*x*x^m*sgn(b*x^2 + a))*c^(3/2)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Mupad [B] (verification not implemented)

Time = 18.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.45

$$\int x^m\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{x^m\left(\frac{3\,a^2\,c\,x^3\,\sqrt{c\,(b\,x^2+a)^2}\,(m^3+13\,m^2+47\,m+35)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{b^2\,c\,x^7\,\sqrt{c\,(b\,x^2+a)^2}\,(m^3+9\,m^2+23\,m+15)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{3\,a\,b\,c\,x^5\,\sqrt{c\,(b\,x^2+}}{m^4+16\,m^3}\right.}{\frac{a}{b}+x^2}$$

```
[In] int(x^m*(c*(a + b*x^2)^2)^(3/2),x)

[Out] (x^m*((3*a^2*c*x^3*(c*(a + b*x^2)^2)^(1/2)*(47*m + 13*m^2 + m^3 + 35))/(176
*m + 86*m^2 + 16*m^3 + m^4 + 105) + (b^2*c*x^7*(c*(a + b*x^2)^2)^(1/2)*(23*
m + 9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105) + (3*a*b*c*x^5
*(c*(a + b*x^2)^2)^(1/2)*(31*m + 11*m^2 + m^3 + 21))/(176*m + 86*m^2 + 16*m
^3 + m^4 + 105) + (a^3*c*x*(c*(a + b*x^2)^2)^(1/2)*(71*m + 15*m^2 + m^3 + 1
05))/(b*(176*m + 86*m^2 + 16*m^3 + m^4 + 105))))/(a/b + x^2)
```

3.228 $\int x^5 \left(c(a+bx^2)^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 143

$$\int x^5\left(c(a+bx^2)^2\right)^{3/2}\,dx = \frac{a^3cx^6\sqrt{c\,(a+bx^2)^2}}{6\,(a+bx^2)} + \frac{3a^2bcx^8\sqrt{c\,(a+bx^2)^2}}{8\,(a+bx^2)} + \frac{3ab^2cx^{10}\sqrt{c\,(a+bx^2)^2}}{10\,(a+bx^2)} + \frac{b^3cx^{12}\sqrt{c\,(a+bx^2)^2}}{12\,(a+bx^2)}$$

```
[Out] 1/6*a^3*c*x^6*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/8*a^2*b*c*x^8*(c*(b*x^2+a)^
2)^(1/2)/(b*x^2+a)+3/10*a*b^2*c*x^10*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/12*b
^3*c*x^12*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1973, 272, 45}

$$\int x^5\left(c(a+bx^2)^2\right)^{3/2}\,dx = \frac{a^3cx^6\sqrt{c\,(a+bx^2)^2}}{6\,(a+bx^2)} + \frac{3a^2bcx^8\sqrt{c\,(a+bx^2)^2}}{8\,(a+bx^2)} + \frac{b^3cx^{12}\sqrt{c\,(a+bx^2)^2}}{12\,(a+bx^2)} + \frac{3ab^2cx^{10}\sqrt{c\,(a+bx^2)^2}}{10\,(a+bx^2)}$$

```
[In] Int[x^5*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^6*Sqrt[c*(a + b*x^2)^2])/(6*(a + b*x^2)) + (3*a^2*b*c*x^8*Sqrt[c*(
a + b*x^2)^2])/(8*(a + b*x^2)) + (3*a*b^2*c*x^10*Sqrt[c*(a + b*x^2)^2])/(10
*(a + b*x^2)) + (b^3*c*x^12*Sqrt[c*(a + b*x^2)^2])/(12*(a + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^2}\right)\int x^5\left(1+\frac{bx^2}{a}\right)^3\,dx}{1+\frac{bx^2}{a}} \\
&= \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^2}\right)\text{Subst}\left(\int x^2\left(1+\frac{bx}{a}\right)^3\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)} \\
&= \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^2}\right)\text{Subst}\left(\int\left(x^2+\frac{3bx^3}{a}+\frac{3b^2x^4}{a^2}+\frac{b^3x^5}{a^3}\right)dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)} \\
&= \frac{a^3cx^6\sqrt{c\left(a+bx^2\right)^2}}{6\left(a+bx^2\right)}+\frac{3a^2bcx^8\sqrt{c\left(a+bx^2\right)^2}}{8\left(a+bx^2\right)}+\frac{3ab^2cx^{10}\sqrt{c\left(a+bx^2\right)^2}}{10\left(a+bx^2\right)}+\frac{b^3cx^{12}\sqrt{c\left(a+bx^2\right)^2}}{12\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.44

$$\int x^5\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{x^6\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(20a^3+45a^2bx^2+36ab^2x^4+10b^3x^6\right)}{120\left(a+bx^2\right)^3}$$

```
[In] Integrate[x^5*(c*(a + b*x^2)^2)^(3/2),x]
[Out] (x^6*(c*(a + b*x^2)^2)^(3/2)*(20*a^3 + 45*a^2*b*x^2 + 36*a*b^2*x^4 + 10*b^3
*x^6))/(120*(a + b*x^2)^3)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{x^{6}\left(10 b^{3} x^{6}+36 b^{2} x^{4} a+45 a^{2} b\,x^{2}+20 a^{3}\right) \left(c \left(b\,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{120 \left(b\,x^{2}+a \right)^{3}}$	60
default	$\frac{x^{6}\left(10 b^{3} x^{6}+36 b^{2} x^{4} a+45 a^{2} b\,x^{2}+20 a^{3}\right) \left(c \left(b\,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{120 \left(b\,x^{2}+a \right)^{3}}$	60
pseudoelliptic	$\frac{x^{6}\left(10 b^{3} x^{6}+36 b^{2} x^{4} a+45 a^{2} b\,x^{2}+20 a^{3}\right) c \sqrt{c \left(b\,x^{2}+a \right)^{2}}}{120 b\,x^{2}+120 a}$	63
trager	$\frac{c\,x^{6}\left(10 b^{3} x^{6}+36 b^{2} x^{4} a+45 a^{2} b\,x^{2}+20 a^{3}\right) \sqrt{b^{2} c\,x^{4}+2 a b c\,x^{2}+c\,a^{2}}}{120 b\,x^{2}+120 a}$	72
risch	$\frac{a^{3} c\,x^{6} \sqrt{c \left(b\,x^{2}+a \right)^{2}}}{6 b\,x^{2}+6 a}+\frac{3 a^{2} b c\,x^{8} \sqrt{c \left(b\,x^{2}+a \right)^{2}}}{8 \left(b\,x^{2}+a \right)}+\frac{3 a\,b^{2} c\,x^{10} \sqrt{c \left(b\,x^{2}+a \right)^{2}}}{10 \left(b\,x^{2}+a \right)}+\frac{b^{3} c\,x^{12} \sqrt{c \left(b\,x^{2}+a \right)^{2}}}{12 b\,x^{2}+12 a}$	128

```
[In] int(x^5*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*x^6*(10*b^3*x^6+36*a*b^2*x^4+45*a^2*b*x^2+20*a^3)*(c*(b*x^2+a)^2)^(3/
2)/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.52

$$\int x^5 \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{(10\, b^3cx^{12} + 36\, ab^2cx^{10} + 45\, a^2bcx^8 + 20\, a^3cx^6)\sqrt{b^2cx^4 + 2\, abcx^2 + a^2c}}{120\, (bx^2 + a)}$$

```
[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/120*(10*b^3*c*x^12 + 36*a*b^2*c*x^10 + 45*a^2*b*c*x^8 + 20*a^3*c*x^6)*sqr
t(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F]

$$\int x^5 \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \int x^5 \left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}} dx$$

```
[In] integrate(x**5*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Integral(x**5*(c*(a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.95

$$\int x^5 \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \frac{\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}}a^2x^2}{8\,b^2} + \frac{\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}}a^3}{8\,b^3} + \frac{\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{5}{2}}x^2}{12\,b^2c} - \frac{7\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{5}{2}}a}{60\,b^3c}$$

```
[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*a^2*x^2/b^2 + 1/8*(b^2*c*x^4 +
2*a*b*c*x^2 + a^2*c)^(3/2)*a^3/b^3 + 1/12*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)
^(5/2)*x^2/(b^2*c) - 7/60*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(5/2)*a/(b^3*c)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int x^5 \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \frac{1}{120}\left(10\,b^3x^{12}\mathrm{sgn}\left(bx^2+a\right) + 36\,ab^2x^{10}\mathrm{sgn}\left(bx^2+a\right) + 45\,a^2bx^8\mathrm{sgn}\left(bx^2+a\right) + 20\,a^3x^6\mathrm{sgn}\left(bx^2+a\right)\right)c^{3/2}$$

```
[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/120*(10*b^3*x^12*sgn(b*x^2 + a) + 36*a*b^2*x^10*sgn(b*x^2 + a) + 45*a^2*b
*x^8*sgn(b*x^2 + a) + 20*a^3*x^6*sgn(b*x^2 + a))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \int x^5 \left(c\left(b\,x^2+a\right)^2\right)^{3/2} dx$$

```
[In] int(x^5*(c*(a + b*x^2)^2)^(3/2),x)
[Out] int(x^5*(c*(a + b*x^2)^2)^(3/2), x)
```

3.229 $\int x^4 \left(c(a+bx^2)^2\right)^{3/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 143

$$\int x^4 \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{a^3cx^5\sqrt{c(a+bx^2)^2}}{5(a+bx^2)} + \frac{3a^2bcx^7\sqrt{c(a+bx^2)^2}}{7(a+bx^2)} + \frac{ab^2cx^9\sqrt{c(a+bx^2)^2}}{3(a+bx^2)} + \frac{b^3cx^{11}\sqrt{c(a+bx^2)^2}}{11(a+bx^2)}$$

```
[Out] 1/5*a^3*c*x^5*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/7*a^2*b*c*x^7*(c*(b*x^2+a)^
2)^(1/2)/(b*x^2+a)+1/3*a*b^2*c*x^9*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/11*b^3
*c*x^11*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1973, 276}

$$\int x^4 \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{a^3cx^5\sqrt{c(a+bx^2)^2}}{5(a+bx^2)} + \frac{3a^2bcx^7\sqrt{c(a+bx^2)^2}}{7(a+bx^2)} + \frac{b^3cx^{11}\sqrt{c(a+bx^2)^2}}{11(a+bx^2)} + \frac{ab^2cx^9\sqrt{c(a+bx^2)^2}}{3(a+bx^2)}$$

```
[In] Int[x^4*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^5*Sqrt[c*(a + b*x^2)^2])/(5*(a + b*x^2)) + (3*a^2*b*c*x^7*Sqrt[c*(
a + b*x^2)^2])/(7*(a + b*x^2)) + (a*b^2*c*x^9*Sqrt[c*(a + b*x^2)^2])/(3*(a
+ b*x^2)) + (b^3*c*x^11*Sqrt[c*(a + b*x^2)^2])/(11*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^2}\right)\int x^4\left(1+\frac{bx^2}{a}\right)^3\,dx}{1+\frac{bx^2}{a}}$$

$$= \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^2}\right)\int\left(x^4+\frac{3bx^6}{a}+\frac{3b^2x^8}{a^2}+\frac{b^3x^{10}}{a^3}\right)\,dx}{1+\frac{bx^2}{a}}$$

$$= \frac{a^3cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{3a^2bcx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}+\frac{ab^2cx^9\sqrt{c\left(a+bx^2\right)^2}}{3\left(a+bx^2\right)}+\frac{b^3cx^{11}\sqrt{c\left(a+bx^2\right)^2}}{11\left(a+bx^2\right)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.44

$$\int x^4\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx = \frac{x^5\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(231a^3+495a^2bx^2+385ab^2x^4+105b^3x^6\right)}{1155\left(a+bx^2\right)^3}$$

```
[In] Integrate[x^4*(c*(a + b*x^2)^2)^(3/2),x]
[Out] (x^5*(c*(a + b*x^2)^2)^(3/2)*(231*a^3 + 495*a^2*b*x^2 + 385*a*b^2*x^4 + 105
*b^3*x^6))/(1155*(a + b*x^2)^3)
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{x^{5}\left(105 b^{3} x^{6}+385 b^{2} x^{4} a+495 a^{2} b \,x^{2}+231 a^{3}\right) \left(c \left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{1155 \left(b \,x^{2}+a \right)^{3}}$	60
default	$\frac{x^{5}\left(105 b^{3} x^{6}+385 b^{2} x^{4} a+495 a^{2} b \,x^{2}+231 a^{3}\right) \left(c \left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{1155 \left(b \,x^{2}+a \right)^{3}}$	60
trager	$\frac{c \,x^{5} \left(105 b^{3} x^{6}+385 b^{2} x^{4} a+495 a^{2} b \,x^{2}+231 a^{3}\right) \sqrt{b^{2} c \,x^{4}+2 a b c \,x^{2}+c \,a^{2}}}{1155 b \,x^{2}+1155 a}$	72
risch	$\frac{a^{3} c \,x^{5} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{5 b \,x^{2}+5 a}+\frac{3 a^{2} b c \,x^{7} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{7 \left(b \,x^{2}+a \right)}+\frac{a \,b^{2} c \,x^{9} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{3 b \,x^{2}+3 a}+\frac{b^{3} c \,x^{11} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{11 b \,x^{2}+11 a}$	128

```
[In] int(x^4*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1155*x^5*(105*b^3*x^6+385*a*b^2*x^4+495*a^2*b*x^2+231*a^3)*(c*(b*x^2+a)^2
)^(3/2)/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.52

$$\int x^4 \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{(105\, b^3cx^{11} + 385\, ab^2cx^9 + 495\, a^2bcx^7 + 231\, a^3cx^5)\sqrt{b^2cx^4 + 2\, abcx^2 + a^2c}}{1155\, (bx^2 + a)}$$

```
[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(105*b^3*c*x^11 + 385*a*b^2*c*x^9 + 495*a^2*b*c*x^7 + 231*a^3*c*x^5)
*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F]

$$\int x^4 \left(c(a+bx^2)^2\right)^{3/2} dx = \int x^{4} \left(c \left(a + b x^{2}\right)^{2}\right)^{\frac{3}{2}}\, dx$$

```
[In] integrate(x**4*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Integral(x**4*(c*(a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.33

$$\int x^4\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{1}{11}\,b^3c^{\frac{3}{2}}x^{11}+\frac{1}{3}\,ab^2c^{\frac{3}{2}}x^9+\frac{3}{7}\,a^2bc^{\frac{3}{2}}x^7+\frac{1}{5}\,a^3c^{\frac{3}{2}}x^5$$

```
[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*c^(3/2)*x^11 + 1/3*a*b^2*c^(3/2)*x^9 + 3/7*a^2*b*c^(3/2)*x^7 + 1/5
*a^3*c^(3/2)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int x^4\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{1}{1155}\,\left(105\,b^3x^{11}\mathrm{sgn}\left(bx^2+a\right)+385\,ab^2x^9\mathrm{sgn}\left(bx^2+a\right)+495\,a^2bx^7\mathrm{sgn}\left(bx^2+a\right)+231\,a^3x^5\right.$$

```
[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1155*(105*b^3*x^11*sgn(b*x^2 + a) + 385*a*b^2*x^9*sgn(b*x^2 + a) + 495*a^
2*b*x^7*sgn(b*x^2 + a) + 231*a^3*x^5*sgn(b*x^2 + a))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\int x^4\,{\left(c\,{\left(b\,x^2+a\right)}^2\right)}^{3/2}\,dx$$

```
[In] int(x^4*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] int(x^4*(c*(a + b*x^2)^2)^(3/2), x)
```

3.230 $\int x^3 \left(c(a+bx^2)^2\right)^{3/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int x^3 \left(c(a+bx^2)^2\right)^{3/2} dx = -\frac{ac(a+bx^2)^3 \sqrt{c\,(a+bx^2)^2}}{8b^2} + \frac{c(a+bx^2)^4 \sqrt{c\,(a+bx^2)^2}}{10b^2}$$

```
[Out] -1/8*a*c*(b*x^2+a)^3*(c*(b*x^2+a)^2)^(1/2)/b^2+1/10*c*(b*x^2+a)^4*(c*(b*x^2
+a)^2)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1973, 272, 45}

$$\int x^3 \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{c(a+bx^2)^4 \sqrt{c\,(a+bx^2)^2}}{10b^2} - \frac{ac(a+bx^2)^3 \sqrt{c\,(a+bx^2)^2}}{8b^2}$$

```
[In] Int[x^3*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] -1/8*(a*c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^2])/b^2 + (c*(a + b*x^2)^4*Sqrt[
c*(a + b*x^2)^2])/(10*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^2}\right)\int x^3\left(1+\frac{bx^2}{a}\right)^3\,dx}{1+\frac{bx^2}{a}} \\
&= \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^2}\right)\text{Subst}\left(\int x\left(1+\frac{bx}{a}\right)^3\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)} \\
&= \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^2}\right)\text{Subst}\left(\int\left(-\frac{a\left(1+\frac{bx}{a}\right)^3}{b}+\frac{a\left(1+\frac{bx}{a}\right)^4}{b}\right)dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)} \\
&= -\frac{ac(a+bx^2)^3\sqrt{c\left(a+bx^2\right)^2}}{8b^2}+\frac{c(a+bx^2)^4\sqrt{c\left(a+bx^2\right)^2}}{10b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int x^3\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{x^4\left(c(a+bx^2)^2\right)^{3/2}\left(10a^3+20a^2bx^2+15ab^2x^4+4b^3x^6\right)}{40\left(a+bx^2\right)^3}$$

```
[In] Integrate[x^3*(c*(a + b*x^2)^2)^(3/2),x]
[Out] (x^4*(c*(a + b*x^2)^2)^(3/2)*(10*a^3 + 20*a^2*b*x^2 + 15*a*b^2*x^4 + 4*b^3*
x^6))/(40*(a + b*x^2)^3)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

method	result	size
gosper	$\frac{x^{4}\left(4b^{3}x^{6}+15b^{2}x^{4}a+20a^{2}b\,x^{2}+10a^{3}\right)\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{40\left(b\,x^{2}+a\right)^{3}}$	60
default	$\frac{x^{4}\left(4b^{3}x^{6}+15b^{2}x^{4}a+20a^{2}b\,x^{2}+10a^{3}\right)\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{40\left(b\,x^{2}+a\right)^{3}}$	60
pseudoelliptic	$\frac{x^{4}\left(4b^{3}x^{6}+15b^{2}x^{4}a+20a^{2}b\,x^{2}+10a^{3}\right)c\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{40b\,x^{2}+40a}$	63
trager	$\frac{c\,x^{4}\left(4b^{3}x^{6}+15b^{2}x^{4}a+20a^{2}b\,x^{2}+10a^{3}\right)\sqrt{b^{2}c\,x^{4}+2abc\,x^{2}+c\,a^{2}}}{40b\,x^{2}+40a}$	72
risch	$\frac{c\sqrt{c\left(b\,x^{2}+a\right)^{2}}\,b^{3}x^{10}}{10b\,x^{2}+10a}+\frac{3c\sqrt{c\left(b\,x^{2}+a\right)^{2}}\,a\,b^{2}x^{8}}{8\left(b\,x^{2}+a\right)}+\frac{c\sqrt{c\left(b\,x^{2}+a\right)^{2}}\,a^{2}b\,x^{6}}{2b\,x^{2}+2a}+\frac{c\sqrt{c\left(b\,x^{2}+a\right)^{2}}\,a^{3}x^{4}}{4b\,x^{2}+4a}$	128

```
[In] int(x^3*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*x^4*(4*b^3*x^6+15*a*b^2*x^4+20*a^2*b*x^2+10*a^3)*(c*(b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\int x^3\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{\left(4\,b^3cx^{10}+15\,ab^2cx^8+20\,a^2bcx^6+10\,a^3cx^4\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{40\,\left(bx^2+a\right)}$$

```
[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(4*b^3*c*x^10 + 15*a*b^2*c*x^8 + 20*a^2*b*c*x^6 + 10*a^3*c*x^4)*sqrt(b
^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F]

$$\int x^3\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\int x^3\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**3*(c*(b*x**2+a)**2)**(3/2),x)
[Out] Integral(x**3*(c*(a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.48

$$\int x^3\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=-\frac{(b^2cx^4+2\,abcx^2+a^2c)^{\frac{3}{2}}ax^2}{8\,b}-\frac{(b^2cx^4+2\,abcx^2+a^2c)^{\frac{3}{2}}a^2}{8\,b^2}+\frac{(b^2cx^4+2\,abcx^2+a^2c)^{\frac{5}{2}}}{10\,b^2c}$$

```
[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
[Out] -1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*a*x^2/b - 1/8*(b^2*c*x^4 + 2*a
*b*c*x^2 + a^2*c)^(3/2)*a^2/b^2 + 1/10*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(5
/2)/(b^2*c)
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int x^3\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{1}{40}\left(4\,b^3x^{10}+15\,ab^2x^8+20\,a^2bx^6+10\,a^3x^4\right)c^{\frac{3}{2}}\mathrm{sgn}\left(bx^2+a\right)$$

```
[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
[Out] 1/40*(4*b^3*x^10 + 15*a*b^2*x^8 + 20*a^2*b*x^6 + 10*a^3*x^4)*c^(3/2)*sgn(b*
x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int x^3 \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \frac{\left(-a^2+3\,a\,b\,x^2+4\,b^2\,x^4\right)\,\left(c\,a^2+2\,c\,a\,b\,x^2+c\,b^2\,x^4\right)^{3/2}}{40\,b^2}$$

```
[In] int(x^3*(c*(a + b*x^2)^2)^(3/2),x)
[Out] ((4*b^2*x^4 - a^2 + 3*a*b*x^2)*(a^2*c + b^2*c*x^4 + 2*a*b*c*x^2)^(3/2))/(40
*b^2)
```

3.231 $\int x^2 \left(c(a+bx^2)^2\right)^{3/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 143

$$\int x^2 \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{a^3cx^3\sqrt{c(a+bx^2)^2}}{3(a+bx^2)} + \frac{3a^2bcx^5\sqrt{c(a+bx^2)^2}}{5(a+bx^2)} + \frac{3ab^2cx^7\sqrt{c(a+bx^2)^2}}{7(a+bx^2)} + \frac{b^3cx^9\sqrt{c(a+bx^2)^2}}{9(a+bx^2)}$$

```
[Out] 1/3*a^3*c*x^3*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/5*a^2*b*c*x^5*(c*(b*x^2+a)^
2)^(1/2)/(b*x^2+a)+3/7*a*b^2*c*x^7*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/9*b^3*
c*x^9*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1973, 276}

$$\int x^2 \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{a^3cx^3\sqrt{c(a+bx^2)^2}}{3(a+bx^2)} + \frac{3a^2bcx^5\sqrt{c(a+bx^2)^2}}{5(a+bx^2)} + \frac{b^3cx^9\sqrt{c(a+bx^2)^2}}{9(a+bx^2)} + \frac{3ab^2cx^7\sqrt{c(a+bx^2)^2}}{7(a+bx^2)}$$

```
[In] Int[x^2*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^3*Sqrt[c*(a + b*x^2)^2])/(3*(a + b*x^2)) + (3*a^2*b*c*x^5*Sqrt[c*(
a + b*x^2)^2])/(5*(a + b*x^2)) + (3*a*b^2*c*x^7*Sqrt[c*(a + b*x^2)^2])/(7*(
a + b*x^2)) + (b^3*c*x^9*Sqrt[c*(a + b*x^2)^2])/(9*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^2}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^3\,dx}{1+\frac{bx^2}{a}}$$

$$= \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^2}\right)\int\left(x^2+\frac{3bx^4}{a}+\frac{3b^2x^6}{a^2}+\frac{b^3x^8}{a^3}\right)dx}{1+\frac{bx^2}{a}}$$

$$= \frac{a^3cx^3\sqrt{c\left(a+bx^2\right)^2}}{3\left(a+bx^2\right)}+\frac{3a^2bcx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{3ab^2cx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}+\frac{b^3cx^9\sqrt{c\left(a+bx^2\right)^2}}{9\left(a+bx^2\right)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.44

$$\int x^2\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx = \frac{x^3\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(105a^3+189a^2bx^2+135ab^2x^4+35b^3x^6\right)}{315\left(a+bx^2\right)^3}$$

```
[In] Integrate[x^2*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^3*(c*(a + b*x^2)^2)^(3/2)*(105*a^3 + 189*a^2*b*x^2 + 135*a*b^2*x^4 + 35*
b^3*x^6))/(315*(a + b*x^2)^3)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{x^{3}\left(35 b^{3} x^{6}+135 b^{2} x^{4} a+189 a^{2} b \,x^{2}+105 a^{3}\right) \left(c \left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{315 \left(b \,x^{2}+a \right)^{3}}$	60
default	$\frac{x^{3}\left(35 b^{3} x^{6}+135 b^{2} x^{4} a+189 a^{2} b \,x^{2}+105 a^{3}\right) \left(c \left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{315 \left(b \,x^{2}+a \right)^{3}}$	60
trager	$\frac{c \,x^{3}\left(35 b^{3} x^{6}+135 b^{2} x^{4} a+189 a^{2} b \,x^{2}+105 a^{3}\right) \sqrt{b^{2} c \,x^{4}+2 a b c \,x^{2}+c \,a^{2}}}{315 b \,x^{2}+315 a}$	72
risch	$\frac{a^{3} c \,x^{3} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{3 b \,x^{2}+3 a}+\frac{3 a^{2} b c \,x^{5} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{5 \left(b \,x^{2}+a \right)}+\frac{3 a \,b^{2} c \,x^{7} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{7 \left(b \,x^{2}+a \right)}+\frac{b^{3} c \,x^{9} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{9 b \,x^{2}+9 a}$	128

```
[In] int(x^2*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/315*x^3*(35*b^3*x^6+135*a*b^2*x^4+189*a^2*b*x^2+105*a^3)*(c*(b*x^2+a)^2)^
(3/2)/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.52

$$\int x^2 \left(c(a+bx^2)^2\right)^{3/2}\,dx = \frac{(35\,b^3cx^9+135\,ab^2cx^7+189\,a^2bcx^5+105\,a^3cx^3)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{315\,(bx^2+a)}$$

```
[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
[Out] 1/315*(35*b^3*c*x^9 + 135*a*b^2*c*x^7 + 189*a^2*b*c*x^5 + 105*a^3*c*x^3)*sq
rt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F]

$$\int x^2 \left(c(a+bx^2)^2\right)^{3/2}\,dx = \int x^2 \left(c(a+bx^2)^2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**2*(c*(b*x**2+a)**2)**(3/2),x)
[Out] Integral(x**2*(c*(a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.33

$$\int x^2\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{1}{9}\,b^3c^{\frac{3}{2}}x^9+\frac{3}{7}\,ab^2c^{\frac{3}{2}}x^7+\frac{3}{5}\,a^2bc^{\frac{3}{2}}x^5+\frac{1}{3}\,a^3c^{\frac{3}{2}}x^3$$

```
[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
[Out] 1/9*b^3*c^(3/2)*x^9 + 3/7*a*b^2*c^(3/2)*x^7 + 3/5*a^2*b*c^(3/2)*x^5 + 1/3*a
^3*c^(3/2)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int x^2\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{1}{315}\left(35\,b^3x^9\mathrm{sgn}\left(bx^2+a\right)+135\,ab^2x^7\mathrm{sgn}\left(bx^2+a\right)+189\,a^2bx^5\mathrm{sgn}\left(bx^2+a\right)+105\,a^3x^3\mathrm{sgn}\left(\right.\right.$$

```
[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
[Out] 1/315*(35*b^3*x^9*sgn(b*x^2 + a) + 135*a*b^2*x^7*sgn(b*x^2 + a) + 189*a^2*b
*x^5*sgn(b*x^2 + a) + 105*a^3*x^3*sgn(b*x^2 + a))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\int x^2\,{\left(c\,{\left(b\,x^2+a\right)}^2\right)}^{3/2}\,dx$$

```
[In] int(x^2*(c*(a + b*x^2)^2)^(3/2),x)
[Out] int(x^2*(c*(a + b*x^2)^2)^(3/2), x)
```

3.232 $\int x\left(c(a+bx^2)^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 32

$$\int x\left(c(a+bx^2)^2\right)^{3/2}\,dx=\frac{c(a+bx^2)^3\sqrt{c\,(a+bx^2)^2}}{8b}$$

```
[Out] 1/8*c*(b*x^2+a)^3*(c*(b*x^2+a)^2)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1605, 15, 30}

$$\int x\left(c(a+bx^2)^2\right)^{3/2}\,dx=\frac{c(a+bx^2)^3\sqrt{c\,(a+bx^2)^2}}{8b}$$

```
[In] Int[x*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^2])/(8*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \left(cx^2\right)^{3/2}\,dx, x, a+bx^2\right)}{2b}\\ &= \frac{\left(c\sqrt{c\left(a+bx^2\right)^2}\right)\text{Subst}\left(\int x^3\,dx, x, a+bx^2\right)}{2b\left(a+bx^2\right)}\\ &= \frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^2}}{8b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx = \frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{8b}$$

```
[In] Integrate[x*(c*(a + b*x^2)^2)^(3/2),x]
[Out] ((a + b*x^2)*(c*(a + b*x^2)^2)^(3/2))/(8*b)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\left(c \left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}} \left(b \,x^{2}+a \right)}{8 b}$	26
risch	$\frac{c \left(b \,x^{2}+a \right)^{3} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{8 b}$	29
gosper	$\frac{x^{2} \left(b^{3} x^{6}+4 b^{2} x^{4} a +6 a^{2} b \,x^{2}+4 a^{3}\right) \left(c \left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{8 \left(b \,x^{2}+a \right)^{3}}$	59
pseudoelliptic	$\frac{x^{2} \left(b^{3} x^{6}+4 b^{2} x^{4} a +6 a^{2} b \,x^{2}+4 a^{3}\right) c \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{8 b \,x^{2}+8 a}$	62
trager	$\frac{c \,x^{2} \left(b^{3} x^{6}+4 b^{2} x^{4} a +6 a^{2} b \,x^{2}+4 a^{3}\right) \sqrt{b^{2} c \,x^{4}+2 a b c \,x^{2}+c \,a^{2}}}{8 b \,x^{2}+8 a}$	71

```
[In] int(x*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/8*(c*(b*x^2+a)^2)^(3/2)*(b*x^2+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.28

$$\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx = \frac{(b^3cx^8 + 4\,ab^2cx^6 + 6\,a^2bcx^4 + 4\,a^3cx^2)\sqrt{b^2cx^4 + 2\,abcx^2 + a^2c}}{8\,(bx^2 + a)}$$

```
[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
[Out] 1/8*(b^3*c*x^8 + 4*a*b^2*c*x^6 + 6*a^2*b*c*x^4 + 4*a^3*c*x^2)*sqrt(b^2*c*x^
4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F]

$$\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx = \int x\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(c*(b*x**2+a)**2)**(3/2),x)
[Out] Integral(x*(c*(a + b*x**2)**2)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(28) = 56.

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.88

$$\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{1}{8}\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}}x^2+\frac{\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}}a}{8\,b}$$

```
[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*x^2 + 1/8*(b^2*c*x^4 + 2*a*b*c*
x^2 + a^2*c)^(3/2)*a/b
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{\left(bx^2+a\right)^4c^{\frac{3}{2}}\operatorname{sgn}\left(bx^2+a\right)}{8\,b}$$

```
[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(b*x^2 + a)^4*c^(3/2)*sgn(b*x^2 + a)/b
```

Mupad [B] (verification not implemented)

Time = 18.47 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{\left(b^2\,x^2+a\,b\right)\,\left(c\,a^2+2\,c\,a\,b\,x^2+c\,b^2\,x^4\right)^{3/2}}{8\,b^2}$$

```
[In] int(x*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] ((a*b + b^2*x^2)*(a^2*c + b^2*c*x^4 + 2*a*b*c*x^2)^(3/2))/(8*b^2)
```

3.233 $\int \left(c\left(a+bx^2\right)^2\right)^{3/2} dx$

Optimal result

Integrand size = 15, antiderivative size = 135

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \frac{a^3cx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{a^2bcx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{3ab^2cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{b^3cx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}$$

```
[Out] a^3*c*x*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+a^2*b*c*x^3*(c*(b*x^2+a)^2)^(1/2)/(
b*x^2+a)+3/5*a*b^2*c*x^5*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/7*b^3*c*x^7*(c*(
b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1973, 200}

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \frac{a^3cx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{a^2bcx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b^3cx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}+\frac{3ab^2cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}$$

```
[In] Int[(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x*Sqrt[c*(a + b*x^2)^2])/(a + b*x^2) + (a^2*b*c*x^3*Sqrt[c*(a + b*x^
2)^2])/(a + b*x^2) + (3*a*b^2*c*x^5*Sqrt[c*(a + b*x^2)^2])/(5*(a + b*x^2))
 + (b^3*c*x^7*Sqrt[c*(a + b*x^2)^2])/(7*(a + b*x^2))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(a^2c\sqrt{c\,(a+bx^2)^2}\right)\int\left(1+\frac{bx^2}{a}\right)^3\,dx}{1+\frac{bx^2}{a}}$$

$$= \frac{\left(a^2c\sqrt{c\,(a+bx^2)^2}\right)\int\left(1+\frac{3bx^2}{a}+\frac{3b^2x^4}{a^2}+\frac{b^3x^6}{a^3}\right)\,dx}{1+\frac{bx^2}{a}}$$

$$= \frac{a^3cx\sqrt{c\,(a+bx^2)^2}}{a+bx^2}+\frac{a^2bcx^3\sqrt{c\,(a+bx^2)^2}}{a+bx^2}+\frac{3ab^2cx^5\sqrt{c\,(a+bx^2)^2}}{5\,(a+bx^2)}+\frac{b^3cx^7\sqrt{c\,(a+bx^2)^2}}{7\,(a+bx^2)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx = \frac{x\left(c\left(a+bx^2\right)^2\right)^{3/2}(35a^3+35a^2bx^2+21ab^2x^4+5b^3x^6)}{35\,(a+bx^2)^3}$$

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2),x]
[Out] (x*(c*(a + b*x^2)^2)^(3/2)*(35*a^3 + 35*a^2*b*x^2 + 21*a*b^2*x^4 + 5*b^3*x^
6))/(35*(a + b*x^2)^3)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.43

method	result	size
gosper	$\frac{x\left(5b^{3}x^{6}+21b^{2}x^{4}a+35a^{2}b\,x^{2}+35a^{3}\right)\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{35\left(b\,x^{2}+a\right)^{3}}$	58
default	$\frac{x\left(5b^{3}x^{6}+21b^{2}x^{4}a+35a^{2}b\,x^{2}+35a^{3}\right)\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{35\left(b\,x^{2}+a\right)^{3}}$	58
trager	$\frac{cx\left(5b^{3}x^{6}+21b^{2}x^{4}a+35a^{2}b\,x^{2}+35a^{3}\right)\sqrt{b^{2}c\,x^{4}+2abc\,x^{2}+c\,a^{2}}}{35b\,x^{2}+35a}$	70
risch	$\frac{a^{3}cx\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{b\,x^{2}+a}+\frac{a^{2}bc\,x^{3}\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{b\,x^{2}+a}+\frac{3a\,b^{2}c\,x^{5}\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{5\left(b\,x^{2}+a\right)}+\frac{b^{3}c\,x^{7}\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{7b\,x^{2}+7a}$	124

```
[In] int((c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*x*(5*b^3*x^6+21*a*b^2*x^4+35*a^2*b*x^2+35*a^3)*(c*(b*x^2+a)^2)^(3/2)/(
b*x^2+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.53

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx = \frac{\left(5\,b^3cx^7+21\,ab^2cx^5+35\,a^2bcx^3+35\,a^3cx\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{35\,\left(bx^2+a\right)}$$

```
[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^3*c*x^7 + 21*a*b^2*c*x^5 + 35*a^2*b*c*x^3 + 35*a^3*c*x)*sqrt(b^2*
c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F]

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx = \int \left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \frac{1}{7}b^3c^{\frac{3}{2}}x^7 + \frac{3}{5}ab^2c^{\frac{3}{2}}x^5 + a^2bc^{\frac{3}{2}}x^3 + a^3c^{\frac{3}{2}}x$$

```
[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
[Out] 1/7*b^3*c^(3/2)*x^7 + 3/5*a*b^2*c^(3/2)*x^5 + a^2*b*c^(3/2)*x^3 + a^3*c^(3/
2)*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.34

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \frac{1}{35}\left(5\,b^3x^7 + 21\,ab^2x^5 + 35\,a^2bx^3 + 35\,a^3x\right)c^{\frac{3}{2}}\mathrm{sgn}\left(bx^2+a\right)$$

```
[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
[Out] 1/35*(5*b^3*x^7 + 21*a*b^2*x^5 + 35*a^2*b*x^3 + 35*a^3*x)*c^(3/2)*sgn(b*x^2
 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \int \left(c\,\left(b\,x^2+a\right)^2\right)^{3/2} \,d x$$

```
[In] int((c*(a + b*x^2)^2)^(3/2),x)
[Out] int((c*(a + b*x^2)^2)^(3/2), x)
```

3.234 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x} dx$

Optimal result

Integrand size = 19, antiderivative size = 139

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x} dx = \frac{3a^2bcx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)} + \frac{3ab^2cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)} + \frac{b^3cx^6\sqrt{c\left(a+bx^2\right)^2}}{6\left(a+bx^2\right)} + \frac{a^3c\sqrt{c\left(a+bx^2\right)^2}\log(x)}{a+bx^2}$$

```
[Out] 3/2*a^2*b*c*x^2*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/4*a*b^2*c*x^4*(c*(b*x^2+a
)^2)^(1/2)/(b*x^2+a)+1/6*b^3*c*x^6*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+a^3*c*ln
(x)*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1973, 272, 45}

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x} dx = \frac{a^3c\log(x)\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2} + \frac{3a^2bcx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)} + \frac{b^3cx^6\sqrt{c\left(a+bx^2\right)^2}}{6\left(a+bx^2\right)} + \frac{3ab^2cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)}$$

```
[In] Int[(c*(a + b*x^2)^2)^(3/2)/x,x]
```

```
[Out] (3*a^2*b*c*x^2*Sqrt[c*(a + b*x^2)^2])/(2*(a + b*x^2)) + (3*a*b^2*c*x^4*Sqrt
[c*(a + b*x^2)^2])/(4*(a + b*x^2)) + (b^3*c*x^6*Sqrt[c*(a + b*x^2)^2])/(6*(
a + b*x^2)) + (a^3*c*Sqrt[c*(a + b*x^2)^2]*Log[x])/(a + b*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(a^2c\sqrt{c\,(a+bx^2)^2}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^3}{x}\,dx}{1+\frac{bx^2}{a}}\\
&= \frac{\left(a^2c\sqrt{c\,(a+bx^2)^2}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^3}{x}\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)}\\
&= \frac{\left(a^2c\sqrt{c\,(a+bx^2)^2}\right)\text{Subst}\left(\int\left(\frac{3b}{a}+\frac{1}{x}+\frac{3b^2x}{a^2}+\frac{b^3x^2}{a^3}\right)\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)}\\
&= \frac{3a^2bcx^2\sqrt{c\,(a+bx^2)^2}}{2\,(a+bx^2)}+\frac{3ab^2cx^4\sqrt{c\,(a+bx^2)^2}}{4\,(a+bx^2)}\\
&\quad+\frac{b^3cx^6\sqrt{c\,(a+bx^2)^2}}{6\,(a+bx^2)}+\frac{a^3c\sqrt{c\,(a+bx^2)^2}\log(x)}{a+bx^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.68

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x}\,dx = \frac{c\left(b\left(2b^2x^4\left(-\sqrt{b^2c}x^2+\sqrt{c\,(a+bx^2)^2}\right)+abx^2\left(-9\sqrt{b^2c}x^2+7\sqrt{c\,(a+bx^2)^2}\right)\right.\right.}{}$$

[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x,x]

[Out] (c*(b*(2*b^2*x^4*(-(Sqrt[b^2*c]*x^2) + Sqrt[c*(a + b*x^2)^2]) + a*b*x^2*(-9 *Sqrt[b^2*c]*x^2 + 7*Sqrt[c*(a + b*x^2)^2]) + a^2*(-18*Sqrt[b^2*c]*x^2 + 11 *Sqrt[c*(a + b*x^2)^2])) + 12*a^3*b*Sqrt[c]*ArcTanh[(Sqrt[b^2*c]*x^2 - Sqrt [c*(a + b*x^2)^2])/(a*Sqrt[c])] - 6*a^3*Sqrt[b^2*c]*Log[x^2*(a*b*c + b^2*c* x^2 - Sqrt[b^2*c]*Sqrt[c*(a + b*x^2)^2])]))/(24*b)

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.42

method	result	size
default	$\frac{\left(c\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\left(2b^3x^6+9b^2x^4a+18a^2b\,x^2+12a^3\ln\left(x\right)\right)}{12\left(b\,x^2+a\right)^3}$	59
pseudoelliptic	$\frac{c\sqrt{c\left(b\,x^2+a\right)^2}\,\left(2b^3x^6+9b^2x^4a+6a^3\ln\left(x^2\right)+18a^2b\,x^2\right)}{12b\,x^2+12a}$	64
risch	$\frac{c\sqrt{c\left(b\,x^2+a\right)^2}\,b\left(\frac{1}{6}b^2x^6+\frac{3}{4}ab\,x^4+\frac{3}{2}a^2x^2\right)}{b\,x^2+a}+\frac{a^3c\ln\left(x\right)\sqrt{c\left(b\,x^2+a\right)^2}}{b\,x^2+a}$	80

[In] int((c*(b*x^2+a)^2)^(3/2)/x,x,method=_RETURNVERBOSE)

[Out] 1/12*(c*(b*x^2+a)^2)^(3/2)*(2*b^3*x^6+9*b^2*x^4*a+18*a^2*b*x^2+12*a^3*ln(x))/(b*x^2+a)^3

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.53

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x}\,dx = \frac{(2\,b^3cx^6+9\,ab^2cx^4+18\,a^2bcx^2+12\,a^3c\log\left(x\right))\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{12\,(bx^2+a)}$$

[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="fricas")

[Out] 1/12*(2*b^3*c*x^6 + 9*a*b^2*c*x^4 + 18*a^2*b*c*x^2 + 12*a^3*c*log(x))*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)

Sympy [F]

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x}\,dx = \int \frac{\left(c(a+bx^2)^2\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((c*(b*x**2+a)**2)**(3/2)/x,x)
[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.23

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x}\,dx = \frac{1}{2}\,(-1)^{2\,b^2cx^2+2\,abc}\,a^3c^{\frac{3}{2}}\log\left(2\,b^2cx^2+2\,abc\right) - \frac{1}{2}\,(-1)^{2\,abcx^2+2\,a^2c}\,a^3c^{\frac{3}{2}}\log\left(2\,abc+\frac{2\,a^2c}{x^2}\right) + \frac{1}{4}\,\sqrt{b^2cx^4+2\,abcx^2+a^2c}abcx^2 + \frac{3}{4}\,\sqrt{b^2cx^4+2\,abcx^2+a^2c}a^2c + \frac{1}{6}\,\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}}$$

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="maxima")
[Out] 1/2*(-1)^(2*b^2*c*x^2 + 2*a*b*c)*a^3*c^(3/2)*log(2*b^2*c*x^2 + 2*a*b*c) - 1
/2*(-1)^(2*a*b*c*x^2 + 2*a^2*c)*a^3*c^(3/2)*log(2*a*b*c + 2*a^2*c/x^2) + 1/
4*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*a*b*c*x^2 + 3/4*sqrt(b^2*c*x^4 + 2*
a*b*c*x^2 + a^2*c)*a^2*c + 1/6*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.53

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x}\,dx = \frac{1}{12}\,\left(2\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+9\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+18\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+6\,a^3\log\left(x\right.\right.$$

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="giac")
[Out] 1/12*(2*b^3*x^6*sgn(b*x^2 + a) + 9*a*b^2*x^4*sgn(b*x^2 + a) + 18*a^2*b*x^2*
sgn(b*x^2 + a) + 6*a^3*log(x^2)*sgn(b*x^2 + a))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x}\,dx=\int \frac{{\left(c\,{\left(b\,x^2+a\right)}^2\right)}^{3/2}}{x} \,d x$$

```
[In] int((c*(a + b*x^2)^2)^(3/2)/x,x)
[Out] int((c*(a + b*x^2)^2)^(3/2)/x, x)
```

3.235 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2}\,dx = -\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{x\left(a+bx^2\right)} + \frac{3a^2bcx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2} + \frac{ab^2cx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2} + \frac{b^3cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}$$

```
[Out] -a^3*c*(c*(b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+3*a^2*b*c*x*(c*(b*x^2+a)^2)^(1/2)/
(b*x^2+a)+a*b^2*c*x^3*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/5*b^3*c*x^5*(c*(b*x
^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1973, 276}

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2}\,dx = -\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{x\left(a+bx^2\right)} + \frac{3a^2bcx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2} + \frac{b^3cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)} + \frac{ab^2cx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}$$

```
[In] Int[(c*(a + b*x^2)^2)^(3/2)/x^2,x]
```

```
[Out] -((a^3*c*Sqrt[c*(a + b*x^2)^2])/(x*(a + b*x^2))) + (3*a^2*b*c*x*Sqrt[c*(a +
 b*x^2)^2])/(a + b*x^2) + (a*b^2*c*x^3*Sqrt[c*(a + b*x^2)^2])/(a + b*x^2) +
 (b^3*c*x^5*Sqrt[c*(a + b*x^2)^2])/(5*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^2}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^3}{x^2}\,dx}{1+\frac{bx^2}{a}}$$

$$= \frac{\left(a^2c\sqrt{c\left(a+bx^2\right)^2}\right)\int\left(\frac{3b}{a}+\frac{1}{x^2}+\frac{3b^2x^2}{a^2}+\frac{b^3x^4}{a^3}\right)dx}{1+\frac{bx^2}{a}}$$

$$= -\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{x\left(a+bx^2\right)}+\frac{3a^2bcx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{ab^2cx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b^3cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.46

$$\int\frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2}\,dx = \frac{\left(c(a+bx^2)^2\right)^{3/2}\left(-5a^3+15a^2bx^2+5ab^2x^4+b^3x^6\right)}{5x\left(a+bx^2\right)^3}$$

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x^2,x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x^4 + b^3*x^6))/(
5*x*(a + b*x^2)^3)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.45

method	result	size
gosper	$-\frac{\left(-b^{3} x^{6}-5 b^{2} x^{4} a-15 a^{2} b \,x^{2}+5 a^{3}\right) \left(c \left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{5 x \left(b \,x^{2}+a \right)^{3}}$	60
default	$-\frac{\left(-b^{3} x^{6}-5 b^{2} x^{4} a-15 a^{2} b \,x^{2}+5 a^{3}\right) \left(c \left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{5 x \left(b \,x^{2}+a \right)^{3}}$	60
risch	$\frac{c \sqrt{c \left(b \,x^{2}+a \right)^{2}}\, b \left(\frac{1}{5} b^{2} x^{5}+a b \,x^{3}+3 a^{2} x \right)}{b \,x^{2}+a}-\frac{a^{3} c \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{x \left(b \,x^{2}+a \right)}$	79
trager	$\frac{c \left(b^{3} x^{5}+b^{3} x^{4}+5 a \,b^{2} x^{3}+b^{3} x^{3}+5 a \,b^{2} x^{2}+b^{3} x^{2}+15 a^{2} b x +5 b^{2} a x +b^{3} x +5 a^{3}\right) \left(x -1\right) \sqrt{b^{2} c \,x^{4}+2 a b c \,x^{2}+c \,a^{2}}}{5 x \left(b \,x^{2}+a \right)}$	114

```
[In] int((c*(b*x^2+a)^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(-b^3*x^6-5*a*b^2*x^4-15*a^2*b*x^2+5*a^3)*(c*(b*x^2+a)^2)^(3/2)/x/(b*x
^2+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.54

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2}\,dx = \frac{(b^3cx^6+5\,ab^2cx^4+15\,a^2bcx^2-5\,a^3c)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{5\,(bx^3+ax)}$$

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*(b^3*c*x^6 + 5*a*b^2*c*x^4 + 15*a^2*b*c*x^2 - 5*a^3*c)*sqrt(b^2*c*x^4 +
 2*a*b*c*x^2 + a^2*c)/(b*x^3 + a*x)
```

Sympy [F]

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2}\,dx = \int \frac{\left(c(a+bx^2)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((c*(b*x**2+a)**2)**(3/2)/x**2,x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.36

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2}\,dx = \frac{b^3c^{\frac{3}{2}}x^6 + 5\,ab^2c^{\frac{3}{2}}x^4 + 15\,a^2bc^{\frac{3}{2}}x^2 - 5\,a^3c^{\frac{3}{2}}}{5\,x}$$

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*(b^3*c^(3/2)*x^6 + 5*a*b^2*c^(3/2)*x^4 + 15*a^2*b*c^(3/2)*x^2 - 5*a^3*c
^(3/2))/x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2}\,dx = \frac{1}{5}\left(b^3x^5\mathrm{sgn}\left(bx^2+a\right) + 5\,ab^2x^3\mathrm{sgn}\left(bx^2+a\right) + 15\,a^2bx\mathrm{sgn}\left(bx^2+a\right) - \frac{5\,a^3\mathrm{sgn}(bx^2}{x}\right.$$

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*(b^3*x^5*sgn(b*x^2 + a) + 5*a*b^2*x^3*sgn(b*x^2 + a) + 15*a^2*b*x*sgn(b
*x^2 + a) - 5*a^3*sgn(b*x^2 + a)/x)*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2}\,dx = \int \frac{\left(c\,{\left(b\,x^2+a\right)}^2\right)^{3/2}}{x^2}\,dx$$

```
[In] int((c*(a + b*x^2)^2)^(3/2)/x^2,x)
```

```
[Out] int((c*(a + b*x^2)^2)^(3/2)/x^2, x)
```

3.236 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 140

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^3} dx = -\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{2x^2\left(a+bx^2\right)} + \frac{3ab^2cx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)} + \frac{b^3cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)} + \frac{3a^2bc\sqrt{c\left(a+bx^2\right)^2}\log(x)}{a+bx^2}$$

```
[Out] -1/2*a^3*c*(c*(b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+3/2*a*b^2*c*x^2*(c*(b*x^2+a)
^2)^(1/2)/(b*x^2+a)+1/4*b^3*c*x^4*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3*a^2*b*c
*ln(x)*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1973, 272, 45}

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^3} dx = -\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{2x^2\left(a+bx^2\right)} + \frac{3a^2bc\log(x)\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2} + \frac{b^3cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)} + \frac{3ab^2cx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)}$$

```
[In] Int[(c*(a + b*x^2)^2)^(3/2)/x^3,x]
```

```
[Out] -1/2*(a^3*c*Sqrt[c*(a + b*x^2)^2])/(x^2*(a + b*x^2)) + (3*a*b^2*c*x^2*Sqrt[
c*(a + b*x^2)^2])/(2*(a + b*x^2)) + (b^3*c*x^4*Sqrt[c*(a + b*x^2)^2])/(4*(a
 + b*x^2)) + (3*a^2*b*c*Sqrt[c*(a + b*x^2)^2]*Log[x])/(a + b*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(a^2 c\sqrt{c\left(a+bx^2\right)^2}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^3}{x^3}\,dx}{1+\frac{bx^2}{a}}$$

$$= \frac{\left(a^2 c\sqrt{c\left(a+bx^2\right)^2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^3}{x^2}\,dx, x, x^2\right)}{2\left(1+\frac{bx^2}{a}\right)}$$

$$= \frac{\left(a^2 c\sqrt{c\left(a+bx^2\right)^2}\right)\text{Subst}\left(\int \left(\frac{3b^2}{a^2}+\frac{1}{x^2}+\frac{3b}{ax}+\frac{b^3x}{a^3}\right)\,dx, x, x^2\right)}{2\left(1+\frac{bx^2}{a}\right)}$$

$$= -\frac{a^3 c\sqrt{c\left(a+bx^2\right)^2}}{2x^2\left(a+bx^2\right)}+\frac{3ab^2cx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)}+\frac{b^3cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)}+\frac{3a^2bc\sqrt{c\left(a+bx^2\right)^2}\log(x)}{a+bx^2}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.69

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^3}\,dx = \frac{1}{16}c\left(\frac{(8a^3+21a^2bx^2-24ab^2x^4-4b^3x^6)\left(abc+b^2cx^2-\sqrt{b^2c}\sqrt{c\left(a+bx^2\right)^2}\right)}{x^2\left(a\sqrt{b^2c}+b\sqrt{b^2c}x^2-b\sqrt{c\left(a+bx^2\right)^2}\right)}\right.$$
$$+24a^2b\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{b^2c}x^2-\sqrt{c\left(a+bx^2\right)^2}}{a\sqrt{c}}\right)$$
$$\left.-12a^2\sqrt{b^2c}\log\left(x^2\left(abc+b^2cx^2-\sqrt{b^2c}\sqrt{c\left(a+bx^2\right)^2}\right)\right)\right)$$

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x^3,x]
```

```
[Out] (c*(((8*a^3 + 21*a^2*b*x^2 - 24*a*b^2*x^4 - 4*b^3*x^6)*(a*b*c + b^2*c*x^2 -
 Sqrt[b^2*c]*Sqrt[c*(a + b*x^2)^2]))/(x^2*(a*Sqrt[b^2*c] + b*Sqrt[b^2*c]*x^
2 - b*Sqrt[c*(a + b*x^2)^2])) + 24*a^2*b*Sqrt[c]*ArcTanh[(Sqrt[b^2*c]*x^2 -
 Sqrt[c*(a + b*x^2)^2])/(a*Sqrt[c])] - 12*a^2*Sqrt[b^2*c]*Log[x^2*(a*b*c +
b^2*c*x^2 - Sqrt[b^2*c]*Sqrt[c*(a + b*x^2)^2])]))/16
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.44

method	result	size
default	$\frac{\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}\left(b^{3}x^{6}+6b^{2}x^{4}a+12a^{2}b\ln\left(x\right)x^{2}-2a^{3}\right)}{4x^{2}\left(b\,x^{2}+a\right)^{3}}$	61
pseudoelliptic	$-\frac{c\left(-\frac{b^{3}x^{6}}{2}-3b^{2}x^{4}a-3a^{2}b\ln\left(x^{2}\right)x^{2}+a^{3}\right)\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{2\left(b\,x^{2}+a\right)x^{2}}$	63
risch	$\frac{c\sqrt{c\left(b\,x^{2}+a\right)^{2}}\,b\left(b\,x^{2}+3a\right)^{2}}{4b\,x^{2}+4a}-\frac{a^{3}c\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{2x^{2}\left(b\,x^{2}+a\right)}+\frac{3a^{2}bc\ln\left(x\right)\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{b\,x^{2}+a}$	101

```
[In] int((c*(b*x^2+a)^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(c*(b*x^2+a)^2)^(3/2)*(b^3*x^6+6*b^2*x^4*a+12*a^2*b*ln(x)*x^2-2*a^3)/x^
2/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.54

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^3}\,dx = \frac{(b^3cx^6+6\,ab^2cx^4+12\,a^2bcx^2\log(x)-2\,a^3c)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{4\,(bx^4+ax^2)}$$

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="fricas")
[Out] 1/4*(b^3*c*x^6 + 6*a*b^2*c*x^4 + 12*a^2*b*c*x^2*log(x) - 2*a^3*c)*sqrt(b^2*
c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^4 + a*x^2)
```

Sympy [F]

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^3}\,dx = \int \frac{\left(c(a+bx^2)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((c*(b*x**2+a)**2)**(3/2)/x**3,x)
[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.26

$$\begin{aligned}\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^3}\,dx &= \frac{3}{2}\,(-1)^{2\,b^2cx^2+2\,abc}\,a^2bc^{\frac{3}{2}}\log\left(2\,b^2cx^2+2\,abc\right)\\&\quad-\frac{3}{2}\,(-1)^{2\,abcx^2+2\,a^2c}\,a^2bc^{\frac{3}{2}}\log\left(2\,abc+\frac{2\,a^2c}{x^2}\right)+\frac{3}{4}\sqrt{b^2cx^4+2\,abcx^2+a^2c}b^2cx^2\\&\quad+\frac{9}{4}\sqrt{b^2cx^4+2\,abcx^2+a^2c}abc-\frac{(b^2cx^4+2\,abcx^2+a^2c)^{\frac{3}{2}}}{2\,x^2}\end{aligned}$$

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="maxima")
[Out] 3/2*(-1)^(2*b^2*c*x^2 + 2*a*b*c)*a^2*b*c^(3/2)*log(2*b^2*c*x^2 + 2*a*b*c) -
 3/2*(-1)^(2*a*b*c*x^2 + 2*a^2*c)*a^2*b*c^(3/2)*log(2*a*b*c + 2*a^2*c/x^2)
+ 3/4*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*b^2*c*x^2 + 9/4*sqrt(b^2*c*x^4
+ 2*a*b*c*x^2 + a^2*c)*a*b*c - 1/2*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)/
x^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^3}\,dx = \frac{1}{4}\left(b^3x^4\mathrm{sgn}\left(bx^2+a\right)+6\,ab^2x^2\mathrm{sgn}\left(bx^2+a\right)+6\,a^2b\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{2\,(3\,a^2$$

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^3*x^4*sgn(b*x^2 + a) + 6*a*b^2*x^2*sgn(b*x^2 + a) + 6*a^2*b*log(x^2)
*sgn(b*x^2 + a) - 2*(3*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^2)*
c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^3}\,dx = \int \frac{\left(c\,{\left(b\,x^2+a\right)}^2\right)^{3/2}}{x^3}\,dx$$

```
[In] int((c*(a + b*x^2)^2)^(3/2)/x^3,x)
```

```
[Out] int((c*(a + b*x^2)^2)^(3/2)/x^3, x)
```

3.237 $\int x^2 \left(c(a+bx^2)^3\right)^{3/2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 253

$$\int x^2 \left(c(a+bx^2)^3\right)^{3/2} \, dx = \frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3} + \frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)} + \frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)} + \frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} - \frac{21a^{9/2}c\sqrt{c\left(a+bx^2\right)^3}\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{1024b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

```
[Out] 7/128*a^3*c*x^3*(c*(b*x^2+a)^3)^(1/2)+21/1024*a^5*c*x*(c*(b*x^2+a)^3)^(1/2)
/b/(b*x^2+a)+21/512*a^4*c*x^3*(c*(b*x^2+a)^3)^(1/2)/(b*x^2+a)+21/320*a^2*c*
x^3*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+3/40*a*c*x^3*(b*x^2+a)^2*(c*(b*x^2+a)^3
)^(1/2)+1/12*c*x^3*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)-21/1024*a^(9/2)*c*arcs
inh(x*b^(1/2)/a^(1/2))*(c*(b*x^2+a)^3)^(1/2)/b^(3/2)/(1+b*x^2/a)^(3/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used

$= \{1973, 285, 327, 221\}$

$$\int x^2\left(c\left(a+bx^2\right)^3\right)^{3/2}dx = -\frac{21a^{9/2}c\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\sqrt{c\left(a+bx^2\right)^3}}{1024b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/2}} + \frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)} + \frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)} + \frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3} + \frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}$$

```
[In] Int[x^2*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (7*a^3*c*x^3*Sqrt[c*(a + b*x^2)^3])/128 + (21*a^5*c*x*Sqrt[c*(a + b*x^2)^3]
)/(1024*b*(a + b*x^2)) + (21*a^4*c*x^3*Sqrt[c*(a + b*x^2)^3])/(512*(a + b*x
^2)) + (21*a^2*c*x^3*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/320 + (3*a*c*x^3*(a
 + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/40 + (c*x^3*(a + b*x^2)^3*Sqrt[c*(a + b*
x^2)^3])/12 - (21*a^(9/2)*c*Sqrt[c*(a + b*x^2)^3]*ArcSinh[(Sqrt[b]*x)/Sqrt[
a]])/(1024*b^(3/2)*(1 + (b*x^2)/a)^(3/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^{9/2}dx}{\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(3a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^{7/2}dx}{4\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(21a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^{5/2}dx}{40\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(21a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^{3/2}dx}{64\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3} + \frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(21a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\int x^2\sqrt{1+\frac{bx^2}{a}}\,dx}{128\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3} + \frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)} + \frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(21a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\int\frac{x^2}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{512\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{7}{128}a^3cx^3\sqrt{c(a+bx^2)^3} + \frac{21a^5cx\sqrt{c(a+bx^2)^3}}{1024b(a+bx^2)} + \frac{21a^4cx^3\sqrt{c(a+bx^2)^3}}{512(a+bx^2)} + \frac{21}{320}a^2cx^3(a+bx^2)\sqrt{c(a+bx^2)^3} + \frac{3}{40}acx^3(a+bx^2)^2\sqrt{c(a+bx^2)^3} + \frac{1}{12}cx^3(a+bx^2)^3\sqrt{c(a+bx^2)^3} - \frac{\left(21a^4c\sqrt{c(a+bx^2)^3}\right)\int\frac{1}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{1024b\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{7}{128}a^3cx^3\sqrt{c(a+bx^2)^3} + \frac{21a^5cx\sqrt{c(a+bx^2)^3}}{1024b(a+bx^2)} + \frac{21a^4cx^3\sqrt{c(a+bx^2)^3}}{512(a+bx^2)} + \frac{21}{320}a^2cx^3(a+bx^2)\sqrt{c(a+bx^2)^3} + \frac{3}{40}acx^3(a+bx^2)^2\sqrt{c(a+bx^2)^3} + \frac{1}{12}cx^3(a+bx^2)^3\sqrt{c(a+bx^2)^3} - \frac{21a^{9/2}c\sqrt{c(a+bx^2)^3}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{1024b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.56

$$\int x^2\left(c(a+bx^2)^3\right)^{3/2}\,dx = \frac{\left(c(a+bx^2)^3\right)^{3/2}\left(\sqrt{b}x\sqrt{a+bx^2}(315a^5+4910a^4bx^2+11432a^3b^2x^4+12144a^2b^3x^6+6272a\cdots\right.}{15360b^{3/2}(a+bx^2)^{9/2}}$$

```
[In] Integrate[x^2*(c*(a + b*x^2)^3)^(3/2),x]
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[a + b*x^2]*(315*a^5 + 4910*a^4*b*x
^2 + 11432*a^3*b^2*x^4 + 12144*a^2*b^3*x^6 + 6272*a*b^4*x^8 + 1280*b^5*x^10
) + 630*a^6*ArcTanh[(Sqrt[b]*x)/(Sqrt[a] - Sqrt[a + b*x^2])]))/(15360*b^(3/
2)*(a + b*x^2)^(9/2))
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.61

method	result
risch	$\frac{x(1280x^{10}b^5+6272a\,x^8b^4+12144a^2x^6b^3+11432a^3x^4b^2+4910x^2a^4b+315a^5)c\sqrt{c(b\,x^2+a)^3}}{15360(b\,x^2+a)b} - \frac{21a^6\ln\left(\frac{bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+ac}\right)c\sqrt{c(b\,x^2+\cdots}}{1024b\sqrt{bc}\,(b\,x^2+a)^2}$
default	$-\frac{\left(c(b\,x^2+a)^3\right)^{\frac{3}{2}}\left(-1280x^7(bc\,x^2+ac)^{\frac{5}{2}}b^3\sqrt{bc}-3712\sqrt{bc}\,(bc\,x^2+ac)^{\frac{5}{2}}a\,b^2x^5-3440\sqrt{bc}\,(bc\,x^2+ac)^{\frac{5}{2}}a^2b\,x^3+315\ln\left(\frac{bcx+\sqrt{bc\,x^2+ac}\cdots}{\sqrt{bc}}\right.\cdots}{15360b(b\,x^2+a)^3(c(b\,x^2+a))^{\frac{3}{2}}c\sqrt{bc}}$

```
[In] int(x^2*(c*(b*x^2+a)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15360*x*(1280*b^5*x^10+6272*a*b^4*x^8+12144*a^2*b^3*x^6+11432*a^3*b^2*x^4
+4910*a^4*b*x^2+315*a^5)/(b*x^2+a)/b*c*(c*(b*x^2+a)^3)^(1/2)-21/1024*a^6/b*
ln(b*c*x/(b*c)^(1/2)+(b*c*x^2+a*c)^(1/2))/(b*c)^(1/2)*c/(b*x^2+a)^2*(c*(b*x
^2+a)^3)^(1/2)*(c*(b*x^2+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.71

$$\int x^2\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx = \left[\frac{315\left(a^6bcx^2+a^7c\right)\sqrt{\frac{c}{b}}\log\left(-\frac{2\,b^2cx^4+3\,abcx^2+a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,bx\sqrt{\frac{c}{b}}}{bx^2+a}\right)+2\,(1280\right.$$

```
[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(315*(a^6*b*c*x^2 + a^7*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 + 3*a*b*c*x
^2 + a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*
sqrt(c/b))/(b*x^2 + a)) + 2*(1280*b^5*c*x^11 + 6272*a*b^4*c*x^9 + 12144*a^2
*b^3*c*x^7 + 11432*a^3*b^2*c*x^5 + 4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt(b^3
*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b), 1/15360*(
315*(a^6*b*c*x^2 + a^7*c)*sqrt(-c/b)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4
+ 3*a^2*b*c*x^2 + a^3*c)*b*x*sqrt(-c/b)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c))
+ (1280*b^5*c*x^11 + 6272*a*b^4*c*x^9 + 12144*a^2*b^3*c*x^7 + 11432*a^3*b^2
*c*x^5 + 4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3
*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b)]
```

Sympy [F]

$$\int x^2\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx = \int x^2\left(c\left(a+bx^2\right)^3\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**2*(c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Integral(x**2*(c*(a + b*x**2)**3)**(3/2), x)
```

Maxima [F]

$$\int x^2 \left(c(a+bx^2)^3\right)^{3/2}\,dx = \int \left((bx^2+a)^3c\right)^{\frac{3}{2}}x^2\,dx$$

```
[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)*x^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.70

$$\int x^2 \left(c(a+bx^2)^3\right)^{3/2}\,dx = \frac{1}{15360}\left(\frac{315\,a^6c\log\left(\left|-\sqrt{bc}x+\sqrt{bcx^2+ac}\right|\right)\mathrm{sgn}(bx^2+a)}{\sqrt{bc}b} + \left(\frac{315\,a^5\mathrm{sgn}(bx^2+a)}{b} + 2\,(245\right.\right.$$

```
[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
[Out] 1/15360*(315*a^6*c*log(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*sgn(b*x^2 +
 a)/(sqrt(b*c)*b) + (315*a^5*sgn(b*x^2 + a)/b + 2*(2455*a^4*sgn(b*x^2 + a)
+ 4*(1429*a^3*b*sgn(b*x^2 + a) + 2*(759*a^2*b^2*sgn(b*x^2 + a) + 8*(10*b^4*
x^2*sgn(b*x^2 + a) + 49*a*b^3*sgn(b*x^2 + a))*x^2)*x^2)*x^2)*x^2)*sqrt(b*c*
x^2 + a*c)*x)*c
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(c(a+bx^2)^3\right)^{3/2}\,dx = \int x^2\,\left(c\,{\left(b\,x^2+a\right)}^3\right)^{3/2} \,d x$$

```
[In] int(x^2*(c*(a + b*x^2)^3)^(3/2),x)
[Out] int(x^2*(c*(a + b*x^2)^3)^(3/2), x)
```

3.238 $\int x\left(c(a+bx^2)^3\right)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 32

$$\int x\left(c(a+bx^2)^3\right)^{3/2}\,dx=\frac{c(a+bx^2)^4\sqrt{c\,(a+bx^2)^3}}{11b}$$

```
[Out] 1/11*c*(b*x^2+a)^4*(c*(b*x^2+a)^3)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1605, 15, 30}

$$\int x\left(c(a+bx^2)^3\right)^{3/2}\,dx=\frac{c(a+bx^2)^4\sqrt{c\,(a+bx^2)^3}}{11b}$$

```
[In] Int[x*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (c*(a + b*x^2)^4*Sqrt[c*(a + b*x^2)^3])/(11*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (cx^3)^{3/2}\, dx, x, a+bx^2\right)}{2b} \\
&= \frac{\left(c\sqrt{c\,(a+bx^2)^3}\right)\text{Subst}\left(\int x^{9/2}\,dx, x, a+bx^2\right)}{2b\,(a+bx^2)^{3/2}} \\
&= \frac{c(a+bx^2)^4\sqrt{c\,(a+bx^2)^3}}{11b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int x\left(c\left(a+bx^2\right)^3\right)^{3/2}\, dx = \frac{(a+bx^2)\left(c(a+bx^2)^3\right)^{3/2}}{11b}$$

```
[In] Integrate[x*(c*(a + b*x^2)^3)^(3/2),x]
[Out] ((a + b*x^2)*(c*(a + b*x^2)^3)^(3/2))/(11*b)
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result
gosper	$\frac{(b\,x^2+a)\left(c(b\,x^2+a)^3\right)^{\frac{3}{2}}}{11b}$
risch	$\frac{c\sqrt{c(b\,x^2+a)^3}\,(x^{10}b^5+5a\,x^8b^4+10a^2x^6b^3+10a^3x^4b^2+5x^2a^4b+a^5)}{11(b\,x^2+a)b}$
trager	$\frac{c(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4)\sqrt{b^3c\,x^6+3a\,b^2c\,x^4+3a^2bc\,x^2+a^3c}}{11b}$
default	$-\frac{\left(c(b\,x^2+a)^3\right)^{\frac{3}{2}}\left(-5x^6(bc\,x^2+ac)^{\frac{5}{2}}b^3-15(bc\,x^2+ac)^{\frac{5}{2}}a\,b^2x^4-15(bc\,x^2+ac)^{\frac{5}{2}}a^2b\,x^2+6(bc\,x^2+ac)^{\frac{5}{2}}a^3-11a^3\left(c(b\,x^2+a)\right)^{\frac{5}{2}}\right)}{55b(b\,x^2+a)^3(c(b\,x^2+a))^{\frac{3}{2}}c}$

```
[In] int(x*(c*(b*x^2+a)^3)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/11*(b*x^2+a)/b*(c*(b*x^2+a)^3)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.72

$$\int x\left(c(a+bx^2)^3\right)^{3/2}\,dx=\frac{(b^4cx^8+4\,ab^3cx^6+6\,a^2b^2cx^4+4\,a^3bcx^2+a^4c)\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}}{11\,b}$$

```
[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
[Out] 1/11*(b^4*c*x^8 + 4*a*b^3*c*x^6 + 6*a^2*b^2*c*x^4 + 4*a^3*b*c*x^2 + a^4*c)*
sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)/b
```

Sympy [F]

$$\int x\left(c(a+bx^2)^3\right)^{3/2}\,dx=\int x\left(c(a+bx^2)^3\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(c*(b*x**2+a)**3)**(3/2),x)
[Out] Integral(x*(c*(a + b*x**2)**3)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(28) = 56.

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

$$\int x\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\frac{\left(b^4c^{\frac{3}{2}}x^8+4\,ab^3c^{\frac{3}{2}}x^6+6\,a^2b^2c^{\frac{3}{2}}x^4+4\,a^3bc^{\frac{3}{2}}x^2+a^4c^{\frac{3}{2}}\right)(bx^2+a)^{\frac{3}{2}}}{11\,b}$$

```
[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*(b^4*c^(3/2)*x^8 + 4*a*b^3*c^(3/2)*x^6 + 6*a^2*b^2*c^(3/2)*x^4 + 4*a^3
*b*c^(3/2)*x^2 + a^4*c^(3/2))*(b*x^2 + a)^(3/2)/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int x\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\frac{(bcx^2+ac)^{\frac{11}{2}}\operatorname{sgn}(bx^2+a)}{11\,bc^4}$$

```
[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/11*(b*c*x^2 + a*c)^(11/2)*sgn(b*x^2 + a)/(b*c^4)
```

Mupad [B] (verification not implemented)

Time = 18.39 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.94

$$\int x\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\sqrt{c\,(b\,x^2+a)^3}\,\left(\frac{a^4\,c}{11\,b}+\frac{4\,a^3\,c\,x^2}{11}+\frac{b^3\,c\,x^8}{11}+\frac{6\,a^2\,b\,c\,x^4}{11}+\frac{4\,a\,b^2\,c\,x^6}{11}\right)$$

```
[In] int(x*(c*(a + b*x^2)^3)^(3/2),x)
```

```
[Out] (c*(a + b*x^2)^3)^(1/2)*((a^4*c)/(11*b) + (4*a^3*c*x^2)/11 + (b^3*c*x^8)/11
 + (6*a^2*b*c*x^4)/11 + (4*a*b^2*c*x^6)/11)
```

3.239 $\int \left(c(a+bx^2)^3\right)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 207

$$\int \left(c(a+bx^2)^3\right)^{3/2}\,dx = \frac{21}{128}a^3cx\sqrt{c\,(a+bx^2)^3} + \frac{63a^4cx\sqrt{c\,(a+bx^2)^3}}{256\,(a+bx^2)} + \frac{21}{160}a^2cx(a+bx^2)\,\sqrt{c\,(a+bx^2)^3} + \frac{9}{80}acx(a+bx^2)^2\,\sqrt{c\,(a+bx^2)^3} + \frac{1}{10}cx(a+bx^2)^3\,\sqrt{c\,(a+bx^2)^3} + \frac{63a^{7/2}c\sqrt{c\,(a+bx^2)^3}\,\text{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256\sqrt{b}\,\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

```
[Out] 21/128*a^3*c*x*(c*(b*x^2+a)^3)^(1/2)+63/256*a^4*c*x*(c*(b*x^2+a)^3)^(1/2)/(
b*x^2+a)+21/160*a^2*c*x*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+9/80*a*c*x*(b*x^2+a
)^2*(c*(b*x^2+a)^3)^(1/2)+1/10*c*x*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)+63/256
*a^(7/2)*c*arcsinh(x*b^(1/2)/a^(1/2))*(c*(b*x^2+a)^3)^(1/2)/(1+b*x^2/a)^(3/
2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{1973, 201, 221\}$

$$\int \left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\frac{63a^{7/2}c\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\sqrt{c\left(a+bx^2\right)^3}}{256\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/2}}$$
$$+\frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)}+\frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3}$$
$$+\frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}$$

```
[In] Int[(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (21*a^3*c*x*Sqrt[c*(a + b*x^2)^3])/128 + (63*a^4*c*x*Sqrt[c*(a + b*x^2)^3])
/(256*(a + b*x^2)) + (21*a^2*c*x*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/160 + (
9*a*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/80 + (c*x*(a + b*x^2)^3*Sqrt[c
*(a + b*x^2)^3])/10 + (63*a^(7/2)*c*Sqrt[c*(a + b*x^2)^3]*ArcSinh[(Sqrt[b]*
x)/Sqrt[a]])/(256*Sqrt[b]*(1 + (b*x^2)/a)^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral}=\frac{\left(a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\int\left(1+\frac{bx^2}{a}\right)^{9/2}dx}{\left(1+\frac{bx^2}{a}\right)^{3/2}}$$
$$=\frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}+\frac{\left(9a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\int\left(1+\frac{bx^2}{a}\right)^{7/2}dx}{10\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(63a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\int\left(1+\frac{bx^2}{a}\right)^{5/2}dx}{80\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}+\frac{\left(21a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\int\left(1+\frac{bx^2}{a}\right)^{3/2}dx}{32\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}+\frac{\left(63a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\int\sqrt{1+\frac{bx^2}{a}}\,dx}{128\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3}+\frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}+\frac{\left(63a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\int\frac{1}{\sqrt{1+\frac{bx^2}{a}}}dx}{256\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3}+\frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}+\frac{63a^{7/2}c\sqrt{c\left(a+bx^2\right)^3}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.59

$$\int \left(c\left(a+bx^2\right)^3\right)^{3/2} dx = \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\sqrt{b}x\sqrt{a+bx^2}(965a^4+1490a^3bx^2+1368a^2b^2x^4+656ab^3x^6+128b^4x^8)-\right.}{1280\sqrt{b}\left(a+bx^2\right)^{9/2}}$$

[In] Integrate[(c*(a + b*x^2)^3)^(3/2),x]

[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[a + b*x^2]*(965*a^4 + 1490*a^3*b*x^2 + 1368*a^2*b^2*x^4 + 656*a*b^3*x^6 + 128*b^4*x^8) - 315*a^5*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]))/(1280*Sqrt[b]*(a + b*x^2)^(9/2))

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.67

method	result
risch	$\frac{x\left(128b^4x^8+656a\,b^3x^6+1368a^2b^2x^4+1490a^3b\,x^2+965a^4\right)c\sqrt{c\left(b\,x^2+a\right)^3}}{1280b\,x^2+1280a}+\frac{63a^5\ln\left(\frac{bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+ac}\right)c\sqrt{c\left(b\,x^2+a\right)^3}\,\sqrt{c\left(b\,x^2+a\right)}}{256\sqrt{bc}\left(b\,x^2+a\right)^2}$
default	$\frac{\left(c\left(b\,x^2+a\right)^3\right)^{\frac{3}{2}}\left(128x^5\left(bc\,x^2+ac\right)^{\frac{5}{2}}b^2\sqrt{bc}+400\left(bc\,x^2+ac\right)^{\frac{5}{2}}\sqrt{bc}\,ab\,x^3+440\left(bc\,x^2+ac\right)^{\frac{5}{2}}\sqrt{bc}\,a^2x+210\left(bc\,x^2+ac\right)^{\frac{3}{2}}\sqrt{bc}\,a^3cx+315\sqrt{bc}\right.}{1280\left(b\,x^2+a\right)^3\left(c\left(b\,x^2+a\right)\right)^{\frac{3}{2}}c\sqrt{bc}}$

[In] int((c*(b*x^2+a)^3)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/1280*x*(128*b^4*x^8+656*a*b^3*x^6+1368*a^2*b^2*x^4+1490*a^3*b*x^2+965*a^4)/(b*x^2+a)*c*(c*(b*x^2+a)^3)^(1/2)+63/256*a^5*ln(b*c*x/(b*c)^(1/2)+(b*c*x^2+a*c)^(1/2))/(b*c)^(1/2)*c/(b*x^2+a)^2*(c*(b*x^2+a)^3)^(1/2)*(c*(b*x^2+a))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.94

$$\int \left(c(a+bx^2)^3\right)^{3/2}\,dx = \left[\frac{315\,(a^5bcx^2+a^6c)\sqrt{\frac{c}{b}}\log\left(-\frac{2\,b^2cx^4+3\,abcx^2+a^2c+2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}bx\sqrt{\frac{c}{b}}}{bx^2+a}\right)+2\,(128\,b}{256}\right.$$

$$-\frac{315\,(a^5bcx^2+a^6c)\sqrt{-\frac{c}{b}}\arctan\left(\frac{\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}bx\sqrt{-\frac{c}{b}}}{b^2cx^4+2\,abcx^2+a^2c}\right)-(128\,b^4cx^9+656\,ab^3cx^7+1368\,a^2b^2c}{1280\,(bx^2+a)}$$

```
[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(315*(a^5*b*c*x^2 + a^6*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 + 3*a*b*c*x^
2 + a^2*c + 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*s
qrt(c/b))/(b*x^2 + a)) + 2*(128*b^4*c*x^9 + 656*a*b^3*c*x^7 + 1368*a^2*b^2*
c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*
a^2*b*c*x^2 + a^3*c))/(b*x^2 + a), -1/1280*(315*(a^5*b*c*x^2 + a^6*c)*sqrt(
-c/b)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*sq
rt(-c/b)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) - (128*b^4*c*x^9 + 656*a*b^3*c*
x^7 + 1368*a^2*b^2*c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*c*x)*sqrt(b^3*c*x^6 +
 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]
```

Sympy [F]

$$\int \left(c(a+bx^2)^3\right)^{3/2}\,dx = \int \left(c(a+bx^2)^3\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2), x)
```

Maxima [F]

$$\int \left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\int \left(\left(bx^2+a\right)^3c\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
[Out] integrate(((b*x^2 + a)^3*c)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.74

$$\int \left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=$$

$$-\frac{1}{1280}\left(\frac{315\,a^5c\log\left(\left|-\sqrt{bc}x+\sqrt{bcx^2+ac}\right|\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{bc}}-\left(965\,a^4\mathrm{sgn}\left(bx^2+a\right)+2\left(745\,a^3b\mathrm{sgn}\left(bx^2+a\right)\right.\right.\right.$$

```
[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
[Out] -1/1280*(315*a^5*c*log(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*sgn(b*x^2 +
 a)/sqrt(b*c) - (965*a^4*sgn(b*x^2 + a) + 2*(745*a^3*b*sgn(b*x^2 + a) + 4*(
171*a^2*b^2*sgn(b*x^2 + a) + 2*(8*b^4*x^2*sgn(b*x^2 + a) + 41*a*b^3*sgn(b*x
^2 + a))*x^2)*x^2)*x^2)*sqrt(b*c*x^2 + a*c)*x)*c
```

Mupad [F(-1)]

Timed out.

$$\int \left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\int {\left(c\,{\left(b\,x^2+a\right)}^3\right)}^{3/2}\,dx$$

```
[In] int((c*(a + b*x^2)^3)^(3/2),x)
[Out] int((c*(a + b*x^2)^3)^(3/2), x)
```

3.240 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x} dx$

Optimal result

Integrand size = 19, antiderivative size = 192

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x} dx = \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3} + \frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} - \frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\operatorname{arctanh}\left(\sqrt{1+\frac{bx^2}{a}}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

```
[Out] 1/3*a^3*c*(c*(b*x^2+a)^3)^(1/2)+a^4*c*(c*(b*x^2+a)^3)^(1/2)/(b*x^2+a)+1/5*a
^2*c*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+1/7*a*c*(b*x^2+a)^2*(c*(b*x^2+a)^3)^(1
/2)+1/9*c*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)-a^3*c*arctanh((1+b*x^2/a)^(1/2)
)*(c*(b*x^2+a)^3)^(1/2)/(1+b*x^2/a)^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1973, 272, 52, 65, 214}

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x} dx = \frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2} - \frac{a^3c\operatorname{arctanh}\left(\sqrt{\frac{bx^2}{a}+1}\right)\sqrt{c\left(a+bx^2\right)^3}}{\left(\frac{bx^2}{a}+1\right)^{3/2}} + \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}$$

```
[In] Int[(c*(a + b*x^2)^3)^(3/2)/x,x]
```

```
[Out] (a^3*c*Sqrt[c*(a + b*x^2)^3])/3 + (a^4*c*Sqrt[c*(a + b*x^2)^3])/(a + b*x^2)
 + (a^2*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/5 + (a*c*(a + b*x^2)^2*Sqrt[c*
(a + b*x^2)^3])/7 + (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/9 - (a^3*c*Sqrt
[c*(a + b*x^2)^3]*ArcTanh[Sqrt[1 + (b*x^2)/a]])/(1 + (b*x^2)/a)^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^{9/2}}{x}\,dx}{\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{\left(a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^{9/2}}{x}\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}+\frac{\left(a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^{7/2}}{x}\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^{5/2}}{x}\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}+\frac{\left(a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^{3/2}}{x}\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}+\frac{\left(a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{bx}{a}}}{x}\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3}+\frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2}+\frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{bx}{a}}}\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3}+\frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2}+\frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(a^4c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{ax^2}{b}}\,dx,x,\sqrt{1+\frac{bx^2}{a}}\right)}{b\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3} + \frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} - \frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\sqrt{1+\frac{bx^2}{a}}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.58

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x}\,dx = \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\sqrt{a+bx^2}\left(563a^4+506a^3bx^2+408a^2b^2x^4+185ab^3x^6+35b^4x^8\right)\right.}{315\left(a+bx^2\right)^{9/2}}$$

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x,x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(563*a^4 + 506*a^3*b*x^2 + 408*a^
2*b^2*x^4 + 185*a*b^3*x^6 + 35*b^4*x^8) - 315*a^(9/2)*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]]))/(315*(a + b*x^2)^(9/2))
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.15

method	result
default	$-\frac{\left(c\left(b\,x^2+a\right)^3\right)^{\frac{3}{2}}\left(-35\sqrt{ac}\,\left(bc\,x^2+ac\right)^{\frac{5}{2}}b^2x^4-115\sqrt{ac}\,\left(bc\,x^2+ac\right)^{\frac{5}{2}}ab\,x^2+315\ln\left(\frac{2ac+2\sqrt{ac}\,\sqrt{bc\,x^2+ac}}{x}\right)a^5c^3+46\sqrt{ac}\,\left(bc\,x^2+ac\right)^{\frac{5}{2}}a\right.}{315\left(b\,x^2+a\right)^3\left(c\left(b\,x^2+a\right)\right)^{\frac{3}{2}}c\sqrt{ac}}$

```
[In] int((c*(b*x^2+a)^3)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(c*(b*x^2+a)^3)^(3/2)*(-35*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*b^2*x^4-1
15*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a*b*x^2+315*ln(2*((a*c)^(1/2)*(b*c*x^2+a
*c)^(1/2)+a*c)/x)*a^5*c^3+46*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a^2-105*(a*c)^
(1/2)*(b*c*x^2+a*c)^(3/2)*a^3*c-315*(a*c)^(1/2)*(b*c*x^2+a*c)^(1/2)*a^4*c^2
-189*a^2*(c*(b*x^2+a))^(5/2)*(a*c)^(1/2))/(b*x^2+a)^3/(c*(b*x^2+a))^(3/2)/c
/(a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.04

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x}\,dx = \left[\frac{315\left(a^4bcx^2+a^5c\right)\sqrt{ac}\log\left(-\frac{b^2cx^4+3\,abcx^2+2\,a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{ac}}{bx^4+ax^2}\right)+\cdots}{\cdots}\right.$$

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/630*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(a*c)*log(-(b^2*c*x^4 + 3*a*b*c*x^2 +
 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(a
*c))/(b*x^4 + a*x^2)) + 2*(35*b^4*c*x^8 + 185*a*b^3*c*x^6 + 408*a^2*b^2*c*x
^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*
c*x^2 + a^3*c))/(b*x^2 + a), 1/315*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(-a*c)*ar
ctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(-a*c)/(b^
2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) + (35*b^4*c*x^8 + 185*a*b^3*c*x^6 + 408*a^2
*b^2*c*x^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 +
3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]
```

Sympy [F]

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x}\,dx = \int \frac{\left(c(a+bx^2)^3\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((c*(b*x**2+a)**3)**(3/2)/x,x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x}\,dx = \int \frac{\left((bx^2+a)^3c\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x, x)
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.96

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x}\,dx = \frac{1}{315}\left(\frac{315\,a^5\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)\mathrm{sgn}(bx^2+a)}{\sqrt{-ac}} + \frac{315\,\sqrt{bcx^2+ac}a^4c^{44}\mathrm{sgn}(bx^2+a)+\ldots}{\ldots}\right.$$

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/315*(315*a^5*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))*sgn(b*x^2 + a)/sqrt(-
a*c) + (315*sqrt(b*c*x^2 + a*c)*a^4*c^44*sgn(b*x^2 + a) + 105*(b*c*x^2 + a*
c)^(3/2)*a^3*c^43*sgn(b*x^2 + a) + 63*(b*c*x^2 + a*c)^(5/2)*a^2*c^42*sgn(b*
x^2 + a) + 45*(b*c*x^2 + a*c)^(7/2)*a*c^41*sgn(b*x^2 + a) + 35*(b*c*x^2 + a
*c)^(9/2)*c^40*sgn(b*x^2 + a))/c^45)*c^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x}\,dx = \int \frac{\left(c\,(b\,x^2+a)^3\right)^{3/2}}{x}\,dx$$

```
[In] int((c*(a + b*x^2)^3)^(3/2)/x,x)
```

```
[Out] int((c*(a + b*x^2)^3)^(3/2)/x, x)
```

3.241 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 208

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}\,dx = \frac{105}{64}a^2bcx\sqrt{c\,(a+bx^2)^3}+\frac{315a^3bcx\sqrt{c\,(a+bx^2)^3}}{128\,(a+bx^2)} + \frac{21}{16}abcx(a+bx^2)\sqrt{c\,(a+bx^2)^3}+\frac{9}{8}bcx(a+bx^2)^2\sqrt{c\,(a+bx^2)^3} - \frac{c(a+bx^2)^3\sqrt{c\,(a+bx^2)^3}}{x}+\frac{315a^{5/2}\sqrt{b}c\sqrt{c\,(a+bx^2)^3}\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

```
[Out] 105/64*a^2*b*c*x*(c*(b*x^2+a)^3)^(1/2)+315/128*a^3*b*c*x*(c*(b*x^2+a)^3)^(1
/2)/(b*x^2+a)+21/16*a*b*c*x*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+9/8*b*c*x*(b*x^
2+a)^2*(c*(b*x^2+a)^3)^(1/2)-c*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)/x+315/128*
a^(5/2)*c*arcsinh(x*b^(1/2)/a^(1/2))*b^(1/2)*(c*(b*x^2+a)^3)^(1/2)/(1+b*x^2
/a)^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used

$= \{1973, 283, 201, 221\}$

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}\,dx = \frac{315a^{5/2}\sqrt{b}\text{carcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\sqrt{c\,(a+bx^2)^3}}{128\left(\frac{bx^2}{a}+1\right)^{3/2}} + \frac{315a^3bcx\sqrt{c\,(a+bx^2)^3}}{128\,(a+bx^2)} + \frac{105}{64}a^2bcx\sqrt{c\,(a+bx^2)^3} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\,(a+bx^2)^3} - \frac{c(a+bx^2)^3\sqrt{c\,(a+bx^2)^3}}{x} + \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\,(a+bx^2)^3}$$

```
[In] Int[(c*(a + b*x^2)^3)^(3/2)/x^2,x]
```

```
[Out] (105*a^2*b*c*x*Sqrt[c*(a + b*x^2)^3])/64 + (315*a^3*b*c*x*Sqrt[c*(a + b*x^2
)^3])/(128*(a + b*x^2)) + (21*a*b*c*x*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/16
 + (9*b*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/8 - (c*(a + b*x^2)^3*Sqrt[
c*(a + b*x^2)^3])/x + (315*a^(5/2)*Sqrt[b]*c*Sqrt[c*(a + b*x^2)^3]*ArcSinh[
(Sqrt[b]*x)/Sqrt[a]])/(128*(1 + (b*x^2)/a)^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^{9/2}}{x^2}\,dx}{\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= -\frac{c(a+bx^2)^3\sqrt{c\left(a+bx^2\right)^3}}{x} + \frac{\left(9a^2bc\sqrt{c\left(a+bx^2\right)^3}\right)\int\left(1+\frac{bx^2}{a}\right)^{7/2}\,dx}{\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c(a+bx^2)^3\sqrt{c\left(a+bx^2\right)^3}}{x} + \frac{\left(63a^2bc\sqrt{c\left(a+bx^2\right)^3}\right)\int\left(1+\frac{bx^2}{a}\right)^{5/2}\,dx}{8\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c(a+bx^2)^3\sqrt{c\left(a+bx^2\right)^3}}{x} + \frac{\left(105a^2bc\sqrt{c\left(a+bx^2\right)^3}\right)\int\left(1+\frac{bx^2}{a}\right)^{3/2}\,dx}{16\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c(a+bx^2)^3\sqrt{c\left(a+bx^2\right)^3}}{x} + \frac{\left(315a^2bc\sqrt{c\left(a+bx^2\right)^3}\right)\int\sqrt{1+\frac{bx^2}{a}}\,dx}{64\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c(a+bx^2)^3\sqrt{c\left(a+bx^2\right)^3}}{x} + \frac{\left(315a^2bc\sqrt{c\left(a+bx^2\right)^3}\right)\int\frac{1}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{128\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c(a+bx^2)^3\sqrt{c\left(a+bx^2\right)^3}}{x} + \frac{315a^{5/2}\sqrt{b}c\sqrt{c\left(a+bx^2\right)^3}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.58

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}\,dx = -\frac{\left(c(a+bx^2)^3\right)^{3/2}\left(\sqrt{a+bx^2}(128a^4-325a^3bx^2-210a^2b^2x^4-88ab^3x^6-16b^4x^8)+315a^4\sqrt{b}x\log\left(-\sqrt{b}x + \right.\right.}{128x\left(a+bx^2\right)^{9/2}}$$

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x^2,x]
```

```
[Out] -1/128*((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(128*a^4 - 325*a^3*b*x^2 -
 210*a^2*b^2*x^4 - 88*a*b^3*x^6 - 16*b^4*x^8) + 315*a^4*Sqrt[b]*x*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]]))/(x*(a + b*x^2)^(9/2))
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{(-16b^4x^8-88a\,b^3x^6-210a^2b^2x^4-325a^3b\,x^2+128a^4)c\sqrt{c(b\,x^2+a)^3}}{128(b\,x^2+a)x}+\frac{315b\,a^4\ln\left(\frac{bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+ac}\right)c\sqrt{c(b\,x^2+a)^3}\,\sqrt{c(b\,x^2+a)}}{128\sqrt{bc}\,(b\,x^2+a)^2}$
default	$-\frac{\left(c(b\,x^2+a)^3\right)^{\frac{3}{2}}\left(-16(bc\,x^2+ac)^{\frac{5}{2}}\sqrt{bc}\,b^2x^4-56(bc\,x^2+ac)^{\frac{5}{2}}\sqrt{bc}\,ab\,x^2-210(bc\,x^2+ac)^{\frac{3}{2}}\sqrt{bc}\,a^2bc\,x^2-315\sqrt{bc\,x^2+ac}\,\sqrt{bc}\,a^3b\,c^2x^2-\right.}{128(b\,x^2+a)^3(c(b\,x^2+a))^{\frac{3}{2}}c\sqrt{bc}\,x}$

```
[In] int((c*(b*x^2+a)^3)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/128/(b*x^2+a)*(-16*b^4*x^8-88*a*b^3*x^6-210*a^2*b^2*x^4-325*a^3*b*x^2+12
8*a^4)/x*c*(c*(b*x^2+a)^3)^(1/2)+315/128*b*a^4*ln(b*c*x/(b*c)^(1/2)+(b*c*x^
2+a*c)^(1/2))/(b*c)^(1/2)*c/(b*x^2+a)^2*(c*(b*x^2+a)^3)^(1/2)*(c*(b*x^2+a))
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.90

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}\,dx = \left[\frac{315\left(a^4bcx^3+a^5cx\right)\sqrt{bc}\log\left(-\frac{2\,b^2cx^4+3\,abcx^2+a^2c+2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{bc}x}{bx^2+a}\right)}{} \right.$$

$$-\frac{315\left(a^4bcx^3+a^5cx\right)\sqrt{-bc}\arctan\left(\frac{\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{-bc}x}{b^2cx^4+2\,abcx^2+a^2c}\right)-\left(16\,b^4cx^8+88\,ab^3cx^6+210\,a^2b^2cx^4\right.}{128\left(bx^3+ax\right)}$$

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/256*(315*(a^4*b*c*x^3 + a^5*c*x)*sqrt(b*c)*log(-(2*b^2*c*x^4 + 3*a*b*c*x
^2 + a^2*c + 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt
(b*c)*x)/(b*x^2 + a)) + 2*(16*b^4*c*x^8 + 88*a*b^3*c*x^6 + 210*a^2*b^2*c*x^
4 + 325*a^3*b*c*x^2 - 128*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c
*x^2 + a^3*c))/(b*x^3 + a*x), -1/128*(315*(a^4*b*c*x^3 + a^5*c*x)*sqrt(-b*c
)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(-b*c)
*x/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) - (16*b^4*c*x^8 + 88*a*b^3*c*x^6 + 21
0*a^2*b^2*c*x^4 + 325*a^3*b*c*x^2 - 128*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x
^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^3 + a*x)]
```

Sympy [F]

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}\,dx = \int \frac{\left(c(a+bx^2)^3\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((c*(b*x**2+a)**3)**(3/2)/x**2,x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}\,dx = \int \frac{\left((bx^2+a)^3c\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.89

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}\,dx = \frac{1}{256}\left(\frac{512\sqrt{bc}a^5c\mathrm{sgn}(bx^2+a)}{\left(\sqrt{bc}x-\sqrt{bcx^2+ac}\right)^2-ac}-315\sqrt{bc}a^4\log\left(\left(\sqrt{bc}x-\sqrt{bcx^2+ac}\right)^2\right)\mathrm{sgn}\right.$$

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/256*(512*sqrt(b*c)*a^5*c*sgn(b*x^2 + a)/((sqrt(b*c)*x - sqrt(b*c*x^2 + a*
c))^2 - a*c) - 315*sqrt(b*c)*a^4*log((sqrt(b*c)*x - sqrt(b*c*x^2 + a*c))^2)
*sgn(b*x^2 + a) + 2*(325*a^3*b*sgn(b*x^2 + a) + 2*(105*a^2*b^2*sgn(b*x^2 +
a) + 4*(2*b^4*x^2*sgn(b*x^2 + a) + 11*a*b^3*sgn(b*x^2 + a))*x^2)*x^2)*sqrt(
b*c*x^2 + a*c)*x)*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}\,dx = \int \frac{\left(c\,{\left(b\,x^2+a\right)}^3\right)^{3/2}}{x^2}\,dx$$

```
[In] int((c*(a + b*x^2)^3)^(3/2)/x^2,x)
```

```
[Out] int((c*(a + b*x^2)^3)^(3/2)/x^2, x)
```

3.242 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 202

$$\begin{aligned}\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^3}\,dx &= \frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}+\frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)} \\ &+\frac{9}{10}abc(a+bx^2)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{14}bc(a+bx^2)^2\sqrt{c\left(a+bx^2\right)^3} \\ &-\frac{c(a+bx^2)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2}-\frac{9a^2bc\sqrt{c\left(a+bx^2\right)^3}\,\mathrm{arctanh}\left(\sqrt{1+\frac{bx^2}{a}}\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/2}}\end{aligned}$$

```
[Out] 3/2*a^2*b*c*(c*(b*x^2+a)^3)^(1/2)+9/2*a^3*b*c*(c*(b*x^2+a)^3)^(1/2)/(b*x^2+
a)+9/10*a*b*c*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+9/14*b*c*(b*x^2+a)^2*(c*(b*x^
2+a)^3)^(1/2)-1/2*c*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)/x^2-9/2*a^2*b*c*arcta
nh((1+b*x^2/a)^(1/2))*(c*(b*x^2+a)^3)^(1/2)/(1+b*x^2/a)^(3/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used

$= \{1973, 272, 43, 52, 65, 214\}$

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^3}\,dx = \frac{9a^3bc\sqrt{c\,(a+bx^2)^3}}{2\,(a+bx^2)} - \frac{9a^2bc\operatorname{arctanh}\left(\sqrt{\frac{bx^2}{a}+1}\right)\sqrt{c\,(a+bx^2)^3}}{2\left(\frac{bx^2}{a}+1\right)^{3/2}} + \frac{3}{2}a^2bc\sqrt{c\,(a+bx^2)^3} + \frac{9}{10}abc(a+bx^2)\sqrt{c\,(a+bx^2)^3} - \frac{c(a+bx^2)^3\sqrt{c\,(a+bx^2)^3}}{2x^2} + \frac{9}{14}bc(a+bx^2)^2\sqrt{c\,(a+bx^2)^3}$$

```
[In] Int[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

```
[Out] (3*a^2*b*c*Sqrt[c*(a + b*x^2)^3])/2 + (9*a^3*b*c*Sqrt[c*(a + b*x^2)^3])/(2*
(a + b*x^2)) + (9*a*b*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/10 + (9*b*c*(a +
 b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/14 - (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3
])/(2*x^2) - (9*a^2*b*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[Sqrt[1 + (b*x^2)/a]])
/(2*(1 + (b*x^2)/a)^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(a^3c\sqrt{c\,(a+bx^2)^3}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^{9/2}}{x^3}\,dx}{\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{\left(a^3c\sqrt{c\,(a+bx^2)^3}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^{9/2}}{x^2}\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= -\frac{c(a+bx^2)^3\sqrt{c\,(a+bx^2)^3}}{2x^2} + \frac{\left(9a^2bc\sqrt{c\,(a+bx^2)^3}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^{7/2}}{x}\,dx,x,x^2\right)}{4\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{9}{14}bc(a+bx^2)^2\sqrt{c\,(a+bx^2)^3} - \frac{c(a+bx^2)^3\sqrt{c\,(a+bx^2)^3}}{2x^2} + \frac{\left(9a^2bc\sqrt{c\,(a+bx^2)^3}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^{5/2}}{x}\,dx,x,x^2\right)}{4\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$= \frac{9}{10}abc(a+bx^2)\sqrt{c\,(a+bx^2)^3} + \frac{9}{14}bc(a+bx^2)^2\sqrt{c\,(a+bx^2)^3} - \frac{c(a+bx^2)^3\sqrt{c\,(a+bx^2)^3}}{2x^2} + \frac{\left(9a^2bc\sqrt{c\,(a+bx^2)^3}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^{3/2}}{x}\,dx,x,x^2\right)}{4\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$=\frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}-\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2}+\frac{\left(9a^2bc\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{bx}{a}}}{x}\,dx,x,x^2\right)}{4\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$=\frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}+\frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)}+\frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}-\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2}+\frac{\left(9a^2bc\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{bx}{a}}}\,dx,x,x^2\right)}{4\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$=\frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}+\frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)}+\frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}-\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2}+\frac{\left(9a^3c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{ax^2}{b}}\,dx,x,\sqrt{1+\frac{bx^2}{a}}\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$=\frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}+\frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)}+\frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}-\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2}-\frac{9a^2bc\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\sqrt{1+\frac{bx^2}{a}}\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.58

$$\int\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^3}\,dx=$$

$$-\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\sqrt{a+bx^2}\left(35a^4-388a^3bx^2-156a^2b^2x^4-58ab^3x^6-10b^4x^8\right)+315a^{7/2}bx^2\text{arctanh}\left(\frac{\sqrt{a+}}{\sqrt{a}}\right.\right.}{70x^2\left(a+bx^2\right)^{9/2}}$$

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

```
[Out] -1/70*((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(35*a^4 - 388*a^3*b*x^2 - 1
56*a^2*b^2*x^4 - 58*a*b^3*x^6 - 10*b^4*x^8) + 315*a^(7/2)*b*x^2*ArcTanh[Sqr
t[a + b*x^2]/Sqrt[a]]))/(x^2*(a + b*x^2)^(9/2))
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{a^4 c\sqrt{c(b\,x^2+a)^3}}{2(b\,x^2+a)x^2}+\frac{\left(\frac{b^4 x^6\sqrt{bc\,x^2+ac}}{7c}+\frac{29b^3 a\,x^4\sqrt{bc\,x^2+ac}}{35c}+\frac{78b^2 a^2 x^2\sqrt{bc\,x^2+ac}}{35c}-\frac{156b\,a^3\sqrt{bc\,x^2+ac}}{35c}-\frac{9b\,a^4\ln\left(\frac{2ac+2\sqrt{ac}\,\sqrt{bc\,x^2+ac}}{x}\right.}{2\sqrt{ac}}\right.}{(b\,x^2+a)^2}$
default	$\frac{\left(c(b\,x^2+a)^3\right)^{\frac{3}{2}}\left(10\sqrt{ac}\,(bc\,x^2+ac)^{\frac{5}{2}}b^2x^4-315\ln\left(\frac{2ac+2\sqrt{ac}\,\sqrt{bc\,x^2+ac}}{x}\right)a^4bc^3x^2+42ba\left(c(b\,x^2+a)\right)^{\frac{5}{2}}x^2\sqrt{ac}-4\sqrt{ac}\,(bc\,x^2+ac)^{\frac{5}{2}}ab\right.}{70(b\,x^2+a)^3\left(c(b\,x^2+a)\right)^{\frac{3}{2}}c\,x^2\sqrt{ac}}$

```
[In] int((c*(b*x^2+a)^3)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^4/(b*x^2+a)/x^2*c*(c*(b*x^2+a)^3)^(1/2)+(1/7*b^4*x^6/c*(b*c*x^2+a*c)
^(1/2)+29/35*b^3*a*x^4/c*(b*c*x^2+a*c)^(1/2)+78/35*b^2*a^2*x^2/c*(b*c*x^2+a
*c)^(1/2)-156/35*b*a^3/c*(b*c*x^2+a*c)^(1/2)-9/2*b*a^4/(a*c)^(1/2)*ln((2*a*
c+2*(a*c)^(1/2)*(b*c*x^2+a*c)^(1/2))/x)+10*b*a^3/c*(c*(b*x^2+a))^(1/2))*c/(
b*x^2+a)^2*(c*(b*x^2+a)^3)^(1/2)*(c*(b*x^2+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.03

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^3}\,dx = \left[\frac{315\left(a^3b^2cx^4+a^4bcx^2\right)\sqrt{ac}\log\left(-\frac{b^2cx^4+3\,abcx^2+2\,a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{ac}}{bx^4+ax^2}\right.}{}\right.$$

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a^3*b^2*c*x^4 + a^4*b*c*x^2)*sqrt(a*c)*log(-(b^2*c*x^4 + 3*a*b
*c*x^2 + 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c
)*sqrt(a*c))/(b*x^4 + a*x^2)) + 2*(10*b^4*c*x^8 + 58*a*b^3*c*x^6 + 156*a^2*
b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*
a^2*b*c*x^2 + a^3*c))/(b*x^4 + a*x^2), 1/70*(315*(a^3*b^2*c*x^4 + a^4*b*c*x
^2)*sqrt(-a*c)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*
c)*sqrt(-a*c)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) + (10*b^4*c*x^8 + 58*a*b^3
*c*x^6 + 156*a^2*b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^4*c)*sqrt(b^3*c*x^6 + 3
*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^4 + a*x^2)]
```

Sympy [F]

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^3}\,dx = \int \frac{\left(c(a+bx^2)^3\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((c*(b*x**2+a)**3)**(3/2)/x**3,x)
[Out] Integral((c*(a + b*x**2)**3)**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^3}\,dx = \int \frac{\left((bx^2+a)^3c\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x^3, x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^3}\,dx = \frac{\left(\frac{315\,a^4b^2c\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{-ac}} - \frac{35\,\sqrt{bcx^2+ac}a^4b\mathrm{sgn}\left(bx^2+a\right)}{x^2} + \frac{2\left(140\,\sqrt{bcx^2+ac}a^3b^2c^{21}\mathrm{sgn}(b\right.}{}\right.}{}$$

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="giac")
[Out] 1/70*(315*a^4*b^2*c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))*sgn(b*x^2 + a)/s
qrt(-a*c) - 35*sqrt(b*c*x^2 + a*c)*a^4*b*sgn(b*x^2 + a)/x^2 + 2*(140*sqrt(b
*c*x^2 + a*c)*a^3*b^2*c^21*sgn(b*x^2 + a) + 35*(b*c*x^2 + a*c)^(3/2)*a^2*b^
2*c^20*sgn(b*x^2 + a) + 14*(b*c*x^2 + a*c)^(5/2)*a*b^2*c^19*sgn(b*x^2 + a)
+ 5*(b*c*x^2 + a*c)^(7/2)*b^2*c^18*sgn(b*x^2 + a))/c^21)*c/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^3}\,dx = \int \frac{\left(c\,\left(b\,x^2+a\right)^3\right)^{3/2}}{x^3}\,dx$$

```
[In] int((c*(a + b*x^2)^3)^(3/2)/x^3,x)
[Out] int((c*(a + b*x^2)^3)^(3/2)/x^3, x)
```

3.243 $\int x^2\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int x^2\left(\frac{c}{a+bx^2}\right)^{3/2}dx=-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}+\frac{\sqrt{a}c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] -c*x*(c/(b*x^2+a))^(1/2)/b+c*arcsinh(x*b^(1/2)/a^(1/2))*a^(1/2)*(c/(b*x^2+a
))^(1/2)*(1+b*x^2/a)^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1973, 294, 221}

$$\int x^2\left(\frac{c}{a+bx^2}\right)^{3/2}dx=\frac{\sqrt{a}c\sqrt{\frac{bx^2}{a}+1}\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\sqrt{\frac{c}{a+bx^2}}}{b^{3/2}}-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}$$

```
[In] Int[x^2*(c/(a + b*x^2))^(3/2),x]
```

```
[Out] -((c*x*Sqrt[c/(a + b*x^2)])/b) + (Sqrt[a]*c*Sqrt[c/(a + b*x^2)]*Sqrt[1 + (b
*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\right)\int\frac{x^2}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{a}\\ &= -\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}+\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\right)\int\frac{1}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{b}\\ &= -\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}+\frac{\sqrt{a}c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int x^2\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx=-\frac{c\sqrt{\frac{c}{a+bx^2}}\left(\sqrt{b}x+2\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}-\sqrt{a+bx^2}}\right)\right)}{b^{3/2}}$$

```
[In] Integrate[x^2*(c/(a + b*x^2))^(3/2),x]
[Out] -((c*Sqrt[c/(a + b*x^2)]*(Sqrt[b]*x + 2*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)
/(Sqrt[a] - Sqrt[a + b*x^2])]))/b^(3/2))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}} \left(b\,x^{2}+a \right) \left(x\,b^{\frac{3}{2}}-\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right) b\sqrt{b\,x^{2}+a}\right)}{b^{\frac{5}{2}}}$	60

```
[In] int(x^2*(c/(b*x^2+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c/(b*x^2+a))^(3/2)*(b*x^2+a)*(x*b^(3/2)-ln(b^(1/2)*x+(b*x^2+a)^(1/2))*b*(
b*x^2+a)^(1/2))/b^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.83

$$\int x^2\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx=\left[-\frac{2\,cx\sqrt{\frac{c}{bx^2+a}}-c\sqrt{\frac{c}{b}}\log\left(-2\,bcx^2-ac-2\,(b^2x^3+abx)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{b}}\right)}{2\,b},\ -\frac{cx\sqrt{\frac{c}{bx^2+a}}+c\sqrt{-\frac{c}{b}}\arctan\left(\frac{bx\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{b}}}{c}\right)}{b}\right]$$

```
[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*c*x*sqrt(c/(b*x^2 + a)) - c*sqrt(c/b)*log(-2*b*c*x^2 - a*c - 2*(b^
2*x^3 + a*b*x)*sqrt(c/(b*x^2 + a))*sqrt(c/b)))/b, -(c*x*sqrt(c/(b*x^2 + a))
 + c*sqrt(-c/b)*arctan(b*x*sqrt(c/(b*x^2 + a))*sqrt(-c/b)/c))/b]
```

Sympy [F]

$$\int x^2\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx=\int x^{2}\left(\frac{c}{a+bx^{2}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**2*(c/(b*x**2+a))**(3/2),x)
```

```
[Out] Integral(x**2*(c/(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int x^2 \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \int x^2 \left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}} dx$$

```
[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="maxima")
[Out] integrate(x^2*(c/(b*x^2 + a))^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int x^2 \left(\frac{c}{a+bx^2}\right)^{3/2} dx = -\left(\frac{cx\text{sgn}(bx^2+a)}{\sqrt{bcx^2+ac}b} + \frac{c\log\left(\left|-\sqrt{bc}x + \sqrt{bcx^2+ac}\right|\right)\text{sgn}(bx^2+a)}{\sqrt{bc}b}\right)c$$

```
[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="giac")
[Out] -(c*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*b) + c*log(abs(-sqrt(b*c)*x + sqr
t(b*c*x^2 + a*c)))*sgn(b*x^2 + a)/(sqrt(b*c)*b))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \int x^2 \left(\frac{c}{b\,x^2+a}\right)^{3/2} dx$$

```
[In] int(x^2*(c/(a + b*x^2))^(3/2),x)
[Out] int(x^2*(c/(a + b*x^2))^(3/2), x)
```

3.244 $\int x\left(\frac{c}{a+bx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2} dx = -\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

```
[Out] -c*(c/(b*x^2+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1605, 15, 30}

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2} dx = -\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

```
[In] Int[x*(c/(a + b*x^2))^(3/2),x]
[Out] -((c*Sqrt[c/(a + b*x^2)])/b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \left(\frac{c}{x}\right)^{3/2} dx, x, a+bx^2\right)}{2b}\\ &= \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}} dx, x, a+bx^2\right)}{2b}\\ &= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2} dx = -\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

```
[In] Integrate[x*(c/(a + b*x^2))^(3/2),x]
[Out] -((c*Sqrt[c/(a + b*x^2)])/b)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
trager	$-\frac{c\sqrt{\frac{c}{b\,x^{2}+a}}}{b}$	20
gosper	$-\frac{\left(b\,x^{2}+a\right)\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}}}{b}$	26
derivativedivides	$-\frac{\left(b\,x^{2}+a\right)\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}}}{b}$	26
default	$-\frac{\left(b\,x^{2}+a\right)\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}}}{b}$	26

```
[In] int(x*(c/(b*x^2+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] -c*(c/(b*x^2+a))^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx=-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

```
[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="fricas")
[Out] -c*sqrt(c/(b*x^2 + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx=\begin{cases}-\frac{a\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{b}-x^2\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}} & \text{for } b\neq 0\\ \frac{x^2\left(\frac{c}{a}\right)^{\frac{3}{2}}}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(c/(b*x**2+a))**(3/2),x)
[Out] Piecewise((-a*(c/(a + b*x**2))**(3/2)/b - x**2*(c/(a + b*x**2))**(3/2), Ne(
b, 0)), (x**2*(c/a)**(3/2)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx=-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

```
[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="maxima")
[Out] -c*sqrt(c/(b*x^2 + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx = -\frac{c^2\mathrm{sgn}(bx^2+a)}{\sqrt{bcx^2+ac}b}$$

```
[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="giac")
[Out] -c^2*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*b)
```

Mupad [B] (verification not implemented)

Time = 19.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx = -\frac{c\sqrt{\frac{c}{b\,x^2+a}}}{b}$$

```
[In] int(x*(c/(a + b*x^2))^(3/2),x)
[Out] -(c*(c/(a + b*x^2))^(1/2))/b
```

3.245 $\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

```
[Out] c*x*(c/(b*x^2+a))^(1/2)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1973, 197}

$$\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

```
[In] Int[(c/(a + b*x^2))^(3/2),x]
```

```
[Out] (c*x*Sqrt[c/(a + b*x^2)])/a
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
```

```
x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\right)\int\frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{a}\\ &= \frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int\left(\frac{c}{a+bx^2}\right)^{3/2}dx=\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

```
[In] Integrate[(c/(a + b*x^2))^(3/2),x]
[Out] (c*x*Sqrt[c/(a + b*x^2)])/a
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
trager	$\frac{cx\sqrt{\frac{c}{b\,x^2+a}}}{a}$	20
gosper	$\frac{(b\,x^2+a)x\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}}{a}$	26
default	$\frac{(b\,x^2+a)x\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}}{a}$	26

```
[In] int((c/(b*x^2+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] c*x*(c/(b*x^2+a))^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \frac{cx\sqrt{\frac{c}{bx^2+a}}}{a}$$

```
[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="fricas")
[Out] c*x*sqrt(c/(b*x^2 + a))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(15) = 30.

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19

$$\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \begin{cases} x\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}} + \frac{bx^3\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{a} & \text{for } a \neq 0 \\ -\frac{x\left(\frac{c}{bx^2}\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate((c/(b*x**2+a))**(3/2),x)
[Out] Piecewise((x*(c/(a + b*x**2))**(3/2) + b*x**3*(c/(a + b*x**2))**(3/2)/a, Ne
(a, 0)), (-x*(c/(b*x**2))**(3/2)/2, True))
```

Maxima [F]

$$\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \int \left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="maxima")
[Out] integrate((c/(b*x^2 + a))^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \frac{c^2 x \operatorname{sgn}(bx^2+a)}{\sqrt{bcx^2+ac}a}$$

```
[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="giac")
[Out] c^2*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a)
```

Mupad [B] (verification not implemented)

Time = 18.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \frac{c\,x\,\sqrt{\frac{c}{b\,x^2+a}}}{a}$$

```
[In] int((c/(a + b*x^2))^(3/2),x)
[Out] (c*x*(c/(a + b*x^2))^(1/2))/a
```

3.246 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx = \frac{c\sqrt{\frac{c}{a+bx^2}}}{a} - \frac{c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\,\text{arctanh}\left(\sqrt{1+\frac{bx^2}{a}}\right)}{a}$$

```
[Out] c*(c/(b*x^2+a))^(1/2)/a-c*arctanh((1+b*x^2/a)^(1/2))*(c/(b*x^2+a))^(1/2)*(1
+b*x^2/a)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1973, 272, 53, 65, 214}

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx = \frac{c\sqrt{\frac{c}{a+bx^2}}}{a} - \frac{c\sqrt{\frac{bx^2}{a}+1}\,\text{arctanh}\left(\sqrt{\frac{bx^2}{a}+1}\right)\sqrt{\frac{c}{a+bx^2}}}{a}$$

```
[In] Int[(c/(a + b*x^2))^(3/2)/x,x]
```

```
[Out] (c*Sqrt[c/(a + b*x^2)])/a - (c*Sqrt[c/(a + b*x^2)]*Sqrt[1 + (b*x^2)/a]*ArcT
anh[Sqrt[1 + (b*x^2)/a]])/a
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\right)\int\frac{1}{x\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{a}\\
&= \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{x\left(1+\frac{bx}{a}\right)^{3/2}}\,dx,x,x^2\right)}{2a}\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a}+\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{bx}{a}}}\,dx,x,x^2\right)}{2a}\\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a}+\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{ax^2}{b}}\,dx,x,\sqrt{1+\frac{bx^2}{a}}\right)}{b}
\end{aligned}$$

$$=\frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\tanh^{-1}\left(\sqrt{1+\frac{bx^2}{a}}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int\frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx=\frac{c\sqrt{\frac{c}{a+bx^2}}\left(\sqrt{a}-\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{a^{3/2}}$$

```
[In] Integrate[(c/(a + b*x^2))^(3/2)/x,x]
[Out] (c*Sqrt[c/(a + b*x^2)]*(Sqrt[a] - Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]]))/a^(3/2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}\left(b\,x^2+a\right)\left(\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)a\sqrt{b\,x^2+a}-a^{\frac{3}{2}}\right)}{a^{\frac{5}{2}}}$	64

```
[In] int((c/(b*x^2+a))^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] -(c/(b*x^2+a))^(3/2)*(b*x^2+a)*(ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a*(b*x^
2+a)^(1/2)-a^(3/2))/a^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.94

$$\int\frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx=\left[\frac{c\sqrt{\frac{c}{a}}\log\left(-\frac{bcx^2+2\,ac-2\,(abx^2+a^2)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{a}}}{x^2}\right)+2\,c\sqrt{\frac{c}{bx^2+a}}}{2\,a},\frac{c\sqrt{-\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{a}}}{c}\right)+\dots}{a}\right.$$

```
[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="fricas")
[Out] [1/2*(c*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c - 2*(a*b*x^2 + a^2)*sqrt(c/(b*x^2 +
 a))*sqrt(c/a))/x^2) + 2*c*sqrt(c/(b*x^2 + a)))/a, (c*sqrt(-c/a)*arctan(a*s
qrt(c/(b*x^2 + a))*sqrt(-c/a)/c) + c*sqrt(c/(b*x^2 + a)))/a]
```

Sympy [F]

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((c/(b*x**2+a))**(3/2)/x,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx = \frac{1}{2}\,c\left(\frac{c\log\left(\frac{a\sqrt{\frac{c}{bx^2+a}}-\sqrt{ac}}{a\sqrt{\frac{c}{bx^2+a}}+\sqrt{ac}}\right)}{\sqrt{ac}a}+\frac{2\sqrt{\frac{c}{bx^2+a}}}{a}\right)$$

```
[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*c*(c*log((a*sqrt(c/(b*x^2 + a)) - sqrt(a*c))/(a*sqrt(c/(b*x^2 + a)) + s
qrt(a*c)))/(sqrt(a*c)*a) + 2*sqrt(c/(b*x^2 + a))/a)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx = c\left(\frac{c\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a}+\frac{c}{\sqrt{bcx^2+ac}a}\right)\operatorname{sgn}\left(bx^2+a\right)$$

```
[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="giac")
```

```
[Out] c*(c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a) + c/(sqrt(b*c*x^
2 + a*c)*a))*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(\frac{c}{b\,x^2+a}\right)^{3/2}}{x}\,dx$$

```
[In] int((c/(a + b*x^2))^(3/2)/x,x)
[Out] int((c/(a + b*x^2))^(3/2)/x, x)
```

3.247 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2} dx = -\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax} - \frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}$$

```
[Out] -c*(c/(b*x^2+a))^(1/2)/a/x-2*b*c*x*(c/(b*x^2+a))^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1973, 277, 197}

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2} dx = -\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2} - \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}$$

```
[In] Int[(c/(a + b*x^2))^(3/2)/x^2,x]
```

```
[Out] -((c*Sqrt[c/(a + b*x^2)])/(a*x)) - (2*b*c*x*Sqrt[c/(a + b*x^2)])/a^2
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\right)\int\frac{1}{x^2\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{a}\\
&= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}-\frac{\left(2bc\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\right)\int\frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{a^2}\\
&= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int\frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx=-\frac{c\sqrt{\frac{c}{a+bx^2}}(a+2bx^2)}{a^2x}$$

```
[In] Integrate[(c/(a + b*x^2))^(3/2)/x^2,x]
[Out] -((c*Sqrt[c/(a + b*x^2)]*(a + 2*b*x^2))/(a^2*x))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{(b\,x^{2}+a)\,(2b\,x^{2}+a)\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}}}{a^{2}x}$	37
default	$-\frac{(b\,x^{2}+a)\,(2b\,x^{2}+a)\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}}}{a^{2}x}$	37
trager	$-\frac{(ac+bc)\,(2b\,x^{2}+a)\sqrt{\frac{c}{b\,x^{2}+a}}}{a^{2}(a+b)x}$	42
risch	$-\frac{(b\,x^{2}+a)\,c\sqrt{\frac{c}{b\,x^{2}+a}}}{a^{2}x}-\frac{bcx\sqrt{\frac{c}{b\,x^{2}+a}}}{a^{2}}$	52

```
[In] int((c/(b*x^2+a))^(3/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -(b*x^2+a)*(2*b*x^2+a)*(c/(b*x^2+a))^(3/2)/a^2/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx=-\frac{(2\,bcx^2+ac)\sqrt{\frac{c}{bx^2+a}}}{a^2x}$$

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="fricas")
[Out] -(2*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a))/(a^2*x)
```

Sympy [F]

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((c/(b*x**2+a))**(3/2)/x**2,x)
[Out] Integral((c/(a + b*x**2))**(3/2)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx = -\frac{2\,b^2c^{\frac{3}{2}}x^4 + 3\,abc^{\frac{3}{2}}x^2 + a^2c^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}a^2x}$$

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -(2*b^2*c^(3/2)*x^4 + 3*a*b*c^(3/2)*x^2 + a^2*c^(3/2))/((b*x^2 + a)^(3/2)*a
^2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.69

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx = -\left(\frac{bcx\mathrm{sgn}(bx^2+a)}{\sqrt{bcx^2+ac}a^2} - \frac{2\sqrt{bc}c\mathrm{sgn}(bx^2+a)}{\left(\left(\sqrt{bc}x-\sqrt{bcx^2+ac}\right)^2-ac\right)a}\right)c$$

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -(b*c*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a^2) - 2*sqrt(b*c)*c*sgn(b*x^2
+ a)/(((sqrt(b*c)*x - sqrt(b*c*x^2 + a*c))^2 - a*c)*a))*c
```

Mupad [B] (verification not implemented)

Time = 19.49 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx = -\frac{\left(\frac{b\,c}{a}+\frac{2\,b^2\,c\,x^2}{a^2}\right)\,\sqrt{\frac{c}{b\,x^2+a}}\,\left(\frac{a}{b}+x^2\right)}{b\,x^3+a\,x}$$

```
[In] int((c/(a + b*x^2))^(3/2)/x^2,x)
```

```
[Out] -(((b*c)/a + (2*b^2*c*x^2)/a^2)*(c/(a + b*x^2))^(1/2)*(a/b + x^2))/(a*x + b
*x^3)
```

3.248 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 104

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx=-\frac{3bc\sqrt{\frac{c}{a+bx^2}}}{2a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{2ax^2}+\frac{3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\,\text{arctanh}\left(\sqrt{1+\frac{bx^2}{a}}\right)}{2a^2}$$

```
[Out] -3/2*b*c*(c/(b*x^2+a))^(1/2)/a^2-1/2*c*(c/(b*x^2+a))^(1/2)/a/x^2+3/2*b*c*ar
ctanh((1+b*x^2/a)^(1/2))*(c/(b*x^2+a))^(1/2)*(1+b*x^2/a)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1973, 272, 44, 53, 65, 214}

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx=\frac{3bc\sqrt{\frac{bx^2}{a}+1}\,\text{arctanh}\left(\sqrt{\frac{bx^2}{a}+1}\right)\sqrt{\frac{c}{a+bx^2}}}{2a^2}-\frac{3bc\sqrt{\frac{c}{a+bx^2}}}{2a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{2ax^2}$$

```
[In] Int[(c/(a + b*x^2))^(3/2)/x^3,x]
```

```
[Out] (-3*b*c*Sqrt[c/(a + b*x^2)])/(2*a^2) - (c*Sqrt[c/(a + b*x^2)])/(2*a*x^2) +
(3*b*c*Sqrt[c/(a + b*x^2)]*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a]]
)/(2*a^2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\right)\int\frac{1}{x^3\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{a}$$

$$= \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{x^2\left(1+\frac{bx}{a}\right)^{3/2}}\,dx,x,x^2\right)}{2a}$$

$$= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{2ax^2} - \frac{\left(3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{x\left(1+\frac{bx}{a}\right)^{3/2}}\,dx, x, x^2\right)}{4a^2}$$

$$= -\frac{3bc\sqrt{\frac{c}{a+bx^2}}}{2a^2} - \frac{c\sqrt{\frac{c}{a+bx^2}}}{2ax^2} - \frac{\left(3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{bx}{a}}}\,dx, x, x^2\right)}{4a^2}$$

$$= -\frac{3bc\sqrt{\frac{c}{a+bx^2}}}{2a^2} - \frac{c\sqrt{\frac{c}{a+bx^2}}}{2ax^2} - \frac{\left(3c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{ax^2}{b}}\,dx, x, \sqrt{1+\frac{bx^2}{a}}\right)}{2a}$$

$$= -\frac{3bc\sqrt{\frac{c}{a+bx^2}}}{2a^2} - \frac{c\sqrt{\frac{c}{a+bx^2}}}{2ax^2} + \frac{3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\tanh^{-1}\left(\sqrt{1+\frac{bx^2}{a}}\right)}{2a^2}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.75

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx = -\frac{c\sqrt{\frac{c}{a+bx^2}}\left(\sqrt{a}(a+3bx^2)-3bx^2\sqrt{a+bx^2}\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{2a^{5/2}x^2}$$

```
[In] Integrate[(c/(a + b*x^2))^(3/2)/x^3,x]
```

```
[Out] -1/2*(c*Sqrt[c/(a + b*x^2)]*(Sqrt[a]*(a + 3*b*x^2) - 3*b*x^2*Sqrt[a + b*x^2
]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/(a^(5/2)*x^2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.78

method	result
default	$\frac{\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}(b\,x^2+a)\left(3\sqrt{b\,x^2+a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)ab\,x^2-3a^{\frac{3}{2}}b\,x^2-a^{\frac{5}{2}}\right)}{2a^{\frac{7}{2}}x^2}$
risch	$-\frac{(b\,x^2+a)c\sqrt{\frac{c}{b\,x^2+a}}}{2a^2x^2} - \frac{b\left(-\frac{3\ln\left(\frac{2ac+2\sqrt{ac}\,\sqrt{bc\,x^2+ac}}{x}\right)}{\sqrt{ac}} - \frac{\sqrt{bc\left(x+\frac{\sqrt{-ab}}{b}\right)^2-2c\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}}{c\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)} + \frac{\sqrt{bc\left(x-\frac{\sqrt{-ab}}{b}\right)^2+2c\sqrt{-ab}\left(x-\sqrt{}\right.}}{c\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}\right.}{2a^2}$

```
[In] int((c/(b*x^2+a))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c/(b*x^2+a))^(3/2)*(b*x^2+a)*(3*(b*x^2+a)^(1/2)*ln(2*(a^(1/2)*(b*x^2+a
)^(1/2)+a)/x)*a*b*x^2-3*a^(3/2)*b*x^2-a^(5/2))/a^(7/2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.68

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx = \left[\frac{3\,bcx^2\sqrt{\frac{c}{a}}\log\left(-\frac{bcx^2+2\,ac+2\,(abx^2+a^2)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{a}}}{x^2}\right)-2\,(3\,bcx^2+ac)\sqrt{\frac{c}{bx^2+a}}}{4\,a^2x^2}, -\frac{3\,bcx^2\sqrt{-\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{a}}}{c}\right)+(3\,bcx^2+ac)\sqrt{\frac{c}{bx^2+a}}}{2\,a^2x^2}\right]$$

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*b*c*x^2*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c + 2*(a*b*x^2 + a^2)*sqrt(c/
(b*x^2 + a))*sqrt(c/a))/x^2) - 2*(3*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a)))/(a^
2*x^2), -1/2*(3*b*c*x^2*sqrt(-c/a)*arctan(a*sqrt(c/(b*x^2 + a))*sqrt(-c/a)/
c) + (3*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a)))/(a^2*x^2)]
```

Sympy [F]

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((c/(b*x**2+a))**(3/2)/x**3,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx = -\frac{1}{4}\,bc\left(\frac{2\,c\sqrt{\frac{c}{bx^2+a}}}{a^2c-\frac{a^3c}{bx^2+a}}+\frac{3\,c\log\left(\frac{a\sqrt{\frac{c}{bx^2+a}}-\sqrt{ac}}{a\sqrt{\frac{c}{bx^2+a}}+\sqrt{ac}}\right)}{\sqrt{ac}a^2}+\frac{4\,\sqrt{\frac{c}{bx^2+a}}}{a^2}\right)$$

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*c*(2*c*sqrt(c/(b*x^2 + a))/(a^2*c - a^3*c/(b*x^2 + a)) + 3*c*log((a*
sqrt(c/(b*x^2 + a)) - sqrt(a*c))/(a*sqrt(c/(b*x^2 + a)) + sqrt(a*c)))/(sqrt
(a*c)*a^2) + 4*sqrt(c/(b*x^2 + a))/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx = -\frac{1}{2}\,c\left(\frac{3\,bc\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^2}+\frac{2\,abc^2-3\,(bcx^2+ac)bc}{\left(\sqrt{bcx^2+ac}ac-(bcx^2+ac)^{\frac{3}{2}}\right)a^2}\right)\operatorname{sgn}\left(bx^2+a\right)$$

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*c*(3*b*c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^2) + (2*
a*b*c^2 - 3*(b*c*x^2 + a*c)*b*c)/((sqrt(b*c*x^2 + a*c)*a*c - (b*c*x^2 + a*c
)^(3/2))*a^2))*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(\frac{c}{b\,x^2+a}\right)^{3/2}}{x^3}\,dx$$

```
[In] int((c/(a + b*x^2))^(3/2)/x^3,x)
```

```
[Out] int((c/(a + b*x^2))^(3/2)/x^3, x)
```

3.249 $\int x^7 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int x^7 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = -\frac{2a^3\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)}{7b^4} + \frac{6a^2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^2}{11b^4} - \frac{2a\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^3}{5b^4} + \frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^4}{19b^4}$$

```
[Out] -2/7*a^3*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^4+6/11*a^2*(b*x^2+a)^2*(c*(b
*x^2+a)^(1/2))^(3/2)/b^4-2/5*a*(b*x^2+a)^3*(c*(b*x^2+a)^(1/2))^(3/2)/b^4+2/
19*(b*x^2+a)^4*(c*(b*x^2+a)^(1/2))^(3/2)/b^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1973, 272, 45}

$$\int x^7 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = -\frac{2a^3(a+bx^2)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^4} + \frac{6a^2(a+bx^2)^2\left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^4} + \frac{2(a+bx^2)^4\left(c\sqrt{a+bx^2}\right)^{3/2}}{19b^4} - \frac{2a(a+bx^2)^3\left(c\sqrt{a+bx^2}\right)^{3/2}}{5b^4}$$

```
[In] Int[x^7*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (-2*a^3*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b^4) + (6*a^2*(c*Sqrt[a +
 b*x^2])^(3/2)*(a + b*x^2)^2)/(11*b^4) - (2*a*(c*Sqrt[a + b*x^2])^(3/2)*(a
+ b*x^2)^3)/(5*b^4) + (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)^4)/(19*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int x^7\left(1+\frac{bx^2}{a}\right)^{3/4}\,dx}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\\
&= \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\text{Subst}\left(\int x^3\left(1+\frac{bx}{a}\right)^{3/4}\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/4}}\\
&= \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\text{Subst}\left(\int\left(-\frac{a^3\left(1+\frac{bx}{a}\right)^{3/4}}{b^3}+\frac{3a^3\left(1+\frac{bx}{a}\right)^{7/4}}{b^3}-\frac{3a^3\left(1+\frac{bx}{a}\right)^{11/4}}{b^3}+\frac{a^3\left(1+\frac{bx}{a}\right)^{15/4}}{b^3}\right)dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/4}}\\
&= -\frac{2a^3\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)}{7b^4}+\frac{6a^2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^2}{11b^4}\\
&\quad-\frac{2a\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^3}{5b^4}+\frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^4}{19b^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

$$\int x^7\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)\left(-128a^3+224a^2bx^2-308ab^2x^4+385b^3x^6\right)}{7315b^4}$$

```
[In] Integrate[x^7*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(-128*a^3 + 224*a^2*b*x^2 - 308*a*
b^2*x^4 + 385*b^3*x^6))/(7315*b^4)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.42

method	result	size
gosper	$-\frac{2\left(b\,x^{2}+a\right)\left(-385b^{3}x^{6}+308b^{2}x^{4}a-224a^{2}b\,x^{2}+128a^{3}\right)\left(c\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{7315b^{4}}$	58

```
[In] int(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7315*(b*x^2+a)*(-385*b^3*x^6+308*a*b^2*x^4-224*a^2*b*x^2+128*a^3)*(c*(b*
x^2+a)^(1/2))^(3/2)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

$$\int x^7\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\left(385\,b^4cx^8+77\,ab^3cx^6-84\,a^2b^2cx^4+96\,a^3bcx^2-128\,a^4c\right)\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}}}{7315\,b^4}$$

```
[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^4*c*x^8 + 77*a*b^3*c*x^6 - 84*a^2*b^2*c*x^4 + 96*a^3*b*c*x^2
- 128*a^4*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b^4
```

Sympy [A] (verification not implemented)

Time = 11.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

$$\int x^7\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\begin{cases}-\frac{256a^4\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{7315b^4}+\frac{192a^3x^2\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{7315b^3}-\frac{24a^2x^4\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{1045b^2}+\frac{2ax^6\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{95b}+\frac{2x^8}{}\\ \frac{x^8\left(\sqrt{a}c\right)^{\frac{3}{2}}}{8}\end{cases}$$

```
[In] integrate(x**7*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Piecewise((-256*a**4*(c*sqrt(a + b*x**2))**(3/2)/(7315*b**4) + 192*a**3*x**
2*(c*sqrt(a + b*x**2))**(3/2)/(7315*b**3) - 24*a**2*x**4*(c*sqrt(a + b*x**2
))**(3/2)/(1045*b**2) + 2*a*x**6*(c*sqrt(a + b*x**2))**(3/2)/(95*b) + 2*x**
8*(c*sqrt(a + b*x**2))**(3/2)/19, Ne(b, 0)), (x**8*(sqrt(a)*c)**(3/2)/8, Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.62

$$\int x^7\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=-\frac{2\left(1045\left(\sqrt{bx^2+a}c\right)^{\frac{7}{2}}a^3c^6-1995\left(\sqrt{bx^2+a}c\right)^{\frac{11}{2}}a^2c^4+1463\left(\sqrt{bx^2+a}c\right)^{\frac{15}{2}}ac^2-385\left(\sqrt{bx^2+a}c\right)^{\frac{19}{2}}\right)}{7315\,b^4c^8}$$

```
[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/7315*(1045*(sqrt(b*x^2 + a)*c)^(7/2)*a^3*c^6 - 1995*(sqrt(b*x^2 + a)*c)^
(11/2)*a^2*c^4 + 1463*(sqrt(b*x^2 + a)*c)^(15/2)*a*c^2 - 385*(sqrt(b*x^2 +
a)*c)^(19/2))/(b^4*c^8)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99

$$\int x^7\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\,c^{\frac{3}{2}}\left(\frac{19\left(77\left(bx^2+a\right)^{\frac{15}{4}}-315\left(bx^2+a\right)^{\frac{11}{4}}a+495\left(bx^2+a\right)^{\frac{7}{4}}a^2-385\left(bx^2+a\right)^{\frac{3}{4}}a^3\right)a}{b^3}+\frac{1155\left(bx^2+a\right)^{\frac{19}{4}}-5}{}\right.}{21945\,b}$$

```
[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/21945*c^(3/2)*(19*(77*(b*x^2 + a)^(15/4) - 315*(b*x^2 + a)^(11/4)*a + 495
*(b*x^2 + a)^(7/4)*a^2 - 385*(b*x^2 + a)^(3/4)*a^3)*a/b^3 + (1155*(b*x^2 +
a)^(19/4) - 5852*(b*x^2 + a)^(15/4)*a + 11970*(b*x^2 + a)^(11/4)*a^2 - 1254
0*(b*x^2 + a)^(7/4)*a^3 + 7315*(b*x^2 + a)^(3/4)*a^4)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 18.88 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.79

$$\int x^7 \left(c\sqrt{a+bx^2}\right)^{3/2}\,dx = \sqrt{c\sqrt{b\,x^2+a}}\left(\frac{2\,c\,x^8\,\sqrt{b\,x^2+a}}{19}-\frac{256\,a^4\,c\,\sqrt{b\,x^2+a}}{7315\,b^4}+\frac{2\,a\,c\,x^6\,\sqrt{b\,x^2+a}}{95\,b}-\frac{24\,a^2\,c\,x^4\,\sqrt{b\,x^2+a}}{1045\,b^2}+\frac{192\,a^3\,c\,x^2\,\sqrt{b\,x^2+a}}{7315\,b^3}\right)$$

```
[In] int(x^7*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

```
[Out] (c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^8*(a + b*x^2)^(1/2))/19 - (256*a^4*c*(a
 + b*x^2)^(1/2))/(7315*b^4) + (2*a*c*x^6*(a + b*x^2)^(1/2))/(95*b) - (24*a^
2*c*x^4*(a + b*x^2)^(1/2))/(1045*b^2) + (192*a^3*c*x^2*(a + b*x^2)^(1/2))/(
7315*b^3))
```

3.250 $\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2a^2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)}{7b^3} - \frac{4a\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^2}{11b^3} + \frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^3}{15b^3}$$

```
[Out] 2/7*a^2*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^3-4/11*a*(b*x^2+a)^2*(c*(b*x^
2+a)^(1/2))^(3/2)/b^3+2/15*(b*x^2+a)^3*(c*(b*x^2+a)^(1/2))^(3/2)/b^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1973, 272, 45}

$$\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2a^2(a+bx^2)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^3} + \frac{2(a+bx^2)^3\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b^3} - \frac{4a(a+bx^2)^2\left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^3}$$

```
[In] Int[x^5*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*a^2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b^3) - (4*a*(c*Sqrt[a + b*
x^2])^(3/2)*(a + b*x^2)^2)/(11*b^3) + (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x
^2)^3)/(15*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int x^5\left(1+\frac{bx^2}{a}\right)^{3/4}\,dx}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\\
&= \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\,\text{Subst}\left(\int x^2\left(1+\frac{bx}{a}\right)^{3/4}\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/4}}\\
&= \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\,\text{Subst}\left(\int\left(\frac{a^2\left(1+\frac{bx}{a}\right)^{3/4}}{b^2}-\frac{2a^2\left(1+\frac{bx}{a}\right)^{7/4}}{b^2}+\frac{a^2\left(1+\frac{bx}{a}\right)^{11/4}}{b^2}\right)dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/4}}\\
&= \frac{2a^2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)}{7b^3}-\frac{4a\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^2}{11b^3}+\frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^3}{15b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.51

$$\int x^5\left(c\sqrt{a+bx^2}\right)^{3/2}dx=\frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)\left(32a^2-56abx^2+77b^2x^4\right)}{1155b^3}$$

```
[In] Integrate[x^5*(c*Sqrt[a + b*x^2])^(3/2),x]
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(32*a^2 - 56*a*b*x^2 + 77*b^2*x^4)
)/(1155*b^3)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{2(b\,x^2+a)(77b^2x^4-56ab\,x^2+32a^2)\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{1155b^3}$	47

```
[In] int(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/1155*(b*x^2+a)*(77*b^2*x^4-56*a*b*x^2+32*a^2)*(c*(b*x^2+a)^(1/2))^(3/2)/b
^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int x^5\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\,(77\,b^3cx^6+21\,ab^2cx^4-24\,a^2bcx^2+32\,a^3c)\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}c}}{1155\,b^3}$$

```
[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
[Out] 2/1155*(77*b^3*c*x^6 + 21*a*b^2*c*x^4 - 24*a^2*b*c*x^2 + 32*a^3*c)*sqrt(b*x
^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b^3
```

Sympy [A] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.14

$$\int x^5\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\begin{cases}\frac{64a^3\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{1155b^3}-\frac{16a^2x^2\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{385b^2}+\frac{2ax^4\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{55b}+\frac{2x^6\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{15} & \text{for } b\neq 0\\ \frac{x^6\left(\sqrt{a}c\right)^{\frac{3}{2}}}{6} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**5*(c*(b*x**2+a)**(1/2))**(3/2),x)
[Out] Piecewise((64*a**3*(c*sqrt(a + b*x**2))**(3/2)/(1155*b**3) - 16*a**2*x**2*(
c*sqrt(a + b*x**2))**(3/2)/(385*b**2) + 2*a*x**4*(c*sqrt(a + b*x**2))**(3/2
)/(55*b) + 2*x**6*(c*sqrt(a + b*x**2))**(3/2)/15, Ne(b, 0)), (x**6*(sqrt(a)
*c)**(3/2)/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2 \left(165 \left(\sqrt{bx^2+a}c\right)^{\frac{7}{2}} a^2c^4 - 210 \left(\sqrt{bx^2+a}c\right)^{\frac{11}{2}} ac^2 + 77 \left(\sqrt{bx^2+a}c\right)^{\frac{15}{2}}\right)}{1155\, b^3c^6}$$

```
[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
[Out] 2/1155*(165*(sqrt(b*x^2 + a)*c)^(7/2)*a^2*c^4 - 210*(sqrt(b*x^2 + a)*c)^(11
/2)*a*c^2 + 77*(sqrt(b*x^2 + a)*c)^(15/2))/(b^3*c^6)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07

$$\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2\, c^{\frac{3}{2}} \left(\frac{5 \left(21 \left(bx^2+a\right)^{\frac{11}{4}} - 66 \left(bx^2+a\right)^{\frac{7}{4}} a + 77 \left(bx^2+a\right)^{\frac{3}{4}} a^2\right) a}{b^2} + \frac{77 \left(bx^2+a\right)^{\frac{15}{4}} - 315 \left(bx^2+a\right)^{\frac{11}{4}} a + 495 \left(bx^2+\right.}{b^2}\right.}{1155\, b}$$

```
[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
[Out] 2/1155*c^(3/2)*(5*(21*(b*x^2 + a)^(11/4) - 66*(b*x^2 + a)^(7/4)*a + 77*(b*x
^2 + a)^(3/4)*a^2)*a/b^2 + (77*(b*x^2 + a)^(15/4) - 315*(b*x^2 + a)^(11/4)*
a + 495*(b*x^2 + a)^(7/4)*a^2 - 385*(b*x^2 + a)^(3/4)*a^3)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 18.94 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.86

$$\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \sqrt{c\sqrt{b\,x^2+a}} \left(\frac{2\,c\,x^6\,\sqrt{b\,x^2+a}}{15} + \frac{64\,a^3\,c\,\sqrt{b\,x^2+a}}{1155\,b^3} + \frac{2\,a\,c\,x^4\,\sqrt{b\,x^2+a}}{55\,b} - \frac{16\,a^2\,c\,x^2\,\sqrt{b\,x^2+a}}{385\,b^2}\right)$$

```
[In] int(x^5*(c*(a + b*x^2)^(1/2))^(3/2),x)
[Out] (c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^6*(a + b*x^2)^(1/2))/15 + (64*a^3*c*(a
+ b*x^2)^(1/2))/(1155*b^3) + (2*a*c*x^4*(a + b*x^2)^(1/2))/(55*b) - (16*a^2
*c*x^2*(a + b*x^2)^(1/2))/(385*b^2))
```

3.251 $\int x^3 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int x^3 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = -\frac{2a\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)}{7b^2} + \frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^2}{11b^2}$$

```
[Out] -2/7*a*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^2+2/11*(b*x^2+a)^2*(c*(b*x^2+a
)^(1/2))^(3/2)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1973, 272, 45}

$$\int x^3 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2(a+bx^2)^2 \left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^2} - \frac{2a(a+bx^2)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^2}$$

```
[In] Int[x^3*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (-2*a*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b^2) + (2*(c*Sqrt[a + b*x^2
])^(3/2)*(a + b*x^2)^2)/(11*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int x^3\left(1+\frac{bx^2}{a}\right)^{3/4}\,dx}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\\
&= \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\operatorname{Subst}\left(\int x\left(1+\frac{bx}{a}\right)^{3/4}\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/4}}\\
&= \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\operatorname{Subst}\left(\int\left(-\frac{a\left(1+\frac{bx}{a}\right)^{3/4}}{b}+\frac{a\left(1+\frac{bx}{a}\right)^{7/4}}{b}\right)\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/4}}\\
&= -\frac{2a\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)}{7b^2}+\frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^2}{11b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.62

$$\int x^3\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)(-4a+7bx^2)}{77b^2}$$

```
[In] Integrate[x^3*(c*Sqrt[a + b*x^2])^(3/2),x]
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(-4*a + 7*b*x^2))/(77*b^2)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{2\left(b\,x^{2}+a\right)\left(-7b\,x^{2}+4a\right)\left(c\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{77b^{2}}$	36

```
[In] int(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/77*(b*x^2+a)*(-7*b*x^2+4*a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int x^3\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\,(7\,b^2cx^4+3\,abcx^2-4\,a^2c)\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}c}}{77\,b^2}$$

```
[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
[Out] 2/77*(7*b^2*c*x^4 + 3*a*b*c*x^2 - 4*a^2*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2
+ a)*c)/b^2
```

Sympy [A] (verification not implemented)

Time = 3.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.32

$$\int x^3\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\begin{cases}-\frac{8a^2\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{77b^2}+\frac{6ax^2\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{77b}+\frac{2x^4\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{11} & \text{for } b\neq 0\\ \frac{x^4\left(\sqrt{a}c\right)^{\frac{3}{2}}}{4} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**3*(c*(b*x**2+a)**(1/2))**(3/2),x)
[Out] Piecewise((-8*a**2*(c*sqrt(a + b*x**2))**(3/2)/(77*b**2) + 6*a*x**2*(c*sqrt
(a + b*x**2))**(3/2)/(77*b) + 2*x**4*(c*sqrt(a + b*x**2))**(3/2)/11, Ne(b,
0)), (x**4*(sqrt(a)*c)**(3/2)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

$$\int x^3 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = -\frac{2\left(11\left(\sqrt{bx^2+a}c\right)^{\frac{7}{2}}ac^2 - 7\left(\sqrt{bx^2+a}c\right)^{\frac{11}{2}}\right)}{77\,b^2c^4}$$

```
[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/77*(11*(sqrt(b*x^2 + a)*c)^(7/2)*a*c^2 - 7*(sqrt(b*x^2 + a)*c)^(11/2))/(
b^2*c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int x^3 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2\left(\frac{11\left(3\left(bx^2+a\right)^{\frac{7}{4}}-7\left(bx^2+a\right)^{\frac{3}{4}}a\right)a}{b} + \frac{21\left(bx^2+a\right)^{\frac{11}{4}}-66\left(bx^2+a\right)^{\frac{7}{4}}a+77\left(bx^2+a\right)^{\frac{3}{4}}a^2}{b}\right)c^{\frac{3}{2}}}{231\,b}$$

```
[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/231*(11*(3*(b*x^2 + a)^(7/4) - 7*(b*x^2 + a)^(3/4)*a)*a/b + (21*(b*x^2 +
a)^(11/4) - 66*(b*x^2 + a)^(7/4)*a + 77*(b*x^2 + a)^(3/4)*a^2)/b)*c^(3/2)/b
```

Mupad [B] (verification not implemented)

Time = 18.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int x^3 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \sqrt{c\sqrt{b\,x^2+a}}\left(\frac{2\,c\,x^4\,\sqrt{b\,x^2+a}}{11} - \frac{8\,a^2\,c\,\sqrt{b\,x^2+a}}{77\,b^2} + \frac{6\,a\,c\,x^2\,\sqrt{b\,x^2+a}}{77\,b}\right)$$

```
[In] int(x^3*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

```
[Out] (c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^4*(a + b*x^2)^(1/2))/11 - (8*a^2*c*(a +
 b*x^2)^(1/2))/(77*b^2) + (6*a*c*x^2*(a + b*x^2)^(1/2))/(77*b))
```

3.252 $\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2c\sqrt{c\sqrt{a+bx^2}}\,(a+bx^2)^{3/2}}{7b}$$

```
[Out] 2/7*c*(b*x^2+a)^(3/2)*(c*(b*x^2+a)^(1/2))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1605, 15, 30}

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2c(a+bx^2)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b}$$

```
[In] Int[x*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \left(c\sqrt{x}\right)^{3/2}\,dx, x, a+bx^2\right)}{2b}\\ &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int x^{3/4}\,dx, x, a+bx^2\right)}{2b\sqrt[4]{a+bx^2}}\\ &= \frac{2c\sqrt{c\sqrt{a+bx^2}}\left(a+bx^2\right)^{3/2}}{7b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx = \frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)}{7b}$$

```
[In] Integrate[x*(c*Sqrt[a + b*x^2])^(3/2),x]
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{2(b\,x^2+a)\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{7b}$	26
derivativedivides	$\frac{2(b\,x^2+a)\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{7b}$	26
default	$\frac{2(b\,x^2+a)\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{7b}$	26

```
[In] int(x*(c*(b*x^2+a)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/7*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\,(bcx^2+ac)\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}c}}{7\,b}$$

```
[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
[Out] 2/7*(b*c*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b
```

Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\begin{cases}\frac{2a\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{7b}+\frac{2x^2\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{7} & \text{for } b\neq 0\\ \frac{x^2\left(\sqrt{a}c\right)^{\frac{3}{2}}}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(c*(b*x**2+a)**(1/2))**(3/2),x)
[Out] Piecewise((2*a*(c*sqrt(a + b*x**2))**(3/2)/(7*b) + 2*x**2*(c*sqrt(a + b*x**
2))**(3/2)/7, Ne(b, 0)), (x**2*(sqrt(a)*c)**(3/2)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\,(bx^2+a)\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}}{7\,b}$$

```
[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
[Out] 2/7*(b*x^2 + a)*(sqrt(b*x^2 + a)*c)^(3/2)/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.47

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\,(bx^2+a)^{\frac{7}{4}}c^{\frac{3}{2}}}{7\,b}$$

```
[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
[Out] 2/7*(b*x^2 + a)^(7/4)*c^(3/2)/b
```

Mupad [B] (verification not implemented)

Time = 18.87 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\,c\,(b\,x^2+a)^{3/2}\,\sqrt{c\,\sqrt{b\,x^2+a}}}{7\,b}$$

```
[In] int(x*(c*(a + b*x^2)^(1/2))^(3/2),x)
[Out] (2*c*(a + b*x^2)^(3/2)*(c*(a + b*x^2)^(1/2))^(1/2))/(7*b)
```

3.253 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 117

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \frac{2}{3}\left(c\sqrt{a+bx^2}\right)^{3/2} + \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\arctan\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}} - \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\operatorname{arctanh}\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

```
[Out] 2/3*(c*(b*x^2+a)^(1/2))^(3/2)+arctan((1+b*x^2/a)^(1/4))*(c*(b*x^2+a)^(1/2))
^(3/2)/(1+b*x^2/a)^(3/4)-arctanh((1+b*x^2/a)^(1/4))*(c*(b*x^2+a)^(1/2))^(3/
2)/(1+b*x^2/a)^(3/4)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1973, 272, 52, 65, 304, 209, 212}

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \frac{\arctan\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{\left(\frac{bx^2}{a}+1\right)^{3/4}} - \frac{\operatorname{arctanh}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{\left(\frac{bx^2}{a}+1\right)^{3/4}} + \frac{2}{3}\left(c\sqrt{a+bx^2}\right)^{3/2}$$

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x,x]
```

```
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2))/3 + ((c*Sqrt[a + b*x^2])^(3/2)*ArcTan[(1 + (b
*x^2)/a)^(1/4)])/(1 + (b*x^2)/a)^(3/4) - ((c*Sqrt[a + b*x^2])^(3/2)*ArcTanh
[(1 + (b*x^2)/a)^(1/4)])/(1 + (b*x^2)/a)^(3/4)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int\frac{\left(1+\frac{bx^2}{a}\right)^{3/4}}{x}\,dx}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\operatorname{Subst}\left(\int\frac{\left(1+\frac{bx}{a}\right)^{3/4}}{x}\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= \frac{2}{3}\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\operatorname{Subst}\left(\int\frac{1}{x\sqrt[4]{1+\frac{bx}{a}}}\,dx,x,x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= \frac{2}{3}\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{\left(2a\left(c\sqrt{a+bx^2}\right)^{3/2}\right)\operatorname{Subst}\left(\int\frac{x^2}{-\frac{a}{b}+\frac{ax^4}{b}}\,dx,x,\sqrt[4]{1+\frac{bx^2}{a}}\right)}{b\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= \frac{2}{3}\left(c\sqrt{a+bx^2}\right)^{3/2}-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\operatorname{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt[4]{1+\frac{bx^2}{a}}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$
$$+\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\operatorname{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt[4]{1+\frac{bx^2}{a}}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= \frac{2}{3}\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\tan^{-1}\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$
$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\tanh^{-1}\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.82

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(2(a+bx^2)^{3/4}+3a^{3/4}\arctan\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)-3a^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\right)}{3\left(a+bx^2\right)^{3/4}}$$

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x,x]
```

```
[Out] ((c*Sqrt[a + b*x^2])^(3/2)*(2*(a + b*x^2)^(3/4) + 3*a^(3/4)*ArcTan[(a + b*x
^2)^(1/4)/a^(1/4)] - 3*a^(3/4)*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)]))/(3*(a +
 b*x^2)^(3/4))
```

Maple [F]

$$\int \frac{\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{x}dx$$

```
[In] int((c*(b*x^2+a)^(1/2))^(3/2)/x,x)
```

```
[Out] int((c*(b*x^2+a)^(1/2))^(3/2)/x,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \text{Timed out}$$

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x,x)
```

```
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \frac{3\,ac^4\left(\frac{2\,\arctan\left(\frac{\sqrt{\sqrt{bx^2+a}c}}{\left(ac^2\right)^{\frac{1}{4}}}\right)}{\left(ac^2\right)^{\frac{1}{4}}}+\frac{\log\left(\frac{\sqrt{\sqrt{bx^2+a}c}-\left(ac^2\right)^{\frac{1}{4}}}{\sqrt{\sqrt{bx^2+a}c}+\left(ac^2\right)^{\frac{1}{4}}}\right)}{\left(ac^2\right)^{\frac{1}{4}}}\right)+4\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}c^2}{6\,c^2}$$

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*c^4*(2*arctan(sqrt(sqrt(b*x^2 + a)*c)/(a*c^2)^(1/4))/(a*c^2)^(1/4)
 + log((sqrt(sqrt(b*x^2 + a)*c) - (a*c^2)^(1/4))/(sqrt(sqrt(b*x^2 + a)*c) +
 (a*c^2)^(1/4)))/(a*c^2)^(1/4)) + 4*(sqrt(b*x^2 + a)*c)^(3/2)*c^2)/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.62

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = -\frac{1}{12}\left(6\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)+6\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(b\right.\right.}{2\left(-a\right)^{\frac{1}{4}}}\right.\right.$$

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/12*(6*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x
^2 + a)^(1/4))/(-a)^(1/4)) + 6*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-a)^(1/4) - 2*(b*x^2 + a)^(1/4))/(-a)^(1/4)) - 3*sqrt(2)*(-a)^(3/4)*lo
g(sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + sqrt(-a)) + 3*sq
rt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a
) + sqrt(-a)) - 8*(b*x^2 + a)^(3/4))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx=\int \frac{\left(c\,\sqrt{b\,x^2+a}\right)^{3/2}}{x}\,dx$$

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x,x)
[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x, x)
```

3.254 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 133

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{2x^2} + \frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2}\arctan\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{4a\left(1+\frac{bx^2}{a}\right)^{3/4}} - \frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2}\operatorname{arctanh}\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{4a\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

```
[Out] -1/2*(c*(b*x^2+a)^(1/2))^(3/2)/x^2+3/4*b*arctan((1+b*x^2/a)^(1/4))*(c*(b*x^
2+a)^(1/2))^(3/2)/a/(1+b*x^2/a)^(3/4)-3/4*b*arctanh((1+b*x^2/a)^(1/4))*(c*(
b*x^2+a)^(1/2))^(3/2)/a/(1+b*x^2/a)^(3/4)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1973, 272, 43, 65, 304, 209, 212}

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = \frac{3b\arctan\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{4a\left(\frac{bx^2}{a}+1\right)^{3/4}} - \frac{3b\text{arctanh}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{4a\left(\frac{bx^2}{a}+1\right)^{3/4}} - \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{2x^2}$$

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x^3,x]
```

```
[Out] -1/2*(c*Sqrt[a + b*x^2])^(3/2)/x^2 + (3*b*(c*Sqrt[a + b*x^2])^(3/2)*ArcTan[
(1 + (b*x^2)/a)^(1/4)])/(4*a*(1 + (b*x^2)/a)^(3/4)) - (3*b*(c*Sqrt[a + b*x^
2])^(3/2)*ArcTanh[(1 + (b*x^2)/a)^(1/4)])/(4*a*(1 + (b*x^2)/a)^(3/4))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{a+bx^2}\right)^{3/2} \int \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}}{x^3} \, dx}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= \frac{\left(c\sqrt{a+bx^2}\right)^{3/2} \operatorname{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{3/4}}{x^2} \, dx, x, x^2\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{2x^2} + \frac{\left(3b\left(c\sqrt{a+bx^2}\right)^{3/2}\right) \operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{1+\frac{bx}{a}}} \, dx, x, x^2\right)}{8a\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{2x^2} + \frac{\left(3\left(c\sqrt{a+bx^2}\right)^{3/2}\right) \operatorname{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{ax^4}{b}} \, dx, x, \sqrt[4]{1+\frac{bx^2}{a}}\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{2x^2} - \frac{\left(3b\left(c\sqrt{a+bx^2}\right)^{3/2}\right) \operatorname{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, \sqrt[4]{1+\frac{bx^2}{a}}\right)}{4a\left(1+\frac{bx^2}{a}\right)^{3/4}} + \frac{\left(3b\left(c\sqrt{a+bx^2}\right)^{3/2}\right) \operatorname{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \sqrt[4]{1+\frac{bx^2}{a}}\right)}{4a\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{2x^2} + \frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2} \tan^{-1}\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{4a\left(1+\frac{bx^2}{a}\right)^{3/4}} - \frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2} \tanh^{-1}\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{4a\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.80

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(2\sqrt[4]{a}(a+bx^2)^{3/4}-3bx^2\arctan\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)+3bx^2\text{arctanh}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\right)}{4\sqrt[4]{a}x^2\left(a+bx^2\right)^{3/4}}$$

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^3,x]
[Out] -1/4*((c*Sqrt[a + b*x^2])^(3/2)*(2*a^(1/4)*(a + b*x^2)^(3/4) - 3*b*x^2*ArcT
an[(a + b*x^2)^(1/4)/a^(1/4)] + 3*b*x^2*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)])
)/(a^(1/4)*x^2*(a + b*x^2)^(3/4))
```

Maple [F]

$$\int \frac{\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{x^3}dx$$

```
[In] int((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x)
[Out] int((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**3,x)
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.04

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = \frac{\left(3\,c^4\left(\frac{2\,\arctan\left(\frac{\sqrt{\sqrt{bx^2+ac}}}{\left(ac^2\right)^{\frac{1}{4}}}\right)}{\left(ac^2\right)^{\frac{1}{4}}} + \frac{\log\left(\frac{\sqrt{\sqrt{bx^2+ac}}-\left(ac^2\right)^{\frac{1}{4}}}{\sqrt{\sqrt{bx^2+ac}}+\left(ac^2\right)^{\frac{1}{4}}}\right)}{\left(ac^2\right)^{\frac{1}{4}}}\right) - \frac{4\left(\sqrt{bx^2+ac}\right)^{\frac{3}{2}}c^4}{(bx^2+a)c^2-ac^2}\right)b}{8\,c^2}$$

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*c^4*(2*arctan(sqrt(sqrt(b*x^2 + a)*c)/(a*c^2)^(1/4))/(a*c^2)^(1/4) +
 log((sqrt(sqrt(b*x^2 + a)*c) - (a*c^2)^(1/4))/(sqrt(sqrt(b*x^2 + a)*c) + (
a*c^2)^(1/4)))/(a*c^2)^(1/4)) - 4*(sqrt(b*x^2 + a)*c)^(3/2)*c^4/((b*x^2 + a
)*c^2 - a*c^2))*b/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.59

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = \frac{\left(\frac{6\,\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}} + \frac{6\,\sqrt{2}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}} + \frac{3\,\sqrt{2}(-\cdots}{\cdots}\right.}{\cdots}$$

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(6*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^2 + a)^
(1/4))/(-a)^(1/4))/(-a)^(1/4) + 6*sqrt(2)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*
(-a)^(1/4) - 2*(b*x^2 + a)^(1/4))/(-a)^(1/4))/(-a)^(1/4) + 3*sqrt(2)*(-a)^(
3/4)*b^2*log(sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + sqrt(
-a))/a + 3*sqrt(2)*b^2*log(-sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x
^2 + a) + sqrt(-a))/(-a)^(1/4) - 8*(b*x^2 + a)^(3/4)*b/x^2)*c^(3/2)/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(c\,\sqrt{b\,x^2+a}\right)^{3/2}}{x^3}\,dx$$

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x^3,x)
[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x^3, x)
```

3.255 $\int x^2\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 152

$$\int x^2\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b}+\frac{2}{9}x^3\left(c\sqrt{a+bx^2}\right)^{3/2}-\frac{4a^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b\,(a+bx^2)}+\frac{4a^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

```
[Out] 2/15*a*x*(c*(b*x^2+a)^(1/2))^(3/2)/b+2/9*x^3*(c*(b*x^2+a)^(1/2))^(3/2)-4/15
*a^2*x*(c*(b*x^2+a)^(1/2))^(3/2)/b/(b*x^2+a)+4/15*a^(3/2)*(cos(1/2*arctan(x
*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(si
n(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*(b*x^2+a)^(1/2))^(3/2)/b^(3/2)
/(1+b*x^2/a)^(3/4)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1973, 285, 327, 233, 202}

$$\int x^2\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{4a^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}}-\frac{4a^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b\,(a+bx^2)}+\frac{2ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b}+\frac{2}{9}x^3\left(c\sqrt{a+bx^2}\right)^{3/2}$$

```
[In] Int[x^2*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*a*x*(c*Sqrt[a + b*x^2])^(3/2))/(15*b) + (2*x^3*(c*Sqrt[a + b*x^2])^(3/2)
)/9 - (4*a^2*x*(c*Sqrt[a + b*x^2])^(3/2))/(15*b*(a + b*x^2)) + (4*a^(3/2)*(
c*Sqrt[a + b*x^2])^(3/2)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b
^(3/2)*(1 + (b*x^2)/a)^(3/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\,dx}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= \frac{2}{9}x^3\left(c\sqrt{a+bx^2}\right)^{3/2} + \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int\frac{x^2}{\sqrt[4]{1+\frac{bx^2}{a}}}dx}{3\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= \frac{2ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b} + \frac{2}{9}x^3\left(c\sqrt{a+bx^2}\right)^{3/2} - \frac{\left(2a\left(c\sqrt{a+bx^2}\right)^{3/2}\right)\int\frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}dx}{15b\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= \frac{2ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b} + \frac{2}{9}x^3\left(c\sqrt{a+bx^2}\right)^{3/2} - \frac{4a^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b\left(a+bx^2\right)} + \frac{\left(2a\left(c\sqrt{a+bx^2}\right)^{3/2}\right)\int\frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}dx}{15b\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= \frac{2ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b} + \frac{2}{9}x^3\left(c\sqrt{a+bx^2}\right)^{3/2} - \frac{4a^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b\left(a+bx^2\right)} + \frac{4a^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.45

$$\int x^2\left(c\sqrt{a+bx^2}\right)^{3/2}dx = \frac{2x\left(c\sqrt{a+bx^2}\right)^{3/2}\left(a+bx^2-\frac{a\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\right)}{9b}$$

```
[In] Integrate[x^2*(c*Sqrt[a + b*x^2])^(3/2),x]
[Out] (2*x*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2 - (a*Hypergeometric2F1[-3/4, 1/2,
 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(3/4)))/(9*b)
```

Maple [F]

$$\int x^{2} \left(c \sqrt{b \,x^{2}+a}\right)^{\frac{3}{2}}d x$$

```
[In] int(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x)
[Out] int(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x)
```

Fricas [F]

$$\int x^{2} \left(c\sqrt{a+bx^{2}}\right)^{3/2} dx = \int \left(\sqrt{bx^{2}+a}c\right)^{\frac{3}{2}} x^{2} dx$$

```
[In] integrate(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)*c*x^2, x)
```

Sympy [F]

$$\int x^{2} \left(c\sqrt{a+bx^{2}}\right)^{3/2} dx = \int x^{2} \left(c\sqrt{a+bx^{2}}\right)^{\frac{3}{2}} dx$$

```
[In] integrate(x**2*(c*(b*x**2+a)**(1/2))**(3/2),x)
[Out] Integral(x**2*(c*sqrt(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int x^{2} \left(c\sqrt{a+bx^{2}}\right)^{3/2} dx = \int \left(\sqrt{bx^{2}+a}c\right)^{\frac{3}{2}} x^{2} dx$$

```
[In] integrate(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(\sqrt{bx^2+ac}\right)^{\frac{3}{2}} x^2 \, dx$$

[In] integrate(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")

[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)*x^2, x)

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \int x^2 \left(c\sqrt{b\,x^2+a}\right)^{3/2} dx$$

[In] int(x^2*(c*(a + b*x^2)^(1/2))^(3/2),x)

[Out] int(x^2*(c*(a + b*x^2)^(1/2))^(3/2), x)

3.256 $\int \left(c\sqrt{a+bx^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 119

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2}{5}x\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{6ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{5\left(a+bx^2\right)}-\frac{6\sqrt{a}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

```
[Out] 2/5*x*(c*(b*x^2+a)^(1/2))^(3/2)+6/5*a*x*(c*(b*x^2+a)^(1/2))^(3/2)/(b*x^2+a)
-6/5*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/
a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*(c*
(b*x^2+a)^(1/2))^(3/2)/(1+b*x^2/a)^(3/4)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1973, 201, 233, 202}

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=-\frac{6\sqrt{a}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}}+\frac{2}{5}x\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{6ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{5\left(a+bx^2\right)}$$

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*x*(c*Sqrt[a + b*x^2])^(3/2))/5 + (6*a*x*(c*Sqrt[a + b*x^2])^(3/2))/(5*(a
 + b*x^2)) - (6*Sqrt[a]*(c*Sqrt[a + b*x^2])^(3/2)*EllipticE[ArcTan[(Sqrt[b]
*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*(1 + (b*x^2)/a)^(3/4))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int\left(1+\frac{bx^2}{a}\right)^{3/4}\,dx}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\\
&= \frac{2}{5}x\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{\left(3\left(c\sqrt{a+bx^2}\right)^{3/2}\right)\int\frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{5\left(1+\frac{bx^2}{a}\right)^{3/4}}\\
&= \frac{2}{5}x\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{6ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{5\left(a+bx^2\right)}-\frac{\left(3\left(c\sqrt{a+bx^2}\right)^{3/2}\right)\int\frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{5\left(1+\frac{bx^2}{a}\right)^{3/4}}\\
&= \frac{2}{5}x\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{6ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{5\left(a+bx^2\right)}-\frac{6\sqrt{a}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.44

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{x\left(c\sqrt{a+bx^2}\right)^{3/2} \text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2),x]
[Out] (x*(c*Sqrt[a + b*x^2])^(3/2)*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^2)/a)
])/(1 + (b*x^2)/a)^(3/4)
```

Maple [F]

$$\int \left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}} dx$$

```
[In] int((c*(b*x^2+a)^(1/2))^(3/2),x)
[Out] int((c*(b*x^2+a)^(1/2))^(3/2),x)
```

Fricas [F]

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}} dx$$

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)*c, x)
```

Sympy [F]

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2),x)
[Out] Integral((c*sqrt(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}} dx$$

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2), x)
```

Giac [F]

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}} dx$$

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(c\sqrt{b\,x^2+a}\right)^{3/2} dx$$

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2),x)
[Out] int((c*(a + b*x^2)^(1/2))^(3/2), x)
```

3.257 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx=-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}+\frac{3bx\left(c\sqrt{a+bx^2}\right)^{3/2}}{a+bx^2}-\frac{3\sqrt{b}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

```
[Out] -(c*(b*x^2+a)^(1/2))^(3/2)/x+3*b*x*(c*(b*x^2+a)^(1/2))^(3/2)/(b*x^2+a)-3*(c
os(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)
))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)*(c*(b*x^2+
a)^(1/2))^(3/2)/(1+b*x^2/a)^(3/4)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1973, 283, 233, 202}

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx=-\frac{3\sqrt{b}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(\frac{bx^2}{a}+1\right)^{3/4}}-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}+\frac{3bx\left(c\sqrt{a+bx^2}\right)^{3/2}}{a+bx^2}$$

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x^2,x]
```

```
[Out] -((c*Sqrt[a + b*x^2])^(3/2)/x) + (3*b*x*(c*Sqrt[a + b*x^2])^(3/2))/(a + b*x
^2) - (3*Sqrt[b]*(c*Sqrt[a + b*x^2])^(3/2)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqr
t[a]]/2, 2])/(Sqrt[a]*(1 + (b*x^2)/a)^(3/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int\frac{\left(1+\frac{bx^2}{a}\right)^{3/4}}{x^2}\,dx}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x} + \frac{\left(3b\left(c\sqrt{a+bx^2}\right)^{3/2}\right)\int\frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{2a\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x} + \frac{3bx\left(c\sqrt{a+bx^2}\right)^{3/2}}{a+bx^2} - \frac{\left(3b\left(c\sqrt{a+bx^2}\right)^{3/2}\right)\int\frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{2a\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x} + \frac{3bx\left(c\sqrt{a+bx^2}\right)^{3/2}}{a+bx^2} - \frac{3\sqrt{b}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.48

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^2,x]
[Out] -(((c*Sqrt[a + b*x^2])^(3/2)*Hypergeometric2F1[-3/4, -1/2, 1/2, -((b*x^2)/a
)])/(x*(1 + (b*x^2)/a)^(3/4)))
```

Maple [F]

$$\int \frac{\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{x^2}dx$$

```
[In] int((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x)
[Out] int((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x)
```

Fricas [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)*c/x^2, x)
```

Sympy [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**2,x)
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(c\,\sqrt{b\,x^2+a}\right)^{3/2}}{x^2}\,dx$$

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x^2,x)
[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x^2, x)
```

3.258 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{3x^3} - \frac{b\left(c\sqrt{a+bx^2}\right)^{3/2}}{2ax} + \frac{b^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{2a\,(a+bx^2)} - \frac{b^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

```
[Out] -1/3*(c*(b*x^2+a)^(1/2))^(3/2)/x^3-1/2*b*(c*(b*x^2+a)^(1/2))^(3/2)/a/x+1/2*
b^2*x*(c*(b*x^2+a)^(1/2))^(3/2)/a/(b*x^2+a)-1/2*b^(3/2)*(cos(1/2*arctan(x*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(
1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*(b*x^2+a)^(1/2))^(3/2)/a^(3/2)/(
1+b*x^2/a)^(3/4)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1973, 283, 331, 233, 202}

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = -\frac{b^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}} + \frac{b^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{2a\,(a+bx^2)} - \frac{b\left(c\sqrt{a+bx^2}\right)^{3/2}}{2ax} - \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{3x^3}$$

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x^4,x]
```

```
[Out] -1/3*(c*Sqrt[a + b*x^2])^(3/2)/x^3 - (b*(c*Sqrt[a + b*x^2])^(3/2))/(2*a*x)
+ (b^2*x*(c*Sqrt[a + b*x^2])^(3/2))/(2*a*(a + b*x^2)) - (b^(3/2)*(c*Sqrt[a
+ b*x^2])^(3/2)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(3/2)*(1
+ (b*x^2)/a)^(3/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int\frac{\left(1+\frac{bx^2}{a}\right)^{3/4}}{x^4}\,dx}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{3x^3} + \frac{\left(b\left(c\sqrt{a+bx^2}\right)^{3/2}\right)\int \frac{1}{x^2\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{2a\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{3x^3} - \frac{b\left(c\sqrt{a+bx^2}\right)^{3/2}}{2ax} + \frac{\left(b^2\left(c\sqrt{a+bx^2}\right)^{3/2}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{4a^2\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{3x^3} - \frac{b\left(c\sqrt{a+bx^2}\right)^{3/2}}{2ax} + \frac{b^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{2a\left(a+bx^2\right)} - \frac{\left(b^2\left(c\sqrt{a+bx^2}\right)^{3/2}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{4a^2\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

$$= -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{3x^3} - \frac{b\left(c\sqrt{a+bx^2}\right)^{3/2}}{2ax} + \frac{b^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{2a\left(a+bx^2\right)} - \frac{b^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2} E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^4,x]
[Out] -1/3*((c*Sqrt[a + b*x^2])^(3/2)*Hypergeometric2F1[-3/2, -3/4, -1/2, -((b*x^
2)/a)])/(x^3*(1 + (b*x^2)/a)^(3/4))
```

Maple [F]

$$\int \frac{\left(c\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{x^{4}}dx$$

```
[In] int((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x)
[Out] int((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   Shoul
dn not  happen
```

Sympy [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**4,x)
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="giac")
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(c\sqrt{b\,x^2+a}\right)^{3/2}}{x^4}\,dx$$

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x^4,x)
[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x^4, x)
```

3.259 $\int \sqrt{(b-x)(-a+x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \sqrt{(b-x)(-a+x)}\,dx = -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} - \frac{1}{8}(a-b)^2\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

```
[Out] -1/8*(a-b)^2*arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))-1/4*(a+b-2*x)*(
-a*b+(a+b)*x-x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1976, 626, 635, 210}

$$\int \sqrt{(b-x)(-a+x)}\,dx = -\frac{1}{8}(a-b)^2\arctan\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right) - \frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}$$

```
[In] Int[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -1/4*((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2]) - ((a - b)^2*ArcTan[(a
+ b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \sqrt{-ab+(a+b)x-x^2}\,dx \\
&= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}+\frac{1}{8}(a-b)^2\int \frac{1}{\sqrt{-ab+(a+b)x-x^2}}\,dx \\
&= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} \\
&\quad +\frac{1}{4}(a-b)^2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,\frac{a+b-2x}{\sqrt{-ab+(a+b)x-x^2}}\right) \\
&= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \sqrt{(b-x)(-a+x)}\,dx = \frac{1}{4}\sqrt{(a-x)(-b+x)}\left(-a-b+2x+\frac{(a-b)^2\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{b-x}}\right)}{\sqrt{b-x}\sqrt{-a+x}}\right)$$

```
[In] Integrate[Sqrt[(b - x)*(-a + x)],x]
[Out] (Sqrt[(a - x)*(-b + x)]*(-a - b + 2*x + ((a - b)^2*ArcTan[Sqrt[-a + x]/Sqrt
[b - x]])/(Sqrt[b - x]*Sqrt[-a + x])))/4
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{(a+b-2x)\sqrt{-ab+(a+b)x-x^2}}{4}-\frac{\left(4ab-(a+b)^2\right)\arctan\left(\frac{x-\frac{a}{2}-\frac{b}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)}{8}$	68
risch	$\frac{(b-x)(a-x)(a+b-2x)}{4\sqrt{-(-b+x)(-a+x)}}-\left(\frac{1}{4}ab-\frac{1}{8}a^2-\frac{1}{8}b^2\right)\arctan\left(\frac{x-\frac{a}{2}-\frac{b}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)$	78

```
[In] int(((b-x)*(-a+x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(a+b-2*x)*(-a*b+(a+b)*x-x^2)^(1/2)-1/8*(4*a*b-(a+b)^2)*arctan((x-1/2*a
-1/2*b)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\int \sqrt{(b-x)(-a+x)}\,dx = -\frac{1}{8}\left(a^2-2\,ab+b^2\right)\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)}{2\left(ab-(a+b)x+x^2\right)}\right) - \frac{1}{4}\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)$$

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
[Out] -1/8*(a^2 - 2*a*b + b^2)*arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b -
2*x)/(a*b - (a + b)*x + x^2)) - 1/4*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2
*x)
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.61

$$\int \sqrt{(b-x)(-a+x)}\,dx = \left(-\frac{ab}{2} + \frac{\left(\frac{a}{4}+\frac{b}{4}\right)(a+b)}{2}\right)\left(\begin{cases} -i\log\left(a+b-2x+2i\sqrt{-ab-x^2+x(a+b)}\right) & \text{for } ab-\frac{(a+b)^2}{4}\neq 0 \\ \frac{\left(-\frac{a}{2}-\frac{b}{2}+x\right)\log\left(-\frac{a}{2}-\frac{b}{2}+x\right)}{\sqrt{-\left(-\frac{a}{2}-\frac{b}{2}+x\right)^2}} & \text{otherwise} \end{cases}\right) + \left(-\frac{a}{4}-\frac{b}{4}+\frac{x}{2}\right)\sqrt{-ab-x^2+x(a+b)}$$

```
[In] integrate(((b-x)*(-a+x))**(1/2),x)
```

```
[Out] (-a*b/2 + (a/4 + b/4)*(a + b)/2)*Piecewise((-I*log(a + b - 2*x + 2*I*sqrt(-
a*b - x**2 + x*(a + b))), Ne(a*b - (a + b)**2/4, 0)), ((-a/2 - b/2 + x)*log
(-a/2 - b/2 + x)/sqrt(-(-a/2 - b/2 + x)**2), True)) + (-a/4 - b/4 + x/2)*sq
rt(-a*b - x**2 + x*(a + b))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{(b-x)(-a+x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \sqrt{(b-x)(-a+x)}\,dx = \frac{1}{8}\left(a^2-2\,ab+b^2\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\operatorname{sgn}(-a+b) - \frac{1}{4}\sqrt{-ab+ax+bx-x^2}(a+b-2\,x)$$

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqr
t(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{(b-x)(-a+x)}\,dx = \int \sqrt{-\left(a-x\right)\,\left(b-x\right)}\,dx$$

```
[In] int((-(a - x)*(b - x))^(1/2),x)
```

```
[Out] int((-(a - x)*(b - x))^(1/2), x)
```

3.260 $\int \sqrt{(1-x^2)(3+x^2)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \frac{1}{3}x\sqrt{3-2x^2-x^4} - \frac{2E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)}{\sqrt{3}} + \frac{4\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] -2/3*EllipticE(x,1/3*I*3^(1/2))*3^(1/2)+4/3*EllipticF(x,1/3*I*3^(1/2))*3^(1
/2)+1/3*x*(-x^4-2*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1976, 1105, 1194, 538, 435, 430}

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \frac{4\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{\sqrt{3}} - \frac{2E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)}{\sqrt{3}} + \frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

```
[In] Int[Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4])/3 - (2*EllipticE[ArcSin[x], -1/3])/Sqrt[3] + (4*E
llipticF[ArcSin[x], -1/3])/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \int \sqrt{3-2x^2-x^4}\,dx$$

$$= \frac{1}{3}x\sqrt{3-2x^2-x^4} + \frac{1}{3}\int \frac{6-2x^2}{\sqrt{3-2x^2-x^4}}\,dx$$

$$= \frac{1}{3}x\sqrt{3-2x^2-x^4} + \frac{2}{3}\int \frac{6-2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx$$

$$= \frac{1}{3}x\sqrt{3-2x^2-x^4} - \frac{2}{3}\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx + 8\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx$$

$$= \frac{1}{3}x\sqrt{3-2x^2-x^4} - \frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}} + \frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \frac{1}{3}\left(x\sqrt{3-2x^2-x^4} - 2iE\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right) - 4i\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right),-3\right)\right)$$

```
[In] Integrate[Sqrt[(1 - x^2)*(3 + x^2)],x]
[Out] (x*Sqrt[3 - 2*x^2 - x^4] - (2*I)*EllipticE[I*ArcSinh[x/Sqrt[3]], -3] - (4*I
)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/3
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(44) = 88.

Time = 1.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.38

method	result	size
default	$\frac{x\sqrt{-x^4-2x^2+3}}{3}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,F\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(F\left(x,\frac{i\sqrt{3}}{3}\right)-E\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{3\sqrt{-x^4-2x^2+3}}$	114
elliptic	$\frac{x\sqrt{-x^4-2x^2+3}}{3}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,F\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(F\left(x,\frac{i\sqrt{3}}{3}\right)-E\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{3\sqrt{-x^4-2x^2+3}}$	114
risch	$-\frac{x(x^2-1)(x^2+3)}{3\sqrt{-(x^2-1)(x^2+3)}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,F\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(F\left(x,\frac{i\sqrt{3}}{3}\right)-E\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{3\sqrt{-x^4-2x^2+3}}$	124

```
[In] int(((-x^2+1)*(x^2+3))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x*(-x^4-2*x^2+3)^(1/2)+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3
)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4
-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \frac{2i\,xE\left(\arcsin\left(\frac{1}{x}\right)\mid -3\right) + 4i\,xF\left(\arcsin\left(\frac{1}{x}\right)\mid -3\right) + \sqrt{-x^4-2\,x^2+3}(x^2+2)}{3\,x}$$

```
[In] integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="fricas")
[Out] 1/3*(2*I*x*elliptic_e(arcsin(1/x), -3) + 4*I*x*elliptic_f(arcsin(1/x), -3)
+ sqrt(-x^4 - 2*x^2 + 3)*(x^2 + 2))/x
```

Sympy [F]

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \int \sqrt{(1-x^2)(x^2+3)}\,dx$$

```
[In] integrate(((-x**2+1)*(x**2+3))**(1/2),x)
[Out] Integral(sqrt((1 - x**2)*(x**2 + 3)), x)
```

Maxima [F]

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \int \sqrt{-(x^2+3)(x^2-1)}\,dx$$

```
[In] integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Giac [F]

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \int \sqrt{-(x^2+3)(x^2-1)}\,dx$$

```
[In] integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \int \sqrt{-(x^2-1)\,(x^2+3)}\,dx$$

```
[In] int((-(x^2 - 1)*(x^2 + 3))^(1/2),x)
[Out] int((-(x^2 - 1)*(x^2 + 3))^(1/2), x)
```

3.261 $\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = -\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

```
[Out] -arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1976, 635, 210}

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = -\arctan\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

```
[In] Int[1/Sqrt[(b - x)*(-a + x)],x]
[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

<u>Rule 635</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{-ab+(a+b)x-x^2}}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{a+b-2x}{\sqrt{-ab+(a+b)x-x^2}}\right)\\ &= -\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \frac{2\sqrt{b-x}\sqrt{-a+x}\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{b-x}}\right)}{\sqrt{(a-x)(-b+x)}}$$

```
[In] Integrate[1/Sqrt[(b - x)*(-a + x)],x]
[Out] (2*Sqrt[b - x]*Sqrt[-a + x]*ArcTan[Sqrt[-a + x]/Sqrt[b - x]])/Sqrt[(a - x)*
(-b + x)]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

method	result	size
default	$\arctan\left(\frac{x-\frac{a}{2}-\frac{b}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)$	28

```
[In] int(1/((b-x)*(-a+x))^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan((x-1/2*a-1/2*b)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = -\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)}{2\,(ab-(a+b)x+x^2)}\right)$$

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
[Out] -arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2*x)/(a*b - (a + b)*x +
x^2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \begin{cases} -i\log\left(a+b-2x+2i\sqrt{-ab-x^2+x\,(a+b)}\right) & \text{for } ab-\frac{(a+b)^2}{4}\neq 0 \\ \frac{\left(-\frac{a}{2}-\frac{b}{2}+x\right)\log\left(-\frac{a}{2}-\frac{b}{2}+x\right)}{\sqrt{-\left(-\frac{a}{2}-\frac{b}{2}+x\right)^2}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/((b-x)*(-a+x))**(1/2),x)
[Out] Piecewise((-I*log(a + b - 2*x + 2*I*sqrt(-a*b - x**2 + x*(a + b))), Ne(a*b
- (a + b)**2/4, 0)), ((-a/2 - b/2 + x)*log(-a/2 - b/2 + x)/sqrt(-(-a/2 - b/
2 + x)**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(28) = 56$.

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \frac{1}{8}\left(a^2-2\,ab+b^2\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\mathrm{sgn}(-a+b) - \frac{1}{4}\sqrt{-ab+ax+bx-x^2}(a+b-2\,x)$$

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqr
t(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \int \frac{1}{\sqrt{-\left(a-x\right)\,\left(b-x\right)}}\,dx$$

```
[In] int(1/(-(a - x)*(b - x))^(1/2),x)
```

```
[Out] int(1/(-(a - x)*(b - x))^(1/2), x)
```

3.262 $\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}} \, dx = \frac{\text{EllipticF}\left(\arcsin(x), -\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1976, 1109, 430}

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}} \, dx = \frac{\text{EllipticF}\left(\arcsin(x), -\frac{1}{3}\right)}{\sqrt{3}}$$

```
[In] Int[1/Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
```

```
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx\\ &= 2\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx\\ &= \frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = -i\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right),-3\right)$$

```
[In] Integrate[1/Sqrt[(1 - x^2)*(3 + x^2)],x]
[Out] (-I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(13) = 26.

Time = 0.50 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.58

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,F\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$	43
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,F\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$	43

```
[In] int(1/((-x^2+1)*(x^2+3))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3
^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \frac{1}{3}\sqrt{3}F(\arcsin(x)\mid -\frac{1}{3})$$

```
[In] integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*elliptic_f(arcsin(x), -1/3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \int \frac{1}{\sqrt{(1-x^2)(x^2+3)}}\,dx$$

```
[In] integrate(1/((-x**2+1)*(x**2+3))**(1/2),x)
[Out] Integral(1/sqrt((1 - x**2)*(x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \int \frac{1}{\sqrt{-(x^2+3)(x^2-1)}}\,dx$$

```
[In] integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \int \frac{1}{\sqrt{-(x^2+3)(x^2-1)}}\,dx$$

```
[In] integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \int \frac{1}{\sqrt{-(x^2-1)\,(x^2+3)}}\,dx$$

```
[In] int(1/(-(x^2 - 1)*(x^2 + 3))^(1/2),x)
[Out] int(1/(-(x^2 - 1)*(x^2 + 3))^(1/2), x)
```

3.263 $\int x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 244

$$\int x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \frac{(11b^2c^2-2abcd-a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{16b^2d^3} - \frac{(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{8bd^3} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)^3}{6bd^2e} - \frac{(bc-ad)(5b^2c^2+2abcd+a^2d^2)\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{5/2}d^{7/2}}$$

```
[Out] 1/6*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)^3/b/d^2/e-1/16*(-a*d+b*c)*(a^2*
d^2+2*a*b*c*d+5*b^2*c^2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1
/2)/e^(1/2))*e^(1/2)/b^(5/2)/d^(7/2)+1/16*(-a^2*d^2-2*a*b*c*d+11*b^2*c^2)*(
d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/d^3-1/8*(a*d+3*b*c)*(d*x^2+c)^2*
(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used

$= \{1981, 1980, 474, 466, 393, 214\}$

$$\int x^5\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = -\frac{\sqrt{e}(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{5/2}d^{7/2}} + \frac{\left(c+dx^2\right)\left(-a^2d^2-2abcd+11b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^2d^3} - \frac{\left(c+dx^2\right)^2(ad+3bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8bd^3} + \frac{\left(c+dx^2\right)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6bd^2e}$$

```
[In] Int[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((11*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c +
d*x^2))/(16*b^2*d^3) - ((3*b*c + a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c
+ d*x^2)^2)/(8*b*d^3) + (((e*(a + b*x^2))/(c + d*x^2))^(3/2)*(c + d*x^2)^3)
/(6*b*d^2*e) - ((b*c - a*d)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[e]*ArcTa
nh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(5
/2)*d^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{\frac{e(a+bx)}{c+dx}}\,dx,x,x^2\right)\\
&= ((bc-ad)e)\text{Subst}\left(\int \frac{x^2(-ae+cx^2)^2}{(be-dx^2)^4}\,dx,x,\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)\\
&= \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)^3}{6bd^2e}\\
&\quad-\frac{(bc-ad)\text{Subst}\left(\int \frac{x^2(-3(2a^2d^2e^2-(bce-ade)^2)+6bc^2dex^2)}{(be-dx^2)^3}\,dx,x,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{6bd^2}\\
&= -\frac{(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{8bd^3}+\frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)^3}{6bd^2e}\\
&\quad+\frac{(bc-ad)\text{Subst}\left(\int \frac{3d(bc-ad)(3bc+ad)e^2+24bc^2d^2ex^2}{(be-dx^2)^2}\,dx,x,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{24bd^4}
\end{aligned}$$

$$= \frac{(11b^2c^2 - 2abcd - a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{16b^2d^3} - \frac{(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{8bd^3} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)^3}{6bd^2e} - \frac{((bc-ad)(5b^2c^2+2abcd+a^2d^2)e)\operatorname{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{16b^2d^3}$$

$$= \frac{(11b^2c^2 - 2abcd - a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{16b^2d^3} - \frac{(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{8bd^3} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)^3}{6bd^2e} - \frac{(bc-ad)(5b^2c^2+2abcd+a^2d^2)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{5/2}d^{7/2}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.81

$$\int x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-b\sqrt{d}(c+dx^2)(3a^2d^2 - 2abd(-2c+dx^2) + b^2(-15c^2+10cdx^2-8d^2x^4)) - \frac{3(bc-ad)^{3/2}(5b^2c^2+2a}{}\right.}{48b^3d^{7/2}}$$

```
[In] Integrate[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(b*Sqrt[d]*(c + d*x^2)*(3*a^2*d^2 - 2*
a*b*d*(-2*c + d*x^2) + b^2*(-15*c^2 + 10*c*d*x^2 - 8*d^2*x^4))) - (3*(b*c -
 a*d)^(3/2)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a
*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a + b*x^2]))/
(48*b^3*d^(7/2))
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{\left(-8b^2d^2x^4-2ab\,d^2x^2+10b^2cd\,x^2+3a^2d^2+4abcd-15b^2c^2\right)\left(d\,x^2+c\right)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{48b^2d^3}+\frac{\left(a^3d^3+a^2bc\,d^2+3d\,c^2b^2a-5b^3c^3\right)\ln\left(\frac{\frac{1}{2}eda+\cdots}{\cdots}\right.}{\cdots}$
default	$\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\left(d\,x^2+c\right)\left(-12\sqrt{bd}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,ab\,d^2x^2-36\sqrt{bd}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,b^2cd\,x^2+3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+a\cdots}}{\cdots}\right.\right.}{\cdots}$

```
[In] int(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(-8*b^2*d^2*x^4-2*a*b*d^2*x^2+10*b^2*c*d*x^2+3*a^2*d^2+4*a*b*c*d-15*b
^2*c^2)*(d*x^2+c)/b^2/d^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/32*(a^3*d^3+a^2*b
*c*d^2+3*a*b^2*c^2*d-5*b^3*c^3)/b^2/d^3*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)/
(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)*(e*(
b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.22

$$\int x^5\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx$$

$$=\left[-\frac{3\left(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3\right)\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)\cdots\right.}{\cdots}\right.$$

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(e/(b*d)
)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d
+ a^2*d^2)*e + 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*
b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) - 4*(8*b^2*d^3
*x^6 + 15*b^2*c^3 - 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*(b^2*c*d^2 - a*b*d^3)*x^4
 + (5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2
 + c)))/(b^2*d^3), 1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d
^3)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(8*b^2*d^3*x^6 + 15*b^2*c^
3 - 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*(b^2*c*d^2 - a*b*d^3)*x^4 + (5*b^2*c^2*d
- 2*a*b*c*d^2 - 3*a^2*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d^3
)]
```

Sympy [F(-1)]

Timed out.

$$\int x^5\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx=\text{Timed out}$$

```
[In] integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^5\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.93

$$\int x^5\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$
$$=\frac{1}{96}\left(2\sqrt{bdex^4+bcex^2+adex^2+ace}\left(2\,x^2\left(\frac{4\,x^2}{d}-\frac{5\,b^2cd-abd^2}{b^2d^3}\right)+\frac{15\,b^2c^2-4\,abcd-3\,a^2d^2}{b^2d^3}\right)+\frac{3\,(5\,b^3\right.$$
$$\left.+c\right)$$

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] 1/96*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)*(2*x^2*(4*x^2/d - (
5*b^2*c*d - a*b*d^2)/(b^2*d^3)) + (15*b^2*c^2 - 4*a*b*c*d - 3*a^2*d^2)/(b^2
*d^3)) + 3*(5*b^3*c^3*e - 3*a*b^2*c^2*d*e - a^2*b*c*d^2*e - a^3*d^3*e)*log(
abs(-b*c*e - a*d*e - 2*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*
e*x^2 + a*d*e*x^2 + a*c*e))))/(sqrt(b*d*e)*b^2*d^3))*sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx=\int x^5\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

```
[In] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.264 $\int x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 161

$$\int x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = -\frac{(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8bd^2}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4d^2}+\frac{(bc-ad)(3bc+ad)\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{3/2}d^{5/2}}$$

```
[Out] 1/8*(-a*d+b*c)*(a*d+3*b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^
(1/2)/e^(1/2))*e^(1/2)/b^(3/2)/d^(5/2)-1/8*(-a*d+5*b*c)*(d*x^2+c)*(e*(b*x^2
+a)/(d*x^2+c))^(1/2)/b/d^2+1/4*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^
2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1981, 1980, 466, 393, 214}

$$\int x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \frac{\sqrt{e}(bc-ad)(ad+3bc)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{3/2}d^{5/2}}+\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^2}-\frac{(c+dx^2)(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8bd^2}$$

```
[In] Int[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] -1/8*((5*b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(b*d^2)
+ (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*d^2) + ((b*c - a*d)*
(3*b*c + a*d)*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(
Sqrt[b]*Sqrt[e])])/(8*b^(3/2)*d^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x\sqrt{\frac{e(a+bx)}{c+dx}}\,dx, x, x^2\right)\\
&= ((bc-ad)e)\text{Subst}\left(\int \frac{x^2(-ae+cx^2)}{(be-dx^2)^3}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)\\
&= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4d^2} - \frac{((bc-ad)e)\text{Subst}\left(\int \frac{(bc-ad)e+4cdx^2}{(be-dx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{4d^2}\\
&= -\frac{(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8bd^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4d^2}\\
&\quad + \frac{((bc-ad)(3bc+ad)e)\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{8bd^2}\\
&= -\frac{(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8bd^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4d^2}\\
&\quad + \frac{(bc-ad)(3bc+ad)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{3/2}d^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00

$$
\int x^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{b}\sqrt{d}\sqrt{a+bx^2}(c+dx^2)\,(-3bc+ad+2bdx^2)+(3b^2c^2-2abcd-a^2d^2)\,\sqrt{c+dx^2}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{8b^{3/2}d^{5/2}\sqrt{a+bx^2}}
$$

```
[In] Integrate[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*
x^2)*(-3*b*c + a*d + 2*b*d*x^2) + (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[c
+ d*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]))/(8*
b^(3/2)*d^(5/2)*Sqrt[a + b*x^2])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.17

method	result
risch	$\frac{(2bd\,x^2+da-3bc)(d\,x^2+c)\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{8b\,d^2}-\frac{(a^2d^2+2abcd-3b^2c^2)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^2}{\sqrt{bde}}+\sqrt{bde\,x^4+(eda+ebc)x^2+ace}\right)\sqrt{\frac{e(b\,x^2+a}{d\,x^2+c}}}{16b\,d^2\sqrt{bde}\,(b\,x^2+a)}$
default	$\frac{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)\left(4\sqrt{bd}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,bd\,x^2-\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+da+bc}{2\sqrt{bd}}\right)a^2d^2-2\ln\left(\frac{2bd\,x^2+2\sqrt{}}{}\right.\right.}{}$

```
[In] int(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*b*d*x^2+a*d-3*b*c)*(d*x^2+c)/b/d^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/1
6*(a^2*d^2+2*a*b*c*d-3*b^2*c^2)/b/d^2*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)/(b
*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)*(e*(b*
x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.53

$$\int x^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$

$$=\left[-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)e-4\,(2\,b^2d^3x\right.}{}\right.$$

$$-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\sqrt{-\frac{e}{bd}}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{bd}}}{2\,(bex^2+ae)}\right)-2\,(2\,bd^2x^4-3\,bc^2+acd-(bcd-}{16\,bd^2}$$

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^4
 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2*b
^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((b*e
*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) - 4*(2*b*d^2*x^4 - 3*b*c^2 + a*c*d
```

```
- (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^2), -1/16*((
3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c
 + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) -
 2*(2*b*d^2*x^4 - 3*b*c^2 + a*c*d - (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c)))/(b*d^2)]
```

Sympy [F(-1)]

Timed out.

$$\int x^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.05

$$\int x^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \frac{1}{16}\left(2\sqrt{bdex^4+bcex^2+adex^2+ace}\left(\frac{2\,x^2}{d}-\frac{3\,bc-ad}{bd^2}\right)-\frac{(3\,b^2c^2e-2\,abcde-a^2d^2e)\log\left(\left|-bce-ade\right.\right.}{}\right.$$
$$\left.+c\right)$$

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)*(2*x^2/d - (3*b*c -
 a*d)/(b*d^2)) - (3*b^2*c^2*e - 2*a*b*c*d*e - a^2*d^2*e)*log(abs(-b*c*e - a
*d*e - 2*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*
x^2 + a*c*e))))/(sqrt(b*d*e)*b*d^2))*sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=\int x^3\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

```
[In] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.265 $\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 103

$$\int x\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{2d}-\frac{(bc-ad)\sqrt{e}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}d^{3/2}}$$

```
[Out] -1/2*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/
2))*e^(1/2)/d^(3/2)/b^(1/2)+1/2*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1981, 1979, 294, 214}

$$\int x\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx=\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d}-\frac{\sqrt{e}(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}d^{3/2}}$$

```
[In] Int[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(2*d) - ((b*c - a*d)*Sqrt[e
]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(
2*Sqrt[b]*d^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \sqrt{\frac{e(a+bx)}{c+dx}}\,dx, x, x^2\right)\\
&= ((bc-ad)e)\text{Subst}\left(\int \frac{x^2}{(be-dx^2)^2}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)\\
&= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{2d} - \frac{((bc-ad)e)\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2d}\\
&= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{2d} - \frac{(bc-ad)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}d^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.28

$$\int x\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{b}\sqrt{d}\sqrt{a+bx^2}(c+dx^2)-(bc-ad)\sqrt{c+dx^2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{2\sqrt{b}d^{3/2}\sqrt{a+bx^2}}$$

```
[In] Integrate[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*
x^2) - (b*c - a*d)*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[
b]*Sqrt[c + d*x^2])]))/(2*Sqrt[b]*d^(3/2)*Sqrt[a + b*x^2])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

method	result
risch	$\frac{(d\,x^2+c)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{2d}+\frac{(da-bc)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^2}{\sqrt{bde}}+\sqrt{bde\,x^4+(eda+ebc)x^2+ace}\right)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,\sqrt{(d\,x^2+c)e(b\,x^2+a)}}{4d\sqrt{bde}\,(b\,x^2+a)}$
default	$\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)\left(a\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+da+bc}{2\sqrt{bd}}\right)d-b\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+da+bc}{2\sqrt{bd}}\right)c+2\sqrt{bd}\cdots}{4\sqrt{(d\,x^2+c)(b\,x^2+a)}\,d\sqrt{bd}}$

```
[In] int(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d+1/4*(a*d-b*c)/d*ln((1/2*e*d*a
+1/2*e*b*c+b*d*e*x^2)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/
2))/(b*d*e)^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/
2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.04

$$\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$$
$$=\left[-\frac{(bc-ad)\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)e+4\,(2\,b^2d^3x^4+b^2c^2d+ab\right.}{8\,d}\right.$$

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*c - a*d)*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2
)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b^2*d^3*x^4 + b^2*c^2*d
+ a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
)*sqrt(e/(b*d))) - 4*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d, 1/4*
((b*c - a*d)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(d*x^2 + c)*sqrt(
(b*e*x^2 + a*e)/(d*x^2 + c)))/d]
```

Sympy [F(-1)]

Timed out.

$$\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx=\text{Timed out}$$

```
[In] integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34

$$\int x\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$
$$=\frac{1}{4}\left(\frac{2\sqrt{bdex^4+bcex^2+adex^2+ace}}{d}+\frac{(bce-ade)\sqrt{bde}\log\left(\left|-2\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+\right.\right.\right.}{bd^2e}\right.$$
$$+c)$$

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)/d + (b*c*e - a*d*e)*
sqrt(b*d*e)*log(abs(-2*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*
e*x^2 + a*c*e))*b*d - sqrt(b*d*e)*b*c - sqrt(b*d*e)*a*d))/(b*d^2*e))*sgn(d*
x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx=\int x\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

```
[In] int(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.266 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 112

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = -\frac{\sqrt{a}\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}} + \frac{\sqrt{b}\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}}$$

```
[Out] -arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))*a^(1/2)*e^(
1/2)/c^(1/2)+arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))
*b^(1/2)*e^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1981, 1980, 492, 214}

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \frac{\sqrt{b}\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}} - \frac{\sqrt{a}\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}}$$

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]
```

```
[Out] -((Sqrt[a]*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqr
t[a]*Sqrt[e])])/Sqrt[c]) + (Sqrt[b]*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b
*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/Sqrt[d]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{\frac{e(a+bx)}{c+dx}}}{x}\,dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{x^2}{(-ae+cx^2)\,(be-dx^2)}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)$$

$$= (ae)\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right) + (be)\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)$$

$$= -\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}} + \frac{\sqrt{b}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx$$
$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\left(-\sqrt{a}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{b}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}}$$

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-(Sqrt[a]*Sqrt[d]*ArcTa
nh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]) + Sqrt[b]*Sqrt[c]*
ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]))/(Sqrt[c]*Sqr
t[d]*Sqrt[a + b*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(84) = 168$.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.60

method	result
default	$-\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)\left(a\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}+2ac}{x^2}\right)\sqrt{bd}-\sqrt{ac}\,\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+d}{2\sqrt{bd}}\right.\right.}{2\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{bd}\,\sqrt{ac}}$

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(a*ln((a*d*x^2+b*c*x^2+2*(a*c)
^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*(b*d)^(1/2)-(a*c)^(1
/2)*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+d*a
+b*c)/(b*d)^(1/2))*b)/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 865, normalized size of antiderivative = 7.72

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \left[\frac{1}{4}\sqrt{\frac{be}{d}}\log\left(8\,b^2d^2ex^4 + 8\,(b^2cd + abd^2)ex^2 + (b^2c^2 + 6\,abcd + a^2d^2)e + 4\,(2\,bd^3x^4 + bc^2d + acd^2 + (3\,bcd^2 + ad^3)x^2)\sqrt{\frac{be}{d}}\sqrt{\frac{bex^2 + ae}{dx^2 + c}}\right) + \frac{1}{4}\sqrt{\frac{ae}{c}}\log\left(\frac{(b^2c^2 + 6\,abcd + a^2d^2)ex^4 + 8\,a^2c^2e + 8\,(abc^2 + a^2cd)ex^2 - 4\,((bc^2d + acd^2)x^4 + 2\,ac^3 + (bc\cdots}{x^4}\right) - \frac{1}{2}\sqrt{-\frac{be}{d}}\arctan\left(\frac{(2\,bdx^2 + bc + ad)\sqrt{-\frac{be}{d}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(b^2ex^2 + abe)}\right) + \frac{1}{4}\sqrt{\frac{ae}{c}}\log\left(\frac{(b^2c^2 + 6\,abcd + a^2d^2)ex^4 + 8\,a^2c^2e + 8\,(abc^2 + a^2cd)ex^2 - 4\,((bc^2d + acd^2)x^4 + 2\,ac^3 + (bc\cdots}{x^4}\right) + \frac{1}{4}\sqrt{\frac{be}{d}}\log\left(8\,b^2d^2ex^4 + 8\,(b^2cd + abd^2)ex^2 + (b^2c^2 + 6\,abcd + a^2d^2)e + 4\,(2\,bd^3x^4 + bc^2d + acd^2 + (3\,bcd^2 + ad^3)x^2)\sqrt{\frac{be}{d}}\sqrt{\frac{bex^2 + ae}{dx^2 + c}}\right), \frac{1}{2}\sqrt{-\frac{ae}{c}}\arctan\left(\frac{((bc + ad)x^2 + 2\,ac\cdots}{2\,(abex^2 \cdots}\right) - \frac{1}{2}\sqrt{-\frac{be}{d}}\arctan\left(\frac{(2\,bdx^2 + bc + ad)\sqrt{-\frac{be}{d}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(b^2ex^2 + abe)}\right)\right]$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b*e/d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c
^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*b*c*d
^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 1/4*sqrt(
a*e/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^
2 + a^2*c*d)*e*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^
2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4), -1/2*sqrt(-b
*e/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) + 1/4*sqrt(a*e/c)*log(((b^2*c^2 + 6*a*b*c
*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c^2
*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*
e*x^2 + a*e)/(d*x^2 + c)))/x^4), 1/2*sqrt(-a*e/c)*arctan(1/2*((b*c + a*d)*x
^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2
*e)) + 1/4*sqrt(b*e/d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 +
(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3
*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))), 1/2*
```

```
sqrt(-a*e/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2*e)) - 1/2*sqrt(-b*e/d)*arctan(1/2*(2
*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e
*x^2 + a*b*e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x}\,dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x, x)
```

3.267 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 127

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)\sqrt{e}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}c^{3/2}}$$

```
[Out] -1/2*(-a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/
2))*e^(1/2)/c^(3/2)/a^(1/2)+1/2*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/
(a-c*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1981, 1980, 294, 214}

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{\sqrt{e}(bc-ad)\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}c^{3/2}}$$

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]
```

```
[Out] ((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*c*(a - (c*(a + b*x^2))/(
c + d*x^2))) - ((b*c - a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(
c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*Sqrt[a]*c^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{\frac{e(a+bx)}{c+dx}}}{x^2}\,dx, x, x^2\right)\\
&= ((bc-ad)e)\text{Subst}\left(\int \frac{x^2}{(-ae+cx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)\\
&= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{((bc-ad)e)\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2c}\\
&= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx$$
$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}(c+dx^2)+(bc-ad)x^2\sqrt{c+dx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{2\sqrt{a}c^{3/2}x^2\sqrt{a+bx^2}}$$

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]
```

```
[Out] -1/2*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*(c
 + d*x^2) + (b*c - a*d)*x^2*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2
])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(Sqrt[a]*c^(3/2)*x^2*Sqrt[a + b*x^2])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{(d\,x^2+c)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{2c\,x^2}+\frac{(da-bc)\ln\left(\frac{2ace+(eda+ebc)x^2+2\sqrt{ace}\,\sqrt{bde\,x^4+(eda+ebc)x^2+ace}}{x^2}\right)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,\sqrt{(d\,x^2+c)e(b\,x^2+a)}}{4c\sqrt{ace}\,(b\,x^2+a)}$
default	$-\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)\left(-2bd\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,x^4\sqrt{ac}-a^2\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}+2ac}{x^2}\right)dc\,x^2+c^2\ln\left(\right.\right.}{}$

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c*(d*x^2+c)/x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/4*(a*d-b*c)/c/(a*c*e)^
(1/2)*ln((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e
)*x^2+a*c*e)^(1/2))/x^2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+
a))^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.62

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx$$
$$= \left[-\frac{(bc-ad)x^2\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2+4\,(2\,a^2c^3+(abc^2d+a^2cd^2)x^4+(abc^3+3\,a^2c^2d)x^2)\sqrt{\frac{bex^2}{dx}}}{x^4}\right.}{8\,cx^2}\right.$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b*c - a*d)*x^2*sqrt(e/(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e
*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*
d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x
^2 + c))*sqrt(e/(a*c)))/x^4) + 4*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c)))/(c*x^2), 1/4*((b*c - a*d)*x^2*sqrt(-e/(a*c))*arctan(1/2*((b*c + a*d)*x
^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e
)) - 2*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(107) = 214$.

Time = 0.39 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = \frac{1}{2}\left(\frac{(bce-ade)\arctan\left(-\frac{\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}c} - \frac{\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right)bc\cdots}{\left(ace-\left(\sqrt{bde}x^2\cdots\right.\right.} \cdots + c\right)$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*((b*c*e - a*d*e)*arctan(-(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2
+ a*d*e*x^2 + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e)*c) - ((sqrt(b*d*e)*x^2 -
sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*b*c*e + (sqrt(b*d*e)*x^2 -
 sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a*d*e + 2*sqrt(b*d*e)*a*c
*e)/((a*c*e - (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a
*c*e))^2)*c))*sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^3}\,dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^3,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^3, x)
```

3.268 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 208

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2}+\frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)(bc+3ad)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}}$$

```
[Out] 1/8*(-a*d+b*c)*(3*a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^
(1/2)/e^(1/2))*e^(1/2)/a^(3/2)/c^(5/2)-1/4*(-a*d+b*c)^2*(e*(b*x^2+a)/(d*x^2
+c))^(1/2)/c^2/(a-c*(b*x^2+a)/(d*x^2+c))^2+1/8*(-5*a*d+b*c)*(-a*d+b*c)*(e*(
b*x^2+a)/(d*x^2+c))^(1/2)/a/c^2/(a-c*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

$= \{1981, 1980, 466, 393, 214\}$

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = \frac{\sqrt{e}(3ad+bc)(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}} - \frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5,x]
```

```
[Out] -1/4*((b*c - a*d)^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c^2*(a - (c*(a + b*
x^2))/(c + d*x^2))^2) + ((b*c - 5*a*d)*(b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c
+ d*x^2)])/(8*a*c^2*(a - (c*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)*(b*c
+ 3*a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[
a]*Sqrt[e])])/(8*a^(3/2)*c^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
```

```
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{\frac{e(a+bx)}{c+dx}}}{x^3}\,dx, x, x^2\right)\\
&= ((bc-ad)e)\text{Subst}\left(\int \frac{x^2(be-dx^2)}{(-ae+cx^2)^3}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)\\
&= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} - \frac{((bc-ad)e)\text{Subst}\left(\int \frac{-((bc-ad)e)+4cdx^2}{(-ae+cx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{4c^2}\\
&= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}\\
&\quad - \frac{((bc-ad)(bc+3ad)e)\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{8ac^2}\\
&= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}\\
&\quad + \frac{(bc-ad)(bc+3ad)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx$$
$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\sqrt{c+dx^2}(-2ac-bcx^2+3adx^2)+(b^2c^2+2abcd-3a^2d^2)\,x^4\text{arctanh}\ldots\right)}{8a^{3/2}c^{5/2}x^4\sqrt{a+bx^2}}$$

[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5,x]

[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - b*c*x^2 + 3*a*d*x^2) + (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(8*a^(3/2)*c^(5/2)*x^4*Sqrt[a + b*x^2])

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{(dx^2+c)(-3adx^2+bcx^2+2ac)\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{8c^2x^4a}-\frac{(3a^2d^2-2abcd-b^2c^2)\ln\left(\frac{2ace+(eda+ebc)x^2+2\sqrt{ace}\sqrt{bde\,x^4+(eda+ebc)x^2+ace}}{x^2}\right)\sqrt{\ldots}}{16ac^2\sqrt{ace}\,(bx^2+a)}$
default	$-\frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\,(dx^2+c)\left(10bd^2\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^6a\sqrt{ac}+2b^2d\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^6c\sqrt{ac}+3a^3\ln\left(\frac{adx^2+bcx^2+2\sqrt{ac}\ldots}{\ldots}\right)\ldots\right)}{\ldots}$

[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x,method=_RETURNVERBOSE)

[Out] -1/8*(d*x^2+c)*(-3*a*d*x^2+b*c*x^2+2*a*c)/c^2/x^4/a*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/16*(3*a^2*d^2-2*a*b*c*d-b^2*c^2)/a/c^2/(a*c*e)^(1/2)*ln((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/x^2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(b*x^2+a)

Fricas [A] (verification not implemented)

none

Time = 0.88 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = \left[-\frac{(b^2c^2+2\,abcd-3\,a^2d^2)x^4\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2-4\,(2\,a^2c^3+(abc^2d+a^2cd^2)x^4+(abc^3 \cdots}{x^4}\right.}{32\,ac^2x^4}\right.$$

$$-\frac{(b^2c^2+2\,abcd-3\,a^2d^2)x^4\sqrt{-\frac{e}{ac}}\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{ac}}}{2\,(bex^2+ae)}\right)+2\,((bcd-3\,ad^2)x^4+2\,ac^2+(b \cdots}{16\,ac^2x^4}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*sqrt(e/(a*c))*log(((b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4
*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)*sq
rt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) + 4*((b*c*d - 3*a*d^2)*
x^4 + 2*a*c^2 + (b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*
c^2*x^4), -1/16*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*sqrt(-e/(a*c))*arcta
n(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(
a*c))/(b*e*x^2 + a*e)) + 2*((b*c*d - 3*a*d^2)*x^4 + 2*a*c^2 + (b*c^2 - a*c*
d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^2*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(184) = 368$.

Time = 0.45 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.64

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = -\frac{1}{8}\left(\frac{(b^2c^2e + 2\,abcde - 3\,a^2d^2e)\arctan\left(-\frac{\sqrt{bde}x^2 - \sqrt{bdex^4+bcex^2+adex^2+ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}ac^2} - \frac{\left(\sqrt{bde}x^2 - \sqrt{bdex^4 + bcex^2}\right.}{} \right.$$
$$+ c)$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/8*((b^2*c^2*e + 2*a*b*c*d*e - 3*a^2*d^2*e)*arctan(-(sqrt(b*d*e)*x^2 - sq
rt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e)*
a*c^2) - ((sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e
))*a*b^2*c^3*e^2 + 10*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e
*x^2 + a*c*e))*a^2*b*c^2*d*e^2 + 5*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*
e*x^2 + a*d*e*x^2 + a*c*e))*a^3*c*d^2*e^2 + 8*sqrt(b*d*e)*a^3*c^2*d*e^2 + (
sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3*b^2*c^
2*e + 2*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))
^3*a*b*c*d*e - 3*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2
+ a*c*e))^3*a^2*d^2*e + 8*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b
*c*e*x^2 + a*d*e*x^2 + a*c*e))^2*a*b*c^2*e)/((a*c*e - (sqrt(b*d*e)*x^2 - sq
rt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^2)^2*a*c^2))*sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^5}\,dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^5,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^5, x)
```

3.269 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 318

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(b^2c^2+2abcd-11a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} - \frac{(bc-ad)\left(b^2c^2+2abcd+5a^2d^2\right)\sqrt{e}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{5/2}c^{7/2}}$$

```
[Out] 1/6*(-a*d+b*c)^3*e^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/a/c^2/(a*e-c*e*(b*x^2+a)
/(d*x^2+c))^3-1/16*(-a*d+b*c)*(5*a^2*d^2+2*a*b*c*d+b^2*c^2)*arctanh(c^(1/2)
*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))*e^(1/2)/a^(5/2)/c^(7/2)+1/8
*(-a*d+b*c)^2*(3*a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^3/(a-c*(b*x^2+a
)/(d*x^2+c))^2-1/16*(-a*d+b*c)*(-11*a^2*d^2+2*a*b*c*d+b^2*c^2)*(e*(b*x^2+a)
/(d*x^2+c))^(1/2)/a^2/c^3/(a-c*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1981, 1980, 474, 466, 393, 214}

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = -\frac{(-11a^2d^2+2abcd+b^2c^2)\,(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{\sqrt{e}(5a^2d^2+2abcd+b^2c^2)\,(bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{5/2}c^{7/2}} + \frac{(3ad+bc)(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{e^2(bc-ad)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3}$$

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]
```

```
[Out] ((b*c - a*d)^2*(b*c + 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(8*a*c^3*(a
 - (c*(a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)*(b^2*c^2 + 2*a*b*c*d - 11
*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(16*a^2*c^3*(a - (c*(a + b*x^2
))/(c + d*x^2))) + ((b*c - a*d)^3*e^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2))/
(6*a*c^2*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^3) - ((b*c - a*d)*(b^2*c^2 +
 2*a*b*c*d + 5*a^2*d^2)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(Sqrt[a]*Sqrt[e])])/(16*a^(5/2)*c^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
```

```
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{\frac{e(a+bx)}{c+dx}}}{x^4} \, dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{x^2(be-dx^2)^2}{(-ae+cx^2)^4} \, dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)\text{Subst}\left(\int \frac{x^2(-3(2b^2c^2e^2-(bce-ade)^2)+6acd^2ex^2)}{(-ae+cx^2)^3} \, dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{6ac^2}$$

$$= \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2}+\frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} - \frac{(bc-ad)\text{Subst}\left(\int\frac{3c(bc-ad)(bc+3ad)e^2-24ac^2d^2ex^2}{(-ae+cx^2)^2}dx,x,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{24ac^4}$$

$$= \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(b^2c^2+2abcd-11a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}+\frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{\left((bc-ad)\left(b^2c^2+2abcd+5a^2d^2\right)e\right)\text{Subst}\left(\int\frac{1}{-ae+cx^2}dx,x,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{16a^2c^3}$$

$$= \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(b^2c^2+2abcd-11a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} - \frac{(bc-ad)\left(b^2c^2+2abcd+5a^2d^2\right)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{5/2}c^{7/2}}$$

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.70

$$\int\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\sqrt{c+dx^2}(3b^2c^2x^4-2abcx^2(c-2dx^2)+a^2(-8c^2+10cdx^2-15d^2x^4))\right.}{48a^{5/2}c^{7/2}x^6\sqrt{a+bx^2}}$$

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2]*(3*b^2*c^2*x^4 - 2*a*b*c*x^2*(c - 2*d*x^2) + a^2*(
-8*c^2 + 10*c*d*x^2 - 15*d^2*x^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d
^2 - 5*a^3*d^3)*x^6*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x
^2])]))/(48*a^(5/2)*c^(7/2)*x^6*Sqrt[a + b*x^2])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{(d\,x^2+c)\left(15a^2d^2x^4-4bdac\,x^4-3b^2c^2x^4-10a^2cd\,x^2+2ab\,c^2x^2+8a^2c^2\right)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{48c^3x^6a^2}+\frac{\left(5a^3d^3-3a^2bc\,d^2-d\,c^2b^2a-b^3c^3\right)\ln\left(\frac{2ac}{}\right.}{}$
default	$-\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)\left(-66\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,a^2b\,d^3x^8-24\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,a\,b^2c\,d^2x^8-6\sqrt{ac}\,\sqrt{bd\,x^4+ad}\right.}{}$

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(d*x^2+c)*(15*a^2*d^2*x^4-4*a*b*c*d*x^4-3*b^2*c^2*x^4-10*a^2*c*d*x^2+
2*a*b*c^2*x^2+8*a^2*c^2)/c^3/x^6/a^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/32*(5*
a^3*d^3-3*a^2*b*c*d^2-a*b^2*c^2*d-b^3*c^3)/a^2/c^3/(a*c*e)^(1/2)*ln((2*a*c*
e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/
2))/x^2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(b*x^2
+a)
```

Fricas [A] (verification not implemented)

none

Time = 2.17 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \left[-\frac{3\,(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3)x^6\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2+4\,(2\,a^2c^3+(abc^2}{x^4}\right.}{}\right.$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*x^6*sqrt(e/(
a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2
 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 +
 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) -
4*((3*b^2*c^2*d + 4*a*b*c*d^2 - 15*a^2*d^3)*x^6 - 8*a^2*c^3 + (3*b^2*c^3 +
2*a*b*c^2*d - 5*a^2*c*d^2)*x^4 - 2*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2
 + a*e)/(d*x^2 + c)))/(a^2*c^3*x^6), 1/96*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2
*b*c*d^2 - 5*a^3*d^3)*x^6*sqrt(-e/(a*c))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*
```

```
c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e)) + 2*((
3*b^2*c^2*d + 4*a*b*c*d^2 - 15*a^2*d^3)*x^6 - 8*a^2*c^3 + (3*b^2*c^3 + 2*a*
b*c^2*d - 5*a^2*c*d^2)*x^4 - 2*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a
*e)/(d*x^2 + c)))/(a^2*c^3*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**7,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(290) = 580$.

Time = 0.41 (sec) , antiderivative size = 1001, normalized size of antiderivative = 3.15

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \frac{1}{48}\left(\frac{3\left(b^3c^3e + ab^2c^2de + 3a^2bcd^2e - 5a^3d^3e\right)\arctan\left(-\frac{\sqrt{bde}x^2 - \sqrt{bdex^4 + bcex^2 + adex^2 + ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}a^2c^3} - \frac{3\left(\sqrt{bde}x^2 - \sqrt{b\cdots}\right.}{\cdots}\right.$$

$$+ c)$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(3*(b^3*c^3*e + a*b^2*c^2*d*e + 3*a^2*b*c*d^2*e - 5*a^3*d^3*e)*arctan(
-(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))/sqrt(-
a*c*e))/(sqrt(-a*c*e)*a^2*c^3) - (3*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c
*e*x^2 + a*d*e*x^2 + a*c*e))*a^2*b^3*c^5*e^3 + 51*(sqrt(b*d*e)*x^2 - sqrt(b
*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a^3*b^2*c^4*d*e^3 + 105*(sqrt(b*
d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a^4*b*c^3*d^2*e
^3 + 33*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))
*a^5*c^2*d^3*e^3 + 16*sqrt(b*d*e)*a^4*b*c^4*d*e^3 + 48*sqrt(b*d*e)*a^5*c^3*
d^2*e^3 + 8*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c
*e))^3*a*b^3*c^4*e^2 + 72*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a
*d*e*x^2 + a*c*e))^3*a^2*b^2*c^3*d*e^2 + 24*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x
^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3*a^3*b*c^2*d^2*e^2 - 40*(sqrt(b*d*e)*
x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3*a^4*c*d^3*e^2 + 48
*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*
c*e))^2*a^2*b^2*c^4*e^2 + 144*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4
 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^2*a^3*b*c^3*d*e^2 - 3*(sqrt(b*d*e)*x^2 -
 sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^5*b^3*c^3*e - 3*(sqrt(b*d
*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^5*a*b^2*c^2*d*e
- 9*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^5*a
^2*b*c*d^2*e + 15*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2
 + a*c*e))^5*a^3*d^3*e)/((a*c*e - (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e
*x^2 + a*d*e*x^2 + a*c*e))^2)^3*a^2*c^3))*sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^7}\,dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^7,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^7, x)
```

3.270 $\int x^4 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\, dx$

Optimal result

Integrand size = 26, antiderivative size = 357

$$\int x^4 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\, dx = \frac{(8b^2c^2-3abcd-2a^2d^2)\, x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15b^2d^2} - \frac{(4bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15bd^2} + \frac{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d} - \frac{\sqrt{c}(8b^2c^2-3abcd-2a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\, E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{c^{3/2}(4bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\, \text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] 1/15*(-2*a^2*d^2-3*a*b*c*d+8*b^2*c^2)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/d
^2-1/15*(-a*d+4*b*c)*x*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^2+1/5*x^
3*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d+1/15*c^(3/2)*(-a*d+4*b*c)*(1/(1
+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^(5/2)/(c*(b*x^2+
a)/a/(d*x^2+c))^(1/2)-1/15*(-2*a^2*d^2-3*a*b*c*d+8*b^2*c^2)*(1/(1+d*x^2/c))
^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b
*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/d^(5/2)/(c*(b*x^2+
a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1986, 489, 596, 545, 429, 506, 422}

$$\int x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = -\frac{\sqrt{c}(-2a^2d^2-3abcd+8b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{x(-2a^2d^2-3abcd+8b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15b^2d^2} + \frac{c^{3/2}(4bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{x(c+dx^2)(4bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15bd^2} + \frac{x^3(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d}$$

```
[In] Int[x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(
15*b^2*d^2) - ((4*b*c - a*d)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2
))/(15*b*d^2) + (x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*d) -
 (Sqrt[c]*(8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x
^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(5/
2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (c^(3/2)*(4*b*c - a*d)*Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= \frac{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d} - \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2(3ac+(4bc-ad)x^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5d\sqrt{a+bx^2}}$$

$$= -\frac{(4bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15bd^2} + \frac{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d} + \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{ac(4bc-ad)+(8b^2c^2-3abcd-2a^2d^2)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15bd^2\sqrt{a+bx^2}}$$

$$= -\frac{(4bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15bd^2} + \frac{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d} + \frac{\left(ac(4bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15bd^2\sqrt{a+bx^2}} + \frac{\left((8b^2c^2-3abcd-2a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15bd^2\sqrt{a+bx^2}}$$

$$= \frac{(8b^2c^2-3abcd-2a^2d^2)\,x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15b^2d^2} - \frac{(4bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15bd^2} + \frac{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d} + \frac{c^{3/2}(4bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\left(c(8b^2c^2-3abcd-2a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{15b^2d^2\sqrt{a+bx^2}}$$

$$= \frac{(8b^2c^2-3abcd-2a^2d^2)\,x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15b^2d^2} - \frac{(4bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15bd^2} + \frac{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d} - \frac{\sqrt{c}(8b^2c^2-3abcd-2a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{c^{3/2}(4bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.71

$$\int x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(-4bc+ad+3bdx^2)+ic(-8b^2c^2+3abcd+2a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{d\ldots}{}}\right.}{15b\sqrt{\frac{b}{a}}c\ldots}$$

```
[In] Integrate[x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(
-4*b*c + a*d + 3*b*d*x^2) + I*c*(-8*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)] - I*c*(-8*b^2*c^2 + 7*a*b*c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*b*Sqrt[b/a
]*d^3*(a + b*x^2))
```

Maple [A] (verified)

Time = 5.21 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.37

method	result
risch	$\frac{x(3bd\,x^2+ad-4bc)(d\,x^2+c)\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{15d^2b}-\frac{\left(\frac{a^2cd\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}-\frac{4abc^2\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\ldots\right.}{\sqrt{-\frac{b}{a}}\sqrt{bde\,x^4+ade\,x^2+bce\,x^2\ldots}}\right.}{\ldots}$
default	$\frac{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}(d\,x^2+c)\left(3\sqrt{-\frac{b}{a}}b^2d^3x^7+4\sqrt{-\frac{b}{a}}abd^3x^5-\sqrt{-\frac{b}{a}}b^2cd^2x^5+\sqrt{-\frac{b}{a}}a^2d^3x^3-4\sqrt{-\frac{b}{a}}b^2c^2dx^3+\sqrt{\frac{b\,x^2+a}{a}}\sqrt{\frac{d\,x^2+c}{c}}F\left(x\sqrt{\ldots}\right.\right.}{\ldots}$

```
[In] int(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*x*(3*b*d*x^2+a*d-4*b*c)*(d*x^2+c)/d^2/b*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-
1/15/b/d^2*(a^2*c*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d
*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e
+b*c*e)/c/b/e)^(1/2))-4*a*b*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2
)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-2*(2*a^2*d^2+3*a*b*c*d-8*b^2*c^2)*a*c*e/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*
e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+
(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/
```

```
e)^(1/2)))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(b*
x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.76

$$\int x^4\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx =$$

$$-\frac{(8\,b^2c^3-3\,abc^2d-2\,a^2cd^2)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(8\,b^2c^3-3\,abc^2d-a^2d^3-2\,(a^2-2\,ab}{}$$

[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out]
```
-1/15*((8*b^2*c^3 - 3*a*b*c^2*d - 2*a^2*c*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*ell
iptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (8*b^2*c^3 - 3*a*b*c^2*d - a^2*d
^3 - 2*(a^2 - 2*a*b)*c*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt
(-c/d)/x), a*d/(b*c)) - (3*b^2*d^3*x^6 + 8*b^2*c^3 - 3*a*b*c^2*d - 2*a^2*c*
d^2 - (b^2*c*d^2 - a*b*d^3)*x^4 + 2*(2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^2
)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int x^4\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \text{Timed out}$$

[In] integrate(x**4*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int x^4\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^4\,dx$$

[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4, x)

Giac [F]

$$\int x^4\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^4\,dx$$

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \int x^4\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

```
[In] int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.271 $\int x^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 266

$$\begin{aligned}\int x^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=&-\frac{(2bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3bd}+\frac{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d}\\&+\frac{\sqrt{c}(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\\&-\frac{c^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\end{aligned}$$

```
[Out] -1/3*(-a*d+2*b*c)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d+1/3*x*(d*x^2+c)*(e*(b
*x^2+a)/(d*x^2+c))^(1/2)/d-1/3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1
/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(e*(b*
x^2+a)/(d*x^2+c))^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+1/3*(-a*d+2
*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b
/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1986, 489, 545, 429, 506, 422}

$$\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = -\frac{c^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{x(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d} - \frac{x(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3bd}$$

```
[In] Int[x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] -1/3*((2*b*c - a*d)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(b*d) + (x*Sqrt[(e
*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*d) + (Sqrt[c]*(2*b*c - a*d)*Sqrt
[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(3*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (c^(3/2)*
Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
```

```
d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}} \\
&= \frac{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d} - \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{ac+(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d\sqrt{a+bx^2}} \\
&= \frac{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d} - \frac{\left(ac\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d\sqrt{a+bx^2}} \\
&\quad - \frac{\left((2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d\sqrt{a+bx^2}}
\end{aligned}
$$

$$= -\frac{(2bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3bd} + \frac{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d} - \frac{c^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\left(c(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}dx}{3bd\sqrt{a+bx^2}}$$

$$= -\frac{(2bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3bd} + \frac{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d} + \frac{\sqrt{c}(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{c^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.78

$$\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2) - ic(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right) + 2ic(-\right.}{3\sqrt{\frac{b}{a}}d^2(a+bx^2)}$$

```
[In] Integrate[x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) -
 I*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*S
qrt[b/a]*d^2*(a + b*x^2))
```

Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.32

method	result
risch	$\frac{x(d\,x^2+c)\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{3d}-\frac{\left(\frac{ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}+\frac{2(ad-2bc)ace\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+e}{cbe}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}\,(ed}\right.}{3d(b\,x^2+a)}$
default	$\frac{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)\left(\sqrt{-\frac{b}{a}}\,b\,d^2x^5+\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,x^3-2ac\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\right.}{3\sqrt{(d\,x^2+c)(b\,x^2+a)}\,d^2\sqrt{-\frac{b}{a}}\,\sqrt{}}$

[In] int(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*x*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d-1/3/d*(a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))+2*(a*d-2*b*c)*a*c*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(b*x^2+a)

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.68

$$\int x^2\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$
$$=\frac{(2\,bc^2-acd)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(2\,bc^2-acd+ad^2)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})+}{3\,bd^2x}$$

[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] 1/3*((2*b*c^2 - a*c*d)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (2*b*c^2 - a*c*d + a*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (b*d^2*x^4 - 2*b*c^2 + a*c*d - (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^2*x)

Sympy [F(-1)]

Timed out.

$$\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^2\,dx$$

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^2\,dx$$

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \int x^2\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

```
[In] int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.272 $\int \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 194

$$\int \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = x\sqrt{\frac{e(a+bx^2)}{c+dx^2}} - \frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ell
ipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b
*x^2+a)/(d*x^2+c))^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+(1/(1+d*x^
2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2)
,(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^(1/2)/(c*(b*x^2
+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1986, 433, 429, 506, 422}

$$\int \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}$$

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)] - (Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= \frac{\left(a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}} + \frac{\left(b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= x\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}} + \frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\left(c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{\sqrt{a+bx^2}}$$

$$= x\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}} - \frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.44

$$\int \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}}$$

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqr
t[-(d/c)]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a])
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\left(aF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bcF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bcE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)}{\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,d}$	184

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] (e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*(a*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a)^
```

```
(1/2),(a*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))/((d*x
^2+c)*(b*x^2+a))^(1/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$\int \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = -\frac{bc^2\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - (bc^2+ad^2)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - (bcdx^2+bc^2)\sqrt{}}{bcdx}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
[Out] -(b*c^2*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)
) - (b*c^2 + a*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x
), a*d/(b*c)) - (b*c*d*x^2 + b*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*c
*d*x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = \int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Giac [F]

$$\int \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=\int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=\int \sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.273 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 239

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2} dx = \frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} - \frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{b\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] d*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c-(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)
/c/x+b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)
^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(
1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1986, 486, 21, 433, 429, 506, 422}

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \frac{b\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c} - \frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cx}$$

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^2,x]
```

```
[Out] (d*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c - (Sqrt[(e*(a + b*x^2))/(c + d*x^
2)]*(c + d*x^2))/(c*x) - (Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]) + (b*Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}} \\
&= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} + \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{bc+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{c\sqrt{a+bx^2}} \\
&= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} + \frac{\left(b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{c\sqrt{a+bx^2}}
\end{aligned}
$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} + \frac{\left(b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}} + \frac{\left(bd\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{c\sqrt{a+bx^2}}$$

$$= \frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} + \frac{b\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\left(d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{\sqrt{a+bx^2}}$$

$$= \frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} - \frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{b\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)\left(-\frac{1}{x} + \frac{b\sqrt{1+\frac{bx^2}{a}}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}(a+bx^2)\sqrt{1+\frac{dx^2}{c}}}\right)}{c}$$

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^2,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)*(-x^(-1) + (b*Sqrt[1 + (b*x^
2)/a]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*(a + b*
x^2)*Sqrt[1 + (d*x^2)/c])))/c
```

Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.80

method	result
default	$-\frac{\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(\sqrt{-\frac{b}{a}}\,b d\,x^{4}-b c\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,x E\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right)+\sqrt{-\frac{b}{a}}\,a d\,x^{2}+\sqrt{-\frac{b}{a}}\,b c\,x^{2}+\sqrt{-\frac{b}{a}}\,a c\right)}{\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,c x\sqrt{-\frac{b}{a}}\,\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}}$
risch	$-\frac{\left(d\,x^{2}+c\right)\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{c x}+\frac{b\left(\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{e d a+e b c}{c b e}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{b d e\,x^{4}+a d e\,x^{2}+b c e\,x^{2}+a c e}}-\frac{2 d a c e\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{e d a+e b c}{c b e}}\right)-E\left(\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{b d e\,x^{4}+a d e\,x^{2}+b c e\,x^{2}+a c e}\,(e d a+e b c}\right)}{c\left(b\,x^{2}+a\right)}$

[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x,method=_RETURNVERBOSE)

[Out] -(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*((-b/a)^(1/2)*b*d*x^4-b*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))+(-b/a)^(1/2)*a*d*x^2+(-b/a)^(1/2)*b*c*x^2+(-b/a)^(1/2)*a*c)/((d*x^2+c)*(b*x^2+a))^(1/2)/c/x/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \frac{bd\sqrt{\frac{ace}{d^2}}x\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - (a+b)d\sqrt{\frac{ace}{d^2}}x\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - (adx^2+ac)\sqrt{\frac{bex}{dx}}}{acx}$$

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="fricas")

[Out] (b*d*sqrt(a*c*e/d^2)*x*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (a + b)*d*sqrt(a*c*e/d^2)*x*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (a*d*x^2 + a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c*x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^2}\,dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^2}\,dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^2}\,dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^2,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^2, x)
```

3.274 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 321

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = \frac{d(bc-2ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3ac^2x} - \frac{\sqrt{d}(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{b\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] 1/3*d*(-2*a*d+b*c)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^2-1/3*(d*x^2+c)*(e*(
b*x^2+a)/(d*x^2+c))^(1/2)/c/x^3-1/3*(-2*a*d+b*c)*(d*x^2+c)*(e*(b*x^2+a)/(d*
x^2+c))^(1/2)/a/c^2/x-1/3*(-2*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1
/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2
)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1
/3*b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1
+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/
c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1986, 486, 597, 545, 429, 506, 422}

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4} dx = -\frac{\sqrt{d}(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{b\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{dx(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2} - \frac{(c+dx^2)(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2x} - \frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3cx^3}$$

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^4,x]
```

```
[Out] (d*(b*c - 2*a*d)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*a*c^2) - (Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*c*x^3) - ((b*c - 2*a*d)*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*a*c^2*x) - (Sqrt[d]*(b*c - 2*a*d)*
Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(3*a*c^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (b*Sq
rt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
```

```
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3cx^3} + \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{bc-2ad-bdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3c\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3ac^2x} - \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{abcd-bd(bc-2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3ac^2\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3ac^2x} - \frac{\left(bd\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3c\sqrt{a+bx^2}} + \frac{\left(bd(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3ac^2\sqrt{a+bx^2}}$$

$$= \frac{d(bc-2ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3ac^2x} - \frac{b\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\left(d(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{3ac\sqrt{a+bx^2}}$$

$$= \frac{d(bc-2ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3ac^2x} - \frac{\sqrt{d}(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{b\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = -\frac{\sqrt{\frac{b}{a}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}(a+bx^2)\left(c+dx^2\right)\left(bcx^2+a(c-2dx^2)\right)-ibc(-bc+2ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(ia\right.\right.}{3bc^2x^3\left(a+bx\right.}$$

[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^4,x]

[Out] -1/3*(Sqrt[b/a]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(b*c*x^2 + a*(c - 2*d*x^2)) - I*b*c*(-(b*c) + 2*a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c^2*x^3*(a + b*x^2))

Maple [A] (verified)

Time = 5.43 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{(d\,x^2+c)\left(-2ad\,x^2+bc\,x^2+ac\right)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{3c^2x^3a}-\frac{db\left(\frac{ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}-\frac{2(2ad-bc)ace\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4}}\right.}{3a}$
default	$-\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)\left(-2\sqrt{-\frac{b}{a}}\,ab\,d^2x^6+\sqrt{-\frac{b}{a}}\,b^2cd\,x^6-bd\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^3ac+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{\right.\right.}{}$

[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x,method=_RETURNVERBOSE)

[Out] -1/3*(d*x^2+c)*(-2*a*d*x^2+b*c*x^2+a*c)/c^2/x^3/a*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*d/a*b/c^2*(a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-2*(2*a*d-b*c)*a*c*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(b*x^2+a)

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx$$
$$= \frac{(b^2cd - 2\,abd^2)\sqrt{\frac{ace}{d^2}}x^3\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - (b^2cd - (a^2 + 2\,ab)d^2)\sqrt{\frac{ace}{d^2}}x^3\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{\right.}\right.}{3\,a^2c^2x^3}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*((b^2*c*d - 2*a*b*d^2)*sqrt(a*c*e/d^2)*x^3*sqrt(-b/a)*elliptic_e(arcsin
(x*sqrt(-b/a)), a*d/(b*c)) - (b^2*c*d - (a^2 + 2*a*b)*d^2)*sqrt(a*c*e/d^2)*
x^3*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((a*b*c*d - 2*
a^2*d^2)*x^4 + a^2*c^2 + (a*b*c^2 - a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x
^2 + c)))/(a^2*c^2*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^4}\,dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^4}\,dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^4}\,dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^4,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^4, x)
```

3.275 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6} dx$

Optimal result

Integrand size = 26, antiderivative size = 424

$$\begin{aligned}\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6} dx = & -\frac{d(2b^2c^2+3abcd-8a^2d^2)\,x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3} \\ & -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5cx^5}-\frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15ac^2x^3} \\ & +\frac{(2b^2c^2+3abcd-8a^2d^2)\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15a^2c^3x} \\ & +\frac{\sqrt{d}(2b^2c^2+3abcd-8a^2d^2)\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^2c^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} \\ & -\frac{b\sqrt{d}(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^2c^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\end{aligned}$$

```
[Out] -1/15*d*(-8*a^2*d^2+3*a*b*c*d+2*b^2*c^2)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^
2/c^3-1/5*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/x^5-1/15*(-4*a*d+b*c)*(
d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^2/x^3+1/15*(-8*a^2*d^2+3*a*b*c*d
+2*b^2*c^2)*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^2/c^3/x+1/15*(-8*a^2*
d^2+3*a*b*c*d+2*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(
x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)
/(d*x^2+c))^(1/2)/a^2/c^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/15*b*(-4*a*
d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
a^2/c^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 486, 597, 545, 429, 506, 422}

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6} \, dx = \frac{\sqrt{d}(-8a^2d^2+3abcd+2b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^2c^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{b\sqrt{d}(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^2c^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{dx(-8a^2d^2+3abcd+2b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3} + \frac{(c+dx^2)(-8a^2d^2+3abcd+2b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3x} - \frac{(c+dx^2)(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15ac^2x^3} - \frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5cx^5}$$

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^6,x]
```

```
[Out] -1/15*(d*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*x*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)])/(a^2*c^3) - (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*c*x^5
) - ((b*c - 4*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(15*a*c^2
*x^3) + ((2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^
2)]*(c + d*x^2))/(15*a^2*c^3*x) + (Sqrt[d]*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d
^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(15*a^2*c^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
- (b*Sqrt[d]*(b*c - 4*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^2*c^(3/2)*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5cx^5} + \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{bc-4ad-3bdx^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5c\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15ac^2x^3} - \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{2b^2c^2+3abcd-8a^2d^2+bd(bc-4ad)x^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15ac^2\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15ac^2x^3} + \frac{(2b^2c^2+3abcd-8a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15a^2c^3x} + \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{-abcd(bc-4ad)-bd(2b^2c^2+3abcd-8a^2d^2)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15a^2c^3\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15ac^2x^3} + \frac{(2b^2c^2+3abcd-8a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15a^2c^3x} - \frac{\left(bd(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15ac^2\sqrt{a+bx^2}} - \frac{\left(bd(2b^2c^2+3abcd-8a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15a^2c^3\sqrt{a+bx^2}}$$

$$= -\frac{d(2b^2c^2+3abcd-8a^2d^2)\,x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15ac^2x^3} + \frac{(2b^2c^2+3abcd-8a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15a^2c^3x} - \frac{b\sqrt{d}(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^2c^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\left(d(2b^2c^2+3abcd-8a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{15a^2c^2\sqrt{a+bx^2}}$$

$$= -\frac{d(2b^2c^2+3abcd-8a^2d^2)\,x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15ac^2x^3} + \frac{(2b^2c^2+3abcd-8a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15a^2c^3x} + \frac{\sqrt{d}(2b^2c^2+3abcd-8a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^2c^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{b\sqrt{d}(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^2c^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.79 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(-2b^2c^2x^4+abcx^2(c-3dx^2)+a^2(3c^2-4cdx^2+8d^2x^4))+ibc(-2b\right.}{}$$

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^6,x]
```

```
[Out] -1/15*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)
*(-2*b^2*c^2*x^4 + a*b*c*x^2*(c - 3*d*x^2) + a^2*(3*c^2 - 4*c*d*x^2 + 8*d^2
*x^4)) + I*b*c*(-2*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)
```

```
*b*c*(-(b^2*c^2) - a*b*c*d + 2*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(a^2*Sqrt[b/a]*c^
3*x^5*(a + b*x^2))
```

Maple [A] (verified)

Time = 6.59 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{(d x^2+c)(8a^2d^2x^4-3bdacx^4-2b^2c^2x^4-4a^2cdx^2+abc^2x^2+3a^2c^2)\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}{15c^3x^5a^2}+\frac{bd\left(\frac{4a^2cd\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ed}{}}\right.}{\sqrt{-\frac{b}{a}}\sqrt{bdex^4+adex^2+bcex^2+ace}}\right.}{}$
default	$-\frac{\sqrt{\frac{e(b x^2+a)}{d x^2+c}}(d x^2+c)\left(8\sqrt{-\frac{b}{a}}a^2bd^3x^8-3\sqrt{-\frac{b}{a}}ab^2cd^2x^8-2\sqrt{-\frac{b}{a}}b^3c^2dx^8+4\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2bcd^2x^5-2\right.}{}$

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(d*x^2+c)*(8*a^2*d^2*x^4-3*a*b*c*d*x^4-2*b^2*c^2*x^4-4*a^2*c*d*x^2+a*
b*c^2*x^2+3*a^2*c^2)/c^3/x^5/a^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/15/a^2*b*d
/c^3*(4*a^2*c*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x
^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c
*e)/c/b/e)^(1/2))-a*b*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2
)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+
(a*d*e+b*c*e)/c/b/e)^(1/2))-2*(8*a^2*d^2-3*a*b*c*d-2*b^2*c^2)*a*c*e/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+
a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e
+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/
2))))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = -\frac{(2\,b^3c^2d+3\,ab^2cd^2-8\,a^2bd^3)\sqrt{\frac{ace}{d^2}}x^5\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(2\,b^3c^2d+(a^2b+3\,ab^2)cd^2-4\,(a^3}{}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*((2*b^3*c^2*d + 3*a*b^2*c*d^2 - 8*a^2*b*d^3)*sqrt(a*c*e/d^2)*x^5*sqrt
(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (2*b^3*c^2*d + (a^2*b
```

```
+ 3*a*b^2)*c*d^2 - 4*(a^3 + 2*a^2*b)*d^3)*sqrt(a*c*e/d^2)*x^5*sqrt(-b/a)*el
liptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((2*a*b^2*c^2*d + 3*a^2*b*c*d^2
 - 8*a^3*d^3)*x^6 - 3*a^3*c^3 + 2*(a*b^2*c^3 + a^2*b*c^2*d - 2*a^3*c*d^2)*x
^4 - (a^2*b*c^3 - a^3*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^3*c
^3*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**6,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^6}\,dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="maxima")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^6}\,dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="giac")
[Out] integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^6}\,dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^6,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^6, x)
```

3.276 $\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 282

$$\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \frac{c^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^4} + \frac{(79b^2c^2-50abcd-5a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{48bd^4} - \frac{(11bc+ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24d^4} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}(c+dx^2)^3}{6bd^2e} - \frac{(bc-ad)\,(35b^2c^2-10abcd-a^2d^2)\,e^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{3/2}d^{9/2}}$$

```
[Out] 1/6*(e*(b*x^2+a)/(d*x^2+c))^(5/2)*(d*x^2+c)^3/b/d^2/e-1/16*(-a*d+b*c)*(-a^2
*d^2-10*a*b*c*d+35*b^2*c^2)*e^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))
^(1/2)/b^(1/2)/e^(1/2))/b^(3/2)/d^(9/2)+c^2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^
2+c))^(1/2)/d^4+1/48*(-5*a^2*d^2-50*a*b*c*d+79*b^2*c^2)*e*(d*x^2+c)*(e*(b*x
^2+a)/(d*x^2+c))^(1/2)/b/d^4-1/24*(a*d+11*b*c)*e*(d*x^2+c)^2*(e*(b*x^2+a)/(
d*x^2+c))^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1981, 1980, 474, 466, 1171, 396, 214}

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = -\frac{e^{3/2}(bc-ad)\left(-a^2d^2-10abcd+35b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{3/2}d^{9/2}} + \frac{e(c+dx^2)\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48bd^4} + \frac{c^2e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^4} - \frac{e(c+dx^2)^2(ad+11bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24d^4} + \frac{(c+dx^2)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6bd^2e}$$

```
[In] Int[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (c^2*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d^4 + ((79*b^2*c^2 -
50*a*b*c*d - 5*a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(4
8*b*d^4) - ((11*b*c + a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^
2)/(24*d^4) + (((e*(a + b*x^2))/(c + d*x^2))^(5/2)*(c + d*x^2)^3)/(6*b*d^2*
e) - ((b*c - a*d)*(35*b^2*c^2 - 10*a*b*c*d - a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt
[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(3/2)*d^(9
/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
```

```
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int x^2\left(\frac{e(a+bx)}{c+dx}\right)^{3/2}dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{x^4(-ae+cx^2)^2}{(be-dx^2)^4}dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)$$

$$=\frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}(c+dx^2)^3}{6bd^2e}-\frac{(bc-ad)\text{Subst}\left(\int\frac{x^4(-6a^2d^2e^2+5(bce-ade)^2+6bc^2dex^2)}{(be-dx^2)^3}\,dx,x,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{6bd^2}$$

$$=-\frac{(11bc+ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24d^4}+\frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}(c+dx^2)^3}{6bd^2e}-\frac{(bc-ad)\text{Subst}\left(\int\frac{-bd(bc-ad)(11bc+ad)e^3-4d^2(bc-ad)(11bc+ad)e^2x^2-24bc^2d^3ex^4}{(be-dx^2)^2}\,dx,x,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{24bd^5}$$

$$=\frac{(79b^2c^2-50abcd-5a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{48bd^4}-\frac{(11bc+ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24d^4}+\frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}(c+dx^2)^3}{6bd^2e}+\frac{(bc-ad)\text{Subst}\left(\int\frac{-3bd(19b^2c^2-10abcd-a^2d^2)e^3-48b^2c^2d^2e^2x^2}{be-dx^2}\,dx,x,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{48b^2d^5e}$$

$$=\frac{c^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^4}+\frac{(79b^2c^2-50abcd-5a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{48bd^4}-\frac{(11bc+ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24d^4}+\frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}(c+dx^2)^3}{6bd^2e}-\frac{((bc-ad)\,(35b^2c^2-10abcd-a^2d^2)\,e^2)\,\text{Subst}\left(\int\frac{1}{be-dx^2}\,dx,x,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{16bd^4}$$

$$=\frac{c^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^4}+\frac{(79b^2c^2-50abcd-5a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{48bd^4}-\frac{(11bc+ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24d^4}+\frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}(c+dx^2)^3}{6bd^2e}-\frac{(bc-ad)\,(35b^2c^2-10abcd-a^2d^2)\,e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{3/2}d^{9/2}}$$

Mathematica [A] (verified)

Time = 4.54 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.79

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(b\sqrt{d}(3a^2d^2(c+dx^2)+2abd(-50c^2-19cdx^2+7d^2x^4)+b^2(105c^3\right.}{}$$

[In] Integrate[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]

[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*(3*a^2*d^2*(c + d*x^2) + 2*a*b*d*(-50*c^2 - 19*c*d*x^2 + 7*d^2*x^4) + b^2*(105*c^3 + 35*c^2*d*x^2 - 14*c*d^2*x^4 + 8*d^3*x^6)) - (3*(b*c - a*d)^(3/2)*(35*b^2*c^2 - 10*a*b*c*d - a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a + b*x^2]))/(48*b^2*d^(9/2))

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.11

method	result
risch	$\frac{(8b^2d^2x^4+14ab\,d^2x^2-22b^2cd\,x^2+3a^2d^2-52abcd+57b^2c^2)(d\,x^2+c)e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{48b\,d^4}-\frac{\left(\frac{(a^2d^2+10abcd-35b^2c^2)(ad-bc)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}eb}{\sqrt{bd}}\right.}{2\sqrt{bde}}\right.}{}$
default	Expression too large to display

[In] int(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/48/b*(8*b^2*d^2*x^4+14*a*b*d^2*x^2-22*b^2*c*d*x^2+3*a^2*d^2-52*a*b*c*d+57*b^2*c^2)*(d*x^2+c)/d^4*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/16/b/d^4*(1/2*(a^2*d^2+10*a*b*c*d-35*b^2*c^2)*(a*d-b*c)*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)+16*c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*b*(b*x^2+a)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))*e/(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 1.29 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.96

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx = \left[\frac{3\left(35\,b^3c^3-45\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)e\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)e\right.}{}\right.$$

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(35*b^3*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*sqrt(e/
(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*e - 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2
 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) + 4*(8*b^
2*d^3*e*x^6 - 14*(b^2*c*d^2 - a*b*d^3)*e*x^4 + (35*b^2*c^2*d - 38*a*b*c*d^2
 + 3*a^2*d^3)*e*x^2 + (105*b^2*c^3 - 100*a*b*c^2*d + 3*a^2*c*d^2)*e)*sqrt((
b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4), 1/96*(3*(35*b^3*c^3 - 45*a*b^2*c^2*d
+ 9*a^2*b*c*d^2 + a^3*d^3)*e*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a
*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(
8*b^2*d^3*e*x^6 - 14*(b^2*c*d^2 - a*b*d^3)*e*x^4 + (35*b^2*c^2*d - 38*a*b*c
*d^2 + 3*a^2*d^3)*e*x^2 + (105*b^2*c^3 - 100*a*b*c^2*d + 3*a^2*c*d^2)*e)*sq
rt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4)]
```

Sympy [F(-1)]

Timed out.

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{2,[0,5,0]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-4,[0,4,0]%%%},0]
:[1,0,
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int x^5 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2} dx$$

```
[In] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.277 $\int x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 199

$$\int x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx=-\frac{c(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3}-\frac{(9bc-5ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8d^3}+\frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4d^3}+\frac{3(bc-ad)(5bc-ad)e^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8\sqrt{b}d^{7/2}}$$

```
[Out] 3/8*(-a*d+b*c)*(-a*d+5*b*c)*e^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))
^(1/2)/b^(1/2)/e^(1/2))/d^(7/2)/b^(1/2)-c*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+
c))^(1/2)/d^3-1/8*(-5*a*d+9*b*c)*e*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
d^3+1/4*b*e*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1981, 1980, 466, 1171, 396, 214}

$$\int x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx=\frac{3e^{3/2}(bc-ad)(5bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8\sqrt{b}d^{7/2}}+\frac{be(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3}-\frac{e(c+dx^2)(9bc-5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8d^3}-\frac{ce(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3}$$

```
[In] Int[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -((c*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d^3) - ((9*b*c - 5*a*
d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*d^3) + (b*e*Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*d^3) + (3*(b*c - a*d)*(5*b*c -
a*d)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*S
qrt[e])])/(8*Sqrt[b]*d^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int x\left(\frac{e(a+bx)}{c+dx}\right)^{3/2}\,dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{x^4(-ae+cx^2)}{(be-dx^2)^3}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4d^3} + \frac{((bc-ad)e)\text{Subst}\left(\int \frac{-b(bc-ad)e^2-4d(bc-ad)ex^2-4cd^2x^4}{(be-dx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{4d^3}$$

$$= -\frac{(9bc-5ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8d^3} + \frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4d^3} - \frac{(bc-ad)\text{Subst}\left(\int \frac{-b(7bc-3ad)e^2-8bcdex^2}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{8bd^3}$$

$$= -\frac{c(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3} - \frac{(9bc-5ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8d^3} + \frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4d^3} + \frac{(3(bc-ad)(5bc-ad)e^2)\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{8d^3}$$

$$= -\frac{c(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3} - \frac{(9bc-5ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8d^3} + \frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4d^3} + \frac{3(bc-ad)(5bc-ad)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8\sqrt{b}d^{7/2}}$$

Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.89

$$\int x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{b}\sqrt{d}\sqrt{a+bx^2}(ad(13c+5dx^2)+b(-15c^2-5cdx^2+2d^2x^4))+3(\right.}{8\sqrt{b}d^{7/2}\sqrt{a+bx^2}}$$

```
[In] Integrate[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*(a*d*
(13*c + 5*d*x^2) + b*(-15*c^2 - 5*c*d*x^2 + 2*d^2*x^4)) + 3*(5*b^2*c^2 - 6*
a*b*c*d + a^2*d^2)*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[
d]*Sqrt[a + b*x^2])]))/(8*Sqrt[b]*d^(7/2)*Sqrt[a + b*x^2])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.27

method	result
risch	$\frac{(2bd\,x^2+5ad-7bc)(d\,x^2+c)e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{8d^3}+\frac{\left(\frac{3(ad-5bc)(ad-bc)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^2}{\sqrt{bde}}+\sqrt{bde\,x^4+(eda+ebc)x^2+ace}\right)}{2\sqrt{bde}}+\frac{8c\left(a^2d^2-2a\right.}{(ad-bc)\sqrt{bde\,x}}\right.}{8d^3(b\,x^2+a)}$
default	$\frac{\left(4\sqrt{bd}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,b\,d^2x^4+3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^2d^3x^2-18\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x}}{2\sqrt{bd}}\right.\right.}{}$

```
[In] int(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*b*d*x^2+5*a*d-7*b*c)*(d*x^2+c)/d^3*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1
/8/d^3*(3/2*(a*d-5*b*c)*(a*d-b*c)*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)/(b*d*e
)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)+8*c*(a^2*d
^2-2*a*b*c*d+b^2*c^2)*(b*x^2+a)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*
c*e)^(1/2))*e/(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a
))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.68 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.10

$$\int x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \left[\frac{3\,(5\,b^2c^2-6\,abcd+a^2d^2)e\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\right.}{}\right.$$

$$-\frac{3\,(5\,b^2c^2-6\,abcd+a^2d^2)e\sqrt{-\frac{e}{bd}}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{bd}}}{2\,(bex^2+ae)}\right)-2\,(2\,bd^2ex^4-5\,(bcd-ad^2)ex^2-(}{16\,d^3}$$

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*sqrt(e/(b*d))*log(8*b^2*d^2*e*
x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(
2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((
b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) + 4*(2*b*d^2*e*x^4 - 5*(b*c*d -
a*d^2)*e*x^2 - (15*b*c^2 - 13*a*c*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/
d^3, -1/16*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*sqrt(-e/(b*d))*arctan(1/2
*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(
b*e*x^2 + a*e)) - 2*(2*b*d^2*e*x^4 - 5*(b*c*d - a*d^2)*e*x^2 - (15*b*c^2 -
13*a*c*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d^3]
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Timed out}$$

```
[In] integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{2,[0,4,0]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-4,[0,3,0]%%%},0]
:[1,0,
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int x^3 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2} dx$$

```
[In] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.278 $\int x \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int x \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \frac{3(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d^2} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)}{2d} - \frac{3\sqrt{b}(bc-ad)e^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2d^{5/2}}$$

```
[Out] 1/2*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)/d-3/2*(-a*d+b*c)*e^(3/2)*arctan
h(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))*b^(1/2)/d^(5/2)+3/
2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1981, 1979, 294, 327, 214}

$$\int x \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = -\frac{3\sqrt{b}e^{3/2}(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2d^{5/2}} + \frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d^2} + \frac{(c+dx^2)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2d}$$

```
[In] Int[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (3*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*d^2) + (((e*(a + b*x
^2))/(c + d*x^2))^(3/2)*(c + d*x^2))/(2*d) - (3*Sqrt[b]*(b*c - a*d)*e^(3/2)
*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2
*d^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \left(\frac{e(a+bx)}{c+dx}\right)^{3/2} dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{x^4}{(be-dx^2)^2}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)}{2d} - \frac{(3(bc-ad)e)\text{Subst}\left(\int \frac{x^2}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2d}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d^2} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)}{2d} - \frac{(3b(bc-ad)e^2)\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2d^2}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d^2} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)}{2d} - \frac{3\sqrt{b}(bc-ad)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2d^{5/2}}$$

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{d}\sqrt{a+bx^2}(3bc-2ad+bdx^2) - 3\sqrt{b}(bc-ad)\sqrt{c+dx^2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{2d^{5/2}\sqrt{a+bx^2}}$$

```
[In] Integrate[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[d]*Sqrt[a + b*x^2]*(3*b*c - 2*a*
d + b*d*x^2) - 3*Sqrt[b]*(b*c - a*d)*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[d]*Sqrt[
a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]))/(2*d^(5/2)*Sqrt[a + b*x^2])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.64

method	result
risch	$\frac{(d\,x^2+c)be\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{2d^2} + \frac{\left(\frac{3b(ad-bc)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^2}{\sqrt{bde}}+\sqrt{bde\,x^4+(eda+ebc)x^2+ace}\right)}{2\sqrt{bde}} - \frac{\left(2a^2d^2-4abcd+2b^2c^2\right)\left(b\,x^2+a\right)}{(ad-bc)\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}\right)e\sqrt{\cdots}}{2d^2(b\,x^2+a)}$
default	$-\frac{\left(-3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)ab\,d^2x^2+3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)b^2cd\,x^2-2\sqrt{bd}\,\sqrt{bd\cdots}\right.}{\cdots}$

```
[In] int(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d^2*(d*x^2+c)*b*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/2/d^2*(3/2*b*(a*d-b*c
)*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)
*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)-(2*a^2*d^2-4*a*b*c*d+2*b^2*c^2)*(b*x^2+a)/
(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))*e/(b*x^2+a)*(e*(b*x^
2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.33

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\left[-\frac{3\,(bc-ad)\sqrt{\frac{be}{d}}e\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)\right.}{}\right.$$

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b*c - a*d)*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d
^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d +
a*c*d^2 + (3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2
+ c))) - 4*(b*d*e*x^2 + (3*b*c - 2*a*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
))/d^2, 1/4*(3*(b*c - a*d)*sqrt(-b*e/d)*e*arctan(1/2*(2*b*d*x^2 + b*c + a*d
)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) + 2*(
b*d*e*x^2 + (3*b*c - 2*a*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d^2]
```

Sympy [F(-1)]

Timed out.

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{2,[0,3,0]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-4,[0,2,0]%%%},0]
:[1,0,
```

Mupad [F(-1)]

Timed out.

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \int x\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}dx$$

```
[In] int(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.279 $\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 151

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x} \, dx = -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} - \frac{a^{3/2}e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}} + \frac{b^{3/2}e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}$$

```
[Out] -a^(3/2)*e^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1
/2))/c^(3/2)+b^(3/2)*e^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
b^(1/2)/e^(1/2))/d^(3/2)-(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1981, 1980, 490, 536, 214}

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x} \, dx = -\frac{a^{3/2}e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}} + \frac{b^{3/2}e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}} - \frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}$$

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d)) - (a^(3/2)*e^(3/
2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/
c^(3/2) + (b^(3/2)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2
)])/(Sqrt[b]*Sqrt[e])])/d^(3/2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\left(\frac{e(a+bx)}{c+dx}\right)^{3/2}}{x}\,dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{x^4}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{((bc-ad)e)\text{Subst}\left(\int \frac{-abe^2+(bc+ad)ex^2}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{cd}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{(a^2e^2)\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{c} + \frac{(b^2e^2)\,\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{d}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} - \frac{a^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}} + \frac{b^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}$$

Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.24

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-a^{3/2}d^{3/2}\sqrt{c+dx^2}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right) + \sqrt{c}\left(\sqrt{d}(-bc+ad)\sqrt{a+bx^2} + \right.\right.}{c^{3/2}d^{3/2}\sqrt{a+bx^2}}$$

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(a^(3/2)*d^(3/2)*Sqrt[c + d*x^2]*Arc
Tanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]) + Sqrt[c]*(Sqrt[
d]*(-(b*c) + a*d)*Sqrt[a + b*x^2] + b^(3/2)*c*Sqrt[c + d*x^2]*ArcTanh[(Sqrt
[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])))/(c^(3/2)*d^(3/2)*Sqrt[a
+ b*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(121) = 242.

Time = 0.14 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.66

method	result
default	$\frac{\left(-\sqrt{bd}\,\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2\sqrt{ac}\,\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}+2ac}{x^{2}}\right)a^{2}d^{2}x^{2}+\ln\left(\frac{2bd\,x^{2}+2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\sqrt{ac}\,b^{2}cd\,x^{2}-\sqrt{bd}}{}$

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)+2*a*c)/x^2)*a^2*d^2*x^2+ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*b^2*c*d*x^2
-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)+2*a*c)/x^2)*a^2*c*d+ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*b^2*c^2+2*(b*d)^(1/2
)*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*a*d-2*(b*d)^(1/2)*((d*x^2+c)*(b*x
^2+a))^(1/2)*(a*c)^(1/2)*b*c)/c/d*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(
a*c)^(1/2)/(b*d)^(1/2)/(b*x^2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.90 (sec) , antiderivative size = 1049, normalized size of antiderivative = 6.95

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \left[\frac{bc\sqrt{\frac{be}{d}}e\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)e+4\,(2\,bd^3x^4+\cdots\right.}{\cdots}\right.$$

$$-\frac{2\,bc\sqrt{-\frac{be}{d}}e\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-\frac{be}{d}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(b^2ex^2+abe)}\right)-ad\sqrt{\frac{ae}{c}}e\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2-4\cdots}{x\cdots}\right.}{4\,cd}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 +
 (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (
```

```
3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + a*
d*sqrt(a*e/c)*e*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e +
8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3
 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) - 4*
(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d), -1/4*(2*b*c*sqrt(-b
*e/d)*e*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e
)/(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) - a*d*sqrt(a*e/c)*e*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b
*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt
((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) + 4*(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c)))/(c*d), 1/4*(2*a*d*sqrt(-a*e/c)*e*arctan(1/2*((b*c + a*d)*x^2
+ 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2*e)
) + b*c*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (
b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*
b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4*(b
*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d), 1/2*(a*d*sqrt(-a*e/c)
*e*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c))/(a*b*e*x^2 + a^2*e)) - b*c*sqrt(-b*e/d)*e*arctan(1/2*(2*b*d*x^2
 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e*x^2 + a
*b*e)) - 2*(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x}\,dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x, x)
```

3.280 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \frac{3(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2} + \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{3\sqrt{a}(bc-ad)e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}}$$

```
[Out] 1/2*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/c/(a-c*(b*x^2+a)/(d*x^2+c))-3/
2*(-a*d+b*c)*e^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/
e^(1/2))*a^(1/2)/c^(5/2)+3/2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1981, 1980, 294, 327, 214}

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = -\frac{3\sqrt{a}e^{3/2}(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}} + \frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2} + \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (3*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*c^2) + ((b*c - a*d)*
((e*(a + b*x^2))/(c + d*x^2))^(3/2))/(2*c*(a - (c*(a + b*x^2))/(c + d*x^2))
) - (3*Sqrt[a]*(b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*c^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\left(\frac{e(a+bx)}{c+dx}\right)^{3/2}}{x^2}\,dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{x^4}{(-ae+cx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(3(bc-ad)e)\text{Subst}\left(\int \frac{x^2}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2c}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2} + \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(3a(bc-ad)e^2)\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2c^2}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2} + \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{3\sqrt{a}(bc-ad)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}}$$

Mathematica [A] (verified)

Time = 3.80 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{c}\sqrt{a+bx^2}(2bcx^2-a(c+3dx^2)) - 3\sqrt{a}(bc-ad)x^2\sqrt{c+dx^2}\text{arctanh}\left(\right.\right.}{2c^{5/2}x^2\sqrt{a+bx^2}}$$

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[c]*Sqrt[a + b*x^2]*(2*b*c*x^2 -
a*(c + 3*d*x^2)) - 3*Sqrt[a]*(b*c - a*d)*x^2*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[
c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(2*c^(5/2)*x^2*Sqrt[a + b*
x^2])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{a(d\,x^2+c)e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{2c^2x^2}-\frac{\left(-\frac{3a(ad-bc)\ln\left(\frac{2ace+(eda+ebc)x^2+2\sqrt{ace}\,\sqrt{bde\,x^4+(eda+ebc)x^2+ace}}{x^2}\right)}{2\sqrt{ace}}-\frac{\left(-2a^2d^2+4abcd-2b^2c^2\right)\left(b\,x^2+\right.}{(ad-bc)\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+}}\right.}{2c^2(b\,x^2+a)}$
default	$-\frac{\left(-2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{ac}\,b\,d^2x^6-3\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}+2ac}{x^2}\right)a^2c\,d^2x^4+3\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{}}{}\right.\right.}{}$

[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x,method=_RETURNVERBOSE)

[Out] -1/2*a/c^2*(d*x^2+c)/x^2*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/2/c^2*(-3/2*a*(a*d-b*c)/(a*c*e)^(1/2)*ln((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/x^2)-(-2*a^2*d^2+4*a*b*c*d-2*b^2*c^2)*(b*x^2+a)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))*e/(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.91 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.12

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx=\left[-\frac{3\,(bc-ad)\sqrt{\frac{ae}{c}}ex^2\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)\,ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)\,ex^2+4\,((bc^2d+acd^2)x^4+2\,ac^3}{x^4}\right.}{8\,c^2x^2}\right.$$

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")

[Out] [-1/8*(3*(b*c - a*d)*sqrt(a*e/c)*e*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) - 4*((2*b*c - 3*a*d)*e*x^2 - a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c^2*x^2), 1/4*(3*(b*c - a*d)*sqrt(-a*e/c)*e*x^2*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2*e)) + 2*((2*b*c - 3*a*d)*e*x^2 - a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c^2*x^2)]

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{2,[0,1,0]%%%},[6,0,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1,1,
1]%%%}
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^3}\,dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3, x)
```

3.281 $\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 256

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = -\frac{d(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^3} - \frac{a(bc-ad)^2e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} + \frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{3(bc-5ad)(bc-ad)e^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8\sqrt{a}c^{7/2}}$$

```
[Out] -3/8*(-5*a*d+b*c)*(-a*d+b*c)*e^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c)
)^(1/2)/a^(1/2)/e^(1/2))/c^(7/2)/a^(1/2)-d*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2
+c))^(1/2)/c^3-1/4*a*(-a*d+b*c)^2*e^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^3/(a*
e-c*e*(b*x^2+a)/(d*x^2+c))^2+1/8*(-9*a*d+5*b*c)*(-a*d+b*c)*e^2*(e*(b*x^2+a)
/(d*x^2+c))^(1/2)/c^3/(a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used

$= \{1981, 1980, 466, 1171, 396, 214\}$

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = -\frac{3e^{3/2}(bc-5ad)(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8\sqrt{a}c^{7/2}} - \frac{ae^3(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} + \frac{e^2(5bc-9ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{de(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^3}$$

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] -((d*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^3) - (a*(b*c - a*d)
^2*e^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*c^3*(a*e - (c*e*(a + b*x^2))/(
c + d*x^2))^2) + ((5*b*c - 9*a*d)*(b*c - a*d)*e^2*Sqrt[(e*(a + b*x^2))/(c +
 d*x^2)])/(8*c^3*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) - (3*(b*c - 5*a*d)*
(b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sq
rt[a]*Sqrt[e])])/(8*Sqrt[a]*c^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
```

```
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\left(\frac{e(a+bx)}{c+dx}\right)^{3/2}}{x^3}\,dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{x^4(be-dx^2)}{(-ae+cx^2)^3}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)$$

$$= -\frac{a(bc-ad)^2e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{((bc-ad)e)\text{Subst}\left(\int \frac{-a(bc-ad)e^2-4c(bc-ad)ex^2+4c^2dx^4}{(-ae+cx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{4c^3}$$

$$= -\frac{a(bc-ad)^2e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} + \frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)\text{Subst}\left(\int \frac{-a(3bc-7ad)e^2+8acdex^2}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{8ac^3}$$

$$
\begin{aligned}
&= -\frac{d(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^3} - \frac{a(bc-ad)^2e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} \\
&\quad + \frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} \\
&\quad + \frac{(3(bc-5ad)(bc-ad)e^2)\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{8c^3} \\
&= -\frac{d(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^3} - \frac{a(bc-ad)^2e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} \\
&\quad + \frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} \\
&\quad - \frac{3(bc-5ad)(bc-ad)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8\sqrt{a}c^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 4.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.73

$$
\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx =
$$

$$
-\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}(bcx^2(5c+13dx^2)+a(2c^2-5cdx^2-15d^2x^4))+3(b^2c^2-6abcd+5a^2d^2)\,x^4\sqrt{c}\right.}{8\sqrt{a}c^{7/2}x^4\sqrt{a+bx^2}}
$$

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] -1/8*(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*
(b*c*x^2*(5*c + 13*d*x^2) + a*(2*c^2 - 5*c*d*x^2 - 15*d^2*x^4)) + 3*(b^2*c^
2 - 6*a*b*c*d + 5*a^2*d^2)*x^4*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*
x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(Sqrt[a]*c^(7/2)*x^4*Sqrt[a + b*x^2])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{(d\,x^2+c)(-7ad\,x^2+5bc\,x^2+2ac)e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{8c^3x^4}+\frac{\left(-\frac{(15a^2d^2-18abcd+3b^2c^2)\ln\left(\frac{2ace+(eda+ebc)x^2+2\sqrt{ace}\,\sqrt{bde\,x^4+(eda+ebc)x^2+}}{x^2}\right.}{2\sqrt{ace}}\right.}{8c^3(b}$
default	Expression too large to display

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(d*x^2+c)*(-7*a*d*x^2+5*b*c*x^2+2*a*c)/c^3/x^4*e*(e*(b*x^2+a)/(d*x^2+c
))^(1/2)+1/8/c^3*(-1/2*(15*a^2*d^2-18*a*b*c*d+3*b^2*c^2)/(a*c*e)^(1/2)*ln((
2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*
e)^(1/2))/x^2)+8*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(b*x^2+a)/(a*d-b*c)/(b*d*e*x
^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))*e/(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c))^(1
/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 2.54 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.70

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \left[\frac{3\,(b^2c^2-6\,abcd+5\,a^2d^2)ex^4\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2-4\,(2\,a}{x^{}}\right.}{}\right.$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*x^4*sqrt(e/(a*c))*log(((b^2*c^
2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2
- 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)
*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) - 4*((13*b*c*d - 15*
a*d^2)*e*x^4 + 2*a*c^2*e + 5*(b*c^2 - a*c*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d
*x^2 + c)))/(c^3*x^4), 1/16*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*x^4*sqrt
(-e/(a*c))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2
 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e)) - 2*((13*b*c*d - 15*a*d^2)*e*x^4 + 2
*a*c^2*e + 5*(b*c^2 - a*c*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c^3
*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{2,[4,1,4]%%%},[2,7,0]%%%}+%%%{%%%{-8,[3,2,4]%%%},[2,6,1]%
%%}+%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^5}\,dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5, x)
```

3.282 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 366

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \frac{d^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^4} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3}$$
$$+\frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(5b^2c^2+50abcd-79a^2d^2\right)e^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48ac^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$
$$+\frac{(bc-ad)\left(b^2c^2+10abcd-35a^2d^2\right)e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{3/2}c^{9/2}}$$

```
[Out] 1/6*(-a*d+b*c)^3*e^2*(e*(b*x^2+a)/(d*x^2+c))^(5/2)/a/c^2/(a*e-c*e*(b*x^2+a)
/(d*x^2+c))^3+1/16*(-a*d+b*c)*(-35*a^2*d^2+10*a*b*c*d+b^2*c^2)*e^(3/2)*arct
anh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))/a^(3/2)/c^(9/2)+
d^2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^4+1/24*(-a*d+b*c)^2*(11*a*
d+b*c)*e^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^4/(a*e-c*e*(b*x^2+a)/(d*x^2+c))^
2-1/48*(-a*d+b*c)*(-79*a^2*d^2+50*a*b*c*d+5*b^2*c^2)*e^2*(e*(b*x^2+a)/(d*x^
2+c))^(1/2)/a/c^4/(a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1981, 1980, 474, 466, 1171, 396, 214}

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = -\frac{e^2(-79a^2d^2+50abcd+5b^2c^2)\,(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48ac^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{e^{3/2}(-35a^2d^2+10abcd+b^2c^2)\,(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{3/2}c^{9/2}} + \frac{d^2e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^4} + \frac{e^3(11ad+bc)(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} + \frac{e^2(bc-ad)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3}$$

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] (d^2*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^4 + ((b*c - a*d)^3*
e^2*((e*(a + b*x^2))/(c + d*x^2))^(5/2))/(6*a*c^2*(a*e - (c*e*(a + b*x^2))/
(c + d*x^2))^3) + ((b*c - a*d)^2*(b*c + 11*a*d)*e^3*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)])/(24*c^4*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)
*(5*b^2*c^2 + 50*a*b*c*d - 79*a^2*d^2)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)
])/(48*a*c^4*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)*(b^2*c^2
 + 10*a*b*c*d - 35*a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(
c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(16*a^(3/2)*c^(9/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\left(\frac{e(a+bx)}{c+dx}\right)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{x^4(be-dx^2)^2}{(-ae+cx^2)^4}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)\text{Subst}\left(\int \frac{x^4(-6b^2c^2e^2+5(bce-ade)^2+6acd^2ex^2)}{(-ae+cx^2)^3}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{6ac^2}$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\text{Subst}\left(\int \frac{ac(bc-ad)(bc+11ad)e^3+4c^2(bc-ad)(bc+11ad)e^2x^2-24ac^3d^2ex^4}{(-ae+cx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{24ac^5}$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(5b^2c^2+50abcd-79a^2d^2\right)e^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48ac^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)\text{Subst}\left(\int \frac{3ac(b^2c^2+10abcd-19a^2d^2)e^3-48a^2c^2d^2e^2x^2}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{48a^2c^5e}$$

$$= \frac{d^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^4} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(5b^2c^2+50abcd-79a^2d^2\right)e^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48ac^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{\left((bc-ad)\left(b^2c^2+10abcd-35a^2d^2\right)e^2\right)\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{16ac^4}$$

$$= \frac{d^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^4} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(5b^2c^2+50abcd-79a^2d^2\right)e^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48ac^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)\left(b^2c^2+10abcd-35a^2d^2\right)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{3/2}c^{9/2}}$$

Mathematica [A] (verified)

Time = 4.76 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.67

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{a}\sqrt{c}\sqrt{a+bx^2}(3b^2c^2x^4(c+dx^2)+2abcx^2(7c^2-19cdx^2-50d^2x^4)+a^2\right.}{}$$

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*(3*
b^2*c^2*x^4*(c + d*x^2) + 2*a*b*c*x^2*(7*c^2 - 19*c*d*x^2 - 50*d^2*x^4) + a
^2*(8*c^3 - 14*c^2*d*x^2 + 35*c*d^2*x^4 + 105*d^3*x^6))) + 3*(b^3*c^3 + 9*a
*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*x^6*Sqrt[c + d*x^2]*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(48*a^(3/2)*c^(9/2)*x^6*S
qrt[a + b*x^2])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{(d\,x^2+c)\left(57a^2d^2x^4-52bdac\,x^4+3b^2c^2x^4-22a^2cd\,x^2+14ab\,c^2x^2+8a^2c^2\right)e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{48c^4x^6a}-\frac{\left(-\frac{\left(35a^3d^3-45a^2bc\,d^2+9d\,c^2b^2a+b^3c^3\right)}{}\right.}{}$
default	Expression too large to display

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(d*x^2+c)*(57*a^2*d^2*x^4-52*a*b*c*d*x^4+3*b^2*c^2*x^4-22*a^2*c*d*x^2
+14*a*b*c^2*x^2+8*a^2*c^2)/c^4/x^6/a*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/16/c
^4/a*(-1/2*(35*a^3*d^3-45*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3)/(a*c*e)^(1/2)*
ln((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+
a*c*e)^(1/2))/x^2)+16*a*d^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(b*x^2+a)/(a*d-b*c)
/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))*e/(b*x^2+a)*(e*(b*x^2+a)/(d*x
^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 9.70 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.57

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \left[\frac{3\,(b^3c^3+9\,ab^2c^2d-45\,a^2bcd^2+35\,a^3d^3)ex^6\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(ab}{}\right.}{}\right.$$

$$-\frac{3\,(b^3c^3+9\,ab^2c^2d-45\,a^2bcd^2+35\,a^3d^3)ex^6\sqrt{-\frac{e}{ac}}\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{ac}}}{2\,(bex^2+ae)}\right)+2\,((3\,b^2c^2d-1}{96\,ac}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(b^3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*x^6*sqr
t(e/(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*
b*c^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*
c^3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^
```

```
4) - 4*((3*b^2*c^2*d - 100*a*b*c*d^2 + 105*a^2*d^3)*e*x^6 + 8*a^2*c^3*e + (
3*b^2*c^3 - 38*a*b*c^2*d + 35*a^2*c*d^2)*e*x^4 + 14*(a*b*c^3 - a^2*c^2*d)*e
*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^4*x^6), -1/96*(3*(b^3*c^3 + 9
*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*x^6*sqrt(-e/(a*c))*arctan(1/2
*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))
/(b*e*x^2 + a*e)) + 2*((3*b^2*c^2*d - 100*a*b*c*d^2 + 105*a^2*d^3)*e*x^6 +
8*a^2*c^3*e + (3*b^2*c^3 - 38*a*b*c^2*d + 35*a^2*c*d^2)*e*x^4 + 14*(a*b*c^3
 - a^2*c^2*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^4*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**7,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{2,[5,1,5]%%%},[2,9,0]%%%}+%%%{%%%{-10,[4,2,5]%%%},[2,8,1]
%%%}+%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^7}\,dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7, x)
```

3.283 $\int x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 391

$$\int x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = -\frac{\left(16ac-\frac{16bc^2}{d}-\frac{a^2d}{b}\right)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d^2} - \frac{ex^3(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} - \frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d^3} + \frac{6bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d^2} - \frac{\sqrt{c}(16b^2c^2-16abcd+a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5bd^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{c^{3/2}(8bc-7ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{5d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -1/5*(16*a*c-16*b*c^2/d-a^2*d/b)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^2-e*x^
3*(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d-1/5*(-7*a*d+8*b*c)*e*x*(d*x^2+c
)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^3+6/5*b*e*x^3*(d*x^2+c)*(e*(b*x^2+a)/(d*x
^2+c))^(1/2)/d^2+1/5*c^(3/2)*(-7*a*d+8*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^
2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))
*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/5*
(a^2*d^2-16*a*b*c*d+16*b^2*c^2)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*E
llipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*
(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1986, 478, 595, 596, 545, 429, 506, 422}

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = -\frac{\sqrt{c}e(a^2d^2-16abcd+16b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5bd^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{ex\left(-\frac{a^2d}{b}+16ac-\frac{16bc^2}{d}\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d^2} + \frac{c^{3/2}e(8bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{5d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{ex(c+dx^2)(8bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d^3} + \frac{6bex^3(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d^2} - \frac{ex^3(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}$$

```
[In] Int[x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -1/5*((16*a*c - (16*b*c^2)/d - (a^2*d)/b)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x
^2)])/d^2 - (e*x^3*(a + b*x^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d - ((8*b
*c - 7*a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*d^3) + (6
*b*e*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*d^2) - (Sqrt[c]*
(16*b^2*c^2 - 16*a*b*c*d + a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b*d^(7/2)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]) + (c^(3/2)*(8*b*c - 7*a*d)*e*Sqrt[(e*(a + b*
x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(5*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^4(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx}{\sqrt{a+bx^2}}\\
&= -\frac{ex^3(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}+\frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2\sqrt{a+bx^2}(3a+6bx^2)}{\sqrt{c+dx^2}}\,dx}{d\sqrt{a+bx^2}}\\
&= -\frac{ex^3(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}+\frac{6bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d^2}\\
&\quad+\frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2(-3a(6bc-5ad)-3b(8bc-7ad)x^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5d^2\sqrt{a+bx^2}}\\
&= -\frac{ex^3(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}-\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d^3}\\
&\quad+\frac{6bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d^2}\\
&\quad-\frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{-3abc(8bc-7ad)-3b(16b^2c^2-16abcd+a^2d^2)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15bd^3\sqrt{a+bx^2}}\\
&= -\frac{ex^3(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}-\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d^3}\\
&\quad+\frac{6bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d^2}\\
&\quad+\frac{\left(ac(8bc-7ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5d^3\sqrt{a+bx^2}}\\
&\quad+\frac{\left((16b^2c^2-16abcd+a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5d^3\sqrt{a+bx^2}}
\end{aligned}$$

$$= \frac{(16b^2c^2-16abcd+a^2d^2)\,ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5bd^3} - \frac{ex^3(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} - \frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d^3} + \frac{6bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d^2} + \frac{c^{3/2}(8bc-7ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\left(c(16b^2c^2-16abcd+a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{5bd^3\sqrt{a+bx^2}}$$

$$= \frac{(16b^2c^2-16abcd+a^2d^2)\,ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5bd^3} - \frac{ex^3(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} - \frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d^3} + \frac{6bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d^2} - \frac{\sqrt{c}(16b^2c^2-16abcd+a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5bd^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{c^{3/2}(8bc-7ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.61 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.68

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(a+bx^2)\left(ad(7c+2dx^2)+b(-8c^2-2cdx^2+d^2x^4)\right)-ic(16b\right.}{}$$

```
[In] Integrate[x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(a*d*(7*c +
 2*d*x^2) + b*(-8*c^2 - 2*c*d*x^2 + d^2*x^4)) - I*c*(16*b^2*c^2 - 16*a*b*c*
d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] + (8*I)*c*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)]))/(5*Sqrt[b/a]*d^4*(a + b*x^2))
```

Maple [A] (verified)

Time = 9.88 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.98

method	result
risch	$\frac{x(bd\,x^2+2ad-3bc)\,(d\,x^2+c)\,e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{5d^3}+\frac{\left(-\frac{2\left(a^2d^2-11abcd+11b^2c^2\right)ace\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)-E\left(x\sqrt{-\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}\,(eda+ebc+e(ad-bc))}}{}$
default	$\frac{\left(\frac{e(b\,x^2+a)}{d\,x^2+c}\right)^{\frac{3}{2}}(d\,x^2+c)\left(\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,b^2d^3x^7+3\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,ab\,d^3x^5-2\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,b^2c\,d\right.}{}$

```
[In] int(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x*(b*d*x^2+2*a*d-3*b*c)*(d*x^2+c)/d^3*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1
/5/d^3*(-2*(a^2*d^2-11*a*b*c*d+11*b^2*c^2)*a*c*e/(-b/a)^(1/2)*(1+b*x^2/a)^(
1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a
+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2
))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))-c*(7*a^2*d^2-1
3*a*b*c*d+5*b^2*c^2)/d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(
b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d*e+b*c*e)/c/b/e)^(1/2))+5*c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d*((b*d*e*x^2+a*
d*e)/c/(a*d-b*c)*x/e/((x^2+c/d)*(b*d*e*x^2+a*d*e))^(1/2)+(1/c-d*a/c/(a*d-b*
c))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2
+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(
1/2))+2*b*d/(a*d-b*c)*a*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2
)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(El
lipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(
1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))))*e/(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c))
^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.70

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx =$$

$$-\frac{(16\,b^2c^3-16\,abc^2d+a^2cd^2)\sqrt{\frac{be}{d}}ex\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(16\,b^2c^3-16\,abc^2d-7\,a^2d^3+(a^2+8\,a}{}$$

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*((16*b^2*c^3 - 16*a*b*c^2*d + a^2*c*d^2)*sqrt(b*e/d)*e*x*sqrt(-c/d)*el
liptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (16*b^2*c^3 - 16*a*b*c^2*d - 7*
a^2*d^3 + (a^2 + 8*a*b)*c*d^2)*sqrt(b*e/d)*e*x*sqrt(-c/d)*elliptic_f(arcsin
(sqrt(-c/d)/x), a*d/(b*c)) - (b^2*d^3*e*x^6 - 2*(b^2*c*d^2 - a*b*d^3)*e*x^4
 + (8*b^2*c^2*d - 9*a*b*c*d^2 + a^2*d^3)*e*x^2 + (16*b^2*c^3 - 16*a*b*c^2*d
 + a^2*c*d^2)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4*x)
```

Sympy [F(-1)]

Timed out.

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \text{Timed out}$$

```
[In] integrate(x**4*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}x^4\,dx$$

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4, x)
```

Giac [F]

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}x^4\,dx$$

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\int x^4\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}\,dx$$

```
[In] int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.284 $\int x^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 310

$$\int x^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = -\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2} - \frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d^2} + \frac{\sqrt{c}(8bc-7ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{c}(4bc-3ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -1/3*(-7*a*d+8*b*c)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^2-e*x*(b*x^2+a)*(e*
(b*x^2+a)/(d*x^2+c))^(1/2)/d+4/3*b*e*x*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1
/2)/d^2+1/3*(-7*a*d+8*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^
2+a)/(d*x^2+c))^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/3*(-3*a*d+4
*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)
/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)
/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1986, 478, 542, 545, 429, 506, 422}

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = -\frac{\sqrt{c}e(4bc-3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}e(8bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{4bex(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2} - \frac{ex(8bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2} - \frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}$$

```
[In] Int[x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -1/3*((8*b*c - 7*a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d^2 - (e*x*(a
+ b*x^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d + (4*b*e*x*Sqrt[(e*(a + b*x^2
))/(c + d*x^2)]*(c + d*x^2))/(3*d^2) + (Sqrt[c]*(8*b*c - 7*a*d)*e*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (Sqrt[c]*(4*b*c
- 3*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/S
qrt[c]], 1 - (b*c)/(a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx}{\sqrt{a+bx^2}} \\ &= -\frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}+\frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}(a+4bx^2)}{\sqrt{c+dx^2}}\,dx}{d\sqrt{a+bx^2}}\end{aligned}$$

$$= -\frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d^2} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{-a(4bc-3ad)-b(8bc-7ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d^2\sqrt{a+bx^2}}$$

$$= -\frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d^2} - \frac{\left(b(8bc-7ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d^2\sqrt{a+bx^2}} - \frac{\left(a(4bc-3ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d^2\sqrt{a+bx^2}}$$

$$= -\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2} - \frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d^2} - \frac{\sqrt{c}(4bc-3ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\left(c(8bc-7ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{3d^2\sqrt{a+bx^2}}$$

$$= -\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2} - \frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d^2} + \frac{\sqrt{c}(8bc-7ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{c}(4bc-3ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.96 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.76

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx =$$

$$-\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(a+bx^2)\left(3ad-b(4c+dx^2)\right)+ibc(-8bc+7ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)\right.\right.}{3\sqrt{\frac{b}{a}}d^3\left(a+bx^2\right)}$$

[In] Integrate[x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]

[Out] -1/3*(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(3*a*d - b*(4*c + d*x^2)) + I*b*c*(-8*b*c + 7*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(8*b^2*c^2 - 11*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*d^3*(a + b*x^2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. 2(350) = 700.

Time = 8.94 (sec) , antiderivative size = 734, normalized size of antiderivative = 2.37

method	result
default	$\frac{\left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{\frac{3}{2}}(d\,x^2+c)\left(\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,b^2d^2x^5+\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^3+\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,b^2cd\,x^3-\right.}{}$
risch	$\frac{bex(d\,x^2+c)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{3d^2}+\frac{\left(-\frac{3c\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{\left(bde\,x^2+eda\right)x}{c(ad-bc)e\sqrt{\left(x^2+\frac{c}{d}\right)\left(bde\,x^2+eda\right)}}+\frac{\left(\frac{1}{c}-\frac{da}{c(ad-bc)}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+c}}\right.}{d}\right.}{}$

[In] int(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/3*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b^2*d^2*x^5+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^3+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^3-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^3+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^3+3*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*d^2-

```
11*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d+8*((d*x^2+c)*(b*x^2+a))^(1/2)
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*b^2*c^2+7*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d-8*((d*x^2+c)*(
b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*b^2*c^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*b
*c*d*x-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a^2*d^2*x+3*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*c*d*x)/(b*x^2+a)^2/d^3/(-b/
a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.74

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \frac{(8\,b^2c^3-7\,abc^2d)\sqrt{\frac{be}{d}}ex\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}) - (8\,b^2c^3-7\,abc^2d+4\,abc}{}$$

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*((8*b^2*c^3 - 7*a*b*c^2*d)*sqrt(b*e/d)*e*x*sqrt(-c/d)*elliptic_e(arcsin
(sqrt(-c/d)/x), a*d/(b*c)) - (8*b^2*c^3 - 7*a*b*c^2*d + 4*a*b*c*d^2 - 3*a^2
*d^3)*sqrt(b*e/d)*e*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)
) + (b^2*c*d^2*e*x^4 - 4*(b^2*c^2*d - a*b*c*d^2)*e*x^2 - (8*b^2*c^3 - 7*a*b
*c^2*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*c*d^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \text{Timed out}$$

```
[In] integrate(x**2*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^2\, dx$$

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^2\, dx$$

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int x^2 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2} dx$$

```
[In] int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.285 $\int \left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 22, antiderivative size = 262

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{(2bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} - \frac{(2bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{b\sqrt{c}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -(-a*d+b*c)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d+(-a*d+2*b*c)*e*x*(e*(b*x^
2+a)/(d*x^2+c))^(1/2)/c/d-(-a*d+2*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(e*(
b*x^2+a)/(d*x^2+c))^(1/2)/d^(3/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+b
*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d
*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^(3
/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1986, 424, 545, 429, 506, 422}

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \frac{b\sqrt{c}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{e(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{ex(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{ex(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}$$

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -(((b*c - a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d)) + ((2*b*c - a*
d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d) - ((2*b*c - a*d)*e*Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(Sqrt[c]*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (b*Sqrt[c
]*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx}{\sqrt{a+bx^2}}\\
&= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}+\frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{abc+b(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{cd\sqrt{a+bx^2}}\\
&= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}+\frac{\left(abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{d\sqrt{a+bx^2}}\\
&\quad+\frac{\left(b(2bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{cd\sqrt{a+bx^2}}
\end{aligned}$$

$$= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{(2bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{b\sqrt{c}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\left((2bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}dx}{d\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{(2bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} - \frac{(2bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{b\sqrt{c}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.79

$$\int\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ibc(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(-bc+a\right.}{\sqrt{\frac{b}{a}}cd^2(}$$

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(I*b*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-
(b*c) + a*d)*(Sqrt[b/a]*d*x*(a + b*x^2) - (2*I)*b*c*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(Sqrt[b/
a]*c*d^2*(a + b*x^2))
```

Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.01

method	result
default	$\frac{\left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{\frac{3}{2}}\left(d\,x^2+c\right)\left(\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^3-\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,b^2cd\,x^3+2\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{}\right.}{}$

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*((b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(-b/a)^(1/2)*a*b*d^2*x^3-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)
*b^2*c*d*x^3+2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c
)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d-2*((d*x^2+c)*(b*x
^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))*b^2*c^2-((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d+2*((
d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(-b/a)^(1/2)*a^2*d^2*x-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a
*b*c*d*x)/(b*x^2+a)^2/d^2/c/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.66

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx =$$

$$-\frac{(2\,bc^2-acd)\sqrt{\frac{be}{d}}ex\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(2\,bc^2-acd+ad^2)\sqrt{\frac{be}{d}}ex\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})}{cd^2x}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -((2*b*c^2 - a*c*d)*sqrt(b*e/d)*e*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)
/x), a*d/(b*c)) - (2*b*c^2 - a*c*d + a*d^2)*sqrt(b*e/d)*e*x*sqrt(-c/d)*elli
ptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (b*c*d*e*x^2 + (2*b*c^2 - a*c*d)*
e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

Giac [F]

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2} dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.286 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 307

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} - \frac{(bc-2ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{c^2dx} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d/x-(-2*a*d+b*c)*e*x*(e*(b*x^
2+a)/(d*x^2+c))^(1/2)/c^2+(-2*a*d+b*c)*e*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^
(1/2)/c^2/d/x+(-2*a*d+b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(e*(b*x^2+a)/(d*
x^2+c))^(1/2)/c^(3/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+b*e*(1/(1+d*x
^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2
),(1-b*c/a/d)^(1/2))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^(1/2)/d^(1/2)/(c*(b*x^
2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1986, 479, 597, 545, 429, 506, 422}

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \frac{e(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{e(c+dx^2)(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2dx} - \frac{ex(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2} - \frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx}$$

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x]
```

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d*x)) - ((b*c - 2*a*
d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^2 + ((b*c - 2*a*d)*e*Sqrt[(e*(a
 + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(c^2*d*x) + ((b*c - 2*a*d)*e*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(c^(3/2)*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (b*e*Sqrt[
(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
```

```
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{(a+bx^2)^{3/2}}{x^2(c+dx^2)^{3/2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} - \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a(bc-2ad)-abdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{cd\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{c^2dx} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a^2bcd-abd(bc-2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{ac^2d\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{c^2dx} + \frac{\left(abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{c\sqrt{a+bx^2}} - \frac{\left(b(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{c^2\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} - \frac{(bc-2ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{c^2dx} + \frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\Big(\mathrm{t}}{\sqrt{c}\sqrt{}} + \frac{\left((bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}dx}{c\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} - \frac{(bc-2ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{c^2dx} + \frac{(bc-2ad)e\sqrt{\frac{e(}{c}}}{c} + \frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.25 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.74

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}dx = -\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}d(a+bx^2)(ac-bcx^2+2adx^2)+ibc(-bc+2ad)x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\right.\right.}{\sqrt{\frac{b}{a}}c^2dx(a+bx^2)}$$

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x]
```

[Out] -((e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*(a + b*x^2)*(a*c - b*c*x^2 + 2*a*d*x^2) + I*b*c*(-(b*c) + 2*a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-(b*c) + a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*c^2*d*x*(a + b*x^2)))

Maple [A] (verified)

Time = 8.05 (sec) , antiderivative size = 670, normalized size of antiderivative = 2.18

method	result
default	$-\frac{\left(\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}\right)^{\frac{3}{2}}\left(d\,x^{2}+c\right)\left(\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{-\frac{b}{a}}\,a b\,d^{2}x^{4}+\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{-\frac{b}{a}}\,a b\,d^{2}x^{4}-\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\right.}{}$
risch	$-\frac{a\left(d\,x^{2}+c\right)e\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{c^{2}x}+\frac{\left(\frac{b^{2}c^{2}\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{d\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}}-\frac{2d\,a^{2}bce\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+}{cb}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}}\right.}{}$

[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x,method=_RETURNVERBOSE)

[Out] -(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^4+(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^4-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^4+((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x-((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x-2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x+((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a^2*d^2*x^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*b*c*d*x^2+(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a^2*d^2*x^2-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*c*d*x^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a^2*c*d)/(b*x^2+a)^2/c^2/x/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.55

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = -\frac{(b^2c-2\,abd)\sqrt{\frac{ace}{d^2}}ex\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(b^2c-(a^2+2\,ab)d)\sqrt{\frac{ace}{d^2}}ex\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)}{ac^2x}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -((b^2*c - 2*a*b*d)*sqrt(a*c*e/d^2)*e*x*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt
(-b/a)), a*d/(b*c)) - (b^2*c - (a^2 + 2*a*b)*d)*sqrt(a*c*e/d^2)*e*x*sqrt(-b
/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (a^2*c*e - (a*b*c - 2*a^2
*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="giac")
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^2}\,dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2, x)
```

3.287 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 383

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{d(7bc-8ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^2dx^3} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^3x} - \frac{\sqrt{d}(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{b(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d/x^3+1/3*d*(-8*a*d+7*b*c)*e*
x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^3+1/3*(-4*a*d+3*b*c)*e*(d*x^2+c)*(e*(b*x^
2+a)/(d*x^2+c))^(1/2)/c^2/d/x^3-1/3*(-8*a*d+7*b*c)*e*(d*x^2+c)*(e*(b*x^2+a)
/(d*x^2+c))^(1/2)/c^3/x+1/3*b*(-4*a*d+3*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x
^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2)
)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^(3/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))
^(1/2)-1/3*(-8*a*d+7*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipti
cE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2
+a)/(d*x^2+c))^(1/2)/c^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1986, 479, 597, 545, 429, 506, 422}

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{be(3bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{d}e(7bc-8ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{e(c+dx^2)(7bc-8ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3x} + \frac{dex(7bc-8ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3} + \frac{e(c+dx^2)(3bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^2dx^3} - \frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3}$$

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x]
```

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d*x^3)) + (d*(7*b*c
- 8*a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*c^3) + ((3*b*c - 4*a*d)*
e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*c^2*d*x^3) - ((7*b*c -
8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*c^3*x) - (Sqrt[d
]*(7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*c^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]) + (b*(3*b*c - 4*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ellipti
cF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*c^(3/2)*Sqrt[d]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{(a+bx^2)^{3/2}}{x^4(c+dx^2)^{3/2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} - \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a(3bc-4ad)+b(2bc-3ad)x^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{cd\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^2dx^3} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a^2d(7bc-8ad)+abd(3bc-4ad)x^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3ac^2d\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^2dx^3} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^3x} - \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{-a^2bcd(3bc-4ad)-a^2bd^2(7bc-8ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a^2c^3d\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^2dx^3} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^3x} + \frac{\left(bd(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3c^3\sqrt{a+bx^2}} + \frac{\left(b(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3c^2\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{d(7bc-8ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^2dx^3} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^3x} + \frac{b(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\left(d(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{3c^2\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{d(7bc-8ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^2dx^3} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^3x} - \frac{\sqrt{d}(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{b(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.63 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.66

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}(a+bx^2)\left(-bcx^2(4c+7dx^2)+a(-c^2+4cdx^2+8d^2x^4)\right)+ibc(-7bc+\right.}{}$$

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*(-(b*c*x^2*(4*c
 + 7*d*x^2)) + a*(-c^2 + 4*c*d*x^2 + 8*d^2*x^4)) + I*b*c*(-7*b*c + 8*a*d)*x
^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)] - (4*I)*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*Sqrt[b/a]*c^
3*x^3*(a + b*x^2))
```

Maple [A] (verified)

Time = 8.28 (sec) , antiderivative size = 790, normalized size of antiderivative = 2.06

method	result
default	$-\frac{\left(\frac{e(b\,x^2+a)}{d\,x^2+c}\right)^{\frac{3}{2}}(d\,x^2+c)\left(-5\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^6+4\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,b^2cd\,x^6-3\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{\right.}{}$
risch	$-\frac{(d\,x^2+c)(-5ad\,x^2+4bc\,x^2+ac)e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{3c^3x^3}-\frac{\left(\frac{abcd\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}-\frac{10a^2b\,d^2ce\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4}}\right.}{}$

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(-5*((d*x^2+c)*(b*x^2+a))^(1/2
)*(-b/a)^(1/2)*a*b*d^2*x^6+4*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b^2*c
*d*x^6-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^6+3*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^6-4*((d*x^2+c)*(b*x
^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))*a*b*c*d*x^3+4*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^
2*x^3+8*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x^3-7*((d*x^2+c)*(b*x^2+
a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*b^2*c^2*x^3-5*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a^2
*d^2*x^4+4*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b^2*c^2*x^4-3*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a^2*d^2*x^4+3*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*c*d*x^4-4*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a
)^(1/2)*a^2*c*d*x^2+5*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*b*c^2*x^2+
((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a^2*c^2)/(b*x^2+a)^2/c^3/x^3/(-b/a
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.56

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{(7\,b^2cd - 8\,abd^2)\sqrt{\frac{ace}{d^2}}ex^3\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - ((3\,ab + 7\,b^2)cd - 4\,(a^2 + 2}{}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*((7*b^2*c*d - 8*a*b*d^2)*sqrt(a*c*e/d^2)*e*x^3*sqrt(-b/a)*elliptic_e(ar
csin(x*sqrt(-b/a)), a*d/(b*c)) - ((3*a*b + 7*b^2)*c*d - 4*(a^2 + 2*a*b)*d^2
)*sqrt(a*c*e/d^2)*e*x^3*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*
c)) - ((7*a*b*c*d - 8*a^2*d^2)*e*x^4 + a^2*c^2*e + 4*(a*b*c^2 - a^2*c*d)*e*
x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^3*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="giac")
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^4}\,dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4, x)
```

3.288 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 480

$$\begin{aligned}\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx =& -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5}\\ &+\frac{d(b^2c^2-16abcd+16a^2d^2)\,ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4}+\frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5c^2dx^5}\\ &-\frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5c^3x^3}-\frac{(b^2c^2-16abcd+16a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5ac^4x}\\ &-\frac{\sqrt{d}(b^2c^2-16abcd+16a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5ac^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\\ &-\frac{b\sqrt{d}(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{5ac^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\end{aligned}$$

```
[Out] -(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d/x^5+1/5*d*(16*a^2*d^2-16*a*
b*c*d+b^2*c^2)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^4+1/5*(-6*a*d+5*b*c)*e
*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^2/d/x^5-1/5*(-8*a*d+7*b*c)*e*(d*
x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^3/x^3-1/5*(16*a^2*d^2-16*a*b*c*d+b^2
*c^2)*e*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^4/x-1/5*(16*a^2*d^2-16*
a*b*c*d+b^2*c^2)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^
2+c))^(1/2)/a/c^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/5*b*(-8*a*d+7*b*c)*
e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*
```

```
x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^(
5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 479, 597, 545, 429, 506, 422}

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}dx = -\frac{\sqrt{d}e(16a^2d^2-16abcd+b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5ac^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{e(c+dx^2)(16a^2d^2-16abcd+b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4x} + \frac{dex(16a^2d^2-16abcd+b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4} - \frac{b\sqrt{d}e(7bc-8ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{5ac^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{e(c+dx^2)(7bc-8ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5c^3x^3} + \frac{e(c+dx^2)(5bc-6ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5c^2dx^5} - \frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5}$$

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x]
```

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d*x^5)) + (d*(b^2*c^
2 - 16*a*b*c*d + 16*a^2*d^2)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(5*a*c^
4) + ((5*b*c - 6*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*c
^2*d*x^5) - ((7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2
))/(5*c^3*x^3) - ((b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*e*Sqrt[(e*(a + b*x^2)
)/(c + d*x^2)]*(c + d*x^2))/(5*a*c^4*x) - (Sqrt[d]*(b^2*c^2 - 16*a*b*c*d +
16*a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*a*c^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]) - (b*Sqrt[d]*(7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*a*c^(5/2)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{(a+bx^2)^{3/2}}{x^6(c+dx^2)^{3/2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} - \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{a(5bc-6ad)+b(4bc-5ad)x^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{cd\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5c^2dx^5} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{3a^2d(7bc-8ad)+3abd(5bc-6ad)x^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5ac^2d\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5c^2dx^5} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5c^3x^3} - \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{-3a^2d(b^2c^2-16abcd+16a^2d^2)+3a^2bd^2(7bc-8ad)x^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15a^2c^3d\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5c^2dx^5} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5c^3x^3} - \frac{(b^2c^2-16abcd+16a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5ac^4x} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{-3a^3bcd^2(7bc-8ad)+3a^2bd^2(b^2c^2-16abcd+16a^2d^2)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15a^3c^4d\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5c^2dx^5} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5c^3x^3} - \frac{(b^2c^2-16abcd+16a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5ac^4x} - \frac{\left(bd(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5c^3\sqrt{a+bx^2}} + \frac{\left(bd(b^2c^2-16abcd+16a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5ac^4\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{d(b^2c^2-16abcd+16a^2d^2)\,ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5c^2dx^5} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5c^3x^3} - \frac{(b^2c^2-16abcd+16a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5ac^4x} - \frac{b\sqrt{d}(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5ac^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\left(d(b^2c^2-16abcd+16a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{5ac^3\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{d(b^2c^2-16abcd+16a^2d^2)\,ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5c^2dx^5} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5c^3x^3} - \frac{(b^2c^2-16abcd+16a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5ac^4x} - \frac{\sqrt{d}(b^2c^2-16abcd+16a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5ac^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{b\sqrt{d}(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5ac^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.23 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.67

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = -\frac{\sqrt{\frac{b}{a}}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}(a+bx^2)\left(b^2c^2x^4(c+dx^2)+abcx^2(2c^2-9cdx^2-16d^2x^4)+a^2(c^3-2c^2dx^2+8cd^2x^4-\right.\right.}{}$$

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x]
```

```
[Out] -1/5*(Sqrt[b/a]*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*
(b^2*c^2*x^4*(c + d*x^2) + a*b*c*x^2*(2*c^2 - 9*c*d*x^2 - 16*d^2*x^4) + a^2
*(c^3 - 2*c^2*d*x^2 + 8*c*d^2*x^4 + 16*d^3*x^6)) + I*b*c*(b^2*c^2 - 16*a*b*
c*d + 16*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(b^2*c^2 - 9*a*b*c*d + 8*a^2*d^2)
*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)]))/(b*c^4*x^5*(a + b*x^2))
```

Maple [A] (verified)

Time = 10.05 (sec) , antiderivative size = 908, normalized size of antiderivative = 1.89

method	result
risch	$-\frac{(d\,x^2+c)(11a^2d^2x^4-11bdac\,x^4+b^2c^2x^4-3a^2cd\,x^2+2ab\,c^2x^2+a^2c^2)e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{5c^4x^5a}+\frac{d\left(-\frac{2b^2a\,c^2\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1}\right.}{\sqrt{-\frac{b}{a}}\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+a}}\right.}{}$
default	Expression too large to display

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(d*x^2+c)*(11*a^2*d^2*x^4-11*a*b*c*d*x^4+b^2*c^2*x^4-3*a^2*c*d*x^2+2*a
*b*c^2*x^2+a^2*c^2)/c^4/x^5/a*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/5/c^4/a*d*(
-2*b^2*a*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+
a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)
/c/b/e)^(1/2))+3*a^2*b*c*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/
2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d*e+b*c*e)/c/b/e)^(1/2))-2*(11*a^2*b*d^2-11*a*b^2*c*d+b^3*c^2)*a*c*e/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e
*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(
a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e
```

```
)^(1/2)))-5*(a^2*d^2-2*a*b*c*d+b^2*c^2)*a*c*((b*d*e*x^2+a*d*e)/c/(a*d-b*c)*
x/e/((x^2+c/d)*(b*d*e*x^2+a*d*e))^(1/2)+(1/c-d*a/c/(a*d-b*c))/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))+2*b*d/(a*d-
b*c)*a*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*
e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^
(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b
*c*e)/c/b/e)^(1/2)))))*e/(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)
*e*(b*x^2+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.61

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \frac{(b^3c^2d-16\,ab^2cd^2+16\,a^2bd^3)\sqrt{\frac{ace}{d^2}}ex^5\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(b^3c^2d-(7\,a^2}{}$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/5*((b^3*c^2*d - 16*a*b^2*c*d^2 + 16*a^2*b*d^3)*sqrt(a*c*e/d^2)*e*x^5*sqrt
(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (b^3*c^2*d - (7*a^2*b
+ 16*a*b^2)*c*d^2 + 8*(a^3 + 2*a^2*b)*d^3)*sqrt(a*c*e/d^2)*e*x^5*sqrt(-b/a)
*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((a*b^2*c^2*d - 16*a^2*b*c*d
^2 + 16*a^3*d^3)*e*x^6 + a^3*c^3*e + (a*b^2*c^3 - 9*a^2*b*c^2*d + 8*a^3*c*d
^2)*e*x^4 + 2*(a^2*b*c^3 - a^3*c^2*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c)))/(a^2*c^4*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \text{Timed out}$$

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**6,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="maxima")
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="giac")
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^6}\,dx$$

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6, x)
```

3.289 $\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx=\frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}(1+x^2)-\arctan\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)$$

```
[Out] -arctan(((-x^2+1)/(x^2+1))^(1/2))+1/2*(x^2+1)*((-x^2+1)/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1981, 1979, 294, 210}

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx=\frac{1}{2}\sqrt{\frac{1-x^2}{x^2+1}}(x^2+1)-\arctan\left(\sqrt{\frac{1-x^2}{x^2+1}}\right)$$

```
[In] Int[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
```

```
[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*(1 + x^2))/2 - ArcTan[Sqrt[(1 - x^2)/(1 + x^2)]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \sqrt{\frac{1-x}{1+x}}\,dx, x, x^2\right) \\
&= -\left(2\text{Subst}\left(\int \frac{x^2}{(-1-x^2)^2}\,dx, x, \sqrt{\frac{1-x^2}{1+x^2}}\right)\right) \\
&= \frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}(1+x^2) + \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x^2}{1+x^2}}\right) \\
&= \frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}(1+x^2) - \tan^{-1}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.86

$$
\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx = \frac{\sqrt{\frac{1-x^2}{1+x^2}}\left(\sqrt{1-x^2}(1+x^2) + 4\sqrt{1+x^2}\arctan\left(\frac{\sqrt{1-x^2}}{\sqrt{2}-\sqrt{1+x^2}}\right)\right)}{2\sqrt{1-x^2}}
$$

```
[In] Integrate[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*(Sqrt[1 - x^2]*(1 + x^2) + 4*Sqrt[1 + x^2]*ArcTa
n[Sqrt[1 - x^2]/(Sqrt[2] - Sqrt[1 + x^2])]))/(2*Sqrt[1 - x^2])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{-\frac{x^{2}-1}{x^{2}+1}}\,\left(x^{2}+1\right)\left(\arcsin\left(x^{2}\right)+\sqrt{-x^{4}+1}\right)}{2\sqrt{-\left(x^{2}+1\right)\left(x^{2}-1\right)}}$	52
risch	$\frac{\left(x^{2}+1\right)\sqrt{-\frac{x^{2}-1}{x^{2}+1}}}{2}-\frac{\arcsin\left(x^{2}\right)\sqrt{-\frac{x^{2}-1}{x^{2}+1}}\,\sqrt{-\left(x^{2}+1\right)\left(x^{2}-1\right)}}{2\left(x^{2}-1\right)}$	68
trager	$\left(\frac{x^{2}}{2}+\frac{1}{2}\right)\sqrt{-\frac{x^{2}-1}{x^{2}+1}}+\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{x^{2}-1}{x^{2}+1}}\,x^{2}+\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{x^{2}-1}{x^{2}+1}}+x^{2}\right)}{2}$	88

```
[In] int(x*((-x^2+1)/(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(-(x^2-1)/(x^2+1))^(1/2)*(x^2+1)*(arcsin(x^2)+(-x^4+1)^(1/2))/(-(x^2+1)
*(x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx=\frac{1}{2}\left(x^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}}-\arctan\left(\frac{\left(x^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}}-1}{x^2}\right)$$

```
[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="fricas")
[Out] 1/2*(x^2 + 1)*sqrt(-(x^2 - 1)/(x^2 + 1)) - arctan(((x^2 + 1)*sqrt(-(x^2 - 1
)/(x^2 + 1)) - 1)/x^2)
```

Sympy [F]

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx=\int x\sqrt{-\frac{(x-1)(x+1)}{x^2+1}}\,dx$$

```
[In] integrate(x*((-x**2+1)/(x**2+1))**(1/2),x)
[Out] Integral(x*sqrt(-(x - 1)*(x + 1)/(x**2 + 1)), x)
```

Maxima [F]

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx = \int x\sqrt{-\frac{x^2-1}{x^2+1}}\,dx$$

```
[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="maxima")
[Out] integrate(x*sqrt(-(x^2 - 1)/(x^2 + 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.35

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx = \frac{1}{2}\sqrt{-x^4+1} + \frac{1}{2}\arcsin\left(x^2\right)$$

```
[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^4 + 1) + 1/2*arcsin(x^2)
```

Mupad [B] (verification not implemented)

Time = 17.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx = -\mathrm{atan}\left(\sqrt{-\frac{x^2-1}{x^2+1}}\right) - \frac{\sqrt{-\frac{x^2-1}{x^2+1}}}{\frac{x^2-1}{x^2+1}-1}$$

```
[In] int(x*(-(x^2 - 1)/(x^2 + 1))^(1/2),x)
[Out] - atan((-(x^2 - 1)/(x^2 + 1))^(1/2)) - (-(x^2 - 1)/(x^2 + 1))^(1/2)/((x^2 -
 1)/(x^2 + 1) - 1)
```

3.290 $\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx=\frac{1}{10}\sqrt{\frac{5-7x^2}{7+5x^2}}(7+5x^2)-\frac{37\arctan\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{7+5x^2}}\right)}{5\sqrt{35}}$$

```
[Out] -37/175*arctan(1/7*35^(1/2)*((-7*x^2+5)/(5*x^2+7))^(1/2))*35^(1/2)+1/10*(5*
x^2+7)*((-7*x^2+5)/(5*x^2+7))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1981, 1979, 294, 210}

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx=\frac{1}{10}\sqrt{\frac{5-7x^2}{5x^2+7}}(5x^2+7)-\frac{37\arctan\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{5x^2+7}}\right)}{5\sqrt{35}}$$

```
[In] Int[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

```
[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(7 + 5*x^2))/10 - (37*ArcTan[Sqrt[5/7]*Sqrt[
(5 - 7*x^2)/(7 + 5*x^2)]])/(5*Sqrt[35])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \sqrt{\frac{5-7x}{7+5x}}\,dx, x, x^2\right)\\
&= -\left(74\text{Subst}\left(\int \frac{x^2}{\left(-7-5x^2\right)^2}\,dx, x, \sqrt{\frac{5-7x^2}{7+5x^2}}\right)\right)\\
&= \frac{1}{10}\sqrt{\frac{5-7x^2}{7+5x^2}}\left(7+5x^2\right)+\frac{37}{5}\text{Subst}\left(\int \frac{1}{-7-5x^2}\,dx, x, \sqrt{\frac{5-7x^2}{7+5x^2}}\right)\\
&= \frac{1}{10}\sqrt{\frac{5-7x^2}{7+5x^2}}\left(7+5x^2\right)-\frac{37\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{7+5x^2}}\right)}{5\sqrt{35}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.65

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx = \frac{\sqrt{\frac{5-7x^2}{7+5x^2}}\left(35\sqrt{5-7x^2}(7+5x^2)+148\sqrt{35}\sqrt{7+5x^2}\arctan\left(\frac{\sqrt{5}\sqrt{5-7x^2}}{\sqrt{74}-\sqrt{7}\sqrt{7+5x^2}}\right)\right)}{350\sqrt{5-7x^2}}$$

```
[In] Integrate[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

```
[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(35*Sqrt[5 - 7*x^2]*(7 + 5*x^2) + 148*Sqrt[3
5]*Sqrt[7 + 5*x^2]*ArcTan[(Sqrt[5]*Sqrt[5 - 7*x^2])/(Sqrt[74] - Sqrt[7]*Sqr
t[7 + 5*x^2])]))/(350*Sqrt[5 - 7*x^2])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

method	result
default	$\frac{\sqrt{-\frac{7x^2-5}{5x^2+7}}\,(5x^2+7)\left(37\sqrt{35}\,\arcsin\left(\frac{35x^2}{37}+\frac{12}{37}\right)+35\sqrt{-35x^4-24x^2+35}\right)}{350\sqrt{-(7x^2-5)(5x^2+7)}}$
risch	$\frac{(5x^2+7)\sqrt{-\frac{7x^2-5}{5x^2+7}}}{10}-\frac{37\sqrt{35}\,\arcsin\left(\frac{35x^2}{37}+\frac{12}{37}\right)\sqrt{-\frac{7x^2-5}{5x^2+7}}\,\sqrt{-(7x^2-5)(5x^2+7)}}{350(7x^2-5)}$
trager	$7\left(\frac{x^2}{14}+\frac{1}{10}\right)\sqrt{-\frac{7x^2-5}{5x^2+7}}-\frac{37\,\mathrm{RootOf}\left(\_Z^2+35\right)\ln\left(35\,\mathrm{RootOf}\left(\_Z^2+35\right)x^2+175\sqrt{-\frac{7x^2-5}{5x^2+7}}\,x^2+12\,\mathrm{RootOf}\left(\_Z^2+35\right)+\right.}{350}$

```
[In] int(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/350*(-(7*x^2-5)/(5*x^2+7))^(1/2)*(5*x^2+7)*(37*35^(1/2)*arcsin(35/37*x^2+
12/37)+35*(-35*x^4-24*x^2+35)^(1/2))/(-(7*x^2-5)*(5*x^2+7))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx = \frac{1}{10}\,(5\,x^2+7)\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}-\frac{37}{350}\,\sqrt{35}\arctan\left(\frac{\sqrt{35}(35\,x^2+12)\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{35\,(7\,x^2-5)}\right)$$

```
[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(5*x^2 + 7)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7)) - 37/350*sqrt(35)*arctan(1/
35*sqrt(35)*(35*x^2 + 12)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(7*x^2 - 5))
```

Sympy [F]

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx=\int x\sqrt{-\frac{7x^2-5}{5x^2+7}}\,dx$$

```
[In] integrate(x*((-7*x**2+5)/(5*x**2+7))**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(7*x**2 - 5)/(5*x**2 + 7)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx=-\frac{37}{175}\sqrt{35}\arctan\left(\frac{1}{7}\sqrt{35}\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}\right)-\frac{37\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{5\left(\frac{5\,(7\,x^2-5)}{5\,x^2+7}-7\right)}$$

```
[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="maxima")
```

```
[Out] -37/175*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))) - 37/5
*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(5*(7*x^2 - 5)/(5*x^2 + 7) - 7)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.42

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx=\frac{37}{350}\sqrt{35}\arcsin\left(\frac{35}{37}\,x^2+\frac{12}{37}\right)+\frac{1}{10}\sqrt{-35\,x^4-24\,x^2+35}$$

```
[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="giac")
```

```
[Out] 37/350*sqrt(35)*arcsin(35/37*x^2 + 12/37) + 1/10*sqrt(-35*x^4 - 24*x^2 + 35
)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx = -\frac{37\sqrt{35}\,\mathrm{atan}\left(\frac{\sqrt{5}\sqrt{7}\sqrt{-\frac{7x^2-5}{5x^2+7}}}{7}\right)}{175} - \frac{37\sqrt{5}\sqrt{7}\sqrt{35}\sqrt{-\frac{7x^2-5}{5x^2+7}}}{1225\left(\frac{5x^2-\frac{25}{7}}{5x^2+7}-1\right)}$$

```
[In] int(x*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2),x)
[Out] - (37*35^(1/2)*atan((5^(1/2)*7^(1/2)*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2))/7))/
175 - (37*5^(1/2)*7^(1/2)*35^(1/2)*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2))/(1225*
((5*x^2 - 25/7)/(5*x^2 + 7) - 1))
```

3.291 $\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\frac{1}{3}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)-\frac{2}{3}\arctan\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)$$

```
[Out] -2/3*arctan(((-x^3+1)/(x^3+1))^(1/2))+1/3*(x^3+1)*((-x^3+1)/(x^3+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1981, 1979, 294, 210}

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\frac{1}{3}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{2}{3}\arctan\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

```
[In] Int[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/3 - (2*ArcTan[Sqrt[(1 - x^3)/(1 + x^3
)]])/3
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \sqrt{\frac{1-x}{1+x}}\,dx, x, x^3\right)\\
&= -\left(\frac{4}{3}\text{Subst}\left(\int \frac{x^2}{\left(-1-x^2\right)^2}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\right)\\
&= \frac{1}{3}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)+\frac{2}{3}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\\
&= \frac{1}{3}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.79

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \frac{\sqrt{\frac{1-x^3}{1+x^3}}\left(\sqrt{1-x^3}(1+x^3)+4\sqrt{1+x^3}\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{2}-\sqrt{1+x^3}}\right)\right)}{3\sqrt{1-x^3}}$$

```
[In] Integrate[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(Sqrt[1 - x^3]*(1 + x^3) + 4*Sqrt[1 + x^3]*ArcTa
n[Sqrt[1 - x^3]/(Sqrt[2] - Sqrt[1 + x^3])]))/(3*Sqrt[1 - x^3])
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

method	result	size
risch	$\frac{\left(x^{3}+1\right) \sqrt{-\frac{x^{3}-1}{x^{3}+1}}}{3}-\frac{\arcsin\left(x^{3}\right) \sqrt{-\frac{x^{3}-1}{x^{3}+1}}\, \sqrt{-\left(x^{3}+1\right) \left(x^{3}-1\right)}}{3 \left(x^{3}-1\right)}$	68
trager	$\left(\frac{x^{3}}{3}+\frac{1}{3}\right) \sqrt{-\frac{x^{3}-1}{x^{3}+1}}+\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right) \ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right) \sqrt{-\frac{x^{3}-1}{x^{3}+1}}\, x^{3}+x^{3}+\operatorname{RootOf}\left(\_Z^{2}+1\right) \sqrt{-\frac{x^{3}-1}{x^{3}+1}}\right)}{3}$	88

```
[In] int(x^2*((-x^3+1)/(x^3+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(x^3+1)*(-(x^3-1)/(x^3+1))^(1/2)-1/3*arcsin(x^3)*(-(x^3-1)/(x^3+1))^(1/
2)*(-(x^3+1)*(x^3-1))^(1/2)/(x^3-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int x^2 \sqrt{\frac{1-x^3}{1+x^3}}\, dx = \frac{1}{3}\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}} - \frac{2}{3}\arctan\left(\frac{\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}-1}{x^3}\right)$$

```
[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 2/3*arctan(((x^3 + 1)*sqrt(-(x^3
 - 1)/(x^3 + 1)) - 1)/x^3)
```

Sympy [F]

$$\int x^2 \sqrt{\frac{1-x^3}{1+x^3}}\, dx = \int x^{2}\sqrt{-\frac{(x-1)\left(x^{2}+x+1\right)}{x^{3}+1}}\, dx$$

```
[In] integrate(x**2*((-x**3+1)/(x**3+1))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(x - 1)*(x**2 + x + 1)/(x**3 + 1)), x)
```

Maxima [F]

$$\int x^2 \sqrt{\frac{1-x^3}{1+x^3}}\,dx = \int x^2 \sqrt{-\frac{x^3-1}{x^3+1}}\,dx$$

```
[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.42

$$\int x^2 \sqrt{\frac{1-x^3}{1+x^3}}\,dx = \frac{1}{3}\left(\sqrt{-x^6+1} + \arcsin\left(x^3\right)\right)\mathrm{sgn}\left(x^3+1\right)$$

```
[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")
[Out] 1/3*(sqrt(-x^6 + 1) + arcsin(x^3))*sgn(x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 16.81 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int x^2 \sqrt{\frac{1-x^3}{1+x^3}}\,dx = -\frac{2\,\mathrm{atan}\left(\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3} - \frac{2\sqrt{-\frac{x^3-1}{x^3+1}}}{\frac{3\,(x^3-1)}{x^3+1}-3}$$

```
[In] int(x^2*(-(x^3 - 1)/(x^3 + 1))^(1/2),x)
[Out] - (2*atan((-(x^3 - 1)/(x^3 + 1))^(1/2)))/3 - (2*(-(x^3 - 1)/(x^3 + 1))^(1/2
))/((3*(x^3 - 1))/(x^3 + 1) - 3)
```

3.292 $\int x^8 \sqrt{\frac{1-x^3}{1+x^3}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int x^8 \sqrt{\frac{1-x^3}{1+x^3}} \, dx = \frac{1}{2}\sqrt{\frac{1-x^3}{1+x^3}}(1+x^3) - \frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}(1+x^3)^2 - \frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}(1+x^3)^3 - \frac{1}{3}\arctan\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)$$

```
[Out] -1/9*((-x^3+1)/(x^3+1))^(3/2)*(x^3+1)^3-1/3*arctan(((-x^3+1)/(x^3+1))^(1/2)
)+1/2*(x^3+1)*((-x^3+1)/(x^3+1))^(1/2)-1/6*(x^3+1)^2*((-x^3+1)/(x^3+1))^(1/
2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1981, 1980, 474, 466, 393, 210}

$$\int x^8 \sqrt{\frac{1-x^3}{1+x^3}} \, dx = -\frac{1}{3}\arctan\left(\sqrt{\frac{1-x^3}{x^3+1}}\right) - \frac{1}{9}\left(\frac{1-x^3}{x^3+1}\right)^{3/2}(x^3+1)^3 - \frac{1}{6}\sqrt{\frac{1-x^3}{x^3+1}}(x^3+1)^2 + \frac{1}{2}\sqrt{\frac{1-x^3}{x^3+1}}(x^3+1)$$

```
[In] Int[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/2 - (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x
^3)^2)/6 - (((1 - x^3)/(1 + x^3))^(3/2)*(1 + x^3)^3)/9 - ArcTan[Sqrt[(1 - x
^3)/(1 + x^3)]]/3
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int x^2\sqrt{\frac{1-x}{1+x}}\,dx, x, x^3\right)$$
$$= -\left(\frac{4}{3}\text{Subst}\left(\int \frac{x^2(-1+x^2)^2}{(-1-x^2)^4}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\right)$$
$$= -\frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}(1+x^3)^3 - \frac{2}{9}\text{Subst}\left(\int \frac{x^2(6-6x^2)}{(-1-x^2)^3}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)$$
$$= -\frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}(1+x^3)^2 - \frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}(1+x^3)^3 + \frac{1}{18}\text{Subst}\left(\int \frac{12-24x^2}{(-1-x^2)^2}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)$$
$$= \frac{1}{2}\sqrt{\frac{1-x^3}{1+x^3}}(1+x^3) - \frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}(1+x^3)^2 - \frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}(1+x^3)^3 + \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)$$
$$= \frac{1}{2}\sqrt{\frac{1-x^3}{1+x^3}}(1+x^3) - \frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}(1+x^3)^2 - \frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}(1+x^3)^3 - \frac{1}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \frac{\sqrt{\frac{1-x^3}{1+x^3}}\left(\sqrt{1-x^3}(4+x^3-x^6+2x^9) - 6\sqrt{1+x^3}\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{1+x^3}}\right)\right)}{18\sqrt{1-x^3}}$$

```
[In] Integrate[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(Sqrt[1 - x^3]*(4 + x^3 - x^6 + 2*x^9) - 6*Sqrt[
1 + x^3]*ArcTan[Sqrt[1 - x^3]/Sqrt[1 + x^3]]))/(18*Sqrt[1 - x^3])
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

method	result
risch	$\frac{(x^3+1)(2x^6-3x^3+4)\sqrt{-\frac{x^3-1}{x^3+1}}}{18}-\frac{\arcsin(x^3)\sqrt{-\frac{x^3-1}{x^3+1}}\sqrt{-(x^3+1)(x^3-1)}}{6(x^3-1)}$
trager	$\frac{(x^3+1)(2x^6-3x^3+4)\sqrt{-\frac{x^3-1}{x^3+1}}}{18}+\frac{\operatorname{RootOf}(\_Z^2+1)\ln\left(\operatorname{RootOf}(\_Z^2+1)\sqrt{-\frac{x^3-1}{x^3+1}}\,x^3+x^3+\operatorname{RootOf}(\_Z^2+1)\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{6}$

```
[In] int(x^8*((-x^3+1)/(x^3+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(x^3+1)*(2*x^6-3*x^3+4)*(-(x^3-1)/(x^3+1))^(1/2)-1/6*arcsin(x^3)*(-(x^
3-1)/(x^3+1))^(1/2)*(-(x^3+1)*(x^3-1))^(1/2)/(x^3-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\frac{1}{18}\left(2\,x^9-x^6+x^3+4\right)\sqrt{-\frac{x^3-1}{x^3+1}}-\frac{1}{3}\arctan\left(\frac{(x^3+1)\sqrt{-\frac{x^3-1}{x^3+1}}-1}{x^3}\right)$$

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*(2*x^9 - x^6 + x^3 + 4)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1/3*arctan(((x^3
+ 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1)/x^3)
```

Sympy [F(-1)]

Timed out.

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x**8*((-x**3+1)/(x**3+1))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\int x^8\sqrt{-\frac{x^3-1}{x^3+1}}\,dx$$

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")
[Out] integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\frac{1}{6}\arcsin\left(x^3\right)\operatorname{sgn}\left(x^3+1\right)+\frac{1}{18}\sqrt{-x^6+1}\left(\left(2x^3\operatorname{sgn}\left(x^3+1\right)-3\operatorname{sgn}\left(x^3+1\right)\right)x^3+4\operatorname{sgn}\left(x^3+1\right)\right)$$

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")
[Out] 1/6*arcsin(x^3)*sgn(x^3 + 1) + 1/18*sqrt(-x^6 + 1)*((2*x^3*sgn(x^3 + 1) - 3
*sgn(x^3 + 1))*x^3 + 4*sgn(x^3 + 1))
```

Mupad [B] (verification not implemented)

Time = 16.90 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\frac{2\sqrt{-\frac{x^3-1}{x^3+1}}}{9}-\frac{\operatorname{atan}\left(\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3}+\frac{x^3\sqrt{-\frac{x^3-1}{x^3+1}}}{18}-\frac{x^6\sqrt{-\frac{x^3-1}{x^3+1}}}{18}+\frac{x^9\sqrt{-\frac{x^3-1}{x^3+1}}}{9}$$

```
[In] int(x^8*(-(x^3 - 1)/(x^3 + 1))^(1/2),x)
[Out] (2*(-(x^3 - 1)/(x^3 + 1))^(1/2))/9 - atan((-(x^3 - 1)/(x^3 + 1))^(1/2))/3 +
 (x^3*(-(x^3 - 1)/(x^3 + 1))^(1/2))/18 - (x^6*(-(x^3 - 1)/(x^3 + 1))^(1/2))
/18 + (x^9*(-(x^3 - 1)/(x^3 + 1))^(1/2))/9
```

3.293 $\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = -\frac{27}{350}\sqrt{\frac{5-7x^5}{7+5x^5}}(7+5x^5)+\frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}(7+5x^5)^2 + \frac{2257\arctan\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{7+5x^5}}\right)}{875\sqrt{35}}$$

```
[Out] 2257/30625*arctan(1/7*35^(1/2)*((-7*x^5+5)/(5*x^5+7))^(1/2))*35^(1/2)-27/35
0*(5*x^5+7)*((-7*x^5+5)/(5*x^5+7))^(1/2)+1/250*(5*x^5+7)^2*((-7*x^5+5)/(5*x
^5+7))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1981, 1980, 466, 393, 210}

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \frac{2257\arctan\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{875\sqrt{35}} + \frac{1}{250}\sqrt{\frac{5-7x^5}{5x^5+7}}(5x^5+7)^2-\frac{27}{350}\sqrt{\frac{5-7x^5}{5x^5+7}}(5x^5+7)$$

```
[In] Int[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

```
[Out] (-27*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(7 + 5*x^5))/350 + (Sqrt[(5 - 7*x^5)/(7
+ 5*x^5)]*(7 + 5*x^5)^2)/250 + (2257*ArcTan[Sqrt[5/7]*Sqrt[(5 - 7*x^5)/(7 +
 5*x^5)]])/(875*Sqrt[35])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{5}\text{Subst}\left(\int x\sqrt{\frac{5-7x}{7+5x}}\,dx, x, x^5\right)$$

$$= -\left(\frac{148}{5}\text{Subst}\left(\int \frac{x^2(-5+7x^2)}{(-7-5x^2)^3}\,dx, x, \sqrt{\frac{5-7x^5}{7+5x^5}}\right)\right)$$

$$
\begin{aligned}
&= \frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)^2 + \frac{37}{125}\text{Subst}\left(\int \frac{-74+140x^2}{(-7-5x^2)^2}\,dx, x, \sqrt{\frac{5-7x^5}{7+5x^5}}\right) \\
&= -\frac{27}{350}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right) + \frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)^2 \\
&\quad - \frac{2257}{875}\text{Subst}\left(\int \frac{1}{-7-5x^2}\,dx, x, \sqrt{\frac{5-7x^5}{7+5x^5}}\right) \\
&= -\frac{27}{350}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right) + \frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)^2 + \frac{2257\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{7+5x^5}}\right)}{875\sqrt{35}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$
\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \frac{\sqrt{\frac{5-7x^5}{7+5x^5}}\left(35\sqrt{5-7x^5}(-602-185x^5+175x^{10}) + 4514\sqrt{35}\sqrt{7+5x^5}\arctan\left(\frac{\sqrt{\frac{25}{7}-5x^5}}{\sqrt{7+5x^5}}\right)\right)}{61250\sqrt{5-7x^5}}
$$

```
[In] Integrate[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

```
[Out] (Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(35*Sqrt[5 - 7*x^5]*(-602 - 185*x^5 + 175*x^
10) + 4514*Sqrt[35]*Sqrt[7 + 5*x^5]*ArcTan[Sqrt[25/7 - 5*x^5]/Sqrt[7 + 5*x^
5]]))/(61250*Sqrt[5 - 7*x^5])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.78 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08

method	result
trager	$\frac{(5x^5+7)(35x^5-86)\sqrt{-\frac{7x^5-5}{5x^5+7}}}{1750} + \frac{2257\,\text{RootOf}\left(\_Z^2+35\right)\ln\left(175\sqrt{-\frac{7x^5-5}{5x^5+7}}\,x^5+35\,\text{RootOf}\left(\_Z^2+35\right)x^5+245\sqrt{-\frac{7x^5-5}{5x^5+7}}+12\,\text{R}\right.}{61250}$
risch	$\frac{(5x^5+7)(35x^5-86)\sqrt{-\frac{7x^5-5}{5x^5+7}}}{1750} + \frac{2257\,\text{RootOf}\left(\_Z^2+35\right)\ln\left(-35\,\text{RootOf}\left(\_Z^2+35\right)x^5+35\sqrt{-35x^{10}-24x^5+35}-12\,\text{RootOf}\left(\_\right.\right.}{61250(7x^5-5)}$

```
[In] int(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1750*(5*x^5+7)*(35*x^5-86)*(-(7*x^5-5)/(5*x^5+7))^(1/2)+2257/61250*RootOf
(_Z^2+35)*ln(175*(-(7*x^5-5)/(5*x^5+7))^(1/2)*x^5+35*RootOf(_Z^2+35)*x^5+24
5*(-(7*x^5-5)/(5*x^5+7))^(1/2)+12*RootOf(_Z^2+35))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \frac{1}{1750}\left(175\,x^{10}-185\,x^5-602\right)\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}} + \frac{2257}{61250}\,\sqrt{35}\arctan\left(\frac{\sqrt{35}(35\,x^5+12)\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{35\,(7\,x^5-5)}\right)$$

```
[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1750*(175*x^10 - 185*x^5 - 602)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7)) + 2257/612
50*sqrt(35)*arctan(1/35*sqrt(35)*(35*x^5 + 12)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7
))/(7*x^5 - 5))
```

Sympy [F(-1)]

Timed out.

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \text{Timed out}$$

```
[In] integrate(x**9*((-7*x**5+5)/(5*x**5+7))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \frac{2257}{30625}\,\sqrt{35}\arctan\left(\frac{1}{7}\,\sqrt{35}\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}\right) - \frac{37\left(675\left(-\frac{7\,x^5-5}{5\,x^5+7}\right)^{\frac{3}{2}}+427\,\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}\right)}{875\left(\frac{25\,(7\,x^5-5)^2}{(5\,x^5+7)^2}-\frac{70\,(7\,x^5-5)}{5\,x^5+7}+49\right)}$$

```
[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="maxima")
```

```
[Out] 2257/30625*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7))) - 3
7/875*(675*(-(7*x^5 - 5)/(5*x^5 + 7))^(3/2) + 427*sqrt(-(7*x^5 - 5)/(5*x^5
+ 7)))/(25*(7*x^5 - 5)^2/(5*x^5 + 7)^2 - 70*(7*x^5 - 5)/(5*x^5 + 7) + 49)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \frac{1}{61250}\left(35\sqrt{-35\,x^{10}-24\,x^5+35}\big(35\,x^5-86\big)-2257\sqrt{35}\arcsin\left(\frac{35}{37}\,x^5+\frac{12}{37}\right)\right)\mathrm{sgn}\big(5\,x^5+7\big)$$

```
[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="giac")
```

```
[Out] 1/61250*(35*sqrt(-35*x^10 - 24*x^5 + 35)*(35*x^5 - 86) - 2257*sqrt(35)*arcs
in(35/37*x^5 + 12/37))*sgn(5*x^5 + 7)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \frac{2257\sqrt{35}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{7}\,\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{7}\right)}{30625}-\frac{43\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{4375}-\frac{37\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,x^5\,\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{12250}+\frac{\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,x^{10}\,\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{350}$$

```
[In] int(x^9*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2),x)
```

```
[Out] (2257*35^(1/2)*atan((5^(1/2)*7^(1/2)*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/7))/
30625 - (43*5^(1/2)*7^(1/2)*35^(1/2)*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/4375
 - (37*5^(1/2)*7^(1/2)*35^(1/2)*x^5*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/12250
 + (5^(1/2)*7^(1/2)*35^(1/2)*x^10*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/350
```

3.294 $\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx=\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{-1+x^2}\arctan\left(\frac{\sqrt{-1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

```
[Out] 1/2*arctan(1/2*(x^2-1)^(1/2)*2^(1/2))*(-x^2/(-x^2+1))^(1/2)*(x^2-1)^(1/2)/x
*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1986, 15, 455, 65, 209}

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx=\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\arctan\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

```
[In] Int[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-(x^2/(1 - x^2))]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sqr
t[2]*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\right)\int\frac{\sqrt{x^2}}{\sqrt{-1+x^2}(1+x^2)}\,dx}{\sqrt{x^2}}\\
&= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\right)\int\frac{x}{\sqrt{-1+x^2}(1+x^2)}\,dx}{x}\\
&= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{-1+x}(1+x)}\,dx,x,x^2\right)}{2x}\\
&= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\right)\operatorname{Subst}\left(\int\frac{1}{2+x^2}\,dx,x,\sqrt{-1+x^2}\right)}{x}\\
&= \frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{-1+x^2}\tan^{-1}\left(\frac{\sqrt{-1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \frac{\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\arctan\left(\frac{\sqrt{-1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

```
[In] Integrate[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
[Out] (Sqrt[x^2/(-1 + x^2)]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sqrt[
2]*x)
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\sqrt{\frac{x^2}{x^2-1}}\,\sqrt{x^2-1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{x^2-1}\,\sqrt{2}}{2}\right)}{2x}$	42
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{-x^3\operatorname{RootOf}\left(\_Z^2+2\right)+4x^2\sqrt{\frac{x^2}{x^2-1}}+3\operatorname{RootOf}\left(\_Z^2+2\right)x-4\sqrt{\frac{x^2}{x^2-1}}}{x\left(x^2+1\right)}\right)}{4}$	75

```
[In] int((x^2/(x^2-1))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*(x^2/(x^2-1))^(1/2)/x*(x^2-1)^(1/2)*2^(1/2)*arctan(1/2*(x^2-1)^(1/2)*2^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2-1)\sqrt{\frac{x^2}{x^2-1}}}{2\,x}\right)$$

```
[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 - 1)*sqrt(x^2/(x^2 - 1))/x)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

```
[In] integrate((x**2/(x**2-1))**(1/2)/(x**2+1),x)
[Out] Integral(sqrt(x**2/(x**2 - 1))/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

```
[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt(x^2/(x^2 - 1))/(x^2 + 1), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-1}\right)\operatorname{sgn}\left(x^2-1\right)\operatorname{sgn}(x) + \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}i\sqrt{2}\right)\operatorname{sgn}(x)$$

```
[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="giac")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 1))*sgn(x^2 - 1)*sgn(x) + 1/2*sqr
t(2)*arctan(1/2*I*sqrt(2))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

```
[In] int((x^2/(x^2 - 1))^(1/2)/(x^2 + 1),x)
[Out] int((x^2/(x^2 - 1))^(1/2)/(x^2 + 1), x)
```

3.295 $\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 68

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2} dx = \frac{\sqrt{-\frac{x^2}{1-a-(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\arctan\left(\frac{\sqrt{-1+a+(1+a)x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

```
[Out] 1/2*arctan(1/2*(-1+a+(1+a)*x^2)^(1/2)*2^(1/2))*(-x^2/(1-a-(1+a)*x^2))^(1/2)
*(-1+a+(1+a)*x^2)^(1/2)/x*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1986, 15, 455, 65, 211}

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2} dx = \frac{\sqrt{-\frac{x^2}{-((a+1)x^2)-a+1}}\sqrt{(a+1)x^2+a-1}\arctan\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

```
[In] Int[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-(x^2/(1 - a - (1 + a)*x^2))]*Sqrt[-1 + a + (1 + a)*x^2]*ArcTan[Sqrt[
-1 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\int \frac{\sqrt{x^2}}{(1+x^2)\sqrt{-1+a+(1+a)x^2}}\,dx}{\sqrt{x^2}}\\
&= \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\int \frac{x}{(1+x^2)\sqrt{-1+a+(1+a)x^2}}\,dx}{x}\\
&= \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1+a+(1+a)x}}\,dx,x,x^2\right)}{2x}\\
&= \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{-1+a}{1+a}+\frac{x^2}{1+a}}\,dx,x,\sqrt{-1+a+(1+a)x^2}\right)}{(1+a)x}\\
&= \frac{\sqrt{-\frac{x^2}{1-a-(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\tan^{-1}\left(\frac{\sqrt{-1+a+(1+a)x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \frac{\sqrt{-1+a+x^2+ax^2}\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\arctan\left(\frac{\sqrt{-1+a+(1+a)x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

```
[In] Integrate[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
[Out] (Sqrt[-1 + a + x^2 + a*x^2]*Sqrt[x^2/(-1 + a + (1 + a)*x^2)]*ArcTan[Sqrt[-1
 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\sqrt{\frac{x^2}{a\,x^2+x^2+a-1}}\,\sqrt{a\,x^2+x^2+a-1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{a\,x^2+x^2+a-1}\,\sqrt{2}}{2}\right)}{2x}$	60

```
[In] int((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*(x^2/(a*x^2+x^2+a-1))^(1/2)/x*(a*x^2+x^2+a-1)^(1/2)*2^(1/2)*arctan(1/2*
(a*x^2+x^2+a-1)^(1/2)*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \frac{1}{4}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}((a+1)x^2+a-3)\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{4\,x}\right)$$

```
[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*((a + 1)*x^2 + a - 3)*sqrt(x^2/((a + 1)*x^2
+ a - 1))/x)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{x^2}{ax^2+a+x^2-1}}}{x^2+1}\,dx$$

```
[In] integrate((x**2/(-1+a+(1+a)*x**2))**(1/2)/(x**2+1),x)
[Out] Integral(sqrt(x**2/(a*x**2 + a + x**2 - 1))/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{x^2+1}\,dx$$

```
[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt(x^2/((a + 1)*x^2 + a - 1))/(x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{ax^2+x^2+a-1}\right)\operatorname{sgn}\left(ax^2+x^2+a-1\right)\operatorname{sgn}(x) - \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{a-1}\right)\operatorname{sgn}(a-1)\operatorname{sgn}(x)$$

```
[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="giac")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + x^2 + a - 1))*sgn(a*x^2 + x^2 +
 a - 1)*sgn(x) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a - 1))*sgn(a - 1)*sgn
(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{x^2}{(a+1)\,x^2+a-1}}}{x^2+1}\,dx$$

```
[In] int((x^2/(a + x^2*(a + 1) - 1))^(1/2)/(x^2 + 1),x)
[Out] int((x^2/(a + x^2*(a + 1) - 1))^(1/2)/(x^2 + 1), x)
```

3.296 $\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 281

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{(b^2c^2+2abcd+5a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{16b^3d^2e} - \frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}{6bd(bc-ad)e} + \frac{(bc-ad)(b^2c^2+2abcd+5a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{7/2}d^{5/2}\sqrt{e}}$$

```
[Out] 1/16*(-a*d+b*c)*(5*a^2*d^2+2*a*b*c*d+b^2*c^2)*arctanh(d^(1/2)*(e*(b*x^2+a)/
(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/b^(7/2)/d^(5/2)/e^(1/2)+1/16*(5*a^2*d^2+2
*a*b*c*d+b^2*c^2)*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^3/d^2/e-1/24*(5
*a*d+3*b*c)*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/d^2/e-1/6*(d*x^2+
c)^3*(a-c*(b*x^2+a)/(d*x^2+c))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d/(-a*d+b*c)
/e
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1981, 1980, 424, 393, 205, 214}

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{(bc-ad)\left(5a^2d^2+2abcd+b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{7/2}d^{5/2}\sqrt{e}} + \frac{(c+dx^2)\left(5a^2d^2+2abcd+b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^3d^2e} - \frac{(c+dx^2)^2(5ad+3bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24b^2d^2e} - \frac{(c+dx^2)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6bde(bc-ad)}$$

```
[In] Int[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d
*x^2))/(16*b^3*d^2*e) - ((3*b*c + 5*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*
(c + d*x^2)^2)/(24*b^2*d^2*e) - (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x
^2)^3*(a - (c*(a + b*x^2))/(c + d*x^2)))/(6*b*d*(b*c - a*d)*e) + ((b*c - a*
d)*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/
(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(7/2)*d^(5/2)*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{\frac{e(a+bx)}{c+dx}}}\,dx, x, x^2\right)\\
&= ((bc-ad)e)\text{Subst}\left(\int \frac{(-ae+cx^2)^2}{(be-dx^2)^4}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)\\
&= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}{6bd(bc-ad)e}\\
&\quad -\frac{(bc-ad)\text{Subst}\left(\int \frac{-a(bc+5ad)e^2+3c(bc+ad)ex^2}{(be-dx^2)^3}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{6bd}\\
&= -\frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24b^2d^2e}-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}{6bd(bc-ad)e}\\
&\quad +\frac{((bc-ad)\left(b^2c^2+2abcd+5a^2d^2\right)e)\text{Subst}\left(\int \frac{1}{(be-dx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{8b^2d^2}
\end{aligned}$$

$$= \frac{(b^2c^2+2abcd+5a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{16b^3d^2e} - \frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}{6bd(bc-ad)e} + \frac{((bc-ad)(b^2c^2+2abcd+5a^2d^2))\operatorname{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{16b^3d^2}$$

$$= \frac{(b^2c^2+2abcd+5a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{16b^3d^2e} - \frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}{6bd(bc-ad)e} + \frac{(bc-ad)(b^2c^2+2abcd+5a^2d^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{7/2}d^{5/2}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\left(\sqrt{d}\sqrt{a+bx^2}\sqrt{\frac{b(c+dx^2)}{bc-ad}}(15a^2d^2-2abd(2c+5dx^2)+b^2(-3c^2+2cdx^2+8d^2x^4))+3\sqrt{bc-ad}(\right.}{48b^3d^{5/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{\frac{b(c+dx^2)}{bc-ad}}}$$

```
[In] Integrate[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]
*(15*a^2*d^2 - 2*a*b*d*(2*c + 5*d*x^2) + b^2*(-3*c^2 + 2*c*d*x^2 + 8*d^2*x^
4)) + 3*Sqrt[b*c - a*d]*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcSinh[(Sqrt[d]*
Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(48*b^3*d^(5/2)*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.86

method	result
risch	$\frac{(8b^2d^2x^4-10ab\,d^2x^2+2b^2cd\,x^2+15a^2d^2-4abcd-3b^2c^2)(b\,x^2+a)}{48b^3d^2\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}-\frac{(5a^3d^3-3a^2bc\,d^2-d\,c^2b^2a-b^3c^3)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^2}{\sqrt{bde}}+\sqrt{bde}\right.}{32b^3d^2\sqrt{bde}\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(}$
default	$\frac{(b\,x^2+a)\left(-36\sqrt{bd}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,ab\,d^2x^2-12\sqrt{bd}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,b^2cd\,x^2-15\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}{2\sqrt{bd}}\right.\right.}{}$

```
[In] int(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(8*b^2*d^2*x^4-10*a*b*d^2*x^2+2*b^2*c*d*x^2+15*a^2*d^2-4*a*b*c*d-3*b^2
*c^2)*(b*x^2+a)/b^3/d^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/32*(5*a^3*d^3-3*a^2
*b*c*d^2-a*b^2*c^2*d-b^3*c^3)/b^3/d^2*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)/(b
*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)/(e*(b*
x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.94

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$= \left[-\frac{3\,(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3)\sqrt{bde}\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)e\right.}{}\right.$$

$$-\frac{3\,(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3)\sqrt{-bde}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(b^2dex^2+abde)}\right)-2\,(8\,b^3d^4x^6-3\,b^3c^3d}{96\,b^4d^3e}$$

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(b*d*e)*
log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d +
```

```
a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b
*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)) - 4*(8*b^3*d^4*x^6 - 3*b^3*c^3*d -
 4*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2*d^4)*x^4 - (b^3*c
^2*d^2 + 14*a*b^2*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c)))/(b^4*d^3*e), -1/96*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d
^3)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*
x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(8*b^3*d^4*x^6 - 3*b^3
*c^3*d - 4*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2*d^4)*x^4
- (b^3*c^2*d^2 + 14*a*b^2*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c)))/(b^4*d^3*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

```
[In] integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{2\sqrt{bdex^4+bcex^2+adex^2+ace}\left(2x^2\left(\frac{4x^2}{be}+\frac{b^2cde-5abd^2e}{b^3d^2e^2}\right)-\frac{3b^2c^2e+4abcde-15a^2d^2e}{b^3d^2e^2}\right)-\frac{3(b^3c^3+ab^2c^2d+3a^2bcd^2-5}{}}{96\,\mathrm{sgn}\left(dx^2+c\right)}$$

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)*(2*x^2*(4*x^2/(b*e)
 + (b^2*c*d*e - 5*a*b*d^2*e)/(b^3*d^2*e^2)) - (3*b^2*c^2*e + 4*a*b*c*d*e -
15*a^2*d^2*e)/(b^3*d^2*e^2)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5
*a^3*d^3)*log(abs(-b*c*e - a*d*e - 2*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*
d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))))/(sqrt(b*d*e)*b^3*d^2))/sgn(d*x^
2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^5}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

```
[In] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.297 $\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 169

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} \, dx = -\frac{(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8b^2de} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4bde} - \frac{(bc-ad)(bc+3ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}}$$

```
[Out] -1/8*(-a*d+b*c)*(3*a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b
^(1/2)/e^(1/2))/b^(5/2)/d^(3/2)/e^(1/2)-1/8*(3*a*d+b*c)*(d*x^2+c)*(e*(b*x^2
+a)/(d*x^2+c))^(1/2)/b^2/d/e+1/4*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
b/d/e
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

$= \{1981, 1980, 393, 205, 214\}$

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{(bc-ad)(3ad+bc)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}} - \frac{(c+dx^2)\,(3ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^2de} + \frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bde}$$

```
[In] Int[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] -1/8*((b*c + 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(b^2*d*e
) + (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*b*d*e) - ((b*c - a
*d)*(b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt
[b]*Sqrt[e])])/(8*b^(5/2)*d^(3/2)*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{\frac{e(a+bx)}{c+dx}}}\,dx, x, x^2\right)\\
&= ((bc-ad)e)\text{Subst}\left(\int \frac{-ae+cx^2}{(be-dx^2)^3}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)\\
&= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4bde} - \frac{((bc-ad)(bc+3ad)e)\text{Subst}\left(\int \frac{1}{(be-dx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{4bd}\\
&= -\frac{(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8b^2de} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4bde}\\
&\quad - \frac{((bc-ad)(bc+3ad))\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{8b^2d}\\
&= -\frac{(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8b^2de} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4bde}\\
&\quad - \frac{(bc-ad)(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{\sqrt{b}\sqrt{d}(a+bx^2)\sqrt{c+dx^2}(-3ad+b(c+2dx^2)) - (b^2c^2+2abcd-3a^2d^2)\sqrt{a+bx^2}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{5/2}d^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

```
[In] Integrate[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[b]*Sqrt[d]*(a + b*x^2)*Sqrt[c + d*x^2]*(-3*a*d + b*(c + 2*d*x^2)) - (
b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a +
b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(5/2)*d^(3/2)*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\left(-2 b d \,x^{2}+3 a d -b c \right) \left(b \,x^{2}+a \right)}{8 b^{2} d \sqrt{\frac{e \left(b \,x^{2}+a \right)}{d \,x^{2}+c}}}+\frac{\left(3 a^{2} d^{2}-2 a b c d -b^{2} c^{2}\right) \ln \left(\frac{\frac{1}{2} e d a +\frac{1}{2} e b c +b d e \,x^{2}}{\sqrt{b d e}}+\sqrt{b d e \,x^{4}+\left(e d a +e b c \right) x^{2}+a c e}\right) \sqrt{\left(d \,x^{2}+c \right) e \left(b \,x^{2}+a \right)}}{16 b^{2} d \sqrt{b d e}\, \sqrt{\frac{e \left(b \,x^{2}+a \right)}{d \,x^{2}+c}}\, \left(d \,x^{2}+c \right)}$
default	$-\frac{\left(b \,x^{2}+a \right) \left(-4 \sqrt{b d}\, \sqrt{b d \,x^{4}+a d \,x^{2}+b c \,x^{2}+a c}\, b d \,x^{2}-3 \ln \left(\frac{2 b d \,x^{2}+2 \sqrt{b d \,x^{4}+a d \,x^{2}+b c \,x^{2}+a c}\, \sqrt{b d}+a d +b c}{2 \sqrt{b d}}\right) a^{2} d^{2}+2 \ln \left(\frac{2 b d \,x^{2}+2 \sqrt{b d \,x^{4}+a d}}{}\right.\right.}{16 \sqrt{\frac{e \left(b x\right.}{d x}}}$

```
[In] int(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-2*b*d*x^2+3*a*d-b*c)*(b*x^2+a)/b^2/d/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1
/16*(3*a^2*d^2-2*a*b*c*d-b^2*c^2)/b^2/d*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)/
(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)/(e*(
b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.44

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \left[-\frac{(b^2c^2+2\,abcd-3\,a^2d^2)\sqrt{bde}\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)e+4\,(2\,bd^2x^4\right.}{3}\right.$$

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4 +
 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d
^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 +
 a*e)/(d*x^2 + c))) - 4*(2*b^2*d^3*x^4 + b^2*c^2*d - 3*a*b*c*d^2 + 3*(b^2*c
```

```
*d^2 - a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e), 1/16*(
(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b*c
+ a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*
e)) + 2*(2*b^2*d^3*x^4 + b^2*c^2*d - 3*a*b*c*d^2 + 3*(b^2*c*d^2 - a*b*d^3)*
x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

```
[In] integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{2\sqrt{bdex^4+bcex^2+adex^2+ace}\left(\frac{2x^2}{be}+\frac{bce-3ade}{b^2de^2}\right)+\frac{(b^2c^2+2abcd-3a^2d^2)\log\left(\left|-bce-ade-2\sqrt{bde}\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex\ldots}\right.\right.\right.}{\sqrt{bde}b^2d}}{16\,\mathrm{sgn}\left(dx^2+c\right)}$$

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)*(2*x^2/(b*e) + (b*c
*e - 3*a*d*e)/(b^2*d*e^2)) + (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*log(abs(-b*c
*e - a*d*e - 2*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 +
a*d*e*x^2 + a*c*e))))/(sqrt(b*d*e)*b^2*d))/sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^3}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

```
[In] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.298 $\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 106

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{2be}+\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}$$

```
[Out] 1/2*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2
))/b^(3/2)/d^(1/2)/e^(1/2)+1/2*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/e
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1981, 1979, 205, 214}

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}+\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be}$$

```
[In] Int[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(2*b*e) + ((b*c - a*d)*ArcT
anh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(3
/2)*Sqrt[d]*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{\frac{e(a+bx)}{c+dx}}}\,dx, x, x^2\right) \\
&= ((bc-ad)e)\text{Subst}\left(\int \frac{1}{(be-dx^2)^2}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right) \\
&= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{2be} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2b} \\
&= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{2be} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.28

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{\sqrt{b}\sqrt{d}(a+bx^2)(c+dx^2)+(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

```
[In] Integrate[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[b]*Sqrt[d]*(a + b*x^2)*(c + d*x^2) + (b*c - a*d)*Sqrt[a + b*x^2]*Sqrt
[c + d*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(
2*b^(3/2)*Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.45

method	result
risch	$\frac{b\,x^2+a}{2b\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}-\frac{(ad-bc)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^2}{\sqrt{bde}}+\sqrt{bde\,x^4+(eda+ebc)x^2+ace}\right)\sqrt{(d\,x^2+c)e(b\,x^2+a)}}{4b\sqrt{bde}\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$-\frac{(b\,x^2+a)\left(d\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a-c\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)b-2\sqrt{bd\,x^4+ad}\right.}{4\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,\sqrt{(d\,x^2+c)(b\,x^2+a)}\,b\sqrt{bd}}$

```
[In] int(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(b*x^2+a)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/4*(a*d-b*c)/b*ln((1/2*e*d*a
+1/2*e*b*c+b*d*e*x^2)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/
2))/(b*d*e)^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/
2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.95

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \left[-\frac{\sqrt{bde}(bc-ad)\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)e-4\,(2\,bd^2x^4+bc^2+acd+\right.}{8\,b^2de}\right.$$

$$\left.-\frac{\sqrt{-bde}(bc-ad)\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(b^2dex^2+abde)}\right)-2\,(bd^2x^2+bcd)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{4\,b^2de}\right]$$

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(b*d*e)*(b*c - a*d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*
e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d
+ (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4
*(b*d^2*x^2 + b*c*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e), -1/4*(sq
rt(-b*d*e)*(b*c - a*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt
((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(b*d^2*x^2 + b*c
*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

```
[In] integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.34

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\frac{2\sqrt{bdex^4+bcex^2+adex^2+ace}}{be} - \frac{\sqrt{bde}(bc-ad)\log\left(\left|-2\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right)bd-\sqrt{bde}bc-\sqrt{bde}ad\right|\right)}{b^2de}}{4\,\mathrm{sgn}\left(dx^2+c\right)}$$

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)/(b*e) - sqrt(b*d*e)*
(b*c - a*d)*log(abs(-2*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*
e*x^2 + a*c*e))*b*d - sqrt(b*d*e)*b*c - sqrt(b*d*e)*a*d))/(b^2*d*e))/sgn(d*
x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

```
[In] int(x/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.299 $\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 112

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}+\frac{\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}$$

```
[Out] -arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))*c^(1/2)/a^(
1/2)/e^(1/2)+arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))
*d^(1/2)/b^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1981, 1980, 400, 214}

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=\frac{\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}$$

```
[In] Int[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqr
t[e])])/(Sqrt[a]*Sqrt[e])) + (Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[b]*Sqrt[e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{e(a+bx)}{c+dx}}}\,dx, x, x^2\right)\\
&= ((bc-ad)e)\text{Subst}\left(\int \frac{1}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)\\
&= c\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)+d\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)\\
&= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.31

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\left(-\sqrt{b}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{a}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{\sqrt{a}\sqrt{b}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

```
[In] Integrate[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-(Sqrt[b]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt
[a]*Sqrt[c + d*x^2])]) + Sqrt[a]*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/
(Sqrt[b]*Sqrt[c + d*x^2])]))/(Sqrt[a]*Sqrt[b]*Sqrt[(e*(a + b*x^2))/(c + d*x
^2)]*Sqrt[c + d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(84) = 168.

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.60

method	result	siz
default	$-\frac{(b\,x^2+a)\left(c\ln\left(\frac{a d\,x^2+b c\,x^2+2\sqrt{a c}\,\sqrt{b d\,x^4+a d\,x^2+b c\,x^2+a c}+2 a c}{x^2}\right)\sqrt{b d}-\ln\left(\frac{2 b d\,x^2+2\sqrt{b d\,x^4+a d\,x^2+b c\,x^2+a c}\,\sqrt{b d}+a d+b c}{2\sqrt{b d}}\right)\sqrt{a c}\,d\right)}{2\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{b d}\,\sqrt{a c}}$	17

```
[In] int(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*x^2+a)*(c*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)+2*a*c)/x^2)*(b*d)^(1/2)-ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*d)/(e*(b*x^
2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 881, normalized size of antiderivative = 7.87

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \left[\frac{1}{4}\sqrt{\frac{d}{be}}\log\left(8\,b^2d^2x^4 + b^2c^2 + 6\,abcd + a^2d^2 + 8\,(b^2cd + abd^2)x^2 + 4\,(2\,b^2d^2x^4 + b^2c^2 + abcd + (3\,b^2cd + abd^2)x^2)\sqrt{\frac{bex^2 + ae}{dx^2 + c}}\sqrt{\frac{d}{be}}\right) + \frac{1}{4}\sqrt{\frac{c}{ae}}\log\left(\frac{(b^2c^2 + 6\,abcd + a^2d^2)x^4 + 8\,a^2c^2 + 8\,(abc^2 + a^2cd)x^2 - 4\,((abcd + a^2d^2)x^4 + 2\,a^2c^2 + (ab\right.\right.$$

$$\left.\frac{}{x^4}\right) - \frac{1}{2}\sqrt{-\frac{d}{be}}\arctan\left(\frac{(2\,bdx^2 + bc + ad)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{d}{be}}}{2\,(bdx^2 + ad)}\right) + \frac{1}{4}\sqrt{\frac{c}{ae}}\log\left(\frac{(b^2c^2 + 6\,abcd + a^2d^2)x^4 + 8\,a^2c^2 + 8\,(abc^2 + a^2cd)x^2 - 4\,((abcd + a^2d^2)x^4 + 2\,a^2c^2 + (ab}{x^4}\right.$$

$$+ \frac{1}{4}\sqrt{\frac{d}{be}}\log\left(8\,b^2d^2x^4 + b^2c^2 + 6\,abcd + a^2d^2 + 8\,(b^2cd + abd^2)x^2 + 4\,(2\,b^2d^2x^4 + b^2c^2 + abcd + (3\,b^2cd + abd^2)x^2)\sqrt{\frac{bex^2 + ae}{dx^2 + c}}\sqrt{\frac{d}{be}}\right), \frac{1}{2}\sqrt{-\frac{c}{ae}}\arctan\left(\frac{((bc + ad)x^2 + 2}{2\,(bc}\right.$$

$$\left. - \frac{1}{2}\sqrt{-\frac{d}{be}}\arctan\left(\frac{(2\,bdx^2 + bc + ad)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{d}{be}}}{2\,(bdx^2 + ad)}\right)\right]$$

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b
^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d +
 a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) + 1/4*sqrt(
c/(a*e))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2
+ a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*
c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4), -1/2*sqrt(
-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) + 1/4*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d
 + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4), 1/2*sqrt(-c/(a*e))*arctan(1/2*((b*c +
a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2
 + a*c)) + 1/4*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*
d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3
*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))),
```

```
1/2*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e
)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) - 1/2*sqrt(-d/(b*e))*arctan(
1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-d/(b*e)
)/(b*d*x^2 + a*d))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{x\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

```
[In] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
[Out] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

3.300 $\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}+\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}$$

```
[Out] 1/2*(-a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2
))/a^(3/2)/c^(1/2)/e^(1/2)+1/2*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/(
a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1981, 1980, 205, 214}

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}+\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

```
[In] Int[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] ((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*a*(a*e - (c*e*(a + b*x^2
))/(c + d*x^2))) + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(3/2)*Sqrt[c]*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{\frac{e(a+bx)}{c+dx}}}\,dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{1}{(-ae+cx^2)^2}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2a}$$

$$= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{-\sqrt{a}\sqrt{c}(a+bx^2)\,(c+dx^2)+(bc-ad)x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{c}x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

```
[In] Integrate[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[c]*(a + b*x^2)*(c + d*x^2)) + (b*c - a*d)*x^2*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^
2])])/(2*a^(3/2)*Sqrt[c]*x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.25

method	result
risch	$-\frac{b\,x^2+a}{2a\,x^2\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}-\frac{(ad-bc)\ln\left(\frac{2ace+(eda+ebc)x^2+2\sqrt{ace}\,\sqrt{bde\,x^4+(eda+ebc)x^2+ace}}{x^2}\right)\sqrt{(d\,x^2+c)e(b\,x^2+a)}}{4a\sqrt{ace}\,\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$-\frac{(b\,x^2+a)\left(-2bd\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,x^4\sqrt{ac}+a^2\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}+2ac}{x^2}\right)dc\,x^2-c^2\ln\left(\frac{ad\,x^2+bc\,x^2+2\cdots}{\cdots}\right.\right.}{4\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\cdots}$

```
[In] int(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*(b*x^2+a)/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/4*(a*d-b*c)/a/(a*c*e)^
(1/2)*ln((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e
)*x^2+a*c*e)^(1/2))/x^2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+
a))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.56

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$= \left[-\frac{\sqrt{ace}(bc-ad)x^2 \log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2-4\,((bcd+ad^2)x^4+2\,ac^2+(bc^2+3\,acd)x^2)\sqrt{ace}\sqrt{\frac{bex^2}{dx^2}}}{x^4}\right.}{8\,a^2cex^2}\right.$$

$$\left. -\frac{\sqrt{-ace}(bc-ad)x^2 \arctan\left(\frac{\sqrt{-ace}((bc+ad)x^2+2\,ac)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(abcex^2+a^2ce)}\right)+2\,(acdx^2+ac^2)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{4\,a^2cex^2}\right]$$

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(a*c*e)*(b*c - a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x
^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c*d + a*d^2)*x^4 + 2
*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c
)))/x^4) + 4*(a*c*d*x^2 + a*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c*
e*x^2), -1/4*(sqrt(-a*c*e)*(b*c - a*d)*x^2*arctan(1/2*sqrt(-a*c*e)*((b*c +
a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*c*e*x^2 + a^2*c*e)
) + 2*(a*c*d*x^2 + a*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c*e*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(110) = 220$.

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.83

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{\frac{(bc-ad)\arctan\left(-\frac{\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}a}+\frac{\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right)bc+\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right.}{\left(ace-\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right)^2\right)a}}{2\,\mathrm{sgn}\left(dx^2+c\right)}$$

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*((b*c - a*d)*arctan(-(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a
*d*e*x^2 + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e)*a) + ((sqrt(b*d*e)*x^2 - sqr
t(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*b*c + (sqrt(b*d*e)*x^2 - sqrt
(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a*d + 2*sqrt(b*d*e)*a*c)/((a*c
*e - (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^2)
*a))/sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{x^3\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

```
[In] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

3.301 $\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 218

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)(3bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{5/2}c^{3/2}\sqrt{e}}$$

```
[Out] -1/8*(-a*d+b*c)*(a*d+3*b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a
^(1/2)/e^(1/2))/a^(5/2)/c^(3/2)/e^(1/2)-1/4*(-a*d+b*c)^2*e*(e*(b*x^2+a)/(d*
x^2+c))^(1/2)/a/c/(a*e-c*e*(b*x^2+a)/(d*x^2+c))^2-1/8*(-a*d+b*c)*(a*d+3*b*c
)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^2/c/(a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

$= \{1981, 1980, 393, 205, 214\}$

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{(ad+3bc)(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{5/2}c^{3/2}\sqrt{e}} - \frac{(ad+3bc)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{e(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

```
[In] Int[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -1/4*((b*c - a*d)^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(a*c*(a*e - (c*e*(
a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)*(3*b*c + a*d)*Sqrt[(e*(a + b*x^2
))/(c + d*x^2)])/(8*a^2*c*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) - ((b*c -
a*d)*(3*b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqr
t[a]*Sqrt[e])])/(8*a^(5/2)*c^(3/2)*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
```

```
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt{\frac{e(a+bx)}{c+dx}}}\,dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{be-dx^2}{(-ae+cx^2)^3}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{((bc-ad)(3bc+ad)e)\text{Subst}\left(\int \frac{1}{(-ae+cx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{4ac}$$

$$= -\frac{(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{((bc-ad)(3bc+ad))\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{8a^2c}$$

$$= -\frac{(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)(3bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{5/2}c^{3/2}\sqrt{e}}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a}\sqrt{c}(a+bx^2)\sqrt{c+dx^2}(3bcx^2-a(2c+dx^2))-(3b^2c^2-2abcd-a^2d^2)\,x^4\sqrt{a+bx^2}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{3/2}x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

```
[In] Integrate[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[a]*Sqrt[c]*(a + b*x^2)*Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(2*c + d*x^2))
- (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*x^4*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sq
rt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*c^(3/2)*x^4*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{(b\,x^2+a)\,(a d\,x^2-3bc\,x^2+2ac)}{8a^2x^4c\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}+\frac{(a^2d^2+2abcd-3b^2c^2)\ln\left(\frac{2ace+(eda+ebc)x^2+2\sqrt{ace}\,\sqrt{bde\,x^4+(eda+ebc)x^2+ace}}{x^2}\right)\sqrt{(d\,x^2+c)e(b\,x^2+}}{16c\,a^2\sqrt{ace}\,\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$-\frac{(b\,x^2+a)\left(2b\,d^2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,x^6a\sqrt{ac}+10b^2d\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,x^6c\sqrt{ac}-a^3\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+b}}{x^2}\right.\right.}{}$

```
[In] int(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(b*x^2+a)*(a*d*x^2-3*b*c*x^2+2*a*c)/a^2/x^4/c/(e*(b*x^2+a)/(d*x^2+c))^
(1/2)+1/16*(a^2*d^2+2*a*b*c*d-3*b^2*c^2)/c/a^2/(a*c*e)^(1/2)*ln((2*a*c*e+(a
*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/
x^2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.88 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.03

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$= \left[-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\sqrt{ace}\,x^4\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)ex^4+8\,a^2c^2e+8\,\left(abc^2+a^2cd\right)ex^2+4\,\left(\left(bcd+ad^2\right)x^4+2\,ac^2+\left(bc^2+3\,\cdots\right.\right.}{x^4}\right.}{32\,a^3c^2ex^4}\cdots\right.$$

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(a*c*e)*x^4*log(((b^2*c^2 + 6
*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*(
(b*c*d + a*d^2)*x^4 + 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*
e*x^2 + a*e)/(d*x^2 + c)))/x^4) + 4*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d^2)*
x^4 - 3*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^3*
c^2*e*x^4), 1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-a*c*e)*x^4*arctan
(1/2*sqrt(-a*c*e)*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c
))/(a*b*c*e*x^2 + a^2*c*e)) - 2*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d^2)*x^4
- 3*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^3*c^2*
e*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(194) = 388.

Time = 0.39 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$= \frac{\frac{(3b^2c^2-2abcd-a^2d^2)\arctan\left(-\frac{\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}a^2c} + \frac{5\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right)ab^2c^3e+10\left(\sqrt{bde}x^2-\sqrt{bdex^4}\right.}{}}{}$$

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(-(sqrt(b*d*e)*x^2 - sqrt(b*d*
e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e)*a^2*c)
+ (5*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a*
b^2*c^3*e + 10*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 +
a*c*e))*a^2*b*c^2*d*e + (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d
*e*x^2 + a*c*e))*a^3*c*d^2*e + 8*sqrt(b*d*e)*a^2*b*c^3*e - 3*(sqrt(b*d*e)*x
^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3*b^2*c^2 + 2*(sqrt(b
*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3*a*b*c*d + (s
qrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3*a^2*d^2
 + 8*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2
+ a*c*e))^2*a^2*c*d)/((a*c*e - (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^
2 + a*d*e*x^2 + a*c*e))^2)^2*a^2*c))/sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=\int \frac{1}{x^5\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

```
[In] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
[Out] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

3.302 $\int \frac{x^4}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} dx$

Optimal result

Integrand size = 26, antiderivative size = 403

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx = \frac{(bc-4ad)x(a+bx^2)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3(a+bx^2)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2b^2c^2+3abcd-8a^2d^2)\,x(a+bx^2)}{15b^3d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \frac{\sqrt{c}(2b^2c^2+3abcd-8a^2d^2)\,(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{c^{3/2}(bc-4ad)\,(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15b^2d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

```
[Out] 1/15*(-4*a*d+b*c)*x*(b*x^2+a)/b^2/d/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/5*x^3*(
b*x^2+a)/b/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/15*(-8*a^2*d^2+3*a*b*c*d+2*b^2*c
^2)*x*(b*x^2+a)/b^3/d/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/15*c^(3/2)*
(-4*a*d+b*c)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*
d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/b^2/d^(3/2)/(d*x^2+c)/
(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/15*(-8*a^2*
d^2+3*a*b*c*d+2*b^2*c^2)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*
EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)/b^
3/d^(3/2)/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))
^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1986, 489, 596, 545, 429, 506, 422}

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx = \frac{\sqrt{c}(a+bx^2)\left(-8a^2d^2+3abcd+2b^2c^2\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3d^{3/2}\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x(a+bx^2)\left(-8a^2d^2+3abcd+2b^2c^2\right)}{15b^3d\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{c^{3/2}(a+bx^2)(bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15b^2d^{3/2}\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x(a+bx^2)(bc-4ad)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3(a+bx^2)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[In] Int[x^4/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((b*c - 4*a*d)*x*(a + b*x^2))/(15*b^2*d*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])
+ (x^3*(a + b*x^2))/(5*b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((2*b^2*c^2 +
 3*a*b*c*d - 8*a^2*d^2)*x*(a + b*x^2))/(15*b^3*d*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)]*(c + d*x^2)) + (Sqrt[c]*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*(a + b*
x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^3*d^(3/
2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*
(c + d*x^2)) - (c^(3/2)*(b*c - 4*a*d)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^2}\int \frac{x^4\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x^3(a+bx^2)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{x^2(3ac+(-bc+4ad)x^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-4ad)x(a+bx^2)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3(a+bx^2)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{-ac(bc-4ad)+(-2b^2c^2-3abcd+8a^2d^2)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-4ad)x(a+bx^2)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3(a+bx^2)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(ac(bc-4ad)\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left((-2b^2c^2-3abcd+8a^2d^2)\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-4ad)x(a+bx^2)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3(a+bx^2)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2b^2c^2+3abcd-8a^2d^2)x(a+bx^2)}{15b^3d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{c^{3/2}(bc-4ad)(a+bx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{\left(c(-2b^2c^2-3abcd+8a^2d^2)\sqrt{a+bx^2}\right)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}dx}{15b^3d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-4ad)x(a+bx^2)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3(a+bx^2)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2b^2c^2+3abcd-8a^2d^2)x(a+bx^2)}{15b^3d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{\sqrt{c}(2b^2c^2+3abcd-8a^2d^2)(a+bx^2)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{c^{3/2}(bc-4ad)(a+bx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.52 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.64

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx = \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(4ad-b(c+3dx^2)) - ic(-2b^2c^2-3abcd+8a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arc}\right.}{15a^2\left(\frac{b}{a}\right)^{5/2}d^2\sqrt{\frac{e(a+}{c+}}}$$

```
[In] Integrate[x^4/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a*d - b*(c + 3*d*x^2))) - I*c*(
-2*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b^2*c^2) - a*b
*c*d + 2*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*(b/a)^(5/2)*d^2*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)]*(c + d*x^2))
```

Maple [A] (verified)

Time = 5.14 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{x\left(-3bd\,x^{2}+4ad-bc\right)\left(b\,x^{2}+a\right)}{15d\,b^{2}\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}+\frac{\left(\frac{4a^{2}cd\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}}-\frac{ab\,c^{2}\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda}{c}}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}}\right.}{}$
default	$-\frac{\left(b\,x^{2}+a\right)\left(-3\sqrt{-\frac{b}{a}}\,b^{2}d^{3}x^{7}+\sqrt{-\frac{b}{a}}\,ab\,d^{3}x^{5}-4\sqrt{-\frac{b}{a}}\,b^{2}c\,d^{2}x^{5}+4\sqrt{-\frac{b}{a}}\,a^{2}d^{3}x^{3}-\sqrt{-\frac{b}{a}}\,b^{2}c^{2}d\,x^{3}+4\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right.\right.}{}$

```
[In] int(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*x*(-3*b*d*x^2+4*a*d-b*c)*(b*x^2+a)/d/b^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2
)+1/15/d/b^2*(4*a^2*c*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/
(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a
*d*e+b*c*e)/c/b/e)^(1/2))-a*b*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x
^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-2*(8*a^2*d^2-3*a*b*c*d-2*b^2*c^2)*a*c*e
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*
c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-
1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/
b/e)^(1/2))))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(
d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{(2\,b^2c^3+3\,abc^2d-8\,a^2cd^2)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(2\,b^2c^3+3\,abc^2d-4\,a^2d^3-(8\,a^2-ab)cd^2)}{}$$

[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] 1/15*((2*b^2*c^3 + 3*a*b*c^2*d - 8*a^2*c*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (2*b^2*c^3 + 3*a*b*c^2*d - 4*a^2*d^3 - (8*a^2 - a*b)*c*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (3*b^2*d^3*x^6 - 2*b^2*c^3 - 3*a*b*c^2*d + 8*a^2*c*d^2 + 4*(b^2*c*d^2 - a*b*d^3)*x^4 - (b^2*c^2*d + 7*a*b*c*d^2 - 8*a^2*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e*x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

[In] integrate(x**4/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] integrate(x^4/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)

Giac [F]

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

```
[In] int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.303 $\int \frac{x^2}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 312

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{x(a+bx^2)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(bc-2ad)x(a+bx^2)}{3b^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{\sqrt{c}(bc-2ad)\,(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{c^{3/2}(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

```
[Out] 1/3*x*(b*x^2+a)/b/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-2*a*d+b*c)*x*(b*x^2+a
)/b^2/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*c^(3/2)*(b*x^2+a)*(1/(1+d
*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1
/2),(1-b*c/a/d)^(1/2))/b/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/
(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-2*a*d+b*c)*(b*x^2+a)*(1/(1+d*x^2/c))^(1
/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/
a/d)^(1/2))*c^(1/2)/b^2/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(
e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1986, 489, 545, 429, 506, 422}

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{\sqrt{c}(a+bx^2)\,(bc-2ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{d}\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{c^{3/2}(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3b\sqrt{d}\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x(a+bx^2)\,(bc-2ad)}{3b^2\,(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x(a+bx^2)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[In] Int[x^2/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (x*(a + b*x^2))/(3*b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((b*c - 2*a*d)*x*
(a + b*x^2))/(3*b^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt[
c]*(b*c - 2*a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(3*b^2*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a
+ b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (c^(3/2)*(a + b*x^2)*EllipticF[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
```

```
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^2}\int \frac{x^2\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x(a+bx^2)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{ac+(-bc+2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x(a+bx^2)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(ac\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} - \frac{\left((-bc+2ad)\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x(a+bx^2)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(bc-2ad)x(a+bx^2)}{3b^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{c^{3/2}(a+bx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \frac{\left(c(-bc+2ad)\sqrt{a+bx^2}\right)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{3b^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x(a+bx^2)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(bc-2ad)x(a+bx^2)}{3b^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{\sqrt{c}(bc-2ad)\,(a+bx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{c^{3/2}(a+bx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.02 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\,(c+dx^2) + ic(-bc+2ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right) - ic(-bc+ad)\sqrt{1-}}{3b\sqrt{\frac{b}{a}}d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

```
[In] Integrate[x^2/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + I*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
 I*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*d*Sqrt[(e*(a + b*x^2))/(c +
 d*x^2)]*(c + d*x^2))
```

Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.12

method	result
risch	$\frac{x(bx^2+a)}{3b\sqrt{\frac{e(bx^2+a)}{dx^2+c}}} - \frac{\left(\frac{ac\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bdex^4+adex^2+bcex^2+ace}} - \frac{2(2ad-bc)ace\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)-E\left(x\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{bdex^4+adex^2+bcex^2+ace}\,(eda+ebc+e(ad}\right.}{3b\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\,(dx^2+c)}$
default	$\frac{(bx^2+a)\left(\sqrt{-\frac{b}{a}}\,bd^2x^5+\sqrt{-\frac{b}{a}}\,ad^2x^3+\sqrt{-\frac{b}{a}}\,bcdx^3+ac\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right.\right.}{3\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\sqrt{(dx^2+c)(bx^2+a)}\,b\sqrt{-\frac{b}{a}}\sqrt{bdx}}$

```
[In] int(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(b*x^2+a)/b/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3/b*(a*c/(-b/a)^(1/2)*(1+
b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1
/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-2*(2*a*d-b*c)*
a*c*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x
^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/
2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*
e)/c/b/e)^(1/2))))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1
/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.59

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{(bc^2-2\,acd)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(bc^2-2\,acd-ad^2)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}}{3\,b^2dex}$$

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*((b*c^2 - 2*a*c*d)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/
d)/x), a*d/(b*c)) - (b*c^2 - 2*a*c*d - a*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*elli
ptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (b*d^2*x^4 + b*c^2 - 2*a*c*d + 2*
(b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^2}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^2}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^2}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

```
[In] int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.304 $\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 252

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{dx(a+bx^2)}{b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{\sqrt{c}\sqrt{d}(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \frac{c^{3/2}(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

```
[Out] d*x*(b*x^2+a)/b/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+c^(3/2)*(b*x^2+a)*(
1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2
/c)^(1/2),(1-b*c/a/d)^(1/2))/a/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^
(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^
2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))
*c^(1/2)*d^(1/2)/b/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(
d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used

$= \{1986, 433, 429, 506, 422\}$

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{c^{3/2}(a+bx^2)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{d}(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{dx(a+bx^2)}{b\,(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[In] Int[1/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (d*x*(a + b*x^2))/(b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt
[c]*Sqrt[d]*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*(c + d*x^2)) + (c^(3/2)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{\left(c\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left(d\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{dx(a+bx^2)}{b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \frac{c^{3/2}(a+bx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{\left(cd\sqrt{a+bx^2}\right)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{dx(a+bx^2)}{b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{\sqrt{c}\sqrt{d}(a+bx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \frac{c^{3/2}(a+bx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{\frac{c+dx^2}{c}}}$$

```
[In] Integrate[1/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
[Out] (Sqrt[(a + b*x^2)/a]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[
-(b/a)]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(c + d*x^2)/c])
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{(b\,x^2+a)c\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)}{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}$	127

```
[In] int(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2
)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx =$$
$$-\frac{c\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - (c+d)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - (dx^2+c)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{bex}$$

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -(c*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) -
(c + d)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)
) - (d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*e*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

```
[In] int(1/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(1/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.305 $\int \frac{1}{x^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 289

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=-\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx(a+bx^2)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}-\frac{\sqrt{c}\sqrt{d}(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}+\frac{\sqrt{c}\sqrt{d}(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

```
[Out] (-b*x^2-a)/a/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+d*x*(b*x^2+a)/a/(d*x^2+c)/(e*(
b*x^2+a)/(d*x^2+c))^(1/2)-(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)
*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d
^(1/2)/a/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^
(1/2)+(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)
/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/a/(d*x^2+c)/(
c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1986, 486, 21, 433, 429, 506, 422}

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{c}\sqrt{d}(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\,(c+dx^2)\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{d}(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\,(c+dx^2)\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{dx(a+bx^2)}{a\,(c+dx^2)\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[In] Int[1/(x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -((a + b*x^2)/(a*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])) + (d*x*(a + b*x^2))/
(a*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt[c]*Sqrt[d]*(a + b
*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
 + (Sqrt[c]*Sqrt[d]*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(a*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)]*(c + d*x^2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}}{x^2\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{ad+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(d\sqrt{a+bx^2}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(d\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left(bd\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{dx(a+bx^2)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \frac{\sqrt{c}\sqrt{d}(a+bx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{\left(cd\sqrt{a+bx^2}\right)\int\frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{dx(a+bx^2)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{\sqrt{c}\sqrt{d}(a+bx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \frac{\sqrt{c}\sqrt{d}(a+bx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.38

$$\int\frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{(a+bx^2)\left(-\frac{1}{x}+\frac{d\sqrt{1+\frac{dx^2}{c}}\,E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{1+\frac{bx^2}{a}}(c+dx^2)}\right)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[In] Integrate[1/(x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] ((a + b*x^2)*(-x^(-1) + (d*Sqrt[1 + (d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)
]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[1 + (b*x^2)/a]*(c + d*x^2))))/(a*Sqr
t[(e*(a + b*x^2))/(c + d*x^2)])
```

Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.03

method	result
default	$-\frac{(b\,x^2+a)\left(\sqrt{-\frac{b}{a}}\,bd\,x^4-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)adx+bc\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,xF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-bc\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+\ldots}{c}}\ldots\right.}{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,\sqrt{(d\,x^2+c)(b\,x^2+a)}\,ax\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\ldots}$
risch	$-\frac{b\,x^2+a}{ax\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}+\frac{d\left(\frac{a\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}-\frac{2bace\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)-E\left(x\sqrt{\ldots}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}\,(eda+ebc+e(\ldots}\ldots\right.}{a\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)}$

```
[In] int(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x^2+a)*((-b/a)^(1/2)*b*d*x^4-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*d*x+b*c*((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*x*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))-b*c*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))+(-b
/a)^(1/2)*a*d*x^2+(-b/a)^(1/2)*b*c*x^2+(-b/a)^(1/2)*a*c)/(e*(b*x^2+a)/(d*x^
2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/a/x/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{b^2cd\sqrt{\frac{ace}{d^2}}x\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - (b^2cd + a^2d^2)\sqrt{\frac{ace}{d^2}}x\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - (abcdx^2 +}{a^2bcex}$$

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (b^2*c*d*sqrt(a*c*e/d^2)*x*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/
(b*c)) - (b^2*c*d + a^2*d^2)*sqrt(a*c*e/d^2)*x*sqrt(-b/a)*elliptic_f(arcsin
(x*sqrt(-b/a)), a*d/(b*c)) - (a*b*c*d*x^2 + a*b*c^2)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c)))/(a^2*b*c*e*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**2/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^2}\,dx$$

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^2}\,dx$$

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{x^2\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

```
[In] int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
[Out] int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

3.306 $\int \frac{1}{x^4\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 375

$$\begin{aligned}\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx &= \frac{-a-bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(2bc-ad)\,(a+bx^2)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{d(2bc-ad)x(a+bx^2)}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\\ &\quad+\frac{\sqrt{d}(2bc-ad)\,(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\\ &\quad-\frac{b\sqrt{c}\sqrt{d}(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\end{aligned}$$

```
[Out] 1/3*(-b*x^2-a)/a/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-a*d+2*b*c)*(b*x^2+
a)/a^2/c/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*d*(-a*d+2*b*c)*x*(b*x^2+a)/a^2
/c/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-a*d+2*b*c)*(b*x^2+a)*(1/(1
+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)/a^2/(d*x^2+c)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^
2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*b*(b*x^2+a)*(1/(1+d*x^2/c))^(
1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c
/a/d)^(1/2))*c^(1/2)*d^(1/2)/a^2/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/
(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1986, 486, 597, 545, 429, 506, 422}

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{b\sqrt{c}\sqrt{d}(a+bx^2)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{d}(a+bx^2)\left(2bc-ad\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(a+bx^2)\left(2bc-ad\right)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{dx(a+bx^2)\left(2bc-ad\right)}{3a^2c\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[In] Int[1/(x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -1/3*(a + b*x^2)/(a*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((2*b*c - a*d)
*(a + b*x^2))/(3*a^2*c*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (d*(2*b*c - a
*d)*x*(a + b*x^2))/(3*a^2*c*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
+ (Sqrt[d]*(2*b*c - a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (b*Sqrt[c]*Sqrt[d]*(a + b*x^2
)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
```

```
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}}{x^4\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{-2bc+ad-bdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)(a+bx^2)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{abcd+bd(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)(a+bx^2)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(bd\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{3a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} - \frac{\left(bd(2bc-ad)\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)(a+bx^2)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(2bc-ad)x(a+bx^2)}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{b\sqrt{c}\sqrt{d}(a+bx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{\left(d(2bc-ad)\sqrt{a+bx^2}\right)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}dx}{3a^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)(a+bx^2)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(2bc-ad)x(a+bx^2)}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{\sqrt{d}(2bc-ad)(a+bx^2)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{b\sqrt{c}\sqrt{d}(a+bx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.42 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$

$$= \frac{-\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(-2bcx^2+a(c+dx^2)) - ibc(-2bc+ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right.\right.}{3a^2\sqrt{\frac{b}{a}}cx^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx}$$

```
[In] Integrate[1/(x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-2*b*c*x^2 + a*(c + d*x^2))) - I*b*c*
(-2*b*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcS
inh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/
(3*a^2*Sqrt[b/a]*c*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Maple [A] (verified)

Time = 6.21 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{(b x^2+a)(a d x^2-2bc x^2+ac)}{3a^2x^3c\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}-\frac{bd\left(\frac{ac\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bde x^4+ade x^2+bce x^2+ace}}+\frac{2(ad-2bc)ace\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{bde x^4+ade x^2+bce x^2}}\right.}{3ca^2\sqrt{\frac{e(b x^2+a)}{d x^2+c}}(d x^2+c)}$
default	$-\frac{(b x^2+a)\left(\sqrt{-\frac{b}{a}}ab d^2x^6-2\sqrt{-\frac{b}{a}}b^2cd x^6+2bd\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^3ac-2\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)b^2\right.}{3\sqrt{\frac{e(b x^2+a}{d x^2+c}}}$

```
[In] int(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x^2+a)*(a*d*x^2-2*b*c*x^2+a*c)/a^2/x^3/c/(e*(b*x^2+a)/(d*x^2+c))^(1
/2)-1/3*b*d/c/a^2*(a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(
b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d*e+b*c*e)/c/b/e)^(1/2))+2*(a*d-2*b*c)*a*c*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b
*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-E
llipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))))/(e*(b*x^2+a)/(d*x
^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{(2\,b^2cd-abd^2)\sqrt{\frac{ace}{d^2}}x^3\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(2\,b^2cd+(a^2-ab)d^2)\sqrt{\frac{ace}{d^2}}x^3\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{\right.\right.}{3\,a^3cex^3}$$

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*((2*b^2*c*d - a*b*d^2)*sqrt(a*c*e/d^2)*x^3*sqrt(-b/a)*elliptic_e(arcsi
n(x*sqrt(-b/a)), a*d/(b*c)) - (2*b^2*c*d + (a^2 - a*b)*d^2)*sqrt(a*c*e/d^2)
```

```
*x^3*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((2*a*b*c*d -
 a^2*d^2)*x^4 - a^2*c^2 + 2*(a*b*c^2 - a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c)))/(a^3*c*e*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**4/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x^4}\,dx$$

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x^4}\,dx$$

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{x^4\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

```
[In] int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
[Out] int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

3.307 $\int \frac{x^5}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 354

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = -\frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{16b^4de^2} - \frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24b^3d(bc-ad)e^2} - \frac{a^2(c+dx^2)^3}{b(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2c^2-2abcd+7a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^3}{6b^2d(bc-ad)^2e^2} - \frac{(bc-ad)\left(b^2c^2+5ad(2bc-7ad)\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{9/2}d^{3/2}e^{3/2}}$$

```
[Out] -1/16*(-a*d+b*c)*(b^2*c^2+5*a*d*(-7*a*d+2*b*c))*arctanh(d^(1/2)*(e*(b*x^2+a
)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/b^(9/2)/d^(3/2)/e^(3/2)-a^2*(d*x^2+c)^3
/b/(-a*d+b*c)^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/16*(b^2*c^2+5*a*d*(-7*a*d
+2*b*c))*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^4/d/e^2-1/24*(b^2*c^2+5*
a*d*(-7*a*d+2*b*c))*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^3/d/(-a*d+b
*c)/e^2+1/6*(7*a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x^2+c)^3*(e*(b*x^2+a)/(d*x^2+c
))^(1/2)/b^2/d/(-a*d+b*c)^2/e^2
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1981, 1980, 473, 393, 205, 214}

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx = \frac{(c+dx^2)^3 (7a^2d^2-2abcd+b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6b^2de^2(bc-ad)^2} - \frac{a^2(c+dx^2)^3}{be(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc-ad)(5ad(2bc-7ad)+b^2c^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{9/2}d^{3/2}e^{3/2}} - \frac{(c+dx^2)^2\left(\frac{5a(2bc-7ad)}{b^2}+\frac{c^2}{d}\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24be^2(bc-ad)} - \frac{(c+dx^2)(5ad(2bc-7ad)+b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^4de^2}$$

```
[In] Int[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -1/16*((b^2*c^2 + 5*a*d*(2*b*c - 7*a*d))*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*
(c + d*x^2))/(b^4*d*e^2) - ((c^2/d + (5*a*(2*b*c - 7*a*d))/b^2)*Sqrt[(e*(a
+ b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(24*b*(b*c - a*d)*e^2) - (a^2*(c + d*
x^2)^3)/(b*(b*c - a*d)^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((b^2*c^2 -
 2*a*b*c*d + 7*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^3)/(6
*b^2*d*(b*c - a*d)^2*e^2) - ((b*c - a*d)*(b^2*c^2 + 5*a*d*(2*b*c - 7*a*d))*
ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16
*b^(9/2)*d^(3/2)*e^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\left(\frac{e(a+bx)}{c+dx}\right)^{3/2}}\,dx, x, x^2\right)\\
&= ((bc-ad)e)\text{Subst}\left(\int \frac{\left(-ae+cx^2\right)^2}{x^2\left(be-dx^2\right)^4}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)\\
&= -\frac{a^2\left(c+dx^2\right)^3}{b(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(bc-ad)\text{Subst}\left(\int \frac{-a(2bc-7ad)e^2+bc^2ex^2}{\left(be-dx^2\right)^4}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{b}
\end{aligned}
$$

$$= -\frac{a^2(c+dx^2)^3}{b(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2c^2-2abcd+7a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^3}{6b^2d(bc-ad)^2e^2} - \frac{((bc-ad)(b^2c^2+5ad(2bc-7ad))e)\operatorname{Subst}\left(\int \frac{1}{(be-dx^2)^3}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{6b^2d}$$

$$= -\frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24b^3d(bc-ad)e^2} - \frac{a^2(c+dx^2)^3}{b(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2c^2-2abcd+7a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^3}{6b^2d(bc-ad)^2e^2} - \frac{((bc-ad)(b^2c^2+5ad(2bc-7ad)))\operatorname{Subst}\left(\int \frac{1}{(be-dx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{8b^3d}$$

$$= -\frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{16b^4de^2} - \frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24b^3d(bc-ad)e^2} - \frac{a^2(c+dx^2)^3}{b(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2c^2-2abcd+7a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^3}{6b^2d(bc-ad)^2e^2} - \frac{((bc-ad)(b^2c^2+5ad(2bc-7ad)))\operatorname{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{16b^4de}$$

$$= -\frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{16b^4de^2} - \frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24b^3d(bc-ad)e^2} - \frac{a^2(c+dx^2)^3}{b(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2c^2-2abcd+7a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^3}{6b^2d(bc-ad)^2e^2} - \frac{(bc-ad)(b^2c^2+5ad(2bc-7ad))\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{9/2}d^{3/2}e^{3/2}}$$

Mathematica [A] (verified)

Time = 4.43 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.70

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{d}\sqrt{\frac{b(c+dx^2)}{bc-ad}}(105a^3d^2+5a^2bd(-20c+7dx^2)+ab^2(3c^2-38cdx^2-14d^2x^4)+b^3x^2(3c}{48b^4d^{3/2}e\sqrt{}}$$

```
[In] Integrate[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(105*a^3*d^2 + 5*a^2*b*d*(-20*c
+ 7*d*x^2) + a*b^2*(3*c^2 - 38*c*d*x^2 - 14*d^2*x^4) + b^3*x^2*(3*c^2 + 14*
c*d*x^2 + 8*d^2*x^4)) - 3*Sqrt[b*c - a*d]*(b^2*c^2 + 10*a*b*c*d - 35*a^2*d^
2)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(48*
b^4*d^(3/2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c -
 a*d)])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.90

method	result
risch	$\frac{(8b^2d^2x^4-22ab\,d^2x^2+14b^2cd\,x^2+57a^2d^2-52abcd+3b^2c^2)(b\,x^2+a)}{48d\,b^4e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}} - \frac{\left(\frac{(35a^2d^2-10abcd-b^2c^2)(ad-bc)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^2}{\sqrt{bde}}+\sqrt{bd}\right.}{2\sqrt{bde}}\right.}{16c}$
default	Expression too large to display

```
[In] int(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/d*(8*b^2*d^2*x^4-22*a*b*d^2*x^2+14*b^2*c*d*x^2+57*a^2*d^2-52*a*b*c*d+3
*b^2*c^2)*(b*x^2+a)/b^4/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/16/d/b^4*(1/2*(35
*a^2*d^2-10*a*b*c*d-b^2*c^2)*(a*d-b*c)*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)/(
b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)-16*a^
2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*d*(d*x^2+c)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*
c*e*x^2+a*c*e)^(1/2))/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a
))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 1.25 (sec) , antiderivative size = 781, normalized size of antiderivative = 2.21

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\,(ab^3c^3+9\,a^2b^2c^2d-45\,a^3bcd^2+35\,a^4d^3+(b^4c^3+9\,ab^3c^2d-45\,a^2b^2cd^2+35\,a^3bd^3)}{}\right.$$

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(a*b^3*c^3 + 9*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^4
*c^3 + 9*a*b^3*c^2*d - 45*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(b*d*e)*lo
g(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d
*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 4*(8*b^4*d^4*x^8 + 3*a*b^3*c^3*d -
 100*a^2*b^2*c^2*d^2 + 105*a^3*b*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^4)*x^6
 + (17*b^4*c^2*d^2 - 52*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^4 + (3*b^4*c^3*d -
35*a*b^3*c^2*d^2 - 65*a^2*b^2*c*d^3 + 105*a^3*b*d^4)*x^2)*sqrt((b*e*x^2 + a
*e)/(d*x^2 + c)))/(b^6*d^2*e^2*x^2 + a*b^5*d^2*e^2), 1/96*(3*(a*b^3*c^3 + 9
*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^4*c^3 + 9*a*b^3*c^2*d - 4
5*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b
*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b
*d*e)) + 2*(8*b^4*d^4*x^8 + 3*a*b^3*c^3*d - 100*a^2*b^2*c^2*d^2 + 105*a^3*b
*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^4)*x^6 + (17*b^4*c^2*d^2 - 52*a*b^3*c*
d^3 + 35*a^2*b^2*d^4)*x^4 + (3*b^4*c^3*d - 35*a*b^3*c^2*d^2 - 65*a^2*b^2*c*
d^3 + 105*a^3*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^6*d^2*e^2*x
^2 + a*b^5*d^2*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{2,[1,0,0]%%%},[2,1,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1,1,
1]%%%}
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx = \int \frac{x^5}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}} \,dx$$

```
[In] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.308 $\int \frac{x^3}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 202

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8b^3e^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4b^2e^2} + \frac{3(bc-5ad)(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{7/2}\sqrt{d}e^{3/2}}$$

```
[Out] 3/8*(-5*a*d+b*c)*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b
^(1/2)/e^(1/2))/b^(7/2)/e^(3/2)/d^(1/2)+a*(-a*d+b*c)/b^3/e/(e*(b*x^2+a)/(d*
x^2+c))^(1/2)+1/8*(-7*a*d+3*b*c)*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^
3/e^2+1/4*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/e^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used

$= \{1981, 1980, 467, 464, 214\}$

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{3(bc-5ad)(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{7/2}\sqrt{d}e^{3/2}} + \frac{(c+dx^2)\,(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^3e^2} + \frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e^2}$$

```
[In] Int[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (a*(b*c - a*d))/(b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((3*b*c - 7*a*d
)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b^3*e^2) + (Sqrt[(e*(a
+ b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*b^2*e^2) + (3*(b*c - 5*a*d)*(b*c -
 a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])
])/(8*b^(7/2)*Sqrt[d]*e^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
```

```
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x}{\left(\frac{e(a+bx)}{c+dx}\right)^{3/2}}\,dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{-ae+cx^2}{x^2\left(be-dx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} - \frac{1}{4}((bc-ad)e)\text{Subst}\left(\int \frac{\frac{4a}{b}-\frac{3(bc-ad)x^2}{b^2e}}{x^2\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8b^3e^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} + \frac{1}{8}((bc-ad)e)\text{Subst}\left(\int \frac{-\frac{8a}{b^2e}+\frac{(3bc-7ad)x^2}{b^3e^2}}{x^2\left(be-dx^2\right)}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8b^3e^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} + \frac{(3(bc-5ad)(bc-ad))\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{8b^3e}$$

$$= \frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8b^3e^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} + \frac{3(bc-5ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{7/2}\sqrt{d}e^{3/2}}$$

Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b}\sqrt{d}\sqrt{c+dx^2}(-15a^2d+ab(13c-5dx^2)+b^2x^2(5c+2dx^2))+3(b^2c^2-6abcd+5a^2}{8b^{7/2}\sqrt{d}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

```
[In] Integrate[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[d]*Sqrt[c + d*x^2]*(-15*a^2*d + a*b*(13*c - 5*d*x^2) + b^2*x^
2*(5*c + 2*d*x^2)) + 3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*Sqrt[a + b*x^2]*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(7/2)*Sqrt
[d]*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{(-2bd\,x^2+7ad-5bc)(b\,x^2+a)}{8b^3e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}+\frac{\left(\frac{3(5ad-bc)(ad-bc)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^2}{\sqrt{bde}}+\sqrt{bde\,x^4+(eda+ebc)x^2+ace}\right)}{2\sqrt{bde}}-\frac{8a\left(a^2d^2-2abcd+b^2c^2\right)}{(ad-bc)\sqrt{bde\,x^4+ade\,x^2+}}\right.}{8b^3e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$-\frac{\left(-4\sqrt{bd}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,b^2d\,x^4-15\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^2b\,d^2x^2+18\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad}}{}\right.\right.}{}$

```
[In] int(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-2*b*d*x^2+7*a*d-5*b*c)*(b*x^2+a)/b^3/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
+1/8/b^3*(3/2*(5*a*d-b*c)*(a*d-b*c)*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)/(b*d
*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)-8*a*(a^2
*d^2-2*a*b*c*d+b^2*c^2)*(d*x^2+c)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+
a*c*e)^(1/2))/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)
/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.70 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.90

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\,(ab^2c^2-6\,a^2bcd+5\,a^3d^2+(b^3c^2-6\,ab^2cd+5\,a^2bd^2)x^2)\sqrt{bde}\log\left(8\,b^2d^2ex^4+8\,(b^2\right.}{}\right.$$

$$-\frac{3\,(ab^2c^2-6\,a^2bcd+5\,a^3d^2+(b^3c^2-6\,ab^2cd+5\,a^2bd^2)x^2)\sqrt{-bde}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(b^2dex^2+abde)}\right)-2}{16\,(b^5de^2x^2+}$$

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(a*b^2*c^2 - 6*a^2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*
a^2*b*d^2)*x^2)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x
^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (
3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 4*(2
*b^3*d^3*x^6 + 13*a*b^2*c^2*d - 15*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5*a*b^2*d^3
)*x^4 + (5*b^3*c^2*d + 8*a*b^2*c*d^2 - 15*a^2*b*d^3)*x^2)*sqrt((b*e*x^2 + a
*e)/(d*x^2 + c)))/(b^5*d*e^2*x^2 + a*b^4*d*e^2), -1/16*(3*(a*b^2*c^2 - 6*a^
2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*sqrt(-b*d*
e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*
x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(2*b^3*d^3*x^6 + 13*a*b^2*c^2*d - 15
*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5*a*b^2*d^3)*x^4 + (5*b^3*c^2*d + 8*a*b^2*c*d
^2 - 15*a^2*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^5*d*e^2*x^2 +
 a*b^4*d*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{2,[1,0,0]%%%},[2,1,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1,1,
1]%%%}
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^3}{{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)}^{3/2}}\,dx$$

```
[In] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.309 $\int \frac{x}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 146

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=-\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{3\sqrt{d}(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}$$

```
[Out] 3/2*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2
))*d^(1/2)/b^(5/2)/e^(3/2)-3/2*(-a*d+b*c)/b^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/
2)+1/2*(d*x^2+c)/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1981, 1979, 296, 331, 214}

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=\frac{3\sqrt{d}(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}-\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[In] Int[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (-3*(b*c - a*d))/(2*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (c + d*x^2)/
(2*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (3*Sqrt[d]*(b*c - a*d)*ArcTanh[
```

```
(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(5/2)*
e^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(\frac{e(a+bx)}{c+dx}\right)^{3/2}}\,dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{1}{x^2\,(be-dx^2)^2}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)$$

$$=\frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(3(bc-ad))\text{Subst}\left(\int\frac{1}{x^2(be-dx^2)}\,dx,x,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2b}$$

$$=-\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(3d(bc-ad))\text{Subst}\left(\int\frac{1}{be-dx^2}\,dx,x,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2b^2e}$$

$$=-\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{3\sqrt{d}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}$$

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int\frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=\frac{\sqrt{b}\sqrt{c+dx^2}(-2bc+3ad+bdx^2)+3\sqrt{d}(bc-ad)\sqrt{a+bx^2}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{5/2}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

```
[In] Integrate[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
[Out] (Sqrt[b]*Sqrt[c + d*x^2]*(-2*b*c + 3*a*d + b*d*x^2) + 3*Sqrt[d]*(b*c - a*d)
*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2]
)])/(2*b^(5/2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.60

method	result
risch	$\frac{(b\,x^2+a)d}{2b^2e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}-\frac{\left(\frac{3d(ad-bc)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^2}{\sqrt{bde}}+\sqrt{bde\,x^4+(eda+ebc)x^2+ace}\right)}{2\sqrt{bde}}+\frac{\left(-2a^2d^2+4abcd-2b^2c^2\right)\left(d\,x^2+c\right)}{(ad-bc)\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}\right)\sqrt{(d\,x^2+c)}}{2b^2e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$-\frac{\left(3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)ab\,d^2x^2-3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)b^2cd\,x^2-2\sqrt{bd}\,\sqrt{bd\,x}\right.}{}$

```
[In] int(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^2*(b*x^2+a)*d/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/2/b^2*(3/2*d*(a*d-b*c
)*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)
*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)+(-2*a^2*d^2+4*a*b*c*d-2*b^2*c^2)*(d*x^2+c)
/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))/e/(e*(b*x^2+a)/(d*x
^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.03

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \left[-\frac{3\left((b^2c-abd)ex^2+(abc-a^2d)e\right)\sqrt{\frac{d}{be}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2\right.}{}\right.$$

$$-\frac{3\left((b^2c-abd)ex^2+(abc-a^2d)e\right)\sqrt{-\frac{d}{be}}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{d}{be}}}{2\,(bdx^2+ad)}\right)-2\left(bd^2x^4-2\,bc^2+3\,acd-\right.}{4\left(b^3e^2x^2+ab^2e^2\right)}$$

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*((b^2*c - a*b*d)*e*x^2 + (a*b*c - a^2*d)*e)*sqrt(d/(b*e))*log(8*b^
2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(
2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) - 4*(b*d^2*x^4 - 2*b*c^2 + 3*a*c*d - (
b*c*d - 3*a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*e^2*x^2 + a*b
^2*e^2), -1/4*(3*((b^2*c - a*b*d)*e*x^2 + (a*b*c - a^2*d)*e)*sqrt(-d/(b*e))
*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(
-d/(b*e))/(b*d*x^2 + a*d)) - 2*(b*d^2*x^4 - 2*b*c^2 + 3*a*c*d - (b*c*d - 3*
a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*e^2*x^2 + a*b^2*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx = \text{Timed out}$$

```
[In] integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{2,[1,0,0]%%%},[2,1,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1,1,
1]%%%}
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.310 $\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 152

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}} + \frac{d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}$$

```
[Out] -c^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))/a^(
3/2)/e^(3/2)+d^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/
e^(1/2))/b^(3/2)/e^(3/2)+(-a*d+b*c)/a/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1981, 1980, 491, 536, 214}

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = -\frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}} + \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}} + \frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[In] Int[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*c - a*d)/(a*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (c^(3/2)*ArcTanh[(S
qrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(a^(3/2)*e^(3
/2)) + (d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b
]*Sqrt[e])])/(b^(3/2)*e^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(\frac{e(a+bx)}{c+dx}\right)^{3/2}}\,dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{1}{x^2\left(-ae+cx^2\right)\left(be-dx^2\right)}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc-ad)\text{Subst}\left(\int \frac{-((bc+ad)e)+cdx^2}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{abe}$$

$$= \frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c^2\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{ae} + \frac{d^2\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{be}$$

$$= \frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}} + \frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}$$

Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{-b^{3/2}c^{3/2}\sqrt{a+bx^2}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{a}\left(\sqrt{b}(bc-ad)\sqrt{c+dx^2}+ad^{3/2}\sqrt{a+bx^2}\ldots\right)}{a^{3/2}b^{3/2}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

```
[In] Integrate[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (-(b^(3/2)*c^(3/2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[
a]*Sqrt[c + d*x^2])]) + Sqrt[a]*(Sqrt[b]*(b*c - a*d)*Sqrt[c + d*x^2] + a*d^
(3/2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d
*x^2])]))/(a^(3/2)*b^(3/2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x
^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(122) = 244.

Time = 0.14 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.64

method	result
default	$-\frac{\left(\sqrt{bd}\,\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}+2ac}{x^2}\right)b^2c^2x^2-\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\sqrt{ac}\,ab\,d^2x^2+\ldots\right.}{\ldots}$

```
[In] int(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)+2*a*c)/x^2)*b^2*c^2*x^2-ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*a*b*d^2*x^2
+(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)+2*a*c)/x^2)*a*b*c^2-ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*a^2*d^2+2*(b*d)^(1/2
)*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*a*d-2*(b*d)^(1/2)*((d*x^2+c)*(b*x
^2+a))^(1/2)*(a*c)^(1/2)*b*c)/a*(b*x^2+a)/b/(a*c)^(1/2)/(b*d)^(1/2)/((d*x^2
+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(122) = 244$.

Time = 0.91 (sec) , antiderivative size = 1293, normalized size of antiderivative = 8.51

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*d*e*x^2 + a^2*d*e)*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6
*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2
 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*s
qrt(d/(b*e))) + (b^2*c*e*x^2 + a*b*c*e)*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d +
 a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)
/(d*x^2 + c))*sqrt(c/(a*e)))/x^4) + 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^2), -1/4*(2*(a
*b*d*e*x^2 + a^2*d*e)*sqrt(-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqr
t((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) - (b^2*c*e*x
^2 + a*b*c*e)*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^
2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2
+ (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)
))/x^4) - 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x
^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^2), 1/4*(2*(b^2*c*e*x^2 + a*b*c*e)*sqrt(
-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2
+ c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) + (a*b*d*e*x^2 + a^2*d*e)*sqrt(d/(b*e
))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2
)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*s
qrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) + 4*(b*c^2 - a*c*d + (b*c*d
 - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^
2), 1/2*((b^2*c*e*x^2 + a*b*c*e)*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2
 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c))
 - (a*b*d*e*x^2 + a^2*d*e)*sqrt(-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d
)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) + 2*(b*
c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^
2*e^2*x^2 + a^2*b*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{2,[1,2,2]%%%},[2,1,3,0]%%%}+%%%{%%%{-4,[2,1,2]%%%},[2,1,2
,1]%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

3.311 $\int \frac{1}{x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 170

$$\int \frac{1}{x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx = -\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{3\sqrt{c}(bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}$$

```
[Out] 3/2*(-a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2
))*c^(1/2)/a^(5/2)/e^(3/2)-3/2*(-a*d+b*c)/a^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/
2)+1/2*(-a*d+b*c)/a/(a*e-c*e*(b*x^2+a)/(d*x^2+c))/(e*(b*x^2+a)/(d*x^2+c))^(
1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used

$= \{1981, 1980, 296, 331, 214\}$

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{3\sqrt{c}(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{5/2}e^{3/2}} - \frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

```
[In] Int[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (-3*(b*c - a*d))/(2*a^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (b*c - a*d)/
(2*a*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(a*e - (c*e*(a + b*x^2))/(c + d*x^2)
)) + (3*Sqrt[c]*(b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^
2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(5/2)*e^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(\frac{e(a+bx)}{c+dx}\right)^{3/2}}\,dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{1}{x^2\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(3(bc-ad))\text{Subst}\left(\int \frac{1}{x^2(-ae+cx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2a}$$

$$= -\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(3c(bc-ad))\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2a^2e}$$

$$= -\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{3\sqrt{c}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}$$

Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{-\sqrt{a}\sqrt{c+dx^2}(3bcx^2+a(c-2dx^2))+3\sqrt{c}(bc-ad)x^2\sqrt{a+bx^2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}ex^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

```
[In] Integrate[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
[Out] (-(Sqrt[a]*Sqrt[c + d*x^2]*(3*b*c*x^2 + a*(c - 2*d*x^2))) + 3*Sqrt[c]*(b*c
- a*d)*x^2*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[
c + d*x^2])])/(2*a^(5/2)*e*x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d
*x^2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.42

method	result
risch	$-\frac{c(b\,x^2+a)}{2a^2x^2e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}+\frac{\left(-\frac{3c(ad-bc)\ln\left(\frac{2ace+(eda+ebc)x^2+2\sqrt{ace}\,\sqrt{bde\,x^4+(eda+ebc)x^2+ace}}{x^2}\right)}{2\sqrt{ace}}+\frac{\left(2a^2d^2-4abcd+2b^2c^2\right)\left(d\,x^2+c\right)}{(ad-bc)\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}\right.}{2a^2e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$\frac{\left(2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{ac}\,b^2d\,x^6-3\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}+2ac}{x^2}\right)a^2bcd\,x^4+3\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x^4}}{x^2}\right.\right.}{}$

[In] int(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/2/a^2*c*(b*x^2+a)/x^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/2/a^2*(-3/2*c*(a*d-b*c)/(a*c*e)^(1/2)*ln((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/x^2)+(2*a^2*d^2-4*a*b*c*d+2*b^2*c^2)*(d*x^2+c)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(d*x^2+c)

Fricas [A] (verification not implemented)

none

Time = 1.29 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.76

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \left[-\frac{3\left((b^2c-abd)ex^4+(abc-a^2d)ex^2\right)\sqrt{\frac{c}{ae}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)}{}\right.}{}\right.$$

$$-\frac{3\left((b^2c-abd)ex^4+(abc-a^2d)ex^2\right)\sqrt{-\frac{c}{ae}}\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{c}{ae}}}{2\,(bcx^2+ac)}\right)+2\left((3\,bcd-2\,ad^2)x^4+a\right.}{4\left(a^2be^2x^4+a^3e^2x^2\right)}$$

[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")

[Out] [-1/8*(3*((b^2*c - a*b*d)*e*x^4 + (a*b*c - a^2*d)*e*x^2)*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^

```
2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqr
t((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4) + 4*((3*b*c*d - 2*a*d^2)
*x^4 + a*c^2 + (3*b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a
^2*b*e^2*x^4 + a^3*e^2*x^2), -1/4*(3*((b^2*c - a*b*d)*e*x^4 + (a*b*c - a^2*
d)*e*x^2)*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2
 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) + 2*((3*b*c*d - 2*a*d^
2)*x^4 + a*c^2 + (3*b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/
(a^2*b*e^2*x^4 + a^3*e^2*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{2,[1,0,0]%%%},[6,1,0,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1,
1,1]%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\, dx = \int \frac{1}{x^3 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\, dx$$

```
[In] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

3.312 $\int \frac{1}{x^5 \left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx = \frac{b(bc-ad)}{a^3 e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)} - \frac{3(bc-ad)(5bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{7/2}\sqrt{c}e^{3/2}}$$

```
[Out] -3/8*(-a*d+b*c)*(-a*d+5*b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
a^(1/2)/e^(1/2))/a^(7/2)/e^(3/2)/c^(1/2)+b*(-a*d+b*c)/a^3/e/(e*(b*x^2+a)/(d
*x^2+c))^(1/2)-1/4*(-a*d+b*c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^2/(a*e-c*e*
(b*x^2+a)/(d*x^2+c))^2-1/8*(-3*a*d+7*b*c)*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c)
)^(1/2)/a^3/(a*e^2-c*e^2*(b*x^2+a)/(d*x^2+c))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

$= \{1981, 1980, 467, 464, 214\}$

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = -\frac{3(5bc-ad)(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{7/2}\sqrt{c}e^{3/2}} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)} + \frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

```
[In] Int[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*(b*c - a*d))/(a^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((b*c - a*d)^2*
Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*a^2*(a*e - (c*e*(a + b*x^2))/(c + d*x
^2))^2) - ((7*b*c - 3*a*d)*(b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(
8*a^3*(a*e^2 - (c*e^2*(a + b*x^2))/(c + d*x^2))) - (3*(b*c - a*d)*(5*b*c -
a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])]
)/(8*a^(7/2)*Sqrt[c]*e^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\left(\frac{e(a+bx)}{c+dx}\right)^{3/2}}\,dx, x, x^2\right)$$

$$= ((bc-ad)e)\text{Subst}\left(\int \frac{be-dx^2}{x^2\left(-ae+cx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{1}{4}((bc-ad)e)\text{Subst}\left(\int \frac{\frac{4b}{a}+\frac{3(bc-ad)x^2}{a^2e}}{x^2\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)} + \frac{1}{8}((bc-ad)e)\text{Subst}\left(\int \frac{\frac{8b}{a^2e}+\frac{(7bc-3ad)x^2}{a^3e^2}}{x^2\left(-ae+cx^2\right)}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)} + \frac{(3(bc-ad)(5bc-ad))\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{8a^3e}$$

$$= \frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)} - \frac{3(bc-ad)(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{7/2}\sqrt{c}e^{3/2}}$$

Mathematica [A] (verified)

Time = 4.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{c}\sqrt{c+dx^2}(15b^2cx^4+abx^2(5c-13dx^2)-a^2(2c+5dx^2))-3(5b^2c^2-6abcd+}{8a^{7/2}\sqrt{c}ex^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

```
[In] Integrate[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[c]*Sqrt[c + d*x^2]*(15*b^2*c*x^4 + a*b*x^2*(5*c - 13*d*x^2) -
 a^2*(2*c + 5*d*x^2)) - 3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x^4*Sqrt[a + b*
x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(7/
2)*Sqrt[c]*e*x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{(b\,x^2+a)\,(5ad\,x^2-7bc\,x^2+2ac)}{8a^3x^4e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}} + \frac{\left(\frac{\left(-3a^2d^2+18abcd-15b^2c^2\right)\ln\left(\frac{2ace+(eda+ebc)x^2+2\sqrt{ace}\,\sqrt{bde\,x^4+(eda+ebc)x^2+ace}}{x^2}\right)}{2\sqrt{ace}} - \frac{8b\left(a^2d^2\right.}{(ad-bc)\sqrt{b}}\right.}{8a^3e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)}$
default	Expression too large to display

```
[In] int(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(b*x^2+a)*(5*a*d*x^2-7*b*c*x^2+2*a*c)/a^3/x^4/e/(e*(b*x^2+a)/(d*x^2+c)
)^(1/2)+1/8/a^3*(1/2*(-3*a^2*d^2+18*a*b*c*d-15*b^2*c^2)/(a*c*e)^(1/2)*ln((2
*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e
)^(1/2))/x^2)-8*b*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x^2+c)/(a*d-b*c)/(b*d*e*x^
4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2
+c)*e*(b*x^2+a))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 3.95 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.40

$$\int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\left((5\,b^3c^2 - 6\,ab^2cd + a^2bd^2)x^6 + (5\,ab^2c^2 - 6\,a^2bcd + a^3d^2)x^4\right)\sqrt{ace}\log\left(\frac{(b^2c^2+6\,a\cdots}{\cdots}\right.}{\cdots}\right.$$

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^6 + (5*a*b^2*c^2 - 6*a^2*
b*c*d + a^3*d^2)*x^4)*sqrt(a*c*e)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^
4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c*d + a*d^2)*x^4 + 2*
a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
))/x^4) + 4*((15*a*b^2*c^2*d - 13*a^2*b*c*d^2)*x^6 - 2*a^3*c^3 + (15*a*b^2*
c^3 - 8*a^2*b*c^2*d - 5*a^3*c*d^2)*x^4 + (5*a^2*b*c^3 - 7*a^3*c^2*d)*x^2)*s
qrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^4*b*c*e^2*x^6 + a^5*c*e^2*x^4), 1/16*(
3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^6 + (5*a*b^2*c^2 - 6*a^2*b*c*d +
 a^3*d^2)*x^4)*sqrt(-a*c*e)*arctan(1/2*sqrt(-a*c*e)*((b*c + a*d)*x^2 + 2*a*
c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*c*e*x^2 + a^2*c*e)) + 2*((15*a*b^
2*c^2*d - 13*a^2*b*c*d^2)*x^6 - 2*a^3*c^3 + (15*a*b^2*c^3 - 8*a^2*b*c^2*d -
 5*a^3*c*d^2)*x^4 + (5*a^2*b*c^3 - 7*a^3*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c)))/(a^4*b*c*e^2*x^6 + a^5*c*e^2*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{2,[1,4,4]%%%},[2,1,7,0]%%%}+%%%{%%%{-8,[2,3,4]%%%},[2,1,6
,1]%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} \, dx = \int \frac{1}{x^5 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}} \, dx$$

```
[In] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

3.313 $\int \frac{x^4}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 453

$$\begin{aligned}\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx &= \frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\\&\quad+\frac{(b^2c^2-16abcd+16a^2d^2)\,x(a+bx^2)}{5b^4e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}-\frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\\&\quad-\frac{\sqrt{c}(b^2c^2-16abcd+16a^2d^2)\,(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5b^4\sqrt{d}e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\\&\quad-\frac{c^{3/2}(7bc-8ad)\,(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{5b^3\sqrt{d}e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\end{aligned}$$

```
[Out] 1/5*(-8*a*d+7*b*c)*x*(b*x^2+a)/b^3/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+6/5*d*x^
3*(b*x^2+a)/b^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/5*(16*a^2*d^2-16*a*b*c*d+
b^2*c^2)*x*(b*x^2+a)/b^4/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-x^3*(d*x
^2+c)/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/5*c^(3/2)*(-8*a*d+7*b*c)*(b*x^2+a
)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*
x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/b^3/e/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x
^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/5*(16*a^2*d^2-16*a*b*c*d+b^2*c
^2)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c
^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)/b^4/e/(d*x^2+c)/d^(1/2)
/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1986, 478, 595, 596, 545, 429, 506, 422}

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = -\frac{\sqrt{c}(a+bx^2)\left(16a^2d^2-16abcd+b^2c^2\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5b^4\sqrt{d}e\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{x(a+bx^2)\left(16a^2d^2-16abcd+b^2c^2\right)}{5b^4e\left(c+dx^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{c^{3/2}(a+bx^2)(7bc-8ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{5b^3\sqrt{d}e\left(c+dx^2\right)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{x(a+bx^2)(7bc-8ad)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[In] Int[x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] ((7*b*c - 8*a*d)*x*(a + b*x^2))/(5*b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])
 + (6*d*x^3*(a + b*x^2))/(5*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((b^
2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*x*(a + b*x^2))/(5*b^4*e*Sqrt[(e*(a + b*x^2
))/(c + d*x^2)]*(c + d*x^2)) - (x^3*(c + d*x^2))/(b*e*Sqrt[(e*(a + b*x^2))/
(c + d*x^2)]) - (Sqrt[c]*(b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*(a + b*x^2)*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b^4*Sqrt[d]*e*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d
*x^2)) - (c^(3/2)*(7*b*c - 8*a*d)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b^3*Sqrt[d]*e*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^2}\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{x^2\sqrt{c+dx^2}(3c+6dx^2)}{\sqrt{a+bx^2}}\,dx}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{x^2(3c(5bc-6ad)+3d(7bc-8ad)x^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{3acd(7bc-8ad)-3d(b^2c^2-16abcd+16a^2d^2)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15b^3de\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(ac(7bc-8ad)\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left((b^2c^2-16abcd+16a^2d^2)\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2c^2-16abcd+16a^2d^2)\,x(a+bx^2)}{5b^4e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{c^{3/2}(7bc-8ad)\,(a+bx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5b^3\sqrt{d}e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{\left(c(b^2c^2-16abcd+16a^2d^2)\sqrt{a+bx^2}\right)\int\frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{5b^4e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2c^2-16abcd+16a^2d^2)\,x(a+bx^2)}{5b^4e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{c}(b^2c^2-16abcd+16a^2d^2)\,(a+bx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5b^4\sqrt{d}e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{c^{3/2}(7bc-8ad)\,(a+bx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5b^3\sqrt{d}e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.55 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.60

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(c+dx^2)\left(-8a^2d+ab(7c-2dx^2)+b^2x^2(2c+dx^2)\right)-ic(b^2c^2-16\right.}{}$$

```
[In] Integrate[x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(c + d*x^2)*(-8*a^2*d + a
*b*(7*c - 2*d*x^2) + b^2*x^2*(2*c + d*x^2)) - I*c*(b^2*c^2 - 16*a*b*c*d + 1
6*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)] + I*c*(b^2*c^2 - 9*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])
)/(5*b^3*Sqrt[b/a]*d*e^2*(a + b*x^2))
```

Maple [A] (verified)

Time = 9.94 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.72

method	result
risch	$-\frac{x\left(-b d\,x^{2}+3 a d-2 b c\right)\left(b\,x^{2}+a\right)}{5 b^{3} e \sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}+\frac{\left(-\frac{2\left(11 a^{2} d^{2}-11 a b c d+b^{2} c^{2}\right) a c e \sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{e d a+e b c}{c b e}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{e d a}{c}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{b d e\,x^{4}+a d e\,x^{2}+b c e\,x^{2}+a c e}\,\left(e d a+e b c+e\left(a d-b c\right)\right)}$
default	$-\frac{\left(b\,x^{2}+a\right)\left(-\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{-\frac{b}{a}}\,b^{2} d^{3} x^{7}+2\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{-\frac{b}{a}}\,a b\,d^{3} x^{5}-3\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{-\frac{b}{a}}\,b^{2} c\,d^{2} x^{5}+5\sqrt{b d\,x^{4}}\right.}{}$

```
[In] int(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b^3*x*(-b*d*x^2+3*a*d-2*b*c)*(b*x^2+a)/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
+1/5/b^3*(-2*(11*a^2*d^2-11*a*b*c*d+b^2*c^2)*a*c*e/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d
*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1
/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))-a*(5*a^2*d^2
-13*a*b*c*d+7*b^2*c^2)/b/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)
/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(
a*d*e+b*c*e)/c/b/e)^(1/2))+5*a^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b*(-(b*d*e*x^2
+b*c*e)/a/(a*d-b*c)*x/e/((x^2+a/b)*(b*d*e*x^2+b*c*e))^(1/2)+(1/a+b*c/a/(a*d
-b*c))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*
x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e
)^(1/2))-2*d*b/(a*d-b*c)*c*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(
1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*
(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a
)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))))/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*
((d*x^2+c)*e*(b*x^2+a))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx =$$

$$-\frac{((b^3c^3-16\,ab^2c^2d+16\,a^2bcd^2)x^3+(ab^2c^3-16\,a^2bc^2d+16\,a^3cd^2)x)\sqrt{\frac{be}{d}}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-((b}{}$$

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(((b^3*c^3 - 16*a*b^2*c^2*d + 16*a^2*b*c*d^2)*x^3 + (a*b^2*c^3 - 16*a^
2*b*c^2*d + 16*a^3*c*d^2)*x)*sqrt(b*e/d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(
-c/d)/x), a*d/(b*c)) - ((b^3*c^3 - 16*a*b^2*c^2*d + 8*a^2*b*d^3 + (16*a^2*b
 - 7*a*b^2)*c*d^2)*x^3 + (a*b^2*c^3 - 16*a^2*b*c^2*d + 8*a^3*d^3 + (16*a^3
- 7*a^2*b)*c*d^2)*x)*sqrt(b*e/d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x)
, a*d/(b*c)) - (b^3*d^3*x^8 + (3*b^3*c*d^2 - 2*a*b^2*d^3)*x^6 + a*b^2*c^3 -
 16*a^2*b*c^2*d + 16*a^3*c*d^2 + (3*b^3*c^2*d - 11*a*b^2*c*d^2 + 8*a^2*b*d^
3)*x^4 + (b^3*c^3 - 8*a*b^2*c^2*d - 8*a^2*b*c*d^2 + 16*a^3*d^3)*x^2)*sqrt((
b*e*x^2 + a*e)/(d*x^2 + c)))/(b^5*d*e^2*x^3 + a*b^4*d*e^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**4/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{x^4}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.314 $\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 378

$$\begin{aligned}\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx &= \frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{d(7bc-8ad)x(a+bx^2)}{3b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\\ &\quad-\frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}(7bc-8ad)\,(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^3e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\\ &\quad+\frac{c^{3/2}(3bc-4ad)\,(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab^2\sqrt{d}e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\end{aligned}$$

```
[Out] 4/3*d*x*(b*x^2+a)/b^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*d*(-8*a*d+7*b*c)*
x*(b*x^2+a)/b^3/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-x*(d*x^2+c)/b/e/(
e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*c^(3/2)*(-4*a*d+3*b*c)*(b*x^2+a)*(1/(1+d*x
^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2
),(1-b*c/a/d)^(1/2))/a/b^2/e/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-8*a*d+7*b*c)*(b*x^2+a)*(1/(1+d*x^2/
c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(
1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/b^3/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))
^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1986, 478, 542, 545, 429, 506, 422}

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = -\frac{\sqrt{c}\sqrt{d}(a+bx^2)\,(7bc-8ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^3e\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c^{3/2}(a+bx^2)\,(3bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab^2\sqrt{d}e\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{dx(a+bx^2)\,(7bc-8ad)}{3b^3e\,(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[In] Int[x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (4*d*x*(a + b*x^2))/(3*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (d*(7*b*c
 - 8*a*d)*x*(a + b*x^2))/(3*b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*
x^2)) - (x*(c + d*x^2))/(b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (Sqrt[c]*
Sqrt[d]*(7*b*c - 8*a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(3*b^3*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a
 + b*x^2))/(c + d*x^2)]*(c + d*x^2)) + (c^(3/2)*(3*b*c - 4*a*d)*(a + b*x^2)
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*b^2*Sqrt[d]*
e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(
c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^2}\int \frac{x^2\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}\,(c+4dx^2)}{\sqrt{a+bx^2}}\,dx}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{c(3bc-4ad)+d(7bc-8ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(d(7bc-8ad)\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left(c(3bc-4ad)\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{d(7bc-8ad)x(a+bx^2)}{3b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c^{3/2}(3bc-4ad)(a+bx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ab^2\sqrt{d}e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{\left(cd(7bc-8ad)\sqrt{a+bx^2}\right)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}dx}{3b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{d(7bc-8ad)x(a+bx^2)}{3b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{d}(7bc-8ad)(a+bx^2)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^3e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{c^{3/2}(3bc-4ad)(a+bx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ab^2\sqrt{d}e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.91 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.58

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}x(c+dx^2)(-3bc+4ad+bdx^2)+ic(-7bc+8ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\right.}{3a^2\left(\frac{b}{a}\right)^5}$$

```
[In] Integrate[x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*x*(c + d*x^2)*(-3*b*c + 4*a*d
 + b*d*x^2) + I*c*(-7*b*c + 8*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
```

```
EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (4*I)*c*(-(b*c) + a*d)*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)]))/(3*a^2*(b/a)^(5/2)*e^2*(a + b*x^2))
```

Maple [A] (verified)

Time = 8.86 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.70

method	result
default	$\frac{(b\,x^2+a)\left(\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,b\,d^2x^5+\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,bcd\,x^3+3\sqrt{bd\,x^4+ad\,x^2+}\right.}{}$
risch	$\frac{dx(b\,x^2+a)}{3b^2e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}-\frac{\left(\frac{3a\left(a^2d^2-2abcd+b^2c^2\right)\left(-\frac{\left(bde\,x^2+ebc\right)x}{a(ad-bc)e\sqrt{\left(x^2+\frac{a}{b}\right)\left(bde\,x^2+ebc\right)}}+\frac{\left(\frac{1}{a}+\frac{bc}{a(ad-bc)}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}\right.}{b}\right.}{}$

```
[In] int(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^2+a)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b*d^2*x^5+((d*x^2+c
)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*d^2*x^3+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a
)^(1/2)*b*c*d*x^3+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*d^2*
x^3-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c*d*x^3+4*((d*x^2+
c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d-4*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2
-8*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d+7*((d*x^2+c)*(b*x^2+a))^(1/2)*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*b*c^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*c*d*x+3*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*c*d*x-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*(-b/a)^(1/2)*b*c^2*x)/b^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(d*x^2+c)^2
/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx =$$

$$-\frac{((7\,b^2c^2-8\,abcd)x^3+(7\,abc^2-8\,a^2cd)x)\sqrt{\frac{be}{d}}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-((7\,b^2c^2-4\,abd^2-(8\,ab-3\,b^2}{}$$

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(((7*b^2*c^2 - 8*a*b*c*d)*x^3 + (7*a*b*c^2 - 8*a^2*c*d)*x)*sqrt(b*e/d)
*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((7*b^2*c^2 - 4*a
*b*d^2 - (8*a*b - 3*b^2)*c*d)*x^3 + (7*a*b*c^2 - 4*a^2*d^2 - (8*a^2 - 3*a*b
)*c*d)*x)*sqrt(b*e/d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)
) - (b^2*d^2*x^6 + (5*b^2*c*d - 4*a*b*d^2)*x^4 + 7*a*b*c^2 - 8*a^2*c*d + (4
*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(
b^4*e^2*x^3 + a*b^3*e^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate(x^2/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.315 $\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 327

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(bc-2ad)x(a+bx^2)}{ab^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \frac{\sqrt{c}\sqrt{d}(bc-2ad)\,(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \frac{c^{3/2}\sqrt{d}(a+bx^2)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{abe\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

```
[Out] (-a*d+b*c)*x/a/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-d*(-2*a*d+b*c)*x*(b*x^2+a)
/a/b^2/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+c^(3/2)*(b*x^2+a)*(1/(1+d*
x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/
2),(1-b*c/a/d)^(1/2))*d^(1/2)/a/b/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+(-2*a*d+b*c)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/
2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a
/d)^(1/2))*c^(1/2)*d^(1/2)/a/b^2/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2
)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1986, 424, 545, 429, 506, 422}

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{d}(a+bx^2)(bc-2ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2e(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c^{3/2}\sqrt{d}(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{abe(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{dx(a+bx^2)(bc-2ad)}{ab^2e(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x(bc-ad)}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(-3/2),x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (d*(b*c - 2*a*d
)*x*(a + b*x^2))/(a*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) +
(Sqrt[c]*Sqrt[d]*(b*c - 2*a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(a*b^2*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) + (c^(3/2)*Sqrt[d]*(a + b*x^2)*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*b*e*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a+bx^2}\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} \\
&= \frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{acd-d(bc-2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} \\
&= \frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(cd\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} - \frac{\left(d(bc-2ad)\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} \\
&= \frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(bc-2ad)x(a+bx^2)}{ab^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} \\
&\quad + \frac{c^{3/2}\sqrt{d}(a+bx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{abe\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} \\
&\quad + \frac{\left(cd(bc-2ad)\sqrt{a+bx^2}\right)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{ab^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}
\end{aligned}
$$

$$= \frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(bc-2ad)x(a+bx^2)}{ab^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \frac{\sqrt{c}\sqrt{d}(bc-2ad)\,(a+bx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \frac{c^{3/2}\sqrt{d}(a+bx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{abe\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-ic(-bc+2ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(bc-ad)\left(\right.\right.}{a^2\left(\frac{b}{a}\right)^{3/2}e^2\,(a+}$$

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(-3/2),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*((-I)*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (
b*c - a*d)*(Sqrt[b/a]*x*(c + d*x^2) - I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(a^2*(b/a)^(3/2)*e
^2*(a + b*x^2))
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.57

method	result
default	$-\frac{(b\,x^2+a)\left(\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,a\,d^2x^3-\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,bcd\,x^3+\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\right.}{}$

```
[In] int(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x^2+a)/b*((b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*d^2*x^3-(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c*d*x^3+((d*x^2+c)*(b*x^2+
a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*a*c*d-((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*
```

```
x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2-2*((d*x^2+c
)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/
a)^(1/2),(a*d/b/c)^(1/2))*a*c*d+((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2+(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*c*d*x-(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c^2*x)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(d*x^2+
c)^2/a/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{((b^2c^2-2\,abcd)x^3+(abc^2-2\,a^2cd)x)\sqrt{\frac{be}{d}}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-((b^2c^2-2\,abc}{}$$

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (((b^2*c^2 - 2*a*b*c*d)*x^3 + (a*b*c^2 - 2*a^2*c*d)*x)*sqrt(b*e/d)*sqrt(-c/
d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((b^2*c^2 - 2*a*b*c*d - a*
b*d^2)*x^3 + (a*b*c^2 - 2*a^2*c*d - a^2*d^2)*x)*sqrt(b*e/d)*sqrt(-c/d)*elli
ptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (a*b*d^2*x^4 + 2*a^2*d^2*x^2 - a*
b*c^2 + 2*a^2*c*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^3*e^2*x^3 + a^2*
b^2*e^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate(((b*x^2 + a)*e/(d*x^2 + c))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(1/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(1/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.316 $\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 380

$$\begin{aligned}\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx &= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(2bc-ad)\left(a+bx^2\right)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\\&\quad+\frac{d(2bc-ad)x(a+bx^2)}{a^2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}-\frac{\sqrt{c}\sqrt{d}(2bc-ad)\left(a+bx^2\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a^2be\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}\\&\quad+\frac{c^{3/2}\sqrt{d}(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a^2e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}\end{aligned}$$

```
[Out] (-a*d+b*c)/a/b/e/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-(-a*d+2*b*c)*(b*x^2+a)/a^2
/b/e/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+d*(-a*d+2*b*c)*x*(b*x^2+a)/a^2/b/e/(d*
x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+c^(3/2)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/
d)^(1/2))*d^(1/2)/a^2/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2
+a)/(d*x^2+c))^(1/2)-(-a*d+2*b*c)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/
c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c
^(1/2)*d^(1/2)/a^2/b/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+
a)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 479, 597, 545, 429, 506, 422}

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{c^{3/2}\sqrt{d}(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a^2e\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{d}(a+bx^2)\,(2bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a^2be\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(a+bx^2)\,(2bc-ad)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{dx(a+bx^2)\,(2bc-ad)}{a^2be\,(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[In] Int[1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*c - a*d)/(a*b*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((2*b*c - a*d)*(a
 + b*x^2))/(a^2*b*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (d*(2*b*c - a*d)
*x*(a + b*x^2))/(a^2*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (
Sqrt[c]*Sqrt[d]*(2*b*c - a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(a^2*b*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) + (c^(3/2)*Sqrt[d]*(a + b*x^2)*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a^2*e*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
```

```
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^2}\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{-c(2bc-ad)-bcdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{abc^2d+bcd(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{a^2bce\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(cd\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{ae\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left(d(2bc-ad)\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx}{a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{d(2bc-ad)x(a+bx^2)}{a^2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{c^{3/2}\sqrt{d}(a+bx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a^2e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{\left(cd(2bc-ad)\sqrt{a+bx^2}\right)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}dx}{a^2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{d(2bc-ad)x(a+bx^2)}{a^2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{\sqrt{c}\sqrt{d}(2bc-ad)(a+bx^2)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a^2be\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{c^{3/2}\sqrt{d}(a+bx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a^2e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.13 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{\frac{b}{a}}(c+dx^2)(ac+2bcx^2-adx^2)+ic(-2bc+ad)x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{d}{}}\right.}{a^2}$$

```
[In] Integrate[1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[b/a]*(c + d*x^2)*(a*c + 2*b*c*x^
2 - a*d*x^2)) + I*c*(-2*b*c + a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
```

```
]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(-(b*c) + a*d)*x
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)]))/(a^2*Sqrt[b/a]*e^2*x*(a + b*x^2))
```

Maple [A] (verified)

Time = 8.12 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.71

method	result
default	$-\frac{(b\,x^2+a)\left(\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,bcd\,x^4-\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,a\,d^2x^4+\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,bcd\,x^4-2\sqrt{(d\right.}{}$
risch	$-\frac{c(b\,x^2+a)}{a^2xe\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}+\frac{\left(\frac{a^2d^2\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{b\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}-\frac{2b\,c^2dae\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)-E\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}\,(eda+ebc}\right.}{}$

```
[In] int(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x^2+a)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b*c*d*x^4-(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*d^2*x^4+(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*(-b/a)^(1/2)*b*c*d*x^4-2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d*x
+2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2*x+((d*x^2+c)*(b*x^2+a))^(1/2)*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*a*c*d*x-2*c^2*b*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*x*((d*x^2+c)*(b*x^2+a))^(1/2)+((d*x^2+c)*(b*x^
2+a))^(1/2)*(-b/a)^(1/2)*a*c*d*x^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)
*b*c^2*x^2-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*c*d*x^2+(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c^2*x^2+((d*x^2+c)*(b*x^2+a))
^(1/2)*(-b/a)^(1/2)*a*c^2)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(d*x^2+c)^2/a^2/x/
(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{((2\,b^3cd-ab^2d^2)x^3+(2\,ab^2cd-a^2bd^2)x)\sqrt{\frac{ace}{d^2}}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-((2}{}$$

[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")

[Out] (((2*b^3*c*d - a*b^2*d^2)*x^3 + (2*a*b^2*c*d - a^2*b*d^2)*x)*sqrt(a*c*e/d^2)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((2*b^3*c*d + (a^2*b - a*b^2)*d^2)*x^3 + (2*a*b^2*c*d + (a^3 - a^2*b)*d^2)*x)*sqrt(a*c*e/d^2)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (2*a*b^2*c^2*x^2 + a^2*b*c^2 + (2*a*b^2*c*d - a^2*b*d^2)*x^4)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^3*b^2*e^2*x^3 + a^4*b*e^2*x)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

[In] integrate(1/x**2/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}x^2}\,dx$$

[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")

[Out] integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2), x)

Giac [F]

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^2\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

3.317 $\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 444

$$
\begin{aligned}
\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx &= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)\,(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} \\
&+ \frac{(8bc-7ad)\,(a+bx^2)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(8bc-7ad)x(a+bx^2)}{3a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} \\
&+ \frac{\sqrt{c}\sqrt{d}(8bc-7ad)\,(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^3e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} \\
&- \frac{\sqrt{c}\sqrt{d}(4bc-3ad)\,(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^3e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}
\end{aligned}
$$

```
[Out] (-a*d+b*c)/a/b/e/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-3*a*d+4*b*c)*(b*x^
2+a)/a^2/b/e/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-7*a*d+8*b*c)*(b*x^2+a)
/a^3/e/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*d*(-7*a*d+8*b*c)*x*(b*x^2+a)/a^3
/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-7*a*d+8*b*c)*(b*x^2+a)*(1/
(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c
)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/a^3/e/(d*x^2+c)/(c*(b*x^2+a)/a/(
d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-3*a*d+4*b*c)*(b*x^2+a)*
(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/a^3/e/(d*x^2+c)/(c*(b*x^2+a)/
a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 479, 597, 545, 429, 506, 422}

$$\int \frac{1}{x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx =$$
$$-\frac{\sqrt{c}\sqrt{d}(a+bx^2)(4bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right), 1-\frac{bc}{ad}\right)}{3a^3 e(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$
$$+\frac{\sqrt{c}\sqrt{d}(a+bx^2)(8bc-7ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^3 e(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(a+bx^2)(8bc-7ad)}{3a^3 ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$
$$-\frac{dx(a+bx^2)(8bc-7ad)}{3a^3 e(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(a+bx^2)(4bc-3ad)}{3a^2 bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

```
[In] Int[1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*c - a*d)/(a*b*e*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((4*b*c - 3*a*d
)*(a + b*x^2))/(3*a^2*b*e*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((8*b*c
- 7*a*d)*(a + b*x^2))/(3*a^3*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (d*(8
*b*c - 7*a*d)*x*(a + b*x^2))/(3*a^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c
+ d*x^2)) + (Sqrt[c]*Sqrt[d]*(8*b*c - 7*a*d)*(a + b*x^2)*EllipticE[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^3*e*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt[c]*Sqrt[d
]*(4*b*c - 3*a*d)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(3*a^3*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x
^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^2}\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$=\frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{a+bx^2}\int\frac{-c(4bc-3ad)-d(3bc-2ad)x^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$=\frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(4bc-3ad)\,(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{a+bx^2}\int\frac{-bc^2(8bc-7ad)-bcd(4bc-3ad)x^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a^2bce\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$=\frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(4bc-3ad)\,(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(8bc-7ad)\,(a+bx^2)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{a+bx^2}\int\frac{abc^2d(4bc-3ad)+b^2c^2d(8bc-7ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a^3bc^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$=\frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(4bc-3ad)\,(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(8bc-7ad)\,(a+bx^2)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\left(bd(8bc-7ad)\sqrt{a+bx^2}\right)\int\frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}-\frac{\left(d(4bc-3ad)\sqrt{a+bx^2}\right)\int\frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$=\frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(4bc-3ad)\,(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(8bc-7ad)\,(a+bx^2)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{d(8bc-7ad)x(a+bx^2)}{3a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}-\frac{\sqrt{c}\sqrt{d}(4bc-3ad)\,(a+bx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^3e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}+\frac{\left(cd(8bc-7ad)\sqrt{a+bx^2}\right)\int\frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{3a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$=\frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(4bc-3ad)\,(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(8bc-7ad)\,(a+bx^2)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{d(8bc-7ad)x(a+bx^2)}{3a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}+\frac{\sqrt{c}\sqrt{d}(8bc-7ad)\,(a+bx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^3e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}-\frac{\sqrt{c}\sqrt{d}(4bc-3ad)\,(a+bx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^3e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.65 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{\frac{b}{a}}(c+dx^2)\left(-8b^2cx^4+a^2(c+4dx^2)+ab(-4cx^2+7dx^4)\right)-ibc(-8\cdots\right.}{\cdots}$$

```
[In] Integrate[1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[b/a]*(c + d*x^2)*(-8*b^2*c*x^4 +
 a^2*(c + 4*d*x^2) + a*b*(-4*c*x^2 + 7*d*x^4))) - I*b*c*(-8*b*c + 7*a*d)*x^
3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)] - I*(8*b^2*c^2 - 11*a*b*c*d + 3*a^2*d^2)*x^3*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3
*a^3*Sqrt[b/a]*e^2*x^3*(a + b*x^2))
```

Maple [A] (verified)

Time = 8.75 (sec) , antiderivative size = 866, normalized size of antiderivative = 1.95

method	result
default	$-\frac{(b\,x^2+a)\left(4\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^6-5\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,b^2cd\,x^6+3\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^6-3\cdots\right.}{\cdots}$
risch	$-\frac{(b\,x^2+a)(4ad\,x^2-5bc\,x^2+ac)}{3a^3x^3e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}+\frac{\left(-\frac{abcd\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}-\frac{8a^2b\,d^2ce\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{\cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\cdots}}\right.}{\cdots}$

```
[In] int(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x^2+a)*(4*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^6-5*((
d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^6+3*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^6-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(-b/a)^(1/2)*b^2*c*d*x^6-3*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*d^2*x^3
+11*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x^3-8*((d*x^2+c)*(b*x^2+a))^
(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*b^2*c^2*x^3-7*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x^3+8
```

```
*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x^3+4*((d*x^2+c)*(b*x^2+a))^(1/
2)*(-b/a)^(1/2)*a^2*d^2*x^4-5*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b^2*
c^2*x^4+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*c*d*x^4-3*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b^2*c^2*x^4+5*((d*x^2+c)*(b*
x^2+a))^(1/2)*(-b/a)^(1/2)*a^2*c*d*x^2-4*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)
^(1/2)*a*b*c^2*x^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a^2*c^2)/(e*(b*
x^2+a)/(d*x^2+c))^(3/2)/(d*x^2+c)^2/a^3/x^3/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = -\frac{((8\,b^4c^2d-7\,ab^3cd^2)x^5+(8\,ab^3c^2d-7\,a^2b^2cd^2)x^3)\sqrt{\frac{ace}{d^2}}\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}\right)-((8\,b^4c^2d-3\,a^3bd^3}{}$$

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(((8*b^4*c^2*d - 7*a*b^3*c*d^2)*x^5 + (8*a*b^3*c^2*d - 7*a^2*b^2*c*d^2
)*x^3)*sqrt(a*c*e/d^2)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c
)) - ((8*b^4*c^2*d - 3*a^3*b*d^3 + (4*a^2*b^2 - 7*a*b^3)*c*d^2)*x^5 + (8*a*
b^3*c^2*d - 3*a^4*d^3 + (4*a^3*b - 7*a^2*b^2)*c*d^2)*x^3)*sqrt(a*c*e/d^2)*s
qrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (a^3*b*c^3 - (8*a*b
^3*c^2*d - 7*a^2*b^2*c*d^2)*x^6 - (8*a*b^3*c^3 - 3*a^2*b^2*c^2*d - 4*a^3*b*
c*d^2)*x^4 - (4*a^2*b^2*c^3 - 5*a^3*b*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x
^2 + c)))/(a^4*b^2*c*e^2*x^5 + a^5*b*c*e^2*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**4/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^4}\,dx$$

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^4}\,dx$$

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^4 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

3.318 $\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 216

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx=-\frac{(b^2+4abc-8a^2c^2)\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^2d^3}-\frac{(b+4ac)\,(c+dx^2)^2\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8ad^3}+\frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{6ad^3}+\frac{b(b^2+4abc+8a^2c^2)\,\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{5/2}d^3}$$

```
[Out] 1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(3/2)/a/d^3+1/16*b*(8*a^2*c^2+4
*a*b*c+b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(5/2)/d^3-
1/16*(-8*a^2*c^2+4*a*b*c+b^2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a
^2/d^3-1/8*(4*a*c+b)*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{1985, 1981, 1980, 474, 466, 393, 214\}$

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{(8a^2c^2-b(4ac+b))\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16a^2d^3}+\frac{b(8a^2c^2+4abc+b^2)\,\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{5/2}d^3}+\frac{(c+dx^2)^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{6ad^3}-\frac{(4ac+b)\,(c+dx^2)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8ad^3}$$

```
[In] Int[x^5*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((8*a^2*c^2 - b*(b + 4*a*c))*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^
2)])/(16*a^2*d^3) - ((b + 4*a*c)*(c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c
+ d*x^2)])/(8*a*d^3) + ((c + d*x^2)^3*((b + a*c + a*d*x^2)/(c + d*x^2))^(3/
2))/(6*a*d^3) + (b*(b^2 + 4*a*b*c + 8*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*
x^2)/(c + d*x^2)]/Sqrt[a]])/(16*a^(5/2)*d^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
```

```
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^5\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{\frac{b+ac+adx}{c+dx}}\,dx,x,x^2\right) \\
&= -\left((bd)\text{Subst}\left(\int \frac{x^2(-b-ac+cx^2)^2}{(ad-dx^2)^4}\,dx,x,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right) \\
&= \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{6ad^3}+\frac{b\text{Subst}\left(\int \frac{x^2(-3(b^2+4abc+2a^2c^2)d^2+6ac^2d^2x^2)}{(ad-dx^2)^3}\,dx,x,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{6ad^2} \\
&= -\frac{(b+4ac)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8ad^3}+\frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{6ad^3} \\
&\quad -\frac{b\text{Subst}\left(\int \frac{-3b(b+4ac)d^3+24ac^2d^3x^2}{(ad-dx^2)^2}\,dx,x,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{24ad^4}
\end{aligned}$$

$$= \frac{(8a^2c^2 - b(b+4ac))(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^2d^3} - \frac{(b+4ac)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8ad^3} + \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{6ad^3} + \frac{(b(b^2+4abc+8a^2c^2))\,\text{Subst}\left(\int \frac{1}{ad-dx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{16a^2d^2}$$

$$= \frac{(8a^2c^2 - b(b+4ac))(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^2d^3} - \frac{(b+4ac)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8ad^3} + \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{6ad^3} + \frac{b(b^2+4abc+8a^2c^2)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{5/2}d^3}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.67

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

$$= \frac{\sqrt{a}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(-3b^2+2ab(-5c+dx^2)+8a^2(c^2-cdx^2+d^2x^4))+3b(b^2+4abc+8a^2c^2)\,\text{arcta}}{48a^{5/2}d^3}$$

```
[In] Integrate[x^5*Sqrt[a + b/(c + d*x^2)],x]
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b^2 + 2*a*b*
(-5*c + d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) + 3*b*(b^2 + 4*a*b*c + 8*
a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(48*a^(5/2
)*d^3)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.10

method	result
risch	$\frac{(8a^2d^2x^4-8a^2cd\,x^2+2abd\,x^2+8a^2c^2-10abc-3b^2)(d\,x^2+c)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{48d^3a^2} + \frac{b(8a^2c^2+4abc+b^2)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+bc+}\right.}{32d^2a^2\sqrt{}}$
default	$\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-48\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,a^2cd\,x^2-12\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,abd\,x^2+2\right.}{}$

```
[In] int(x^5*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/d^3*(8*a^2*d^2*x^4-8*a^2*c*d*x^2+2*a*b*d*x^2+8*a^2*c^2-10*a*b*c-3*b^2)
*(d*x^2+c)/a^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/32*b/d^2*(8*a^2*c^2+4*a*
b*c+b^2)/a^2*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^2+b*c+(2*a*c*d
+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)
*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.96

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

$$= \left[\frac{3\,(8\,a^2bc^2+4\,ab^2c+b^3)\sqrt{a}\log\Big(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2+4\,(2\,ad^2x^4+(4\,ac+\right.$$

$$-\frac{3\,(8\,a^2bc^2+4\,ab^2c+b^3)\sqrt{-a}\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(8\,a^3d^3x^6+2\,a^2bd^2x^4+8\,a^3c^3-}{96\,a^3d^3}$$

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2
*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c +
b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) +
4*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10*a^2*b*c^2 - 3*a*b^2*c -
 (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d
^3), -1/96*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^
2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 +
a^2*c + a*b)) - 2*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10*a^2*b*c
^2 - 3*a*b^2*c - (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/(a^3*d^3)]
```

Sympy [F]

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

```
[In] integrate(x**5*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.52

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx=$$

$$-\frac{3\,(8\,a^2bc^2-4\,ab^2c-b^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\,(6\,a^3bc^2-ab^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\,(8\,a^4bc^2+4\,a^3b^2c+a^2b^3)\sqrt{\cdots}}{48\left(a^5d^3-\frac{3\,(adx^2+ac+b)a^4d^3}{dx^2+c}+\frac{3\,(adx^2+ac+b)^2a^3d^3}{(dx^2+c)^2}-\frac{(adx^2+ac+b)^3a^2d^3}{(dx^2+c)^3}\right)}$$

$$-\frac{(8\,a^2c^2+4\,abc+b^2)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{5}{2}}d^3}$$

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(8*a^2*b*c^2 - 4*a*b^2*c - b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^
(5/2) - 8*(6*a^3*b*c^2 - a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3
*(8*a^4*b*c^2 + 4*a^3*b^2*c + a^2*b^3)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^5*d^3 - 3*(a*d*x^2 + a*c + b)*a^4*d^3/(d*x^2 + c) + 3*(a*d*x^2 + a*c
+ b)^2*a^3*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a^2*d^3/(d*x^2 + c)^3)
 - 1/32*(8*a^2*c^2 + 4*a*b*c + b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/
2)*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.01

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$=\frac{1}{96}\left(2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\,x^2\left(\frac{4\,x^2}{d}-\frac{4\,a^2cd^3-abd^3}{a^2d^5}\right)+\frac{8\,a^2c^2d^2-10\,abcd^2-3\,b^2d^2}{a^2d^5}\right)\right.$$
$$\left.+c\right)$$

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2
/d - (4*a^2*c*d^3 - a*b*d^3)/(a^2*d^5)) + (8*a^2*c^2*d^2 - 10*a*b*c*d^2 - 3
*b^2*d^2)/(a^2*d^5)) - 3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*log(abs(2*a*c*d +
2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))
*sqrt(a)*abs(d) + b*d))/(a^(5/2)*d^2*abs(d)))*sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^5\,\sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

```
[In] int(x^5*(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^5*(a + b/(c + d*x^2))^(1/2), x)
```

3.319 $\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 141

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{(b-4ac)\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8ad^2}+\frac{(c+dx^2)^2\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4d^2}-\frac{b(b+4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}$$

```
[Out] -1/8*b*(4*a*c+b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)
/d^2+1/8*(-4*a*c+b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2+1/4*(
d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1985, 1981, 1980, 466, 393, 214}

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=-\frac{b(4ac+b)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}+\frac{(c+dx^2)^2\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4d^2}+\frac{(b-4ac)\,(c+dx^2)\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8ad^2}$$

```
[In] Int[x^3*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((b - 4*a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*a*d^2) +
 ((c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*d^2) - (b*(b + 4*
a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*a^(3/2)*d^2
)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\text{integral} = \int x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int x\sqrt{\frac{b+ac+adx}{c+dx}}\,dx, x, x^2\right)$$
$$= -\left((bd)\text{Subst}\left(\int \frac{x^2(-b-ac+cx^2)}{(ad-dx^2)^3}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right)$$
$$= \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4d^2} + \frac{b\text{Subst}\left(\int \frac{-bd+4cdx^2}{(ad-dx^2)^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{4d}$$
$$= \frac{(b-4ac)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8ad^2} + \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4d^2} - \frac{(b(b+4ac))\text{Subst}\left(\int \frac{1}{ad-dx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{8ad}$$
$$= \frac{(b-4ac)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8ad^2} + \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4d^2} - \frac{b(b+4ac)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(b-2ac+2adx^2)}{8ad^2} - \frac{b(b+4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}$$

```
[In] Integrate[x^3*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b - 2*a*c + 2*a*d*x^2))
/(8*a*d^2) - (b*(b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/S
qrt[a]])/(8*a^(3/2)*d^2)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{\left(-2ad\,x^{2}+2ac-b\right)\left(d\,x^{2}+c\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}{8d^{2}a}-\frac{b(4ac+b)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^{2}x^{2}}{\sqrt{a\,d^{2}}}+\sqrt{a\,c^{2}+bc+(2acd+bd)x^{2}+a\,d^{2}x^{4}}\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}\,\sqrt{(}}{16da\sqrt{a\,d^{2}}\,(ad\,x^{2}+ac+b)}$
default	$-\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}\,(d\,x^{2}+c)\left(-4\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}\,\sqrt{a\,d^{2}}\,ad\,x^{2}+4\ln\left(\frac{2a\,d^{2}x^{2}+2acd+2\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}\,\sqrt{a}}{2\sqrt{a\,d^{2}}}\right.\right.$

```
[In] int(x^3*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d^2*(-2*a*d*x^2+2*a*c-b)*(d*x^2+c)/a*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)
-1/16*b/d*(4*a*c+b)/a*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^2+b*c
+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.30

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

$$=\left[\frac{(4\,abc+b^2)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2-4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac\right.}{32\,a^2d^2}\right.$$

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c +
 a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2
+ b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4 +
a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*
d^2), 1/16*((4*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqr
t(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*
(2*a^2*d^2*x^4 + a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/(a^2*d^2)]
```

Sympy [F]

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

```
[In] integrate(x**3*(a+b/(d*x**2+c))**(1/2),x)
[Out] Integral(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.55

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=-\frac{(4\,abc-b^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc+ab^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^3d^2-\frac{2\,(adx^2+ac+b)a^2d^2}{dx^2+c}+\frac{(adx^2+ac+b)^2ad^2}{(dx^2+c)^2}\right)}+\frac{(4\,ac+b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{3}{2}}d^2}$$

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] -1/8*((4*a*b*c - b^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c
+ a*b^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d^2 - 2*(a*d*x^2 + a*c
 + b)*a^2*d^2/(d*x^2 + c) + (a*d*x^2 + a*c + b)^2*a*d^2/(d*x^2 + c)^2) + 1/
16*(4*a*c + b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sq
rt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.12

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$=\frac{1}{16}\left(2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{d}-\frac{2\,acd-bd}{ad^3}\right)+\frac{(4\,abc+b^2)\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x^2\right.\right.\right.}{}\right.$$

$$+c)$$

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/d - (2
*a*c*d - b*d)/(a*d^3)) + (4*a*b*c + b^2)*log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x
^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d)
+ b*d))/(a^(3/2)*d*abs(d)))*sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^3\,\sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

```
[In] int(x^3*(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^3*(a + b/(c + d*x^2))^(1/2), x)
```

3.320 $\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

```
[Out] 1/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/d/a^(1/2)+1/2*(d*x^2+c)*(a+b/(
d*x^2+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1605, 248, 43, 65, 214}

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}d}+\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}$$

```
[In] Int[x*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*d) + (b*ArcTanh[Sqrt[a + b/(c + d*
x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{a+\frac{b}{x}}\,dx, x, c+dx^2\right)}{2d}\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \frac{1}{c+dx^2}\right)}{2d}\\
&= \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{4d}\\
&= \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{c+dx^2}}\right)}{2d}\\
&= \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{(c+dx^2)\sqrt{\frac{b+a(c+dx^2)}{c+dx^2}}}{2d}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+a\left(c+dx^2\right)}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

```
[In] Integrate[x*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[(b + a*(c + d*x^2))/(c + d*x^2)])/(2*d) + (b*ArcTanh[Sqrt
[(b + a*(c + d*x^2))/(c + d*x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(57) = 114$.

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.99

method	result
derivativedivides	$\frac{\sqrt{\frac{a\left(d\,x^2+c\right)+b}{d\,x^2+c}}\,(d\,x^2+c)\left(2\sqrt{a(d\,x^2+c)^2+b(d\,x^2+c)}\,\sqrt{a}+b\ln\left(\frac{2\sqrt{a\left(d\,x^2+c\right)^2+b\left(d\,x^2+c\right)}\,\sqrt{a}+2a\left(d\,x^2+c\right)+b}{2\sqrt{a}}\right)\right)}{4d\sqrt{(a(d\,x^2+c)+b)(d\,x^2+c)}\,\sqrt{a}}$
risch	$\frac{(d\,x^2+c)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{2d}+\frac{b\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+bc+(2acd+bd)x^2+a\,d^2x^4}\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,\sqrt{(ad\,x^2+ac+b)(d}}{4\sqrt{a\,d^2}\,(ad\,x^2+ac+b)}$
default	$\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(b\ln\left(\frac{2a\,d^2x^2+2acd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)d+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+}\right.}{4\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{a\,d^2}\,d}$

```
[In] int(x*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*((a*(d*x^2+c)+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(2*(a*(d*x^2+c)^2+b*(d*x^
2+c))^(1/2)*a^(1/2)+b*ln(1/2*(2*(a*(d*x^2+c)^2+b*(d*x^2+c))^(1/2)*a^(1/2)+2
*a*(d*x^2+c)+b)/a^(1/2)))/((a*(d*x^2+c)+b)*(d*x^2+c))^(1/2)/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.87

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx = \left[\frac{\sqrt{a}b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2+4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc)\sqrt{a}\cdots\right.}{8\,ad}\right.$$
$$\left. -\frac{\sqrt{-a}b\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(adx^2+ac)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,ad}\right]$$

[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] [1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d), -1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d)]

Sympy [F]

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx = \int x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

[In] integrate(x*(a+b/(d*x**2+c))**(1/2),x)

[Out] Integral(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(57) = 114.

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.83

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx=-\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ad-\frac{(adx^2+ac+b)d}{dx^2+c}\right)}-\frac{b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\sqrt{a}d}$$

```
[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d - (a*d*x^2 + a*c + b)*d/(
d*x^2 + c)) - 1/4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/
(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(sqrt(a)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(57) = 114.

Time = 0.43 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.78

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx=-\frac{1}{4}\left(\frac{b\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}|d|+bd\right|\right)}{\sqrt{a}|d|}-\frac{2\sqrt{ad^2x^4+2\,a\cdots}}{\cdots}\right.$$
$$+c)$$

```
[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(b*log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2
 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) + b*d))/(sqrt(a)*abs(d)) - 2*sqrt
(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/d)*sgn(d*x^2 + c)
```

Mupad [B] (verification not implemented)

Time = 17.77 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.74

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{\frac{b\,(d\,x^2+c)+a\,{(d\,x^2+c)}^2}{{(d\,x^2+c)}^2}}\,(d\,x^2+c)\left(\frac{b\,\ln\left(\frac{\frac{b}{2}+a\,(d\,x^2+c)+\sqrt{a}\,\sqrt{b\,(d\,x^2+c)+a\,{(d\,x^2+c)}^2}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b\,(d\,x^2+c)+a\,{(d\,x^2+c)}^2}}+2\right)}{4\,d}$$

```
[In] int(x*(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] (((b*(c + d*x^2) + a*(c + d*x^2)^2)/(c + d*x^2)^2)^(1/2)*(c + d*x^2)*((b*lo
g((b/2 + a*(c + d*x^2) + a^(1/2)*(b*(c + d*x^2) + a*(c + d*x^2)^2)^(1/2))/a
^(1/2)))/(a^(1/2)*(b*(c + d*x^2) + a*(c + d*x^2)^2)^(1/2)) + 2))/(4*d)
```

3.321 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x} \, dx = \sqrt{a}\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right) - \frac{\sqrt{b+ac}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{\sqrt{c}}$$

```
[Out] arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))*a^(1/2)-arctanh(c^(1/2)*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*(a*c+b)^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1985, 1981, 1980, 492, 214}

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x} \, dx = \sqrt{a}\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right) - \frac{\sqrt{ac+b}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{c}}$$

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x,x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]] - (Sqrt[b +
a*c]*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]]
)/Sqrt[c]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{x} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{x} \, dx, x, x^2\right) \\
&= -\left((bd)\text{Subst}\left(\int \frac{x^2}{\left(-b-ac+cx^2\right)\left(ad-dx^2\right)} \, dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right)
\end{aligned}
$$

$$= -\left((-b-ac)\text{Subst}\left(\int \frac{1}{-b-ac+cx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right)$$
$$+ (ad)\text{Subst}\left(\int \frac{1}{ad-dx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)$$

$$= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right) - \frac{\sqrt{b+ac}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = -\frac{\sqrt{-b-ac}\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{\sqrt{c}} + \sqrt{a}\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)$$

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x,x]
```

```
[Out] -((Sqrt[-b - a*c]*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sq
rt[-b - a*c]])/Sqrt[c]) + Sqrt[a]*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x
^2)]/Sqrt[a]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(80) = 160.

Time = 0.10 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.45

method	result
default	$-\frac{\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}\,(d x^2+c)\left(-\ln\left(\frac{2a d^2x^2+2acd+2\sqrt{a d^2x^4+2acd x^2+bd x^2+a c^2+bc}\,\sqrt{a d^2}+bd}{2\sqrt{a d^2}}\right)acd+\sqrt{a c^2+bc}\,\ln\left(\frac{2acd x^2+bd x^2+2a c^2+}{}\right.\right.}{2\sqrt{(ad x^2+ac+b)(d x^2+c)}\,c\sqrt{a d^2}}$

```
[In] int((a+b/(d*x^2+c))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-ln(1/2*(2*a*d^2*x^2+2*a*
c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a
*d^2)^(1/2))*a*c*d+(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c
^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*(
a*d^2)^(1/2))/((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/c/(a*d^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(80) = 160$.

Time = 0.35 (sec) , antiderivative size = 927, normalized size of antiderivative = 9.66

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \left[\frac{1}{4}\sqrt{a}\log\left(8a^2d^2x^4+8a^2c^2+8\left(2a^2c+ab\right)dx^2+8abc+b^2 + 4\left(2ad^2x^4+(4ac+b)dx^2+2ac^2+bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right) + \frac{1}{4}\sqrt{\frac{ac+b}{c}}\log\left(\frac{(8a^2c^2+8abc+b^2)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8(2a^2c^3+3abc^2+b^2c)dx^2-4(\ldots}{x^4}\right) - \frac{1}{2}\sqrt{-a}\arctan\left(\frac{(2adx^2+2ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(a^2dx^2+a^2c+ab)}\right) + \frac{1}{4}\sqrt{\frac{ac+b}{c}}\log\left(\frac{(8a^2c^2+8abc+b^2)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8(2a^2c^3+3abc^2+b^2c)dx^2-4(\ldots}{x^4}\right) + \frac{1}{4}\sqrt{a}\log\left(8a^2d^2x^4+8a^2c^2+8\left(2a^2c+ab\right)dx^2+8abc+b^2 + 4\left(2ad^2x^4+(4ac+b)dx^2+2ac^2+bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right), -\frac{1}{2}\sqrt{-a}\arctan\left(\frac{(2adx^2+2ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(a^2dx^2+a^2c+ab)}\right) + \frac{1}{2}\sqrt{-\frac{ac+b}{c}}\arctan\left(\frac{((2ac+b)dx^2+2ac^2+2bc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\sqrt{-\frac{ac+b}{c}}}{2(a^2c^2+(a^2c+ab)dx^2+2abc+b^2)}\right)\right]$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*
b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqr
t((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 1/4*sqrt((a*c + b)/c)*log(((8*a^2*c^2
 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c
^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*
c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt
((a*c + b)/c))/x^4), -1/2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(
-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 1/4*
sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16
```

```
*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^
2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d
*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4), 1/2*sqrt(-(a*c + b)/c
)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c +
 b^2)) + 1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^
2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqr
t(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))), -1/2*sqrt(-a)*arctan(1/2*(2*a*
d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^
2 + a^2*c + a*b)) + 1/2*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 +
2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(
a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2))]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x}\,dx$$

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x,x)
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.66

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \frac{(ac+b)\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}} - \frac{1}{2}\sqrt{a}\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")
[Out] 1/2*(a*c + b)*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)
*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/sqrt((a
*c + b)*c) - 1/2*sqrt(a)*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x}\,dx$$

```
[In] int((a + b/(c + d*x^2))^(1/2)/x,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x, x)
```

3.322 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3} dx = -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2cx^2} + \frac{bd\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2c^{3/2}\sqrt{b+ac}}$$

```
[Out] 1/2*b*d*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/c^
(3/2)/(a*c+b)^(1/2)-1/2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1985, 1981, 1980, 294, 214}

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3} dx = \frac{bd\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2c^{3/2}\sqrt{ac+b}} - \frac{(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2cx^2}$$

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^3,x]
```

```
[Out] -1/2*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*x^2) + (b*d*Arc
Tanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(2*c^(
3/2)*Sqrt[b + a*c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{x^3}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{x^2}\,dx, x, x^2\right)\\
&= -\left((bd)\text{Subst}\left(\int \frac{x^2}{\left(-b-ac+cx^2\right)^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right)\\
&= -\frac{\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2cx^2} - \frac{(bd)\text{Subst}\left(\int \frac{1}{-b-ac+cx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{2c}
\end{aligned}
$$

$$= -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2cx^2} + \frac{bd\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2c^{3/2}\sqrt{b+ac}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx = -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2cx^2} - \frac{bd\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{2c^{3/2}\sqrt{-b-ac}}$$

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^3,x]
```

```
[Out] -1/2*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*x^2) - (b*d*Arc
Tan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(2*c^(
3/2)*Sqrt[-b - a*c])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(88) = 176.

Time = 0.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{(d\,x^2+c)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{2c\,x^2}+\frac{bd\ln\left(\frac{2a\,c^2+2bc+(2acd+bd)x^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,c^2+bc+(2acd+bd)x^2+a\,d^2x^4}}{x^2}\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,\sqrt{(ad\,x^2+ac+}}{4c\sqrt{a\,c^2+bc}\,(ad\,x^2+ac+b)}$
default	$-\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-2a\,d^2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,x^4\sqrt{a\,c^2+bc}-\ln\left(\frac{2acd\,x^2+bd\,x^2+2a\,c^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,d^2x^4+2acd}}{x^2}\right.\right.}{}$

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^2+1/4*b*d/c/(a*c^2+b*c
)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+
(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*
((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(88) = 176$.

Time = 0.33 (sec) , antiderivative size = 433, normalized size of antiderivative = 4.16

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx$$
$$= \left[\frac{\sqrt{ac^2+bc}bdx^2\log\left(\frac{(8a^2c^2+8abc+b^2)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8(2a^2c^3+3abc^2+b^2c)dx^2+4((2ac+b)d^2x^4+2ac^3+(4ac^2+3bc)}{x^4}\right.}{8(ac^3+bc^2)x^2}\right.$$
$$-\frac{\sqrt{-ac^2-bc}bdx^2\arctan\left(\frac{((2ac+b)dx^2+2ac^2+2bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(a^2c^3+2abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)+2(ac^3+(ac^2+bc)dx^2+bc^2)\sqrt{\frac{adx^2}{dx}}}{4(ac^3+bc^2)x^2}$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 +
8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^
2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*s
qrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3 + (
a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^3
+ b*c^2)*x^2), -1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1/2*((2*a*c + b)*d*x
^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(a*c^3 + (
a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^3
+ b*c^2)*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx = \int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^3}\,dx$$

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx = -\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ac^2+bc-\frac{(adx^2+ac+b)c^2}{dx^2+c}\right)} - \frac{bd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}c}$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*c^2 + b*c - (a*d*x^2 + a*
c + b)*c^2/(d*x^2 + c)) - 1/4*b*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a
*c + b)*c)))/(sqrt((a*c + b)*c)*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(88) = 176.

Time = 0.39 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.70

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx =$$

$$-\frac{1}{2}\left(\frac{bd\arctan\left(-\frac{\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{\sqrt{-ac^2-bc}c} + \frac{2\,a^{\frac{3}{2}}c^2|d| + 2\left(\sqrt{ad^2}x^2 - \sqrt{ad^2x^4 + 2\,acdx^2 + bdx}\right.}{\left(ac^2 - \left(\sqrt{ad^2}\right.\right.}\right.$$

$$+\,c)$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(b*d*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2
 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/(sqrt(-a*c^2 - b*c)*c) + (2*a^(3/2)*c^
2*abs(d) + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*
c^2 + b*c))*a*c*d + 2*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^
4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b*d)/((a*c^2 - (sqrt(a*d^2)*x^2 -
 sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)*c))*sgn(d*
x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx=\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^3}\,dx$$

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^3,x)
[Out] int((a + b/(c + d*x^2))^(1/2)/x^3, x)
```

3.323 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx=\frac{(5b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^2(b+ac)x^2}-\frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c^2x^4}-\frac{b(3b+4ac)d^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8c^{5/2}(b+ac)^{3/2}}$$

```
[Out] -1/8*b*(4*a*c+3*b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a
*c+b)^(1/2))/c^(5/2)/(a*c+b)^(3/2)+1/8*(4*a*c+5*b)*d*(d*x^2+c)*((a*d*x^2+a*
c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)/x^2-1/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x
^2+c))^(1/2)/c^2/x^4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1985, 1981, 1980, 466, 393, 214}

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx=-\frac{bd^2(4ac+3b)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{5/2}(ac+b)^{3/2}}+\frac{d(4ac+5b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8c^2x^2(ac+b)}-\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2x^4}$$

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

```
[Out] ((5*b + 4*a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*c^2*
(b + a*c)*x^2) - ((c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*c
^2*x^4) - (b*(3*b + 4*a*c)*d^2*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)])/Sqrt[b + a*c]])/(8*c^(5/2)*(b + a*c)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{x^5}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{x^3}\,dx, x, x^2\right) \\
&= -\left((bd)\text{Subst}\left(\int \frac{x^2(ad-dx^2)}{(-b-ac+cx^2)^3}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right) \\
&= -\frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c^2x^4} + \frac{(bd)\text{Subst}\left(\int \frac{bd+4cdx^2}{(-b-ac+cx^2)^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{4c^2} \\
&= \frac{(5b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c^2x^4} \\
&\quad + \frac{(b(3b+4ac)d^2)\,\text{Subst}\left(\int \frac{1}{-b-ac+cx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{8c^2(b+ac)} \\
&= \frac{(5b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c^2x^4} \\
&\quad - \frac{b(3b+4ac)d^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8c^{5/2}(b+ac)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx = -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(b(2c-3dx^2)+2ac(c-dx^2))}{8c^2(b+ac)x^4} - \frac{b(3b+4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8c^{5/2}(-b-ac)^{3/2}}$$

[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^5,x]

[Out] -1/8*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*(2*c - 3*d*x^2) + 2*a*c*(c - d*x^2)))/(c^2*(b + a*c)*x^4) - (b*(3*b + 4*a*c)*d^2*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*c^(5/2)*(-b - a*c)^(3/2))

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{(d x^2+c)(-2acd x^2-3bd x^2+2a c^2+2bc)\sqrt{\frac{ad x^2+ac+b}{d x^2+c}}}{8c^2x^4(ac+b)}-\frac{d^2b(4ac+3b)\ln\left(\frac{2a c^2+2bc+(2acd+bd)x^2+2\sqrt{a c^2+bc}\sqrt{a c^2+bc+(2acd+bd)x^2}}{x^2}\right.}{16c^2(ac+b)\sqrt{a c^2+bc}(ad x^2+}$
default	$-\frac{\sqrt{\frac{ad x^2+ac+b}{d x^2+c}}(d x^2+c)\left(12a^2d^3\sqrt{a d^2x^4+2acd x^2+bd x^2+a c^2+bc}x^6c(a c^2+bc)^{\frac{3}{2}}+4\ln\left(\frac{2acd x^2+bd x^2+2a c^2+2\sqrt{a c^2+bc}\sqrt{a d^2x^4+2a}}{x^2}\right.\right.}{}$

[In] int((a+b/(d*x^2+c))^(1/2)/x^5,x,method=_RETURNVERBOSE)

[Out] -1/8*(d*x^2+c)*(-2*a*c*d*x^2-3*b*d*x^2+2*a*c^2+2*b*c)/c^2/x^4/(a*c+b)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/16*d^2*b*(4*a*c+3*b)/c^2/(a*c+b)/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.32

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$$

$$=\left[\frac{(4\,abc+3\,b^2)\sqrt{ac^2+bc}d^2x^4\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2-4\,((2\,ac+b)d^2x^4+2}{x^4}\right.}{}\right.$$

[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")

[Out] [1/32*((4*a*b*c + 3*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*

```
b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)
*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/
x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + 5*a*b*c^2 + 3*b^2*c)*d^2*x^4 + 4*a*b*c^4
 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x^4), 1/16*((4*a*b*c + 3*b^2)*sqrt(-
a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(
-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 +
(a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 5*a*b*c^2 +
 3*b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*d*x^2)*sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x^4)]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx=\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^5}\,dx$$

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**5,x)
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(154) = 308$.

Time = 0.30 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx = \frac{(4\,abc+3\,b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,(ac^3+bc^2)\sqrt{(ac+b)c}} - \frac{(4\,abc^2+5\,b^2c)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc^2+7\,ab^2c+3\,b^3)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2+\frac{(ac^5+bc^4)(adx^2+ac+b)^2}{(dx^2+c)^2}-\frac{2\,(a^2c^5+2\,abc^4+b^2c^3)(adx^2+ac+b)}{dx^2+c}\right)}$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")
[Out] 1/16*(4*a*b*c + 3*b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - s
qrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)
*c)))/((a*c^3 + b*c^2)*sqrt((a*c + b)*c)) - 1/8*((4*a*b*c^2 + 5*b^2*c)*d^2*
((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c^2 + 7*a*b^2*c + 3*b^3)
```

```
*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^
2*c^3 + b^3*c^2 + (a*c^5 + b*c^4)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 2*(
a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(154) = 308$.

Time = 0.43 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.10

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$$
$$=\frac{1}{8}\left(\frac{(4\,abcd^2+3\,b^2d^2)\arctan\left(-\frac{\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{(ac^3+bc^2)\sqrt{-ac^2-bc}}+\frac{8\,a^{\frac{7}{2}}c^5d|d|+16\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+\cdots}\right.}{\cdots}\right.$$
$$+c)$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*((4*a*b*c*d^2 + 3*b^2*d^2)*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/((a*c^3 + b*c^2)*
sqrt(-a*c^2 - b*c)) + (8*a^(7/2)*c^5*d*abs(d) + 16*(sqrt(a*d^2)*x^2 - sqrt(
a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^4*d^2 + 8*(sqrt(a*d
^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(5/2)*
c^3*d*abs(d) + 24*a^(5/2)*b*c^4*d*abs(d) + 36*(sqrt(a*d^2)*x^2 - sqrt(a*d^2
*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c^3*d^2 + 8*(sqrt(a*d^2)
*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*b*c
^2*d*abs(d) + 24*a^(3/2)*b^2*c^3*d*abs(d) - 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2
*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a*b*c*d^2 + 25*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*c^2*d^2
+ 8*sqrt(a)*b^3*c^2*d*abs(d) - 3*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*
d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*b^2*d^2 + 5*(sqrt(a*d^2)*x^2 - sqrt(a*d^2
*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b^3*c*d^2)/((a*c^3 + b*c^2)*(a
*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 +
b*c))^2 + b*c)^2))*sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx = \int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^5}\,dx$$

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^5,x)
[Out] int((a + b/(c + d*x^2))^(1/2)/x^5, x)
```

3.324 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7} dx$

Optimal result

Integrand size = 21, antiderivative size = 265

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7} dx = -\frac{(11b^2+20abc+8a^2c^2)\,d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16c^3(b+ac)^2x^2} + \frac{(3b+4ac)d(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^3(b+ac)x^4} - \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{6c^2(b+ac)x^6} + \frac{b(5b^2+12abc+8a^2c^2)\,d^3\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{16c^{7/2}(b+ac)^{5/2}}$$

```
[Out] -1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(3/2)/c^2/(a*c+b)/x^6+1/16*b*(
8*a^2*c^2+12*a*b*c+5*b^2)*d^3*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(
1/2)/(a*c+b)^(1/2))/c^(7/2)/(a*c+b)^(5/2)-1/16*(8*a^2*c^2+20*a*b*c+11*b^2)*
d^2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/(a*c+b)^2/x^2+1/8*(4*a*
c+3*b)*d*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/(a*c+b)/x^4
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{1985, 1981, 1980, 474, 466, 393, 214\}$

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx = \frac{bd^3(8a^2c^2+12abc+5b^2)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{16c^{7/2}(ac+b)^{5/2}} - \frac{d^2(8a^2c^2+20abc+11b^2)\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16c^3x^2(ac+b)^2} + \frac{d(4ac+3b)\,(c+dx^2)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8c^3x^4(ac+b)} - \frac{(c+dx^2)^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{6c^2x^6(ac+b)}$$

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^7,x]
```

```
[Out] -1/16*((11*b^2 + 20*a*b*c + 8*a^2*c^2)*d^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*
x^2)/(c + d*x^2)])/(c^3*(b + a*c)^2*x^2) + ((3*b + 4*a*c)*d*(c + d*x^2)^2*S
qrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*c^3*(b + a*c)*x^4) - ((c + d*x^2)^
3*((b + a*c + a*d*x^2)/(c + d*x^2))^(3/2))/(6*c^2*(b + a*c)*x^6) + (b*(5*b^
2 + 12*a*b*c + 8*a^2*c^2)*d^3*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c
+ d*x^2)])/Sqrt[b + a*c]])/(16*c^(7/2)*(b + a*c)^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{x^7}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{x^4}\,dx, x, x^2\right)$$

$$= -\left((bd)\text{Subst}\left(\int \frac{x^2(ad-dx^2)^2}{(-b-ac+cx^2)^4}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right)$$

$$= -\frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{6c^2(b+ac)x^6} - \frac{(bd)\text{Subst}\left(\int \frac{x^2(3(b^2-2a^2c^2)d^2+6c(b+ac)d^2x^2)}{(-b-ac+cx^2)^3}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{6c^2(b+ac)}$$

$$= \frac{(3b+4ac)d(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^3(b+ac)x^4} - \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{6c^2(b+ac)x^6} + \frac{(bd)\text{Subst}\left(\int \frac{-3bc(3b+4ac)d^2-24c^2(b+ac)d^2x^2}{(-b-ac+cx^2)^2}dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{24c^4(b+ac)}$$

$$= -\frac{(11b^2+20abc+8a^2c^2)\,d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16c^3(b+ac)^2x^2} + \frac{(3b+4ac)d(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^3(b+ac)x^4} - \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{6c^2(b+ac)x^6} - \frac{(b(5b^2+12abc+8a^2c^2)\,d^3)\,\text{Subst}\left(\int \frac{1}{-b-ac+cx^2}dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{16c^3(b+ac)^2}$$

$$= -\frac{(11b^2+20abc+8a^2c^2)\,d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16c^3(b+ac)^2x^2} + \frac{(3b+4ac)d(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^3(b+ac)x^4} - \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{6c^2(b+ac)x^6} + \frac{b(5b^2+12abc+8a^2c^2)\,d^3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{16c^{7/2}(b+ac)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx = -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(8a^2c^2(c^2-cdx^2+d^2x^4)+2abc(8c^2-9cdx^2+13d^2x^4)+b^2(8c^2-10cdx^2+15d^2x^4)\right)}{48c^3(b+ac)^2x^6} - \frac{b(5b^2+12abc+8a^2c^2)\,d^3\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{16c^{7/2}(-b-ac)^{5/2}}$$

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^7,x]
```

```
[Out] -1/48*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(8*a^2*c^2*(c^2 -
c*d*x^2 + d^2*x^4) + 2*a*b*c*(8*c^2 - 9*c*d*x^2 + 13*d^2*x^4) + b^2*(8*c^2
- 10*c*d*x^2 + 15*d^2*x^4)))/(c^3*(b + a*c)^2*x^6) - (b*(5*b^2 + 12*a*b*c +
 8*a^2*c^2)*d^3*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt
[-b - a*c]])/(16*c^(7/2)*(-b - a*c)^(5/2))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{(d\,x^2+c)\left(8a^2c^2d^2x^4+26ac\,d^2b\,x^4-8a^2c^3d\,x^2+15b^2d^2x^4-18ab\,c^2d\,x^2+8a^2c^4-10b^2cd\,x^2+16ab\,c^3+8b^2c^2\right)\sqrt{\frac{a\,d\,x^2+ac+b}{d\,x^2+c}}}{48c^3x^6(ac+b)^2}+\frac{d^3b(8c$
default	Expression too large to display

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(d*x^2+c)*(8*a^2*c^2*d^2*x^4+26*a*b*c*d^2*x^4-8*a^2*c^3*d*x^2+15*b^2*
d^2*x^4-18*a*b*c^2*d*x^2+8*a^2*c^4-10*b^2*c*d*x^2+16*a*b*c^3+8*b^2*c^2)/c^3
/x^6/(a*c+b)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/32*d^3*b*(8*a^2*c^2+12*a
*b*c+5*b^2)/(a*c+b)^2/c^3/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)
*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2
)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*
x^2+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.64 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.85

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx = \left[\frac{3\,(8\,a^2bc^2+12\,ab^2c+5\,b^3)\sqrt{ac^2+bc}d^3x^6\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2+4}{x^4}\right.}{}\right.$$

$$-\frac{3\,(8\,a^2bc^2+12\,ab^2c+5\,b^3)\sqrt{-ac^2-bc}d^3x^6\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2c^3+2\,abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)+2\,(8\,a^3c^7}{}$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*log(
((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 +
```

```
 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3
 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c
 + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 34*a^2*b*c^3 + 41*a*
b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4 + (8*
a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^2*x^4 - 2*(a^2*b*c^5 + 2*a*b^2*c^4
+ b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^7 + 3*a^2*
b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6), -1/96*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5
*b^3)*sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 +
2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 +
2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 + (8*a^3*c^4 +
 34*a^2*b*c^3 + 41*a*b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*
c^5 + 8*b^3*c^4 + (8*a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^2*x^4 - 2*(a^2
*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/((a^3*c^7 + 3*a^2*b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6)]
```

Sympy [F]

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^7}\, dx = \int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^7}\, dx$$

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(241) = 482$.

Time = 0.31 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^7}\, dx = -\frac{(8\,a^2bc^2 + 12\,ab^2c + 5\,b^3)d^3 \log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} - \sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} + \sqrt{(ac+b)c}}\right)}{32\,(a^2c^5 + 2\,abc^4 + b^2c^3)\sqrt{(ac+b)c}} - \frac{3\,(8\,a^2bc^4 + 20\,ab^2c^3 + 11\,b^3c^2)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}} - 8\,(6\,a^3bc^4 + 18\,a^2b^2c^3 + 17\,ab^3c^2 + 5\,b^4c)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)}{48\left(a^5c^8 + 5\,a^4bc^7 + 10\,a^3b^2c^6 + 10\,a^2b^3c^5 + 5\,ab^4c^4 + b^5c^3 - \frac{(a^2c^8+2\,abc^7+b^2c^6)(adx^2+ac+b)^3}{(dx^2+c)^3} + \frac{3\,(a^3c^8+3\,a^2bc^7}{}\right.}$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/32*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*d^3*log((c*sqrt((a*d*x^2 + a*c + b
)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
) + sqrt((a*c + b)*c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*sqrt((a*c + b)*c))
```

```
 - 1/48*(3*(8*a^2*b*c^4 + 20*a*b^2*c^3 + 11*b^3*c^2)*d^3*((a*d*x^2 + a*c +
b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^4 + 18*a^2*b^2*c^3 + 17*a*b^3*c^2 + 5*
b^4*c)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^4*b*c^4 + 28*a^
3*b^2*c^3 + 37*a^2*b^3*c^2 + 22*a*b^4*c + 5*b^5)*d^3*sqrt((a*d*x^2 + a*c +
b)/(d*x^2 + c)))/(a^5*c^8 + 5*a^4*b*c^7 + 10*a^3*b^2*c^6 + 10*a^2*b^3*c^5 +
 5*a*b^4*c^4 + b^5*c^3 - (a^2*c^8 + 2*a*b*c^7 + b^2*c^6)*(a*d*x^2 + a*c + b
)^3/(d*x^2 + c)^3 + 3*(a^3*c^8 + 3*a^2*b*c^7 + 3*a*b^2*c^6 + b^3*c^5)*(a*d*
x^2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(a^4*c^8 + 4*a^3*b*c^7 + 6*a^2*b^2*c^6 +
 4*a*b^3*c^5 + b^4*c^4)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. $2(241) = 482$.

Time = 0.42 (sec) , antiderivative size = 1414, normalized size of antiderivative = 5.34

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/48*(3*(8*a^2*b*c^2*d^3 + 12*a*b^2*c*d^3 + 5*b^3*d^3)*arctan(-(sqrt(a*d^2
)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2
- b*c))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*sqrt(-a*c^2 - b*c)) + (64*a^(11/2)
*c^8*d^2*abs(d) + 192*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))*a^5*c^7*d^3 + 192*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(9/2)*c^6*d^2*abs(d) + 304*a^(9/2
)*b*c^7*d^2*abs(d) + 64*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b
*d*x^2 + a*c^2 + b*c))^3*a^4*c^5*d^3 + 744*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^
4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^4*b*c^6*d^3 + 528*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(7/2)*b*c^
5*d^2*abs(d) + 576*a^(7/2)*b^2*c^6*d^2*abs(d) + 64*(sqrt(a*d^2)*x^2 - sqrt(
a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^3*b*c^4*d^3 + 1116*(s
qrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3
*b^2*c^5*d^3 + 480*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^
2 + a*c^2 + b*c))^2*a^(5/2)*b^2*c^4*d^2*abs(d) + 544*a^(5/2)*b^3*c^5*d^2*ab
s(d) + 24*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
 + b*c))^5*a^2*b*c^2*d^3 - 96*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x
^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*b^2*c^3*d^3 + 801*(sqrt(a*d^2)*x^2 - sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b^3*c^4*d^3 + 144*(
sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*
a^(3/2)*b^3*c^3*d^2*abs(d) + 256*a^(3/2)*b^4*c^4*d^2*abs(d) + 36*(sqrt(a*d^
2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^5*a*b^2*c*d
^3 - 136*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
+ b*c))^3*a*b^3*c^2*d^3 + 270*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x
```

```
^2 + b*d*x^2 + a*c^2 + b*c))*a*b^4*c^3*d^3 + 48*sqrt(a)*b^5*c^3*d^2*abs(d)
+ 15*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*
c))^5*b^3*d^3 - 40*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^
2 + a*c^2 + b*c))^3*b^4*c*d^3 + 33*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*
c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b^5*c^2*d^3)/((a^2*c^5 + 2*a*b*c^4 + b^2*
c^3)*(a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a
*c^2 + b*c))^2 + b*c)^3))*sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx=\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^7}\,dx$$

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^7,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x^7, x)
```

3.325 $\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 368

$$\begin{aligned}\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx &= -\frac{(2b^2+7abc-3a^2c^2)\,x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15a^2d^2} \\ &+\frac{(b-3ac)x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15ad^2}+\frac{x^3(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d} \\ &+\frac{\sqrt{c}(2b^2+7abc-3a^2c^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^2d^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ &-\frac{c^{3/2}(b-3ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{15ad^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

```
[Out] -1/15*(-3*a^2*c^2+7*a*b*c+2*b^2)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d^
2+1/15*(-3*a*c+b)*x*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2+1/5*x
^3*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d-1/15*c^(3/2)*(-3*a*c+b)*(1
/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^(5/2)/(c*
(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/15*(-3*a^2*c^2+7*a*b*c+2*b^2)*(1
/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d
^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 489, 596, 545, 429, 506, 422}

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\sqrt{c}(-3a^2c^2+7abc+2b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^2d^{5/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} - \frac{x(-3a^2c^2+7abc+2b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{15a^2d^2} - \frac{c^{3/2}(b-3ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{15ad^{5/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{x(b-3ac)(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{15ad^2} + \frac{x^3(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5d}$$

```
[In] Int[x^4*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -1/15*((2*b^2 + 7*a*b*c - 3*a^2*c^2)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
])/(a^2*d^2) + ((b - 3*a*c)*x*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x
^2)])/(15*a*d^2) + (x^3*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/
(5*d) + (Sqrt[c]*(2*b^2 + 7*a*b*c - 3*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(c
+ d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(15*a^2*d^(5
/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) - (c^(3/2)*(b -
3*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/S
qrt[c]], b/(b + a*c)])/(15*a*d^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c
)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \int x^4\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,dx$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{x^4\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+ac+adx^2}}$$

$$= \frac{x^3(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d} - \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{x^2(3c(b+ac)-(b-3ac)dx^2)}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5d\sqrt{b+ac+adx^2}}$$

$$= \frac{(b-3ac)x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15ad^2} + \frac{x^3(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d} + \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{-c(b-3ac)(b+ac)d-(2b^2+7abc-3a^2c^2)d^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{15ad^3\sqrt{b+ac+adx^2}}$$

$$= \frac{(b-3ac)x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15ad^2} + \frac{x^3(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d} - \frac{\left(c(b-3ac)(b+ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{15ad^2\sqrt{b+ac+adx^2}} - \frac{\left((2b^2+7abc-3a^2c^2)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{15ad\sqrt{b+ac+adx^2}}$$

$$= -\frac{(2b^2+7abc-3a^2c^2)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15a^2d^2} + \frac{(b-3ac)x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15ad^2} + \frac{x^3(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d} - \frac{c^{3/2}(b-3ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15ad^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{\left(c(2b^2+7abc-3a^2c^2)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{15a^2d^2\sqrt{b+ac+adx^2}}$$

$$= -\frac{(2b^2 + 7abc - 3a^2c^2)\, x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15a^2d^2} + \frac{(b-3ac)x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15ad^2} + \frac{x^3(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d} + \frac{\sqrt{c}(2b^2+7abc-3a^2c^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^2d^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{c^{3/2}(b-3ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15ad^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.80

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

$$= \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\left(b^2-2ab(c-2dx^2)-3a^2(c^2-d^2x^4)\right)+i(2b^3+9ab^2c+4a^2bc^2-3a^3c^3)\sqrt{b-\cdots}\right.}{15a^2c\cdots}$$

```
[In] Integrate[x^4*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(b^2 - 2*
a*b*(c - 2*d*x^2) - 3*a^2*(c^2 - d^2*x^4)) + I*(2*b^3 + 9*a*b^2*c + 4*a^2*b
*c^2 - 3*a^3*c^3)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*E
llipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - (2*I)*b*(b^2 + 4*a*b*c
+ 3*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(15*a^2*c^2*(d/c)^(5/2)*(b +
 a*(c + d*x^2)))
```

Maple [A] (verified)

Time = 6.52 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.81

method	result
default	$-\frac{\left(-3\sqrt{-\frac{ad}{ac+b}}\,a^2d^3x^7-3\sqrt{-\frac{ad}{ac+b}}\,a^2c\,d^2x^5-4\sqrt{-\frac{ad}{ac+b}}\,ab\,d^2x^5+3\sqrt{-\frac{ad}{ac+b}}\,a^2c^2d\,x^3-2\sqrt{-\frac{ad}{ac+b}}\,abcd\,x^3-3\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2}{c}}\right.}{}$
risch	$-\frac{x\left(-3ad\,x^2+3ac-b\right)\left(d\,x^2+c\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{15d^2a}+\frac{\left(\frac{3a^2c^3\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}-\frac{b^2c\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{d\,x^2}{c}}}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4}}\right.}{}$

[In] int(x^4*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/15*(-3*(-a*d/(a*c+b))^(1/2)*a^2*d^3*x^7-3*(-a*d/(a*c+b))^(1/2)*a^2*c*d^2*x^5-4*(-a*d/(a*c+b))^(1/2)*a*b*d^2*x^5+3*(-a*d/(a*c+b))^(1/2)*a^2*c^2*d*x^3-2*(-a*d/(a*c+b))^(1/2)*a*b*c*d*x^3-3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^3+3*(-a*d/(a*c+b))^(1/2)*a^2*c^3*x-(-a*d/(a*c+b))^(1/2)*b^2*d*x^3-9*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2+7*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2+2*(-a*d/(a*c+b))^(1/2)*a*b*c^2*x-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c+2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c-(-a*d/(a*c+b))^(1/2)*b^2*c*x)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/a/((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.65

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx = -\frac{(3\,a^2c^3-7\,abc^2-2\,b^2c)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-(3\,a^2c^3-7\,abc^2-2\,b^2c+(3\,a^2c^2+2\,abc}{}$$

[In] integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] -1/15*((3*a^2*c^3 - 7*a*b*c^2 - 2*b^2*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (3*a^2*c^3 - 7*a*b*c^2 - 2*b^2*c + (3*a^2*c^2 + 2*a*b*c - b^2)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (3*a^2*d^3*x^6 + a*b*d^2*x^4 + 3*a^2*c^3 - 7*a*b*c^2 - 2*(3*a*b*c + b^2)*d*x^2 - 2*b^2*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d^3*x)

Sympy [F]

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^4\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

```
[In] integrate(x**4*(a+b/(d*x**2+c))**(1/2),x)
[Out] Integral(x**4*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [F]

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{dx^2+c}}x^4\,dx$$

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{dx^2+c}}x^4\,dx$$

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^4\,\sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

```
[In] int(x^4*(a + b/(c + d*x^2))^(1/2),x)
[Out] int(x^4*(a + b/(c + d*x^2))^(1/2), x)
```

3.326 $\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 282

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{(b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3ad}+\frac{x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}-\frac{\sqrt{c}(b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3ad^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}-\frac{c^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

```
[Out] 1/3*(-a*c+b)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d+1/3*x*(d*x^2+c)*((a*d*
x^2+a*c+b)/(d*x^2+c))^(1/2)/d-1/3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)
^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a
*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c)
)^(1/2)-1/3*(-a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^
(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)
/(d*x^2+c))^(1/2)/a/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1985, 1986, 489, 545, 429, 506, 422}

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx=-\frac{c^{3/2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-\frac{\sqrt{c}(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3ad^{3/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+\frac{x(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3ad}+\frac{x(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3d}$$

```
[In] Int[x^2*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((b - a*c)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*a*d) + (x*(c + d*x^2
)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*d) - (Sqrt[c]*(b - a*c)*Sqrt[(b
 + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b
+ a*c)])/(3*a*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]
) - (c^(3/2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], b/(b + a*c)])/(3*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b +
a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
```

```
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,dx \\
&= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{x^2\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+ac+adx^2}} \\
&= \frac{x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d} - \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{c(b+ac)-(b-ac)dx^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3d\sqrt{b+ac+adx^2}} \\
&= \frac{x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d} + \frac{\left((b-ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3\sqrt{b+ac+adx^2}} \\
&\quad - \frac{\left(c(b+ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3d\sqrt{b+ac+adx^2}}
\end{aligned}
$$

$$= \frac{(b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3ad} + \frac{x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d} - \frac{c^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{\left(c(b-ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{3ad\sqrt{b+ac+adx^2}}$$

$$= \frac{(b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3ad} + \frac{x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d} - \frac{\sqrt{c}(b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3ad^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{c^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.86

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\left(b+a(c+dx^2)\right)+i(-b^2+a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{a}{b+}\right.\right.}{3ad\sqrt{\frac{d}{c}}\left(b+a\left(c+dx^2\right)\right)}$$

```
[In] Integrate[x^2*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(b + a*(c
 + d*x^2)) + I*(-b^2 + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1
+ (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] + I*b*(b +
a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Ar
cSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(3*a*d*Sqrt[d/c]*(b + a*(c + d*x^2))
)
```

Maple [A] (verified)

Time = 5.92 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.44

method	result
default	$\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a\, d^{2} x^{5}+2\sqrt{-\frac{ad}{ac+b}}\, acd\, x^{3}+\sqrt{-\frac{ad}{ac+b}}\, bd\, x^{3}-\sqrt{\frac{ad\, x^{2}+ac+b}{ac+b}}\, \sqrt{\frac{d\, x^{2}+c}{c}}\, E\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{\frac{ac+b}{ac}}\right) a\, c^{2}+\sqrt{-\frac{ad}{ac+b}}\, a\, c^{2} x-2\sqrt{\frac{ac}{}}\right.}{3d\sqrt{a\, d^{2} x^{4}+2acd\, x^{2}+bd\, x^{2}+a\, c^{2}}}$
risch	$\frac{x\left(d\, x^{2}+c\right)\sqrt{\frac{ad\, x^{2}+ac+b}{d\, x^{2}+c}}}{3d}-\frac{\left(\frac{a\, c^{2}\sqrt{1+\frac{ad\, x^{2}}{ac+b}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, F\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^{2} x^{4}+2acd\, x^{2}+bd\, x^{2}+a\, c^{2}+bc}}+\frac{bc\sqrt{1+\frac{ad\, x^{2}}{ac+b}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, F\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{-1+\frac{2acd}{dc}}\right.}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^{2} x^{4}+2acd\, x^{2}+bd\, x^{2}+a\, c^{2}+bc}}\right.}{}$

```
[In] int(x^2*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-a*d/(a*c+b))^(1/2)*a*d^2*x^5+2*(-a*d/(a*c+b))^(1/2)*a*c*d*x^3+(-a*d/
(a*c+b))^(1/2)*b*d*x^3-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2+(-a*d/(a*c+b))^
(1/2)*a*c^2*x-2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipti
cF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c+((a*d*x^2+a*c+b)/(a*c+b)
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^
(1/2))*b*c+(-a*d/(a*c+b))^(1/2)*b*c*x)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1
/2)/((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.59

$$\int x^{2}\sqrt{a+\frac{b}{c+dx^{2}}}\,dx$$
$$=\frac{(ac^{2}-bc)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac})-(ac^{2}-bc+(ac+b)d)\sqrt{a}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac})}{3\,ad^{2}x}$$

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a*c^2 - b*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a
*c + b)/(a*c)) - (a*c^2 - b*c + (a*c + b)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_
f(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) + (a*d^2*x^4 + b*d*x^2 - a*c^2 + b
*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d^2*x)
```

Sympy [F]

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

```
[In] integrate(x**2*(a+b/(d*x**2+c))**(1/2),x)
[Out] Integral(x**2*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [F]

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{dx^2+c}}x^2\,dx$$

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{dx^2+c}}x^2\,dx$$

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^2\,\sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

```
[In] int(x^2*(a + b/(c + d*x^2))^(1/2),x)
[Out] int(x^2*(a + b/(c + d*x^2))^(1/2), x)
```

3.327 $\int \sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 213

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx = x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}-\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}+\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

```
[Out] x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)
*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*(
(a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+
c))^(1/2)+(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/
2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))
^(1/2)/d^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1985, 1986, 433, 429, 506, 422}

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}$$

```
[In] Int[Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)] - (Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)
/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]
*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (Sqrt[c]*Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b +
 a*c)])/(Sqrt[d]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,dx \\
&= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+ac+adx^2}} \\
&= \frac{\left((b+ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{b+ac+adx^2}} \\
&\quad+\frac{\left(ad\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{b+ac+adx^2}} \\
&= x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}+\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\
&\quad-\frac{\left(c\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{\sqrt{b+ac+adx^2}} \\
&= x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}-\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\
&\quad+\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 8.87 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.46

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{ac}{b+ac}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{b+ac+adx^2}{b+ac}}}$$

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)],x]
[Out] (Sqrt[(c + d*x^2)/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcSin
[Sqrt[-(d/c)]*x], (a*c)/(b + a*c)])/(Sqrt[-(d/c)]*Sqrt[(b + a*c + a*d*x^2)/
(b + a*c)])
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\left(acE\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)+F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)b\right)\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,(d\,x^2+c)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{-\frac{ad}{ac+b}}\,\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}}$	199

```
[In] int((a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*c*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))+EllipticF(x*(-a*
d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b)*((d*x^2+c)/c)^(1/2)*((a*d*x^2+a*c+
b)/(a*c+b))^(1/2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/((a*d*x^2+a*c+b)*(d
*x^2+c))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.69

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx = -\frac{a^{\frac{3}{2}}c^2x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}) - (ac^2+(ac+b)d)\sqrt{a}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}) - (acdx^2+c}{acdx}$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -(a^(3/2)*c^2*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)
) - (a*c^2 + (a*c + b)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)
/x), (a*c + b)/(a*c)) - (a*c*d*x^2 + a*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2
 + c)))/(a*c*d*x)
```

Sympy [F]

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx = \int \sqrt{a+\frac{b}{c+dx^2}}\,dx$$

```
[In] integrate((a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/(c + d*x**2)), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{dx^2+c}}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a + b/(d*x^2 + c)), x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{dx^2+c}}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a + b/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

```
[In] int((a + b/(c + d*x^2))^(1/2),x)
[Out] int((a + b/(c + d*x^2))^(1/2), x)
```

3.328 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 265

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2} \, dx = \frac{dx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} - \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx} - \frac{\sqrt{d}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{a\sqrt{c}\sqrt{d}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

```
[Out] d*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c-(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/c/x-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*d^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)/c^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+a*(1/(1+d*x^2
/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),
(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b
)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1985, 1986, 486, 21, 433, 429, 506, 422}

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \frac{a\sqrt{c}\sqrt{d}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{(ac+b)\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} - \frac{\sqrt{d}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{dx\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c} - \frac{(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{cx}$$

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^2,x]
```

```
[Out] (d*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c - ((c + d*x^2)*Sqrt[(b + a*c
+ a*d*x^2)/(c + d*x^2)])/(c*x) - (Sqrt[d]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x
^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[c]*Sqrt[(c*
(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (a*Sqrt[c]*Sqrt[d]*Sqrt[(b
 + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b
+ a*c)])/((b + a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{x^2}\,dx$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^2\sqrt{c+dx^2}}\,dx}{\sqrt{b+ac+adx^2}}$$

$$= -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx} + \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{acd+ad^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{c\sqrt{b+ac+adx^2}}$$

$$= -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx} + \frac{\left(ad\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{c\sqrt{b+ac+adx^2}}$$

$$= -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx} + \frac{\left(ad\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{b+ac+adx^2}} + \frac{\left(ad^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{c\sqrt{b+ac+adx^2}}$$

$$= \frac{dx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} - \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx} + \frac{a\sqrt{c}\sqrt{d}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{\left(d\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{\sqrt{b+ac+adx^2}}$$

$$= \frac{dx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} - \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx} - \frac{\sqrt{d}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{a\sqrt{c}\sqrt{d}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [A] (verified)

Time = 9.54 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.51

$$\int\frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-\frac{1}{x}-\frac{dx}{c} + \frac{ad\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\,E\left(\arcsin\left(\sqrt{-\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)}{\sqrt{-\frac{ad}{b+ac}}\,(b+a(c+dx^2))}\right)$$

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^2,x]
```

```
[Out] Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-x^(-1) - (d*x)/c + (a*d*Sqrt[(b + a
*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[ArcSin[Sqrt[-((a*d)/
(b + a*c))]*x], 1 + b/(a*c)])/(Sqrt[-((a*d)/(b + a*c))]*(b + a*(c + d*x^2))
))
```

Maple [A] (verified)

Time = 6.06 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.03

method	result
default	$-\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a\, d^{2} x^{4}-a d c \sqrt{\frac{a d\, x^{2}+a c+b}{a c+b}}\, \sqrt{\frac{d\, x^{2}+c}{c}}\, x E\left(x \sqrt{-\frac{a d}{a c+b}}, \sqrt{\frac{a c+b}{a c}}\right)+2 \sqrt{-\frac{a d}{a c+b}}\, a c d\, x^{2}+\sqrt{-\frac{a d}{a c+b}}\, b d\, x^{2}+\sqrt{-\frac{a d}{a c+b}}\, a\, c^{2}+\sqrt{-\frac{a}{}}\cdots\right.}{\sqrt{a\, d^{2} x^{4}+2 a c d\, x^{2}+b d\, x^{2}+a\, c^{2}+b c}\, \sqrt{-\frac{a d}{a c+b}}\, x c \sqrt{\left(a d\, x^{2}+a c+b\right)\left(d\, x^{2}+c\right)}}$
risch	$-\frac{\left(d\, x^{2}+c\right) \sqrt{\frac{a d\, x^{2}+a c+b}{d\, x^{2}+c}}}{c x}+\frac{a d\left(\frac{c \sqrt{1+\frac{a d\, x^{2}}{a c+b}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, F\left(x \sqrt{-\frac{a d}{a c+b}}, \sqrt{-1+\frac{2 a c d+b d}{d c a}}\right)}{\sqrt{-\frac{a d}{a c+b}}\, \sqrt{a\, d^{2} x^{4}+2 a c d\, x^{2}+b d\, x^{2}+a\, c^{2}+b c}}-\frac{2 d\left(a\, c^{2}+b c\right) \sqrt{1+\frac{a d\, x^{2}}{a c+b}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \left(F\left(x \sqrt{-\frac{a d}{a c+b}}, \cdots\right.\right.}{\sqrt{-\frac{a d}{a c+b}}\, \sqrt{a\, d^{2} x^{4}+2 a c d\, x^{2}\cdots}}\right.}{c\left(a d\, x^{2}+a c+b\right)\cdots}$

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -((-a*d/(a*c+b))^(1/2)*a*d^2*x^4-a*d*c*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*
x^2+c)/c)^(1/2)*x*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))+2*(
-a*d/(a*c+b))^(1/2)*a*c*d*x^2+(-a*d/(a*c+b))^(1/2)*b*d*x^2+(-a*d/(a*c+b))^(
1/2)*a*c^2+(-a*d/(a*c+b))^(1/2)*b*c)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^
(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/
x/c/((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\, dx = \frac{a\sqrt{-\frac{ad}{ac+b}}d^2x\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+b}{ac}) - (ad^2+(ac+b)d)\sqrt{-\frac{ad}{ac+b}}x\sqrt{\frac{ac^2+bc}{d^2}}F(\arcsin\left(\sqrt{-\frac{ad}{ac+}}\cdots\right.}{(ac^2+bc)x}$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (a*sqrt(-a*d/(a*c + b))*d^2*x*sqrt((a*c^2 + b*c)/d^2)*elliptic_e(arcsin(sqr
t(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - (a*d^2 + (a*c + b)*d)*sqrt(-a*d/(a
*c + b))*x*sqrt((a*c^2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x
), (a*c + b)/(a*c)) - ((a*c + b)*d*x^2 + a*c^2 + b*c)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/((a*c^2 + b*c)*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^2}\,dx$$

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**2,x)
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^2}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^2}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^2}\,dx$$

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^2,x)
[Out] int((a + b/(c + d*x^2))^(1/2)/x^2, x)
```

3.329 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 362

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4} dx =& -\frac{(2b+ac)d^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)} - \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3cx^3} \\ &+ \frac{(2b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)x} \\ &+ \frac{(2b+ac)d^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ &- \frac{ad^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3\sqrt{c}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

```
[Out] -1/3*(a*c+2*b)*d^2*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)-1/3*(d*x
^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^3+1/3*(a*c+2*b)*d*(d*x^2+c)*((a
*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)/x+1/3*(a*c+2*b)*d^(3/2)*(1/(1+d*
x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/
2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(3/2)/(a*c+b)/(c*
(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/3*a*d^(3/2)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b
))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)/c^(1/2)/(c*(a*d*x^2+a*c
+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 486, 597, 545, 429, 506, 422}

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx = \frac{d^{3/2}(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(ac+b)\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} - \frac{ad^{3/2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3\sqrt{c}(ac+b)\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} - \frac{d^2x(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3c^2(ac+b)} + \frac{d(ac+2b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3c^2x(ac+b)} - \frac{\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3cx^3}$$

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^4,x]
```

```
[Out] -1/3*((2*b + a*c)*d^2*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c^2*(b + a*
c)) - ((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c*x^3) + ((2*b
 + a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c^2*(b + a*
c)*x) + ((2*b + a*c)*d^(3/2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*c^(3/2)*(b + a*c)*Sqrt[(c*(
b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) - (a*d^(3/2)*Sqrt[(b + a*c +
a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/
(3*Sqrt[c]*(b + a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{x^4}\,dx$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^4\sqrt{c+dx^2}}\,dx}{\sqrt{b+ac+adx^2}}$$

$$= -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3cx^3} + \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{-((2b+ac)d)-ad^2x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3c\sqrt{b+ac+adx^2}}$$

$$= -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3cx^3} + \frac{(2b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)x} - \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{ac(b+ac)d^2+a(2b+ac)d^3x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3c^2(b+ac)\sqrt{b+ac+adx^2}}$$

$$= -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3cx^3} + \frac{(2b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)x} - \frac{\left(ad^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3c\sqrt{b+ac+adx^2}} - \frac{\left(a(2b+ac)d^3\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3c^2(b+ac)\sqrt{b+ac+adx^2}}$$

$$= -\frac{(2b+ac)d^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)} - \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3cx^3} + \frac{(2b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)x} - \frac{ad^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3\sqrt{c}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{\left((2b+ac)d^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{3c(b+ac)\sqrt{b+ac+adx^2}}$$

$$= -\frac{(2b+ac)d^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)} - \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3cx^3} + \frac{(2b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)x} + \frac{(2b+ac)d^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{ad^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3\sqrt{c}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.74 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx = -\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}(c+dx^2)\left(b^2(c-2dx^2)+a^2c(c^2-d^2x^4)+2ab(c^2-cdx^2-d^2x^4)\right)-i(2b^2+3abc+a^2c\right.}{3c^2(}$$

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^4,x]
```

```
[Out] -1/3*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*(c + d*x^2)*(b^2*(c
- 2*d*x^2) + a^2*c*(c^2 - d^2*x^4) + 2*a*b*(c^2 - c*d*x^2 - d^2*x^4)) - I*(
2*b^2 + 3*a*b*c + a^2*c^2)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt
[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] + (2*I)*
b*(b + a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]
*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(c^2*(b + a*c)*Sqrt[d
/c]*x^3*(b + a*(c + d*x^2)))
```

Maple [A] (verified)

Time = 7.00 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.57

method	result
risch	$-\frac{\left(d\,x^{2}+c\right)\left(-acd\,x^{2}-2bd\,x^{2}+a\,c^{2}+bc\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}{3c^{2}x^{3}(ac+b)}-\frac{a\,d^{2}\left(\frac{a\,c^{2}\sqrt{1+\frac{ad\,x^{2}}{ac+b}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}}+\frac{bc\sqrt{1+\frac{ad\,x^{2}}{ac+b}}}{\sqrt{-\frac{ad}{ac+}}}\right.}{}$
default	$-\frac{\left(-\sqrt{-\frac{ad}{ac+b}}\,a^{2}c\,d^{3}x^{6}-2\sqrt{-\frac{ad}{ac+b}}\,ab\,d^{3}x^{6}+\sqrt{\frac{ad\,x^{2}+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,E\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a^{2}c^{2}d^{2}x^{3}-\sqrt{-\frac{ad}{ac+b}}\,a^{2}c^{2}d^{2}x^{4}-\sqrt{\frac{ad}{}}\right.}{}$

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(d*x^2+c)*(-a*c*d*x^2-2*b*d*x^2+a*c^2+b*c)/c^2/x^3/(a*c+b)*((a*d*x^2+a
*c+b)/(d*x^2+c))^(1/2)-1/3*a*d^2/c^2/(a*c+b)*(a*c^2/(-a*d/(a*c+b))^(1/2)*(1
+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^
(1/2))+b*c/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/
2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b
))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-2*(a*c*d+2*b*d)*(a*c^2+b*c)/(-a*d/
(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d/(a*c+b
))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b))^(1/2),(
-1+(2*a*c*d+b*d)/d/c/a)^(1/2))))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^
2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx = -\frac{(a^2c+2\,ab)\sqrt{-\frac{ad}{ac+b}}d^3x^3\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right)\mid\frac{ac+b}{ac})-((a^2c+2\,ab)d^3+(a^2c^2+2\,abc+b^2)d}{}$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*((a^2*c + 2*a*b)*sqrt(-a*d/(a*c + b))*d^3*x^3*sqrt((a*c^2 + b*c)/d^2)*
elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^2*c + 2*a
*b)*d^3 + (a^2*c^2 + 2*a*b*c + b^2)*d^2)*sqrt(-a*d/(a*c + b))*x^3*sqrt((a*c
^2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c))
```

```
- ((a^2*c^2 + 3*a*b*c + 2*b^2)*d^2*x^4 - a^2*c^4 - 2*a*b*c^3 - b^2*c^2 + (a
*b*c^2 + b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^4 + 2
*a*b*c^3 + b^2*c^2)*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx=\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^4}\,dx$$

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**4,x)
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx=\int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^4}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx=\int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^4}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx = \int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^4}\,dx$$

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^4,x)
[Out] int((a + b/(c + d*x^2))^(1/2)/x^4, x)
```

3.330 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 466

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx &= \frac{(8b^2+13abc+3a^2c^2)\,d^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^3(b+ac)^2} \\ &\quad - \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5cx^5} + \frac{(4b+3ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^2(b+ac)x^3} \\ &\quad - \frac{(8b^2+13abc+3a^2c^2)\,d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^3(b+ac)^2x} \\ &\quad - \frac{(8b^2+13abc+3a^2c^2)\,d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15c^{5/2}(b+ac)^2\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ &\quad + \frac{a(4b+3ac)d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{15c^{3/2}(b+ac)^2\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

```
[Out] 1/15*(3*a^2*c^2+13*a*b*c+8*b^2)*d^3*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3
/(a*c+b)^2-1/5*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^5+1/15*(3*a*
c+4*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)/x^3-1/15*(
3*a^2*c^2+13*a*b*c+8*b^2)*d^2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c
^3/(a*c+b)^2/x-1/15*(3*a^2*c^2+13*a*b*c+8*b^2)*d^(5/2)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b
))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(5/2)/(a*c+b)^2/(c*(a*d*x^2+a
*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/15*a*(3*a*c+4*b)*d^(5/2)*(1/(1+d*x^2/c))^(
1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*
c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(3/2)/(a*c+b)^2/(c*(a*d*x^
2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1985, 1986, 486, 597, 545, 429, 506, 422}

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx = -\frac{d^{5/2}(3a^2c^2+13abc+8b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15c^{5/2}(ac+b)^2\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{d^3x(3a^2c^2+13abc+8b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{15c^3(ac+b)^2} - \frac{d^2(3a^2c^2+13abc+8b^2)(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{15c^3x(ac+b)^2} + \frac{ad^{5/2}(3ac+4b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{15c^{3/2}(ac+b)^2\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{d(3ac+4b)(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{15c^2x^3(ac+b)} - \frac{(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5cx^5}$$

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^6,x]
```

```
[Out] ((8*b^2 + 13*a*b*c + 3*a^2*c^2)*d^3*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]
)/(15*c^3*(b + a*c)^2) - ((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]
)/(5*c*x^5) + ((4*b + 3*a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*
x^2)])/(15*c^2*(b + a*c)*x^3) - ((8*b^2 + 13*a*b*c + 3*a^2*c^2)*d^2*(c + d*
x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(15*c^3*(b + a*c)^2*x) - ((8*b^
2 + 13*a*b*c + 3*a^2*c^2)*d^(5/2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(15*c^(5/2)*(b + a*c)^2*S
qrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (a*(4*b + 3*a*c)*d^
(5/2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], b/(b + a*c)])/(15*c^(3/2)*(b + a*c)^2*Sqrt[(c*(b + a*c + a*d*x^2))/
((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
```

```
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{x^6}\,dx\\
&= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^6\sqrt{c+dx^2}}\,dx}{\sqrt{b+ac+adx^2}}\\
&= -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5cx^5}+\frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{-((4b+3ac)d)-3ad^2x^2}{x^4\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5c\sqrt{b+ac+adx^2}}\\
&= -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5cx^5}+\frac{(4b+3ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^2(b+ac)x^3}\\
&\quad-\frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{-((8b^2+13abc+3a^2c^2)d^2)-a(4b+3ac)d^3x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{15c^2(b+ac)\sqrt{b+ac+adx^2}}\\
&= -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5cx^5}+\frac{(4b+3ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^2(b+ac)x^3}\\
&\quad-\frac{(8b^2+13abc+3a^2c^2)d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^3(b+ac)^2x}\\
&\quad+\frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{ac(b+ac)(4b+3ac)d^3+a(8b^2+13abc+3a^2c^2)d^4x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{15c^3(b+ac)^2\sqrt{b+ac+adx^2}}\\
&= -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5cx^5}+\frac{(4b+3ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^2(b+ac)x^3}\\
&\quad-\frac{(8b^2+13abc+3a^2c^2)d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^3(b+ac)^2x}\\
&\quad+\frac{\left(a(4b+3ac)d^3\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{15c^2(b+ac)\sqrt{b+ac+adx^2}}\\
&\quad+\frac{\left(a(8b^2+13abc+3a^2c^2)d^4\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{15c^3(b+ac)^2\sqrt{b+ac+adx^2}}
\end{aligned}
$$

$$= \frac{(8b^2+13abc+3a^2c^2)\,d^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^3(b+ac)^2} - \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5cx^5} + \frac{(4b+3ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^2(b+ac)x^3} - \frac{(8b^2+13abc+3a^2c^2)\,d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^3(b+ac)^2x} + \frac{a(4b+3ac)d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15c^{3/2}(b+ac)^2\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{\left((8b^2+13abc+3a^2c^2)\,d^3\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{15c^2(b+ac)^2\sqrt{b+ac+adx^2}}$$

$$= \frac{(8b^2+13abc+3a^2c^2)\,d^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^3(b+ac)^2} - \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5cx^5} + \frac{(4b+3ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^2(b+ac)x^3} - \frac{(8b^2+13abc+3a^2c^2)\,d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^3(b+ac)^2x} - \frac{(8b^2+13abc+3a^2c^2)\,d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15c^{5/2}(b+ac)^2\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{a(4b+3ac)d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15c^{3/2}(b+ac)^2\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.32 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.86

$$\int\frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx = -\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}(c+dx^2)\left(b^3(3c^2-4cdx^2+8d^2x^4)+3a^3c^2(c^3+d^3x^6)+ab^2(9c^3-8c^2dx^2+17cd^2x^4+\right.\right.}{}$$

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^6,x]
```

```
[Out] -1/15*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*(c + d*x^2)*(b^3*(3
*c^2 - 4*c*d*x^2 + 8*d^2*x^4) + 3*a^3*c^2*(c^3 + d^3*x^6) + a*b^2*(9*c^3 -
8*c^2*d*x^2 + 17*c*d^2*x^4 + 8*d^3*x^6) + a^2*b*c*(9*c^3 - 4*c^2*d*x^2 + 9*
c*d^2*x^4 + 13*d^3*x^6)) + I*(8*b^3 + 21*a*b^2*c + 16*a^2*b*c^2 + 3*a^3*c^3
)*d^3*x^5*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE
[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - I*b*(8*b^2 + 17*a*b*c + 9*a^2*c
^2)*d^3*x^5*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(c^3*(b + a*c)^2*Sqrt[d/c]*x^
5*(b + a*(c + d*x^2)))
```

Maple [A] (verified)

Time = 7.87 (sec) , antiderivative size = 778, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{\left(d\,x^{2}+c\right)\left(3a^{2}c^{2}d^{2}x^{4}+13ac\,d^{2}b\,x^{4}-3a^{2}c^{3}d\,x^{2}+8b^{2}d^{2}x^{4}-7ab\,c^{2}d\,x^{2}+3a^{2}c^{4}-4b^{2}cd\,x^{2}+6ab\,c^{3}+3b^{2}c^{2}\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}{15c^{3}x^{5}\left(ac+b\right)^{2}}+\frac{a\,d^{3}\left(\frac{3a^{2}}{\cdots}\right.}{\cdots}$
default	$-\frac{\left(3\sqrt{-\frac{ad}{ac+b}}\,a^{3}c^{2}d^{4}x^{8}+13\sqrt{-\frac{ad}{ac+b}}\,a^{2}bc\,d^{4}x^{8}-3\sqrt{\frac{ad\,x^{2}+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,E\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a^{3}c^{3}d^{3}x^{5}+3\sqrt{-\frac{ad}{ac+b}}\,a^{3}c^{3}d^{3}x^{6}-\cdots\right.}{\cdots}$

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(d*x^2+c)*(3*a^2*c^2*d^2*x^4+13*a*b*c*d^2*x^4-3*a^2*c^3*d*x^2+8*b^2*d
^2*x^4-7*a*b*c^2*d*x^2+3*a^2*c^4-4*b^2*c*d*x^2+6*a*b*c^3+3*b^2*c^2)/c^3/x^5
/(a*c+b)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/15*a*d^3/(a*c+b)^2/c^3*(3*a^
2*c^3/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1
/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+4*b^2*c/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*
c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+7
*a*b*c^2/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)
/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))
^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-2*(3*a^2*c^2*d+13*a*b*c*d+8*b^2*d)*(
a*c^2+b*c)/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/
2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(Ellipti
cF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d
/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))))*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx = \frac{(3\,a^3c^2+13\,a^2bc+8\,ab^2)\sqrt{-\frac{ad}{ac+b}}d^4x^5\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right)\mid\frac{ac+b}{ac})-((3\,a^3c^2+13\,a^2bc+8\,ab^2)d^4}{}$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*((3*a^3*c^2 + 13*a^2*b*c + 8*a*b^2)*sqrt(-a*d/(a*c + b))*d^4*x^5*sqrt(
(a*c^2 + b*c)/d^2)*elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*
c)) - ((3*a^3*c^2 + 13*a^2*b*c + 8*a*b^2)*d^4 + (3*a^3*c^3 + 10*a^2*b*c^2 +
 11*a*b^2*c + 4*b^3)*d^3)*sqrt(-a*d/(a*c + b))*x^5*sqrt((a*c^2 + b*c)/d^2)*
elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((3*a^3*c^3 +
 16*a^2*b*c^2 + 21*a*b^2*c + 8*b^3)*d^3*x^6 + 3*a^3*c^6 + 9*a^2*b*c^5 + 9*a
*b^2*c^4 + 2*(3*a^2*b*c^3 + 5*a*b^2*c^2 + 2*b^3*c)*d^2*x^4 + 3*b^3*c^3 - (a
^2*b*c^4 + 2*a*b^2*c^3 + b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/((a^3*c^6 + 3*a^2*b*c^5 + 3*a*b^2*c^4 + b^3*c^3)*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx = \int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^6}\,dx$$

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx = \int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^6}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx=\int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^6}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="giac")
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx=\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^6}\,dx$$

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^6,x)
[Out] int((a + b/(c + d*x^2))^(1/2)/x^6, x)
```

3.331 $\int x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 21, antiderivative size = 249

$$\int x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = -\frac{bc^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d^3} - \frac{(5b^2+60abc-24a^2c^2)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{48ad^3} - \frac{(b+12ac)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24d^3} + \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{5/2}}{6ad^3} - \frac{b(b^2+12abc-24a^2c^2)\,\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{3/2}d^3}$$

```
[Out] 1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(5/2)/a/d^3-1/16*b*(-24*a^2*c^2
+12*a*b*c+b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)/d
^3-b*c^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^3-1/48*(-24*a^2*c^2+60*a*b*c+5
*b^2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^3-1/24*(12*a*c+b)*(d*
x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used

= {1985, 1981, 1980, 474, 466, 1171, 396, 214}

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=-\frac{(-24a^2c^2+60abc+5b^2)\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{48ad^3}-\frac{b(-24a^2c^2+12abc+b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{3/2}d^3}-\frac{bc^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{d^3}+\frac{(c+dx^2)^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{5/2}}{6ad^3}-\frac{(12ac+b)\,(c+dx^2)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{24d^3}$$

```
[In] Int[x^5*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((b*c^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/d^3) - ((5*b^2 + 60*a*b*c -
 24*a^2*c^2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(48*a*d^3)
- ((b + 12*a*c)*(c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(24*d^
3) + ((c + d*x^2)^3*((b + a*c + a*d*x^2)/(c + d*x^2))^(5/2))/(6*a*d^3) - (b
*(b^2 + 12*a*b*c - 24*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
]/Sqrt[a]])/(16*a^(3/2)*d^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
```

```
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\text{integral} = \int x^5\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int x^2\left(\frac{b+ac+adx}{c+dx}\right)^{3/2}\,dx, x, x^2\right)$$

$$= -\left((bd)\text{Subst}\left(\int \frac{x^4\left(-b-ac+cx^2\right)^2}{\left(ad-dx^2\right)^4}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right)$$

$$= \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{5/2}}{6ad^3} + \frac{b\text{Subst}\left(\int \frac{x^4(-((b^2+12abc+6a^2c^2)d^2)+6ac^2d^2x^2)}{(ad-dx^2)^3}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{6ad^2}$$

$$= -\frac{(b+12ac)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24d^3} + \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{5/2}}{6ad^3} + \frac{b\text{Subst}\left(\int \frac{ab(b+12ac)d^4+4b(b+12ac)d^4x^2-24ac^2d^4x^4}{(ad-dx^2)^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{24ad^5}$$

$$= -\frac{(5b^2+60abc-24a^2c^2)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{48ad^3} - \frac{(b+12ac)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24d^3} + \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{5/2}}{6ad^3} - \frac{b\text{Subst}\left(\int \frac{3a(b^2+12abc-8a^2c^2)d^4-48a^2c^2d^4x^2}{ad-dx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{48a^2d^6}$$

$$= -\frac{bc^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d^3} - \frac{(5b^2+60abc-24a^2c^2)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{48ad^3} - \frac{(b+12ac)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24d^3} + \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{5/2}}{6ad^3} - \frac{(b(b^2+12abc-24a^2c^2))\text{Subst}\left(\int \frac{1}{ad-dx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{16ad^2}$$

$$= -\frac{bc^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d^3} - \frac{(5b^2+60abc-24a^2c^2)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{48ad^3} - \frac{(b+12ac)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24d^3} + \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{5/2}}{6ad^3} - \frac{b(b^2+12abc-24a^2c^2)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{3/2}d^3}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.60

$$\int x^5\left(a + \frac{b}{c+dx^2}\right)^{3/2} dx = \frac{\sqrt{a}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(3b^2(c+dx^2)-2ab(47c^2+16cdx^2-7d^2x^4)+8a^2(c^3+d^3x^6))-3b(b^2+}{48a^{3/2}d^3}$$

```
[In] Integrate[x^5*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b^2*(c + d*x^2) - 2*a*b*(
47*c^2 + 16*c*d*x^2 - 7*d^2*x^4) + 8*a^2*(c^3 + d^3*x^6)) - 3*b*(b^2 + 12*a
*b*c - 24*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/
(48*a^(3/2)*d^3)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.18

method	result
risch	$\frac{\left(8a^2d^2x^4-8a^2cd\,x^2+14abd\,x^2+8a^2c^2-46abc+3b^2\right)\left(d\,x^2+c\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{48a\,d^3}+\frac{b\left(-\frac{16a\,c^2\left(ad\,x^2+ac+b\right)}{d\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}+\frac{\left(24a^2c^2-12abc\right.}{}\right.}{}$
default	Expression too large to display

```
[In] int(x^5*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/a/d^3*(8*a^2*d^2*x^4-8*a^2*c*d*x^2+14*a*b*d*x^2+8*a^2*c^2-46*a*b*c+3*b
^2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/16*b/a/d^2*(-16*a*c^2*(a*
d*x^2+a*c+b)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+1/2*(24*a^2*
c^2-12*a*b*c-b^2)*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^2+b*c+(2*
a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))
^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.71

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\left[\frac{3\left(24\,a^2bc^2-12\,ab^2c-b^3\right)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2+\right.}{}\right.$$

$$-\frac{3\left(24\,a^2bc^2-12\,ab^2c-b^3\right)\sqrt{-a}\arctan\left(\frac{\left(2\,adx^2+2\,ac+b\right)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2dx^2+a^2c+ab\right)}\right)-2\left(8\,a^3d^3x^6+14\,a^2bd^2x^4+8\,a^3c^3-\right.}{96\,a^2d^3}$$

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a
^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c
+ b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
+ 4*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c^3 - 94*a^2*b*c^2 + 3*a*b^2*
c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a
^2*d^3), -1/96*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(-a)*arctan(1/2*(2*
a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*
x^2 + a^2*c + a*b)) - 2*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c^3 - 94*
a^2*b*c^2 + 3*a*b^2*c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/(a^2*d^3)]
```

Sympy [F]

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\int x^5\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**5*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.48

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=-\frac{bc^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d^3}-\frac{3\left(8\,a^2bc^2-20\,ab^2c+b^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\left(6\,a^3bc^2-12\,a^2b^2c-ab^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\left(8\,a^4bc^2-12\,a^3b^2c\right.}{48\left(a^4d^3-\frac{3\,(adx^2+ac+b)a^3d^3}{dx^2+c}+\frac{3\,(adx^2+ac+b)^2a^2d^3}{(dx^2+c)^2}-\frac{(adx^2+ac+b)^3ad^3}{(dx^2+c)^3}\right)}-\frac{\left(24\,a^2c^2-12\,abc-b^2\right)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{3}{2}}d^3}$$

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -b*c^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/d^3 - 1/48*(3*(8*a^2*b*c^2 - 2
0*a*b^2*c + b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^2 -
 12*a^2*b^2*c - a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^4*b
```

```
*c^2 - 12*a^3*b^2*c - a^2*b^3)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*
d^3 - 3*(a*d*x^2 + a*c + b)*a^3*d^3/(d*x^2 + c) + 3*(a*d*x^2 + a*c + b)^2*a
^2*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a*d^3/(d*x^2 + c)^3) - 1/32*(2
4*a^2*c^2 - 12*a*b*c - b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(229) = 458$.

Time = 0.80 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.08

$$\int x^5 \left(a + \frac{b}{c+dx^2}\right)^{3/2} dx = \frac{1}{48}\sqrt{ad^2x^4 + 2\,acdx^2 + bdx^2 + ac^2 + bc}\left(2\left(\frac{4\,ax^2\mathrm{sgn}(dx^2+c)}{d} - \frac{4\,a^3cd^6\mathrm{sgn}(dx^2+c) - }{a^2d^8}\right.\right.$$
$$- \frac{(24\,a^2bc^2\mathrm{sgn}(dx^2+c) - 12\,ab^2c\mathrm{sgn}(dx^2+c) - b^3\mathrm{sgn}(dx^2+c))\log\left(\left|2\,a^2c^3d + 6\left(\sqrt{ad^2}x^2 - \sqrt{ad^2x^4 + 2\,ac}\right.\right.\right.}{}$$

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*(4*a*x^2*sgn(
d*x^2 + c)/d - (4*a^3*c*d^6*sgn(d*x^2 + c) - 7*a^2*b*d^6*sgn(d*x^2 + c))/(a
^2*d^8))*x^2 + (8*a^3*c^2*d^5*sgn(d*x^2 + c) - 46*a^2*b*c*d^5*sgn(d*x^2 + c
) + 3*a*b^2*d^5*sgn(d*x^2 + c))/(a^2*d^8)) - 1/96*(24*a^2*b*c^2*sgn(d*x^2 +
 c) - 12*a*b^2*c*sgn(d*x^2 + c) - b^3*sgn(d*x^2 + c))*log(abs(2*a^2*c^3*d +
 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)
)*a^(3/2)*c^2*abs(d) + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 +
b*d*x^2 + a*c^2 + b*c))^2*a*c*d + a*b*c^2*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d
^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*sqrt(a)*abs(d) + 2*(sqrt(a
*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*
b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*
c^2 + b*c))^2*b*d))/(a^(3/2)*d^2*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a + \frac{b}{c+dx^2}\right)^{3/2} dx = \int x^5\,\left(a + \frac{b}{d\,x^2+c}\right)^{3/2} dx$$

```
[In] int(x^5*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^5*(a + b/(c + d*x^2))^(3/2), x)
```

3.332 $\int x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 21, antiderivative size = 172

$$\int x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{bc\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d^2} + \frac{(5b-4ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8d^2} + \frac{a\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4d^2} + \frac{3b(b-4ac)\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}d^2}$$

```
[Out] 3/8*b*(-4*a*c+b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/d^2/a^(
1/2)+b*c*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2+1/8*(-4*a*c+5*b)*(d*x^2+c)*(
(a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2+1/4*a*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*
x^2+c))^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1985, 1981, 1980, 466, 1171, 396, 214}

$$\int x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{3b(b-4ac)\operatorname{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}d^2} + \frac{a\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4d^2} + \frac{(5b-4ac)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8d^2} + \frac{bc\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{d^2}$$

```
[In] Int[x^3*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (b*c*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/d^2 + ((5*b - 4*a*c)*(c + d*x^2
)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*d^2) + (a*(c + d*x^2)^2*Sqrt[(b
 + a*c + a*d*x^2)/(c + d*x^2)])/(4*d^2) + (3*b*(b - 4*a*c)*ArcTanh[Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*Sqrt[a]*d^2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int x\left(\frac{b+ac+adx}{c+dx}\right)^{3/2}\,dx, x, x^2\right) \\
&= -\left((bd)\text{Subst}\left(\int \frac{x^4(-b-ac+cx^2)}{(ad-dx^2)^3}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right) \\
&= \frac{a(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4d^2} - \frac{b\text{Subst}\left(\int \frac{abd^2+4bd^2x^2-4cd^2x^4}{(ad-dx^2)^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{4d^2} \\
&= \frac{(5b-4ac)\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8d^2} + \frac{a(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4d^2} \\
&\quad + \frac{b\text{Subst}\left(\int \frac{a(3b-4ac)d^2-8acd^2x^2}{ad-dx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{8ad^3} \\
&= \frac{bc\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d^2} + \frac{(5b-4ac)\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8d^2} + \frac{a(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4d^2} \\
&\quad + \frac{(3b(b-4ac))\text{Subst}\left(\int \frac{1}{ad-dx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{8d} \\
&= \frac{bc\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d^2} + \frac{(5b-4ac)\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8d^2} \\
&\quad + \frac{a(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4d^2} + \frac{3b(b-4ac)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(13bc-2ac^2+5bdx^2+2ad^2x^4\right)}{8d^2}-\frac{3b(-b+4ac)\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}d^2}$$

```
[In] Integrate[x^3*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(13*b*c - 2*a*c^2 + 5*b*d*x^2 + 2*a*
d^2*x^4))/(8*d^2) - (3*b*(-b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c +
 d*x^2)]/Sqrt[a]])/(8*Sqrt[a]*d^2)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{\left(-2ad\,x^2+2ac-5b\right)\left(d\,x^2+c\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{8d^2}-\frac{b\left(\frac{(12ac-3b)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+bc+(2acd+bd)x^2+a\,d^2x^4}\right)}{2\sqrt{a\,d^2}}-\frac{8c\left(ad\,x^2\right.}{d\sqrt{a\,d^2x^4+2acd\,x}}\right.}{8d(ad\,x^2+ac+b)}$
default	$\left(4\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,a\,d^2x^4-12\ln\left(\frac{2a\,d^2x^2+2acd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)abc\,d^2x^2+3\ln\left(\frac{2}{}\right.\right.$

```
[In] int(x^3*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d^2*(-2*a*d*x^2+2*a*c-5*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)
-1/8*b/d*(1/2*(12*a*c-3*b)*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^
2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)-8*c*(a*d*x^2+a*c+b)
/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))*((a*d*x^2+a*c+b)/(d*x^2
+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.95

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx = \left[\frac{3\,(4\,abc-b^2)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2-4\,(2\,ad^2x^4+\right.}{}\right.$$

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a*b*c - b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c
 + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^
2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4
+ 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/(a*d^2), 1/16*(3*(4*a*b*c - b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c +
 b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b
)) + 2*(2*a^2*d^2*x^4 + 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c)))/(a*d^2)]
```

Sympy [F]

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx = \int x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}dx$$

```
[In] integrate(x**3*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.44

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\frac{bc\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d^2}+\frac{3\,(4\,ac-b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,\sqrt{a}d^2}$$
$$-\frac{(4\,abc-5\,b^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc-3\,ab^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^2d^2-\frac{2\,(adx^2+ac+b)ad^2}{dx^2+c}+\frac{(adx^2+ac+b)^2d^2}{(dx^2+c)^2}\right)}$$

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] b*c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/d^2 + 3/16*(4*a*c - b)*b*log(-(sq
rt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c))))/(sqrt(a)*d^2) - 1/8*((4*a*b*c - 5*b^2)*((a*d*x^2 + a
*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c - 3*a*b^2)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c)))/(a^2*d^2 - 2*(a*d*x^2 + a*c + b)*a*d^2/(d*x^2 + c) + (a*d*x^
2 + a*c + b)^2*d^2/(d*x^2 + c)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(156) = 312.

Time = 0.74 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.76

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\frac{1}{8}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,ax^2\mathrm{sgn}(dx^2+c)}{d}-\frac{2\,a^2cd^2\mathrm{sgn}(dx^2+c)-5\,abc}{ad^4}\right.$$
$$+\frac{(4\,abc\mathrm{sgn}(dx^2+c)-b^2\mathrm{sgn}(dx^2+c))\log\left(\left|2\,a^2c^3d+6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^{\frac{3}{2}}\right.\right.}{}$$
$$+\frac{(4\,abcd^2|d|\mathrm{sgn}(dx^2+c)-b^2d^2|d|\mathrm{sgn}(dx^2+c))\log(96)}{8\,\sqrt{a}d^5}$$

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*a*x^2*sgn(d*x^
2 + c)/d - (2*a^2*c*d^2*sgn(d*x^2 + c) - 5*a*b*d^2*sgn(d*x^2 + c))/(a*d^4))
 + 1/16*(4*a*b*c*sgn(d*x^2 + c) - b^2*sgn(d*x^2 + c))*log(abs(2*a^2*c^3*d +
 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)
)*a^(3/2)*c^2*abs(d) + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 +
b*d*x^2 + a*c^2 + b*c))^2*a*c*d + a*b*c^2*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d
^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*sqrt(a)*abs(d) + 2*(sqrt(a
```

```
*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*
b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*
c^2 + b*c))^2*b*d))/(sqrt(a)*d*abs(d)) + 1/8*(4*a*b*c*d^2*abs(d)*sgn(d*x^2
+ c) - b^2*d^2*abs(d)*sgn(d*x^2 + c))*log(96)/(sqrt(a)*d^5)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\int x^3\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}dx$$

```
[In] int(x^3*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^3*(a + b/(c + d*x^2))^(3/2), x)
```

3.333 $\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}+\frac{3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

```
[Out] 1/2*(d*x^2+c)*(a+b/(d*x^2+c))^(3/2)/d+3/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a
^(1/2))*a^(1/2)/d-3/2*b*(a+b/(d*x^2+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1605, 248, 43, 52, 65, 214}

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\frac{3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}+\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}$$

```
[In] Int[x*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (-3*b*Sqrt[a + b/(c + d*x^2)])/(2*d) + ((c + d*x^2)*(a + b/(c + d*x^2))^(3/
2))/(2*d) + (3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(a+\frac{b}{x}\right)^{3/2}\,dx, x, c+dx^2\right)}{2d}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, \frac{1}{c+dx^2}\right)}{2d}$$

$$= \frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d} - \frac{(3b)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{c+dx^2}\right)}{4d}$$

$$= -\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d} + \frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d} - \frac{(3ab)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{4d}$$

$$= -\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d} + \frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d} - \frac{(3a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{c+dx^2}}\right)}{2d}$$

$$= -\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d} + \frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d} + \frac{3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(-2b+a(c+dx^2)) + 3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

```
[In] Integrate[x*(a + b/(c + d*x^2))^(3/2),x]
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b + a*(c + d*x^2)) + 3*Sqrt[a]*b
*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(78) = 156.

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.99

method	result
derivativedivides	$\frac{\sqrt{\frac{a\left(dx^2+c\right)+b}{dx^2+c}}\left(6a^{\frac{3}{2}}\sqrt{a(dx^2+c)^2+b(dx^2+c)}\,(dx^2+c)^2+3\ln\left(\frac{2\sqrt{a\left(dx^2+c\right)^2+b\left(dx^2+c\right)}\,\sqrt{a}+2a\left(dx^2+c\right)+b}{2\sqrt{a}}\right)ab(dx^2+\right.}{4d(dx^2+c)\sqrt{(a(dx^2+c)+b)(dx^2+c)}\,\sqrt{a}}$
risch	$\frac{(dx^2+c)a\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2d}+\frac{b\left(\frac{3a\ln\left(\frac{acd+\frac{1}{2}bd+ad^2x^2}{\sqrt{a\,d^2}}+\sqrt{ac^2+bc+(2acd+bd)x^2+ad^2x^4}\right)}{2\sqrt{a\,d^2}}-\frac{2\left(adx^2+ac+b\right)}{d\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2}}\right.}{2adx^2+2ac+2b}$
default	$\left(3\ln\left(\frac{2ad^2x^2+2acd+2\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)abd^2x^2+2\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\,\sqrt{a\,d^2}\,ad\right.$

```
[In] int(x*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*((a*(d*x^2+c)+b)/(d*x^2+c))^(1/2)*(6*a^(3/2)*(a*(d*x^2+c)^2+b*(d*x^2+
c))^(1/2)*(d*x^2+c)^2+3*ln(1/2*(2*(a*(d*x^2+c)^2+b*(d*x^2+c))^(1/2)*a^(1/2)
+2*a*(d*x^2+c)+b)/a^(1/2))*a*b*(d*x^2+c)^2-4*(a*(d*x^2+c)^2+b*(d*x^2+c))^(3
/2)*a^(1/2))/(d*x^2+c)/((a*(d*x^2+c)+b)*(d*x^2+c))^(1/2)/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.86

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\left[\frac{3\sqrt{a}b\log\left(8a^2d^2x^4+8a^2c^2+8(2a^2c+ab)dx^2+8abc+b^2+4(2ad^2x^4+(4ac+b)\right.}{8d}\right.$$

$$\left.-\frac{3\sqrt{-a}b\arctan\left(\frac{(2adx^2+2ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(a^2dx^2+a^2c+ab)}\right)-2(adx^2+ac-2b)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4d}\right]$$

```
[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 +
 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c - 2*b)*sqrt((a*
```

```
d*x^2 + a*c + b)/(d*x^2 + c)))/d, -1/4*(3*sqrt(-a)*b*arctan(1/2*(2*a*d*x^2
+ 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^
2*c + a*b)) - 2*(a*d*x^2 + a*c - 2*b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/d]
```

Sympy [F]

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int x\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}} dx$$

```
[In] integrate(x*(a+b/(d*x**2+c))**(3/2),x)
[Out] Integral(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.66

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = -\frac{ab\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ad-\frac{(adx^2+ac+b)d}{dx^2+c}\right)} - \frac{3\sqrt{a}b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4d} - \frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d}$$

```
[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] -1/2*a*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d - (a*d*x^2 + a*c + b)*d
/(d*x^2 + c)) - 3/4*sqrt(a)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/d - b*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(78) = 156.

Time = 0.72 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.05

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=-\frac{\sqrt{a}b|d|\log(24)\,\mathrm{sgn}(dx^2+c)}{2\,d^2}$$

$$-\frac{\sqrt{a}b\log\left(\left|2\,a^2c^3d+6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^{\frac{3}{2}}c^2|d|+6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+\cdots}\right.\right.\right.}{\cdots}$$

$$+\frac{\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}a\,\mathrm{sgn}(dx^2+c)}{2\,d}$$

```
[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*b*abs(d)*log(24)*sgn(d*x^2 + c)/d^2 - 1/4*sqrt(a)*b*log(abs(2*
a^2*c^3*d + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a
*c^2 + b*c))*a^(3/2)*c^2*abs(d) + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a
*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a*c*d + a*b*c^2*d + 2*(sqrt(a*d^2)*x^2
 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*sqrt(a)*abs(d)
+ 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c
))*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b
*d*x^2 + a*c^2 + b*c))^2*b*d))*sgn(d*x^2 + c)/abs(d) + 1/2*sqrt(a*d^2*x^4 +
 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*a*sgn(d*x^2 + c)/d
```

Mupad [B] (verification not implemented)

Time = 18.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=-\frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}\left(d\,x^2+c\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,(d\,x^2+c)}{b}\right)}{d\left(\frac{a\,(d\,x^2+c)}{b}+1\right)^{3/2}}$$

```
[In] int(x*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] -((a + b/(c + d*x^2))^(3/2)*(c + d*x^2)*hypergeom([-3/2, -1/2], 1/2, -(a*(c
 + d*x^2))/b))/(d*((a*(c + d*x^2))/b + 1)^(3/2))
```

3.334 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x} dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x} dx = \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} + a^{3/2}\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right) - \frac{(b+ac)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{c^{3/2}}$$

```
[Out] a^(3/2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))-(a*c+b)^(3/2)*ar
ctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/c^(3/2)+b*((
a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1985, 1981, 1980, 490, 536, 214}

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x} dx = a^{3/2}\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right) - \frac{(ac+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{c^{3/2}} + \frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}$$

[In] Int[(a + b/(c + d*x^2))^(3/2)/x,x]

[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c + a^(3/2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]] - ((b + a*c)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/c^(3/2)

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{x}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\left(\frac{b+ac+adx}{c+dx}\right)^{3/2}}{x}\,dx, x, x^2\right) \\
&= -\left((bd)\text{Subst}\left(\int \frac{x^4}{(-b-ac+cx^2)(ad-dx^2)}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right) \\
&= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} - \frac{b\text{Subst}\left(\int \frac{-a(b+ac)d+(b+2ac)dx^2}{(-b-ac+cx^2)(ad-dx^2)}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{c} \\
&= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} + \frac{(b+ac)^2\text{Subst}\left(\int \frac{1}{-b-ac+cx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{c} \\
&\quad + \left(a^2d\right)\text{Subst}\left(\int \frac{1}{ad-dx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right) \\
&= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} + a^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right) - \frac{(b+ac)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{c^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.10

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx &= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} \\
&\quad - \frac{(-b-ac)^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{-b-ac}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{b+ac}\right)}{c^{3/2}} + a^{3/2}\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)
\end{aligned}
$$

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x,x]
[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c - ((-b - a*c)^(3/2)*ArcTan[(Sqr
t[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b + a*c)])/c^(3
/2) + a^(3/2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(108) = 216$.

Time = 0.13 (sec) , antiderivative size = 652, normalized size of antiderivative = 5.17

method	result
default	$\frac{\left(\ln\left(\frac{2 a \,d^{2} x^{2}+2 a c d +2 \sqrt{a \,d^{2} x^{4}+2 a c d \,x^{2}+b d \,x^{2}+a \,c^{2}+b c}\, \sqrt{a \,d^{2}}+b d}{2 \sqrt{a \,d^{2}}}\right) a^{2} c^{2} d^{2} x^{2}-\sqrt{a \,d^{2}}\, \sqrt{a \,c^{2}+b c}\, \ln\left(\frac{2 a c d \,x^{2}+b d \,x^{2}+2 a \,c^{2}+2 \sqrt{a \,c^{2}+b c}\, \sqrt{a \,d^{2}}}{x^{2}}\right.\right.}{}$

```
[In] int((a+b/(d*x^2+c))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*c^2*d^2*x^2-(a*d^2)^(1/2)*(a*
c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x
^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*c*d*x^2-(a*d^2)^(1/2)
*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b*d*x^2+ln(1/2*(2*a
*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^
(1/2)+b*d)/(a*d^2)^(1/2))*a^2*c^3*d-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a
*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*c^2-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a
*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b*c+2*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(a*d^
2)^(1/2)*b*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2)/c^2/((a*d*x^2
+a*c+b)*(d*x^2+c))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 1073, normalized size of antiderivative = 8.52

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx = \left[\frac{a^{\frac{3}{2}}c\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2+4\,(2\,ad^2x^4+(4\,ac+b)c\right.}{}\right.$$

$$-\frac{2\sqrt{-a}ac\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-(ac+b)\sqrt{\frac{ac+b}{c}}\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+}{}\right.}{4\,c}$$

$$-\frac{\sqrt{-a}ac\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-(ac+b)\sqrt{-\frac{ac+b}{c}}\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\sqrt{-}}{2\,(a^2c^2+(a^2c+ab)dx^2+2\,abc+b^2)}\right.}{2\,c}$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8
*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + (a*c + b)*sqrt((a*c + b)/c)*log(((
8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8
*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c
^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c))*sqrt((a*c + b)/c))/x^4) + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/c
, -1/4*(2*sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a*c + b)*sqrt((
a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c
^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*
c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4) - 4*b*sqrt((a*d*x^2 + a*c +
b)/(d*x^2 + c)))/c, 1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2
*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*
c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 2*(a*c + b)*sqr
t(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x
^2 + 2*a*b*c + b^2)) + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/c, -1/2*(
sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*
c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a*c + b)*sqrt(-(a*c + b)/
c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b
)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c
+ b^2)) - 2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/c]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx=\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

[In] integrate((a+b/(d*x**2+c))**(3/2)/x,x)

[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x, x)

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.60

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx=-\frac{1}{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)+\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c}+\frac{(a^2c^2+2\,abc+b^2)\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\,\sqrt{(ac+b)c}c}$$

[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")

[Out] -1/2*a^(3/2)*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))) + b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/c + 1/2*(a^2*c^2 + 2*a*b*c + b^2)*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*c)

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx=\text{Exception raised: TypeError}$$

[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x}\,dx$$

```
[In] int((a + b/(c + d*x^2))^(3/2)/x,x)
[Out] int((a + b/(c + d*x^2))^(3/2)/x, x)
```

3.335 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx=-\frac{3bd\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2c^2}-\frac{(c+dx^2)\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{2cx^2}+\frac{3b\sqrt{b+ac}\,d\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2c^{5/2}}$$

```
[Out] -1/2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(3/2)/c/x^2+3/2*b*d*arctanh(c^(1
/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*(a*c+b)^(1/2)/c^(5/2)-
3/2*b*d*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1985, 1981, 1980, 294, 327, 214}

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx=\frac{3bd\sqrt{ac+b}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2c^{5/2}}-\frac{3bd\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2c^2}-\frac{(c+dx^2)\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{2cx^2}$$

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (-3*b*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*c^2) - ((c + d*x^2)*((b +
 a*c + a*d*x^2)/(c + d*x^2))^(3/2))/(2*c*x^2) + (3*b*Sqrt[b + a*c]*d*ArcTan
h[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(2*c^(5/2
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{x^3}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\left(\frac{b+ac+adx}{c+dx}\right)^{3/2}}{x^2}\,dx, x, x^2\right)\\
&= -\left((bd)\text{Subst}\left(\int \frac{x^4}{\left(-b-ac+cx^2\right)^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right)\\
&= -\frac{(c+dx^2)\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{2cx^2} - \frac{(3bd)\text{Subst}\left(\int \frac{x^2}{-b-ac+cx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{2c}\\
&= -\frac{3bd\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2c^2} - \frac{(c+dx^2)\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{2cx^2}\\
&\quad - \frac{(3b(b+ac)d)\text{Subst}\left(\int \frac{1}{-b-ac+cx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{2c^2}\\
&= -\frac{3bd\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2c^2} - \frac{(c+dx^2)\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{2cx^2} + \frac{3b\sqrt{b+ac}\,d\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2c^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$
\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx = -\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(ac(c+dx^2)+b(c+3dx^2)\right)}{2c^2x^2} + \frac{3b\sqrt{-b-ac}\,d\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{2c^{5/2}}
$$

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^3,x]
[Out] -1/2*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*c*(c + d*x^2) + b*(c + 3*d*x
^2)))/(c^2*x^2) + (3*b*Sqrt[-b - a*c]*d*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d
*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(2*c^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(118) = 236.

Time = 0.19 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{(ac+b)(dx^2+c)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2c^2x^2} - \frac{bd\left(-\frac{(3ac+3b)\ln\left(\frac{2ac^2+2bc+(2acd+bd)x^2+2\sqrt{ac^2+bc}\sqrt{ac^2+bc+(2acd+bd)x^2+ad^2x^4}}{x^2}\right)}{2\sqrt{ac^2+bc}} + \frac{2c}{\sqrt{ad^2x^4+2}}\right.}{2c^2(adx^2+ac+b)}$
default	$-\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(-2\sqrt{ac^2+bc}\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\,ad^3x^6-3\ln\left(\frac{2acdx^2+bdx^2+2ac^2+2\sqrt{ac^2+bc}\sqrt{ad^2x^4+2acdx^2+bdx^2}}{x^2}\right.\right.}{}$

[In] int((a+b/(d*x^2+c))^(3/2)/x^3,x,method=_RETURNVERBOSE)

[Out] -1/2*(a*c+b)/c^2*(d*x^2+c)/x^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/2*b*d/c^2*(-1/2*(3*a*c+3*b)/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)+2*(a*d*x^2+a*c+b)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.93

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \left[\frac{3\,bdx^2\sqrt{\frac{ac+b}{c}}\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2+4\,((2\,ac^2+b}{x^4}\right.}{}\right.$$

$$\left.-\frac{3\,bdx^2\sqrt{-\frac{ac+b}{c}}\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\sqrt{-\frac{ac+b}{c}}}{2\,(a^2c^2+(a^2c+ab)dx^2+2\,abc+b^2)}\right)+2\,((ac+3\,b)dx^2+ac^2+bc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,c^2x^2}\right]$$

[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")

[Out] [1/8*(3*b*d*x^2*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*

```
d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4) - 4*((
a*c + 3*b)*d*x^2 + a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c^2
*x^2), -1/4*(3*b*d*x^2*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2
*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a
^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)) + 2*((a*c + 3*b)*d*x^2 + a*c
^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c^2*x^2)]
```

Sympy [F]

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^3}\, dx = \int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^3}\, dx$$

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**3,x)
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.46

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^3}\, dx = -\frac{(abc+b^2)d\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ac^3+bc^2-\frac{(adx^2+ac+b)c^3}{dx^2+c}\right)} - \frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^2} - \frac{3\,(abc+b^2)d\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}c^2}$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")
[Out] -1/2*(a*b*c + b^2)*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*c^3 + b*c^2 -
 (a*d*x^2 + a*c + b)*c^3/(d*x^2 + c)) - b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2
 + c))/c^2 - 3/4*(a*b*c + b^2)*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*
c + b)*c)))/(sqrt((a*c + b)*c)*c^2)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")
[Out] undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^3}\,dx$$

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^3,x)
[Out] int((a + b/(c + d*x^2))^(3/2)/x^3, x)
```

3.336 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 205

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx=\frac{bd^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^3}+\frac{(9b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^3x^2}-\frac{(b+ac)\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c^3x^4}-\frac{3b(5b+4ac)d^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8c^{7/2}\sqrt{b+ac}}$$

```
[Out] -3/8*b*(4*a*c+5*b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a
*c+b)^(1/2))/c^(7/2)/(a*c+b)^(1/2)+b*d^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/
c^3+1/8*(4*a*c+9*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/x^2-1
/4*(a*c+b)*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/x^4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1985, 1981, 1980, 466, 1171, 396, 214}

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx=-\frac{3bd^2(4ac+5b)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{7/2}\sqrt{ac+b}}+\frac{bd^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^3}+\frac{d(4ac+9b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8c^3x^2}-\frac{(ac+b)\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^3x^4}$$

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] (b*d^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c^3 + ((9*b + 4*a*c)*d*(c + d
*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*c^3*x^2) - ((b + a*c)*(c +
d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*c^3*x^4) - (3*b*(5*b + 4
*a*c)*d^2*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b +
a*c]])/(8*c^(7/2)*Sqrt[b + a*c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\text{integral} = \int \frac{\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{x^5}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{\left(\frac{b+ac+adx}{c+dx}\right)^{3/2}}{x^3}\,dx, x, x^2\right)$$

$$= -\left((bd)\text{Subst}\left(\int \frac{x^4(ad-dx^2)}{(-b-ac+cx^2)^3}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right)$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c^3x^4} + \frac{(bd)\text{Subst}\left(\int \frac{b(b+ac)d+4bcdx^2+4c^2dx^4}{(-b-ac+cx^2)^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{4c^3}$$

$$= \frac{(9b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^3x^2} - \frac{(b+ac)\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c^3x^4} + \frac{(bd)\text{Subst}\left(\int \frac{(b+ac)(7b+4ac)d+8c(b+ac)dx^2}{-b-ac+cx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{8c^3(b+ac)}$$

$$= \frac{bd^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^3} + \frac{(9b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^3x^2} - \frac{(b+ac)\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c^3x^4} + \frac{(3b(5b+4ac)d^2)\,\text{Subst}\left(\int \frac{1}{-b-ac+cx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{8c^3}$$

$$= \frac{bd^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^3} + \frac{(9b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^3x^2} - \frac{(b+ac)\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c^3x^4} - \frac{3b(5b+4ac)d^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8c^{7/2}\sqrt{b+ac}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-2bc^2-2ac^3+5bcdx^2+15bd^2x^4+2acd^2x^4\right)}{8c^3x^4} + \frac{3b(5b+4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8c^{7/2}\sqrt{-b-ac}}$$

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b*c^2 - 2*a*c^3 + 5*b*c*d*x^2 +
15*b*d^2*x^4 + 2*a*c*d^2*x^4))/(8*c^3*x^4) + (3*b*(5*b + 4*a*c)*d^2*ArcTan[
(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*c^(7/2)
*Sqrt[-b - a*c])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{(d\,x^2+c)\left(-2acd\,x^2-7bd\,x^2+2a\,c^2+2bc\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{8c^3x^4} + \frac{b\,d^2\left(-\frac{(12ac+15b)\ln\left(\frac{2a\,c^2+2bc+(2acd+bd)x^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,c^2+bc+(2acd-}{x^2}\right.}{2\sqrt{a\,c^2+bc}}\right.}{}$
default	Expression too large to display

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(d*x^2+c)*(-2*a*c*d*x^2-7*b*d*x^2+2*a*c^2+2*b*c)/c^3/x^4*((a*d*x^2+a*c
+b)/(d*x^2+c))^(1/2)+1/8*b*d^2/c^3*(-1/2*(12*a*c+15*b)/(a*c^2+b*c)^(1/2)*ln
((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b
*d)*x^2+a*d^2*x^4)^(1/2))/x^2)+8*(a*d*x^2+a*c+b)/(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(
d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.72

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \left[\frac{3\,(4\,abc+5\,b^2)\sqrt{ac^2+bcd}^2x^4\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)\,d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,ab}{}\right.}{}\right.$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a*b*c + 5*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*
b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*
a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*
c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^2 + 15*b^2*c)*d^2*x^4 + 4*a*b
*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/((a*c^5 + b*c^4)*x^4), 1/16*(3*(4*a*b*c + 5*b^2)*sqrt(-a*c^2 -
b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 -
 b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2
 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^2 + 15*b^2
*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^5 + b*c^4)*x^4)]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**5,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.53

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \frac{bd^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^3} + \frac{3\,(4\,abc+5\,b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,\sqrt{(ac+b)c}c^3} - \frac{(4\,abc^2+9\,b^2c)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}} - (4\,a^2bc^2+11\,ab^2c+7\,b^3)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^2c^5+2\,abc^4+b^2c^3+\frac{(adx^2+ac+b)^2c^5}{(dx^2+c)^2}-\frac{2\,(ac^5+bc^4)(adx^2+ac+b)}{dx^2+c}\right)}$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] b*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/c^3 + 3/16*(4*a*b*c + 5*b^2)*d^
2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*
c^3) - 1/8*((4*a*b*c^2 + 9*b^2*c)*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/
2) - (4*a^2*b*c^2 + 11*a*b^2*c + 7*b^3)*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2
 + c)))/(a^2*c^5 + 2*a*b*c^4 + b^2*c^3 + (a*d*x^2 + a*c + b)^2*c^5/(d*x^2 +
 c)^2 - 2*(a*c^5 + b*c^4)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^5}\,dx$$

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^5,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x^5, x)
```

3.337 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 21, antiderivative size = 292

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7} dx = -\frac{bd^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^4} - \frac{(79b^2+108abc+24a^2c^2)\,d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{48c^4(b+ac)x^2} + \frac{(11b+12ac)d(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24c^4x^4} - \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{5/2}}{6c^2(b+ac)x^6} + \frac{b(35b^2+60abc+24a^2c^2)\,d^3\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{16c^{9/2}(b+ac)^{3/2}}$$

```
[Out] -1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(5/2)/c^2/(a*c+b)/x^6+1/16*b*(
24*a^2*c^2+60*a*b*c+35*b^2)*d^3*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))
^(1/2)/(a*c+b)^(1/2))/c^(9/2)/(a*c+b)^(3/2)-b*d^3*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/c^4-1/48*(24*a^2*c^2+108*a*b*c+79*b^2)*d^2*(d*x^2+c)*((a*d*x^2+a*c
+b)/(d*x^2+c))^(1/2)/c^4/(a*c+b)/x^2+1/24*(12*a*c+11*b)*d*(d*x^2+c)^2*((a*d
*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^4/x^4
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1985, 1981, 1980, 474, 466, 1171, 396, 214}

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \frac{bd^3(24a^2c^2+60abc+35b^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{16c^{9/2}(ac+b)^{3/2}} - \frac{d^2(24a^2c^2+108abc+79b^2)\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{48c^4x^2(ac+b)} - \frac{bd^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^4} + \frac{d(12ac+11b)\,(c+dx^2)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{24c^4x^4} - \frac{(c+dx^2)^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{5/2}}{6c^2x^6(ac+b)}$$

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] -((b*d^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c^4) - ((79*b^2 + 108*a*b*c
 + 24*a^2*c^2)*d^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(48*c
^4*(b + a*c)*x^2) + ((11*b + 12*a*c)*d*(c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^
2)/(c + d*x^2)])/(24*c^4*x^4) - ((c + d*x^2)^3*((b + a*c + a*d*x^2)/(c + d*
x^2))^(5/2))/(6*c^2*(b + a*c)*x^6) + (b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*d^
3*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(
16*c^(9/2)*(b + a*c)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
```

```
(IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\text{integral} = \int \frac{\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{x^7}\,dx$$

$$=\frac{1}{2}\text{Subst}\left(\int\frac{\left(\frac{b+ac+adx}{c+dx}\right)^{3/2}}{x^4}\,dx,x,x^2\right)$$

$$=-\left((bd)\text{Subst}\left(\int\frac{x^4(ad-dx^2)^2}{(-b-ac+cx^2)^4}\,dx,x,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right)$$

$$=-\frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{5/2}}{6c^2(b+ac)x^6}-\frac{(bd)\text{Subst}\left(\int\frac{x^4((5b^2-6a^2c^2)d^2+6c(b+ac)d^2x^2)}{(-b-ac+cx^2)^3}\,dx,x,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{6c^2(b+ac)}$$

$$=\frac{(11b+12ac)d(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24c^4x^4}-\frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{5/2}}{6c^2(b+ac)x^6}+\frac{(bd)\text{Subst}\left(\int\frac{-bc(b+ac)(11b+12ac)d^2-4bc^2(11b+12ac)d^2x^2-24c^3(b+ac)d^2x^4}{(-b-ac+cx^2)^2}\,dx,x,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{24c^5(b+ac)}$$

$$=-\frac{(79b^2+108abc+24a^2c^2)\,d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{48c^4(b+ac)x^2}+\frac{(11b+12ac)d(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24c^4x^4}-\frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{5/2}}{6c^2(b+ac)x^6}+\frac{(bd)\text{Subst}\left(\int\frac{-3c(b+ac)(19b^2+28abc+8a^2c^2)d^2-48c^2(b+ac)^2d^2x^2}{-b-ac+cx^2}\,dx,x,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{48c^5(b+ac)^2}$$

$$=-\frac{bd^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^4}-\frac{(79b^2+108abc+24a^2c^2)\,d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{48c^4(b+ac)x^2}+\frac{(11b+12ac)d(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24c^4x^4}-\frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{5/2}}{6c^2(b+ac)x^6}-\frac{(b(35b^2+60abc+24a^2c^2)\,d^3)\,\text{Subst}\left(\int\frac{1}{-b-ac+cx^2}\,dx,x,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{16c^4(b+ac)}$$

$$=-\frac{bd^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^4}-\frac{(79b^2+108abc+24a^2c^2)\,d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{48c^4(b+ac)x^2}+\frac{(11b+12ac)d(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24c^4x^4}-\frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{5/2}}{6c^2(b+ac)x^6}+\frac{b(35b^2+60abc+24a^2c^2)\,d^3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{16c^{9/2}(b+ac)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx = -\frac{\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(8a^2c^2(c^3+d^3x^6)+2abc(8c^3-7c^2dx^2+16cd^2x^4+55d^3x^6)+b^2(8c^3-14c^2dx^2+35cd^2x^4+105d^3x^6\right.}{(b+ac)x^6}}{48c^{9/2}}$$

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] (-((Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(8*a^2*c^2*(c^3 + d^3*x^6
) + 2*a*b*c*(8*c^3 - 7*c^2*d*x^2 + 16*c*d^2*x^4 + 55*d^3*x^6) + b^2*(8*c^3
- 14*c^2*d*x^2 + 35*c*d^2*x^4 + 105*d^3*x^6)))/((b + a*c)*x^6)) + (3*b*(35*
b^2 + 60*a*b*c + 24*a^2*c^2)*d^3*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(
c + d*x^2)])/Sqrt[-b - a*c]])/(-b - a*c)^(3/2))/(48*c^(9/2))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{(dx^2+c)(8a^2c^2d^2x^4+62acd^2bx^4-8a^2c^3dx^2+57b^2d^2x^4-30abc^2dx^2+8a^2c^4-22b^2cdx^2+16abc^3+8b^2c^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{48c^4x^6(ac+b)} - \frac{d^3b\ldots}{\ldots}$
default	Expression too large to display

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(d*x^2+c)*(8*a^2*c^2*d^2*x^4+62*a*b*c*d^2*x^4-8*a^2*c^3*d*x^2+57*b^2*
d^2*x^4-30*a*b*c^2*d*x^2+8*a^2*c^4-22*b^2*c*d*x^2+16*a*b*c^3+8*b^2*c^2)/c^4
/x^6/(a*c+b)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/16*d^3*b/c^4/(a*c+b)*(-1/2
*(24*a^2*c^2+60*a*b*c+35*b^2)/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+
b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))
/x^2)+16*(a*c+b)*(a*d*x^2+a*c+b)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/
(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.88 (sec) , antiderivative size = 733, normalized size of antiderivative = 2.51

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \left[\frac{3\left(24\,a^2bc^2+60\,ab^2c+35\,b^3\right)\sqrt{ac^2+bc}d^3x^6\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c\cdots}{\cdots}\right)\cdots}{\cdots}\right.$$

$$-\frac{3\left(24\,a^2bc^2+60\,ab^2c+35\,b^3\right)\sqrt{-ac^2-bc}d^3x^6\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2c^3+2\,abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)+2\,(8\,a^3c^7\cdots}{\cdots}$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*lo
g(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2
 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c
^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 118*a^2*b*c^3 + 21
5*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4
+ (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2*x^4 - 14*(a^2*b*c^5 + 2*a*
b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^7
+ 2*a*b*c^6 + b^2*c^5)*x^6), -1/96*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*
sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)
*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*
c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 + (8*a^3*c^4 + 118*a
^2*b*c^3 + 215*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5
 + 8*b^3*c^4 + (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2*x^4 - 14*(a^2
*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/((a^2*c^7 + 2*a*b*c^6 + b^2*c^5)*x^6)]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx=\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**7,x)
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(266) = 532$.

Time = 0.32 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.83

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx=$$

$$-\frac{(24\,a^2bc^2+60\,ab^2c+35\,b^3)d^3\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{32\,(ac^5+bc^4)\sqrt{(ac+b)c}}-\frac{bd^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^4}$$

$$-\frac{3\,(8\,a^2bc^4+36\,ab^2c^3+29\,b^3c^2)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\,(6\,a^3bc^4+30\,a^2b^2c^3+41\,ab^3c^2+17\,b^4c)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}}{48\left(a^4c^8+4\,a^3bc^7+6\,a^2b^2c^6+4\,ab^3c^5+b^4c^4-\frac{(ac^8+bc^7)(adx^2+ac+b)^3}{(dx^2+c)^3}+\frac{3\,(a^2c^8+2\,abc^7+b^2c^6)}{(dx^2+c)}\right.}$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")
[Out] -1/32*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*d^3*log((c*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)) + sqrt((a*c + b)*c)))/((a*c^5 + b*c^4)*sqrt((a*c + b)*c)) - b*d^3*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))/c^4 - 1/48*(3*(8*a^2*b*c^4 + 36*a*b^2*c^3
+ 29*b^3*c^2)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^4
+ 30*a^2*b^2*c^3 + 41*a*b^3*c^2 + 17*b^4*c)*d^3*((a*d*x^2 + a*c + b)/(d*x^2
 + c))^(3/2) + 3*(8*a^4*b*c^4 + 44*a^3*b^2*c^3 + 83*a^2*b^3*c^2 + 66*a*b^4*
c + 19*b^5)*d^3*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^8 + 4*a^3*b*c
^7 + 6*a^2*b^2*c^6 + 4*a*b^3*c^5 + b^4*c^4 - (a*c^8 + b*c^7)*(a*d*x^2 + a*c
 + b)^3/(d*x^2 + c)^3 + 3*(a^2*c^8 + 2*a*b*c^7 + b^2*c^6)*(a*d*x^2 + a*c +
b)^2/(d*x^2 + c)^2 - 3*(a^3*c^8 + 3*a^2*b*c^7 + 3*a*b^2*c^6 + b^3*c^5)*(a*d
*x^2 + a*c + b)/(d*x^2 + c))
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^7}\,dx$$

[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="giac")

[Out] undef

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^7}\,dx$$

[In] int((a + b/(c + d*x^2))^(3/2)/x^7,x)

[Out] int((a + b/(c + d*x^2))^(3/2)/x^7, x)

3.338 $\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 405

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{(b^2-14abc+a^2c^2)\,x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5ad^2}+\frac{(7b-ac)x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d^2}+\frac{6ax^3(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d}-\frac{x^3(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d}-\frac{\sqrt{c}(b^2-14abc+a^2c^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5ad^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}-\frac{c^{3/2}(7b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{5d^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

```
[Out] 1/5*(a^2*c^2-14*a*b*c+b^2)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2+1/5*(-
a*c+7*b)*x*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2+6/5*a*x^3*(d*x^2
+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d-x^3*(a*d*x^2+a*c+b)*((a*d*x^2+a*c+b
)/(d*x^2+c))^(1/2)/d-1/5*c^(3/2)*(-a*c+7*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/
c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*(
(a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+
c))^(1/2)-1/5*(a^2*c^2-14*a*b*c+b^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2
)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x
^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1985, 1986, 478, 595, 596, 545, 429, 506, 422}

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx = -\frac{\sqrt{c}(a^2c^2-14abc+b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5ad^{5/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{x(a^2c^2-14abc+b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5ad^2} - \frac{c^{3/2}(7b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{5d^{5/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{x(7b-ac)(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5d^2} + \frac{6ax^3(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5d} - \frac{x^3(ac+adx^2+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{d}$$

```
[In] Int[x^4*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] ((b^2 - 14*a*b*c + a^2*c^2)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*a*d
^2) + ((7*b - a*c)*x*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*
d^2) + (6*a*x^3*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*d) -
(x^3*(b + a*c + a*d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/d - (Sqrt[c
]*(b^2 - 14*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(5*a*d^(5/2)*Sqrt[(c*(b + a*c
+ a*d*x^2))/((b + a*c)*(c + d*x^2))]) - (c^(3/2)*(7*b - a*c)*Sqrt[(b + a*c
+ a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)]
)/(5*d^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^4\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2} dx \\
&= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{x^4(b+ac+adx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx}{\sqrt{b+ac+adx^2}} \\
&= -\frac{x^3(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d}+\frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{x^2\sqrt{b+ac+adx^2}\,(3(b+ac)+6adx^2)}{\sqrt{c+dx^2}}\,dx}{d\sqrt{b+ac+adx^2}} \\
&= \frac{6ax^3(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d}-\frac{x^3(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d} \\
&\quad+\frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{x^2(3(5b-ac)(b+ac)d+3a(7b-ac)d^2x^2)}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5d^2\sqrt{b+ac+adx^2}} \\
&= \frac{(7b-ac)x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d^2}+\frac{6ax^3(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d} \\
&\quad-\frac{x^3(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d} \\
&\quad-\frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{3ac(7b-ac)(b+ac)d^2-3a(b^2-14abc+a^2c^2)d^3x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{15ad^4\sqrt{b+ac+adx^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(7b-ac)x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d^2} + \frac{6ax^3(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d} \\
&\quad - \frac{x^3(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d} \\
&\quad - \frac{\left(c(7b-ac)(b+ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5d^2\sqrt{b+ac+adx^2}} \\
&\quad + \frac{\left((b^2-14abc+a^2c^2)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5d\sqrt{b+ac+adx^2}} \\
&= \frac{(b^2-14abc+a^2c^2)\,x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5ad^2} + \frac{(7b-ac)x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d^2} \\
&\quad + \frac{6ax^3(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d} - \frac{x^3(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d} \\
&\quad - \frac{c^{3/2}(7b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5d^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\
&\quad - \frac{\left(c(b^2-14abc+a^2c^2)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{5ad^2\sqrt{b+ac+adx^2}} \\
&= \frac{(b^2-14abc+a^2c^2)\,x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5ad^2} + \frac{(7b-ac)x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d^2} \\
&\quad + \frac{6ax^3(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d} - \frac{x^3(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d} \\
&\quad - \frac{\sqrt{c}(b^2-14abc+a^2c^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5ad^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\
&\quad - \frac{c^{3/2}(7b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5d^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.63 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.77

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x\left(-a^2(c-dx^2)\left(c+dx^2\right)^2+b^2(7c+2dx^2)+3ab(2c^2+3cdx^2+d^2x^4)\right)\right.}{}$$

[In] Integrate[x^4*(a + b/(c + d*x^2))^(3/2),x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(-(a^2*(c - d*x^2)*(c + d*x^2)^2) + b^2*(7*c + 2*d*x^2) + 3*a*b*(2*c^2 + 3*c*d*x^2 + d^2*x^4)) - I*(b^3 - 13*a*b^2*c - 13*a^2*b*c^2 + a^3*c^3)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] + I*b*(b^2 - 6*a*b*c - 7*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(5*a*c^2*(d/c)^(5/2)*(b + a*(c + d*x^2)))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. 2(441) = 882.

Time = 11.57 (sec) , antiderivative size = 1065, normalized size of antiderivative = 2.63

method	result	size
risch	Expression too large to display	1065
default	Expression too large to display	1101

[In] int(x^4*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/5/d^2*x*(-a*d*x^2+a*c-2*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/5/d^2*(a^2*c^3/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-7*b^2*c/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-a*b*c^2/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+5*b^2*c^2*((a*d^2*x^2+a*c*d+b*d)/c/b*x/d/((x^2+c/d)*(a*d^2*x^2+a*c*d+b*d))^(1/2)+(1/c-(a*c*d+b*d)/c/b/d)/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),

```
(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+2*a*d/b/c*(a*c^2+b*c)/(-a*d/(a*c+b))^(1/2)*
(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2
*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d
)/d/c/a)^(1/2))))-2*(a^2*c^2*d-9*a*b*c*d+b^2*d)*(a*c^2+b*c)/(-a*d/(a*c+b))^
(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+
b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d/(a*c+b))^(1/2),
(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c
*d+b*d)/d/c/a)^(1/2))))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*
(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.58

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx =$$

$$-\frac{(a^2c^3-14\,abc^2+b^2c)\sqrt{a}x\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac}\right)-(a^2c^3-14\,abc^2+b^2c+(a^2c^2-6\,abc-7\,b^2)d)}{5}$$

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*((a^2*c^3 - 14*a*b*c^2 + b^2*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsin
(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (a^2*c^3 - 14*a*b*c^2 + b^2*c + (a^2*c^2
 - 6*a*b*c - 7*b^2)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x)
, (a*c + b)/(a*c)) - (a^2*d^3*x^6 + 2*a*b*d^2*x^4 + a^2*c^3 - 14*a*b*c^2 -
(7*a*b*c - b^2)*d*x^2 + b^2*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d^
3*x)
```

Sympy [F]

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx = \int x^4\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}dx$$

```
[In] integrate(x**4*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**4*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [F]

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\int\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^4\,dx$$

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^4, x)
```

Giac [F]

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\int\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^4\,dx$$

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\int x^4\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}\,dx$$

```
[In] int(x^4*(a + b/(c + d*x^2))^(3/2),x)
[Out] int(x^4*(a + b/(c + d*x^2))^(3/2), x)
```

3.339 $\int x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 21, antiderivative size = 331

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{(7b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}+\frac{4ax(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d} - \frac{x(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d} - \frac{\sqrt{c}(7b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{\sqrt{c}(3b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

```
[Out] 1/3*(-a*c+7*b)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d+4/3*a*x*(d*x^2+c)*((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d-x*(a*d*x^2+a*c+b)*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/d-1/3*(-a*c+7*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE
(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a
*c+b)/(d*x^2+c))^(1/2)/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+
1/3*(-a*c+3*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/
c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)/(d*x^
2+c))^(1/2)/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 478, 542, 545, 429, 506, 422}

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx = \frac{\sqrt{c}(3b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} - \frac{\sqrt{c}(7b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{4ax(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3d} - \frac{x(ac+adx^2+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{d} + \frac{x(7b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3d}$$

```
[In] Int[x^2*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] ((7*b - a*c)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*d) + (4*a*x*(c + d
*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*d) - (x*(b + a*c + a*d*x^2)
*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/d - (Sqrt[c]*(7*b - a*c)*Sqrt[(b +
a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a
*c)])/(3*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (
Sqrt[c]*(3*b - a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))
/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
```

```
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \int x^2 \left(\frac{b + ac + adx^2}{c + dx^2}\right)^{3/2} dx$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{x^2(b+ac+adx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx}{\sqrt{b+ac+adx^2}}$$

$$=-\frac{x(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d}+\frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{\sqrt{b+ac+adx^2}(b+ac+4adx^2)}{\sqrt{c+dx^2}}\,dx}{d\sqrt{b+ac+adx^2}}$$

$$=\frac{4ax(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}-\frac{x(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d}+\frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{(3b-ac)(b+ac)d+a(7b-ac)d^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3d^2\sqrt{b+ac+adx^2}}$$

$$=\frac{4ax(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}-\frac{x(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d}+\frac{\left(a(7b-ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3\sqrt{b+ac+adx^2}}+\frac{\left((3b-ac)(b+ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3d\sqrt{b+ac+adx^2}}$$

$$=\frac{(7b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}+\frac{4ax(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}-\frac{x(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d}+\frac{\sqrt{c}(3b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}-\frac{\left(c(7b-ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{3d\sqrt{b+ac+adx^2}}$$

$$=\frac{(7b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}+\frac{4ax(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}-\frac{x(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d}-\frac{\sqrt{c}(7b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}+\frac{\sqrt{c}(3b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.42 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.77

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx = \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}x\left(-3b^2-2ab(c+dx^2)+a^2(c+dx^2)^2\right)+i(-7b^2-6abc+a^2c^2)\sqrt{\frac{b+}{}}\right.}{}$$

```
[In] Integrate[x^2*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*x*(-3*b^2 - 2*a*b*(c + d*
x^2) + a^2*(c + d*x^2)^2) + I*(-7*b^2 - 6*a*b*c + a^2*c^2)*Sqrt[(b + a*c +
a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (
a*c)/(b + a*c)] + (4*I)*b*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqr
t[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(3*d*
Sqrt[d/c]*(b + a*(c + d*x^2)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. 2(371) = 742.

Time = 9.35 (sec) , antiderivative size = 820, normalized size of antiderivative = 2.48

method	result
default	$\frac{\left(\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,a^2d^2x^5+2\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,a^2cd\,x^3+\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,abd\right.}{}$
risch	Expression too large to display

```
[In] int(x^2*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*d^2*x^5+2*(
(a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c*d*x^3+((a*d*x^2
+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d*x^3-((a*d*x^2+a*c+b)*(d
*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
E(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2-3*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d*x^3+((a*d*x^2+a*c+b
)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c^2*x-5*((a*d*x^2+a*c+b)*(d*x^2
+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*
(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c+7*((a*d*x^2+a*c+b)*(d*x^2+c
))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-
```

```
a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c+((a*d*x^2+a*c+b)*(d*x^2+c))^(
1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*x+3*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d
*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(
1/2),((a*c+b)/a/c)^(1/2))*b^2-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*x-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*x)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d/(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*d*x^2
+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.62

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx = \frac{(a^2c^3-7\,abc^2)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac}) - (a^2c^3-7\,abc^2+(a^2c^2-2\,abc-3\,b^2)}{3\,ac}$$

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a^2*c^3 - 7*a*b*c^2)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d
)/x), (a*c + b)/(a*c)) - (a^2*c^3 - 7*a*b*c^2 + (a^2*c^2 - 2*a*b*c - 3*b^2)
*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c))
+ (a^2*c*d^2*x^4 + 4*a*b*c*d*x^2 - a^2*c^3 + 7*a*b*c^2)*sqrt((a*d*x^2 + a*c
 + b)/(d*x^2 + c)))/(a*c*d^2*x)
```

Sympy [F]

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx = \int x^2\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**2*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**2*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [F]

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\int \left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^2\,dx$$

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\int \left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^2\,dx$$

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\int x^2\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}\,dx$$

```
[In] int(x^2*(a + b/(c + d*x^2))^(3/2),x)
[Out] int(x^2*(a + b/(c + d*x^2))^(3/2), x)
```

3.340 $\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 17, antiderivative size = 260

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{bx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} - \frac{(b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} + \frac{(b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{a\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

```
[Out] b*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c-(-a*c+b)*x*((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)/c+(-a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^
(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)/c^(1/2)/d^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+a*(1/
(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c
)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^(1/2
)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1985, 1986, 424, 545, 429, 506, 422}

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{a\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} - \frac{x(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c} + \frac{bx\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}$$

```
[In] Int[(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (b*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c - ((b - a*c)*x*Sqrt[(b + a*c
+ a*d*x^2)/(c + d*x^2)])/c + ((b - a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2
)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[c]*Sqrt[d]*Sq
rt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (a*Sqrt[c]*Sqrt[(b +
 a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b +
a*c)])/(Sqrt[d]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2} dx \\
&= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{(b+ac+adx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx}{\sqrt{b+ac+adx^2}} \\
&= \frac{bx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} + \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{ac(b+ac)d-a(b-ac)d^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{cd\sqrt{b+ac+adx^2}} \\
&= \frac{bx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} + \frac{\left(a(b+ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{b+ac+adx^2}} \\
&\quad - \frac{\left(a(b-ac)d\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{c\sqrt{b+ac+adx^2}}
\end{aligned}$$

$$= \frac{bx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} - \frac{(b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} + \frac{a\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$
$$+ \frac{\left((b-ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{\sqrt{b+ac+adx^2}}$$
$$= \frac{bx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} - \frac{(b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} + \frac{(b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$
$$+ \frac{a\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.42 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.88

$$\int \left(a + \frac{b}{c+dx^2}\right)^{3/2} dx = \frac{\sqrt{\frac{d}{c}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b\sqrt{\frac{d}{c}}x(b+a(c+dx^2)) + i(b^2-a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\right.\right.}{d\,(b+a\,(c}$$

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[d/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*Sqrt[d/c]*x*(b + a*(c +
 d*x^2)) + I*(b^2 - a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (
d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - I*b*(b + a*c
)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSi
nh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(d*(b + a*(c + d*x^2)))
```

Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.98

method	result
default	$\frac{\left(\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a^2c^2+\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{-\frac{ad}{ac+b}}\,abd\,x\right.}{}$

```
[In] int((a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

[Out] (((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2+(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d*x^3+2*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c-((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c+(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*x+(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*x)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/c/(a*d*x^2+a*c+b)

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.62

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx =$$
$$-\frac{(ac^2-bc)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}) - (ac^2-bc+(ac+b)d)\sqrt{a}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}) - }{cdx}$$

[In] integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")

[Out] -((a*c^2 - b*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (a*c^2 - b*c + (a*c + b)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (a*c*d*x^2 + a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c*d*x)

Sympy [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int \left(a+\frac{b}{c+dx^2}\right)^{\frac{3}{2}} dx$$

[In] integrate((a+b/(d*x**2+c))**(3/2),x)

[Out] Integral((a + b/(c + d*x**2))**(3/2), x)

Maxima [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int \left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a + b/(d*x^2 + c))^(3/2), x)
```

Giac [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int \left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate((a + b/(d*x^2 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int \left(a+\frac{b}{d\,x^2+c}\right)^{3/2} dx$$

```
[In] int((a + b/(c + d*x^2))^(3/2),x)
[Out] int((a + b/(c + d*x^2))^(3/2), x)
```

3.341 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 312

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx} + \frac{(2b+ac)dx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^2} - \frac{(2b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^2x} - \frac{(2b+ac)\sqrt{d}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{c^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{a\sqrt{d}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

```
[Out] b*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x+(a*c+2*b)*d*x*((a*d*x^2+a*c+b)/(d*x
^2+c))^(1/2)/c^2-(a*c+2*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/
x-(a*c+2*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(
1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*d^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/c^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+a*(1/(1+d*x^2/
c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(
b/(a*c+b))^(1/2))*d^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(1/2)/(c*(a*d
*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1985, 1986, 479, 597, 545, 429, 506, 422}

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx = -\frac{\sqrt{d}(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{c^{3/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{a\sqrt{d}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{dx(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^2} - \frac{(ac+2b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^2x} + \frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{cx}$$

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^2,x]
```

```
[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*x) + ((2*b + a*c)*d*x*Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)])/c^2 - ((2*b + a*c)*(c + d*x^2)*Sqrt[(b + a*c
 + a*d*x^2)/(c + d*x^2)])/(c^2*x) - ((2*b + a*c)*Sqrt[d]*Sqrt[(b + a*c + a*
d*x^2)/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(c
^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (a*Sqrt[d]*
Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, b/(b + a*c)])/(Sqrt[c]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2
))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
```

```
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \int \frac{\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{x^2}\,dx$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{(b+ac+adx^2)^{3/2}}{x^2(c+dx^2)^{3/2}}\,dx}{\sqrt{b+ac+adx^2}}$$

$$=\frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx}-\frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{-((b+ac)(2b+ac)d)-a(b+ac)d^2x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{cd\sqrt{b+ac+adx^2}}$$

$$=\frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx}-\frac{(2b+ac)\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^2x}+\frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{ac(b+ac)^2d^2+a(b+ac)(2b+ac)d^3x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{c^2(b+ac)d\sqrt{b+ac+adx^2}}$$

$$=\frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx}-\frac{(2b+ac)\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^2x}+\frac{\left(a(b+ac)d\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{c\sqrt{b+ac+adx^2}}+\frac{\left(a(2b+ac)d^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{c^2\sqrt{b+ac+adx^2}}$$

$$=\frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx}+\frac{(2b+ac)dx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^2}-\frac{(2b+ac)\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^2x}+\frac{a\sqrt{d}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}-\frac{\left((2b+ac)d\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int\frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{c\sqrt{b+ac+adx^2}}$$

$$=\frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx}+\frac{(2b+ac)dx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^2}-\frac{(2b+ac)\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^2x}-\frac{(2b+ac)\sqrt{d}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{c^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}+\frac{a\sqrt{d}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.45 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx =$$

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}\left(2ab(c+dx^2)^2+a^2c(c+dx^2)^2+b^2(c+2dx^2)\right)+i(2b^2+3abc+a^2c^2)\,dx\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1\ldots}\right.}{c^2\sqrt{\frac{d}{c}}x\,(b+\ldots}$$

[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^2,x]

[Out] -((Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*(2*a*b*(c + d*x^2)^2 + a^2*c*(c + d*x^2)^2 + b^2*(c + 2*d*x^2)) + I*(2*b^2 + 3*a*b*c + a^2*c^2)*d*x*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - (2*I)*b*(b + a*c)*d*x*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(c^2*Sqrt[d/c]*x*(b + a*(c + d*x^2))))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 872 vs. 2(360) = 720.

Time = 9.40 (sec) , antiderivative size = 873, normalized size of antiderivative = 2.80

method	result
default	$-\frac{\left(\sqrt{(adx^2+ac+b)(dx^2+c)}\sqrt{-\frac{ad}{ac+b}}\,a^2cd^2x^4+\sqrt{(adx^2+ac+b)(dx^2+c)}\sqrt{-\frac{ad}{ac+b}}\,abd^2x^4-\sqrt{(adx^2+ac+b)(dx^2+c)}\sqrt{\frac{adx^2+ac+b}{ac+b}}\ldots\right.}{\ldots}$
risch	$-\frac{(ac+b)(dx^2+c)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^2x}+\frac{d\left(\frac{a^2c^2\sqrt{1+\frac{adx^2}{ac+b}}\sqrt{1+\frac{dx^2}{c}}\,F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}}-\frac{2a^2cd(ac^2+bc)\sqrt{1+\frac{adx^2}{ac+b}}\sqrt{1+\frac{dx^2}{c}}\left(F\ldots\right.}{\sqrt{-\frac{ad}{ac+b}}\sqrt{ad^2\ldots}}\right.}{\ldots}$

[In] int((a+b/(d*x^2+c))^(3/2)/x^2,x,method=_RETURNVERBOSE)

[Out] -(((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c*d^2*x^4+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d^2*x^4-((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2*d*x+(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d^2*x^4+2*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c^2*d*x^2+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c*d*x-2*((a*d*

```
x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(
1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c*d*x+3*((a*
d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*d*x^2+(a*d^2*x^4+2
*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*d*x^2+((a*d*
x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c^3+((a*d*x^2+a*c+b)*(
d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*d*x^2+(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*d*x^2+2*((a*d*x^2+a*c+b)*(d*x^
2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c^2+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*
(-a*d/(a*c+b))^(1/2)*b^2*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/x/c^2/(a*d*x^2+a*c+
b)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx = (a^2c+2\,ab)\sqrt{-\frac{ad}{ac+b}}d^2x\sqrt{\frac{ac^2+bc}{d^2}}E\left(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+b}{ac}\right) - \left((a^2c+2\,ab)d^2+(a^2 \ldots\right.$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] ((a^2*c + 2*a*b)*sqrt(-a*d/(a*c + b))*d^2*x*sqrt((a*c^2 + b*c)/d^2)*ellipti
c_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^2*c + 2*a*b)*d^2
 + (a^2*c^2 + 2*a*b*c + b^2)*d)*sqrt(-a*d/(a*c + b))*x*sqrt((a*c^2 + b*c)/d
^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - (a^2*c^3
+ 2*a*b*c^2 + (a^2*c^2 + 3*a*b*c + 2*b^2)*d*x^2 + b^2*c)*sqrt((a*d*x^2 + a*
c + b)/(d*x^2 + c)))/((a*c^3 + b*c^2)*x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**2,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^2}\,dx$$

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^2,x)
[Out] int((a + b/(c + d*x^2))^(3/2)/x^2, x)
```

3.342 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 388

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4} \, dx = \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^3} - \frac{(8b+ac)d^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^3} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2x^3} + \frac{(8b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^3x} + \frac{(8b+ac)d^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{a(4b+ac)d^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3c^{3/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

```
[Out] b*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^3-1/3*(a*c+8*b)*d^2*x*((a*d*x^2+a*c
+b)/(d*x^2+c))^(1/2)/c^3-1/3*(a*c+4*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/c^2/x^3+1/3*(a*c+8*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)
/c^3/x+1/3*(a*c+8*b)*d^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)
/(d*x^2+c))^(1/2)/c^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/3*a
*(a*c+4*b)*d^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/c^(3/2)/(a*c+b)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 479, 597, 545, 429, 506, 422}

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx = -\frac{ad^{3/2}(ac+4b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3c^{3/2}(ac+b)\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{d^{3/2}(ac+8b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{5/2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} - \frac{d^2x(ac+8b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3c^3} + \frac{d(ac+8b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3c^3x} - \frac{(ac+4b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3c^2x^3} + \frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{cx^3}$$

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^4,x]
```

```
[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*x^3) - ((8*b + a*c)*d^2*x*Sqrt
[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c^3) - ((4*b + a*c)*(c + d*x^2)*Sqrt[
(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c^2*x^3) + ((8*b + a*c)*d*(c + d*x^2)*
Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c^3*x) + ((8*b + a*c)*d^(3/2)*Sqr
t[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b
/(b + a*c)])/(3*c^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2)
)]) - (a*(4*b + a*c)*d^(3/2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*c^(3/2)*(b + a*c)*Sqrt[(c*(
b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
```

```
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \int \frac{\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{x^4}\,dx$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{(b+ac+adx^2)^{3/2}}{x^4(c+dx^2)^{3/2}}\,dx}{\sqrt{b+ac+adx^2}}$$

$$= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^3} - \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{-((b+ac)(4b+ac)d)-a(3b+ac)d^2x^2}{x^4\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{cd\sqrt{b+ac+adx^2}}$$

$$= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^3} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2x^3} + \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{-(b+ac)^2(8b+ac)d^2-a(b+ac)(4b+ac)d^3x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3c^2(b+ac)d\sqrt{b+ac+adx^2}}$$

$$= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^3} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2x^3} + \frac{(8b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^3x} - \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{ac(b+ac)^2(4b+ac)d^3+a(b+ac)^2(8b+ac)d^4x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3c^3(b+ac)^2d\sqrt{b+ac+adx^2}}$$

$$= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^3} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2x^3} + \frac{(8b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^3x} - \frac{\left(a(4b+ac)d^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3c^2\sqrt{b+ac+adx^2}} - \frac{\left(a(8b+ac)d^3\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3c^3\sqrt{b+ac+adx^2}}$$

$$= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^3} - \frac{(8b+ac)d^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^3} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2x^3} + \frac{(8b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^3x} - \frac{a(4b+ac)d^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{\left((8b+ac)d^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{3c^2\sqrt{b+ac+adx^2}}$$

$$= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^3} - \frac{(8b+ac)d^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^3} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2x^3} + \frac{(8b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^3x} + \frac{(8b+ac)d^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{a(4b+ac)d^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.92 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-b^2c^2-2abc^3-a^2c^4+4b^2cdx^2+3abc^2dx^2-a^2c^3dx^2+8b^2d^2x^4+13a\right.}{}$$

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^4,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(b^2*c^2) - 2*a*b*c^3 - a^2*c^4 +
4*b^2*c*d*x^2 + 3*a*b*c^2*d*x^2 - a^2*c^3*d*x^2 + 8*b^2*d^2*x^4 + 13*a*b*c*
d^2*x^4 + a^2*c^2*d^2*x^4 + 8*a*b*d^3*x^6 + a^2*c*d^3*x^6 + I*c*(8*b^2 + 9*
a*b*c + a^2*c^2)*d*Sqrt[d/c]*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1
 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - I*b*c*(8
*b + 5*a*c)*d*Sqrt[d/c]*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d
*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(3*c^3*x^3*(b
 + a*(c + d*x^2)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. 2(424) = 848.

Time = 10.72 (sec) , antiderivative size = 1037, normalized size of antiderivative = 2.67

method	result	size
default	Expression too large to display	1037
risch	Expression too large to display	1122

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c*d^3*x^6
-5*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d^3*x^6+((a*d
*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2*d^2*x^3
-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b
*d^3*x^6-((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c^2*d^2
*x^4-4*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b
*c*d^2*x^3+8*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2
))*a*b*c*d^2*x^3-10*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*
a*b*c*d^2*x^4-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+
b))^(1/2)*a*b*c*d^2*x^4+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1
/2)*a^2*c^3*d*x^2-5*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*
b^2*d^2*x^4-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b)
)^(1/2)*b^2*d^2*x^4-3*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2
)*a*b*c^2*d*x^2+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*
c^4-4*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*c*d*x^2+2*
((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c^3+((a*d*x^2+a*
c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*c^2)*((a*d*x^2+a*c+b)/(d*x^2
+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(
1/2)/x^3/c^3/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.73

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx =$$

$$-\frac{(a^2c+8\,ab)\sqrt{-\frac{ad}{ac+b}}d^3x^3\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right)\mid\frac{ac+b}{ac}) - ((a^2c+8\,ab)d^3+(a^2c^2+5\,abc+4\,b^2)d^2)\sqrt{}}{}$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*((a^2*c + 8*a*b)*sqrt(-a*d/(a*c + b))*d^3*x^3*sqrt((a*c^2 + b*c)/d^2)*
elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^2*c + 8*a
*b)*d^3 + (a^2*c^2 + 5*a*b*c + 4*b^2)*d^2)*sqrt(-a*d/(a*c + b))*x^3*sqrt((a
*c^2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)
) - ((a^2*c^2 + 9*a*b*c + 8*b^2)*d^2*x^4 - a^2*c^4 - 2*a*b*c^3 - b^2*c^2 +
4*(a*b*c^2 + b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^4 +
 b*c^3)*x^3)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**4,x)
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="giac")
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^4}\,dx$$

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^4,x)
[Out] int((a + b/(c + d*x^2))^(3/2)/x^4, x)
```

3.343 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6} dx$

Optimal result

Integrand size = 21, antiderivative size = 494

$$\begin{aligned}\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6} dx = & \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^5} + \frac{(16b^2+16abc+a^2c^2)\, d^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^4(b+ac)} \\ & - \frac{(6b+ac)\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^2x^5} + \frac{(8b+ac)d(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^3x^3} \\ & - \frac{(16b^2+16abc+a^2c^2)\, d^2(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^4(b+ac)x} \\ & - \frac{(16b^2+16abc+a^2c^2)\, d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5c^{7/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ & + \frac{a(8b+ac)d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right), \frac{b}{b+ac}\right)}{5c^{5/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

```
[Out] b*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^5+1/5*(a^2*c^2+16*a*b*c+16*b^2)*d^3
*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^4/(a*c+b)-1/5*(a*c+6*b)*(d*x^2+c)*((
a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/x^5+1/5*(a*c+8*b)*d*(d*x^2+c)*((a*d*x^2
+a*c+b)/(d*x^2+c))^(1/2)/c^3/x^3-1/5*(a^2*c^2+16*a*b*c+16*b^2)*d^2*(d*x^2+c
)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^4/(a*c+b)/x-1/5*(a^2*c^2+16*a*b*c+16*
b^2)*d^(5/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)/c^(7/2)/(a*c+b)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/5*a*(a*c+8*
b)*d^(5/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1
/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/
c^(5/2)/(a*c+b)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1985, 1986, 479, 597, 545, 429, 506, 422}

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx = -\frac{d^{5/2}(a^2c^2+16abc+16b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5c^{7/2}(ac+b)\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{d^3x(a^2c^2+16abc+16b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5c^4(ac+b)} - \frac{d^2(a^2c^2+16abc+16b^2)(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5c^4x(ac+b)} + \frac{ad^{5/2}(ac+8b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{5c^{5/2}(ac+b)\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{d(ac+8b)(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5c^3x^3} - \frac{(ac+6b)(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5c^2x^5} + \frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{cx^5}$$

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^6,x]
```

```
[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*x^5) + ((16*b^2 + 16*a*b*c + a
^2*c^2)*d^3*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*c^4*(b + a*c)) - ((
6*b + a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*c^2*x^5) +
 ((8*b + a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*c^3*x
^3) - ((16*b^2 + 16*a*b*c + a^2*c^2)*d^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^
2)/(c + d*x^2)])/(5*c^4*(b + a*c)*x) - ((16*b^2 + 16*a*b*c + a^2*c^2)*d^(5/
2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], b/(b + a*c)])/(5*c^(7/2)*(b + a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b +
a*c)*(c + d*x^2))]) + (a*(8*b + a*c)*d^(5/2)*Sqrt[(b + a*c + a*d*x^2)/(c +
d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(5*c^(5/2)*(b
+ a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \int \frac{\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{x^6}\,dx$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{(b+ac+adx^2)^{3/2}}{x^6(c+dx^2)^{3/2}}\,dx}{\sqrt{b+ac+adx^2}}$$

$$= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^5} - \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{-((b+ac)(6b+ac)d)-a(5b+ac)d^2x^2}{x^6\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{cd\sqrt{b+ac+adx^2}}$$

$$= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^5} - \frac{(6b+ac)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^2x^5} + \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{-3(b+ac)^2(8b+ac)d^2-3a(b+ac)(6b+ac)d^3x^2}{x^4\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5c^2(b+ac)d\sqrt{b+ac+adx^2}}$$

$$= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^5} - \frac{(6b+ac)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^2x^5} + \frac{(8b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^3x^3} - \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{-3(b+ac)^2(16b^2+16abc+a^2c^2)d^3-3a(b+ac)^2(8b+ac)d^4x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{15c^3(b+ac)^2d\sqrt{b+ac+adx^2}}$$

$$= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^5} - \frac{(6b+ac)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^2x^5} + \frac{(8b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^3x^3} - \frac{(16b^2+16abc+a^2c^2)d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^4(b+ac)x} + \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{3ac(b+ac)^3(8b+ac)d^4+3a(b+ac)^2(16b^2+16abc+a^2c^2)d^5x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{15c^4(b+ac)^3d\sqrt{b+ac+adx^2}}$$

$$= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^5} - \frac{(6b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^2x^5} + \frac{(8b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^3x^3}$$
$$- \frac{\left(16b^2+16abc+a^2c^2\right)d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^4(b+ac)x}$$
$$+ \frac{\left(a(8b+ac)d^3\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5c^3\sqrt{b+ac+adx^2}}$$
$$+ \frac{\left(a\left(16b^2+16abc+a^2c^2\right)d^4\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5c^4(b+ac)\sqrt{b+ac+adx^2}}$$

$$= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^5} + \frac{\left(16b^2+16abc+a^2c^2\right)d^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^4(b+ac)}$$
$$- \frac{(6b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^2x^5} + \frac{(8b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^3x^3}$$
$$- \frac{\left(16b^2+16abc+a^2c^2\right)d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^4(b+ac)x}$$
$$+ \frac{a(8b+ac)d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5c^{5/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$
$$- \frac{\left(\left(16b^2+16abc+a^2c^2\right)d^3\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{5c^3(b+ac)\sqrt{b+ac+adx^2}}$$

$$= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^5} + \frac{\left(16b^2+16abc+a^2c^2\right)d^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^4(b+ac)}$$
$$- \frac{(6b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^2x^5} + \frac{(8b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^3x^3}$$
$$- \frac{\left(16b^2+16abc+a^2c^2\right)d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^4(b+ac)x}$$
$$- \frac{\left(16b^2+16abc+a^2c^2\right)d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5c^{7/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$
$$+ \frac{a(8b+ac)d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5c^{5/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.23 (sec) , antiderivative size = 472, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx =$$

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b^3c^3+3ab^2c^4+3a^2bc^5+a^3c^6-2b^3c^2dx^2-3ab^2c^3dx^2+a^3c^5dx^2+8b^3cd^2x^4+13ab^2c^2d^2x^4\right.}{}$$

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^6,x]
```

```
[Out] -1/5*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b^3*c^3 + 3*a*b^2*c^4 + 3*a^2*
b*c^5 + a^3*c^6 - 2*b^3*c^2*d*x^2 - 3*a*b^2*c^3*d*x^2 + a^3*c^5*d*x^2 + 8*b
^3*c*d^2*x^4 + 13*a*b^2*c^2*d^2*x^4 + 5*a^2*b*c^3*d^2*x^4 + 16*b^3*d^3*x^6
+ 40*a*b^2*c*d^3*x^6 + 24*a^2*b*c^2*d^3*x^6 + a^3*c^3*d^3*x^6 + 16*a*b^2*d^
4*x^8 + 16*a^2*b*c*d^4*x^8 + a^3*c^2*d^4*x^8 + I*c*(16*b^3 + 32*a*b^2*c + 1
7*a^2*b*c^2 + a^3*c^3)*d^2*Sqrt[d/c]*x^5*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)
]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] -
(8*I)*b*c*(2*b^2 + 3*a*b*c + a^2*c^2)*d^2*Sqrt[d/c]*x^5*Sqrt[(b + a*c + a*d
*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c
)/(b + a*c)]))/(c^4*(b + a*c)*x^5*(b + a*(c + d*x^2)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1169 vs. 2(526) = 1052.

Time = 11.58 (sec) , antiderivative size = 1170, normalized size of antiderivative = 2.37

method	result	size
risch	Expression too large to display	1170
default	Expression too large to display	1666

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(d*x^2+c)*(a^2*c^2*d^2*x^4+11*a*b*c*d^2*x^4-a^2*c^3*d*x^2+11*b^2*d^2*x
^4-4*a*b*c^2*d*x^2+a^2*c^4-3*b^2*c*d*x^2+2*a*b*c^3+b^2*c^2)/c^4/x^5/(a*c+b)
*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/5/c^4*d^3/(a*c+b)*(c^3*a^3/(-a*d/(a*c+
b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+
b*d)/d/c/a)^(1/2))+4*a^2*b*c^2/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/
2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*Elli
pticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+3*a*b^2*c/(-a*
d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2
```

```
*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2
*a*c*d+b*d)/d/c/a)^(1/2))-5*b^2*c*(a*c+b)*((a*d^2*x^2+a*c*d+b*d)/c/b*x/d/((
x^2+c/d)*(a*d^2*x^2+a*c*d+b*d))^(1/2)+(1/c-(a*c*d+b*d)/c/b/d)/(-a*d/(a*c+b)
)^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*
d)/d/c/a)^(1/2))+2*a*d/b/c*(a*c^2+b*c)/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*
x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1
/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c
/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))
))-2*(a^3*c^2*d+11*a^2*b*c*d+11*a*b^2*d)*(a*c^2+b*c)/(-a*d/(a*c+b))^(1/2)*(
1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*
a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)
/d/c/a)^(1/2))))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+
c))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \frac{(a^3c^2+16\,a^2bc+16\,ab^2)\sqrt{-\frac{ad}{ac+b}}d^4x^5\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+b}{ac}) - ((a^3c^2+$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/5*((a^3*c^2 + 16*a^2*b*c + 16*a*b^2)*sqrt(-a*d/(a*c + b))*d^4*x^5*sqrt((a
*c^2 + b*c)/d^2)*elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)
) - ((a^3*c^2 + 16*a^2*b*c + 16*a*b^2)*d^4 + (a^3*c^3 + 10*a^2*b*c^2 + 17*a
*b^2*c + 8*b^3)*d^3)*sqrt(-a*d/(a*c + b))*x^5*sqrt((a*c^2 + b*c)/d^2)*ellip
tic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^3*c^3 + 17*a^2
*b*c^2 + 32*a*b^2*c + 16*b^3)*d^3*x^6 + a^3*c^6 + 3*a^2*b*c^5 + 3*a*b^2*c^4
 + (7*a^2*b*c^3 + 15*a*b^2*c^2 + 8*b^3*c)*d^2*x^4 + b^3*c^3 - 2*(a^2*b*c^4
+ 2*a*b^2*c^3 + b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^
2*c^6 + 2*a*b*c^5 + b^2*c^4)*x^5)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx=\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**6,x)
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="maxima")
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="giac")
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^6}\,dx$$

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^6,x)
[Out] int((a + b/(c + d*x^2))^(3/2)/x^6, x)
```

3.344 $\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}} dx$

Optimal result

Integrand size = 21, antiderivative size = 225

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}} dx = \frac{(5b^2+12abc+8a^2c^2)\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^3d^3} - \frac{(5b+8ac)\,(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24a^2d^3} + \frac{x^2(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{6ad^2} - \frac{b(5b^2+12abc+8a^2c^2)\,\mathrm{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{7/2}d^3}$$

```
[Out] -1/16*b*(8*a^2*c^2+12*a*b*c+5*b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)/a^(1/2))/a^(7/2)/d^3+1/16*(8*a^2*c^2+12*a*b*c+5*b^2)*(d*x^2+c)*((a*d*x^2+
a*c+b)/(d*x^2+c))^(1/2)/a^3/d^3-1/24*(8*a*c+5*b)*(d*x^2+c)^2*((a*d*x^2+a*c+
b)/(d*x^2+c))^(1/2)/a^2/d^3+1/6*x^2*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))
^(1/2)/a/d^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{1985, 1981, 1980, 424, 393, 205, 214\}$

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{(8ac+5b)\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{24a^2d^3} - \frac{b(8a^2c^2+12abc+5b^2)\,\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{7/2}d^3} + \frac{(8a^2c^2+12abc+5b^2)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16a^3d^3} + \frac{x^2\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6ad^2}$$

```
[In] Int[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((5*b^2 + 12*a*b*c + 8*a^2*c^2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d
*x^2)])/(16*a^3*d^3) - ((5*b + 8*a*c)*(c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2
)/(c + d*x^2)])/(24*a^2*d^3) + (x^2*(c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/
(c + d*x^2)])/(6*a*d^2) - (b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*ArcTanh[Sqrt[(b
 + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(16*a^(7/2)*d^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^5}{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{\frac{b+ac+adx}{c+dx}}}\,dx, x, x^2\right)\\
&= -\left((bd)\text{Subst}\left(\int \frac{(-b-ac+cx^2)^2}{(ad-dx^2)^4}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right)\\
&= \frac{x^2(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{6ad^2}+\frac{b\text{Subst}\left(\int \frac{-((b+ac)(5b+6ac)d)+3c(b+2ac)dx^2}{(ad-dx^2)^3}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{6ad}
\end{aligned}$$

$$= -\frac{(5b+8ac)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24a^2d^3} + \frac{x^2(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{6ad^2} - \frac{(b(5b^2+12abc+8a^2c^2))\,\text{Subst}\left(\int \frac{1}{(ad-dx^2)^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{8a^2d}$$

$$= \frac{(5b^2+12abc+8a^2c^2)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^3d^3} - \frac{(5b+8ac)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24a^2d^3} + \frac{x^2(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{6ad^2} - \frac{(b(5b^2+12abc+8a^2c^2))\,\text{Subst}\left(\int \frac{1}{ad-dx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{16a^3d^2}$$

$$= \frac{(5b^2+12abc+8a^2c^2)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^3d^3} - \frac{(5b+8ac)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24a^2d^3} + \frac{x^2(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{6ad^2} - \frac{b(5b^2+12abc+8a^2c^2)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{7/2}d^3}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.66

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \frac{\sqrt{a}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(15b^2+2ab(13c-5dx^2)+8a^2(c^2-cdx^2+d^2x^4)) - 3b(5b^2+12abc+8a^2c^2)\,\text{arc}}{48a^{7/2}d^3}$$

```
[In] Integrate[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(15*b^2 + 2*a*b*
(13*c - 5*d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) - 3*b*(5*b^2 + 12*a*b*c
 + 8*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(48*a
^(7/2)*d^3)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.06

method	result
risch	$\frac{\left(8a^{2}d^{2}x^{4}-8a^{2}cd\,x^{2}-10abd\,x^{2}+8a^{2}c^{2}+26abc+15b^{2}\right)\left(ad\,x^{2}+ac+b\right)}{48d^{3}a^{3}\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}-\frac{b\left(8a^{2}c^{2}+12abc+5b^{2}\right)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^{2}x^{2}}{\sqrt{a\,d^{2}}}+\sqrt{a\,c^{2}+bc+(2acd}\right.}{32d^{2}a^{3}\sqrt{a\,d^{2}}\,\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}$
default	$\frac{\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(-48\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}\,\sqrt{a\,d^{2}}\,a^{2}cd\,x^{2}-36\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}\,\sqrt{a\,d^{2}}\,abd\,x^{2}-24\right.}{}$

[In] int(x^5/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/48/d^3*(8*a^2*d^2*x^4-8*a^2*c*d*x^2-10*a*b*d*x^2+8*a^2*c^2+26*a*b*c+15*b^2)*(a*d*x^2+a*c+b)/a^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/32*b/d^2*(8*a^2*c^2+12*a*b*c+5*b^2)/a^3*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.89

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$=\left[\frac{3\left(8\,a^{2}bc^{2}+12\,ab^{2}c+5\,b^{3}\right)\sqrt{a}\log\left(8\,a^{2}d^{2}x^{4}+8\,a^{2}c^{2}+8\left(2\,a^{2}c+ab\right)dx^{2}+8\,abc+b^{2}-4\left(2\,ad^{2}x^{4}+(4\,ac\right.\right.}{}\right.$$

[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] [1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^3*c^3 + 26*a^2*b*c^2 + 15*a*b^2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^3), 1/96*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^3*c^3 + 26*a^2*b*c^2 + 15*a*b^2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^3)]

Sympy [F]

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^5}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

[In] integrate(x**5/(a+b/(d*x**2+c))**(1/2),x)

[Out] Integral(x**5/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.51

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{3\,(8\,a^2bc^2+12\,ab^2c+5\,b^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}} - 8\,(6\,a^3bc^2+12\,a^2b^2c+5\,ab^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}} + 3\,(8\,a^4bc^2+20}{48\left(a^6d^3 - \frac{3\,(adx^2+ac+b)a^5d^3}{dx^2+c} + \frac{3\,(adx^2+ac+b)^2a^4d^3}{(dx^2+c)^2} - \frac{(adx^2+ac+b)^3a^3d^3}{(dx^2+c)^3}\right)} + \frac{(8\,a^2c^2+12\,abc+5\,b^2)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{7}{2}}d^3}$$

[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] -1/48*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^2 + 12*a^2*b^2*c + 5*a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^4*b*c^2 + 20*a^3*b^2*c + 11*a^2*b^3)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^6*d^3 - 3*(a*d*x^2 + a*c + b)*a^5*d^3/(d*x^2 + c) + 3*(a*d*x^2 + a*c + b)^2*a^4*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a^3*d^3/(d*x^2 + c)^3) + 1/32*(8*a^2*c^2 + 12*a*b*c + 5*b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(7/2)*d^3)

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\,x^2\left(\frac{4\,x^2}{ad}-\frac{4\,a^2cd^3+5\,abd^3}{a^3d^5}\right)+\frac{8\,a^2c^2d^2+26\,abcd^2+15\,b^2d^2}{a^3d^5}\right)+\frac{3\,(8\,a^2bc^2+12\,ab^2c\cdots}{\cdots}}{96\,\mathrm{sgn}\left(dx^2+c\right)}$$

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2
/(a*d) - (4*a^2*c*d^3 + 5*a*b*d^3)/(a^3*d^5)) + (8*a^2*c^2*d^2 + 26*a*b*c*d
^2 + 15*b^2*d^2)/(a^3*d^5)) + 3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*log(abs(
2*a*c*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c
^2 + b*c))*sqrt(a)*abs(d) + b*d))/(a^(7/2)*d^2*abs(d)))/sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^5}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

```
[In] int(x^5/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^5/(a + b/(c + d*x^2))^(1/2), x)
```

3.345 $\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{(3b+4ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8a^2d^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4ad^2}+\frac{b(3b+4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{5/2}d^2}$$

```
[Out] 1/8*b*(4*a*c+3*b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(5/2
)/d^2-1/8*(4*a*c+3*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d^2+1
/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1985, 1981, 1980, 393, 205, 214}

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{b(4ac+3b)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{5/2}d^2}-\frac{(4ac+3b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8a^2d^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4ad^2}$$

```
[In] Int[x^3/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -1/8*((3*b + 4*a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(a^2
*d^2) + ((c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*a*d^2) + (
b*(3*b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*
a^(5/2)*d^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^3}{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{\frac{b+ac+adx}{c+dx}}}\,dx, x, x^2\right)\\
&= -\left((bd)\text{Subst}\left(\int \frac{-b-ac+cx^2}{(ad-dx^2)^3}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right)\\
&= \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4ad^2} + \frac{(b(3b+4ac))\text{Subst}\left(\int \frac{1}{(ad-dx^2)^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{4a}\\
&= -\frac{(3b+4ac)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8a^2d^2} + \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4ad^2}\\
&\quad + \frac{(b(3b+4ac))\text{Subst}\left(\int \frac{1}{ad-dx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{8a^2d}\\
&= -\frac{(3b+4ac)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8a^2d^2} + \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4ad^2}\\
&\quad + \frac{b(3b+4ac)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{5/2}d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

$$
\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{a}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(-3b-2ac+2adx^2) + b(3b+4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{5/2}d^2}
$$

```
[In] Integrate[x^3/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b - 2*a*c +
2*a*d*x^2) + b*(3*b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/
Sqrt[a]])/(8*a^(5/2)*d^2)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{\left(-2ad\,x^{2}+2ac+3b\right)\left(ad\,x^{2}+ac+b\right)}{8d^{2}a^{2}\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}+\frac{b(4ac+3b)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^{2}x^{2}}{\sqrt{a\,d^{2}}}+\sqrt{a\,c^{2}+bc+(2acd+bd)x^{2}+a\,d^{2}x^{4}}\right)\sqrt{(ad\,x^{2}+ac+b)(d\,x^{2}+c)}}{16d\,a^{2}\sqrt{a\,d^{2}}\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}\,(d\,x^{2}+c)}$
default	$-\frac{\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}\,(d\,x^{2}+c)\left(-4\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}\,\sqrt{a\,d^{2}}\,ad\,x^{2}-4\ln\left(\frac{2a\,d^{2}x^{2}+2acd+2\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}\,\sqrt{a}}{2\sqrt{a\,d^{2}}}\right.\right.}{}$

[In] int(x^3/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/8/d^2*(-2*a*d*x^2+2*a*c+3*b)*(a*d*x^2+a*c+b)/a^2/((a*d*x^2+a*c+b)/(d*x^2
+c))^(1/2)+1/16*b/d*(4*a*c+3*b)/a^2*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1
/2)+(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)/((a*d*x^2+
a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.25

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \left[\frac{(4\,abc+3\,b^2)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2+4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,a\right.}{32\,a^3d^2}\right.$$

$$-\frac{(4\,abc+3\,b^2)\sqrt{-a}\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(2\,a^2d^2x^4-3\,abdx^2-2\,a^2c^2-3\,abc)\sqrt{\frac{adx^2}{da}}}{16\,a^3d^2}$$

[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] [1/32*((4*a*b*c + 3*b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c
 + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^
2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4

```
- 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
/(a^3*d^2), -1/16*((4*a*b*c + 3*b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c
 + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a
*b)) - 2*(2*a^2*d^2*x^4 - 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a*d*x^2
+ a*c + b)/(d*x^2 + c)))/(a^3*d^2)]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{x^3}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

```
[In] integrate(x**3/(a+b/(d*x**2+c))**(1/2),x)
[Out] Integral(x**3/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.51

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=-\frac{(4\,abc+3\,b^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc+5\,ab^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^4d^2-\frac{2\,(adx^2+ac+b)a^3d^2}{dx^2+c}+\frac{(adx^2+ac+b)^2a^2d^2}{(dx^2+c)^2}\right)}-\frac{(4\,ac+3\,b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{5}{2}}d^2}$$

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] -1/8*((4*a*b*c + 3*b^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*
c + 5*a*b^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^2 - 2*(a*d*x^2 +
 a*c + b)*a^3*d^2/(d*x^2 + c) + (a*d*x^2 + a*c + b)^2*a^2*d^2/(d*x^2 + c)^2
) - 1/16*(4*a*c + 3*b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/2)*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.12

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{ad}-\frac{2\,acd+3\,bd}{a^2d^3}\right)-\frac{(4\,abc+3\,b^2)\log\left(\left|2\,acd+2\,\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}|d|+bd\right|\right)}{a^{\frac{5}{2}}d|d|}}{16\,\mathrm{sgn}\left(dx^2+c\right)}$$

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a*d)
- (2*a*c*d + 3*b*d)/(a^2*d^3)) - (4*a*b*c + 3*b^2)*log(abs(2*a*c*d + 2*(sqr
t(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(
a)*abs(d) + b*d))/(a^(5/2)*d*abs(d)))/sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^3}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

```
[In] int(x^3/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^3/(a + b/(c + d*x^2))^(1/2), x)
```

3.346 $\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}-\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

```
[Out] -1/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)/d+1/2*(d*x^2+c)*(a+b/
(d*x^2+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1605, 248, 44, 65, 214}

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}-\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

```
[In] Int[x/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*a*d) - (b*ArcTanh[Sqrt[a + b/(c +
d*x^2)]/Sqrt[a]])/(2*a^(3/2)*d)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx, x, c+dx^2\right)}{2d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{2d}\\
&= \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad} + \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{4ad}\\
&= \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{c+dx^2}}\right)}{2ad}
\end{aligned}$$

$$=\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int\frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{(c+dx^2)\sqrt{\frac{b+a(c+dx^2)}{c+dx^2}}}{2ad}-\frac{b\text{arctanh}\left(\frac{\sqrt{\frac{b+a(c+dx^2)}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

```
[In] Integrate[x/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[(b + a*(c + d*x^2))/(c + d*x^2)])/(2*a*d) - (b*ArcTanh[Sq
rt[(b + a*(c + d*x^2))/(c + d*x^2)]/Sqrt[a]])/(2*a^(3/2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(60) = 120$.

Time = 1.02 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{\sqrt{\frac{a\left(d\,x^2+c\right)+b}{d\,x^2+c}}\,(d\,x^2+c)\left(2\sqrt{(a(d\,x^2+c)+b)(d\,x^2+c)}\,\sqrt{a}-b\ln\left(\frac{2\sqrt{\left(a\left(d\,x^2+c\right)+b\right)\left(d\,x^2+c\right)}\,\sqrt{a}+2a\left(d\,x^2+c\right)+b}{2\sqrt{a}}\right)\right)}{4d\sqrt{(a(d\,x^2+c)+b)(d\,x^2+c)}\,a^{\frac{3}{2}}}$
risch	$\frac{ad\,x^2+ac+b}{2da\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}-\frac{\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+bc+(2acd+bd)x^2+a\,d^2x^4}\right)b\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}}{4a\sqrt{a\,d^2}\,\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-b\ln\left(\frac{2a\,d^2x^2+2acd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)d+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2}}{4\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,ad\sqrt{a\,d^2}}$

```
[In] int(x/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*((a*(d*x^2+c)+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(2*((a*(d*x^2+c)+b)*(d*x^
2+c))^(1/2)*a^(1/2)-b*ln(1/2*(2*((a*(d*x^2+c)+b)*(d*x^2+c))^(1/2)*a^(1/2)+2
*a*(d*x^2+c)+b)/a^(1/2)))/((a*(d*x^2+c)+b)*(d*x^2+c))^(1/2)/a^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.71

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \left[\frac{\sqrt{a}b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2-4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc)\sqrt{a}\right.}{8\,a^2d}\right.$$

```
[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8
*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2 +
a*c + b)/(d*x^2 + c)))/(a^2*d), 1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 + 2*a
*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c +
 a*b)) + 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d)]
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

```
[In] integrate(x/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(60) = 120.

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.79

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2d-\frac{(adx^2+ac+b)ad}{dx^2+c}\right)}+\frac{b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,a^{\frac{3}{2}}d}$$

```
[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d - (a*d*x^2 + a*c + b)*a
*d/(d*x^2 + c)) + 1/4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(60) = 120$.

Time = 0.37 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.78

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\frac{b\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}|d|+bd\right|\right)}{a^{\frac{3}{2}}|d|}+\frac{2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{ad}}{4\,\mathrm{sgn}\left(dx^2+c\right)}$$

```
[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(b*log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2
+ b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) + b*d))/(a^(3/2)*abs(d)) + 2*sqrt(
a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/(a*d))/sgn(d*x^2 + c)
```

Mupad [B] (verification not implemented)

Time = 18.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.54

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{\frac{a\,(d\,x^2+c)}{b}+1}\,(d\,x^2+c)\,\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,(d\,x^2+c)}}{2\,a\,(d\,x^2+c)}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{d\,x^2+c}\,1\mathrm{i}}{\sqrt{b}}\right)\,3\mathrm{i}}{2\,a^{3/2}\,(d\,x^2+c)^{3/2}}\right)}{3\,d\,\sqrt{a+\frac{b}{d\,x^2+c}}}$$

```
[In] int(x/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] (((a*(c + d*x^2))/b + 1)^(1/2)*(c + d*x^2)*((b^(3/2)*asin((a^(1/2)*(c + d*x
^2)^(1/2)*1i)/b^(1/2))*3i)/(2*a^(3/2)*(c + d*x^2)^(3/2)) + (3*b^(1/2)*(b +
a*(c + d*x^2))^(1/2))/(2*a*(c + d*x^2))))/(3*d*(a + b/(c + d*x^2))^(1/2))
```

3.347 $\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{\sqrt{b+ac}}$$

```
[Out] arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(1/2)-arctanh(c^(1/2)*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*c^(1/2)/(a*c+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1985, 1981, 1980, 400, 214}

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{ac+b}}$$

```
[In] Int[1/(x*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/Sqrt[a] - (Sqrt[c]*A
rcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/Sqrt
[b + a*c]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{\frac{b+ac+adx}{c+dx}}}\,dx, x, x^2\right) \\
&= -\left((bd)\text{Subst}\left(\int \frac{1}{(-b-ac+cx^2)\,(ad-dx^2)}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right) \\
&= c\text{Subst}\left(\int \frac{1}{-b-ac+cx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right) \\
&\quad + d\text{Subst}\left(\int \frac{1}{ad-dx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)
\end{aligned}
$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{\sqrt{b+ac}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{\sqrt{-b-ac}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Integrate[1/(x*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a
*c]])/Sqrt[-b - a*c] + ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a
]]/Sqrt[a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(80) = 160$.

Time = 0.08 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.26

method	result
default	$-\frac{\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}\,(d x^2+c)\left(-\ln\left(\frac{2a d^2x^2+2acd+2\sqrt{a d^2x^4+2acd x^2+bd x^2+a c^2+bc}\,\sqrt{a d^2}+bd}{2\sqrt{a d^2}}\right)acd+\sqrt{a c^2+bc}\,\ln\left(\frac{2acd x^2+bd x^2+2a c^2+2\sqrt{}}{}\right.\right.}{2\sqrt{(ad x^2+ac+b)(d x^2+c}}$

```
[In] int(1/x/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-ln(1/2*(2*a*d^2*x^2+2*a*
c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a
*d^2)^(1/2))*a*c*d+(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c
^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*(
a*d^2)^(1/2)-b*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*d)/((a*d*x^2+a*c+b)*(d*
x^2+c))^(1/2)/(a*c+b)/(a*d^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(80) = 160.

Time = 0.36 (sec) , antiderivative size = 972, normalized size of antiderivative = 10.12

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \left[\frac{a\sqrt{\frac{c}{ac+b}}\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)\,dx^2-4\,((2\,a^2c^2+3\,abc+b^2)d^2x^4+2\,a^2c^4+4\,abc^3+2}{x^4}\right.}{} \right.$$

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*
c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*
((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 +
(4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))*sqrt(c/(a*c + b)))/x^4) + sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*
a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2
*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a, 1/4*(a*sqr
t(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*
b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2
 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3
+ 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/
(a*c + b)))/x^4) - 2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*s
qrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)))/a, 1/4*(2*
a*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*
c)) + sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a
*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sq
rt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a, 1/2*(a*sqrt(-c/(a*c + b))*arctan(1
/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c)) - sqrt(-a)*arctan(1/2*(2*
a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*
x^2 + a^2*c + a*b)))/a]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

```
[In] integrate(1/x/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.61

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{c\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}} - \frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{2\sqrt{a}}$$

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/sqrt((a*c + b)*
c) - 1/2*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) +
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/sqrt(a)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

```
[In] int(1/(x*(a + b/(c + d*x^2))^(1/2)),x)
[Out] int(1/(x*(a + b/(c + d*x^2))^(1/2)), x)
```

3.348 $\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx=-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2(b+ac)x^2}-\frac{bd\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2\sqrt{c}(b+ac)^{3/2}}$$

```
[Out] -1/2*b*d*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/(
a*c+b)^(3/2)/c^(1/2)-1/2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b
)/x^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1985, 1981, 1980, 205, 214}

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx=-\frac{bd\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2\sqrt{c}(ac+b)^{3/2}}-\frac{(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2x^2(ac+b)}$$

```
[In] Int[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -1/2*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/((b + a*c)*x^2) -
(b*d*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]]
)/(2*Sqrt[c]*(b + a*c)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{\frac{b+ac+adx}{c+dx}}}\,dx, x, x^2\right)\\ &= -\left((bd)\text{Subst}\left(\int \frac{1}{(-b-ac+cx^2)^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right)\end{aligned}$$

$$
= -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2(b+ac)x^2} + \frac{(bd)\text{Subst}\left(\int \frac{1}{-b-ac+cx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{2(b+ac)}
$$

$$
= -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2(b+ac)x^2} - \frac{bd\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2\sqrt{c}(b+ac)^{3/2}}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06

$$
\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{1}{2}\left(-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{(b+ac)x^2} - \frac{bd\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{\sqrt{c}(-b-ac)^{3/2}}\right)
$$

```
[In] Integrate[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] (-(((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/((b + a*c)*x^2)) - (
b*d*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])
/(Sqrt[c]*(-b - a*c)^(3/2)))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(92) = 184.

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{a d\,x^2+a c+b}{2(a c+b)x^2\sqrt{\frac{a d\,x^2+a c+b}{d\,x^2+c}}}-\frac{b d\ln\left(\frac{2a\,c^2+2b c+(2a c d+b d)x^2+2\sqrt{a\,c^2+b c}\,\sqrt{a\,c^2+b c+(2a c d+b d)x^2+a\,d^2x^4}}{x^2}\right)\sqrt{(a d\,x^2+a c+b)(d\,x^2+c)}}{4(a c+b)\sqrt{a\,c^2+b c}\,\sqrt{\frac{a d\,x^2+a c+b}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$-\frac{\sqrt{\frac{a d\,x^2+a c+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-2a\,d^2\sqrt{a\,d^2x^4+2a c d\,x^2+b d\,x^2+a\,c^2+b c}\,x^4\sqrt{a\,c^2+b c}+\ln\left(\frac{2a c d\,x^2+b d\,x^2+2a\,c^2+2\sqrt{a\,c^2+b c}\,\sqrt{a\,d^2x^4+2a c d\,x^2}}{x^2}\right.\right.}{}$

```
[In] int(1/x^3/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a*c+b)*(a*d*x^2+a*c+b)/x^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/4*b*d/
(a*c+b)/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)
^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)/((a*d*x^2+a*c+b)
/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(92) = 184$.

Time = 0.36 (sec) , antiderivative size = 451, normalized size of antiderivative = 4.18

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \left[\frac{\sqrt{ac^2+bc}\,bdx^2\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2-4\,((2\,ac+b)d^2x^4+2\,ac^3+(4\,ac^2+3\,b}{x^4}\right.}{8\,(a^2c^3+2\,abc^2+b^2c)x^2}\right.$$

[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] [1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*x^2), 1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*(a*c^3 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*x^2)]

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

[In] integrate(1/x**3/(a+b/(d*x**2+c))**(1/2),x)

[Out] Integral(1/(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.60

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2c^2+2\,abc+b^2-\frac{(adx^2+ac+b)(ac^2+bc)}{dx^2+c}\right)} + \frac{bd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}(ac+b)}$$

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^2 + 2*a*b*c + b^2 - (
a*d*x^2 + a*c + b)*(a*c^2 + b*c)/(d*x^2 + c)) + 1/4*b*d*log((c*sqrt((a*d*x^
2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/
(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*(a*c + b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(92) = 184$.

Time = 0.40 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.70

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\frac{bd\arctan\left(-\frac{\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{\sqrt{-ac^2-bc}(ac+b)} - \frac{2\,a^{\frac{3}{2}}c^2|d|+2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)acd+2\sqrt{a}bc|d|+\left(\sqrt{ad^2}x^2-\sqrt{}\right.}{\left(ac^2-\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2+bc\right)}}{2\,\mathrm{sgn}\left(dx^2+c\right)}$$

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(b*d*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2
+ a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/(sqrt(-a*c^2 - b*c)*(a*c + b)) - (2*a^(
3/2)*c^2*abs(d) + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x
^2 + a*c^2 + b*c))*a*c*d + 2*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a
*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b*d)/((a*c^2 - (sqrt(a*d^2
)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)*(a*
c + b)))/sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{x^3\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

```
[In] int(1/(x^3*(a + b/(c + d*x^2))^(1/2)),x)
[Out] int(1/(x^3*(a + b/(c + d*x^2))^(1/2)), x)
```

3.349 $\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 177

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{(b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c(b+ac)^2x^2} - \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c(b+ac)x^4} + \frac{b(b+4ac)d^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8c^{3/2}(b+ac)^{5/2}}$$

```
[Out] 1/8*b*(4*a*c+b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+
b)^(1/2))/c^(3/2)/(a*c+b)^(5/2)+1/8*(4*a*c+b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/
(d*x^2+c))^(1/2)/c/(a*c+b)^2/x^2-1/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/c/(a*c+b)/x^4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1985, 1981, 1980, 393, 205, 214}

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{bd^2(4ac+b)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{3/2}(ac+b)^{5/2}} + \frac{d(4ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8cx^2(ac+b)^2} - \frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4cx^4(ac+b)}$$

```
[In] Int[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] ((b + 4*a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*c*(b +
 a*c)^2*x^2) - ((c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*c*(
b + a*c)*x^4) + (b*(b + 4*a*c)*d^2*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2
)/(c + d*x^2)])/Sqrt[b + a*c]])/(8*c^(3/2)*(b + a*c)^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^5\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt{\frac{b+ac+adx}{c+dx}}}\,dx, x, x^2\right)$$

$$= -\left((bd)\text{Subst}\left(\int \frac{ad-dx^2}{(-b-ac+cx^2)^3}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right)$$

$$= -\frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c(b+ac)x^4} + \frac{(b(b+4ac)d^2)\,\text{Subst}\left(\int \frac{1}{(-b-ac+cx^2)^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{4c(b+ac)}$$

$$= \frac{(b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c(b+ac)^2x^2} - \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c(b+ac)x^4} - \frac{(b(b+4ac)d^2)\,\text{Subst}\left(\int \frac{1}{-b-ac+cx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{8c(b+ac)^2}$$

$$= \frac{(b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c(b+ac)^2x^2} - \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c(b+ac)x^4} + \frac{b(b+4ac)d^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8c^{3/2}(b+ac)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(2ac(c-dx^2)+b(2c+dx^2))}{8c(b+ac)^2x^4} - \frac{b(b+4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8c^{3/2}(-b-ac)^{5/2}}$$

```
[In] Integrate[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]
```

[Out] -1/8*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*a*c*(c - d*x^2) + b*(2*c + d*x^2)))/(c*(b + a*c)^2*x^4) - (b*(b + 4*a*c)*d^2*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*c^(3/2)*(-b - a*c)^(5/2))

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{(a d\,x^2+a c+b)\left(-2 a c d\,x^2+b d\,x^2+2 a\,c^2+2 b c\right)}{8(a c+b)^2 x^4 c\sqrt{\frac{a d\,x^2+a c+b}{d\,x^2+c}}}+\frac{d^2 b(4 a c+b)\ln\left(\frac{2 a\,c^2+2 b c+(2 a c d+b d)x^2+2\sqrt{a\,c^2+b c}\,\sqrt{a\,c^2+b c+(2 a c d+b d)x^2+a\,d^2x^4}}{x^2}\right.}{16(a c+b)^2 c\sqrt{a\,c^2+b c}\,\sqrt{\frac{a d\,x^2+a c+b}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$-\frac{\sqrt{\frac{a d\,x^2+a c+b}{d\,x^2+c}}\,(d\,x^2+c)\left(12 a^2 d^3\sqrt{a\,d^2x^4+2 a c d\,x^2+b d\,x^2+a\,c^2+b c}\,x^6 c(a\,c^2+b c)^{\frac{3}{2}}-4\ln\left(\frac{2 a c d\,x^2+b d\,x^2+2 a\,c^2+2\sqrt{a\,c^2+b c}\,\sqrt{a\,d^2x^4+}}{x^2}\right.\right.}{}$

[In] int(1/x^5/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/8*(a*d*x^2+a*c+b)*(-2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c)/(a*c+b)^2/x^4/c/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/16*d^2*b*(4*a*c+b)/(a*c+b)^2/c/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 593, normalized size of antiderivative = 3.35

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$=\left[\frac{(4\,abc+b^2)\sqrt{ac^2+bc}d^2x^4\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2+4\,((2\,ac+b)d^2x^4+2}{x^4}\right.}{32\,(a^3}\right.$$

$$-\frac{(4\,abc+b^2)\sqrt{-ac^2-bc}d^2x^4\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2c^3+2\,abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)+2\,(2\,a^2c^5-(2\,a^2c^3+a}{16\,(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2)x^4}$$

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b*c
+ b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*
c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d
*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^
4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b
^2*c^3 + 3*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*x^4), -1/16*((4*a*b*c +
b^2)*sqrt(-a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2
*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2
*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(2*a^2*c^5 - (2*a^2*c^3 +
a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 + 3*(a*b*c^3 + b^2*c^2)*d*
x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b
^2*c^3 + b^3*c^2)*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

```
[In] integrate(1/x**5/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(157) = 314$.

Time = 0.30 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.03

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx=-\frac{(4\,abc+b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,(a^2c^3+2\,abc^2+b^2c)\sqrt{(ac+b)c}} - \frac{(4\,abc^2+b^2c)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc^2+3\,ab^2c-b^3)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^4c^5+4\,a^3bc^4+6\,a^2b^2c^3+4\,ab^3c^2+b^4c+\frac{(a^2c^5+2\,abc^4+b^2c^3)(adx^2+ac+b)^2}{(dx^2+c)^2}-\frac{2\,(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2)(adx^2+}{dx^2+c}\right.}$$

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(4*a*b*c + b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sq
rt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*
```

```
c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*sqrt((a*c + b)*c)) - 1/8*((4*a*b*c^2 +
b^2*c)*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c^2 + 3*a*b^2
*c - b^3)*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^5 + 4*a^3*b*c^4
 + 6*a^2*b^2*c^3 + 4*a*b^3*c^2 + b^4*c + (a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*(a
*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 2*(a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3
+ b^3*c^2)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(157) = 314$.

Time = 0.41 (sec) , antiderivative size = 778, normalized size of antiderivative = 4.40

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{\frac{(4\,abcd^2+b^2d^2)\arctan\left(-\frac{\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{(a^2c^3+2\,abc^2+b^2c)\sqrt{-ac^2-bc}} - \frac{8\,a^{\frac{7}{2}}c^5d|d|+16\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^3c^4d^2-\cdots}{\cdots}}{\cdots}$$

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((4*a*b*c*d^2 + b^2*d^2)*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2
*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/((a^2*c^3 + 2*a*b*
c^2 + b^2*c)*sqrt(-a*c^2 - b*c)) - (8*a^(7/2)*c^5*d*abs(d) + 16*(sqrt(a*d^2
)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^4*d^2
+ 8*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c
))^2*a^(5/2)*c^3*d*abs(d) + 16*a^(5/2)*b*c^4*d*abs(d) + 28*(sqrt(a*d^2)*x^2
 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c^3*d^2 + 1
6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))
^2*a^(3/2)*b*c^2*d*abs(d) + 8*a^(3/2)*b^2*c^3*d*abs(d) + 4*(sqrt(a*d^2)*x^2
 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a*b*c*d^2 + 13*
(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a
*b^2*c^2*d^2 + 8*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2
+ a*c^2 + b*c))^2*sqrt(a)*b^2*c*d*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^
4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*b^2*d^2 + (sqrt(a*d^2)*x^2 - sq
rt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b^3*c*d^2)/((a^2*c^3 +
 2*a*b*c^2 + b^2*c)*(a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^
2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)^2))/sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x^5\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

```
[In] int(1/(x^5*(a + b/(c + d*x^2))^(1/2)),x)
[Out] int(1/(x^5*(a + b/(c + d*x^2))^(1/2)), x)
```

3.350 $\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 443

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{(4b+3ac)x(b+ac+adx^2)}{15a^2d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{x^3(b+ac+adx^2)}{5ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(8b^2+13abc+3a^2c^2)\,x(b+ac+adx^2)}{15a^3d^2\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{\sqrt{c}(8b^2+13abc+3a^2c^2)\,(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^3d^{5/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}+\frac{c^{3/2}(4b+3ac)\,(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{15a^2d^{5/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

```
[Out] -1/15*(3*a*c+4*b)*x*(a*d*x^2+a*c+b)/a^2/d^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/
2)+1/5*x^3*(a*d*x^2+a*c+b)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/15*(3*a^
2*c^2+13*a*b*c+8*b^2)*x*(a*d*x^2+a*c+b)/a^3/d^2/(d*x^2+c)/((a*d*x^2+a*c+b)/
(d*x^2+c))^(1/2)+1/15*c^(3/2)*(3*a*c+4*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(
1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*
c+b))^(1/2))/a^2/d^(5/2)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*
d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/15*(3*a^2*c^2+13*a*b*c+8*b^2)*(a*d*
x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1
/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^3/d^(5/2)/(d*x^2+c)/((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 489, 596, 545, 429, 506, 422}

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{c^{3/2}(3ac+4b)\left(ac+adx^2+b\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{15a^2d^{5/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} - \frac{x(3ac+4b)\left(ac+adx^2+b\right)}{15a^2d^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} - \frac{\sqrt{c}(3a^2c^2+13abc+8b^2)\left(ac+adx^2+b\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^3d^{5/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{x(3a^2c^2+13abc+8b^2)\left(ac+adx^2+b\right)}{15a^3d^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} + \frac{x^3\left(ac+adx^2+b\right)}{5ad\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[In] Int[x^4/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -1/15*((4*b + 3*a*c)*x*(b + a*c + a*d*x^2))/(a^2*d^2*Sqrt[(b + a*c + a*d*x^
2)/(c + d*x^2)]) + (x^3*(b + a*c + a*d*x^2))/(5*a*d*Sqrt[(b + a*c + a*d*x^2
)/(c + d*x^2)]) + ((8*b^2 + 13*a*b*c + 3*a^2*c^2)*x*(b + a*c + a*d*x^2))/(1
5*a^3*d^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - (Sqrt[c]*(8*
b^2 + 13*a*b*c + 3*a^2*c^2)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], b/(b + a*c)])/(15*a^3*d^(5/2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x
^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (
c^(3/2)*(4*b + 3*a*c)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], b/(b + a*c)])/(15*a^2*d^(5/2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^4}{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\,dx\\
&= \frac{\sqrt{b+ac+adx^2}\int \frac{x^4\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&= \frac{x^3(b+ac+adx^2)}{5ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{b+ac+adx^2}\int \frac{x^2(3c(b+ac)+(4b+3ac)dx^2)}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5ad\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&= -\frac{(4b+3ac)x(b+ac+adx^2)}{15a^2d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{x^3(b+ac+adx^2)}{5ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&\quad + \frac{\sqrt{b+ac+adx^2}\int \frac{c(b+ac)(4b+3ac)d+(8b^2+13abc+3a^2c^2)d^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{15a^2d^3\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&= -\frac{(4b+3ac)x(b+ac+adx^2)}{15a^2d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{x^3(b+ac+adx^2)}{5ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&\quad + \frac{\left(c(b+ac)(4b+3ac)\sqrt{b+ac+adx^2}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{15a^2d^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&\quad + \frac{\left((8b^2+13abc+3a^2c^2)\sqrt{b+ac+adx^2}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{15a^2d\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&= -\frac{(4b+3ac)x(b+ac+adx^2)}{15a^2d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{x^3(b+ac+adx^2)}{5ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&\quad + \frac{(8b^2+13abc+3a^2c^2)\,x(b+ac+adx^2)}{15a^3d^2\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&\quad + \frac{c^{3/2}(4b+3ac)\,(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^2d^{5/2}\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\\
&\quad - \frac{\left(c(8b^2+13abc+3a^2c^2)\sqrt{b+ac+adx^2}\right)\int \frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{15a^3d^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}
\end{aligned}
$$

$$= -\frac{(4b+3ac)x(b+ac+adx^2)}{15a^2d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{x^3(b+ac+adx^2)}{5ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(8b^2+13abc+3a^2c^2)\,x(b+ac+adx^2)}{15a^3d^2\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{c}(8b^2+13abc+3a^2c^2)\,(b+ac+adx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^3d^{5/2}\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{c^{3/2}(4b+3ac)\,(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^2d^{5/2}\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.78 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\,(4b^2+ab(7c+dx^2)+3a^2(c^2-d^2x^4))+i(8b^3+21ab^2c+16a^2bc^2+3a^3c\right.}{}$$

```
[In] Integrate[x^4/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -1/15*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(4*
b^2 + a*b*(7*c + d*x^2) + 3*a^2*(c^2 - d^2*x^4)) + I*(8*b^3 + 21*a*b^2*c +
16*a^2*b*c^2 + 3*a^3*c^3)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x
^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - I*b*(8*b^2 + 17
*a*b*c + 9*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]
*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(a^3*c^2*(d/c)^(5/2)*
(b + a*(c + d*x^2)))
```

Maple [A] (verified)

Time = 8.06 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.50

method	result
default	$-\frac{\left(-3\sqrt{-\frac{ad}{ac+b}}\, a^{2} d^{3} x^{7}-3\sqrt{-\frac{ad}{ac+b}}\, a^{2} c\, d^{2} x^{5}+\sqrt{-\frac{ad}{ac+b}}\, ab\, d^{2} x^{5}+3\sqrt{-\frac{ad}{ac+b}}\, a^{2} c^{2} d\, x^{3}+8\sqrt{-\frac{ad}{ac+b}}\, abcd\, x^{3}-3\sqrt{\frac{a d\, x^{2}+ac+b}{ac+b}}\, \sqrt{\frac{d\, x^{2}+c}{c}}\,\right.}{}$
risch	$-\frac{x\left(-3ad\, x^{2}+3ac+4b\right)\left(ad\, x^{2}+ac+b\right)}{15d^{2} a^{2} \sqrt{\frac{ad\, x^{2}+ac+b}{d\, x^{2}+c}}}+\frac{\left(\frac{3a^{2} c^{3} \sqrt{1+\frac{ad\, x^{2}}{ac+b}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, F\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^{2} x^{4}+2acd\, x^{2}+bd\, x^{2}+a\, c^{2}+bc}}+\frac{4b^{2} c\sqrt{1+\frac{ad\, x^{2}}{ac+b}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, F\left(x\sqrt{-\right.}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^{2} x^{4}+2acd\, x^{2}}}\right.}{}$

```
[In] int(x^4/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(-3*(-a*d/(a*c+b))^(1/2)*a^2*d^3*x^7-3*(-a*d/(a*c+b))^(1/2)*a^2*c*d^2
*x^5+(-a*d/(a*c+b))^(1/2)*a*b*d^2*x^5+3*(-a*d/(a*c+b))^(1/2)*a^2*c^2*d*x^3+
8*(-a*d/(a*c+b))^(1/2)*a*b*c*d*x^3-3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^3
+3*(-a*d/(a*c+b))^(1/2)*a^2*c^3*x+4*(-a*d/(a*c+b))^(1/2)*b^2*d*x^3+6*((a*d*
x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1
/2),((a*c+b)/a/c)^(1/2))*a*b*c^2-13*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2+
7*(-a*d/(a*c+b))^(1/2)*a*b*c^2*x+4*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c-8*(
(a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b
))^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c+4*(-a*d/(a*c+b))^(1/2)*b^2*c*x)*(d*x^2+
c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d^2/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.54

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\, dx =$$

$$-\frac{(3\, a^2c^3 + 13\, abc^2 + 8\, b^2c)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}) - (3\, a^2c^3 + 13\, abc^2 + 8\, b^2c + (3\, a^2c^2 + 7\, abc}{}$$

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*((3*a^2*c^3 + 13*a*b*c^2 + 8*b^2*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(a
rcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (3*a^2*c^3 + 13*a*b*c^2 + 8*b^2*c +
 (3*a^2*c^2 + 7*a*b*c + 4*b^2)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin(sq
rt(-c/d)/x), (a*c + b)/(a*c)) - (3*a^2*d^3*x^6 - 4*a*b*d^2*x^4 + 3*a^2*c^3
+ 13*a*b*c^2 + (9*a*b*c + 8*b^2)*d*x^2 + 8*b^2*c)*sqrt((a*d*x^2 + a*c + b)/
(d*x^2 + c)))/(a^3*d^3*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

```
[In] integrate(x**4/(a+b/(d*x**2+c))**(1/2),x)
[Out] Integral(x**4/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/sqrt(a + b/(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(a + b/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

```
[In] int(x^4/(a + b/(c + d*x^2))^(1/2),x)
[Out] int(x^4/(a + b/(c + d*x^2))^(1/2), x)
```

3.351 $\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 354

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{x(b+ac+adx^2)}{3ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{(2b+ac)x(b+ac+adx^2)}{3a^2d\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{\sqrt{c}(2b+ac)\,(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2d^{3/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}-\frac{c^{3/2}(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3ad^{3/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

```
[Out] 1/3*x*(a*d*x^2+a*c+b)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(a*c+2*b)*x
*(a*d*x^2+a*c+b)/a^2/d/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*c^(3
/2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(
1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a/d^(3/2)/(d*x^2+c)/((a*d
*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/
3*(a*c+2*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipti
cE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^2/d^(3/
2)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(
d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1985, 1986, 489, 545, 429, 506, 422}

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{c}(ac+2b)\left(ac+adx^2+b\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2d^{3/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} - \frac{x(ac+2b)\left(ac+adx^2+b\right)}{3a^2d\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} - \frac{c^{3/2}\left(ac+adx^2+b\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3ad^{3/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{x\left(ac+adx^2+b\right)}{3ad\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[In] Int[x^2/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (x*(b + a*c + a*d*x^2))/(3*a*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - ((2
*b + a*c)*x*(b + a*c + a*d*x^2))/(3*a^2*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x
^2)/(c + d*x^2)]) + (Sqrt[c]*(2*b + a*c)*(b + a*c + a*d*x^2)*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*a^2*d^(3/2)*(c + d*x^2)*Sqrt[(b +
 a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d
*x^2))]) - (c^(3/2)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], b/(b + a*c)])/(3*a*d^(3/2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*
x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\,dx$$

$$= \frac{\sqrt{b+ac+adx^2}\int \frac{x^2\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= \frac{x(b+ac+adx^2)}{3ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{b+ac+adx^2}\int \frac{c(b+ac)+(2b+ac)dx^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3ad\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= \frac{x(b+ac+adx^2)}{3ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\left((2b+ac)\sqrt{b+ac+adx^2}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3a\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\left(c(b+ac)\sqrt{b+ac+adx^2}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3ad\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= \frac{x(b+ac+adx^2)}{3ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(2b+ac)x(b+ac+adx^2)}{3a^2d\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{c^{3/2}(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3ad^{3/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{\left(c(2b+ac)\sqrt{b+ac+adx^2}\right)\int \frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{3a^2d\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= \frac{x(b+ac+adx^2)}{3ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(2b+ac)x(b+ac+adx^2)}{3a^2d\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\sqrt{c}(2b+ac)\,(b+ac+adx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2d^{3/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{c^{3/2}(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3ad^{3/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.62 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\,(b+a(c+dx^2)) + i(2b^2+3abc+a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\right.\right.\right.}{3a^2d\sqrt{\frac{d}{c}}\,(b+a\,(c+dx^2)}$$

```
[In] Integrate[x^2/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(b + a*(c
 + d*x^2)) + I*(2*b^2 + 3*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*
c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]
```

```
- (2*I)*b*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]
*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(3*a^2*d*Sqrt[d/c]*(b
 + a*(c + d*x^2)))
```

Maple [A] (verified)

Time = 6.00 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.16

method	result
default	$\frac{\left(\sqrt{-\frac{ad}{ac+b}}\,a\,d^2x^5+2\sqrt{-\frac{ad}{ac+b}}\,acd\,x^3+\sqrt{-\frac{ad}{ac+b}}\,bd\,x^3-\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a\,c^2+\sqrt{-\frac{ad}{ac+b}}\,a\,c^2x+\sqrt{\frac{ad\,x^2}{ac}}\right.}{3d\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}$
risch	$\frac{x(ad\,x^2+ac+b)}{3ad\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}-\frac{\left(\frac{a\,c^2\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}+\frac{bc\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}-\right.}{3da\sqrt{}}$

```
[In] int(x^2/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-a*d/(a*c+b))^(1/2)*a*d^2*x^5+2*(-a*d/(a*c+b))^(1/2)*a*c*d*x^3+(-a*d/
(a*c+b))^(1/2)*b*d*x^3-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2+(-a*d/(a*c+b))^
(1/2)*a*c^2*x+((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF
(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c-2*((a*d*x^2+a*c+b)/(a*c+b)
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^
(1/2))*b*c+(-a*d/(a*c+b))^(1/2)*b*c*x)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1
/2)/a/((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.47

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{(ac^2+2\,bc)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-(ac^2+2\,bc+(ac+b)d)\sqrt{a}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac}}{3\,a^2d^2x}$$

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a*c^2 + 2*b*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x),
(a*c + b)/(a*c)) - (a*c^2 + 2*b*c + (a*c + b)*d)*sqrt(a)*x*sqrt(-c/d)*ellip
```

```
tic_f(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) + (a*d^2*x^4 - 2*b*d*x^2 - a*c
^2 - 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d^2*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a + \frac{b}{c+dx^2}}}\,dx = \int \frac{x^2}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

```
[In] integrate(x**2/(a+b/(d*x**2+c))**(1/2),x)
[Out] Integral(x**2/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a + \frac{b}{c+dx^2}}}\,dx = \int \frac{x^2}{\sqrt{a + \frac{b}{dx^2+c}}}\,dx$$

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(a + b/(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a + \frac{b}{c+dx^2}}}\,dx = \int \frac{x^2}{\sqrt{a + \frac{b}{dx^2+c}}}\,dx$$

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(a + b/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a + \frac{b}{c+dx^2}}}\,dx = \int \frac{x^2}{\sqrt{a + \frac{b}{d\,x^2+c}}}\,dx$$

```
[In] int(x^2/(a + b/(c + d*x^2))^(1/2),x)
[Out] int(x^2/(a + b/(c + d*x^2))^(1/2), x)
```

3.352 $\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 286

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{x(b+ac+adx^2)}{a(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{c}(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a\sqrt{d}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{c^{3/2}(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{(b+ac)\sqrt{d}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

```
[Out] x*(a*d*x^2+a*c+b)/a/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+c^(3/2)*(a*
d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/(a*c+b)/(d*x^2+c)/d^(1/2)/((a*d*
x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-(a*
d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a/(d*x^2+c)/d^(1/2)/((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used

$= \{1985, 1986, 433, 429, 506, 422\}$

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{c^{3/2}(ac+adx^2+b)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\,(c+dx^2)\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} - \frac{\sqrt{c}(ac+adx^2+b)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a\sqrt{d}\,(c+dx^2)\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{x(ac+adx^2+b)}{a\,(c+dx^2)\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[In] Int[1/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (x*(b + a*c + a*d*x^2))/(a*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
]) - (Sqrt[c]*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/
(b + a*c)])/(a*Sqrt[d]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sq
rt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (c^(3/2)*(b + a*c +
a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*Sq
rt[d]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c +
a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\,dx \\
&= \frac{\sqrt{b+ac+adx^2}\int \frac{\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\
&= \frac{\left(c\sqrt{b+ac+adx^2}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{\left(d\sqrt{b+ac+adx^2}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\
&= \frac{x(b+ac+adx^2)}{a\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{c^{3/2}(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(b+ac)\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\
&\quad -\frac{\left(c\sqrt{b+ac+adx^2}\right)\int \frac{\sqrt{b+ac+adx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{a\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\
&= \frac{x(b+ac+adx^2)}{a\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{\sqrt{c}(b+ac+adx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\
&\quad +\frac{c^{3/2}(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(b+ac)\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 7.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{\sqrt{\frac{b+ac+adx^2}{b+ac}}E\left(\arcsin\left(\sqrt{-\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)}{\sqrt{-\frac{ad}{b+ac}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{1+\frac{dx^2}{c}}}$$

```
[In] Integrate[1/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*EllipticE[ArcSin[Sqrt[-((a*d)/(b + a*c
))]*x], 1 + b/(a*c)])/(Sqrt[-((a*d)/(b + a*c))]*Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)]*Sqrt[1 + (d*x^2)/c])
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{E\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,c(d\,x^2+c)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{-\frac{ad}{ac+b}}\,\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}}$	164

```
[In] int(1/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)/c)^(1/2)*(
(a*d*x^2+a*c+b)/(a*c+b))^(1/2)*c*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/((a*
d*x^2+a*c+b)*(d*x^2+c))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=$$

$$-\frac{\sqrt{a}cx\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-\sqrt{a}(c+d)x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-(dx^2+c)\sqrt{\frac{adx^2+ac-}{dx^2+c}}}{adx}$$

```
[In] integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(a)*c*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c))
- sqrt(a)*(c + d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), (a*c + b)/(
a*c)) - (d*x^2 + c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

```
[In] integrate(1/(a+b/(d*x**2+c))**(1/2),x)
[Out] Integral(1/sqrt(a + b/(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

```
[In] integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a + b/(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

```
[In] integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a + b/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

```
[In] int(1/(a + b/(c + d*x^2))^(1/2),x)
[Out] int(1/(a + b/(c + d*x^2))^(1/2), x)
```

3.353 $\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 343

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx &= -\frac{b+ac+adx^2}{(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{dx(b+ac+adx^2)}{(b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&\quad-\frac{\sqrt{c}\sqrt{d}(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\\
&\quad+\frac{\sqrt{c}\sqrt{d}(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{(b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

```
[Out] (-a*d*x^2-a*c-b)/(a*c+b)/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+d*x*(a*d*x^2+a
*c+b)/(a*c+b)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-(a*d*x^2+a*c+b)*(
1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2
/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)/(a*c+b)/(d*x^2+c)/((a*d*x^2+a*
c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+(a*d*x^2+
a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)/(a*c+b)/(d*x^2+c)/((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1985, 1986, 486, 21, 433, 429, 506, 422}

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{c}\sqrt{d}(ac+adx^2+b)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} - \frac{\sqrt{c}\sqrt{d}(ac+adx^2+b)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} - \frac{ac+adx^2+b}{x(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} + \frac{dx(ac+adx^2+b)}{(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[In] Int[1/(x^2*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -((b + a*c + a*d*x^2)/((b + a*c)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]))
+ (d*x*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)
/(c + d*x^2)]) - (Sqrt[c]*Sqrt[d]*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*
x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) +
(Sqrt[c]*Sqrt[d]*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 b/(b + a*c)])/((b + a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]
*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\,dx$$

$$= \frac{\sqrt{b+ac+adx^2}\int \frac{\sqrt{c+dx^2}}{x^2\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b+ac+adx^2}{(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\sqrt{b+ac+adx^2}\int \frac{(b+ac)d+ad^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{(b+ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b+ac+adx^2}{(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\left(d\sqrt{b+ac+adx^2}\right)\int \frac{\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{(b+ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b+ac+adx^2}{(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\left(d\sqrt{b+ac+adx^2}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\left(ad^2\sqrt{b+ac+adx^2}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{(b+ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b+ac+adx^2}{(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{dx(b+ac+adx^2)}{(b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\sqrt{c}\sqrt{d}(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{\left(cd\sqrt{b+ac+adx^2}\right)\int \frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{(b+ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b+ac+adx^2}{(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{dx(b+ac+adx^2)}{(b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{d}(b+ac+adx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{d}(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [A] (verified)

Time = 8.74 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{(b+ac)x} + \frac{d\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{ac}{b+ac}\right)}{(b+ac)\sqrt{-\frac{d}{c}}\sqrt{\frac{b+ac+adx^2}{b+ac}}}$$

```
[In] Integrate[1/(x^2*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -(((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/((b + a*c)*x)) + (d*S
qrt[(c + d*x^2)/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcSin[S
qrt[-(d/c)]*x], (a*c)/(b + a*c)])/((b + a*c)*Sqrt[-(d/c)]*Sqrt[(b + a*c + a
*d*x^2)/(b + a*c)])
```

Maple [A] (verified)

Time = 6.06 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.01

method	result
default	$-\frac{\left(\sqrt{-\frac{ad}{ac+b}}\,a\,d^{2}x^{4}-adc\sqrt{\frac{ad\,x^{2}+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,xE\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)+2\sqrt{-\frac{ad}{ac+b}}\,acd\,x^{2}-\sqrt{\frac{ad\,x^{2}+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,F\left(x\sqrt{-\frac{ad}{ac+b}}\right.\right.}{\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}\,\sqrt{-\frac{ad}{ac+b}}\,x(ac+b)\sqrt{(ad\,x^{2}+}}$
risch	$-\frac{ad\,x^{2}+ac+b}{(ac+b)x\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}+\frac{d\left(\frac{b\sqrt{1+\frac{ad\,x^{2}}{ac+b}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}}+\frac{ac\sqrt{1+\frac{ad\,x^{2}}{ac+b}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right.}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}}\right.}{(ac-}$

```
[In] int(1/x^2/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((-a*d/(a*c+b))^(1/2)*a*d^2*x^4-a*d*c*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*
x^2+c)/c)^(1/2)*x*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))+2*(
-a*d/(a*c+b))^(1/2)*a*c*d*x^2-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*d*x+(-a*d/(a
*c+b))^(1/2)*b*d*x^2+(-a*d/(a*c+b))^(1/2)*a*c^2+(-a*d/(a*c+b))^(1/2)*b*c)*(
d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/x/(a*c+b)/((a*d*x^2+a*c+b)*(d*x^2+c))^
(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$
$$=\frac{a^2c\sqrt{-\frac{ad}{ac+b}}d^2x\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right)\mid\frac{ac+b}{ac})-(a^2cd^2+(a^2c^2+2\,abc+b^2)d)\sqrt{-\frac{ad}{ac+b}}x\sqrt{\frac{ac^2+bc}{d^2}}F(a}{(a^3c^3+2\,a^2bc^2+ab^2c)x}$$

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (a^2*c*sqrt(-a*d/(a*c + b))*d^2*x*sqrt((a*c^2 + b*c)/d^2)*elliptic_e(arcsin
(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - (a^2*c*d^2 + (a^2*c^2 + 2*a*b*
```

```
c + b^2)*d)*sqrt(-a*d/(a*c + b))*x*sqrt((a*c^2 + b*c)/d^2)*elliptic_f(arcsi
n(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - (a^2*c^3 + a*b*c^2 + (a^2*c^2
 + a*b*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^3 + 2*a^2*b
*c^2 + a*b^2*c)*x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

```
[In] integrate(1/x**2/(a+b/(d*x**2+c))**(1/2),x)
[Out] Integral(1/(x**2*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}\,x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}\,x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{x^2\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

```
[In] int(1/(x^2*(a + b/(c + d*x^2))^(1/2)),x)
[Out] int(1/(x^2*(a + b/(c + d*x^2))^(1/2)), x)
```

3.354 $\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 435

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx &= \frac{-b-ac-adx^2}{3(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(b-ac)d(b+ac+adx^2)}{3c(b+ac)^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\ &\quad + \frac{(b-ac)d^2x(b+ac+adx^2)}{3c(b+ac)^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\ &\quad - \frac{(b-ac)d^{3/2}(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3\sqrt{c}(b+ac)^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ &\quad - \frac{a\sqrt{c}d^{3/2}(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3(b+ac)^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

```
[Out] 1/3*(-a*d*x^2-a*c-b)/(a*c+b)/x^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(-a*
c+b)*d*(a*d*x^2+a*c+b)/c/(a*c+b)^2/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/3*
(-a*c+b)*d^2*x*(a*d*x^2+a*c+b)/c/(a*c+b)^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^
2+c))^(1/2)-1/3*(-a*c+b)*d^(3/2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d
*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/
2))/(a*c+b)^2/(d*x^2+c)/c^(1/2)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x
^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/3*a*d^(3/2)*(a*d*x^2+a*c+b)*(1/(1+d*x^
2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2)
,(b/(a*c+b))^(1/2))*c^(1/2)/(a*c+b)^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))
^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1985, 1986, 486, 597, 545, 429, 506, 422}

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{a\sqrt{c}d^{3/2}(ac+adx^2+b)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3(ac+b)^2(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} - \frac{d^{3/2}(b-ac)(ac+adx^2+b)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3\sqrt{c}(ac+b)^2(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{d^2x(b-ac)(ac+adx^2+b)}{3c(ac+b)^2(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} - \frac{d(b-ac)(ac+adx^2+b)}{3cx(ac+b)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} - \frac{ac+adx^2+b}{3x^3(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[In] Int[1/(x^4*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -1/3*(b + a*c + a*d*x^2)/((b + a*c)*x^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2
)]) - ((b - a*c)*d*(b + a*c + a*d*x^2))/(3*c*(b + a*c)^2*x*Sqrt[(b + a*c +
a*d*x^2)/(c + d*x^2)]) + ((b - a*c)*d^2*x*(b + a*c + a*d*x^2))/(3*c*(b + a*
c)^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - ((b - a*c)*d^(3/2
)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/
(3*Sqrt[c]*(b + a*c)^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sq
rt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) - (a*Sqrt[c]*d^(3/2)*(
b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*
(b + a*c)^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b +
a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^4\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\,dx$$

$$= \frac{\sqrt{b+ac+adx^2}\int \frac{\sqrt{c+dx^2}}{x^4\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b+ac+adx^2}{3(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\sqrt{b+ac+adx^2}\int \frac{(b-ac)d-ad^2x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3(b+ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b+ac+adx^2}{3(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(b-ac)d(b+ac+adx^2)}{3c(b+ac)^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{b+ac+adx^2}\int \frac{ac(b+ac)d^2-a(b-ac)d^3x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3c(b+ac)^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b+ac+adx^2}{3(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(b-ac)d(b+ac+adx^2)}{3c(b+ac)^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\left(ad^2\sqrt{b+ac+adx^2}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3(b+ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\left(a(b-ac)d^3\sqrt{b+ac+adx^2}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3c(b+ac)^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b+ac+adx^2}{3(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(b-ac)d(b+ac+adx^2)}{3c(b+ac)^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(b-ac)d^2x(b+ac+adx^2)}{3c(b+ac)^2\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{a\sqrt{c}d^{3/2}(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\mid\frac{b}{b+ac}\right)}{3(b+ac)^2\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{\left((b-ac)d^2\sqrt{b+ac+adx^2}\right)\int \frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{3(b+ac)^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b+ac+adx^2}{3(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(b-ac)d(b+ac+adx^2)}{3c(b+ac)^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(b-ac)d^2x(b+ac+adx^2)}{3c(b+ac)^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(b-ac)d^{3/2}(b+ac+adx^2)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3\sqrt{c}(b+ac)^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{a\sqrt{c}d^{3/2}(b+ac+adx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(b+ac)^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.85 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{\sqrt{\frac{d}{c}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}(c+dx^2)\left(b^2(c+dx^2)+a^2c(c^2-d^2x^4)+ab(2c^2+cdx^2+d^2x^4)\right)+i(b^2-a^2c^2)d^2\cdots\right.}{3(b\cdots}$$

```
[In] Integrate[1/(x^4*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -1/3*(Sqrt[d/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*(c + d*x^2
)*(b^2*(c + d*x^2) + a^2*c*(c^2 - d^2*x^4) + a*b*(2*c^2 + c*d*x^2 + d^2*x^4
)) + I*(b^2 - a^2*c^2)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 +
 (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - I*b*(b + a
*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/((b + a*c)^2*d*x^3*(b + a*(c
+ d*x^2)))
```

Maple [A] (verified)

Time = 7.00 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.31

method	result
risch	$-\frac{(ad\,x^2+ac+b)(-acd\,x^2+bd\,x^2+a\,c^2+bc)}{3(ac+b)^2x^3c\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}-\frac{d^2a\left(\frac{a\,c^2\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}+\frac{bc\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F(\cdots}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4+2\cdots}}\right.}{}$
default	$-\frac{\left(-\sqrt{-\frac{ad}{ac+b}}\,a^2c\,d^3x^6+\sqrt{-\frac{ad}{ac+b}}\,ab\,d^3x^6+\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a^2c^2d^2x^3-\sqrt{-\frac{ad}{ac+b}}\,a^2c^2d^2x^4+2\sqrt{\frac{ad\,x^2}{a}}\cdots\right.}{}$

[In] int(1/x^4/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/3*(a*d*x^2+a*c+b)*(-a*c*d*x^2+b*d*x^2+a*c^2+b*c)/(a*c+b)^2/x^3/c/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*d^2*a/(a*c+b)^2/c*(a*c^2/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+b*c/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-2*(a*c*d-b*d)*(a*c^2+b*c)/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))))/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{(a^2c-ab)\sqrt{-\frac{ad}{ac+b}}d^3x^3\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+b}{ac}) - ((a^2c-ab)d^3+(a^2c^2+2\,abc+b^2)d^2)\sqrt{\cdots}}{3\,(a^3c^4+\cdots}$$

[In] integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] -1/3*((a^2*c - a*b)*sqrt(-a*d/(a*c + b))*d^3*x^3*sqrt((a*c^2 + b*c)/d^2)*elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^2*c - a*b)*d^3 + (a^2*c^2 + 2*a*b*c + b^2)*d^2)*sqrt(-a*d/(a*c + b))*x^3*sqrt((a*c^2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^2*c^2 - b^2)*d^2*x^4 - a^2*c^4 - 2*a*b*c^3 - b^2*c^2 - 2*(a*b*c^2 + b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^4 + 3*a^2*b*c^3 + 3*a*b^2*c^2 + b^3*c)*x^3)

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x^4\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

```
[In] integrate(1/x**4/(a+b/(d*x**2+c))**(1/2),x)
[Out] Integral(1/(x**4*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}\,x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}\,x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x^4\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

```
[In] int(1/(x^4*(a + b/(c + d*x^2))^(1/2)),x)
[Out] int(1/(x^4*(a + b/(c + d*x^2))^(1/2)), x)
```

3.355 $\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 310

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{(b+ac)^2\left(c+dx^2\right)^3}{ab^2d^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(35b^2+60abc+24a^2c^2)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^4d^3} - \frac{(35b^2+60abc+24a^2c^2)\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24a^3bd^3} + \frac{(7b^2+12abc+6a^2c^2)\left(c+dx^2\right)^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{6a^2b^2d^3} - \frac{b(35b^2+60abc+24a^2c^2)\,\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{9/2}d^3}$$

```
[Out] -1/16*b*(24*a^2*c^2+60*a*b*c+35*b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1
/2)/a^(1/2))/a^(9/2)/d^3-(a*c+b)^2*(d*x^2+c)^3/a/b^2/d^3/((a*d*x^2+a*c+b)/(
d*x^2+c))^(1/2)+1/16*(24*a^2*c^2+60*a*b*c+35*b^2)*(d*x^2+c)*((a*d*x^2+a*c+b
)/(d*x^2+c))^(1/2)/a^4/d^3-1/24*(24*a^2*c^2+60*a*b*c+35*b^2)*(d*x^2+c)^2*((
a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^3/b/d^3+1/6*(6*a^2*c^2+12*a*b*c+7*b^2)*(d
*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/b^2/d^3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1985, 1981, 1980, 473, 393, 205, 214}

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{(6a^2c^2+12abc+7b^2)\,(c+dx^2)^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a^2b^2d^3} - \frac{b(24a^2c^2+60abc+35b^2)\,\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{9/2}d^3} + \frac{(24a^2c^2+60abc+35b^2)\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16a^4d^3} - \frac{(24a^2c^2+60abc+35b^2)\,(c+dx^2)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{24a^3bd^3} - \frac{(ac+b)^2\,(c+dx^2)^3}{ab^2d^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[In] Int[x^5/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -(((b + a*c)^2*(c + d*x^2)^3)/(a*b^2*d^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^
2)])) + ((35*b^2 + 60*a*b*c + 24*a^2*c^2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x
^2)/(c + d*x^2)])/(16*a^4*d^3) - ((35*b^2 + 60*a*b*c + 24*a^2*c^2)*(c + d*x
^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(24*a^3*b*d^3) + ((7*b^2 + 12*
a*b*c + 6*a^2*c^2)*(c + d*x^2)^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(6*
a^2*b^2*d^3) - (b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*ArcTanh[Sqrt[(b + a*c +
a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(16*a^(9/2)*d^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^5}{\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\left(\frac{b+ac+adx}{c+dx}\right)^{3/2}}\,dx, x, x^2\right)\\ &= -\left((bd)\text{Subst}\left(\int \frac{\left(-b-ac+cx^2\right)^2}{x^2\left(ad-dx^2\right)^4}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right)\end{aligned}$$

$$= -\frac{(b+ac)^2 (c+dx^2)^3}{ab^2d^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{b\text{Subst}\left(\int \frac{(b+ac)(7b+5ac)d+ac^2dx^2}{(ad-dx^2)^4}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{a}$$

$$= -\frac{(b+ac)^2 (c+dx^2)^3}{ab^2d^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(7b^2+12abc+6a^2c^2)(c+dx^2)^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{6a^2b^2d^3} - \frac{(b(35b^2+60abc+24a^2c^2))\,\text{Subst}\left(\int \frac{1}{(ad-dx^2)^3}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{6a^2}$$

$$= -\frac{(b+ac)^2 (c+dx^2)^3}{ab^2d^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(35b^2+60abc+24a^2c^2)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24a^3bd^3} + \frac{(7b^2+12abc+6a^2c^2)(c+dx^2)^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{6a^2b^2d^3} - \frac{(b(35b^2+60abc+24a^2c^2))\,\text{Subst}\left(\int \frac{1}{(ad-dx^2)^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{8a^3d}$$

$$= -\frac{(b+ac)^2 (c+dx^2)^3}{ab^2d^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(35b^2+60abc+24a^2c^2)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^4d^3} - \frac{(35b^2+60abc+24a^2c^2)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24a^3bd^3} + \frac{(7b^2+12abc+6a^2c^2)(c+dx^2)^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{6a^2b^2d^3} - \frac{(b(35b^2+60abc+24a^2c^2))\,\text{Subst}\left(\int \frac{1}{ad-dx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{16a^4d^2}$$

$$= -\frac{(b+ac)^2 (c+dx^2)^3}{ab^2d^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(35b^2+60abc+24a^2c^2)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^4d^3} - \frac{(35b^2+60abc+24a^2c^2)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24a^3bd^3} + \frac{(7b^2+12abc+6a^2c^2)(c+dx^2)^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{6a^2b^2d^3} - \frac{b(35b^2+60abc+24a^2c^2)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{9/2}d^3}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.59

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\frac{\sqrt{a}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(105b^3+5ab^2(43c+7dx^2)+2a^2b(59c^2+16cdx^2-7d^2x^4)+8a^3(c^3+d^3x^6)\right)}{b+a(c+dx^2)} - 3b(35b^2+6}{48a^{9/2}d^3}$$

```
[In] Integrate[x^5/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] ((Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(105*b^3 + 5*a*
b^2*(43*c + 7*d*x^2) + 2*a^2*b*(59*c^2 + 16*c*d*x^2 - 7*d^2*x^4) + 8*a^3*(c
^3 + d^3*x^6)))/(b + a*(c + d*x^2)) - 3*b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*
ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(48*a^(9/2)*d^3)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.98

method	result
risch	$\frac{(8a^2d^2x^4-8a^2cdx^2-22abdx^2+8a^2c^2+62abc+57b^2)(adx^2+ac+b)}{48d^3a^4\sqrt{\frac{adx^2+ac+b}{dx^2+c}}} - \frac{b\left(\frac{(24a^2c^2+60abc+35b^2)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+bc+(2acd\right.}{2\sqrt{a\,d^2}}\right.}{16a^4d^2}$
default	Expression too large to display

```
[In] int(x^5/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/d^3*(8*a^2*d^2*x^4-8*a^2*c*d*x^2-22*a*b*d*x^2+8*a^2*c^2+62*a*b*c+57*b^
2)*(a*d*x^2+a*c+b)/a^4/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/16*b/a^4/d^2*(1/
2*(24*a^2*c^2+60*a*b*c+35*b^2)*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(
a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)-16*(a^2*c^2+2*a
*b*c+b^2)*(d*x^2+c)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 675, normalized size of antiderivative = 2.18

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\,(24\,a^3bc^3+84\,a^2b^2c^2+95\,ab^3c+35\,b^4+(24\,a^3bc^2+60\,a^2b^2c+35\,ab^3)dx^2)\sqrt{a}\log\right.$$

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(24*a^3*b*c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b*
c^2 + 60*a^2*b^2*c + 35*a*b^3)*d*x^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2
 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d
*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(8
*a^4*d^4*x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c^3 +
(18*a^3*b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (8*a^4*
c^3 + 150*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^2 + a*c
 + b)/(d*x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3), 1/96*(3*(24*a^3*b*
c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b*c^2 + 60*a^2*b^2*c +
 35*a*b^3)*d*x^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(8*a^4*d^4
*x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c^3 + (18*a^3*
b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (8*a^4*c^3 + 15
0*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3)]
```

Sympy [F]

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^5}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**5/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**5/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.25

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{48\,a^5bc^2+96\,a^4b^2c+48\,a^3b^3-\frac{3\,(24\,a^2bc^2+60\,ab^2c+35\,b^3)\,(adx^2+ac+b)^3}{(dx^2+c)^3}+\frac{8\,(24\,a^3bc^2+60\,a^2b^2c+35\,c}{(dx^2+c)}}{48\left(a^7d^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-3\,a^6d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\,a^5d^3\left(\frac{adx^2+ac+}{dx^2+c}\right.\right.} + \frac{(24\,a^2c^2+60\,abc+35\,b^2)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{9}{2}}d^3}$$

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/48*(48*a^5*b*c^2 + 96*a^4*b^2*c + 48*a^3*b^3 - 3*(24*a^2*b*c^2 + 60*a*b^2
*c + 35*b^3)*(a*d*x^2 + a*c + b)^3/(d*x^2 + c)^3 + 8*(24*a^3*b*c^2 + 60*a^2
*b^2*c + 35*a*b^3)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(56*a^4*b*c^2 +
132*a^3*b^2*c + 77*a^2*b^3)*(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^7*d^3*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c)) - 3*a^6*d^3*((a*d*x^2 + a*c + b)/(d*x^2 +
c))^(3/2) + 3*a^5*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - a^4*d^3*((a
*d*x^2 + a*c + b)/(d*x^2 + c))^(7/2)) + 1/32*(24*a^2*c^2 + 60*a*b*c + 35*b^
2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(9/2)*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. 2(288) = 576.

Time = 0.68 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.14

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{1}{48}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\,x^2\left(\frac{4\,x^2}{a^2d\,\mathrm{sgn}\left(dx^2+c\right)}-\frac{4\,a^{11}cd^6\mathrm{sgn}(dx^2+\right.\right. + \frac{(24\,a^2bc^2+60\,ab^2c+35\,b^3)\log\left(\left|2\,a^3c^3d+6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^{\frac{5}{2}}c^2|d|+6\right.\right.}{} + \frac{\left(24\,a^{\frac{13}{2}}bc^2d^3|d|\mathrm{sgn}(dx^2+c)+60\,a^{\frac{11}{2}}b^2cd^3|d|\mathrm{sgn}(dx^2+c)+35\,a^{\frac{9}{2}}b^3d^3|d|\mathrm{sgn}(dx^2+c)\right)\log\left(|a|\right)}{96\,a^9d^7}$$

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2/(a
^2*d*sgn(d*x^2 + c)) - (4*a^11*c*d^6*sgn(d*x^2 + c) + 11*a^10*b*d^6*sgn(d*x
```

```
^2 + c))/(a^13*d^8)) + (8*a^11*c^2*d^5*sgn(d*x^2 + c) + 62*a^10*b*c*d^5*sgn
(d*x^2 + c) + 57*a^9*b^2*d^5*sgn(d*x^2 + c))/(a^13*d^8)) + 1/96*(24*a^2*b*c
^2 + 60*a*b^2*c + 35*b^3)*log(abs(2*a^3*c^3*d + 6*(sqrt(a*d^2)*x^2 - sqrt(a
*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^(5/2)*c^2*abs(d) + 6*(sq
rt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^
2*c*d + 5*a^2*b*c^2*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 +
 b*d*x^2 + a*c^2 + b*c))^3*a^(3/2)*abs(d) + 10*(sqrt(a*d^2)*x^2 - sqrt(a*d^
2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^(3/2)*b*c*abs(d) + 5*(sqrt(
a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a*b*d
 + 4*a*b^2*c*d + 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^
2 + a*c^2 + b*c))*sqrt(a)*b^2*abs(d) + b^3*d))/(a^(9/2)*d^2*abs(d)*sgn(d*x^
2 + c)) + 1/96*(24*a^(13/2)*b*c^2*d^3*abs(d)*sgn(d*x^2 + c) + 60*a^(11/2)*b
^2*c*d^3*abs(d)*sgn(d*x^2 + c) + 35*a^(9/2)*b^3*d^3*abs(d)*sgn(d*x^2 + c))*
log(abs(a))/(a^9*d^7)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{x^5}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(x^5/(a + b/(c + d*x^2))^(3/2),x)
[Out] int(x^5/(a + b/(c + d*x^2))^(3/2), x)
```

3.356 $\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 187

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{b(b+ac)}{a^3d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(7b+4ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8a^3d^2} + \frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4a^2d^2} + \frac{3b(5b+4ac)\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{7/2}d^2}$$

```
[Out] 3/8*b*(4*a*c+5*b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(7/2
)/d^2-b*(a*c+b)/a^3/d^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/8*(4*a*c+7*b)*(
d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^3/d^2+1/4*(d*x^2+c)^2*((a*d*x^
2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1985, 1981, 1980, 467, 464, 214}

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{3b(4ac+5b)\operatorname{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{7/2}d^2} - \frac{(4ac+7b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8a^3d^2} - \frac{b(ac+b)}{a^3d^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} + \frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2d^2}$$

```
[In] Int[x^3/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((b*(b + a*c))/(a^3*d^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) - ((7*b +
4*a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*a^3*d^2) + ((c
 + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*a^2*d^2) + (3*b*(5*b
+ 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*a^(7/2)
*d^2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3}{\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\left(\frac{b+ac+adx}{c+dx}\right)^{3/2}}\,dx, x, x^2\right) \\
&= -\left((bd)\text{Subst}\left(\int \frac{-b-ac+cx^2}{x^2\left(ad-dx^2\right)^3}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right) \\
&= \frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4a^2d^2} + \frac{1}{4}(bd)\text{Subst}\left(\int \frac{\frac{4(b+ac)}{ad}+\frac{3bx^2}{a^2d}}{x^2\left(ad-dx^2\right)^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right) \\
&= -\frac{(7b+4ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8a^3d^2} + \frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4a^2d^2} \\
&\quad - \frac{1}{8}(bd)\text{Subst}\left(\int \frac{-\frac{8(b+ac)}{a^2d^2}-\frac{(7b+4ac)x^2}{a^3d^2}}{x^2\left(ad-dx^2\right)}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right) \\
&= -\frac{b(b+ac)}{a^3d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(7b+4ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8a^3d^2} \\
&\quad + \frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4a^2d^2} + \frac{(3b(5b+4ac))\text{Subst}\left(\int \frac{1}{ad-dx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{8a^3d} \\
&= -\frac{b(b+ac)}{a^3d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(7b+4ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8a^3d^2} \\
&\quad + \frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4a^2d^2} + \frac{3b(5b+4ac)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{7/2}d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{-\frac{\sqrt{a}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(15b^2+ab(17c+5dx^2)+2a^2(c^2-d^2x^4)\right)}{b+a(c+dx^2)}+3b(5b+4ac)\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right.}{8a^{7/2}d^2}$$

[In] Integrate[x^3/(a + b/(c + d*x^2))^(3/2),x]

[Out] (-((Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(15*b^2 + a*b
*(17*c + 5*d*x^2) + 2*a^2*(c^2 - d^2*x^4)))/(b + a*(c + d*x^2))) + 3*b*(5*b
 + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*a^(7/2
)*d^2)

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{(-2adx^2+2ac+7b)(adx^2+ac+b)}{8d^2a^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}+\frac{b\left(\frac{(12ac+15b)\ln\left(\frac{acd+\frac{1}{2}bd+ad^2x^2}{\sqrt{ad^2}}+\sqrt{ac^2+bc+(2acd+bd)x^2+ad^2x^4}\right)}{2\sqrt{ad^2}}-\frac{8(ac+b)(dx^2+}{d\sqrt{ad^2x^4+2acdx^2+bd}}\right.}{8a^3d\sqrt{\frac{adx^2+ac+b}{dx^2+c}}(dx^2+c)}$
default	$-\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}(dx^2+c)\left(-4\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\sqrt{ad^2}a^2d^2x^4-12\ln\left(\frac{2ad^2x^2+2acd+2\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+}}{2\sqrt{ad^2}}\right.\right.}{}$

[In] int(x^3/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/8/d^2*(-2*a*d*x^2+2*a*c+7*b)*(a*d*x^2+a*c+b)/a^3/((a*d*x^2+a*c+b)/(d*x^2
+c))^(1/2)+1/8*b/a^3/d*(1/2*(12*a*c+15*b)*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d
^2)^(1/2)+(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)-8*(a
c+b)(d*x^2+c)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/((a*d*x^
2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.89

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\,(4\,a^2bc^2+9\,ab^2c+(4\,a^2bc+5\,ab^2)dx^2+5\,b^3)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c\right.}{}\right.$$

$$-\frac{3\,(4\,a^2bc^2+9\,ab^2c+(4\,a^2bc+5\,ab^2)dx^2+5\,b^3)\sqrt{-a}\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(2\,a^3d^3x^6+}{16\,(a^5d^3x^2+(a^5c+}$$

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b^2)*d*x^2 + 5*b^3)*sq
rt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b
^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^3*d^3*x^6 + (2*a^3*c - 5*a^2*b)*d^2*x
^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^2*c - (2*a^3*c^2 + 22*a^2*b*c + 15*a
*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^5*d^3*x^2 + (a^5*c +
 a^4*b)*d^2), -1/16*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b^2)*d*x
^2 + 5*b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - 2*(2*a^3*d^3*x^6 +
 (2*a^3*c - 5*a^2*b)*d^2*x^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^2*c - (2*a
^3*c^2 + 22*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^5*d^3*x^2 + (a^5*c + a^4*b)*d^2)]
```

Sympy [F]

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^3}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**3/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.40

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{8\,a^3bc+8\,a^2b^2+\frac{3\,(adx^2+ac+b)^2(4\,abc+5\,b^2)}{(dx^2+c)^2}-\frac{5\,(4\,a^2bc+5\,ab^2)(adx^2+ac+b)}{dx^2+c}}{8\left(a^5d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-2\,a^4d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+a^3d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}\right)} - \frac{3\,(4\,ac+5\,b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{7}{2}}d^2}$$

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^3*b*c + 8*a^2*b^2 + 3*(a*d*x^2 + a*c + b)^2*(4*a*b*c + 5*b^2)/(d*
x^2 + c)^2 - 5*(4*a^2*b*c + 5*a*b^2)*(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^5*
d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - 2*a^4*d^2*((a*d*x^2 + a*c + b)/
(d*x^2 + c))^(3/2) + a^3*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2)) - 3/1
6*(4*a*c + 5*b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(s
qrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(7/2)*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(169) = 338.

Time = 0.61 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.95

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{1}{8}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{a^2d\mathrm{sgn}\left(dx^2+c\right)}-\frac{2\,a^6cd^2+7\,a^5bd^2}{a^8d^4\mathrm{sgn}\left(dx^2+c\right)}\right) - \frac{(4\,abc+5\,b^2)\log\left(\left|2\,a^3c^3d+6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^{\frac{5}{2}}c^2|d|+6\left(\sqrt{ad^2}x^2-\right.\right.\right.}{} - \frac{\left(4\,a^{\frac{9}{2}}bcd^2|d|\mathrm{sgn}(dx^2+c)+5\,a^{\frac{7}{2}}b^2d^2|d|\mathrm{sgn}(dx^2+c)\right)\log\left(|a|\right)}{16\,a^7d^5}$$

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a^2*d*sgn
(d*x^2 + c)) - (2*a^6*c*d^2 + 7*a^5*b*d^2)/(a^8*d^4*sgn(d*x^2 + c))) - 1/16
*(4*a*b*c + 5*b^2)*log(abs(2*a^3*c^3*d + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^
```

```
4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^(5/2)*c^2*abs(d) + 6*(sqrt(a*d^
2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^2*c*d +
 5*a^2*b*c^2*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^
2 + a*c^2 + b*c))^3*a^(3/2)*abs(d) + 10*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^(3/2)*b*c*abs(d) + 5*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a*b*d + 4*a*
b^2*c*d + 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c
^2 + b*c))*sqrt(a)*b^2*abs(d) + b^3*d))/(a^(7/2)*d*abs(d)*sgn(d*x^2 + c)) -
 1/16*(4*a^(9/2)*b*c*d^2*abs(d)*sgn(d*x^2 + c) + 5*a^(7/2)*b^2*d^2*abs(d)*s
gn(d*x^2 + c))*log(abs(a))/(a^7*d^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{x^3}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(x^3/(a + b/(c + d*x^2))^(3/2),x)
[Out] int(x^3/(a + b/(c + d*x^2))^(3/2), x)
```

3.357 $\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 100

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\frac{3b}{2a^2d\sqrt{a+\frac{b}{c+dx^2}}}+\frac{c+dx^2}{2ad\sqrt{a+\frac{b}{c+dx^2}}}-\frac{3b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}$$

```
[Out] -3/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/a^(5/2)/d+3/2*b/a^2/d/(a+b/(d
*x^2+c))^(1/2)+1/2*(d*x^2+c)/a/d/(a+b/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1605, 248, 44, 53, 65, 214}

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=-\frac{3b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}+\frac{3b}{2a^2d\sqrt{a+\frac{b}{c+dx^2}}}+\frac{c+dx^2}{2ad\sqrt{a+\frac{b}{c+dx^2}}}$$

```
[In] Int[x/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (3*b)/(2*a^2*d*Sqrt[a + b/(c + d*x^2)]) + (c + d*x^2)/(2*a*d*Sqrt[a + b/(c
+ d*x^2)]) - (3*b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*a^(5/2)*d)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx, x, c+dx^2\right)}{2d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{c+dx^2}\right)}{2d}$$

$$= \frac{c+dx^2}{2ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{c+dx^2}\right)}{4ad}$$

$$= \frac{3b}{2a^2d\sqrt{a+\frac{b}{c+dx^2}}} + \frac{c+dx^2}{2ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{4a^2d}$$

$$= \frac{3b}{2a^2d\sqrt{a+\frac{b}{c+dx^2}}} + \frac{c+dx^2}{2ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{3\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{c+dx^2}}\right)}{2a^2d}$$

$$= \frac{3b}{2a^2d\sqrt{a+\frac{b}{c+dx^2}}} + \frac{c+dx^2}{2ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{(c+dx^2)\sqrt{\frac{b+a(c+dx^2)}{c+dx^2}}(3b+a(c+dx^2))}{2a^2d\,(b+a\,(c+dx^2))} - \frac{3b\text{arctanh}\left(\frac{\sqrt{\frac{b+a\left(c+dx^2\right)}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}$$

```
[In] Integrate[x/(a + b/(c + d*x^2))^(3/2),x]
[Out] ((c + d*x^2)*Sqrt[(b + a*(c + d*x^2))/(c + d*x^2)]*(3*b + a*(c + d*x^2)))/(
2*a^2*d*(b + a*(c + d*x^2))) - (3*b*ArcTanh[Sqrt[(b + a*(c + d*x^2))/(c + d
*x^2)]/Sqrt[a]])/(2*a^(5/2)*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(84) = 168.

Time = 1.14 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.17

method	result
risch	$\frac{a d x^2+a c+b}{2 d a^2 \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}-\frac{b\left(\frac{3 \ln\left(\frac{a c d+\frac{1}{2} b d+a d^2 x^2}{\sqrt{a d^2}}+\sqrt{a c^2+b c+(2 a c d+b d) x^2+a d^2 x^4}\right)}{2\sqrt{a d^2}}-\frac{2\left(d x^2+c\right)}{d\sqrt{a d^2 x^4+2 a c d x^2+b d x^2+a c^2+b c}}\right)\sqrt{(}}{2 a^2 \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}\,(d x^2+c)}$
derivativedivides	$-\frac{\sqrt{\frac{a\left(d x^2+c\right)+b}{d x^2+c}}\,(d x^2+c)\left(-6\sqrt{(a(d x^2+c)+b)(d x^2+c)}\,a^{\frac{5}{2}}\,(d x^2+c)^2+3\ln\left(\frac{2\sqrt{\left(a\left(d x^2+c\right)+b\right)\left(d x^2+c\right)}\,\sqrt{a}+2a\left(d x^2+c\right)}{2\sqrt{a}}\right.\right.}{}$
default	$\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}\,(d x^2+c)\left(-3\ln\left(\frac{2 a d^2 x^2+2 a c d+2\sqrt{a d^2 x^4+2 a c d x^2+b d x^2+a c^2+b c}\,\sqrt{a d^2}+b d}{2\sqrt{a d^2}}\right)a b d^2 x^2+2\sqrt{a d^2 x^4+2 a c d x^2+}\right.$

[In] int(x/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2/d/a^2*(a*d*x^2+a*c+b)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/2*b/a^2*(3/2*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)-2*(d*x^2+c)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(84) = 168$.

Time = 0.34 (sec) , antiderivative size = 395, normalized size of antiderivative = 3.95

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\left(abdx^2+abc+b^2\right)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2-4\left(\right.\right.}{}\right.$$

[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")

[Out] [1/8*(3*(a*b*d*x^2 + a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a^2*d^2*x^4 + a^2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d), 1/4*(3*(a*b*d*x^2 + a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(a^2*d^2*x^4 + a^2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d)]

Sympy [F]

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{x}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+b/(d*x**2+c))**(3/2),x)
[Out] Integral(x/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.61

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\frac{2\,ab-\frac{3\,(adx^2+ac+b)b}{dx^2+c}}{2\left(a^3d\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-a^2d\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}\right)}+\frac{3\,b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,a^{\frac{5}{2}}d}$$

```
[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] 1/2*(2*a*b - 3*(a*d*x^2 + a*c + b)*b/(d*x^2 + c))/(a^3*d*sqrt((a*d*x^2 + a*
c + b)/(d*x^2 + c)) - a^2*d*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2)) + 3/4*
b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a
*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/2)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 18.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.61

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\left(\frac{a\,(d\,x^2+c)}{b}+1\right)^{3/2}(d\,x^2+c)\ {}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,(d\,x^2+c)}{b}\right)}{5\,d\,\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}$$

```
[In] int(x/(a + b/(c + d*x^2))^(3/2),x)
[Out] (((a*(c + d*x^2))/b + 1)^(3/2)*(c + d*x^2)*hypergeom([3/2, 5/2], 7/2, -(a*(
c + d*x^2))/b))/(5*d*(a + b/(c + d*x^2))^(3/2))
```

3.358 $\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 134

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{b}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{(b+ac)^{3/2}}$$

```
[Out] arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)-c^(3/2)*arctanh(
c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/(a*c+b)^(3/2)-b/a/
(a*c+b)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1985, 1981, 1980, 491, 536, 214}

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{(ac+b)^{3/2}} - \frac{b}{a(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[In] Int[1/(x*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -(b/(a*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) + ArcTanh[Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/a^(3/2) - (c^(3/2)*ArcTanh[(Sqrt[c]*
Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(b + a*c)^(3/2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(\frac{b+ac+adx}{c+dx}\right)^{3/2}}\,dx, x, x^2\right) \\
&= -\left((bd)\text{Subst}\left(\int \frac{1}{x^2\left(-b-ac+cx^2\right)\left(ad-dx^2\right)}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right) \\
&= -\frac{b}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{b\text{Subst}\left(\int \frac{-((b+2ac)d)+cdx^2}{(-b-ac+cx^2)(ad-dx^2)}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{a(b+ac)} \\
&= -\frac{b}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{c^2\text{Subst}\left(\int \frac{1}{-b-ac+cx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{b+ac} \\
&\quad + \frac{d\text{Subst}\left(\int \frac{1}{ad-dx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{a} \\
&= -\frac{b}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{(b+ac)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04

$$
\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{b}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{c^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{(-b-ac)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}
$$

```
[In] Integrate[1/(x*(a + b/(c + d*x^2))^(3/2)),x]

[Out] -(b/(a*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) - (c^(3/2)*ArcTan[
(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(-b - a*c)
^(3/2) + ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/a^(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. $2(116) = 232$.

Time = 0.13 (sec) , antiderivative size = 1015, normalized size of antiderivative = 7.57

method	result	size
default	Expression too large to display	1015

```
[In] int(1/x/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a*(-ln(1/2*(2*a*d^2*x^2+2*
a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/
(a*d^2)^(1/2))*a^3*c^2*d^2*x^2-2*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2
*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b
*c*d^2*x^2+(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^
(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*(a*d^2)^(
1/2)*a^2*c*d*x^2-ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^3*c^3*d-ln(1/2*(2*a
*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^
(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*d^2*x^2-3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*
d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/
2))*a^2*b*c^2*d+(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c^2+
b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*(a*d
^2)^(1/2)*a^2*c^2-3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*c*d+(a*c^2+b
*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*(a*d^2)^(1/2)*a*b*c+2*((a*d*
x^2+a*c+b)*(d*x^2+c))^(1/2)*(a*d^2)^(1/2)*a*b*c-ln(1/2*(2*a*d^2*x^2+2*a*c*d
+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^
2)^(1/2))*b^3*d+2*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(a*d^2)^(1/2)*b^2)/((a*
d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*c+b)^2/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(116) = 232$.

Time = 0.47 (sec) , antiderivative size = 1477, normalized size of antiderivative = 11.02

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2
*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4
```

```
 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c))) + (a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8*a^2
*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a
^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4
+ 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x
^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) - 4*(a*b*
d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c
+ a^2*b^2 + (a^4*c + a^3*b)*d*x^2), -1/4*(2*(a^2*c^2 + (a^2*c + a*b)*d*x^2
+ 2*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a^3*c*d*x^2
+ a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2
*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*
c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 +
 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) + 4*(a*b*d*x^2 + a*b*c)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*
b)*d*x^2), 1/4*(2*(a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(-c/(a*c + b))*arct
an(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^
2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c)) + (a^2*c^2 + (a^2*c +
 a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a
^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*
a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) - 4*(a*b*d*x^2
+ a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*
b^2 + (a^4*c + a^3*b)*d*x^2), -1/2*((a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*
c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2
 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a^3*c*d*x^2 + a^3*c^
2 + a^2*b*c)*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2
*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 +
 a*c^2 + b*c)) + 2*(a*b*d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*b)*d*x^2)]
```

Sympy [F]

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(a+b/(d*x**2+c))**(3/2),x)
[Out] Integral(1/(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.50

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{c^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}(ac+b)} - \frac{b}{(a^2c+ab)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}} - \frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{2\,a^{\frac{3}{2}}}$$

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(
c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c +
b)*c)*(a*c + b)) - b/((a^2*c + a*b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
- 1/2*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqr
t((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a^(3/2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(1/(x*(a + b/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x*(a + b/(c + d*x^2))^(3/2)), x)
```

3.359 $\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{3bd}{2(b+ac)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{3b\sqrt{c}d\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2(b+ac)^{5/2}}$$

```
[Out] -3/2*b*d*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*c
^(1/2)/(a*c+b)^(5/2)+3/2*b*d/(a*c+b)^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/
2*(-d*x^2-c)/(a*c+b)/x^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1985, 1981, 1980, 296, 331, 214}

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{3b\sqrt{c}d\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2(ac+b)^{5/2}} + \frac{3bd}{2(ac+b)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} - \frac{c+dx^2}{2x^2(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[In] Int[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] (3*b*d)/(2*(b + a*c)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - (c + d*x^2)
/(2*(b + a*c)*x^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - (3*b*Sqrt[c]*d*A
rcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(2*(
b + a*c)^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(\frac{b+ac+adx}{c+dx}\right)^{3/2}}\,dx, x, x^2\right) \\
&= -\left((bd)\text{Subst}\left(\int \frac{1}{x^2\left(-b-ac+cx^2\right)^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right) \\
&= -\frac{c+dx^2}{2(b+ac)x^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(3bd)\text{Subst}\left(\int \frac{1}{x^2(-b-ac+cx^2)}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{2(b+ac)} \\
&= \frac{3bd}{2(b+ac)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\
&\quad + \frac{(3bcd)\text{Subst}\left(\int \frac{1}{-b-ac+cx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{2(b+ac)^2} \\
&= \frac{3bd}{2(b+ac)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{3b\sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2(b+ac)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

$$
\begin{aligned}
\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx &= -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(b(c-2dx^2)+ac(c+dx^2))}{2(b+ac)^2x^2\left(b+a\left(c+dx^2\right)\right)} \\
&\quad + \frac{3b\sqrt{c}d\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{2(-b-ac)^{5/2}}
\end{aligned}
$$

```
[In] Integrate[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
[Out] -1/2*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*(c - 2*d*x^2) +
a*c*(c + d*x^2)))/((b + a*c)^2*x^2*(b + a*(c + d*x^2))) + (3*b*Sqrt[c]*d*Ar
cTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(2*(-
b - a*c)^(5/2))
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{c(ad\,x^2+ac+b)}{2(ac+b)^2 x^2 \sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}+\frac{db\left(-\frac{3c\ln\left(\frac{2a\,c^2+2bc+(2acd+bd)x^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,c^2+bc+(2acd+bd)x^2+a\,d^2x^4}}{x^2}\right)}{2\sqrt{a\,c^2+bc}}+\frac{2d\,x^2+2c}{\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x}}\right.}{2(ac+b)^2\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)}$
default	Expression too large to display

```
[In] int(1/x^3/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a*c+b)^2*c*(a*d*x^2+a*c+b)/x^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/2*
d*b/(a*c+b)^2*(-3/2*c/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2
+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)+2*
(d*x^2+c)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/((a*d*x^2+a*c+b)
/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 599, normalized size of antiderivative = 4.10

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\left(abd^2x^4+(abc+b^2)dx^2\right)\sqrt{\frac{c}{ac+b}}\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c\right.}{}\right.$$

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*d*x^2)*sqrt(c/(a*c + b))*log(((8*a^2*c
^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2
*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 +
2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) - 4*((a*c -
 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d*x^4 + (a^3*c^3 + 3*a^2*
b*c^2 + 3*a*b^2*c + b^3)*x^2), 1/4*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*d*x^2)*s
qrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c))
 - 2*((a*c - 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^2)*sqrt((a*
```

```
d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d*x^4 + (a^3*
c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**3/(a+b/(d*x**2+c))**(3/2),x)
[Out] Integral(1/(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\frac{3\,bcd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\,(a^2c^2+2\,abc+b^2)\sqrt{(ac+b)c}}+\frac{\frac{3\,(adx^2+ac+b)bcd}{dx^2+c}-2\,(abc+b^2)d}{2\left((a^2c^3+2\,abc^2+b^2c)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(a^3c^3+3\,a^2bc^2+3\,ab^2c+b^3)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)}$$

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] 3/4*b*c*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))
/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/((a^2*c^2 +
 2*a*b*c + b^2)*sqrt((a*c + b)*c)) + 1/2*(3*(a*d*x^2 + a*c + b)*b*c*d/(d*x^
2 + c) - 2*(a*b*c + b^2)*d)/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*((a*d*x^2 + a*c
+ b)/(d*x^2 + c))^(3/2) - (a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*sqrt((a
*d*x^2 + a*c + b)/(d*x^2 + c)))
```

Giac [F]

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}} x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^3 \left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^3*(a + b/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x^3*(a + b/(c + d*x^2))^(3/2)), x)
```

3.360 $\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 212

$$\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{abd^2}{(b+ac)^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(3b-4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8(b+ac)^3x^2} - \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4(b+ac)^2x^4} - \frac{3b(b-4ac)d^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8\sqrt{c}(b+ac)^{7/2}}$$

```
[Out] -3/8*b*(-4*a*c+b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*
c+b)^(1/2))/(a*c+b)^(7/2)/c^(1/2)-a*b*d^2/(a*c+b)^3/((a*d*x^2+a*c+b)/(d*x^2
+c))^(1/2)-1/8*(-4*a*c+3*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(
a*c+b)^3/x^2-1/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^2/x^
4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{1985, 1981, 1980, 467, 464, 214\}$

$$\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{3bd^2(b-4ac)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8\sqrt{c}(ac+b)^{7/2}} - \frac{abd^2}{(ac+b)^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} - \frac{d(3b-4ac)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8x^2(ac+b)^3} - \frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4x^4(ac+b)^2}$$

```
[In] Int[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -((a*b*d^2)/((b + a*c)^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) - ((3*b -
4*a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*(b + a*c)^3*
x^2) - ((c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*(b + a*c)^2
*x^4) - (3*b*(b - 4*a*c)*d^2*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c +
 d*x^2)])/Sqrt[b + a*c]])/(8*Sqrt[c]*(b + a*c)^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
```

```
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^5 \left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3 \left(\frac{b+ac+adx}{c+dx}\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\left((bd)\text{Subst}\left(\int \frac{ad-dx^2}{x^2\left(-b-ac+cx^2\right)^3}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)\right)$$

$$= -\frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4(b+ac)^2x^4} + \frac{1}{4}(bd)\text{Subst}\left(\int \frac{\frac{4ad}{b+ac}-\frac{3bdx^2}{(b+ac)^2}}{x^2\left(-b-ac+cx^2\right)^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)$$

$$= -\frac{(3b-4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8(b+ac)^3x^2} - \frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4(b+ac)^2x^4} - \frac{1}{8}(bd)\text{Subst}\left(\int \frac{\frac{8ad}{(b+ac)^2}-\frac{(3b-4ac)dx^2}{(b+ac)^3}}{x^2\left(-b-ac+cx^2\right)}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)$$

$$= -\frac{abd^2}{(b+ac)^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(3b-4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8(b+ac)^3x^2} - \frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4(b+ac)^2x^4} + \frac{\left(3b(b-4ac)d^2\right)\text{Subst}\left(\int \frac{1}{-b-ac+cx^2}\,dx, x, \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\right)}{8(b+ac)^3}$$

$$= -\frac{abd^2}{(b+ac)^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(3b-4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8(b+ac)^3x^2} - \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4(b+ac)^2x^4} - \frac{3b(b-4ac)d^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8\sqrt{c}(b+ac)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b^2(2c+5dx^2)+2a^2c(c^2-d^2x^4)+ab(4c^2+5cdx^2+13d^2x^4)\right)}{8(b+ac)^3x^4\left(b+a\left(c+dx^2\right)\right)} - \frac{3b(b-4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8\sqrt{c}(-b-ac)^{7/2}}$$

```
[In] Integrate[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -1/8*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b^2*(2*c + 5*d*x^2
) + 2*a^2*c*(c^2 - d^2*x^4) + a*b*(4*c^2 + 5*c*d*x^2 + 13*d^2*x^4)))/((b +
a*c)^3*x^4*(b + a*(c + d*x^2))) - (3*b*(b - 4*a*c)*d^2*ArcTan[(Sqrt[c]*Sqrt
[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*Sqrt[c]*(-b - a*c)^(
7/2))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{(ad\,x^2+ac+b)(-2acd\,x^2+5bd\,x^2+2a\,c^2+2bc)}{8(ac+b)^3x^4\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}} - \frac{d^2b\left(-\frac{(12ac-3b)\ln\left(\frac{2a\,c^2+2bc+(2acd+bd)x^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,c^2+bc+(2acd+bd)x^2+a\,d}}{x^2}\right.}{2\sqrt{a\,c^2+bc}}\right.}{8(ac+b)^3\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}$
default	Expression too large to display

```
[In] int(1/x^5/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(a*d*x^2+a*c+b)*(-2*a*c*d*x^2+5*b*d*x^2+2*a*c^2+2*b*c)/(a*c+b)^3/x^4/(
(a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/8*d^2*b/(a*c+b)^3*(-1/2*(12*a*c-3*b)/(a*
c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c
^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)+8*a*(d*x^2+c)/(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d
*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(192) = 384$.

Time = 0.73 (sec) , antiderivative size = 961, normalized size of antiderivative = 4.53

$$\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\left((4a^2bc-ab^2)d^3x^6+(4a^2bc^2+3ab^2c-b^3)d^2x^4\right)\sqrt{ac^2+bc}\log\left(\frac{(8a^2c^2+8abc+b\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{3\left((4a^2bc-ab^2)d^3x^6+(4a^2bc^2+3ab^2c-b^3)d^2x^4\right)\sqrt{-ac^2-bc}\arctan\left(\frac{((2ac+b)dx^2+2ac^2+2bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2\cdots}{dx\cdots}}}{2(a^2c^3+2abc^2+(a^2c^2+abc)dx^2+b^2c)\cdots}\right.}{16\left((a^5c^5+4a^4bc\cdots\right.}$$

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*((4*a^2*b*c - a*b^2)*d^3*x^6 + (4*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2
*x^4)*sqrt(a*c^2 + b*c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^
4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((
2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^
2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) + 4*((2*a^3*c^3 - 11*a
^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6 - 6*a^2*b*c^5 - 6*a*b^2*c^4 + (2
*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 - 5*b^3*c)*d^2*x^4 - 2*b^3*c^3 - (2*
a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 + 7*b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4*b*c^4 + 6*a^3*b^2*c^3 + 4*a^2*b^3*c
^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4*b*c^5 + 10*a^3*b^2*c^4 + 10*a^2*b^3*
c^3 + 5*a*b^4*c^2 + b^5*c)*x^4), -1/16*(3*((4*a^2*b*c - a*b^2)*d^3*x^6 + (4
*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2*x^4)*sqrt(-a*c^2 - b*c)*arctan(1/2*((2*a*
c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*
((2*a^3*c^3 - 11*a^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6 - 6*a^2*b*c^5
- 6*a*b^2*c^4 + (2*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 - 5*b^3*c)*d^2*x^4
 - 2*b^3*c^3 - (2*a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 + 7*b^3*c^2)*d*x^2)
```

```
*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4*b*c^4 + 6*a^3*b^2
*c^3 + 4*a^2*b^3*c^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4*b*c^5 + 10*a^3*b^2
*c^4 + 10*a^2*b^3*c^3 + 5*a*b^4*c^2 + b^5*c)*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^5 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**5/(a+b/(d*x**2+c))**(3/2),x)
[Out] Integral(1/(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(192) = 384$.

Time = 0.32 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.12

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{3\,(4\,abc-b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,(a^3c^3+3\,a^2bc^2+3\,ab^2c+b^3)\sqrt{(ac+b)c}} - \frac{8\,(a^3bc^2+2\,a^2b^2c+ab^3)d^2+\frac{3\,(4\,abc^2-b^2c)\,(adx^2+ac+b)^2d^2}{(dx^2+c)^2}-\frac{5\,(4\,a^2}{}}{8\left((a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-2\,(a^4c^5+4\,a^3bc^4+6\,a^2b^2c^3+4\,ab^3c^2+b^4c)\left(\frac{adx^2+ac+b}{dx^2+c}\right)\right.}$$

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] -3/16*(4*a*b*c - b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sq
rt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*
c)))/((a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*sqrt((a*c + b)*c)) - 1/8*(8
*(a^3*b*c^2 + 2*a^2*b^2*c + a*b^3)*d^2 + 3*(4*a*b*c^2 - b^2*c)*(a*d*x^2 + a
*c + b)^2*d^2/(d*x^2 + c)^2 - 5*(4*a^2*b*c^2 + 3*a*b^2*c - b^3)*(a*d*x^2 +
a*c + b)*d^2/(d*x^2 + c))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*
((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 2*(a^4*c^5 + 4*a^3*b*c^4 + 6*a^2*
b^2*c^3 + 4*a*b^3*c^2 + b^4*c)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + (a
^5*c^5 + 5*a^4*b*c^4 + 10*a^3*b^2*c^3 + 10*a^2*b^3*c^2 + 5*a*b^4*c + b^5)*s
qrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
```

Giac [F]

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}} x^5}\,dx$$

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^5 \left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^5*(a + b/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x^5*(a + b/(c + d*x^2))^(3/2)), x)
```

3.361 $\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 482

$$\begin{aligned}
\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx &= -\frac{x^3(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{(8b+ac)x(b+ac+adx^2)}{5a^3d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&\quad+\frac{6x^3(b+ac+adx^2)}{5a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(16b^2+16abc+a^2c^2)\,x(b+ac+adx^2)}{5a^4d^2\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&\quad-\frac{\sqrt{c}(16b^2+16abc+a^2c^2)\,(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5a^4d^{5/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\\
&\quad+\frac{c^{3/2}(8b+ac)\,(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{5a^3d^{5/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

```
[Out] -x^3*(d*x^2+c)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/5*(a*c+8*b)*x*(a*d*x
^2+a*c+b)/a^3/d^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+6/5*x^3*(a*d*x^2+a*c+b)
/a^2/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/5*(a^2*c^2+16*a*b*c+16*b^2)*x*(a
*d*x^2+a*c+b)/a^4/d^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/5*c^(3/
2)*(a*c+8*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a^3/d^(5/2)/(d*x
^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c
))^(1/2)-1/5*(a^2*c^2+16*a*b*c+16*b^2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b
))^(1/2))*c^(1/2)/a^4/d^(5/2)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(
c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1985, 1986, 478, 595, 596, 545, 429, 506, 422}

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{c^{3/2}(ac+8b)\left(ac+adx^2+b\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{5a^3d^{5/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} - \frac{x(ac+8b)\left(ac+adx^2+b\right)}{5a^3d^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} + \frac{6x^3\left(ac+adx^2+b\right)}{5a^2d\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} - \frac{\sqrt{c}(a^2c^2+16abc+16b^2)\left(ac+adx^2+b\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5a^4d^{5/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{x(a^2c^2+16abc+16b^2)\left(ac+adx^2+b\right)}{5a^4d^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} - \frac{x^3\left(c+dx^2\right)}{ad\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[In] Int[x^4/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((x^3*(c + d*x^2))/(a*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) - ((8*b +
a*c)*x*(b + a*c + a*d*x^2))/(5*a^3*d^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
]) + (6*x^3*(b + a*c + a*d*x^2))/(5*a^2*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x
^2)]) + ((16*b^2 + 16*a*b*c + a^2*c^2)*x*(b + a*c + a*d*x^2))/(5*a^4*d^2*(c
 + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - (Sqrt[c]*(16*b^2 + 16*a*
b*c + a^2*c^2)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b
/(b + a*c)])/(5*a^4*d^(5/2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2
)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (c^(3/2)*(8*b +
 a*c)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c
)])/(5*a^3*d^(5/2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(
c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^4}{\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}\,dx$$

$$= \frac{\sqrt{b+ac+adx^2}\int \frac{x^4(c+dx^2)^{3/2}}{(b+ac+adx^2)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{x^3(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\sqrt{b+ac+adx^2}\int \frac{x^2\sqrt{c+dx^2}(3c+6dx^2)}{\sqrt{b+ac+adx^2}}\,dx}{ad\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{x^3(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{6x^3(b+ac+adx^2)}{5a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\sqrt{b+ac+adx^2}\int \frac{x^2(-3c(6b+ac)d-3(8b+ac)d^2x^2)}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5a^2d^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{x^3(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(8b+ac)x(b+ac+adx^2)}{5a^3d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{6x^3(b+ac+adx^2)}{5a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{b+ac+adx^2}\int \frac{-3c(b+ac)(8b+ac)d^2-3(16b^2+16abc+a^2c^2)d^3x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{15a^3d^4\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{x^3(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(8b+ac)x(b+ac+adx^2)}{5a^3d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{6x^3(b+ac+adx^2)}{5a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\left(c(b+ac)(8b+ac)\sqrt{b+ac+adx^2}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5a^3d^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\left((16b^2+16abc+a^2c^2)\sqrt{b+ac+adx^2}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{5a^3d\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{x^3(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(8b+ac)x(b+ac+adx^2)}{5a^3d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{6x^3(b+ac+adx^2)}{5a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(16b^2+16abc+a^2c^2)\,x(b+ac+adx^2)}{5a^4d^2\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{c^{3/2}(8b+ac)\,(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5a^3d^{5/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{\left(c(16b^2+16abc+a^2c^2)\sqrt{b+ac+adx^2}\right)\int\frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{5a^4d^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{x^3(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(8b+ac)x(b+ac+adx^2)}{5a^3d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{6x^3(b+ac+adx^2)}{5a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(16b^2+16abc+a^2c^2)\,x(b+ac+adx^2)}{5a^4d^2\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{c}(16b^2+16abc+a^2c^2)\,(b+ac+adx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5a^4d^{5/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{c^{3/2}(8b+ac)\,(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5a^3d^{5/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.61 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.61

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\,(8b^2+ab(9c+2dx^2)+a^2(c^2-d^2x^4))+i(16b^3+32ab^2c+17a^2bc^2+a^3c^3)\sqrt{\right.}{5a^4}$$

```
[In] Integrate[x^4/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -1/5*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(8*b
^2 + a*b*(9*c + 2*d*x^2) + a^2*(c^2 - d^2*x^4)) + I*(16*b^3 + 32*a*b^2*c +
17*a^2*b*c^2 + a^3*c^3)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2
)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - (8*I)*b*(2*b^2 +
```

```
3*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*
EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(a^4*c^2*(d/c)^(5/2)*(
b + a*(c + d*x^2)))
```

Maple [A] (verified)

Time = 11.98 (sec) , antiderivative size = 879, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{x(-ad\,x^2+ac+3b)(ad\,x^2+ac+b)}{5d^2a^3\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}+\frac{\left(-\frac{2d(a^2c^2+11abc+11b^2)(a\,c^2+bc)\sqrt{1+\frac{ad\,x^2}{ac+b}}\sqrt{1+\frac{d\,x^2}{c}}\left(F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)-E\left(x\sqrt{\right.\right.}{\sqrt{-\frac{ad}{ac+b}}\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,(2acd+2bd)}\right.}{}$
default	Expression too large to display

```
[In] int(x^4/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/d^2*x*(-a*d*x^2+a*c+3*b)*(a*d*x^2+a*c+b)/a^3/((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)+1/5/a^3/d^2*(-2*d*(a^2*c^2+11*a*b*c+11*b^2)*(a*c^2+b*c)/(-a*d/(a*c
+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d/(a*c+b))^(
1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b))^(1/2),(-1+(
2*a*c*d+b*d)/d/c/a)^(1/2)))+(a^3*c^3+4*a^2*b*c^2-2*a*b^2*c-5*b^3)/a/(-a*d/(
a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*
c*d+b*d)/d/c/a)^(1/2))+5*b^2*(a^2*c^2+2*a*b*c+b^2)/a*(-(a*d^2*x^2+a*c*d)/(a
*c+b)/b*x/d/((x^2+(a*c+b)/a/d)*(a*d^2*x^2+a*c*d))^(1/2)+(1/(a*c+b)+a*c/(a*c
+b)/b)/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(
1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-2/b*a*d/(a*c+b)*(a*c^2+b*c)/(-a*d/(a*c
+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d/(a*c+b))^(
1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b))^(1/2),(-1+(
2*a*c*d+b*d)/d/c/a)^(1/2)))))/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a
*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx =$$

$$-\frac{((a^3c^3+16\,a^2bc^2+16\,ab^2c)dx^3+(a^3c^4+17\,a^2bc^3+32\,ab^2c^2+16\,b^3c)x)\sqrt{a}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})}{}$$

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(((a^3*c^3 + 16*a^2*b*c^2 + 16*a*b^2*c)*d*x^3 + (a^3*c^4 + 17*a^2*b*c^
3 + 32*a*b^2*c^2 + 16*b^3*c)*x)*sqrt(a)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-
c/d)/x), (a*c + b)/(a*c)) - (((a^3*c^2 + 9*a^2*b*c + 8*a*b^2)*d^2 + (a^3*c^
3 + 16*a^2*b*c^2 + 16*a*b^2*c)*d)*x^3 + (a^3*c^4 + 17*a^2*b*c^3 + 32*a*b^2*
c^2 + 16*b^3*c + (a^3*c^3 + 10*a^2*b*c^2 + 17*a*b^2*c + 8*b^3)*d)*x)*sqrt(a
)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (a^3*d^4*x
^8 + (a^3*c - 2*a^2*b)*d^3*x^6 + a^3*c^4 + (5*a^2*b*c + 8*a*b^2)*d^2*x^4 +
17*a^2*b*c^3 + 32*a*b^2*c^2 + 16*b^3*c + (a^3*c^3 + 24*a^2*b*c^2 + 40*a*b^2
*c + 16*b^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^5*d^4*x^3 + (
a^5*c + a^4*b)*d^3*x)
```

Sympy [F]

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**4/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(a + b/(d*x^2 + c))^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate(x^4/(a + b/(d*x^2 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(x^4/(a + b/(c + d*x^2))^(3/2),x)
[Out] int(x^4/(a + b/(c + d*x^2))^(3/2), x)
```

3.362 $\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 409

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{x(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{4x(b+ac+adx^2)}{3a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(8b+ac)x(b+ac+adx^2)}{3a^3d\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\sqrt{c}(8b+ac)\,(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^3d^{3/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{c^{3/2}(4b+ac)\,(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3a^2(b+ac)d^{3/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

```
[Out] -x*(d*x^2+c)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+4/3*x*(a*d*x^2+a*c+b)/a^
2/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(a*c+8*b)*x*(a*d*x^2+a*c+b)/a^3/d
/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*c^(3/2)*(a*c+4*b)*(a*d*x^2
+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)
/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a^2/(a*c+b)/d^(3/2)/(d*x^2+c)/((a*d*x
^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/3*
(a*c+8*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE
(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^3/d^(3/2)
/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*
x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1985, 1986, 478, 542, 545, 429, 506, 422}

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{c}(ac+8b)\left(ac+adx^2+b\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^3d^{3/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} - \frac{x(ac+8b)\left(ac+adx^2+b\right)}{3a^3d\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} - \frac{c^{3/2}(ac+4b)\left(ac+adx^2+b\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3a^2d^{3/2}(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{4x\left(ac+adx^2+b\right)}{3a^2d\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} - \frac{x\left(c+dx^2\right)}{ad\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[In] Int[x^2/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((x*(c + d*x^2))/(a*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) + (4*x*(b +
a*c + a*d*x^2))/(3*a^2*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - ((8*b + a
*c)*x*(b + a*c + a*d*x^2))/(3*a^3*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)]) + (Sqrt[c]*(8*b + a*c)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*a^3*d^(3/2)*(c + d*x^2)*Sqrt[(b + a*c +
 a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))
]) - (c^(3/2)*(4*b + a*c)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], b/(b + a*c)])/(3*a^2*(b + a*c)*d^(3/2)*(c + d*x^2)*Sqrt[(b + a*c
+ a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2)
)])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}\,dx$$

$$= \frac{\sqrt{b+ac+adx^2}\int \frac{x^2(c+dx^2)^{3/2}}{(b+ac+adx^2)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{x(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\sqrt{b+ac+adx^2}\int \frac{\sqrt{c+dx^2}(c+4dx^2)}{\sqrt{b+ac+adx^2}}\,dx}{ad\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{x(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{4x(b+ac+adx^2)}{3a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\sqrt{b+ac+adx^2}\int \frac{-c(4b+ac)d-(8b+ac)d^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3a^2d^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{x(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{4x(b+ac+adx^2)}{3a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\left((8b+ac)\sqrt{b+ac+adx^2}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3a^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\left(c(4b+ac)\sqrt{b+ac+adx^2}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3a^2d\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{x(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{4x(b+ac+adx^2)}{3a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(8b+ac)x(b+ac+adx^2)}{3a^3d\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{c^{3/2}(4b+ac)\,(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2(b+ac)d^{3/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{\left(c(8b+ac)\sqrt{b+ac+adx^2}\right)\int \frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{3a^3d\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{x(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{4x(b+ac+adx^2)}{3a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(8b+ac)x(b+ac+adx^2)}{3a^3d\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\sqrt{c}(8b+ac)\,(b+ac+adx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^3d^{3/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{c^{3/2}(4b+ac)\,(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2(b+ac)d^{3/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.41 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\left(4b+a(c+dx^2)\right)+i(8b^2+9abc+a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1\cdots\right.}{3a^3\cdots}$$

```
[In] Integrate[x^2/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(4*b + a*
(c + d*x^2)) + I*(8*b^2 + 9*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b +
a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)
] - I*b*(8*b + 5*a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/
c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(3*a^3*d*Sqrt[d/c]*
(b + a*(c + d*x^2)))
```

Maple [A] (verified)

Time = 9.20 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.63

method	result
default	$\frac{\left(\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,a\,d^2x^5+2\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,acd\,x^3+\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,bd\,x^3-\cdots\right.}{\cdots}$
risch	$\frac{x\left(ad\,x^2+ac+b\right)}{3a^2d\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}-\frac{\left(-\frac{2d(ac+5b)\left(a\,c^2+bc\right)\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)-E\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)\right)}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,(2acd+2bd)}+\frac{(a^2\cdots}{\cdots}\right.}{\cdots}$

```
[In] int(x^2/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*d^2*x^5+2*((a
*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*c*d*x^3+((a*d*x^2+a*c
+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*b*d*x^3-((a*d*x^2+a*c+b)*(d*x^2+c
))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-
a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2+3*(-a*d/(a*c+b))^(1/2)*(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b*d*x^3+((a*d*x^2+a*c+b)*(d*x^2+c
))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*c^2*x+4*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(
(a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b
))^(1/2),((a*c+b)/a/c)^(1/2))*b*c-8*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d
```

```
*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(
1/2),((a*c+b)/a/c)^(1/2))*b*c+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+
b))^(1/2)*b*c*x+3*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2)*b*c*x)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d/(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{((a^2c^2+8\,abc)dx^3+(a^2c^3+9\,abc^2+8\,b^2c)x)\sqrt{a}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-((\cdots}{\cdots}$$

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(((a^2*c^2 + 8*a*b*c)*d*x^3 + (a^2*c^3 + 9*a*b*c^2 + 8*b^2*c)*x)*sqrt(a
)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (((a^2*c +
 4*a*b)*d^2 + (a^2*c^2 + 8*a*b*c)*d)*x^3 + (a^2*c^3 + 9*a*b*c^2 + 8*b^2*c +
 (a^2*c^2 + 5*a*b*c + 4*b^2)*d)*x)*sqrt(a)*sqrt(-c/d)*elliptic_f(arcsin(sqr
t(-c/d)/x), (a*c + b)/(a*c)) + (a^2*d^3*x^6 + (a^2*c - 4*a*b)*d^2*x^4 - a^2
*c^3 - 9*a*b*c^2 - (a^2*c^2 + 13*a*b*c + 8*b^2)*d*x^2 - 8*b^2*c)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^3*x^3 + (a^4*c + a^3*b)*d^2*x)
```

Sympy [F]

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**2/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/(a + b/(d*x^2 + c))^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate(x^2/(a + b/(d*x^2 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(x^2/(a + b/(c + d*x^2))^(3/2),x)
[Out] int(x^2/(a + b/(c + d*x^2))^(3/2), x)
```

3.363 $\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 356

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{bx}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(2b+ac)x(b+ac+adx^2)}{a^2(b+ac)\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$
$$-\frac{\sqrt{c}(2b+ac)\,(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a^2(b+ac)\sqrt{d}\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$
$$+\frac{c^{3/2}(b+ac+adx^2)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{a(b+ac)\sqrt{d}\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

```
[Out] -b*x/a/(a*c+b)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+(a*c+2*b)*x*(a*d*x^2+a*c+b
)/a^2/(a*c+b)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+c^(3/2)*(a*d*x^2+
a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a/(a*c+b)/(d*x^2+c)/d^(1/2)/((a*d*x^2+
a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-(a*c+2*
b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^2/(a*c+b)/(d*x^2
+c)/d^(1/2)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d
*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1985, 1986, 424, 545, 429, 506, 422}

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=-\frac{\sqrt{c}(ac+2b)\left(ac+adx^2+b\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a^2\sqrt{d}(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+\frac{x(ac+2b)\left(ac+adx^2+b\right)}{a^2(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{c^{3/2}\left(ac+adx^2+b\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{a\sqrt{d}(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-\frac{bx}{a(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[In] Int[(a + b/(c + d*x^2))^(-3/2),x]
```

```
[Out] -((b*x)/(a*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) + ((2*b + a*c)
*x*(b + a*c + a*d*x^2))/(a^2*(b + a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)
/(c + d*x^2)]) - (Sqrt[c]*(2*b + a*c)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a^2*(b + a*c)*Sqrt[d]*(c + d*x^2)*Sqrt
[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(
c + d*x^2))]) + (c^(3/2)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/S
qrt[c]], b/(b + a*c)])/(a*(b + a*c)*Sqrt[d]*(c + d*x^2)*Sqrt[(b + a*c + a*d
*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}\,dx\\
&= \frac{\sqrt{b+ac+adx^2}\int \frac{\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&= -\frac{bx}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{\sqrt{b+ac+adx^2}\int \frac{c(b+ac)d+(2b+ac)d^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{a(b+ac)d\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}
\end{aligned}$$

$$= -\frac{bx}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\left(c\sqrt{b+ac+adx^2}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{a\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\left((2b+ac)d\sqrt{b+ac+adx^2}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{a(b+ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{bx}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(2b+ac)x(b+ac+adx^2)}{a^2(b+ac)\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{c^{3/2}(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a(b+ac)\sqrt{d}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{\left(c(2b+ac)\sqrt{b+ac+adx^2}\right)\int \frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{a^2(b+ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{bx}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(2b+ac)x(b+ac+adx^2)}{a^2(b+ac)\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{c}(2b+ac)\,(b+ac+adx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a^2(b+ac)\sqrt{d}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{c^{3/2}(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a(b+ac)\sqrt{d}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.43 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{i\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-iab\sqrt{\frac{d}{c}}x(c+dx^2)+(2b^2+3abc+a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{ac}{b+ac}\right)-2\right.}{a^2(b+ac)\sqrt{\frac{d}{c}}\,(b+a\,(c+dx^2))}$$

```
[In] Integrate[(a + b/(c + d*x^2))^(-3/2),x]
```

```
[Out] ((-I)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((-I)*a*b*Sqrt[d/c]*x*(c + d*x^
2) + (2*b^2 + 3*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1
 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - 2*b*(b +
```

a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(a^2*(b + a*c)*Sqrt[d/c]*(b + a*(c
+ d*x^2)))

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.31

method	result
default	$-\frac{\left(-\sqrt{\left(a d \,x^{2}+a c +b \right) \left(d \,x^{2}+c \right)}\, \sqrt{\frac{a d \,x^{2}+a c +b}{a c +b}}\, \sqrt{\frac{d \,x^{2}+c}{c}}\, E\left(x \sqrt{-\frac{a d}{a c +b}}, \sqrt{\frac{a c +b}{a c}}\right) a \,c^{2}+\sqrt{-\frac{a d}{a c +b}}\, \sqrt{a \,d^{2} x^{4}+2 a c d \,x^{2}+b d \,x^{2}+a \,c^{2}+b c}\, b d \cdots}{\cdots}$

[In] int(1/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -(-((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2+(
-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b*d*x^3
+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c-2*((a
*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c
)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c+(-a*d/(a*
c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b*c*x)/a*((a*d*
x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
/(-a*d/(a*c+b))^(1/2)/(a*c+b)/(a*d*x^2+a*c+b)

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx =$$

$$-\frac{((a^2c^2+2\,abc)dx^3+(a^2c^3+3\,abc^2+2\,b^2c)x)\sqrt{a}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}) - (((a^2c+ab)d^2+(a^2c^2 \cdots}{\cdots}$$

[In] integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")

[Out] -(((a^2*c^2 + 2*a*b*c)*d*x^3 + (a^2*c^3 + 3*a*b*c^2 + 2*b^2*c)*x)*sqrt(a)*s
qrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (((a^2*c + a*
b)*d^2 + (a^2*c^2 + 2*a*b*c)*d)*x^3 + (a^2*c^3 + 3*a*b*c^2 + 2*b^2*c + (a^2
*c^2 + 2*a*b*c + b^2)*d)*x)*sqrt(a)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)
/x), (a*c + b)/(a*c)) - ((a^2*c + a*b)*d^2*x^4 + a^2*c^3 + 3*a*b*c^2 + 2*(a
^2*c^2 + 2*a*b*c + b^2)*d*x^2 + 2*b^2*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/((a^4*c + a^3*b)*d^2*x^3 + (a^4*c^2 + 2*a^3*b*c + a^2*b^2)*d*x)

Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b/(d*x**2+c))**(3/2),x)
[Out] Integral((a + b/(c + d*x**2))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a + b/(d*x^2 + c))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate((a + b/(d*x^2 + c))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(1/(a + b/(c + d*x^2))^(3/2),x)
[Out] int(1/(a + b/(c + d*x^2))^(3/2), x)
```

3.364 $\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 410

$$\begin{aligned}\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx &= -\frac{b}{a(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\ &+ \frac{(b-ac)\left(b+ac+adx^2\right)}{a(b+ac)^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(b-ac)dx(b+ac+adx^2)}{a(b+ac)^2\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\ &+ \frac{\sqrt{c}(b-ac)\sqrt{d}(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a(b+ac)^2\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ &+ \frac{c^{3/2}\sqrt{d}(b+ac+adx^2)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{(b+ac)^2\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

```
[Out] -b/a/(a*c+b)/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+(-a*c+b)*(a*d*x^2+a*c+b)/a
/(a*c+b)^2/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-(-a*c+b)*d*x*(a*d*x^2+a*c+b)
/a/(a*c+b)^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+c^(3/2)*(a*d*x^2+a
*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*d^(1/2)/(a*c+b)^2/(d*x^2+c)/((a*d*x^2+a
*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+(-a*c+b)
*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2
)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)/a/(a*c+b)^2/
(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x
^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 479, 597, 545, 429, 506, 422}

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{c^{3/2}\sqrt{d}(ac+adx^2+b)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{(ac+b)^2\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{d}(b-ac)\,(ac+adx^2+b)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a(ac+b)^2\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}} + \frac{(b-ac)\,(ac+adx^2+b)}{ax(ac+b)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} - \frac{dx(b-ac)\,(ac+adx^2+b)}{a(ac+b)^2\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} - \frac{b}{ax(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[In] Int[1/(x^2*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -(b/(a*(b + a*c)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) + ((b - a*c)*(b
+ a*c + a*d*x^2))/(a*(b + a*c)^2*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) -
 ((b - a*c)*d*x*(b + a*c + a*d*x^2))/(a*(b + a*c)^2*(c + d*x^2)*Sqrt[(b + a
*c + a*d*x^2)/(c + d*x^2)]) + (Sqrt[c]*(b - a*c)*Sqrt[d]*(b + a*c + a*d*x^2
)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*(b + a*c)^2*(c +
d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/(
(b + a*c)*(c + d*x^2))]) + (c^(3/2)*Sqrt[d]*(b + a*c + a*d*x^2)*EllipticF[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)^2*(c + d*x^2)*Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c +
d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
```

```
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}\,dx$$

$$= \frac{\sqrt{b+ac+adx^2}\int \frac{(c+dx^2)^{3/2}}{x^2(b+ac+adx^2)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{b+ac+adx^2}\int \frac{c(b-ac)d-acd^2x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{a(b+ac)d\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(b-ac)\left(b+ac+adx^2\right)}{a(b+ac)^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\sqrt{b+ac+adx^2}\int \frac{ac^2(b+ac)d^2-ac(b-ac)d^3x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{ac(b+ac)^2d\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(b-ac)\left(b+ac+adx^2\right)}{a(b+ac)^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\left(cd\sqrt{b+ac+adx^2}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{(b+ac)\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\left((b-ac)d^2\sqrt{b+ac+adx^2}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{(b+ac)^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(b-ac)\left(b+ac+adx^2\right)}{a(b+ac)^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(b-ac)dx(b+ac+adx^2)}{a(b+ac)^2\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{c^{3/2}\sqrt{d}(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|\frac{b}{b+ac}\right)}{(b+ac)^2\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{\left(c(b-ac)d\sqrt{b+ac+adx^2}\right)\int \frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{a(b+ac)^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(b-ac)(b+ac+adx^2)}{a(b+ac)^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(b-ac)dx(b+ac+adx^2)}{a(b+ac)^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\sqrt{c}(b-ac)\sqrt{d}(b+ac+adx^2)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a(b+ac)^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{c^{3/2}\sqrt{d}(b+ac+adx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(b+ac)^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.51 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}(c+dx^2)\left(b(c-dx^2)+ac(c+dx^2)\right)+i(-b^2+a^2c^2)\,dx\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsin}\right.\right.}{a(b+ac)^2\sqrt{\frac{d}{c}}x\left(b+a\left(c\right.\right.}$$

```
[In] Integrate[1/(x^2*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -((Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*(c + d*x^2)*(b*(c - d
*x^2) + a*c*(c + d*x^2)) + I*(-b^2 + a^2*c^2)*d*x*Sqrt[(b + a*c + a*d*x^2)/
(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b +
 a*c)] + I*b*(b + a*c)*d*x*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*
x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(a*(b + a*c)^2
*Sqrt[d/c]*x*(b + a*(c + d*x^2))))
```

Maple [A] (verified)

Time = 11.04 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.67

method	result
default	$-\frac{\left(\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,ac\,d^2x^4-a\,c^2d\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)x\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}-\right.}{}$
risch	Expression too large to display

```
[In] int(1/x^2/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*c*d^2*x^4-a*c^2*
d*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*
c+b))^(1/2),((a*c+b)/a/c)^(1/2))*x*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)-(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b*d^2*x^4+2*
((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*c^2*d*x^2-2*((a*d*
x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(
1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c*d*x+((a*d*x^
2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c*d*x+((a*d*x^2+
a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*b*c*d*x^2-(a*d^2*x^4+2*a*c*d*x
^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b*c*d*x^2+((a*d*x^2+a*c+b)
*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*c^3+((a*d*x^2+a*c+b)*(d*x^2+c))^(1
/2)*(-a*d/(a*c+b))^(1/2)*b*c^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/x/(a*c+b)^2/(a*
d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\left((a^3c-a^2b)d^3x^3+(a^3c^2-ab^2)d^2x\right)\sqrt{-\frac{ad}{ac+b}}\sqrt{\frac{ac^2+bc}{d^2}}E\left(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+b}{ac}\right) -}{}$$

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (((a^3*c - a^2*b)*d^3*x^3 + (a^3*c^2 - a*b^2)*d^2*x)*sqrt(-a*d/(a*c + b))*s
qrt((a*c^2 + b*c)/d^2)*elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)
/(a*c)) - (((a^3*c - a^2*b)*d^3 + (a^3*c^2 + 2*a^2*b*c + a*b^2)*d^2)*x^3 +
((a^3*c^2 - a*b^2)*d^2 + (a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*d)*x)*sq
rt(-a*d/(a*c + b))*sqrt((a*c^2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c
 + b))*x), (a*c + b)/(a*c)) - (a^3*c^4 + (a^3*c^2 - a*b^2)*d^2*x^4 + 2*a^2*
b*c^3 + a*b^2*c^2 + 2*(a^3*c^3 + a^2*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c)))/((a^5*c^3 + 3*a^4*b*c^2 + 3*a^3*b^2*c + a^2*b^3)*d*x^3 + (a^
5*c^4 + 4*a^4*b*c^3 + 6*a^3*b^2*c^2 + 4*a^2*b^3*c + a*b^4)*x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**2/(a+b/(d*x**2+c))**(3/2),x)
[Out] Integral(1/(x**2*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^2*(a + b/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x^2*(a + b/(c + d*x^2))^(3/2)), x)
```

3.365 $\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 490

$$\begin{aligned}
\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx &= -\frac{b}{a(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(3b-ac)\left(b+ac+adx^2\right)}{3a(b+ac)^2x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\
&\quad -\frac{(7b-ac)d(b+ac+adx^2)}{3(b+ac)^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(7b-ac)d^2x(b+ac+adx^2)}{3(b+ac)^3\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\
&\quad -\frac{\sqrt{c}(7b-ac)d^{3/2}(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(b+ac)^3\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\
&\quad +\frac{\sqrt{c}(3b-ac)d^{3/2}(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3(b+ac)^3\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

```
[Out] -b/a/(a*c+b)/x^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/3*(-a*c+3*b)*(a*d*x^2+
a*c+b)/a/(a*c+b)^2/x^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(-a*c+7*b)*d*(
a*d*x^2+a*c+b)/(a*c+b)^3/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/3*(-a*c+7*b)
*d^2*x*(a*d*x^2+a*c+b)/(a*c+b)^3/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)-1/3*(-a*c+7*b)*d^(3/2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1
/2)/(a*c+b)^3/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b
)/(a*c+b)/(d*x^2+c))^(1/2)+1/3*(-a*c+3*b)*d^(3/2)*(a*d*x^2+a*c+b)*(1/(1+d*x
^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2
),(b/(a*c+b))^(1/2))*c^(1/2)/(a*c+b)^3/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1985, 1986, 479, 597, 545, 429, 506, 422}

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{c}d^{3/2}(3b-ac)\left(ac+adx^2+b\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3(ac+b)^3\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}$$
$$-\frac{\sqrt{c}d^{3/2}(7b-ac)\left(ac+adx^2+b\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(ac+b)^3\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}$$
$$+\frac{d^2x(7b-ac)\left(ac+adx^2+b\right)}{3(ac+b)^3\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{d(7b-ac)\left(ac+adx^2+b\right)}{3x(ac+b)^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{(3b-ac)\left(ac+adx^2+b\right)}{3ax^3(ac+b)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$
$$-\frac{b}{ax^3(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

```
[In] Int[1/(x^4*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -(b/(a*(b + a*c)*x^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) + ((3*b - a*c)
*(b + a*c + a*d*x^2))/(3*a*(b + a*c)^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*
x^2)]) - ((7*b - a*c)*d*(b + a*c + a*d*x^2))/(3*(b + a*c)^3*x*Sqrt[(b + a*c
 + a*d*x^2)/(c + d*x^2)]) + ((7*b - a*c)*d^2*x*(b + a*c + a*d*x^2))/(3*(b +
 a*c)^3*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - (Sqrt[c]*(7*b
- a*c)*d^(3/2)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b
/(b + a*c)])/(3*(b + a*c)^3*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2
)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (Sqrt[c]*(3*b -
 a*c)*d^(3/2)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/
(b + a*c)])/(3*(b + a*c)^3*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^4\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}\,dx$$

$$= \frac{\sqrt{b+ac+adx^2}\int \frac{(c+dx^2)^{3/2}}{x^4(b+ac+adx^2)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{b+ac+adx^2}\int \frac{c(3b-ac)d+(2b-ac)d^2x^2}{x^4\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{a(b+ac)d\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(3b-ac)\left(b+ac+adx^2\right)}{3a(b+ac)^2x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\sqrt{b+ac+adx^2}\int \frac{ac^2(7b-ac)d^2+ac(3b-ac)d^3x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3ac(b+ac)^2d\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(3b-ac)\left(b+ac+adx^2\right)}{3a(b+ac)^2x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(7b-ac)d(b+ac+adx^2)}{3(b+ac)^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{b+ac+adx^2}\int \frac{-ac^2(3b-ac)(b+ac)d^3-a^2c^2(7b-ac)d^4x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3ac^2(b+ac)^3d\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(3b-ac)\left(b+ac+adx^2\right)}{3a(b+ac)^2x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(7b-ac)d(b+ac+adx^2)}{3(b+ac)^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\left((3b-ac)d^2\sqrt{b+ac+adx^2}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3(b+ac)^2\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\left(a(7b-ac)d^3\sqrt{b+ac+adx^2}\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{3(b+ac)^3\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}$$

$$
\begin{aligned}
&= -\frac{b}{a(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(3b-ac)\left(b+ac+adx^2\right)}{3a(b+ac)^2x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\
&\quad - \frac{(7b-ac)d(b+ac+adx^2)}{3(b+ac)^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(7b-ac)d^2x(b+ac+adx^2)}{3(b+ac)^3\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\
&\quad + \frac{\sqrt{c}(3b-ac)d^{3/2}(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(b+ac)^3\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\
&\quad - \frac{\left(c(7b-ac)d^2\sqrt{b+ac+adx^2}\right)\int\frac{\sqrt{b+ac+adx^2}}{(c+dx^2)^{3/2}}\,dx}{3(b+ac)^3\sqrt{c+dx^2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b}{a(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(3b-ac)\left(b+ac+adx^2\right)}{3a(b+ac)^2x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\
&\quad - \frac{(7b-ac)d(b+ac+adx^2)}{3(b+ac)^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(7b-ac)d^2x(b+ac+adx^2)}{3(b+ac)^3\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\
&\quad - \frac{\sqrt{c}(7b-ac)d^{3/2}(b+ac+adx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(b+ac)^3\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\
&\quad + \frac{\sqrt{c}(3b-ac)d^{3/2}(b+ac+adx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(b+ac)^3\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.79 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.62

$$
\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx =
$$

$$
-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}(c+dx^2)\left(b^2(c+4dx^2)+a^2c(c^2-d^2x^4)+ab(2c^2+4cdx^2+7d^2x^4)\right)+i(7b^2+6abc-a^2c\right.}{3(b+}
$$

```
[In] Integrate[1/(x^4*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -1/3*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*(c + d*x^2)*(b^2*(c
+ 4*d*x^2) + a^2*c*(c^2 - d^2*x^4) + a*b*(2*c^2 + 4*c*d*x^2 + 7*d^2*x^4)) +
 I*(7*b^2 + 6*a*b*c - a^2*c^2)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*
Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - (4
```

```
*I)*b*(b + a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2
)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/((b + a*c)^3*Sqrt
[d/c]*x^3*(b + a*(c + d*x^2)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. 2(526) = 1052.

Time = 10.81 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.20

method	result	size
default	Expression too large to display	1080
risch	Expression too large to display	1142

```
[In] int(1/x^4/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c*d^3*x^6
+4*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d^3*x^6+((a*d
*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2*d^2*x^3
+3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b
*d^3*x^6-((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c^2*d^2
*x^4+5*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b
*c*d^2*x^3-7*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2
))*a*b*c*d^2*x^3+8*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a
*b*c*d^2*x^4-3*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1
/2))*b^2*d^2*x^3+3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a
*c+b))^(1/2)*a*b*c*d^2*x^4+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))
^(1/2)*a^2*c^3*d*x^2+4*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/
2)*b^2*d^2*x^4+6*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b
*c^2*d*x^2+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c^4+5
*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*c*d*x^2+2*((a*d
*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c^3+((a*d*x^2+a*c+b)*
(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*c^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^
(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/
x^3/(a*c+b)^3/(a*d*x^2+a*c+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{((a^4c^2-7\,a^3bc)d^4x^5+(a^4c^3-6\,a^3bc^2-7\,a^2b^2c)d^3x^3)\sqrt{-\frac{ad}{ac+b}}\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right)\mid\frac{ac+b}{ac})-(((a^4}{}$$

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(((a^4*c^2 - 7*a^3*b*c)*d^4*x^5 + (a^4*c^3 - 6*a^3*b*c^2 - 7*a^2*b^2*c
)*d^3*x^3)*sqrt(-a*d/(a*c + b))*sqrt((a*c^2 + b*c)/d^2)*elliptic_e(arcsin(s
qrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - (((a^4*c^2 - 7*a^3*b*c)*d^4 + (a
^4*c^3 - a^3*b*c^2 - 5*a^2*b^2*c - 3*a*b^3)*d^3)*x^5 + ((a^4*c^3 - 6*a^3*b*
c^2 - 7*a^2*b^2*c)*d^3 + (a^4*c^4 - 6*a^2*b^2*c^2 - 8*a*b^3*c - 3*b^4)*d^2)
*x^3)*sqrt(-a*d/(a*c + b))*sqrt((a*c^2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-
a*d/(a*c + b))*x), (a*c + b)/(a*c)) + (a^4*c^6 - (a^4*c^3 - 6*a^3*b*c^2 - 7
*a^2*b^2*c)*d^3*x^6 + 3*a^3*b*c^5 + 3*a^2*b^2*c^4 + a*b^3*c^3 - (a^4*c^4 -
10*a^3*b*c^3 - 15*a^2*b^2*c^2 - 4*a*b^3*c)*d^2*x^4 + (a^4*c^5 + 7*a^3*b*c^4
 + 11*a^2*b^2*c^3 + 5*a*b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
)))/((a^6*c^5 + 4*a^5*b*c^4 + 6*a^4*b^2*c^3 + 4*a^3*b^3*c^2 + a^2*b^4*c)*d*
x^5 + (a^6*c^6 + 5*a^5*b*c^5 + 10*a^4*b^2*c^4 + 10*a^3*b^3*c^3 + 5*a^2*b^4*
c^2 + a*b^5*c)*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^4 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**4/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(1/(x**4*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^4 \left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^4*(a + b/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x^4*(a + b/(c + d*x^2))^(3/2)), x)
```

3.366 $\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx=-\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9}+\frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4}+\frac{3\sqrt{ax^{23}}\operatorname{arcsinh}\left(x^{5/2}\right)}{20x^{23/2}}$$

```
[Out] 3/20*arcsinh(x^(5/2))*(a*x^23)^(1/2)/x^(23/2)-3/20*(a*x^23)^(1/2)*(x^5+1)^(
1/2)/x^9+1/10*(a*x^23)^(1/2)*(x^5+1)^(1/2)/x^4
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 327, 335, 281, 221}

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx=\frac{3\sqrt{ax^{23}}\operatorname{arcsinh}\left(x^{5/2}\right)}{20x^{23/2}}-\frac{3\sqrt{x^5+1}\sqrt{ax^{23}}}{20x^9}+\frac{\sqrt{x^5+1}\sqrt{ax^{23}}}{10x^4}$$

```
[In] Int[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
```

```
[Out] (-3*Sqrt[a*x^23]*Sqrt[1 + x^5])/(20*x^9) + (Sqrt[a*x^23]*Sqrt[1 + x^5])/(10
*x^4) + (3*Sqrt[a*x^23]*ArcSinh[x^(5/2)])/(20*x^(23/2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ax^{23}}\int \frac{x^{23/2}}{\sqrt{1+x^5}}\,dx}{x^{23/2}}\\
&= \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} - \frac{\left(3\sqrt{ax^{23}}\right)\int \frac{x^{13/2}}{\sqrt{1+x^5}}\,dx}{4x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{\left(3\sqrt{ax^{23}}\right)\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx}{8x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{\left(3\sqrt{ax^{23}}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right)}{4x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{\left(3\sqrt{ax^{23}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right)}{20x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{3\sqrt{ax^{23}}\sinh^{-1}\left(x^{5/2}\right)}{20x^{23/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{ax^{23}}\left(x^{5/2}\sqrt{1+x^5}(-3+2x^5)+3\log\left(x^{5/2}+\sqrt{1+x^5}\right)\right)}{20x^{23/2}}$$

```
[In] Integrate[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
[Out] (Sqrt[a*x^23]*(x^(5/2)*Sqrt[1 + x^5]*(-3 + 2*x^5) + 3*Log[x^(5/2) + Sqrt[1
+ x^5]]))/(20*x^(23/2))
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

method	result	size
meijerg	$\frac{\sqrt{a\,x^{23}}\left(-\frac{\sqrt{\pi}\,x^{\frac{5}{2}}\left(-10x^5+15\right)\sqrt{x^5+1}}{20}+\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{4}\right)}{5x^{\frac{23}{2}}\sqrt{\pi}}$	48
risch	$\frac{(2x^5-3)\sqrt{x^5+1}\,\sqrt{a\,x^{23}}}{20x^9}+\frac{3\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)\sqrt{a\,x^{23}}\,\sqrt{ax(x^5+1)}}{20\sqrt{a}\,x^{12}\sqrt{x^5+1}}$	64

```
[In] int((a*x^23)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/5*(a*x^23)^(1/2)/x^(23/2)/Pi^(1/2)*(-1/20*Pi^(1/2)*x^(5/2)*(-10*x^5+15)*(
x^5+1)^(1/2)+3/4*Pi^(1/2)*arcsinh(x^(5/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \left[\frac{3\sqrt{a}x^9\log\left(-\frac{8\,ax^{19}+8\,ax^{14}+ax^9+4\sqrt{ax^{23}}(2\,x^5+1)\sqrt{x^5+1}\sqrt{a}}{x^9}\right)+4\sqrt{ax^{23}}(2\,x^5-3)\sqrt{x^5+1}}{80\,x^9},\ -\frac{3\sqrt{-a}x^9\arctan\left(\frac{\sqrt{ax^{23}}(2\,x^5+1)\sqrt{x^5+1}\sqrt{-a}}{2\,(ax^{19}+ax^{14})}\right)-2\sqrt{ax^{23}}(2\,x^5-3)\sqrt{x^5+1}}{40\,x^9}\right]$$

```
[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/80*(3*sqrt(a)*x^9*log(-(8*a*x^19 + 8*a*x^14 + a*x^9 + 4*sqrt(a*x^23)*(2*
x^5 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^9) + 4*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x^5
+ 1))/x^9, -1/40*(3*sqrt(-a)*x^9*arctan(1/2*sqrt(a*x^23)*(2*x^5 + 1)*sqrt(x
^5 + 1)*sqrt(-a)/(a*x^19 + a*x^14)) - 2*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x^5 +
 1))/x^9]
```

Sympy [F]

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^{23}}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

```
[In] integrate((a*x**23)**(1/2)/(x**5+1)**(1/2),x)
[Out] Integral(sqrt(a*x**23)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^{23}}}{\sqrt{x^5+1}}\,dx$$

```
[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^23)/sqrt(x^5 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(55) = 110$.

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{a^6x^5+a^6}\sqrt{ax}a^6x^2\left(\frac{2x^5}{a^8}-\frac{3}{a^8}\right)\mathrm{sgn}(x)}{20\,|a|} - \frac{3\left(\frac{a^{\frac{5}{2}}\log\left(-\sqrt{ax}a^{\frac{5}{2}}x^2+\sqrt{a^6x^5+a^6}\right)\mathrm{sgn}(x)}{|a|}-\frac{a^{\frac{5}{2}}\log\left(a^2|a|\right)\mathrm{sgn}(x)}{|a|}\right)a^3}{20\,|a|^4}$$

```
[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
[Out] 1/20*sqrt(a^6*x^5 + a^6)*sqrt(a*x)*a^6*x^2*(2*x^5/a^8 - 3/a^8)*sgn(x)/abs(a
) - 3/20*(a^(5/2)*log(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6))*sgn(x)/
abs(a) - a^(5/2)*log(a^2*abs(a))*sgn(x)/abs(a))*a^3/abs(a)^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{a\,x^{23}}}{\sqrt{x^5+1}}\,dx$$

```
[In] int((a*x^23)^(1/2)/(x^5 + 1)^(1/2),x)
[Out] int((a*x^23)^(1/2)/(x^5 + 1)^(1/2), x)
```

3.367 $\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\operatorname{arcsinh}\left(x^{5/2}\right)}{5x^{13/2}}$$

```
[Out] -1/5*arcsinh(x^(5/2))*(a*x^13)^(1/2)/x^(13/2)+1/5*(a*x^13)^(1/2)*(x^5+1)^(1
/2)/x^4
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {15, 327, 335, 281, 221}

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{x^5+1}\sqrt{ax^{13}}}{5x^4} - \frac{\sqrt{ax^{13}}\operatorname{arcsinh}\left(x^{5/2}\right)}{5x^{13/2}}$$

```
[In] Int[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
```

```
[Out] (Sqrt[a*x^13]*Sqrt[1 + x^5])/(5*x^4) - (Sqrt[a*x^13]*ArcSinh[x^(5/2)])/(5*x
^(13/2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ax^{13}}\int \frac{x^{13/2}}{\sqrt{1+x^5}}\,dx}{x^{13/2}}\\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx}{2x^{13/2}}\\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right)}{x^{13/2}}\\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right)}{5x^{13/2}}\\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{ax^{13}}\left(x^{5/2}\sqrt{1+x^5}-\log\left(x^{5/2}+\sqrt{1+x^5}\right)\right)}{5x^{13/2}}$$

```
[In] Integrate[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
```

```
[Out] (Sqrt[a*x^13]*(x^(5/2)*Sqrt[1 + x^5] - Log[x^(5/2) + Sqrt[1 + x^5]]))/(5*x^
(13/2))
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

method	result	size
meijerg	$\frac{\sqrt{a\,x^{13}}\left(\sqrt{\pi}\,x^{\frac{5}{2}}\sqrt{x^5+1}-\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)\right)}{5x^{\frac{13}{2}}\sqrt{\pi}}$	40
risch	$\frac{\sqrt{a\,x^{13}}\,\sqrt{x^5+1}}{5x^4}-\frac{\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)\sqrt{a\,x^{13}}\,\sqrt{ax(x^5+1)}}{5\sqrt{a}\,x^7\sqrt{x^5+1}}$	57

```
[In] int((a*x^13)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(a*x^13)^(1/2)/x^(13/2)/Pi^(1/2)*(Pi^(1/2)*x^(5/2)*(x^5+1)^(1/2)-Pi^(1/
2)*arcsinh(x^(5/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(36) = 72.

Time = 0.37 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.06

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \left[\frac{\sqrt{a}x^4\log\left(-\frac{8\,ax^{14}+8\,ax^9+ax^4-4\,\sqrt{ax^{13}}\,(2\,x^5+1)\sqrt{x^5+1}\sqrt{a}}{x^4}\right)+4\,\sqrt{ax^{13}}\sqrt{x^5+1}}{20\,x^4}, \frac{\sqrt{-a}x^4\arctan\left(\frac{\sqrt{ax^{13}}\,(2\,x^5+1)\sqrt{x^5+}}{2\,(ax^{14}+ax^9)}\right.}{10\,x^4}\right.$$

```
[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/20*(sqrt(a)*x^4*log(-(8*a*x^14 + 8*a*x^9 + a*x^4 - 4*sqrt(a*x^13)*(2*x^5
 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^4) + 4*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4, 1/10
*(sqrt(-a)*x^4*arctan(1/2*sqrt(a*x^13)*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(-a)/(
a*x^14 + a*x^9)) + 2*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4]
```

Sympy [F]

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^{13}}}{\sqrt{(x+1)\,(x^4-x^3+x^2-x+1)}}\,dx$$

```
[In] integrate((a*x**13)**(1/2)/(x**5+1)**(1/2),x)
[Out] Integral(sqrt(a*x**13)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^{13}}}{\sqrt{x^5+1}}\,dx$$

```
[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^13)/sqrt(x^5 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \frac{a^{\frac{11}{2}}\log\left(-\sqrt{ax}a^{\frac{5}{2}}x^2+\sqrt{a^6x^5+a^6}\right)}{5\,|a|^5} + \frac{\sqrt{a^6x^5+a^6}\sqrt{ax}x^2}{5\,a^2|a|}$$

```
[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
[Out] 1/5*a^(11/2)*log(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6))/abs(a)^5 + 1
/5*sqrt(a^6*x^5 + a^6)*sqrt(a*x)*x^2/(a^2*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{a\,x^{13}}}{\sqrt{x^5+1}}\,dx$$

```
[In] int((a*x^13)^(1/2)/(x^5 + 1)^(1/2),x)
[Out] int((a*x^13)^(1/2)/(x^5 + 1)^(1/2), x)
```

3.368 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \frac{2\sqrt{ax^3}\operatorname{arcsinh}\left(x^{5/2}\right)}{5x^{3/2}}$$

```
[Out] 2/5*arcsinh(x^(5/2))*(a*x^3)^(1/2)/x^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {15, 335, 281, 221}

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \frac{2\sqrt{ax^3}\operatorname{arcsinh}\left(x^{5/2}\right)}{5x^{3/2}}$$

```
[In] Int[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
[Out] (2*Sqrt[a*x^3]*ArcSinh[x^(5/2)])/(5*x^(3/2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx}{x^{3/2}}\\
&= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right)}{x^{3/2}}\\
&= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right)}{5x^{3/2}}\\
&= \frac{2\sqrt{ax^3}\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \frac{2\sqrt{ax^3}\log\left(x^{5/2}+\sqrt{1+x^5}\right)}{5x^{3/2}}$$

```
[In] Integrate[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
[Out] (2*Sqrt[a*x^3]*Log[x^(5/2) + Sqrt[1 + x^5]])/(5*x^(3/2))
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result	size
meijerg	$\frac{2\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)\sqrt{a\,x^{3}}}{5x^{\frac{3}{2}}}$	17

```
[In] int((a*x^3)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*arcsinh(x^(5/2))*(a*x^3)^(1/2)/x^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(16) = 32.

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 4.08

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \left[\frac{1}{10}\,\sqrt{a}\log\left(-8\,ax^{10}-8\,ax^5-4\left(2\,x^6+x\right)\sqrt{x^5+1}\sqrt{ax^3}\sqrt{a}-a\right),\right.$$
$$\left.-\frac{1}{5}\,\sqrt{-a}\arctan\left(\frac{\left(2\,x^5+1\right)\sqrt{x^5+1}\sqrt{ax^3}\sqrt{-a}}{2\,\left(ax^9+ax^4\right)}\right)\right]$$

```
[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
[Out] [1/10*sqrt(a)*log(-8*a*x^10 - 8*a*x^5 - 4*(2*x^6 + x)*sqrt(x^5 + 1)*sqrt(a*
x^3)*sqrt(a) - a), -1/5*sqrt(-a)*arctan(1/2*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(
a*x^3)*sqrt(-a)/(a*x^9 + a*x^4))]
```

Sympy [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

```
[In] integrate((a*x**3)**(1/2)/(x**5+1)**(1/2),x)
[Out] Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^5+1}}\,dx$$

```
[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^3)/sqrt(x^5 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(16) = 32$.

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.42

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = -\frac{2\,a^{\frac{3}{2}}\log\left(-\sqrt{ax}a^{\frac{5}{2}}x^2+\sqrt{a^6x^5+a^6}\right)\mathrm{sgn}(x)}{5\,|a|} + \frac{2\,a^{\frac{3}{2}}\log\left(a^2|a|\right)\mathrm{sgn}(x)}{5\,|a|}$$

```
[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
[Out] -2/5*a^(3/2)*log(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6))*sgn(x)/abs(a
) + 2/5*a^(3/2)*log(a^2*abs(a))*sgn(x)/abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{a\,x^3}}{\sqrt{x^5+1}}\,dx$$

```
[In] int((a*x^3)^(1/2)/(x^5 + 1)^(1/2),x)
[Out] int((a*x^3)^(1/2)/(x^5 + 1)^(1/2), x)
```

3.369 $\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 23

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}} \, dx = -\frac{2}{5}\sqrt{\frac{a}{x^7}}x\sqrt{1+x^5}$$

```
[Out] -2/5*x*(a/x^7)^(1/2)*(x^5+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 270}

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}} \, dx = -\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

```
[In] Int[Sqrt[a/x^7]/Sqrt[1 + x^5],x]
```

```
[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{\frac{a}{x^7}}x^{7/2}\right)\int \frac{1}{x^{7/2}\sqrt{1+x^5}}\,dx\\ &= -\frac{2}{5}\sqrt{\frac{a}{x^7}}x\sqrt{1+x^5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = -\frac{2}{5}\sqrt{\frac{a}{x^7}}x\sqrt{1+x^5}$$

```
[In] Integrate[Sqrt[a/x^7]/Sqrt[1 + x^5],x]
[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
meijerg	$-\frac{2x\sqrt{\frac{a}{x^7}}\,\sqrt{x^5+1}}{5}$	18
risch	$-\frac{2x\sqrt{\frac{a}{x^7}}\,\sqrt{x^5+1}}{5}$	18
gosper	$-\frac{2x(x+1)\left(x^4-x^3+x^2-x+1\right)\sqrt{\frac{a}{x^7}}}{5\sqrt{x^5+1}}$	37

```
[In] int((a/x^7)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/5*x*(a/x^7)^(1/2)*(x^5+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = -\frac{2}{5}\,\sqrt{x^5+1}x\sqrt{\frac{a}{x^7}}$$

```
[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
[Out] -2/5*sqrt(x^5 + 1)*x*sqrt(a/x^7)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{(x+1)\,(x^4-x^3+x^2-x+1)}}\,dx$$

```
[In] integrate((a/x**7)**(1/2)/(x**5+1)**(1/2),x)
[Out] Integral(sqrt(a/x**7)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(17) = 34.

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = -\frac{2\left(\sqrt{a}x^6+\sqrt{a}x\right)}{5\,\sqrt{x^4-x^3+x^2-x+1}\sqrt{x+1}x^{\frac{7}{2}}}$$

```
[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
[Out] -2/5*(sqrt(a)*x^6 + sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x + 1)*sqrt(x + 1)*x
^(7/2))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = -\frac{2\,a^4\left(\frac{\sqrt{a+\frac{a}{x^5}}}{a^3}-\frac{1}{a^{\frac{5}{2}}}\right)}{5\,|a|}$$

```
[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
[Out] -2/5*a^4*(sqrt(a + a/x^5)/a^3 - 1/a^(5/2))/abs(a)
```

Mupad [B] (verification not implemented)

Time = 18.65 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = -\frac{2\,x\,\sqrt{x^5+1}\,\sqrt{\frac{a}{x^7}}}{5}$$

```
[In] int((a/x^7)^(1/2)/(x^5 + 1)^(1/2),x)
[Out] -(2*x*(x^5 + 1)^(1/2)*(a/x^7)^(1/2))/5
```

3.370 $\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}} \, dx = -\frac{2}{15}\sqrt{\frac{a}{x^{17}}}x\sqrt{1+x^5}+\frac{4}{15}\sqrt{\frac{a}{x^{17}}}x^6\sqrt{1+x^5}$$

```
[Out] -2/15*x*(a/x^17)^(1/2)*(x^5+1)^(1/2)+4/15*x^6*(a/x^17)^(1/2)*(x^5+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {15, 277, 270}

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}} \, dx = \frac{4}{15}x^6\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}-\frac{2}{15}x\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

```
[In] Int[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
```

```
[Out] (-2*Sqrt[a/x^17]*x*Sqrt[1 + x^5])/15 + (4*Sqrt[a/x^17]*x^6*Sqrt[1 + x^5])/1
5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\frac{a}{x^{17}}}x^{17/2}\right)\int\frac{1}{x^{17/2}\sqrt{1+x^5}}\,dx\\
&= -\frac{2}{15}\sqrt{\frac{a}{x^{17}}}x\sqrt{1+x^5}-\frac{1}{3}\left(2\sqrt{\frac{a}{x^{17}}}x^{17/2}\right)\int\frac{1}{x^{7/2}\sqrt{1+x^5}}\,dx\\
&= -\frac{2}{15}\sqrt{\frac{a}{x^{17}}}x\sqrt{1+x^5}+\frac{4}{15}\sqrt{\frac{a}{x^{17}}}x^6\sqrt{1+x^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

$$\int\frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx=\frac{2}{15}\sqrt{\frac{a}{x^{17}}}x\sqrt{1+x^5}\left(-1+2x^5\right)$$

```
[In] Integrate[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
[Out] (2*Sqrt[a/x^17]*x*Sqrt[1 + x^5]*(-1 + 2*x^5))/15
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

method	result	size
meijerg	$-\frac{2\sqrt{\frac{a}{x^{17}}}\,x\left(-2x^5+1\right)\sqrt{x^5+1}}{15}$	25
risch	$\frac{2\sqrt{\frac{a}{x^{17}}}\,x\left(2x^{10}+x^5-1\right)}{15\sqrt{x^5+1}}$	28
gosper	$\frac{2x(x+1)\left(x^4-x^3+x^2-x+1\right)\left(2x^5-1\right)\sqrt{\frac{a}{x^{17}}}}{15\sqrt{x^5+1}}$	44

```
[In] int((a/x^17)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(a/x^17)^(1/2)*x*(-2*x^5+1)*(x^5+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx = \frac{2}{15}\left(2\,x^6-x\right)\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

```
[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
[Out] 2/15*(2*x^6 - x)*sqrt(x^5 + 1)*sqrt(a/x^17)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

```
[In] integrate((a/x**17)**(1/2)/(x**5+1)**(1/2),x)
[Out] Integral(sqrt(a/x**17)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx = \frac{2\left(2\sqrt{a}x^{11}+\sqrt{a}x^6-\sqrt{a}x\right)}{15\sqrt{x^4-x^3+x^2-x+1}\sqrt{x+1}x^{\frac{17}{2}}}$$

```
[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
[Out] 2/15*(2*sqrt(a)*x^11 + sqrt(a)*x^6 - sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x +
 1)*sqrt(x + 1)*x^(17/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 17.85 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{\frac{a}{x^{17}}}\left(\frac{4\,x^{11}}{15}+\frac{2\,x^6}{15}-\frac{2\,x}{15}\right)}{\sqrt{x^5+1}}$$

```
[In] int((a/x^17)^(1/2)/(x^5 + 1)^(1/2),x)
```

```
[Out] ((a/x^17)^(1/2)*((2*x^6)/15 - (2*x)/15 + (4*x^11)/15))/(x^5 + 1)^(1/2)
```

3.371 $\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 37

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = -\frac{\sqrt{ax^6}\arctan(x)}{2x^3} + \frac{\sqrt{ax^6}\operatorname{arctanh}(x)}{2x^3}$$

```
[Out] -1/2*arctan(x)*(a*x^6)^(1/2)/x^3+1/2*arctanh(x)*(a*x^6)^(1/2)/x^3
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {15, 304, 209, 212}

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = \frac{\sqrt{ax^6}\operatorname{arctanh}(x)}{2x^3} - \frac{\sqrt{ax^6}\arctan(x)}{2x^3}$$

```
[In] Int[Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] -1/2*(Sqrt[a*x^6]*ArcTan[x])/x^3 + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}\,dx}{x^3}\\ &= \frac{\sqrt{ax^6}\int \frac{1}{1-x^2}\,dx}{2x^3}-\frac{\sqrt{ax^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\ &= -\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = \frac{\sqrt{ax^6}(-\arctan(x)+\operatorname{arctanh}(x))}{2x^3}$$

```
[In] Integrate[Sqrt[a*x^6]/(x*(1 - x^4)),x]
[Out] (Sqrt[a*x^6]*(-ArcTan[x] + ArcTanh[x]))/(2*x^3)
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$\frac{\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{4}}}{\sqrt{a}}\right)}{2}$	18
default	$-\frac{\sqrt{a\,x^{6}}\,(\ln(x-1)-\ln(x+1)+2\arctan(x))}{4x^{3}}$	28
meijerg	$-\frac{\sqrt{a\,x^{6}}\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)+2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{3}{4}}}$	44
risch	$-\frac{\sqrt{a\,x^{6}}\,\ln(x-1)}{4x^{3}}-\frac{i\sqrt{a\,x^{6}}\,\ln(x+i)}{4x^{3}}+\frac{i\sqrt{a\,x^{6}}\,\ln(x-i)}{4x^{3}}+\frac{\sqrt{a\,x^{6}}\,\ln(x+1)}{4x^{3}}$	70

```
[In] int((a*x^6)^(1/2)/x/(-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*a^(1/2)*arctanh((a*x^4)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^6}}{x\,(1-x^4)}\,dx=-\frac{\sqrt{ax^6}\left(2\arctan\left(x\right)-\log\left(\frac{x+1}{x-1}\right)\right)}{4\,x^3}$$

```
[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="fricas")
[Out] -1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

Sympy [F]

$$\int \frac{\sqrt{ax^6}}{x\,(1-x^4)}\,dx=-\int \frac{\sqrt{ax^6}}{x^5-x}\,dx$$

```
[In] integrate((a*x**6)**(1/2)/x/(-x**4+1),x)
[Out] -Integral(sqrt(a*x**6)/(x**5 - x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = -\frac{1}{2}\sqrt{a}\arctan(x) + \frac{1}{4}\sqrt{a}\log(x+1) - \frac{1}{4}\sqrt{a}\log(x-1)$$

```
[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")
[Out] -1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = -\frac{1}{4}\left(2\arctan(x)\operatorname{sgn}(x) - \log(|x+1|)\operatorname{sgn}(x) + \log(|x-1|)\operatorname{sgn}(x)\right)\sqrt{a}$$

```
[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="giac")
[Out] -1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))
*sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = -\int \frac{\sqrt{a\,x^6}}{x\,(x^4-1)}\,dx$$

```
[In] int(-(a*x^6)^(1/2)/(x*(x^4 - 1)),x)
[Out] -int((a*x^6)^(1/2)/(x*(x^4 - 1)), x)
```

3.372 $\int \frac{\sqrt{ax^6}}{x-x^5} dx$

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \frac{\sqrt{ax^6}}{x-x^5} dx = -\frac{\sqrt{ax^6}\arctan(x)}{2x^3} + \frac{\sqrt{ax^6}\text{arctanh}(x)}{2x^3}$$

```
[Out] -1/2*arctan(x)*(a*x^6)^(1/2)/x^3+1/2*arctanh(x)*(a*x^6)^(1/2)/x^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 1598, 304, 209, 212}

$$\int \frac{\sqrt{ax^6}}{x-x^5} dx = \frac{\sqrt{ax^6}\text{arctanh}(x)}{2x^3} - \frac{\sqrt{ax^6}\arctan(x)}{2x^3}$$

```
[In] Int[Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] -1/2*(Sqrt[a*x^6]*ArcTan[x])/x^3 + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ax^6}\int \frac{x^3}{x-x^5}\,dx}{x^3} \\
&= \frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}\,dx}{x^3} \\
&= \frac{\sqrt{ax^6}\int \frac{1}{1-x^2}\,dx}{2x^3} - \frac{\sqrt{ax^6}\int \frac{1}{1+x^2}\,dx}{2x^3} \\
&= -\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3} + \frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = \frac{\sqrt{ax^6}(-\arctan(x)+\operatorname{arctanh}(x))}{2x^3}$$

```
[In] Integrate[Sqrt[a*x^6]/(x - x^5),x]
[Out] (Sqrt[a*x^6]*(-ArcTan[x] + ArcTanh[x]))/(2*x^3)
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$\frac{\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{4}}}{\sqrt{a}}\right)}{2}$	18
default	$-\frac{\sqrt{a\,x^{6}}\,(\ln(x-1)-\ln(x+1)+2\arctan(x))}{4x^{3}}$	28
meijerg	$-\frac{\sqrt{a\,x^{6}}\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)+2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{3}{4}}}$	44
risch	$-\frac{\sqrt{a\,x^{6}}\,\ln(x-1)}{4x^{3}}-\frac{i\sqrt{a\,x^{6}}\,\ln(x+i)}{4x^{3}}+\frac{i\sqrt{a\,x^{6}}\,\ln(x-i)}{4x^{3}}+\frac{\sqrt{a\,x^{6}}\,\ln(x+1)}{4x^{3}}$	70

```
[In] int((a*x^6)^(1/2)/(-x^5+x),x,method=_RETURNVERBOSE)
[Out] 1/2*a^(1/2)*arctanh((a*x^4)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = -\frac{\sqrt{ax^6}\left(2\arctan(x)-\log\left(\frac{x+1}{x-1}\right)\right)}{4\,x^3}$$

```
[In] integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="fricas")
[Out] -1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

Sympy [F]

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = -\int \frac{\sqrt{ax^6}}{x^5-x}\,dx$$

```
[In] integrate((a*x**6)**(1/2)/(-x**5+x),x)
[Out] -Integral(sqrt(a*x**6)/(x**5 - x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = -\frac{1}{2}\,\sqrt{a}\arctan\left(x\right) + \frac{1}{4}\,\sqrt{a}\log\left(x+1\right) - \frac{1}{4}\,\sqrt{a}\log\left(x-1\right)$$

```
[In] integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")
[Out] -1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = -\frac{1}{4}\left(2\,\arctan\left(x\right)\operatorname{sgn}(x) - \log\left(|x+1|\right)\operatorname{sgn}(x) + \log\left(|x-1|\right)\operatorname{sgn}(x)\right)\sqrt{a}$$

```
[In] integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="giac")
[Out] -1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))
*sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = \int \frac{\sqrt{a\,x^6}}{x-x^5}\,dx$$

```
[In] int((a*x^6)^(1/2)/(x - x^5),x)
[Out] int((a*x^6)^(1/2)/(x - x^5), x)
```

3.373 $\int \frac{(ax^6)^{3/2}}{x(1-x^4)} dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)} dx = -\frac{a\sqrt{ax^6}}{x^2} - \frac{1}{5}ax^2\sqrt{ax^6} + \frac{a\sqrt{ax^6}\arctan(x)}{2x^3} + \frac{a\sqrt{ax^6}\operatorname{arctanh}(x)}{2x^3}$$

```
[Out] -a*(a*x^6)^(1/2)/x^2-1/5*a*x^2*(a*x^6)^(1/2)+1/2*a*arctan(x)*(a*x^6)^(1/2)/
x^3+1/2*a*arctanh(x)*(a*x^6)^(1/2)/x^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {15, 308, 218, 212, 209}

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)} dx = \frac{a\sqrt{ax^6}\arctan(x)}{2x^3} + \frac{a\sqrt{ax^6}\operatorname{arctanh}(x)}{2x^3} - \frac{1}{5}ax^2\sqrt{ax^6} - \frac{a\sqrt{ax^6}}{x^2}$$

```
[In] Int[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

```
[Out] -((a*Sqrt[a*x^6])/x^2) - (a*x^2*Sqrt[a*x^6])/5 + (a*Sqrt[a*x^6]*ArcTan[x])/
(2*x^3) + (a*Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(a\sqrt{ax^6}\right)\int \frac{x^8}{1-x^4}\,dx}{x^3}\\
&= \frac{\left(a\sqrt{ax^6}\right)\int \left(-1-x^4+\frac{1}{1-x^4}\right)\,dx}{x^3}\\
&= -\frac{a\sqrt{ax^6}}{x^2}-\frac{1}{5}ax^2\sqrt{ax^6}+\frac{\left(a\sqrt{ax^6}\right)\int \frac{1}{1-x^4}\,dx}{x^3}\\
&= -\frac{a\sqrt{ax^6}}{x^2}-\frac{1}{5}ax^2\sqrt{ax^6}+\frac{\left(a\sqrt{ax^6}\right)\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\left(a\sqrt{ax^6}\right)\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= -\frac{a\sqrt{ax^6}}{x^2}-\frac{1}{5}ax^2\sqrt{ax^6}+\frac{a\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{a\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.48

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\frac{a\sqrt{ax^6}(2x(5+x^4)-5\arctan(x)-5\text{arctanh}(x))}{10x^3}$$

```
[In] Integrate[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
[Out] -1/10*(a*Sqrt[a*x^6]*(2*x*(5 + x^4) - 5*ArcTan[x] - 5*ArcTanh[x]))/x^3
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

method	result	size
pseudoelliptic	$-\frac{a\left(-\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a\,x^4}}{\sqrt{a}}\right)+\sqrt{a\,x^4}\right)}{2}$	29
default	$-\frac{(a\,x^6)^{\frac{3}{2}}(4x^5+5\ln(x-1)-5\ln(x+1)-10\arctan(x)+20x)}{20x^9}$	38
meijerg	$-\frac{(a\,x^6)^{\frac{3}{2}}(-1)^{\frac{3}{4}}\left(-\frac{4x(-1)^{\frac{1}{4}}(9x^4+45)}{45}-\frac{x(-1)^{\frac{1}{4}}\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{(x^4)^{\frac{1}{4}}}\right)}{4x^9}$	70
risch	$-\frac{a\,x^2\sqrt{a\,x^6}}{5}-\frac{a\sqrt{a\,x^6}}{x^2}+\frac{a\sqrt{a\,x^6}\,\ln(x+1)}{4x^3}-\frac{a\sqrt{a\,x^6}\,\ln(x-1)}{4x^3}-\frac{ia\sqrt{a\,x^6}\,\ln(x-i)}{4x^3}+\frac{ia\sqrt{a\,x^6}\,\ln(x+i)}{4x^3}$	100

```
[In] int((a*x^6)^(3/2)/x/(-x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/2*a*(-a^(1/2)*arctanh((a*x^4)^(1/2)/a^(1/2))+(a*x^4)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.58

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\frac{\sqrt{ax^6}\left(4\,ax^5+20\,ax-10\,a\arctan(x)-5\,a\log\left(\frac{x+1}{x-1}\right)\right)}{20\,x^3}$$

```
[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="fricas")
[Out] -1/20*sqrt(a*x^6)*(4*a*x^5 + 20*a*x - 10*a*arctan(x) - 5*a*log((x + 1)/(x -
 1)))/x^3
```

Sympy [F]

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\int \frac{(ax^6)^{\frac{3}{2}}}{x^5-x}\,dx$$

```
[In] integrate((a*x**6)**(3/2)/x/(-x**4+1),x)
[Out] -Integral((a*x**6)**(3/2)/(x**5 - x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\frac{1}{5}\,a^{\frac{3}{2}}x^5 - a^{\frac{3}{2}}x + \frac{1}{2}\,a^{\frac{3}{2}}\arctan(x) + \frac{1}{4}\,a^{\frac{3}{2}}\log(x+1) - \frac{1}{4}\,a^{\frac{3}{2}}\log(x-1)$$

```
[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="maxima")
[Out] -1/5*a^(3/2)*x^5 - a^(3/2)*x + 1/2*a^(3/2)*arctan(x) + 1/4*a^(3/2)*log(x +
1) - 1/4*a^(3/2)*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.59

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\frac{1}{20}\left(4\,x^5\mathrm{sgn}(x) + 20\,x\mathrm{sgn}(x) - 10\,\arctan(x)\,\mathrm{sgn}(x) - 5\,\log(|x+1|)\,\mathrm{sgn}(x) + 5\,\log(|x-1|)\,\mathrm{sgn}(x)\right)a^{\frac{3}{2}}$$

```
[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="giac")
[Out] -1/20*(4*x^5*sgn(x) + 20*x*sgn(x) - 10*arctan(x)*sgn(x) - 5*log(abs(x + 1))
*sgn(x) + 5*log(abs(x - 1))*sgn(x))*a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\int \frac{(a\,x^6)^{3/2}}{x\,(x^4-1)}\,dx$$

```
[In] int(-(a*x^6)^(3/2)/(x*(x^4 - 1)),x)
[Out] -int((a*x^6)^(3/2)/(x*(x^4 - 1)), x)
```

3.374 $\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 49

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right) dx=\frac{\arctan(x)}{2}+\frac{\sqrt{ax^6}\arctan(x)}{2x^3}+\frac{\text{arctanh}(x)}{2}-\frac{\sqrt{ax^6}\text{arctanh}(x)}{2x^3}$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(a*x^6)^(1/2)/x^3-1/2*arctanh(x)
*(a*x^6)^(1/2)/x^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {218, 212, 209, 15, 304}

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right) dx=\frac{\sqrt{ax^6}\arctan(x)}{2x^3}-\frac{\sqrt{ax^6}\text{arctanh}(x)}{2x^3}+\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}$$

```
[In] Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6]
*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{1-x^4}\,dx - \int \frac{\sqrt{ax^6}}{x\,(1-x^4)}\,dx \\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx - \frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}\,dx}{x^3} \\
&= \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x) - \frac{\sqrt{ax^6}\int \frac{1}{1-x^2}\,dx}{2x^3} + \frac{\sqrt{ax^6}\int \frac{1}{1+x^2}\,dx}{2x^3} \\
&= \frac{1}{2}\tan^{-1}(x) + \frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3} + \frac{1}{2}\tanh^{-1}(x) - \frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right) dx = \frac{\left(x^3+\sqrt{ax^6}\right)\arctan(x)+\left(x^3-\sqrt{ax^6}\right)\text{arctanh}(x)}{2x^3}$$

```
[In] Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
[Out] ((x^3 + Sqrt[a*x^6])*ArcTan[x] + (x^3 - Sqrt[a*x^6])*ArcTanh[x])/(2*x^3)
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}+\frac{\sqrt{a\,x^6}\,(\ln(x-1)-\ln(x+1)+2\arctan(x))}{4x^3}$	37
meijerg	$-\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}+\frac{\sqrt{a\,x^6}\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}$	82
risch	$\frac{i\ln(x+i)x^3-i\ln(x-i)x^3-\ln(x-1)x^3+\ln(x+1)x^3+i\sqrt{a\,x^6}\,\ln(x+i)-i\sqrt{a\,x^6}\,\ln(x-i)+\sqrt{a\,x^6}\,\ln(x-1)-\sqrt{a\,x^6}\,\ln(x+1)}{4x^3}$	101

```
[In] int(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/4*(a*x^6)^(1/2)*(ln(x-1)-ln(x+1)+2*arctan(x)
)/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(37) = 74.

Time = 0.29 (sec) , antiderivative size = 256, normalized size of antiderivative = 5.22

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right) dx$$
$$= \left[\frac{x^3\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}\log\left(\frac{(a-1)x^4-(a-1)x^2-2\left(x^3-\sqrt{ax^6}\right)\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}}{x^4+x^2}\right)+x^3\log(x+1)-x^3\log(x-1)-\cdots}{4\,x^3}\right.$$

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] [1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x^4 - (a -
1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))/(x
^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - lo
g(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arctan(-
(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)/((a - 1)*x^2))
+ x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/
x^3]
```

Sympy [F]

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\right)dx=-\int \frac{x}{x^5-x}\,dx-\int \left(-\frac{\sqrt{ax^6}}{x^5-x}\right)dx$$

```
[In] integrate(1/(-x**4+1)-(a*x**6)**(1/2)/x/(-x**4+1),x)
[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\right)dx=\frac{1}{2}\sqrt{a}\arctan\left(x\right)-\frac{1}{4}\sqrt{a}\log\left(x+1\right)+\frac{1}{4}\sqrt{a}\log\left(x-1\right)+\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(x+1\right)-\frac{1}{4}\log\left(x-1\right)$$

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")
[Out] 1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) + 1
/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\right)dx=\frac{1}{4}\left(2\arctan\left(x\right)\operatorname{sgn}(x)-\log\left(|x+1|\right)\operatorname{sgn}(x)+\log\left(|x-1|\right)\operatorname{sgn}(x)\right)\sqrt{a}+\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(|x+1|\right)-\frac{1}{4}\log\left(|x-1|\right)$$

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))*
sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\,(1-x^4)}\right)\,dx=\int \frac{\sqrt{a\,x^6}}{x\,\left(x^4-1\right)}-\frac{1}{x^4-1}\,dx$$

```
[In] int((a*x^6)^(1/2)/(x*(x^4 - 1)) - 1/(x^4 - 1),x)
```

```
[Out] int((a*x^6)^(1/2)/(x*(x^4 - 1)) - 1/(x^4 - 1), x)
```

3.375 $\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 49

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx=\frac{\arctan(x)}{2}+\frac{\sqrt{ax^6}\arctan(x)}{2x^3}+\frac{\operatorname{arctanh}(x)}{2}-\frac{\sqrt{ax^6}\operatorname{arctanh}(x)}{2x^3}$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(a*x^6)^(1/2)/x^3-1/2*arctanh(x)
*(a*x^6)^(1/2)/x^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {218, 212, 209, 15, 1598, 304}

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx=\frac{\sqrt{ax^6}\arctan(x)}{2x^3}-\frac{\sqrt{ax^6}\operatorname{arctanh}(x)}{2x^3}+\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}$$

```
[In] Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6]
*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{1-x^4}\,dx - \int \frac{\sqrt{ax^6}}{x-x^5}\,dx \\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx - \frac{\sqrt{ax^6}\int \frac{x^3}{x-x^5}\,dx}{x^3} \\
&= \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x) - \frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}\,dx}{x^3} \\
&= \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x) - \frac{\sqrt{ax^6}\int \frac{1}{1-x^2}\,dx}{2x^3} + \frac{\sqrt{ax^6}\int \frac{1}{1+x^2}\,dx}{2x^3} \\
&= \frac{1}{2}\tan^{-1}(x) + \frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3} + \frac{1}{2}\tanh^{-1}(x) - \frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right) dx = \frac{\left(x^3+\sqrt{ax^6}\right)\arctan(x)+\left(x^3-\sqrt{ax^6}\right)\operatorname{arctanh}(x)}{2x^3}$$

```
[In] Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
[Out] ((x^3 + Sqrt[a*x^6])*ArcTan[x] + (x^3 - Sqrt[a*x^6])*ArcTanh[x])/(2*x^3)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

method	result	siz
default	$\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}+\frac{\sqrt{a\,x^6}\,(\ln(x-1)-\ln(x+1)+2\arctan(x))}{4x^3}$	37
meijerg	$-\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}+\frac{\sqrt{a\,x^6}\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}$	82
risch	$\frac{i\ln(x+i)x^3-i\ln(x-i)x^3-\ln(x-1)x^3+\ln(x+1)x^3+i\sqrt{a\,x^6}\,\ln(x+i)-i\sqrt{a\,x^6}\,\ln(x-i)+\sqrt{a\,x^6}\,\ln(x-1)-\sqrt{a\,x^6}\,\ln(x+1)}{4x^3}$	10

```
[In] int(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/4*(a*x^6)^(1/2)*(ln(x-1)-ln(x+1)+2*arctan(x)
)/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(37) = 74.

Time = 0.32 (sec) , antiderivative size = 256, normalized size of antiderivative = 5.22

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right) dx$$

$$= \left[\frac{x^3\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}\log\left(\frac{(a-1)x^4-(a-1)x^2-2\left(x^3-\sqrt{ax^6}\right)\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}}{x^4+x^2}\right)+x^3\log(x+1)-x^3\log(x-1)-}{4\,x^3}\right.$$

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="fricas")
```

```
[Out] [1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x^4 - (a -
1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))/(x
^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - lo
g(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arctan(-
(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)/((a - 1)*x^2))
+ x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/
x^3]
```

Sympy [F]

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x-x^5}\right) dx = -\int \frac{x}{x^5-x}\, dx - \int \left(-\frac{\sqrt{ax^6}}{x^5-x}\right) dx$$

```
[In] integrate(1/(-x**4+1)-(a*x**6)**(1/2)/(-x**5+x),x)
[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x-x^5}\right) dx = \frac{1}{2}\sqrt{a}\arctan(x) - \frac{1}{4}\sqrt{a}\log(x+1) + \frac{1}{4}\sqrt{a}\log(x-1) + \frac{1}{2}\arctan(x) + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")
[Out] 1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) + 1
/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x-x^5}\right) dx = \frac{1}{4}\left(2\arctan(x)\operatorname{sgn}(x) - \log(|x+1|)\operatorname{sgn}(x) + \log(|x-1|)\operatorname{sgn}(x)\right)\sqrt{a} + \frac{1}{2}\arctan(x) + \frac{1}{4}\log(|x+1|) - \frac{1}{4}\log(|x-1|)$$

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="giac")
```

```
[Out] 1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))*
sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right) dx = \int -\frac{1}{x^4-1}-\frac{\sqrt{a\,x^6}}{x-x^5}\,dx$$

```
[In] int(- 1/(x^4 - 1) - (a*x^6)^(1/2)/(x - x^5),x)
```

```
[Out] int(- 1/(x^4 - 1) - (a*x^6)^(1/2)/(x - x^5), x)
```

3.376 $\int \frac{\sqrt{ax^3}}{x-x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx=-\frac{\sqrt{ax^3}\arctan\left(\sqrt{x}\right)}{x^{3/2}}+\frac{\sqrt{ax^3}\text{arctanh}\left(\sqrt{x}\right)}{x^{3/2}}$$

```
[Out] -arctan(x^(1/2))*(a*x^3)^(1/2)/x^(3/2)+arctanh(x^(1/2))*(a*x^3)^(1/2)/x^(3/
2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {15, 1598, 335, 304, 209, 212}

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx=\frac{\sqrt{ax^3}\text{arctanh}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{ax^3}\arctan\left(\sqrt{x}\right)}{x^{3/2}}$$

```
[In] Int[Sqrt[a*x^3]/(x - x^3),x]
```

```
[Out] -((Sqrt[a*x^3]*ArcTan[Sqrt[x]])/x^(3/2)) + (Sqrt[a*x^3]*ArcTanh[Sqrt[x]])/x
^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{x-x^3}\,dx}{x^{3/2}}\\
&= \frac{\sqrt{ax^3}\int \frac{\sqrt{x}}{1-x^2}\,dx}{x^{3/2}}\\
&= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\sqrt{x}\right)}{x^{3/2}}\\
&= \frac{\sqrt{ax^3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{ax^3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}\\
&= -\frac{\sqrt{ax^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\frac{\sqrt{ax^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx = \frac{\sqrt{ax^3}\left(-\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)\right)}{x^{3/2}}$$

```
[In] Integrate[Sqrt[a*x^3]/(x - x^3),x]
[Out] (Sqrt[a*x^3]*(-ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]))/x^(3/2)
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\left(\operatorname{arctanh}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)+\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)\sqrt{a}$	26
default	$\frac{\sqrt{a\,x^3}\,\sqrt{a}\left(\operatorname{arctanh}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)-\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)}{x\sqrt{ax}}$	43
meijerg	$-\frac{\sqrt{a\,x^3}\left(\ln\left(1-(x^2)^{\frac{1}{4}}\right)-\ln\left(1+(x^2)^{\frac{1}{4}}\right)+2\arctan\left((x^2)^{\frac{1}{4}}\right)\right)}{2(x^2)^{\frac{3}{4}}}$	44

```
[In] int((a*x^3)^(1/2)/(-x^3+x),x,method=_RETURNVERBOSE)
[Out] (arctanh((a*x)^(1/2)/a^(1/2))+arctan((a*x)^(1/2)/a^(1/2)))*a^(1/2)
```

Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(32) = 64.

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.89

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx = \left[-\sqrt{a}\arctan\left(\frac{\sqrt{ax^3}}{\sqrt{a}x}\right)+\frac{1}{2}\sqrt{a}\log\left(\frac{ax^2+ax+2\sqrt{ax^3}\sqrt{a}}{x^2-x}\right),\right.$$
$$\left.-\sqrt{-a}\arctan\left(\frac{\sqrt{ax^3}\sqrt{-a}}{ax}\right)+\frac{1}{2}\sqrt{-a}\log\left(\frac{ax^2-ax-2\sqrt{ax^3}\sqrt{-a}}{x^2+x}\right)\right]$$

```
[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="fricas")
[Out] [-sqrt(a)*arctan(sqrt(a*x^3)/(sqrt(a)*x)) + 1/2*sqrt(a)*log((a*x^2 + a*x +
2*sqrt(a*x^3)*sqrt(a))/(x^2 - x)), -sqrt(-a)*arctan(sqrt(a*x^3)*sqrt(-a)/(a
*x)) + 1/2*sqrt(-a)*log((a*x^2 - a*x - 2*sqrt(a*x^3)*sqrt(-a))/(x^2 + x))]
```

Sympy [F]

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx = -\int \frac{\sqrt{ax^3}}{x^3-x}\,dx$$

```
[In] integrate((a*x**3)**(1/2)/(-x**3+x),x)
[Out] -Integral(sqrt(a*x**3)/(x**3 - x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx = -\sqrt{a}\arctan\left(\sqrt{x}\right) + \frac{1}{2}\sqrt{a}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\sqrt{a}\log\left(\sqrt{x}-1\right)$$

```
[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="maxima")
[Out] -sqrt(a)*arctan(sqrt(x)) + 1/2*sqrt(a)*log(sqrt(x) + 1) - 1/2*sqrt(a)*log(s
qrt(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx = -\frac{\left(\frac{a^2\arctan\left(\frac{\sqrt{ax}}{\sqrt{-a}}\right)}{\sqrt{-a}} + a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)\operatorname{sgn}(x)}{a}$$

```
[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="giac")
[Out] -(a^2*arctan(sqrt(a*x)/sqrt(-a))/sqrt(-a) + a^(3/2)*arctan(sqrt(a*x)/sqrt(a
)))*sgn(x)/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^3}}{x - x^3} \, dx = \int \frac{\sqrt{a\,x^3}}{x - x^3} \,dx$$

```
[In] int((a*x^3)^(1/2)/(x - x^3),x)
[Out] int((a*x^3)^(1/2)/(x - x^3), x)
```

3.377 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^4}\sqrt{1+x^2}}{2x} - \frac{\sqrt{ax^4}\text{arcsinh}(x)}{2x^2}$$

```
[Out] -1/2*arcsinh(x)*(a*x^4)^(1/2)/x^2+1/2*(a*x^4)^(1/2)*(x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {15, 327, 221}

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{x^2+1}\sqrt{ax^4}}{2x} - \frac{\sqrt{ax^4}\text{arcsinh}(x)}{2x^2}$$

```
[In] Int[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
[Out] (Sqrt[a*x^4]*Sqrt[1 + x^2])/(2*x) - (Sqrt[a*x^4]*ArcSinh[x])/(2*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{ax^4}\int \frac{x^2}{\sqrt{1+x^2}}\,dx}{x^2}\\ &= \frac{\sqrt{ax^4}\sqrt{1+x^2}}{2x} - \frac{\sqrt{ax^4}\int \frac{1}{\sqrt{1+x^2}}\,dx}{2x^2}\\ &= \frac{\sqrt{ax^4}\sqrt{1+x^2}}{2x} - \frac{\sqrt{ax^4}\sinh^{-1}(x)}{2x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^4}\left(x\sqrt{1+x^2}+\log\left(-x+\sqrt{1+x^2}\right)\right)}{2x^2}$$

```
[In] Integrate[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
[Out] (Sqrt[a*x^4]*(x*Sqrt[1 + x^2] + Log[-x + Sqrt[1 + x^2]]))/(2*x^2)
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\sqrt{a\,x^{4}}\,\left(x\sqrt{x^{2}+1}-\operatorname{arcsinh}\left(x\right)\right)}{2x^{2}}$	27
meijerg	$\frac{\sqrt{a\,x^{4}}\,\left(\sqrt{\pi}\,x\sqrt{x^{2}+1}-\sqrt{\pi}\,\operatorname{arcsinh}\left(x\right)\right)}{2x^{2}\sqrt{\pi}}$	36
risch	$\frac{\sqrt{a\,x^{4}}\,\sqrt{x^{2}+1}}{2x}-\frac{\ln\left(x\sqrt{a}+\sqrt{a\,x^{2}+a}\right)\sqrt{a\,x^{4}}\,\sqrt{\left(x^{2}+1\right)a}}{2\sqrt{a}\,x^{2}\sqrt{x^{2}+1}}$	68

```
[In] int((a*x^4)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a*x^4)^(1/2)*(x*(x^2+1)^(1/2)-arcsinh(x))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^4}\sqrt{x^2+1}x + \sqrt{ax^4}\log\left(-x+\sqrt{x^2+1}\right)}{2\,x^2}$$

```
[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*(sqrt(a*x^4)*sqrt(x^2 + 1)*x + sqrt(a*x^4)*log(-x + sqrt(x^2 + 1)))/x^2
```

Sympy [F]

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax^4}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((a*x**4)**(1/2)/(x**2+1)**(1/2),x)
[Out] Integral(sqrt(a*x**4)/sqrt(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax^4}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^4)/sqrt(x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \frac{1}{2}\left(\sqrt{x^2+1}x + \log\left(-x+\sqrt{x^2+1}\right)\right)\sqrt{a}$$

```
[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*(sqrt(x^2 + 1)*x + log(-x + sqrt(x^2 + 1)))*sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{a\,x^4}}{\sqrt{x^2+1}}\,dx$$

```
[In] int((a*x^4)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((a*x^4)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.378 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx=\frac{2\sqrt{ax^3}\sqrt{1+x^2}}{3x}-\frac{\sqrt{ax^3}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{3x^{3/2}\sqrt{1+x^2}}$$

```
[Out] 2/3*(a*x^3)^(1/2)*(x^2+1)^(1/2)/x-1/3*(1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/2
)/cos(2*arctan(x^(1/2)))*EllipticF(sin(2*arctan(x^(1/2))),1/2*2^(1/2))*(a*x
^3)^(1/2)*((x^2+1)/(1+x)^2)^(1/2)/x^(3/2)/(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {15, 327, 335, 226}

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx=\frac{2\sqrt{x^2+1}\sqrt{ax^3}}{3x}-\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\sqrt{ax^3}\,\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{3x^{3/2}\sqrt{x^2+1}}$$

```
[In] Int[Sqrt[a*x^3]/Sqrt[1 + x^2],x]
```

```
[Out] (2*Sqrt[a*x^3]*Sqrt[1 + x^2])/(3*x) - (Sqrt[a*x^3]*(1 + x)*Sqrt[(1 + x^2)/(
1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/(3*x^(3/2)*Sqrt[1 + x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{1+x^2}}\,dx}{x^{3/2}}\\
&= \frac{2\sqrt{ax^3}\sqrt{1+x^2}}{3x} - \frac{\sqrt{ax^3}\int \frac{1}{\sqrt{x}\sqrt{1+x^2}}\,dx}{3x^{3/2}}\\
&= \frac{2\sqrt{ax^3}\sqrt{1+x^2}}{3x} - \frac{\left(2\sqrt{ax^3}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \sqrt{x}\right)}{3x^{3/2}}\\
&= \frac{2\sqrt{ax^3}\sqrt{1+x^2}}{3x} - \frac{\sqrt{ax^3}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{3x^{3/2}\sqrt{1+x^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \frac{2\sqrt{ax^3}\left(\sqrt{1+x^2} - \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^2\right)\right)}{3x}$$

```
[In] Integrate[Sqrt[a*x^3]/Sqrt[1 + x^2],x]
[Out] (2*Sqrt[a*x^3]*(Sqrt[1 + x^2] - Hypergeometric2F1[1/4, 1/2, 5/4, -x^2]))/(3
*x)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.27

method	result	size
meijerg	$\frac{2\sqrt{a\,x^{3}}\,x\,{}_{2}F_{1}\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};-x^{2}\right)}{5}$	22
default	$-\frac{\sqrt{a\,x^{3}}\,\left(i\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\,F\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)-2x^{3}-2x\right)}{3x^{2}\sqrt{x^{2}+1}}$	76
risch	$\frac{2\sqrt{a\,x^{3}}\,\sqrt{x^{2}+1}}{3x}-\frac{i\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{i(x-i)}\,\sqrt{ix}\,F\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)\sqrt{a\,x^{3}}\,\sqrt{ax(x^{2}+1)}}{3\sqrt{a\,x^{3}+ax}\,x^{2}\sqrt{x^{2}+1}}$	104

```
[In] int((a*x^3)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*(a*x^3)^(1/2)*x*hypergeom([1/2,5/4],[9/4],-x^2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = -\frac{2\left(\sqrt{a}x\mathrm{weierstrassPInverse}(-4,0,x)-\sqrt{ax^3}\sqrt{x^2+1}\right)}{3\,x}$$

```
[In] integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] -2/3*(sqrt(a)*x*weierstrassPInverse(-4, 0, x) - sqrt(a*x^3)*sqrt(x^2 + 1))/
x
```

Sympy [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((a*x**3)**(1/2)/(x**2+1)**(1/2),x)
[Out] Integral(sqrt(a*x**3)/sqrt(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{a\,x^3}}{\sqrt{x^2+1}}\,dx$$

```
[In] int((a*x^3)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((a*x^3)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.379 $\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}} \, dx = \frac{\sqrt{ax^2}\sqrt{1+x^2}}{x}$$

```
[Out] (a*x^2)^(1/2)*(x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 267}

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}} \, dx = \frac{\sqrt{x^2+1}\sqrt{ax^2}}{x}$$

```
[In] Int[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{ax^2}\int \frac{x}{\sqrt{1+x^2}}\,dx}{x}$$
$$= \frac{\sqrt{ax^2}\sqrt{1+x^2}}{x}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^2}\sqrt{1+x^2}}{x}$$

```
[In] Integrate[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{\sqrt{a\,x^2}\,\sqrt{x^2+1}}{x}$	19
default	$\frac{\sqrt{a\,x^2}\,\sqrt{x^2+1}}{x}$	19
risch	$\frac{\sqrt{a\,x^2}\,\sqrt{x^2+1}}{x}$	19
meijerg	$\frac{\sqrt{a\,x^2}\,\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x^2+1}\right)}{2x\sqrt{\pi}}$	34

```
[In] int((a*x^2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] (a*x^2)^(1/2)*(x^2+1)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^2}\sqrt{x^2+1}}{x}$$

```
[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(a*x^2)*sqrt(x^2 + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^2}\sqrt{x^2+1}}{x}$$

```
[In] integrate((a*x**2)**(1/2)/(x**2+1)**(1/2),x)
[Out] sqrt(a*x**2)*sqrt(x**2 + 1)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{a}x^2+\sqrt{a}}{\sqrt{x^2+1}}$$

```
[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] (sqrt(a)*x^2 + sqrt(a))/sqrt(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \left(\sqrt{x^2+1}\mathrm{sgn}(x)-\mathrm{sgn}(x)\right)\sqrt{a}$$

```
[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] (sqrt(x^2 + 1)*sgn(x) - sgn(x))*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 16.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{a}\,\sqrt{x^2+1}\,\sqrt{x^2}}{x}$$

```
[In] int((a*x^2)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] (a^(1/2)*(x^2 + 1)^(1/2)*(x^2)^(1/2))/x
```

3.380 $\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 131

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \frac{2\sqrt{ax}\sqrt{1+x^2}}{1+x} - \frac{2\sqrt{a}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}E\left(2\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}} + \frac{\sqrt{a}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right),\frac{1}{2}\right)}{\sqrt{1+x^2}}$$

```
[Out] 2*(a*x)^(1/2)*(x^2+1)^(1/2)/(1+x)-2*(1+x)*(cos(2*arctan((a*x)^(1/2)/a^(1/2)
))^2)^(1/2)/cos(2*arctan((a*x)^(1/2)/a^(1/2)))*EllipticE(sin(2*arctan((a*x)
^(1/2)/a^(1/2))),1/2*2^(1/2))*a^(1/2)*((x^2+1)/(1+x)^2)^(1/2)/(x^2+1)^(1/2)
+(1+x)*(cos(2*arctan((a*x)^(1/2)/a^(1/2)))^2)^(1/2)/cos(2*arctan((a*x)^(1/2
)/a^(1/2)))*EllipticF(sin(2*arctan((a*x)^(1/2)/a^(1/2))),1/2*2^(1/2))*a^(1/
2)*((x^2+1)/(1+x)^2)^(1/2)/(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {335, 311, 226, 1210}

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{a}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right),\frac{1}{2}\right)}{\sqrt{x^2+1}} - \frac{2\sqrt{a}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}E\left(2\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}} + \frac{2\sqrt{x^2+1}\sqrt{ax}}{x+1}$$

```
[In] Int[Sqrt[a*x]/Sqrt[1 + x^2],x]
```

```
[Out] (2*Sqrt[a*x]*Sqrt[1 + x^2])/(1 + x) - (2*Sqrt[a]*(1 + x)*Sqrt[(1 + x^2)/(1
+ x)^2]*EllipticE[2*ArcTan[Sqrt[a*x]/Sqrt[a]], 1/2])/Sqrt[1 + x^2] + (Sqrt[
a]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[a*x]/Sqrt[a]],
 1/2])/Sqrt[1 + x^2]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \sqrt{ax}\right)}{a}\\
&= 2\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \sqrt{ax}\right) - 2\text{Subst}\left(\int \frac{1-\frac{x^2}{a}}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \sqrt{ax}\right)\\
&= \frac{2\sqrt{ax}\sqrt{1+x^2}}{1+x} - \frac{2\sqrt{a}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}}\\
&\quad + \frac{\sqrt{a}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}} \, dx = \frac{2}{3} x\sqrt{ax} \, \text{Hypergeometric2F1} \left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -x^2\right)$$

```
[In] Integrate[Sqrt[a*x]/Sqrt[1 + x^2],x]
[Out] (2*x*Sqrt[a*x]*Hypergeometric2F1[1/2, 3/4, 7/4, -x^2])/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.15

method	result	size
meijerg	$\frac{2\sqrt{ax}\, x\, {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-x^2\right)}{3}$	20
default	$\frac{\sqrt{ax}\,\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\,\left(2E\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)-F\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)\right)}{\sqrt{x^2+1}\,x}$	81
elliptic	$\frac{i\sqrt{ax}\,\sqrt{ax(x^2+1)}\,\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{i(x-i)}\,\sqrt{ix}\,\left(-2iE\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)+iF\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)\right)}{\sqrt{x^2+1}\,x\sqrt{a\,x^3+ax}}$	104

```
[In] int((a*x)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(a*x)^(1/2)*x*hypergeom([1/2,3/4],[7/4],-x^2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}} \, dx = -2\sqrt{a}\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, x))$$

```
[In] integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, x))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{a}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| x^2e^{i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((a*x)**(1/2)/(x**2+1)**(1/2),x)
[Out] sqrt(a)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**2*exp_polar(I*pi))
/(2*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x)/sqrt(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x)/sqrt(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{a\,x}}{\sqrt{x^2+1}}\,dx$$

```
[In] int((a*x)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((a*x)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.381 $\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx=\frac{\sqrt{\frac{a}{x}}\sqrt{x}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{\sqrt{1+x^2}}$$

```
[Out] (1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/2)/cos(2*arctan(x^(1/2)))*EllipticF(sin
(2*arctan(x^(1/2))),1/2*2^(1/2))*(a/x)^(1/2)*x^(1/2)*((x^2+1)/(1+x)^2)^(1/2
)/(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {15, 335, 226}

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx=\frac{\sqrt{x}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\sqrt{\frac{a}{x}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

```
[In] Int[Sqrt[a/x]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a/x]*Sqrt[x]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqr
t[x]], 1/2])/Sqrt[1 + x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{\frac{a}{x}}\sqrt{x}\right)\int\frac{1}{\sqrt{x}\sqrt{1+x^2}}\,dx\\ &= \left(2\sqrt{\frac{a}{x}}\sqrt{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+x^4}}\,dx,x,\sqrt{x}\right)\\ &= \frac{\sqrt{\frac{a}{x}}\sqrt{x}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.50

$$\int\frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx = 2\sqrt{\frac{a}{x}}x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^2\right)$$

```
[In] Integrate[Sqrt[a/x]/Sqrt[1 + x^2],x]
[Out] 2*Sqrt[a/x]*x*Hypergeometric2F1[1/4, 1/2, 5/4, -x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.41

method	result	size
meijerg	$2\sqrt{\frac{a}{x}}\, x_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^2\right)$	22
default	$\frac{i\sqrt{\frac{a}{x}}\,\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\,F\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^2+1}}$	62

[In] int((a/x)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*(a/x)^(1/2)*x*hypergeom([1/4,1/2],[5/4],-x^2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx = 2\sqrt{a}\text{weierstrassPInverse}(-4,0,x)$$

[In] integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(a)*weierstrassPInverse(-4, 0, x)

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

[In] integrate((a/x)**(1/2)/(x**2+1)**(1/2),x)

[Out] Integral(sqrt(a/x)/sqrt(x**2 + 1), x)

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

[In] integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a/x)/sqrt(x^2 + 1), x)

Giac [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a/x)/sqrt(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

```
[In] int((a/x)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((a/x)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.382 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}} \, dx = -\sqrt{\frac{a}{x^2}} x \operatorname{arctanh}\left(\sqrt{1+x^2}\right)$$

```
[Out] -x*arctanh((x^2+1)^(1/2))*(a/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {15, 272, 65, 213}

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}} \, dx = x \left(-\sqrt{\frac{a}{x^2}}\right) \operatorname{arctanh}\left(\sqrt{x^2+1}\right)$$

```
[In] Int[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
```

```
[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\frac{a}{x^2}}x\right)\int \frac{1}{x\sqrt{1+x^2}}\,dx\\
&= \frac{1}{2}\left(\sqrt{\frac{a}{x^2}}x\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^2\right)\\
&= \left(\sqrt{\frac{a}{x^2}}x\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^2}\right)\\
&= -\sqrt{\frac{a}{x^2}}x\tanh^{-1}\left(\sqrt{1+x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = -\sqrt{\frac{a}{x^2}}x\text{arctanh}\left(\sqrt{1+x^2}\right)$$

```
[In] Integrate[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$-\sqrt{\frac{a}{x^{2}}}\, x \,\operatorname{arctanh}\left(\frac{1}{\sqrt{x^{2}+1}}\right)$	19
meijerg	$\frac{\sqrt{\frac{a}{x^{2}}}\, x \left(-2 \sqrt{\pi}\, \ln\left(\frac{1}{2}+\frac{\sqrt{x^{2}+1}}{2}\right)+\left(-2 \ln \left(2\right)+2 \ln \left(x \right)\right) \sqrt{\pi}\right)}{2 \sqrt{\pi}}$	45

```
[In] int((a/x^2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(a/x^2)^(1/2)*x*arctanh(1/(x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(18) = 36.

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.45

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = \left[x\sqrt{\frac{a}{x^2}}\log\left(\frac{\sqrt{x^2+1}-1}{x}\right), 2\sqrt{-a}\arctan\left(-\frac{\sqrt{-a}x^2\sqrt{\frac{a}{x^2}}-\sqrt{x^2+1}\sqrt{-a}x\sqrt{\frac{a}{x^2}}}{a}\right)\right]$$

```
[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] [x*sqrt(a/x^2)*log((sqrt(x^2 + 1) - 1)/x), 2*sqrt(-a)*arctan(-(sqrt(-a)*x^2
*sqrt(a/x^2) - sqrt(x^2 + 1)*sqrt(-a)*x*sqrt(a/x^2))/a)]
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((a/x**2)**(1/2)/(x**2+1)**(1/2),x)
[Out] Integral(sqrt(a/x**2)/sqrt(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a/x^2)/sqrt(x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = -\frac{1}{2}\sqrt{a}\left(\log\left(\sqrt{x^2+1}+1\right)-\log\left(\sqrt{x^2+1}-1\right)\right)\operatorname{sgn}(x)$$

```
[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(a)*(log(sqrt(x^2 + 1) + 1) - log(sqrt(x^2 + 1) - 1))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

```
[In] int((a/x^2)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((a/x^2)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.383 $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^2} + \frac{2\sqrt{\frac{a}{x^3}}x^2\sqrt{1+x^2}}{1+x} - \frac{2\sqrt{\frac{a}{x^3}}x^{3/2}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}E\left(2\arctan\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}} + \frac{\sqrt{\frac{a}{x^3}}x^{3/2}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{\sqrt{1+x^2}}$$

```
[Out] -2*x*(a/x^3)^(1/2)*(x^2+1)^(1/2)+2*x^2*(a/x^3)^(1/2)*(x^2+1)^(1/2)/(1+x)-2*
x^(3/2)*(1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/2)/cos(2*arctan(x^(1/2)))*Ellip
ticE(sin(2*arctan(x^(1/2))),1/2*2^(1/2))*(a/x^3)^(1/2)*((x^2+1)/(1+x)^2)^(1
/2)/(x^2+1)^(1/2)+x^(3/2)*(1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/2)/cos(2*arct
an(x^(1/2)))*EllipticF(sin(2*arctan(x^(1/2))),1/2*2^(1/2))*(a/x^3)^(1/2)*((
x^2+1)/(1+x)^2)^(1/2)/(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used

$= \{15, 331, 335, 311, 226, 1210\}$

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = \frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}x^{3/2}\sqrt{\frac{a}{x^3}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{\sqrt{x^2+1}} - \frac{2(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}x^{3/2}\sqrt{\frac{a}{x^3}}E\left(2\arctan\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}} + \frac{2\sqrt{x^2+1}x^2\sqrt{\frac{a}{x^3}}}{x+1} - 2\sqrt{x^2+1}x\sqrt{\frac{a}{x^3}}$$

```
[In] Int[Sqrt[a/x^3]/Sqrt[1 + x^2],x]
```

```
[Out] -2*Sqrt[a/x^3]*x*Sqrt[1 + x^2] + (2*Sqrt[a/x^3]*x^2*Sqrt[1 + x^2])/(1 + x)
- (2*Sqrt[a/x^3]*x^(3/2)*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticE[2*ArcT
an[Sqrt[x]], 1/2])/Sqrt[1 + x^2] + (Sqrt[a/x^3]*x^(3/2)*(1 + x)*Sqrt[(1 + x
^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/Sqrt[1 + x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\sqrt{\frac{a}{x^3}}x^{3/2}\right)\int \frac{1}{x^{3/2}\sqrt{1+x^2}}\,dx \\
&= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^2}+\left(\sqrt{\frac{a}{x^3}}x^{3/2}\right)\int \frac{\sqrt{x}}{\sqrt{1+x^2}}\,dx \\
&= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^2}+\left(2\sqrt{\frac{a}{x^3}}x^{3/2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx,x,\sqrt{x}\right) \\
&= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^2}+\left(2\sqrt{\frac{a}{x^3}}x^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx,x,\sqrt{x}\right) \\
&\quad -\left(2\sqrt{\frac{a}{x^3}}x^{3/2}\right)\text{Subst}\left(\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx,x,\sqrt{x}\right) \\
&= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^2}+\frac{2\sqrt{\frac{a}{x^3}}x^2\sqrt{1+x^2}}{1+x} \\
&\quad -\frac{2\sqrt{\frac{a}{x^3}}x^{3/2}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}E\left(2\tan^{-1}\left(\sqrt{x}\right)\middle|\frac{1}{2}\right)}{\sqrt{1+x^2}} \\
&\quad +\frac{\sqrt{\frac{a}{x^3}}x^{3/2}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)\middle|\frac{1}{2}\right)}{\sqrt{1+x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.17

$$
\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = -2\sqrt{\frac{a}{x^3}}x\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-x^2\right)
$$

```
[In] Integrate[Sqrt[a/x^3]/Sqrt[1 + x^2],x]
[Out] -2*Sqrt[a/x^3]*x*Hypergeometric2F1[-1/4, 1/2, 3/4, -x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.14

method	result	size
meijerg	$-2\sqrt{\frac{a}{x^{3}}}\, x\,{}_{2}F_{1}\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-x^{2}\right)$	22
default	$\frac{\sqrt{\frac{a}{x^{3}}}\, x \left(2\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\, E\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)-\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\, F\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)-2x^{2}-2\right)}{\sqrt{x^{2}+1}}$	116
risch	$-2x\sqrt{\frac{a}{x^{3}}}\,\sqrt{x^{2}+1}+\frac{i\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{i(x-i)}\,\sqrt{ix}\,\left(-2iE\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)+iF\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)\right)\sqrt{\frac{a}{x^{3}}}\, x\sqrt{ax(x^{2}+1)}}{\sqrt{a\,x^{3}+ax}\,\sqrt{x^{2}+1}}$	122

```
[In] int((a/x^3)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(a/x^3)^(1/2)*x*hypergeom([-1/4,1/2],[3/4],-x^2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = -2\sqrt{x^2+1}x\sqrt{\frac{a}{x^3}}-2\sqrt{a}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,x))$$

```
[In] integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(x^2 + 1)*x*sqrt(a/x^3) - 2*sqrt(a)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, x))
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((a/x**3)**(1/2)/(x**2+1)**(1/2),x)
[Out] Integral(sqrt(a/x**3)/sqrt(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

```
[In] int((a/x^3)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((a/x^3)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.384 $\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^2}$$

```
[Out] -x*(a/x^4)^(1/2)*(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 270}

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

```
[In] Int[Sqrt[a/x^4]/Sqrt[1 + x^2],x]
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(\sqrt{\frac{a}{x^4}}x^2\right)\int \frac{1}{x^2\sqrt{1+x^2}}\,dx$$
$$= -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^2}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^2}$$

```
[In] Integrate[Sqrt[a/x^4]/Sqrt[1 + x^2],x]
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-x\sqrt{\frac{a}{x^4}}\,\sqrt{x^2+1}$	18
default	$-x\sqrt{\frac{a}{x^4}}\,\sqrt{x^2+1}$	18
meijerg	$-x\sqrt{\frac{a}{x^4}}\,\sqrt{x^2+1}$	18
risch	$-x\sqrt{\frac{a}{x^4}}\,\sqrt{x^2+1}$	18

```
[In] int((a/x^4)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -x*(a/x^4)^(1/2)*(x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -x^2\sqrt{\frac{a}{x^4}} - \sqrt{x^2+1}x\sqrt{\frac{a}{x^4}}$$

```
[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] -x^2*sqrt(a/x^4) - sqrt(x^2 + 1)*x*sqrt(a/x^4)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((a/x**4)**(1/2)/(x**2+1)**(1/2),x)
[Out] Integral(sqrt(a/x**4)/sqrt(x**2 + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -\frac{\sqrt{a}x^2+\sqrt{a}}{\sqrt{x^2+1}x}$$

```
[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] -(sqrt(a)*x^2 + sqrt(a))/(sqrt(x^2 + 1)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = \frac{2\sqrt{a}}{\left(x-\sqrt{x^2+1}\right)^2-1}$$

```
[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(a)/((x - sqrt(x^2 + 1))^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 16.62 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -\sqrt{a}\,x\,\sqrt{x^2+1}\,\sqrt{\frac{1}{x^4}}$$

```
[In] int((a/x^4)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] -a^(1/2)*x*(x^2 + 1)^(1/2)*(1/x^4)^(1/2)
```

3.385 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{ax^4}\sqrt{1+x^3}}{3x^2}$$

```
[Out] 2/3*(a*x^4)^(1/2)*(x^3+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 267}

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{x^3+1}\sqrt{ax^4}}{3x^2}$$

```
[In] Int[Sqrt[a*x^4]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{ax^4}\int \frac{x^2}{\sqrt{1+x^3}}\,dx}{x^2}\\ &= \frac{2\sqrt{ax^4}\sqrt{1+x^3}}{3x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{ax^4}\sqrt{1+x^3}}{3x^2}$$

```
[In] Integrate[Sqrt[a*x^4]/Sqrt[1 + x^3],x]
[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{2\sqrt{a\,x^4}\,\sqrt{x^3+1}}{3x^2}$	20
risch	$\frac{2\sqrt{a\,x^4}\,\sqrt{x^3+1}}{3x^2}$	20
gosper	$\frac{2(x+1)(x^2-x+1)\sqrt{a\,x^4}}{3x^2\sqrt{x^3+1}}$	31
meijerg	$\frac{\sqrt{a\,x^4}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x^3+1}\right)}{3x^2\sqrt{\pi}}$	34

```
[In] int((a*x^4)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(a*x^4)^(1/2)*(x^3+1)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\,\sqrt{ax^4}\sqrt{x^3+1}}{3\,x^2}$$

```
[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(a*x^4)*sqrt(x^3 + 1)/x^2
```

Sympy [F]

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^4}}{\sqrt{(x+1)\,(x^2-x+1)}}\,dx$$

```
[In] integrate((a*x**4)**(1/2)/(x**3+1)**(1/2),x)
[Out] Integral(sqrt(a*x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\,(\sqrt{a}x^3+\sqrt{a})}{3\,\sqrt{x^2-x+1}\sqrt{x+1}}$$

```
[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] 2/3*(sqrt(a)*x^3 + sqrt(a))/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2}{3}\,\sqrt{x^3+1}\sqrt{a}$$

```
[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(x^3 + 1)*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 16.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a x^4}}{\sqrt{1+x^3}}\,dx = \frac{2\,\sqrt{a}\,\sqrt{x^3+1}\,\sqrt{x^4}}{3\,x^2}$$

```
[In] int((a*x^4)^(1/2)/(x^3 + 1)^(1/2),x)
[Out] (2*a^(1/2)*(x^3 + 1)^(1/2)*(x^4)^(1/2))/(3*x^2)
```

3.386 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 292

$$\begin{aligned}\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx &= \frac{\left(1+\sqrt{3}\right)\sqrt{ax^3}\sqrt{1+x^3}}{x\left(1+\left(1+\sqrt{3}\right)x\right)} \\ &- \frac{\sqrt[4]{3}\sqrt{ax^3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}E\left(\arccos\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{x\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}} \\ &- \frac{\left(1-\sqrt{3}\right)\sqrt{ax^3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}x\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}\end{aligned}$$

```
[Out] (1+3^(1/2))*(a*x^3)^(1/2)*(x^3+1)^(1/2)/x/(1+x*(1+3^(1/2)))-3^(1/4)*(1+x)*(
(1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2)/(1+x*(1-3^(1/2)))*(1+x*(1+3^
(1/2)))*EllipticE((1-(1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2),1/4*6^(
1/2)+1/4*2^(1/2))*(a*x^3)^(1/2)*((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)/x/(x^
3+1)^(1/2)/(x*(1+x)/(1+x*(1+3^(1/2)))^2)^(1/2)-1/6*(1+x)*((1+x*(1-3^(1/2)))
^2/(1+x*(1+3^(1/2)))^2)^(1/2)/(1+x*(1-3^(1/2)))*(1+x*(1+3^(1/2)))*EllipticF
((1-(1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))
*(1-3^(1/2))*(a*x^3)^(1/2)*((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/x/
(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {15, 335, 314, 231, 1895}

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx$$
$$= -\frac{\left(1-\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{ax^3}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}x\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$
$$-\frac{\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{ax^3}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{x\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$
$$+\frac{\left(1+\sqrt{3}\right)\sqrt{x^3+1}\sqrt{ax^3}}{x\left(\left(1+\sqrt{3}\right)x+1\right)}$$

```
[In] Int[Sqrt[a*x^3]/Sqrt[1 + x^3],x]
```

```
[Out] ((1 + Sqrt[3])*Sqrt[a*x^3]*Sqrt[1 + x^3])/(x*(1 + (1 + Sqrt[3])*x)) - (3^(1
/4)*Sqrt[a*x^3]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*Ellipti
cE[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(
x*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3]) - ((1 - Sqrt[3])
*Sqrt[a*x^3]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[
ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(2*3
^(1/4)*x*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{1+x^3}}\,dx}{x^{3/2}} \\
&= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)}{x^{3/2}} \\
&= -\frac{\sqrt{ax^3}\text{Subst}\left(\int \frac{-1+\sqrt{3}-2x^4}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)}{x^{3/2}} + \frac{\left(\left(-1+\sqrt{3}\right)\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)}{x^{3/2}}
\end{aligned}$$

$$= \frac{\left(1+\sqrt{3}\right)\sqrt{ax^3}\sqrt{1+x^3}}{x\left(1+\left(1+\sqrt{3}\right)x\right)} - \frac{\sqrt[4]{3}\sqrt{ax^3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}E\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{x\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}} - \frac{\left(1-\sqrt{3}\right)\sqrt{ax^3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}x\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \frac{2}{5}x\sqrt{ax^3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-x^3\right)$$

```
[In] Integrate[Sqrt[a*x^3]/Sqrt[1 + x^3],x]
[Out] (2*x*Sqrt[a*x^3]*Hypergeometric2F1[1/2, 5/6, 11/6, -x^3])/5
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 2.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.08

method	result	size
meijerg	$\frac{2\sqrt{a\,x^3}\,x\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-x^3\right)}{5}$	22
default	Expression too large to display	1521

```
[In] int((a*x^3)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*(a*x^3)^(1/2)*x*hypergeom([1/2,5/6],[11/6],-x^3)
```

Fricas [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

Sympy [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{(x+1)\,(x^2-x+1)}}\,dx$$

```
[In] integrate((a*x**3)**(1/2)/(x**3+1)**(1/2),x)
[Out] Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{a\,x^3}}{\sqrt{x^3+1}}\,dx$$

```
[In] int((a*x^3)^(1/2)/(x^3 + 1)^(1/2),x)
[Out] int((a*x^3)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.387 $\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}} dx$

Optimal result

Integrand size = 19, antiderivative size = 260

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}} dx = \frac{2\sqrt{ax^2}\sqrt{1+x^3}}{x\left(1+\sqrt{3}+x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{ax^2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{x\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{2}\sqrt{ax^2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 2*(a*x^2)^(1/2)*(x^3+1)^(1/2)/x/(1+x+3^(1/2))+2/3*(1+x)*EllipticF((1+x-3^(1
/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(a*x^2)^(1/2)*((x^2-x+1)/(1+x+3^(
1/2))^2)^(1/2)*3^(3/4)/x/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-3^(1/4
)*(1+x)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(a*x^2)^(1/2)*
(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/x/(x^3+1)^(1/2)
/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {15, 309, 224, 1891}

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{ax^2}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{ax^2}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{x\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} + \frac{2\sqrt{x^3+1}\sqrt{ax^2}}{x\left(x+\sqrt{3}+1\right)}$$

```
[In] Int[Sqrt[a*x^2]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a*x^2]*Sqrt[1 + x^3])/(x*(1 + Sqrt[3] + x)) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*Sqrt[a*x^2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE
[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(x*Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2]*Sqrt[a*x^2]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*x*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
```

```
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ax^2}\int \frac{x}{\sqrt{1+x^3}}\,dx}{x}\\
&= \frac{\sqrt{ax^2}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{x} + \frac{\left(\left(-1+\sqrt{3}\right)\sqrt{ax^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{x}\\
&= \frac{2\sqrt{ax^2}\sqrt{1+x^3}}{x\left(1+\sqrt{3}+x\right)}\\
&\quad - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{ax^2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{x\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\\
&\quad + \frac{2\sqrt{2}\sqrt{ax^2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \frac{1}{2}x\sqrt{ax^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

```
[In] Integrate[Sqrt[a*x^2]/Sqrt[1 + x^3],x]
[Out] (x*Sqrt[a*x^2]*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.08

method	result
meijerg	$\frac{\sqrt{a\,x^2}\,x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)}{2}$
default	$\frac{\sqrt{a\,x^2}\left(i\sqrt{3}-3\right)\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\left(iE\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{3+i\sqrt{3}}}\right)\sqrt{3}-iF\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{3+i\sqrt{3}}}\right)\sqrt{3}+3E\left(\sqrt{-\right.\right.}{2x\sqrt{x^3+1}}$

```
[In] int((a*x^2)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a*x^2)^(1/2)*x*hypergeom([1/2,2/3],[5/3],-x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.07

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = -\frac{2\sqrt{ax^2}\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))}{x}$$

```
[In] integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(a*x^2)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x))/x
```

Sympy [F]

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^2}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

```
[In] integrate((a*x**2)**(1/2)/(x**3+1)**(1/2),x)
[Out] Integral(sqrt(a*x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^2}}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^2)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^2}}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^2)/sqrt(x^3 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{a\,x^2}}{\sqrt{x^3+1}}\,dx$$

```
[In] int((a*x^2)^(1/2)/(x^3 + 1)^(1/2),x)
[Out] int((a*x^2)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.388 $\int \frac{\sqrt{ax}}{\sqrt{1+x^3}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}} \, dx = \frac{2}{3}\sqrt{a}\text{arcsinh}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

```
[Out] 2/3*arcsinh((a*x)^(3/2)/a^(3/2))*a^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {335, 281, 221}

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}} \, dx = \frac{2}{3}\sqrt{a}\text{arcsinh}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

```
[In] Int[Sqrt[a*x]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a]*ArcSinh[(a*x)^(3/2)/a^(3/2)])/3
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^6}{a^3}}}\,dx, x, \sqrt{ax}\right)}{a}\\ &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a^3}}}\,dx, x, (ax)^{3/2}\right)}{3a}\\ &= \frac{2}{3}\sqrt{a}\sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{ax}\log\left(x^{3/2}+\sqrt{1+x^3}\right)}{3\sqrt{x}}$$

```
[In] Integrate[Sqrt[a*x]/Sqrt[1 + x^3],x]
[Out] (2*Sqrt[a*x]*Log[x^(3/2) + Sqrt[1 + x^3]])/(3*Sqrt[x])
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

method	result
meijerg	$\frac{2\sqrt{ax}\,\operatorname{arcsinh}\left(x^{\frac{3}{2}}\right)}{3\sqrt{x}}$
default	$\frac{2\sqrt{ax}\,\sqrt{x^3+1}\,\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(x^3+1\right)a}}{x^2\sqrt{a}}\right)}{3\sqrt{x(x^3+1)a}}$
elliptic	$-\frac{2\sqrt{ax}\,\sqrt{x(x^3+1)a}\,\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x+1)}}\,(x+1)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(x+1)}}\,\sqrt{-\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x+1)}}\,\left(-F\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x+1)}},\sqrt{\ldots}\right.\right.}{\sqrt{x^3+1}\,x\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{ax(x+1)\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}\ldots\right.}}$

```
[In] int((a*x)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(a*x)^(1/2)/x^(1/2)*arcsinh(x^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(15) = 30$.

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.70

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \left[\frac{1}{6}\sqrt{a}\log\left(-8\,ax^6 - 8\,ax^3 - 4\left(2\,x^4 + x\right)\sqrt{x^3+1}\sqrt{ax}\sqrt{a} - a\right),\ -\frac{1}{3}\sqrt{-a}\arctan\left(\frac{2\sqrt{x^3+1}\sqrt{ax}\sqrt{-a}x}{2\,ax^3 + a}\right)\right]$$

```
[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] [1/6*sqrt(a)*log(-8*a*x^6 - 8*a*x^3 - 4*(2*x^4 + x)*sqrt(x^3 + 1)*sqrt(a*x)
*sqrt(a) - a), -1/3*sqrt(-a)*arctan(2*sqrt(x^3 + 1)*sqrt(a*x)*sqrt(-a)*x/(2
*a*x^3 + a))]
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{a}\operatorname{asinh}\left(x^{\frac{3}{2}}\right)}{3}$$

```
[In] integrate((a*x)**(1/2)/(x**3+1)**(1/2),x)
[Out] 2*sqrt(a)*asinh(x**(3/2))/3
```

Maxima [F]

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x)/sqrt(x^3 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = -\frac{2\,a^{\frac{5}{2}}\log\left(-\sqrt{ax}a^{\frac{3}{2}}x+\sqrt{a^4x^3+a^4}\right)}{3\,|a|^2}$$

```
[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] -2/3*a^(5/2)*log(-sqrt(a*x)*a^(3/2)*x + sqrt(a^4*x^3 + a^4))/abs(a)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{a\,x}}{\sqrt{x^3+1}}\,dx$$

```
[In] int((a*x)^(1/2)/(x^3 + 1)^(1/2),x)
[Out] int((a*x)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.389 $\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \frac{\sqrt{\frac{a}{x}}x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 1/3*x*(1+x)*((1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2)/(1+x*(1-3^(1/2)
))*(1+x*(1+3^(1/2)))*EllipticF((1-(1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a/x)^(1/2)*((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^
(1/2)*3^(3/4)/(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {15, 335, 231}

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \frac{x(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{\frac{a}{x}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

```
[In] Int[Sqrt[a/x]/Sqrt[1 + x^3],x]
```

```
[Out] (Sqrt[a/x]*x*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[
ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(
1/4)*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{\frac{a}{x}}\sqrt{x}\right)\int\frac{1}{\sqrt{x}\sqrt{1+x^3}}\,dx\\ &= \left(2\sqrt{\frac{a}{x}}\sqrt{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+x^6}}\,dx,x,\sqrt{x}\right)\\ &= \frac{\sqrt{\frac{a}{x}}x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.23

$$\int\frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx=2\sqrt{\frac{a}{x}}x\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-x^3\right)$$

```
[In] Integrate[Sqrt[a/x]/Sqrt[1 + x^3],x]
[Out] 2*Sqrt[a/x]*x*Hypergeometric2F1[1/6, 1/2, 7/6, -x^3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.19

method	result
meijerg	$2\sqrt{\frac{a}{x}}\, x\, {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-x^3\right)$
default	$\frac{4\sqrt{\frac{a}{x}}\, x\sqrt{x^3+1}\,\left(1+i\sqrt{3}\right)\sqrt{\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x+1)}}\,(x+1)^2\sqrt{\frac{i\sqrt{3}+2x-1}{\left(-1+i\sqrt{3}\right)(x+1)}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{\left(1+i\sqrt{3}\right)(x+1)}}\,F\left(\sqrt{\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x+1)}},\sqrt{\frac{\left(i\sqrt{3}-3\right)\left(1+i\sqrt{3}\right)}{\left(-1+i\sqrt{3}\right)\left(3+i\sqrt{3}\right)}}\right)}{\sqrt{(x^3+1)x}\,\left(3+i\sqrt{3}\right)\sqrt{-x(x+1)\left(i\sqrt{3}+2x-1\right)\left(i\sqrt{3}-2x+1\right)}}$

```
[In] int((a/x)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(a/x)^(1/2)*x*hypergeom([1/6,1/2],[7/6],-x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = -2\sqrt{a}\,\text{weierstrassPInverse}\left(0,-4,\frac{1}{x}\right)$$

```
[In] integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(a)*weierstrassPInverse(0, -4, 1/x)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

```
[In] integrate((a/x)**(1/2)/(x**3+1)**(1/2),x)
[Out] Integral(sqrt(a/x)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a/x)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a/x)/sqrt(x^3 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

```
[In] int((a/x)^(1/2)/(x^3 + 1)^(1/2),x)
[Out] int((a/x)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.390 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}} dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}} dx = -\frac{2}{3}\sqrt{\frac{a}{x^2}} x \operatorname{arctanh}\left(\sqrt{1+x^3}\right)$$

[Out] -2/3*x*arctanh((x^3+1)^(1/2))*(a/x^2)^(1/2)

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {15, 272, 65, 213}

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}} dx = -\frac{2}{3} x \sqrt{\frac{a}{x^2}} \operatorname{arctanh}\left(\sqrt{x^3+1}\right)$$

```
[In] Int[Sqrt[a/x^2]/Sqrt[1 + x^3],x]
[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\frac{a}{x^2}}x\right)\int \frac{1}{x\sqrt{1+x^3}}\,dx \\ &= \frac{1}{3}\left(\sqrt{\frac{a}{x^2}}x\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right) \\ &= \frac{1}{3}\left(2\sqrt{\frac{a}{x^2}}x\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right) \\ &= -\frac{2}{3}\sqrt{\frac{a}{x^2}}x\tanh^{-1}\left(\sqrt{1+x^3}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = -\frac{2}{3}\sqrt{\frac{a}{x^2}}x\text{arctanh}\left(\sqrt{1+x^3}\right)$$

```
[In] Integrate[Sqrt[a/x^2]/Sqrt[1 + x^3],x]
[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{2x\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)\sqrt{\frac{a}{x^2}}}{3}$	19
meijerg	$\frac{\sqrt{\frac{a}{x^2}}\,x\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}\right)}{3\sqrt{\pi}}$	45

```
[In] int((a/x^2)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*x*arctanh((x^3+1)^(1/2))*(a/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(18) = 36.

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = \left[\frac{1}{3}\,x\sqrt{\frac{a}{x^2}}\log\left(\frac{x^3-2\sqrt{x^3+1}+2}{x^3}\right),\ \frac{2}{3}\,\sqrt{-a}\arctan\left(\frac{\sqrt{x^3+1}\sqrt{-a}x\sqrt{\frac{a}{x^2}}}{ax^3+a}\right)\right]$$

```
[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] [1/3*x*sqrt(a/x^2)*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3), 2/3*sqrt(-a)*arcta
n(sqrt(x^3 + 1)*sqrt(-a)*x*sqrt(a/x^2)/(a*x^3 + a))]
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{(x+1)\,(x^2-x+1)}}\,dx$$

```
[In] integrate((a/x**2)**(1/2)/(x**3+1)**(1/2),x)
[Out] Integral(sqrt(a/x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a/x^2)/sqrt(x^3 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = -\frac{1}{3}\,\sqrt{a}\Big(\log\left(\sqrt{x^3+1}+1\right) - \log\left(\left|\sqrt{x^3+1}-1\right|\right)\Big)\mathrm{sgn}(x)$$

```
[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] -1/3*sqrt(a)*(log(sqrt(x^3 + 1) + 1) - log(abs(sqrt(x^3 + 1) - 1)))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}}\,dx$$

```
[In] int((a/x^2)^(1/2)/(x^3 + 1)^(1/2),x)
[Out] int((a/x^2)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.391 $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 312

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^3}+\frac{2\left(1+\sqrt{3}\right)\sqrt{\frac{a}{x^3}}x^2\sqrt{1+x^3}}{1+\left(1+\sqrt{3}\right)x} - \frac{2\sqrt[4]{3}\sqrt{\frac{a}{x^3}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}E\left(\arccos\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}} - \frac{\left(1-\sqrt{3}\right)\sqrt{\frac{a}{x^3}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -2*x*(a/x^3)^(1/2)*(x^3+1)^(1/2)+2*x^2*(1+3^(1/2))*(a/x^3)^(1/2)*(x^3+1)^(1
/2)/(1+x*(1+3^(1/2)))-2*3^(1/4)*x^2*(1+x)*((1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1
/2)))^2)^(1/2)/(1+x*(1-3^(1/2)))*(1+x*(1+3^(1/2)))*EllipticE((1-(1+x*(1-3^(
1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a/x^3)^(1/2)*
((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)/(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3^(1/2
)))^2)^(1/2)-1/3*x^2*(1+x)*((1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2)/
(1+x*(1-3^(1/2)))*(1+x*(1+3^(1/2)))*EllipticF((1-(1+x*(1-3^(1/2)))^2/(1+x*(
1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(a/x^3)^(1/2)*((x
^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3
^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {15, 331, 335, 314, 231, 1895}

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx$$
$$= -\frac{(1-\sqrt{3})(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}x^2\sqrt{\frac{a}{x^3}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right),\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$
$$-\frac{2\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}x^2\sqrt{\frac{a}{x^3}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$
$$-2\sqrt{x^3+1}x\sqrt{\frac{a}{x^3}}+\frac{2\left(1+\sqrt{3}\right)\sqrt{x^3+1}x^2\sqrt{\frac{a}{x^3}}}{\left(1+\sqrt{3}\right)x+1}$$

```
[In] Int[Sqrt[a/x^3]/Sqrt[1 + x^3],x]
```

```
[Out] -2*Sqrt[a/x^3]*x*Sqrt[1 + x^3] + (2*(1 + Sqrt[3])*Sqrt[a/x^3]*x^2*Sqrt[1 +
x^3])/(1 + (1 + Sqrt[3])*x) - (2*3^(1/4)*Sqrt[a/x^3]*x^2*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticE[ArcCos[(1 + (1 - Sqrt[3])*x)/(1
 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3]
)*x)^2]*Sqrt[1 + x^3]) - ((1 - Sqrt[3])*Sqrt[a/x^3]*x^2*(1 + x)*Sqrt[(1 - x
 + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1 + (1 - Sqrt[3])*x)/(1
+ (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(1/4)*Sqrt[(x*(1 + x))/(1 + (1 +
Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\frac{a}{x^3}}x^{3/2}\right)\int \frac{1}{x^{3/2}\sqrt{1+x^3}}\,dx \\
&= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^3}+\left(2\sqrt{\frac{a}{x^3}}x^{3/2}\right)\int \frac{x^{3/2}}{\sqrt{1+x^3}}\,dx \\
&= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^3}+\left(4\sqrt{\frac{a}{x^3}}x^{3/2}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^6}}\,dx,x,\sqrt{x}\right)
\end{aligned}$$

$$= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^3} - \left(2\sqrt{\frac{a}{x^3}}x^{3/2}\right)\text{Subst}\left(\int \frac{-1+\sqrt{3}-2x^4}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)$$
$$+ \left(2\left(-1+\sqrt{3}\right)\sqrt{\frac{a}{x^3}}x^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)$$
$$= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^3} + \frac{2\left(1+\sqrt{3}\right)\sqrt{\frac{a}{x^3}}x^2\sqrt{1+x^3}}{1+\left(1+\sqrt{3}\right)x}$$
$$- \frac{2\sqrt[4]{3}\sqrt{\frac{a}{x^3}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}E\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}$$
$$- \frac{\left(1-\sqrt{3}\right)\sqrt{\frac{a}{x^3}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = -2\sqrt{\frac{a}{x^3}}x\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-x^3\right)$$

```
[In] Integrate[Sqrt[a/x^3]/Sqrt[1 + x^3],x]
[Out] -2*Sqrt[a/x^3]*x*Hypergeometric2F1[-1/6, 1/2, 5/6, -x^3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 2.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.07

method	result
meijerg	$-2\sqrt{\frac{a}{x^3}}\, x\, {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; -x^3\right)$
risch	$-2x\sqrt{\frac{a}{x^3}}\,\sqrt{x^3+1} + \frac{2\left(x\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x+1)}}\,(x+1)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(x+1)}}\,\sqrt{-\frac{x-}{\left(\frac{1}{2}+i\right.}}\right.}{}$
default	Expression too large to display

[In] int((a/x^3)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2*(a/x^3)^(1/2)*x*hypergeom([-1/6,1/2],[5/6],-x^3)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.04

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\, dx = 2\sqrt{a}\text{weierstrassZeta}\left(0, -4, \text{weierstrassPInverse}\left(0, -4, \frac{1}{x}\right)\right)$$

[In] integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(a)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, 1/x))

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\, dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\, dx$$

[In] integrate((a/x**3)**(1/2)/(x**3+1)**(1/2),x)

[Out] Integral(sqrt(a/x**3)/sqrt((x + 1)*(x**2 - x + 1)), x)

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

```
[In] int((a/x^3)^(1/2)/(x^3 + 1)^(1/2),x)
[Out] int((a/x^3)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.392 $\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}} dx$

Optimal result

Integrand size = 19, antiderivative size = 281

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}} dx = -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^3} + \frac{\sqrt{\frac{a}{x^4}}x^2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{\frac{a}{x^4}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{\sqrt{2}\sqrt{\frac{a}{x^4}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -x*(a/x^4)^(1/2)*(x^3+1)^(1/2)+x^2*(a/x^4)^(1/2)*(x^3+1)^(1/2)/(1+x+3^(1/2)
)+1/3*x^2*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2
)*(a/x^4)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1
+x)/(1+x+3^(1/2))^2)^(1/2)-1/2*3^(1/4)*x^2*(1+x)*EllipticE((1+x-3^(1/2))/(1
+x+3^(1/2)),I*3^(1/2)+2*I)*(a/x^4)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+
1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {15, 331, 309, 224, 1891}

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = \frac{\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}x^2\sqrt{\frac{a}{x^4}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}x^2\sqrt{\frac{a}{x^4}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} - \sqrt{x^3+1}x\sqrt{\frac{a}{x^4}} + \frac{\sqrt{x^3+1}x^2\sqrt{\frac{a}{x^4}}}{x+\sqrt{3}+1}$$

```
[In] Int[Sqrt[a/x^4]/Sqrt[1 + x^3],x]
```

```
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^3]) + (Sqrt[a/x^4]*x^2*Sqrt[1 + x^3])/(1 + Sqrt[
3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[a/x^4]*x^2*(1 + x)*Sqrt[(1 - x +
x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] +
 x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
+ (Sqrt[2]*Sqrt[a/x^4]*x^2*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*
EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
```

```
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\sqrt{\frac{a}{x^4}}x^2\right)\int \frac{1}{x^2\sqrt{1+x^3}}\,dx \\
&= -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^3}+\frac{1}{2}\left(\sqrt{\frac{a}{x^4}}x^2\right)\int \frac{x}{\sqrt{1+x^3}}\,dx \\
&= -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^3}+\frac{1}{2}\left(\sqrt{\frac{a}{x^4}}x^2\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx+\frac{1}{2}\left(\left(-1+\sqrt{3}\right)\sqrt{\frac{a}{x^4}}x^2\right)\int \frac{1}{\sqrt{1+x^3}}\,dx \\
&= -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^3}+\frac{\sqrt{\frac{a}{x^4}}x^2\sqrt{1+x^3}}{1+\sqrt{3}+x} \\
&\quad -\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{\frac{a}{x^4}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} \\
&\quad +\frac{\sqrt{2}\sqrt{\frac{a}{x^4}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = -\sqrt{\frac{a}{x^4}}x\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-x^3\right)$$

```
[In] Integrate[Sqrt[a/x^4]/Sqrt[1 + x^3],x]
[Out] -(Sqrt[a/x^4]*x*Hypergeometric2F1[-1/3, 1/2, 2/3, -x^3])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.08

method	result
meijerg	$-\sqrt{\frac{a}{x^4}}\,x\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-x^3\right)$
risch	$-x\sqrt{\frac{a}{x^4}}\,\sqrt{x^3+1}-\frac{i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)E\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right.\right.}{3\sqrt{a\,x^3+a}\,\sqrt{x^3+1}}$
default	$\frac{\sqrt{\frac{a}{x^4}}\,x\left(i\sqrt{3}\,\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,F\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{3+i\sqrt{3}}}\right)x-6\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,E\left(\sqrt{-\frac{2(x+}{i\sqrt{3}-}}\right.\right.}{2\sqrt{x^3+1}}$

```
[In] int((a/x^4)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(a/x^4)^(1/2)*x*hypergeom([-1/3,1/2],[2/3],-x^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = -x^2\sqrt{\frac{a}{x^4}}\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x)) - \sqrt{x^3+1}x\sqrt{\frac{a}{x^4}}$$

```
[In] integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] -x^2*sqrt(a/x^4)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) - sq
rt(x^3 + 1)*x*sqrt(a/x^4)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

```
[In] integrate((a/x**4)**(1/2)/(x**3+1)**(1/2),x)
[Out] Integral(sqrt(a/x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

```
[In] integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

```
[In] int((a/x^4)^(1/2)/(x^3 + 1)^(1/2),x)
[Out] int((a/x^4)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.393 $\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} dx$

Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} dx = \frac{x\sqrt{ax^{2n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{1}{n},2+\frac{1}{n},-x^n\right)}{1+n}$$

```
[Out] x*hypergeom([1/2, 1+1/n],[2+1/n],-x^n)*(a*x^(2*n))^(1/2)/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {15, 371}

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} dx = \frac{x\sqrt{ax^{2n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{1}{n},2+\frac{1}{n},-x^n\right)}{n+1}$$

```
[In] Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n],x]
```

```
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(1
 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \left(x^{-n}\sqrt{ax^{2n}}\right)\int \frac{x^n}{\sqrt{1+x^n}}\,dx$$
$$= \frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{1+n}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \frac{x\sqrt{ax^{2n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{1}{n},2+\frac{1}{n},-x^n\right)}{1+n}$$

```
[In] Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n],x]
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(1
 + n)
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
meijerg	$\frac{x\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)\sqrt{x^{2n}a}}{1+n}$	36

```
[In] int((x^(2*n)*a)^(1/2)/(1+x^n)^(1/2),x,method=_RETURNVERBOSE)
[Out] x*hypergeom([1/2,1+1/n],[2+1/n],-x^n)*(x^(2*n)*a)^(1/2)/(1+n)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx$$

```
[In] integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2),x)
[Out] Integral(sqrt(a*x**(2*n))/sqrt(x**n + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{2\,n}}}{\sqrt{x^n+1}}\,dx$$

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{2\,n}}}{\sqrt{x^n+1}}\,dx$$

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{a\,x^{2\,n}}}{\sqrt{x^n+1}}\,dx$$

```
[In] int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2),x)
[Out] int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2), x)
```

3.394 $\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}} dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}} dx = \frac{2x\sqrt{ax^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-x^n\right)}{2+n}$$

```
[Out] 2*x*hypergeom([1/2, 1/2+1/n],[3/2+1/n],-x^n)*(a*x^n)^(1/2)/(2+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 371}

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}} dx = \frac{2x\sqrt{ax^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-x^n\right)}{n+2}$$

```
[In] Int[Sqrt[a*x^n]/Sqrt[1 + x^n],x]
```

```
[Out] (2*x*Sqrt[a*x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3 + 2/n)/2, -x^n])/(2
 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(x^{-n/2}\sqrt{ax^n}\right)\int \frac{x^{n/2}}{\sqrt{1+x^n}}\,dx\\ &= \frac{2x\sqrt{ax^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{2+n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \frac{2x\sqrt{ax^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n},\frac{3}{2}+\frac{1}{n},-x^n\right)}{2+n}$$

```
[In] Integrate[Sqrt[a*x^n]/Sqrt[1 + x^n],x]
[Out] (2*x*Sqrt[a*x^n]*Hypergeometric2F1[1/2, 1/2 + n^(-1), 3/2 + n^(-1), -x^n])/
(2 + n)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

method	result	size
meijerg	$\frac{2x{}_2F_1\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n};\frac{3}{2}+\frac{1}{n};-x^n\right)\sqrt{a\,x^n}}{2+n}$	35

```
[In] int((a*x^n)^(1/2)/(1+x^n)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*x*hypergeom([1/2,1/2+1/n],[3/2+1/n],-x^n)*(a*x^n)^(1/2)/(2+n)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

```
[In] integrate((a*x**n)**(1/2)/(1+x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**n)/sqrt(x**n + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

```
[In] integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^n)/sqrt(x^n + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

```
[In] integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^n)/sqrt(x^n + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{a\,x^n}}{\sqrt{x^n+1}}\,dx$$

```
[In] int((a*x^n)^(1/2)/(x^n + 1)^(1/2),x)
[Out] int((a*x^n)^(1/2)/(x^n + 1)^(1/2), x)
```

3.395 $\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx=\frac{4x\sqrt{ax^{n/2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right),\frac{1}{4}\left(5+\frac{4}{n}\right),-x^n\right)}{4+n}$$

```
[Out] 4*x*hypergeom([1/2, 1/4+1/n],[5/4+1/n],-x^n)*(a*x^(1/2*n))^(1/2)/(4+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {15, 371}

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx=\frac{4x\sqrt{ax^{n/2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right),\frac{1}{4}\left(5+\frac{4}{n}\right),-x^n\right)}{n+4}$$

```
[In] Int[Sqrt[a*x^(n/2)]/Sqrt[1 + x^n],x]
```

```
[Out] (4*x*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, (1 + 4/n)/4, (5 + 4/n)/4, -x^n]
)/(4 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \left(x^{-n/4}\sqrt{ax^{n/2}}\right)\int \frac{x^{n/4}}{\sqrt{1+x^n}}\,dx$$
$$= \frac{4x\sqrt{ax^{n/2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right);\frac{1}{4}\left(5+\frac{4}{n}\right);-x^n\right)}{4+n}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \frac{4x\sqrt{ax^{n/2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}+\frac{1}{n},\frac{5}{4}+\frac{1}{n},-x^n\right)}{4+n}$$

```
[In] Integrate[Sqrt[a*x^(n/2)]/Sqrt[1 + x^n],x]
[Out] (4*x*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, 1/4 + n^(-1), 5/4 + n^(-1), -x^
n])/(4 + n)
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.71

method	result	size
meijerg	$\frac{4x\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}+\frac{1}{n};\frac{5}{4}+\frac{1}{n};-x^n\right)\sqrt{a\,x^{\frac{n}{2}}}}{4+n}$	37

```
[In] int((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x,method=_RETURNVERBOSE)
[Out] 4*x*hypergeom([1/2,1/4+1/n],[5/4+1/n],-x^n)*(a*x^(1/2*n))^(1/2)/(4+n)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{\frac{n}{2}}}}{\sqrt{x^n+1}}\,dx$$

```
[In] integrate((a*x**(1/2*n))**(1/2)/(1+x**n)**(1/2),x)
[Out] Integral(sqrt(a*x**(n/2))/sqrt(x**n + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{\frac{1}{2}\,n}}}{\sqrt{x^n+1}}\,dx$$

```
[In] integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{\frac{1}{2}\,n}}}{\sqrt{x^n+1}}\,dx$$

```
[In] integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{a\,x^{n/2}}}{\sqrt{x^n+1}}\,dx$$

```
[In] int((a*x^(n/2))^(1/2)/(x^n + 1)^(1/2),x)
[Out] int((a*x^(n/2))^(1/2)/(x^n + 1)^(1/2), x)
```

3.396 $\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)\,dx$

Optimal result

Integrand size = 54, antiderivative size = 34

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)\,dx=\frac{2x^{1-n}\sqrt{ax^{2n}}\sqrt{1+x^n}}{2+n}$$

```
[Out] 2*x^(1-n)*(a*x^(2*n))^(1/2)*(1+x^n)^(1/2)/(2+n)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {15, 371, 251}

$$\begin{aligned}&\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)\,dx\\&=\frac{2x^{1-n}\sqrt{ax^{2n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-x^n\right)}{n+2}\\&\quad+\frac{x\sqrt{ax^{2n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{1}{n},2+\frac{1}{n},-x^n\right)}{n+1}\end{aligned}$$

```
[In] Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*Sqrt[1
 + x^n]),x]
```

```
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(1
 + n) + (2*x^(1 - n)*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, n^(-1), 1 + n^(
-1), -x^n])/(2 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\int \frac{x^{-n}\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx}{2+n} + \int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx \\ &= \left(x^{-n}\sqrt{ax^{2n}}\right)\int \frac{x^n}{\sqrt{1+x^n}}\,dx + \frac{\left(2x^{-n}\sqrt{ax^{2n}}\right)\int \frac{1}{\sqrt{1+x^n}}\,dx}{2+n} \\ &= \frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{1+n} + \frac{2x^{1-n}\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-x^n\right)}{2+n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} + \frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right) dx = \frac{2ax^{1+n}\sqrt{1+x^n}}{(2+n)\sqrt{ax^{2n}}}$$

```
[In] Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*
Sqrt[1 + x^n]),x]
[Out] (2*a*x^(1 + n)*Sqrt[1 + x^n])/((2 + n)*Sqrt[a*x^(2*n)])
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{2x\sqrt{1+x^{n}}\,\sqrt{x^{2n}a}\,x^{-n}}{2+n}$	30
meijerg	$\frac{x\,{}_{2}F_{1}\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^{n}\right)\sqrt{x^{2n}a}}{1+n}+\frac{2\sqrt{x^{2n}a}\,x^{1-n}\,{}_{2}F_{1}\left(\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-x^{n}\right)}{2+n}$	77

```
[In] int((x^(2*n)*a)^(1/2)/(1+x^n)^(1/2)+2*(x^(2*n)*a)^(1/2)/(2+n)/(x^n)/(1+x^n)
^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x*(1+x^n)^(1/2)/(2+n)*((x^n)^2*a)^(1/2)/(x^n)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(
1+x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)dx=\frac{\int \frac{2\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx+\int \frac{n\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx+\int \frac{2x^{-n}\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx}{n+2}$$

```
[In] integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2)+2*(a*x**(2*n))**(1/2)/(2+n)/(
x**n)/(1+x**n)**(1/2),x)
```

```
[Out] (Integral(2*sqrt(a*x**(2*n))/sqrt(x**n + 1), x) + Integral(n*sqrt(a*x**(2*n
))/sqrt(x**n + 1), x) + Integral(2*sqrt(a*x**(2*n))/(x**n*sqrt(x**n + 1)),
x))/(n + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.53

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} + \frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right) dx = \frac{2\sqrt{a}\sqrt{x^n+1}x}{n+2}$$

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(
1+x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*sqrt(x^n + 1)*x/(n + 2)
```

Giac [F]

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} + \frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right) dx = \int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}} + \frac{2\sqrt{ax^{2n}}}{(n+2)\sqrt{x^n+1}x^n}\, dx$$

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(
1+x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1) + 2*sqrt(a*x^(2*n))/((n + 2)*sqrt(x
^n + 1)*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 16.70 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} + \frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right) dx = \frac{\sqrt{a\,x^{2\,n}}\,\left(\frac{2\,x}{n+2}+\frac{2\,x^{n+1}}{n+2}\right)}{x^n\,\sqrt{x^n+1}}$$

```
[In] int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2) + (2*(a*x^(2*n))^(1/2))/(x^n*(x^n + 1
)^(1/2)*(n + 2)),x)
```

```
[Out] ((a*x^(2*n))^(1/2)*((2*x)/(n + 2) + (2*x^(n + 1))/(n + 2)))/(x^n*(x^n + 1)^
(1/2))
```

3.397 $\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 114

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \frac{2\sqrt{-e^2+df}\sqrt{ax}\sqrt{\frac{e(e+fx)}{e^2-df}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{-e^2+df}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

```
[Out] 2*EllipticE(f^(1/2)*(e*x+d)^(1/2)/(d*f-e^2)^(1/2),(1-e^2/d/f)^(1/2))*(d*f-e
^2)^(1/2)*(a*x)^(1/2)*(e*(f*x+e)/(-d*f+e^2))^(1/2)/e/f^(1/2)/(-e*x/d)^(1/2)
/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {115, 114}

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \frac{2\sqrt{ax}\sqrt{df-e^2}\sqrt{\frac{e(e+fx)}{e^2-df}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{df-e^2}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

```
[In] Int[Sqrt[a*x]/(Sqrt[d + e*x]*Sqrt[e + f*x]),x]
```

```
[Out] (2*Sqrt[-e^2 + d*f]*Sqrt[a*x]*Sqrt[(e*(e + f*x))/(e^2 - d*f)]*EllipticE[Arc
Sin[(Sqrt[f]*Sqrt[d + e*x])/Sqrt[-e^2 + d*f]], 1 - e^2/(d*f)])/(e*Sqrt[f]*S
qrt[-((e*x)/d)]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
```

```
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{ax}\sqrt{\frac{e(e+fx)}{e^2-df}}\right)\int \frac{\sqrt{-\frac{ex}{d}}}{\sqrt{d+ex}\sqrt{\frac{e^2}{e^2-df}+\frac{efx}{e^2-df}}}\,dx}{\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}\\ &= \frac{2\sqrt{-e^2+df}\sqrt{ax}\sqrt{\frac{e(e+fx)}{e^2-df}}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{-e^2+df}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.79 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = -\frac{2ie\sqrt{ax}\sqrt{1+\frac{fx}{e}}\left(E\left(i\text{arcsinh}\left(\sqrt{\frac{ex}{d}}\right)|\frac{df}{e^2}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{ex}{d}}\right),\frac{df}{e^2}\right)\right)}{f\sqrt{\frac{ex}{d+ex}}\sqrt{d+ex}\sqrt{e+fx}}$$

```
[In] Integrate[Sqrt[a*x]/(Sqrt[d + e*x]*Sqrt[e + f*x]),x]
```

```
[Out] ((-2*I)*e*Sqrt[a*x]*Sqrt[1 + (f*x)/e]*(EllipticE[I*ArcSinh[Sqrt[(e*x)/d]],
(d*f)/e^2] - EllipticF[I*ArcSinh[Sqrt[(e*x)/d]], (d*f)/e^2]))/(f*Sqrt[(e*x)
/(d + e*x)]*Sqrt[d + e*x]*Sqrt[e + f*x])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.68

method	result
default	$-\frac{2\left(dF\left(\sqrt{\frac{fx+e}{e}},\sqrt{-\frac{e^2}{df-e^2}}\right)f-E\left(\sqrt{\frac{fx+e}{e}},\sqrt{-\frac{e^2}{df-e^2}}\right)df+E\left(\sqrt{\frac{fx+e}{e}},\sqrt{-\frac{e^2}{df-e^2}}\right)e^2\right)\sqrt{-\frac{fx}{e}}\sqrt{\frac{(ex+d)f}{df-e^2}}\sqrt{\frac{fx+e}{e}}\sqrt{ax}\sqrt{ex+d}\sqrt{f}}{f^2x(ef\,x^2+dfx+e^2x+ed)}$
elliptic	$\frac{2\sqrt{ax}\sqrt{(ex+d)(fx+e)ax}\,e\sqrt{\frac{\left(x+\frac{e}{f}\right)f}{e}}\sqrt{\frac{x+\frac{d}{e}}{-\frac{e}{f}+\frac{d}{e}}}\sqrt{-\frac{fx}{e}}\left(\left(-\frac{e}{f}+\frac{d}{e}\right)E\left(\sqrt{\frac{\left(x+\frac{e}{f}\right)f}{e}},\sqrt{-\frac{e}{f\left(-\frac{e}{f}+\frac{d}{e}\right)}}\right)-\frac{dF\left(\sqrt{\frac{\left(x+\frac{e}{f}\right)f}{e}},\sqrt{-\frac{e}{f\left(-\frac{e}{f}+\frac{d}{e}\right)}}\right.}{e}\right.}{\sqrt{ex+d}\sqrt{fx+e}\,xf\sqrt{aef\,x^3+adf\,x^2+a\,e^2x^2+edax}}$

[In] int((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2*(d*EllipticF(((f*x+e)/e)^(1/2),(-e^2/(d*f-e^2))^(1/2))*f-EllipticE(((f*x+e)/e)^(1/2),(-e^2/(d*f-e^2))^(1/2))*d*f+EllipticE(((f*x+e)/e)^(1/2),(-e^2/(d*f-e^2))^(1/2))*e^2)*(-f*x/e)^(1/2)*((e*x+d)*f/(d*f-e^2))^(1/2)*((f*x+e)/e)^(1/2)*(a*x)^(1/2)*(e*x+d)^(1/2)*(f*x+e)^(1/2)/f^2/x/(e*f*x^2+d*f*x+e^2*x+d*e)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = -\frac{2\left(3\sqrt{aef}ef\,\text{weierstrassZeta}\left(\frac{4\left(e^4-de^2f+d^2f^2\right)}{3e^2f^2},-\frac{4\left(2e^6-3de^4f-3d^2e^2f^2+2d^3f^3\right)}{27e^3f^3},\text{weierstrassPInverse}\left(\frac{4\left(e^4-de^2f\right.}{3e^2f}\right.\right.\right.}{}$$

[In] integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")

[Out] -2/3*(3*sqrt(a*e*f)*e*f*weierstrassZeta(4/3*(e^4 - d*e^2*f + d^2*f^2)/(e^2*f^2), -4/27*(2*e^6 - 3*d*e^4*f - 3*d^2*e^2*f^2 + 2*d^3*f^3)/(e^3*f^3), weierstrassPInverse(4/3*(e^4 - d*e^2*f + d^2*f^2)/(e^2*f^2), -4/27*(2*e^6 - 3*d*e^4*f - 3*d^2*e^2*f^2 + 2*d^3*f^3)/(e^3*f^3), 1/3*(3*e*f*x + e^2 + d*f)/(e*f))) + sqrt(a*e*f)*(e^2 + d*f)*weierstrassPInverse(4/3*(e^4 - d*e^2*f + d^2*f^2)/(e^2*f^2), -4/27*(2*e^6 - 3*d*e^4*f - 3*d^2*e^2*f^2 + 2*d^3*f^3)/(e^3*f^3), 1/3*(3*e*f*x + e^2 + d*f)/(e*f)))/(e^2*f^2)

Sympy [F]

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx$$

```
[In] integrate((a*x)**(1/2)/(e*x+d)**(1/2)/(f*x+e)**(1/2),x)
[Out] Integral(sqrt(a*x)/(sqrt(d + e*x)*sqrt(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{ex+d}\sqrt{fx+e}}\,dx$$

```
[In] integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{ex+d}\sqrt{fx+e}}\,dx$$

```
[In] integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{a\,x}}{\sqrt{e+f\,x}\,\sqrt{d+e\,x}}\,dx$$

```
[In] int((a*x)^(1/2)/((e + f*x)^(1/2)*(d + e*x)^(1/2)),x)
[Out] int((a*x)^(1/2)/((e + f*x)^(1/2)*(d + e*x)^(1/2)), x)
```

3.398 $\int (ax^m)^r \, dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int (ax^m)^r \, dx = \frac{x(ax^m)^r}{1+mr}$$

```
[Out] x*(a*x^m)^r/(m*r+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {15, 30}

$$\int (ax^m)^r \, dx = \frac{x(ax^m)^r}{mr+1}$$

```
[In] Int[(a*x^m)^r,x]
```

```
[Out] (x*(a*x^m)^r)/(1 + m*r)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\text{integral} = \left(x^{-mr}(ax^m)^r\right)\int x^{mr}\,dx$$
$$= \frac{x(ax^m)^r}{1+mr}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (ax^m)^r\,dx = \frac{x(ax^m)^r}{1+mr}$$

```
[In] Integrate[(a*x^m)^r,x]
[Out] (x*(a*x^m)^r)/(1 + m*r)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x(x^m a)^r}{mr+1}$	17
parallelrisch	$\frac{x(x^m a)^r}{mr+1}$	17
norman	$\frac{x\,\mathrm{e}^{r\ln\left(\mathrm{e}^{m\ln(x)}a\right)}}{mr+1}$	21

```
[In] int((x^m*a)^r,x,method=_RETURNVERBOSE)
[Out] x*(x^m*a)^r/(m*r+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (ax^m)^r\,dx = \frac{xe^{(mr\log(x)+r\log(a))}}{mr+1}$$

```
[In] integrate((a*x^m)^r,x, algorithm="fricas")
[Out] x*e^(m*r*log(x) + r*log(a))/(m*r + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int (ax^m)^r \, dx = \begin{cases} \frac{x(ax^m)^r}{mr+1} & \text{for } m \neq -\frac{1}{r} \\ x\left(ax^{-\frac{1}{r}}\right)^r \log(x) & \text{otherwise} \end{cases}$$

```
[In] integrate((a*x**m)**r,x)
[Out] Piecewise((x*(a*x**m)**r/(m*r + 1), Ne(m, -1/r)), (x*(a/x**(1/r))**r*log(x)
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (ax^m)^r \, dx = \frac{a^r x(x^m)^r}{mr+1}$$

```
[In] integrate((a*x^m)^r,x, algorithm="maxima")
[Out] a^r*x*(x^m)^r/(m*r + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (ax^m)^r \, dx = \frac{xe^{(mr\log(x)+r\log(a))}}{mr+1}$$

```
[In] integrate((a*x^m)^r,x, algorithm="giac")
[Out] x*e^(m*r*log(x) + r*log(a))/(m*r + 1)
```

Mupad [B] (verification not implemented)

Time = 17.50 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (ax^m)^r \, dx = \frac{x\,(a\,x^m)^r}{m\,r+1}$$

```
[In] int((a*x^m)^r,x)
[Out] (x*(a*x^m)^r)/(m*r + 1)
```

3.399 $\int (ax^m)^r (bx^n)^s \, dx$

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int (ax^m)^r (bx^n)^s \, dx = \frac{x(ax^m)^r (bx^n)^s}{1 + mr + ns}$$

```
[Out] x*(a*x^m)^r*(b*x^n)^s/(m*r+n*s+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 30}

$$\int (ax^m)^r (bx^n)^s \, dx = \frac{x(ax^m)^r (bx^n)^s}{mr + ns + 1}$$

```
[In] Int[(a*x^m)^r*(b*x^n)^s,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \left(x^{-mr}(ax^m)^r\right)\int x^{mr}(bx^n)^s\,dx \\ &= \left(x^{-mr-ns}(ax^m)^r\,(bx^n)^s\right)\int x^{mr+ns}\,dx \\ &= \frac{x(ax^m)^r\,(bx^n)^s}{1+mr+ns} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (ax^m)^r\,(bx^n)^s\,dx = \frac{x(ax^m)^r\,(bx^n)^s}{1+mr+ns}$$

```
[In] Integrate[(a*x^m)^r*(b*x^n)^s,x]
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result
gosper	$\frac{x(x^m a)^r (b\,x^n)^s}{mr+ns+1}$
parallelrisch	$\frac{x(x^m a)^r (b\,x^n)^s}{mr+ns+1}$
risch	$\frac{b^s (x^n)^s (x^m)^r a^r x\,\mathrm{e}^{\frac{i\pi\left(-\operatorname{csgn}(ib\,x^n)^3 s+\operatorname{csgn}(ib\,x^n)^2 \operatorname{csgn}(ib)s+\operatorname{csgn}(ib\,x^n)^2 \operatorname{csgn}(ix^n)s-\operatorname{csgn}(ib\,x^n)\operatorname{csgn}(ib)\operatorname{csgn}(ix^n)s+\operatorname{csgn}(ix^m)\operatorname{csg}}{2}}}{mr+ns+1}$

```
[In] int((x^m*a)^r*(b*x^n)^s,x,method=_RETURNVERBOSE)
[Out] x*(x^m*a)^r*(b*x^n)^s/(m*r+n*s+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int (ax^m)^r (bx^n)^s \, dx = \frac{xe^{(mr\log(x)+ns\log(x)+r\log(a)+s\log(b))}}{mr+ns+1}$$

```
[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="fricas")
[Out] x*e^(m*r*log(x) + n*s*log(x) + r*log(a) + s*log(b))/(m*r + n*s + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(22) = 44.

Time = 11.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.31

$$\int (ax^m)^r (bx^n)^s \, dx = \begin{cases} \frac{x(ax^m)^r(bx^n)^s}{mr+ns+1} & \text{for } m \neq -\frac{ns+1}{r} \\ \begin{cases} \frac{x\left(ax^{-\frac{ns}{r}-\frac{1}{r}}\right)^r(bx^n)^s}{ns+r\left(-\frac{ns}{r}-\frac{1}{r}\right)+1} & \text{for } ns+r\left(-\frac{ns}{r}-\frac{1}{r}\right) \neq -1 \\ x\left(ax^{-\frac{ns}{r}-\frac{1}{r}}\right)^r (bx^n)^s \log(x) & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

```
[In] integrate((a*x**m)**r*(b*x**n)**s,x)
[Out] Piecewise((x*(a*x**m)**r*(b*x**n)**s/(m*r + n*s + 1), Ne(m, -(n*s + 1)/r)),
 (Piecewise((x*(a*x**(-n*s/r - 1/r))**r*(b*x**n)**s/(n*s + r*(-n*s/r - 1/r)
 + 1), Ne(n*s + r*(-n*s/r - 1/r), -1)), (x*(a*x**(-n*s/r - 1/r))**r*(b*x**n
)**s*log(x), True)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int (ax^m)^r (bx^n)^s \, dx = \frac{a^r b^s x e^{(r\log(x^m)+s\log(x^n))}}{mr+ns+1}$$

```
[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="maxima")
[Out] a^r*b^s*x*e^(r*log(x^m) + s*log(x^n))/(m*r + n*s + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int (ax^m)^r (bx^n)^s \, dx = \frac{xe^{(mr\log(x)+ns\log(x)+r\log(a)+s\log(b))}}{mr+ns+1}$$

```
[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="giac")
[Out] x*e^(m*r*log(x) + n*s*log(x) + r*log(a) + s*log(b))/(m*r + n*s + 1)
```

Mupad [B] (verification not implemented)

Time = 16.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (ax^m)^r (bx^n)^s \, dx = \frac{x\,(a\,x^m)^r\,(b\,x^n)^s}{m\,r+n\,s+1}$$

```
[In] int((a*x^m)^r*(b*x^n)^s,x)
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(m*r + n*s + 1)
```

3.400 $\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx$

Optimal result

Integrand size = 22, antiderivative size = 36

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx = \frac{x(ax^m)^r (bx^n)^s (cx^p)^t}{1 + mr + ns + pt}$$

```
[Out] x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t/(m*r+n*s+p*t+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 30}

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx = \frac{x(ax^m)^r (bx^n)^s (cx^p)^t}{mr + ns + pt + 1}$$

```
[In] Int[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(x^{-mr}(ax^m)^r\right)\int x^{mr}(bx^n)^s\,(cx^p)^t\;dx\\ &= \left(x^{-mr-ns}(ax^m)^r\,(bx^n)^s\right)\int x^{mr+ns}(cx^p)^t\;dx\\ &= \left(x^{-mr-ns-pt}(ax^m)^r\,(bx^n)^s\,(cx^p)^t\right)\int x^{mr+ns+pt}\,dx\\ &= \frac{x(ax^m)^r\,(bx^n)^s\,(cx^p)^t}{1+mr+ns+pt}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (ax^m)^r\,(bx^n)^s\,(cx^p)^t\;dx = \frac{x(ax^m)^r\,(bx^n)^s\,(cx^p)^t}{1+mr+ns+pt}$$

```
[In] Integrate[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

Maple [A] (verified)

Time = 49.96 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

method	result
gosper	$\frac{x(x^m a)^r (b\,x^n)^s (c\,x^p)^t}{mr+ns+pt+1}$
parallelrisch	$\frac{x(x^m a)^r (b\,x^n)^s (c\,x^p)^t}{mr+ns+pt+1}$
risch	$(x^p)^t c^t (x^n)^s b^s (x^m)^r a^r x\,\mathrm{e}^{\frac{i\pi\left(-\operatorname{csgn}(ic\,x^p)^3 t+\operatorname{csgn}(ic\,x^p)^2\operatorname{csgn}(ic)t+\operatorname{csgn}(ic\,x^p)^2\operatorname{csgn}(ix^p)t-\operatorname{csgn}(ic\,x^p)\operatorname{csgn}(ic)\operatorname{csgn}(ix^p)t+\operatorname{csgn}(ix^r}{}}$

```
[In] int((x^m*a)^r*(b*x^n)^s*(c*x^p)^t,x,method=_RETURNVERBOSE)
[Out] x*(x^m*a)^r*(b*x^n)^s*(c*x^p)^t/(m*r+n*s+p*t+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx = \frac{xe^{(mr\log(x)+ns\log(x)+pt\log(x)+r\log(a)+s\log(b)+t\log(c))}}{mr+ns+pt+1}$$

```
[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="fricas")
[Out] x*e^(m*r*log(x) + n*s*log(x) + p*t*log(x) + r*log(a) + s*log(b) + t*log(c))
/(m*r + n*s + p*t + 1)
```

Sympy [F(-1)]

Timed out.

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx = \text{Timed out}$$

```
[In] integrate((a*x**m)**r*(b*x**n)**s*(c*x**p)**t,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx = \frac{a^r b^s c^t x e^{(r\log(x^m)+s\log(x^n)+t\log(x^p))}}{mr+ns+pt+1}$$

```
[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="maxima")
[Out] a^r*b^s*c^t*x*e^(r*log(x^m) + s*log(x^n) + t*log(x^p))/(m*r + n*s + p*t + 1
)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx = \frac{xe^{(mr\log(x)+ns\log(x)+pt\log(x)+r\log(a)+s\log(b)+t\log(c))}}{mr+ns+pt+1}$$

```
[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="giac")
[Out] x*e^(m*r*log(x) + n*s*log(x) + p*t*log(x) + r*log(a) + s*log(b) + t*log(c))
/(m*r + n*s + p*t + 1)
```

Mupad [B] (verification not implemented)

Time = 16.58 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx = \frac{x \left(a\,x^m\right)^r \left(b\,x^n\right)^s \left(c\,x^p\right)^t}{m\,r+n\,s+p\,t+1}$$

```
[In] int((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x)
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(m*r + n*s + p*t + 1)
```

3.401 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(a-c)} + \frac{2(a+bx)^{7/2}}{7b^3(a-c)} - \frac{2c^2(c+bx)^{3/2}}{3b^3(a-c)} + \frac{4c(c+bx)^{5/2}}{5b^3(a-c)} - \frac{2(c+bx)^{7/2}}{7b^3(a-c)}$$

```
[Out] 2/3*a^2*(b*x+a)^(3/2)/b^3/(a-c)-4/5*a*(b*x+a)^(5/2)/b^3/(a-c)+2/7*(b*x+a)^(
7/2)/b^3/(a-c)-2/3*c^2*(b*x+c)^(3/2)/b^3/(a-c)+4/5*c*(b*x+c)^(5/2)/b^3/(a-c
)-2/7*(b*x+c)^(7/2)/b^3/(a-c)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2129, 45}

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)} - \frac{2c^2(bx+c)^{3/2}}{3b^3(a-c)} + \frac{2(a+bx)^{7/2}}{7b^3(a-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(a-c)} - \frac{2(bx+c)^{7/2}}{7b^3(a-c)} + \frac{4c(bx+c)^{5/2}}{5b^3(a-c)}$$

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(a - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(a -
 c)) + (2*(a + b*x)^(7/2))/(7*b^3*(a - c)) - (2*c^2*(c + b*x)^(3/2))/(3*b^3
*(a - c)) + (4*c*(c + b*x)^(5/2))/(5*b^3*(a - c)) - (2*(c + b*x)^(7/2))/(7*
b^3*(a - c))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2129

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[-d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\int x^2\sqrt{a+bx}\,dx}{-ab+bc}+\frac{b\int x^2\sqrt{c+bx}\,dx}{-ab+bc}\\
&= -\frac{b\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx}{-ab+bc}+\frac{b\int\left(\frac{c^2\sqrt{c+bx}}{b^2}-\frac{2c(c+bx)^{3/2}}{b^2}+\frac{(c+bx)^{5/2}}{b^2}\right)dx}{-ab+bc}\\
&= \frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(a-c)}+\frac{2(a+bx)^{7/2}}{7b^3(a-c)}\\
&\quad -\frac{2c^2(c+bx)^{3/2}}{3b^3(a-c)}+\frac{4c(c+bx)^{5/2}}{5b^3(a-c)}-\frac{2(c+bx)^{7/2}}{7b^3(a-c)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

$$
\int\frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2\left(8a^3\sqrt{a+bx}-4a^2bx\sqrt{a+bx}+3ab^2x^2\sqrt{a+bx}-8c^3\sqrt{c+bx}+4bc^2x\sqrt{c+bx}-3b^2cx^2\sqrt{c+bx}+15b^3\right.}{105b^3(a-c)}
$$

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (2*(8*a^3*Sqrt[a + b*x] - 4*a^2*b*x*Sqrt[a + b*x] + 3*a*b^2*x^2*Sqrt[a + b*
x] - 8*c^3*Sqrt[c + b*x] + 4*b*c^2*x*Sqrt[c + b*x] - 3*b^2*c*x^2*Sqrt[c + b
*x] + 15*b^3*x^3*(Sqrt[a + b*x] - Sqrt[c + b*x])))/(105*b^3*(a - c))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4(bx+a)^{\frac{5}{2}}a}{5}+\frac{2(bx+a)^{\frac{3}{2}}a^{2}}{3}}{(a-c)b^{3}}-\frac{2\left(\frac{(bx+c)^{\frac{7}{2}}}{7}-\frac{2c(bx+c)^{\frac{5}{2}}}{5}+\frac{c^{2}(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)b^{3}}$	90

```
[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/(a-c)/b^3*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*(b*x+a)^(3/2)*a^2)-2
/(a-c)/b^3*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2*(b*x+c)^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$$
$$=\frac{2\left((15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}-(15\,b^3x^3+3\,b^2cx^2-4\,bc^2x+8\,c^3)\sqrt{bx+c}\right)}{105\,(ab^3-b^3c)}$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
[Out] 2/105*((15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a) - (15*b
^3*x^3 + 3*b^2*c*x^2 - 4*b*c^2*x + 8*c^3)*sqrt(b*x + c))/(a*b^3 - b^3*c)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\int \frac{x^2}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\int \frac{x^2}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(123) = 246$.

Time = 0.36 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.65

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = -\frac{2}{105}\left(\left(3\,(bx+a)\left(\frac{5\,(a^2b^9-2\,ab^9c+b^9c^2)(bx+a)}{a^3b^{12}-3\,a^2b^{12}c+3\,ab^{12}c^2-b^{12}c^3}-\frac{15\,a^3b^9-31\,a^2b^9c+17\,ab^9c^2-b^9c^3}{a^3b^{12}-3\,a^2b^{12}c+3\,ab^{12}c^2-b^{12}c^3}\right)+\frac{45\,a^4}{} \right.\right. + \frac{2\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)}{105\,(ab^3-b^3c)}$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
[Out] -2/105*((3*(b*x + a)*(5*(a^2*b^9 - 2*a*b^9*c + b^9*c^2)*(b*x + a)/(a^3*b^12
 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3) - (15*a^3*b^9 - 31*a^2*b^9*c + 1
7*a*b^9*c^2 - b^9*c^3)/(a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))
 + (45*a^4*b^9 - 96*a^3*b^9*c + 53*a^2*b^9*c^2 + 2*a*b^9*c^3 - 4*b^9*c^4)/(
a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))*(b*x + a) - (15*a^5*b^9
 - 33*a^4*b^9*c + 17*a^3*b^9*c^2 - 3*a^2*b^9*c^3 + 12*a*b^9*c^4 - 8*b^9*c^5
)/(a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))*sqrt(b*x + c) + 2/10
5*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2)/(a*b
^3 - b^3*c)
```

Mupad [B] (verification not implemented)

Time = 16.57 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2\,x^3\,\sqrt{a+b\,x}}{7\,(a-c)}-\frac{2\,x^3\,\sqrt{c+b\,x}}{7\,(a-c)}+\frac{16\,a^3\,\sqrt{a+b\,x}}{105\,b^3\,(a-c)}-\frac{16\,c^3\,\sqrt{c+b\,x}}{105\,b^3\,(a-c)}+\frac{2\,a\,x^2\,\sqrt{a+b\,x}}{35\,b\,(a-c)}-\frac{8\,a^2\,x\,\sqrt{a+b\,x}}{105\,b^2\,(a-c)}-\frac{2\,c\,x^2\,\sqrt{c+b\,x}}{35\,b\,(a-c)}+\frac{8\,c^2\,x\,\sqrt{c+b\,x}}{105\,b^2\,(a-c)}$$

```
[In] int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

```
[Out] (2*x^3*(a + b*x)^(1/2))/(7*(a - c)) - (2*x^3*(c + b*x)^(1/2))/(7*(a - c)) +
 (16*a^3*(a + b*x)^(1/2))/(105*b^3*(a - c)) - (16*c^3*(c + b*x)^(1/2))/(105
*b^3*(a - c)) + (2*a*x^2*(a + b*x)^(1/2))/(35*b*(a - c)) - (8*a^2*x*(a + b*
x)^(1/2))/(105*b^2*(a - c)) - (2*c*x^2*(c + b*x)^(1/2))/(35*b*(a - c)) + (8
*c^2*x*(c + b*x)^(1/2))/(105*b^2*(a - c))
```

3.402 $\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}+\frac{2(a+bx)^{5/2}}{5b^2(a-c)}+\frac{2c(c+bx)^{3/2}}{3b^2(a-c)}-\frac{2(c+bx)^{5/2}}{5b^2(a-c)}$$

```
[Out] -2/3*a*(b*x+a)^(3/2)/b^2/(a-c)+2/5*(b*x+a)^(5/2)/b^2/(a-c)+2/3*c*(b*x+c)^(3
/2)/b^2/(a-c)-2/5*(b*x+c)^(5/2)/b^2/(a-c)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2129, 45}

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\frac{2(a+bx)^{5/2}}{5b^2(a-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}-\frac{2(bx+c)^{5/2}}{5b^2(a-c)}+\frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(a - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(a - c)
) + (2*c*(c + b*x)^(3/2))/(3*b^2*(a - c)) - (2*(c + b*x)^(5/2))/(5*b^2*(a -
 c))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2129

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[-d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\int x\sqrt{a+bx}\,dx}{-ab+bc}+\frac{b\int x\sqrt{c+bx}\,dx}{-ab+bc}\\
&= -\frac{b\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{-ab+bc}+\frac{b\int\left(-\frac{c\sqrt{c+bx}}{b}+\frac{(c+bx)^{3/2}}{b}\right)dx}{-ab+bc}\\
&= -\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}+\frac{2(a+bx)^{5/2}}{5b^2(a-c)}+\frac{2c(c+bx)^{3/2}}{3b^2(a-c)}-\frac{2(c+bx)^{5/2}}{5b^2(a-c)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\begin{aligned}
&\int\frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx\\
&= -\frac{2\sqrt{a+bx}(2a^2+ac-3c^2-a(c+bx)+6c(c+bx)-3(c+bx)^2)}{15b^2(a-c)}\\
&\quad+\frac{2\left(5c(c+bx)^{3/2}-3(c+bx)^{5/2}\right)}{15b^2(a-c)}
\end{aligned}$$

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
[Out] (-2*Sqrt[a + b*x]*(2*a^2 + a*c - 3*c^2 - a*(c + b*x) + 6*c*(c + b*x) - 3*(c
 + b*x)^2))/(15*b^2*(a - c)) + (2*(5*c*(c + b*x)^(3/2) - 3*(c + b*x)^(5/2))
)/(15*b^2*(a - c))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2(bx+a)^{\frac{3}{2}}a}{3}}{(a-c)b^2}-\frac{2\left(\frac{(bx+c)^{\frac{5}{2}}}{5}-\frac{c(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)b^2}$	66

```
[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a-c)/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-2/(a-c)/b^2*(1/5*(b*x+c
)^(5/2)-1/3*c*(b*x+c)^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2\left((3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}-(3\,b^2x^2+bcx-2\,c^2)\sqrt{bx+c}\right)}{15\,(ab^2-b^2c)}$$

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/15*((3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a) - (3*b^2*x^2 + b*c*x - 2*c^
2)*sqrt(b*x + c))/(a*b^2 - b^2*c)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \int \frac{x}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

```
[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \int \frac{x}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(79) = 158$.

Time = 0.35 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.17

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = -\frac{2\left(\left((bx+a)\left(\frac{3\,(ab^2-b^2c)(bx+a)}{a^2b^3-2\,ab^3c+b^3c^2}-\frac{6\,a^2b^2-7\,ab^2c+b^2c^2}{a^2b^3-2\,ab^3c+b^3c^2}\right)+\frac{3\,a^3b^2-4\,a^2b^2c-ab^2c^2+2\,b^2c^3}{a^2b^3-2\,ab^3c+b^3c^2}\right)\sqrt{bx+c}-\frac{3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}\,a}{ab-bc}\right)}{15\,b}$$

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/15*(((b*x + a)*(3*(a*b^2 - b^2*c)*(b*x + a)/(a^2*b^3 - 2*a*b^3*c + b^3*c
^2) - (6*a^2*b^2 - 7*a*b^2*c + b^2*c^2)/(a^2*b^3 - 2*a*b^3*c + b^3*c^2)) +
(3*a^3*b^2 - 4*a^2*b^2*c - a*b^2*c^2 + 2*b^2*c^3)/(a^2*b^3 - 2*a*b^3*c + b^
3*c^2))*sqrt(b*x + c) - (3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/(a*b - b*
c))/b
```

Mupad [B] (verification not implemented)

Time = 16.94 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.36

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2\,x^2\,\sqrt{a+b\,x}}{5\,(a-c)}-\frac{2\,x^2\,\sqrt{c+b\,x}}{5\,(a-c)}-\frac{4\,a^2\,\sqrt{a+b\,x}}{15\,b^2\,(a-c)}+\frac{4\,c^2\,\sqrt{c+b\,x}}{15\,b^2\,(a-c)}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{15\,b\,(a-c)}-\frac{2\,c\,x\,\sqrt{c+b\,x}}{15\,b\,(a-c)}$$

```
[In] int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

```
[Out] (2*x^2*(a + b*x)^(1/2))/(5*(a - c)) - (2*x^2*(c + b*x)^(1/2))/(5*(a - c)) -
 (4*a^2*(a + b*x)^(1/2))/(15*b^2*(a - c)) + (4*c^2*(c + b*x)^(1/2))/(15*b^2
*(a - c)) + (2*a*x*(a + b*x)^(1/2))/(15*b*(a - c)) - (2*c*x*(c + b*x)^(1/2)
)/(15*b*(a - c))
```

3.403 $\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(c+bx)^{3/2}}{3b(a-c)}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b/(a-c)-2/3*(b*x+c)^(3/2)/b/(a-c)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {6821}

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(bx+c)^{3/2}}{3b(a-c)}$$

```
[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b*(a - c)) - (2*(c + b*x)^(3/2))/(3*b*(a - c))
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\int \left(\sqrt{a+bx}-\sqrt{c+bx}\right)\,dx}{a-c}\\&=\frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(c+bx)^{3/2}}{3b(a-c)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\frac{2\left((a+bx)^{3/2}-(c+bx)^{3/2}\right)}{3b(a-c)}$$

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
[Out] (2*((a + b*x)^(3/2) - (c + b*x)^(3/2)))/(3*b*(a - c))
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3b(a-c)}-\frac{2(bx+c)^{\frac{3}{2}}}{3b(a-c)}$	40

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/3*(b*x+a)^(3/2)/b/(a-c)-2/3*(b*x+c)^(3/2)/b/(a-c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\frac{2\left((bx+a)^{\frac{3}{2}}-(bx+c)^{\frac{3}{2}}\right)}{3\,(ab-bc)}$$

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
[Out] 2/3*((b*x + a)^(3/2) - (b*x + c)^(3/2))/(a*b - b*c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(32) = 64.

Time = 0.36 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.89

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \begin{cases} \frac{2a}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} + \frac{4bx}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} + \frac{2c}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} + \frac{2\sqrt{a+bx}\sqrt{bx+c}}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} & \text{for } b\neq 0 \\ \frac{x}{\sqrt{a}+\sqrt{c}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Piecewise((2*a/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) + 4*b*x/(3*b*sqrt(a
+ b*x) + 3*b*sqrt(b*x + c)) + 2*c/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) +
 2*sqrt(a + b*x)*sqrt(b*x + c)/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)), Ne(
b, 0)), (x/(sqrt(a) + sqrt(c)), True))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \int \frac{1}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = -\frac{2}{3}\sqrt{bx+c}\left(\frac{(bx+a)b}{ab^2-b^2c} - \frac{ab-bc}{ab^2-b^2c}\right) + \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,(ab-bc)}$$

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*x + c)*((b*x + a)*b/(a*b^2 - b^2*c) - (a*b - b*c)/(a*b^2 - b^2*
c)) + 2/3*(b*x + a)^(3/2)/(a*b - b*c)
```

Mupad [B] (verification not implemented)

Time = 16.85 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\frac{2\,x\,\sqrt{a+b\,x}}{3\,\left(a-c\right)}-\frac{2\,x\,\sqrt{c+b\,x}}{3\,\left(a-c\right)}+\frac{2\,a\,\sqrt{a+b\,x}}{3\,b\,\left(a-c\right)}-\frac{2\,c\,\sqrt{c+b\,x}}{3\,b\,\left(a-c\right)}$$

```
[In] int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

```
[Out] (2*x*(a + b*x)^(1/2))/(3*(a - c)) - (2*x*(c + b*x)^(1/2))/(3*(a - c)) + (2*
a*(a + b*x)^(1/2))/(3*b*(a - c)) - (2*c*(c + b*x)^(1/2))/(3*b*(a - c))
```

3.404 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=\frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}-\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{a-c}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/(a-c)+2*arctanh((b*x+c)^(1/2)/c^(
1/2))*c^(1/2)/(a-c)+2*(b*x+a)^(1/2)/(a-c)-2*(b*x+c)^(1/2)/(a-c)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2129, 52, 65, 214}

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=-\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}+\frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{bx+c}}{a-c}$$

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (2*Sqrt[a + b*x])/(a - c) - (2*Sqrt[c + b*x])/(a - c) - (2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(a - c) + (2*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]]
)/(a - c)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2129

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[-d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\int \frac{\sqrt{a+bx}}{x}\,dx}{-ab+bc}+\frac{b\int \frac{\sqrt{c+bx}}{x}\,dx}{-ab+bc}\\
&= \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}+\frac{a\int \frac{1}{x\sqrt{a+bx}}\,dx}{a-c}-\frac{c\int \frac{1}{x\sqrt{c+bx}}\,dx}{a-c}\\
&= \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}+\frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b(a-c)}\\
&\quad-\frac{(2c)\text{Subst}\left(\int \frac{1}{-\frac{c}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{c+bx}\right)}{b(a-c)}\\
&= \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{a-c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.65

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \frac{2\left(\sqrt{a+bx}-\sqrt{c+bx}-\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}\arctan\left(\frac{\sqrt{a+bx}-\sqrt{c+bx}}{\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}}\right)-\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}\arctan\left(\frac{\sqrt{a+bx}-\sqrt{c+bx}}{\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}}\right)\right)}{a-c}$$

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (2*(Sqrt[a + b*x] - Sqrt[c + b*x] - Sqrt[-(Sqrt[a] - Sqrt[c])^2]*ArcTan[(Sq
rt[a + b*x] - Sqrt[c + b*x])/Sqrt[-(Sqrt[a] - Sqrt[c])^2]] - Sqrt[-(Sqrt[a]
 + Sqrt[c])^2]*ArcTan[(Sqrt[a + b*x] - Sqrt[c + b*x])/Sqrt[-(Sqrt[a] + Sqrt
[c])^2]]))/(a - c)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a-c}-\frac{2\sqrt{bx+c}-2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$	73

```
[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-1/(a-c)*
(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.28

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \left[-\frac{\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{c}\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)-2\sqrt{bx+a}+2\sqrt{bx+c}}{a-c},\right.$$
$$-\frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}+2\sqrt{bx+c}}{a-c},\ \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+}}{a}\right.}{}$$

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(c)*log((b*x
- 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/(a
 - c), -(2*sqrt(-c)*arctan(sqrt(b*x + c)*sqrt(-c)/c) + sqrt(a)*log((b*x + 2
*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/(a -
c), (2*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(c)*log((b*x - 2*sqr
t(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a) - 2*sqrt(b*x + c))/(a - c),
2*(sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-c)*arctan(sqrt(b*x + c
)*sqrt(-c)/c) + sqrt(b*x + a) - sqrt(b*x + c))/(a - c)]
```

Sympy [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}\,dx$$

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

Maxima [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)}\,dx$$

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(81) = 162$.

Time = 0.51 (sec) , antiderivative size = 1015, normalized size of antiderivative = 10.46

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}(a-c)}$$

$$-\frac{2\left(a^4c-a^3c^2-a^2c^3+ac^4+2\left(ac^2+\sqrt{ac}c^2\right)(a-c)^2\mathrm{sgn}(-a+c)-2\left(ac^2+\sqrt{ac}ac\right)(a-c)^2+\left(a^2c^2-\right.\right.}{}$$

$$+\frac{2\left(a^4c-a^3c^2-a^2c^3+ac^4-2\left(ac^2-\sqrt{ac}c^2\right)(a-c)^2\mathrm{sgn}(-a+c)-2\left(ac^2+\sqrt{ac}ac\right)(a-c)^2+\left(a^2c^2-\right.\right.}{}$$

$$+\frac{2\sqrt{bx+a}}{a-c}-\frac{2\sqrt{bx+c}}{a-c}$$

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(a - c)) - 2*(a^4*c - a^3*c^2
- a^2*c^3 + a*c^4 + 2*(a*c^2 + sqrt(a*c)*c^2)*(a - c)^2*sgn(-a + c) - 2*(a*
c^2 + sqrt(a*c)*a*c)*(a - c)^2 + (a^2*c^2 - 2*a*c^3 + c^4 + (a^2*c - 2*a*c^
2 + c^3)*sqrt(a*c))*abs(-a + c)*sgn(-a + c) - (a^3*c - 2*a^2*c^2 + a*c^3 +
(a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c) - (a^4*c - a^3*c^2 - a^2*c^3
 + a*c^4 - (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*sgn(-a + c) + (a^4 -
a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))
/sqrt(-(a^2 - c^2 + sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(a
 - c)))/(a - c)))/((sqrt(-a)*a^4 - a^4*sqrt(-c) - 4*sqrt(-a)*a^3*c + 4*a^3*
sqrt(-c)*c + 6*sqrt(-a)*a^2*c^2 - 6*a^2*sqrt(-c)*c^2 - 4*sqrt(-a)*a*c^3 + 4
*a*sqrt(-c)*c^3 + sqrt(-a)*c^4 - sqrt(-c)*c^4)*abs(-a + c)) + 2*(a^4*c - a^
3*c^2 - a^2*c^3 + a*c^4 - 2*(a*c^2 - sqrt(a*c)*c^2)*(a - c)^2*sgn(-a + c) -
 2*(a*c^2 + sqrt(a*c)*a*c)*(a - c)^2 + (a^2*c^2 - 2*a*c^3 + c^4 - (a^2*c -
2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c)*sgn(-a + c) + (a^3*c - 2*a^2*c^2 + a*
c^3 + (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c) + (a^4*c - a^3*c^2 - a
^2*c^3 + a*c^4 + (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*sgn(-a + c) + (
a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*arctan(-(sqrt(b*x + a) - sqrt(b*x
 + c))/sqrt(-(a^2 - c^2 - sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c
^3)*(a - c)))/(a - c)))/((sqrt(-a)*a^4 - a^4*sqrt(-c) - 4*sqrt(-a)*a^3*c +
4*a^3*sqrt(-c)*c + 6*sqrt(-a)*a^2*c^2 - 6*a^2*sqrt(-c)*c^2 - 4*sqrt(-a)*a*c
^3 + 4*a*sqrt(-c)*c^3 + sqrt(-a)*c^4 - sqrt(-c)*c^4)*abs(-a + c)) + 2*sqrt(
b*x + a)/(a - c) - 2*sqrt(b*x + c)/(a - c)
```

Mupad [B] (verification not implemented)

Time = 32.29 (sec) , antiderivative size = 2983, normalized size of antiderivative = 30.75

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \text{Too large to display}$$

```
[In] int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))),x)
[Out] (atan((a^2*c^(5/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^
3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/
2)*2i + a^(5/2)*c^2*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^
(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 +
 a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3 + a^3
*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c -
 2*a^2*c^2)^(1/2)*2i - a^2*c^2*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^
(1/2)*4i - a^(3/2)*c^(5/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2
)*2i - a^(5/2)*c^(3/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i
)/(2*a^5*c^(3/2) - 4*a^4*c^(5/2) + 2*a^(5/2)*c^4 + 2*a^3*c^(7/2) - 4*a^(7/2
)*c^3 + 2*a^(9/2)*c^2 - 2*a^2*c^4*(a + b*x)^(1/2) + 4*a^3*c^3*(a + b*x)^(1/
2) - 2*a^4*c^2*(a + b*x)^(1/2) - 2*a^(3/2)*c^(9/2)*(a + b*x)^(1/2) + 2*a^(5
/2)*c^(7/2)*(a + b*x)^(1/2) + 2*a^(7/2)*c^(5/2)*(a + b*x)^(1/2) - 2*a^(9/2)
*c^(3/2)*(a + b*x)^(1/2) + 2*a^2*c^4*(c + b*x)^(1/2) - 4*a^3*c^3*(c + b*x)^
(1/2) + 2*a^4*c^2*(c + b*x)^(1/2)))*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*
c^2)^(1/2)*2i - 4*a^(3/2)*c - 8*a*c^(3/2) + atan((a^2*c^(5/2)*(a*c^3 + a^3*
c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i
- a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^2*(a*c^3 + a^3
*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2
)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a
^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/
2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^2*c^2*(
c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*4i - a^(3/2)*c^(5/2)*(c +
b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(5/2)*c^(3/2)*(c + b*x)
^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i)/(2*a^5*c^(3/2) - 4*a^4*c^(5/2)
 + 2*a^(5/2)*c^4 + 2*a^3*c^(7/2) - 4*a^(7/2)*c^3 + 2*a^(9/2)*c^2 - 2*a^2*c^
4*(a + b*x)^(1/2) + 4*a^3*c^3*(a + b*x)^(1/2) - 2*a^4*c^2*(a + b*x)^(1/2) -
 2*a^(3/2)*c^(9/2)*(a + b*x)^(1/2) + 2*a^(5/2)*c^(7/2)*(a + b*x)^(1/2) + 2*
a^(7/2)*c^(5/2)*(a + b*x)^(1/2) - 2*a^(9/2)*c^(3/2)*(a + b*x)^(1/2) + 2*a^2
*c^4*(c + b*x)^(1/2) - 4*a^3*c^3*(c + b*x)^(1/2) + 2*a^4*c^2*(c + b*x)^(1/2
)))*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + 4*a^(3/2)*c^(1/2
)*(c + b*x)^(1/2) - 3*a*c^(3/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^
(1/2) - c^(1/2))) - 3*a^(3/2)*c*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^
(1/2) - c^(1/2))) + 8*a*c*(c + b*x)^(1/2) - c^(1/2)*atan((a^2*c^(5/2)*(a*c^
3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(
1/2)*2i - a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^2*(a*c
```

^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^2*c^2*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*4i - a^(3/2)*c^(5/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(5/2)*c^(3/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i)/(2*a^5*c^(3/2) - 4*a^4*c^(5/2) + 2*a^(5/2)*c^4 + 2*a^3*c^(7/2) - 4*a^(7/2)*c^3 + 2*a^(9/2)*c^2 - 2*a^2*c^4*(a + b*x)^(1/2) + 4*a^3*c^3*(a + b*x)^(1/2) - 2*a^4*c^2*(a + b*x)^(1/2) - 2*a^(3/2)*c^(9/2)*(a + b*x)^(1/2) + 2*a^(5/2)*c^(7/2)*(a + b*x)^(1/2) + 2*a^(7/2)*c^(5/2)*(a + b*x)^(1/2) - 2*a^(9/2)*c^(3/2)*(a + b*x)^(1/2) + 2*a^2*c^4*(c + b*x)^(1/2) - 4*a^3*c^3*(c + b*x)^(1/2) + 2*a^4*c^2*(c + b*x)^(1/2)))*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^2*c^(1/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2))) + a^(3/2)*c^(1/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a + b*x)^(1/2) + a^(3/2)*c^(1/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(c + b*x)^(1/2) + 2*a*c*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a + b*x)^(1/2) + 2*a*c*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(c + b*x)^(1/2))/(a^(1/2)*c^(1/2)*(a^(1/2) - c^(1/2))*(a^(1/2) + c^(1/2))^2*((a + b*x)^(1/2) + (c + b*x)^(1/2) - a^(1/2) - c^(1/2))) - (c^2*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2))) - 4*c^(3/2)*(c + b*x)^(1/2) + 4*c^2 + atan((a^2*c^(5/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^2*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^2*c^2*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*4i - a^(3/2)*c^(5/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(5/2)*c^(3/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i)/(2*a^5*c^(3/2) - 4*a^4*c^(5/2) + 2*a^(5/2)*c^4 + 2*a^3*c^(7/2) - 4*a^(7/2)*c^3 + 2*a^(9/2)*c^2 - 2*a^2*c^4*(a + b*x)^(1/2) + 4*a^3*c^3*(a + b*x)^(1/2) - 2*a^4*c^2*(a + b*x)^(1/2) - 2*a^(3/2)*c^(9/2)*(a + b*x)^(1/2) + 2*a^(5/2)*c^(7/2)*(a + b*x)^(1/2) + 2*a^(7/2)*c^(5/2)*(a + b*x)^(1/2) - 2*a^(9/2)*c^(3/2)*(a + b*x)^(1/2) + 2*a^2*c^4*(c + b*x)^(1/2) - 4*a^3*c^3*(c + b*x)^(1/2) + 2*a^4*c^2*(c + b*x)^(1/2)))*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - c^(3/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a + b*x)^(1/2) - c^(3/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(c + b*x)^(1/2))/(c^(1/2)*(a^(1/2) - c^(1/2))*(a^(1/2) + c^(1/2))^2*((a + b*x)^(1/2) + (c + b*x)^(1/2) - a^(1/2) - c^(1/2)))

3.405 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=-\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{(a-c)\sqrt{c}}$$

```
[Out] -b*arctanh((b*x+a)^(1/2)/a^(1/2))/(a-c)/a^(1/2)+b*arctanh((b*x+c)^(1/2)/c^(
1/2))/(a-c)/c^(1/2)-(b*x+a)^(1/2)/(a-c)/x+(b*x+c)^(1/2)/(a-c)/x
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2129, 43, 65, 214}

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}(a-c)}-\frac{\sqrt{a+bx}}{x(a-c)}+\frac{\sqrt{bx+c}}{x(a-c)}$$

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] -(Sqrt[a + b*x]/((a - c)*x)) + Sqrt[c + b*x]/((a - c)*x) - (b*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(Sqrt[a]*(a - c)) + (b*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(
(a - c)*Sqrt[c])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2129

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[-d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\int \frac{\sqrt{a+bx}}{x^2}\,dx}{-ab+bc}+\frac{b\int \frac{\sqrt{c+bx}}{x^2}\,dx}{-ab+bc}\\
&= -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}+\frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2(a-c)}-\frac{b\int \frac{1}{x\sqrt{c+bx}}\,dx}{2(a-c)}\\
&= -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a-c}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{c+bx}\right)}{a-c}\\
&= -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{(a-c)\sqrt{c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2 \left(\sqrt{a+bx}+\sqrt{c+bx}\right)} \, dx = \frac{\sqrt{a}\sqrt{c}(-\sqrt{a+bx}+\sqrt{c+bx})+b\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}x\arctan\left(\frac{\sqrt{a+bx}-\sqrt{c+bx}}{\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}}\right)-b\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}x\arctan\left(\right.}{\sqrt{a}(a-c)\sqrt{c}x}$$

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (Sqrt[a]*Sqrt[c]*(-Sqrt[a + b*x] + Sqrt[c + b*x]) + b*Sqrt[-(Sqrt[a] - Sqrt
[c])^2]*x*ArcTan[(Sqrt[a + b*x] - Sqrt[c + b*x])/Sqrt[-(Sqrt[a] - Sqrt[c])^
2]] - b*Sqrt[-(Sqrt[a] + Sqrt[c])^2]*x*ArcTan[(Sqrt[a + b*x] - Sqrt[c + b*x
])/Sqrt[-(Sqrt[a] + Sqrt[c])^2]])/(Sqrt[a]*(a - c)*Sqrt[c]*x)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2b\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{a-c}-\frac{2b\left(-\frac{\sqrt{bx+c}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{a-c}$	88

```
[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a-c)*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
))-2/(a-c)*b*(-1/2*(b*x+c)^(1/2)/x/b-1/2/c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1
/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.87

$$\int \frac{1}{x^2 \left(\sqrt{a+bx}+\sqrt{c+bx}\right)} \, dx = \left[-\frac{\sqrt{a}bcx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+ab\sqrt{c}x\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)+2\sqrt{bx+a}ac-2\sqrt{bx+c}ac}{2\left(a^2c-ac^2\right)x},\right.$$

$$-\frac{2ab\sqrt{-c}x\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+\sqrt{a}bcx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}ac-2\sqrt{bx+c}ac}{2\left(a^2c-ac^2\right)x}, \frac{2\sqrt{-a}bcx}{}$$

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + a*b*sqr
t(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a)*a*c -
 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x), -1/2*(2*a*b*sqrt(-c)*x*arctan(sq
rt(b*x + c)*sqrt(-c)/c) + sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a)
+ 2*a)/x) + 2*sqrt(b*x + a)*a*c - 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x),
 1/2*(2*sqrt(-a)*b*c*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - a*b*sqrt(c)*x*log
((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a)*a*c + 2*sqrt(b*
x + c)*a*c)/((a^2*c - a*c^2)*x), (sqrt(-a)*b*c*x*arctan(sqrt(b*x + a)*sqrt(
-a)/a) - a*b*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c) - sqrt(b*x + a)*a*
c + sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}\,dx$$

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)}\,dx$$

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(87) = 174$.

Time = 2.44 (sec) , antiderivative size = 1192, normalized size of antiderivative = 11.57

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=\text{Too large to display}$$

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(a - c)) + (2*(a*c^2 - sqrt(a*c)
*c^2)*(a - c)^2*b*sgn(2*a - 2*c) + 2*(a*c^2 + sqrt(a*c)*a*c)*(a - c)^2*b +
```

```
(a^2*c^2 - 2*a*c^3 + c^4 - (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*b*abs(a - c)*
sgn(2*a - 2*c) + (a^3*c - 2*a^2*c^2 + a*c^3 - (a^2*c - 2*a*c^2 + c^3)*sqrt(
a*c))*b*abs(a - c) - (a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 + (a^3*c - a^2*c^2
- a*c^3 + c^4)*sqrt(a*c))*b*sgn(2*a - 2*c) - (a^4*c - a^3*c^2 - a^2*c^3 + a
*c^4 + (a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*b)*arctan(-(sqrt(b*x + a)
 - sqrt(b*x + c))/sqrt(-(a^2 - c^2 + sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c +
3*a*c^2 - c^3)*(a - c)))/(a - c)))/((sqrt(-a)*a^4*c - a^4*sqrt(-c)*c - 4*sq
rt(-a)*a^3*c^2 + 4*a^3*sqrt(-c)*c^2 + 6*sqrt(-a)*a^2*c^3 - 6*a^2*sqrt(-c)*c
^3 - 4*sqrt(-a)*a*c^4 + 4*a*sqrt(-c)*c^4 + sqrt(-a)*c^5 - sqrt(-c)*c^5)*abs
(a - c)) - (2*(a*c^2 - sqrt(a*c)*c^2)*(a - c)^2*b*sgn(2*a - 2*c) - 2*(a*c^2
 + sqrt(a*c)*a*c)*(a - c)^2*b + (a^2*c^2 - 2*a*c^3 + c^4 + (a^2*c - 2*a*c^2
 + c^3)*sqrt(a*c))*b*abs(a - c)*sgn(2*a - 2*c) - (a^3*c - 2*a^2*c^2 + a*c^3
 - (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*b*abs(a - c) - (a^4*c - a^3*c^2 - a^2
*c^3 + a*c^4 + (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*b*sgn(2*a - 2*c)
+ (a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 + (a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt
(a*c))*b)*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))/sqrt(-(a^2 - c^2 - sqrt((
a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(a - c)))/(a - c)))/((sqrt(-
a)*a^4*c - a^4*sqrt(-c)*c - 4*sqrt(-a)*a^3*c^2 + 4*a^3*sqrt(-c)*c^2 + 6*sqr
t(-a)*a^2*c^3 - 6*a^2*sqrt(-c)*c^3 - 4*sqrt(-a)*a*c^4 + 4*a*sqrt(-c)*c^4 +
sqrt(-a)*c^5 - sqrt(-c)*c^5)*abs(a - c)) - 2*(b*(sqrt(b*x + a) - sqrt(b*x +
 c))^3 - a*b*(sqrt(b*x + a) - sqrt(b*x + c)) + b*c*(sqrt(b*x + a) - sqrt(b*
x + c)))/(((sqrt(b*x + a) - sqrt(b*x + c))^4 - 2*a*(sqrt(b*x + a) - sqrt(b*
x + c))^2 - 2*c*(sqrt(b*x + a) - sqrt(b*x + c))^2 + a^2 - 2*a*c + c^2)*(a -
 c)) - sqrt(b*x + a)/((a - c)*x)
```

Mupad [B] (verification not implemented)

Time = 35.04 (sec) , antiderivative size = 2642, normalized size of antiderivative = 25.65

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))),x)
```

```
[Out] (b*atan(((b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(
1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a^3*b*c^(7/2) -
a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c
^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5*b*c^(3/2) + 2*a
^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 -
2*a^4*c^4 + a^5*c^3)) - (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)*c^(11/2) - a^(
7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5 - 2*a^4*c^4 + a
^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*c^5 + 16*a^4*c^4
 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4
*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(
1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)
```

*c^(7/2) - a^(7/2)*c^(3/2))))*1i)/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/
2) - a^(7/2)*c^(3/2))) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a
*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((
a^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2
*a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5
*b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1
/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2
)*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5
 - 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*
c^5 + 16*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*
(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/
2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a
^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))))*1i)/(2*(2*a^2*c^3 - 2*a^3*c^2
 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))))/(((a^(3/2)*b^2*c^(7/2))/2 - a^(5/2)
*b^2*c^(5/2) + (a^(7/2)*b^2*c^(3/2))/2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) - (
((a + b*x)^(1/2) - a^(1/2))*(a^(3/2)*b^2*c^(7/2) - 2*a^(5/2)*b^2*c^(5/2) +
a^(7/2)*b^2*c^(3/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a
^5*c^3)) - (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c
^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a^3*b*c^(7/2)
 - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*a^4*c^4 + a^5
*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5*b*c^(3/2) + 2
*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5
 - 2*a^4*c^4 + a^5*c^3)) - (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)*c^(11/2) - a
^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5 - 2*a^4*c^4 +
 a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*c^5 + 16*a^4*c
^4 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a
^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a
^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/
2)*c^(7/2) - a^(7/2)*c^(3/2)))))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2
) - a^(7/2)*c^(3/2))) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*
c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a
^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*
a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5*
b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/
2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)
*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5
 - 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*c
^5 + 16*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(
a^3*c^5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2
))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^
3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2)))))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a
^(3/2)*c^(7/2) - a^(7/2)*c^(3/2)))))*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3
/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))
^(1/2)*1i)/(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2)) - ((
a^(1/2)*b)/(4*(a*c - a^2)) - (b*c^(1/2))/(4*(a*c - c^2)) - (((a^(1/2)*((a^2

```
*b)/4 - (b*c^2)/4 + (a*b*c)/4))/(a^3*c - a^2*c^2) - (c^(1/2)*((b*c^2)/4 - (
a^2*b)/4 + (a*b*c)/4))/(a*c^3 - a^2*c^2))*((a + b*x)^(1/2) - a^(1/2))^2)/((
c + b*x)^(1/2) - c^(1/2))^2 + (((a^(1/2)*((a*b)/4 - (3*b*c)/4))/(a*c^2 - a^
2*c) - (c^(1/2)*((3*a*b)/4 - (b*c)/4))/(a*c^2 - a^2*c))*((a + b*x)^(1/2) -
a^(1/2)))/((c + b*x)^(1/2) - c^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))/((c + b
*x)^(1/2) - c^(1/2)) + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(
1/2))^3 - ((a + c)*((a + b*x)^(1/2) - a^(1/2))^2)/(a^(1/2)*c^(1/2)*((c + b*
x)^(1/2) - c^(1/2))^2)) - log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2)
- c^(1/2)))*(b/(2*a^(1/2)*c) - (b*(a^(1/2) + c^(1/2)))/(2*c*(a - c))) - (b*
((a + b*x)^(1/2) - a^(1/2)))/(4*a^(1/2)*c^(1/2)*(a^(1/2) - c^(1/2))*((c + b
*x)^(1/2) - c^(1/2)))
```

3.406 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 228

$$\begin{aligned}\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx &= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}-\frac{(4ac-5(a+c)^2)\sqrt{a+bx}\sqrt{c+bx}}{32b^3(a-c)}\\ &\quad+\frac{(4ac-5(a+c)^2)(a+bx)^{3/2}\sqrt{c+bx}}{16b^3(a-c)^2}\\ &\quad+\frac{5(a+c)(a+bx)^{3/2}(c+bx)^{3/2}}{12b^3(a-c)^2}-\frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2}\\ &\quad-\frac{(4ac-5(a+c)^2)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{32b^3}\end{aligned}$$

```
[Out] 1/3*(a+c)*x^3/(a-c)^2+1/2*b*x^4/(a-c)^2+5/12*(a+c)*(b*x+a)^(3/2)*(b*x+c)^(3
/2)/b^3/(a-c)^2-1/2*x*(b*x+a)^(3/2)*(b*x+c)^(3/2)/b^2/(a-c)^2-1/32*(4*a*c-5
*(a+c)^2)*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/b^3+1/16*(4*a*c-5*(a+c)^2)*(
b*x+a)^(3/2)*(b*x+c)^(1/2)/b^3/(a-c)^2-1/32*(4*a*c-5*(a+c)^2)*(b*x+a)^(1/2)
*(b*x+c)^(1/2)/b^3/(a-c)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

$= \{6821, 92, 81, 52, 65, 223, 212\}$

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = -\frac{(4ac-5(a+c)^2)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{32b^3} + \frac{5(a+c)(a+bx)^{3/2}(bx+c)^{3/2}}{12b^3(a-c)^2} + \frac{(4ac-5(a+c)^2)\,(a+bx)^{3/2}\sqrt{bx+c}}{16b^3(a-c)^2} - \frac{(4ac-5(a+c)^2)\sqrt{a+bx}\sqrt{bx+c}}{32b^3(a-c)} - \frac{x(a+bx)^{3/2}(bx+c)^{3/2}}{2b^2(a-c)^2} + \frac{bx^4}{2(a-c)^2} + \frac{x^3(a+c)}{3(a-c)^2}$$

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] ((a + c)*x^3)/(3*(a - c)^2) + (b*x^4)/(2*(a - c)^2) - ((4*a*c - 5*(a + c)^2
)*Sqrt[a + b*x]*Sqrt[c + b*x])/(32*b^3*(a - c)) + ((4*a*c - 5*(a + c)^2)*(a
 + b*x)^(3/2)*Sqrt[c + b*x])/(16*b^3*(a - c)^2) + (5*(a + c)*(a + b*x)^(3/2
)*(c + b*x)^(3/2))/(12*b^3*(a - c)^2) - (x*(a + b*x)^(3/2)*(c + b*x)^(3/2))
/(2*b^2*(a - c)^2) - ((4*a*c - 5*(a + c)^2)*ArcTanh[Sqrt[a + b*x]/Sqrt[c +
b*x]])/(32*b^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(a\left(1+\frac{c}{a}\right)x^2+2bx^3-2x^2\sqrt{a+bx}\sqrt{c+bx}\right)\,dx}{(a-c)^2}\\
&= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}-\frac{2\int x^2\sqrt{a+bx}\sqrt{c+bx}\,dx}{(a-c)^2}\\
&= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}-\frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2}-\frac{\int \sqrt{a+bx}\sqrt{c+bx}\left(-ac-\frac{5}{2}b(a+c)x\right)\,dx}{2b^2(a-c)^2}\\
&= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{5(a+c)(a+bx)^{3/2}(c+bx)^{3/2}}{12b^3(a-c)^2}\\
&\quad -\frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2}+\frac{\left(4ac-5(a+c)^2\right)\int \sqrt{a+bx}\sqrt{c+bx}\,dx}{8b^2(a-c)^2}
\end{aligned}$$

$$= \frac{(a+c)x^3}{3(a-c)^2} + \frac{bx^4}{2(a-c)^2} + \frac{(4ac-5(a+c)^2)(a+bx)^{3/2}\sqrt{c+bx}}{16b^3(a-c)^2} + \frac{5(a+c)(a+bx)^{3/2}(c+bx)^{3/2}}{12b^3(a-c)^2} - \frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2} + \frac{(5a^2+6ac+5c^2)\int \frac{\sqrt{a+bx}}{\sqrt{c+bx}}\,dx}{32b^2(a-c)}$$

$$= \frac{(a+c)x^3}{3(a-c)^2} + \frac{bx^4}{2(a-c)^2} + \frac{(5a^2+6ac+5c^2)\sqrt{a+bx}\sqrt{c+bx}}{32b^3(a-c)} + \frac{(4ac-5(a+c)^2)(a+bx)^{3/2}\sqrt{c+bx}}{16b^3(a-c)^2} + \frac{5(a+c)(a+bx)^{3/2}(c+bx)^{3/2}}{12b^3(a-c)^2} - \frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2} + \frac{(5a^2+6ac+5c^2)\int \frac{1}{\sqrt{a+bx}\sqrt{c+bx}}\,dx}{64b^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2} + \frac{bx^4}{2(a-c)^2} + \frac{(5a^2+6ac+5c^2)\sqrt{a+bx}\sqrt{c+bx}}{32b^3(a-c)} + \frac{(4ac-5(a+c)^2)(a+bx)^{3/2}\sqrt{c+bx}}{16b^3(a-c)^2} + \frac{5(a+c)(a+bx)^{3/2}(c+bx)^{3/2}}{12b^3(a-c)^2} - \frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2} + \frac{(5a^2+6ac+5c^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{-a+c+x^2}}\,dx, x, \sqrt{a+bx}\right)}{32b^3}$$

$$= \frac{(a+c)x^3}{3(a-c)^2} + \frac{bx^4}{2(a-c)^2} + \frac{(5a^2+6ac+5c^2)\sqrt{a+bx}\sqrt{c+bx}}{32b^3(a-c)} + \frac{(4ac-5(a+c)^2)(a+bx)^{3/2}\sqrt{c+bx}}{16b^3(a-c)^2} + \frac{5(a+c)(a+bx)^{3/2}(c+bx)^{3/2}}{12b^3(a-c)^2} - \frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2} + \frac{(5a^2+6ac+5c^2)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{32b^3}$$

$$= \frac{(a+c)x^3}{3(a-c)^2} + \frac{bx^4}{2(a-c)^2} + \frac{(5a^2+6ac+5c^2)\sqrt{a+bx}\sqrt{c+bx}}{32b^3(a-c)} + \frac{(4ac-5(a+c)^2)(a+bx)^{3/2}\sqrt{c+bx}}{16b^3(a-c)^2} + \frac{5(a+c)(a+bx)^{3/2}(c+bx)^{3/2}}{12b^3(a-c)^2} - \frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2} + \frac{(5a^2+6ac+5c^2)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{32b^3}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = -\frac{\sqrt{a+bx}\sqrt{c+bx}(15a^3+15c^3-10bc^2x+8b^2cx^2+48b^3x^3-a^2(7c+10bx)+a(-7c^2+4bcx+8b^2x^2))}{96b^3(a}$$

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] -1/96*(Sqrt[a + b*x]*Sqrt[c + b*x]*(15*a^3 + 15*c^3 - 10*b*c^2*x + 8*b^2*c*
x^2 + 48*b^3*x^3 - a^2*(7*c + 10*b*x) + a*(-7*c^2 + 4*b*c*x + 8*b^2*x^2)) -
 16*(-c^4 + 2*b^3*c*x^3 + 3*b^4*x^4 + 2*a*(c^3 + b^3*x^3)) + 3*(a - c)^2*(5
*a^2 + 6*a*c + 5*c^2)*Log[Sqrt[a + b*x] - Sqrt[c + b*x]])/(b^3*(a - c)^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.65

method	result
default	$\frac{a\,x^3}{3(a-c)^2}+\frac{c\,x^3}{3(a-c)^2}+\frac{b\,x^4}{2(a-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{bx+c}\left(96\ \mathrm{csgn}(b)x^3b^3\sqrt{b^2x^2+abx+bcx+ac}+16\ \mathrm{csgn}(b)x^2a\,b^2\sqrt{b^2x^2+abx+bcx+ac}\right.}{}$

```
[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(a-c)^2*a*x^3+1/3/(a-c)^2*c*x^3+1/2*b*x^4/(a-c)^2-1/192/(a-c)^2*(b*x+a)
^(1/2)*(b*x+c)^(1/2)*(96*csgn(b)*x^3*b^3*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+16
*csgn(b)*x^2*a*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+16*csgn(b)*x^2*b^2*c*(b^
2*x^2+a*b*x+b*c*x+a*c)^(1/2)-20*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*x*a
^2*b+8*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*x*a*b*c-20*(b^2*x^2+a*b*x+b*
c*x+a*c)^(1/2)*csgn(b)*x*b*c^2+30*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a
^3-14*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a^2*c-14*(b^2*x^2+a*b*x+b*c*x
+a*c)^(1/2)*csgn(b)*a*c^2+30*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*c^3-15
*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*csgn(b))*a^4+
12*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*csgn(b))*a^
3*c+6*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*csgn(b))
*a^2*c^2+12*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*cs
gn(b))*a*c^3-15*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c
)*csgn(b))*c^4)*csgn(b)/b^3/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{96\,b^4x^4 + 64\,(ab^3+b^3c)x^3 - 2\,(48\,b^3x^3 + 15\,a^3 - 7\,a^2c - 7\,ac^2 + 15\,c^3 + 8\,(ab^2+b^2c)x^2 - 2\,(5\,a^2b - 2\,abc -}{192\,(a^2b^3 -}$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/192*(96*b^4*x^4 + 64*(a*b^3 + b^3*c)*x^3 - 2*(48*b^3*x^3 + 15*a^3 - 7*a^2
*c - 7*a*c^2 + 15*c^3 + 8*(a*b^2 + b^2*c)*x^2 - 2*(5*a^2*b - 2*a*b*c + 5*b*
c^2)*x)*sqrt(b*x + a)*sqrt(b*x + c) - 3*(5*a^4 - 4*a^3*c - 2*a^2*c^2 - 4*a*
c^3 + 5*c^4)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c))/(a^2*b^3
- 2*a*b^3*c + b^3*c^2)
```

Sympy [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. 2(194) = 388.

Time = 0.36 (sec) , antiderivative size = 797, normalized size of antiderivative = 3.50

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx =$$

$$-\frac{1}{96}\left(2\left(4\,(bx+a)\left(\frac{6\,(a^5b^9-5\,a^4b^9c+10\,a^3b^9c^2-10\,a^2b^9c^3+5\,ab^9c^4-b^9c^5)(bx+a)}{a^7b^{12}-7\,a^6b^{12}c+21\,a^5b^{12}c^2-35\,a^4b^{12}c^3+35\,a^3b^{12}c^4-21\,a^2b^{12}c^5+7\,ab^{12}c^6-b^{12}}\right.\right.\right.$$

$$+\frac{3\,(bx+a)^4-10\,(bx+a)^3a+12\,(bx+a)^2a^2-6\,(bx+a)a^3+2\,(bx+a)^3c-6\,(bx+a)^2ac+6\,(bx+a)}{6\,(a^2b^3-2\,ab^3c+b^3c^2)}$$

$$-\frac{(5\,a^2+6\,ac+5\,c^2)\log\left(\left|-\sqrt{bx+a}+\sqrt{bx+c}\right|\right)}{32\,b^3}$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/96*(2*(4*(b*x + a)*(6*(a^5*b^9 - 5*a^4*b^9*c + 10*a^3*b^9*c^2 - 10*a^2*b
^9*c^3 + 5*a*b^9*c^4 - b^9*c^5)*(b*x + a)/(a^7*b^12 - 7*a^6*b^12*c + 21*a^5
*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c^5 + 7*a*b^12*
c^6 - b^12*c^7) - (17*a^6*b^9 - 86*a^5*b^9*c + 175*a^4*b^9*c^2 - 180*a^3*b^
9*c^3 + 95*a^2*b^9*c^4 - 22*a*b^9*c^5 + b^9*c^6)/(a^7*b^12 - 7*a^6*b^12*c +
 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c^5 + 7*
a*b^12*c^6 - b^12*c^7)) + (59*a^7*b^9 - 301*a^6*b^9*c + 615*a^5*b^9*c^2 - 6
25*a^4*b^9*c^3 + 305*a^3*b^9*c^4 - 39*a^2*b^9*c^5 - 19*a*b^9*c^6 + 5*b^9*c^
7)/(a^7*b^12 - 7*a^6*b^12*c + 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^
12*c^4 - 21*a^2*b^12*c^5 + 7*a*b^12*c^6 - b^12*c^7))*(b*x + a) - 3*(5*a^8*b
^9 - 24*a^7*b^9*c + 44*a^6*b^9*c^2 - 40*a^5*b^9*c^3 + 30*a^4*b^9*c^4 - 40*a
^3*b^9*c^5 + 44*a^2*b^9*c^6 - 24*a*b^9*c^7 + 5*b^9*c^8)/(a^7*b^12 - 7*a^6*b
^12*c + 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c
^5 + 7*a*b^12*c^6 - b^12*c^7))*sqrt(b*x + a)*sqrt(b*x + c) + 1/6*(3*(b*x +
a)^4 - 10*(b*x + a)^3*a + 12*(b*x + a)^2*a^2 - 6*(b*x + a)*a^3 + 2*(b*x + a
)^3*c - 6*(b*x + a)^2*a*c + 6*(b*x + a)*a^2*c)/(a^2*b^3 - 2*a*b^3*c + b^3*c
^2) - 1/32*(5*a^2 + 6*a*c + 5*c^2)*log(abs(-sqrt(b*x + a) + sqrt(b*x + c)))
/b^3
```

Mupad [B] (verification not implemented)

Time = 147.00 (sec) , antiderivative size = 1358, normalized size of antiderivative = 5.96

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

```
[Out] (x^3*(a + c))/(3*(a - c)^2) - ((((a + b*x)^(1/2) - a^(1/2))^15*((3*a*c)/8 +
 (5*a^2)/16 + (5*c^2)/16))/(b^3*((c + b*x)^(1/2) - c^(1/2))^15) + (((a + b*
x)^(1/2) - a^(1/2))^3*((23*a*c^3)/12 + (23*a^3*c)/12 - (115*a^4)/48 - (115*
c^4)/48 + (349*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2*b^3 + b^3*c
^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))^13*((23*a*c^3)/12 + (23*a^3
*c)/12 - (115*a^4)/48 - (115*c^4)/48 + (349*a^2*c^2)/8))/(((c + b*x)^(1/2)
- c^(1/2))^13*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2
))^5*((3917*a*c^3)/12 + (3917*a^3*c)/12 + (383*a^4)/48 + (383*c^4)/48 + (72
79*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^5*(a^2*b^3 + b^3*c^2 - 2*a*b^3
*c)) + (((a + b*x)^(1/2) - a^(1/2))^11*((3917*a*c^3)/12 + (3917*a^3*c)/12 +
 (383*a^4)/48 + (383*c^4)/48 + (7279*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/
2))^11*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))^7*((
17567*a*c^3)/12 + (17567*a^3*c)/12 + (2789*a^4)/48 + (2789*c^4)/48 + (28213
*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^7*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c
)) + (((a + b*x)^(1/2) - a^(1/2))^9*((17567*a*c^3)/12 + (17567*a^3*c)/12 +
(2789*a^4)/48 + (2789*c^4)/48 + (28213*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(
1/2))^9*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))*((3
*a*c)/8 + (5*a^2)/16 + (5*c^2)/16))/(b^3*((c + b*x)^(1/2) - c^(1/2))) - (a^
(1/2)*c^(1/2)*(192*a*c^2 + 192*a^2*c)*((a + b*x)^(1/2) - a^(1/2))^4)/(((c +
 b*x)^(1/2) - c^(1/2))^4*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) - (a^(1/2)*c^(1/2
)*(192*a*c^2 + 192*a^2*c)*((a + b*x)^(1/2) - a^(1/2))^12)/(((c + b*x)^(1/2)
 - c^(1/2))^12*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) - (a^(1/2)*c^(1/2)*((a + b*
x)^(1/2) - a^(1/2))^6*((5120*a*c^2)/3 + (5120*a^2*c)/3 + 256*a^3 + 256*c^3)
)/(((c + b*x)^(1/2) - c^(1/2))^6*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) - (a^(1/2
)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^10*((5120*a*c^2)/3 + (5120*a^2*c)/3 +
 256*a^3 + 256*c^3))/(((c + b*x)^(1/2) - c^(1/2))^10*(a^2*b^3 + b^3*c^2 - 2
*a*b^3*c)) - (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^8*((10112*a*c^2)/
3 + (10112*a^2*c)/3 + 512*a^3 + 512*c^3))/(((c + b*x)^(1/2) - c^(1/2))^8*(a
^2*b^3 + b^3*c^2 - 2*a*b^3*c)))/((28*((a + b*x)^(1/2) - a^(1/2))^4)/((c + b
*x)^(1/2) - c^(1/2))^4 - (8*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2)
 - c^(1/2))^2 - (56*((a + b*x)^(1/2) - a^(1/2))^6)/((c + b*x)^(1/2) - c^(1/
2))^6 + (70*((a + b*x)^(1/2) - a^(1/2))^8)/((c + b*x)^(1/2) - c^(1/2))^8 -
(56*((a + b*x)^(1/2) - a^(1/2))^10)/((c + b*x)^(1/2) - c^(1/2))^10 + (28*((
a + b*x)^(1/2) - a^(1/2))^12)/((c + b*x)^(1/2) - c^(1/2))^12 - (8*((a + b*x
)^(1/2) - a^(1/2))^14)/((c + b*x)^(1/2) - c^(1/2))^14 + ((a + b*x)^(1/2) -
a^(1/2))^16/((c + b*x)^(1/2) - c^(1/2))^16 + 1) + (b*x^4)/(2*(a - c)^2) + (
atanh(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(6*a*c + 5*a
^2 + 5*c^2))/(16*b^3)
```

3.407 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\sqrt{c+bx}}{4b^2(a-c)} + \frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2} - \frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}-\frac{(a+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{4b^2}$$

```
[Out] 1/2*(a+c)*x^2/(a-c)^2+2/3*b*x^3/(a-c)^2-2/3*(b*x+a)^(3/2)*(b*x+c)^(3/2)/b^2
/(a-c)^2-1/4*(a+c)*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/b^2+1/2*(a+c)*(b*x+
a)^(3/2)*(b*x+c)^(1/2)/b^2/(a-c)^2-1/4*(a+c)*(b*x+a)^(1/2)*(b*x+c)^(1/2)/b^
2/(a-c)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6821, 81, 52, 65, 223, 212}

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = -\frac{(a+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{4b^2}-\frac{2(a+bx)^{3/2}(bx+c)^{3/2}}{3b^2(a-c)^2} + \frac{(a+c)(a+bx)^{3/2}\sqrt{bx+c}}{2b^2(a-c)^2} - \frac{(a+c)\sqrt{a+bx}\sqrt{bx+c}}{4b^2(a-c)}+\frac{2bx^3}{3(a-c)^2}+\frac{x^2(a+c)}{2(a-c)^2}$$

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] ((a + c)*x^2)/(2*(a - c)^2) + (2*b*x^3)/(3*(a - c)^2) - ((a + c)*Sqrt[a + b
*x]*Sqrt[c + b*x])/(4*b^2*(a - c)) + ((a + c)*(a + b*x)^(3/2)*Sqrt[c + b*x]
)/(2*b^2*(a - c)^2) - (2*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(3*b^2*(a - c)^2)
 - ((a + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(4*b^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \left(a\left(1+\frac{c}{a}\right)x+2bx^2-2x\sqrt{a+bx}\sqrt{c+bx}\right)\,dx}{(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{2\int x\sqrt{a+bx}\sqrt{c+bx}\,dx}{(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}+\frac{(a+c)\int\sqrt{a+bx}\sqrt{c+bx}\,dx}{b(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2} - \frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}-\frac{(a+c)\int\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\,dx}{4b(a-c)}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2} - \frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}-\frac{(a+c)\int\frac{1}{\sqrt{a+bx}\sqrt{c+bx}}\,dx}{8b}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2} - \frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}-\frac{(a+c)\text{Subst}\left(\int\frac{1}{\sqrt{-a+c+x^2}}\,dx,x,\sqrt{a+bx}\right)}{4b^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2} - \frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}-\frac{(a+c)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{4b^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2} - \frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}-\frac{(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{4b^2}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.81

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\frac{2(c+bx)(-3ac+c^2+3abx-bcx+4b^2x^2)}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}(3a^2+3c^2-2bcx-8b^2x^2-2a(c+bx))}{(a-c)^2}+3(a+c)\log\left(\sqrt{a+bx}-\sqrt{c+bx}\right)}{12b^2}$$

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] ((2*(c + b*x)*(-3*a*c + c^2 + 3*a*b*x - b*c*x + 4*b^2*x^2))/(a - c)^2 + (Sq
rt[a + b*x]*Sqrt[c + b*x]*(3*a^2 + 3*c^2 - 2*b*c*x - 8*b^2*x^2 - 2*a*(c + b
*x)))/(a - c)^2 + 3*(a + c)*Log[Sqrt[a + b*x] - Sqrt[c + b*x]])/(12*b^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.61

method	result
default	$\frac{x^2 a}{2(a-c)^2}+\frac{x^2 c}{2(a-c)^2}+\frac{2b\,x^3}{3(a-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{bx+c}\left(16\ \mathrm{csgn}(b)x^2b^2\sqrt{b^2x^2+abx+bcx+ac}+4\ \mathrm{csgn}(b)\sqrt{b^2x^2+abx+bcx+ac}\,xab+4\ \mathrm{c}\right.}{}$

```
[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/(a-c)^2*a+1/2*x^2/(a-c)^2*c+2/3*b*x^3/(a-c)^2-1/24/(a-c)^2*(b*x+a)^
(1/2)*(b*x+c)^(1/2)*(16*csgn(b)*x^2*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+4*c
sgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*x*a*b+4*csgn(b)*(b^2*x^2+a*b*x+b*c*x
+a*c)^(1/2)*x*b*c-6*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*a^2+4*csgn(b)*(
b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*a*c-6*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2
)*c^2+3*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*csgn(b
))*a^3-3*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*csgn(
b))*a^2*c-3*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*cs
gn(b))*a*c^2+3*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)
*csgn(b))*c^3)*csgn(b)/b^2/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$= \frac{16\,b^3x^3 + 12\,(ab^2+b^2c)x^2 - 2\,(8\,b^2x^2 - 3\,a^2 + 2\,ac - 3\,c^2 + 2\,(ab+bc)x)\sqrt{bx+a}\sqrt{bx+c} + 3\,(a^3 - a^2c -}{24\,(a^2b^2 - 2\,ab^2c + b^2c^2)}$$

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
[Out] 1/24*(16*b^3*x^3 + 12*(a*b^2 + b^2*c)*x^2 - 2*(8*b^2*x^2 - 3*a^2 + 2*a*c -
3*c^2 + 2*(a*b + b*c)*x)*sqrt(b*x + a)*sqrt(b*x + c) + 3*(a^3 - a^2*c - a*c
^2 + c^3)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c))/(a^2*b^2 - 2
*a*b^2*c + b^2*c^2)
```

Sympy [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{x}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

```
[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
[Out] Integral(x/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

Maxima [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{x}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(137) = 274$.

Time = 0.34 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.70

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = -\frac{\left(2\,(bx+a)\left(\frac{4\,(a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3)(bx+a)}{a^5b^3-5\,a^4b^3c+10\,a^3b^3c^2-10\,a^2b^3c^3+5\,ab^3c^4-b^3c^5}-\frac{7\,a^4b^2-22\,a^3b^2c+24\,a^2b^2c^2-10\,ab^2c^3+b^2c^4}{a^5b^3-5\,a^4b^3c+10\,a^3b^3c^2-10\,a^2b^3c^3+5\,ab^3c^4-b^3c^5}\right)+\frac{3\,(a^5b^2-3\,a}{a^5b^3-5\,a^4}\right.}{}$$

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/12*((2*(b*x + a)*(4*(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)*(b*x
 + a)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*b^3*c^3 + 5*a*b^3*c^
4 - b^3*c^5) - (7*a^4*b^2 - 22*a^3*b^2*c + 24*a^2*b^2*c^2 - 10*a*b^2*c^3 +
b^2*c^4)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*b^3*c^3 + 5*a*b^3
*c^4 - b^3*c^5)) + 3*(a^5*b^2 - 3*a^4*b^2*c + 2*a^3*b^2*c^2 + 2*a^2*b^2*c^3
 - 3*a*b^2*c^4 + b^2*c^5)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*
b^3*c^3 + 5*a*b^3*c^4 - b^3*c^5))*sqrt(b*x + a)*sqrt(b*x + c) - 3*(a + c)*l
og(abs(-sqrt(b*x + a) + sqrt(b*x + c)))/b - 2*(4*(b*x + a)^3 - 9*(b*x + a)^
2*a + 6*(b*x + a)*a^2 + 3*(b*x + a)^2*c - 6*(b*x + a)*a*c)/(a^2*b - 2*a*b*c
 + b*c^2))/b
```

Mupad [B] (verification not implemented)

Time = 75.36 (sec) , antiderivative size = 1012, normalized size of antiderivative = 6.13

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)\left(\frac{a}{2}+\frac{c}{2}\right)}{b^2\left(\sqrt{c+b\,x}-\sqrt{c}\right)}+\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{11}\left(\frac{a}{2}+\frac{c}{2}\right)}{b^2\left(\sqrt{c+b\,x}-\sqrt{c}\right)^{11}}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3\left(\frac{17\,a^3}{6}+\frac{101\,a^2\,c}{2}+\frac{101\,a\,c^2}{2}+\frac{17\,c^3}{6}\right)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^3\left(a^2\,b^2-2\,a\,b^2\,c+b^2\,c^2\right)}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^9\left(\frac{17\,a^3}{6}+\frac{101\,a^2\,c}{2}+\right.}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^9\left(a^2\,b^2-2\,a\,b\right.}}{}$$

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}\right)(a+c)}{2\,b^2}+\frac{x^2\,(a+c)}{2\,(a-c)^2}+\frac{2\,b\,x^3}{3\,(a-c)^2}$$

```
[In] int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

```
[Out] ((((a + b*x)^(1/2) - a^(1/2))*(a/2 + c/2))/(b^2*((c + b*x)^(1/2) - c^(1/2))
) + (((a + b*x)^(1/2) - a^(1/2))^11*(a/2 + c/2))/(b^2*((c + b*x)^(1/2) - c^
(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^3*((101*a*c^2)/2 + (101*a^2*c)/2
+ (17*a^3)/6 + (17*c^3)/6))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2*b^2 + b^2*c
```

```
^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^9*((101*a*c^2)/2 + (101*a^2
*c)/2 + (17*a^3)/6 + (17*c^3)/6))/(((c + b*x)^(1/2) - c^(1/2))^9*(a^2*b^2 +
 b^2*c^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^5*(269*a*c^2 + 269*a^
2*c + 19*a^3 + 19*c^3))/(((c + b*x)^(1/2) - c^(1/2))^5*(a^2*b^2 + b^2*c^2 -
 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^7*(269*a*c^2 + 269*a^2*c + 19*a
^3 + 19*c^3))/(((c + b*x)^(1/2) - c^(1/2))^7*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c
)) + (16*a^(3/2)*c^(3/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(((c + b*x)^(1/2) -
 c^(1/2))^2*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) + (16*a^(3/2)*c^(3/2)*((a + b*
x)^(1/2) - a^(1/2))^10)/(((c + b*x)^(1/2) - c^(1/2))^10*(a^2*b^2 + b^2*c^2
- 2*a*b^2*c)) + (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^4*(192*a*c + 6
4*a^2 + 64*c^2))/(((c + b*x)^(1/2) - c^(1/2))^4*(a^2*b^2 + b^2*c^2 - 2*a*b^
2*c)) + (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^8*(192*a*c + 64*a^2 +
64*c^2))/(((c + b*x)^(1/2) - c^(1/2))^8*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) +
(a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^6*((1312*a*c)/3 + 128*a^2 + 12
8*c^2))/(((c + b*x)^(1/2) - c^(1/2))^6*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)))/((
15*((a + b*x)^(1/2) - a^(1/2))^4)/((c + b*x)^(1/2) - c^(1/2))^4 - (6*((a +
b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2 - (20*((a + b*x)^(1/
2) - a^(1/2))^6)/((c + b*x)^(1/2) - c^(1/2))^6 + (15*((a + b*x)^(1/2) - a^(
1/2))^8)/((c + b*x)^(1/2) - c^(1/2))^8 - (6*((a + b*x)^(1/2) - a^(1/2))^10)
/((c + b*x)^(1/2) - c^(1/2))^10 + ((a + b*x)^(1/2) - a^(1/2))^12/((c + b*x)
^(1/2) - c^(1/2))^12 + 1) - (atanh(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(
1/2) - c^(1/2)))*(a + c))/(2*b^2) + (x^2*(a + c))/(2*(a - c)^2) + (2*b*x^3)
/(3*(a - c)^2)
```

3.408 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=\frac{(a-c)^2}{8b\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^4}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{2b}$$

```
[Out] 1/2*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/b+1/8*(a-c)^2/b/((b*x+a)^(1/2)+(b*
x+c)^(1/2))^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.81, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6821, 52, 65, 223, 212}

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{2b}+\frac{bx^2}{(a-c)^2}-\frac{(a+bx)^{3/2}\sqrt{bx+c}}{b(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{bx+c}}{2b(a-c)}+\frac{x(a+c)}{(a-c)^2}$$

```
[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

```
[Out] ((a + c)*x)/(a - c)^2 + (b*x^2)/(a - c)^2 + (Sqrt[a + b*x]*Sqrt[c + b*x])/(
2*b*(a - c)) - ((a + b*x)^(3/2)*Sqrt[c + b*x])/(b*(a - c)^2) + ArcTanh[Sqrt
[a + b*x]/Sqrt[c + b*x]]/(2*b)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(a\left(1+\frac{c}{a}\right)+2bx-2\sqrt{a+bx}\sqrt{c+bx}\right)\,dx}{(a-c)^2} \\
&= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}-\frac{2\int \sqrt{a+bx}\sqrt{c+bx}\,dx}{(a-c)^2} \\
&= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\int \frac{\sqrt{a+bx}}{\sqrt{c+bx}}\,dx}{2(a-c)} \\
&= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{1}{4}\int \frac{1}{\sqrt{a+bx}\sqrt{c+bx}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&=\frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}}{2b(a-c)} \\
&\quad-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\text{Subst}\left(\int\frac{1}{\sqrt{-a+c+x^2}}\,dx,x,\sqrt{a+bx}\right)}{2b}
\end{aligned}
$$

$$
=\frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{2b}
$$

$$
=\frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{2b}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$
\int\frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx
=-\frac{-\frac{2(a+bx)(c+bx)}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}(a+c+2bx)}{(a-c)^2}+\log\left(\sqrt{a+bx}-\sqrt{c+bx}\right)}{2b}
$$

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

```
[Out] -1/2*((-2*(a + b*x)*(c + b*x))/(a - c)^2 + (Sqrt[a + b*x]*Sqrt[c + b*x]*(a
+ c + 2*b*x))/(a - c)^2 + Log[Sqrt[a + b*x] - Sqrt[c + b*x]])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(51) = 102.

Time = 0.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.92

method	result
default	$\frac{xa}{(a-c)^2}+\frac{xc}{(a-c)^2}+\frac{x^2b}{(a-c)^2}-\frac{2\left(\frac{\sqrt{bx+a}\,(bx+c)^{\frac{3}{2}}}{2b}-\frac{(-ab+bc)\left(\frac{\sqrt{bx+c}\,\sqrt{bx+a}}{b}-\frac{(ab-bc)\sqrt{(bx+a)(bx+c)}\,\ln\left(\frac{\frac{1}{2}ab+\frac{1}{2}bc+b^2x}{\sqrt{b^2}}+\sqrt{b^2x^2+(ab}\right.}{2b\sqrt{bx+c}\,\sqrt{bx+a}\,\sqrt{b^2}}\right.}{4b}\right.}{(a-c)^2}$

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/(a-c)^2*a+x/(a-c)^2*c+x^2/(a-c)^2*b-2/(a-c)^2*(1/2/b*(b*x+a)^(1/2)*(b*x+c
)^(3/2)-1/4*(-a*b+b*c)/b*(1/b*(b*x+c)^(1/2)*(b*x+a)^(1/2)-1/2*(a*b-b*c)/b*(
```

```
(b*x+a)*(b*x+c))^(1/2)/(b*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*b*c+b^2*
x)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(1/2))/(b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(51) = 102$.

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.63

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{4\,b^2x^2-2\,(2\,bx+a+c)\sqrt{bx+a}\sqrt{bx+c}+4\,(ab+bc)x-(a^2-2\,ac+c^2)\log\left(-2\,bx+2\,\sqrt{bx+a}\sqrt{bx+\right.}{4\,(a^2b-2\,abc+bc^2)}$$

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*x^2 - 2*(2*b*x + a + c)*sqrt(b*x + a)*sqrt(b*x + c) + 4*(a*b + b
*c)*x - (a^2 - 2*a*c + c^2)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a
- c))/(a^2*b - 2*a*b*c + b*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(48) = 96$.

Time = 0.42 (sec) , antiderivative size = 388, normalized size of antiderivative = 6.16

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \begin{cases} \frac{2a\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}}+\frac{a}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}}+\frac{4bx\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}}+\frac{2bx}{4ab+8b^2x+4bc+8b\sqrt{a+bx}} \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^2} \end{cases}$$

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((2*a*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*c
 + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + a/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqr
t(a + b*x)*sqrt(b*x + c)) + 4*b*x*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b
 + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 2*b*x/(4*a*b + 8*b
**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 2*c*log(sqrt(a + b*x) +
sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c))
 + c/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 4*sqrt(
a + b*x)*sqrt(b*x + c)*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x
 + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(
c))**2, True))
```

Maxima [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{1}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
[Out] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(51) = 102$.

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.00

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = -\frac{1}{2}\sqrt{bx+a}\sqrt{bx+c}\left(\frac{2\,(ab-bc)(bx+a)}{a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3}-\frac{a^2b-2\,abc+bc^2}{a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3}\right) + \frac{(bx+a)^2-(bx+a)a+(bx+a)c}{a^2b-2\,abc+bc^2} - \frac{\log\left(\left|-\sqrt{bx+a}+\sqrt{bx+c}\right|\right)}{2\,b}$$

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
[Out] -1/2*sqrt(b*x + a)*sqrt(b*x + c)*(2*(a*b - b*c)*(b*x + a)/(a^3*b^2 - 3*a^2*
b^2*c + 3*a*b^2*c^2 - b^2*c^3) - (a^2*b - 2*a*b*c + b*c^2)/(a^3*b^2 - 3*a^2
*b^2*c + 3*a*b^2*c^2 - b^2*c^3)) + ((b*x + a)^2 - (b*x + a)*a + (b*x + a)*c
)/(a^2*b - 2*a*b*c + b*c^2) - 1/2*log(abs(-sqrt(b*x + a) + sqrt(b*x + c)))/
b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.75

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{b\,x^2}{(a-c)^2} + \frac{x\,(a+c)}{(a-c)^2} + \frac{\ln\left(a+c+2\,\sqrt{a+b\,x}\,\sqrt{c+b\,x}+2\,b\,x\right)\,(a\,b-b\,c)^2}{4\,b^3\,(a-c)^2} - \frac{2\,\sqrt{a+b\,x}\,\sqrt{c+b\,x}\,\left(\frac{x}{2}+\frac{a\,b+b\,c}{4\,b^2}\right)}{(a-c)^2}$$

```
[In] int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
[Out] (b*x^2)/(a - c)^2 + (x*(a + c))/(a - c)^2 + (log(a + c + 2*(a + b*x)^(1/2)*
(c + b*x)^(1/2) + 2*b*x)*(a*b - b*c)^2)/(4*b^3*(a - c)^2) - (2*(a + b*x)^(1
/2)*(c + b*x)^(1/2)*(x/2 + (a*b + b*c)/(4*b^2)))/(a - c)^2
```

3.409 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2}-\frac{2(a+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}$$

```
[Out] 2*b*x/(a-c)^2-2*(a+c)*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/(a-c)^2+(a+c)*ln
(x)/(a-c)^2+4*arctanh(c^(1/2)*(b*x+a)^(1/2)/a^(1/2)/(b*x+c)^(1/2))*a^(1/2)*
c^(1/2)/(a-c)^2-2*(b*x+a)^(1/2)*(b*x+c)^(1/2)/(a-c)^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6821, 103, 163, 65, 223, 212, 95, 214}

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = -\frac{2(a+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{bx+c}}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}$$

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (2*b*x)/(a - c)^2 - (2*Sqrt[a + b*x]*Sqrt[c + b*x])/(a - c)^2 - (2*(a + c)*
ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 + (4*Sqrt[a]*Sqrt[c]*ArcTan
```

```
h[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(a - c)^2 + ((a + c)*Lo
g[x])/(a - c)^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \left(2b + \frac{a\left(1+\frac{c}{a}\right)}{x} - \frac{2\sqrt{a+bx}\sqrt{c+bx}}{x}\right) dx}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2} + \frac{(a+c)\log(x)}{(a-c)^2} - \frac{2\int \frac{\sqrt{a+bx}\sqrt{c+bx}}{x}\,dx}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2} - \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2} + \frac{(a+c)\log(x)}{(a-c)^2} + \frac{2\int \frac{-ac-\frac{1}{2}b(a+c)x}{x\sqrt{a+bx}\sqrt{c+bx}}\,dx}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2} - \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2} + \frac{(a+c)\log(x)}{(a-c)^2} - \frac{(2ac)\int \frac{1}{x\sqrt{a+bx}\sqrt{c+bx}}\,dx}{(a-c)^2} - \frac{(b(a+c))\int \frac{1}{\sqrt{a+bx}\sqrt{c+bx}}\,dx}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2} - \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2} + \frac{(a+c)\log(x)}{(a-c)^2} - \frac{(4ac)\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2} - \frac{(2(a+c))\text{Subst}\left(\int \frac{1}{\sqrt{-a+c+x^2}}\,dx, x, \sqrt{a+bx}\right)}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2} - \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2} + \frac{4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{(a-c)^2} + \frac{(a+c)\log(x)}{(a-c)^2} - \frac{(2(a+c))\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2} - \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2} - \frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2} + \frac{4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{(a-c)^2} + \frac{(a+c)\log(x)}{(a-c)^2}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\log\left(\sqrt{a}\sqrt{c}+bx-\sqrt{a+bx}\sqrt{c+bx}\right)}{\left(\sqrt{a}+\sqrt{c}\right)^2} + \frac{2\left(c+bx-\sqrt{a+bx}\sqrt{c+bx}\right)+\left(\sqrt{a}+\sqrt{c}\right)^2\log\left(\sqrt{a}\sqrt{c}-bx+\sqrt{a+bx}\sqrt{c+bx}\right)}{(a-c)^2}$$

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] Log[Sqrt[a]*Sqrt[c] + b*x - Sqrt[a + b*x]*Sqrt[c + b*x]]/(Sqrt[a] + Sqrt[c]
)^2 + (2*(c + b*x - Sqrt[a + b*x]*Sqrt[c + b*x]) + (Sqrt[a] + Sqrt[c])^2*Lo
g[Sqrt[a]*Sqrt[c] - b*x + Sqrt[a + b*x]*Sqrt[c + b*x]])/(a - c)^2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.94

method	result
default	$\frac{a\ln(x)}{(a-c)^2}+\frac{c\ln(x)}{(a-c)^2}+\frac{2bx}{(a-c)^2}+\frac{\sqrt{bx+a}\,\sqrt{bx+c}\left(2\ \mathrm{csgn}(b)\ln\left(\frac{abx+bcx+2\sqrt{ac}\,\sqrt{b^2x^2+abx+bcx+ac}+2ac}{x}\right)ac-2\ \mathrm{csgn}(b)\sqrt{b^2x^2+abx+}\right.}{}$

```
[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a-c)^2*a*ln(x)+1/(a-c)^2*c*ln(x)+2*b*x/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*(
b*x+c)^(1/2)*(2*csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+
a*c)^(1/2)+2*a*c)/x)*a*c-2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*(a*c)^(1
/2)-ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*csgn(b))*(
a*c)^(1/2)*a-ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*c
sgn(b))*(a*c)^(1/2)*c)*csgn(b)/(a*c)^(1/2)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.18

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$= \left[\frac{2\,bx+(a+c)\log\left(-2\,bx+2\,\sqrt{bx+a}\sqrt{bx+c}-a-c\right)+(a+c)\log\left(x\right)+2\,\sqrt{ac}\log\left(\frac{2\,a^2c+2\,ac^2+2\,(2\,ac+\sqrt{}\right.}{a^2-2\,ac+c^2}\right.$$

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + (a
+ c)*log(x) + 2*sqrt(a*c)*log((2*a^2*c + 2*a*c^2 + 2*(2*a*c + sqrt(a*c)*(a
+ c))*sqrt(b*x + a)*sqrt(b*x + c) + (a^2*b + 2*a*b*c + b*c^2)*x + 2*(2*a*c
+ (a*b + b*c)*x)*sqrt(a*c))/x) - 2*sqrt(b*x + a)*sqrt(b*x + c))/(a^2 - 2*a*
c + c^2), (2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a -
 c) + (a + c)*log(x) - 4*sqrt(-a*c)*arctan(-(sqrt(-a*c)*b*x - sqrt(-a*c)*sq
rt(b*x + a)*sqrt(b*x + c))/(a*c)) - 2*sqrt(b*x + a)*sqrt(b*x + c))/(a^2 - 2
*a*c + c^2)]
```

Sympy [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

Maxima [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.46

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{4\,ac\arctan\left(\frac{\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-a-c}{2\sqrt{-ac}}\right)}{(a^2-2\,ac+c^2)\sqrt{-ac}} - \frac{2\,(a^2-2\,ac+c^2)\sqrt{bx+a}\sqrt{bx+c}}{a^4-4\,a^3c+6\,a^2c^2-4\,ac^3+c^4} + \frac{(a+c)\log\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2\right)}{a^2-2\,ac+c^2} + \frac{(a+c)\log\left(|bx|\right)}{a^2-2\,ac+c^2} + \frac{2\,(bx+a)}{a^2-2\,ac+c^2}$$

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 4*a*c*arctan(1/2*((sqrt(b*x + a) - sqrt(b*x + c))^2 - a - c)/sqrt(-a*c))/((
a^2 - 2*a*c + c^2)*sqrt(-a*c)) - 2*(a^2 - 2*a*c + c^2)*sqrt(b*x + a)*sqrt(b
*x + c)/(a^4 - 4*a^3*c + 6*a^2*c^2 - 4*a*c^3 + c^4) + (a + c)*log((sqrt(b*x
 + a) - sqrt(b*x + c))^2)/(a^2 - 2*a*c + c^2) + (a + c)*log(abs(b*x))/(a^2
- 2*a*c + c^2) + 2*(b*x + a)/(a^2 - 2*a*c + c^2)
```

Mupad [B] (verification not implemented)

Time = 25.54 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.94

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{2\,b\,x}{(a-c)^2} - \ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}+1\right)\left(\frac{4\,c}{(a-c)^2}+\frac{2}{a-c}\right) - \frac{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3(4\,a+4\,c)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^3(a^2-2\,a\,c+c^2)} + \frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)(4\,a+4\,c)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)(a^2-2\,a\,c+c^2)} - \frac{16\sqrt{a}\sqrt{c}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^2(a^2-2\,a\,c+c^2)}}{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^4} - \frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^2}+1} + \frac{2\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}-1\right)(a+c)}{(a-c)^2} + \frac{\ln(x)\,(a+c)}{a^2-2\,a\,c+c^2} + \frac{2\sqrt{a}\sqrt{c}\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}\right)}{(a-c)^2} - \frac{2\sqrt{a}\sqrt{c}\,\ln\left(\frac{a\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{c+b\,x}-\sqrt{c}} - \sqrt{a}\sqrt{c} + \frac{c\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{c+b\,x}-\sqrt{c}} - \frac{\sqrt{a}\sqrt{c}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^2}\right)}{a^2-2\,a\,c+c^2}$$

```
[In] int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))^2),x)
```

```
[Out] (2*b*x)/(a - c)^2 - log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1
/2)) + 1)*((4*c)/(a - c)^2 + 2/(a - c)) - ((((a + b*x)^(1/2) - a^(1/2))^3*(
4*a + 4*c))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2 - 2*a*c + c^2)) + (((a + b*
x)^(1/2) - a^(1/2))*(4*a + 4*c))/(((c + b*x)^(1/2) - c^(1/2))*(a^2 - 2*a*c
+ c^2)) - (16*a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(((c + b*x)^(1
/2) - c^(1/2))^2*(a^2 - 2*a*c + c^2)))/(((a + b*x)^(1/2) - a^(1/2))^4/((c +
 b*x)^(1/2) - c^(1/2))^4 - (2*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/
2) - c^(1/2))^2 + 1) + (2*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2)
- c^(1/2)) - 1)*(a + c))/(a - c)^2 + (log(x)*(a + c))/(a^2 - 2*a*c + c^2) +
 (2*a^(1/2)*c^(1/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1
/2))))/(a - c)^2 - (2*a^(1/2)*c^(1/2)*log((a*((a + b*x)^(1/2) - a^(1/2)))/(
(c + b*x)^(1/2) - c^(1/2)) - a^(1/2)*c^(1/2) + (c*((a + b*x)^(1/2) - a^(1/2
)))/((c + b*x)^(1/2) - c^(1/2)) - (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/
2))^2)/((c + b*x)^(1/2) - c^(1/2))^2))/(a^2 - 2*a*c + c^2)
```

3.410 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2x}-\frac{4b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{2b(a+c)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{\sqrt{a}(a-c)^2\sqrt{c}}+\frac{2b\log(x)}{(a-c)^2}$$

```
[Out] (-a-c)/(a-c)^2/x-4*b*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/(a-c)^2+2*b*ln(x)
/(a-c)^2+2*b*(a+c)*arctanh(c^(1/2)*(b*x+a)^(1/2)/a^(1/2)/(b*x+c)^(1/2))/(a-
c)^2/a^(1/2)/c^(1/2)+2*(b*x+a)^(1/2)*(b*x+c)^(1/2)/(a-c)^2/x
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6821, 99, 163, 65, 223, 212, 95, 214}

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{2b(a+c)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{\sqrt{a}\sqrt{c}(a-c)^2}-\frac{4b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{2\sqrt{a+bx}\sqrt{bx+c}}{x(a-c)^2}+\frac{2b\log(x)}{(a-c)^2}-\frac{a+c}{x(a-c)^2}$$

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] -((a + c)/((a - c)^2*x)) + (2*Sqrt[a + b*x]*Sqrt[c + b*x])/((a - c)^2*x) -
(4*b*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 + (2*b*(a + c)*ArcTanh
```

```
[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(Sqrt[a]*(a - c)^2*Sqrt[
c]) + (2*b*Log[x])/(a - c)^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \left(\frac{a\left(1+\frac{c}{a}\right)}{x^2} + \frac{2b}{x} - \frac{2\sqrt{a+bx}\sqrt{c+bx}}{x^2}\right) dx}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2 x} + \frac{2b\log(x)}{(a-c)^2} - \frac{2\int \frac{\sqrt{a+bx}\sqrt{c+bx}}{x^2}\,dx}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2 x} + \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2 x} + \frac{2b\log(x)}{(a-c)^2} - \frac{2\int \frac{\frac{1}{2}b(a+c)+b^2x}{x\sqrt{a+bx}\sqrt{c+bx}}\,dx}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2 x} + \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2 x} + \frac{2b\log(x)}{(a-c)^2} - \frac{(2b^2)\int \frac{1}{\sqrt{a+bx}\sqrt{c+bx}}\,dx}{(a-c)^2} - \frac{(b(a+c))\int \frac{1}{x\sqrt{a+bx}\sqrt{c+bx}}\,dx}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2 x} + \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2 x} + \frac{2b\log(x)}{(a-c)^2} - \frac{(4b)\text{Subst}\left(\int \frac{1}{\sqrt{-a+c+x^2}}\,dx, x, \sqrt{a+bx}\right)}{(a-c)^2} - \frac{(2b(a+c))\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2 x} + \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2 x} + \frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{\sqrt{a}(a-c)^2\sqrt{c}} + \frac{2b\log(x)}{(a-c)^2} - \frac{(4b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2 x} + \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2 x} - \frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2} + \frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{\sqrt{a}(a-c)^2\sqrt{c}} + \frac{2b\log(x)}{(a-c)^2}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\frac{2b(a+c)\operatorname{arctanh}\left(\frac{-bx+\sqrt{a+bx}\sqrt{c+bx}}{\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{c}}-\frac{a+c-2bx-2\sqrt{a+bx}\sqrt{c+bx}-2bx\log\left(bx\left(a+c+2bx-2\sqrt{a+bx}\sqrt{c+bx}\right)\right)}{x}}{(a-c)^2}$$

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] ((2*b*(a + c)*ArcTanh[(-(b*x) + Sqrt[a + b*x]*Sqrt[c + b*x])/(Sqrt[a]*Sqrt[
c])])/(Sqrt[a]*Sqrt[c]) - (a + c - 2*b*x - 2*Sqrt[a + b*x]*Sqrt[c + b*x] -
2*b*x*Log[b*x*(a + c + 2*b*x - 2*Sqrt[a + b*x]*Sqrt[c + b*x])])/x)/(a - c)^
2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.94

method	result
default	$-\frac{a}{x(a-c)^2}-\frac{c}{x(a-c)^2}+\frac{2b\ln(x)}{(a-c)^2}+\frac{\sqrt{bx+a}\,\sqrt{bx+c}\left(\operatorname{csgn}(b)\ln\left(\frac{abx+bcx+2\sqrt{ac}\,\sqrt{b^2x^2+abx+bcx+ac}+2ac}{x}\right)xab+\operatorname{csgn}(b)\ln\left(abx+\cdots\right.\right.}{\cdots}$

```
[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x/(a-c)^2*a-1/x/(a-c)^2*c+2*b*ln(x)/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*(b*x
+c)^(1/2)*(csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+a*c)^
(1/2)+2*a*c)/x)*x*a*b+csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+
b*c*x+a*c)^(1/2)+2*a*c)/x)*x*b*c-2*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2
)*csgn(b)+2*b*x+a+c)*csgn(b))*x*b*(a*c)^(1/2)+2*csgn(b)*(b^2*x^2+a*b*x+b*c*
x+a*c)^(1/2)*(a*c)^(1/2))*csgn(b)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)/x/(a*c)^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.60

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$= \left[\frac{2\,abcx\log\left(-2\,bx+2\,\sqrt{bx+a}\sqrt{bx+c}-a-c\right)+2\,abcx\log\left(x\right)+2\,abcx+(ab+bc)\sqrt{ac}x\log\left(\frac{2\,a^2c+2\,ac^2\cdots}{\cdots}\right.}{\left(a^3c-2\,a^2c^2+\cdots\right.}\right.$$

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*a*b*c*x*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + 2*a*b*c*x
*log(x) + 2*a*b*c*x + (a*b + b*c)*sqrt(a*c)*x*log((2*a^2*c + 2*a*c^2 + 2*(2
*a*c + sqrt(a*c)*(a + c))*sqrt(b*x + a)*sqrt(b*x + c) + (a^2*b + 2*a*b*c +
b*c^2)*x + 2*(2*a*c + (a*b + b*c)*x)*sqrt(a*c))/x) + 2*sqrt(b*x + a)*sqrt(b
*x + c)*a*c - a^2*c - a*c^2)/((a^3*c - 2*a^2*c^2 + a*c^3)*x), (2*a*b*c*x*lo
g(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + 2*a*b*c*x*log(x) + 2*a*
b*c*x - 2*(a*b + b*c)*sqrt(-a*c)*x*arctan(-(sqrt(-a*c)*b*x - sqrt(-a*c)*sqr
t(b*x + a)*sqrt(b*x + c))/(a*c)) + 2*sqrt(b*x + a)*sqrt(b*x + c)*a*c - a^2*
c - a*c^2)/((a^3*c - 2*a^2*c^2 + a*c^3)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(121) = 242$.

Time = 0.77 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.21

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{2\,b\log\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2\right)}{a^2-2\,ac+c^2} + \frac{2\,b\log\left(|bx|\right)}{a^2-2\,ac+c^2} + \frac{2\,(ab+bc)\arctan\left(\frac{\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-a-c}{2\sqrt{-ac}}\right)}{(a^2-2\,ac+c^2)\sqrt{-ac}} - \frac{4\left(ab\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2+bc\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-a^2b+2\,abc-bc^2\right)}{\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^4-2\,a\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-2\,c\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2+a^2-2\,ac+c^2\right)(a^2} - \frac{2\,(bx+a)b-ab+bc}{(a^2-2\,ac+c^2)bx}$$

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*b*log((sqrt(b*x + a) - sqrt(b*x + c))^2)/(a^2 - 2*a*c + c^2) + 2*b*log(ab
s(b*x))/(a^2 - 2*a*c + c^2) + 2*(a*b + b*c)*arctan(1/2*((sqrt(b*x + a) - sq
rt(b*x + c))^2 - a - c)/sqrt(-a*c))/((a^2 - 2*a*c + c^2)*sqrt(-a*c)) - 4*(a
*b*(sqrt(b*x + a) - sqrt(b*x + c))^2 + b*c*(sqrt(b*x + a) - sqrt(b*x + c))^
2 - a^2*b + 2*a*b*c - b*c^2)/(((sqrt(b*x + a) - sqrt(b*x + c))^4 - 2*a*(sqr
t(b*x + a) - sqrt(b*x + c))^2 - 2*c*(sqrt(b*x + a) - sqrt(b*x + c))^2 + a^2
 - 2*a*c + c^2)*(a^2 - 2*a*c + c^2)) - (2*(b*x + a)*b - a*b + b*c)/((a^2 -
2*a*c + c^2)*b*x)
```

Mupad [B] (verification not implemented)

Time = 44.38 (sec) , antiderivative size = 7637, normalized size of antiderivative = 54.16

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))^2),x)
```

```
[Out] (2*b*log(x))/(a^2 - 2*a*c + c^2) - ((((a + b*x)^(1/2) - a^(1/2))^2*((a^2*b)
/2 + (b*c^2)/2 - (3*a*b*c)/2))/(((c + b*x)^(1/2) - c^(1/2))^2*(a*c^3 + a^3*
c - 2*a^2*c^2)) - b/(2*(a^2 - 2*a*c + c^2)) + (a^(1/2)*c^(1/2)*((a*b)/2 + (
b*c)/2)*((a + b*x)^(1/2) - a^(1/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a*c^3 +
a^3*c - 2*a^2*c^2)))/((((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2
)) + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(1/2))^3 - ((a + c)
*((a + b*x)^(1/2) - a^(1/2))^2)/(a^(1/2)*c^(1/2)*((c + b*x)^(1/2) - c^(1/2)
```

)^2)) + (b*atan(((b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11 - 32*a^6*b^3*c^10 - 8*a^7*b^3*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6 + 8*a^11*b^3*c^5 + 4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*b*((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 - 192*a^8*b*c^12 + 504*a^9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*a^12*b*c^8 + 12*a^13*b*c^7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)*c^(33/2) + 27*a^(13/2)*c^(31/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^(27/2) - 27*a^(19/2)*c^(25/2) + 6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) - 27*a^(25/2)*c^(19/2) + 50*a^(27/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^(31/2)*c^(13/2) - 8*a^(33/2)*c^(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) - (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^4*c^18 - 47*a^5*c^17 + 268*a^6*c^16 - 982*a^7*c^15 + 2564*a^8*c^14 - 4993*a^9*c^13 + 7404*a^10*c^12 - 8436*a^11*c^11 + 7404*a^12*c^10 - 4993*a^13*c^9 + 2564*a^14*c^8 - 982*a^15*c^7 + 268*a^16*c^6 - 47*a^17*c^5 + 4*a^18*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 + (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^(7/2)*b*c^(33/2) - 43*a^(9/2)*b*c^(31/2) + 231*a^(11/2)*b*c^(29/2) - 749*a^(13/2)*b*c^(27/2) + 1505*a^(15/2)*b*c^(25/2) - 1770*a^(17/2)*b*c^(23/2) + 822*a^(19/2)*b*c^(21/2) + 822*a^(21/2)*b*c^(19/2) - 1770*a^(23/2)*b*c^(17/2) + 1505*a^(25/2)*b*c^(15/2) - 749*a^(27/2)*b*c^(13/2) + 231*a^(29/2)*b*c^(11/2) - 43*a^(31/2)*b*c^(9/2) + 4*a^(33/2)*b*c^(7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (4*(a^(7/2)*b^2*c^(29/2) + 12*a^(9/2)*b^2*c^(27/2) - 100*a^(11/2)*b^2*c^(25/2) + 285*a^(13/2)*b^2*c^(23/2) - 390*a^(15/2)*b^2*c^(21/2) + 192*a^(17/2)*b^2*c^(19/2) + 192*a^(19/2)*b^2*c^(17/2) - 390*a^(21/2)*b^2*c^(15/2) + 285*a^(23/2)*b^2*c^(13/2) - 100*a^(25/2)*b^2*c^(11/2) + 12*a^(27/2)*b^2*c^(9/2) + a^(29/2)*b^2*c^(7/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (2*((a + b*x)^(1/2) - a^(1/2))*(73*a^4*b^2*c^14 - 570*a^5*b^2*c^13 + 2053*a^6*b^2*c^12 - 4568*a^7*b^2*c^11 + 7090*a^8*b^2*c^10 - 8156*a^9*b^2*c^9 + 7090*a^10*b^2*c^8 - 4568*a^11*b^2*c^7 + 2053*a^12*b^2*c^6 - 570*a^13*b^2*c^5 + 73*a^14*b^2*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^3*c^(25/2) - 427*a^(9/2)*b^3*c^(23/2) + 1256*a^(11/2)*b^3*c^(21/2) - 1856*a^(13/2)*b^3*c^(19/2) + 962*a^(15/2)*b^3*c^(17/2) + 962*a^(17/2)*b^3*c^(15/2) - 1856*a^(19/2)*b^3*c^(13/2) + 1256*a^(21/2)*b^3*c^(11/2) - 427*a^(23/2)*b^3*c^(9/2) + 65*a^(25/2)*b^3*c^(7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7)))*4i)/(a - c

)^2 + (b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11 - 32*a^6*b^3*c^10 - 8*a^7*b^3
*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6 + 8*a^11*b^3*c^5 +
4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a
^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(
a^(7/2)*b^2*c^(29/2) + 12*a^(9/2)*b^2*c^(27/2) - 100*a^(11/2)*b^2*c^(25/2)
+ 285*a^(13/2)*b^2*c^(23/2) - 390*a^(15/2)*b^2*c^(21/2) + 192*a^(17/2)*b^2*
c^(19/2) + 192*a^(19/2)*b^2*c^(17/2) - 390*a^(21/2)*b^2*c^(15/2) + 285*a^(2
3/2)*b^2*c^(13/2) - 100*a^(25/2)*b^2*c^(11/2) + 12*a^(27/2)*b^2*c^(9/2) + a
^(29/2)*b^2*c^(7/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 +
70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*
((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 - 192*a^8*b*c^12 + 504*a^
9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*a^12*b*c^8 + 12*a^13*b*c^
7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 5
6*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15
*c^7) - (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)*c^(33/2) + 27*a^(13/2)*c^(3
1/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^(27/2) - 27*a^(19/2)*c^(25/2) +
6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) - 27*a^(25/2)*c^(19/2) + 50*a^(2
7/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^(31/2)*c^(13/2) - 8*a^(33/2)*c^
(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*
c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) -
(2*((a + b*x)^(1/2) - a^(1/2))*(4*a^4*c^18 - 47*a^5*c^17 + 268*a^6*c^16 -
982*a^7*c^15 + 2564*a^8*c^14 - 4993*a^9*c^13 + 7404*a^10*c^12 - 8436*a^11*c
^11 + 7404*a^12*c^10 - 4993*a^13*c^9 + 2564*a^14*c^8 - 982*a^15*c^7 + 268*a
^16*c^6 - 47*a^17*c^5 + 4*a^18*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15
- 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 +
28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 + (2*((a + b*x)^(1/2) - a
^(1/2))*(4*a^(7/2)*b*c^(33/2) - 43*a^(9/2)*b*c^(31/2) + 231*a^(11/2)*b*c^(2
9/2) - 749*a^(13/2)*b*c^(27/2) + 1505*a^(15/2)*b*c^(25/2) - 1770*a^(17/2)*b
*c^(23/2) + 822*a^(19/2)*b*c^(21/2) + 822*a^(21/2)*b*c^(19/2) - 1770*a^(23/
2)*b*c^(17/2) + 1505*a^(25/2)*b*c^(15/2) - 749*a^(27/2)*b*c^(13/2) + 231*a^
(29/2)*b*c^(11/2) - 43*a^(31/2)*b*c^(9/2) + 4*a^(33/2)*b*c^(7/2)))/(((c + b
x)^(1/2) - c^(1/2))(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 +
70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a -
c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*a^4*b^2*c^14 - 570*a^5*b^2*c^13 +
2053*a^6*b^2*c^12 - 4568*a^7*b^2*c^11 + 7090*a^8*b^2*c^10 - 8156*a^9*b^2*c
^9 + 7090*a^10*b^2*c^8 - 4568*a^11*b^2*c^7 + 2053*a^12*b^2*c^6 - 570*a^13*b
^2*c^5 + 73*a^14*b^2*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c
^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^
9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(6
5*a^(7/2)*b^3*c^(25/2) - 427*a^(9/2)*b^3*c^(23/2) + 1256*a^(11/2)*b^3*c^(21
/2) - 1856*a^(13/2)*b^3*c^(19/2) + 962*a^(15/2)*b^3*c^(17/2) + 962*a^(17/2)
*b^3*c^(15/2) - 1856*a^(19/2)*b^3*c^(13/2) + 1256*a^(21/2)*b^3*c^(11/2) - 4
27*a^(23/2)*b^3*c^(9/2) + 65*a^(25/2)*b^3*c^(7/2)))/(((c + b*x)^(1/2) - c^(
1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 -
56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7)))*4i)/(a - c)^2)/((8*(1

4*a^(7/2)*b^4*c^(21/2) - 14*a^(9/2)*b^4*c^(19/2) - 42*a^(11/2)*b^4*c^(17/2) + 42*a^(13/2)*b^4*c^(15/2) + 42*a^(15/2)*b^4*c^(13/2) - 42*a^(17/2)*b^4*c^(11/2) - 14*a^(19/2)*b^4*c^(9/2) + 14*a^(21/2)*b^4*c^(7/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11 - 32*a^6*b^3*c^10 - 8*a^7*b^3*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6 + 8*a^11*b^3*c^5 + 4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*b*((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 - 192*a^8*b*c^12 + 504*a^9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*a^12*b*c^8 + 12*a^13*b*c^7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)*c^(33/2) + 27*a^(13/2)*c^(31/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^(27/2) - 27*a^(19/2)*c^(25/2) + 6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) - 27*a^(25/2)*c^(19/2) + 50*a^(27/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^(31/2)*c^(13/2) - 8*a^(33/2)*c^(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) - (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^4*c^18 - 47*a^5*c^17 + 268*a^6*c^16 - 982*a^7*c^15 + 2564*a^8*c^14 - 4993*a^9*c^13 + 7404*a^10*c^12 - 8436*a^11*c^11 + 7404*a^12*c^10 - 4993*a^13*c^9 + 2564*a^14*c^8 - 982*a^15*c^7 + 268*a^16*c^6 - 47*a^17*c^5 + 4*a^18*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 + (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^(7/2)*b*c^(33/2) - 43*a^(9/2)*b*c^(31/2) + 231*a^(11/2)*b*c^(29/2) - 749*a^(13/2)*b*c^(27/2) + 1505*a^(15/2)*b*c^(25/2) - 1770*a^(17/2)*b*c^(23/2) + 822*a^(19/2)*b*c^(21/2) + 822*a^(21/2)*b*c^(19/2) - 1770*a^(23/2)*b*c^(17/2) + 1505*a^(25/2)*b*c^(15/2) - 749*a^(27/2)*b*c^(13/2) + 231*a^(29/2)*b*c^(11/2) - 43*a^(31/2)*b*c^(9/2) + 4*a^(33/2)*b*c^(7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (4*(a^(7/2)*b^2*c^(29/2) + 12*a^(9/2)*b^2*c^(27/2) - 100*a^(11/2)*b^2*c^(25/2) + 285*a^(13/2)*b^2*c^(23/2) - 390*a^(15/2)*b^2*c^(21/2) + 192*a^(17/2)*b^2*c^(19/2) + 192*a^(19/2)*b^2*c^(17/2) - 390*a^(21/2)*b^2*c^(15/2) + 285*a^(23/2)*b^2*c^(13/2) - 100*a^(25/2)*b^2*c^(11/2) + 12*a^(27/2)*b^2*c^(9/2) + a^(29/2)*b^2*c^(7/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (2*((a + b*x)^(1/2) - a^(1/2))*(73*a^4*b^2*c^14 - 570*a^5*b^2*c^13 + 2053*a^6*b^2*c^12 - 4568*a^7*b^2*c^11 + 7090*a^8*b^2*c^10 - 8156*a^9*b^2*c^9 + 7090*a^10*b^2*c^8 - 4568*a^11*b^2*c^7 + 2053*a^12*b^2*c^6 - 570*a^13*b^2*c^5 + 73*a^14*b^2*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^3*c^(25/2) - 427*a^(9/2)*b^3*c^(23/2) + 1256*a^(11/2)*b^3*c^(21/2) - 1856*a^(13/2)*b^3*c^(19/2) + 962*a^(15/

2)*b^3*c^(17/2) + 962*a^(17/2)*b^3*c^(15/2) - 1856*a^(19/2)*b^3*c^(13/2) + 1256*a^(21/2)*b^3*c^(11/2) - 427*a^(23/2)*b^3*c^(9/2) + 65*a^(25/2)*b^3*c^(7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (4*b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11 - 32*a^6*b^3*c^10 - 8*a^7*b^3*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6 + 8*a^11*b^3*c^5 + 4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(a^(7/2)*b^2*c^(29/2) + 12*a^(9/2)*b^2*c^(27/2) - 100*a^(11/2)*b^2*c^(25/2) + 285*a^(13/2)*b^2*c^(23/2) - 390*a^(15/2)*b^2*c^(21/2) + 192*a^(17/2)*b^2*c^(19/2) + 192*a^(19/2)*b^2*c^(17/2) - 390*a^(21/2)*b^2*c^(15/2) + 285*a^(23/2)*b^2*c^(13/2) - 100*a^(25/2)*b^2*c^(11/2) + 12*a^(27/2)*b^2*c^(9/2) + a^(29/2)*b^2*c^(7/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 - 192*a^8*b*c^12 + 504*a^9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*a^12*b*c^8 + 12*a^13*b*c^7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) - (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)*c^(33/2) + 27*a^(13/2)*c^(31/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^(27/2) - 27*a^(19/2)*c^(25/2) + 6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) - 27*a^(25/2)*c^(19/2) + 50*a^(27/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^(31/2)*c^(13/2) - 8*a^(33/2)*c^(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) - (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^4*c^18 - 47*a^5*c^17 + 268*a^6*c^16 - 982*a^7*c^15 + 2564*a^8*c^14 - 4993*a^9*c^13 + 7404*a^10*c^12 - 8436*a^11*c^11 + 7404*a^12*c^10 - 4993*a^13*c^9 + 2564*a^14*c^8 - 982*a^15*c^7 + 268*a^16*c^6 - 47*a^17*c^5 + 4*a^18*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 + (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^(7/2)*b*c^(33/2) - 43*a^(9/2)*b*c^(31/2) + 231*a^(11/2)*b*c^(29/2) - 749*a^(13/2)*b*c^(27/2) + 1505*a^(15/2)*b*c^(25/2) - 1770*a^(17/2)*b*c^(23/2) + 822*a^(19/2)*b*c^(21/2) + 822*a^(21/2)*b*c^(19/2) - 1770*a^(23/2)*b*c^(17/2) + 1505*a^(25/2)*b*c^(15/2) - 749*a^(27/2)*b*c^(13/2) + 231*a^(29/2)*b*c^(11/2) - 43*a^(31/2)*b*c^(9/2) + 4*a^(33/2)*b*c^(7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*a^4*b^2*c^14 - 570*a^5*b^2*c^13 + 2053*a^6*b^2*c^12 - 4568*a^7*b^2*c^11 + 7090*a^8*b^2*c^10 - 8156*a^9*b^2*c^9 + 7090*a^10*b^2*c^8 - 4568*a^11*b^2*c^7 + 2053*a^12*b^2*c^6 - 570*a^13*b^2*c^5 + 73*a^14*b^2*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^3*c^(25/2) - 427*a^(9/2)*b^3*c^(23/2) + 1256*a^(11/2)*b^3*c^(21/2) - 1856*a^(13/2)*b^3*c^(19/2) + 962*a^(15/2)*b^3*c^(17

```
/2) + 962*a^(17/2)*b^3*c^(15/2) - 1856*a^(19/2)*b^3*c^(13/2) + 1256*a^(21/2
)*b^3*c^(11/2) - 427*a^(23/2)*b^3*c^(9/2) + 65*a^(25/2)*b^3*c^(7/2)))/(((c
+ b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12
 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a
 - c)^2 + (4*((a + b*x)^(1/2) - a^(1/2))*(224*a^5*b^4*c^9 - 112*a^4*b^4*c^1
0 + 112*a^6*b^4*c^8 - 448*a^7*b^4*c^7 + 112*a^8*b^4*c^6 + 224*a^9*b^4*c^5 -
 112*a^10*b^4*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 2
8*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a
^14*c^8 + a^15*c^7))))*8i)/(a - c)^2 - (log((a*((a + b*x)^(1/2) - a^(1/2)))
/((c + b*x)^(1/2) - c^(1/2)) - a^(1/2)*c^(1/2) + (c*((a + b*x)^(1/2) - a^(1
/2)))/((c + b*x)^(1/2) - c^(1/2)) - (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(
1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2)*(a^(1/2)*b*c^(3/2) + a^(3/2)*b*c^(1
/2)))/(a*c^3 + a^3*c - 2*a^2*c^2) - (a + c)/(x*(a^2 - 2*a*c + c^2)) + (log(
((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2)
 + a^(3/2)*b*c^(1/2)))/(a*c^3 + a^3*c - 2*a^2*c^2) + (b*((a + b*x)^(1/2) -
a^(1/2)))/(2*(a - c)^2*((c + b*x)^(1/2) - c^(1/2)))
```

3.411 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 375

$$\begin{aligned}\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} dx = &-\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3} \\ &+\frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3}-\frac{4a(a+3c)(a+bx)^{5/2}}{5b^3(a-c)^3} \\ &-\frac{24a(a+bx)^{7/2}}{7b^3(a-c)^3}+\frac{2(a+3c)(a+bx)^{7/2}}{7b^3(a-c)^3}+\frac{8(a+bx)^{9/2}}{9b^3(a-c)^3} \\ &+\frac{8c^3(c+bx)^{3/2}}{3b^3(a-c)^3}-\frac{2c^2(3a+c)(c+bx)^{3/2}}{3b^3(a-c)^3} \\ &-\frac{24c^2(c+bx)^{5/2}}{5b^3(a-c)^3}+\frac{4c(3a+c)(c+bx)^{5/2}}{5b^3(a-c)^3} \\ &+\frac{24c(c+bx)^{7/2}}{7b^3(a-c)^3}-\frac{2(3a+c)(c+bx)^{7/2}}{7b^3(a-c)^3}-\frac{8(c+bx)^{9/2}}{9b^3(a-c)^3}\end{aligned}$$

```
[Out] -8/3*a^3*(b*x+a)^(3/2)/b^3/(a-c)^3+2/3*a^2*(a+3*c)*(b*x+a)^(3/2)/b^3/(a-c)^
3+24/5*a^2*(b*x+a)^(5/2)/b^3/(a-c)^3-4/5*a*(a+3*c)*(b*x+a)^(5/2)/b^3/(a-c)^
3-24/7*a*(b*x+a)^(7/2)/b^3/(a-c)^3+2/7*(a+3*c)*(b*x+a)^(7/2)/b^3/(a-c)^3+8/
9*(b*x+a)^(9/2)/b^3/(a-c)^3+8/3*c^3*(b*x+c)^(3/2)/b^3/(a-c)^3-2/3*c^2*(3*a+
c)*(b*x+c)^(3/2)/b^3/(a-c)^3-24/5*c^2*(b*x+c)^(5/2)/b^3/(a-c)^3+4/5*c*(3*a+
c)*(b*x+c)^(5/2)/b^3/(a-c)^3+24/7*c*(b*x+c)^(7/2)/b^3/(a-c)^3-2/7*(3*a+c)*(
b*x+c)^(7/2)/b^3/(a-c)^3-8/9*(b*x+c)^(9/2)/b^3/(a-c)^3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6821, 45}

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = -\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{8c^3(bx+c)^{3/2}}{3b^3(a-c)^3}-\frac{24c^2(bx+c)^{5/2}}{5b^3(a-c)^3}-\frac{2c^2(3a+c)(bx+c)^{3/2}}{3b^3(a-c)^3}+\frac{8(a+bx)^{9/2}}{9b^3(a-c)^3}+\frac{2(a+3c)(a+bx)^{7/2}}{7b^3(a-c)^3}-\frac{24a(a+bx)^{7/2}}{7b^3(a-c)^3}-\frac{4a(a+3c)(a+bx)^{5/2}}{5b^3(a-c)^3}-\frac{8(bx+c)^{9/2}}{9b^3(a-c)^3}+\frac{24c(bx+c)^{7/2}}{7b^3(a-c)^3}-\frac{2(3a+c)(bx+c)^{7/2}}{7b^3(a-c)^3}+\frac{4c(3a+c)(bx+c)^{5/2}}{5b^3(a-c)^3}$$

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (-8*a^3*(a + b*x)^(3/2))/(3*b^3*(a - c)^3) + (2*a^2*(a + 3*c)*(a + b*x)^(3/
2))/(3*b^3*(a - c)^3) + (24*a^2*(a + b*x)^(5/2))/(5*b^3*(a - c)^3) - (4*a*(
a + 3*c)*(a + b*x)^(5/2))/(5*b^3*(a - c)^3) - (24*a*(a + b*x)^(7/2))/(7*b^3
*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(7/2))/(7*b^3*(a - c)^3) + (8*(a + b*x
)^(9/2))/(9*b^3*(a - c)^3) + (8*c^3*(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (2
*c^2*(3*a + c)*(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (24*c^2*(c + b*x)^(5/2)
)/(5*b^3*(a - c)^3) + (4*c*(3*a + c)*(c + b*x)^(5/2))/(5*b^3*(a - c)^3) + (
24*c*(c + b*x)^(7/2))/(7*b^3*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(7/2))/(7*
b^3*(a - c)^3) - (8*(c + b*x)^(9/2))/(9*b^3*(a - c)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \left(a\left(1+\frac{3c}{a}\right)x^2\sqrt{a+bx}+4bx^3\sqrt{a+bx}-3a\left(1+\frac{c}{3a}\right)x^2\sqrt{c+bx}-4bx^3\sqrt{c+bx}\right)\,dx}{(a-c)^3}\\
&= \frac{(4b)\int x^3\sqrt{a+bx}\,dx}{(a-c)^3}-\frac{(4b)\int x^3\sqrt{c+bx}\,dx}{(a-c)^3}-\frac{(3a+c)\int x^2\sqrt{c+bx}\,dx}{(a-c)^3}+\frac{(a+3c)\int x^2\sqrt{a+bx}\,dx}{(a-c)^3}\\
&= \frac{(4b)\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx}{(a-c)^3}\\
&\quad-\frac{(4b)\int\left(-\frac{c^3\sqrt{c+bx}}{b^3}+\frac{3c^2(c+bx)^{3/2}}{b^3}-\frac{3c(c+bx)^{5/2}}{b^3}+\frac{(c+bx)^{7/2}}{b^3}\right)dx}{(a-c)^3}\\
&\quad-\frac{(3a+c)\int\left(\frac{c^2\sqrt{c+bx}}{b^2}-\frac{2c(c+bx)^{3/2}}{b^2}+\frac{(c+bx)^{5/2}}{b^2}\right)dx}{(a-c)^3}\\
&\quad+\frac{(a+3c)\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx}{(a-c)^3}\\
&= -\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3}\\
&\quad-\frac{4a(a+3c)(a+bx)^{5/2}}{5b^3(a-c)^3}-\frac{24a(a+bx)^{7/2}}{7b^3(a-c)^3}+\frac{2(a+3c)(a+bx)^{7/2}}{7b^3(a-c)^3}\\
&\quad+\frac{8(a+bx)^{9/2}}{9b^3(a-c)^3}+\frac{8c^3(c+bx)^{3/2}}{3b^3(a-c)^3}-\frac{2c^2(3a+c)(c+bx)^{3/2}}{3b^3(a-c)^3}-\frac{24c^2(c+bx)^{5/2}}{5b^3(a-c)^3}\\
&\quad+\frac{4c(3a+c)(c+bx)^{5/2}}{5b^3(a-c)^3}+\frac{24c(c+bx)^{7/2}}{7b^3(a-c)^3}-\frac{2(3a+c)(c+bx)^{7/2}}{7b^3(a-c)^3}-\frac{8(c+bx)^{9/2}}{9b^3(a-c)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.37

$$
\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx
= \frac{2\left((a+bx)^{3/2}\left(-40a^3+12a^2(6c+5bx)-3abx(36c+25bx)+5b^2x^2(27c+28bx)\right)+(c+bx)^{3/2}\left(-9a(8c^2\right.\right.}{315b^3(a-c)^3}
$$

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (2*((a + b*x)^(3/2)*(-40*a^3 + 12*a^2*(6*c + 5*b*x) - 3*a*b*x*(36*c + 25*b*
x) + 5*b^2*x^2*(27*c + 28*b*x)) + (c + b*x)^(3/2)*(-9*a*(8*c^2 - 12*b*c*x +
 15*b^2*x^2) + 5*(8*c^3 - 12*b*c^2*x + 15*b^2*c*x^2 - 28*b^3*x^3))))/(315*b
^3*(a - c)^3)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.78

method	result
default	$\frac{2a\left(\frac{(bx+a)^{\frac{7}{2}}}{7}-\frac{2(bx+a)^{\frac{5}{2}}a}{5}+\frac{(bx+a)^{\frac{3}{2}}a^2}{3}\right)}{(a-c)^3b^3}+\frac{6c\left(\frac{(bx+a)^{\frac{7}{2}}}{7}-\frac{2(bx+a)^{\frac{5}{2}}a}{5}+\frac{(bx+a)^{\frac{3}{2}}a^2}{3}\right)}{(a-c)^3b^3}-\frac{6a\left(\frac{(bx+c)^{\frac{7}{2}}}{7}-\frac{2c(bx+c)^{\frac{5}{2}}}{5}+\frac{c^2(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)^3b^3}-$

```
[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a-c)^3*a/b^3*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*(b*x+a)^(3/2)*a^
2)+6/(a-c)^3*c/b^3*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*(b*x+a)^(3/2)
*a^2)-6/(a-c)^3*a/b^3*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2*(b*x+c
)^(3/2))-2/(a-c)^3*c/b^3*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2*(b*
x+c)^(3/2))+8/(a-c)^3/b^3*(1/9*(b*x+a)^(9/2)-3/7*a*(b*x+a)^(7/2)+3/5*a^2*(b
*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a^3)-8/(a-c)^3/b^3*(1/9*(b*x+c)^(9/2)-3/7*c*(
b*x+c)^(7/2)+3/5*c^2*(b*x+c)^(5/2)-1/3*c^3*(b*x+c)^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{2\left((140\,b^4x^4-40\,a^4+72\,a^3c+5\,(13\,ab^3+27\,b^3c)x^3-3\,(5\,a^2b^2-9\,ab^2c)x^2+4\,(5\,a^3b-9\,a^2bc)x)\sqrt{bx+a}\right.}{315\,(a^3b^3-3\,a^2b^3c}$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/315*((140*b^4*x^4 - 40*a^4 + 72*a^3*c + 5*(13*a*b^3 + 27*b^3*c)*x^3 - 3*(
5*a^2*b^2 - 9*a*b^2*c)*x^2 + 4*(5*a^3*b - 9*a^2*b*c)*x)*sqrt(b*x + a) - (14
0*b^4*x^4 + 72*a*c^3 - 40*c^4 + 5*(27*a*b^3 + 13*b^3*c)*x^3 + 3*(9*a*b^2*c
- 5*b^2*c^2)*x^2 - 4*(9*a*b*c^2 - 5*b*c^3)*x)*sqrt(b*x + c))/(a^3*b^3 - 3*a
^2*b^3*c + 3*a*b^3*c^2 - b^3*c^3)
```

Sympy [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**3, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1447 vs. $2(319) = 638$.

Time = 0.69 (sec) , antiderivative size = 1447, normalized size of antiderivative = 3.86

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
[Out] -2/315*(((5*(b*x + a)*(28*(a^9*b^12 - 9*a^8*b^12*c + 36*a^7*b^12*c^2 - 84*a
^6*b^12*c^3 + 126*a^5*b^12*c^4 - 126*a^4*b^12*c^5 + 84*a^3*b^12*c^6 - 36*a^
2*b^12*c^7 + 9*a*b^12*c^8 - b^12*c^9)*(b*x + a)/(a^12*b^15 - 12*a^11*b^15*c
 + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^
5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c
^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15*c^12) - (85*a^10*b^12 - 778*a
^9*b^12*c + 3177*a^8*b^12*c^2 - 7608*a^7*b^12*c^3 + 11802*a^6*b^12*c^4 - 12
348*a^5*b^12*c^5 + 8778*a^4*b^12*c^6 - 4152*a^3*b^12*c^7 + 1233*a^2*b^12*c^
8 - 202*a*b^12*c^9 + 13*b^12*c^10)/(a^12*b^15 - 12*a^11*b^15*c + 66*a^10*b^
15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^5 + 924*a^6*b
^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c^9 + 66*a^2*b
^15*c^10 - 12*a*b^15*c^11 + b^15*c^12)) + 3*(145*a^11*b^12 - 1361*a^10*b^12
*c + 5719*a^9*b^12*c^2 - 14151*a^8*b^12*c^3 + 22794*a^7*b^12*c^4 - 24906*a^
6*b^12*c^5 + 18606*a^5*b^12*c^6 - 9294*a^4*b^12*c^7 + 2901*a^3*b^12*c^8 - 4
```

69*a^2*b^12*c^9 + 11*a*b^12*c^10 + 5*b^12*c^11)/(a^12*b^15 - 12*a^11*b^15*c + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15*c^12))*(b*x + a) - (155*a^12*b^12 - 1536*a^11*b^12*c + 6855*a^10*b^12*c^2 - 18170*a^9*b^12*c^3 + 31770*a^8*b^12*c^4 - 38520*a^7*b^12*c^5 + 33222*a^6*b^12*c^6 - 20700*a^5*b^12*c^7 + 9495*a^4*b^12*c^8 - 3320*a^3*b^12*c^9 + 915*a^2*b^12*c^10 - 186*a*b^12*c^11 + 20*b^12*c^12)/(a^12*b^15 - 12*a^11*b^15*c + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15*c^12))*(b*x + a) + (5*a^13*b^12 - 83*a^12*b^12*c + 543*a^11*b^12*c^2 - 1925*a^10*b^12*c^3 + 4070*a^9*b^12*c^4 - 4950*a^8*b^12*c^5 + 2046*a^7*b^12*c^6 + 3894*a^6*b^12*c^7 - 8415*a^5*b^12*c^8 + 8305*a^4*b^12*c^9 - 5005*a^3*b^12*c^10 + 1887*a^2*b^12*c^11 - 412*a*b^12*c^12 + 40*b^12*c^13)/(a^12*b^15 - 12*a^11*b^15*c + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15*c^12))*sqrt(b*x + c) + 2/315*(140*(b*x + a)^(9/2) - 495*(b*x + a)^(7/2)*a + 630*(b*x + a)^(5/2)*a^2 - 315*(b*x + a)^(3/2)*a^3 + 135*(b*x + a)^(7/2)*c - 378*(b*x + a)^(5/2)*a*c + 315*(b*x + a)^(3/2)*a^2*c)/(a^3*b^3 - 3*a^2*b^3*c + 3*a*b^3*c^2 - b^3*c^3)

Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.41

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{x^3\left(\frac{64\,b\,c}{9\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)\sqrt{c+b\,x}}{7\,b} - \frac{x^3\left(\frac{64\,a\,b}{9\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)\sqrt{a+b\,x}}{7\,b} - \frac{8\,c^2\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{6\,c\left(\frac{64\,b\,c}{9\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{c+b\,x}}{15\,b^3} - \frac{x^2\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{6\,c\left(\frac{64\,b\,c}{9\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{c+b\,x}}{5\,b} + \frac{8\,a^2\left(\frac{2\,(a^2+3\,c\,a)}{(a-c)^3}+\frac{6\,a\left(\frac{64\,a\,b}{9\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{a+b\,x}}{15\,b^3} + \frac{x^2\left(\frac{2\,(a^2+3\,c\,a)}{(a-c)^3}+\frac{6\,a\left(\frac{64\,a\,b}{9\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{a+b\,x}}{5\,b} + \frac{8\,b\,x^4\sqrt{a+b\,x}}{9\,(a-c)^3} - \frac{8\,b\,x^4\sqrt{c+b\,x}}{9\,(a-c)^3} + \frac{4\,c\,x\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{6\,c\left(\frac{64\,b\,c}{9\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{c+b\,x}}{15\,b^2} - \frac{4\,a\,x\left(\frac{2\,(a^2+3\,c\,a)}{(a-c)^3}+\frac{6\,a\left(\frac{64\,a\,b}{9\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{a+b\,x}}{15\,b^2}$$

```
[In] int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

```
[Out] (x^3*((64*b*c)/(9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3)*(c + b*x)^(1/2)
)/(7*b) - (x^3*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3)*(a +
b*x)^(1/2))/(7*b) - (8*c^2*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c)/(9*(
a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(15*b^3)
- (x^2*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c)/(9*(a - c)^3) - (2*b*(3*
a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(5*b) + (8*a^2*((2*(3*a*c + a
^2))/(a - c)^3 + (6*a*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3
))/(7*b))*(a + b*x)^(1/2))/(15*b^3) + (x^2*((2*(3*a*c + a^2))/(a - c)^3 + (
6*a*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(7*b))*(a + b*x
)^(1/2))/(5*b) + (8*b*x^4*(a + b*x)^(1/2))/(9*(a - c)^3) - (8*b*x^4*(c + b*
x)^(1/2))/(9*(a - c)^3) + (4*c*x*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c
```

```
)/(9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(15
*b^2) - (4*a*x*((2*(3*a*c + a^2))/(a - c)^3 + (6*a*((64*a*b)/(9*(a - c)^3)
- (2*b*(5*a + 3*c))/(a - c)^3))/(7*b))*(a + b*x)^(1/2))/(15*b^2)
```

3.412 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 261

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3}+\frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3}+\frac{8(a+bx)^{7/2}}{7b^2(a-c)^3}-\frac{8c^2(c+bx)^{3/2}}{3b^2(a-c)^3}+\frac{2c(3a+c)(c+bx)^{3/2}}{3b^2(a-c)^3}+\frac{16c(c+bx)^{5/2}}{5b^2(a-c)^3}-\frac{2(3a+c)(c+bx)^{5/2}}{5b^2(a-c)^3}-\frac{8(c+bx)^{7/2}}{7b^2(a-c)^3}$$

```
[Out] 8/3*a^2*(b*x+a)^(3/2)/b^2/(a-c)^3-2/3*a*(a+3*c)*(b*x+a)^(3/2)/b^2/(a-c)^3-1
6/5*a*(b*x+a)^(5/2)/b^2/(a-c)^3+2/5*(a+3*c)*(b*x+a)^(5/2)/b^2/(a-c)^3+8/7*(
b*x+a)^(7/2)/b^2/(a-c)^3-8/3*c^2*(b*x+c)^(3/2)/b^2/(a-c)^3+2/3*c*(3*a+c)*(b
*x+c)^(3/2)/b^2/(a-c)^3+16/5*c*(b*x+c)^(5/2)/b^2/(a-c)^3-2/5*(3*a+c)*(b*x+c
)^(5/2)/b^2/(a-c)^3-8/7*(b*x+c)^(7/2)/b^2/(a-c)^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used

$= \{6821, 45\}$

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{8c^2(bx+c)^{3/2}}{3b^2(a-c)^3} + \frac{8(a+bx)^{7/2}}{7b^2(a-c)^3} + \frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3} - \frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3} - \frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{8(bx+c)^{7/2}}{7b^2(a-c)^3} + \frac{16c(bx+c)^{5/2}}{5b^2(a-c)^3} - \frac{2(3a+c)(bx+c)^{5/2}}{5b^2(a-c)^3} + \frac{2c(3a+c)(bx+c)^{3/2}}{3b^2(a-c)^3}$$

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (8*a^2*(a + b*x)^(3/2))/(3*b^2*(a - c)^3) - (2*a*(a + 3*c)*(a + b*x)^(3/2))
/(3*b^2*(a - c)^3) - (16*a*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (2*(a + 3*c
)*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (8*(a + b*x)^(7/2))/(7*b^2*(a - c)^3
) - (8*c^2*(c + b*x)^(3/2))/(3*b^2*(a - c)^3) + (2*c*(3*a + c)*(c + b*x)^(3
/2))/(3*b^2*(a - c)^3) + (16*c*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (2*(3*a
 + c)*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (8*(c + b*x)^(7/2))/(7*b^2*(a -
c)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \left(a\left(1+\frac{3c}{a}\right)x\sqrt{a+bx}+4bx^2\sqrt{a+bx}-3a\left(1+\frac{c}{3a}\right)x\sqrt{c+bx}-4bx^2\sqrt{c+bx}\right)dx}{(a-c)^3}$$

$$= \frac{(4b)\int x^2\sqrt{a+bx}\,dx}{(a-c)^3} - \frac{(4b)\int x^2\sqrt{c+bx}\,dx}{(a-c)^3} - \frac{(3a+c)\int x\sqrt{c+bx}\,dx}{(a-c)^3} + \frac{(a+3c)\int x\sqrt{a+bx}\,dx}{(a-c)^3}$$

$$= \frac{(4b)\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx}{(a-c)^3} - \frac{(4b)\int\left(\frac{c^2\sqrt{c+bx}}{b^2}-\frac{2c(c+bx)^{3/2}}{b^2}+\frac{(c+bx)^{5/2}}{b^2}\right)dx}{(a-c)^3} - \frac{(3a+c)\int\left(-\frac{c\sqrt{c+bx}}{b}+\frac{(c+bx)^{3/2}}{b}\right)dx}{(a-c)^3} + \frac{(a+3c)\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{(a-c)^3}$$

$$= \frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3} + \frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3} + \frac{8(a+bx)^{7/2}}{7b^2(a-c)^3} - \frac{8c^2(c+bx)^{3/2}}{3b^2(a-c)^3} + \frac{2c(3a+c)(c+bx)^{3/2}}{3b^2(a-c)^3} + \frac{16c(c+bx)^{5/2}}{5b^2(a-c)^3} - \frac{2(3a+c)(c+bx)^{5/2}}{5b^2(a-c)^3} - \frac{8(c+bx)^{7/2}}{7b^2(a-c)^3}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.36

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{2\left((c+bx)^{3/2}\left(-6c^2+9bcx-20b^2x^2+7a(2c-3bx)\right)+(a+bx)^{3/2}\left(6a^2-a(14c+9bx)+bx(21c+20bx\right)\right)}{35b^2(a-c)^3}$$

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (2*((c + b*x)^(3/2)*(-6*c^2 + 9*b*c*x - 20*b^2*x^2 + 7*a*(2*c - 3*b*x)) + (
a + b*x)^(3/2)*(6*a^2 - a*(14*c + 9*b*x) + b*x*(21*c + 20*b*x))))/(35*b^2*(
a - c)^3)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.85

method	result
default	$\frac{2a\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{(bx+a)^{\frac{3}{2}}a}{3}\right)}{(a-c)^3b^2}+\frac{6c\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{(bx+a)^{\frac{3}{2}}a}{3}\right)}{(a-c)^3b^2}-\frac{6a\left(\frac{(bx+c)^{\frac{5}{2}}}{5}-\frac{c(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)^3b^2}-\frac{2c\left(\frac{(bx+c)^{\frac{5}{2}}}{5}-\frac{c(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)^3b^2}+\frac{8(bx+a)}{7}$

```
[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a-c)^3*a/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)+6/(a-c)^3*c/b^2*(1/
5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-6/(a-c)^3*a/b^2*(1/5*(b*x+c)^(5/2)-1/3
*c*(b*x+c)^(3/2))-2/(a-c)^3*c/b^2*(1/5*(b*x+c)^(5/2)-1/3*c*(b*x+c)^(3/2))+8
/(a-c)^3/b^2*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*(b*x+a)^(3/2)*a^2)-
8/(a-c)^3/b^2*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2*(b*x+c)^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.61

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{2\left((20\,b^3x^3+6\,a^3-14\,a^2c+(11\,ab^2+21\,b^2c)x^2-(3\,a^2b-7\,abc)x)\sqrt{bx+a}-(20\,b^3x^3-14\,ac^2+6\,c^3+(2\right.}{35\,(a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3)}$$

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/35*((20*b^3*x^3 + 6*a^3 - 14*a^2*c + (11*a*b^2 + 21*b^2*c)*x^2 - (3*a^2*b
 - 7*a*b*c)*x)*sqrt(b*x + a) - (20*b^3*x^3 - 14*a*c^2 + 6*c^3 + (21*a*b^2 +
 11*b^2*c)*x^2 + (7*a*b*c - 3*b*c^2)*x)*sqrt(b*x + c))/(a^3*b^2 - 3*a^2*b^2
*c + 3*a*b^2*c^2 - b^2*c^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 942 vs. 2(240) = 480.

Time = 0.75 (sec) , antiderivative size = 942, normalized size of antiderivative = 3.61

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \begin{cases} \frac{12a^2}{35ab^2\sqrt{a+bx}+105ab^2\sqrt{bx+c}+140b^3x\sqrt{a+bx}+140b^3x\sqrt{bx+c}+105b^2c\sqrt{a+bx}+35b^2c\sqrt{bx+c}} + \frac{54ab}{35ab^2\sqrt{a+bx}+105ab^2\sqrt{bx+c}+140b^3x\sqrt{a+bx}+} \\ \frac{x^2}{2\left(\sqrt{a}+\sqrt{c}\right)^3} \end{cases}$$

```
[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Piecewise((12*a**2/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 14
0*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x
) + 35*b**2*c*sqrt(b*x + c)) + 54*a*b*x/(35*a*b**2*sqrt(a + b*x) + 105*a*b*
*2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 10
5*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 44*a*c/(35*a*b**2*sqrt(
a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x
*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 36*a
*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x
 + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sq
rt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 40*b**2*x**2/(35*a*b**2*sqrt(a + b
*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt
(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 54*b*c*x/
(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b
```

```
*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(
b*x + c)) + 30*b*x*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 1
05*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x +
c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 12*c**2/(35*a*b*
*2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 14
0*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)
) + 36*c*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*
sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b
**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)), Ne(b, 0)), (x**2/(2*(sqrt(a
) + sqrt(c))**3), True))
```

Maxima [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(221) = 442$.

Time = 0.35 (sec) , antiderivative size = 866, normalized size of antiderivative = 3.32

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=$$
$$-\frac{2\left(\left(\left(\left(bx+a\right)\left(\frac{20\,(a^6b^3-6\,a^5b^3c+15\,a^4b^3c^2-20\,a^3b^3c^3+15\,a^2b^3c^4-6\,ab^3c^5+b^3c^6)(bx+a)}{a^9b^4-9\,a^8b^4c+36\,a^7b^4c^2-84\,a^6b^4c^3+126\,a^5b^4c^4-126\,a^4b^4c^5+84\,a^3b^4c^6-36\,a^2b^4c^7+9\,ab^4c^8-b^4c^9}-\frac{39\,a^7b^3-2}{a^9b^4-9\,a^8b^4c+}\right.\right.\right.\right.}{}$$

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
[Out] -2/35*((((b*x + a)*(20*(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3
*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6)*(b*x + a)/(a^9*b^4 - 9*a^8*b
^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5
+ 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9) - (39*a^7*b^3 -
245*a^6*b^3*c + 651*a^5*b^3*c^2 - 945*a^4*b^3*c^3 + 805*a^3*b^3*c^4 - 399*a
^2*b^3*c^5 + 105*a*b^3*c^6 - 11*b^3*c^7)/(a^9*b^4 - 9*a^8*b^4*c + 36*a^7*b^
4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*b^4*c^6
 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9)) + 3*(6*a^8*b^3 - 41*a^7*b^3*c +
 119*a^6*b^3*c^2 - 189*a^5*b^3*c^3 + 175*a^4*b^3*c^4 - 91*a^3*b^3*c^5 + 21*
a^2*b^3*c^6 + a*b^3*c^7 - b^3*c^8)/(a^9*b^4 - 9*a^8*b^4*c + 36*a^7*b^4*c^2
```

```
- 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*b^4*c^6 - 36*
a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9))*(b*x + a) + (a^9*b^3 - 2*a^8*b^3*c -
20*a^7*b^3*c^2 + 112*a^6*b^3*c^3 - 266*a^5*b^3*c^4 + 364*a^4*b^3*c^5 - 308*
a^3*b^3*c^6 + 160*a^2*b^3*c^7 - 47*a*b^3*c^8 + 6*b^3*c^9)/(a^9*b^4 - 9*a^8*
b^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5
 + 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9))*sqrt(b*x + c)
- (20*(b*x + a)^(7/2) - 49*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 + 21*
(b*x + a)^(5/2)*c - 35*(b*x + a)^(3/2)*a*c)/(a^3*b - 3*a^2*b*c + 3*a*b*c^2
- b*c^3))/b
```

Mupad [B] (verification not implemented)

Time = 16.83 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.48

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{x^2\left(\frac{48\,b\,c}{7\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)\sqrt{c+b\,x}}{5\,b} - \frac{x^2\left(\frac{48\,a\,b}{7\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)\sqrt{a+b\,x}}{5\,b} - \frac{2\,a\left(\frac{2\,a\,(a+3\,c)}{(a-c)^3}+\frac{4\,a\left(\frac{48\,a\,b}{7\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b^2} + \frac{8\,b\,x^3\sqrt{a+b\,x}}{7\,(a-c)^3} + \frac{2\,c\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{4\,c\left(\frac{48\,b\,c}{7\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{c+b\,x}}{3\,b^2} - \frac{8\,b\,x^3\sqrt{c+b\,x}}{7\,(a-c)^3} + \frac{x\left(\frac{2\,a\,(a+3\,c)}{(a-c)^3}+\frac{4\,a\left(\frac{48\,a\,b}{7\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b} - \frac{x\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{4\,c\left(\frac{48\,b\,c}{7\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{c+b\,x}}{3\,b}$$

```
[In] int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

```
[Out] (x^2*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3)*(c + b*x)^(1/2)
)/(5*b) - (x^2*((48*a*b)/(7*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3)*(a +
b*x)^(1/2))/(5*b) - (2*a*((2*a*(a + 3*c))/(a - c)^3 + (4*a*((48*a*b)/(7*(a
- c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(5*b))*(a + b*x)^(1/2))/(3*b^2) + (
8*b*x^3*(a + b*x)^(1/2))/(7*(a - c)^3) + (2*c*((2*c*(3*a + c))/(a - c)^3 +
```

```
(4*c*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(5*b))*(c + b*
x)^(1/2))/(3*b^2) - (8*b*x^3*(c + b*x)^(1/2))/(7*(a - c)^3) + (x*((2*a*(a +
 3*c))/(a - c)^3 + (4*a*((48*a*b)/(7*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)
^3))/(5*b))*(a + b*x)^(1/2))/(3*b) - (x*((2*c*(3*a + c))/(a - c)^3 + (4*c*(
(48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(5*b))*(c + b*x)^(1/
2))/(3*b)
```

3.413 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} \, dx = \frac{(a-c)^2}{10b\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^5} - \frac{1}{2b\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}$$

```
[Out] 1/10*(a-c)^2/b/((b*x+a)^(1/2)+(b*x+c)^(1/2))^5-1/2/b/((b*x+a)^(1/2)+(b*x+c)
^(1/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 151 vs. 2(64) = 128.

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.36, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6821, 45}

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} \, dx = \frac{8(a+bx)^{5/2}}{5b(a-c)^3} + \frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3} - \frac{8a(a+bx)^{3/2}}{3b(a-c)^3} - \frac{8(bx+c)^{5/2}}{5b(a-c)^3} + \frac{8c(bx+c)^{3/2}}{3b(a-c)^3} - \frac{2(3a+c)(bx+c)^{3/2}}{3b(a-c)^3}$$

```
[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

```
[Out] (-8*a*(a + b*x)^(3/2))/(3*b*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(3/2))/(3*b
*(a - c)^3) + (8*(a + b*x)^(5/2))/(5*b*(a - c)^3) + (8*c*(c + b*x)^(3/2))/(
3*b*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(3/2))/(3*b*(a - c)^3) - (8*(c + b*
x)^(5/2))/(5*b*(a - c)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \left(a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}+4bx\sqrt{a+bx}-3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}-4bx\sqrt{c+bx}\right)dx}{(a-c)^3}$$

$$= \frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)(c+bx)^{3/2}}{3b(a-c)^3}+\frac{(4b)\int x\sqrt{a+bx}\,dx}{(a-c)^3}-\frac{(4b)\int x\sqrt{c+bx}\,dx}{(a-c)^3}$$

$$= \frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)(c+bx)^{3/2}}{3b(a-c)^3} + \frac{(4b)\int \left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{(a-c)^3}-\frac{(4b)\int \left(-\frac{c\sqrt{c+bx}}{b}+\frac{(c+bx)^{3/2}}{b}\right)dx}{(a-c)^3}$$

$$= -\frac{8a(a+bx)^{3/2}}{3b(a-c)^3}+\frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}+\frac{8(a+bx)^{5/2}}{5b(a-c)^3} + \frac{8c(c+bx)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)(c+bx)^{3/2}}{3b(a-c)^3}-\frac{8(c+bx)^{5/2}}{5b(a-c)^3}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{2\left((-5a+c-4bx)(c+bx)^{3/2}+(a+bx)^{3/2}(-a+5c+4bx)\right)}{5b(a-c)^3}$$

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
[Out] (2*((-5*a + c - 4*b*x)*(c + b*x)^(3/2) + (a + b*x)^(3/2)*(-a + 5*c + 4*b*x)
))/(5*b*(a - c)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(52) = 104$.

Time = 0.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.28

method	result	size
default	$\frac{2a(bx+a)^{\frac{3}{2}}}{3(a-c)^{3}b}+\frac{2c(bx+a)^{\frac{3}{2}}}{(a-c)^{3}b}-\frac{2a(bx+c)^{\frac{3}{2}}}{(a-c)^{3}b}-\frac{2c(bx+c)^{\frac{3}{2}}}{3(a-c)^{3}b}+\frac{\frac{8(bx+a)^{\frac{5}{2}}}{5}-\frac{8(bx+a)^{\frac{3}{2}}a}{3}}{(a-c)^{3}b}-\frac{8\left(\frac{(bx+c)^{\frac{5}{2}}}{5}-\frac{c(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)^{3}b}$	146

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(a-c)^3*a*(b*x+a)^(3/2)/b+2/(a-c)^3*c*(b*x+a)^(3/2)/b-2/(a-c)^3*a*(b*x+
c)^(3/2)/b-2/3/(a-c)^3*c*(b*x+c)^(3/2)/b+8/(a-c)^3/b*(1/5*(b*x+a)^(5/2)-1/3
*(b*x+a)^(3/2)*a)-8/(a-c)^3/b*(1/5*(b*x+c)^(5/2)-1/3*c*(b*x+c)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(52) = 104$.

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.66

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^{3}}\,dx$$
$$=\frac{2\left((4\,b^{2}x^{2}-a^{2}+5\,ac+(3\,ab+5\,bc)x)\sqrt{bx+a}-(4\,b^{2}x^{2}+5\,ac-c^{2}+(5\,ab+3\,bc)x)\sqrt{bx+c}\right)}{5\left(a^{3}b-3\,a^{2}bc+3\,abc^{2}-bc^{3}\right)}$$

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/5*((4*b^2*x^2 - a^2 + 5*a*c + (3*a*b + 5*b*c)*x)*sqrt(b*x + a) - (4*b^2*x
^2 + 5*a*c - c^2 + (5*a*b + 3*b*c)*x)*sqrt(b*x + c))/(a^3*b - 3*a^2*b*c + 3
*a*b*c^2 - b*c^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(48) = 96$.

Time = 0.73 (sec) , antiderivative size = 384, normalized size of antiderivative = 6.00

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^{3}}\,dx$$
$$=\begin{cases}-\frac{2a}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^{2}x\sqrt{a+bx}+20b^{2}x\sqrt{bx+c}+15bc\sqrt{a+bx}+5bc\sqrt{bx+c}}-\frac{4bx}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^{2}x\sqrt{a+bx}+20b^{2}x\sqrt{bx+c}+} \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^{3}}\end{cases}$$

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Piecewise((-2*a/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqr
t(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*
x + c)) - 4*b*x/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqr
t(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*
x + c)) - 2*c/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqrt(
a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*x
+ c)) - 6*sqrt(a + b*x)*sqrt(b*x + c)/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*
x + c) + 20*b**2*x*sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a
+ b*x) + 5*b*c*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(c))**3, True))
```

Maxima [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \int \frac{1}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(52) = 104$.

Time = 0.35 (sec) , antiderivative size = 427, normalized size of antiderivative = 6.67

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = -\frac{2}{5}\left((bx+a)\left(\frac{4\,(a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3)(bx+a)}{a^6b^3-6\,a^5b^3c+15\,a^4b^3c^2-20\,a^3b^3c^3+15\,a^2b^3c^4-6\,ab^3c^5+b^3c^6} - \frac{3\,(a^4b^2-}{a^6b^3-6\,a^5b^3c+15} \right.\right. + \frac{2\left(4\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a+5\,(bx+a)^{\frac{3}{2}}c\right)}{5\,(a^3b-3\,a^2bc+3\,abc^2-bc^3)}$$

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/5*((b*x + a)*(4*(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)*(b*x + a
)/(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c^4
 - 6*a*b^3*c^5 + b^3*c^6) - 3*(a^4*b^2 - 4*a^3*b^2*c + 6*a^2*b^2*c^2 - 4*a*
b^2*c^3 + b^2*c^4)/(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3
 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6)) - (a^5*b^2 - 5*a^4*b^2*c + 10*a
^3*b^2*c^2 - 10*a^2*b^2*c^3 + 5*a*b^2*c^4 - b^2*c^5)/(a^6*b^3 - 6*a^5*b^3*c
 + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6
))*sqrt(b*x + c) + 2/5*(4*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a + 5*(b*x +
a)^(3/2)*c)/(a^3*b - 3*a^2*b*c + 3*a*b*c^2 - b*c^3)
```

Mupad [B] (verification not implemented)

Time = 16.64 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.94

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\left(\frac{2\,(a^2+3\,c\,a)}{(a-c)^3}+\frac{2\,a\,\left(\frac{32\,a\,b}{5\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{3\,b}\right)\,\sqrt{a+b\,x}}{b} - \frac{\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{2\,c\,\left(\frac{32\,b\,c}{5\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{3\,b}\right)\,\sqrt{c+b\,x}}{b} + \frac{8\,b\,x^2\,\sqrt{a+b\,x}}{5\,(a-c)^3} - \frac{8\,b\,x^2\,\sqrt{c+b\,x}}{5\,(a-c)^3} - \frac{x\,\left(\frac{32\,a\,b}{5\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)\,\sqrt{a+b\,x}}{3\,b} + \frac{x\,\left(\frac{32\,b\,c}{5\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)\,\sqrt{c+b\,x}}{3\,b}$$

```
[In] int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

```
[Out] (((2*(3*a*c + a^2))/(a - c)^3 + (2*a*((32*a*b)/(5*(a - c)^3) - (2*b*(5*a +
3*c))/(a - c)^3))/(3*b))*(a + b*x)^(1/2))/b - (((2*c*(3*a + c))/(a - c)^3 +
 (2*c*((32*b*c)/(5*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(3*b))*(c + b
*x)^(1/2))/b + (8*b*x^2*(a + b*x)^(1/2))/(5*(a - c)^3) - (8*b*x^2*(c + b*x)
^(1/2))/(5*(a - c)^3) - (x*((32*a*b)/(5*(a - c)^3) - (2*b*(5*a + 3*c))/(a -
 c)^3)*(a + b*x)^(1/2))/(3*b) + (x*((32*b*c)/(5*(a - c)^3) - (2*b*(3*a + 5*
c))/(a - c)^3)*(c + b*x)^(1/2))/(3*b)
```

3.414 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{2\sqrt{a}(a+3c)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{2\sqrt{c}(3a+c)\operatorname{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{(a-c)^3}$$

```
[Out] 8/3*(b*x+a)^(3/2)/(a-c)^3-8/3*(b*x+c)^(3/2)/(a-c)^3-2*(a+3*c)*arctanh((b*x+
a)^(1/2)/a^(1/2))*a^(1/2)/(a-c)^3+2*(3*a+c)*arctanh((b*x+c)^(1/2)/c^(1/2))*
c^(1/2)/(a-c)^3+2*(a+3*c)*(b*x+a)^(1/2)/(a-c)^3-2*(3*a+c)*(b*x+c)^(1/2)/(a-
c)^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6821, 52, 65, 214}

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = -\frac{2\sqrt{a}(a+3c)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{2\sqrt{c}(3a+c)\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}+\frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}-\frac{8(bx+c)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{bx+c}}{(a-c)^3}$$

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (2*(a + 3*c)*Sqrt[a + b*x])/(a - c)^3 + (8*(a + b*x)^(3/2))/(3*(a - c)^3) -
 (2*(3*a + c)*Sqrt[c + b*x])/(a - c)^3 - (8*(c + b*x)^(3/2))/(3*(a - c)^3)
- (2*Sqrt[a]*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c)^3 + (2*Sqrt[
c]*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(a - c)^3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(4b\sqrt{a+bx}+\frac{a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}}{x}-4b\sqrt{c+bx}-\frac{3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}}{x}\right)dx}{(a-c)^3}\\
&= \frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{(3a+c)\int \frac{\sqrt{c+bx}}{x}\,dx}{(a-c)^3}+\frac{(a+3c)\int \frac{\sqrt{a+bx}}{x}\,dx}{(a-c)^3}\\
&= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}\\
&\quad -\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{(c(3a+c))\int \frac{1}{x\sqrt{c+bx}}\,dx}{(a-c)^3}+\frac{(a(a+3c))\int \frac{1}{x\sqrt{a+bx}}\,dx}{(a-c)^3}
\end{aligned}$$

$$= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3} + \frac{8(a+bx)^{3/2}}{3(a-c)^3} - \frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3} - \frac{8(c+bx)^{3/2}}{3(a-c)^3} - \frac{(2c(3a+c))\text{Subst}\left(\int \frac{1}{-\frac{c}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{c+bx}\right)}{b(a-c)^3} + \frac{(2a(a+3c))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b(a-c)^3}$$

$$= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3} + \frac{8(a+bx)^{3/2}}{3(a-c)^3} - \frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3} - \frac{8(c+bx)^{3/2}}{3(a-c)^3} - \frac{2\sqrt{a}(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3} + \frac{2\sqrt{c}(3a+c)\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{(a-c)^3}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.55

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{2}{3}\left(-\frac{\sqrt{c+bx}(9a+7c+4bx)}{(a-c)^3} + \frac{\sqrt{a+bx}(7a+9c+4bx)}{(a-c)^3} - \frac{3\left(\sqrt{a}-\sqrt{c}\right)\arctan\left(\frac{-\sqrt{a+bx}+\sqrt{c+bx}}{\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}}\right)}{\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}\left(\sqrt{a}+\sqrt{c}\right)^3} - \frac{3\left(\sqrt{a}+\sqrt{c}\right)\arctan\left(\frac{-\sqrt{a+bx}+\sqrt{c+bx}}{\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}}\right)}{\left(\sqrt{a}-\sqrt{c}\right)^3\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}}\right)$$

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (2*(-((Sqrt[c + b*x]*(9*a + 7*c + 4*b*x))/(a - c)^3) + (Sqrt[a + b*x]*(7*a
+ 9*c + 4*b*x))/(a - c)^3 - (3*(Sqrt[a] - Sqrt[c])*ArcTan[(-Sqrt[a + b*x] +
 Sqrt[c + b*x])/Sqrt[-(Sqrt[a] - Sqrt[c])^2]])/(Sqrt[-(Sqrt[a] - Sqrt[c])^2
]*(Sqrt[a] + Sqrt[c])^3) - (3*(Sqrt[a] + Sqrt[c])*ArcTan[(-Sqrt[a + b*x] +
Sqrt[c + b*x])/Sqrt[-(Sqrt[a] + Sqrt[c])^2]])/((Sqrt[a] - Sqrt[c])^3*Sqrt[-
(Sqrt[a] + Sqrt[c])^2])))/3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

method	result
default	$\frac{a\left(2\sqrt{bx+a}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{(a-c)^3}+\frac{8(bx+a)^{\frac{3}{2}}}{3(a-c)^3}-\frac{8(bx+c)^{\frac{3}{2}}}{3(a-c)^3}+\frac{3c\left(2\sqrt{bx+a}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{(a-c)^3}-\frac{3a\left(2\sqrt{bx+c}-2\sqrt{c}\ \right.}{(a-}$

[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)

[Out] 1/(a-c)^3*a*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))+8/3*(b*x+a)^(3/2)/(a-c)^3-8/3*(b*x+c)^(3/2)/(a-c)^3+3/(a-c)^3*c*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-3/(a-c)^3*a*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))-1/(a-c)^3*c*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 516, normalized size of antiderivative = 3.29

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$
$$=\left[-\frac{3\,(a+3\,c)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+3\,(3\,a+c)\sqrt{c}\log\left(\frac{bx-2\,\sqrt{bx+c}\sqrt{c}+2\,c}{x}\right)-2\,(4\,bx+7\,a+9\,c)\sqrt{bx+c}}{3\,(a^3-3\,a^2c+3\,ac^2-c^3)}\right.$$
$$-\frac{6\,(3\,a+c)\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+3\,(a+3\,c)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(4\,bx+7\,a+9\,c)\sqrt{bx+a}}{3\,(a^3-3\,a^2c+3\,ac^2-c^3)}$$

[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")

[Out] [-1/3*(3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) + 2*(4*b*x + 9*a + 7*c)*sqrt(b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), -1/3*(6*(3*a + c)*sqrt(-c)*arctan(sqrt(b*x + c)*sqrt(-c)/c) + 3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) + 2*(4*b*x + 9*a + 7*c)*sqrt(b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 1/3*(6*sqrt(-a)*(a + 3*c)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) - 2*(4*b*x + 9*a + 7*c)*sqrt(b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 2/3*(3*sqrt(-a)*(a + 3*c)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(3*a + c)*sqrt(-c)*arctan(sqrt(b*x + c)*sqrt(-c)/c) + (4*b*x + 7*a + 9*c)*sqrt(b*x + a) - (4*b*x + 9*a + 7*c)*sqrt(b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)]

Sympy [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

Maxima [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2649 vs. $2(133) = 266$.

Time = 0.93 (sec) , antiderivative size = 2649, normalized size of antiderivative = 16.87

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
[Out] -2/3*sqrt(b*x + c)*(4*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(b*x + a)/(a^6 - 6*a^
5*c + 15*a^4*c^2 - 20*a^3*c^3 + 15*a^2*c^4 - 6*a*c^5 + c^6) + (5*a^4 - 8*a^
3*c - 6*a^2*c^2 + 16*a*c^3 - 7*c^4)/(a^6 - 6*a^5*c + 15*a^4*c^2 - 20*a^3*c^
3 + 15*a^2*c^4 - 6*a*c^5 + c^6)) + 2*(a^2 + 3*a*c)*arctan(sqrt(b*x + a)/sqr
t(-a))/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*sqrt(-a)) + 2/3*(4*(b*x + a)^(3/2)*
a^6 + 3*sqrt(b*x + a)*a^7 - 24*(b*x + a)^(3/2)*a^5*c - 9*sqrt(b*x + a)*a^6*
c + 60*(b*x + a)^(3/2)*a^4*c^2 - 9*sqrt(b*x + a)*a^5*c^2 - 80*(b*x + a)^(3/
2)*a^3*c^3 + 75*sqrt(b*x + a)*a^4*c^3 + 60*(b*x + a)^(3/2)*a^2*c^4 - 135*sq
rt(b*x + a)*a^3*c^4 - 24*(b*x + a)^(3/2)*a*c^5 + 117*sqrt(b*x + a)*a^2*c^5
+ 4*(b*x + a)^(3/2)*c^6 - 51*sqrt(b*x + a)*a*c^6 + 9*sqrt(b*x + a)*c^7)/(a^
9 - 9*a^8*c + 36*a^7*c^2 - 84*a^6*c^3 + 126*a^5*c^4 - 126*a^4*c^5 + 84*a^3*
c^6 - 36*a^2*c^7 + 9*a*c^8 - c^9) + 2*(3*a^9*c - 14*a^8*c^2 + 22*a^7*c^3 -
6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2*a^2*c^8 + a*c^9 - 2*(3*
a^2*c^2 + a*c^3 - (3*a*c^2 + c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3
```

)^2*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 2*(3*a^2*c^2 + a*c^3 + (3*a^2*c + a*c^2)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2 + (3*a^5*c^2 - 11*a^4*c^3 + 14*a^3*c^4 - 6*a^2*c^5 - a*c^6 + c^7 - (3*a^5*c - 11*a^4*c^2 + 14*a^3*c^3 - 6*a^2*c^4 - a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) + (3*a^6*c - 11*a^5*c^2 + 14*a^4*c^3 - 6*a^3*c^4 - a^2*c^5 + a*c^6 + (3*a^5*c - 11*a^4*c^2 + 14*a^3*c^3 - 6*a^2*c^4 - a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) + (3*a^9*c - 14*a^8*c^2 + 22*a^7*c^3 - 6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2*a^2*c^8 + a*c^9 + (3*a^8*c - 14*a^7*c^2 + 22*a^6*c^3 - 6*a^5*c^4 - 20*a^4*c^5 + 22*a^3*c^6 - 6*a^2*c^7 - 2*a*c^8 + c^9)*sqrt(a*c))*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) + (3*a^9 - 14*a^8*c + 22*a^7*c^2 - 6*a^6*c^3 - 20*a^5*c^4 + 22*a^4*c^5 - 6*a^3*c^6 - 2*a^2*c^7 + a*c^8)*sqrt(a*c))*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 + sqrt((a^4 - 2*a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*c^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/((sqrt(-a)*a^8 - a^8*sqrt(-c) - 8*sqrt(-a)*a^7*c + 8*a^7*sqrt(-c)*c + 28*sqrt(-a)*a^6*c^2 - 28*a^6*sqrt(-c)*c^2 - 56*sqrt(-a)*a^5*c^3 + 56*a^5*sqrt(-c)*c^3 + 70*sqrt(-a)*a^4*c^4 - 70*a^4*sqrt(-c)*c^4 - 56*sqrt(-a)*a^3*c^5 + 56*a^3*sqrt(-c)*c^5 + 28*sqrt(-a)*a^2*c^6 - 28*a^2*sqrt(-c)*c^6 - 8*sqrt(-a)*a*c^7 + 8*a*sqrt(-c)*c^7 + sqrt(-a)*c^8 - sqrt(-c)*c^8)*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)) + 2*(3*a^9*c - 14*a^8*c^2 + 22*a^7*c^3 - 6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2*a^2*c^8 + a*c^9 + 2*(3*a^2*c^2 + a*c^3 + (3*a*c^2 + c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 2*(3*a^2*c^2 + a*c^3 + (3*a^2*c + a*c^2)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2 - (3*a^5*c^2 - 11*a^4*c^3 + 14*a^3*c^4 - 6*a^2*c^5 - a*c^6 + c^7 + (3*a^5*c - 11*a^4*c^2 + 14*a^3*c^3 - 6*a^2*c^4 - a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) + (3*a^6*c - 11*a^5*c^2 + 14*a^4*c^3 - 6*a^3*c^4 - a^2*c^5 + a*c^6 + (3*a^5*c - 11*a^4*c^2 + 14*a^3*c^3 - 6*a^2*c^4 - a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) - (3*a^9*c - 14*a^8*c^2 + 22*a^7*c^3 - 6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2*a^2*c^8 + a*c^9 - (3*a^8*c - 14*a^7*c^2 + 22*a^6*c^3 - 6*a^5*c^4 - 20*a^4*c^5 + 22*a^3*c^6 - 6*a^2*c^7 - 2*a*c^8 + c^9)*sqrt(a*c))*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) + (3*a^9 - 14*a^8*c + 22*a^7*c^2 - 6*a^6*c^3 - 20*a^5*c^4 + 22*a^4*c^5 - 6*a^3*c^6 - 2*a^2*c^7 + a*c^8)*sqrt(a*c))*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 - sqrt((a^4 - 2*a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*c^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/((sqrt(-a)*a^8 - a^8*sqrt(-c) - 8*sqrt(-a)*a^7*c + 8*a^7*sqrt(-c)*c + 28*sqrt(-a)*a^6*c^2 - 28*a^6*sqrt(-c)*c^2 - 56*sqrt(-a)*a^5*c^3 + 56*a^5*sqrt(-c)*c^3 + 70*sqrt(-a)*a^4*c^4 - 70*a^4*sqrt(-c)*c^4 - 56*sqrt(-a)*a^3*c^5 + 56*a^3*sqrt(-c)*c^5 + 28*sqrt(-a)*a^2*c^6 - 28*a^2*sqrt(-c)*c^6 - 8*sqrt(-a)*a*c^7 + 8*a*sqrt(-c)*c^7 + sqrt(-a)*c^8 - sqrt(-c)*c^8)*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3))

Mupad [B] (verification not implemented)

Time = 42.36 (sec) , antiderivative size = 4060, normalized size of antiderivative = 25.86

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\text{Too large to display}$$

```
[In] int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))^3),x)
[Out] ((((a^(1/2)*(16*a + 16*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) + (c^(1/2)*(16*a
 + 16*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3))*((a + b*x)^(1/2) - a^(1/2)))/((c
 + b*x)^(1/2) - c^(1/2)) + (((a^(1/2)*(12*a + 20*c))/(3*a*c^2 - 3*a^2*c + a
^3 - c^3) + (c^(1/2)*(20*a + 12*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3))*((a +
b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2 + (a^(1/2)*((28*a)/3
 + 12*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) + (c^(1/2)*(12*a + (28*c)/3))/(3*
a*c^2 - 3*a^2*c + a^3 - c^3))/((3*((a + b*x)^(1/2) - a^(1/2)))/((c + b*x)^(
1/2) - c^(1/2)) + (3*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1
/2))^2 + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(1/2))^3 + 1) +
 (log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a*(a^(1/2)
+ 3*c^(1/2)) + c*(3*a^(1/2) + c^(1/2))))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) +
(atan(((((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/
2) + a^(3/2)*c^(1/2)))^(1/2)*((6*a*c^(11/2) - 6*a^(11/2)*c + 2*a^(3/2)*c^5
- 2*a^5*c^(3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/2) + 16*a^(
9/2)*c^2)/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3
 - 6*a^6*c^2) + (((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^(5/2)*c^(11/
2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2) - 5*a^(13/2
)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*
a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(a*c^9 +
 a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*a^6*c^4 + 22
*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 +
15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*((a^(1/2)*c^(
3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2))
)^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3
/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3
+ 5*a^5*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(3*a^(3/2)*c^7 - 3*a^7*c^(3/2
) + 8*a^6*c^(5/2) - 8*a^(5/2)*c^6 - 6*a^5*c^(7/2) + 6*a^(7/2)*c^5 + a^3*c^(
11/2) - a^(11/2)*c^3))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 +
15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a^(1/2)*c^3
- 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2)*
1i)/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2) - (((
a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/
2)*c^(1/2)))^(1/2)*((((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^(5/2)*c^
(11/2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2) - 5*a^(
13/2)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 -
 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(a*c
```

^9 + a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*a^6*c^4
+ 22*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^
7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*((a^(1/2)
*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1
/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*
c^(3/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*
c^3 + 5*a^5*c^2) - (6*a*c^(11/2) - 6*a^(11/2)*c + 2*a^(3/2)*c^5 - 2*a^5*c^(
3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/2) + 16*a^(9/2)*c^2)/(
a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^
2) + (2*((a + b*x)^(1/2) - a^(1/2))*(3*a^(3/2)*c^7 - 3*a^7*c^(3/2) + 8*a^6*
c^(5/2) - 8*a^(5/2)*c^6 - 6*a^5*c^(7/2) + 6*a^(7/2)*c^5 + a^3*c^(11/2) - a^
(11/2)*c^3))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 + 15*a^4*c^6
 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a^(1/2)*c^3 - 3*a^(5/2
)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2)*1i)/(a*c^6
 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2))/((2*(a^(1/2)*c
^(9/2) - 4*a^(3/2)*c^(7/2) + 6*a^(5/2)*c^(5/2) - 4*a^(7/2)*c^(3/2) + a^(9/2
)*c^(1/2)))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c
^3 - 6*a^6*c^2) - (((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^
(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((6*a*c^(11/2) - 6*a^(11/2)*c + 2*a
^(3/2)*c^5 - 2*a^5*c^(3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/
2) + 16*a^(9/2)*c^2)/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 +
 15*a^5*c^3 - 6*a^6*c^2) + (((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^(
5/2)*c^(11/2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2)
 - 5*a^(13/2)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^
3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/2
))*(a*c^9 + a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*a
^6*c^4 + 22*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6
*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*((
a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/
2)*c^(1/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) +
 2*a^2*c^(3/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 -
10*a^4*c^3 + 5*a^5*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(3*a^(3/2)*c^7 - 3
*a^7*c^(3/2) + 8*a^6*c^(5/2) - 8*a^(5/2)*c^6 - 6*a^5*c^(7/2) + 6*a^(7/2)*c^
5 + a^3*c^(11/2) - a^(11/2)*c^3))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6
*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a
^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^
(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^
2) - (((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2)
 + a^(3/2)*c^(1/2)))^(1/2)*((((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^
(5/2)*c^(11/2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2)
 - 5*a^(13/2)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a
^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/
2))*(a*c^9 + a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*
a^6*c^4 + 22*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 -
6*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2))*(

```
(a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3
/2)*c^(1/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2)
+ 2*a^2*c^(3/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 -
 10*a^4*c^3 + 5*a^5*c^2) - (6*a*c^(11/2) - 6*a^(11/2)*c + 2*a^(3/2)*c^5 - 2
*a^5*c^(3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/2) + 16*a^(9/2
)*c^2)/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 -
6*a^6*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(3*a^(3/2)*c^7 - 3*a^7*c^(3/2)
+ 8*a^6*c^(5/2) - 8*a^(5/2)*c^6 - 6*a^5*c^(7/2) + 6*a^(7/2)*c^5 + a^3*c^(11
/2) - a^(11/2)*c^3))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 + 15
*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a^(1/2)*c^3 -
3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2))/(
a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2) + (4*((a +
 b*x)^(1/2) - a^(1/2))*(6*a^3*c^4 - a^6*c - 5*a^2*c^5 - a*c^6 + 6*a^4*c^3 -
 5*a^5*c^2 + 3*a^(3/2)*c^(11/2) + 4*a^(5/2)*c^(9/2) - 14*a^(7/2)*c^(7/2) +
4*a^(9/2)*c^(5/2) + 3*a^(11/2)*c^(3/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*
c^8 - 6*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^
2))))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2)
 + a^(3/2)*c^(1/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c
^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2)*2i)/(a*c^6 - a^6*c - 5*a^2*c^5 + 10
*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2)
```

3.415 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}-\frac{3b(3a+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)^3}-\frac{3b(a+3c)\text{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{\sqrt{c}(-a+c)^3}$$

```
[Out] -3*b*(3*a+c)*arctanh((b*x+a)^(1/2)/a^(1/2))/(a-c)^3/a^(1/2)-3*b*(a+3*c)*arc
tanh((b*x+c)^(1/2)/c^(1/2))/(-a+c)^3/c^(1/2)+8*b*(b*x+a)^(1/2)/(a-c)^3-(a+3
*c)*(b*x+a)^(1/2)/(a-c)^3/x-8*b*(b*x+c)^(1/2)/(a-c)^3+(3*a+c)*(b*x+c)^(1/2)
/(a-c)^3/x
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.38, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6821, 43, 65, 214, 52}

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=-\frac{b(a+3c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)^3}-\frac{8\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{b(3a+c)\text{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}(a-c)^3}+\frac{8b\sqrt{c}\text{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{(a-c)^3}+\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{8b\sqrt{bx+c}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{x(a-c)^3}+\frac{(3a+c)\sqrt{bx+c}}{x(a-c)^3}$$

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (8*b*Sqrt[a + b*x])/(a - c)^3 - ((a + 3*c)*Sqrt[a + b*x])/((a - c)^3*x) - (
8*b*Sqrt[c + b*x])/(a - c)^3 + ((3*a + c)*Sqrt[c + b*x])/((a - c)^3*x) - (8
*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c)^3 - (b*(a + 3*c)*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(a - c)^3) + (8*b*Sqrt[c]*ArcTanh[Sqrt[c
+ b*x]/Sqrt[c]])/(a - c)^3 + (b*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(
(a - c)^3*Sqrt[c])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}}{x^2}+\frac{4b\sqrt{a+bx}}{x}-\frac{3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}}{x^2}-\frac{4b\sqrt{c+bx}}{x}\right)dx}{(a-c)^3}\\
&= \frac{(4b)\int \frac{\sqrt{a+bx}}{x}\,dx}{(a-c)^3}-\frac{(4b)\int \frac{\sqrt{c+bx}}{x}\,dx}{(a-c)^3}-\frac{(3a+c)\int \frac{\sqrt{c+bx}}{x^2}\,dx}{(a-c)^3}+\frac{(a+3c)\int \frac{\sqrt{a+bx}}{x^2}\,dx}{(a-c)^3}\\
&= \frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}+\frac{(4ab)\int \frac{1}{x\sqrt{a+bx}}\,dx}{(a-c)^3}\\
&\quad-\frac{(4bc)\int \frac{1}{x\sqrt{c+bx}}\,dx}{(a-c)^3}-\frac{(b(3a+c))\int \frac{1}{x\sqrt{c+bx}}\,dx}{2(a-c)^3}+\frac{(b(a+3c))\int \frac{1}{x\sqrt{a+bx}}\,dx}{2(a-c)^3}\\
&= \frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}\\
&\quad+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}+\frac{(8a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{(a-c)^3}\\
&\quad-\frac{(8c)\text{Subst}\left(\int \frac{1}{-\frac{c}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{c+bx}\right)}{(a-c)^3}-\frac{(3a+c)\text{Subst}\left(\int \frac{1}{-\frac{c}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{c+bx}\right)}{(a-c)^3}\\
&\quad+\frac{(a+3c)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{(a-c)^3}
\end{aligned}$$

$$= \frac{8b\sqrt{a+bx}}{(a-c)^3} - \frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x} - \frac{8b\sqrt{c+bx}}{(a-c)^3} + \frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x} - \frac{8\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3} - \frac{b(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)^3} + \frac{8b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{(a-c)^3} + \frac{b(3a+c)\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{(a-c)^3\sqrt{c}}$$

Mathematica [A] (verified)

Time = 10.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$
$$= \frac{b\left(8\sqrt{a+bx} - 8\sqrt{c+bx} - 8\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + 8\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right) - \frac{(a+3c)\left(a+bx+bx\sqrt{1+\frac{bx}{a}}\text{arctanh}\left(\sqrt{\right.\right.}}{bx\sqrt{a+bx}}\right.}{(a-c)^3}$$

[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]

[Out] (b*(8*Sqrt[a + b*x] - 8*Sqrt[c + b*x] - 8*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]] + 8*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]] - ((a + 3*c)*(a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]]))/(b*x*Sqrt[a + b*x]) + ((3*a + c)*(c + b*x + b*x*Sqrt[1 + (b*x)/c]*ArcTanh[Sqrt[1 + (b*x)/c]]))/(b*x*Sqrt[c + b*x])))/(a - c)^3

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.56

method	result
default	$\frac{2ab\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(a-c)^3}+\frac{6cb\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(a-c)^3}-\frac{6ab\left(-\frac{\sqrt{bx+c}}{2xb}-\frac{\text{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{(a-c)^3}-\frac{2cb\left(-\frac{\sqrt{bx}}{2x}\right.}{}$

[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)

[Out] 2/(a-c)^3*a*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))+6/(a-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))-6/(a-c)^3*a*b*(-1/2*(b*x+c)^(1/2)/x/b-1/2/c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))-2/(a-c)^3*c*b*(-1/2*(b*x+c)^(1/2)/x/b-1/2/c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))+4/(a-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-4/(a-c)^3*b*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 675, normalized size of antiderivative = 4.17

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$
$$= \left[-\frac{3\,(3\,abc+bc^2)\sqrt{a}x\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+3\,(a^2b+3\,abc)\sqrt{c}x\log\left(\frac{bx-2\,\sqrt{bx+c}\sqrt{c}+2\,c}{x}\right)-2\,(8\,abcx-a^2c}{2\,(a^4c-3\,a^3c^2+3\,a^2c^3-ac^4)x}\right.$$
$$-\frac{6\,(a^2b+3\,abc)\sqrt{-c}x\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+3\,(3\,abc+bc^2)\sqrt{a}x\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(8\,abcx-a^2c-}{2\,(a^4c-3\,a^3c^2+3\,a^2c^3-ac^4)x}$$

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2
*a)/x) + 3*(a^2*b + 3*a*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) +
 2*c)/x) - 2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x - 3
*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x),
 -1/2*(6*(a^2*b + 3*a*b*c)*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c) + 3*
(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) -
2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x - 3*a^2*c - a*
c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), 1/2*(6*(3*
a*b*c + b*c^2)*sqrt(-a)*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(a^2*b + 3*a
*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*(8*a*b*c*x
 - a^2*c - 3*a*c^2)*sqrt(b*x + a) - 2*(8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*
x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), (3*(3*a*b*c + b*c^2)*s
qrt(-a)*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(a^2*b + 3*a*b*c)*sqrt(-c)*x
*arctan(sqrt(b*x + c)*sqrt(-c)/c) + (8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x
+ a) - (8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3
*a^2*c^3 - a*c^4)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2594 vs. $2(142) = 284$.

Time = 7.56 (sec) , antiderivative size = 2594, normalized size of antiderivative = 16.01

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
[Out] 8*sqrt(b*x + a)*b/(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 8*sqrt(b*x + c)*b/(a^3
- 3*a^2*c + 3*a*c^2 - c^3) + 3*(3*a*b + b*c)*arctan(sqrt(b*x + a)/sqrt(-a))
/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*sqrt(-a)) - 3*(2*(a^2*c^2 + 3*a*c^3 + (a*
c^2 + 3*c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b*sgn(-2*a^3 + 6*
a^2*c - 6*a*c^2 + 2*c^3) - 2*(a^2*c^2 + 3*a*c^3 + (a^2*c + 3*a*c^2)*sqrt(a*
c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b + (a^5*c^2 - a^4*c^3 - 6*a^3*c^4 +
14*a^2*c^5 - 11*a*c^6 + 3*c^7 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*c^4 -
 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)*sgn(-2*
a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^6*c - a^5*c^2 - 6*a^4*c^3 + 14*a^3*c^
4 - 11*a^2*c^5 + 3*a*c^6 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*c^4 - 11*a
*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) - (a^9*c - 2
*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 1
4*a^2*c^8 + 3*a*c^9 + (a^8*c - 2*a^7*c^2 - 6*a^6*c^3 + 22*a^5*c^4 - 20*a^4*
c^5 - 6*a^3*c^6 + 22*a^2*c^7 - 14*a*c^8 + 3*c^9)*sqrt(a*c))*b*sgn(-2*a^3 +
6*a^2*c - 6*a*c^2 + 2*c^3) + (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 -
20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 + (a^9 - 2*a^8*c
 - 6*a^7*c^2 + 22*a^6*c^3 - 20*a^5*c^4 - 6*a^4*c^5 + 22*a^3*c^6 - 14*a^2*c^
7 + 3*a*c^8)*sqrt(a*c))*b)*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))/sqrt(-(a
^4 - 2*a^3*c + 2*a*c^3 - c^4 + sqrt((a^4 - 2*a^3*c + 2*a*c^3 - c^4)^2 - (a^
5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*c^4 - c^5)*(a^3 - 3*a^2*c + 3*a
*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/((sqrt(-a)*a^8*c - a^8*sqrt
(-c)*c - 8*sqrt(-a)*a^7*c^2 + 8*a^7*sqrt(-c)*c^2 + 28*sqrt(-a)*a^6*c^3 - 28
*a^6*sqrt(-c)*c^3 - 56*sqrt(-a)*a^5*c^4 + 56*a^5*sqrt(-c)*c^4 + 70*sqrt(-a)
*a^4*c^5 - 70*a^4*sqrt(-c)*c^5 - 56*sqrt(-a)*a^3*c^6 + 56*a^3*sqrt(-c)*c^6
```

```
+ 28*sqrt(-a)*a^2*c^7 - 28*a^2*sqrt(-c)*c^7 - 8*sqrt(-a)*a*c^8 + 8*a*sqrt(-
c)*c^8 + sqrt(-a)*c^9 - sqrt(-c)*c^9)*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3))
- 3*(2*(a^2*c^2 + 3*a*c^3 - (a*c^2 + 3*c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a
*c^2 - c^3)^2*b*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) + 2*(a^2*c^2 + 3*a*
c^3 + (a^2*c + 3*a*c^2)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b - (a
^5*c^2 - a^4*c^3 - 6*a^3*c^4 + 14*a^2*c^5 - 11*a*c^6 + 3*c^7 + (a^5*c - a^4
*c^2 - 6*a^3*c^3 + 14*a^2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3
*a^2*c - 3*a*c^2 + c^3)*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^6*c -
a^5*c^2 - 6*a^4*c^3 + 14*a^3*c^4 - 11*a^2*c^5 + 3*a*c^6 - (a^5*c - a^4*c^2
- 6*a^3*c^3 + 14*a^2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*
c - 3*a*c^2 + c^3) - (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c
^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 + (a^8*c - 2*a^7*c^2 - 6
*a^6*c^3 + 22*a^5*c^4 - 20*a^4*c^5 - 6*a^3*c^6 + 22*a^2*c^7 - 14*a*c^8 + 3*
c^9)*sqrt(a*c))*b*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^9*c - 2*a^8*
c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2
*c^8 + 3*a*c^9 + (a^9 - 2*a^8*c - 6*a^7*c^2 + 22*a^6*c^3 - 20*a^5*c^4 - 6*a
^4*c^5 + 22*a^3*c^6 - 14*a^2*c^7 + 3*a*c^8)*sqrt(a*c))*b)*arctan(-(sqrt(b*x
 + a) - sqrt(b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 - sqrt((a^4 - 2
*a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*
c^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3
)))/((sqrt(-a)*a^8*c - a^8*sqrt(-c)*c - 8*sqrt(-a)*a^7*c^2 + 8*a^7*sqrt(-c)
*c^2 + 28*sqrt(-a)*a^6*c^3 - 28*a^6*sqrt(-c)*c^3 - 56*sqrt(-a)*a^5*c^4 + 56
*a^5*sqrt(-c)*c^4 + 70*sqrt(-a)*a^4*c^5 - 70*a^4*sqrt(-c)*c^5 - 56*sqrt(-a)
*a^3*c^6 + 56*a^3*sqrt(-c)*c^6 + 28*sqrt(-a)*a^2*c^7 - 28*a^2*sqrt(-c)*c^7
- 8*sqrt(-a)*a*c^8 + 8*a*sqrt(-c)*c^8 + sqrt(-a)*c^9 - sqrt(-c)*c^9)*abs(-a
^3 + 3*a^2*c - 3*a*c^2 + c^3)) - 2*(3*a*b*(sqrt(b*x + a) - sqrt(b*x + c))^3
 + b*c*(sqrt(b*x + a) - sqrt(b*x + c))^3 - 3*a^2*b*(sqrt(b*x + a) - sqrt(b*
x + c)) + 2*a*b*c*(sqrt(b*x + a) - sqrt(b*x + c)) + b*c^2*(sqrt(b*x + a) -
sqrt(b*x + c)))/(((sqrt(b*x + a) - sqrt(b*x + c))^4 - 2*a*(sqrt(b*x + a) -
sqrt(b*x + c))^2 - 2*c*(sqrt(b*x + a) - sqrt(b*x + c))^2 + a^2 - 2*a*c + c^
2)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)) - (sqrt(b*x + a)*a*b + 3*sqrt(b*x + a)*
b*c)/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*b*x)
```

Mupad [B] (verification not implemented)

Time = 51.54 (sec) , antiderivative size = 4681, normalized size of antiderivative = 28.90

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))^3),x)
```

```
[Out] (b*atan(((b*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c
^(3/2) + a^(3/2)*c^(1/2))))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*b*c^6 - 24*a
^5*b*c^(9/2) + 24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4 - 3
```

*a^2*b*c^(15/2) + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(6*a^(3/2)*b*c^8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7/2)*b*c^6 - 48*a^5*b*c^(9/2) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) - (3*b*((a^(5/2)*c^(19/2) - 5*a^(7/2)*c^(17/2) + 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a^(13/2)*c^(11/2) + 9*a^(15/2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/2))/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a^4*c^8 - 164*a^5*c^7 + 200*a^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3 + 4*a^10*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2))/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2)))*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2)*3i)/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2)) + (b*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*b*c^6 - 24*a^5*b*c^(9/2) + 24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4 - 3*a^2*b*c^(15/2) + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(6*a^(3/2)*b*c^8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7/2)*b*c^6 - 48*a^5*b*c^(9/2) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) + (3*b*((a^(5/2)*c^(19/2) - 5*a^(7/2)*c^(17/2) + 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a^(13/2)*c^(11/2) + 9*a^(15/2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/2))/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a^4*c^8 - 164*a^5*c^7 + 200*a^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3 + 4*a^10*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2))/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2)))*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2)*3i)/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2)))/(((9*a^(3/2)*b^2*c^(11/2))/2 - 18*a^(5/2)*b^2*c^(9/2) + 27*a^(7/2)*b^2*c^(7/2) - 18*a^(9/2)*b^2*c^(5/2) + (9*a^(11/2)*b^2*c^(3/2))/2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(72*a^3*b^2*c^4 - 72*a^2*b^2*c^5 + 72*a^4*b^2*c^3 - 72*a^5*b^2*c^2 + 27*a^(3/2)*b^2*c^(11/2) + 36*a^(5/2)*b^2*c^(9/2) - 126*a^(7/2)*b^2*c^(7/2) + 36*a^(9/2)*b^2*c^(5/2) +

27*a^(11/2)*b^2*c^(3/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^
8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) - (3*b*((a
^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2
)*c^(1/2)))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*b*c^6 - 24*a^5*b*c^(9/2) +
24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4 - 3*a^2*b*c^(15/2)
 + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^
7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(6*a^(3/2)*b*c^
8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7/2)*b*c^6 - 48*a^5*b*c^(9/2
) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4))/(2*((c + b*x
)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*
c^5 - 6*a^8*c^4 + a^9*c^3)) - (3*b*((a^(5/2)*c^(19/2) - 5*a^(7/2)*c^(17/2)
+ 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a^(13/2)*c^(11/2) + 9*a^(15/
2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/2))/(a^3*c^9 - 6*a^4*c^8 +
15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) - (((a + b*x)^(
1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a^4*c^8 - 164*a^5*c^7 + 200*a
^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3 + 4*a^10*c^2))/(2*((c + b*x)
^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c
^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2
*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a*c^(7/2) + a^(7/2)*c - 3
*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2))/(2*(a^2*c^7
- 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2)))*(a*c^(7/2) +
 a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2)
)/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2))
 + (3*b*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/
2) + a^(3/2)*c^(1/2)))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*b*c^6 - 24*a^5*b
*c^(9/2) + 24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4 - 3*a^2
*b*c^(15/2) + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*
c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(6*a
^(3/2)*b*c^8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7/2)*b*c^6 - 48*a
^5*b*c^(9/2) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4))/(
2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^
6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) + (3*b*((a^(5/2)*c^(19/2) - 5*a^(7/2
)*c^(17/2) + 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a^(13/2)*c^(11/2)
 + 9*a^(15/2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/2))/(a^3*c^9 - 6
*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) - ((
(a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a^4*c^8 - 164*a^5*
c^7 + 200*a^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3 + 4*a^10*c^2))/(2
*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6
 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*
c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a*c^(7/2) + a^
(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2))/(
2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2)))*(
a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a
^(5/2)*c^2))/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3
- a^7*c^2))))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)

```
*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) -
 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2)*3i)/(a^2*c^7 - 5*a^3*c^6 +
10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2) - (log(((a + b*x)^(1/2) - a^
(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(3*a^2*b*c^(1/2) + 3*a^(1/2)*b*c^2 + a*
c*(9*a^(1/2)*b + 9*b*c^(1/2))))/(2*a*c^4 - 2*a^4*c - 6*a^2*c^3 + 6*a^3*c^2)
 - ((a^(1/2)*((3*a*b)/4 + (b*c)/4))/(a*c^3 + 3*a^3*c - a^4 - 3*a^2*c^2) - (
c^(1/2)*((a*b)/4 + (3*b*c)/4))/(3*a*c^3 + a^3*c - c^4 - 3*a^2*c^2) - (((a^(
1/2)*((3*a^3*b)/4 - (b*c^3)/4 - (a*b*c^2)/2 + 17*a^2*b*c))/(a^5*c - a^2*c^4
 + 3*a^3*c^3 - 3*a^4*c^2) + (c^(1/2)*((a^3*b)/4 - (3*b*c^3)/4 - 17*a*b*c^2
+ (a^2*b*c)/2))/(a*c^5 - 3*a^2*c^4 + 3*a^3*c^3 - a^4*c^2))*((a + b*x)^(1/2)
 - a^(1/2))^3)/((c + b*x)^(1/2) - c^(1/2))^3 + (((a^(1/2)*((b*c^3)/4 - a^3*
b + (75*a*b*c^2)/4 + 15*a^2*b*c))/(a^5*c - a^2*c^4 + 3*a^3*c^3 - 3*a^4*c^2)
 - (c^(1/2)*((a^3*b)/4 - b*c^3 + 15*a*b*c^2 + (75*a^2*b*c)/4))/(a*c^5 - 3*a
^2*c^4 + 3*a^3*c^3 - a^4*c^2))*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1
/2) - c^(1/2))^2 + (((a^(1/2)*((a^2*b)/4 - 2*b*c^2 + (67*a*b*c)/4))/(a*c^4
- a^4*c - 3*a^2*c^3 + 3*a^3*c^2) + (c^(1/2)*((b*c^2)/4 - 2*a^2*b + (67*a*b*
c)/4))/(a*c^4 - a^4*c - 3*a^2*c^3 + 3*a^3*c^2))*((a + b*x)^(1/2) - a^(1/2))
)/((c + b*x)^(1/2) - c^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2
) - c^(1/2)) + ((a + b*x)^(1/2) - a^(1/2))^4/((c + b*x)^(1/2) - c^(1/2))^4
- (((a + c)/(a^(1/2)*c^(1/2)) - 1)*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x
)^(1/2) - c^(1/2))^2 - (((a + c)/(a^(1/2)*c^(1/2)) - 1)*((a + b*x)^(1/2) -
a^(1/2))^3)/((c + b*x)^(1/2) - c^(1/2))^3) - (b*((a + b*x)^(1/2) - a^(1/2))
)/(4*a^(1/2)*c^(1/2)*(a^(1/2) - c^(1/2))^3*((c + b*x)^(1/2) - c^(1/2)))
```

3.416 $\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx=-\frac{2x^{3/2}}{3}+\frac{2}{3}(1+x)^{3/2}$$

```
[Out] -2/3*x^(3/2)+2/3*(1+x)^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2131, 30, 32}

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx=\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

```
[In] Int[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2131

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Dist[-b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\text{integral} = -\int \sqrt{x}\,dx + \int \sqrt{1+x}\,dx$$
$$= -\frac{2x^{3/2}}{3} + \frac{2}{3}(1+x)^{3/2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = -\frac{2x^{3/2}}{3} + \frac{2}{3}(1+x)^{3/2}$$

```
[In] Integrate[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{2x^{\frac{3}{2}}}{3}+\frac{2(x+1)^{\frac{3}{2}}}{3}$	14
meijerg	$-\frac{\frac{4\sqrt{\pi}\,x^{\frac{3}{2}}}{3}-\frac{2\sqrt{\pi}\,x^{\frac{3}{2}}\left(2+\frac{2}{x}\right)\sqrt{1+\frac{1}{x}}}{3}}{2\sqrt{\pi}}$	37

```
[In] int(1/(x^(1/2)+(x+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -2/3*x^(3/2)+2/3*(x+1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}} - \frac{2}{3}\,x^{\frac{3}{2}}$$

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(17) = 34.

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \frac{2\sqrt{x}\sqrt{x+1}}{3\sqrt{x}+3\sqrt{x+1}} + \frac{4x}{3\sqrt{x}+3\sqrt{x+1}} + \frac{2}{3\sqrt{x}+3\sqrt{x+1}}$$

```
[In] integrate(1/(x**(1/2)+(1+x)**(1/2)),x)
[Out] 2*sqrt(x)*sqrt(x + 1)/(3*sqrt(x) + 3*sqrt(x + 1)) + 4*x/(3*sqrt(x) + 3*sqrt
(x + 1)) + 2/(3*sqrt(x) + 3*sqrt(x + 1))
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$$

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x + 1) + sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}} - \frac{2}{3}\,x^{\frac{3}{2}}$$

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 16.61 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx=\frac{2\,x\,\sqrt{x+1}}{3}+\frac{2\,\sqrt{x+1}}{3}-\frac{2\,x^{3/2}}{3}$$

```
[In] int(1/((x + 1)^(1/2) + x^(1/2)),x)
[Out] (2*x*(x + 1)^(1/2))/3 + (2*(x + 1)^(1/2))/3 - (2*x^(3/2))/3
```

3.417 $\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = -\frac{2}{3}(-1+x)^{3/2}+\frac{2x^{3/2}}{3}$$

```
[Out] -2/3*(-1+x)^(3/2)+2/3*x^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2131, 30, 32}

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = \frac{2x^{3/2}}{3}-\frac{2}{3}(x-1)^{3/2}$$

```
[In] Int[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

```
[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2131

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Dist[-b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\int \sqrt{-1+x}\,dx + \int \sqrt{x}\,dx \\ &= -\frac{2}{3}(-1+x)^{3/2} + \frac{2x^{3/2}}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = -\frac{2}{3}(-1+x)^{3/2} + \frac{2x^{3/2}}{3}$$

```
[In] Integrate[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{2(x-1)^{\frac{3}{2}}}{3}+\frac{2x^{\frac{3}{2}}}{3}$	14
meijerg	$-\frac{i\left(\frac{4i\sqrt{\pi}\,x^{\frac{3}{2}}}{3}-\frac{2i\sqrt{\pi}\,x^{\frac{3}{2}}\left(2-\frac{2}{x}\right)\sqrt{1-\frac{1}{x}}}{3}\right)}{2\sqrt{\pi}}$	42

```
[In] int(1/((x-1)^(1/2)+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -2/3*(x-1)^(3/2)+2/3*x^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = -\frac{2}{3}\,(x-1)^{\frac{3}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}$$

```
[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="fricas")
[Out] -2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(17) = 34.

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = \frac{2\sqrt{x}\sqrt{x-1}}{3\sqrt{x}+3\sqrt{x-1}} + \frac{4x}{3\sqrt{x}+3\sqrt{x-1}} - \frac{2}{3\sqrt{x}+3\sqrt{x-1}}$$

```
[In] integrate(1/((-1+x)**(1/2)+x**(1/2)),x)
[Out] 2*sqrt(x)*sqrt(x - 1)/(3*sqrt(x) + 3*sqrt(x - 1)) + 4*x/(3*sqrt(x) + 3*sqrt
(x - 1)) - 2/(3*sqrt(x) + 3*sqrt(x - 1))
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x-1}+\sqrt{x}}\,dx$$

```
[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x - 1) + sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = -\frac{2}{3}\,(x-1)^{\frac{3}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}$$

```
[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="giac")
[Out] -2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 16.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = \frac{2\,\sqrt{x-1}}{3}-\frac{2\,x\,\sqrt{x-1}}{3}+\frac{2\,x^{3/2}}{3}$$

```
[In] int(1/((x - 1)^(1/2) + x^(1/2)),x)
[Out] (2*(x - 1)^(1/2))/3 - (2*x*(x - 1)^(1/2))/3 + (2*x^(3/2))/3
```

3.418 $\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = -\frac{1}{3}(-1+x)^{3/2}+\frac{1}{3}(1+x)^{3/2}$$

```
[Out] -1/3*(-1+x)^(3/2)+1/3*(1+x)^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6821}

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \frac{1}{3}(x+1)^{3/2}-\frac{1}{3}(x-1)^{3/2}$$

```
[In] Int[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]
[Out] -1/3*(-1 + x)^(3/2) + (1 + x)^(3/2)/3
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int\left(\sqrt{-1+x}-\sqrt{1+x}\right)\,dx\right)\\ &= -\frac{1}{3}(-1+x)^{3/2}+\frac{1}{3}(1+x)^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = -\frac{1}{3}(-1+x)^{3/2}+\frac{1}{3}(1+x)^{3/2}$$

```
[In] Integrate[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]
[Out] -1/3*(-1 + x)^(3/2) + (1 + x)^(3/2)/3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{(x-1)^{\frac{3}{2}}}{3}+\frac{(x+1)^{\frac{3}{2}}}{3}$	16

```
[In] int(1/((x-1)^(1/2)+(x+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/3*(x-1)^(3/2)+1/3*(x+1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \frac{1}{3}\,(x+1)^{\frac{3}{2}}-\frac{1}{3}\,(x-1)^{\frac{3}{2}}$$

```
[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(15) = 30.

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \frac{4x}{3\sqrt{x-1}+3\sqrt{x+1}}+\frac{2\sqrt{x-1}\sqrt{x+1}}{3\sqrt{x-1}+3\sqrt{x+1}}$$

```
[In] integrate(1/((-1+x)**(1/2)+(1+x)**(1/2)),x)
[Out] 4*x/(3*sqrt(x - 1) + 3*sqrt(x + 1)) + 2*sqrt(x - 1)*sqrt(x + 1)/(3*sqrt(x -
 1) + 3*sqrt(x + 1))
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \int \frac{1}{\sqrt{x+1}+\sqrt{x-1}}\,dx$$

```
[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x + 1) + sqrt(x - 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \frac{1}{3}\,(x+1)^{\frac{3}{2}} - \frac{1}{3}\,(x-1)^{\frac{3}{2}}$$

```
[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
[Out] 1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \frac{(x+1)^{3/2}}{3} - \frac{(x-1)^{3/2}}{3}$$

```
[In] int(1/((x - 1)^(1/2) + (x + 1)^(1/2)),x)
[Out] (x + 1)^(3/2)/3 - (x - 1)^(3/2)/3
```

3.419 $\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 38

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{x^4}{2}-\frac{2}{3}\left(1-x^2\right)^{3/2}+\frac{2}{5}\left(1-x^2\right)^{5/2}$$

```
[Out] 1/2*x^4-2/3*(-x^2+1)^(3/2)+2/5*(-x^2+1)^(5/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6874, 272, 45}

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{x^4}{2}+\frac{2}{5}\left(1-x^2\right)^{5/2}-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

```
[In] Int[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
[Out] x^4/2 - (2*(1 - x^2)^(3/2))/3 + (2*(1 - x^2)^(5/2))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(2x^3 + 2x^3\sqrt{1-x^2}\right)\,dx\\ &= \frac{x^4}{2} + 2\int x^3\sqrt{1-x^2}\,dx\\ &= \frac{x^4}{2} + \text{Subst}\left(\int \sqrt{1-x}x\,dx, x, x^2\right)\\ &= \frac{x^4}{2} + \text{Subst}\left(\int \left(\sqrt{1-x} - (1-x)^{3/2}\right)\,dx, x, x^2\right)\\ &= \frac{x^4}{2} - \frac{2}{3}\left(1-x^2\right)^{3/2} + \frac{2}{5}\left(1-x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx = \frac{1}{30}\left(-1+x^2\right)\left(15+8\sqrt{1-x^2}+3x^2\left(5+4\sqrt{1-x^2}\right)\right)$$

```
[In] Integrate[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
[Out] ((-1 + x^2)*(15 + 8*Sqrt[1 - x^2] + 3*x^2*(5 + 4*Sqrt[1 - x^2])))/30
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x^4}{2}+\frac{2\sqrt{1-x}\,\sqrt{x+1}\,(x^2-1)\,(3x^2+2)}{15}$	33

```
[In] int(x^3*((1-x)^(1/2)+(x+1)^(1/2))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x^4+2/15*(1-x)^(1/2)*(x+1)^(1/2)*(x^2-1)*(3*x^2+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{1}{2}\,x^4+\frac{2}{15}\,(3\,x^4-x^2-2)\sqrt{x+1}\sqrt{-x+1}$$

```
[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
[Out] 1/2*x^4 + 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [F]

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\int x^3\left(\sqrt{1-x}+\sqrt{x+1}\right)^2\,dx$$

```
[In] integrate(x**3*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
[Out] Integral(x**3*(sqrt(1 - x) + sqrt(x + 1))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{1}{2}\,x^4-\frac{2}{5}\,(-x^2+1)^{\frac{3}{2}}x^2-\frac{4}{15}\,(-x^2+1)^{\frac{3}{2}}$$

```
[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
[Out] 1/2*x^4 - 2/5*(-x^2 + 1)^(3/2)*x^2 - 4/15*(-x^2 + 1)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(28) = 56.

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

$$\begin{aligned}&\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx\\&=\frac{1}{2}\,x^4+\frac{1}{60}\,((2\,(3\,(4\,x-17)(x+1)+133)(x+1)-295)(x+1)+195)\sqrt{x+1}\sqrt{-x+1}\\&\quad+\frac{1}{12}\,((2\,(3\,x-10)(x+1)+43)(x+1)-39)\sqrt{x+1}\sqrt{-x+1}\end{aligned}$$

```
[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*x^4 + 1/60*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 19
5)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 3
9)*sqrt(x + 1)*sqrt(-x + 1)
```

Mupad [B] (verification not implemented)

Time = 16.51 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{x^4}{2}-\frac{\sqrt{1-x}\,\left(-\frac{2\,x^5}{5}-\frac{2\,x^4}{5}+\frac{2\,x^3}{15}+\frac{2\,x^2}{15}+\frac{4\,x}{15}+\frac{4}{15}\right)}{\sqrt{x+1}}$$

```
[In] int(x^3*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] x^4/2 - ((1 - x)^(1/2)*((4*x)/15 + (2*x^2)/15 + (2*x^3)/15 - (2*x^4)/5 - (2
*x^5)/5 + 4/15))/(x + 1)^(1/2)
```

3.420 $\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 48

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{2x^3}{3}-\frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{\arcsin(x)}{4}$$

```
[Out] 2/3*x^3+1/4*arcsin(x)-1/4*x*(-x^2+1)^(1/2)+1/2*x^3*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6874, 285, 327, 222}

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{\arcsin(x)}{4}+\frac{2x^3}{3}-\frac{1}{4}\sqrt{1-x^2}x+\frac{1}{2}\sqrt{1-x^2}x^3$$

```
[In] Int[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] (2*x^3)/3 - (x*Sqrt[1 - x^2])/4 + (x^3*Sqrt[1 - x^2])/2 + ArcSin[x]/4
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(2x^2+2x^2\sqrt{1-x^2}\right)\,dx\\
&= \frac{2x^3}{3}+2\int x^2\sqrt{1-x^2}\,dx\\
&= \frac{2x^3}{3}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\,dx\\
&= \frac{2x^3}{3}-\frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{4}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{2x^3}{3}-\frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{4}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx = \frac{1}{12}\left(8-3x\sqrt{1-x^2}+x^3\left(8+6\sqrt{1-x^2}\right)+12\arctan\left(\frac{-\sqrt{2}+\sqrt{1+x}}{\sqrt{1-x}}\right)\right)$$

```
[In] Integrate[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
[Out] (8 - 3*x*Sqrt[1 - x^2] + x^3*(8 + 6*Sqrt[1 - x^2]) + 12*ArcTan[(-Sqrt[2] +
Sqrt[1 + x])/Sqrt[1 - x]])/12
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

method	result	size
default	$\frac{2x^3}{3}+\frac{\sqrt{1-x}\,\sqrt{x+1}\,\left(2x^3\sqrt{-x^2+1}-x\sqrt{-x^2+1}+\arcsin(x)\right)}{4\sqrt{-x^2+1}}$	59

```
[In] int(x^2*((1-x)^(1/2)+(x+1)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^3+1/4*(1-x)^(1/2)*(x+1)^(1/2)*(2*x^3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2)+
arcsin(x))/(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{2}{3}\,x^3+\frac{1}{4}\,(2\,x^3-x)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{2}\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

```
[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 2/3*x^3 + 1/4*(2*x^3 - x)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*arctan((sqrt(x + 1
)*sqrt(-x + 1) - 1)/x)
```

Sympy [F]

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\int x^2\left(\sqrt{1-x}+\sqrt{x+1}\right)^2\,dx$$

```
[In] integrate(x**2*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Integral(x**2*(sqrt(1 - x) + sqrt(x + 1))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{2}{3}\,x^3-\frac{1}{2}\,\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{1}{4}\,\sqrt{-x^2+1}x+\frac{1}{4}\,\arcsin\left(x\right)$$

```
[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
[Out] 2/3*x^3 - 1/2*(-x^2 + 1)^(3/2)*x + 1/4*sqrt(-x^2 + 1)*x + 1/4*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(36) = 72.

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\begin{aligned}&\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx\\&=\frac{2}{3}\,x^3+\frac{1}{12}\,\left(\left(2\,\left(3\,x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\sqrt{-x+1}\\&\quad+\frac{1}{3}\,\left(\left(2\,x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)\end{aligned}$$

```
[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
[Out] 2/3*x^3 + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(
-x + 1) + 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*arcsin
(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [B] (verification not implemented)

Time = 29.81 (sec) , antiderivative size = 563, normalized size of antiderivative = 11.73

$$\begin{aligned}&\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx\\&=\frac{\frac{4\,\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{28\,{\left(\sqrt{1-x}-1\right)}^3}{{\left(\sqrt{x+1}-1\right)}^3}+\frac{28\,{\left(\sqrt{1-x}-1\right)}^5}{{\left(\sqrt{x+1}-1\right)}^5}-\frac{4\,{\left(\sqrt{1-x}-1\right)}^7}{{\left(\sqrt{x+1}-1\right)}^7}}{\frac{4\,{\left(\sqrt{1-x}-1\right)}^2}{{\left(\sqrt{x+1}-1\right)}^2}+\frac{6\,{\left(\sqrt{1-x}-1\right)}^4}{{\left(\sqrt{x+1}-1\right)}^4}+\frac{4\,{\left(\sqrt{1-x}-1\right)}^6}{{\left(\sqrt{x+1}-1\right)}^6}+\frac{{\left(\sqrt{1-x}-1\right)}^8}{{\left(\sqrt{x+1}-1\right)}^8}+1}-\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)\\&\quad-\frac{\frac{3\,\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}+\frac{23\,{\left(\sqrt{1-x}-1\right)}^3}{{\left(\sqrt{x+1}-1\right)}^3}-\frac{333\,{\left(\sqrt{1-x}-1\right)}^5}{{\left(\sqrt{x+1}-1\right)}^5}+\frac{671\,{\left(\sqrt{1-x}-1\right)}^7}{{\left(\sqrt{x+1}-1\right)}^7}-\frac{671\,{\left(\sqrt{1-x}-1\right)}^9}{{\left(\sqrt{x+1}-1\right)}^9}+\frac{333\,{\left(\sqrt{1-x}-1\right)}^{11}}{{\left(\sqrt{x+1}-1\right)}^{11}}-\frac{23\,{\left(\sqrt{1-x}-1\right)}^{13}}{{\left(\sqrt{x+1}-1\right)}^{13}}-}{\frac{8\,{\left(\sqrt{1-x}-1\right)}^2}{{\left(\sqrt{x+1}-1\right)}^2}+\frac{28\,{\left(\sqrt{1-x}-1\right)}^4}{{\left(\sqrt{x+1}-1\right)}^4}+\frac{56\,{\left(\sqrt{1-x}-1\right)}^6}{{\left(\sqrt{x+1}-1\right)}^6}+\frac{70\,{\left(\sqrt{1-x}-1\right)}^8}{{\left(\sqrt{x+1}-1\right)}^8}+\frac{56\,{\left(\sqrt{1-x}-1\right)}^{10}}{{\left(\sqrt{x+1}-1\right)}^{10}}+\frac{28\,{\left(\sqrt{1-x}-1\right)}^{12}}{{\left(\sqrt{x+1}-1\right)}^{12}}+\frac{8\,{\left(\sqrt{1-x}-1\right)}^{14}}{{\left(\sqrt{x+1}-1\right)}^{14}}+}\\&\quad+\frac{2\,x^3}{3}\end{aligned}$$

```
[In] int(x^2*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] ((4*((1 - x)^(1/2) - 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/(
(x + 1)^(1/2) - 1)^3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (
4*((1 - x)^(1/2) - 1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/
((x + 1)^(1/2) - 1)^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (
4*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x
+ 1)^(1/2) - 1)^8 + 1) - atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((
3*((1 - x)^(1/2) - 1))/((x + 1)^(1/2) - 1) + (23*((1 - x)^(1/2) - 1)^3)/((x
 + 1)^(1/2) - 1)^3 - (333*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 + (6
71*((1 - x)^(1/2) - 1)^7)/((x + 1)^(1/2) - 1)^7 - (671*((1 - x)^(1/2) - 1)^
9)/((x + 1)^(1/2) - 1)^9 + (333*((1 - x)^(1/2) - 1)^11)/((x + 1)^(1/2) - 1)
^11 - (23*((1 - x)^(1/2) - 1)^13)/((x + 1)^(1/2) - 1)^13 - (3*((1 - x)^(1/2
) - 1)^15)/((x + 1)^(1/2) - 1)^15)/((8*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2
) - 1)^2 + (28*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (56*((1 - x)^
(1/2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + (70*((1 - x)^(1/2) - 1)^8)/((x + 1)^(
1/2) - 1)^8 + (56*((1 - x)^(1/2) - 1)^10)/((x + 1)^(1/2) - 1)^10 + (28*((1
- x)^(1/2) - 1)^12)/((x + 1)^(1/2) - 1)^12 + (8*((1 - x)^(1/2) - 1)^14)/((x
 + 1)^(1/2) - 1)^14 + ((1 - x)^(1/2) - 1)^16/((x + 1)^(1/2) - 1)^16 + 1) +
(2*x^3)/3
```

3.421 $\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] x^2-2/3*(-x^2+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6874, 267}

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

```
[In] Int[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x^2 - (2*(1 - x^2)^(3/2))/3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(2x + 2x\sqrt{1-x^2}\right)\,dx\\ &= x^2 + 2\int x\sqrt{1-x^2}\,dx\\ &= x^2 - \frac{2}{3}\left(1-x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx = \frac{1}{3}(-1+x)(1+x)\left(3+2\sqrt{1-x^2}\right)$$

```
[In] Integrate[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
[Out] ((-1 + x)*(1 + x)*(3 + 2*Sqrt[1 - x^2]))/3
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

method	result	size
default	$x^2+\frac{2\sqrt{1-x}\,\sqrt{x+1}\,(x^2-1)}{3}$	24

```
[In] int(x*((1-x)^(1/2)+(x+1)^(1/2))^2,x,method=_RETURNVERBOSE)
[Out] x^2+2/3*(1-x)^(1/2)*(x+1)^(1/2)*(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx = x^2+\frac{2}{3}\left(x^2-1\right)\sqrt{x+1}\sqrt{-x+1}$$

```
[In] integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
[Out] x^2 + 2/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [F]

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\int x\left(\sqrt{1-x}+\sqrt{x+1}\right)^2\,dx$$

```
[In] integrate(x*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
[Out] Integral(x*(sqrt(1 - x) + sqrt(x + 1))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=x^2-\frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
[Out] x^2 - 2/3*(-x^2 + 1)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(15) = 30.

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.68

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=(x+1)^2+\frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}+\sqrt{x+1}(x-2)\sqrt{-x+1}-2x-2$$

```
[In] integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
[Out] (x + 1)^2 + 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x +
 1)*(x - 2)*sqrt(-x + 1) - 2*x - 2
```

Mupad [B] (verification not implemented)

Time = 16.82 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=x^2-\frac{\sqrt{1-x}\,\left(-\frac{2\,x^3}{3}-\frac{2\,x^2}{3}+\frac{2\,x}{3}+\frac{2}{3}\right)}{\sqrt{x+1}}$$

```
[In] int(x*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
[Out] x^2 - ((1 - x)^(1/2)*((2*x)/3 - (2*x^2)/3 - (2*x^3)/3 + 2/3))/(x + 1)^(1/2)
```

3.422 $\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = 2x+x\sqrt{1-x^2}+\arcsin(x)$$

```
[Out] 2*x+arcsin(x)+x*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6874, 201, 222}

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = \arcsin(x)+\sqrt{1-x^2}x+2x$$

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] 2*x + x*Sqrt[1 - x^2] + ArcSin[x]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(2+2\sqrt{1-x^2}\right)\,dx \\
&= 2x+2\int \sqrt{1-x^2}\,dx \\
&= 2x+x\sqrt{1-x^2}+\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= 2x+x\sqrt{1-x^2}+\sin^{-1}(x)
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 45 vs. 2(19) = 38.

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$
\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx = 2+x\left(2+\sqrt{1-x^2}\right)+4\arctan\left(\frac{-\sqrt{2}+\sqrt{1+x}}{\sqrt{1-x}}\right)
$$

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
[Out] 2 + x*(2 + Sqrt[1 - x^2]) + 4*ArcTan[(-Sqrt[2] + Sqrt[1 + x])/Sqrt[1 - x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(17) = 34.

Time = 0.97 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.05

method	result	size
default	$2x+\sqrt{1-x}\,(x+1)^{\frac{3}{2}}-\sqrt{1-x}\,\sqrt{x+1}+\frac{\sqrt{(1-x)(x+1)}\,\arcsin(x)}{\sqrt{x+1}\,\sqrt{1-x}}$	58

```
[In] int(((1-x)^(1/2)+(x+1)^(1/2))^2,x,method=_RETURNVERBOSE)
[Out] 2*x+(1-x)^(1/2)*(x+1)^(3/2)-(1-x)^(1/2)*(x+1)^(1/2)+((1-x)*(x+1))^(1/2)/(x+
1)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(17) = 34$.

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.11

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\sqrt{x+1}x\sqrt{-x+1}+2\,x-2\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
[Out] sqrt(x + 1)*x*sqrt(-x + 1) + 2*x - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/
x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(15) = 30$.

Time = 0.90 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=2x+4\sqrt{1-x}\left(\frac{(x+1)^{\frac{3}{2}}}{4}-\frac{\sqrt{x+1}}{4}\right)+2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)$$

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2,x)
[Out] 2*x + 4*sqrt(1 - x)*((x + 1)**(3/2)/4 - sqrt(x + 1)/4) + 2*asin(sqrt(2)*sqr
t(x + 1)/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\sqrt{-x^2+1}x+2\,x+\arcsin\left(x\right)$$

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
[Out] sqrt(-x^2 + 1)*x + 2*x + arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = \sqrt{x+1}(x-2)\sqrt{-x+1}+2x+2\sqrt{x+1}\sqrt{-x+1} + 2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)+2$$

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
[Out] sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + 2*x + 2*sqrt(x + 1)*sqrt(-x + 1) + 2*arc
sin(1/2*sqrt(2)*sqrt(x + 1)) + 2
```

Mupad [B] (verification not implemented)

Time = 21.70 (sec) , antiderivative size = 206, normalized size of antiderivative = 10.84

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = 2x-4\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \frac{\frac{4(\sqrt{1-x}-1)}{\sqrt{x+1}-1}-\frac{28(\sqrt{1-x}-1)^3}{(\sqrt{x+1}-1)^3}+\frac{28(\sqrt{1-x}-1)^5}{(\sqrt{x+1}-1)^5}-\frac{4(\sqrt{1-x}-1)^7}{(\sqrt{x+1}-1)^7}}{\frac{4(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2}+\frac{6(\sqrt{1-x}-1)^4}{(\sqrt{x+1}-1)^4}+\frac{4(\sqrt{1-x}-1)^6}{(\sqrt{x+1}-1)^6}+\frac{(\sqrt{1-x}-1)^8}{(\sqrt{x+1}-1)^8}+1}$$

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
[Out] 2*x - 4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((4*((1 - x)^(1/2)
- 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^
3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (4*((1 - x)^(1/2) -
1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)
^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2) -
1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)^8 +
 1)
```

3.423 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x} dx$

Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x} dx = 2\sqrt{1-x^2} - 2\text{arctanh}\left(\sqrt{1-x^2}\right) + 2\log(x)$$

```
[Out] -2*arctanh((-x^2+1)^(1/2))+2*ln(x)+2*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6874, 272, 52, 65, 212}

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x} dx = -2\text{arctanh}\left(\sqrt{1-x^2}\right) + 2\sqrt{1-x^2} + 2\log(x)$$

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

```
[Out] 2*Sqrt[1 - x^2] - 2*ArcTanh[Sqrt[1 - x^2]] + 2*Log[x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}\right) dx \\
&= 2\log(x)+2\int \frac{\sqrt{1-x^2}}{x}\, dx \\
&= 2\log(x)+\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\, dx, x, x^2\right) \\
&= 2\sqrt{1-x^2}+2\log(x)+\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\, dx, x, x^2\right) \\
&= 2\sqrt{1-x^2}+2\log(x)-2\text{Subst}\left(\int \frac{1}{1-x^2}\, dx, x, \sqrt{1-x^2}\right) \\
&= 2\sqrt{1-x^2}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\log(x)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 72 vs. 2(32) = 64.

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.25

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = 2\Big(\sqrt{1-x^2}+2\log\left(\sqrt{2}-\sqrt{1+x}\right) + 2\log\left(\sqrt{1-x}-\sqrt{1+x}\right)-2\log\left(-2+\sqrt{2}\sqrt{1+x}\right)\Big)$$

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

```
[Out] 2*(Sqrt[1 - x^2] + 2*Log[Sqrt[2] - Sqrt[1 + x]] + 2*Log[Sqrt[1 - x] - Sqrt[
1 + x]] - 2*Log[-2 + Sqrt[2]*Sqrt[1 + x]])
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

method	result	size
default	$2\ln\left(x\right)+\frac{2\sqrt{1-x}\,\sqrt{x+1}\,\left(\sqrt{-x^{2}+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)\right)}{\sqrt{-x^{2}+1}}$	51

```
[In] int(((1-x)^(1/2)+(x+1)^(1/2))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*ln(x)+2*(1-x)^(1/2)*(x+1)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1/
(-x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = 2\,\sqrt{x+1}\sqrt{-x+1}+2\,\log\left(x\right)+2\,\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 1)*sqrt(-x + 1) + 2*log(x) + 2*log((sqrt(x + 1)*sqrt(-x + 1) - 1
)/x)
```

Sympy [F]

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx=\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x}\,dx$$

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x,x)
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx=2\sqrt{-x^2+1}+2\log\left(x\right)-2\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="maxima")
[Out] 2*sqrt(-x^2 + 1) + 2*log(x) - 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(28) = 56.

Time = 0.40 (sec) , antiderivative size = 130, normalized size of antiderivative = 4.06

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx=2\sqrt{x+1}\sqrt{-x+1}+2\log\left(\sqrt{x+1}+1\right)+2\log\left(\left|\sqrt{x+1}-1\right|\right)-2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}+2\right|\right)+2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-2\right|\right)$$

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="giac")
[Out] 2*sqrt(x + 1)*sqrt(-x + 1) + 2*log(sqrt(x + 1) + 1) + 2*log(abs(sqrt(x + 1)
 - 1)) - 2*log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(sqr
t(2) - sqrt(-x + 1)) + 2)) + 2*log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1
) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 2))
```

Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.81

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = 2\,\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-2\,\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)+2\,\ln\left(x\right)+\frac{16\,\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2\,\left(\frac{2\,\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+1\right)}$$

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x,x)
```

```
[Out] 2*log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - 2*log(((1 - x)^(1/
2) - 1)/((x + 1)^(1/2) - 1)) + 2*log(x) + (16*((1 - x)^(1/2) - 1)^2)/(((x +
 1)^(1/2) - 1)^2*((2*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + ((1 - x
)^(1/2) - 1)^4/((x + 1)^(1/2) - 1)^4 + 1))
```

3.424 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 26

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2} dx = -\frac{2}{x}-\frac{2\sqrt{1-x^2}}{x}-2\arcsin(x)$$

```
[Out] -2/x-2*arcsin(x)-2*(-x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6874, 283, 222}

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2} dx = -2\arcsin(x)-\frac{2\sqrt{1-x^2}}{x}-\frac{2}{x}$$

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]
[Out] -2/x - (2*Sqrt[1 - x^2])/x - 2*ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{2}{x^2}+\frac{2\sqrt{1-x^2}}{x^2}\right)dx\\ &= -\frac{2}{x}+2\int \frac{\sqrt{1-x^2}}{x^2}\,dx\\ &= -\frac{2}{x}-\frac{2\sqrt{1-x^2}}{x}-2\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{2}{x}-\frac{2\sqrt{1-x^2}}{x}-2\sin^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = -\frac{2\left(1+\sqrt{1-x^2}-4x\arctan\left(\frac{\sqrt{1+x}}{\sqrt{2}-\sqrt{1-x}}\right)\right)}{x}$$

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]
[Out] (-2*(1 + Sqrt[1 - x^2] - 4*x*ArcTan[Sqrt[1 + x]/(Sqrt[2] - Sqrt[1 - x])]))/
x
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(24) = 48.

Time = 0.92 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

method	result	size
default	$-\frac{2}{x}+\frac{2\left(-\arcsin(x)x-\sqrt{-x^2+1}\right)\sqrt{1-x}\,\sqrt{x+1}}{x\sqrt{-x^2+1}}$	50

```
[In] int(((1-x)^(1/2)+(x+1)^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
```

[Out] -2/x+2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(1-x)^(1/2)*(x+1)^(1/2)/x/(-x^2+1)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = \frac{2\left(2\,x\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x}$$

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="fricas")

[Out] 2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1) - 1)/x

Sympy [F]

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = \int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^2}\,dx$$

[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**2,x)

[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = -\frac{2\sqrt{-x^2+1}}{x}-\frac{2}{x}-2\arcsin\left(x\right)$$

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="maxima")

[Out] -2*sqrt(-x^2 + 1)/x - 2/x - 2*arcsin(x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(24) = 48$.

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 5.73

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = -2\pi - \frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2 - 4} - \frac{2}{x} - 4\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1} - 1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="giac")
```

```
[Out] -2*pi - 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sq
rt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2)
- sqrt(-x + 1)))^2 - 4) - 2/x - 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqrt(-
x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))
```

Mupad [B] (verification not implemented)

Time = 17.69 (sec) , antiderivative size = 120, normalized size of antiderivative = 4.62

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = 8\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \frac{\frac{5\left(\sqrt{1-x}-1\right)^2}{2\left(\sqrt{x+1}-1\right)^2} - \frac{1}{2}}{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1} - \frac{\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}} - \frac{\sqrt{1-x}-1}{2\left(\sqrt{x+1}-1\right)} - \frac{2}{x}$$

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^2,x)
```

```
[Out] 8*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((5*((1 - x)^(1/2) - 1)^2
)/(2*((x + 1)^(1/2) - 1)^2) - 1/2)/(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)
 - ((1 - x)^(1/2) - 1)^3/((x + 1)^(1/2) - 1)^3) - ((1 - x)^(1/2) - 1)/(2*((
x + 1)^(1/2) - 1)) - 2/x
```

3.425 $\int \frac{(\sqrt{1-x}+\sqrt{1+x})^2}{x^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 34

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3} \, dx = -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}+\operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

```
[Out] -1/x^2+arctanh((-x^2+1)^(1/2))-(-x^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6874, 272, 43, 65, 212}

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3} \, dx = \operatorname{arctanh}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{x^2}$$

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]
```

```
[Out] -x^(-2) - Sqrt[1 - x^2]/x^2 + ArcTanh[Sqrt[1 - x^2]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{2}{x^3}+\frac{2\sqrt{1-x^2}}{x^3}\right)\,dx\\
&= -\frac{1}{x^2}+2\int \frac{\sqrt{1-x^2}}{x^3}\,dx\\
&= -\frac{1}{x^2}+\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx,x,x^2\right)\\
&= -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 147 vs. 2(34) = 68.

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 4.32

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx = 2\text{arctanh}\left(\frac{2-\sqrt{2}+2\sqrt{1-x}+2\sqrt{1+x}-\sqrt{2}\sqrt{1+x}}{-2+\sqrt{2}+\sqrt{2}\sqrt{1+x}}\right)+\log\left(\sqrt{2}-\sqrt{1+x}\right) - \frac{1+\sqrt{1-x^2}+x^2\log\left(-2-\sqrt{2}+\sqrt{1-x}+\sqrt{1+x}+\sqrt{2}\sqrt{1+x}\right)}{x^2}$$

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]
```

```
[Out] 2*ArcTanh[(2 - Sqrt[2] + 2*Sqrt[1 - x] + 2*Sqrt[1 + x] - Sqrt[2]*Sqrt[1 + x
])/(-2 + Sqrt[2] + Sqrt[2]*Sqrt[1 + x])] + Log[Sqrt[2] - Sqrt[1 + x]] - (1
+ Sqrt[1 - x^2] + x^2*Log[-2 - Sqrt[2] + Sqrt[1 - x] + Sqrt[1 + x] + Sqrt[2
]*Sqrt[1 + x]])/x^2
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

method	result	size
default	$-\frac{1}{x^2}+\frac{\sqrt{1-x}\,\sqrt{x+1}\,\left(\text{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)x^2-\sqrt{-x^2+1}\right)}{x^2\sqrt{-x^2+1}}$	58

```
[In] int(((1-x)^(1/2)+(x+1)^(1/2))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x^2+(1-x)^(1/2)*(x+1)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x^2+1)^(1/2
))/x^2/(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx = -\frac{x^2\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+\sqrt{x+1}\sqrt{-x+1}+1}{x^2}$$

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="fricas")
```

```
[Out] -(x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1)
/x^2
```

Sympy [F]

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx = \int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^3}\,dx$$

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**3,x)
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx = -\sqrt{-x^2+1} - \frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^2} - \frac{1}{x^2} + \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1) - (-x^2 + 1)^(3/2)/x^2 - 1/x^2 + log(2*sqrt(-x^2 + 1)/abs(x
) + 2/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(30) = 60.

Time = 0.40 (sec) , antiderivative size = 235, normalized size of antiderivative = 6.91

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx = \frac{4\left(\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^3 + \frac{4\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}} - \frac{4\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2 - 4\right)^2} - \frac{1}{x^2} + \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} + 2\right|\right) - \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} - 2\right|\right)$$

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="giac")
[Out] 4*(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x +
 1)))^3 + 4*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 4*sqrt(x + 1)/(sqrt(2) -
 sqrt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(
2) - sqrt(-x + 1)))^2 - 4)^2 - 1/x^2 + log(abs(-(sqrt(2) - sqrt(-x + 1))/sq
rt(x + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) + 2)) - log(abs(-(sqrt(2)
- sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 2))
```

Mupad [B] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 189, normalized size of antiderivative = 5.56

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx = \ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right) + \frac{\left(\sqrt{1-x}-1\right)^2}{16\left(\sqrt{x+1}-1\right)^2} - \frac{\frac{\left(\sqrt{1-x}-1\right)^2}{8\left(\sqrt{x+1}-1\right)^2}+\frac{15\left(\sqrt{1-x}-1\right)^4}{16\left(\sqrt{x+1}-1\right)^4}-\frac{1}{16}}{\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-\frac{2\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}} - \frac{1}{x^2}$$

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^3,x)
```

```
[Out] log(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - log(((1 - x)^(1/2) - 1)^2/((
x + 1)^(1/2) - 1)^2 - 1) + ((1 - x)^(1/2) - 1)^2/(16*((x + 1)^(1/2) - 1)^2)
 - (((1 - x)^(1/2) - 1)^2/(8*((x + 1)^(1/2) - 1)^2) + (15*((1 - x)^(1/2) -
1)^4)/(16*((x + 1)^(1/2) - 1)^4) - 1/16)/(((1 - x)^(1/2) - 1)^2/((x + 1)^(1
/2) - 1)^2 - (2*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + ((1 - x)^(1/
2) - 1)^6/((x + 1)^(1/2) - 1)^6) - 1/x^2
```

3.426 $\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3(b-c)c^3}+\frac{4a(a+cx)^{5/2}}{5(b-c)c^3}-\frac{2(a+cx)^{7/2}}{7(b-c)c^3}$$

```
[Out] 2/3*a^2*(b*x+a)^(3/2)/b^3/(b-c)-4/5*a*(b*x+a)^(5/2)/b^3/(b-c)+2/7*(b*x+a)^(
7/2)/b^3/(b-c)-2/3*a^2*(c*x+a)^(3/2)/(b-c)/c^3+4/5*a*(c*x+a)^(5/2)/(b-c)/c^
3-2/7*(c*x+a)^(7/2)/(b-c)/c^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2128, 45}

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}-\frac{2(a+cx)^{7/2}}{7c^3(b-c)}+\frac{4a(a+cx)^{5/2}}{5c^3(b-c)}$$

```
[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(b - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(b -
 c)) + (2*(a + b*x)^(7/2))/(7*b^3*(b - c)) - (2*a^2*(a + c*x)^(3/2))/(3*(b
- c)*c^3) + (4*a*(a + c*x)^(5/2))/(5*(b - c)*c^3) - (2*(a + c*x)^(7/2))/(7*
(b - c)*c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2128

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int x^2\sqrt{a+bx}\,dx}{b-c} - \frac{\int x^2\sqrt{a+cx}\,dx}{b-c} \\
&= \frac{\int \left(\frac{a^2\sqrt{a+bx}}{b^2} - \frac{2a(a+bx)^{3/2}}{b^2} + \frac{(a+bx)^{5/2}}{b^2}\right)dx}{b-c} - \frac{\int \left(\frac{a^2\sqrt{a+cx}}{c^2} - \frac{2a(a+cx)^{3/2}}{c^2} + \frac{(a+cx)^{5/2}}{c^2}\right)dx}{b-c} \\
&= \frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(b-c)} + \frac{2(a+bx)^{7/2}}{7b^3(b-c)} \\
&\quad - \frac{2a^2(a+cx)^{3/2}}{3(b-c)c^3} + \frac{4a(a+cx)^{5/2}}{5(b-c)c^3} - \frac{2(a+cx)^{7/2}}{7(b-c)c^3}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1832 vs. $2(147) = 294$.

Time = 5.98 (sec) , antiderivative size = 1832, normalized size of antiderivative = 12.46

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = -\frac{2a^3(b-c)^2(a+cx)\left(15b^3\sqrt{a-\frac{ab}{c}}c^6x^5\left(bx-\sqrt{a+bx}\sqrt{a+cx}\right)+3ab^2c^5x^4\left(109b\sqrt{a-\frac{ab}{c}}\sqrt{a+bx}\sqrt{a-\right.\right.}{}}$$

```
[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (-2*a^3*(b - c)^2*(a + c*x)*(15*b^3*Sqrt[a - (a*b)/c]*c^6*x^5*(b*x - Sqrt[a
 + b*x]*Sqrt[a + c*x]) + 3*a*b^2*c^5*x^4*(109*b*Sqrt[a - (a*b)/c]*Sqrt[a +
b*x]*Sqrt[a + c*x] - 120*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x]*Sqrt[a + c*x] +
7*b*c*x*(18*Sqrt[a - (a*b)/c] - 5*Sqrt[a + b*x] + 5*Sqrt[a + c*x]) - 5*b^2*
```

x*(22*Sqrt[a - (a*b)/c] - 7*Sqrt[a + b*x] + 7*Sqrt[a + c*x])) + a^6*(b^3*c*
(-378*Sqrt[a - (a*b)/c] + 966*Sqrt[a + b*x] - 200*Sqrt[a + c*x]) + b^4*(15*
Sqrt[a - (a*b)/c] - 105*Sqrt[a + b*x] + 8*Sqrt[a + c*x]) + 32*c^4*(35*Sqrt[
a - (a*b)/c] - 35*Sqrt[a + b*x] + 16*Sqrt[a + c*x]) - 4*b*c^3*(595*Sqrt[a -
 (a*b)/c] - 735*Sqrt[a + b*x] + 288*Sqrt[a + c*x]) + b^2*c^2*(1631*Sqrt[a -
 (a*b)/c] - 2681*Sqrt[a + b*x] + 832*Sqrt[a + c*x])) + a^3*c^3*x^2*(-960*Sq
rt[a - (a*b)/c]*c^3*Sqrt[a + b*x]*Sqrt[a + c*x] + b^4*x*(-960*Sqrt[a - (a*b
)/c] + 945*Sqrt[a + b*x] - 791*Sqrt[a + c*x]) + 20*b*c^2*(132*Sqrt[a - (a*b
)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(119*Sqrt[a - (a*b)/c] - 91*Sqrt[a +
 b*x] + 84*Sqrt[a + c*x])) + b^2*(-2376*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x]*S
qrt[a + c*x] - 14*c^2*x*(379*Sqrt[a - (a*b)/c] - 319*Sqrt[a + b*x] + 288*Sq
rt[a + c*x])) + b^3*(693*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + 7*
c*x*(558*Sqrt[a - (a*b)/c] - 513*Sqrt[a + b*x] + 449*Sqrt[a + c*x]))) + a^2
*b*c^4*x^3*(-1200*Sqrt[a - (a*b)/c]*c^2*Sqrt[a + b*x]*Sqrt[a + c*x] + b^3*x
*(855*Sqrt[a - (a*b)/c] - 630*Sqrt[a + b*x] + 609*Sqrt[a + c*x]) + b^2*(-78
5*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] - 21*c*x*(116*Sqrt[a - (a*b
)/c] - 71*Sqrt[a + b*x] + 69*Sqrt[a + c*x])) + b*(1968*Sqrt[a - (a*b)/c]*c*
Sqrt[a + b*x]*Sqrt[a + c*x] + c^2*x*(1631*Sqrt[a - (a*b)/c] - 861*Sqrt[a +
b*x] + 840*Sqrt[a + c*x]))) + a^5*(-30*b^4*c*x*(11*Sqrt[a - (a*b)/c] - 21*S
qrt[a + b*x] + 6*Sqrt[a + c*x]) + b^2*(-448*Sqrt[a - (a*b)/c]*c^2*Sqrt[a +
b*x]*Sqrt[a + c*x] + c^3*x*(-5306*Sqrt[a - (a*b)/c] + 5866*Sqrt[a + b*x] -
3040*Sqrt[a + c*x])) - 32*c^4*(16*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a +
c*x] + c*x*(35*Sqrt[a - (a*b)/c] - 35*Sqrt[a + b*x] + 24*Sqrt[a + c*x])) +
4*b^3*c*(14*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(609*Sqrt[a
 - (a*b)/c] - 819*Sqrt[a + b*x] + 341*Sqrt[a + c*x])) + b*(896*Sqrt[a - (a*
b)/c]*c^3*Sqrt[a + b*x]*Sqrt[a + c*x] + c^4*x*(4340*Sqrt[a - (a*b)/c] - 434
0*Sqrt[a + b*x] + 2624*Sqrt[a + c*x]))) + a^4*c^2*x*(b^4*x*(855*Sqrt[a - (a
*b)/c] - 945*Sqrt[a + b*x] + 547*Sqrt[a + c*x]) + b^3*(-364*Sqrt[a - (a*b)/
c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(-3906*Sqrt[a - (a*b)/c] + 4011*Sqrt[a
 + b*x] - 2699*Sqrt[a + c*x])) + 32*c^3*(24*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]
*Sqrt[a + c*x] + 5*c*x*(7*Sqrt[a - (a*b)/c] - 7*Sqrt[a + b*x] + 6*Sqrt[a +
c*x])) - 4*b*c^2*(496*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(
1085*Sqrt[a - (a*b)/c] - 1085*Sqrt[a + b*x] + 876*Sqrt[a + c*x])) + b^2*(15
68*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x]*Sqrt[a + c*x] + c^2*x*(6286*Sqrt[a - (
a*b)/c] - 6286*Sqrt[a + b*x] + 4696*Sqrt[a + c*x])))))/(105*c^3*(a*(b - c)
+ Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x])^7)

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4(bx+a)^{\frac{5}{2}}a}{5}+\frac{2(bx+a)^{\frac{3}{2}}a^2}{3}}{(b-c)b^3}-\frac{2\left(\frac{(cx+a)^{\frac{7}{2}}}{7}-\frac{2(cx+a)^{\frac{5}{2}}a}{5}+\frac{(cx+a)^{\frac{3}{2}}a^2}{3}\right)}{(b-c)c^3}$	90

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b-c)/b^3*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*(b*x+a)^(3/2)*a^2)-2
/(b-c)/c^3*(1/7*(c*x+a)^(7/2)-2/5*(c*x+a)^(5/2)*a+1/3*(c*x+a)^(3/2)*a^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2\left((15\,b^3c^3x^3+3\,ab^2c^3x^2-4\,a^2bc^3x+8\,a^3c^3)\sqrt{bx+a}-(15\,b^3c^3x^3+3\,ab^3c^2x^2-4\,a^2b^3cx+8\,a^3b^3)\sqrt{cx+a}\right)}{105\,(b^4c^3-b^3c^4)}$$

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/105*((15*b^3*c^3*x^3 + 3*a*b^2*c^3*x^2 - 4*a^2*b*c^3*x + 8*a^3*c^3)*sqrt(
b*x + a) - (15*b^3*c^3*x^3 + 3*a*b^3*c^2*x^2 - 4*a^2*b^3*c*x + 8*a^3*b^3)*s
qrt(c*x + a))/(b^4*c^3 - b^3*c^4)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{x^3}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")

[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a)), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(123) = 246$.

Time = 0.39 (sec) , antiderivative size = 451, normalized size of antiderivative = 3.07

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = -\frac{2}{105}\sqrt{ab^2+(bx+a)bc-abc}\left(\left(3\,(bx+a)\left(\frac{5\,(b^{17}c^5|b|-2\,b^{16}c^6|b|+b^{15}c^7|b|)(bx+a)}{b^{23}c^5-3\,b^{22}c^6+3\,b^{21}c^7-b^{20}c^8}+\frac{ab^{18}c^4|b|-17\,a}{b^{23}c^5}\right.\right.\right.$$
$$+\frac{2\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)}{105\,(b^4-b^3c)}$$

[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")

```
[Out] -2/105*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((3*(b*x + a)*(5*(b^17*c^5*abs(b
) - 2*b^16*c^6*abs(b) + b^15*c^7*abs(b))*(b*x + a)/(b^23*c^5 - 3*b^22*c^6 +
 3*b^21*c^7 - b^20*c^8) + (a*b^18*c^4*abs(b) - 17*a*b^17*c^5*abs(b) + 31*a*
b^16*c^6*abs(b) - 15*a*b^15*c^7*abs(b))/(b^23*c^5 - 3*b^22*c^6 + 3*b^21*c^7
 - b^20*c^8)) - (4*a^2*b^19*c^3*abs(b) - 2*a^2*b^18*c^4*abs(b) - 53*a^2*b^1
7*c^5*abs(b) + 96*a^2*b^16*c^6*abs(b) - 45*a^2*b^15*c^7*abs(b))/(b^23*c^5 -
 3*b^22*c^6 + 3*b^21*c^7 - b^20*c^8))*(b*x + a) + (8*a^3*b^20*c^2*abs(b) -
12*a^3*b^19*c^3*abs(b) + 3*a^3*b^18*c^4*abs(b) - 17*a^3*b^17*c^5*abs(b) + 3
3*a^3*b^16*c^6*abs(b) - 15*a^3*b^15*c^7*abs(b))/(b^23*c^5 - 3*b^22*c^6 + 3*
b^21*c^7 - b^20*c^8)) + 2/105*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a +
35*(b*x + a)^(3/2)*a^2)/(b^4 - b^3*c)
```

Mupad [B] (verification not implemented)

Time = 17.61 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.22

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2\,x^3\,\sqrt{a+b\,x}}{7\,\left(b-c\right)}-\frac{2\,x^3\,\sqrt{a+c\,x}}{7\,\left(b-c\right)}+\frac{16\,a^3\,\sqrt{a+b\,x}}{105\,b^3\,\left(b-c\right)}-\frac{16\,a^3\,\sqrt{a+c\,x}}{105\,c^3\,\left(b-c\right)}+\frac{2\,a\,x^2\,\sqrt{a+b\,x}}{35\,b\,\left(b-c\right)}-\frac{8\,a^2\,x\,\sqrt{a+b\,x}}{105\,b^2\,\left(b-c\right)}-\frac{2\,a\,x^2\,\sqrt{a+c\,x}}{35\,c\,\left(b-c\right)}+\frac{8\,a^2\,x\,\sqrt{a+c\,x}}{105\,c^2\,\left(b-c\right)}$$

```
[In] int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] (2*x^3*(a + b*x)^(1/2))/(7*(b - c)) - (2*x^3*(a + c*x)^(1/2))/(7*(b - c)) +
 (16*a^3*(a + b*x)^(1/2))/(105*b^3*(b - c)) - (16*a^3*(a + c*x)^(1/2))/(105
*c^3*(b - c)) + (2*a*x^2*(a + b*x)^(1/2))/(35*b*(b - c)) - (8*a^2*x*(a + b*
x)^(1/2))/(105*b^2*(b - c)) - (2*a*x^2*(a + c*x)^(1/2))/(35*c*(b - c)) + (8
*a^2*x*(a + c*x)^(1/2))/(105*c^2*(b - c))
```

3.427 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = -\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}+\frac{2(a+bx)^{5/2}}{5b^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3(b-c)c^2}-\frac{2(a+cx)^{5/2}}{5(b-c)c^2}$$

```
[Out] -2/3*a*(b*x+a)^(3/2)/b^2/(b-c)+2/5*(b*x+a)^(5/2)/b^2/(b-c)+2/3*a*(c*x+a)^(3
/2)/(b-c)/c^2-2/5*(c*x+a)^(5/2)/(b-c)/c^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2128, 45}

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2(a+bx)^{5/2}}{5b^2(b-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}-\frac{2(a+cx)^{5/2}}{5c^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3c^2(b-c)}$$

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(b - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(b - c)
) + (2*a*(a + c*x)^(3/2))/(3*(b - c)*c^2) - (2*(a + c*x)^(5/2))/(5*(b - c)*
c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2128

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int x\sqrt{a+bx}\,dx}{b-c} - \frac{\int x\sqrt{a+cx}\,dx}{b-c}\\ &= \frac{\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{b-c} - \frac{\int\left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right)dx}{b-c}\\ &= -\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}+\frac{2(a+bx)^{5/2}}{5b^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3(b-c)c^2}-\frac{2(a+cx)^{5/2}}{5(b-c)c^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{6b^2c^2x^2\left(\sqrt{a+bx}-\sqrt{a+cx}\right)+2abcx\left(c\sqrt{a+bx}-b\sqrt{a+cx}\right)+a^2\left(-4c^2\sqrt{a+bx}+4b^2\sqrt{a+cx}\right)}{15b^2(b-c)c^2}$$

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
[Out] (6*b^2*c^2*x^2*(Sqrt[a + b*x] - Sqrt[a + c*x]) + 2*a*b*c*x*(c*Sqrt[a + b*x]
 - b*Sqrt[a + c*x]) + a^2*(-4*c^2*Sqrt[a + b*x] + 4*b^2*Sqrt[a + c*x]))/(15
*b^2*(b - c)*c^2)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2(bx+a)^{\frac{3}{2}}a}{3}}{(b-c)b^2}-\frac{2\left(\frac{(cx+a)^{\frac{5}{2}}}{5}-\frac{(cx+a)^{\frac{3}{2}}a}{3}\right)}{(b-c)c^2}$	66

```
[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b-c)/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-2/(b-c)/c^2*(1/5*(c*x+a
)^(5/2)-1/3*(c*x+a)^(3/2)*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2\left((3\,b^2c^2x^2+abc^2x-2\,a^2c^2)\sqrt{bx+a}-(3\,b^2c^2x^2+ab^2cx-2\,a^2b^2)\sqrt{cx+a}\right)}{15\,(b^3c^2-b^2c^3)}$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
[Out] 2/15*((3*b^2*c^2*x^2 + a*b*c^2*x - 2*a^2*c^2)*sqrt(b*x + a) - (3*b^2*c^2*x^
2 + a*b^2*c*x - 2*a^2*b^2)*sqrt(c*x + a))/(b^3*c^2 - b^2*c^3)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

```
[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{x^2}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(79) = 158.

Time = 0.37 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.68

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = -\frac{2}{15}\sqrt{ab^2+(bx+a)bc-abc}\left((bx+a)\left(\frac{3\,(b^9c^3|b|-b^8c^4|b|)(bx+a)}{b^{14}c^3-2\,b^{13}c^4+b^{12}c^5}+\frac{ab^{10}c^2|b|-7\,ab^9c^3|b|+6\,ab^8c^4|b}{b^{14}c^3-2\,b^{13}c^4+b^{12}c^5}\right.\right.$$
$$+\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)}{15\,(b^3-b^2c)}$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/15*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((b*x + a)*(3*(b^9*c^3*abs(b) - b
^8*c^4*abs(b))*(b*x + a)/(b^14*c^3 - 2*b^13*c^4 + b^12*c^5) + (a*b^10*c^2*a
bs(b) - 7*a*b^9*c^3*abs(b) + 6*a*b^8*c^4*abs(b))/(b^14*c^3 - 2*b^13*c^4 + b
^12*c^5)) - (2*a^2*b^11*c*abs(b) - a^2*b^10*c^2*abs(b) - 4*a^2*b^9*c^3*abs(
b) + 3*a^2*b^8*c^4*abs(b))/(b^14*c^3 - 2*b^13*c^4 + b^12*c^5)) + 2/15*(3*(b
*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/(b^3 - b^2*c)
```

Mupad [B] (verification not implemented)

Time = 16.54 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.36

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2\,x^2\,\sqrt{a+b\,x}}{5\,\left(b-c\right)}-\frac{2\,x^2\,\sqrt{a+c\,x}}{5\,\left(b-c\right)}-\frac{4\,a^2\,\sqrt{a+b\,x}}{15\,b^2\,\left(b-c\right)}+\frac{4\,a^2\,\sqrt{a+c\,x}}{15\,c^2\,\left(b-c\right)}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{15\,b\,\left(b-c\right)}-\frac{2\,a\,x\,\sqrt{a+c\,x}}{15\,c\,\left(b-c\right)}$$

```
[In] int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] (2*x^2*(a + b*x)^(1/2))/(5*(b - c)) - (2*x^2*(a + c*x)^(1/2))/(5*(b - c)) -
 (4*a^2*(a + b*x)^(1/2))/(15*b^2*(b - c)) + (4*a^2*(a + c*x)^(1/2))/(15*c^2
*(b - c)) + (2*a*x*(a + b*x)^(1/2))/(15*b*(b - c)) - (2*a*x*(a + c*x)^(1/2)
)/(15*c*(b - c))
```

3.428 $\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3(b-c)c}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b/(b-c)-2/3*(c*x+a)^(3/2)/(b-c)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2128, 32}

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3c(b-c)}$$

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
[Out] (2*(a + b*x)^(3/2))/(3*b*(b - c)) - (2*(a + c*x)^(3/2))/(3*(b - c)*c)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2128

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d, 
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \sqrt{a+bx}\,dx}{b-c} - \frac{\int \sqrt{a+cx}\,dx}{b-c}$$
$$= \frac{2(a+bx)^{3/2}}{3b(b-c)} - \frac{2(a+cx)^{3/2}}{3(b-c)c}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.51

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2ac\sqrt{a+bx}+2bcx\sqrt{a+bx}-2ab\sqrt{a+cx}-2bcx\sqrt{a+cx}}{3b^2c-3bc^2}$$

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
[Out] (2*a*c*Sqrt[a + b*x] + 2*b*c*x*Sqrt[a + b*x] - 2*a*b*Sqrt[a + c*x] - 2*b*c*
x*Sqrt[a + c*x])/(3*b^2*c - 3*b*c^2)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3b(b-c)} - \frac{2(cx+a)^{\frac{3}{2}}}{3(b-c)c}$	40

```
[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/3*(b*x+a)^(3/2)/b/(b-c)-2/3*(c*x+a)^(3/2)/(b-c)/c
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2\left((bcx+ac)\sqrt{bx+a}-(bcx+ab)\sqrt{cx+a}\right)}{3\left(b^2c-bc^2\right)}$$

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
[Out] 2/3*((b*c*x + a*c)*sqrt(b*x + a) - (b*c*x + a*b)*sqrt(c*x + a))/(b^2*c - b*
c^2)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

```
[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{x}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(39) = 78$.

Time = 0.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.28

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = -\frac{2\left(\left(\frac{(bx+a)b^2c|b|}{b^5c-b^4c^2}+\frac{ab^3|b|-ab^2c|b|}{b^5c-b^4c^2}\right)\sqrt{ab^2+(bx+a)bc-abc}-\frac{(bx+a)^{\frac{3}{2}}}{b-c}\right)}{3\,b}$$

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
[Out] -2/3*(((b*x + a)*b^2*c*abs(b)/(b^5*c - b^4*c^2) + (a*b^3*abs(b) - a*b^2*c*a
bs(b))/(b^5*c - b^4*c^2))*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c) - (b*x + a)^(
3/2)/(b - c))/b
```

Mupad [B] (verification not implemented)

Time = 16.55 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\frac{2\,x\,\sqrt{a+b\,x}}{3\,\left(b-c\right)}-\frac{2\,x\,\sqrt{a+c\,x}}{3\,\left(b-c\right)}+\frac{2\,a\,\sqrt{a+b\,x}}{3\,b\,\left(b-c\right)}-\frac{2\,a\,\sqrt{a+c\,x}}{3\,c\,\left(b-c\right)}$$

```
[In] int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] (2*x*(a + b*x)^(1/2))/(3*(b - c)) - (2*x*(a + c*x)^(1/2))/(3*(b - c)) + (2*
a*(a + b*x)^(1/2))/(3*b*(b - c)) - (2*a*(a + c*x)^(1/2))/(3*c*(b - c))
```

3.429 $\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/(b-c)+2*arctanh((c*x+a)^(1/2)/a^(
1/2))*a^(1/2)/(b-c)+2*(b*x+a)^(1/2)/(b-c)-2*(c*x+a)^(1/2)/(b-c)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6822, 52, 65, 214}

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=-\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}$$

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

```
[Out] (2*Sqrt[a + b*x])/(b - c) - (2*Sqrt[a + c*x])/(b - c) - (2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(b - c) + (2*Sqrt[a]*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]
)/(b - c)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{\sqrt{a+bx}}{x}-\frac{\sqrt{a+cx}}{x}\right) dx}{b-c}\\
&= \frac{\int \frac{\sqrt{a+bx}}{x} dx}{b-c}-\frac{\int \frac{\sqrt{a+cx}}{x} dx}{b-c}\\
&= \frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}+\frac{a\int \frac{1}{x\sqrt{a+bx}} dx}{b-c}-\frac{a\int \frac{1}{x\sqrt{a+cx}} dx}{b-c}\\
&= \frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}+\frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+bx}\right)}{b(b-c)}\\
&\quad -\frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}} dx, x, \sqrt{a+cx}\right)}{(b-c)c}\\
&= \frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2\left(\sqrt{a+bx}-\sqrt{a+cx}\right)}{b-c} - \frac{4\sqrt{a-\frac{ab}{c}}\sqrt{c}\arctan\left(\frac{\sqrt{b-c}\sqrt{a+cx}}{\sqrt{c}\left(-\sqrt{a-\frac{ab}{c}}+\sqrt{a+bx}+\sqrt{a+cx}\right)}\right)}{(b-c)^{3/2}}$$

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

```
[Out] (2*(Sqrt[a + b*x] - Sqrt[a + c*x]))/(b - c) - (4*Sqrt[a - (a*b)/c]*Sqrt[c]*
ArcTan[(Sqrt[b - c]*Sqrt[a + c*x])/(Sqrt[c]*(-Sqrt[a - (a*b)/c] + Sqrt[a +
b*x] + Sqrt[a + c*x]))])/(b - c)^(3/2)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{2\sqrt{bx+a}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b-c}-\frac{2\sqrt{cx+a}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{b-c}$	73

```
[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-1/(b-c)*
(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.63

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \left[-\frac{\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}+2\sqrt{cx+a}}{b-c}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{bx+}\right.\right.}{}\right.$$

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*log((c*x
- 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(c*x + a))/(b
```

```
 - c), 2*(sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-a)*arctan(sqrt(
c*x + a)*sqrt(-a)/a) + sqrt(b*x + a) - sqrt(c*x + a))/(b - c)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
[Out] Integral(1/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{1}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. $2(81) = 162$.

Time = 0.54 (sec) , antiderivative size = 1093, normalized size of antiderivative = 11.27

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
[Out] -2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*abs(b)/(b^3 - b^2*c) + 2*a*arctan(sq
rt(b*x + a)/sqrt(-a))/(sqrt(-a)*(b - c)) + 2*sqrt(b*x + a)/(b - c) - 2*(2*(
a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(b - c) + 2*(a*b^
3 - a*b^2*c)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5 - 3*a^2*b^4*c
 + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*sqrt(-a*b*c)*abs(a*b^2 - a*b*c)*abs(b)*sgn(
b - c) + (a^2*b^6 - 3*a^2*b^5*c + 3*a^2*b^4*c^2 - a^2*b^3*c^3)*sqrt(-a)*abs
(a*b^2 - a*b*c)*abs(b) + (a^3*b^7*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b
^3*c^5)*sqrt(-a)*abs(b)*sgn(b - c) + (a^3*b^7 - 2*a^3*b^6*c + 2*a^3*b^4*c^3
 - a^3*b^3*c^4)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqr
t(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^3 - a*b*c^2 + sqrt((a*b^3 - a*
b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*(b - c)))/
```

```
(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*
a^2*abs(a*b^2 - a*b*c)) + 2*(2*(a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt
(-a)*abs(b)*sgn(b - c) + 2*(a*b^3 - a*b^2*c)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)
*abs(b) + (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*sqrt(-a*b*c
)*abs(a*b^2 - a*b*c)*abs(b)*sgn(b - c) + (a^2*b^6 - 3*a^2*b^5*c + 3*a^2*b^4
*c^2 - a^2*b^3*c^3)*sqrt(-a)*abs(a*b^2 - a*b*c)*abs(b) + (a^3*b^7*c - 2*a^3
*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a)*abs(b)*sgn(b - c) + (a^3*b
^7 - 2*a^3*b^6*c + 2*a^3*b^4*c^3 - a^3*b^3*c^4)*sqrt(-a*b*c)*abs(b))*arctan
(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*
b^3 - a*b*c^2 - sqrt((a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b
^3*c^2 - a^2*b^2*c^3)*(b - c)))/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10
*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^2*abs(a*b^2 - a*b*c))
```

Mupad [B] (verification not implemented)

Time = 18.16 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.20

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = -\frac{2\sqrt{a}\,c\left(\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+\frac{\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}\right)-2\sqrt{a}\,b\left(\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)-\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+4\right)}{(b-c)\left(b-\frac{c\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}\right)}$$

```
[In] int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] -(2*a^(1/2)*c*((2*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))
+ (log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^
(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2) - 2*a^(1/2)*b*(log(((a +
 b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2))) - (2*((a + b*x)^(1/2) -
 a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + 4))/((b - c)*(b - (c*((a + b*x)^(1
/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2))
```

3.430 $\int \frac{1}{x(\sqrt{a+bx}+\sqrt{a+cx})}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = -\frac{\sqrt{a+bx}}{(b-c)x}+\frac{\sqrt{a+cx}}{(b-c)x}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}+\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}$$

```
[Out] -b*arctanh((b*x+a)^(1/2)/a^(1/2))/(b-c)/a^(1/2)+c*arctanh((c*x+a)^(1/2)/a^(
1/2))/(b-c)/a^(1/2)-(b*x+a)^(1/2)/(b-c)/x+(c*x+a)^(1/2)/(b-c)/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2128, 43, 65, 214}

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = -\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}+\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}-\frac{\sqrt{a+bx}}{x(b-c)}+\frac{\sqrt{a+cx}}{x(b-c)}$$

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] -(Sqrt[a + b*x]/((b - c)*x)) + Sqrt[a + c*x]/((b - c)*x) - (b*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)) + (c*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(
Sqrt[a]*(b - c))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2128

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt{a+bx}}{x^2}\,dx}{b-c} - \frac{\int \frac{\sqrt{a+cx}}{x^2}\,dx}{b-c}\\
&= -\frac{\sqrt{a+bx}}{(b-c)x} + \frac{\sqrt{a+cx}}{(b-c)x} + \frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2(b-c)} - \frac{c\int \frac{1}{x\sqrt{a+cx}}\,dx}{2(b-c)}\\
&= -\frac{\sqrt{a+bx}}{(b-c)x} + \frac{\sqrt{a+cx}}{(b-c)x} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b-c}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx}\right)}{b-c}\\
&= -\frac{\sqrt{a+bx}}{(b-c)x} + \frac{\sqrt{a+cx}}{(b-c)x} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)} + \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.17 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$
$$=\frac{-\frac{a}{\sqrt{a+bx}}-\frac{bx}{\sqrt{a+bx}}+\frac{a}{\sqrt{a+cx}}+\frac{cx}{\sqrt{a+cx}}-\frac{bx\sqrt{1+\frac{bx}{a}}\operatorname{arctanh}\left(\sqrt{1+\frac{bx}{a}}\right)}{\sqrt{a+bx}}+\frac{cx\sqrt{1+\frac{cx}{a}}\operatorname{arctanh}\left(\sqrt{1+\frac{cx}{a}}\right)}{\sqrt{a+cx}}}{bx-cx}$$

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] (-(a/Sqrt[a + b*x]) - (b*x)/Sqrt[a + b*x] + a/Sqrt[a + c*x] + (c*x)/Sqrt[a
+ c*x] - (b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[a + b*x] +
 (c*x*Sqrt[1 + (c*x)/a]*ArcTanh[Sqrt[1 + (c*x)/a]])/Sqrt[a + c*x])/(b*x - c
*x)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2b\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{b-c}-\frac{2c\left(-\frac{\sqrt{cx+a}}{2cx}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{b-c}$	88

```
[In] int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b-c)*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
))-2/(b-c)*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1
/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.77

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$
$$=\left[-\frac{\sqrt{a}bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}cx\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}a-2\sqrt{cx+a}a}{2(ab-ac)x},\frac{\sqrt{-a}bx\operatorname{arcta}}{}\right.$$

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*c
*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a)*a - 2*sqr
t(c*x + a)*a)/((a*b - a*c)*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/
a) - sqrt(-a)*c*x*arctan(sqrt(c*x + a)*sqrt(-a)/a) - sqrt(b*x + a)*a + sqrt
(c*x + a)*a)/((a*b - a*c)*x)]
```

Sympy [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx=\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

```
[In] integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx=\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{cx+a}\right)}\,dx$$

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(87) = 174$.

Time = 2.75 (sec) , antiderivative size = 1402, normalized size of antiderivative = 13.61

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx=\text{Too large to display}$$

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
[Out] b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(b - c)) - 2*((sqrt(b*c)*sqrt(b*
x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a*b^2*c*abs(b) - (sqrt(b*c)*s
qrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a*b*c^2*abs(b) + (sqrt(
b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*c*abs(b))/((a^2
*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
+ (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
+ (b*x + a)*b*c - a*b*c))^4)*(b - c)) - sqrt(b*x + a)/((b - c)*x) + (2*(a*b
```

```
^3*c^2 - a*b^2*c^3)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + 2*(
a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5*c - 3
*a^2*b^4*c^2 + 3*a^2*b^3*c^3 - a^2*b^2*c^4)*sqrt(-a*b*c)*abs(-a*b^2 + a*b*c
)*abs(b)*sgn(-2*b + 2*c) + (a^2*b^6*c - 3*a^2*b^5*c^2 + 3*a^2*b^4*c^3 - a^2
*b^3*c^4)*sqrt(-a)*abs(-a*b^2 + a*b*c)*abs(b) + (a^3*b^7*c^2 - 2*a^3*b^6*c^
3 + 2*a^3*b^4*c^5 - a^3*b^3*c^6)*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + (a^3*b^7
*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a*b*c)*abs(b))*arct
an(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(
a*b^3 - a*b*c^2 + sqrt((a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2
*b^3*c^2 - a^2*b^2*c^3)*(b - c)))/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 -
10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^3*abs(-a*b^2 + a*b*c)) - (2*(a*b^3*c^2
- a*b^2*c^3)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + 2*(a*b^3*c
 - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5*c - 3*a^2*b^
4*c^2 + 3*a^2*b^3*c^3 - a^2*b^2*c^4)*sqrt(-a*b*c)*abs(-a*b^2 + a*b*c)*abs(b
)*sgn(-2*b + 2*c) + (a^2*b^6*c - 3*a^2*b^5*c^2 + 3*a^2*b^4*c^3 - a^2*b^3*c^
4)*sqrt(-a)*abs(-a*b^2 + a*b*c)*abs(b) + (a^3*b^7*c^2 - 2*a^3*b^6*c^3 + 2*a
^3*b^4*c^5 - a^3*b^3*c^6)*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + (a^3*b^7*c - 2*
a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a*b*c)*abs(b))*arctan(-(sq
rt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^3 -
 a*b*c^2 - sqrt((a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^
2 - a^2*b^2*c^3)*(b - c)))/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*
c^3 + 5*b^4*c^4 - b^3*c^5)*a^3*abs(-a*b^2 + a*b*c))
```

Mupad [B] (verification not implemented)

Time = 24.65 (sec) , antiderivative size = 1637, normalized size of antiderivative = 15.89

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \text{Too large to display}$$

```
[In] int(1/(x*((a + b*x)^(1/2) + (a + c*x)^(1/2))),x)
```

```
[Out] (2*a*b - 2*a*c + a*c*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(
1/2))) - 2*a^(1/2)*b*(a + c*x)^(1/2) + 2*a^(1/2)*c*(a + b*x)^(1/2) + a*b*at
an((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1
i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a +
 b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(
1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*2i - a*c*atan((b^3*(a + b*x)^(1/2)*1i
- b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2
)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a + c*x)
^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x
)^(1/2)))*2i + a*b*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/
2))) + b*atan((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b
*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i
)/(b^3*(a + b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2
```

)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*(a + c*x)^(
1/2)*2i - c*atan((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a
+ b*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)
1i)/(b^3(a + b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(
1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*(a + c*x
)^(1/2)*2i + b*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))
*(a + b*x)^(1/2)*(a + c*x)^(1/2) + c*log(((a + b*x)^(1/2) - a^(1/2))/((a +
c*x)^(1/2) - a^(1/2)))*(a + b*x)^(1/2)*(a + c*x)^(1/2) - a^(1/2)*b*atan((b^
3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^
(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^
(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b
*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*2i - a^(1/2)*b*atan((b^3*(a
+ b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)
*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2)
 - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2
+ b*c^2*(a + c*x)^(1/2)))*(a + c*x)^(1/2)*2i + a^(1/2)*c*atan((b^3*(a + b*x
)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*
1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^
3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c
^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*2i + a^(1/2)*c*atan((b^3*(a + b*x)^(1/
2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*1i +
a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a
+ c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a
 + c*x)^(1/2)))*(a + c*x)^(1/2)*2i - a^(1/2)*b*log(((a + b*x)^(1/2) - a^(1/
2))/((a + c*x)^(1/2) - a^(1/2)))*(a + b*x)^(1/2) - a^(1/2)*b*log(((a + b*x)
^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(a + c*x)^(1/2) - a^(1/2)*c*
log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(a + b*x)^(1/2
) - a^(1/2)*c*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*
(a + c*x)^(1/2))/(2*a^(1/2)*(b - c)*((a + b*x)^(1/2) - a^(1/2))*((a + c*x)^
(1/2) - a^(1/2)))

3.431 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = -\frac{\sqrt{a+bx}}{2(b-c)x^2}-\frac{b\sqrt{a+bx}}{4a(b-c)x}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{c\sqrt{a+cx}}{4a(b-c)x}+\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2\operatorname{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}$$

```
[Out] 1/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)/(b-c)-1/4*c^2*arctanh((c*x+a
)^(1/2)/a^(1/2))/a^(3/2)/(b-c)-1/2*(b*x+a)^(1/2)/(b-c)/x^2-1/4*b*(b*x+a)^(1
/2)/a/(b-c)/x+1/2*(c*x+a)^(1/2)/(b-c)/x^2+1/4*c*(c*x+a)^(1/2)/a/(b-c)/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2128, 43, 44, 65, 214}

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2\operatorname{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{\sqrt{a+bx}}{2x^2(b-c)}+\frac{\sqrt{a+cx}}{2x^2(b-c)}-\frac{b\sqrt{a+bx}}{4ax(b-c)}+\frac{c\sqrt{a+cx}}{4ax(b-c)}$$

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] -1/2*Sqrt[a + b*x]/((b - c)*x^2) - (b*Sqrt[a + b*x])/(4*a*(b - c)*x) + Sqrt
[a + c*x]/(2*(b - c)*x^2) + (c*Sqrt[a + c*x])/(4*a*(b - c)*x) + (b^2*ArcTan
```

```
h[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(3/2)*(b - c)) - (c^2*ArcTanh[Sqrt[a + c*x]/
Sqrt[a]])/(4*a^(3/2)*(b - c))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2128

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt{a+bx}}{x^3}\,dx}{b-c} - \frac{\int \frac{\sqrt{a+cx}}{x^3}\,dx}{b-c} \\
&= -\frac{\sqrt{a+bx}}{2(b-c)x^2} + \frac{\sqrt{a+cx}}{2(b-c)x^2} + \frac{b\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4(b-c)} - \frac{c\int \frac{1}{x^2\sqrt{a+cx}}\,dx}{4(b-c)} \\
&= -\frac{\sqrt{a+bx}}{2(b-c)x^2} - \frac{b\sqrt{a+bx}}{4a(b-c)x} + \frac{\sqrt{a+cx}}{2(b-c)x^2} + \frac{c\sqrt{a+cx}}{4a(b-c)x} - \frac{b^2\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a(b-c)} + \frac{c^2\int \frac{1}{x\sqrt{a+cx}}\,dx}{8a(b-c)}
\end{aligned}
$$

$$= -\frac{\sqrt{a+bx}}{2(b-c)x^2} - \frac{b\sqrt{a+bx}}{4a(b-c)x} + \frac{\sqrt{a+cx}}{2(b-c)x^2} + \frac{c\sqrt{a+cx}}{4a(b-c)x} - \frac{b\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{4a(b-c)} + \frac{c\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx}\right)}{4a(b-c)}$$

$$= -\frac{\sqrt{a+bx}}{2(b-c)x^2} - \frac{b\sqrt{a+bx}}{4a(b-c)x} + \frac{\sqrt{a+cx}}{2(b-c)x^2} + \frac{c\sqrt{a+cx}}{4a(b-c)x} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)} - \frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \frac{-2b^2(a+bx)^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{2},3,\frac{5}{2},1+\frac{bx}{a}\right)+2c^2(a+cx)^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{2},3,\frac{5}{2},1+\frac{cx}{a}\right)}{3a^3(b-c)}$$

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
[Out] (-2*b^2*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x)/a] + 2*c^2
*(a + c*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (c*x)/a])/(3*a^3*(b - c
))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{2b^2\left(\frac{-\frac{(bx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx+a}}{8}}{x^2b^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{b-c}-\frac{2c^2\left(\frac{-\frac{(cx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{cx+a}}{8}}{c^2x^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{b-c}$	120

```
[In] int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/(b-c)*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/x^2/b^2+1/8/a^(3/2)*a
rctanh((b*x+a)^(1/2)/a^(1/2)))-2/(b-c)*c^2*((-1/8/a*(c*x+a)^(3/2)-1/8*(c*x+
a)^(1/2))/c^2/x^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$
$$= \left[-\frac{\sqrt{a}b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}c^2x^2\log\left(\frac{cx+2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)+2\left(abx+2a^2\right)\sqrt{bx+a}-2\left(acx+2a^2\right.}{8\left(a^2b-a^2c\right)x^2}\right.$$
$$-\frac{\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{-a}c^2x^2\arctan\left(\frac{\sqrt{cx+a}\sqrt{-a}}{a}\right)+\left(abx+2a^2\right)\sqrt{bx+a}-\left(acx+2a^2\right)\sqrt{cx}}{4\left(a^2b-a^2c\right)x^2}$$

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(
a)*c^2*x^2*log((c*x + 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(a*b*x + 2*a^2)
*sqrt(b*x + a) - 2*(a*c*x + 2*a^2)*sqrt(c*x + a))/((a^2*b - a^2*c)*x^2), -1
/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-a)*c^2*x^2*ar
ctan(sqrt(c*x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a) - (a*c*x + 2
*a^2)*sqrt(c*x + a))/((a^2*b - a^2*c)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{cx+a}\right)}\,dx$$

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1895 vs. $2(139) = 278$.

Time = 6.13 (sec) , antiderivative size = 1895, normalized size of antiderivative = 11.08

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/((a*b - a*c)*sqrt(-a)) - 1/2*((sqrt
(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*b^6*c^2*abs(
b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*
b^5*c^3*abs(b) + 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c -
a*b*c))*a^3*b^4*c^4*abs(b) - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
 a)*b*c - a*b*c))*a^3*b^3*c^5*abs(b) + 7*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*
b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^4*c^2*abs(b) - 10*(sqrt(b*c)*sqrt(b*x
 + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^3*c^3*abs(b) + 3*(sqrt
(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^2*c^4*ab
s(b) + 7*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^5*
a*b^2*c^2*abs(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c
- a*b*c))^5*a*b*c^3*abs(b) + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
 a)*b*c - a*b*c))^7*c^2*abs(b))/((a^2*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(
sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*
(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (
sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4)^2*(a*b -
a*c)) - 1/4*((b*x + a)^(3/2)*b^2 + sqrt(b*x + a)*a*b^2)/((a*b - a*c)*b^2*x^
2) - 1/4*(2*(a*b^3*c^3 - a*b^2*c^4)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a)*abs(b)*s
gn(8*a*b - 8*a*c) + 2*(a*b^3*c^2 - a*b^2*c^3)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a
*b*c)*abs(b) + (a^3*b^5*c^2 - 3*a^3*b^4*c^3 + 3*a^3*b^3*c^4 - a^3*b^2*c^5)*
sqrt(-a*b*c)*abs(a^2*b^2 - a^2*b*c)*abs(b)*sgn(8*a*b - 8*a*c) + (a^3*b^6*c^
2 - 3*a^3*b^5*c^3 + 3*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a)*abs(a^2*b^2 - a^2
*b*c)*abs(b) + (a^5*b^7*c^3 - 2*a^5*b^6*c^4 + 2*a^5*b^4*c^6 - a^5*b^3*c^7)*
sqrt(-a)*abs(b)*sgn(8*a*b - 8*a*c) + (a^5*b^7*c^2 - 2*a^5*b^6*c^3 + 2*a^5*b
^4*c^5 - a^5*b^3*c^6)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a)
 - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a^2*b^3 - a^2*b*c^2 + sqrt((
a^2*b^3 - a^2*b*c^2)^2 - (a^3*b^5 - 3*a^3*b^4*c + 3*a^3*b^3*c^2 - a^3*b^2*c
^3)*(a*b - a*c)))/(a*b - a*c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 +
 5*b^4*c^4 - b^3*c^5)*a^5*abs(a^2*b^2 - a^2*b*c)) + 1/4*(2*(a*b^3*c^3 - a*b
^2*c^4)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a)*abs(b)*sgn(8*a*b - 8*a*c) + 2*(a*b^3
*c^2 - a*b^2*c^3)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^3*b^5*c^2
- 3*a^3*b^4*c^3 + 3*a^3*b^3*c^4 - a^3*b^2*c^5)*sqrt(-a*b*c)*abs(a^2*b^2 - a
^2*b*c)*abs(b)*sgn(8*a*b - 8*a*c) + (a^3*b^6*c^2 - 3*a^3*b^5*c^3 + 3*a^3*b^
4*c^4 - a^3*b^3*c^5)*sqrt(-a)*abs(a^2*b^2 - a^2*b*c)*abs(b) + (a^5*b^7*c^3
- 2*a^5*b^6*c^4 + 2*a^5*b^4*c^6 - a^5*b^3*c^7)*sqrt(-a)*abs(b)*sgn(8*a*b -
```

```
8*a*c) + (a^5*b^7*c^2 - 2*a^5*b^6*c^3 + 2*a^5*b^4*c^5 - a^5*b^3*c^6)*sqrt(-
a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))/sqrt(-(a^2*b^3 - a^2*b*c^2 - sqrt((a^2*b^3 - a^2*b*c^2)^2 - (a^
3*b^5 - 3*a^3*b^4*c + 3*a^3*b^3*c^2 - a^3*b^2*c^3)*(a*b - a*c)))/(a*b - a*c
)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^5*ab
s(a^2*b^2 - a^2*b*c))
```

Mupad [B] (verification not implemented)

Time = 26.55 (sec) , antiderivative size = 1610, normalized size of antiderivative = 9.42

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (a + c*x)^(1/2))),x)
```

```
[Out] ((a^(3/2)*b^3)/(16*(a^3*c^2 - a^3*b*c)) + (a^(3/2)*((a + b*x)^(1/2) - a^(1/
2))^2*((b*c^2)/4 - (7*b^2*c)/16 + b^3/4))/((a^3*c^2 - a^3*b*c)*((a + c*x)^(
1/2) - a^(1/2))^2) - (a^(3/2)*((b^2*c)/16 + b^3/16)*((a + b*x)^(1/2) - a^(1
/2)))/((a^3*c^2 - a^3*b*c)*((a + c*x)^(1/2) - a^(1/2))) + ((b^2/8 - c^2/8)*
((a + b*x)^(1/2) - a^(1/2))^3)/(a^(3/2)*c*((a + c*x)^(1/2) - a^(1/2))^3))/(
((a + b*x)^(1/2) - a^(1/2))^4/((a + c*x)^(1/2) - a^(1/2))^4 - ((b + c)*((a
+ b*x)^(1/2) - a^(1/2))^3)/(c*((a + c*x)^(1/2) - a^(1/2))^3) + (b*((a + b*x
)^(1/2) - a^(1/2))^2)/(c*((a + c*x)^(1/2) - a^(1/2))^2)) - (((c*(b + c))/(4
*a^(3/2)*(b - c)) - (c*(b^2 - c^2))/(4*a^(3/2)*(b - c)^2))*((a + b*x)^(1/2)
 - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (log(((a + b*x)^(1/2) - a^(1/2))
/((a + c*x)^(1/2) - a^(1/2)))*(a^(3/2)*b^2 + a^(3/2)*c^2))/(8*a^3*b - 8*a^3
*c) + (atan((((b + c)*(((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3 -
 a^6*b*c^2)) - (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3 - 64*a^6*c^3 + 128*
a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a^
(1/2)))))/(8*a^3) - (16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2))
+ ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6*
b*c^2)*((a + c*x)^(1/2) - a^(1/2))))*1i)/(8*a^3) - ((b + c)*((16*a^3*b^4 +
16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2)) + ((b + c)*((64*a^6*b^3 - 64*a^6*b
*c^2)/(64*(a^6*c^3 - a^6*b*c^2)) - (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3
 - 64*a^6*c^3 + 128*a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*(
(a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) - ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)
^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a^(1/2))))*
1i)/(8*a^3))/(((b + c)*(((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3
- a^6*b*c^2)) - (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3 - 64*a^6*c^3 + 128
*a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a
^(1/2)))))/(8*a^3) - (16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2))
 + ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6
*b*c^2)*((a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) - (b*c^4 - b^5)/(32*(a^6*c^3
 - a^6*b*c^2)) + ((b + c)*((16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b
```

```
*c^2)) + ((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3 - a^6*b*c^2)) -
 (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3 - 64*a^6*c^3 + 128*a^6*b*c^2 - 12
8*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a^(1/2)))))/(8*a
^3) - ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 -
a^6*b*c^2)*((a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) + (((a + b*x)^(1/2) - a^(
1/2))*(b*c^4 - b^4*c + b^2*c^3 - b^3*c^2))/(16*(a^6*c^3 - a^6*b*c^2)*((a +
c*x)^(1/2) - a^(1/2)))))*(a^(3/2)*b + a^(3/2)*c)*1i)/(4*a^3) + (c^2*((a + b
*x)^(1/2) - a^(1/2))^2)/(16*a^(3/2)*(b - c)*((a + c*x)^(1/2) - a^(1/2))^2)
```

3.432 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 195

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} dx = \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2(b-c)c^2} + \frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c} - \frac{a^3(b+c)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}$$

```
[Out] a*x^2/(b-c)^2+1/3*(b+c)*x^3/(b-c)^2-2/3*(b*x+a)^(3/2)*(c*x+a)^(3/2)/b/(b-c)
^2/c-1/4*a^3*(b+c)*arctanh(c^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(c*x+a)^(1/2))/b^(
5/2)/c^(5/2)+1/2*a*(b+c)*(b*x+a)^(3/2)*(c*x+a)^(1/2)/b^2/(b-c)^2/c+1/4*a^2*
(b+c)*(b*x+a)^(1/2)*(c*x+a)^(1/2)/b^2/(b-c)/c^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6822, 81, 52, 65, 223, 212}

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} dx = -\frac{a^3(b+c)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2c^2(b-c)} + \frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2c(b-c)^2}+\frac{ax^2}{(b-c)^2} - \frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3bc(b-c)^2}+\frac{x^3(b+c)}{3(b-c)^2}$$

```
[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (a*x^2)/(b - c)^2 + ((b + c)*x^3)/(3*(b - c)^2) + (a^2*(b + c)*Sqrt[a + b*x
]*Sqrt[a + c*x])/(4*b^2*(b - c)*c^2) + (a*(b + c)*(a + b*x)^(3/2)*Sqrt[a +
c*x])/(2*b^2*(b - c)^2*c) - (2*(a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*b*(b - c
)^2*c) - (a^3*(b + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x
])])/(4*b^(5/2)*c^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
```

```
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \left(2ax+b\left(1+\frac{c}{b}\right)x^2-2x\sqrt{a+bx}\sqrt{a+cx}\right)\,dx}{(b-c)^2}\\
&= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}-\frac{2\int x\sqrt{a+bx}\sqrt{a+cx}\,dx}{(b-c)^2}\\
&= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c}+\frac{(a(b+c))\int\sqrt{a+bx}\sqrt{a+cx}\,dx}{b(b-c)^2c}\\
&= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}\\
&\quad -\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c}+\frac{\left(a^2(b+c)\right)\int\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\,dx}{4b^2(b-c)c}\\
&= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}\\
&\quad -\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c}-\frac{\left(a^3(b+c)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{a+cx}}\,dx}{8b^2c^2}\\
&= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}\\
&\quad -\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c}-\frac{\left(a^3(b+c)\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a-\frac{ac}{b}+\frac{cx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{4b^3c^2}\\
&= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}\\
&\quad -\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c}-\frac{\left(a^3(b+c)\right)\operatorname{Subst}\left(\int\frac{1}{1-\frac{cx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4b^3c^2}\\
&= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}\\
&\quad -\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c}-\frac{a^3(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

$$= \frac{\frac{c\sqrt{a+bx}\sqrt{a+cx}\left(a^2(3b^2-2bc+3c^2)-2abc(b+c)x-8b^2c^2x^2\right)}{b^2(b-c)^2}+\frac{4\left(a^3(b-2c)+3ac^3x^2+c^3(b+c)x^3\right)}{(b-c)^2}+\frac{6a^3\sqrt{c}(b+c)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{a+cx}}{\sqrt{c}\left(\sqrt{a-\frac{ab}{c}}-\sqrt{a+\cdots}\right.}\right.}{b^{5/2}}}{12c^3}$$

```
[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] ((c*Sqrt[a + b*x]*Sqrt[a + c*x]*(a^2*(3*b^2 - 2*b*c + 3*c^2) - 2*a*b*c*(b +
 c)*x - 8*b^2*c^2*x^2))/(b^2*(b - c)^2) + (4*(a^3*(b - 2*c) + 3*a*c^3*x^2 +
 c^3*(b + c)*x^3))/(b - c)^2 + (6*a^3*Sqrt[c]*(b + c)*ArcTanh[(Sqrt[b]*Sqrt
[a + c*x])/(Sqrt[c]*(Sqrt[a - (a*b)/c] - Sqrt[a + b*x]))])/b^(5/2))/(12*c^3
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(161) = 322.

Time = 0.04 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.65

method	result
default	$\frac{b\,x^3}{3(b-c)^2}+\frac{c\,x^3}{3(b-c)^2}+\frac{a\,x^2}{(b-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(16x^2b^2c^2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}+3\ln\left(\frac{2bcx+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}+ab+\cdots}{2\sqrt{bc}}\right.\right.}{\cdots}$

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(b-c)^2*b*x^3+1/3/(b-c)^2*c*x^3+a*x^2/(b-c)^2-1/24/(b-c)^2*(b*x+a)^(1/2
)*(c*x+a)^(1/2)*(16*x^2*b^2*c^2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)
+3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(
b*c)^(1/2))*a^3*b^3-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*
c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b^2*c-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x
+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b*c^2+3*ln(1/2*(2*b
*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^
3*c^3+4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*x*a*b^2*c+4*(b*c*x^2+a*
b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*x*a*b*c^2-6*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2
)*(b*c)^(1/2)*a^2*b^2+4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*a^2*b*c
-6*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*a^2*c^2)/(b*c*x^2+a*b*x+a*c*
x+a^2)^(1/2)/b^2/c^2/(b*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.46

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \left[\frac{24\,ab^3c^3x^2 + 8\,(b^4c^3 + b^3c^4)x^3 + 3\,(a^3b^3 - a^3b^2c - a^3bc^2 + a^3c^3)\sqrt{bc}\log\left(ab^2 + 2\,abc + ac^2 + 2\left(2\,bc - \sqrt{b\right.\right.}{}\right.$$

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [1/24*(24*a*b^3*c^3*x^2 + 8*(b^4*c^3 + b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2*
c - a^3*b*c^2 + a^3*c^3)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c -
 sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x - 2*(
2*b*c*x + a*b + a*c)*sqrt(b*c)) - 2*(8*b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^2*b^
2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3*c^2 + a*b^2*c^3)*x)*sqrt(b*x + a)*sqrt(c*x +
 a))/(b^5*c^3 - 2*b^4*c^4 + b^3*c^5), 1/12*(12*a*b^3*c^3*x^2 + 4*(b^4*c^3 +
 b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2*c - a^3*b*c^2 + a^3*c^3)*sqrt(-b*c)*ar
ctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) - (8*
b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^2*b^2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3*c^2 + a*
b^2*c^3)*x)*sqrt(b*x + a)*sqrt(c*x + a))/(b^5*c^3 - 2*b^4*c^4 + b^3*c^5)]
```

Sympy [F]

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{x^3}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(161) = 322$.

Time = 0.80 (sec) , antiderivative size = 511, normalized size of antiderivative = 2.62

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{1}{12}\sqrt{ab^2+(bx+a)bc-abc}\left(2\,(bx+a)\left(\frac{4\,(b^{11}c^4|b|-3\,b^{10}c^5|b|+3\,b^9c^6|b|-b^8c^7|b|)(bx+a)}{b^{17}c^4-5\,b^{16}c^5+10\,b^{15}c^6-10\,b^{14}c^7+5\,b^{13}c^8-b^{12}c^9}+\frac{ab^{12}c^3}{b}\right.\right.$$
$$+\frac{(bx+a)^3b-3\,(bx+a)a^2b+(bx+a)^3c-3\,(bx+a)^2ac+3\,(bx+a)a^2c}{3\,(b^5-2\,b^4c+b^3c^2)}$$
$$+\frac{(a^3b|b|+a^3c|b|)\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{ab^2+(bx+a)bc-abc}\right|\right)}{4\sqrt{bc}b^3c^2}$$

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/12*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(2*(b*x + a)*(4*(b^11*c^4*abs(b)
- 3*b^10*c^5*abs(b) + 3*b^9*c^6*abs(b) - b^8*c^7*abs(b))*(b*x + a)/(b^17*c^
4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14*c^7 + 5*b^13*c^8 - b^12*c^9) + (a*b^
12*c^3*abs(b) - 10*a*b^11*c^4*abs(b) + 24*a*b^10*c^5*abs(b) - 22*a*b^9*c^6*
abs(b) + 7*a*b^8*c^7*abs(b))/(b^17*c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14
*c^7 + 5*b^13*c^8 - b^12*c^9)) - 3*(a^2*b^13*c^2*abs(b) - 3*a^2*b^12*c^3*ab
s(b) + 2*a^2*b^11*c^4*abs(b) + 2*a^2*b^10*c^5*abs(b) - 3*a^2*b^9*c^6*abs(b)
 + a^2*b^8*c^7*abs(b))/(b^17*c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14*c^7 +
 5*b^13*c^8 - b^12*c^9))*sqrt(b*x + a) + 1/3*((b*x + a)^3*b - 3*(b*x + a)*a
^2*b + (b*x + a)^3*c - 3*(b*x + a)^2*a*c + 3*(b*x + a)*a^2*c)/(b^5 - 2*b^4*
c + b^3*c^2) + 1/4*(a^3*b*abs(b) + a^3*c*abs(b))*log(abs(-sqrt(b*c)*sqrt(b*
x + a) + sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)))/(sqrt(b*c)*b^3*c^2)
```

Mupad [B] (verification not implemented)

Time = 45.03 (sec) , antiderivative size = 1107, normalized size of antiderivative = 5.68

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \frac{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6\left(128\,a^3\,b^3\,c+\frac{1312\,a^3\,b^2\,c^2}{3}+128\,a^3\,b\,c^3\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^6}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^7\left(19\,a^3\,b^3\,c+269\,a^3\,b^2\,c^2+269\,a^3\,b\,c^3+19\,a^3\,c^4\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^7}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5}{}}{}$$
$$+\frac{x^3\,(b+c)}{3\,(b-c)^2}+\frac{a\,x^2}{(b-c)^2}-\frac{a^3\,\mathrm{atanh}\left(\frac{\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{a+c\,x}-\sqrt{a}\right)}\right)\,(b+c)}{2\,b^{5/2}\,c^{5/2}}$$

```
[In] int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)

[Out] ((((a + b*x)^(1/2) - a^(1/2))^6*(128*a^3*b*c^3 + 128*a^3*b^3*c + (1312*a^3*
b^2*c^2)/3))/((a + c*x)^(1/2) - a^(1/2))^6 - (((a + b*x)^(1/2) - a^(1/2))^7
*(19*a^3*c^4 + 269*a^3*b*c^3 + 19*a^3*b^3*c + 269*a^3*b^2*c^2))/((a + c*x)^
(1/2) - a^(1/2))^7 - (((a + b*x)^(1/2) - a^(1/2))^5*(19*a^3*b^4 + 19*a^3*b*
c^3 + 269*a^3*b^3*c + 269*a^3*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^5 + (((
a + b*x)^(1/2) - a^(1/2))^4*(64*a^3*b^4 + 192*a^3*b^3*c + 64*a^3*b^2*c^2))/
((a + c*x)^(1/2) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^8*(64*a^3*c^4
+ 192*a^3*b*c^3 + 64*a^3*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^8 + (16*a^3*
b^4*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 + (16*a^3*
c^4*((a + b*x)^(1/2) - a^(1/2))^10)/((a + c*x)^(1/2) - a^(1/2))^10 + (((a +
 b*x)^(1/2) - a^(1/2))^11*(a^3*c^6 - a^3*b*c^5 - a^3*b^2*c^4 + a^3*b^3*c^3)
)/(2*b^2*((a + c*x)^(1/2) - a^(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^3*(
17*a^3*b^5 + 303*a^3*b^4*c + 17*a^3*b^2*c^3 + 303*a^3*b^3*c^2))/(6*c*((a +
c*x)^(1/2) - a^(1/2))^3) - (((a + b*x)^(1/2) - a^(1/2))^9*(17*a^3*c^5 + 303
*a^3*b*c^4 + 303*a^3*b^2*c^3 + 17*a^3*b^3*c^2))/(6*b*((a + c*x)^(1/2) - a^(
1/2))^9) + ((a^3*b + a^3*c)*((a + b*x)^(1/2) - a^(1/2))*(b^5 - 2*b^4*c + b^
3*c^2))/(2*c^2*((a + c*x)^(1/2) - a^(1/2))))/(b^8 - 2*b^7*c + b^6*c^2 + (((
a + b*x)^(1/2) - a^(1/2))^12*(c^8 - 2*b*c^7 + b^2*c^6))/((a + c*x)^(1/2) -
a^(1/2))^12 - (((a + b*x)^(1/2) - a^(1/2))^2*(6*b^7*c + 6*b^5*c^3 - 12*b^6*
c^2))/((a + c*x)^(1/2) - a^(1/2))^2 - (((a + b*x)^(1/2) - a^(1/2))^10*(6*b*
c^7 - 12*b^2*c^6 + 6*b^3*c^5))/((a + c*x)^(1/2) - a^(1/2))^10 + (((a + b*x)
^(1/2) - a^(1/2))^4*(15*b^4*c^4 - 30*b^5*c^3 + 15*b^6*c^2))/((a + c*x)^(1/2
) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^8*(15*b^2*c^6 - 30*b^3*c^5 +
15*b^4*c^4))/((a + c*x)^(1/2) - a^(1/2))^8 - (((a + b*x)^(1/2) - a^(1/2))^6
*(20*b^3*c^5 - 40*b^4*c^4 + 20*b^5*c^3))/((a + c*x)^(1/2) - a^(1/2))^6) + (
x^3*(b + c))/(3*(b - c)^2) + (a*x^2)/(b - c)^2 - (a^3*atanh((c^(1/2)*((a +
b*x)^(1/2) - a^(1/2)))/(b^(1/2)*((a + c*x)^(1/2) - a^(1/2))))*(b + c))/(2*b
^(5/2)*c^(5/2))
```

3.433 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 142

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}$$

```
[Out] 2*a*x/(b-c)^2+1/2*(b+c)*x^2/(b-c)^2+1/2*a^2*arctanh(c^(1/2)*(b*x+a)^(1/2)/b
^(1/2)/(c*x+a)^(1/2))/b^(3/2)/c^(3/2)-(b*x+a)^(3/2)*(c*x+a)^(1/2)/b/(b-c)^2
-1/2*a*(b*x+a)^(1/2)*(c*x+a)^(1/2)/b/(b-c)/c
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6822, 52, 65, 223, 212}

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}+\frac{2ax}{(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2bc(b-c)}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{x^2(b+c)}{2(b-c)^2}$$

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (2*a*x)/(b - c)^2 + ((b + c)*x^2)/(2*(b - c)^2) - (a*Sqrt[a + b*x]*Sqrt[a +
 c*x])/(2*b*(b - c)*c) - ((a + b*x)^(3/2)*Sqrt[a + c*x])/(b*(b - c)^2) + (a
```

```
^2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(2*b^(3/2)*c^(
3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \left(2a+b\left(1+\frac{c}{b}\right)x-2\sqrt{a+bx}\sqrt{a+cx}\right)\,dx}{(b-c)^2}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{2\int \sqrt{a+bx}\sqrt{a+cx}\,dx}{(b-c)^2}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}-\frac{a\int \frac{\sqrt{a+bx}}{\sqrt{a+cx}}\,dx}{2b(b-c)}$$

$$= \frac{2ax}{(b-c)^2} + \frac{(b+c)x^2}{2(b-c)^2} - \frac{a\sqrt{a+bx}\sqrt{a+cx}}{2b(b-c)c} - \frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2} + \frac{a^2\int \frac{1}{\sqrt{a+bx}\sqrt{a+cx}}\,dx}{4bc}$$

$$= \frac{2ax}{(b-c)^2} + \frac{(b+c)x^2}{2(b-c)^2} - \frac{a\sqrt{a+bx}\sqrt{a+cx}}{2b(b-c)c} - \frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2} + \frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{ac}{b}+\frac{cx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{2b^2c}$$

$$= \frac{2ax}{(b-c)^2} + \frac{(b+c)x^2}{2(b-c)^2} - \frac{a\sqrt{a+bx}\sqrt{a+cx}}{2b(b-c)c} - \frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2} + \frac{a^2\text{Subst}\left(\int \frac{1}{1-\frac{cx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2b^2c}$$

$$= \frac{2ax}{(b-c)^2} + \frac{(b+c)x^2}{2(b-c)^2} - \frac{a\sqrt{a+bx}\sqrt{a+cx}}{2b(b-c)c} - \frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2} + \frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{a^2b(b-3c) - bc^2x\left(bx+cx-2\sqrt{a+bx}\sqrt{a+cx}\right) + ac\big(-4bcx + b\sqrt{a+bx}\sqrt{a+cx} + c\sqrt{a+bx}\sqrt{a+c}}{2b(b-c)^2c^2} - \frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{a+cx}}{\sqrt{c}\left(\sqrt{a-\frac{ab}{c}}-\sqrt{a+bx}\right)}\right)}{b^{3/2}c^{3/2}}$$

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] -1/2*(a^2*b*(b - 3*c) - b*c^2*x*(b*x + c*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x])
 + a*c*(-4*b*c*x + b*Sqrt[a + b*x]*Sqrt[a + c*x] + c*Sqrt[a + b*x]*Sqrt[a +
 c*x]))/(b*(b - c)^2*c^2) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[a + c*x])/(Sqrt[c]*(
Sqrt[a - (a*b)/c] - Sqrt[a + b*x]))])/(b^(3/2)*c^(3/2))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.32

method	result
default	$\frac{x^2 b}{2(b-c)^2}+\frac{x^2 c}{2(b-c)^2}+\frac{2ax}{(b-c)^2}-\frac{2\left(\frac{\sqrt{bx+a}\,(cx+a)^{\frac{3}{2}}}{2c}-\frac{(ab-ac)\left(\frac{\sqrt{cx+a}\,\sqrt{bx+a}}{b}-\frac{(-ab+ac)\sqrt{(bx+a)(cx+a)}\,\ln\left(\frac{\frac{1}{2}ab+\frac{1}{2}ac+bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+}\right.}{2b\sqrt{cx+a}\,\sqrt{bx+a}\,\sqrt{bc}}\right.}{4c}\right.}{(b-c)^2}$

```
[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/(b-c)^2*b+1/2*x^2/(b-c)^2*c+2*a*x/(b-c)^2-2/(b-c)^2*(1/2/c*(b*x+a)^
(1/2)*(c*x+a)^(3/2)-1/4*(a*b-a*c)/c*(1/b*(c*x+a)^(1/2)*(b*x+a)^(1/2)-1/2*(-
a*b+a*c)/b*((b*x+a)*(c*x+a))^(1/2)/(c*x+a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+
1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b+a*c)*x+a^2)^(1/2))/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.62

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$=\left[\frac{8\,ab^2c^2x+2\,(b^3c^2+b^2c^3)x^2+(a^2b^2-2\,a^2bc+a^2c^2)\sqrt{bc}\log\left(ab^2+2\,abc+ac^2+2\left(2\,bc+\sqrt{bc}(b+c)\right)\sqrt{\right.}{4\,(b^4c^2-2\,b}\right.$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(8*a*b^2*c^2*x + 2*(b^3*c^2 + b^2*c^3)*x^2 + (a^2*b^2 - 2*a^2*b*c + a^
2*c^2)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c + sqrt(b*c)*(b + c)
)*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x + 2*(2*b*c*x + a*b + a*
c)*sqrt(b*c)) - 2*(2*b^2*c^2*x + a*b^2*c + a*b*c^2)*sqrt(b*x + a)*sqrt(c*x
+ a))/(b^4*c^2 - 2*b^3*c^3 + b^2*c^4), 1/2*(4*a*b^2*c^2*x + (b^3*c^2 + b^2*
c^3)*x^2 - (a^2*b^2 - 2*a^2*b*c + a^2*c^2)*sqrt(-b*c)*arctan((sqrt(-b*c)*sq
rt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) - (2*b^2*c^2*x + a*b^2*c
 + a*b*c^2)*sqrt(b*x + a)*sqrt(c*x + a))/(b^4*c^2 - 2*b^3*c^3 + b^2*c^4)]
```

Sympy [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)

[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)

Maxima [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")

[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(116) = 232.

Time = 0.83 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.92

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{1}{2}\sqrt{ab^2+(bx+a)bc-abc}\sqrt{bx+a}\left(\frac{2\left(b^4c^2|b|-b^3c^3|b|\right)(bx+a)}{b^9c^2-3\,b^8c^3+3\,b^7c^4-b^6c^5}+\frac{ab^5c|b|-2\,ab^4c^2|b|+ab^3c^3|b|}{b^9c^2-3\,b^8c^3+3\,b^7c^4-b^6c^5}\right) - \frac{a^2|b|\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{ab^2+(bx+a)bc-abc}\right|\right)}{2\sqrt{bc}b^2c} + \frac{(bx+a)^2b+2\,(bx+a)ab+(bx+a)^2c-2\,(bx+a)ac}{2\left(b^4-2\,b^3c+b^2c^2\right)}$$

[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")

[Out] -1/2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*sqrt(b*x + a)*(2*(b^4*c^2*abs(b) - b^3*c^3*abs(b))*(b*x + a)/(b^9*c^2 - 3*b^8*c^3 + 3*b^7*c^4 - b^6*c^5) + (a*b^5*c*abs(b) - 2*a*b^4*c^2*abs(b) + a*b^3*c^3*abs(b))/(b^9*c^2 - 3*b^8*c^3 + 3*b^7*c^4 - b^6*c^5)) - 1/2*a^2*abs(b)*log(abs(-sqrt(b*c)*sqrt(b*x + a) + sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)))/(sqrt(b*c)*b^2*c) + 1/2*((b*x + a)^2*b + 2*(b*x + a)*a*b + (b*x + a)^2*c - 2*(b*x + a)*a*c)/(b^4 - 2*b^3*c + b^2*c^2)

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{2\,a\,x}{{\left(b-c\right)}^2}+\frac{x^2\,\left(b+c\right)}{2\,{\left(b-c\right)}^2}-\frac{2\,\left(\frac{x}{2}+\frac{a\,b+a\,c}{4\,b\,c}\right)\,\sqrt{a+b\,x}\,\sqrt{a+c\,x}}{{\left(b-c\right)}^2} + \frac{\ln\left(a\,b+a\,c+2\,b\,c\,x+2\,\sqrt{b}\,\sqrt{c}\,\sqrt{a+b\,x}\,\sqrt{a+c\,x}\right)\,{\left(a\,b-a\,c\right)}^2}{4\,b^{3/2}\,c^{3/2}\,{\left(b-c\right)}^2}$$

```
[In] int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

```
[Out] (2*a*x)/(b - c)^2 + (x^2*(b + c))/(2*(b - c)^2) - (2*(x/2 + (a*b + a*c)/(4*
b*c))*(a + b*x)^(1/2)*(a + c*x)^(1/2))/(b - c)^2 + (log(a*b + a*c + 2*b*c*x
 + 2*b^(1/2)*c^(1/2)*(a + b*x)^(1/2)*(a + c*x)^(1/2))*(a*b - a*c)^2)/(4*b^(
3/2)*c^(3/2)*(b - c)^2)
```

3.434 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} dx = \frac{(b+c)x}{(b-c)^2} - \frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2} + \frac{4a\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2} - \frac{2a(b+c)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}(b-c)^2\sqrt{c}} + \frac{2a\log(x)}{(b-c)^2}$$

```
[Out] (b+c)*x/(b-c)^2+4*a*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/(b-c)^2+2*a*ln(x)/
(b-c)^2-2*a*(b+c)*arctanh(c^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(c*x+a)^(1/2))/(b-c
)^2/b^(1/2)/c^(1/2)-2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/(b-c)^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6822, 103, 163, 65, 223, 212, 95, 214}

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} dx = \frac{4a\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2} - \frac{2a(b+c)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}\sqrt{c}(b-c)^2} - \frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2} + \frac{2a\log(x)}{(b-c)^2} + \frac{x(b+c)}{(b-c)^2}$$

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] ((b + c)*x)/(b - c)^2 - (2*Sqrt[a + b*x]*Sqrt[a + c*x])/(b - c)^2 + (4*a*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (2*a*(b + c)*ArcTanh[(Sqrt[
```

```
c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(Sqrt[b]*(b - c)^2*Sqrt[c]) + (
2*a*Log[x])/(b - c)^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(b\left(1+\frac{c}{b}\right)+\frac{2a}{x}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x}\right)dx}{(b-c)^2}\\
&= \frac{(b+c)x}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\sqrt{a+cx}}{x}\,dx}{(b-c)^2}\\
&= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}+\frac{2\int \frac{-a^2-\frac{1}{2}a(b+c)x}{x\sqrt{a+bx}\sqrt{a+cx}}\,dx}{(b-c)^2}\\
&= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}\\
&\quad-\frac{(2a^2)\int \frac{1}{x\sqrt{a+bx}\sqrt{a+cx}}\,dx}{(b-c)^2}-\frac{(a(b+c))\int \frac{1}{\sqrt{a+bx}\sqrt{a+cx}}\,dx}{(b-c)^2}\\
&= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}\\
&\quad-\frac{(2a(b+c))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{ac}{b}+\frac{cx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b(b-c)^2}\\
&= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}\\
&\quad+\frac{2a\log(x)}{(b-c)^2}-\frac{(2a(b+c))\text{Subst}\left(\int \frac{1}{1-\frac{cx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{b(b-c)^2}\\
&= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}\\
&\quad-\frac{2a(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}(b-c)^2\sqrt{c}}+\frac{2a\log(x)}{(b-c)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.88

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \frac{4a\sqrt{c}(b+c)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{a+cx}}{\sqrt{c}\left(\sqrt{a-\frac{ab}{c}}-\sqrt{a+bx}\right)}\right)+\sqrt{b}\left(a(b+c)+c\left(bx+cx-2\sqrt{a+bx}\sqrt{a+cx}\right)+8ac\text{arctanh}\right.}{\sqrt{b}(b-c)^2c}$$

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (4*a*Sqrt[c]*(b + c)*ArcTanh[(Sqrt[b]*Sqrt[a + c*x])/(Sqrt[c]*(Sqrt[a - (a*
b)/c] - Sqrt[a + b*x]))] + Sqrt[b]*(a*(b + c) + c*(b*x + c*x - 2*Sqrt[a + b
*x]*Sqrt[a + c*x]) + 8*a*c*ArcTanh[(-(a*b) - b*c*x + c*Sqrt[a + c*x]*(Sqrt[
a - (a*b)/c] - Sqrt[a + b*x]))/(a*(b - 2*c) - b*c*x + 2*Sqrt[a - (a*b)/c]*c
*Sqrt[a + b*x] + Sqrt[a - (a*b)/c]*c*Sqrt[a + c*x] - c*Sqrt[a + b*x]*Sqrt[a
 + c*x])]))/(Sqrt[b]*(b - c)^2*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.97

method	result
default	$\frac{xb}{(b-c)^2}+\frac{xc}{(b-c)^2}+\frac{2a\ln(x)}{(b-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(\text{csgn}(a)\ln\left(\frac{2bcx+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}+ab+ac}{2\sqrt{bc}}\right)ab+\text{csgn}(a)\ln\left(\frac{2bcx+2\sqrt{bc\,x^2}\right.\right.}{}$

```
[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/(b-c)^2*b+x/(b-c)^2*c+2*a*ln(x)/(b-c)^2-1/(b-c)^2*(b*x+a)^(1/2)*(c*x+a)^(
1/2)*(csgn(a)*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)
+a*b+a*c)/(b*c)^(1/2))*a*b+csgn(a)*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a
^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a*c+2*csgn(a)*(b*c)^(1/2)*(b*c*
x^2+a*b*x+a*c*x+a^2)^(1/2)-2*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2
)+b*x+c*x+2*a)/x)*(b*c)^(1/2)*a)*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/(b
*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.56

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \left[\frac{2\,abc\log\left(x\right) - 2\,abc\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\sqrt{cx+a}+2\,a}{x}\right) - 2\,\sqrt{bx+a}\sqrt{cx+a}bc + (ab+ac)\sqrt{bc}\log\left(ab^2+2\,\right.}{b^3c-}\right.$$

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*a*b*c*log(x) - 2*a*b*c*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a)
+ 2*a)/x) - 2*sqrt(b*x + a)*sqrt(c*x + a)*b*c + (a*b + a*c)*sqrt(b*c)*log(a
*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c
*x + a) + 2*(b^2*c + b*c^2)*x - 2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b^2*c
 + b*c^2)*x)/(b^3*c - 2*b^2*c^2 + b*c^3), (2*a*b*c*log(x) - 2*a*b*c*log(-((
b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) - 2*sqrt(b*x + a)*sqrt(c
*x + a)*b*c + 2*(a*b + a*c)*sqrt(-b*c)*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqr
t(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) + (b^2*c + b*c^2)*x)/(b^3*c - 2*b^2*c^2
 + b*c^3)]
```

Sympy [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

```
[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

Maxima [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{x}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(115) = 230.

Time = 0.92 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.66

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\frac{a(b+c)|b|\log\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2\right)}{(b^2-2\,bc+c^2)\sqrt{bc}}+\frac{2\,a|b|\log\left(\left|\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2-\left(b^2+bc+2\,\sqrt{bc}b\right)a\right|\right)}{b^2-2\,bc+c^2}-\frac{2\,a|b|\log}{}}{}$$

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] (a*(b + c)*abs(b)*log((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c
 - a*b*c))^2)/((b^2 - 2*b*c + c^2)*sqrt(b*c)) + 2*a*abs(b)*log(abs((sqrt(b*
c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2 - (b^2 + b*c + 2*
sqrt(b*c)*b)*a))/(b^2 - 2*b*c + c^2) - 2*a*abs(b)*log(abs((sqrt(b*c)*sqrt(b
*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2 - (b^2 + b*c - 2*sqrt(b*c)
*b)*a))/(b^2 - 2*b*c + c^2) + 2*a*b*log(abs(b*x))/(b^2 - 2*b*c + c^2) - 2*s
qrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(b^2*abs(b) - 2*b*c*abs(b) + c^2*abs(b))
*sqrt(b*x + a)/(b^5 - 4*b^4*c + 6*b^3*c^2 - 4*b^2*c^3 + b*c^4) + ((b*x + a)
*b + (b*x + a)*c)/(b^2 - 2*b*c + c^2))/b
```

Mupad [B] (verification not implemented)

Time = 34.87 (sec) , antiderivative size = 5098, normalized size of antiderivative = 37.76

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

```
[Out] (2*a*log(x))/(b^2 - 2*b*c + c^2) - (((4*a*c^2 + 4*a*b*c)*((a + b*x)^(1/2) -
 a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + ((4*a*b^2 + 4*a*b*c)*((a + b*x
)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (16*a*b*c*((a + b*x)^(1/2
) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2)/(b^4 - 2*b^3*c + b^2*c^2 - (
((a + b*x)^(1/2) - a^(1/2))^2*(2*b*c^3 + 2*b^3*c - 4*b^2*c^2))/((a + c*x)^(
1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^4*(c^4 - 2*b*c^3 + b^2*c^2
))/((a + c*x)^(1/2) - a^(1/2))^4) - (2*a*log((((a + b*x)^(1/2) - (a + c*x)^
(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))/(
(a + c*x)^(1/2) - a^(1/2))))/(b^2 - 2*b*c + c^2) + (2*a*log(((a + b*x)^(1/2
) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2))))/(b - c)^2 + (x*(b + c))/(b - c)^
2 + (a*atan(((a*(b*c)^(1/2)*(b + c)*((2*((a + b*x)^(1/2) - a^(1/2))*(32*a^3
*b^2*c^10 - 64*a^3*b^3*c^9 + 8*a^3*b^4*c^8 + 240*a^3*b^5*c^7 + 8*a^3*b^6*c^
```

6 - 64*a^3*b^7*c^5 + 32*a^3*b^8*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4
*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) - (4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 +
44*a^3*b^6*c^6 + 4*a^3*b^7*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^
2) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*a^2*b^3*c^11 + 2*a^2*b^4*c^10 - 18*a^2
*b^5*c^9 + 12*a^2*b^6*c^8 + 12*a^2*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5
+ 4*a^2*b^10*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a +
b*x)^(1/2) - a^(1/2))*(16*a^2*b^2*c^12 - 32*a^2*b^3*c^11 + 36*a^2*b^4*c^10
- 64*a^2*b^5*c^9 + 88*a^2*b^6*c^8 - 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^
2*b^9*c^5 + 16*a^2*b^10*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c -
4*b*c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((4*(a*b^4*c^12 + 7
*a*b^5*c^11 - 27*a*b^6*c^10 + 19*a*b^7*c^9 + 19*a*b^8*c^8 - 27*a*b^9*c^7 +
7*a*b^10*c^6 + a*b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (
2*((a + b*x)^(1/2) - a^(1/2))*(8*a*b^3*c^13 - 54*a*b^4*c^12 + 212*a*b^5*c^1
1 - 490*a*b^6*c^10 + 648*a*b^7*c^9 - 490*a*b^8*c^8 + 212*a*b^9*c^7 - 54*a*b
^10*c^6 + 8*a*b^11*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*
c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*b^5*c^13 - b^4*c^
14 - 5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4
*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a
+ b*x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6
*c^12 + 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11
*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3
*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b
^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2))*2i)/(b*c^3 + b^3*c - 2*b^2
c^2) - (a(b*c)^(1/2)*(b + c)*((4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 + 44*a^3
*b^6*c^6 + 4*a^3*b^7*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2
*((a + b*x)^(1/2) - a^(1/2))*(32*a^3*b^2*c^10 - 64*a^3*b^3*c^9 + 8*a^3*b^4*
c^8 + 240*a^3*b^5*c^7 + 8*a^3*b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3*b^8*c^4))/(
((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) +
(2*a*(b*c)^(1/2)*(b + c)*((4*(4*a^2*b^3*c^11 + 2*a^2*b^4*c^10 - 18*a^2*b^5*
c^9 + 12*a^2*b^6*c^8 + 12*a^2*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5 + 4*
a^2*b^10*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^
(1/2) - a^(1/2))*(16*a^2*b^2*c^12 - 32*a^2*b^3*c^11 + 36*a^2*b^4*c^10 - 64*
a^2*b^5*c^9 + 88*a^2*b^6*c^8 - 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^2*b^9
*c^5 + 16*a^2*b^10*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*
c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((2*((a + b*x)^(1/2) - a
^(1/2))*(8*a*b^3*c^13 - 54*a*b^4*c^12 + 212*a*b^5*c^11 - 490*a*b^6*c^10 + 6
48*a*b^7*c^9 - 490*a*b^8*c^8 + 212*a*b^9*c^7 - 54*a*b^10*c^6 + 8*a*b^11*c^5
))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)
) - (4*(a*b^4*c^12 + 7*a*b^5*c^11 - 27*a*b^6*c^10 + 19*a*b^7*c^9 + 19*a*b^8
*c^8 - 27*a*b^9*c^7 + 7*a*b^10*c^6 + a*b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3
+ c^4 + 6*b^2*c^2) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*b^5*c^13 - b^4*c^14 -
5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12
*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*
x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12
+ 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7

- 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2))*2i)/(b*c^3 + b^3*c - 2*b^2*c^2))/((4*((a + b*x)^(1/2) - a^(1/2))*(128*a^4*b^3*c^7 + 256*a^4*b^4*c^6 + 128* a^4*b^5*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) - (8*(16*a^4*b^3*c^7 + 56*a^4*b^4*c^6 + 56*a^4*b^5*c^5 + 16*a^ 4*b^6*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*a*(b*c)^(1/2)* (b + c)*((2*((a + b*x)^(1/2) - a^(1/2))*(32*a^3*b^2*c^10 - 64*a^3*b^3*c^9 + 8*a^3*b^4*c^8 + 240*a^3*b^5*c^7 + 8*a^3*b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3* b^8*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b ^2*c^2)) - (4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 + 44*a^3*b^6*c^6 + 4*a^3*b^7* c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*a^2*b^3*c^11 + 2*a^2*b^4*c^10 - 18*a^2*b^5*c^9 + 12*a^2*b^6*c^8 + 12*a^2*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5 + 4*a^2*b^10*c^4))/(b^4 - 4 *b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(16*a^ 2*b^2*c^12 - 32*a^2*b^3*c^11 + 36*a^2*b^4*c^10 - 64*a^2*b^5*c^9 + 88*a^2*b^ 6*c^8 - 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^2*b^9*c^5 + 16*a^2*b^10*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((4*(a*b^4*c^12 + 7*a*b^5*c^11 - 27*a*b^6*c^10 + 19*a*b^7*c^9 + 19*a*b^8*c^8 - 27*a*b^9*c^7 + 7*a*b^10*c^6 + a*b^11*c^5))/ (b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(8*a*b^3*c^13 - 54*a*b^4*c^12 + 212*a*b^5*c^11 - 490*a*b^6*c^10 + 648*a*b ^7*c^9 - 490*a*b^8*c^8 + 212*a*b^9*c^7 - 54*a*b^10*c^6 + 8*a*b^11*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) + (2 *a*(b*c)^(1/2)*(b + c)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4* b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^7*c^11 - 618*b ^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12*c^6 + 4*b^13* c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c ^2))))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 + 44*a^3*b^6*c^6 + 4*a^3*b^7*c^5))/(b ^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))* (32*a^3*b^2*c^10 - 64*a^3*b^3*c^9 + 8*a^3*b^4*c^8 + 240*a^3*b^5*c^7 + 8*a^3 *b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3*b^8*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((4* (4*a^2*b^3*c^11 + 2*a^2*b^4*c^10 - 18*a^2*b^5*c^9 + 12*a^2*b^6*c^8 + 12*a^2 *b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5 + 4*a^2*b^10*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(16*a^2*b^2* c^12 - 32*a^2*b^3*c^11 + 36*a^2*b^4*c^10 - 64*a^2*b^5*c^9 + 88*a^2*b^6*c^8 - 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^2*b^9*c^5 + 16*a^2*b^10*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) + (2* a*(b*c)^(1/2)*(b + c)*((2*((a + b*x)^(1/2) - a^(1/2))*(8*a*b^3*c^13 - 54*a* b^4*c^12 + 212*a*b^5*c^11 - 490*a*b^6*c^10 + 648*a*b^7*c^9 - 490*a*b^8*c^8

```
+ 212*a*b^9*c^7 - 54*a*b^10*c^6 + 8*a*b^11*c^5))/(((a + c*x)^(1/2) - a^(1/2
))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) - (4*(a*b^4*c^12 + 7*a*b^5*
c^11 - 27*a*b^6*c^10 + 19*a*b^7*c^9 + 19*a*b^8*c^8 - 27*a*b^9*c^7 + 7*a*b^1
0*c^6 + a*b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*a*(b*
c)^(1/2)*(b + c)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^12 + b^7*c^11 + b^8*c
^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^
3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^
15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^7*c^11 - 618*b^8*c^1
0 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/
(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))
/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c
 - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2)))*(b*c)^(1/2)*(b + c)*4i)/(b*c^
3 + b^3*c - 2*b^2*c^2)
```

3.435 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} dx = -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{2(b+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2} - \frac{4\sqrt{b}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}$$

```
[Out] -2*a/(b-c)^2/x+2*(b+c)*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/(b-c)^2+(b+c)*l
n(x)/(b-c)^2-4*arctanh(c^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(c*x+a)^(1/2))*b^(1/2)
*c^(1/2)/(b-c)^2+2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/(b-c)^2/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6822, 99, 163, 65, 223, 212, 95, 214}

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} dx = \frac{2(b+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\sqrt{b}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{(b-c)^2} - \frac{2a}{x(b-c)^2}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}$$

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

```
[Out] (-2*a)/((b - c)^2*x) + (2*Sqrt[a + b*x]*Sqrt[a + c*x])/((b - c)^2*x) + (2*(
b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (4*Sqrt[b]*Sqrt[c]
```

```
*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(b - c)^2 + ((b
+ c)*Log[x])/(b - c)^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{2a}{x^2}+\frac{b\left(1+\frac{c}{b}\right)}{x}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^2}\right)dx}{(b-c)^2}\\
&= -\frac{2a}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{2\int\frac{\sqrt{a+bx}\sqrt{a+cx}}{x^2}\,dx}{(b-c)^2}\\
&= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{2\int\frac{\frac{1}{2}a(b+c)+bcx}{x\sqrt{a+bx}\sqrt{a+cx}}\,dx}{(b-c)^2}\\
&= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}\\
&\quad-\frac{(2bc)\int\frac{1}{\sqrt{a+bx}\sqrt{a+cx}}\,dx}{(b-c)^2}-\frac{(a(b+c))\int\frac{1}{x\sqrt{a+bx}\sqrt{a+cx}}\,dx}{(b-c)^2}\\
&= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}\\
&\quad-\frac{(4c)\text{Subst}\left(\int\frac{1}{\sqrt{a-\frac{ac}{b}+\frac{cx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{(b-c)^2}\\
&\quad-\frac{(2a(b+c))\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}\\
&= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}\\
&\quad+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{(4c)\text{Subst}\left(\int\frac{1}{1-\frac{cx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}
\end{aligned}
$$

$$= -\frac{2a}{(b-c)^2x} + \frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x} + \frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2} - \frac{4\sqrt{b}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{(b-c)^2} + \frac{(b+c)\log(x)}{(b-c)^2}$$

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.73

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{-2a-2cx+2\sqrt{a+bx}\sqrt{a+cx}+8\sqrt{b}\sqrt{c}x\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{a+cx}}{\sqrt{c}\left(\sqrt{a-\frac{ab}{c}}-\sqrt{a+bx}\right)}\right)+4(b+c)x\operatorname{arctanh}\left(\frac{}{a(b-2c)-bcx\cdots}\right)}{(b-c)^2x}$$

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

```
[Out] (-2*a - 2*c*x + 2*Sqrt[a + b*x]*Sqrt[a + c*x] + 8*Sqrt[b]*Sqrt[c]*x*ArcTanh
[(Sqrt[b]*Sqrt[a + c*x])/(Sqrt[c]*(Sqrt[a - (a*b)/c] - Sqrt[a + b*x]))] + 4
*(b + c)*x*ArcTanh[(-(a*b) - b*c*x + c*Sqrt[a + c*x]*(Sqrt[a - (a*b)/c] - S
qrt[a + b*x]))/(a*(b - 2*c) - b*c*x + 2*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x] +
 Sqrt[a - (a*b)/c]*c*Sqrt[a + c*x] - c*Sqrt[a + b*x]*Sqrt[a + c*x])])/((b -
 c)^2*x)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.95

method	result
default	$\frac{b\ln(x)}{(b-c)^2}+\frac{c\ln(x)}{(b-c)^2}-\frac{2a}{(b-c)^2x}+\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(-2\,\operatorname{csgn}(a)\ln\left(\frac{2bcx+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}+ab+ac}{2\sqrt{bc}}\right)xbc+\ln\left(\frac{a\left(2\,\operatorname{csgn}(a)\sqrt{bc}\cdots\right.}{}\right.\right.}{}$

```
[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b-c)^2*b*ln(x)+1/(b-c)^2*c*ln(x)-2*a/(b-c)^2/x+1/(b-c)^2*(b*x+a)^(1/2)*(
c*x+a)^(1/2)*(-2*csgn(a)*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*
(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*x*b*c+ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*
x+a^2)^(1/2)+b*x+c*x+2*a)/x)*x*b*(b*c)^(1/2)+ln(a*(2*csgn(a)*(b*c*x^2+a*b*x
+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)*x*c*(b*c)^(1/2)+2*csgn(a)*(b*c)^(1/2)*(b*
```

```
c*x^2+a*b*x+a*c*x+a^2)^(1/2))*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x/(b*
c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.30

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \left[\frac{2\,(b+c)x\log\left(x\right) - 2\,(b+c)x\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\sqrt{cx+a}+2\,a}{x}\right) + 4\,\sqrt{bc}x\log\left(ab^2+2\,abc+ac^2+2\left(2\,bc-\right.\right.}{2\,(b^2-}\right.$$

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b + c)*x*log(x) - 2*(b + c)*x*log(-((b + c)*x - 2*sqrt(b*x + a)*sq
rt(c*x + a) + 2*a)/x) + 4*sqrt(b*c)*x*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*
c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x -
2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b + c)*x + 4*sqrt(b*x + a)*sqrt(c*x +
 a) - 4*a)/((b^2 - 2*b*c + c^2)*x), 1/2*(2*(b + c)*x*log(x) - 2*(b + c)*x*l
og(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) + 8*sqrt(-b*c)*x*a
rctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) + (b
 + c)*x + 4*sqrt(b*x + a)*sqrt(c*x + a) - 4*a)/((b^2 - 2*b*c + c^2)*x)]
```

Sympy [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{1}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(118) = 236$.

Time = 0.97 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.17

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{2\sqrt{bc}|b|\log\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2\right)}{b^3-2\,b^2c+bc^2} + \frac{2\sqrt{bc}(b+c)|b|\arctan\left(-\frac{ab^2+abc-\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\sqrt{-bc}ab}\right)}{(b^2-2\,bc+c^2)\sqrt{-bc}b} + \frac{(b+c)\log\left(|bx|\right)}{b^2-2\,bc+c^2} - \frac{4\left(\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2 a(b+c)|b| - (b^3-2\,b^2c \cdots\right.}{\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^4 - 2\,(b^2+bc)\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)\cdots\right.} - \frac{(bx+a)b+ab+(bx+a)c-ac}{(b^2-2\,bc+c^2)bx}$$

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*sqrt(b*c)*abs(b)*log((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))^2)/(b^3 - 2*b^2*c + b*c^2) + 2*sqrt(b*c)*(b + c)*abs(b)*arctan(
-1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c
 - a*b*c))^2)/(sqrt(-b*c)*a*b))/((b^2 - 2*b*c + c^2)*sqrt(-b*c)*b) + (b + c
)*log(abs(b*x))/(b^2 - 2*b*c + c^2) - 4*(sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a)
 - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*(b + c)*abs(b) - (b^3 - 2*b^2*c
 + b*c^2)*sqrt(b*c)*a^2*abs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))^4 - 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*
b^2 + (b*x + a)*b*c - a*b*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)*(b^2 - 2
*b*c + c^2)) - ((b*x + a)*b + a*b + (b*x + a)*c - a*c)/((b^2 - 2*b*c + c^2)
*b*x)
```

Mupad [B] (verification not implemented)

Time = 33.21 (sec) , antiderivative size = 4285, normalized size of antiderivative = 31.05

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

```
[Out] (atan((((b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c^
10 + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4 -
 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^12 - 36
*b^7*c^10 + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b*c^3
 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^1
2 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^
13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2)
- a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^
7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12
*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 +
 c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^14
 - 27*b^4*c^13 + 99*b^5*c^12 - 175*b^6*c^11 + 99*b^7*c^10 + 99*b^8*c^9 - 17
5*b^9*c^8 + 99*b^10*c^7 - 27*b^11*c^6 + 4*b^12*c^5))/(((a + c*x)^(1/2) - a^
(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a +
b*x)^(1/2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*c^10 - 596*b^7*
c^9 + 503*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)^(1/2) - a^(1/2)
)*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (4*(4*b^5*c^10
 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*b*
c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(65*b^4*c^11 - 167*
b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + 65*b^9*c^6))/(((a + c*
x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))*4i)/(b -
c)^2 - ((b*c)^(1/2)*((4*(4*b^5*c^10 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7
+ 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*
((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c^10 + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^
9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2
) + (4*(b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^12
+ b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^13
*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) -
a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^7*
c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12*c
^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c
^4 + 6*b^2*c^2))))/(b - c)^2 - (4*(4*b^5*c^12 - 36*b^7*c^10 + 64*b^8*c^9 -
36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*
((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^14 - 27*b^4*c^13 + 99*b^5*c^12 - 175*b
^6*c^11 + 99*b^7*c^10 + 99*b^8*c^9 - 175*b^9*c^8 + 99*b^10*c^7 - 27*b^11*c^
6 + 4*b^12*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^
```

4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*c^10 - 596*b^7*c^9 + 503*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(65*b^4*c^11 - 167*b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + 65*b^9*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))*4i)/(b - c)^2)/((4*(b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c^10 + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^12 - 36*b^7*c^10 + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^14 - 27*b^4*c^13 + 99*b^5*c^12 - 175*b^6*c^11 + 99*b^7*c^10 + 99*b^8*c^9 - 175*b^9*c^8 + 99*b^10*c^7 - 27*b^11*c^6 + 4*b^12*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*c^10 - 596*b^7*c^9 + 503*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (4*(4*b^5*c^10 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(65*b^4*c^11 - 167*b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + 65*b^9*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (8*(14*b^5*c^9 + 42*b^6*c^8 + 42*b^7*c^7 + 14*b^8*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^10 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c^10 + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (4*(4*b^5*c^12 - 36*b^7*c^10 + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^14 - 27*b^4*c^13 + 99*b^5*c^12 - 175*b^6*c^11 + 99*b^7*c^10 + 99*b^8*c^9 - 175*b^9*c^8 + 99*b^10*c^7 - 27*b^11*c^6 + 4*b^12*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*c^10 - 596*b^7*c^9 + 50

3*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(65*b^4*c^11 - 167*b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + 65*b^9*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 + (4*((a + b*x)^(1/2) - a^(1/2))*(112*b^5*c^9 + 224*b^6*c^8 + 112*b^7*c^7))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))*(b*c)^(1/2)*8i)/(b - c)^2 - (((b*c + b^2)*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - b^2 + (((a + b*x)^(1/2) - a^(1/2))^2*(b^2 - 3*b*c + c^2))/((a + c*x)^(1/2) - a^(1/2))^2)/((((a + b*x)^(1/2) - a^(1/2))^3*(2*b^2*c - 4*b*c^2 + 2*c^3))/((a + c*x)^(1/2) - a^(1/2))^3 + (((a + b*x)^(1/2) - a^(1/2))^2*(2*b*c^2 + 2*b^2*c - 2*b^3 - 2*c^3))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))*(2*b*c^2 - 4*b^2*c + 2*b^3))/((a + c*x)^(1/2) - a^(1/2))) + (log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(b + c))/(b - c)^2 + (log(x)*(b + c))/(b^2 - 2*b*c + c^2) - (log((((a + b*x)^(1/2) - (a + c*x)^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))/((a + c*x)^(1/2) - a^(1/2)))*(b + c))/(b^2 - 2*b*c + c^2) - (2*a)/(x*(b^2 - 2*b*c + c^2)) + (c*((a + b*x)^(1/2) - a^(1/2)))/(2*(b - c)^2*((a + c*x)^(1/2) - a^(1/2)))

3.436 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\sqrt{a+cx}}{2a(b-c)x}+\frac{\sqrt{a+bx}(a+cx)^{3/2}}{a(b-c)^2x^2}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}$$

```
[Out] -a/(b-c)^2/x^2+(-b-c)/(b-c)^2/x-1/2*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/a+
(c*x+a)^(3/2)*(b*x+a)^(1/2)/a/(b-c)^2/x^2+1/2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/a
/(b-c)/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6822, 96, 95, 214}

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}+\frac{\sqrt{a+bx}(a+cx)^{3/2}}{ax^2(b-c)^2}-\frac{a}{x^2(b-c)^2}+\frac{\sqrt{a+bx}\sqrt{a+cx}}{2ax(b-c)}-\frac{b+c}{x(b-c)^2}$$

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] -(a/((b - c)^2*x^2)) - (b + c)/((b - c)^2*x) + (Sqrt[a + b*x]*Sqrt[a + c*x]
)/(2*a*(b - c)*x) + (Sqrt[a + b*x]*(a + c*x)^(3/2))/(a*(b - c)^2*x^2) - Arc
Tanh[Sqrt[a + b*x]/Sqrt[a + c*x]]/(2*a)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \left(\frac{2a}{x^3} + \frac{b\left(1+\frac{c}{b}\right)}{x^2} - \frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^3}\right) dx}{(b-c)^2}$$

$$= -\frac{a}{(b-c)^2x^2} - \frac{b+c}{(b-c)^2x} - \frac{2\int \frac{\sqrt{a+bx}\sqrt{a+cx}}{x^3}\,dx}{(b-c)^2}$$

$$= -\frac{a}{(b-c)^2x^2} - \frac{b+c}{(b-c)^2x} + \frac{\sqrt{a+bx}(a+cx)^{3/2}}{a(b-c)^2x^2} - \frac{\int \frac{\sqrt{a+cx}}{x^2\sqrt{a+bx}}\,dx}{2(b-c)}$$

$$= -\frac{a}{(b-c)^2x^2} - \frac{b+c}{(b-c)^2x} + \frac{\sqrt{a+bx}\sqrt{a+cx}}{2a(b-c)x} + \frac{\sqrt{a+bx}(a+cx)^{3/2}}{a(b-c)^2x^2} + \frac{1}{4}\int \frac{1}{x\sqrt{a+bx}\sqrt{a+cx}}\,dx$$

$$= -\frac{a}{(b-c)^2x^2} - \frac{b+c}{(b-c)^2x} + \frac{\sqrt{a+bx}\sqrt{a+cx}}{2a(b-c)x} + \frac{\sqrt{a+bx}(a+cx)^{3/2}}{a(b-c)^2x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)$$

$$= -\frac{a}{(b-c)^2x^2} - \frac{b+c}{(b-c)^2x} + \frac{\sqrt{a+bx}\sqrt{a+cx}}{2a(b-c)x} + \frac{\sqrt{a+bx}(a+cx)^{3/2}}{a(b-c)^2x^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{-2a^2+(b+c)x\sqrt{a+bx}\sqrt{a+cx}+2a\left(-bx-cx+\sqrt{a+bx}\sqrt{a+cx}\right)-(b-c)^2x^2\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a+bx}}\right)}{2a(b-c)^2x^2}$$

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] (-2*a^2 + (b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + 2*a*(-(b*x) - c*x + Sqrt[
a + b*x]*Sqrt[a + c*x]) - (b - c)^2*x^2*ArcTanh[Sqrt[a + c*x]/Sqrt[a + b*x]
])/(2*a*(b - c)^2*x^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.54

method	result
default	$-\frac{b}{x(b-c)^2} - \frac{c}{x(b-c)^2} - \frac{a}{(b-c)^2x^2} + \frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(-\ln\left(\frac{a\left(2\ \text{csgn}(a)\sqrt{bc\,x^2+abx+acx+a^2}+bx+cx+2a\right)}{x}\right)x^2b^2+2\ln\left(\frac{a\left(2\ \text{csgn}(\right.}{}\right.\right.}{}$

```
[In] int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x/(b-c)^2*b-1/x/(b-c)^2*c-a/(b-c)^2/x^2+1/4/(b-c)^2*(b*x+a)^(1/2)*(c*x+a
)^(1/2)/a*(-ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)
*x^2*b^2+2*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)*
x^2*b*c-ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)*x^2
*c^2+2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*b+2*csgn(a)*(b*c*x^2+a*b*x
+a*c*x+a^2)^(1/2)*x*c+4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2))*csgn(a)/
(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \frac{4\,(b^2-2\,bc+c^2)x^2\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\sqrt{cx+a}+2\,a}{x}\right)+(b^2+2\,bc+c^2)x^2+8\,((b+c)x+2\,a)\sqrt{bx+a}\sqrt{cx+}}{16\,(ab^2-2\,abc+ac^2)x^2}$$

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(b^2 - 2*b*c + c^2)*x^2*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x
+ a) + 2*a)/x) + (b^2 + 2*b*c + c^2)*x^2 + 8*((b + c)*x + 2*a)*sqrt(b*x + a
)*sqrt(c*x + a) - 16*a^2 - 16*(a*b + a*c)*x)/((a*b^2 - 2*a*b*c + a*c^2)*x^2
)
```

Sympy [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

```
[In] integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(107) = 214.

Time = 2.84 (sec) , antiderivative size = 532, normalized size of antiderivative = 4.33

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{\sqrt{bc}|b|\arctan\left(-\frac{ab^2+abc-\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\sqrt{-bc}ab}\right)}{2\sqrt{-bc}ab}$$

$$-\frac{(b^2+6\,bc+c^2)\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^6|b|-(3\,b^4+5\,b^3c+5\,b^2c^2+3\,bc^3)\sqrt{bc}\left(\right.}{\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+}\right.\right.}$$

$$-\frac{(bx+a)b^2+(bx+a)bc-abc}{(b^2-2\,bc+c^2)b^2x^2}$$

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(b*c)*abs(b)*arctan(-1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a)
 - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/(sqrt(-b*c)*a*
b) - ((b^2 + 6*b*c + c^2)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 +
 (b*x + a)*b*c - a*b*c))^6*abs(b) - (3*b^4 + 5*b^3*c + 5*b^2*c^2 + 3*b*c^3)
*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^
4*a*abs(b) + (3*b^6 - 4*b^5*c + 2*b^4*c^2 - 4*b^3*c^3 + 3*b^2*c^4)*sqrt(b*c
)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a^2*abs
(b) - (b^8 - 3*b^7*c + 2*b^6*c^2 + 2*b^5*c^3 - 3*b^4*c^4 + b^3*c^5)*sqrt(b*
c)*a^3*abs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*
b*c))^4 - 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b
*c - a*b*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)^2*(b^2 - 2*b*c + c^2)) -
((b*x + a)*b^2 + (b*x + a)*b*c - a*b*c)/((b^2 - 2*b*c + c^2)*b^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 30.04 (sec) , antiderivative size = 787, normalized size of antiderivative = 6.40

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\ln\left(\frac{\left(\sqrt{a+b\,x}-\sqrt{a+c\,x}\right)\left(b-\frac{c\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{4\,a}$$

$$-\frac{\frac{b^4}{2}+\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4\left(-\frac{b^4}{2}+4\,b^3\,c+\frac{3\,b^2\,c^2}{2}+4\,b\,c^3-\frac{c^4}{2}\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^4}-\frac{\left(2\,b^4+2\,c\,b^3\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}-\frac{\left(b^2\,c^2+b\,c^3\right)\left(\sqrt{\cdots}\right.}{\left(\sqrt{a+c\,x}\cdots\right.}}{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4\left(8\,a\,b^4+16\,a\,b^3\,c-48\,a\,b^2\,c^2+16\,a\,b\,c^3+8\,a\,c^4\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^4}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3\left(16\,a\,b^4-16\,a\,b^3\,c-16\,a\,b^2\,c^2+16\,a\,b\,c^3\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^3}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5\cdots}{\cdots}}$$

$$-\frac{\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{4\,a}-\frac{a+x\,(b+c)}{x^2\,\left(b^2-2\,b\,c+c^2\right)}$$

$$-\frac{c^2\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{16\,a\,(b-c)^2\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}+\frac{c\,(b+c)\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{8\,a\,(b-c)^2\,\left(\sqrt{a+c\,x}-\sqrt{a}\right)}$$

```
[In] int(1/(x*((a + b*x)^(1/2) + (a + c*x)^(1/2))^2),x)
```

```
[Out] log((((a + b*x)^(1/2) - (a + c*x)^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2)
))/((a + c*x)^(1/2) - a^(1/2))))/((a + c*x)^(1/2) - a^(1/2)))/(4*a) - (b^4/
2 + (((a + b*x)^(1/2) - a^(1/2))^4*(4*b*c^3 + 4*b^3*c - b^4/2 - c^4/2 + (3*
b^2*c^2)/2))/((a + c*x)^(1/2) - a^(1/2))^4 - ((2*b^3*c + 2*b^4)*((a + b*x)^
(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - ((b*c^3 + b^2*c^2)*((a + b*
x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^(1/2))^5 + (((a + b*x)^(1/2) -
a^(1/2))^2*(6*b^3*c + (5*b^4)/2 + (5*b^2*c^2)/2))/((a + c*x)^(1/2) - a^(1/2
))^2 - (((a + b*x)^(1/2) - a^(1/2))^3*(b*c^3 + 6*b^3*c + b^4 + 6*b^2*c^2))/
((a + c*x)^(1/2) - a^(1/2))^3)/((((a + b*x)^(1/2) - a^(1/2))^4*(8*a*b^4 + 8
*a*c^4 - 48*a*b^2*c^2 + 16*a*b*c^3 + 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2
))^4 - (((a + b*x)^(1/2) - a^(1/2))^3*(16*a*b^4 - 16*a*b^2*c^2 + 16*a*b*c^3
 - 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2))^3 - (((a + b*x)^(1/2) - a^(1/2)
)^5*(16*a*c^4 - 16*a*b^2*c^2 - 16*a*b*c^3 + 16*a*b^3*c))/((a + c*x)^(1/2) -
 a^(1/2))^5 + (((a + b*x)^(1/2) - a^(1/2))^2*(8*a*b^4 + 8*a*b^2*c^2 - 16*a*
b^3*c))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^6*(8*a
*c^4 + 8*a*b^2*c^2 - 16*a*b*c^3))/((a + c*x)^(1/2) - a^(1/2))^6) - log(((a
+ b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))/(4*a) - (a + x*(b + c)
)/(x^2*(b^2 - 2*b*c + c^2)) - (c^2*((a + b*x)^(1/2) - a^(1/2))^2)/(16*a*(b
- c)^2*((a + c*x)^(1/2) - a^(1/2))^2) + (c*(b + c)*((a + b*x)^(1/2) - a^(1/
2)))/(8*a*(b - c)^2*((a + c*x)^(1/2) - a^(1/2)))
```

3.437 $\int \frac{1}{x^2(\sqrt{a+bx}+\sqrt{a+cx})^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2(b-c)x}-\frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2x^2}+\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2(b-c)^2x^3}+\frac{(b+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}$$

```
[Out] -2/3*a/(b-c)^2/x^3+1/2*(-b-c)/(b-c)^2/x^2+2/3*(b*x+a)^(3/2)*(c*x+a)^(3/2)/a
^2/(b-c)^2/x^3+1/4*(b+c)*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/a^2-1/2*(b+c)
*(c*x+a)^(3/2)*(b*x+a)^(1/2)/a^2/(b-c)^2/x^2-1/4*(b+c)*(b*x+a)^(1/2)*(c*x+a
)^(1/2)/a^2/(b-c)/x
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6822, 98, 96, 95, 214}

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{(b+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}+\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2x^3(b-c)^2}-\frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2x^2(b-c)^2}-\frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2x(b-c)}-\frac{2a}{3x^3(b-c)^2}-\frac{b+c}{2x^2(b-c)^2}$$

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] (-2*a)/(3*(b - c)^2*x^3) - (b + c)/(2*(b - c)^2*x^2) - ((b + c)*Sqrt[a + b*
x]*Sqrt[a + c*x])/(4*a^2*(b - c)*x) - ((b + c)*Sqrt[a + b*x]*(a + c*x)^(3/2
))/(2*a^2*(b - c)^2*x^2) + (2*(a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*a^2*(b -
c)^2*x^3) + ((b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(4*a^2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \left(\frac{2a}{x^4} + \frac{b\left(1+\frac{c}{b}\right)}{x^3} - \frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^4}\right) dx}{(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3} - \frac{b+c}{2(b-c)^2x^2} - \frac{2\int \frac{\sqrt{a+bx}\sqrt{a+cx}}{x^4}\,dx}{(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3} - \frac{b+c}{2(b-c)^2x^2} + \frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2(b-c)^2x^3} + \frac{(b+c)\int \frac{\sqrt{a+bx}\sqrt{a+cx}}{x^3}\,dx}{a(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3} - \frac{b+c}{2(b-c)^2x^2} - \frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2x^2} + \frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2(b-c)^2x^3} + \frac{(b+c)\int \frac{\sqrt{a+cx}}{x^2\sqrt{a+bx}}\,dx}{4a(b-c)}$$

$$= -\frac{2a}{3(b-c)^2x^3} - \frac{b+c}{2(b-c)^2x^2} - \frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2(b-c)x} - \frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2x^2} + \frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2(b-c)^2x^3} - \frac{(b+c)\int \frac{1}{x\sqrt{a+bx}\sqrt{a+cx}}\,dx}{8a}$$

$$= -\frac{2a}{3(b-c)^2x^3} - \frac{b+c}{2(b-c)^2x^2} - \frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2(b-c)x} - \frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2x^2} + \frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2(b-c)^2x^3} - \frac{(b+c)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a}$$

$$= -\frac{2a}{3(b-c)^2x^3} - \frac{b+c}{2(b-c)^2x^2} - \frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2(b-c)x} - \frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2x^2} + \frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2(b-c)^2x^3} + \frac{(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}$$

Mathematica [A] (verified)

Time = 10.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \frac{-8a^3 + 2a(b+c)x\sqrt{a+bx}\sqrt{a+cx} + \left(-3b^2+2bc-3c^2\right)x^2\sqrt{a+bx}\sqrt{a+cx} + a^2\left(-6bx-6cx+8\sqrt{a+\right.}{12a^2(b-c)^2x^3}$$

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] (-8*a^3 + 2*a*(b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + (-3*b^2 + 2*b*c - 3*c
^2)*x^2*Sqrt[a + b*x]*Sqrt[a + c*x] + a^2*(-6*b*x - 6*c*x + 8*Sqrt[a + b*x]
*Sqrt[a + c*x]) + 3*(b - c)^2*(b + c)*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*
x]])/(12*a^2*(b - c)^2*x^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.63

method	result
default	$-\frac{b}{2x^2(b-c)^2}-\frac{c}{2x^2(b-c)^2}-\frac{2a}{3(b-c)^2x^3}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(-3\ln\left(\frac{a\left(2\,\mathrm{csgn}(a)\sqrt{bc\,x^2+abx+acx+a^2}+bx+cx+2a\right)}{x}\right)x^3b^3+3\ln\left(\frac{a\left(2\right.}{}\right.\right.}{}$

```
[In] int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2/(b-c)^2*b-1/2/x^2/(b-c)^2*c-2/3*a/(b-c)^2/x^3-1/24/(b-c)^2*(b*x+a)
^(1/2)*(c*x+a)^(1/2)/a^2*(-3*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2
)+b*x+c*x+2*a)/x)*x^3*b^3+3*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)
+b*x+c*x+2*a)/x)*x^3*b^2*c+3*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2
)+b*x+c*x+2*a)/x)*x^3*b*c^2-3*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/
2)+b*x+c*x+2*a)/x)*x^3*c^3+6*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x^2*b^
2-4*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x^2*b*c+6*csgn(a)*(b*c*x^2+a*b*
x+a*c*x+a^2)^(1/2)*x^2*c^2-4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*b-
4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*c-16*(b*c*x^2+a*b*x+a*c*x+a^2
)^(1/2)*a^2*csgn(a))*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx =$$
$$-\frac{12\,(b^3-b^2c-bc^2+c^3)x^3\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\sqrt{cx+a}+2\,a}{x}\right)+(5\,b^3+3\,b^2c+3\,bc^2+5\,c^3)x^3+64\,a^3+8\,((3}{96\,(a^2b^2-2\,a^2bc+a^2c^2)x^3}$$

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/96*(12*(b^3 - b^2*c - b*c^2 + c^3)*x^3*log(-((b + c)*x - 2*sqrt(b*x + a)
*sqrt(c*x + a) + 2*a)/x) + (5*b^3 + 3*b^2*c + 3*b*c^2 + 5*c^3)*x^3 + 64*a^3
 + 8*((3*b^2 - 2*b*c + 3*c^2)*x^2 - 8*a^2 - 2*(a*b + a*c)*x)*sqrt(b*x + a)*
sqrt(c*x + a) + 48*(a^2*b + a^2*c)*x)/((a^2*b^2 - 2*a^2*b*c + a^2*c^2)*x^3)
```

Sympy [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx=\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx=\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(146) = 292$.

Time = 2.46 (sec) , antiderivative size = 802, normalized size of antiderivative = 4.61

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$=\frac{\sqrt{bc}(b+c)|b|\arctan\left(-\frac{ab^2+abc-\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\sqrt{-bc}ab}\right)}{4\sqrt{-bc}a^2b}$$
$$+\frac{3\left(b^3-b^2c-bc^2+c^3\right)\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^{10}|b|-3\left(5b^5+22b^3c^2+5bc^4\right)\sqrt{}}{}$$
$$-\frac{3(bx+a)b^3+ab^3+3(bx+a)b^2c-3ab^2c}{6\left(b^2-2bc+c^2\right)b^3x^3}$$

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
[Out] 1/4*sqrt(b*c)*(b + c)*abs(b)*arctan(-1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b
*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/(sqrt(-
b*c)*a^2*b) + 1/6*(3*(b^3 - b^2*c - b*c^2 + c^3)*sqrt(b*c)*(sqrt(b*c)*sqrt(
b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^10*abs(b) - 3*(5*b^5 + 22*b
```

```
^3*c^2 + 5*b*c^4)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
a)*b*c - a*b*c))^8*a*abs(b) + 2*(15*b^7 - b^6*c + 18*b^5*c^2 + 18*b^4*c^3 -
 b^3*c^4 + 15*b^2*c^5)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b
*x + a)*b*c - a*b*c))^6*a^2*abs(b) - 6*(5*b^9 - 6*b^8*c - 5*b^7*c^2 + 12*b^
6*c^3 - 5*b^5*c^4 - 6*b^4*c^5 + 5*b^3*c^6)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x +
a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4*a^3*abs(b) + 3*(5*b^11 - 17*b^1
0*c + 21*b^9*c^2 - 9*b^8*c^3 - 9*b^7*c^4 + 21*b^6*c^5 - 17*b^5*c^6 + 5*b^4*
c^7)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*
c))^2*a^4*abs(b) - (3*b^13 - 20*b^12*c + 60*b^11*c^2 - 108*b^10*c^3 + 130*b
^9*c^4 - 108*b^8*c^5 + 60*b^7*c^6 - 20*b^6*c^7 + 3*b^5*c^8)*sqrt(b*c)*a^5*a
bs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4
- 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b
*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)^3*(b^2 - 2*b*c + c^2)*a) - 1/6*(3
*(b*x + a)*b^3 + a*b^3 + 3*(b*x + a)*b^2*c - 3*a*b^2*c)/((b^2 - 2*b*c + c^2
)*b^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 48.12 (sec) , antiderivative size = 1290, normalized size of antiderivative = 7.41

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx=\text{Too large to display}$$

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (a + c*x)^(1/2))^2),x)
```

```
[Out] (log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(b + c))/(8*a
^2) - ((((a + b*x)^(1/2) - a^(1/2))^7*(3*b^5*c - 15*b*c^5 + 3*c^6 + 3*b^2*c
^4 + 3*b^3*c^3 - 15*b^4*c^2))/((a + c*x)^(1/2) - a^(1/2))^7 - (((a + b*x)^(
1/2) - a^(1/2))^5*(26*b^5*c - b*c^5 - b^6 + 26*b^2*c^4 + 4*b^3*c^3 + 4*b^4*
c^2))/((a + c*x)^(1/2) - a^(1/2))^5 - b^6/3 + ((b^5*c + b^6)*((a + b*x)^(1/
2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (((a + b*x)^(1/2) - a^(1/2))^8
*(c^6 - 6*b*c^5 + 7*b^2*c^4 - 6*b^3*c^3 + b^4*c^2))/((a + c*x)^(1/2) - a^(1
/2))^8 + (((a + b*x)^(1/2) - a^(1/2))^6*(6*b*c^5 + 6*b^5*c - (5*b^6)/3 - (5
*c^6)/3 + 30*b^2*c^4 - 24*b^3*c^3 + 30*b^4*c^2))/((a + c*x)^(1/2) - a^(1/2)
)^6 - (((17*b^6)/3 + (17*b^3*c^3)/3)*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c
*x)^(1/2) - a^(1/2))^3 + (((a + b*x)^(1/2) - a^(1/2))^2*(b^6 - 4*b^5*c + b^
4*c^2))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^4*(18*
b^5*c + 5*b^6 + 5*b^2*c^4 + 18*b^3*c^3 - 6*b^4*c^2))/((a + c*x)^(1/2) - a^(
1/2))^4)/((((a + b*x)^(1/2) - a^(1/2))^5*(96*a^2*b^5 + 96*a^2*b*c^4 + 96*a^
2*b^4*c + 96*a^2*b^2*c^3 - 384*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^5
- (((a + b*x)^(1/2) - a^(1/2))^8*(96*a^2*c^5 - 96*a^2*b*c^4 - 96*a^2*b^2*c^
3 + 96*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^8 - (((a + b*x)^(1/2) - a^
(1/2))^6*(32*a^2*b^5 + 32*a^2*c^5 + 224*a^2*b*c^4 + 224*a^2*b^4*c - 256*a^2
*b^2*c^3 - 256*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^6 - (((a + b*x)^(1
/2) - a^(1/2))^4*(96*a^2*b^5 - 96*a^2*b^4*c + 96*a^2*b^2*c^3 - 96*a^2*b^3*c
```

^2))/((a + c*x)^(1/2) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^7*(96*a^2*c^5 + 96*a^2*b*c^4 + 96*a^2*b^4*c - 384*a^2*b^2*c^3 + 96*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^7 + (((a + b*x)^(1/2) - a^(1/2))^3*(32*a^2*b^5 - 64*a^2*b^4*c + 32*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^3 + (((a + b*x)^(1/2) - a^(1/2))^9*(32*a^2*c^5 - 64*a^2*b*c^4 + 32*a^2*b^2*c^3))/((a + c*x)^(1/2) - a^(1/2))^9) - (((c*(8*b*c + 3*b^2 + 3*c^2))/(16*a^2*(b - c)^2) - (c*(17*b*c + 4*b^2 + 4*c^2))/(32*a^2*(b - c)^2))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (log((((a + b*x)^(1/2) - (a + c*x)^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))/((a + c*x)^(1/2) - a^(1/2)))*(b + c))/(8*a^2) - ((2*a)/3 + x*(b/2 + c/2))/(x^3*(b^2 - 2*b*c + c^2)) + (c^3*((a + b*x)^(1/2) - a^(1/2))^3)/(96*a^2*(b - c)^2*((a + c*x)^(1/2) - a^(1/2))^3)

3.438 $\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 277

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = -\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3}+\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3}+\frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3} -\frac{4a(b+3c)(a+bx)^{5/2}}{5b^3(b-c)^3}+\frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3} +\frac{8a^2(a+cx)^{3/2}}{3(b-c)^3c^2}-\frac{2a^2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c^3}-\frac{8a(a+cx)^{5/2}}{5(b-c)^3c^2} +\frac{4a(3b+c)(a+cx)^{5/2}}{5(b-c)^3c^3}-\frac{2(3b+c)(a+cx)^{7/2}}{7(b-c)^3c^3}$$

```
[Out] -8/3*a^2*(b*x+a)^(3/2)/b^2/(b-c)^3+2/3*a^2*(b+3*c)*(b*x+a)^(3/2)/b^3/(b-c)^
3+8/5*a*(b*x+a)^(5/2)/b^2/(b-c)^3-4/5*a*(b+3*c)*(b*x+a)^(5/2)/b^3/(b-c)^3+2
/7*(b+3*c)*(b*x+a)^(7/2)/b^3/(b-c)^3+8/3*a^2*(c*x+a)^(3/2)/(b-c)^3/c^2-2/3*
a^2*(3*b+c)*(c*x+a)^(3/2)/(b-c)^3/c^3-8/5*a*(c*x+a)^(5/2)/(b-c)^3/c^2+4/5*a
*(3*b+c)*(c*x+a)^(5/2)/(b-c)^3/c^3-2/7*(3*b+c)*(c*x+a)^(7/2)/(b-c)^3/c^3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used

$= \{6822, 45\}$

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3} - \frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3} - \frac{2a^2(3b+c)(a+cx)^{3/2}}{3c^3(b-c)^3} + \frac{8a^2(a+cx)^{3/2}}{3c^2(b-c)^3} + \frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3} - \frac{4a(b+3c)(a+bx)^{5/2}}{5b^3(b-c)^3} + \frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{2(3b+c)(a+cx)^{7/2}}{7c^3(b-c)^3} + \frac{4a(3b+c)(a+cx)^{5/2}}{5c^3(b-c)^3} - \frac{8a(a+cx)^{5/2}}{5c^2(b-c)^3}$$

```
[In] Int[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (-8*a^2*(a + b*x)^(3/2))/(3*b^2*(b - c)^3) + (2*a^2*(b + 3*c)*(a + b*x)^(3/
2))/(3*b^3*(b - c)^3) + (8*a*(a + b*x)^(5/2))/(5*b^2*(b - c)^3) - (4*a*(b +
 3*c)*(a + b*x)^(5/2))/(5*b^3*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(7/2))/(7
*b^3*(b - c)^3) + (8*a^2*(a + c*x)^(3/2))/(3*(b - c)^3*c^2) - (2*a^2*(3*b +
 c)*(a + c*x)^(3/2))/(3*(b - c)^3*c^3) - (8*a*(a + c*x)^(5/2))/(5*(b - c)^3
*c^2) + (4*a*(3*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^3) - (2*(3*b + c)*(a
 + c*x)^(7/2))/(7*(b - c)^3*c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

integral

$$= \frac{\int \left(4ax\sqrt{a+bx}+b\left(1+\frac{3c}{b}\right)x^2\sqrt{a+bx}-4ax\sqrt{a+cx}-3b\left(1+\frac{c}{3b}\right)x^2\sqrt{a+cx}\right)\,dx}{(b-c)^3}$$

$$= \frac{(4a)\int x\sqrt{a+bx}\,dx}{(b-c)^3} - \frac{(4a)\int x\sqrt{a+cx}\,dx}{(b-c)^3} - \frac{(3b+c)\int x^2\sqrt{a+cx}\,dx}{(b-c)^3} + \frac{(b+3c)\int x^2\sqrt{a+bx}\,dx}{(b-c)^3}$$

$$= \frac{(4a)\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{(b-c)^3} - \frac{(4a)\int\left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right)dx}{(b-c)^3} - \frac{(3b+c)\int\left(\frac{a^2\sqrt{a+cx}}{c^2}-\frac{2a(a+cx)^{3/2}}{c^2}+\frac{(a+cx)^{5/2}}{c^2}\right)dx}{(b-c)^3} + \frac{(b+3c)\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx}{(b-c)^3}$$

$$= -\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3} + \frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3} + \frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{4a(b+3c)(a+bx)^{5/2}}{5b^3(b-c)^3} + \frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3} + \frac{8a^2(a+cx)^{3/2}}{3(b-c)^3c^2} - \frac{2a^2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c^3} - \frac{8a(a+cx)^{5/2}}{5(b-c)^3c^2} + \frac{4a(3b+c)(a+cx)^{5/2}}{5(b-c)^3c^3} - \frac{2(3b+c)(a+cx)^{7/2}}{7(b-c)^3c^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2321 vs. $2(277) = 554$.

Time = 6.82 (sec) , antiderivative size = 2321, normalized size of antiderivative = 8.38

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (-2*(a + c*x)*(5*a^3*b^3*Sqrt[a - (a*b)/c]*c^6*x^5*(b^2*x + 3*b*c*x - 3*b*S
qrt[a + b*x]*Sqrt[a + c*x] - c*Sqrt[a + b*x]*Sqrt[a + c*x]) + a^9*(b^4*c*(-
139*Sqrt[a - (a*b)/c] + 413*Sqrt[a + b*x] - 216*Sqrt[a + c*x]) + b^5*(5*Sqr
t[a - (a*b)/c] - 35*Sqrt[a + b*x] + 8*Sqrt[a + c*x]) - 32*c^5*(35*Sqrt[a -
(a*b)/c] - 35*Sqrt[a + b*x] + 32*Sqrt[a + c*x]) + 7*b^3*c^2*(125*Sqrt[a - (
a*b)/c] - 251*Sqrt[a + b*x] + 176*Sqrt[a + c*x]) + 4*b*c^4*(665*Sqrt[a - (a
*b)/c] - 805*Sqrt[a + b*x] + 704*Sqrt[a + c*x]) - b^2*c^3*(2289*Sqrt[a - (a
*b)/c] - 3479*Sqrt[a + b*x] + 2816*Sqrt[a + c*x])) + a^4*b^2*c^5*x^4*(-120*
Sqrt[a - (a*b)/c]*c^2*Sqrt[a + b*x]*Sqrt[a + c*x] - 5*b^3*x*(22*Sqrt[a - (a
*b)/c] - 7*Sqrt[a + b*x] + 21*Sqrt[a + c*x]) + b*(-279*Sqrt[a - (a*b)/c]*c*
Sqrt[a + b*x]*Sqrt[a + c*x] + 7*c^2*x*(54*Sqrt[a - (a*b)/c] - 15*Sqrt[a + b
*x] + 5*Sqrt[a + c*x])) + b^2*(327*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a +
 c*x] + c*x*(-176*Sqrt[a - (a*b)/c] + 70*Sqrt[a + b*x] + 70*Sqrt[a + c*x]))
) + a^5*b*c^4*x^3*(-400*Sqrt[a - (a*b)/c]*c^3*Sqrt[a + b*x]*Sqrt[a + c*x] +
 b^4*x*(285*Sqrt[a - (a*b)/c] - 210*Sqrt[a + b*x] + 609*Sqrt[a + c*x]) + b^
3*(-785*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(-573*Sqrt[a -
(a*b)/c] + 63*Sqrt[a + b*x] - 1442*Sqrt[a + c*x])) + b*(-1216*Sqrt[a - (a*b
)/c]*c^2*Sqrt[a + b*x]*Sqrt[a + c*x] + 7*c^3*x*(233*Sqrt[a - (a*b)/c] - 123
```

```
*Sqrt[a + b*x] + 40*Sqrt[a + c*x])) + b^2*(2313*Sqrt[a - (a*b)/c]*c*Sqrt[a
+ b*x]*Sqrt[a + c*x] + c^2*x*(-1183*Sqrt[a - (a*b)/c] + 1008*Sqrt[a + b*x]
+ 553*Sqrt[a + c*x]))) + a^6*c^3*x^2*(-320*Sqrt[a - (a*b)/c]*c^4*Sqrt[a + b
*x]*Sqrt[a + c*x] + b^5*x*(-320*Sqrt[a - (a*b)/c] + 315*Sqrt[a + b*x] - 791
*Sqrt[a + c*x]) - 14*b^2*c^2*(-388*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a +
 c*x] + c*x*(99*Sqrt[a - (a*b)/c] - 159*Sqrt[a + b*x] - 136*Sqrt[a + c*x]))
 + 20*b*c^3*(-116*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + 7*c*x*(17
*Sqrt[a - (a*b)/c] - 13*Sqrt[a + b*x] + 4*Sqrt[a + c*x])) + b^4*(693*Sqrt[a
 - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + 6*c*x*(323*Sqrt[a - (a*b)/c] - 23
8*Sqrt[a + b*x] + 665*Sqrt[a + c*x])) + b^3*(-3517*Sqrt[a - (a*b)/c]*c*Sqrt
[a + b*x]*Sqrt[a + c*x] - 7*c^2*x*(356*Sqrt[a - (a*b)/c] - 101*Sqrt[a + b*x
] + 809*Sqrt[a + c*x]))) + a^7*c^2*x*(b^5*x*(285*Sqrt[a - (a*b)/c] - 315*Sq
rt[a + b*x] + 547*Sqrt[a + c*x]) + b^2*(-5120*Sqrt[a - (a*b)/c]*c^2*Sqrt[a
+ b*x]*Sqrt[a + c*x] + c^3*x*(-5474*Sqrt[a - (a*b)/c] + 3794*Sqrt[a + b*x]
- 9256*Sqrt[a + c*x])) + b^4*(-364*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a +
 c*x] + c*x*(-2043*Sqrt[a - (a*b)/c] + 2128*Sqrt[a + b*x] - 3758*Sqrt[a + c
*x])) + 32*c^4*(-48*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + 5*c*x*(
7*Sqrt[a - (a*b)/c] - 7*Sqrt[a + b*x] + 2*Sqrt[a + c*x])) + 7*b^3*c*(328*Sq
rt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(776*Sqrt[a - (a*b)/c] -
701*Sqrt[a + b*x] + 1317*Sqrt[a + c*x])) + b*(4736*Sqrt[a - (a*b)/c]*c^3*Sq
rt[a + b*x]*Sqrt[a + c*x] + c^4*x*(700*Sqrt[a - (a*b)/c] + 420*Sqrt[a + b*x
] + 2928*Sqrt[a + c*x]))) + a^8*(-10*b^5*c*x*(11*Sqrt[a - (a*b)/c] - 21*Sqr
t[a + b*x] + 18*Sqrt[a + c*x]) + 32*c^5*(32*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]
*Sqrt[a + c*x] + c*x*(35*Sqrt[a - (a*b)/c] - 35*Sqrt[a + b*x] + 48*Sqrt[a +
 c*x])) - 14*b^3*c^2*(40*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*
x*(283*Sqrt[a - (a*b)/c] - 353*Sqrt[a + b*x] + 412*Sqrt[a + c*x])) + 2*b^4*
c*(28*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(549*Sqrt[a - (a*
b)/c] - 819*Sqrt[a + b*x] + 862*Sqrt[a + c*x])) - 4*b*c^4*(576*Sqrt[a - (a*
b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(1155*Sqrt[a - (a*b)/c] - 1155*Sqrt
[a + b*x] + 1504*Sqrt[a + c*x])) + b^2*(1792*Sqrt[a - (a*b)/c]*c^3*Sqrt[a +
 b*x]*Sqrt[a + c*x] + c^4*x*(6454*Sqrt[a - (a*b)/c] - 7014*Sqrt[a + b*x] +
8704*Sqrt[a + c*x])))))/(35*c^3*(a*(b - c) + Sqrt[a - (a*b)/c]*c*Sqrt[a + b
*x])^7)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.89

method	result
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4(bx+a)^{\frac{5}{2}}a}{5}+\frac{2(bx+a)^{\frac{3}{2}}a^2}{3}}{(b-c)^3 b^2}+\frac{8a\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{(bx+a)^{\frac{3}{2}}a}{3}\right)}{(b-c)^3 b^2}-\frac{8a\left(\frac{(cx+a)^{\frac{5}{2}}}{5}-\frac{(cx+a)^{\frac{3}{2}}a}{3}\right)}{(b-c)^3 c^2}+\frac{6c\left(\frac{(bx+a)^{\frac{7}{2}}}{7}-\frac{2(bx+a)^{\frac{5}{2}}a}{5}+\right.}{(b-c)^3 b^3}$

```
[In] int(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b-c)^3/b^2*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*(b*x+a)^(3/2)*a^2)
+8/(b-c)^3*a/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-8/(b-c)^3*a/c^2*(1
/5*(c*x+a)^(5/2)-1/3*(c*x+a)^(3/2)*a)+6/(b-c)^3*c/b^3*(1/7*(b*x+a)^(7/2)-2/
5*(b*x+a)^(5/2)*a+1/3*(b*x+a)^(3/2)*a^2)-6/(b-c)^3*b/c^3*(1/7*(c*x+a)^(7/2)
-2/5*(c*x+a)^(5/2)*a+1/3*(c*x+a)^(3/2)*a^2)-2/(b-c)^3/c^2*(1/7*(c*x+a)^(7/2
)-2/5*(c*x+a)^(5/2)*a+1/3*(c*x+a)^(3/2)*a^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = -\frac{2\left(\left(16\,a^3bc^3-8\,a^3c^4-5\left(b^4c^3+3\,b^3c^4\right)x^3-\left(29\,ab^3c^3+3\,ab^2c^4\right)x^2-4\left(2\,a^2b^2c^3-a^2bc^4\right)x\right)\sqrt{bx+a}+(8}{35\left(b^6c^3-3\,b^5c^4+3\right.}$$

```
[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -2/35*((16*a^3*b*c^3 - 8*a^3*c^4 - 5*(b^4*c^3 + 3*b^3*c^4)*x^3 - (29*a*b^3*
c^3 + 3*a*b^2*c^4)*x^2 - 4*(2*a^2*b^2*c^3 - a^2*b*c^4)*x)*sqrt(b*x + a) + (
8*a^3*b^4 - 16*a^3*b^3*c + 5*(3*b^4*c^3 + b^3*c^4)*x^3 + (3*a*b^4*c^2 + 29*
a*b^3*c^3)*x^2 - 4*(a^2*b^4*c - 2*a^2*b^3*c^2)*x)*sqrt(c*x + a))/(b^6*c^3 -
 3*b^5*c^4 + 3*b^4*c^5 - b^3*c^6)
```

Sympy [F]

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

```
[In] integrate(x**4/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**4/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

Maxima [F]

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{x^4}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

```
[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
[Out] integrate(x^4/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(237) = 474$.

Time = 1.06 (sec) , antiderivative size = 932, normalized size of antiderivative = 3.36

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
[Out] -2/35*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(((b*x + a)*(5*(3*b^22*c^5*abs(b)
 - 17*b^21*c^6*abs(b) + 39*b^20*c^7*abs(b) - 45*b^19*c^8*abs(b) + 25*b^18*c
^9*abs(b) - 3*b^17*c^10*abs(b) - 3*b^16*c^11*abs(b) + b^15*c^12*abs(b))*(b*
x + a)/(b^29*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 -
126*b^24*c^10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14) + (3
*a*b^23*c^4*abs(b) - 34*a*b^22*c^5*abs(b) + 126*a*b^21*c^6*abs(b) - 210*a*b
^20*c^7*abs(b) + 140*a*b^19*c^8*abs(b) + 42*a*b^18*c^9*abs(b) - 126*a*b^17*
c^10*abs(b) + 74*a*b^16*c^11*abs(b) - 15*a*b^15*c^12*abs(b))/(b^29*c^5 - 9*
b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 - 126*b^24*c^10 + 84*b^
23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14)) - (4*a^2*b^24*c^3*abs(b)
 - 26*a^2*b^23*c^4*abs(b) + 85*a^2*b^22*c^5*abs(b) - 203*a^2*b^21*c^6*abs(b
) + 385*a^2*b^20*c^7*abs(b) - 539*a^2*b^19*c^8*abs(b) + 511*a^2*b^18*c^9*ab
s(b) - 305*a^2*b^17*c^10*abs(b) + 103*a^2*b^16*c^11*abs(b) - 15*a^2*b^15*c^
12*abs(b))/(b^29*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^
9 - 126*b^24*c^10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14))
*(b*x + a) + (8*a^3*b^25*c^2*abs(b) - 60*a^3*b^24*c^3*abs(b) + 187*a^3*b^23
*c^4*abs(b) - 296*a^3*b^22*c^5*abs(b) + 196*a^3*b^21*c^6*abs(b) + 112*a^3*b
^20*c^7*abs(b) - 350*a^3*b^19*c^8*abs(b) + 328*a^3*b^18*c^9*abs(b) - 164*a^
3*b^17*c^10*abs(b) + 44*a^3*b^16*c^11*abs(b) - 5*a^3*b^15*c^12*abs(b))/(b^2
9*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 - 126*b^24*c^
10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14)) + 2/35*(5*(b*x
 + a)^(7/2)*b + 14*(b*x + a)^(5/2)*a*b - 35*(b*x + a)^(3/2)*a^2*b + 15*(b*x
 + a)^(7/2)*c - 42*(b*x + a)^(5/2)*a*c + 35*(b*x + a)^(3/2)*a^2*c)/(b^6 - 3
*b^5*c + 3*b^4*c^2 - b^3*c^3)
```

Mupad [B] (verification not implemented)

Time = 17.55 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.55

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{x^2\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)\sqrt{a+c\,x}}{5\,c} - \frac{2\,a\left(\frac{8\,a^2}{(b-c)^3}-\frac{4\,a\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}-\frac{12\,a\,\left(b^2+3\,c\,b\right)}{7\,b\,(b-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b^2} + \frac{x\left(\frac{8\,a^2}{(b-c)^3}-\frac{4\,a\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}-\frac{12\,a\,\left(b^2+3\,c\,b\right)}{7\,b\,(b-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b} + \frac{2\,a\left(\frac{8\,a^2}{(b-c)^3}+\frac{4\,a\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)}{5\,c}\right)\sqrt{a+c\,x}}{3\,c^2} + \frac{x^2\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}-\frac{12\,a\,\left(b^2+3\,c\,b\right)}{7\,b\,(b-c)^3}\right)\sqrt{a+b\,x}}{5\,b} - \frac{x\left(\frac{8\,a^2}{(b-c)^3}+\frac{4\,a\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)}{5\,c}\right)\sqrt{a+c\,x}}{3\,c} - \frac{2\,x^3\,(3\,b+c)\,\sqrt{a+c\,x}}{7\,(b-c)^3} + \frac{2\,x^3\,\left(b^2+3\,c\,b\right)\sqrt{a+b\,x}}{7\,b\,(b-c)^3}$$

```
[In] int(x^4/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (x^2*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^3)*(a + c*
x)^(1/2))/(5*c) - (2*a*((8*a^2)/(b - c)^3 - (4*a*((2*a*(5*b + 3*c))/(b - c)
^3 - (12*a*(3*b*c + b^2))/(7*b*(b - c)^3)))/(5*b))*(a + b*x)^(1/2))/(3*b^2)
 + (x*((8*a^2)/(b - c)^3 - (4*a*((2*a*(5*b + 3*c))/(b - c)^3 - (12*a*(3*b*c
 + b^2))/(7*b*(b - c)^3)))/(5*b))*(a + b*x)^(1/2))/(3*b) + (2*a*((8*a^2)/(b
 - c)^3 + (4*a*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^
3))/(5*c))*(a + c*x)^(1/2))/(3*c^2) + (x^2*((2*a*(5*b + 3*c))/(b - c)^3 - (
12*a*(3*b*c + b^2))/(7*b*(b - c)^3))*(a + b*x)^(1/2))/(5*b) - (x*((8*a^2)/(
b - c)^3 + (4*a*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)
^3))/(5*c))*(a + c*x)^(1/2))/(3*c) - (2*x^3*(3*b + c)*(a + c*x)^(1/2))/(7*(
b - c)^3) + (2*x^3*(3*b*c + b^2)*(a + b*x)^(1/2))/(7*b*(b - c)^3)
```

3.439 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 163

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} dx = \frac{8a(a+bx)^{3/2}}{3b(b-c)^3} - \frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3} + \frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{8a(a+cx)^{3/2}}{3(b-c)^3c} + \frac{2a(3b+c)(a+cx)^{3/2}}{3(b-c)^3c^2} - \frac{2(3b+c)(a+cx)^{5/2}}{5(b-c)^3c^2}$$

```
[Out] 8/3*a*(b*x+a)^(3/2)/b/(b-c)^3-2/3*a*(b+3*c)*(b*x+a)^(3/2)/b^2/(b-c)^3+2/5*(
b+3*c)*(b*x+a)^(5/2)/b^2/(b-c)^3-8/3*a*(c*x+a)^(3/2)/(b-c)^3/c+2/3*a*(3*b+c
)*(c*x+a)^(3/2)/(b-c)^3/c^2-2/5*(3*b+c)*(c*x+a)^(5/2)/(b-c)^3/c^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6822, 45}

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} dx = \frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3} - \frac{2(3b+c)(a+cx)^{5/2}}{5c^2(b-c)^3} + \frac{2a(3b+c)(a+cx)^{3/2}}{3c^2(b-c)^3} + \frac{8a(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a(a+cx)^{3/2}}{3c(b-c)^3}$$

```
[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (8*a*(a + b*x)^(3/2))/(3*b*(b - c)^3) - (2*a*(b + 3*c)*(a + b*x)^(3/2))/(3*
b^2*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(5/2))/(5*b^2*(b - c)^3) - (8*a*(a
+ c*x)^(3/2))/(3*(b - c)^3*c) + (2*a*(3*b + c)*(a + c*x)^(3/2))/(3*(b - c)^
3*c^2) - (2*(3*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \left(4a\sqrt{a+bx}+b\left(1+\frac{3c}{b}\right)x\sqrt{a+bx}-4a\sqrt{a+cx}-3b\left(1+\frac{c}{3b}\right)x\sqrt{a+cx}\right)dx}{(b-c)^3}$$

$$= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3(b-c)^3c}-\frac{(3b+c)\int x\sqrt{a+cx}\,dx}{(b-c)^3}+\frac{(b+3c)\int x\sqrt{a+bx}\,dx}{(b-c)^3}$$

$$= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3(b-c)^3c}-\frac{(3b+c)\int\left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right)dx}{(b-c)^3}+\frac{(b+3c)\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{(b-c)^3}$$

$$= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3}+\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3(b-c)^3c}+\frac{2a(3b+c)(a+cx)^{3/2}}{3(b-c)^3c^2}-\frac{2(3b+c)(a+cx)^{5/2}}{5(b-c)^3c^2}$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \frac{2\sqrt{a-\frac{ab}{c}+\frac{b(a+cx)}{c}}\left(a^2b^3-4a^2b^2c+5a^2bc^2-2a^2c^3-2ab^3(a+cx)+ab^2c(a+cx)+abc^2(a+cx)+b^3(a\right.}{5b^2(b-c)^3c^2}$$
$$+\frac{2\left(5ab(a+cx)^{3/2}-5ac(a+cx)^{3/2}-3b(a+cx)^{5/2}-c(a+cx)^{5/2}\right)}{5(b-c)^3c^2}$$

```
[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(a^2*b^3 - 4*a^2*b^2*c + 5*a^2*b*c^2
 - 2*a^2*c^3 - 2*a*b^3*(a + c*x) + a*b^2*c*(a + c*x) + a*b*c^2*(a + c*x) +
b^3*(a + c*x)^2 + 3*b^2*c*(a + c*x)^2))/(5*b^2*(b - c)^3*c^2) + (2*(5*a*b*(
a + c*x)^(3/2) - 5*a*c*(a + c*x)^(3/2) - 3*b*(a + c*x)^(5/2) - c*(a + c*x)^
(5/2)))/(5*(b - c)^3*c^2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.06

method	result
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2(bx+a)^{\frac{3}{2}}a}{3}}{(b-c)^3 b}+\frac{8a(bx+a)^{\frac{3}{2}}}{3b(b-c)^3}-\frac{8a(cx+a)^{\frac{3}{2}}}{3(b-c)^3 c}+\frac{6c\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{(bx+a)^{\frac{3}{2}}a}{3}\right)}{(b-c)^3 b^2}-\frac{6b\left(\frac{(cx+a)^{\frac{5}{2}}}{5}-\frac{(cx+a)^{\frac{3}{2}}a}{3}\right)}{(b-c)^3 c^2}-\frac{2\left(\frac{(cx+a)}{5}\right.}{(}$

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b-c)^3/b*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)+8/3*a*(b*x+a)^(3/2)/b/(
b-c)^3-8/3*a*(c*x+a)^(3/2)/(b-c)^3/c+6/(b-c)^3*c/b^2*(1/5*(b*x+a)^(5/2)-1/3
*(b*x+a)^(3/2)*a)-6/(b-c)^3*b/c^2*(1/5*(c*x+a)^(5/2)-1/3*(c*x+a)^(3/2)*a)-2
/(b-c)^3/c*(1/5*(c*x+a)^(5/2)-1/3*(c*x+a)^(3/2)*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$=\frac{2\left((6\,a^2bc^2-2\,a^2c^3+(b^3c^2+3\,b^2c^3)x^2+(7\,ab^2c^2+abc^3)x)\sqrt{bx+a}+(2\,a^2b^3-6\,a^2b^2c-(3\,b^3c^2+b^2c^3)x^2\right.}{5\left(b^5c^2-3\,b^4c^3+3\,b^3c^4-b^2c^5\right)}$$

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/5*((6*a^2*b*c^2 - 2*a^2*c^3 + (b^3*c^2 + 3*b^2*c^3)*x^2 + (7*a*b^2*c^2 +
a*b*c^3)*x)*sqrt(b*x + a) + (2*a^2*b^3 - 6*a^2*b^2*c - (3*b^3*c^2 + b^2*c^3
)*x^2 - (a*b^3*c + 7*a*b^2*c^2)*x)*sqrt(c*x + a))/(b^5*c^2 - 3*b^4*c^3 + 3*
b^3*c^4 - b^2*c^5)
```

Sympy [F]

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

Maxima [F]

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{x^3}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(139) = 278$.

Time = 1.02 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.94

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx =$$

$$-\frac{2}{5}\sqrt{ab^2+(bx+a)bc-abc}\left((bx+a)\left(\frac{(3\,b^{12}c^3|b|-8\,b^{11}c^4|b|+6\,b^{10}c^5|b|-b^8c^7|b|)(bx+a)}{b^{18}c^3-6\,b^{17}c^4+15\,b^{16}c^5-20\,b^{15}c^6+15\,b^{14}c^7-6\,b^{13}c^8+b^{12}c^9}\right.\right.$$

$$+\frac{2\left((bx+a)^{\frac{5}{2}}b+5\,(bx+a)^{\frac{3}{2}}ab+3\,(bx+a)^{\frac{5}{2}}c-5\,(bx+a)^{\frac{3}{2}}ac\right)}{5\,(b^5-3\,b^4c+3\,b^3c^2-b^2c^3)}$$

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
[Out] -2/5*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((b*x + a)*((3*b^12*c^3*abs(b) - 8
*b^11*c^4*abs(b) + 6*b^10*c^5*abs(b) - b^8*c^7*abs(b))*(b*x + a)/(b^18*c^3
- 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^12*
c^9) + (a*b^13*c^2*abs(b) - 2*a*b^12*c^3*abs(b) - 2*a*b^11*c^4*abs(b) + 8*a
*b^10*c^5*abs(b) - 7*a*b^9*c^6*abs(b) + 2*a*b^8*c^7*abs(b))/(b^18*c^3 - 6*b
^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^12*c^9))
 - (2*a^2*b^14*c*abs(b) - 11*a^2*b^13*c^2*abs(b) + 25*a^2*b^12*c^3*abs(b) -
 30*a^2*b^11*c^4*abs(b) + 20*a^2*b^10*c^5*abs(b) - 7*a^2*b^9*c^6*abs(b) + a
^2*b^8*c^7*abs(b))/(b^18*c^3 - 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*
b^14*c^7 - 6*b^13*c^8 + b^12*c^9)) + 2/5*((b*x + a)^(5/2)*b + 5*(b*x + a)^(
3/2)*a*b + 3*(b*x + a)^(5/2)*c - 5*(b*x + a)^(3/2)*a*c)/(b^5 - 3*b^4*c + 3*
b^3*c^2 - b^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 16.86 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.64

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\left(\frac{8\,a^2}{(b-c)^3}+\frac{2\,a\,\left(\frac{8\,a\,(b+3\,c)}{5\,(b-c)^3}-\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}\right)}{3\,b}\right)\sqrt{a+b\,x}}{b}$$

$$-\frac{\left(\frac{8\,a^2}{(b-c)^3}+\frac{2\,a\,\left(\frac{8\,a\,(3\,b+c)}{5\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)}{3\,c}\right)\sqrt{a+c\,x}}{c}$$

$$-\frac{x\,\left(\frac{8\,a\,(b+3\,c)}{5\,(b-c)^3}-\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}\right)\sqrt{a+b\,x}}{3\,b}$$

$$+\frac{x\,\left(\frac{8\,a\,(3\,b+c)}{5\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)\sqrt{a+c\,x}}{3\,c}$$

$$+\frac{2\,x^2\,(b+3\,c)\,\sqrt{a+b\,x}}{5\,(b-c)^3}-\frac{2\,x^2\,(3\,b+c)\,\sqrt{a+c\,x}}{5\,(b-c)^3}$$

```
[In] int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (((8*a^2)/(b - c)^3 + (2*a*((8*a*(b + 3*c))/(5*(b - c)^3) - (2*a*(5*b + 3*c
))/(b - c)^3))/(3*b))*(a + b*x)^(1/2))/b - (((8*a^2)/(b - c)^3 + (2*a*((8*a
*(3*b + c))/(5*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^3))/(3*c))*(a + c*x)^
(1/2))/c - (x*((8*a*(b + 3*c))/(5*(b - c)^3) - (2*a*(5*b + 3*c))/(b - c)^3)
*(a + b*x)^(1/2))/(3*b) + (x*((8*a*(3*b + c))/(5*(b - c)^3) - (2*a*(3*b + 5
*c))/(b - c)^3)*(a + c*x)^(1/2))/(3*c) + (2*x^2*(b + 3*c)*(a + b*x)^(1/2))/
(5*(b - c)^3) - (2*x^2*(3*b + c)*(a + c*x)^(1/2))/(5*(b - c)^3)
```

3.440 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} dx = \frac{8a\sqrt{a+bx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c}-\frac{8a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

```
[Out] 2/3*(b+3*c)*(b*x+a)^(3/2)/b/(b-c)^3-2/3*(3*b+c)*(c*x+a)^(3/2)/(b-c)^3/c-8*a
^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))/(b-c)^3+8*a^(3/2)*arctanh((c*x+a)^(1/
2)/a^(1/2))/(b-c)^3+8*a*(b*x+a)^(1/2)/(b-c)^3-8*a*(c*x+a)^(1/2)/(b-c)^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6822, 52, 65, 214}

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} dx = -\frac{8a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a\sqrt{a+bx}}{(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3c(b-c)^3}$$

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (8*a*Sqrt[a + b*x])/(b - c)^3 + (2*(b + 3*c)*(a + b*x)^(3/2))/(3*b*(b - c)^
3) - (8*a*Sqrt[a + c*x])/(b - c)^3 - (2*(3*b + c)*(a + c*x)^(3/2))/(3*(b -
c)^3*c) - (8*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (8*a^(3/2)
*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)^3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \left(b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}+\frac{4a\sqrt{a+bx}}{x}-3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}-\frac{4a\sqrt{a+cx}}{x}\right)dx}{(b-c)^3}$$

$$= \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c}+\frac{(4a)\int \frac{\sqrt{a+bx}}{x}\,dx}{(b-c)^3}-\frac{(4a)\int \frac{\sqrt{a+cx}}{x}\,dx}{(b-c)^3}$$

$$= \frac{8a\sqrt{a+bx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c}+\frac{(4a^2)\int \frac{1}{x\sqrt{a+bx}}\,dx}{(b-c)^3}-\frac{(4a^2)\int \frac{1}{x\sqrt{a+cx}}\,dx}{(b-c)^3}$$

$$= \frac{8a\sqrt{a+bx}}{(b-c)^3} + \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a\sqrt{a+cx}}{(b-c)^3} - \frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c} + \frac{(8a^2)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b(b-c)^3} - \frac{(8a^2)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx}\right)}{(b-c)^3c}$$

$$= \frac{8a\sqrt{a+bx}}{(b-c)^3} + \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a\sqrt{a+cx}}{(b-c)^3} - \frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c} - \frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3} + \frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1004 vs. $2(155) = 310$.

Time = 5.57 (sec) , antiderivative size = 1004, normalized size of antiderivative = 6.48

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

$$= \frac{-2a\sqrt{b-c}\left(b\sqrt{a-\frac{ab}{c}}c^3x^2\left(b^2x+3bcx-3b\sqrt{a+bx}\sqrt{a+cx}-c\sqrt{a+bx}\sqrt{a+cx}\right)+a^3\left(bc^2\left(12\sqrt{a-\frac{ab}{c}}\right.\right.\right.}{}$$

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (-2*a*Sqrt[b - c]*(b*Sqrt[a - (a*b)/c]*c^3*x^2*(b^2*x + 3*b*c*x - 3*b*Sqrt[
a + b*x]*Sqrt[a + c*x] - c*Sqrt[a + b*x]*Sqrt[a + c*x]) + a^3*(b*c^2*(12*Sq
rt[a - (a*b)/c] - 3*Sqrt[a + b*x] - 53*Sqrt[a + c*x]) + b^2*c*(-15*Sqrt[a -
 (a*b)/c] + 24*Sqrt[a + b*x] - 2*Sqrt[a + c*x]) + b^3*(Sqrt[a - (a*b)/c] -
3*Sqrt[a + b*x] + 3*Sqrt[a + c*x]) + 2*c^3*(9*Sqrt[a - (a*b)/c] - 9*Sqrt[a
+ b*x] + 26*Sqrt[a + c*x])) + a*c^2*x*(-4*Sqrt[a - (a*b)/c]*c^2*Sqrt[a + b*
x]*Sqrt[a + c*x] + b^3*x*(-3*Sqrt[a - (a*b)/c] + 3*Sqrt[a + b*x] - 9*Sqrt[a
 + c*x]) + b*(-22*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x]*Sqrt[a + c*x] + 3*c^2*x
*(6*Sqrt[a - (a*b)/c] - 3*Sqrt[a + b*x] + Sqrt[a + c*x])) + b^2*(6*Sqrt[a -
 (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(9*Sqrt[a - (a*b)/c] + 6*Sqrt[a
 + b*x] + 6*Sqrt[a + c*x]))) + a^2*(-3*b^3*c*x*(Sqrt[a - (a*b)/c] + 2*Sqrt[
a + c*x]) + b^2*(9*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x]*Sqrt[a + c*x] + c^2*x*
(-9*Sqrt[a - (a*b)/c] + 30*Sqrt[a + b*x] - 44*Sqrt[a + c*x])) + 2*c^3*(-26*
Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(9*Sqrt[a - (a*b)/c] -
9*Sqrt[a + b*x] + 2*Sqrt[a + c*x])) + b*(27*Sqrt[a - (a*b)/c]*c^2*Sqrt[a +
b*x]*Sqrt[a + c*x] + c^3*x*(30*Sqrt[a - (a*b)/c] - 12*Sqrt[a + b*x] + 46*Sq
rt[a + c*x])))) - 48*a^3*c^(3/2)*(-b + c)*(b*c*x*(3*Sqrt[a - (a*b)/c] - Sqr
t[a + b*x]) + a*(-(b*Sqrt[a - (a*b)/c]) + 4*Sqrt[a - (a*b)/c]*c + 3*b*Sqrt[
```

```
a + b*x] - 4*c*Sqrt[a + b*x]))*ArcTan[(Sqrt[b - c]*Sqrt[a + c*x])/(Sqrt[c]*
(-Sqrt[a - (a*b)/c] + Sqrt[a + b*x] + Sqrt[a + c*x]))])/(3*(b - c)^(5/2)*c*
(a*(b - c) + Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x])^3)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

method	result
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3(b-c)^3}+\frac{2c(bx+a)^{\frac{3}{2}}}{(b-c)^3 b}-\frac{2b(cx+a)^{\frac{3}{2}}}{(b-c)^3 c}-\frac{2(cx+a)^{\frac{3}{2}}}{3(b-c)^3}+\frac{4a\left(2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{(b-c)^3}-\frac{4a\left(2\sqrt{cx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\right.\right.}{(b-c)^3}$

```
[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(b-c)^3*(b*x+a)^(3/2)+2/(b-c)^3*c*(b*x+a)^(3/2)/b-2/(b-c)^3*b*(c*x+a)^(
3/2)/c-2/3/(b-c)^3*(c*x+a)^(3/2)+4*a/(b-c)^3*(2*(b*x+a)^(1/2)-2*a^(1/2)*arc
tanh((b*x+a)^(1/2)/a^(1/2)))-4*a/(b-c)^3*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh
((c*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.07

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$=\left[-\frac{2\left(6\,a^{\frac{3}{2}}bc\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+6\,a^{\frac{3}{2}}bc\log\left(\frac{cx-2\,\sqrt{cx+a}\sqrt{a}+2\,a}{x}\right)-(13\,abc+3\,ac^2+(b^2c+3\,bc^2)x)\sqrt{bx}\right.}{3\,(b^4c-3\,b^3c^2+3\,b^2c^3-bc^4)}\right.$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-2/3*(6*a^(3/2)*b*c*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 6*a^(3/
2)*b*c*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - (13*a*b*c + 3*a*c^2 +
 (b^2*c + 3*b*c^2)*x)*sqrt(b*x + a) + (3*a*b^2 + 13*a*b*c + (3*b^2*c + b*c^
2)*x)*sqrt(c*x + a))/(b^4*c - 3*b^3*c^2 + 3*b^2*c^3 - b*c^4), 2/3*(12*sqrt(
-a)*a*b*c*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 12*sqrt(-a)*a*b*c*arctan(sqrt(
c*x + a)*sqrt(-a)/a) + (13*a*b*c + 3*a*c^2 + (b^2*c + 3*b*c^2)*x)*sqrt(b*x
+ a) - (3*a*b^2 + 13*a*b*c + (3*b^2*c + b*c^2)*x)*sqrt(c*x + a))/(b^4*c - 3
*b^3*c^2 + 3*b^2*c^3 - b*c^4)]
```

Sympy [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

```
[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2374 vs. $2(131) = 262$.

Time = 1.58 (sec) , antiderivative size = 2374, normalized size of antiderivative = 15.32

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
[Out] -2/3*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((3*b^7*c*abs(b) - 8*b^6*c^2*abs(b
) + 6*b^5*c^3*abs(b) - b^3*c^5*abs(b))*(b*x + a)/(b^12*c - 6*b^11*c^2 + 15*
b^10*c^3 - 20*b^9*c^4 + 15*b^8*c^5 - 6*b^7*c^6 + b^6*c^7) + (3*a*b^8*abs(b)
 + a*b^7*c*abs(b) - 22*a*b^6*c^2*abs(b) + 30*a*b^5*c^3*abs(b) - 13*a*b^4*c^
4*abs(b) + a*b^3*c^5*abs(b))/(b^12*c - 6*b^11*c^2 + 15*b^10*c^3 - 20*b^9*c^
4 + 15*b^8*c^5 - 6*b^7*c^6 + b^6*c^7)) + 8*a^2*arctan(sqrt(b*x + a)/sqrt(-a
))/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*sqrt(-a)) + 2/3*((b*x + a)^(3/2)*b^9 +
12*sqrt(b*x + a)*a*b^9 - 3*(b*x + a)^(3/2)*b^8*c - 72*sqrt(b*x + a)*a*b^8*c
 - 3*(b*x + a)^(3/2)*b^7*c^2 + 180*sqrt(b*x + a)*a*b^7*c^2 + 25*(b*x + a)^(
3/2)*b^6*c^3 - 240*sqrt(b*x + a)*a*b^6*c^3 - 45*(b*x + a)^(3/2)*b^5*c^4 + 1
80*sqrt(b*x + a)*a*b^5*c^4 + 39*(b*x + a)^(3/2)*b^4*c^5 - 72*sqrt(b*x + a)*
a*b^4*c^5 - 17*(b*x + a)^(3/2)*b^3*c^6 + 12*sqrt(b*x + a)*a*b^3*c^6 + 3*(b*
x + a)^(3/2)*b^2*c^7)/(b^12 - 9*b^11*c + 36*b^10*c^2 - 84*b^9*c^3 + 126*b^8
*c^4 - 126*b^7*c^5 + 84*b^6*c^6 - 36*b^5*c^7 + 9*b^4*c^8 - b^3*c^9) - 8*(2*
```

(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3*c - a*b^2*c^2)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3 - a*b^2*c)*sqrt(-a*b*c)*abs(b) + (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*sqrt(-a*b*c)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^8 - 5*a^2*b^7*c + 10*a^2*b^6*c^2 - 10*a^2*b^5*c^3 + 5*a^2*b^4*c^4 - a^2*b^3*c^5)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^11*c - 6*a^3*b^10*c^2 + 14*a^3*b^9*c^3 - 14*a^3*b^8*c^4 + 14*a^3*b^6*c^6 - 14*a^3*b^5*c^7 + 6*a^3*b^4*c^8 - a^3*b^3*c^9)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^11 - 6*a^3*b^10*c + 14*a^3*b^9*c^2 - 14*a^3*b^8*c^3 + 14*a^3*b^6*c^5 - 14*a^3*b^5*c^6 + 6*a^3*b^4*c^7 - a^3*b^3*c^8)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 + sqrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^12 - 9*b^11*c + 36*b^10*c^2 - 84*b^9*c^3 + 126*b^8*c^4 - 126*b^7*c^5 + 84*b^6*c^6 - 36*b^5*c^7 + 9*b^4*c^8 - b^3*c^9)*a*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)) + 8*(2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3*c - a*b^2*c^2)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3 - a*b^2*c)*sqrt(-a*b*c)*abs(b) + (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*sqrt(-a*b*c)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^8 - 5*a^2*b^7*c + 10*a^2*b^6*c^2 - 10*a^2*b^5*c^3 + 5*a^2*b^4*c^4 - a^2*b^3*c^5)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^11*c - 6*a^3*b^10*c^2 + 14*a^3*b^9*c^3 - 14*a^3*b^8*c^4 + 14*a^3*b^6*c^6 - 14*a^3*b^5*c^7 + 6*a^3*b^4*c^8 - a^3*b^3*c^9)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^11 - 6*a^3*b^10*c + 14*a^3*b^9*c^2 - 14*a^3*b^8*c^3 + 14*a^3*b^6*c^5 - 14*a^3*b^5*c^6 + 6*a^3*b^4*c^7 - a^3*b^3*c^8)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 - sqrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^12 - 9*b^11*c + 36*b^10*c^2 - 84*b^9*c^3 + 126*b^8*c^4 - 126*b^7*c^5 + 84*b^6*c^6 - 36*b^5*c^7 + 9*b^4*c^8 - b^3*c^9)*a*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3))

Mupad [B] (verification not implemented)

Time = 20.35 (sec) , antiderivative size = 762, normalized size of antiderivative = 4.92

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \frac{4\,a^{3/2}\,b^4-\frac{4\,a^{3/2}\,c^4\,\left(\frac{4\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^3}-\frac{15\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^4}+\frac{24\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^5}+\frac{6\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^6}\right)}{3}-\frac{4\,a^{3/2}\,b^2\,c^2\,\left(\frac{24\,\left(\sqrt{a+b\right.}{\sqrt{a+c\,x}}\right.}{}}{}$$

```
[In] int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (4*a^(3/2)*b^4 - (4*a^(3/2)*c^4*((4*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*
x)^(1/2) - a^(1/2))^3 - (15*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2)
 - a^(1/2))^4 + (24*((a + b*x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^(1/
2))^5 + (6*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a
 + b*x)^(1/2) - a^(1/2))^6)/((a + c*x)^(1/2) - a^(1/2))^6))/3 - (4*a^(3/2)*
b^2*c^2*((24*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + (12
*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 + (12*((a + b
*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 - (15*((a + b*x)^(1/2
) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4 + (18*log(((a + b*x)^(1/2) -
a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^2)/((a +
c*x)^(1/2) - a^(1/2))^2 - 3))/3 + (4*a^(3/2)*b*c^3*((6*((a + b*x)^(1/2) - a
^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - (12*((a + b*x)^(1/2) - a^(1/2))^
3)/((a + c*x)^(1/2) - a^(1/2))^3 + (66*((a + b*x)^(1/2) - a^(1/2))^4)/((a +
 c*x)^(1/2) - a^(1/2))^4 - (24*((a + b*x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1
/2) - a^(1/2))^5 + (18*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a
^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4))/3 +
 (4*a^(3/2)*b^3*c*(6*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(
1/2))) - (24*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + (6*
((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - (4*((a + b*x
)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + 26))/3)/(c*(b - c)^3*
(b - (c*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2)^3)
```

3.441 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}-\frac{6\sqrt{a}(b+c)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{6\sqrt{a}(b+c)\operatorname{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

```
[Out] -6*(b+c)*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/(b-c)^3+6*(b+c)*arctanh((c*
x+a)^(1/2)/a^(1/2))*a^(1/2)/(b-c)^3+2*(b+3*c)*(b*x+a)^(1/2)/(b-c)^3-4*a*(b*
x+a)^(1/2)/(b-c)^3/x-2*(3*b+c)*(c*x+a)^(1/2)/(b-c)^3+4*a*(c*x+a)^(1/2)/(b-c
)^3/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.42, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6822, 43, 65, 214, 52}

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = -\frac{2\sqrt{a}(b+3c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3} - \frac{4\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3} + \frac{4\sqrt{a}c\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3} + \frac{2\sqrt{a}(3b+c)\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3} - \frac{4a\sqrt{a+bx}}{x(b-c)^3} + \frac{4a\sqrt{a+cx}}{x(b-c)^3} + \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3} - \frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}$$

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*(b + 3*c)*Sqrt[a + b*x])/(b - c)^3 - (4*a*Sqrt[a + b*x])/((b - c)^3*x) -
 (2*(3*b + c)*Sqrt[a + c*x])/(b - c)^3 + (4*a*Sqrt[a + c*x])/((b - c)^3*x)
- (4*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 - (2*Sqrt[a]*(b +
3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (4*Sqrt[a]*c*ArcTanh[Sqrt[
a + c*x]/Sqrt[a]])/(b - c)^3 + (2*Sqrt[a]*(3*b + c)*ArcTanh[Sqrt[a + c*x]/S
qrt[a]])/(b - c)^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(\frac{4a\sqrt{a+bx}}{x^2}+\frac{b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}}{x}-\frac{4a\sqrt{a+cx}}{x^2}-\frac{3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}}{x}\right)dx}{(b-c)^3}\\
&= \frac{(4a)\int \frac{\sqrt{a+bx}}{x^2}\,dx}{(b-c)^3}-\frac{(4a)\int \frac{\sqrt{a+cx}}{x^2}\,dx}{(b-c)^3}-\frac{(3b+c)\int \frac{\sqrt{a+cx}}{x}\,dx}{(b-c)^3}+\frac{(b+3c)\int \frac{\sqrt{a+bx}}{x}\,dx}{(b-c)^3}\\
&= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}\\
&\quad+\frac{4a\sqrt{a+cx}}{(b-c)^3x}+\frac{(2ab)\int \frac{1}{x\sqrt{a+bx}}\,dx}{(b-c)^3}-\frac{(2ac)\int \frac{1}{x\sqrt{a+cx}}\,dx}{(b-c)^3}\\
&\quad-\frac{(a(3b+c))\int \frac{1}{x\sqrt{a+cx}}\,dx}{(b-c)^3}+\frac{(a(b+3c))\int \frac{1}{x\sqrt{a+bx}}\,dx}{(b-c)^3}\\
&= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}\\
&\quad+\frac{(4a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{(b-c)^3}-\frac{(4a)\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx}\right)}{(b-c)^3}\\
&\quad-\frac{(2a(3b+c))\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx}\right)}{(b-c)^3c}\\
&\quad+\frac{(2a(b+3c))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b(b-c)^3}
\end{aligned}$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3} - \frac{4a\sqrt{a+bx}}{(b-c)^3x} - \frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3} + \frac{4a\sqrt{a+cx}}{(b-c)^3x} - \frac{4\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3} - \frac{2\sqrt{a}(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3} + \frac{4\sqrt{a}c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3} + \frac{2\sqrt{a}(3b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 690 vs. $2(157) = 314$.

Time = 10.68 (sec) , antiderivative size = 690, normalized size of antiderivative = 4.39

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

$$= \frac{2\sqrt{b-c}\left(b\sqrt{a-\frac{ab}{c}}c^2x\left(bx+cx-2\sqrt{a+bx}\sqrt{a+cx}\right)+a\left(4b\sqrt{a-\frac{ab}{c}}c\sqrt{a+bx}\sqrt{a+cx}+6\sqrt{a-\frac{ab}{c}}c^2\sqrt{\right.\right.}}{}$$

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*Sqrt[b - c]*(b*Sqrt[a - (a*b)/c]*c^2*x*(b*x + c*x - 2*Sqrt[a + b*x]*Sqrt
[a + c*x]) + a*(4*b*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x]*Sqrt[a + c*x] + 6*Sqr
t[a - (a*b)/c]*c^2*Sqrt[a + b*x]*Sqrt[a + c*x] + 2*c^3*x*(Sqrt[a - (a*b)/c]
 - Sqrt[a + b*x]) + b^2*c*x*(Sqrt[a - (a*b)/c] + Sqrt[a + b*x] - 5*Sqrt[a +
 c*x]) + b*c^2*x*(7*Sqrt[a - (a*b)/c] - 5*Sqrt[a + b*x] - Sqrt[a + c*x])) +
 a^2*(2*c^2*(Sqrt[a - (a*b)/c] - Sqrt[a + b*x] - 3*Sqrt[a + c*x]) + b*c*(6*
Sqrt[a - (a*b)/c] - 5*Sqrt[a + b*x] - Sqrt[a + c*x]) + b^2*(Sqrt[a + b*x] +
 Sqrt[a + c*x]))) - 12*a*Sqrt[c]*(b + c)*(2*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*
x]*Sqrt[a + c*x] + 2*a*c*(Sqrt[a - (a*b)/c] - Sqrt[a + b*x] - Sqrt[a + c*x]
) + b*c*x*(2*Sqrt[a - (a*b)/c] - Sqrt[a + b*x] - Sqrt[a + c*x]) + a*b*(Sqrt
[a + b*x] + Sqrt[a + c*x]))*ArcTan[(Sqrt[b - c]*Sqrt[a + c*x])/(Sqrt[c]*(-S
qrt[a - (a*b)/c] + Sqrt[a + b*x] + Sqrt[a + c*x]))]])/((b - c)^(5/2)*c*(a*(b
 - c) + Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x])*(a + b*x - Sqrt[a - (a*b)/c]*Sqr
t[a + b*x] - Sqrt[a - (a*b)/c]*Sqrt[a + c*x] + Sqrt[a + b*x]*Sqrt[a + c*x])
)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.51

method	result
default	$\frac{b\left(2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{(b-c)^3}+\frac{8ab\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(b-c)^3}-\frac{8ac\left(-\frac{\sqrt{cx+a}}{2cx}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(b-c)^3}+\frac{3c\left(2\sqrt{bx+c\right.}}{}$

```
[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))+8*a/
(b-c)^3*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
))-8*a/(b-c)^3*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)/
a^(1/2)))+3/(b-c)^3*c*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1
/2)))-3/(b-c)^3*b*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2))
)-1/(b-c)^3*c*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.66

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$=\left[-\frac{3\sqrt{a}(b+c)x\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+3\sqrt{a}(b+c)x\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)-2\left((b+3c)x-2a\right)\sqrt{bx+a}}{(b^3-3b^2c+3bc^2-c^3)x}\right.$$

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-(3*sqrt(a)*(b + c)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*sqr
t(a)*(b + c)*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*((b + 3*c)*
x - 2*a)*sqrt(b*x + a) + 2*((3*b + c)*x - 2*a)*sqrt(c*x + a))/((b^3 - 3*b^2
*c + 3*b*c^2 - c^3)*x), 2*(3*sqrt(-a)*(b + c)*x*arctan(sqrt(b*x + a)*sqrt(-
a)/a) - 3*sqrt(-a)*(b + c)*x*arctan(sqrt(c*x + a)*sqrt(-a)/a) + ((b + 3*c)*
x - 2*a)*sqrt(b*x + a) - ((3*b + c)*x - 2*a)*sqrt(c*x + a))/((b^3 - 3*b^2*c
 + 3*b*c^2 - c^3)*x)]
```

Sympy [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

```
[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

Maxima [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \int \frac{x}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2318 vs. $2(137) = 274$.

Time = 9.20 (sec) , antiderivative size = 2318, normalized size of antiderivative = 14.76

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
[Out] -2*(sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(3*b*abs(b) + c*abs(b))/(b^4 - 3*b^
3*c + 3*b^2*c^2 - b*c^3) + 2*sqrt(b*x + a)*a*b/((b^3 - 3*b^2*c + 3*b*c^2 -
c^3)*x) - 3*(a*b^2 + a*b*c)*arctan(sqrt(b*x + a)/sqrt(-a))/((b^3 - 3*b^2*c
+ 3*b*c^2 - c^3)*sqrt(-a)) - (sqrt(b*x + a)*b^2 + 3*sqrt(b*x + a)*b*c)/(b^3
 - 3*b^2*c + 3*b*c^2 - c^3) + 4*((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b
*x + a)*b*c - a*b*c))*a^2*b^3*c*abs(b) - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*
b^2 + (b*x + a)*b*c - a*b*c))*a^2*b^2*c^2*abs(b) + (sqrt(b*c)*sqrt(b*x + a)
 - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a*b*c*abs(b))/((a^2*b^4 - 2*a^2*b
^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b
*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))^4)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)) + 3*(2*(a*b^4*c - a*b^2*c^3
)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*sqrt(-a)*abs(b)*sgn(b^3 - 3
*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(
```

a*b^4 - a*b^2*c^2)*sqrt(-a*b*c)*abs(b) + (a^2*b^8 - 4*a^2*b^7*c + 5*a^2*b^6*c^2 - 5*a^2*b^4*c^4 + 4*a^2*b^3*c^5 - a^2*b^2*c^6)*sqrt(-a*b*c)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^9 - 4*a^2*b^8*c + 5*a^2*b^7*c^2 - 5*a^2*b^5*c^4 + 4*a^2*b^4*c^5 - a^2*b^3*c^6)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^12*c - 5*a^3*b^11*c^2 + 8*a^3*b^10*c^3 - 14*a^3*b^8*c^5 + 14*a^3*b^7*c^6 - 8*a^3*b^5*c^8 + 5*a^3*b^4*c^9 - a^3*b^3*c^10)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^12 - 5*a^3*b^11*c + 8*a^3*b^10*c^2 - 14*a^3*b^8*c^4 + 14*a^3*b^7*c^5 - 8*a^3*b^5*c^7 + 5*a^3*b^4*c^8 - a^3*b^3*c^9)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 + sqrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^11 - 9*b^10*c + 36*b^9*c^2 - 84*b^8*c^3 + 126*b^7*c^4 - 126*b^6*c^5 + 84*b^5*c^6 - 36*b^4*c^7 + 9*b^3*c^8 - b^2*c^9)*a^2*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)) - 3*(2*(a*b^4*c - a*b^2*c^3)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^4 - a*b^2*c^2)*sqrt(-a*b*c)*abs(b) + (a^2*b^8 - 4*a^2*b^7*c + 5*a^2*b^6*c^2 - 5*a^2*b^4*c^4 + 4*a^2*b^3*c^5 - a^2*b^2*c^6)*sqrt(-a*b*c)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^9 - 4*a^2*b^8*c + 5*a^2*b^7*c^2 - 5*a^2*b^5*c^4 + 4*a^2*b^4*c^5 - a^2*b^3*c^6)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^12*c - 5*a^3*b^11*c^2 + 8*a^3*b^10*c^3 - 14*a^3*b^8*c^5 + 14*a^3*b^7*c^6 - 8*a^3*b^5*c^8 + 5*a^3*b^4*c^9 - a^3*b^3*c^10)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^12 - 5*a^3*b^11*c + 8*a^3*b^10*c^2 - 14*a^3*b^8*c^4 + 14*a^3*b^7*c^5 - 8*a^3*b^5*c^7 + 5*a^3*b^4*c^8 - a^3*b^3*c^9)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 - sqrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^11 - 9*b^10*c + 36*b^9*c^2 - 84*b^8*c^3 + 126*b^7*c^4 - 126*b^6*c^5 + 84*b^5*c^6 - 36*b^4*c^7 + 9*b^3*c^8 - b^2*c^9)*a^2*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)))/b

Mupad [B] (verification not implemented)

Time = 21.49 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.56

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \frac{2\sqrt{a}\,b^2\left(\sqrt{a+c\,x}-\sqrt{a}\right)\left(\frac{8\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}-\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}+\frac{3\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+1\right)-2\sqrt{a}\,c^2\left(\sqrt{a\right.}}{}$$

```
[In] int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (2*a^(1/2)*b^2*((a + c*x)^(1/2) - a^(1/2))*((8*((a + b*x)^(1/2) - a^(1/2)))
/((a + c*x)^(1/2) - a^(1/2)) - (2*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)
^(1/2) - a^(1/2))^2 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) -
 a^(1/2)))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + 1) -
2*a^(1/2)*c^2*((a + c*x)^(1/2) - a^(1/2))*((2*((a + b*x)^(1/2) - a^(1/2))^2
)/((a + c*x)^(1/2) - a^(1/2))^2 - ((a + b*x)^(1/2) - a^(1/2))^4/((a + c*x)^
(1/2) - a^(1/2))^4 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) -
a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3) + 2
*a^(1/2)*b*c*((a + c*x)^(1/2) - a^(1/2))*((8*((a + b*x)^(1/2) - a^(1/2)))/(
(a + c*x)^(1/2) - a^(1/2)) - (14*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^
(1/2) - a^(1/2))^2 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) -
a^(1/2)))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (3*log
(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2)
- a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3))/((b - c)^3*((a + b*x)^(1/2) -
 a^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2)
)^2))
```

3.442 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 164

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} dx = -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2} - \frac{(2b+3c)\sqrt{a+bx}}{(b-c)^3x} + \frac{2a\sqrt{a+cx}}{(b-c)^3x^2} + \frac{(3b+2c)\sqrt{a+cx}}{(b-c)^3x} - \frac{3bc\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3} + \frac{3bc\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}$$

```
[Out] -3*b*c*arctanh((b*x+a)^(1/2)/a^(1/2))/(b-c)^3/a^(1/2)+3*b*c*arctanh((c*x+a)
^(1/2)/a^(1/2))/(b-c)^3/a^(1/2)-2*a*(b*x+a)^(1/2)/(b-c)^3/x^2-(2*b+3*c)*(b*
x+a)^(1/2)/(b-c)^3/x+2*a*(c*x+a)^(1/2)/(b-c)^3/x^2+(3*b+2*c)*(c*x+a)^(1/2)/
(b-c)^3/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.68, number of steps used = 16, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6822, 43, 44, 65, 214}

$$\begin{aligned}\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = {} & \frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}-\frac{c^2\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}\\ & -\frac{b(b+3c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}+\frac{c(3b+c)\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}\\ & -\frac{2a\sqrt{a+bx}}{x^2(b-c)^3}+\frac{2a\sqrt{a+cx}}{x^2(b-c)^3}-\frac{b\sqrt{a+bx}}{x(b-c)^3}\\ & -\frac{(b+3c)\sqrt{a+bx}}{x(b-c)^3}+\frac{c\sqrt{a+cx}}{x(b-c)^3}+\frac{(3b+c)\sqrt{a+cx}}{x(b-c)^3}\end{aligned}$$

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/((b - c)^3*x^2) - (b*Sqrt[a + b*x])/((b - c)^3*x) - ((
b + 3*c)*Sqrt[a + b*x])/((b - c)^3*x) + (2*a*Sqrt[a + c*x])/((b - c)^3*x^2)
 + (c*Sqrt[a + c*x])/((b - c)^3*x) + ((3*b + c)*Sqrt[a + c*x])/((b - c)^3*x
) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) - (b*(b + 3*c)
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) - (c^2*ArcTanh[Sqrt[a
+ c*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) + (c*(3*b + c)*ArcTanh[Sqrt[a + c*x]/S
qrt[a]])/(Sqrt[a]*(b - c)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \left(\frac{4a\sqrt{a+bx}}{x^3} + \frac{b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}}{x^2} - \frac{4a\sqrt{a+cx}}{x^3} - \frac{3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}}{x^2}\right) dx}{(b-c)^3}$$

$$= \frac{(4a)\int \frac{\sqrt{a+bx}}{x^3}\,dx}{(b-c)^3} - \frac{(4a)\int \frac{\sqrt{a+cx}}{x^3}\,dx}{(b-c)^3} - \frac{(3b+c)\int \frac{\sqrt{a+cx}}{x^2}\,dx}{(b-c)^3} + \frac{(b+3c)\int \frac{\sqrt{a+bx}}{x^2}\,dx}{(b-c)^3}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2} - \frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x} + \frac{2a\sqrt{a+cx}}{(b-c)^3x^2} + \frac{(3b+c)\sqrt{a+cx}}{(b-c)^3x} + \frac{(ab)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{(b-c)^3} - \frac{(ac)\int \frac{1}{x^2\sqrt{a+cx}}\,dx}{(b-c)^3} - \frac{(c(3b+c))\int \frac{1}{x\sqrt{a+cx}}\,dx}{2(b-c)^3} + \frac{(b(b+3c))\int \frac{1}{x\sqrt{a+bx}}\,dx}{2(b-c)^3}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2} - \frac{b\sqrt{a+bx}}{(b-c)^3x} - \frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x} + \frac{2a\sqrt{a+cx}}{(b-c)^3x^2} + \frac{c\sqrt{a+cx}}{(b-c)^3x} + \frac{(3b+c)\sqrt{a+cx}}{(b-c)^3x} - \frac{b^2\int \frac{1}{x\sqrt{a+bx}}\,dx}{2(b-c)^3} + \frac{c^2\int \frac{1}{x\sqrt{a+cx}}\,dx}{2(b-c)^3} - \frac{(3b+c)\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx}\right)}{(b-c)^3} + \frac{(b+3c)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{(b-c)^3}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2} - \frac{b\sqrt{a+bx}}{(b-c)^3x} - \frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x} + \frac{2a\sqrt{a+cx}}{(b-c)^3x^2} + \frac{c\sqrt{a+cx}}{(b-c)^3x} + \frac{(3b+c)\sqrt{a+cx}}{(b-c)^3x} - \frac{b(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3} + \frac{c(3b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3} - \frac{b\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{(b-c)^3} + \frac{c\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx}\right)}{(b-c)^3}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2} - \frac{b\sqrt{a+bx}}{(b-c)^3x} - \frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x} + \frac{2a\sqrt{a+cx}}{(b-c)^3x^2} + \frac{c\sqrt{a+cx}}{(b-c)^3x} + \frac{(3b+c)\sqrt{a+cx}}{(b-c)^3x} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3} - \frac{b(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3} - \frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3} + \frac{c(3b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{-\frac{3(b+3c)\left(a+bx+bx\sqrt{1+\frac{bx}{a}}\text{arctanh}\left(\sqrt{1+\frac{bx}{a}}\right)\right)}{x\sqrt{a+bx}} + \frac{3(3b+c)\left(a+cx+cx\sqrt{1+\frac{cx}{a}}\text{arctanh}\left(\sqrt{1+\frac{cx}{a}}\right)\right)}{x\sqrt{a+cx}} - \frac{8b^2(a+bx)^{3/2}\,\text{Hypergeometric}}{a^2}}{3(b-c)^3}$$

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

```
[Out] ((-3*(b + 3*c)*(a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])
)/(x*Sqrt[a + b*x]) + (3*(3*b + c)*(a + c*x + c*x*Sqrt[1 + (c*x)/a]*ArcTanh
[Sqrt[1 + (c*x)/a]]))/(x*Sqrt[a + c*x]) - (8*b^2*(a + b*x)^(3/2)*Hypergeome
tric2F1[3/2, 3, 5/2, 1 + (b*x)/a])/a^2 + (8*c^2*(a + c*x)^(3/2)*Hypergeomet
ric2F1[3/2, 3, 5/2, 1 + (c*x)/a])/a^2)/(3*(b - c)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(144) = 288.

Time = 0.05 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.83

method	result
default	$\frac{2b^2\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(b-c)^3}+\frac{8a\,b^2\left(\frac{-\frac{(bx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx+a}}{8}}{x^2b^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{(b-c)^3}-\frac{8a\,c^2\left(\frac{-\frac{(cx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{cx+a}}{8}}{c^2x^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx}}{\sqrt{}}\right.}{8a^{\frac{3}{2}}}\right.}{(b-c)^3}$

```
[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b-c)^3*b^2*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^
(1/2))+8/(b-c)^3*a*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/x^2/b^2+1/
8/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-8/(b-c)^3*a*c^2*((-1/8/a*(c*x+a)^
(3/2)-1/8*(c*x+a)^(1/2))/c^2/x^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2))
)+6/(b-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/
a^(1/2))-6/(b-c)^3*b*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^
(1/2)/a^(1/2)))-2/(b-c)^3*c^2*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((
c*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.81

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \left[-\frac{3\sqrt{a}bcx^2\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+3\sqrt{a}bcx^2\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)+2\left(2a^2+(2ab+3ac)x\right)\sqrt{bx+a}-}{2\left(ab^3-3ab^2c+3abc^2-ac^3\right)x^2}\right.$$

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(3*sqrt(a)*b*c*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*s
qrt(a)*b*c*x^2*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(2*a^2 + (2
*a*b + 3*a*c)*x)*sqrt(b*x + a) - 2*(2*a^2 + (3*a*b + 2*a*c)*x)*sqrt(c*x + a
))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2), (3*sqrt(-a)*b*c*x^2*arcta
n(sqrt(b*x + a)*sqrt(-a)/a) - 3*sqrt(-a)*b*c*x^2*arctan(sqrt(c*x + a)*sqrt(
-a)/a) - (2*a^2 + (2*a*b + 3*a*c)*x)*sqrt(b*x + a) + (2*a^2 + (3*a*b + 2*a*
c)*x)*sqrt(c*x + a))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2)]
```

Sympy [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
[Out] Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
[Out] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2766 vs. $2(144) = 288$.

Time = 17.84 (sec) , antiderivative size = 2766, normalized size of antiderivative = 16.87

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
[Out] 3*b*c*arctan(sqrt(b*x + a)/sqrt(-a))/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*sqrt(
-a)) - 2*(3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))
*a^3*b^7*c*abs(b) - 7*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c
 - a*b*c))*a^3*b^6*c^2*abs(b) + 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))*a^3*b^5*c^3*abs(b) + 3*(sqrt(b*c)*sqrt(b*x + a) - sq
rt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*b^4*c^4*abs(b) - 2*(sqrt(b*c)*sqrt(b
*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*b^3*c^5*abs(b) - 3*(sqrt
(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^5*c*abs(
b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^
2*b^3*c^3*abs(b) + 6*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c
- a*b*c))^3*a^2*b^2*c^4*abs(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 +
(b*x + a)*b*c - a*b*c))^5*a*b^3*c*abs(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqr
t(a*b^2 + (b*x + a)*b*c - a*b*c))^5*a*b^2*c^2*abs(b) - 6*(sqrt(b*c)*sqrt(b*
x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^5*a*b*c^3*abs(b) + 3*(sqrt(b*
```

c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^7*b*c*abs(b) + 2*(s
qrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^7*c^2*abs(b))
/((a^2*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(
a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt
(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt(
a*b^2 + (b*x + a)*b*c - a*b*c))^4)^2*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)) - 3*(
2*(a*b^3*c^2 - a*b^2*c^3)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*sqr
t(-a)*abs(b)*sgn(2*b^3 - 6*b^2*c + 6*b*c^2 - 2*c^3) + 2*(a*b^4 - 3*a*b^3*c
+ 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3*c - a*b^2*c^2)*sqrt(-a*b*c)*abs(b) + (a^2
*b^7*c - 5*a^2*b^6*c^2 + 10*a^2*b^5*c^3 - 10*a^2*b^4*c^4 + 5*a^2*b^3*c^5 -
a^2*b^2*c^6)*sqrt(-a*b*c)*abs(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)*ab
s(b)*sgn(2*b^3 - 6*b^2*c + 6*b*c^2 - 2*c^3) + (a^2*b^8*c - 5*a^2*b^7*c^2 +
10*a^2*b^6*c^3 - 10*a^2*b^5*c^4 + 5*a^2*b^4*c^5 - a^2*b^3*c^6)*sqrt(-a)*abs
(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)*abs(b) + (a^3*b^11*c^2 - 6*a^3*
b^10*c^3 + 14*a^3*b^9*c^4 - 14*a^3*b^8*c^5 + 14*a^3*b^6*c^7 - 14*a^3*b^5*c^
8 + 6*a^3*b^4*c^9 - a^3*b^3*c^10)*sqrt(-a)*abs(b)*sgn(2*b^3 - 6*b^2*c + 6*b
*c^2 - 2*c^3) + (a^3*b^11*c - 6*a^3*b^10*c^2 + 14*a^3*b^9*c^3 - 14*a^3*b^8*
c^4 + 14*a^3*b^6*c^6 - 14*a^3*b^5*c^7 + 6*a^3*b^4*c^8 - a^3*b^3*c^9)*sqrt(-
a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 + sqrt((a*b^5
- 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^
5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*
c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^11 - 9*b^10*c + 36*b^9*c
^2 - 84*b^8*c^3 + 126*b^7*c^4 - 126*b^6*c^5 + 84*b^5*c^6 - 36*b^4*c^7 + 9*b
^3*c^8 - b^2*c^9)*a^3*abs(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)) + 3*(
2*(a*b^3*c^2 - a*b^2*c^3)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*sqr
t(-a)*abs(b)*sgn(2*b^3 - 6*b^2*c + 6*b*c^2 - 2*c^3) + 2*(a*b^4 - 3*a*b^3*c
+ 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3*c - a*b^2*c^2)*sqrt(-a*b*c)*abs(b) + (a^2
*b^7*c - 5*a^2*b^6*c^2 + 10*a^2*b^5*c^3 - 10*a^2*b^4*c^4 + 5*a^2*b^3*c^5 -
a^2*b^2*c^6)*sqrt(-a*b*c)*abs(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)*ab
s(b)*sgn(2*b^3 - 6*b^2*c + 6*b*c^2 - 2*c^3) + (a^2*b^8*c - 5*a^2*b^7*c^2 +
10*a^2*b^6*c^3 - 10*a^2*b^5*c^4 + 5*a^2*b^4*c^5 - a^2*b^3*c^6)*sqrt(-a)*abs
(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)*abs(b) + (a^3*b^11*c^2 - 6*a^3*
b^10*c^3 + 14*a^3*b^9*c^4 - 14*a^3*b^8*c^5 + 14*a^3*b^6*c^7 - 14*a^3*b^5*c^
8 + 6*a^3*b^4*c^9 - a^3*b^3*c^10)*sqrt(-a)*abs(b)*sgn(2*b^3 - 6*b^2*c + 6*b
*c^2 - 2*c^3) + (a^3*b^11*c - 6*a^3*b^10*c^2 + 14*a^3*b^9*c^3 - 14*a^3*b^8*
c^4 + 14*a^3*b^6*c^6 - 14*a^3*b^5*c^7 + 6*a^3*b^4*c^8 - a^3*b^3*c^9)*sqrt(-
a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 - sqrt((a*b^5
- 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^
5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*
c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^11 - 9*b^10*c + 36*b^9*c
^2 - 84*b^8*c^3 + 126*b^7*c^4 - 126*b^6*c^5 + 84*b^5*c^6 - 36*b^4*c^7 + 9*b
^3*c^8 - b^2*c^9)*a^3*abs(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)) - (2*
(b*x + a)^(3/2)*b^2 + 3*(b*x + a)^(3/2)*b*c - 3*sqrt(b*x + a)*a*b*c)/((b^3

```
- 3*b^2*c + 3*b*c^2 - c^3)*b^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 19.71 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.75

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \frac{c^2\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{4\sqrt{a}\,(b-c)^3\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2} - \frac{\left(\frac{\sqrt{a}\,b^2}{4\,(a\,b^3-3\,a\,b^2\,c+3\,a\,b\,c^2-a\,c^3)} - \frac{\sqrt{a}\,(b^2+c\,b)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)(a\,b^3-3\,a\,b^2\,c+3\,a\,b\,c^2-a\,c^3)}\right)\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}$$
$$+ \frac{3\,b\,c\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{\sqrt{a}\,\left(b^3-3\,b^2\,c+3\,b\,c^2-c^3\right)} - \frac{c\,(b+c)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a}\,(b-c)^3\left(\sqrt{a+c\,x}-\sqrt{a}\right)}$$

```
[In] int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (c^2*((a + b*x)^(1/2) - a^(1/2))^2)/(4*a^(1/2)*(b - c)^3*((a + c*x)^(1/2) -
 a^(1/2))^2) - (((a^(1/2)*b^2)/(4*(a*b^3 - a*c^3 + 3*a*b*c^2 - 3*a*b^2*c))
- (a^(1/2)*(b*c + b^2)*((a + b*x)^(1/2) - a^(1/2)))/(((a + c*x)^(1/2) - a^(
1/2))*(a*b^3 - a*c^3 + 3*a*b*c^2 - 3*a*b^2*c)))*((a + c*x)^(1/2) - a^(1/2))
^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (3*b*c*log(((a + b*x)^(1/2) - a^(1/2))/
((a + c*x)^(1/2) - a^(1/2))))/(a^(1/2)*(3*b*c^2 - 3*b^2*c + b^3 - c^3)) - (
c*(b + c)*((a + b*x)^(1/2) - a^(1/2)))/(a^(1/2)*(b - c)^3*((a + c*x)^(1/2)
- a^(1/2)))
```

3.443 $\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 31

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{\arcsin(x)}{2}$$

```
[Out] x-1/2*x^2+1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6820, 201, 222}

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=\frac{\arcsin(x)}{2}-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}x+x$$

```
[In] Int[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] x - x^2/2 + (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(1-x+\sqrt{1-x^2}\right) dx \\ &= x-\frac{x^2}{2}+\int \sqrt{1-x^2}\,dx \\ &= x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right) dx = x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}-\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

```
[In] Integrate[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
[Out] x - x^2/2 + (x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(23) = 46.

Time = 0.94 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

method	result	size
default	$x-\frac{x^2}{2}+\frac{\sqrt{1-x}\,(x+1)^{\frac{3}{2}}}{2}-\frac{\sqrt{1-x}\,\sqrt{x+1}}{2}+\frac{\sqrt{(1-x)(x+1)}\,\arcsin(x)}{2\sqrt{x+1}\,\sqrt{1-x}}$	63

```
[In] int((1-x)^(1/2)*((1-x)^(1/2)+(x+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] x-1/2*x^2+1/2*(1-x)^(1/2)*(x+1)^(3/2)-1/2*(1-x)^(1/2)*(x+1)^(1/2)+1/2*((1-x
)*(x+1))^(1/2)/(x+1)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{1}{2}\,x^2+\frac{1}{2}\,\sqrt{x+1}x\sqrt{-x+1}+x-\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

```
[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + x - arctan((sqrt(x + 1)*sqrt(-x
 + 1) - 1)/x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(22) = 44$.

Time = 1.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{(1-x)^2}{2}-2\sqrt{x+1}\left(\frac{(1-x)^{\frac{3}{2}}}{4}-\frac{\sqrt{1-x}}{4}\right)-\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{1-x}}{2}\right)$$

```
[In] integrate((1-x)**(1/2)*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -(1 - x)**2/2 - 2*sqrt(x + 1)*((1 - x)**(3/2)/4 - sqrt(1 - x)/4) - asin(sqr
t(2)*sqrt(1 - x)/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{1}{2}\,x^2+\frac{1}{2}\,\sqrt{-x^2+1}x+x+\frac{1}{2}\,\arcsin\left(x\right)$$

```
[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + 1/2*sqrt(-x^2 + 1)*x + x + 1/2*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(23) = 46$.

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{1}{2}\,(x-1)^2+\frac{1}{2}\,(x+2)\sqrt{x+1}\sqrt{-x+1} - \sqrt{x+1}\sqrt{-x+1}-\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{-x+1}\right)$$

```
[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
[Out] -1/2*(x - 1)^2 + 1/2*(x + 2)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-x
 + 1) - arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [B] (verification not implemented)

Time = 21.80 (sec) , antiderivative size = 209, normalized size of antiderivative = 6.74

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$= x-2\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\frac{2\,(\sqrt{1-x}-1)}{\sqrt{x+1}-1}-\frac{14\,(\sqrt{1-x}-1)^3}{(\sqrt{x+1}-1)^3}+\frac{14\,(\sqrt{1-x}-1)^5}{(\sqrt{x+1}-1)^5}-\frac{2\,(\sqrt{1-x}-1)^7}{(\sqrt{x+1}-1)^7}}{\frac{4\,(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2}+\frac{6\,(\sqrt{1-x}-1)^4}{(\sqrt{x+1}-1)^4}+\frac{4\,(\sqrt{1-x}-1)^6}{(\sqrt{x+1}-1)^6}+\frac{(\sqrt{1-x}-1)^8}{(\sqrt{x+1}-1)^8}+1}-\frac{x^2}{2}$$

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))*(1 - x)^(1/2),x)
[Out] x - 2*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((2*((1 - x)^(1/2) -
1))/((x + 1)^(1/2) - 1) - (14*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^3
+ (14*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (2*((1 - x)^(1/2) - 1)
^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2
 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2) - 1)
^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)^8 + 1
) - x^2/2
```

3.444 $\int x^3 \left(-\sqrt{1-x}-\sqrt{1+x}\right) \left(\sqrt{1-x}+\sqrt{1+x}\right) \, dx$

Optimal result

Integrand size = 42, antiderivative size = 38

$$\int x^3 \left(-\sqrt{1-x}-\sqrt{1+x}\right) \left(\sqrt{1-x}+\sqrt{1+x}\right) \, dx = -\frac{x^4}{2}+\frac{2}{3}\left(1-x^2\right)^{3/2}-\frac{2}{5}\left(1-x^2\right)^{5/2}$$

```
[Out] -1/2*x^4+2/3*(-x^2+1)^(3/2)-2/5*(-x^2+1)^(5/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6820, 6874, 272, 45}

$$\int x^3 \left(-\sqrt{1-x}-\sqrt{1+x}\right) \left(\sqrt{1-x}+\sqrt{1+x}\right) \, dx = -\frac{x^4}{2}-\frac{2}{5}\left(1-x^2\right)^{5/2}+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

```
[In] Int[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -1/2*x^4 + (2*(1 - x^2)^(3/2))/3 - (2*(1 - x^2)^(5/2))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx \\
&= -\int \left(2x^3+2x^3\sqrt{1-x^2}\right)\,dx \\
&= -\frac{x^4}{2}-2\int x^3\sqrt{1-x^2}\,dx \\
&= -\frac{x^4}{2}-\text{Subst}\left(\int \sqrt{1-x}x\,dx,x,x^2\right) \\
&= -\frac{x^4}{2}-\text{Subst}\left(\int \left(\sqrt{1-x}-(1-x)^{3/2}\right)\,dx,x,x^2\right) \\
&= -\frac{x^4}{2}+\frac{2}{3}\left(1-x^2\right)^{3/2}-\frac{2}{5}\left(1-x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{1}{30}\left(-1+x^2\right)\left(15+8\sqrt{1-x^2}+3x^2\left(5+4\sqrt{1-x^2}\right)\right)$$

```
[In] Integrate[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
[Out] -1/30*((-1 + x^2)*(15 + 8*Sqrt[1 - x^2] + 3*x^2*(5 + 4*Sqrt[1 - x^2])))
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{x^4}{2}-\frac{2\sqrt{1-x}\,\sqrt{x+1}\,(x^2-1)\,(3x^2+2)}{15}$	33

```
[In] int(x^3*(-(1-x)^(1/2)-(x+1)^(1/2))*((1-x)^(1/2)+(x+1)^(1/2)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2*x^4-2/15*(1-x)^(1/2)*(x+1)^(1/2)*(x^2-1)*(3*x^2+2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\frac{1}{2}\,x^4-\frac{2}{15}\,\left(3\,x^4-x^2-2\right)\sqrt{x+1}\sqrt{-x+1}$$

```
[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="fricas")
```

```
[Out] -1/2*x^4 - 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [F]

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-\int 2x^3\,dx-\int 2x^3\sqrt{1-x}\sqrt{x+1}\,dx$$

```
[In] integrate(x**3*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -Integral(2*x**3, x) - Integral(2*x**3*sqrt(1 - x)*sqrt(x + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\frac{1}{2}\,x^4+\frac{2}{5}\,(-x^2+1)^{\frac{3}{2}}x^2+\frac{4}{15}\,(-x^2+1)^{\frac{3}{2}}$$

```
[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="maxima")
```

```
[Out] -1/2*x^4 + 2/5*(-x^2 + 1)^(3/2)*x^2 + 4/15*(-x^2 + 1)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(28) = 56.

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\frac{1}{2}\,x^4-\frac{1}{60}\,((2\,(3\,(4\,x-17)(x+1)+133)(x+1)-295)(x+1)+195)\sqrt{x+1}\sqrt{-x+1}$$
$$-\frac{1}{12}\,((2\,(3\,x-10)(x+1)+43)(x+1)-39)\sqrt{x+1}\sqrt{-x+1}$$

```
[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="giac")
```

```
[Out] -1/2*x^4 - 1/60*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 1
95)*sqrt(x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) -
39)*sqrt(x + 1)*sqrt(-x + 1)
```

Mupad [B] (verification not implemented)

Time = 17.65 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=\sqrt{1-x}\left(\frac{4\,\sqrt{x+1}}{15}+\frac{2\,x^2\,\sqrt{x+1}}{15}-\frac{2\,x^4\,\sqrt{x+1}}{5}\right)-\frac{x^4}{2}$$

```
[In] int(-x^3*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] (1 - x)^(1/2)*((4*(x + 1)^(1/2))/15 + (2*x^2*(x + 1)^(1/2))/15 - (2*x^4*(x
+ 1)^(1/2))/5) - x^4/2
```

3.445 $\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

Optimal result

Integrand size = 42, antiderivative size = 48

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{2x^3}{3}+\frac{1}{4}x\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{\arcsin(x)}{4}$$

```
[Out] -2/3*x^3-1/4*arcsin(x)+1/4*x*(-x^2+1)^(1/2)-1/2*x^3*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {6820, 6874, 285, 327, 222}

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{\arcsin(x)}{4}-\frac{2x^3}{3}+\frac{1}{4}\sqrt{1-x^2}x-\frac{1}{2}\sqrt{1-x^2}x^3$$

```
[In] Int[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
[Out] (-2*x^3)/3 + (x*Sqrt[1 - x^2])/4 - (x^3*Sqrt[1 - x^2])/2 - ArcSin[x]/4
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx\\
&= -\int \left(2x^2+2x^2\sqrt{1-x^2}\right)\,dx\\
&= -\frac{2x^3}{3}-2\int x^2\sqrt{1-x^2}\,dx\\
&= -\frac{2x^3}{3}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2x^3}{3}+\frac{1}{4}x\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{4}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2x^3}{3}+\frac{1}{4}x\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{4}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.42

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=\frac{1}{12}\left(-8+3x\sqrt{1-x^2}-x^3\left(8+6\sqrt{1-x^2}\right)-12\arctan\left(\frac{-\sqrt{2}+\sqrt{1+x}}{\sqrt{1-x}}\right)\right)$$

```
[In] Integrate[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] (-8 + 3*x*Sqrt[1 - x^2] - x^3*(8 + 6*Sqrt[1 - x^2]) - 12*ArcTan[(-Sqrt[2] +
 Sqrt[1 + x])/Sqrt[1 - x]])/12
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

method	result	size
default	$-\frac{2x^3}{3}-\frac{\sqrt{1-x}\,\sqrt{x+1}\,\left(2x^3\sqrt{-x^2+1}-x\sqrt{-x^2+1}+\arcsin(x)\right)}{4\sqrt{-x^2+1}}$	59

```
[In] int(x^2*(-(1-x)^(1/2)-(x+1)^(1/2))*((1-x)^(1/2)+(x+1)^(1/2)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3*x^3-1/4*(1-x)^(1/2)*(x+1)^(1/2)*(2*x^3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2)
+arcsin(x))/(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\frac{2}{3}\,x^3-\frac{1}{4}\,(2\,x^3-x)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

```
[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="fricas")
```

```
[Out] -2/3*x^3 - 1/4*(2*x^3 - x)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*arctan((sqrt(x +
1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F]

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-\int 2x^2\,dx-\int 2x^2\sqrt{1-x}\sqrt{x+1}\,dx$$

```
[In] integrate(x**2*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
[Out] -Integral(2*x**2, x) - Integral(2*x**2*sqrt(1 - x)*sqrt(x + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\frac{2}{3}x^3+\frac{1}{2}\left(-x^2+1\right)^{\frac{3}{2}}x-\frac{1}{4}\sqrt{-x^2+1}x-\frac{1}{4}\arcsin(x)$$

```
[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="maxima")
[Out] -2/3*x^3 + 1/2*(-x^2 + 1)^(3/2)*x - 1/4*sqrt(-x^2 + 1)*x - 1/4*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(36) = 72.

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\frac{2}{3}x^3-\frac{1}{12}\left((2(3x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1}$$
$$-\frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{2}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

```
[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="giac")
[Out] -2/3*x^3 - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt
(-x + 1) - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*arcsi
n(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [B] (verification not implemented)

Time = 26.49 (sec) , antiderivative size = 381, normalized size of antiderivative = 7.94

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=\operatorname{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)$$
$$-\frac{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}-\frac{35\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{273\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{715\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}+\frac{715\left(\sqrt{1-x}-1\right)^9}{\left(\sqrt{x+1}-1\right)^9}-\frac{273\left(\sqrt{1-x}-1\right)^{11}}{\left(\sqrt{x+1}-1\right)^{11}}+\frac{35\left(\sqrt{1-x}-1\right)^{13}}{\left(\sqrt{x+1}-1\right)^{13}}-\cdots}{\frac{8\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{28\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{56\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{70\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+\frac{56\left(\sqrt{1-x}-1\right)^{10}}{\left(\sqrt{x+1}-1\right)^{10}}+\frac{28\left(\sqrt{1-x}-1\right)^{12}}{\left(\sqrt{x+1}-1\right)^{12}}+\frac{8\left(\sqrt{1-x}-1\right)^{14}}{\left(\sqrt{x+1}-1\right)^{14}}+\cdots}$$
$$-\frac{2\,x^3}{3}$$

```
[In] int(-x^2*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - (((1 - x)^(1/2) - 1)/((x +
1)^(1/2) - 1) - (35*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^3 + (273*((1
 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (715*((1 - x)^(1/2) - 1)^7)/((x
 + 1)^(1/2) - 1)^7 + (715*((1 - x)^(1/2) - 1)^9)/((x + 1)^(1/2) - 1)^9 - (2
73*((1 - x)^(1/2) - 1)^11)/((x + 1)^(1/2) - 1)^11 + (35*((1 - x)^(1/2) - 1)
^13)/((x + 1)^(1/2) - 1)^13 - ((1 - x)^(1/2) - 1)^15/((x + 1)^(1/2) - 1)^15
)/((8*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + (28*((1 - x)^(1/2) - 1
)^4)/((x + 1)^(1/2) - 1)^4 + (56*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1/2) - 1)
^6 + (70*((1 - x)^(1/2) - 1)^8)/((x + 1)^(1/2) - 1)^8 + (56*((1 - x)^(1/2)
- 1)^10)/((x + 1)^(1/2) - 1)^10 + (28*((1 - x)^(1/2) - 1)^12)/((x + 1)^(1/2
) - 1)^12 + (8*((1 - x)^(1/2) - 1)^14)/((x + 1)^(1/2) - 1)^14 + ((1 - x)^(1
/2) - 1)^16/((x + 1)^(1/2) - 1)^16 + 1) - (2*x^3)/3
```

3.446 $\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right) dx$

Optimal result

Integrand size = 40, antiderivative size = 21

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right) dx = -x^2+\frac{2}{3}(1-x^2)^{3/2}$$

```
[Out] -x^2+2/3*(-x^2+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {6820, 6874, 267}

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right) dx = \frac{2}{3}(1-x^2)^{3/2}-x^2$$

```
[In] Int[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
[Out] -x^2 + (2*(1 - x^2)^(3/2))/3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx \\
&= -\int \left(2x+2x\sqrt{1-x^2}\right)\,dx \\
&= -x^2-2\int x\sqrt{1-x^2}\,dx \\
&= -x^2+\frac{2}{3}\left(1-x^2\right)^{3/2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{1}{3}(-1+x)(1+x)\left(3+2\sqrt{1-x^2}\right)$$

```
[In] Integrate[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
[Out] -1/3*((-1 + x)*(1 + x)*(3 + 2*Sqrt[1 - x^2]))
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

method	result	size
default	$-x^{2}-\frac{2\sqrt{1-x}\,\sqrt{x+1}\,\left(x^{2}-1\right)}{3}$	26

```
[In] int(x*(-(1-x)^(1/2)-(x+1)^(1/2))*((1-x)^(1/2)+(x+1)^(1/2)),x,method=_RETURN
VERBOSE)
[Out] -x^2-2/3*(1-x)^(1/2)*(x+1)^(1/2)*(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-x^2-\frac{2}{3}\left(x^2-1\right)\sqrt{x+1}\sqrt{-x+1}$$

```
[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="fricas")
[Out] -x^2 - 2/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [F]

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-\int 2x\,dx-\int 2x\sqrt{1-x}\sqrt{x+1}\,dx$$

```
[In] integrate(x*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
[Out] -Integral(2*x, x) - Integral(2*x*sqrt(1 - x)*sqrt(x + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-x^2+\frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="maxima")
[Out] -x^2 + 2/3*(-x^2 + 1)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(17) = 34.

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.57

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-(x+1)^2-\frac{1}{3}\left((2\,x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}-\sqrt{x+1}(x-2)\sqrt{-x+1}+2\,x+2$$

```
[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="giac")
```

```
[Out] -(x + 1)^2 - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x
+ 1)*(x - 2)*sqrt(-x + 1) + 2*x + 2
```

Mupad [B] (verification not implemented)

Time = 17.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-x^2-\frac{2\left(x^2-1\right)\sqrt{1-x}\,\sqrt{x+1}}{3}$$

```
[In] int(-x*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] - x^2 - (2*(x^2 - 1)*(1 - x)^(1/2)*(x + 1)^(1/2))/3
```

3.447 $\int \left(-\sqrt{1-x}-\sqrt{1+x}\right) \left(\sqrt{1-x}+\sqrt{1+x}\right) \, dx$

Optimal result

Integrand size = 39, antiderivative size = 22

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right) \left(\sqrt{1-x}+\sqrt{1+x}\right) \, dx = -2x-x\sqrt{1-x^2}-\arcsin(x)$$

```
[Out] -2*x-arcsin(x)-x*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {6820, 6874, 201, 222}

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right) \left(\sqrt{1-x}+\sqrt{1+x}\right) \, dx = -\arcsin(x)-\sqrt{1-x^2}x-2x$$

```
[In] Int[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -2*x - x*Sqrt[1 - x^2] - ArcSin[x]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx \\
&= -\int \left(2+2\sqrt{1-x^2}\right) dx \\
&= -2x-2\int \sqrt{1-x^2}\,dx \\
&= -2x-x\sqrt{1-x^2}-\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -2x-x\sqrt{1-x^2}-\sin^{-1}(x)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right) dx = -2-x\left(2+\sqrt{1-x^2}\right)-4\arctan\left(\frac{-\sqrt{2}+\sqrt{1+x}}{\sqrt{1-x}}\right)$$

```
[In] Integrate[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
[Out] -2 - x*(2 + Sqrt[1 - x^2]) - 4*ArcTan[(-Sqrt[2] + Sqrt[1 + x])/Sqrt[1 - x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.94 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.68

method	result	size
default	$-2x-\sqrt{1-x}\,(x+1)^{\frac{3}{2}}+\sqrt{1-x}\,\sqrt{x+1}-\frac{\sqrt{(1-x)(x+1)}\,\arcsin(x)}{\sqrt{x+1}\,\sqrt{1-x}}$	59

```
[In] int((-(1-x)^(1/2)-(x+1)^(1/2))*((1-x)^(1/2)+(x+1)^(1/2)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*x-(1-x)^(1/2)*(x+1)^(3/2)+(1-x)^(1/2)*(x+1)^(1/2)-((1-x)*(x+1))^(1/2)/(x
+1)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\sqrt{x+1}x\sqrt{-x+1}-2\,x+2\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="fricas")
```

```
[Out] -sqrt(x + 1)*x*sqrt(-x + 1) - 2*x + 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)
/x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 1.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-2x-4\sqrt{1-x}\left(\frac{(x+1)^{\frac{3}{2}}}{4}-\frac{\sqrt{x+1}}{4}\right)-2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-2$$

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -2*x - 4*sqrt(1 - x)*((x + 1)**(3/2)/4 - sqrt(x + 1)/4) - 2*asin(sqrt(2)*sq
rt(x + 1)/2) - 2
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\sqrt{-x^2+1}x-2\,x-\arcsin\left(x\right)$$

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="maxima")
[Out] -sqrt(-x^2 + 1)*x - 2*x - arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(20) = 40.

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$= -\sqrt{x+1}(x-2)\sqrt{-x+1}-2\,x-2\,\sqrt{x+1}\sqrt{-x+1}-2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)-2$$

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="giac")
[Out] -sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - 2*x - 2*sqrt(x + 1)*sqrt(-x + 1) - 2*ar
csin(1/2*sqrt(2)*sqrt(x + 1)) - 2
```

Mupad [B] (verification not implemented)

Time = 17.97 (sec) , antiderivative size = 205, normalized size of antiderivative = 9.32

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$= 4\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-2\,x+\frac{\frac{4\,\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{28\,{\left(\sqrt{1-x}-1\right)}^3}{{\left(\sqrt{x+1}-1\right)}^3}+\frac{28\,{\left(\sqrt{1-x}-1\right)}^5}{{\left(\sqrt{x+1}-1\right)}^5}-\frac{4\,{\left(\sqrt{1-x}-1\right)}^7}{{\left(\sqrt{x+1}-1\right)}^7}}{\frac{4\,{\left(\sqrt{1-x}-1\right)}^2}{{\left(\sqrt{x+1}-1\right)}^2}+\frac{6\,{\left(\sqrt{1-x}-1\right)}^4}{{\left(\sqrt{x+1}-1\right)}^4}+\frac{4\,{\left(\sqrt{1-x}-1\right)}^6}{{\left(\sqrt{x+1}-1\right)}^6}+\frac{{\left(\sqrt{1-x}-1\right)}^8}{{\left(\sqrt{x+1}-1\right)}^8}+1}$$

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
[Out] 4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - 2*x + ((4*((1 - x)^(1/2)
- 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^
3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (4*((1 - x)^(1/2) -
1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)
^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2) -
1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)^8 +
 1)
```

3.448 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 32

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx = -2\sqrt{1-x^2}+2\text{arctanh}\left(\sqrt{1-x^2}\right)-2\log(x)$$

```
[Out] 2*arctanh((-x^2+1)^(1/2))-2*ln(x)-2*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6820, 6874, 272, 52, 65, 212}

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx = 2\text{arctanh}\left(\sqrt{1-x^2}\right)-2\sqrt{1-x^2}-2\log(x)$$

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]
```

```
[Out] -2*Sqrt[1 - x^2] + 2*ArcTanh[Sqrt[1 - x^2]] - 2*Log[x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx \\
&= -\int \left(\frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}\right)dx \\
&= -2\log(x)-2\int \frac{\sqrt{1-x^2}}{x}\,dx \\
&= -2\log(x)-\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx,x,x^2\right) \\
&= -2\sqrt{1-x^2}-2\log(x)-\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx,x,x^2\right)
\end{aligned}$$

$$= -2\sqrt{1-x^2} - 2\log(x) + 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right)$$
$$= -2\sqrt{1-x^2} + 2\tanh^{-1}\left(\sqrt{1-x^2}\right) - 2\log(x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 72 vs. $2(32) = 64$.

Time = 0.01 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.25

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$$
$$= -2\Big(\sqrt{1-x^2} + 2\log\left(\sqrt{2}-\sqrt{1+x}\right) + 2\log\left(\sqrt{1-x}-\sqrt{1+x}\right) - 2\log\left(-2+\sqrt{2}\sqrt{1+x}\right)\Big)$$

```
[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]
```

```
[Out] -2*(Sqrt[1 - x^2] + 2*Log[Sqrt[2] - Sqrt[1 + x]] + 2*Log[Sqrt[1 - x] - Sqrt
[1 + x]] - 2*Log[-2 + Sqrt[2]*Sqrt[1 + x]])
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

method	result	size
default	$-2\ln(x) - \frac{2\sqrt{1-x}\,\sqrt{x+1}\,\left(\sqrt{-x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)\right)}{\sqrt{-x^2+1}}$	51

```
[In] int((-(1-x)^(1/2)-(x+1)^(1/2))*((1-x)^(1/2)+(x+1)^(1/2))/x,x,method=_RETURN
VERBOSE)
```

```
[Out] -2*ln(x)-2*(1-x)^(1/2)*(x+1)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1
/(-x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx = -2\,\sqrt{x+1}\sqrt{-x+1}-2\,\log\left(x\right)-2\,\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorit
hm="fricas")
```

```
[Out] -2*sqrt(x + 1)*sqrt(-x + 1) - 2*log(x) - 2*log((sqrt(x + 1)*sqrt(-x + 1) -
1)/x)
```

Sympy [F]

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx = -\int \frac{2}{x}\,dx-\int \frac{2\sqrt{1-x}\sqrt{x+1}}{x}\,dx$$

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x,x)
```

```
[Out] -Integral(2/x, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx = -2\,\sqrt{-x^2+1}-2\,\log\left(x\right)+2\,\log\left(\frac{2\,\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorit
hm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1) - 2*log(x) + 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(28) = 56$.

Time = 0.34 (sec) , antiderivative size = 130, normalized size of antiderivative = 4.06

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$$
$$= -2\sqrt{x+1}\sqrt{-x+1} - 2\log\left(\sqrt{x+1}+1\right) - 2\log\left(\left|\sqrt{x+1}-1\right|\right)$$
$$+ 2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} + 2\right|\right)$$
$$- 2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} - 2\right|\right)$$

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorit
hm="giac")
```

```
[Out] -2*sqrt(x + 1)*sqrt(-x + 1) - 2*log(sqrt(x + 1) + 1) - 2*log(abs(sqrt(x + 1
) - 1)) + 2*log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(sq
rt(2) - sqrt(-x + 1)) + 2)) - 2*log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x +
1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 2))
```

Mupad [B] (verification not implemented)

Time = 18.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.81

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$$
$$= 2\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - 2\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2} - 1\right)$$
$$- 2\ln(x) - \frac{16\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2\left(\frac{2\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2} + \frac{\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4} + 1\right)}$$

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x,x)
```

```
[Out] 2*log(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - 2*log(((1 - x)^(1/2) - 1)^
2/((x + 1)^(1/2) - 1)^2 - 1) - 2*log(x) - (16*((1 - x)^(1/2) - 1)^2)/(((x +
 1)^(1/2) - 1)^2*((2*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + ((1 - x
)^(1/2) - 1)^4/((x + 1)^(1/2) - 1)^4 + 1))
```

3.449 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 26

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx = \frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}+2\arcsin(x)$$

```
[Out] 2/x+2*arcsin(x)+2*(-x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6820, 6874, 283, 222}

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx = 2\arcsin(x)+\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}$$

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]
[Out] 2/x + (2*Sqrt[1 - x^2])/x + 2*ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx \\
&= -\int \left(\frac{2}{x^2}+\frac{2\sqrt{1-x^2}}{x^2}\right)\,dx \\
&= \frac{2}{x}-2\int \frac{\sqrt{1-x^2}}{x^2}\,dx \\
&= \frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}+2\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}+2\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx=\frac{2\left(1+\sqrt{1-x^2}-4x\arctan\left(\frac{\sqrt{1+x}}{\sqrt{2}-\sqrt{1-x}}\right)\right)}{x}$$

```
[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]
[Out] (2*(1 + Sqrt[1 - x^2] - 4*x*ArcTan[Sqrt[1 + x]/(Sqrt[2] - Sqrt[1 - x])]))/x
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(24) = 48$.

Time = 1.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

method	result	size
default	$\frac{2}{x}-\frac{2\left(-\arcsin\left(x\right)x-\sqrt{-x^{2}+1}\right)\sqrt{1-x}\,\sqrt{x+1}}{x\sqrt{-x^{2}+1}}$	50

```
[In] int((-(1-x)^(1/2)-(x+1)^(1/2))*((1-x)^(1/2)+(x+1)^(1/2))/x^2,x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/x-2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(1-x)^(1/2)*(x+1)^(1/2)/x/(-x^2+1)^(1/2
)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx = -\frac{2\left(2\,x\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x}$$

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algor
ithm="fricas")
```

```
[Out] -2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1)
 - 1)/x
```

Sympy [F]

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx = -\int \frac{2}{x^2}\,dx-\int \frac{2\sqrt{1-x}\sqrt{x+1}}{x^2}\,dx$$

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**2,x)
```

```
[Out] -Integral(2/x**2, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx = \frac{2\sqrt{-x^2+1}}{x}+\frac{2}{x}+2\arcsin\left(x\right)$$

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algor
ithm="maxima")
```

```
[Out] 2*sqrt(-x^2 + 1)/x + 2/x + 2*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(24) = 48.

Time = 0.35 (sec) , antiderivative size = 149, normalized size of antiderivative = 5.73

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx = 2\pi+\frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2-4}+\frac{2}{x}+4\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algor
ithm="giac")
```

```
[Out] 2*pi + 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) -
 sqrt(-x + 1)))^2 - 4) + 2/x + 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqrt(-x
 + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))
```

Mupad [B] (verification not implemented)

Time = 17.87 (sec) , antiderivative size = 118, normalized size of antiderivative = 4.54

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx = \frac{\frac{5\left(\sqrt{1-x}-1\right)^2}{2\left(\sqrt{x+1}-1\right)^2}-\frac{1}{2}}{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}-\frac{\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}}-8\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)+\frac{\sqrt{1-x}-1}{2\left(\sqrt{x+1}-1\right)}+\frac{2}{x}$$

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^2,x)
```

```
[Out] ((5*((1 - x)^(1/2) - 1)^2)/(2*((x + 1)^(1/2) - 1)^2) - 1/2)/((((1 - x)^(1/2)
 - 1)/((x + 1)^(1/2) - 1) - ((1 - x)^(1/2) - 1)^3/((x + 1)^(1/2) - 1)^3) -
8*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) + ((1 - x)^(1/2) - 1)/(2*((
x + 1)^(1/2) - 1)) + 2/x
```

3.450 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 33

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx=\frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}-\operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

```
[Out] 1/x^2-arctanh((-x^2+1)^(1/2))+(-x^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6820, 6874, 272, 43, 65, 212}

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx=-\operatorname{arctanh}\left(\sqrt{1-x^2}\right)+\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{x^2}$$

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]
```

```
[Out] x^(-2) + Sqrt[1 - x^2]/x^2 - ArcTanh[Sqrt[1 - x^2]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx\\
&= -\int \left(\frac{2}{x^3}+\frac{2\sqrt{1-x^2}}{x^3}\right)dx\\
&= \frac{1}{x^2}-2\int \frac{\sqrt{1-x^2}}{x^3}\,dx\\
&= \frac{1}{x^2}-\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx,x,x^2\right)\\
&= \frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx,x,x^2\right)\\
&= \frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= \frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 148 vs. 2(33) = 66.

Time = 0.03 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.48

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx = -2\operatorname{arctanh}\left(\frac{2-\sqrt{2}+2\sqrt{1-x}+2\sqrt{1+x}-\sqrt{2}\sqrt{1+x}}{-2+\sqrt{2}+\sqrt{2}\sqrt{1+x}}\right)-\log\left(\sqrt{2}-\sqrt{1+x}\right) + \frac{1+\sqrt{1-x^2}+x^2\log\left(-2-\sqrt{2}+\sqrt{1-x}+\sqrt{1+x}+\sqrt{2}\sqrt{1+x}\right)}{x^2}$$

```
[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]
```

```
[Out] -2*ArcTanh[(2 - Sqrt[2] + 2*Sqrt[1 - x] + 2*Sqrt[1 + x] - Sqrt[2]*Sqrt[1 +
x])/(-2 + Sqrt[2] + Sqrt[2]*Sqrt[1 + x])] - Log[Sqrt[2] - Sqrt[1 + x]] + (1
 + Sqrt[1 - x^2] + x^2*Log[-2 - Sqrt[2] + Sqrt[1 - x] + Sqrt[1 + x] + Sqrt[
2]*Sqrt[1 + x]])/x^2
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

method	result	size
default	$\frac{1}{x^2}-\frac{\sqrt{1-x}\,\sqrt{x+1}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)x^2-\sqrt{-x^2+1}\right)}{x^2\sqrt{-x^2+1}}$	57

```
[In] int((-(1-x)^(1/2)-(x+1)^(1/2))*((1-x)^(1/2)+(x+1)^(1/2))/x^3,x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/x^2-(1-x)^(1/2)*(x+1)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x^2+1)^(1/2)
)/x^2/(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx = \frac{x^2\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+\sqrt{x+1}\sqrt{-x+1}+1}{x^2}$$

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algor
ithm="fricas")
```

```
[Out] (x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1)/
x^2
```

Sympy [F]

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx = -\int \frac{2}{x^3}\,dx - \int \frac{2\sqrt{1-x}\sqrt{x+1}}{x^3}\,dx$$

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**3,x)
```

```
[Out] -Integral(2/x**3, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx = \sqrt{-x^2+1} + \frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^2} + \frac{1}{x^2} - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algor
ithm="maxima")
```

```
[Out] sqrt(-x^2 + 1) + (-x^2 + 1)^(3/2)/x^2 + 1/x^2 - log(2*sqrt(-x^2 + 1)/abs(x)
 + 2/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(29) = 58.

Time = 0.42 (sec) , antiderivative size = 233, normalized size of antiderivative = 7.06

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx = -\frac{4\left(\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^3+\frac{4\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}-\frac{4\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2-4\right)^2} + \frac{1}{x^2} - \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}+2\right|\right) + \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-2\right|\right)$$

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algor
ithm="giac")
```

```
[Out] -4*(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x
+ 1)))^3 + 4*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 4*sqrt(x + 1)/(sqrt(2)
- sqrt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt
(2) - sqrt(-x + 1)))^2 - 4)^2 + 1/x^2 - log(abs(-(sqrt(2) - sqrt(-x + 1))/s
qrt(x + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) + 2)) + log(abs(-(sqrt(2)
 - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 2))
```

Mupad [B] (verification not implemented)

Time = 19.13 (sec) , antiderivative size = 186, normalized size of antiderivative = 5.64

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx = \ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right) - \ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \frac{\left(\sqrt{1-x}-1\right)^2}{16\left(\sqrt{x+1}-1\right)^2} + \frac{\frac{\left(\sqrt{1-x}-1\right)^2}{8\left(\sqrt{x+1}-1\right)^2}+\frac{15\left(\sqrt{1-x}-1\right)^4}{16\left(\sqrt{x+1}-1\right)^4}-\frac{1}{16}}{\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-\frac{2\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}} + \frac{1}{x^2}$$

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^3,x)
```

```
[Out] log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - log(((1 - x)^(1/2) -
 1)/((x + 1)^(1/2) - 1)) - ((1 - x)^(1/2) - 1)^2/(16*((x + 1)^(1/2) - 1)^2)
 + (((1 - x)^(1/2) - 1)^2/(8*((x + 1)^(1/2) - 1)^2) + (15*((1 - x)^(1/2) -
1)^4)/(16*((x + 1)^(1/2) - 1)^4) - 1/16)/(((1 - x)^(1/2) - 1)^2/((x + 1)^(1
/2) - 1)^2 - (2*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + ((1 - x)^(1/
2) - 1)^6/((x + 1)^(1/2) - 1)^6) + 1/x^2
```

3.451 $\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 28

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx=\sqrt{1-x^2}-\operatorname{arctanh}\left(\sqrt{1-x^2}\right)+\log(x)$$

```
[Out] -arctanh((-x^2+1)^(1/2))+ln(x)+(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2128, 6820, 14, 272, 52, 65, 212}

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx=-\operatorname{arctanh}\left(\sqrt{1-x^2}\right)+\sqrt{1-x^2}+\log(x)$$

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
[Out] Sqrt[1 - x^2] - ArcTanh[Sqrt[1 - x^2]] + Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2128

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \frac{\sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx+\frac{1}{2}\int \frac{\sqrt{1+x}\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx\\ &= \frac{1}{2}\int \frac{1-x+\sqrt{1-x^2}}{x}\,dx+\frac{1}{2}\int \frac{1+x+\sqrt{1-x^2}}{x}\,dx\\ &= \frac{1}{2}\int \left(-1+\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right)\,dx+\frac{1}{2}\int \left(1+\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right)\,dx\end{aligned}$$

$$= \log(x) + 2\left(\frac{1}{2}\int \frac{\sqrt{1-x^2}}{x}\,dx\right)$$

$$= \log(x) + 2\left(\frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx, x, x^2\right)\right)$$

$$= \log(x) + 2\left(\frac{\sqrt{1-x^2}}{2} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, x^2\right)\right)$$

$$= \log(x) + 2\left(\frac{\sqrt{1-x^2}}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right)\right)$$

$$= 2\left(\frac{\sqrt{1-x^2}}{2} - \frac{1}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)\right) + \log(x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx = \sqrt{1-x^2} - 2\log\left(-2+\sqrt{2-2x}\right) + 2\log\left(\sqrt{2}-\sqrt{1-x}\right) + 2\log\left(-\sqrt{1-x}+\sqrt{1+x}\right)$$

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 - x^2] - 2*Log[-2 + Sqrt[2 - 2*x]] + 2*Log[Sqrt[2] - Sqrt[1 - x]] +
2*Log[-Sqrt[1 - x] + Sqrt[1 + x]]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

method	result	size
default	$\ln(x) + \frac{\sqrt{1-x}\,\sqrt{x+1}\left(\sqrt{-x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)\right)}{\sqrt{-x^2+1}}$	48

```
[In] int(((1-x)^(1/2)+(x+1)^(1/2))/(-(1-x)^(1/2)+(x+1)^(1/2)),x,method=_RETURNVE
RBOSE)
```

```
[Out] ln(x)+(1-x)^(1/2)*(x+1)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1/(-x^
2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx=\sqrt{x+1}\sqrt{-x+1}+\log\left(x\right)+\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="fricas")
```

```
[Out] sqrt(x + 1)*sqrt(-x + 1) + log(x) + log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F]

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx=-\int \frac{\sqrt{1-x}}{\sqrt{1-x}-\sqrt{x+1}}\,dx-\int \frac{\sqrt{x+1}}{\sqrt{1-x}-\sqrt{x+1}}\,dx$$

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))/(-(1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -Integral(sqrt(1 - x)/(sqrt(1 - x) - sqrt(x + 1)), x) - Integral(sqrt(x + 1
)/(sqrt(1 - x) - sqrt(x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx=\int \frac{\sqrt{x+1}+\sqrt{-x+1}}{\sqrt{x+1}-\sqrt{-x+1}}\,dx$$

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="maxima")
```

```
[Out] integrate((sqrt(x + 1) + sqrt(-x + 1))/(sqrt(x + 1) - sqrt(-x + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(24) = 48$.

Time = 0.37 (sec) , antiderivative size = 123, normalized size of antiderivative = 4.39

$$\begin{aligned}\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx&=\sqrt{x+1}\sqrt{-x+1}+\log\left(\sqrt{x+1}+1\right)+\log\left(\left|\sqrt{x+1}-1\right|\right)\\&\quad-\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}+2\right|\right)\\&\quad+\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-2\right|\right)\end{aligned}$$

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="giac")
```

```
[Out] sqrt(x + 1)*sqrt(-x + 1) + log(sqrt(x + 1) + 1) + log(abs(sqrt(x + 1) - 1))
 - log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) - s
qrt(-x + 1)) + 2)) + log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x
 + 1)/(sqrt(2) - sqrt(-x + 1)) - 2))
```

Mupad [B] (verification not implemented)

Time = 19.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.32

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx=\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)+\ln\left(x\right)-\frac{8\left(x-2\sqrt{x+1}+2\right)\left(x+2\sqrt{1-x}-2\right)}{\left(2\sqrt{x+1}+2\sqrt{1-x}-4\right)^2}$$

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))/((x + 1)^(1/2) - (1 - x)^(1/2)),x)
```

```
[Out] log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - log(((1 - x)^(1/2) -
 1)/((x + 1)^(1/2) - 1)) + log(x) - (8*(x - 2*(x + 1)^(1/2) + 2)*(x + 2*(1
- x)^(1/2) - 2))/(2*(x + 1)^(1/2) + 2*(1 - x)^(1/2) - 4)^2
```

3.452 $\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 33

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx=\frac{x^2}{2}-\frac{1}{2}\sqrt{-1+x}x\sqrt{1+x}+\frac{\operatorname{arccosh}(x)}{2}$$

```
[Out] 1/2*x^2+1/2*arccosh(x)-1/2*x*(-1+x)^(1/2)*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2129, 6874, 38, 54}

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx=\frac{\operatorname{arccosh}(x)}{2}+\frac{x^2}{2}-\frac{1}{2}\sqrt{x-1}\sqrt{x+1}x$$

```
[In] Int[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

```
[Out] x^2/2 - (Sqrt[-1 + x]*x*Sqrt[1 + x])/2 + ArcCosh[x]/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
```

```
- d, 0] && GtQ[a, 0]
```

Rule 2129

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[-d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \sqrt{-1+x}\left(-\sqrt{-1+x}+\sqrt{1+x}\right)\,dx\right) \\
&\quad +\frac{1}{2}\int \sqrt{1+x}\left(-\sqrt{-1+x}+\sqrt{1+x}\right)\,dx \\
&= \frac{1}{2}\int \left(1+x-\sqrt{-1+x}\sqrt{1+x}\right)\,dx-\frac{1}{2}\int \left(1-x+\sqrt{-1+x}\sqrt{1+x}\right)\,dx \\
&= \frac{x^2}{2}-2\left(\frac{1}{2}\int \sqrt{-1+x}\sqrt{1+x}\,dx\right) \\
&= \frac{x^2}{2}-2\left(\frac{1}{4}\sqrt{-1+x}x\sqrt{1+x}-\frac{1}{4}\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx\right) \\
&= \frac{x^2}{2}-2\left(\frac{1}{4}\sqrt{-1+x}x\sqrt{1+x}-\frac{1}{4}\cosh^{-1}(x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx=\frac{1}{2}\left(-1+x^2-\sqrt{-1+x}x\sqrt{1+x}-2\log\left(\sqrt{-1+x}-\sqrt{1+x}\right)\right)$$

```
[In] Integrate[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
[Out] (-1 + x^2 - Sqrt[-1 + x]*x*Sqrt[1 + x] - 2*Log[Sqrt[-1 + x] - Sqrt[1 + x]])
/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(23) = 46$.

Time = 0.92 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

method	result	size
default	$-\frac{\sqrt{x-1}\,(x+1)^{\frac{3}{2}}}{2}+\frac{\sqrt{x-1}\,\sqrt{x+1}}{2}+\frac{\sqrt{(x-1)(x+1)}\,\ln\left(x+\sqrt{x^{2}-1}\right)}{2\sqrt{x+1}\,\sqrt{x-1}}+\frac{x^{2}}{2}$	62

```
[In] int((-(x-1)^(1/2)+(x+1)^(1/2))/((x-1)^(1/2)+(x+1)^(1/2)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2*(x-1)^(1/2)*(x+1)^(3/2)+1/2*(x-1)^(1/2)*(x+1)^(1/2)+1/2*((x-1)*(x+1))^
(1/2)/(x+1)^(1/2)/(x-1)^(1/2)*ln(x+(x^2-1)^(1/2))+1/2*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = -\frac{1}{2}\sqrt{x+1}\sqrt{x-1}x+\frac{1}{2}x^{2}-\frac{1}{2}\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)$$

```
[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="fricas")
```

```
[Out] -1/2*sqrt(x + 1)*sqrt(x - 1)*x + 1/2*x^2 - 1/2*log(sqrt(x + 1)*sqrt(x - 1)
- x)
```

Sympy [A] (verification not implemented)

Time = 11.61 (sec) , antiderivative size = 223, normalized size of antiderivative = 6.76

$$\begin{aligned}&\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx\\&=-\frac{(x-1)^{\frac{5}{2}}}{4\sqrt{x+1}}-\frac{3(x-1)^{\frac{3}{2}}}{4\sqrt{x+1}}-\frac{\sqrt{x-1}}{2\sqrt{x+1}}+\frac{(x-1)^{2}}{4}\\&\quad+\begin{cases}\frac{(x+1)^{2}}{4}+\frac{\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2}-\frac{(x+1)^{\frac{5}{2}}}{4\sqrt{x-1}}+\frac{3(x+1)^{\frac{3}{2}}}{4\sqrt{x-1}}-\frac{\sqrt{x+1}}{2\sqrt{x-1}} & \text{for } |x+1|>2\\\frac{(x+1)^{2}}{4}-\frac{i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2}+\frac{i(x+1)^{\frac{5}{2}}}{4\sqrt{1-x}}-\frac{3i(x+1)^{\frac{3}{2}}}{4\sqrt{1-x}}+\frac{i\sqrt{x+1}}{2\sqrt{1-x}} & \text{otherwise}\end{cases}\\&\quad+\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{x-1}}{2}\right)}{2}\end{aligned}$$

```
[In] integrate((-(-1+x)**(1/2)+(1+x)**(1/2))/((-1+x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -(x - 1)**(5/2)/(4*sqrt(x + 1)) - 3*(x - 1)**(3/2)/(4*sqrt(x + 1)) - sqrt(x
 - 1)/(2*sqrt(x + 1)) + (x - 1)**2/4 + Piecewise(((x + 1)**2/4 + acosh(sqrt
(2)*sqrt(x + 1)/2)/2 - (x + 1)**(5/2)/(4*sqrt(x - 1)) + 3*(x + 1)**(3/2)/(4
*sqrt(x - 1)) - sqrt(x + 1)/(2*sqrt(x - 1)), Abs(x + 1) > 2), ((x + 1)**2/4
 - I*asin(sqrt(2)*sqrt(x + 1)/2)/2 + I*(x + 1)**(5/2)/(4*sqrt(1 - x)) - 3*I
*(x + 1)**(3/2)/(4*sqrt(1 - x)) + I*sqrt(x + 1)/(2*sqrt(1 - x)), True)) + a
sinh(sqrt(2)*sqrt(x - 1)/2)/2
```

Maxima [F]

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx=\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\,dx$$

```
[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="maxima")
```

```
[Out] integrate((sqrt(x + 1) - sqrt(x - 1))/(sqrt(x + 1) + sqrt(x - 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx=\frac{1}{2}\,(x+1)^2-\frac{1}{2}\,\sqrt{x+1}\sqrt{x-1}x-x-\log\left(\sqrt{x+1}-\sqrt{x-1}\right)-1$$

```
[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="giac")
```

```
[Out] 1/2*(x + 1)^2 - 1/2*sqrt(x + 1)*sqrt(x - 1)*x - x - log(sqrt(x + 1) - sqrt(
x - 1)) - 1
```

Mupad [B] (verification not implemented)

Time = 27.82 (sec) , antiderivative size = 200, normalized size of antiderivative = 6.06

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \operatorname{acosh}(x) - 2\operatorname{atanh}\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right) + \frac{\frac{14\left(\sqrt{x-1}-\mathrm{i}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{14\left(\sqrt{x-1}-\mathrm{i}\right)^5}{\left(\sqrt{x+1}-1\right)^5}+\frac{2\left(\sqrt{x-1}-\mathrm{i}\right)^7}{\left(\sqrt{x+1}-1\right)^7}+\frac{2\left(\sqrt{x-1}-\mathrm{i}\right)}{\sqrt{x+1}-1}}{1+\frac{6\left(\sqrt{x-1}-\mathrm{i}\right)^4}{\left(\sqrt{x+1}-1\right)^4}-\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^8}{\left(\sqrt{x+1}-1\right)^8}-\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}} + \frac{x^2}{2}$$

```
[In] int(-((x - 1)^(1/2) - (x + 1)^(1/2))/((x - 1)^(1/2) + (x + 1)^(1/2)),x)
```

```
[Out] acosh(x) - 2*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)) + ((14*((x - 1
)^(1/2) - 1i)^3)/((x + 1)^(1/2) - 1)^3 + (14*((x - 1)^(1/2) - 1i)^5)/((x +
1)^(1/2) - 1)^5 + (2*((x - 1)^(1/2) - 1i)^7)/((x + 1)^(1/2) - 1)^7 + (2*((x
 - 1)^(1/2) - 1i))/((x + 1)^(1/2) - 1))/((6*((x - 1)^(1/2) - 1i)^4)/((x + 1
)^(1/2) - 1)^4 - (4*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 - (4*((x
- 1)^(1/2) - 1i)^6)/((x + 1)^(1/2) - 1)^6 + ((x - 1)^(1/2) - 1i)^8/((x + 1)
^(1/2) - 1)^8 + 1) + x^2/2
```

3.453 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n\,dx$

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n\,dx = \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)} + \frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}{d}\right)}{2d^2e(1+n)}$$

```
[Out] 1/2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)+1/2*a*f^2*hypergeom([2, 1
+n],[2+n],(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))/d)*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)
)^(1+n)/d^2/e/(1+n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used

$= \{2142, 961, 66\}$

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n\,dx$$
$$=\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}{d}\right)}{2d^2e(n+1)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^n,x]
```

```
[Out] (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n)) + (a*f^2*(d + e
*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n,
(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])/d])/(2*d^2*e*(1 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral}=\frac{\text{Subst}\left(\int\frac{x^n\left(d^2+af^2-2dx+x^2\right)}{(d-x)^2}\,dx,x,d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int\left(x^n+\frac{af^2x^n}{(d-x)^2}\right)\,dx,x,d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}+\frac{(af^2)\,\text{Subst}\left(\int\frac{x^n}{(d-x)^2}\,dx,x,d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)} + \frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}\,{}_2F_1\left(2,1+n;2+n;\frac{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}{d}\right)}{2d^2e(1+n)}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.71

$$\int\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n\,dx = \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}\left(d^2+af^2\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}{d}\right)\right)}{2d^2e(1+n)}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^n,x]
[Out] ((d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)*(d^2 + a*f^2*Hypergeometric2
F1[2, 1 + n, 2 + n, (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])/d]))/(2*d^2*e*(1
+ n))
```

Maple [F]

$$\int\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx$$

```
[In] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x)
[Out] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x)
```

Fricas [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^n dx$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)^n, x)
```

Sympy [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx$$

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**n,x)
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**n, x)
```

Maxima [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^n dx$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^n, x)
```

Giac [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^n dx$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n\,dx=\int \left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^n\,dx$$

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^n,x)
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^n, x)
```

3.454 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx$

Optimal result

Integrand size = 25, antiderivative size = 175

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx = -\frac{ad^3f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{adf^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{e}+\frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}{4e}+\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^4}{8e}+\frac{3ad^2f^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

```
[Out] 3/2*a*d^2*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e-1/2*a*d^3*f^2/e/(e*x+f*(a+e
^2*x^2/f^2)^(1/2))+a*d*f^2*(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e+1/4*a*f^2*(d+e*x
+f*(a+e^2*x^2/f^2)^(1/2))^2/e+1/8*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^4/e
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used

$= \{2142, 907\}$

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx = -\frac{ad^3f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} + \frac{3ad^2f^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e} + \frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^4}{8e} + \frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{4e} + \frac{adf^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]
```

```
[Out] -1/2*(a*d^3*f^2)/(e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*d*f^2*(e*x + f*
Sqrt[a + (e^2*x^2)/f^2]))/e + (a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^
2)/(4*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^4/(8*e) + (3*a*d^2*f^2*Log
[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^3(d^2+af^2-2dx+x^2)}{(d-x)^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(2adf^2+\frac{ad^3f^2}{(d-x)^2}-\frac{3ad^2f^2}{d-x}+af^2x+x^3\right)\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{ad^3f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{adf^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{e} + \frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}{4e} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^4}{8e} + \frac{3ad^2f^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10

$$\int\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx$$
$$= \frac{ex(2d^3+6adf^2+3d^2ex+3aef^2x+4de^2x^2+2e^3x^3)+\sqrt{a+\frac{e^2x^2}{f^2}}(2af^3(2d+ex)+efx(3d^2+4dex+2e}{2e}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (e*x*(2*d^3 + 6*a*d*f^2 + 3*d^2*e*x + 3*a*e*f^2*x + 4*d*e^2*x^2 + 2*e^3*x^3
) + Sqrt[a + (e^2*x^2)/f^2]*(2*a*f^3*(2*d + e*x) + e*f*x*(3*d^2 + 4*d*e*x +
 2*e^2*x^2)) - 3*a*d^2*f^2*Log[e*(Sqrt[a]*f + e*x - f*Sqrt[a + (e^2*x^2)/f^
2])] + 3*a*d^2*f^2*Log[-(Sqrt[a]*f) + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*
e)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(157) = 314.

Time = 0.93 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.95

method	result
default	$f^3\left(\frac{x\left(a+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{a+\frac{e^2x^2}{f^2}}}{2}+\frac{a\ln\left(\frac{e^2x}{f^2\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2\sqrt{\frac{e^2}{f^2}}}\right)}{4}\right)+3f^2\left(\frac{e^3x^4}{4f^2}+\frac{de^2x^3}{3f^2}+\frac{aex^2}{2}+adx\right)+3f$

```
[In] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] f^3*(1/4*x*(a+e^2*x^2/f^2)^(3/2)+3/4*a*(1/2*x*(a+e^2*x^2/f^2)^(1/2)+1/2*a*l
n(e^2*x/f^2/(e^2/f^2)^(1/2)+(a+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)))+3*f^2*
```

```
(1/4*e^3/f^2*x^4+1/3*d*e^2/f^2*x^3+1/2*a*e*x^2+a*d*x)+3*f*(d^2*(1/2*x*(a+e^
2*x^2/f^2)^(1/2)+1/2*a*ln(e^2*x/f^2/(e^2/f^2)^(1/2)+(a+e^2*x^2/f^2)^(1/2))/
(e^2/f^2)^(1/2))+e^2*(1/4*x*(a+e^2*x^2/f^2)^(3/2)/e^2*f^2-1/4*a/e^2*f^2*(1/
2*x*(a+e^2*x^2/f^2)^(1/2)+1/2*a*ln(e^2*x/f^2/(e^2/f^2)^(1/2)+(a+e^2*x^2/f^2
)^(1/2))/(e^2/f^2)^(1/2)))+2/3/e*d*f^2*((e^2*x^2+a*f^2)/f^2)^(3/2))+1/4*(e*
x+d)^4/e
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.92

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx$$
$$=\frac{2\,e^4x^4+4\,de^3x^3-3\,ad^2f^2\log\left(-ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)+3\,(ae^2f^2+d^2e^2)x^2+2\,(3\,adef^2+d^3e)x+(2\,e^3fx^3}{2\,e}$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^4*x^4 + 4*d*e^3*x^3 - 3*a*d^2*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f
^2)/f^2)) + 3*(a*e^2*f^2 + d^2*e^2)*x^2 + 2*(3*a*d*e*f^2 + d^3*e)*x + (2*e^
3*f*x^3 + 4*d*e^2*f*x^2 + 4*a*d*f^3 + (2*a*e*f^3 + 3*d^2*e*f)*x)*sqrt((e^2*
x^2 + a*f^2)/f^2))/e
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.62

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3\,dx = 3adf^2x+\frac{3aef^2x^2}{2} + af^3\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(\frac{2e^2x}{f^2}+2\sqrt{\frac{e^2}{f^2}}\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{\frac{e^2x^2}{f^2}}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{a+\frac{e^2x^2}{f^2}}}{2} & \text{for } \frac{e^2}{f^2}\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right)+d^3x + \frac{3d^2ex^2}{2}+3d^2f\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(\frac{2e^2x}{f^2}+2\sqrt{\frac{e^2}{f^2}}\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{\frac{e^2x^2}{f^2}}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{a+\frac{e^2x^2}{f^2}}}{2} & \text{for } \frac{e^2}{f^2}\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + 2de^2x^3+6def\left(\begin{cases}\sqrt{a+\frac{e^2x^2}{f^2}}\left(\frac{af^2}{3e^2}+\frac{x^2}{3}\right) & \text{for } \frac{e^2}{f^2}\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right)+e^3x^4 + 4e^2f\left(\begin{cases}-\frac{a^2f^2\left(\begin{cases}\frac{\log\left(\frac{2e^2x}{f^2}+2\sqrt{\frac{e^2}{f^2}}\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{\frac{e^2x^2}{f^2}}} & \text{otherwise}\end{cases}\right)}{8e^2}+\sqrt{a+\frac{e^2x^2}{f^2}}\left(\frac{af^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for } \frac{e^2}{f^2}\neq 0\\ \frac{\sqrt{a}x^3}{3} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] 3*a*d*f**2*x + 3*a*e*f**2*x**2/2 + a*f**3*Piecewise((a*Piecewise((log(2*e**
2*x/f**2 + 2*sqrt(e**2/f**2)*sqrt(a + e**2*x**2/f**2))/sqrt(e**2/f**2), Ne(
a, 0)), (x*log(x)/sqrt(e**2*x**2/f**2), True))/2 + x*sqrt(a + e**2*x**2/f**
2)/2, Ne(e**2/f**2, 0)), (sqrt(a)*x, True)) + d**3*x + 3*d**2*e*x**2/2 + 3*
d**2*f*Piecewise((a*Piecewise((log(2*e**2*x/f**2 + 2*sqrt(e**2/f**2)*sqrt(a
 + e**2*x**2/f**2))/sqrt(e**2/f**2), Ne(a, 0)), (x*log(x)/sqrt(e**2*x**2/f*
*2), True))/2 + x*sqrt(a + e**2*x**2/f**2)/2, Ne(e**2/f**2, 0)), (sqrt(a)*x
```

```
, True)) + 2*d*e**2*x**3 + 6*d*e*f*Piecewise((sqrt(a + e**2*x**2/f**2)*(a*f
**2/(3*e**2) + x**2/3), Ne(e**2/f**2, 0)), (sqrt(a)*x**2/2, True)) + e**3*x
**4 + 4*e**2*f*Piecewise((-a**2*f**2*Piecewise((log(2*e**2*x/f**2 + 2*sqrt(
e**2/f**2)*sqrt(a + e**2*x**2/f**2))/sqrt(e**2/f**2), Ne(a, 0)), (x*log(x)/
sqrt(e**2*x**2/f**2), True))/(8*e**2) + sqrt(a + e**2*x**2/f**2)*(a*f**2*x/
(8*e**2) + x**3/4), Ne(e**2/f**2, 0)), (sqrt(a)*x**3/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.67

$$
\begin{aligned}
&\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx \\
&= \frac{1}{4}e^3x^4 + \frac{3\left(\frac{e^2x^2}{f^2}+a\right)^2 f^4}{4e} \\
&\quad - \frac{3}{8}\left(\frac{a^2f^3\,\mathrm{arsinh}\left(\frac{e^2x}{\sqrt{ae^2}f}\right)}{\sqrt{e^2}e^2} - \frac{2\left(\frac{e^2x^2}{f^2}+a\right)^{\frac{3}{2}}f^2x}{e^2} + \frac{\sqrt{\frac{e^2x^2}{f^2}+a}\,af^2x}{e^2}\right)e^2f \\
&\quad + \frac{1}{8}\left(\frac{3a^2f\,\mathrm{arsinh}\left(\frac{e^2x}{\sqrt{ae^2}f}\right)}{\sqrt{e^2}} + 2\left(\frac{e^2x^2}{f^2}+a\right)^{\frac{3}{2}}x + 3\sqrt{\frac{e^2x^2}{f^2}+a}\,ax\right)f^3 \\
&\quad + d^3x + \frac{3}{2}\left(ex^2 + \left(\frac{af\,\mathrm{arsinh}\left(\frac{e^2x}{\sqrt{ae^2}f}\right)}{\sqrt{e^2}} + \sqrt{\frac{e^2x^2}{f^2}+a}\,x\right)f\right)d^2 \\
&\quad + \left(e^2x^3 + \frac{2\left(\frac{e^2x^2}{f^2}+a\right)^{\frac{3}{2}}f^3}{e} + \left(\frac{e^2x^3}{f^2}+3\,ax\right)f^2\right)d
\end{aligned}
$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 1/4*e^3*x^4 + 3/4*(e^2*x^2/f^2 + a)^2*f^4/e - 3/8*(a^2*f^3*arcsinh(e^2*x/(s
qrt(a*e^2)*f))/(sqrt(e^2)*e^2) - 2*(e^2*x^2/f^2 + a)^(3/2)*f^2*x/e^2 + sqrt
(e^2*x^2/f^2 + a)*a*f^2*x/e^2)*e^2*f + 1/8*(3*a^2*f*arcsinh(e^2*x/(sqrt(a*e
^2)*f))/sqrt(e^2) + 2*(e^2*x^2/f^2 + a)^(3/2)*x + 3*sqrt(e^2*x^2/f^2 + a)*a
*x)*f^3 + d^3*x + 3/2*(e*x^2 + (a*f*arcsinh(e^2*x/(sqrt(a*e^2)*f))/sqrt(e^2
) + sqrt(e^2*x^2/f^2 + a)*x)*f)*d^2 + (e^2*x^3 + 2*(e^2*x^2/f^2 + a)^(3/2)*
f^3/e + (e^2*x^3/f^2 + 3*a*x)*f^2)*d
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx$$
$$= e^3x^4 + \frac{3}{2}aef^2x^2 + 2de^2x^3 + 3adf^2x + \frac{3}{2}d^2ex^2 - \frac{3ad^2f|f|\log\left(\left|-x|e| + \sqrt{e^2x^2+af^2}\right|\right)}{2|e|} + d^3x + \frac{1}{2}\sqrt{e^2x^2+af^2}\left(\frac{4adf|f|}{e} + \left(2\left(\frac{e^2x|f|}{f} + \frac{2de|f|}{f}\right)x + \frac{2ae^4f^4|f| + 3d^2e^4f^2|f|}{e^4f^3}\right)x\right)$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] e^3*x^4 + 3/2*a*e*f^2*x^2 + 2*d*e^2*x^3 + 3*a*d*f^2*x + 3/2*d^2*e*x^2 - 3/2
*a*d^2*f*abs(f)*log(abs(-x*abs(e) + sqrt(e^2*x^2 + a*f^2)))/abs(e) + d^3*x
+ 1/2*sqrt(e^2*x^2 + a*f^2)*(4*a*d*f*abs(f)/e + (2*(e^2*x*abs(f)/f + 2*d*e*
abs(f)/f)*x + (2*a*e^4*f^4*abs(f) + 3*d^2*e^4*f^2*abs(f))/(e^4*f^3))*x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx = \int \left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^3 dx$$

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3,x)
```

```
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3, x)
```

3.455 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx$

Optimal result

Integrand size = 25, antiderivative size = 136

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = -\frac{ad^2f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{af^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}+\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}{6e}+\frac{adf^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{e}$$

```
[Out] a*d*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e-1/2*a*d^2*f^2/e/(e*x+f*(a+e^2*x^2
/f^2)^(1/2))+1/2*a*f^2*(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e+1/6*(d+e*x+f*(a+e^2*
x^2/f^2)^(1/2))^3/e
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2142, 907}

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = -\frac{ad^2f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3}{6e}+\frac{adf^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]
```

```
[Out] -1/2*(a*d^2*f^2)/(e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*(e*x + f*Sq
rt[a + (e^2*x^2)/f^2]))/(2*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3/(6*
e) + (a*d*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2\left(d^2+af^2-2dx+x^2\right)}{(d-x)^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} \\
&= \frac{\text{Subst}\left(\int \left(af^2+\frac{ad^2f^2}{(d-x)^2}-\frac{2adf^2}{d-x}+x^2\right)\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} \\
&= -\frac{ad^2f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{af^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} \\
&\quad +\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}{6e}+\frac{adf^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.83

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = d^2x+af^2x+dex^2+\frac{2e^2x^3}{3} + \frac{\sqrt{a+\frac{e^2x^2}{f^2}}(2af^3+efx(3d+2ex))}{3e} + \frac{2adf^2\operatorname{arctanh}\left(\frac{f\left(-\sqrt{a}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{ex}\right)}{e}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]
```

```
[Out] d^2*x + a*f^2*x + d*e*x^2 + (2*e^2*x^3)/3 + (Sqrt[a + (e^2*x^2)/f^2]*(2*a*f
^3 + e*f*x*(3*d + 2*e*x)))/(3*e) + (2*a*d*f^2*ArcTanh[(f*(-Sqrt[a] + Sqrt[a
 + (e^2*x^2)/f^2]))/(e*x)])/e
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{e^2x^3}{3}+a\,f^2x+2f\left(d\left(\frac{x\sqrt{a+\frac{e^2x^2}{f^2}}}{2}+\frac{a\ln\left(\frac{e^2x}{f^2\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2\sqrt{\frac{e^2}{f^2}}}\right)+\frac{f^2\left(\frac{e^2x^2+a\,f^2}{f^2}\right)^{\frac{3}{2}}}{3e}\right)+\frac{(ex+d)^3}{3e}$	124

```
[In] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e^2*x^3+a*f^2*x+2*f*(d*(1/2*x*(a+e^2*x^2/f^2)^(1/2)+1/2*a*ln(e^2*x/f^2/
(e^2/f^2)^(1/2)+(a+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2))+1/3/e*f^2*((e^2*x^2
+a*f^2)/f^2)^(3/2))+1/3*(e*x+d)^3/e
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = \frac{2\,e^3x^3+3\,de^2x^2-3\,adf^2\log\left(-ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)+3\,(aef^2+d^2e)x+(2\,e^2fx^2+2\,af^3+3\,defx)\sqrt{\frac{e^2x^2+af^2}{f^2}}}{3\,e}$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*e^3*x^3 + 3*d*e^2*x^2 - 3*a*d*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f^2
)/f^2)) + 3*(a*e*f^2 + d^2*e)*x + (2*e^2*f*x^2 + 2*a*f^3 + 3*d*e*f*x)*sqrt(
(e^2*x^2 + a*f^2)/f^2))/e
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.36

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = af^2x+d^2x+dex^2 + 2df\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(\frac{2e^2x}{f^2}+2\sqrt{\frac{e^2}{f^2}}\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{\frac{e^2x^2}{f^2}}} & \text{otherwise} \end{cases}\right)}{2}+\frac{x\sqrt{a+\frac{e^2x^2}{f^2}}}{2} & \text{for } \frac{e^2}{f^2}\neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right) + \frac{2e^2x^3}{3} + 2ef\left(\begin{cases} \sqrt{a+\frac{e^2x^2}{f^2}}\left(\frac{af^2}{3e^2}+\frac{x^2}{3}\right) & \text{for } \frac{e^2}{f^2}\neq 0 \\ \frac{\sqrt{a}x^2}{2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] a*f**2*x + d**2*x + d*e*x**2 + 2*d*f*Piecewise((a*Piecewise((log(2*e**2*x/f
**2 + 2*sqrt(e**2/f**2)*sqrt(a + e**2*x**2/f**2))/sqrt(e**2/f**2), Ne(a, 0)
), (x*log(x)/sqrt(e**2*x**2/f**2), True))/2 + x*sqrt(a + e**2*x**2/f**2)/2,
 Ne(e**2/f**2, 0)), (sqrt(a)*x, True)) + 2*e**2*x**3/3 + 2*e*f*Piecewise((s
qrt(a + e**2*x**2/f**2)*(a*f**2/(3*e**2) + x**2/3), Ne(e**2/f**2, 0)), (sqr
t(a)*x**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = \frac{1}{3}e^2x^3+\frac{2\left(\frac{e^2x^2}{f^2}+a\right)^{\frac{3}{2}}f^3}{3e}+\frac{1}{3}\left(\frac{e^2x^3}{f^2}+3ax\right)f^2+d^2x+\left(ex^2+\left(\frac{af\,\mathrm{arsinh}\left(\frac{e^2x}{\sqrt{ae^2}f}\right)}{\sqrt{e^2}}+\sqrt{\frac{e^2x^2}{f^2}+ax}\right)f\right)d$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/3*e^2*x^3 + 2/3*(e^2*x^2/f^2 + a)^(3/2)*f^3/e + 1/3*(e^2*x^3/f^2 + 3*a*x)
*f^2 + d^2*x + (e*x^2 + (a*f*arcsinh(e^2*x/(sqrt(a*e^2)*f))/sqrt(e^2) + sqr
t(e^2*x^2/f^2 + a)*x)*f)*d
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = \frac{2}{3}e^2x^3+af^2x+dex^2-\frac{adf|f|\log\left(\left|-x|e|+\sqrt{e^2x^2+af^2}\right|\right)}{|e|}+d^2x+\frac{1}{3}\sqrt{e^2x^2+af^2}\left(\left(\frac{2ex|f|}{f}+\frac{3d|f|}{f}\right)x+\frac{2af|f|}{e}\right)$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2/3*e^2*x^3 + a*f^2*x + d*e*x^2 - a*d*f*abs(f)*log(abs(-x*abs(e) + sqrt(e^2
*x^2 + a*f^2)))/abs(e) + d^2*x + 1/3*sqrt(e^2*x^2 + a*f^2)*((2*e*x*abs(f)/f
 + 3*d*abs(f)/f)*x + 2*a*f*abs(f)/e)
```

Mupad [B] (verification not implemented)

Time = 18.79 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.54

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2\,dx$$
$$=\begin{cases} x\left(d+\sqrt{a}\,f\right)^2 \\ x\left(d^2+a\,f^2\right)+\frac{2\,e^2\,x^3}{3}+d\,e\,x^2+\frac{2\,a\,f^3\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}{e}-\frac{2\,f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\,\left(2\,a\,f^2-e^2\,x^2\right)}{3\,e}+d\,f\,x\,\sqrt{a+\frac{e^2\,x^2}{f^2}}+\frac{2\,a\,d\,f\,\ln\left(x\,\sqrt{\right.}\right.}{\sqrt{}} \end{cases}$$

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2,x)
```

```
[Out] piecewise(e == 0, x*(d + a^(1/2)*f)^2, e ~= 0, x*(a*f^2 + d^2) + (2*e^2*x^3
)/3 + d*e*x^2 + (2*a*f^3*(a + (e^2*x^2)/f^2)^(1/2))/e - (2*f*(a + (e^2*x^2)
/f^2)^(1/2)*(2*a*f^2 - e^2*x^2))/(3*e) + d*f*x*(a + (e^2*x^2)/f^2)^(1/2) +
(2*a*d*f*log(x*(e^2/f^2)^(1/2) + (a + (e^2*x^2)/f^2)^(1/2)))/(e^2/f^2)^(1/2
) - (a*d*e^2*log(2*x*(e^2/f^2)^(1/2) + 2*(a + (e^2*x^2)/f^2)^(1/2)))/(f*(e^
2/f^2)^(3/2)))
```

3.456 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\,dx=dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\text{arctanh}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}$$

```
[Out] d*x+1/2*e*x^2+1/2*a*f^2*arctanh(e*x/f/(a+e^2*x^2/f^2)^(1/2))/e+1/2*f*x*(a+e
^2*x^2/f^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {201, 223, 212}

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\,dx=\frac{af^2\text{arctanh}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+dx+\frac{ex^2}{2}$$

```
[In] Int[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[a + (e^2*x^2)/f^2])/2 + (a*f^2*ArcTanh[(e*x)/(f
*Sqrt[a + (e^2*x^2)/f^2])])/(2*e)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= dx + \frac{ex^2}{2} + f\int \sqrt{a + \frac{e^2x^2}{f^2}}\,dx \\
&= dx + \frac{ex^2}{2} + \frac{1}{2}fx\sqrt{a + \frac{e^2x^2}{f^2}} + \frac{1}{2}(af)\int \frac{1}{\sqrt{a + \frac{e^2x^2}{f^2}}}\,dx \\
&= dx + \frac{ex^2}{2} + \frac{1}{2}fx\sqrt{a + \frac{e^2x^2}{f^2}} + \frac{1}{2}(af)\text{Subst}\left(\int \frac{1}{1 - \frac{e^2x^2}{f^2}}\,dx, x, \frac{x}{\sqrt{a + \frac{e^2x^2}{f^2}}}\right) \\
&= dx + \frac{ex^2}{2} + \frac{1}{2}fx\sqrt{a + \frac{e^2x^2}{f^2}} + \frac{af^2\tanh^{-1}\left(\frac{ex}{f\sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{2e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)dx = dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\text{arctanh}\left(\frac{-\frac{\sqrt{a}f}{e}+\frac{f\sqrt{a+\frac{e^2x^2}{f^2}}}{e}}{x}\right)}{e}$$

```
[In] Integrate[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]
[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[a + (e^2*x^2)/f^2])/2 + (a*f^2*ArcTanh[(-((Sqrt
[a]*f)/e) + (f*Sqrt[a + (e^2*x^2)/f^2])/e)/x])/e
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

method	result	size
default	$dx+\frac{e\,x^{2}}{2}+\frac{fx\sqrt{a+\frac{e^{2}x^{2}}{f^{2}}}}{2}+\frac{fa\ln\left(\frac{e^{2}x}{f^{2}\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{a+\frac{e^{2}x^{2}}{f^{2}}}\right)}{2\sqrt{\frac{e^{2}}{f^{2}}}}$	75
parts	$dx+\frac{e\,x^{2}}{2}+\frac{fx\sqrt{a+\frac{e^{2}x^{2}}{f^{2}}}}{2}+\frac{fa\ln\left(\frac{e^{2}x}{f^{2}\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{a+\frac{e^{2}x^{2}}{f^{2}}}\right)}{2\sqrt{\frac{e^{2}}{f^{2}}}}$	75

```
[In] int(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] d*x+1/2*e*x^2+1/2*f*x*(a+e^2*x^2/f^2)^(1/2)+1/2*f*a*ln(e^2*x/f^2/(e^2/f^2)^
(1/2)+(a+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

$$\int\left(d+ex+f\sqrt{a+\frac{e^{2}x^{2}}{f^{2}}}\right)\,dx$$
$$=\frac{e^{2}x^{2}-af^{2}\log\left(-ex+f\sqrt{\frac{e^{2}x^{2}+af^{2}}{f^{2}}}\right)+efx\sqrt{\frac{e^{2}x^{2}+af^{2}}{f^{2}}}+2\,dex}{2\,e}$$

```
[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(e^2*x^2 - a*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)) + e*f*x*sqrt
((e^2*x^2 + a*f^2)/f^2) + 2*d*e*x)/e
```

Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int\left(d+ex+f\sqrt{a+\frac{e^{2}x^{2}}{f^{2}}}\right)\,dx=dx+\frac{ex^{2}}{2}+f\left(\frac{\sqrt{a}x\sqrt{1+\frac{e^{2}x^{2}}{af^{2}}}}{2}+\frac{af\,\mathrm{asinh}\left(\frac{ex}{\sqrt{a}f}\right)}{2e}\right)$$

```
[In] integrate(d+e*x+f*(a+e**2*x**2/f**2)**(1/2),x)
[Out] d*x + e*x**2/2 + f*(sqrt(a)*x*sqrt(1 + e**2*x**2/(a*f**2))/2 + a*f*asinh(e*
x/(sqrt(a)*f))/(2*e))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\,dx=\frac{1}{2}\,ex^2+\frac{1}{2}\left(\frac{af\,\mathrm{arsinh}\left(\frac{e^2x}{\sqrt{ae^2}f}\right)}{\sqrt{e^2}}+\sqrt{\frac{e^2x^2}{f^2}+a}x\right)f+dx$$

```
[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*e*x^2 + 1/2*(a*f*arcsinh(e^2*x/(sqrt(a*e^2)*f))/sqrt(e^2) + sqrt(e^2*x^
2/f^2 + a)*x)*f + d*x
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\,dx=\frac{1}{2}\,ex^2+dx-\frac{\left(\frac{af^2\log\left(\left|-x|e|+\sqrt{e^2x^2+af^2}\right|\right)}{|e|}-\sqrt{e^2x^2+af^2}x\right)|f|}{2\,f}$$

```
[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*e*x^2 + d*x - 1/2*(a*f^2*log(abs(-x*abs(e) + sqrt(e^2*x^2 + a*f^2)))/ab
s(e) - sqrt(e^2*x^2 + a*f^2)*x)*abs(f)/f
```

Mupad [B] (verification not implemented)

Time = 17.64 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.00

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\,dx$$
$$=\begin{cases} x\,\left(d+\sqrt{a}\,f\right) & \text{if}\ \ e=0\\ d\,x+\frac{e\,x^2}{2}+\frac{f\,x\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}{2}+\frac{a\,e^2\,\ln\left(x\,\sqrt{\frac{e^2}{f^2}}+\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}{f\,{\left(\frac{e^2}{f^2}\right)}^{3/2}}-\frac{a\,e^2\,\ln\left(2\,x\,\sqrt{\frac{e^2}{f^2}}+2\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}{2\,f\,{\left(\frac{e^2}{f^2}\right)}^{3/2}} & \text{if}\ \ e\ne 0 \end{cases}$$

```
[In] int(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, x*(d + a^(1/2)*f), e ~= 0, d*x + (e*x^2)/2 + (f*x*(a + (e
^2*x^2)/f^2)^(1/2))/2 + (a*e^2*log(x*(e^2/f^2)^(1/2) + (a + (e^2*x^2)/f^2)^
(1/2)))/(f*(e^2/f^2)^(3/2)) - (a*e^2*log(2*x*(e^2/f^2)^(1/2) + 2*(a + (e^2*
x^2)/f^2)^(1/2)))/(2*f*(e^2/f^2)^(3/2)))
```

3.457 $\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx=-\frac{af^2}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}-\frac{af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^2e}+\frac{\left(1+\frac{af^2}{d^2}\right)\log\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

```
[Out] -1/2*a*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/d^2/e+1/2*(1+a*f^2/d^2)*ln(d+e*x
+f*(a+e^2*x^2/f^2)^(1/2))/e-1/2*a*f^2/d/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2142, 907}

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx=-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^2e}+\frac{\left(\frac{af^2}{d^2}+1\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2e}-\frac{af^2}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] -1/2*(a*f^2)/(d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*S
qrt[a + (e^2*x^2)/f^2]])/(2*d^2*e) + ((1 + (a*f^2)/d^2)*Log[d + e*x + f*Sqr
t[a + (e^2*x^2)/f^2]])/(2*e)
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} \\
&= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d(d-x)^2}+\frac{af^2}{d^2(d-x)}+\frac{d^2+af^2}{d^2x}\right)\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} \\
&= -\frac{af^2}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}-\frac{af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^2e} \\
&\quad +\frac{\left(1+\frac{af^2}{d^2}\right)\log\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 337 vs. 2(117) = 234.

Time = 0.86 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.88

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$
$$= \frac{2de^2x-2def\sqrt{a+\frac{e^2x^2}{f^2}}-\left(af^2\left(e-\sqrt{\frac{e^2}{f^2}}f\right)+d^2\left(e+\sqrt{\frac{e^2}{f^2}}f\right)\right)\log\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)+\sqrt{\frac{e^2}{f^2}}f(d^2}{}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] (2*d*e^2*x - 2*d*e*f*Sqrt[a + (e^2*x^2)/f^2] - (a*f^2*(e - Sqrt[e^2/f^2]*f)
 + d^2*(e + Sqrt[e^2/f^2]*f))*Log[-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f
^2]] + Sqrt[e^2/f^2]*f*(d^2 + a*f^2)*Log[a*f + d*(-(Sqrt[e^2/f^2]*x) + Sqrt
[a + (e^2*x^2)/f^2])] + e*(d^2 + a*f^2)*Log[d*e*(a*f + d*(-(Sqrt[e^2/f^2]*x
) + Sqrt[a + (e^2*x^2)/f^2]))] - Sqrt[e^2/f^2]*f*(d^2 + a*f^2)*Log[d + f*(-
(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2])] + e*(d^2 + a*f^2)*Log[d^2*e*(
d + f*(-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]))])/(4*d^2*e^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(105) = 210$.

Time = 0.06 (sec) , antiderivative size = 651, normalized size of antiderivative = 5.56

method	result
default	$-\frac{f\left(\frac{\sqrt{\frac{4e^2\left(x+\frac{-a f^2+d^2}{2ed}\right)^2}{f^2}+\frac{4e\left(a f^2-d^2\right)\left(x+\frac{-a f^2+d^2}{2ed}\right)}{d f^2}+\frac{a^2 f^4+2a d^2 f^2+d^4}{d^2 f^2}}}{2}+\frac{e\left(a f^2-d^2\right)\ln\left(\frac{\frac{e\left(a f^2-d^2\right)}{2d f^2}+\frac{e^2\left(x+\frac{-a f^2+d^2}{2ed}\right)}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{\frac{e^2(\cdots}{}}\right.}{}\right.}{}$

```
[In] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*f/e/d*(1/2*(4*e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+4*e*(a*f^2-d^2)/d/f^2
*(x+1/2*(-a*f^2+d^2)/e/d)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)+1/2*e*(a
*f^2-d^2)/d/f^2*ln((1/2*e*(a*f^2-d^2)/d/f^2+e^2/f^2*(x+1/2*(-a*f^2+d^2)/e/d
))/(e^2/f^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+e*(a*f^2-d^2)/d/f^2*
(x+1/2*(-a*f^2+d^2)/e/d)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(e^2
/f^2)^(1/2)-1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2/((a^2*f^4+2*a*d^2*f^2+d^4
)/d^2/f^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+e*(a*f^2-d^2)/d/
f^2*(x+1/2*(-a*f^2+d^2)/e/d)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*
(4*e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+
d^2)/e/d)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/e/d
)))+1/2*ln(a*f^2-2*d*e*x-d^2)/e-e*(-1/2/e/d*x+1/4*(-a*f^2+d^2)/e^2/d^2*ln(-
a*f^2+2*d*e*x+d^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.60

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \frac{2\,dex-2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}+(af^2+d^2)\log\left(af^2-dex+df\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)+(af^2+d^2)\log\left(-af^2+2\,dex+d^2\right.}{4\,d^2e}$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*e*x - 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + (a*f^2 + d^2)*log(a*f^2
- d*e*x + d*f*sqrt((e^2*x^2 + a*f^2)/f^2)) + (a*f^2 + d^2)*log(-a*f^2 + 2*d
*e*x + d^2) - (a*f^2 + d^2)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) - d) +
 (a*f^2 - d^2)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)))/(d^2*e)
```

Sympy [F]

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2)),x)
```

```
[Out] Integral(1/(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

Maxima [F]

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \int \frac{1}{ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(105) = 210.

Time = 0.50 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.56

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$
$$= \frac{x}{2\,d} + \frac{(af^2+d^2)\log\left(|-af^2+2\,dex+d^2|\right)}{4\,d^2e} - \frac{\sqrt{e^2x^2+af^2}|f|}{2\,def} + \frac{(af^2|f|-d^2|f|)\log\left(\left|-x|e|+\sqrt{e^2x^2+af^2}\right|\right)}{4\,d^2f|e|} - \frac{(a^2e^2f^4|f|+2\,ad^2e^2f^2|f|+d^4e^2|f|)\log\left(\frac{\left|aef^2-d^2e-2\left(x|e|-\sqrt{e^2x^2+af^2}\right)d|e|-|aef^2+d^2e|\right|}{\left|aef^2-d^2e-2\left(x|e|-\sqrt{e^2x^2+af^2}\right)d|e|+|aef^2+d^2e|\right|}\right)}{4\,d^2ef|aef^2+d^2e||e|}$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x/d + 1/4*(a*f^2 + d^2)*log(abs(-a*f^2 + 2*d*e*x + d^2))/(d^2*e) - 1/2*
sqrt(e^2*x^2 + a*f^2)*abs(f)/(d*e*f) + 1/4*(a*f^2*abs(f) - d^2*abs(f))*log(
abs(-x*abs(e) + sqrt(e^2*x^2 + a*f^2)))/(d^2*f*abs(e)) - 1/4*(a^2*e^2*f^4*a
bs(f) + 2*a*d^2*e^2*f^2*abs(f) + d^4*e^2*abs(f))*log(abs(a*e*f^2 - d^2*e -
2*(x*abs(e) - sqrt(e^2*x^2 + a*f^2))*d*abs(e) - abs(a*e*f^2 + d^2*e))/abs(a
*e*f^2 - d^2*e - 2*(x*abs(e) - sqrt(e^2*x^2 + a*f^2))*d*abs(e) + abs(a*e*f^
2 + d^2*e)))/(d^2*e*f*abs(a*e*f^2 + d^2*e)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \int \frac{1}{d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}\,dx$$

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2)),x)
```

```
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2)), x)
```

3.458 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 151

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx = -\frac{af^2}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{1+\frac{af^2}{d^2}}{2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{d^3e} + \frac{af^2\log\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{d^3e}$$

```
[Out] -a*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/d^3/e+a*f^2*ln(d+e*x+f*(a+e^2*x^2/f^
2)^(1/2))/d^3/e-1/2*a*f^2/d^2/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))+1/2*(-1-a*f^2
/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used

$= \{2142, 907\}$

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx = -\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^3e} + \frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^3e} - \frac{af^2}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} - \frac{\frac{af^2}{d^2}+1}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2),x]
[Out] -1/2*(a*f^2)/(d^2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/
(2*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqrt[a + (
e^2*x^2)/f^2]])/(d^3*e) + (a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/
(d^3*e)
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} \\
&= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d^2(d-x)^2}+\frac{2af^2}{d^3(d-x)}+\frac{d^2+af^2}{d^2x^2}+\frac{2af^2}{d^3x}\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} \\
&= -\frac{af^2}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{1+\frac{af^2}{d^2}}{2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} \\
&\quad - \frac{af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{d^3e} + \frac{af^2\log\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{d^3e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.51

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx$$
$$= \frac{-d\left(d^3+d^2ex+af^2\left(ex-f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+dex\left(-ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)+af^2(-d^2+af^2-2dex)\log\left(-\right.}{d^3e}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] (-(d*(d^3 + d^2*e*x + a*f^2*(e*x - f*Sqrt[a + (e^2*x^2)/f^2]) + d*e*x*(-(e*
x) + f*Sqrt[a + (e^2*x^2)/f^2]))) + a*f^2*(-d^2 + a*f^2 - 2*d*e*x)*Log[-(Sq
rt[a]*f) + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + a*f^2*(d^2 - a*f^2 + 2*d*e*x)
*Log[-(a*f^2) + d*(e*x - f*Sqrt[a + (e^2*x^2)/f^2]) + Sqrt[a]*f*(d + e*x +
f*Sqrt[a + (e^2*x^2)/f^2])]])/(d^3*e*(d^2 - a*f^2 + 2*d*e*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3181 vs. 2(140) = 280.

Time = 0.08 (sec) , antiderivative size = 3182, normalized size of antiderivative = 21.07

method	result	size
default	Expression too large to display	3182

```
[In] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d/(a*f^2-2*d*e*x-d^2)/e+1/2*a*f^2/(a*f^2-2*d*e*x-d^2)/e/d+2*e^2*(1/4/e^
2/d^2*x+1/4/e^3/d^3*(a*f^2-d^2)*ln(-a*f^2+2*d*e*x+d^2)-1/8*(a^2*f^4-2*a*d^2
*f^2+d^4)/e^3/d^3/(-a*f^2+2*d*e*x+d^2))+2*e*d*(1/4/e^2/d^2*ln(-a*f^2+2*d*e*
x+d^2)-1/4*(a*f^2-d^2)/(-a*f^2+2*d*e*x+d^2)/e^2/d^2)-1/2/d*f/e^2*(-4/(a^2*f
^4+2*a*d^2*f^2+d^4)*d^2*f^2/(x+1/2*(-a*f^2+d^2)/e/d)*(e^2*(x+1/2*(-a*f^2+d^
2)/e/d)^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+1/4*(a^2*f^4+2*a
*d^2*f^2+d^4)/d^2/f^2)^(3/2)+2*e*(a*f^2-d^2)*d/(a^2*f^4+2*a*d^2*f^2+d^4)*(1
/2*(4*e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f
^2+d^2)/e/d)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)+1/2*e*(a*f^2-d^2)/d/f
^2*ln((1/2*e*(a*f^2-d^2)/d/f^2+e^2/f^2*(x+1/2*(-a*f^2+d^2)/e/d))/(e^2/f^2)^
(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^
2+d^2)/e/d)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(e^2/f^2)^(1/2)-1
/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1
/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-
a*f^2+d^2)/e/d)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4*e^2*(x+1/2
*(-a*f^2+d^2)/e/d)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+(a^
```

2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/e/d)))+8*e^2/(a^
2*f^4+2*a*d^2*f^2+d^4)*d^2*(1/4*(2*e^2/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+e*(a*f^
2-d^2)/d/f^2)/e^2*f^2*(e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+e*(a*f^2-d^2)/d/f
^2*(x+1/2*(-a*f^2+d^2)/e/d)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)+1/
8*(e^2/f^4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2-e^2*(a*f^2-d^2)^2/d^2/f^4)/e^2*f^2
*ln((1/2*e*(a*f^2-d^2)/d/f^2+e^2/f^2*(x+1/2*(-a*f^2+d^2)/e/d))/(e^2/f^2)^(1
/2)+(e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+
d^2)/e/d)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(e^2/f^2)^(1/2)))-2
*e*f*(1/4/e^2/d^2*(1/2*(4*e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+4*e*(a*f^2-d^2
)/d/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)+1
/2*e*(a*f^2-d^2)/d/f^2*ln((1/2*e*(a*f^2-d^2)/d/f^2+e^2/f^2*(x+1/2*(-a*f^2+d
^2)/e/d))/(e^2/f^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+e*(a*f^2-d^2)
/d/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2
))/(e^2/f^2)^(1/2)-1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2/((a^2*f^4+2*a*d^2*
f^2+d^4)/d^2/f^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+e*(a*f^2-
d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)
^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(
-a*f^2+d^2)/e/d)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d
^2)/e/d)))+1/8/e^3/d^3*(a*f^2-d^2)*(-4/(a^2*f^4+2*a*d^2*f^2+d^4)*d^2*f^2/(x
+1/2*(-a*f^2+d^2)/e/d)*(e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+e*(a*f^2-d^2)/d/
f^2*(x+1/2*(-a*f^2+d^2)/e/d)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(3/2)+2
e(a*f^2-d^2)*d/(a^2*f^4+2*a*d^2*f^2+d^4)*(1/2*(4*e^2*(x+1/2*(-a*f^2+d^2)/
e/d)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+(a^2*f^4+2*a*d^2*
f^2+d^4)/d^2/f^2)^(1/2)+1/2*e*(a*f^2-d^2)/d/f^2*ln((1/2*e*(a*f^2-d^2)/d/f^2
+e^2/f^2*(x+1/2*(-a*f^2+d^2)/e/d))/(e^2/f^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)
/e/d)^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+1/4*(a^2*f^4+2*a*d
^2*f^2+d^4)/d^2/f^2)^(1/2))/(e^2/f^2)^(1/2)-1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d
^2/f^2/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f
^2+d^4)/d^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+1/2*((a^2*f^4+
2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+4*e*(
a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2
)^(1/2))/(x+1/2*(-a*f^2+d^2)/e/d)))+8*e^2/(a^2*f^4+2*a*d^2*f^2+d^4)*d^2*(1/
4*(2*e^2/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+e*(a*f^2-d^2)/d/f^2)/e^2*f^2*(e^2*(x+
1/2*(-a*f^2+d^2)/e/d)^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+1/
4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)+1/8*(e^2/f^4*(a^2*f^4+2*a*d^2*f^
2+d^4)/d^2-e^2*(a*f^2-d^2)^2/d^2/f^4)/e^2*f^2*ln((1/2*e*(a*f^2-d^2)/d/f^2+e
^2/f^2*(x+1/2*(-a*f^2+d^2)/e/d))/(e^2/f^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/e
/d)^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+1/4*(a^2*f^4+2*a*d^2
*f^2+d^4)/d^2/f^2)^(1/2))/(e^2/f^2)^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(139) = 278$.

Time = 0.38 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.88

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx = \frac{a^2f^4 - 2\,d^2e^2x^2 + ad^2f^2 - 2\,d^3ex + (a^2f^4 - 2\,adef^2x - ad^2f^2)\log\left(-aef^2x + 2\,de^2x^2 + adf^2 + (af^3 - 2\right.}{}$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*f^4 - 2*d^2*e^2*x^2 + a*d^2*f^2 - 2*d^3*e*x + (a^2*f^4 - 2*a*d*e*f
^2*x - a*d^2*f^2)*log(-a*e*f^2*x + 2*d*e^2*x^2 + a*d*f^2 + (a*f^3 - 2*d*e*f
*x)*sqrt((e^2*x^2 + a*f^2)/f^2)) + (a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*lo
g(-a*f^2 + 2*d*e*x + d^2) - (a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*log(-e*x
+ f*sqrt((e^2*x^2 + a*f^2)/f^2) - d) - 2*(a*d*f^3 - d^2*e*f*x)*sqrt((e^2*x^
2 + a*f^2)/f^2))/(a*d^3*e*f^2 - 2*d^4*e^2*x - d^5*e)
```

Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx = \int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx = \int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^2}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx = \frac{af^2\log\left(\left|-af^2+2\,dex+d^2\right|\right)}{2\,d^3e} + \frac{af|f|\log\left(\left|-x|e|+\sqrt{e^2x^2+af^2}\right|\right)}{2\,d^3|e|} + \frac{x}{2\,d^2} - \frac{\sqrt{e^2x^2+af^2}|f|}{2\,d^2ef} + \frac{a^2f^4+2\,ad^2f^2+d^4}{4\,(af^2-2\,dex-d^2)d^3e}$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*a*f^2*log(abs(-a*f^2 + 2*d*e*x + d^2))/(d^3*e) + 1/2*a*f*abs(f)*log(abs
(-x*abs(e) + sqrt(e^2*x^2 + a*f^2)))/(d^3*abs(e)) + 1/2*x/d^2 - 1/2*sqrt(e^
2*x^2 + a*f^2)*abs(f)/(d^2*e*f) + 1/4*(a^2*f^4 + 2*a*d^2*f^2 + d^4)/((a*f^2
 - 2*d*e*x - d^2)*d^3*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx = \int \frac{1}{\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^2}\,dx$$

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2,x)
```

```
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2, x)
```

3.459 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3} dx = -\frac{af^2}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{1+\frac{af^2}{d^2}}{4e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2} - \frac{af^2}{d^3e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{3af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^4e} + \frac{3af^2\log\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^4e}$$

```
[Out] -3/2*a*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/d^4/e+3/2*a*f^2*ln(d+e*x+f*(a+e^
2*x^2/f^2)^(1/2))/d^4/e-1/2*a*f^2/d^3/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))+1/4*(
-1-a*f^2/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2-a*f^2/d^3/e/(d+e*x+f*(a+e
^2*x^2/f^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2142, 907}

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx = -\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^4e} + \frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2d^4e} - \frac{af^2}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} - \frac{af^2}{d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)} - \frac{\frac{af^2}{d^2}+1}{4e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -1/2*(a*f^2)/(d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/
(4*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2) - (a*f^2)/(d^3*e*(d + e*x + f
*Sqrt[a + (e^2*x^2)/f^2])) - (3*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])
/(2*d^4*e) + (3*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^4*e)
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^3}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d^3(d-x)^2} + \frac{3af^2}{d^4(d-x)} + \frac{d^2+af^2}{d^2x^3} + \frac{2af^2}{d^3x^2} + \frac{3af^2}{d^4x}\right) dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{af^2}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{1+\frac{af^2}{d^2}}{4e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2} - \frac{af^2}{d^3e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{3af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^4e} + \frac{3af^2\log\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^4e}$$

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.44

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3} dx = -\frac{\frac{d\sqrt{a+\frac{e^2x^2}{f^2}}(3a^2f^5+d^2efx(3d+4ex)-adf^3(5d+9ex))}{(d^2-af^2+2dex)^2} + \frac{d(2d^5+6d^4ex-3a^2ef^4x+3d^3e^2x^2+9ade^2f^2x^2+d^2(3aef^2x-4e^3x^3))}{(d^2-af^2+2dex)^2} + 3af^2 \text{lo}}{}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -1/2*((d*Sqrt[a + (e^2*x^2)/f^2]*(3*a^2*f^5 + d^2*e*f*x*(3*d + 4*e*x) - a*d
*f^3*(5*d + 9*e*x)))/(d^2 - a*f^2 + 2*d*e*x)^2 + (d*(2*d^5 + 6*d^4*e*x - 3*
a^2*e*f^4*x + 3*d^3*e^2*x^2 + 9*a*d*e^2*f^2*x^2 + d^2*(3*a*e*f^2*x - 4*e^3*
x^3)))/(d^2 - a*f^2 + 2*d*e*x)^2 + 3*a*f^2*Log[-(Sqrt[a]*f) + e*x + f*Sqrt[
a + (e^2*x^2)/f^2]] - 3*a*f^2*Log[-(a*f^2) + d*(e*x - f*Sqrt[a + (e^2*x^2)/
f^2]) + Sqrt[a]*f*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])])/(d^4*e)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11351 vs. 2(176) = 352.

Time = 0.13 (sec) , antiderivative size = 11352, normalized size of antiderivative = 58.82

method	result	size
default	Expression too large to display	11352

```
[In] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(175) = 350.

Time = 0.55 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.78

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx$$
$$= \frac{5\,a^3f^6 + 8\,d^3e^3x^3 - 6\,a^2d^2f^4 - 3\,ad^4f^2 + 2\,(ad^2e^2f^2 + 5\,d^4e^2)x^2 - 2\,(7\,a^2def^4 + ad^3ef^2 - 2\,d^5e)x + 3\,(a^3f}{}$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(5*a^3*f^6 + 8*d^3*e^3*x^3 - 6*a^2*d^2*f^4 - 3*a*d^4*f^2 + 2*(a*d^2*e^2
*f^2 + 5*d^4*e^2)*x^2 - 2*(7*a^2*d*e*f^4 + a*d^3*e*f^2 - 2*d^5*e)*x + 3*(a^
3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a^2*d*e*f^4 -
a*d^3*e*f^2)*x)*log(-a*e*f^2*x + 2*d*e^2*x^2 + a*d*f^2 + (a*f^3 - 2*d*e*f*x
)*sqrt((e^2*x^2 + a*f^2)/f^2)) + 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d
^2*f^4 + a*d^4*f^2 - 4*(a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-a*f^2 + 2*d*e*x
+ d^2) - 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(
a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) - d)
 - 2*(3*a^2*d*f^5 + 4*d^3*e^2*f*x^2 - 5*a*d^3*f^3 - 3*(3*a*d^2*e*f^3 - d^4*
e*f)*x)*sqrt((e^2*x^2 + a*f^2)/f^2))/(a^2*d^4*e*f^4 + 4*d^6*e^3*x^2 - 2*a*d
^6*e*f^2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)*x)
```

Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^3}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3), x)
```

Giac [A] (verification not implemented)

none

Time = 1.22 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \frac{3\,af^2\log\left(\left|-af^2+2\,dex+d^2\right|\right)}{4\,d^4e} + \frac{3\,af|f|\log\left(\left|-x|e|+\sqrt{e^2x^2+af^2}\right|\right)}{4\,d^4|e|} + \frac{x}{2\,d^3} - \frac{\sqrt{e^2x^2+af^2}|f|}{2\,d^3ef} + \frac{5\,a^3f^6-3\,a^2d^2f^4-9\,ad^4f^2-d^6-12\,(a^2def^4+ad^3ef^2)x}{8\,(af^2-2\,dex-d^2)^2d^4e}$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
[Out] 3/4*a*f^2*log(abs(-a*f^2 + 2*d*e*x + d^2))/(d^4*e) + 3/4*a*f*abs(f)*log(abs
(-x*abs(e) + sqrt(e^2*x^2 + a*f^2)))/(d^4*abs(e)) + 1/2*x/d^3 - 1/2*sqrt(e^
2*x^2 + a*f^2)*abs(f)/(d^3*e*f) + 1/8*(5*a^3*f^6 - 3*a^2*d^2*f^4 - 9*a*d^4*
f^2 - d^6 - 12*(a^2*d*e*f^4 + a*d^3*e*f^2)*x)/((a*f^2 - 2*d*e*x - d^2)^2*d^
4*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \int \frac{1}{\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^3}\,dx$$

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3,x)
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3, x)
```

3.460 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

Optimal result

Integrand size = 27, antiderivative size = 225

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \frac{2adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{7/2}}{7e} - \frac{5ad^{3/2}f^2\text{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2e}$$

```
[Out] -5/2*a*d^(3/2)*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/e
+1/3*a*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2)/e+1/7*(d+e*x+f*(a+e^2*x^2/
f^2)^(1/2))^(7/2)/e+2*a*d*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e-1/2*a
*d^2*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e/(e*x+f*(a+e^2*x^2/f^2)^(1/
2))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2142, 911, 1271, 1824, 212}

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = -\frac{5ad^{3/2}f^2\text{arctanh}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e} - \frac{ad^2f^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} + \frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e} + \frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e} + \frac{2adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (2*a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e - (a*d^2*f^2*Sqrt[d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])
) + (a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/(3*e) + (d + e*x +
f*Sqrt[a + (e^2*x^2)/f^2])^(7/2)/(7*e) - (5*a*d^(3/2)*f^2*ArcTanh[Sqrt[d +
e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
```

```
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^{5/2}(d^2+af^2-2dx+x^2)}{(d-x)^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}\\
&= \frac{\text{Subst}\left(\int \frac{x^6(d^2+af^2-2dx^2+x^4)}{(d-x^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}\\
&= -\frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}\\
&\quad+\frac{\text{Subst}\left(\int \frac{-ad^2f^2-2adf^2x^2-2af^2x^4+2dx^6-2x^8}{d-x^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}\\
&= -\frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}\\
&\quad+\frac{\text{Subst}\left(\int \left(4adf^2+2af^2x^2+2x^6-\frac{5ad^2f^2}{d-x^2}\right)dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}
\end{aligned}$$

$$= \frac{2adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{7/2}}{7e} - \frac{(5ad^2f^2)\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}$$

$$= \frac{2adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{7/2}}{7e} - \frac{5ad^{3/2}f^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2e}$$

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.94

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \frac{\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\left(20a^2f^4+6(d+2ex)^3\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+af^2\left(-3d^2+4ex\left(19ex+13f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+4d\left(38 \cdots\right.\right.\right.}{ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{42e}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] ((Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]*(20*a^2*f^4 + 6*(d + 2*e*x)^3*(
e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*(-3*d^2 + 4*e*x*(19*e*x + 13*f*Sqr
t[a + (e^2*x^2)/f^2]) + 4*d*(38*e*x + 29*f*Sqrt[a + (e^2*x^2)/f^2]))))/(e*x
 + f*Sqrt[a + (e^2*x^2)/f^2]) - 105*a*d^(3/2)*f^2*ArcTanh[Sqrt[d + e*x + f*
Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(42*e)
```

Maple [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}dx$$

[In] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)

[Out] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.85

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}dx = \left[\frac{105\,ad^{\frac{3}{2}}f^2\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+\right.}{}\right.\right.$$

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")

[Out] [1/84*(105*a*d^(3/2)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(24*e^3*x^3 + 36*d*e^2*x^2 + 116*a*d*f^2 + 6*d^3 + (32*a*e*f^2 + 39*d^2*e)*x + (24*e^2*f*x^2 + 20*a*f^3 + 36*d*e*f*x - 3*d^2*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/e, 1/42*(105*a*sqrt(-d)*d*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d) + (24*e^3*x^3 + 36*d*e^2*x^2 + 116*a*d*f^2 + 6*d^3 + (32*a*e*f^2 + 39*d^2*e)*x + (24*e^2*f*x^2 + 20*a*f^3 + 36*d*e*f*x - 3*d^2*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/e]

Sympy [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}} dx$$

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(5/2), x)
```

Maxima [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}} dx$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

Giac [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}} dx$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \int \left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^{5/2} dx$$

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

3.461 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx$

Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}{5e} - \frac{3a\sqrt{d}f^2\operatorname{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2e}$$

```
[Out] -3/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))*d^(1/2)/e
+1/5*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2)/e+a*f^2*(d+e*x+f*(a+e^2*x^2/f^2)
^(1/2))^(1/2)/e-1/2*a*d*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e/(e*x+f*
(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2142, 911, 1271, 1824, 212}

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}dx = -\frac{3a\sqrt{d}f^2\text{arctanh}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{5e}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}-\frac{adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e - (a*d*f^2*Sqrt[d + e*x
 + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2)/(5*e) - (3*a*Sqrt[d]*f^2*ArcTanh[
Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
```

```
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^{3/2}(d^2+af^2-2dx+x^2)}{(d-x)^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} \\
&= \frac{\text{Subst}\left(\int \frac{x^4(d^2+af^2-2dx^2+x^4)}{(d-x^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e} \\
&= -\frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{adf^2+2af^2x^2-2dx^4+2x^6}{d-x^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e} \\
&= -\frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} \\
&\quad - \frac{\text{Subst}\left(\int \left(-2af^2-2x^4+\frac{3adf^2}{d-x^2}\right)\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}
\end{aligned}$$

$$= \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}{5e} - \frac{(3adf^2)\,\text{Subst}\left(\int\frac{1}{d-x^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}$$

$$= \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}{5e} - \frac{3a\sqrt{d}f^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2e}$$

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.93

$$\int\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}dx = \frac{\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\left(2(d+2ex)^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+af^2\left(-d+16ex+12f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{ex+f\sqrt{a+\frac{e^2x^2}{f^2}}} - 15a\sqrt{d}f^2\text{arct}}{10e}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] ((Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]*(2*(d + 2*e*x)^2*(e*x + f*Sqrt[
a + (e^2*x^2)/f^2]) + a*f^2*(-d + 16*e*x + 12*f*Sqrt[a + (e^2*x^2)/f^2])))/
(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) - 15*a*Sqrt[d]*f^2*ArcTanh[Sqrt[d + e*x +
 f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(10*e)
```

Maple [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}} dx$$

[In] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)

[Out] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.84

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \left[\frac{15\,a\sqrt{d}f^2\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+\cdots}\right.}{\cdots}\right.$$

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")

[Out] [1/20*(15*a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(4*e^2*x^2 + 12*a*f^2 + 9*d*e*x + 2*d^2 + (4*e*f*x - d*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/e, 1/10*(15*a*sqrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d) + (4*e^2*x^2 + 12*a*f^2 + 9*d*e*x + 2*d^2 + (4*e*f*x - d*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/e]

Sympy [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(3/2), x)
```

Maxima [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}} dx$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

Giac [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}} dx$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^{3/2} dx$$

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

3.462 $\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e}-\frac{af^2\operatorname{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2\sqrt{d}e}$$

```
[Out] -1/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/e/d^(1/2)
+1/3*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2)/e-1/2*a*f^2*(d+e*x+f*(a+e^2*x^2/
f^2)^(1/2))^(1/2)/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2142, 911, 1271, 1167, 212}

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx=-\frac{af^2\text{arctanh}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2\sqrt{d}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}$$

```
[In] Int[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] -1/2*(a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(e*(e*x + f*Sqrt[a +
 (e^2*x^2)/f^2])) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)/(3*e) - (a*
f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*Sqrt[d]*
e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
```

```
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{x}(d^2+af^2-2dx+x^2)}{(d-x)^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} \\
&= \frac{\text{Subst}\left(\int \frac{x^2(d^2+af^2-2dx^2+x^4)}{(d-x^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e} \\
&= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \frac{-af^2+2dx^2-2x^4}{d-x^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e} \\
&= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \left(2x^2-\frac{af^2}{d-x^2}\right)dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e} \\
&= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} \\
&\quad - \frac{(af^2)\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}
\end{aligned}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2\sqrt{d}e}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \frac{\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\left(-af^2+2(d+2ex)\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{ex+f\sqrt{a+\frac{e^2x^2}{f^2}}} - \frac{3af^2\text{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{\sqrt{d}}}{6e}$$

```
[In] Integrate[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] ((Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]*(-(a*f^2) + 2*(d + 2*e*x)*(e*x
+ f*Sqrt[a + (e^2*x^2)/f^2])))/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) - (3*a*f^2
*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/Sqrt[d])/(6*e)
```

Maple [F]

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$

```
[In] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

```
[Out] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.05

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$

$$= \left[\frac{3\,a\sqrt{d}f^2\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)+2\,(}{12\,de}\right.$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/f
^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*
sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(5*d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)
/f^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(d*e), 1/6*(3
*a*sqrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-
d)/d) + (5*d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x + f*sq
rt((e^2*x^2 + a*f^2)/f^2) + d))/(d*e)]
```

Sympy [F]

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

Maxima [F]

$$\int \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\, dx = \int \sqrt{ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d}\, dx$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

Giac [F]

$$\int \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\, dx = \int \sqrt{ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d}\, dx$$

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\, dx = \int \sqrt{d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}}\, dx$$

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

3.463 $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx=\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e}-\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{af^2\operatorname{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{3/2}e}$$

```
[Out] 1/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/d^(3/2)/e+
(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e-1/2*a*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^
(1/2))^(1/2)/d/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used

$= \{2142, 911, 1171, 396, 212\}$

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx = \frac{af^2\operatorname{arctanh}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}e} - \frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} + \frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

```
[In] Int[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/e - (a*f^2*Sqrt[d + e*x + f*Sqrt[
a + (e^2*x^2)/f^2]])/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*Arc
Tanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
```

```
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2\sqrt{x}}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} \\
&= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{(d-x^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e} \\
&= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{-2d^2-af^2+2dx^2}{d-x^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2de} \\
&= \frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} \\
&\quad + \frac{(af^2)\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2de} \\
&= \frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{3/2}e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx = \frac{\frac{\sqrt{d}\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\left(-af^2+2d\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}+af^2\operatorname{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{3/2}e}$$

```
[In] Integrate[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] ((Sqrt[d]*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]*(-(a*f^2) + 2*d*(e*x +
f*Sqrt[a + (e^2*x^2)/f^2])))/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*ArcT
anh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)*e)
```

Maple [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$$

```
[In] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.03

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx = \left[\frac{a\sqrt{d}f^2\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}-2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)+2\left(d\right.}{4\,d^2e}, -\frac{a\sqrt{-d}f^2\arctan\left(\frac{\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\sqrt{-d}}{d}\right)-\left(dex-df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\,d^2\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{2\,d^2e}\right]$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2)
 - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqr
t((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2)
 + 2*d^2)*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(d^2*e), -1/2*(a*s
qrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d
) - (d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^
2*x^2 + a*f^2)/f^2) + d))/(d^2*e)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx = \int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx = \int \frac{1}{\sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx = \int \frac{1}{\sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx = \int \frac{1}{\sqrt{d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}}\,dx$$

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

3.464 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 158

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = -\frac{1+\frac{af^2}{d^2}}{e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{3af^2\operatorname{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{5/2}e}$$

```
[Out] 3/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/d^(5/2)/e+
(-1-a*f^2/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)-1/2*a*f^2*(d+e*x+f*(
a+e^2*x^2/f^2)^(1/2))^(1/2)/d^2/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used

= {2142, 911, 1273, 464, 212}

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \frac{3af^2\operatorname{arctanh}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{5/2}e} - \frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} - \frac{\frac{af^2}{d^2}+1}{e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] -((1 + (a*f^2)/d^2)/(e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])) - (a*f^2
*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^2*e*(e*x + f*Sqrt[a + (e^2
*x^2)/f^2])) + (3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/S
qrt[d]])/(2*d^(5/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^{3/2}}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{x^2(d-x^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{-2d(d^2+af^2)+(2d^2-af^2)x^2}{x^2(d-x^2)}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2d^2e}$$

$$= -\frac{d^2+af^2}{d^2e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{(3af^2)\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2d^2e}$$

$$= -\frac{d^2+af^2}{d^2e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{3af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{5/2}e}$$

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \frac{-\frac{\sqrt{d}\left(2d^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+af^2\left(d+3ex+3f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} + 3af^2\operatorname{arctanh}\left(\sqrt{d+ex+\ldots}\right.}{2d^{5/2}e}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] (-((Sqrt[d]*(2*d^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*(d + 3*e*x + 3
*f*Sqrt[a + (e^2*x^2)/f^2])))/((e*x + f*Sqrt[a + (e^2*x^2)/f^2])*Sqrt[d + e
*x + f*Sqrt[a + (e^2*x^2)/f^2]])) + 3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a
 + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(5/2)*e)
```

Maple [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.08

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \left[\frac{3\,(a^2f^4-2\,adef^2x-ad^2f^2)\sqrt{d}\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}-2\right.}{}\right.$$

$$-\frac{3\,(a^2f^4-2\,adef^2x-ad^2f^2)\sqrt{-d}\arctan\left(\frac{\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\sqrt{-d}}{d}\right)+\left(2\,d^2e^2x^2-2\,ad^2f^2-2\,d^4-(3\,adef^2\right.}{2\,(ad^3ef^2-2\,d^4e^2x-d^5e)}$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*sqrt(d)*log(a*f^2 - 2*d*e*x +
 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x
^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) - 2*(2*d^2
*e^2*x^2 - 2*a*d^2*f^2 - 2*d^4 - (3*a*d*e*f^2 + d^3*e)*x + (3*a*d*f^3 - 2*d
^2*e*f*x + d^3*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 +
 a*f^2)/f^2) + d))/(a*d^3*e*f^2 - 2*d^4*e^2*x - d^5*e), -1/2*(3*(a^2*f^4 -
2*a*d*e*f^2*x - a*d^2*f^2)*sqrt(-d)*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f
^2)/f^2) + d)*sqrt(-d)/d) + (2*d^2*e^2*x^2 - 2*a*d^2*f^2 - 2*d^4 - (3*a*d*e
*f^2 + d^3*e)*x + (3*a*d*f^3 - 2*d^2*e*f*x + d^3*f)*sqrt((e^2*x^2 + a*f^2)/
f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(a*d^3*e*f^2 - 2*d^4*e
^2*x - d^5*e)]
```

Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx=\int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx=\int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx=\int \frac{1}{\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^{3/2}}\,dx$$

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

3.465 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 199

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = -\frac{1+\frac{af^2}{d^2}}{3e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{2af^2}{d^3e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{5af^2\operatorname{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{7/2}e}$$

```
[Out] 5/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/d^(7/2)/e+
1/3*(-1-a*f^2/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2)-2*a*f^2/d^3/e/(d
+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)-1/2*a*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2
))^(1/2)/d^3/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2142, 911, 1273, 1275, 212}

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}dx = \frac{5af^2\text{arctanh}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{7/2}e} - \frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} - \frac{2af^2}{d^3e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}} - \frac{\frac{af^2}{d^2}+1}{3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] -1/3*(1 + (a*f^2)/d^2)/(e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)) - (2
*a*f^2)/(d^3*e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]) - (a*f^2*Sqrt[d +
 e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2
])) + (5*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/
(2*d^(7/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^{5/2}}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{x^4(d-x^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \frac{2d^2(d^2+af^2)-2d(d^2-af^2)x^2+af^2x^4}{x^4(d-x^2)}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2d^3e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \left(\frac{2(d^3+adf^2)}{x^4}+\frac{4af^2}{x^2}+\frac{5af^2}{d-x^2}\right)dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2d^3e}$$

$$= -\frac{d^2+af^2}{3d^2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{2af^2}{d^3e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{(5af^2)\,\text{Subst}\left(\int\frac{1}{d-x^2}\,dx,x,\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2d^3e}$$

$$= -\frac{d^2+af^2}{3d^2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{2af^2}{d^3e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{5af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{7/2}e}$$

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.05

$$\int\frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \frac{-\frac{\sqrt{d}\left(15a^2f^4+2d^3\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+af^2\left(3d^2+20d\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+30ex\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)\right)}{\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}+15af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{6d^{7/2}e}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] (-((Sqrt[d]*(15*a^2*f^4 + 2*d^3*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*(
3*d^2 + 20*d*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + 30*e*x*(e*x + f*Sqrt[a + (
e^2*x^2)/f^2]))))/((e*x + f*Sqrt[a + (e^2*x^2)/f^2])*(d + e*x + f*Sqrt[a +
(e^2*x^2)/f^2])^(3/2))) + 15*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x
^2)/f^2]]/Sqrt[d]])/(6*d^(7/2)*e)
```

Maple [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}dx$$

[In] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)

[Out] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(171) = 342.

Time = 0.66 (sec) , antiderivative size = 812, normalized size of antiderivative = 4.08

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}dx = \left[\frac{15\,(a^3f^6+4\,ad^2e^2f^2x^2-2\,a^2d^2f^4+ad^4f^2-4\,(a^2def^4-ad^3ef^2)x)\sqrt{d}}{\quad}\right.$$

$$-\frac{15\,(a^3f^6+4\,ad^2e^2f^2x^2-2\,a^2d^2f^4+ad^4f^2-4\,(a^2def^4-ad^3ef^2)x)\sqrt{-d}\arctan\left(\frac{\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\sqrt{-d}}{d}\right)}{\quad}$$

[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")

```
[Out] [1/12*(15*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a
^2*d*e*f^4 - a*d^3*e*f^2)*x)*sqrt(d)*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*
x^2 + a*f^2)/f^2) - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))
*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(12*d^3*e^3*x^3 + 10*a^
2*d^2*f^4 - 16*a*d^4*f^2 - 2*d^6 - 8*(5*a*d^2*e^2*f^2 - d^4*e^2)*x^2 + (15*
a^2*d*e*f^4 - 46*a*d^3*e*f^2 - d^5*e)*x - (15*a^2*d*f^5 + 12*d^3*e^2*f*x^2
- 22*a*d^3*f^3 - d^5*f - 8*(5*a*d^2*e*f^3 - d^4*e*f)*x)*sqrt((e^2*x^2 + a*f
^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(a^2*d^4*e*f^4 + 4
*d^6*e^3*x^2 - 2*a*d^6*e*f^2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)*x), -1/6
*(15*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a^2*d*
e*f^4 - a*d^3*e*f^2)*x)*sqrt(-d)*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)
/f^2) + d)*sqrt(-d)/d) - (12*d^3*e^3*x^3 + 10*a^2*d^2*f^4 - 16*a*d^4*f^2 -
2*d^6 - 8*(5*a*d^2*e^2*f^2 - d^4*e^2)*x^2 + (15*a^2*d*e*f^4 - 46*a*d^3*e*f^
```

```
2 - d^5*e)*x - (15*a^2*d*f^5 + 12*d^3*e^2*f*x^2 - 22*a*d^3*f^3 - d^5*f - 8*
(5*a*d^2*e*f^3 - d^4*e*f)*x)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt
((e^2*x^2 + a*f^2)/f^2) + d))/(a^2*d^4*e*f^4 + 4*d^6*e^3*x^2 - 2*a*d^6*e*f^
2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)*x)]
```

Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx=\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx=\int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx=\int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \int \frac{1}{\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^{5/2}}\,dx$$

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

3.466 $\int \sqrt{x-\sqrt{-4+x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \sqrt{x-\sqrt{-4+x^2}}\,dx=\frac{4}{\sqrt{x-\sqrt{-4+x^2}}}+\frac{1}{3}\left(x-\sqrt{-4+x^2}\right)^{3/2}$$

```
[Out] 1/3*(x-(x^2-4)^(1/2))^(3/2)+4/(x-(x^2-4)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2142, 14}

$$\int \sqrt{x-\sqrt{-4+x^2}}\,dx=\frac{1}{3}\left(x-\sqrt{x^2-4}\right)^{3/2}+\frac{4}{\sqrt{x-\sqrt{x^2-4}}}$$

```
[In] Int[Sqrt[x - Sqrt[-4 + x^2]],x]
[Out] 4/Sqrt[x - Sqrt[-4 + x^2]] + (x - Sqrt[-4 + x^2])^(3/2)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
```

```
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{-4+x^2}{x^{3/2}}\,dx, x, x-\sqrt{-4+x^2}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{4}{x^{3/2}}+\sqrt{x}\right)\,dx, x, x-\sqrt{-4+x^2}\right)$$
$$= \frac{4}{\sqrt{x-\sqrt{-4+x^2}}}+\frac{1}{3}\left(x-\sqrt{-4+x^2}\right)^{3/2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \sqrt{x-\sqrt{-4+x^2}}\,dx = \frac{4}{\sqrt{x-\sqrt{-4+x^2}}}+\frac{1}{3}\left(x-\sqrt{-4+x^2}\right)^{3/2}$$

```
[In] Integrate[Sqrt[x - Sqrt[-4 + x^2]],x]
[Out] 4/Sqrt[x - Sqrt[-4 + x^2]] + (x - Sqrt[-4 + x^2])^(3/2)/3
```

Maple [F]

$$\int \sqrt{x-\sqrt{x^2-4}}dx$$

```
[In] int((x-(x^2-4)^(1/2))^(1/2),x)
[Out] int((x-(x^2-4)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \sqrt{x-\sqrt{-4+x^2}}\,dx = \frac{2}{3}\left(2\,x+\sqrt{x^2-4}\right)\sqrt{x-\sqrt{x^2-4}}$$

```
[In] integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/3*(2*x + sqrt(x^2 - 4))*sqrt(x - sqrt(x^2 - 4))
```

Sympy [F]

$$\int \sqrt{x-\sqrt{-4+x^2}}\,dx = \int \sqrt{x-\sqrt{x^2-4}}\,dx$$

```
[In] integrate((x-(x**2-4)**(1/2))**(1/2),x)
[Out] Integral(sqrt(x - sqrt(x**2 - 4)), x)
```

Maxima [F]

$$\int \sqrt{x-\sqrt{-4+x^2}}\,dx = \int \sqrt{x-\sqrt{x^2-4}}\,dx$$

```
[In] integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x - sqrt(x^2 - 4)), x)
```

Giac [F]

$$\int \sqrt{x-\sqrt{-4+x^2}}\,dx = \int \sqrt{x-\sqrt{x^2-4}}\,dx$$

```
[In] integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x - sqrt(x^2 - 4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x-\sqrt{-4+x^2}}\,dx = \int \sqrt{x-\sqrt{x^2-4}}\,dx$$

```
[In] int((x - (x^2 - 4)^(1/2))^(1/2),x)
[Out] int((x - (x^2 - 4)^(1/2))^(1/2), x)
```

3.467 $\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx = -\frac{b^2c}{a\sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}}+\frac{\left(ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)^{3/2}}{3a}$$

```
[Out] 1/3*(a*x+b*(c+a^2*x^2/b^2)^(1/2))^(3/2)/a-b^2*c/a/(a*x+b*(c+a^2*x^2/b^2)^(1
/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2142, 14}

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx = \frac{\left(b\sqrt{\frac{a^2x^2}{b^2}+c}+ax\right)^{3/2}}{3a}-\frac{b^2c}{a\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}$$

```
[In] Int[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]
```

```
[Out] -((b^2*c)/(a*Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]])) + (a*x + b*Sqrt[c + (a
^2*x^2)/b^2])^(3/2)/(3*a)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{b^2c+x^2}{x^{3/2}}\,dx, x, ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)}{2a}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^2c}{x^{3/2}}+\sqrt{x}\right)\,dx, x, ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)}{2a}$$

$$= -\frac{b^2c}{a\sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}}+\frac{\left(ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)^{3/2}}{3a}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx = -\frac{b^2c}{a\sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}}+\frac{\left(ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)^{3/2}}{3a}$$

```
[In] Integrate[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]
[Out] -((b^2*c)/(a*Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]])) + (a*x + b*Sqrt[c + (a
^2*x^2)/b^2])^(3/2)/(3*a)
```

Maple [F]

$$\int \sqrt{ax + b\sqrt{c + \frac{a^2x^2}{b^2}}}dx$$

[In] int((a*x+b*(c+a^2/b^2*x^2)^(1/2))^(1/2),x)

[Out] int((a*x+b*(c+a^2/b^2*x^2)^(1/2))^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \sqrt{ax + b\sqrt{c + \frac{a^2x^2}{b^2}}}\,dx = \frac{2\left(2\,ax - b\sqrt{\frac{a^2x^2+b^2c}{b^2}}\right)\sqrt{ax + b\sqrt{\frac{a^2x^2+b^2c}{b^2}}}}{3\,a}$$

[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 2/3*(2*a*x - b*sqrt((a^2*x^2 + b^2*c)/b^2))*sqrt(a*x + b*sqrt((a^2*x^2 + b^2*c)/b^2))/a

Sympy [F]

$$\int \sqrt{ax + b\sqrt{c + \frac{a^2x^2}{b^2}}}\,dx = \int \sqrt{ax + b\sqrt{\frac{a^2x^2}{b^2} + c}}\,dx$$

[In] integrate((a*x+b*(c+a**2*x**2/b**2)**(1/2))**(1/2),x)

[Out] Integral(sqrt(a*x + b*sqrt(a**2*x**2/b**2 + c)), x)

Maxima [F]

$$\int \sqrt{ax + b\sqrt{c + \frac{a^2x^2}{b^2}}}\,dx = \int \sqrt{ax + \sqrt{\frac{a^2x^2}{b^2} + c}b}\,dx$$

[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b), x)

Giac [F]

$$\int \sqrt{ax + b\sqrt{c + \frac{a^2x^2}{b^2}}}\,dx = \int \sqrt{ax + \sqrt{\frac{a^2x^2}{b^2} + c}\,b}\,dx$$

[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ax + b\sqrt{c + \frac{a^2x^2}{b^2}}}\,dx = \int \sqrt{a\,x + b\,\sqrt{c + \frac{a^2\,x^2}{b^2}}}\,dx$$

[In] int((a*x + b*(c + (a^2*x^2)/b^2)^(1/2))^(1/2),x)

[Out] int((a*x + b*(c + (a^2*x^2)/b^2)^(1/2))^(1/2), x)

3.468 $\int \sqrt{1+\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = -\frac{2x^3}{3\left(1+\sqrt{1-x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1-x^2}}}$$

```
[Out] -2/3*x^3/(1+(-x^2+1)^(1/2))^(3/2)+2*x/(1+(-x^2+1)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2154}

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \frac{2x}{\sqrt{\sqrt{1-x^2}+1}}-\frac{2x^3}{3\left(\sqrt{1-x^2}+1\right)^{3/2}}$$

```
[In] Int[Sqrt[1 + Sqrt[1 - x^2]],x]
[Out] (-2*x^3)/(3*(1 + Sqrt[1 - x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 - x^2]]
```

Rule 2154

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\text{integral} = -\frac{2x^3}{3\left(1+\sqrt{1-x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1-x^2}}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \frac{2x\left(2+\sqrt{1-x^2}\right)}{3\sqrt{1+\sqrt{1-x^2}}}$$

[In] Integrate[Sqrt[1 + Sqrt[1 - x^2]],x]

[Out] (2*x*(2 + Sqrt[1 - x^2]))/(3*Sqrt[1 + Sqrt[1 - x^2]])

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

method	result	size
meijerg	$\frac{i\left(\frac{32 i \sqrt{\pi}\,\sqrt{2}\,x^{3} \cos\left(\frac{3 \arcsin(x)}{2}\right)}{3}-\frac{8 i \sqrt{\pi}\,\sqrt{2}\,\left(-\frac{4}{3} x^{4}+\frac{2}{3} x^{2}+\frac{2}{3}\right) \sin\left(\frac{3 \arcsin(x)}{2}\right)}{\sqrt{-x^{2}+1}}\right)}{8 \sqrt{\pi}}$	60

[In] int((1+(-x^2+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/8*I/Pi^(1/2)*(32/3*I*Pi^(1/2)*2^(1/2)*x^3*cos(3/2*arcsin(x))-8*I*Pi^(1/2)*2^(1/2)*(-4/3*x^4+2/3*x^2+2/3)*sin(3/2*arcsin(x))/(-x^2+1)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \frac{2\left(x^2-\sqrt{-x^2+1}+1\right)\sqrt{\sqrt{-x^2+1}+1}}{3\,x}$$

[In] integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 2/3*(x^2 - sqrt(-x^2 + 1) + 1)*sqrt(sqrt(-x^2 + 1) + 1)/x

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 415, normalized size of antiderivative = 9.22

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx$$
$$= \begin{cases} -\frac{\sqrt{2}ix^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12i\pi\sqrt{i\sqrt{x^2-1}+1}} - \frac{3\sqrt{2}x\sqrt{x^2-1}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12i\pi\sqrt{i\sqrt{x^2-1}+1}} + \frac{3\sqrt{2}ix\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12i\pi\sqrt{i\sqrt{x^2-1}+1}} \\ \frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}} - \frac{3\sqrt{2}x\sqrt{1-x^2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}} - \frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}} \end{cases}$$

```
[In] integrate((1+(-x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Piecewise((-sqrt(2)*I*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 - 1)*sqr
t(I*sqrt(x**2 - 1) + 1) - 12*I*pi*sqrt(I*sqrt(x**2 - 1) + 1)) - 3*sqrt(2)*x
*sqrt(x**2 - 1)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 - 1)*sqrt(I*sqrt(x*
*2 - 1) + 1) - 12*I*pi*sqrt(I*sqrt(x**2 - 1) + 1)) + 3*sqrt(2)*I*x*gamma(-1
/4)*gamma(1/4)/(12*pi*sqrt(x**2 - 1)*sqrt(I*sqrt(x**2 - 1) + 1) - 12*I*pi*s
qrt(I*sqrt(x**2 - 1) + 1)), Abs(x**2) > 1), (sqrt(2)*x**3*gamma(-1/4)*gamma
(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sqrt(1 -
x**2) + 1)) - 3*sqrt(2)*x*sqrt(1 - x**2)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt
(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sqrt(1 - x**2) + 1)) - 3*s
qrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2) +
 1) + 12*pi*sqrt(sqrt(1 - x**2) + 1)), True))
```

Maxima [F]

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \int \sqrt{\sqrt{-x^2+1}+1}\,dx$$

```
[In] integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(-x^2 + 1) + 1), x)
```

Giac [F]

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \int \sqrt{\sqrt{-x^2+1}+1}\,dx$$

```
[In] integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(-x^2 + 1) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \int \sqrt{\sqrt{1-x^2}+1}\,dx$$

```
[In] int(((1 - x^2)^(1/2) + 1)^(1/2),x)
[Out] int(((1 - x^2)^(1/2) + 1)^(1/2), x)
```

3.469 $\int \sqrt{1+\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}} + \frac{2x}{\sqrt{1+\sqrt{1+x^2}}}$$

```
[Out] 2/3*x^3/(1+(x^2+1)^(1/2))^(3/2)+2*x/(1+(x^2+1)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2154}

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \frac{2x}{\sqrt{\sqrt{x^2+1}+1}} + \frac{2x^3}{3\left(\sqrt{x^2+1}+1\right)^{3/2}}$$

```
[In] Int[Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] (2*x^3)/(3*(1 + Sqrt[1 + x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 + x^2]]
```

Rule 2154

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\text{integral} = \frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}} + \frac{2x}{\sqrt{1+\sqrt{1+x^2}}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \frac{2x\left(2+\sqrt{1+x^2}\right)}{3\sqrt{1+\sqrt{1+x^2}}}$$

```
[In] Integrate[Sqrt[1 + Sqrt[1 + x^2]],x]
[Out] (2*x*(2 + Sqrt[1 + x^2]))/(3*Sqrt[1 + Sqrt[1 + x^2]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

method	result	size
meijerg	$-\frac{-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^3\cosh\left(\frac{3\,\mathrm{arcsinh}(x)}{2}\right)}{3}-\frac{8\sqrt{\pi}\,\sqrt{2}\left(-\frac{4}{3}x^4-\frac{2}{3}x^2+\frac{2}{3}\right)\sinh\left(\frac{3\,\mathrm{arcsinh}(x)}{2}\right)}{\sqrt{x^2+1}}}{8\sqrt{\pi}}$	55

```
[In] int((1+(x^2+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/8/Pi^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(x))-8*Pi^(1/2)*2
^(1/2)*(-4/3*x^4-2/3*x^2+2/3)*sinh(3/2*arcsinh(x))/(x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \frac{2\left(x^2+\sqrt{x^2+1}-1\right)\sqrt{\sqrt{x^2+1}+1}}{3\,x}$$

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/3*(x^2 + sqrt(x^2 + 1) - 1)*sqrt(sqrt(x^2 + 1) + 1)/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(36) = 72$.

Time = 0.66 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.80

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = -\frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}} - \frac{3\sqrt{2}x\sqrt{x^2+1}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}} - \frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}$$

```
[In] integrate((1+(x**2+1)**(1/2))**(1/2),x)
[Out] -sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 +
 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*sqrt(x**2 + 1)*gam
ma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*
sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(
x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1))
```

Maxima [F]

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \int \sqrt{\sqrt{x^2+1}+1}\,dx$$

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

Giac [F]

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \int \sqrt{\sqrt{x^2+1}+1}\,dx$$

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \int \sqrt{\sqrt{x^2+1}+1}\,dx$$

```
[In] int(((x^2 + 1)^(1/2) + 1)^(1/2),x)
[Out] int(((x^2 + 1)^(1/2) + 1)^(1/2), x)
```

3.470 $\int \sqrt{5+\sqrt{25+x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx=\frac{2x^3}{3\left(5+\sqrt{25+x^2}\right)^{3/2}}+\frac{10x}{\sqrt{5+\sqrt{25+x^2}}}$$

```
[Out] 2/3*x^3/(5+(x^2+25)^(1/2))^(3/2)+10*x/(5+(x^2+25)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2154}

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx=\frac{10x}{\sqrt{\sqrt{x^2+25}+5}}+\frac{2x^3}{3\left(\sqrt{x^2+25}+5\right)^{3/2}}$$

```
[In] Int[Sqrt[5 + Sqrt[25 + x^2]],x]
```

```
[Out] (2*x^3)/(3*(5 + Sqrt[25 + x^2])^(3/2)) + (10*x)/Sqrt[5 + Sqrt[25 + x^2]]
```

Rule 2154

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\text{integral}=\frac{2x^3}{3\left(5+\sqrt{25+x^2}\right)^{3/2}}+\frac{10x}{\sqrt{5+\sqrt{25+x^2}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx=\frac{2x\left(10+\sqrt{25+x^2}\right)}{3\sqrt{5+\sqrt{25+x^2}}}$$

```
[In] Integrate[Sqrt[5 + Sqrt[25 + x^2]],x]
[Out] (2*x*(10 + Sqrt[25 + x^2]))/(3*Sqrt[5 + Sqrt[25 + x^2]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

method	result	size
meijerg	$-\frac{5\sqrt{5}\left(-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^3\cosh\left(\frac{3\,\operatorname{arcsinh}\left(\frac{x}{5}\right)}{2}\right)}{375}-\frac{8\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{4}{1875}x^4-\frac{2}{75}x^2+\frac{2}{3}\right)\sinh\left(\frac{3\,\operatorname{arcsinh}\left(\frac{x}{5}\right)}{2}\right)}{\sqrt{\frac{x^2}{25}+1}}\right)}{8\sqrt{\pi}}$	64

```
[In] int((5+(x^2+25)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] -5/8*5^(1/2)/Pi^(1/2)*(-32/375*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(1/5*x)
)-8*Pi^(1/2)*2^(1/2)*(-4/1875*x^4-2/75*x^2+2/3)*sinh(3/2*arcsinh(1/5*x))/(1
/25*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx=\frac{2\left(x^2+5\sqrt{x^2+25}-25\right)\sqrt{\sqrt{x^2+25}+5}}{3\,x}$$

```
[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/3*(x^2 + 5*sqrt(x^2 + 25) - 25)*sqrt(sqrt(x^2 + 25) + 5)/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(36) = 72$.

Time = 0.67 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.80

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx = -\frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}} - \frac{15\sqrt{2}x\sqrt{x^2+25}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}} - \frac{75\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}}$$

```
[In] integrate((5+(x**2+25)**(1/2))**(1/2),x)
```

```
[Out] -sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2
+ 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 15*sqrt(2)*x*sqrt(x**2 + 25
)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5) +
 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 75*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12
*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25)
+ 5))
```

Maxima [F]

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx = \int \sqrt{\sqrt{x^2+25}+5}\,dx$$

```
[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

Giac [F]

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx = \int \sqrt{\sqrt{x^2+25}+5}\,dx$$

```
[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx = \int \sqrt{\sqrt{x^2+25}+5}\,dx$$

```
[In] int(((x^2 + 25)^(1/2) + 5)^(1/2),x)
[Out] int(((x^2 + 25)^(1/2) + 5)^(1/2), x)
```

3.471 $\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 66

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx=\frac{2b^2cx^3}{3\left(a+b\sqrt{\frac{a^2}{b^2}+cx^2}\right)^{3/2}}+\frac{2ax}{\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}}$$

```
[Out] 2/3*b^2*c*x^3/(a+b*(a^2/b^2+c*x^2)^(1/2))^(3/2)+2*a*x/(a+b*(a^2/b^2+c*x^2)^
(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2154}

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx=\frac{2ax}{\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}+\frac{2b^2cx^3}{3\left(b\sqrt{\frac{a^2}{b^2}+cx^2}+a\right)^{3/2}}$$

```
[In] Int[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
```

```
[Out] (2*b^2*c*x^3)/(3*(a + b*Sqrt[a^2/b^2 + c*x^2])^(3/2)) + (2*a*x)/Sqrt[a + b*
Sqrt[a^2/b^2 + c*x^2]]
```

Rule 2154

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
```

```
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\text{integral} = \frac{2b^2cx^3}{3\left(a+b\sqrt{\frac{a^2}{b^2}+cx^2}\right)^{3/2}} + \frac{2ax}{\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}}$$

Mathematica [A] (verified)

Time = 4.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx = \frac{2\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\left(-a^2+b^2cx^2+ab\sqrt{\frac{a^2}{b^2}+cx^2}\right)}{3b^2cx}$$

```
[In] Integrate[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
[Out] (2*Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]]*(-a^2 + b^2*c*x^2 + a*b*Sqrt[a^2/b^2 +
 c*x^2]))/(3*b^2*c*x)
```

Maple [F]

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+c\,x^2}}dx$$

```
[In] int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)
[Out] int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx = \frac{2\left(b^2cx^2+ab\sqrt{\frac{b^2cx^2+a^2}{b^2}}-a^2\right)\sqrt{b\sqrt{\frac{b^2cx^2+a^2}{b^2}}+a}}{3\,b^2cx}$$

```
[In] integrate((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/3*(b^2*c*x^2 + a*b*sqrt((b^2*c*x^2 + a^2)/b^2) - a^2)*sqrt(b*sqrt((b^2*c*
x^2 + a^2)/b^2) + a)/(b^2*c*x)
```

Sympy [F]

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx = \int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx$$

```
[In] integrate((a+b*(a**2/b**2+c*x**2)**(1/2))**(1/2),x)
[Out] Integral(sqrt(a + b*sqrt(a**2/b**2 + c*x**2)), x)
```

Maxima [F]

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx = \int \sqrt{\sqrt{cx^2+\frac{a^2}{b^2}}b+a}\,dx$$

```
[In] integrate((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

Giac [F]

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx = \int \sqrt{\sqrt{cx^2+\frac{a^2}{b^2}}b+a}\,dx$$

```
[In] integrate((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx = \int \sqrt{a+b\,\sqrt{c\,x^2+\frac{a^2}{b^2}}}\,dx$$

```
[In] int((a + b*(c*x^2 + a^2/b^2)^(1/2))^(1/2),x)
[Out] int((a + b*(c*x^2 + a^2/b^2)^(1/2))^(1/2), x)
```

3.472 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n \, dx$

Optimal result

Integrand size = 28, antiderivative size = 166

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n \, dx = \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)} + \frac{f^2(4ae^2-b^2f^2)\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n} \text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{2e\left(d+ex+f\sqrt{a+bx}\right.}{2de-bf^2}\right.}{2e\,(2de-bf^2)^2\,(1+n)}$$

```
[Out] 1/2*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)+1/2*f^2*(-b^2*f^2+4*a
*e^2)*hypergeom([2, 1+n],[2+n],2*e*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-b*
f^2+2*d*e))*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1+n)/e/(-b*f^2+2*d*e)^2/(1
+n)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2141, 961, 66}

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx$$

$$=\frac{f^2(4ae^2-b^2f^2)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+}\right.}{2de-bf^2}\right.}{2e(n+1)\left(2de-bf^2\right)^2}$$

$$+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^n,x]
```

```
[Out] (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n)) + (f^2*(4
*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(1 + n)*Hyper
geometric2F1[2, 1 + n, 2 + n, (2*e*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]))/(2*d*e - b*f^2)])/(2*e*(2*d*e - b*f^2)^2*(1 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^n(d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2)}{(-2de+bf^2+2ex)^2}\,dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{x^n}{4e}+\frac{(4ae^2f^2-b^2f^4)\,x^n}{4e\,(2de-bf^2-2ex)^2}\right)dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)} + \frac{(4ae^2f^2-b^2f^4)\,\text{Subst}\left(\int \frac{x^n}{(2de-bf^2-2ex)^2}\,dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)} + \frac{f^2(4ae^2-b^2f^2)\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n}{}_2F_1\left(2,1+n;2+n;\frac{2e\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{2de-bf^2}\right)}{2e\,(2de-bf^2)^2\,(1+n)}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.81

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx$$

$$= \frac{\left(d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)^{1+n}\left((-2de+bf^2)^2+(4ae^2f^2-b^2f^4)\,\text{Hypergeometric2F1}\left(2,1+n,2\right.\right.}{2e\,(-2de+bf^2)^2\,(1+n)}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^n,x]
[Out] ((d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(1 + n)*((-2*d*e + b*f^2)^2 +
(4*a*e^2*f^2 - b^2*f^4)*Hypergeometric2F1[2, 1 + n, 2 + n, (2*e*(d + e*x +
f*Sqrt[a + x*(b + (e^2*x)/f^2)]))/(2*d*e - b*f^2)]))/(2*e*(-2*d*e + b*f^2)^
2*(1 + n))
```

Maple [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx$$

```
[In] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x)
[Out] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x)
```

Fricas [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx = \text{Timed out}$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-2)]

Exception generated.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**n,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^n dx$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^n, x)
```

Giac [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^n dx$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^n dx$$

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^n,x)
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^n, x)
```

3.473 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx$

Optimal result

Integrand size = 28, antiderivative size = 303

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx = \frac{f^2(2de-bf^2)(4ae^2-b^2f^2)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^4} + \frac{f^2(4ae^2-b^2f^2)\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}{16e^3} + \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^4}{8e} - \frac{f^2(2de-bf^2)^3(4ae^2-b^2f^2)}{32e^5\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{3f^2(2de-bf^2)^2(4ae^2-b^2f^2)\log\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{32e^5}$$

```
[Out] 3/32*f^2*(-b*f^2+2*d*e)^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^
2+e^2*x)/f^2)^(1/2)))/e^5+1/8*f^2*(-b*f^2+2*d*e)*(-b^2*f^2+4*a*e^2)*(e*x+f*
(a+b*x+e^2*x^2/f^2)^(1/2))/e^4+1/16*f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+
e^2*x^2/f^2)^(1/2))^2/e^3+1/8*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^4/e-1/32*
f^2*(-b*f^2+2*d*e)^3*(-b^2*f^2+4*a*e^2)/e^5/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e
^2*x)/f^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2141, 907}

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx$$
$$= -\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)^3}{32e^5\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)} + \frac{3f^2(4ae^2-b^2f^2)(2de-bf^2)^2\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{32e^5} + \frac{f^2(4ae^2-b^2f^2)(2de-bf^2)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+ex\right)}{8e^4} + \frac{f^2(4ae^2-b^2f^2)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{16e^3} + \frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^4}{8e}$$

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]))/(8*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*
x^2)/f^2])^2)/(16*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^4/(8*e
) - (f^2*(2*d*e - b*f^2)^3*(4*a*e^2 - b^2*f^2))/(32*e^5*(b*f^2 + 2*e*(e*x +
 f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (3*f^2*(2*d*e - b*f^2)^2*(4*a*e^2
 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(
32*e^5)
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^3(d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2)}{(-2de+bf^2+2ex)^2}\,dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{f^2(2de-bf^2)\,(4ae^2-b^2f^2)}{16e^4}+\frac{f^2(4ae^2-b^2f^2)\,x}{16e^3}+\frac{x^3}{4e}+\frac{f^2(2de-bf^2)^3\,(4ae^2-b^2f^2)}{32e^4\,(2de-bf^2-2ex)^2}-\frac{3(4ae^2-b^2f^2)\,(2def-bf^3)^2}{32e^4\,(2de-bf^2-2ex)}\right)dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$= \frac{f^2(2de-bf^2)\,(4ae^2-b^2f^2)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^4}+\frac{f^2(4ae^2-b^2f^2)\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}{16e^3}+\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^4}{8e}-\frac{f^2(2de-bf^2)^3\,(4ae^2-b^2f^2)}{32e^5\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}+\frac{3f^2(2de-bf^2)^2\,(4ae^2-b^2f^2)\log\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{32e^5}$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.86

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx$$
$$=\frac{1}{16}\left(8x\left(2d^3+3d^2ex+ex\left(3af^2+2x\left(bf^2+e^2x\right)\right)+d\left(6af^2+x\left(3bf^2+4e^2x\right)\right)\right)\right.$$
$$+\frac{\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}(3b^3f^7-2b^2ef^5(6d+ex)+4be^2f^3(3d^2-2af^2+2dex+2e^2x^2)+8e^3f(2af^2(2d+e$$
$$\left.+\frac{3(4ae^2-b^2f^2)\left(-2def+bf^3\right)^2\operatorname{arctanh}\left(\frac{ex}{f\left(-\sqrt{a}+\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}\right)}{e^5}\right)$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (8*x*(2*d^3 + 3*d^2*e*x + e*x*(3*a*f^2 + 2*x*(b*f^2 + e^2*x)) + d*(6*a*f^2
+ x*(3*b*f^2 + 4*e^2*x))) + (Sqrt[a + x*(b + (e^2*x)/f^2)]*(3*b^3*f^7 - 2*b
^2*e*f^5*(6*d + e*x) + 4*b*e^2*f^3*(3*d^2 - 2*a*f^2 + 2*d*e*x + 2*e^2*x^2)
+ 8*e^3*f*(2*a*f^2*(2*d + e*x) + e*x*(3*d^2 + 4*d*e*x + 2*e^2*x^2))))/e^4 +
 (3*(4*a*e^2 - b^2*f^2)*(-2*d*e*f + b*f^3)^2*ArcTanh[(e*x)/(f*(-Sqrt[a] + S
qrt[a + x*(b + (e^2*x)/f^2)]))])/e^5)/16
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(283) = 566$.

Time = 1.26 (sec) , antiderivative size = 809, normalized size of antiderivative = 2.67

method	result
default	$f^3\left(\frac{\left(b+\frac{2e^2x}{f^2}\right)f^2\left(a+bx+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}}{8e^2}+\frac{3\left(\frac{4e^2a}{f^2}-b^2\right)f^2\left(\frac{\left(b+\frac{2e^2x}{f^2}\right)f^2\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{\left(\frac{4e^2a}{f^2}-b^2\right)f^2\ln\left(\frac{\frac{b}{2}+\frac{e^2x}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^2\sqrt{\frac{e^2}{f^2}}}\right)}{16e^2}\right)$

```
[In] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] f^3*(1/8*(b+2*e^2*x/f^2)/e^2*f^2*(a+b*x+e^2*x^2/f^2)^(3/2)+3/16*(4*e^2/f^2*
a-b^2)/e^2*f^2*(1/4*(b+2*e^2*x/f^2)/e^2*f^2*(a+b*x+e^2*x^2/f^2)^(1/2)+1/8*(
4*e^2/f^2*a-b^2)/e^2*f^2*ln((1/2*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+(a+b*x+e^2*x^
2/f^2)^(1/2))/(e^2/f^2)^(1/2)))+3*f^2*(1/4*e^3/f^2*x^4+1/3*(d*e^2/f^2+b*e)*
x^3+1/2*(a*e+b*d)*x^2+a*d*x)+3*f*(d^2*(1/4*(b+2*e^2*x/f^2)/e^2*f^2*(a+b*x+e
^2*x^2/f^2)^(1/2)+1/8*(4*e^2/f^2*a-b^2)/e^2*f^2*ln((1/2*b+e^2*x/f^2)/(e^2/f
^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2))+e^2*(1/4*x*(a+b*x+e^2
*x^2/f^2)^(3/2)/e^2*f^2-5/8*b/e^2*f^2*(1/3*(a+b*x+e^2*x^2/f^2)^(3/2)/e^2*f^
2-1/2*b/e^2*f^2*(1/4*(b+2*e^2*x/f^2)/e^2*f^2*(a+b*x+e^2*x^2/f^2)^(1/2)+1/8*
(4*e^2/f^2*a-b^2)/e^2*f^2*ln((1/2*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+(a+b*x+e^2*x
^2/f^2)^(1/2))/(e^2/f^2)^(1/2)))-1/4*a/e^2*f^2*(1/4*(b+2*e^2*x/f^2)/e^2*f^2
*(a+b*x+e^2*x^2/f^2)^(1/2)+1/8*(4*e^2/f^2*a-b^2)/e^2*f^2*ln((1/2*b+e^2*x/f^
2)/(e^2/f^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)))+2*e*d*(1/3*
(a+b*x+e^2*x^2/f^2)^(3/2)/e^2*f^2-1/2*b/e^2*f^2*(1/4*(b+2*e^2*x/f^2)/e^2*f^
2*(a+b*x+e^2*x^2/f^2)^(1/2)+1/8*(4*e^2/f^2*a-b^2)/e^2*f^2*ln((1/2*b+e^2*x/f
^2)/(e^2/f^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2))))+1/4*(e*x+
d)^4/e
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.14

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx$$
$$= \frac{32\,e^8x^4+32\,(be^6f^2+2\,de^7)x^3+48\,(d^2e^6+(bde^5+ae^6)f^2)x^2+32\,(3\,ade^5f^2+d^3e^5)x+3\,(b^4f^8-16\,ad^2}{}$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(32*e^8*x^4 + 32*(b*e^6*f^2 + 2*d*e^7)*x^3 + 48*(d^2*e^6 + (b*d*e^5 +
a*e^6)*f^2)*x^2 + 32*(3*a*d*e^5*f^2 + d^3*e^5)*x + 3*(b^4*f^8 - 16*a*d^2*e^
4*f^2 - 4*(b^3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)*lo
g(-b*f^2 - 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(3*b^
3*e*f^7 + 16*e^7*f*x^3 - 4*(3*b^2*d*e^2 + 2*a*b*e^3)*f^5 + 4*(3*b*d^2*e^3 +
 8*a*d*e^4)*f^3 + 8*(b*e^5*f^3 + 4*d*e^6*f)*x^2 - 2*(b^2*e^3*f^5 - 12*d^2*e
^5*f - 4*(b*d*e^4 + 2*a*e^5)*f^3)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))
/e^5
```

Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 1363, normalized size of antiderivative = 4.50

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx = \text{Too large to display}$$

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] 3*a*d*f**2*x + 3*a*e*f**2*x**2/2 + a*f**3*Piecewise(((a/2 - b**2*f**2/(8*e*
*2))*Piecewise((log(b + 2*e**2*x/f**2 + 2*sqrt(e**2/f**2)*sqrt(a + b*x + e*
*2*x**2/f**2))/sqrt(e**2/f**2), Ne(a - b**2*f**2/(4*e**2), 0)), ((b*f**2/(2
*e**2) + x)*log(b*f**2/(2*e**2) + x)/sqrt(e**2*(b*f**2/(2*e**2) + x)**2/f**
2), True)) + (b*f**2/(4*e**2) + x/2)*sqrt(a + b*x + e**2*x**2/f**2), Ne(e**
2/f**2, 0)), (2*(a + b*x)**(3/2)/(3*b), Ne(b, 0)), (sqrt(a)*x, True)) + 3*b
*d*f**2*x**2/2 + b*e*f**2*x**3 + b*f**3*Piecewise(((-a*b*f**2/(12*e**2) - b
*f**2*(a/3 - b**2*f**2/(8*e**2))/(2*e**2))*Piecewise((log(b + 2*e**2*x/f**2
 + 2*sqrt(e**2/f**2)*sqrt(a + b*x + e**2*x**2/f**2))/sqrt(e**2/f**2), Ne(a
- b**2*f**2/(4*e**2), 0)), ((b*f**2/(2*e**2) + x)*log(b*f**2/(2*e**2) + x)/
sqrt(e**2*(b*f**2/(2*e**2) + x)**2/f**2), True)) + sqrt(a + b*x + e**2*x**2
/f**2)*(b*f**2*x/(12*e**2) + x**2/3 + f**2*(a/3 - b**2*f**2/(8*e**2))/e**2)
, Ne(e**2/f**2, 0)), (2*(-a*(a + b*x)**(3/2)/3 + (a + b*x)**(5/2)/5)/b**2,
```

```
Ne(b, 0)), (sqrt(a)*x**2/2, True)) + d**3*x + 3*d**2*e*x**2/2 + 3*d**2*f*Pi
ecewise(((a/2 - b**2*f**2/(8*e**2))*Piecewise((log(b + 2*e**2*x/f**2 + 2*sq
rt(e**2/f**2)*sqrt(a + b*x + e**2*x**2/f**2))/sqrt(e**2/f**2), Ne(a - b**2*
f**2/(4*e**2), 0)), ((b*f**2/(2*e**2) + x)*log(b*f**2/(2*e**2) + x)/sqrt(e*
*2*(b*f**2/(2*e**2) + x)**2/f**2), True)) + (b*f**2/(4*e**2) + x/2)*sqrt(a
+ b*x + e**2*x**2/f**2), Ne(e**2/f**2, 0)), (2*(a + b*x)**(3/2)/(3*b), Ne(b
, 0)), (sqrt(a)*x, True)) + 2*d*e**2*x**3 + 6*d*e*f*Piecewise(((-a*b*f**2/(
12*e**2) - b*f**2*(a/3 - b**2*f**2/(8*e**2))/(2*e**2))*Piecewise((log(b + 2
*e**2*x/f**2 + 2*sqrt(e**2/f**2)*sqrt(a + b*x + e**2*x**2/f**2))/sqrt(e**2/
f**2), Ne(a - b**2*f**2/(4*e**2), 0)), ((b*f**2/(2*e**2) + x)*log(b*f**2/(2
*e**2) + x)/sqrt(e**2*(b*f**2/(2*e**2) + x)**2/f**2), True)) + sqrt(a + b*x
 + e**2*x**2/f**2)*(b*f**2*x/(12*e**2) + x**2/3 + f**2*(a/3 - b**2*f**2/(8*
e**2))/e**2), Ne(e**2/f**2, 0)), (2*(-a*(a + b*x)**(3/2)/3 + (a + b*x)**(5/
2)/5)/b**2, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + e**3*x**4 + 4*e**2*f*Piece
wise(((-a*f**2*(a/4 - 5*b**2*f**2/(48*e**2))/(2*e**2) - b*f**2*(-a*b*f**2/(
12*e**2) - 3*b*f**2*(a/4 - 5*b**2*f**2/(48*e**2))/(4*e**2))/(2*e**2))*Piece
wise((log(b + 2*e**2*x/f**2 + 2*sqrt(e**2/f**2)*sqrt(a + b*x + e**2*x**2/f*
*2))/sqrt(e**2/f**2), Ne(a - b**2*f**2/(4*e**2), 0)), ((b*f**2/(2*e**2) + x
)*log(b*f**2/(2*e**2) + x)/sqrt(e**2*(b*f**2/(2*e**2) + x)**2/f**2), True))
 + sqrt(a + b*x + e**2*x**2/f**2)*(b*f**2*x**2/(24*e**2) + x**3/4 + f**2*x*
(a/4 - 5*b**2*f**2/(48*e**2))/(2*e**2) + f**2*(-a*b*f**2/(12*e**2) - 3*b*f*
*2*(a/4 - 5*b**2*f**2/(48*e**2))/(4*e**2))/e**2), Ne(e**2/f**2, 0)), (2*(a*
*2*(a + b*x)**(3/2)/3 - 2*a*(a + b*x)**(5/2)/5 + (a + b*x)**(7/2)/7)/b**3,
Ne(b, 0)), (sqrt(a)*x**3/3, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2*f^2-4*a*e^2>0)', see `assume?`
for mor
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.31

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx$$
$$= bef^2x^3 + e^3x^4 + \frac{3}{2}bdf^2x^2 + \frac{3}{2}aef^2x^2 + 2de^2x^3 + 3adf^2x + \frac{3}{2}d^2ex^2 + d^3x + \frac{1}{16}\sqrt{bf^2x + e^2x^2 + af^2}\left(2\left(4\left(\frac{2e^2x|f|}{f} + \frac{be^6f^4|f| + 4de^7f^2|f|}{e^6f^3}\right)x - \frac{b^2e^4f^6|f| - 4bde^5f^4|f| - 8ae^6\right.\right.$$
$$+ \frac{3\left(b^4f^7|f| - 4b^3def^5|f| - 4ab^2e^2f^5|f| + 4b^2d^2e^2f^3|f| + 16abde^3f^3|f| - 16ad^2e^4f|f|\right)\log\left(\left|-bf^2 - 2\right.\right.}{32e^4|e|}$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] b*e*f^2*x^3 + e^3*x^4 + 3/2*b*d*f^2*x^2 + 3/2*a*e*f^2*x^2 + 2*d*e^2*x^3 + 3
*a*d*f^2*x + 3/2*d^2*e*x^2 + d^3*x + 1/16*sqrt(b*f^2*x + e^2*x^2 + a*f^2)*(
2*(4*(2*e^2*x*abs(f)/f + (b*e^6*f^4*abs(f) + 4*d*e^7*f^2*abs(f))/(e^6*f^3))
*x - (b^2*e^4*f^6*abs(f) - 4*b*d*e^5*f^4*abs(f) - 8*a*e^6*f^4*abs(f) - 12*d
^2*e^6*f^2*abs(f))/(e^6*f^3))*x + (3*b^3*e^2*f^8*abs(f) - 12*b^2*d*e^3*f^6*
abs(f) - 8*a*b*e^4*f^6*abs(f) + 12*b*d^2*e^4*f^4*abs(f) + 32*a*d*e^5*f^4*ab
s(f))/(e^6*f^3)) + 3/32*(b^4*f^7*abs(f) - 4*b^3*d*e*f^5*abs(f) - 4*a*b^2*e^
2*f^5*abs(f) + 4*b^2*d^2*e^2*f^3*abs(f) + 16*a*b*d*e^3*f^3*abs(f) - 16*a*d^
2*e^4*f*abs(f))*log(abs(-b*f^2 - 2*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f
^2))*abs(e)))/(e^4*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx = \int \left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^3 dx$$

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3,x)
```

```
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3, x)
```

3.474 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx$

Optimal result

Integrand size = 28, antiderivative size = 237

$$\begin{aligned}
&\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx \\
&= \frac{f^2(4ae^2-b^2f^2)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^3}+\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}{6e} \\
&\quad -\frac{f^2(2de-bf^2)^2(4ae^2-b^2f^2)}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} \\
&\quad +\frac{f^2(2de-bf^2)(4ae^2-b^2f^2)\log\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{8e^4}
\end{aligned}$$

```
[Out] 1/8*f^2*(-b*f^2+2*d*e)*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e
^2*x)/f^2)^(1/2)))/e^4+1/8*f^2*(-b^2*f^2+4*a*e^2)*(e*x+f*(a+b*x+e^2*x^2/f^2
)^(1/2))/e^3+1/6*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3/e-1/16*f^2*(-b*f^2+2
*d*e)^2*(-b^2*f^2+4*a*e^2)/e^4/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1
/2)))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2141, 907}

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx$$
$$= -\frac{f^2(4ae^2-b^2f^2)\left(2de-bf^2\right)^2}{16e^4\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)} + \frac{f^2(4ae^2-b^2f^2)\left(2de-bf^2\right)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{8e^4} + \frac{f^2(4ae^2-b^2f^2)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+ex\right)}{8e^3} + \frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^3}{6e}$$

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]
```

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]))/(8*e^3) +
 (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3/(6*e) - (f^2*(2*d*e - b*f^2)
^2*(4*a*e^2 - b^2*f^2))/(16*e^4*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 +
e^2*x))/f^2]))) + (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*
(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(8*e^4)
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^2(d^2e-(bd-ae)f^2-(2de-bf^2)x+ex^2)}{(-2de+bf^2+2ex)^2}\,dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{4ae^2f^2-b^2f^4}{16e^3}+\frac{x^2}{4e}+\frac{(4ae^2-b^2f^2)(2def-bf^3)^2}{16e^3(2de-bf^2-2ex)^2}-\frac{f^2(2de-bf^2)(4ae^2-b^2f^2)}{8e^3(2de-bf^2-2ex)}\right)dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$= \frac{f^2(4ae^2-b^2f^2)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^3}+\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}{6e}-\frac{f^2(2de-bf^2)^2(4ae^2-b^2f^2)}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}+\frac{f^2(2de-bf^2)(4ae^2-b^2f^2)\log\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{8e^4}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.74

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx$$

$$= \frac{1}{12}\left(2x\left(6d^2+6af^2+6dex+x\left(3bf^2+4e^2x\right)\right)+\frac{\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\left(-3b^2f^5+2bef^3(3d+ex)+4e^2f(2af^2+ex(3d+2ex))\right)}{e^3}+\frac{3f^2(-2de+bf^2)(-4ae^2+b^2f^2)\,\text{arctanh}\left(\frac{ex}{f\left(-\sqrt{a}+\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}\right)}{e^4}\right)$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]
```

```
[Out] (2*x*(6*d^2 + 6*a*f^2 + 6*d*e*x + x*(3*b*f^2 + 4*e^2*x)) + (Sqrt[a + x*(b +
 (e^2*x)/f^2)]*(-3*b^2*f^5 + 2*b*e*f^3*(3*d + e*x) + 4*e^2*f*(2*a*f^2 + e*x
*(3*d + 2*e*x))))/e^3 + (3*f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2)*ArcTan
h[(e*x)/(f*(-Sqrt[a] + Sqrt[a + x*(b + (e^2*x)/f^2)]))])/e^4)/12
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.27

method	result
default	$\frac{b x^2 f^2}{2}+\frac{e^2 x^3}{3}+a f^2 x+2 f \left(d \left(\frac{\left(b+\frac{2 e^2 x}{f^2}\right) f^2 \sqrt{a+b x+\frac{e^2 x^2}{f^2}}}{4 e^2}+\frac{\left(\frac{4 e^2 a}{f^2}-b^2\right) f^2 \ln \left(\frac{\frac{b}{2}+\frac{e^2 x}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+b x+\frac{e^2 x^2}{f^2}}\right)}{8 e^2 \sqrt{\frac{e^2}{f^2}}}\right)+e \left(\right.\right.$

```
[In] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*x^2*f^2+1/3*e^2*x^3+a*f^2*x+2*f*(d*(1/4*(b+2*e^2*x/f^2)/e^2*f^2*(a+b*
x+e^2*x^2/f^2)^(1/2)+1/8*(4*e^2/f^2*a-b^2)/e^2*f^2*ln((1/2*b+e^2*x/f^2)/(e^
2/f^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2))+e*(1/3*(a+b*x+e^2*
x^2/f^2)^(3/2)/e^2*f^2-1/2*b/e^2*f^2*(1/4*(b+2*e^2*x/f^2)/e^2*f^2*(a+b*x+e^
2*x^2/f^2)^(1/2)+1/8*(4*e^2/f^2*a-b^2)/e^2*f^2*ln((1/2*b+e^2*x/f^2)/(e^2/f^
2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2))))+1/3*(e*x+d)^3/e
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.92

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx$$
$$=\frac{16 e^6x^3+12 (be^4f^2+2 de^5)x^2+24 (ae^4f^2+d^2e^4)x-3 (b^3f^6+8 ade^3f^2-2 (b^2de+2 abe^2)f^4)\log \left(-bf\right.}{}$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*e^6*x^3 + 12*(b*e^4*f^2 + 2*d*e^5)*x^2 + 24*(a*e^4*f^2 + d^2*e^4)*
x - 3*(b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*log(-b*f^2 -
```

```
2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 2*(3*b^2*e*f^5 - 8
*e^5*f*x^2 - 2*(3*b*d*e^2 + 4*a*e^3)*f^3 - 2*(b*e^3*f^3 + 6*d*e^4*f)*x)*sqr
t((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/e^4
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.07

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx = af^2x + \frac{bf^2x^2}{2} + d^2x + dex^2 + 2df\begin{cases} \left(\frac{a}{2}-\frac{b^2f^2}{8e^2}\right)\left(\begin{cases} \frac{\log\left(b+\frac{2e^2x}{f^2}+2\sqrt{\frac{e^2}{f^2}}\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}} & \text{for } a-\frac{b^2f^2}{4e^2}\neq 0 \\ \frac{\left(\frac{bf^2}{2e^2}+x\right)\log\left(\frac{bf^2}{2e^2}+x\right)}{\sqrt{\frac{e^2\left(\frac{bf^2}{2e^2}+x\right)^2}{f^2}}} & \text{otherwise} \end{cases}\right) + \left(\frac{bf^2}{4e^2}+\frac{x}{2}\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}} & \text{fo} \\ \frac{2(a+bx)^{\frac{3}{2}}}{3b} & \text{fo} \\ \sqrt{a}x & \text{o} \end{cases} + \frac{2e^2x^3}{3} + 2ef\begin{cases} \left(-\frac{abf^2}{12e^2}-\frac{bf^2\left(\frac{a}{3}-\frac{b^2f^2}{8e^2}\right)}{2e^2}\right)\left(\begin{cases} \frac{\log\left(b+\frac{2e^2x}{f^2}+2\sqrt{\frac{e^2}{f^2}}\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}} & \text{for } a-\frac{b^2f^2}{4e^2}\neq 0 \\ \frac{\left(\frac{bf^2}{2e^2}+x\right)\log\left(\frac{bf^2}{2e^2}+x\right)}{\sqrt{\frac{e^2\left(\frac{bf^2}{2e^2}+x\right)^2}{f^2}}} & \text{otherwise} \end{cases}\right) + \sqrt{a+bx+\frac{e^2x^2}{f^2}} \\ \frac{2\left(-\frac{a(a+bx)^{\frac{3}{2}}}{3}+\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2} \\ \frac{\sqrt{a}x^2}{2} \end{cases}$$

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] a*f**2*x + b*f**2*x**2/2 + d**2*x + d*e*x**2 + 2*d*f*Piecewise(((a/2 - b**2
*f**2/(8*e**2))*Piecewise((log(b + 2*e**2*x/f**2 + 2*sqrt(e**2/f**2)*sqrt(a
 + b*x + e**2*x**2/f**2))/sqrt(e**2/f**2), Ne(a - b**2*f**2/(4*e**2), 0)),
((b*f**2/(2*e**2) + x)*log(b*f**2/(2*e**2) + x)/sqrt(e**2*(b*f**2/(2*e**2)
+ x)**2/f**2), True)) + (b*f**2/(4*e**2) + x/2)*sqrt(a + b*x + e**2*x**2/f*
*2), Ne(e**2/f**2, 0)), (2*(a + b*x)**(3/2)/(3*b), Ne(b, 0)), (sqrt(a)*x, T
rue)) + 2*e**2*x**3/3 + 2*e*f*Piecewise(((-a*b*f**2/(12*e**2) - b*f**2*(a/3
```

```
 - b**2*f**2/(8*e**2))/(2*e**2))*Piecewise((log(b + 2*e**2*x/f**2 + 2*sqrt(
e**2/f**2)*sqrt(a + b*x + e**2*x**2/f**2))/sqrt(e**2/f**2), Ne(a - b**2*f**
2/(4*e**2), 0)), ((b*f**2/(2*e**2) + x)*log(b*f**2/(2*e**2) + x)/sqrt(e**2*
(b*f**2/(2*e**2) + x)**2/f**2), True)) + sqrt(a + b*x + e**2*x**2/f**2)*(b*
f**2*x/(12*e**2) + x**2/3 + f**2*(a/3 - b**2*f**2/(8*e**2))/e**2), Ne(e**2/
f**2, 0)), (2*(-a*(a + b*x)**(3/2)/3 + (a + b*x)**(5/2)/5)/b**2, Ne(b, 0)),
 (sqrt(a)*x**2/2, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2*f^2-4*a*e^2>0)', see `assume?`
for mor
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2\,dx = \frac{1}{2}\,bf^2x^2+\frac{2}{3}\,e^2x^3+af^2x+dex^2+d^2x + \frac{1}{12}\,\sqrt{bf^2x+e^2x^2+af^2}\left(2\left(\frac{4\,ex|f|}{f}+\frac{be^3f^3|f|+6\,de^4f|f|}{e^4f^2}\right)x-\frac{3\,b^2ef^5|f|-6\,bde^2f^3|f|-8\,ae^3f^3|f|}{e^4f^2}\right) - \frac{(b^3f^5|f|-2\,b^2def^3|f|-4\,abe^2f^3|f|+8\,ade^3f|f|)\log\left(\left|-bf^2-2\left(x|e|-\sqrt{bf^2x+e^2x^2+af^2}\right)|e|\right|\right)}{8\,e^3|e|}$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*f^2*x^2 + 2/3*e^2*x^3 + a*f^2*x + d*e*x^2 + d^2*x + 1/12*sqrt(b*f^2*x
 + e^2*x^2 + a*f^2)*(2*(4*e*x*abs(f)/f + (b*e^3*f^3*abs(f) + 6*d*e^4*f*abs(
f))/(e^4*f^2))*x - (3*b^2*e*f^5*abs(f) - 6*b*d*e^2*f^3*abs(f) - 8*a*e^3*f^3
*abs(f))/(e^4*f^2)) - 1/8*(b^3*f^5*abs(f) - 2*b^2*d*e*f^3*abs(f) - 4*a*b*e^
2*f^3*abs(f) + 8*a*d*e^3*f*abs(f))*log(abs(-b*f^2 - 2*(x*abs(e) - sqrt(b*f^
2*x + e^2*x^2 + a*f^2))*abs(e)))/(e^3*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx = \int \left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^2 dx$$

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2,x)
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2, x)
```

3.475 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right) dx$

Optimal result

Integrand size = 26, antiderivative size = 118

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right) dx = dx+\frac{ex^2}{2}+\frac{f(bf^2+2e^2x)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{f^2(4ae^2-b^2f^2)\operatorname{arctanh}\left(\frac{bf^2+2e^2x}{2ef\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{8e^3}$$

```
[Out] d*x+1/2*e*x^2+1/8*f^2*(-b^2*f^2+4*a*e^2)*arctanh(1/2*(b*f^2+2*e^2*x)/e/f/(a
+b*x+e^2*x^2/f^2)^(1/2))/e^3+1/4*f*(b*f^2+2*e^2*x)*(a+b*x+e^2*x^2/f^2)^(1/2
)/e^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {626, 635, 212}

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right) dx = \frac{f^2(4ae^2-b^2f^2)\operatorname{arctanh}\left(\frac{bf^2+2e^2x}{2ef\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{8e^3}+\frac{f(bf^2+2e^2x)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+dx+\frac{ex^2}{2}$$

```
[In] Int[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*(b*f^2 + 2*e^2*x)*Sqrt[a + b*x + (e^2*x^2)/f^2])/(4*e^
2) + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(b*f^2 + 2*e^2*x)/(2*e*f*Sqrt[a + b*x
 + (e^2*x^2)/f^2])])/(8*e^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = dx + \frac{ex^2}{2} + f\int \sqrt{a+bx+\frac{e^2x^2}{f^2}}\,dx$$

$$= dx+\frac{ex^2}{2}+\frac{f(bf^2+2e^2x)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{1}{8}\left(f\left(4a-\frac{b^2f^2}{e^2}\right)\right)\int \frac{1}{\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$$

$$= dx+\frac{ex^2}{2}+\frac{f(bf^2+2e^2x)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2} + \frac{1}{4}\left(f\left(4a-\frac{b^2f^2}{e^2}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{4e^2}{f^2}-x^2}\,dx, x, \frac{b+\frac{2e^2x}{f^2}}{\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)$$

$$= dx+\frac{ex^2}{2}+\frac{f(bf^2+2e^2x)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{bf^2+2e^2x}{2ef\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{8e^3}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.53

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right) dx$$
$$= \frac{8de^3x+4e^4x^2+2bef^3\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+4e^3fx\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+(-4ae^2f^2+b^2f^4)\log\left(e^3\left(\sqrt{a}f+\right.\right.}{8e^3}$$

```
[In] Integrate[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2],x]
```

```
[Out] (8*d*e^3*x + 4*e^4*x^2 + 2*b*e*f^3*Sqrt[a + x*(b + (e^2*x)/f^2)] + 4*e^3*f*
x*Sqrt[a + x*(b + (e^2*x)/f^2)] + (-4*a*e^2*f^2 + b^2*f^4)*Log[e^3*(Sqrt[a]
*f + e*x - f*Sqrt[a + x*(b + (e^2*x)/f^2)])] + (4*a*e^2*f^2 - b^2*f^4)*Log[
-(Sqrt[a]*f) + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(8*e^3)
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04

method	result	size
default	$dx+\frac{e\,x^2}{2}+f\left(\frac{\left(b+\frac{2e^2x}{f^2}\right)f^2\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{\left(\frac{4e^2a}{f^2}-b^2\right)f^2\ln\left(\frac{\frac{b}{2}+\frac{e^2x}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^2\sqrt{\frac{e^2}{f^2}}}\right)$	123
parts	$dx+\frac{e\,x^2}{2}+f\left(\frac{\left(b+\frac{2e^2x}{f^2}\right)f^2\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{\left(\frac{4e^2a}{f^2}-b^2\right)f^2\ln\left(\frac{\frac{b}{2}+\frac{e^2x}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^2\sqrt{\frac{e^2}{f^2}}}\right)$	123

```
[In] int(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d*x+1/2*e*x^2+f*(1/4*(b+2*e^2*x/f^2)/e^2*f^2*(a+b*x+e^2*x^2/f^2)^(1/2)+1/8*
(4*e^2/f^2*a-b^2)/e^2*f^2*ln((1/2*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+(a+b*x+e^2*x
^2/f^2)^(1/2))/(e^2/f^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\,dx$$
$$=\frac{4\,e^4x^2+8\,de^3x+(b^2f^4-4\,ae^2f^2)\log\left(-bf^2-2\,e^2x+2\,ef\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}\right)+2\,(bef^3+2\,e^3fx)\sqrt{\frac{bf^2x+e^2x}{f^2}}}{8\,e^3}$$

```
[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*e^4*x^2 + 8*d*e^3*x + (b^2*f^4 - 4*a*e^2*f^2)*log(-b*f^2 - 2*e^2*x +
 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(b*e*f^3 + 2*e^3*f*x)*sqr
t((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.67

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\,dx = dx+\frac{ex^2}{2}$$
$$+f\left(\begin{cases}\left(\frac{a}{2}-\frac{b^2f^2}{8e^2}\right)\left(\begin{cases}\frac{\log\left(b+\frac{2e^2x}{f^2}+2\sqrt{\frac{e^2}{f^2}}\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}} & \text{for}\ a-\frac{b^2f^2}{4e^2}\neq 0\\ \frac{\left(\frac{bf^2}{2e^2}+x\right)\log\left(\frac{bf^2}{2e^2}+x\right)}{\sqrt{\frac{e^2\left(\frac{bf^2}{2e^2}+x\right)^2}{f^2}}} & \text{otherwise}\end{cases}\right)+\left(\frac{bf^2}{4e^2}+\frac{x}{2}\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}} & \text{for}\\ \frac{2(a+bx)^{\frac{3}{2}}}{3b} & \text{for}\\ \sqrt{a}x & \text{oth}\end{cases}\right.$$

```
[In] integrate(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] d*x + e*x**2/2 + f*Piecewise(((a/2 - b**2*f**2/(8*e**2))*Piecewise((log(b +
 2*e**2*x/f**2 + 2*sqrt(e**2/f**2)*sqrt(a + b*x + e**2*x**2/f**2))/sqrt(e**
2/f**2), Ne(a - b**2*f**2/(4*e**2), 0)), ((b*f**2/(2*e**2) + x)*log(b*f**2/
(2*e**2) + x)/sqrt(e**2*(b*f**2/(2*e**2) + x)**2/f**2), True)) + (b*f**2/(4
*e**2) + x/2)*sqrt(a + b*x + e**2*x**2/f**2), Ne(e**2/f**2, 0)), (2*(a + b*
x)**(3/2)/(3*b), Ne(b, 0)), (sqrt(a)*x, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right) dx = \text{Exception raised: ValueError}$$

```
[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2*f^2-4*a*e^2>0)', see `assume?`
for mor
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.01

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right) dx = \frac{1}{2}\,ex^2 + dx + \frac{\left(2\,\sqrt{bf^2x + e^2x^2 + af^2}\left(\frac{bf^2}{e^2} + 2\,x\right) + \frac{(b^2f^4 - 4\,ae^2f^2)\log\left(\left|-bf^2 - 2\,\left(x|e| - \sqrt{bf^2x + e^2x^2 + af^2}\right)|e|\right|\right)}{e^2|e|}\right)|f|}{8\,f}$$

```
[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*e*x^2 + d*x + 1/8*(2*sqrt(b*f^2*x + e^2*x^2 + a*f^2)*(b*f^2/e^2 + 2*x)
+ (b^2*f^4 - 4*a*e^2*f^2)*log(abs(-b*f^2 - 2*(x*abs(e) - sqrt(b*f^2*x + e^2
*x^2 + a*f^2))*abs(e)))/(e^2*abs(e)))*abs(f)/f
```

Mupad [F(-1)]

Timed out.

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right) dx = \int d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}\,dx$$

```
[In] int(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2),x)
```

```
[Out] int(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2), x)
```

3.476 $\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 215

$$\begin{aligned}&\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx\\&=-\frac{f^2(4ae^2-b^2f^2)}{2e\,(2de-bf^2)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}\\&\quad+\frac{2(d^2e-bdf^2+aef^2)\log\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{(2de-bf^2)^2}\\&\quad-\frac{f^2(4ae^2-b^2f^2)\log\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{2e\,(2de-bf^2)^2}\end{aligned}$$

```
[Out] 2*(a*e*f^2-b*d*f^2+d^2*e)*ln(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-b*f^2+2*d
*e)^2-1/2*f^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2
)^(1/2)))/e/(-b*f^2+2*d*e)^2-1/2*f^2*(-b^2*f^2+4*a*e^2)/e/(-b*f^2+2*d*e)/(b
*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2141, 907}

$$\begin{aligned}
&\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx \\
&= -\frac{f^2(4ae^2-b^2f^2)}{2e(2de-bf^2)\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)} \\
&\quad -\frac{f^2(4ae^2-b^2f^2)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{2e(2de-bf^2)^2} \\
&\quad +\frac{2(aef^2-bdf^2+d^2e)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{(2de-bf^2)^2}
\end{aligned}$$

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] -1/2*(f^2*(4*a*e^2 - b^2*f^2))/(e*(2*d*e - b*f^2)*(b*f^2 + 2*e*(e*x + f*Sqr
t[a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*(d^2*e - b*d*f^2 + a*e*f^2)*Log[d +
e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^2 - (f^2*(4*a*e^2 -
 b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*
e*(2*d*e - b*f^2)^2)
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2}{x\,(-2de+bf^2+2ex)^2}\,dx,x,d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$= 2\text{Subst}\left(\int\left(\frac{d^2e-bdf^2+aef^2}{(2de-bf^2)^2\,x}+\frac{4ae^2f^2-b^2f^4}{2\,(2de-bf^2)\,(2de-bf^2-2ex)^2}+\frac{4ae^2f^2-b^2f^4}{2\,(2de-bf^2)^2\,(2de-bf^2-2ex)}\right)dx,x,d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$= -\frac{f^2(4ae^2-b^2f^2)}{2e\,(2de-bf^2)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}+\frac{2(d^2e-bdf^2+aef^2)\log\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{(2de-bf^2)^2}-\frac{f^2(4ae^2-b^2f^2)\log\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{2e\,(2de-bf^2)^2}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.24

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$$

$$= \frac{2e^2(2de-bf^2)\,x+2ef(-2de+bf^2)\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}-(-2de+bf^2)^2\log\left(e\left(\sqrt{a}f+ex-f\sqrt{a+x\left(b\right.\right.\right.}}{}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] (2*e^2*(2*d*e - b*f^2)*x + 2*e*f*(-2*d*e + b*f^2)*Sqrt[a + x*(b + (e^2*x)/f
^2)] - (-2*d*e + b*f^2)^2*Log[e*(Sqrt[a]*f + e*x - f*Sqrt[a + x*(b + (e^2*x
)/f^2)])] - (4*a*e^2*f^2 - b^2*f^4)*Log[-(Sqrt[a]*f) + e*x + f*Sqrt[a + x*(
b + (e^2*x)/f^2)]] + 4*e*(d^2*e - b*d*f^2 + a*e*f^2)*Log[-(a*f^2) + d*e*x -
 b*f^2*x - d*f*Sqrt[a + x*(b + (e^2*x)/f^2)] + Sqrt[a]*f*(d + e*x + f*Sqrt[
a + x*(b + (e^2*x)/f^2)])])/(2*e*(-2*d*e + b*f^2)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1262 vs. 2(205) = 410.

Time = 0.08 (sec) , antiderivative size = 1263, normalized size of antiderivative = 5.87

method	result	size
default	Expression too large to display	1263

```
[In] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x,method=_RETURNVERBOSE)
[Out] f/(b*f^2-2*d*e)*((e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2
*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))
+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e
^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*
e^2)/f^2/(b*f^2-2*d*e)*ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2
)/f^2/(b*f^2-2*d*e)+e^2/f^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e)))/(e^2/f^2)^(1/2)+
(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-
2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a
*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*
d*e)^2)^(1/2))/(e^2/f^2)^(1/2)-(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d
^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2/((a^2*e^2*f^4-2*a*b*d
*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)
^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*
d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*
d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((a^2*e^2*f^4-2*
a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2
*d*e)^2)^(1/2)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f
^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(
a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2
)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e))))-d*ln((b*f^2-2
*d*e)*x+a*f^2-d^2)/(b*f^2-2*d*e)-e*(1/(b*f^2-2*d*e)*x+(-a*f^2+d^2)/(b*f^2-2
*d*e)^2*ln(b*f^2*x+a*f^2-2*d*e*x-d^2))
```

Fricas [A] (verification not implemented)

none

Time = 2.52 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.73

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx =$$

$$-\frac{2\,(be^2f^2-2\,de^3)x-2\,(d^2e^2-(bde-ae^2)f^2)\log\Big((bd-2\,ae)f^2-(bef^2-2\,de^2)x+(bf^3-2\,def)\sqrt{\frac{b}{}}}{}$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*e^2*f^2 - 2*d*e^3)*x - 2*(d^2*e^2 - (b*d*e - a*e^2)*f^2)*log((b*
d - 2*a*e)*f^2 - (b*e*f^2 - 2*d*e^2)*x + (b*f^3 - 2*d*e*f)*sqrt((b*f^2*x +
e^2*x^2 + a*f^2)/f^2)) - 2*(d^2*e^2 - (b*d*e - a*e^2)*f^2)*log(a*f^2 - d^2
+ (b*f^2 - 2*d*e)*x) + (b^2*f^4 + 2*d^2*e^2 - 2*(b*d*e + a*e^2)*f^2)*log(-b
*f^2 - 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(d^2*e^2
- (b*d*e - a*e^2)*f^2)*log(-e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) -
 d) - 2*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(b^2*e
*f^4 - 4*b*d*e^2*f^2 + 4*d^2*e^3)
```

Sympy [F]

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = \int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2)),x)
[Out] Integral(1/(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

Maxima [F]

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = \int \frac{1}{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}\,dx = \int \frac{1}{d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}}\,dx$$

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2)),x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2)), x)
```

3.477 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 266

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx = -\frac{2(d^2e-bdf^2+aef^2)}{(2de-bf^2)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)} - \frac{f^2(4ae^2-b^2f^2)}{(2de-bf^2)^2\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{2f^2(4ae^2-b^2f^2)\log\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{(2de-bf^2)^3} - \frac{2f^2(4ae^2-b^2f^2)\log\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{(2de-bf^2)^3}$$

```
[Out] 2*f^2*(-b^2*f^2+4*a*e^2)*ln(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-b*f^2+2*d*
e)^3-2*f^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(
1/2)))/(-b*f^2+2*d*e)^3-2*(a*e*f^2-b*d*f^2+d^2*e)/(-b*f^2+2*d*e)^2/(d+e*x+f
*(a+b*x+e^2*x^2/f^2)^(1/2))-f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2+2*d*e)^2/(b*f^2+
2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2141, 907}

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx$$
$$=\frac{2f^2(4ae^2-b^2f^2)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{(2de-bf^2)^3}-\frac{f^2(4ae^2-b^2f^2)}{(2de-bf^2)^2\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{2f^2(4ae^2-b^2f^2)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{(2de-bf^2)^3}-\frac{2(aef^2-bdf^2+d^2e)}{(2de-bf^2)^2\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}$$

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] (-2*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b
*x + (e^2*x^2)/f^2])) - (f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^2*(b*f^2
 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*f^2*(4*a*e^2 - b^
2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^3 -
(2*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^
2*x))/f^2])])/(2*d*e - b*f^2)^3
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2}{x^2\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$=2\text{Subst}\left(\int \left(\frac{d^2e-bdf^2+aef^2}{\left(2de-bf^2\right)^2x^2}+\frac{4ae^2f^2-b^2f^4}{\left(2de-bf^2\right)^3x}+\frac{4ae^3f^2-b^2ef^4}{\left(2de-bf^2\right)^2\left(2de-bf^2-2ex\right)^2}+\frac{2(4ae^3f^2-b^2ef^4)}{\left(2de-bf^2\right)^3\left(2de-bf^2-2ex\right)}\right)dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$=-\frac{2(d^2e-bdf^2+aef^2)}{\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}-\frac{f^2(4ae^2-b^2f^2)}{\left(2de-bf^2\right)^2\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}+\frac{2f^2(4ae^2-b^2f^2)\log\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{\left(2de-bf^2\right)^3}-\frac{2f^2(4ae^2-b^2f^2)\log\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{\left(2de-bf^2\right)^3}$$

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.61

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx$$

$$=\frac{2(bf^3(-d+ex)+2ef(af^2-dex))\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}{\left(-2de+bf^2\right)^2\left(d^2+2dex-f^2(a+bx)\right)}$$

$$-\frac{2(4a^2e^3f^2x+bdx(2d^2e^2-2bdef^2+b^2f^4+2de^3x-be^2f^2x)+a(4d^3e^2+2be^3f^2x^2+d^2(-4bef^2+4e^3x)}{(bd-2ae)\left(-2de+bf^2\right)^2\left(-d^2-2dex+f^2(a+bx)\right)}$$

$$-\frac{2(4ae^2f^2-b^2f^4)\log\left(-\sqrt{a}f+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}{\left(2de-bf^2\right)^3}$$

$$+\frac{2(4ae^2f^2-b^2f^4)\log\left(-af^2+dex-bf^2x-df\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+\sqrt{a}f\left(d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right.\right.}{\left(2de-bf^2\right)^3}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] (2*(b*f^3*(-d + e*x) + 2*e*f*(a*f^2 - d*e*x))*Sqrt[a + x*(b + (e^2*x)/f^2)]
)/((-2*d*e + b*f^2)^2*(d^2 + 2*d*e*x - f^2*(a + b*x))) - (2*(4*a^2*e^3*f^2*
x + b*d*x*(2*d^2*e^2 - 2*b*d*e*f^2 + b^2*f^4 + 2*d*e^3*x - b*e^2*f^2*x) + a
*(4*d^3*e^2 + 2*b*e^3*f^2*x^2 + d^2*(-4*b*e*f^2 + 4*e^3*x) + d*(b^2*f^4 - 6
*b*e^2*f^2*x - 4*e^4*x^2))))/((b*d - 2*a*e)*(-2*d*e + b*f^2)^2*(-d^2 - 2*d*
e*x + f^2*(a + b*x))) - (2*(4*a*e^2*f^2 - b^2*f^4)*Log[-(Sqrt[a]*f) + e*x +
 f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(2*d*e - b*f^2)^3 + (2*(4*a*e^2*f^2 - b^
2*f^4)*Log[-(a*f^2) + d*e*x - b*f^2*x - d*f*Sqrt[a + x*(b + (e^2*x)/f^2)] +
 Sqrt[a]*f*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/(2*d*e - b*f^2)^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6302 vs. $2(258) = 516$.

Time = 0.11 (sec) , antiderivative size = 6303, normalized size of antiderivative = 23.70

method	result	size
default	Expression too large to display	6303

```
[In] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(251) = 502$.

Time = 1.67 (sec) , antiderivative size = 826, normalized size of antiderivative = 3.11

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx =$$

$$-\frac{ab^2f^6+(3\,b^2d^2-14\,abde+8\,a^2e^2)f^4-2\,(bd^3e-4\,ad^2e^2)f^2-4\,(b^2e^2f^4-4\,bde^3f^2+4\,d^2e^4)x^2+(b^3f$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*f^6 + (3*b^2*d^2 - 14*a*b*d*e + 8*a^2*e^2)*f^4 - 2*(b*d^3*e - 4
*a*d^2*e^2)*f^2 - 4*(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4)*x^2 + (b^3*f^
6 - 8*b^2*d*e*f^4 + 20*b*d^2*e^2*f^2 - 16*d^3*e^3)*x - 2*(a*b^2*f^6 + 4*a*d
^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*
d*e + 2*a*b*e^2)*f^4)*x)*log(-4*a*d*e^2*f^2 - (b^2*d - 4*a*b*e)*f^4 + 4*(b*
e^3*f^2 - 2*d*e^4)*x^2 + (3*b^2*e*f^4 - 4*(2*b*d*e^2 - a*e^3)*f^2)*x - (b^2
*f^5 - 4*(b*d*e - a*e^2)*f^3 + 4*(b*e^2*f^3 - 2*d*e^3*f)*x)*sqrt((b*f^2*x +
```

```
 e^2*x^2 + a*f^2)/f^2)) - 2*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2
*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*log(
a*f^2 - d^2 + (b*f^2 - 2*d*e)*x) + 2*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^
2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4
)*x)*log(-e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - d) - 4*((b^2*d -
2*a*b*e)*f^5 - 2*(b*d^2*e - 2*a*d*e^2)*f^3 - (b^2*e*f^5 - 4*b*d*e^2*f^3 + 4
*d^2*e^3*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(a*b^3*f^8 + 8*d^5*e^
3 - (b^3*d^2 + 6*a*b^2*d*e)*f^6 + 6*(b^2*d^3*e + 2*a*b*d^2*e^2)*f^4 - 4*(3*
b*d^4*e^2 + 2*a*d^3*e^3)*f^2 + (b^4*f^8 - 8*b^3*d*e*f^6 + 24*b^2*d^2*e^2*f^
4 - 32*b*d^3*e^3*f^2 + 16*d^4*e^4)*x)
```

Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx = \int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx = \int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^2}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. 2(251) = 502.

Time = 6.66 (sec) , antiderivative size = 1618, normalized size of antiderivative = 6.08

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
[Out] 2*e^2*x/(b^2*f^4 - 4*b*d*e*f^2 + 4*d^2*e^2) + 1/5*(b^2*e*f^3*abs(f) - 4*a*e
^3*f*abs(f))*log(abs((x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^4*b^3*f^6
```

```
 - 2*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^2*b^3*d^2*f^6 + b^3*d^4*f
^6 + 4*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^2*a*b^2*d*e*f^6 - 4*a*b
^2*d^3*e*f^6 + 4*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^2*a^2*b*e^2*f
^6 + 4*a^2*b*d^2*e^2*f^6 + 4*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^3
*a*b^2*f^6*abs(e) - 4*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))*a*b^2*d^
2*f^6*abs(e) + 8*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))*a^2*b*d*e*f^6
*abs(e) - 4*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^4*b^2*d*e*f^4 + 4*
(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^2*b^2*d^3*e*f^4 + 8*(x*abs(e)
- sqrt(b*f^2*x + e^2*x^2 + a*f^2))^4*a*b*e^2*f^4 - 24*(x*abs(e) - sqrt(b*f^
2*x + e^2*x^2 + a*f^2))^2*a*b*d^2*e^2*f^4 + 16*(x*abs(e) - sqrt(b*f^2*x + e
^2*x^2 + a*f^2))^2*a^2*d*e^3*f^4 + 2*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a
*f^2))^5*b^2*f^4*abs(e) - 8*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^3*
b^2*d^2*f^4*abs(e) + 6*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))*b^2*d^4
*f^4*abs(e) - 16*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))*a*b*d^3*e*f^4
*abs(e) + 8*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^3*a^2*e^2*f^4*abs(
e) + 8*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))*a^2*d^2*e^2*f^4*abs(e)
- 4*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^4*b*d^2*e^2*f^2 + 12*(x*ab
s(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^2*b*d^4*e^2*f^2 - 16*(x*abs(e) - sq
rt(b*f^2*x + e^2*x^2 + a*f^2))^4*a*d*e^3*f^2 - 16*(x*abs(e) - sqrt(b*f^2*x
+ e^2*x^2 + a*f^2))^2*a*d^3*e^3*f^2 - 8*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2
+ a*f^2))^5*b*d*e*f^2*abs(e) + 16*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^
2))^3*b*d^3*e*f^2*abs(e) - 32*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^
3*a*d^2*e^2*f^2*abs(e) + 16*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^4*
d^3*e^3 + 8*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^5*d^2*e^2*abs(e) +
 8*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^3*d^4*e^2*abs(e)))/(b^3*f^6
*abs(e) - 6*b^2*d*e*f^4*abs(e) + 12*b*d^2*e^2*f^2*abs(e) - 8*d^3*e^3*abs(e)
) + (b^2*f^4 - 4*a*e^2*f^2)*log(abs(b*f^2*x + a*f^2 - 2*d*e*x - d^2))/(b^3*
f^6 - 6*b^2*d*e*f^4 + 12*b*d^2*e^2*f^2 - 8*d^3*e^3) - 2*(b^3*e*f^6*abs(f) -
 6*b^2*d*e^2*f^4*abs(f) + 12*b*d^2*e^3*f^2*abs(f) - 8*d^3*e^4*abs(f))*sqrt(
b*f^2*x + e^2*x^2 + a*f^2)/(b^5*f^11 - 10*b^4*d*e*f^9 + 40*b^3*d^2*e^2*f^7
- 80*b^2*d^3*e^3*f^5 + 80*b*d^4*e^4*f^3 - 32*d^5*e^5*f) - 2*(b^2*d^2*f^4 -
2*a*b*d*e*f^4 + a^2*e^2*f^4 - 2*b*d^3*e*f^2 + 2*a*d^2*e^2*f^2 + d^4*e^2)/((
b*f^2*x + a*f^2 - 2*d*e*x - d^2)*(b*f^2 - 2*d*e)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx=\int \frac{1}{\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^2}\,dx$$

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2,x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2, x)
```

3.478 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 330

$$\begin{aligned}
\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx
&= -\frac{d^2e-bdf^2+aef^2}{(2de-bf^2)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2} \\
&\quad -\frac{2f^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)} \\
&\quad -\frac{2ef^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} \\
&\quad +\frac{6ef^2(4ae^2-b^2f^2)\log\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{(2de-bf^2)^4} \\
&\quad -\frac{6ef^2(4ae^2-b^2f^2)\log\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{(2de-bf^2)^4}
\end{aligned}$$

```
[Out] 6*e*f^2*(-b^2*f^2+4*a*e^2)*ln(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-b*f^2+2*
d*e)^4-6*e*f^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^
2)^(1/2)))/(-b*f^2+2*d*e)^4+(-a*e*f^2+b*d*f^2-d^2*e)/(-b*f^2+2*d*e)^2/(d+e*
x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2-2*f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2+2*d*e)^3/
(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))-2*e*f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2+2*d*
e)^3/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2141, 907}

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx$$
$$= -\frac{2f^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}$$
$$+\frac{6ef^2(4ae^2-b^2f^2)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{(2de-bf^2)^4}$$
$$-\frac{2ef^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}$$
$$-\frac{6ef^2(4ae^2-b^2f^2)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{(2de-bf^2)^4}$$
$$-\frac{aef^2-bdf^2+d^2e}{(2de-bf^2)^2\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^2}$$

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -((d^2*e - b*d*f^2 + a*e*f^2)/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b*x
+ (e^2*x^2)/f^2])^2)) - (2*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^3*(d +
 e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])) - (2*e*f^2*(4*a*e^2 - b^2*f^2))/((
2*d*e - b*f^2)^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])))
 + (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]])/(2*d*e - b*f^2)^4 - (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x
+ f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*d*e - b*f^2)^4
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
```

```
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2}{x^3\,(-2de+bf^2+2ex)^2}\,dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{d^2e-bdf^2+aef^2}{(2de-bf^2)^2\,x^3}+\frac{4ae^2f^2-b^2f^4}{(2de-bf^2)^3\,x^2}+\frac{3(4ae^3f^2-b^2ef^4)}{(2de-bf^2)^4\,x}+\frac{2(4ae^4f^2-b^2e^2f^4)}{(2de-bf^2)^3\,(2de-bf^2-2ex)^2}+\frac{6(4ae^4f^2-b^2e^2f^4)}{(2de-bf^2)^4\,(2de-bf^2-2ex)}\right)dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$= -\frac{d^2e-bdf^2+aef^2}{(2de-bf^2)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}-\frac{2f^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}-\frac{2ef^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}+\frac{6ef^2(4ae^2-b^2f^2)\log\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{(2de-bf^2)^4}-\frac{6ef^2(4ae^2-b^2f^2)\log\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{(2de-bf^2)^4}$$

Mathematica [A] (verified)

Time = 10.51 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx = -\frac{\frac{(-2de+bf^2)^2(d^2e-bdf^2+aef^2)}{\left(d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)^2}+\frac{2f^2(-2de+bf^2)(-4ae^2+b^2f^2)}{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}+\frac{2ef^2(2de-bf^2)(4ae^2-b^2f^2)}{bf^2+2e\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}-6ef^2(4ae^2-b^2f^2)\log}{(-2de+}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3),x]
[Out] -((((-2*d*e + b*f^2)^2*(d^2*e - b*d*f^2 + a*e*f^2))/(d + e*x + f*Sqrt[a + x
*(b + (e^2*x)/f^2)])^2 + (2*f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2))/(d +
 e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + (2*e*f^2*(2*d*e - b*f^2)*(4*a*e^2
 - b^2*f^2))/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - 6*e*f^
2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]] + 6*e*
f^2*(4*a*e^2 - b^2*f^2)*Log[-(b*f^2) - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)
/f^2)])])/(-2*d*e + b*f^2)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 29132 vs. 2(320) = 640.

Time = 0.29 (sec) , antiderivative size = 29133, normalized size of antiderivative = 88.28

method	result	size
default	Expression too large to display	29133

```
[In] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1954 vs. 2(311) = 622.

Time = 12.00 (sec) , antiderivative size = 1954, normalized size of antiderivative = 5.92

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] ((3*a*b^3*d - 4*a^2*b^2*e)*f^8 - (b^3*d^3 + 4*a*b^2*d^2*e + 10*a^2*b*d*e^2
- 20*a^3*e^3)*f^6 - 4*(b^2*d^4*e - 8*a*b*d^3*e^2 + 6*a^2*d^2*e^3)*f^4 - 4*(
b^3*e^3*f^6 - 6*b^2*d*e^4*f^4 + 12*b*d^2*e^5*f^2 - 8*d^3*e^6)*x^3 + 2*(b*d^
5*e^2 - 6*a*d^4*e^3)*f^2 - (b^4*e*f^8 - 2*a*b^2*e^3*f^6 - 40*d^4*e^5 - 2*(1
1*b^2*d^2*e^3 - 4*a*b*d*e^4)*f^4 + 8*(7*b*d^3*e^4 - a*d^2*e^5)*f^2)*x^2 + (
16*d^5*e^4 + (3*b^4*d - 5*a*b^3*e)*f^8 - (7*b^3*d^2*e + 10*a*b^2*d*e^2 - 28
*a^2*b*e^3)*f^6 + 2*(5*b^2*d^3*e^2 + 22*a*b*d^2*e^3 - 28*a^2*d*e^4)*f^4 - 8
*(3*b*d^4*e^3 + a*d^3*e^4)*f^2)*x - 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*
(a*b^2*d^2*e + 2*a^3*e^3)*f^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^
8 - 16*a*d^2*e^5*f^2 - 4*(b^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a
*b*d*e^4)*f^4)*x^2 + 2*(a*b^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^
2*d*e^2 + 4*a^2*b*e^3)*f^6 + 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*
f^4)*x)*log(-4*a*d*e^2*f^2 - (b^2*d - 4*a*b*e)*f^4 + 4*(b*e^3*f^2 - 2*d*e^4
)*x^2 + (3*b^2*e*f^4 - 4*(2*b*d*e^2 - a*e^3)*f^2)*x - (b^2*f^5 - 4*(b*d*e -
 a*e^2)*f^3 + 4*(b*e^2*f^3 - 2*d*e^3*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)
/f^2)) - 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*(a*b^2*d^2*e + 2*a^3*e^3)*f
^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^8 - 16*a*d^2*e^5*f^2 - 4*(b
^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a*b*d*e^4)*f^4)*x^2 + 2*(a*b
^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^2*d*e^2 + 4*a^2*b*e^3)*f^6
+ 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*f^4)*x)*log(a*f^2 - d^2 + (
b*f^2 - 2*d*e)*x) + 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*(a*b^2*d^2*e + 2
*a^3*e^3)*f^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^8 - 16*a*d^2*e^5
*f^2 - 4*(b^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a*b*d*e^4)*f^4)*x
^2 + 2*(a*b^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^2*d*e^2 + 4*a^2*
b*e^3)*f^6 + 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*f^4)*x)*log(-e*x
 + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - d) - 2*(a*b^3*f^9 - 3*(a*b^2*d
*e + 2*a^2*b*e^2)*f^7 - 3*(b^2*d^3*e - 4*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^5 + 2
*(3*b*d^4*e^2 - 10*a*d^3*e^3)*f^3 - 2*(b^3*e^2*f^7 - 6*b^2*d*e^3*f^5 + 12*b
*d^2*e^4*f^3 - 8*d^3*e^5*f)*x^2 + (b^4*f^9 + 12*d^4*e^4*f - 3*(b^3*d*e + 3*
a*b^2*e^2)*f^7 + 3*(b^2*d^2*e^2 + 12*a*b*d*e^3)*f^5 - 4*(2*b*d^3*e^3 + 9*a*
d^2*e^4)*f^3)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(a^2*b^4*f^12 + 16*
d^8*e^4 - 2*(a*b^4*d^2 + 4*a^2*b^3*d*e)*f^10 + (b^4*d^4 + 16*a*b^3*d^3*e +
24*a^2*b^2*d^2*e^2)*f^8 - 8*(b^3*d^5*e + 6*a*b^2*d^4*e^2 + 4*a^2*b*d^3*e^3)
*f^6 + 8*(3*b^2*d^6*e^2 + 8*a*b*d^5*e^3 + 2*a^2*d^4*e^4)*f^4 - 32*(b*d^7*e^
3 + a*d^6*e^4)*f^2 + (b^6*f^12 - 12*b^5*d*e*f^10 + 60*b^4*d^2*e^2*f^8 - 160
*b^3*d^3*e^3*f^6 + 240*b^2*d^4*e^4*f^4 - 192*b*d^5*e^5*f^2 + 64*d^6*e^6)*x^
2 + 2*(a*b^5*f^12 + 32*d^7*e^5 - (b^5*d^2 + 10*a*b^4*d*e)*f^10 + 10*(b^4*d^
3*e + 4*a*b^3*d^2*e^2)*f^8 - 40*(b^3*d^4*e^2 + 2*a*b^2*d^3*e^3)*f^6 + 80*(b
^2*d^5*e^3 + a*b*d^4*e^4)*f^4 - 16*(5*b*d^6*e^4 + 2*a*d^5*e^5)*f^2)*x)
```

Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^3}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \int \frac{1}{\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^3}\,dx$$

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3,x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3, x)
```

3.479 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

Optimal result

Integrand size = 30, antiderivative size = 370

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \frac{f^2(2de-bf^2)(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^4} + \frac{f^2(4ae^2-b^2f^2)\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{12e^3} + \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{7/2}}{7e} - \frac{f^2(2de-bf^2)^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{5f^2(2de-bf^2)^{3/2}(4ae^2-b^2f^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{16\sqrt{2}e^{9/2}}$$

```
[Out] -5/32*f^2*(-b*f^2+2*d*e)^(3/2)*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(
d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/e^(9/2)*2^(1
/2)+1/12*f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2)/e
^3+1/7*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(7/2)/e+1/4*f^2*(-b*f^2+2*d*e)*(
-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e^4-1/16*f^2*(-
b*f^2+2*d*e)^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)
/e^4/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2141, 911, 1271, 1824, 214}

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = -\frac{5f^2(4ae^2-b^2f^2)\left(2de-bf^2\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{16\sqrt{2}e^{9/2}} + \frac{f^2(4ae^2-b^2f^2)\left(2de-bf^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^4} - \frac{f^2(4ae^2-b^2f^2)\left(2de-bf^2\right)^2\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{16e^4\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)} + \frac{f^2(4ae^2-b^2f^2)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{12e^3} + \frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e}$$

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e
^2*x^2)/f^2]])/(4*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x
 + (e^2*x^2)/f^2])^(3/2))/(12*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2])^(7/2)/(7*e) - (f^2*(2*d*e - b*f^2)^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x
 + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(16*e^4*(b*f^2 + 2*e*(e*x + f*Sqrt[a +
 (x*(b*f^2 + e^2*x))/f^2]))) - (5*f^2*(2*d*e - b*f^2)^(3/2)*(4*a*e^2 - b^2*
f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2
]])/Sqrt[2*d*e - b*f^2]])/(16*Sqrt[2]*e^(9/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^{5/2}(d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2)}{(-2de+bf^2+2ex)^2}\,dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$=4\text{Subst}\left(\int \frac{x^6(d^2e-(bd-ae)f^2+(-2de+bf^2)\,x^2+ex^4)}{(-2de+bf^2+2ex^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)$$

$$=-\frac{f^2(2de-bf^2)^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}-\frac{\text{Subst}\left(\int\frac{-ef^2(2de-bf^2)^2(4ae^2-b^2f^2)-4e^2f^2(2de-bf^2)(4ae^2-b^2f^2)x^2-8e^3f^2(4ae^2-b^2f^2)x^4+16e^4(2de-bf^2)x^6-32e^5x^8}{-2de+bf^2+2ex^2}\right.}{16e^5}$$

$$=-\frac{f^2(2de-bf^2)^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}-\frac{\text{Subst}\left(\int\left(-4ef^2(2de-bf^2)(4ae^2-b^2f^2)-4e^2f^2(4ae^2-b^2f^2)x^2-16e^4x^6-\frac{5(16ad^2e^5f^2-4b^2d\right.\right.}{16e^5}$$

$$=\frac{f^2(2de-bf^2)(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^4}+\frac{f^2(4ae^2-b^2f^2)\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{12e^3}+\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{7/2}}{7e}-\frac{f^2(2de-bf^2)^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}+\frac{\left(5f^2(2de-bf^2)^2(4ae^2-b^2f^2)\right)\text{Subst}\left(\int\frac{1}{-2de+bf^2+2ex^2}\,dx,x,\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{16e^4}$$

$$
\begin{aligned}
&= \frac{f^2(2de-bf^2)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^4} \\
&+ \frac{f^2(4ae^2-b^2f^2)\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{12e^3} \\
&+ \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{7/2}}{7e} \\
&- \frac{f^2(2de-bf^2)^2\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} \\
&- \frac{5f^2(2de-bf^2)^{3/2}\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{16\sqrt{2}e^{9/2}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 985 vs. 2(370) = 740.

Time = 4.55 (sec) , antiderivative size = 985, normalized size of antiderivative = 2.66

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \frac{1}{32}\left(\frac{2\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}\left(105b^4f^8+28b^3ef^6\left(-10d+3ex+5f\right.\right.}{}\right.$$

$$-\frac{20b^2d^2f^4\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{e^{5/2}\sqrt{-de+\frac{bf^2}{2}}}$$

$$-\frac{80abdf^4\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{e^{3/2}\sqrt{-de+\frac{bf^2}{2}}}$$

$$+\frac{20b^3df^6\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{e^{7/2}\sqrt{-de+\frac{bf^2}{2}}}$$

$$+\frac{20ab^2f^6\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{e^{5/2}\sqrt{-de+\frac{bf^2}{2}}}-\frac{5b^4f^8\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{e^{9/2}\sqrt{-de+\frac{bf^2}{2}}}$$

$$\left.+\frac{80\sqrt{2}ad^2f^2\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{\sqrt{e}\sqrt{-2de+bf^2}}\right)$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]
[Out] ((2*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]*(105*b^4*f^8 + 28*b^3*e
*f^6*(-10*d + 3*e*x + 5*f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*b^2*e^2*f^4*(2
```

```
1*d^2 - 119*a*f^2 + 16*e*x*(2*e*x - f*Sqrt[a + x*(b + (e^2*x)/f^2)]) - 2*d*
(31*e*x + 49*f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + 16*b*e^3*f^2*(3*d^3 + 79*a
*d*f^2 + 36*d*e*x*(2*e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 9*d^2*(3*e*x
+ f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*(15*e^3*x^3 - 8*a*f^3*Sqrt[a + x*(b
+ (e^2*x)/f^2)] + 9*e^2*f*x^2*Sqrt[a + x*(b + (e^2*x)/f^2)])) + 16*e^4*(20*
a^2*f^4 + 6*(d + 2*e*x)^3*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + a*f^2*(
-3*d^2 + 4*e*x*(19*e*x + 13*f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*d*(38*e*x
+ 29*f*Sqrt[a + x*(b + (e^2*x)/f^2)])))))/(21*e^4*(b*f^2 + 2*e*(e*x + f*Sqr
t[a + x*(b + (e^2*x)/f^2)]))) - (20*b^2*d^2*f^4*ArcTan[(Sqrt[2]*Sqrt[e]*Sqr
t[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(e^(5/
2)*Sqrt[-(d*e) + (b*f^2)/2]) - (80*a*b*d*f^4*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d
 + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(e^(3/2)*
Sqrt[-(d*e) + (b*f^2)/2]) + (20*b^3*d*f^6*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d +
e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(e^(7/2)*Sqr
t[-(d*e) + (b*f^2)/2]) + (20*a*b^2*f^6*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x
 + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(e^(5/2)*Sqrt[-
(d*e) + (b*f^2)/2]) - (5*b^4*f^8*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*S
qrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(e^(9/2)*Sqrt[-(d*e)
+ (b*f^2)/2]) + (80*Sqrt[2]*a*d^2*f^2*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x
+ f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(Sqrt[e]*Sqrt[-2
*d*e + b*f^2]))/32
```

Maple [F]

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}} dx$$

```
[In] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)
[Out] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 923, normalized size of antiderivative = 2.49

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \left[\frac{105\sqrt{\frac{1}{2}}(b^3f^6+8\,ade^3f^2-2\,(b^2de+2\,abe^2)f^4)\sqrt{-\frac{bf^2-2\,de}{e}}\log\left(-b^2f^4+4\,(\right.}{}\right.$$

$$-\frac{105\sqrt{\frac{1}{2}}(b^3f^6+8\,ade^3f^2-2\,(b^2de+2\,abe^2)f^4)\sqrt{\frac{bf^2-2\,de}{e}}\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{ex+f\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}+de}\sqrt{\frac{bf^2-2\,de}{e}}}{bf^2-2\,de}\right)-}{}$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/672*(105*sqrt(1/2)*(b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^
4)*sqrt(-(b*f^2 - 2*d*e)/e)*log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2
*f^2 - 2*d*e^3)*x + 4*(2*sqrt(1/2)*e^2*f*sqrt(-(b*f^2 - 2*d*e)/e)*sqrt((b*f
^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(1/2)*(b*e*f^2 + 2*e^3*x)*sqrt(-(b*f^2 -
 2*d*e)/e))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d) + 4*(b*
e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(105*b^3*f^6
+ 192*e^6*x^3 + 48*d^3*e^3 - 56*(5*b^2*d*e + 6*a*b*e^2)*f^4 + 4*(21*b*d^2*e
^2 + 232*a*d*e^3)*f^2 + 144*(b*e^4*f^2 + 2*d*e^5)*x^2 + 2*(7*b^2*e^2*f^4 +
156*d^2*e^4 - 4*(3*b*d*e^3 - 32*a*e^4)*f^2)*x - 2*(35*b^2*e*f^5 - 96*e^5*f*
x^2 + 12*d^2*e^3*f - 4*(21*b*d*e^2 + 20*a*e^3)*f^3 - 24*(b*e^3*f^3 + 6*d*e^
4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x +
 e^2*x^2 + a*f^2)/f^2) + d))/e^4, -1/336*(105*sqrt(1/2)*(b^3*f^6 + 8*a*d*e^
3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*sqrt((b*f^2 - 2*d*e)/e)*arctan(2*sqrt(
1/2)*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*e*sqrt((b*f^2
- 2*d*e)/e)/(b*f^2 - 2*d*e)) - (105*b^3*f^6 + 192*e^6*x^3 + 48*d^3*e^3 - 56
*(5*b^2*d*e + 6*a*b*e^2)*f^4 + 4*(21*b*d^2*e^2 + 232*a*d*e^3)*f^2 + 144*(b*
e^4*f^2 + 2*d*e^5)*x^2 + 2*(7*b^2*e^2*f^4 + 156*d^2*e^4 - 4*(3*b*d*e^3 - 32
*a*e^4)*f^2)*x - 2*(35*b^2*e*f^5 - 96*e^5*f*x^2 + 12*d^2*e^3*f - 4*(21*b*d*
e^2 + 20*a*e^3)*f^3 - 24*(b*e^3*f^3 + 6*d*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2
 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/e^
4]
```

Sympy [F]

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{5/2} dx = \int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}} dx$$

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(5/2), x)
```

Maxima [F]

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{5/2} dx = \int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{5}{2}} dx$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

Giac [F]

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{5/2} dx = \int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{5}{2}} dx$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{5/2} dx = \int \left(d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}\right)^{5/2} dx$$

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

3.480 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx$

Optimal result

Integrand size = 30, antiderivative size = 302

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \frac{f^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^3} + \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}{5e} - \frac{f^2(2de-bf^2)(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{8e^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{3f^2\sqrt{2de-bf^2}(4ae^2-b^2f^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{8\sqrt{2}e^{7/2}}$$

```
[Out] -3/16*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^
2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))*(-b*f^2+2*d*e)^(1/2)/e^(7/2)*2^(1
/2)+1/5*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2)/e+1/4*f^2*(-b^2*f^2+4*a*e
^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e^3-1/8*f^2*(-b*f^2+2*d*e)*(-
b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e^3/(b*f^2+2*e*(
e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2141, 911, 1271, 1824, 214}

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = -\frac{3f^2(4ae^2-b^2f^2)\sqrt{2de-bf^2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{8\sqrt{2}e^{7/2}} + \frac{f^2(4ae^2-b^2f^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^3} - \frac{f^2(4ae^2-b^2f^2)(2de-bf^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{8e^3\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)} + \frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{5e}$$

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(
4*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2)/(5*e) - (f^2*(2*
d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]])/(8*e^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) -
(3*f^2*Sqrt[2*d*e - b*f^2]*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqr
t[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(8*Sqrt
[2]*e^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^{3/2}(d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2)}{(-2de+bf^2+2ex)^2}\,dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$= 4\text{Subst}\left(\int \frac{x^4(d^2e-(bd-ae)f^2+(-2de+bf^2)\,x^2+ex^4)}{(-2de+bf^2+2ex^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)$$

$$= -\frac{f^2(2de-bf^2)\,(4ae^2-b^2f^2)\,\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{8e^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}$$

$$-\frac{\text{Subst}\left(\int \frac{-ef^2(2de-bf^2)(4ae^2-b^2f^2)-4e^2f^2(4ae^2-b^2f^2)x^2+8e^3(2de-bf^2)x^4-16e^4x^6}{-2de+bf^2+2ex^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\cdots}}\right.}{8e^4}$$

$$= -\frac{f^2(2de-bf^2)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{8e^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}$$

$$-\frac{\text{Subst}\left(\int\left(-2ef^2(4ae^2-b^2f^2)-8e^3x^4-\frac{3\left(8ade^4f^2-2b^2de^2f^4-4abe^3f^4+b^3ef^6\right)}{-2de+bf^2+2ex^2}\right)dx,x,\sqrt{d+ex+f\sqrt{}}\right.}{8e^4}$$

$$= \frac{f^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^3}+\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}{5e}$$

$$-\frac{f^2(2de-bf^2)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{8e^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}$$

$$+\frac{\left(3f^2(2de-bf^2)\left(4ae^2-b^2f^2\right)\right)\text{Subst}\left(\int\frac{1}{-2de+bf^2+2ex^2}dx,x,\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{8e^3}$$

$$= \frac{f^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^3}+\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}{5e}$$

$$-\frac{f^2(2de-bf^2)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{8e^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}$$

$$-\frac{3f^2\sqrt{2de-bf^2}(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{8\sqrt{2}e^{7/2}}$$

Mathematica [A] (verified)

Time = 2.57 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.47

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \frac{\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}\left(-15b^3f^6-2b^2ef^4\left(-5d+6ex+10f\sqrt{a+\right.\right.}{}$$

$$-\frac{3af^2\sqrt{-de+\frac{bf^2}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{2e^{3/2}}$$

$$+\frac{3b^2f^4\sqrt{-de+\frac{bf^2}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{8e^{7/2}}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]*(-15*b^3*f^6 - 2*b^2*e*f^4
*(-5*d + 6*e*x + 10*f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*b*e^2*f^2*(2*d^2 +
 17*a*f^2 + 8*e*x*(2*e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*d*(3*e*x +
f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + 8*e^3*(2*(d + 2*e*x)^2*(e*x + f*Sqrt[a
+ x*(b + (e^2*x)/f^2)]) + a*f^2*(-d + 16*e*x + 12*f*Sqrt[a + x*(b + (e^2*x)
/f^2)]))))/(40*e^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))) -
 (3*a*f^2*Sqrt[-(d*e) + (b*f^2)/2]*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f
*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(2*e^(3/2)) + (3*b^
2*f^4*Sqrt[-(d*e) + (b*f^2)/2]*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqr
t[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(8*e^(7/2))
```

Maple [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}} dx$$

```
[In] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

```
[Out] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 657, normalized size of antiderivative = 2.18

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \left[-\frac{15\sqrt{\frac{1}{2}}(b^2f^4-4\,ae^2f^2)\sqrt{-\frac{bf^2-2\,de}{e}}\log\left(-b^2f^4+4\,(bde-ae^2)f^2-4\,(be^2f^2\right.}{}\right.$$

[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")

[Out] [-1/80*(15*sqrt(1/2)*(b^2*f^4 - 4*a*e^2*f^2)*sqrt(-(b*f^2 - 2*d*e)/e)*log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x + 4*(2*sqrt(1/2)*e^2*f*sqrt(-(b*f^2 - 2*d*e)/e)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(1/2)*(b*e*f^2 + 2*e^3*x)*sqrt(-(b*f^2 - 2*d*e)/e))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(15*b^2*f^4 - 16*e^4*x^2 - 8*d^2*e^2 - 2*(5*b*d*e + 24*a*e^2)*f^2 + 2*(b*e^2*f^2 - 18*d*e^3)*x - 2*(5*b*e*f^3 + 8*e^3*f*x - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/e^3, 1/40*(15*sqrt(1/2)*(b^2*f^4 - 4*a*e^2*f^2)*sqrt((b*f^2 - 2*d*e)/e)*arctan(2*sqrt(1/2)*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*e*sqrt((b*f^2 - 2*d*e)/e)/(b*f^2 - 2*d*e)) - (15*b^2*f^4 - 16*e^4*x^2 - 8*d^2*e^2 - 2*(5*b*d*e + 24*a*e^2)*f^2 + 2*(b*e^2*f^2 - 18*d*e^3)*x - 2*(5*b*e*f^3 + 8*e^3*f*x - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/e^3]

Sympy [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}} dx$$

[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)

[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(3/2), x)

Maxima [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}} dx$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

Giac [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}} dx$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^{3/2} dx$$

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

3.481 $\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 233

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{f^2(4ae^2-b^2f^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{4\sqrt{2}e^{5/2}\sqrt{2de-bf^2}}$$

```
[Out] -1/8*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^2
/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/e^(5/2)*2^(1/2)/(-b*f^2+2*d*e)^(1/
2)+1/3*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2)/e-1/4*f^2*(4*a-b^2*f^2/e^2
)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e
^2*x)/f^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2141, 911, 1271, 1167, 214}

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = -\frac{f^2(4ae^2-b^2f^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{4\sqrt{2}e^{5/2}\sqrt{2de-bf^2}} - \frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)} + \frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}$$

```
[In] Int[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2)/(3*e) - (f^2*(4*a - (b^2*
f^2)/e^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(4*(b*f^2 + 2*e*
(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) - (f^2*(4*a*e^2 - b^2*f^2)*Ar
cTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqr
t[2*d*e - b*f^2]])/(4*Sqrt[2]*e^(5/2)*Sqrt[2*d*e - b*f^2])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{\sqrt{x}(d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2)}{(-2de+bf^2+2ex)^2}\,dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$= 4\text{Subst}\left(\int \frac{x^2(d^2e-(bd-ae)f^2+(-2de+bf^2)\,x^2+ex^4)}{(-2de+bf^2+2ex^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)$$

$$= -\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int \frac{-ef^2(4ae^2-b^2f^2)+4e^2(2de-bf^2)x^2-8e^3x^4}{-2de+bf^2+2ex^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{4e^3}$$

$$= -\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int\left(-4e^2x^2-\frac{ef^2(4ae^2-b^2f^2)}{-2de+bf^2+2ex^2}\right)dx, x, \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{4e^3}$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{1}{4}\left(f^2\left(4a-\frac{b^2f^2}{e^2}\right)\right)\text{Subst}\left(\int\frac{1}{-2de+bf^2+2ex^2}dx, x, \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{4\sqrt{2}e^{5/2}\sqrt{2de-bf^2}}$$

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.40

$$\int\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = \frac{\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}\left(3b^2f^4+4bef^2\left(d+3ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+4e^2\left(-af^2+2(d+2\right.\right.}{12e^2\left(bf^2+2e\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)} + \frac{af^2\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{\sqrt{2}\sqrt{e}\sqrt{-2de+bf^2}} - \frac{b^2f^4\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{4e^{5/2}\sqrt{-4de+2bf^2}}$$

```
[In] Integrate[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

[Out] (Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]*(3*b^2*f^4 + 4*b*e*f^2*(d + 3*e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*e^2*(-(a*f^2) + 2*(d + 2*e*x)*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))))/(12*e^2*(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))) + (a*f^2*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e + b*f^2]) - (b^2*f^4*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(4*e^(5/2)*Sqrt[-4*d*e + 2*b*f^2])

Maple [F]

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}dx$$

[In] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)

[Out] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 692, normalized size of antiderivative = 2.97

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = \left[-\frac{3\,(b^2f^4-4\,ae^2f^2)\sqrt{-2\,bef^2+4\,de^2}\log\left(-b^2f^4+4\,(bde-ae^2)f^2-4\,(be^2f^2-2\,de^3)x-2\left(2\sqrt{-2\,bef\right.\right.}{}\right.$$

[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")

[Out] [-1/48*(3*(b^2*f^4 - 4*a*e^2*f^2)*sqrt(-2*b*e*f^2 + 4*d*e^2)*log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*sqrt(-2*b*e*f^2 + 4*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(-2*b*e*f^2 + 4*d*e^2)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 4*(3*b^2*e*f^4 - 2*b*d*e^2*f^2 - 8*d^2*e^3 + 10*(b*e^3*f^2 - 2*d*e^4)*x - 2*(b*e^2*f^3 - 2*d*e^3*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(b*e^3*f^2 - 2*d*e^4), 1/24*(3*

```
(b^2*f^4 - 4*a*e^2*f^2)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt(e*x + f*s
qrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e^2)*f*sqrt
((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e*x + d))/(a
*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) + 2*(3*b^2*e*f^4 - 2*b*d*e^2*f^2 -
 8*d^2*e^3 + 10*(b*e^3*f^2 - 2*d*e^4)*x - 2*(b*e^2*f^3 - 2*d*e^3*f)*sqrt((b
*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^
2)/f^2) + d))/(b*e^3*f^2 - 2*d*e^4)]
```

Sympy [F]

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx=\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$$

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
[Out] Integral(sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

Maxima [F]

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx=\int \sqrt{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d}\,dx$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

Giac [F]

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx=\int \sqrt{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d}\,dx$$

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = \int \sqrt{d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}}\,dx$$

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

3.482 $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 244

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx = \frac{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{e} - \frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{f^2(4ae^2-b^2f^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}$$

```
[Out] 1/4*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^2/
f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/e^(3/2)/(-b*f^2+2*d*e)^(3/2)*2^(1/2
)+(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e-1/2*f^2*(4*a*e-b^2*f^2/e)*(d+
e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)/(b*f^2+2*e*(e*x+f*(a+
x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2141, 911, 1171, 396, 214}

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx = \frac{f^2(4ae^2-b^2f^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\,(2de-bf^2)^{3/2}} - \frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2\,(2de-bf^2)\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)} + \frac{\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

```
[In] Int[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]/e - (f^2*(4*a*e - (b^2*f^2)
/e)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*(2*d*e - b*f^2)*(b*
f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (f^2*(4*a*e^2 - b
^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]])/Sqrt[2*d*e - b*f^2]])/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2}{\sqrt{x}\,(-2de+bf^2+2ex)^2}\,dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$=4\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2+(-2de+bf^2)\,x^2+ex^4}{(-2de+bf^2+2ex^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)$$

$$=-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{2\,(2de-bf^2)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{2\text{Subst}\left(\int \frac{\frac{1}{4}\left(-8d^2e+8bdf^2-4aef^2-\frac{b^2f^4}{e}\right)+(2de-bf^2)x^2}{-2de+bf^2+2ex^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{2de-bf^2}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{e} - \frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{\left(f^2(4ae^2-b^2f^2)\right)\text{Subst}\left(\int\frac{1}{-2de+bf^2+2ex^2}\,dx,x,\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{2e\left(2de-bf^2\right)}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{e} - \frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$$

$$= \frac{\frac{2\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}\left(-b^2f^4-4bef^2\left(-d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+4e^2\left(-af^2+2dex+2df\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}{(2de-bf^2)\left(bf^2+2e\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)} + \frac{4\sqrt{2}ae^2f^2\arctan\left(\cdots\right.}{(\cdots}}{4e^{3/2}}$$

```
[In] Integrate[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] ((2*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]*(-(b^2*f^4) - 4
*b*e*f^2*(-d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*e^2*(-(a*f^2) + 2
*d*e*x + 2*d*f*Sqrt[a + x*(b + (e^2*x)/f^2)])))/((2*d*e - b*f^2)*(b*f^2 + 2
*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))) + (4*Sqrt[2]*a*e^2*f^2*ArcTan[
(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d
*e + b*f^2]])/(-2*d*e + b*f^2)^(3/2) - (Sqrt[2]*b^2*f^4*ArcTan[(Sqrt[2]*Sqr
t[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]]
)/(-2*d*e + b*f^2)^(3/2))/(4*e^(3/2))
```

Maple [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$$

[In] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)

[Out] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 716, normalized size of antiderivative = 2.93

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$$
$$= \left[\frac{(b^2f^4 - 4\,ae^2f^2)\sqrt{-2\,bef^2 + 4\,de^2}\log\left(-b^2f^4 + 4\,(bde - ae^2)f^2 - 4\,(be^2f^2 - 2\,de^3)x + 2\left(2\sqrt{-2\,bef^2\right.\right.}{} \right.$$

[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas"
)

[Out] [1/8*((b^2*f^4 - 4*a*e^2*f^2)*sqrt(-2*b*e*f^2 + 4*d*e^2)*log(-b^2*f^4 + 4*(
b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x + 2*(2*sqrt(-2*b*e*f^2 + 4*d
*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(-2*b*e*f^2 + 4*d*e^2
)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d
) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 4*(b^2
*e*f^4 - 6*b*d*e^2*f^2 + 8*d^2*e^3 - 2*(b*e^3*f^2 - 2*d*e^4)*x + 2*(b*e^2*f
^3 - 2*d*e^3*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b
*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e
^4), 1/4*((b^2*f^4 - 4*a*e^2*f^2)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt
(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e
^2)*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e*
x + d))/(a*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) + 2*(b^2*e*f^4 - 6*b*d*e
^2*f^2 + 8*d^2*e^3 - 2*(b*e^3*f^2 - 2*d*e^4)*x + 2*(b*e^2*f^3 - 2*d*e^3*f)*
sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2
 + a*f^2)/f^2) + d))/(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4)]

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx = \int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx = \int \frac{1}{\sqrt{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx = \int \frac{1}{\sqrt{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx = \int \frac{1}{\sqrt{d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}}}\,dx$$

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

3.483 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 269

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = -\frac{4(d^2e-bdf^2+aef^2)}{(2de-bf^2)^2\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}} - \frac{f^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{(2de-bf^2)^2\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{3f^2(4ae^2-b^2f^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}\,(2de-bf^2)^{5/2}}$$

```
[Out] 3/2*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^2/
f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/(-b*f^2+2*d*e)^(5/2)*2^(1/2)/e^(1/2
)-4*(a*e*f^2-b*d*f^2+d^2*e)/(-b*f^2+2*d*e)^2/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(
1/2))^(1/2)-f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2
)/(-b*f^2+2*d*e)^2/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2141, 911, 1273, 464, 214}

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \frac{3f^2(4ae^2-b^2f^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}\,(2de-bf^2)^{5/2}} - \frac{f^2(4ae^2-b^2f^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{(2de-bf^2)^2\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)} - \frac{4(aef^2-bdf^2+d^2e)}{(2de-bf^2)^2\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}$$

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*Sqrt[d + e*x + f*Sqrt[a
 + b*x + (e^2*x^2)/f^2]]) - (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[
a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^2*(b*f^2 + 2*e*(e*x + f*Sqrt[a
+ (x*(b*f^2 + e^2*x))/f^2]))) + (3*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]
*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^
2]])/(Sqrt[2]*Sqrt[e]*(2*d*e - b*f^2)^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2}{x^{3/2}\,(-2de+bf^2+2ex)^2}\,dx, x, d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$= 4\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2+(-2de+bf^2)\,x^2+ex^4}{x^2\,(-2de+bf^2+2ex^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)$$

$$= -\frac{f^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{(2de-bf^2)^2\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int \frac{8e^2(2de-bf^2)(d^2e-bdf^2+aef^2)-2e^2(8d^2e^2-8bdef^2-4ae^2f^2+3b^2f^4)x^2}{x^2(-2de+bf^2+2ex^2)}\,dx, x, \sqrt{d+ex+f\sqrt{a+bx+\cdots}}\right.}{2e^2\,(2de-bf^2)^2}$$

$$= -\frac{4(d^2e - bdf^2 + aef^2)}{(2de - bf^2)^2 \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}} - \frac{f^2(4ae^2 - b^2f^2)\sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2de - bf^2)^2\left(bf^2 + 2e\left(ex + f\sqrt{a + \frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{(3f^2(4ae^2 - b^2f^2))\,\text{Subst}\left(\int \frac{1}{-2de+bf^2+2ex^2}\,dx, x, \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}\right)}{(2de - bf^2)^2}$$

$$= -\frac{4(d^2e - bdf^2 + aef^2)}{(2de - bf^2)^2 \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}} - \frac{f^2(4ae^2 - b^2f^2)\sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2de - bf^2)^2\left(bf^2 + 2e\left(ex + f\sqrt{a + \frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{3f^2(4ae^2 - b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}\,(2de - bf^2)^{5/2}}$$

Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \frac{b^2f^4\left(5d + ex + f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right)}\right) - 4bef^2\left(d^2 + af^2 - 2d\left(ex\right.\right.}{(-2de + bf^2)^2\sqrt{d + ex}} - \frac{6\sqrt{2}ae^{3/2}f^2\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{(-2de + bf^2)^{5/2}} + \frac{3b^2f^4\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{\sqrt{2}\sqrt{e}\,(-2de + bf^2)^{5/2}}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] (b^2*f^4*(5*d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) - 4*b*e*f^2*(d^2 + a
*f^2 - 2*d*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - 4*e^2*(2*d^2*(e*x + f
*Sqrt[a + x*(b + (e^2*x)/f^2)]) + a*f^2*(d + 3*e*x + 3*f*Sqrt[a + x*(b + (e
^2*x)/f^2)])))/((-2*d*e + b*f^2)^2*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)
/f^2)]]*(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))) - (6*Sqrt[2]
*a*e^(3/2)*f^2*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^
2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(-2*d*e + b*f^2)^(5/2) + (3*b^2*f^4*Arc
Tan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[
-2*d*e + b*f^2]])/(Sqrt[2]*Sqrt[e]*(-2*d*e + b*f^2)^(5/2))
```

Maple [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(239) = 478$.

Time = 0.79 (sec) , antiderivative size = 1456, normalized size of antiderivative = 5.41

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*(3*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6
 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*sqrt(-2*b*e*f^2 + 4*d*e^
2)*log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*
sqrt(-2*b*e*f^2 + 4*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt
(-2*b*e*f^2 + 4*d*e^2)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*
x^2 + a*f^2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 +
a*f^2)/f^2)) + 4*(a*b^2*e*f^6 - 8*d^4*e^3 - (5*b^2*d^2*e - 2*a*b*d*e^2)*f^4
 + 2*(7*b*d^3*e^2 - 4*a*d^2*e^3)*f^2 + 2*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d
^2*e^5)*x^2 + (b^3*e*f^6 - 4*d^3*e^4 - 2*(4*b^2*d*e^2 - 3*a*b*e^3)*f^4 + 2*
(7*b*d^2*e^3 - 6*a*d*e^4)*f^2)*x + 2*(2*d^3*e^3*f + (2*b^2*d*e - 3*a*b*e^2)
*f^5 - (5*b*d^2*e^2 - 6*a*d*e^3)*f^3 - (b^2*e^2*f^5 - 4*b*d*e^3*f^3 + 4*d^2
*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*
x + e^2*x^2 + a*f^2)/f^2) + d))/(a*b^3*e*f^8 + 8*d^5*e^4 - (b^3*d^2*e + 6*a
```

```
*b^2*d*e^2)*f^6 + 6*(b^2*d^3*e^2 + 2*a*b*d^2*e^3)*f^4 - 4*(3*b*d^4*e^3 + 2*
a*d^3*e^4)*f^2 + (b^4*e*f^8 - 8*b^3*d*e^2*f^6 + 24*b^2*d^2*e^3*f^4 - 32*b*d
^3*e^4*f^2 + 16*d^4*e^5)*x), -1/2*(3*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^
2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4
)*x)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt(e*x + f*sqrt((b*f^2*x + e^2*
x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e^2)*f*sqrt((b*f^2*x + e^2*x^2
 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e*x + d))/(a*e*f^2 - d^2*e + (b
*e*f^2 - 2*d*e^2)*x)) - 2*(a*b^2*e*f^6 - 8*d^4*e^3 - (5*b^2*d^2*e - 2*a*b*d
*e^2)*f^4 + 2*(7*b*d^3*e^2 - 4*a*d^2*e^3)*f^2 + 2*(b^2*e^3*f^4 - 4*b*d*e^4*
f^2 + 4*d^2*e^5)*x^2 + (b^3*e*f^6 - 4*d^3*e^4 - 2*(4*b^2*d*e^2 - 3*a*b*e^3)
*f^4 + 2*(7*b*d^2*e^3 - 6*a*d*e^4)*f^2)*x + 2*(2*d^3*e^3*f + (2*b^2*d*e - 3
*a*b*e^2)*f^5 - (5*b*d^2*e^2 - 6*a*d*e^3)*f^3 - (b^2*e^2*f^5 - 4*b*d*e^3*f^
3 + 4*d^2*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqr
t((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a*b^3*e*f^8 + 8*d^5*e^4 - (b^3*d^
2*e + 6*a*b^2*d*e^2)*f^6 + 6*(b^2*d^3*e^2 + 2*a*b*d^2*e^3)*f^4 - 4*(3*b*d^4
*e^3 + 2*a*d^3*e^4)*f^2 + (b^4*e*f^8 - 8*b^3*d*e^2*f^6 + 24*b^2*d^2*e^3*f^4
 - 32*b*d^3*e^4*f^2 + 16*d^4*e^5)*x)]
```

Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx=\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx=\int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima"
)
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \int \frac{1}{\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^{3/2}}\,dx$$

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

3.484 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 335

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = -\frac{4(d^2e-bdf^2+aef^2)}{3\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{4f^2(4ae^2-b^2f^2)}{\left(2de-bf^2\right)^3\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}} - \frac{2ef^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\left(2de-bf^2\right)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{5\sqrt{2}\sqrt{e}f^2(4ae^2-b^2f^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{\left(2de-bf^2\right)^{7/2}}$$

```
[Out] 5*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^2/f^
2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))*2^(1/2)*e^(1/2)/(-b*f^2+2*d*e)^(7/2)-
4/3*(a*e*f^2-b*d*f^2+d^2*e)/(-b*f^2+2*d*e)^2/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(
1/2))^(3/2)-4*f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2+2*d*e)^3/(d+e*x+f*(a+b*x+e^2*x
^2/f^2)^(1/2))^(1/2)-2*e*f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2
)^(1/2))^(1/2)/(-b*f^2+2*d*e)^3/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(
1/2)))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2141, 911, 1273, 1275, 214}

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \frac{5\sqrt{2}\sqrt{e}f^2(4ae^2-b^2f^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{(2de-bf^2)^{7/2}} - \frac{4f^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}} - \frac{2ef^2(4ae^2-b^2f^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{(2de-bf^2)^3\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)} - \frac{4(aef^2-bdf^2+d^2e)}{3(2de-bf^2)^2\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}$$

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/(3*(2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a +
 b*x + (e^2*x^2)/f^2])^(3/2)) - (4*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2
)^3*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]) - (2*e*f^2*(4*a*e^2 -
b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^
3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (5*Sqrt[2]*S
qrt[e]*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sq
rt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(2*d*e - b*f^2)^(7/2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2}{x^{5/2}\,(-2de+bf^2+2ex)^2}\,dx, x, d+ex + f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)$$

$$= 4\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2+(-2de+bf^2)\,x^2+ex^4}{x^4\,(-2de+bf^2+2ex^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)$$

$$= -\frac{2ef^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{(2de-bf^2)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int \frac{8e^2(2de-bf^2)^2(d^2e-bdf^2+aef^2)-8e^2(2de-bf^2)(2d^2e^2-2bdef^2-2ae^2f^2+b^2f^4)x^2+4e^3f^2(4ae^2-b^2f^2)x^4}{x^4(-2de+bf^2+2ex^2)}\,dx, x,\right.}{2e^2(2de-bf^2)^3}$$

$$= -\frac{2ef^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{(2de-bf^2)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int\left(-\frac{8e^2(2de-bf^2)(d^2e-bdf^2+aef^2)}{x^4}-\frac{8(4ae^4f^2-b^2e^2f^4)}{x^2}-\frac{20(4ae^5f^2-b^2e^3f^4)}{2de-bf^2-2ex^2}\right)dx, x, \sqrt{d+ex+f\right.}{2e^2(2de-bf^2)^3}$$

$$= -\frac{4(d^2e-bdf^2+aef^2)}{3(2de-bf^2)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{4f^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}} - \frac{2ef^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{(2de-bf^2)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{(10ef^2(4ae^2-b^2f^2))\,\text{Subst}\left(\int\frac{1}{2de-bf^2-2ex^2}dx, x, \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{(2de-bf^2)^3}$$

$$= -\frac{4(d^2e-bdf^2+aef^2)}{3(2de-bf^2)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{4f^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}} - \frac{2ef^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{(2de-bf^2)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{5\sqrt{2}\sqrt{e}f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{(2de-bf^2)^{7/2}}$$

Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.66

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \frac{2b^3f^6\left(4d+21ex+6f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+2b^2ef^4\left(9d^2+17af^2\right.}{}$$

$$+\frac{20\sqrt{2}ae^{5/2}f^2\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{(-2de+bf^2)^{7/2}}$$

$$-\frac{5\sqrt{2}b^2\sqrt{e}f^4\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{(-2de+bf^2)^{7/2}}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] (2*b^3*f^6*(4*d + 21*e*x + 6*f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 2*b^2*e*f^4
*(9*d^2 + 17*a*f^2 + 14*d*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 30*e*x*
(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - 8*b*e^2*f^2*(d^3 + 7*a*d*f^2 - 3
*d^2*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 5*a*f^2*(4*e*x + f*Sqrt[a +
x*(b + (e^2*x)/f^2)])) - 8*e^3*(15*a^2*f^4 + 2*d^3*(e*x + f*Sqrt[a + x*(b +
 (e^2*x)/f^2)]) + a*f^2*(3*d^2 + 20*d*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)
]) + 30*e*x*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))))/(3*(2*d*e - b*f^2)^3
*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(3/2)*(b*f^2 + 2*e*(e*x + f*Sq
rt[a + x*(b + (e^2*x)/f^2)]))) + (20*Sqrt[2]*a*e^(5/2)*f^2*ArcTan[(Sqrt[2]*
Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^
2]])/(-2*d*e + b*f^2)^(7/2) - (5*Sqrt[2]*b^2*Sqrt[e]*f^4*ArcTan[(Sqrt[2]*Sq
rt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]
])/(-2*d*e + b*f^2)^(7/2)
```

Maple [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1223 vs. 2(298) = 596.

Time = 1.96 (sec) , antiderivative size = 2514, normalized size of antiderivative = 7.50

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas"
)
```

```
[Out] [-1/6*(15*sqrt(2)*(a^2*b^2*f^8 - 4*a*d^4*e^2*f^2 - 2*(a*b^2*d^2 + 2*a^3*e^2
)*f^6 + (b^2*d^4 + 8*a^2*d^2*e^2)*f^4 + (b^4*f^8 - 16*a*d^2*e^4*f^2 - 4*(b^
3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)*x^2 + 2*(a*b^3*
f^8 - 8*a*d^3*e^3*f^2 - (b^3*d^2 + 2*a*b^2*d*e + 4*a^2*b*e^2)*f^6 + 2*(b^2*
d^3*e + 2*a*b*d^2*e^2 + 4*a^2*d*e^3)*f^4)*x)*sqrt(-e/(b*f^2 - 2*d*e))*log(-
b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*sqrt(2)*
(b*e*f^3 - 2*d*e^2*f)*sqrt(-e/(b*f^2 - 2*d*e))*sqrt((b*f^2*x + e^2*x^2 + a*
f^2)/f^2) - sqrt(2)*(b^2*f^4 - 2*b*d*e*f^2 + 2*(b*e^2*f^2 - 2*d*e^3)*x)*sqr
t(-e/(b*f^2 - 2*d*e)))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) +
 d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 4*(4
*d^5*e^2 + (8*a*b^2*d - 5*a^2*b*e)*f^6 - 2*(2*b^2*d^3 + a*b*d^2*e + 10*a^2*
d*e^2)*f^4 - 6*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d^2*e^5)*x^3 - (9*b*d^4*e -
 32*a*d^3*e^2)*f^2 + (3*b^3*e*f^6 - 16*d^3*e^4 + 4*(b^2*d*e^2 - 10*a*b*e^3)
*f^4 - 4*(3*b*d^2*e^3 - 20*a*d*e^4)*f^2)*x^2 + 2*(d^4*e^3 + (4*b^3*d - a*b^
2*e)*f^6 - (7*b^2*d^2*e + 6*a*b*d*e^2 + 15*a^2*e^3)*f^4 - 2*(5*b*d^3*e^2 -
23*a*d^2*e^3)*f^2)*x - 2*(3*a*b^2*f^7 + d^4*e^2*f - (b^2*d^2 + 2*a*b*d*e +
15*a^2*e^2)*f^5 - 2*(3*b*d^3*e - 11*a*d^2*e^2)*f^3 - 3*(b^2*e^2*f^5 - 4*b*d
*e^3*f^3 + 4*d^2*e^4*f)*x^2 + (3*b^3*f^7 + 40*a*d*e^3*f^3 - 8*d^3*e^3*f - 4
*(b^2*d*e + 5*a*b*e^2)*f^5)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(
e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a^2*b^3*f^10 - 8*d^7*e
^3 - 2*(a*b^3*d^2 + 3*a^2*b^2*d*e)*f^8 + (b^3*d^4 + 12*a*b^2*d^3*e + 12*a^2
*b*d^2*e^2)*f^6 - 2*(3*b^2*d^5*e + 12*a*b*d^4*e^2 + 4*a^2*d^3*e^3)*f^4 + 4*
(3*b*d^6*e^2 + 4*a*d^5*e^3)*f^2 + (b^5*f^10 - 10*b^4*d*e*f^8 + 40*b^3*d^2*e
^2*f^6 - 80*b^2*d^3*e^3*f^4 + 80*b*d^4*e^4*f^2 - 32*d^5*e^5)*x^2 + 2*(a*b^4
*f^10 - 16*d^6*e^4 - (b^4*d^2 + 8*a*b^3*d*e)*f^8 + 8*(b^3*d^3*e + 3*a*b^2*d
^2*e^2)*f^6 - 8*(3*b^2*d^4*e^2 + 4*a*b*d^3*e^3)*f^4 + 16*(2*b*d^5*e^3 + a*d
^4*e^4)*f^2)*x), 1/3*(15*sqrt(2)*(a^2*b^2*f^8 - 4*a*d^4*e^2*f^2 - 2*(a*b^2*
d^2 + 2*a^3*e^2)*f^6 + (b^2*d^4 + 8*a^2*d^2*e^2)*f^4 + (b^4*f^8 - 16*a*d^2*
e^4*f^2 - 4*(b^3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)*
x^2 + 2*(a*b^3*f^8 - 8*a*d^3*e^3*f^2 - (b^3*d^2 + 2*a*b^2*d*e + 4*a^2*b*e^2
)*f^6 + 2*(b^2*d^3*e + 2*a*b*d^2*e^2 + 4*a^2*d*e^3)*f^4)*x)*sqrt(e/(b*f^2 -
 2*d*e))*arctan(1/2*(sqrt(2)*(b*f^3 - 2*d*e*f)*sqrt(e/(b*f^2 - 2*d*e))*sqrt
((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2)*(b*d*f^2 - 2*d^2*e + (b*e*f^2 -
```

```
2*d*e^2)*x)*sqrt(e/(b*f^2 - 2*d*e)))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2
+ a*f^2)/f^2) + d)/(a*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) + 2*(4*d^5*e^
2 + (8*a*b^2*d - 5*a^2*b*e)*f^6 - 2*(2*b^2*d^3 + a*b*d^2*e + 10*a^2*d*e^2)*
f^4 - 6*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d^2*e^5)*x^3 - (9*b*d^4*e - 32*a*d
^3*e^2)*f^2 + (3*b^3*e*f^6 - 16*d^3*e^4 + 4*(b^2*d*e^2 - 10*a*b*e^3)*f^4 -
4*(3*b*d^2*e^3 - 20*a*d*e^4)*f^2)*x^2 + 2*(d^4*e^3 + (4*b^3*d - a*b^2*e)*f^
6 - (7*b^2*d^2*e + 6*a*b*d*e^2 + 15*a^2*e^3)*f^4 - 2*(5*b*d^3*e^2 - 23*a*d^
2*e^3)*f^2)*x - 2*(3*a*b^2*f^7 + d^4*e^2*f - (b^2*d^2 + 2*a*b*d*e + 15*a^2*
e^2)*f^5 - 2*(3*b*d^3*e - 11*a*d^2*e^2)*f^3 - 3*(b^2*e^2*f^5 - 4*b*d*e^3*f^
3 + 4*d^2*e^4*f)*x^2 + (3*b^3*f^7 + 40*a*d*e^3*f^3 - 8*d^3*e^3*f - 4*(b^2*d
*e + 5*a*b*e^2)*f^5)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f
*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a^2*b^3*f^10 - 8*d^7*e^3 - 2*
(a*b^3*d^2 + 3*a^2*b^2*d*e)*f^8 + (b^3*d^4 + 12*a*b^2*d^3*e + 12*a^2*b*d^2*
e^2)*f^6 - 2*(3*b^2*d^5*e + 12*a*b*d^4*e^2 + 4*a^2*d^3*e^3)*f^4 + 4*(3*b*d^
6*e^2 + 4*a*d^5*e^3)*f^2 + (b^5*f^10 - 10*b^4*d*e*f^8 + 40*b^3*d^2*e^2*f^6
- 80*b^2*d^3*e^3*f^4 + 80*b*d^4*e^4*f^2 - 32*d^5*e^5)*x^2 + 2*(a*b^4*f^10 -
16*d^6*e^4 - (b^4*d^2 + 8*a*b^3*d*e)*f^8 + 8*(b^3*d^3*e + 3*a*b^2*d^2*e^2)
*f^6 - 8*(3*b^2*d^4*e^2 + 4*a*b*d^3*e^3)*f^4 + 16*(2*b*d^5*e^3 + a*d^4*e^4)
*f^2)*x)]
```

Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx=\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx=\int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima"
)
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx=\int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx=\int \frac{1}{\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^{5/2}}\,dx$$

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

3.485 $\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal result

Integrand size = 21, antiderivative size = 164

$$\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n \, dx = -\frac{a^5\left(x+\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)} - \frac{5a^4\left(x+\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)} - \frac{5a^3\left(x+\sqrt{a+x^2}\right)^{-1+n}}{16(1-n)} + \frac{5a^2\left(x+\sqrt{a+x^2}\right)^{1+n}}{16(1+n)} + \frac{5a\left(x+\sqrt{a+x^2}\right)^{3+n}}{32(3+n)} + \frac{\left(x+\sqrt{a+x^2}\right)^{5+n}}{32(5+n)}$$

```
[Out] -1/32*a^5*(x+(x^2+a)^(1/2))^(-5+n)/(5-n)-5/32*a^4*(x+(x^2+a)^(1/2))^(-3+n)/
(3-n)-5/16*a^3*(x+(x^2+a)^(1/2))^(-1+n)/(1-n)+5/16*a^2*(x+(x^2+a)^(1/2))^(1
+n)/(1+n)+5/32*a*(x+(x^2+a)^(1/2))^(3+n)/(3+n)+1/32*(x+(x^2+a)^(1/2))^(5+n)
/(5+n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2147, 276}

$$\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n \, dx = -\frac{a^5\left(\sqrt{a+x^2}+x\right)^{n-5}}{32(5-n)} - \frac{5a^4\left(\sqrt{a+x^2}+x\right)^{n-3}}{32(3-n)} - \frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n-1}}{16(1-n)} + \frac{5a^2\left(\sqrt{a+x^2}+x\right)^{n+1}}{16(n+1)} + \frac{5a\left(\sqrt{a+x^2}+x\right)^{n+3}}{32(n+3)} + \frac{\left(\sqrt{a+x^2}+x\right)^{n+5}}{32(n+5)}$$

```
[In] Int[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -1/32*(a^5*(x + Sqrt[a + x^2])^(-5 + n))/(5 - n) - (5*a^4*(x + Sqrt[a + x^2
])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x + Sqrt[a + x^2])^(-1 + n))/(16*(1 - n
)) + (5*a^2*(x + Sqrt[a + x^2])^(1 + n))/(16*(1 + n)) + (5*a*(x + Sqrt[a +
x^2])^(3 + n))/(32*(3 + n)) + (x + Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{32}\text{Subst}\left(\int x^{-6+n}\left(a+x^2\right)^5\,dx,x,x+\sqrt{a+x^2}\right)\\
&= \frac{1}{32}\text{Subst}\left(\int \left(a^5x^{-6+n}+5a^4x^{-4+n}+10a^3x^{-2+n}+10a^2x^n+5ax^{2+n}+x^{4+n}\right)\,dx,x,x+\sqrt{a+x^2}\right)\\
&= -\frac{a^5\left(x+\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)}-\frac{5a^4\left(x+\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)}-\frac{5a^3\left(x+\sqrt{a+x^2}\right)^{-1+n}}{16(1-n)}\\
&\quad+\frac{5a^2\left(x+\sqrt{a+x^2}\right)^{1+n}}{16(1+n)}+\frac{5a\left(x+\sqrt{a+x^2}\right)^{3+n}}{32(3+n)}+\frac{\left(x+\sqrt{a+x^2}\right)^{5+n}}{32(5+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.84

$$\int \left(a+x^2\right)^2\left(x+\sqrt{a+x^2}\right)^n\,dx=\frac{1}{32}\left(x+\sqrt{a+x^2}\right)^{-5+n}\left(\frac{a^5}{-5+n}+\frac{5a^4\left(x+\sqrt{a+x^2}\right)^2}{-3+n}+\frac{10a^3\left(x+\sqrt{a+x^2}\right)^4}{-1+n}+\frac{10a^2\left(x+\sqrt{a+x^2}\right)^6}{1+n}+\frac{5a\left(x+\sqrt{a+x^2}\right)^8}{3+n}+\frac{\left(x+\sqrt{a+x^2}\right)^{10}}{5+n}\right)$$

```
[In] Integrate[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-5 + n)*(a^5/(-5 + n) + (5*a^4*(x + Sqrt[a + x^2])^2)
/(-3 + n) + (10*a^3*(x + Sqrt[a + x^2])^4)/(-1 + n) + (10*a^2*(x + Sqrt[a +
 x^2])^6)/(1 + n) + (5*a*(x + Sqrt[a + x^2])^8)/(3 + n) + (x + Sqrt[a + x^2
])^10/(5 + n)))/32
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.96 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.32

method	result
meijerg	$\frac{2^{n} x^{5+n} {}_{3}F_{2}\left(-\frac{n}{2},-\frac{5}{2}-\frac{n}{2},\frac{1}{2}-\frac{n}{2};1-n,-\frac{3}{2}-\frac{n}{2};-\frac{a}{x^{2}}\right)}{5+n}+\frac{2^{1+n} a\, x^{3+n} {}_{3}F_{2}\left(-\frac{n}{2},-\frac{3}{2}-\frac{n}{2},\frac{1}{2}-\frac{n}{2};1-n,-\frac{1}{2}-\frac{n}{2};-\frac{a}{x^{2}}\right)}{3+n}+\frac{a^{\frac{5}{2}+\frac{n}{2}} n\left(\frac{8\sqrt{\pi}\, x^{1+\cdots}}{\cdots}\right.}{\cdots}$

```
[In] int((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 2^n/(5+n)*x^(5+n)*hypergeom([-1/2*n,-5/2-1/2*n,1/2-1/2*n],[1-n,-3/2-1/2*n],
-a/x^2)+2^(1+n)*a/(3+n)*x^(3+n)*hypergeom([-1/2*n,-3/2-1/2*n,1/2-1/2*n],[1-
n,-1/2-1/2*n],-a/x^2)+1/4*a^(5/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(1+n)/n*x^(1
+n)*a^(-1/2-1/2*n)*(a/x^2*n+n-1)/(-2+2*n)*((1+a/x^2)^(1/2)+1)^(-1+n)+4*Pi^(
1/2)/(1+n)/n*x^(1+n)*a^(-1/2-1/2*n)*(1+a/x^2)^(1/2)*((1+a/x^2)^(1/2)+1)^(-1
+n))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96

$$\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n dx = -\frac{\left(5\,(n^4-10\,n^2+9)x^5+10\,(an^4-16\,an^2+15\,a)x^3+5\,(a^2n^4-22\,a^2n^2+45\,a^2)x-(a^2n^5-30\,a^2n^3+(\cdots\right.}{n^6-35\,n^4+259\,n^2-\cdots}$$

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5*(a^2*n^4
- 22*a^2*n^2 + 45*a^2)*x - (a^2*n^5 - 30*a^2*n^3 + (n^5 - 10*n^3 + 9*n)*x^4
 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*x^2)*sqrt(x^2 + a))*(x + sqrt(
x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2 - 225)
```

Sympy [F(-1)]

Timed out.

$$\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n dx = \text{Timed out}$$

```
[In] integrate((x**2+a)**2*(x+(x**2+a)**(1/2))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n dx = \int (x^2+a)^2 \left(x+\sqrt{x^2+a}\right)^n dx$$

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)
```

Giac [F]

$$\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n dx = \int (x^2+a)^2 \left(x+\sqrt{x^2+a}\right)^n dx$$

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n dx = \int (x^2+a)^2 \left(x+\sqrt{x^2+a}\right)^n dx$$

```
[In] int((a + x^2)^2*(x + (a + x^2)^(1/2))^n,x)
[Out] int((a + x^2)^2*(x + (a + x^2)^(1/2))^n, x)
```

3.486 $\int (a+x^2) \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int (a+x^2) \left(x+\sqrt{a+x^2}\right)^n \, dx = -\frac{a^3\left(x+\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)} - \frac{3a^2\left(x+\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)} + \frac{3a\left(x+\sqrt{a+x^2}\right)^{1+n}}{8(1+n)} + \frac{\left(x+\sqrt{a+x^2}\right)^{3+n}}{8(3+n)}$$

```
[Out] -1/8*a^3*(x+(x^2+a)^(1/2))^(-3+n)/(3-n)-3/8*a^2*(x+(x^2+a)^(1/2))^(-1+n)/(1
-n)+3/8*a*(x+(x^2+a)^(1/2))^(1+n)/(1+n)+1/8*(x+(x^2+a)^(1/2))^(3+n)/(3+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2147, 276}

$$\int (a+x^2) \left(x+\sqrt{a+x^2}\right)^n \, dx = -\frac{a^3\left(\sqrt{a+x^2}+x\right)^{n-3}}{8(3-n)} - \frac{3a^2\left(\sqrt{a+x^2}+x\right)^{n-1}}{8(1-n)} + \frac{3a\left(\sqrt{a+x^2}+x\right)^{n+1}}{8(n+1)} + \frac{\left(\sqrt{a+x^2}+x\right)^{n+3}}{8(n+3)}$$

```
[In] Int[(a + x^2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -1/8*(a^3*(x + Sqrt[a + x^2])^(-3 + n))/(3 - n) - (3*a^2*(x + Sqrt[a + x^2]
)^(-1 + n))/(8*(1 - n)) + (3*a*(x + Sqrt[a + x^2])^(1 + n))/(8*(1 + n)) + (
x + Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{8}\text{Subst}\left(\int x^{-4+n}\left(a+x^2\right)^3\,dx,x,x+\sqrt{a+x^2}\right)\\ &= \frac{1}{8}\text{Subst}\left(\int \left(a^3x^{-4+n}+3a^2x^{-2+n}+3ax^n+x^{2+n}\right)\,dx,x,x+\sqrt{a+x^2}\right)\\ &= -\frac{a^3\left(x+\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)}-\frac{3a^2\left(x+\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)}\\ &\quad+\frac{3a\left(x+\sqrt{a+x^2}\right)^{1+n}}{8(1+n)}+\frac{\left(x+\sqrt{a+x^2}\right)^{3+n}}{8(3+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \left(a+x^2\right)\left(x+\sqrt{a+x^2}\right)^n\,dx=\frac{1}{8}\left(x+\sqrt{a+x^2}\right)^{-3+n}\left(\frac{a^3}{-3+n}+\frac{3a^2\left(x+\sqrt{a+x^2}\right)^2}{-1+n}+\frac{3a\left(x+\sqrt{a+x^2}\right)^4}{1+n}+\frac{\left(x+\sqrt{a+x^2}\right)^6}{3+n}\right)$$

```
[In] Integrate[(a + x^2)*(x + Sqrt[a + x^2])^n,x]
[Out] ((x + Sqrt[a + x^2])^(-3 + n)*(a^3/(-3 + n) + (3*a^2*(x + Sqrt[a + x^2])^2)
/(-1 + n) + (3*a*(x + Sqrt[a + x^2])^4)/(1 + n) + (x + Sqrt[a + x^2])^6/(3
+ n)))/8
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.55

method	result
meijerg	$\frac{2^{n} x^{3+n} {}_{3}F_{2}\left(-\frac{n}{2},-\frac{3}{2}-\frac{n}{2},\frac{1}{2}-\frac{n}{2};1-n,-\frac{1}{2}-\frac{n}{2};-\frac{a}{x^{2}}\right)}{3+n}+\frac{a^{\frac{3}{2}+\frac{n}{2}} n \left(\frac{8\sqrt{\pi}\, x^{1+n} a^{-\frac{1}{2}-\frac{n}{2}} \left(\frac{a n}{x^{2}}+n-1\right) \left(\sqrt{1+\frac{a}{x^{2}}}+1\right)^{-1+n}}{(1+n) n (-2+2n)}+\frac{4\sqrt{\pi}\, x^{1+n} a^{-\frac{1}{2}-\frac{n}{2}}}{}\right.}{4\sqrt{\pi}}$

```
[In] int((x^2+a)*(x+(x^2+a)^(1/2))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 2^n/(3+n)*x^(3+n)*hypergeom([-1/2*n,-3/2-1/2*n,1/2-1/2*n],[1-n,-1/2-1/2*n],
-a/x^2)+1/4*a^(3/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(1+n)/n*x^(1+n)*a^(-1/2-1/
2*n)*(a/x^2*n+n-1)/(-2+2*n)*((1+a/x^2)^(1/2)+1)^(-1+n)+4*Pi^(1/2)/(1+n)/n*x
^(1+n)*a^(-1/2-1/2*n)*(1+a/x^2)^(1/2)*((1+a/x^2)^(1/2)+1)^(-1+n))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int (a+x^2) \left(x+\sqrt{a+x^2}\right)^n \, dx = -\frac{\left(3\,(n^2-1)x^3+3\,(an^2-3\,a)x-(an^3+(n^3-n)x^2-7\,an)\sqrt{x^2+a}\right)\left(x+\sqrt{x^2+a}\right)^n}{n^4-10\,n^2+9}$$

```
[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x - (a*n^3 + (n^3 - n)*x^2 - 7*a*n)*sqr
t(x^2 + a))*(x + sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2237 vs. 2(85) = 170.

Time = 14.58 (sec) , antiderivative size = 15302, normalized size of antiderivative = 141.69

$$\int (a+x^2) \left(x+\sqrt{a+x^2}\right)^n \, dx = \text{Too large to display}$$

```
[In] integrate((x**2+a)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] a*Piecewise((2*a**(9/2)*a**(n/2 + 1/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/
sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma
```

(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 -
n/2)) - 2*a**(9/2)*a**(n/2 + 1/2)*n*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(
1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2)
- 2*a**(7/2)*x**2*gamma(1 - n/2)) - 2*a**(7/2)*a**(n/2 + 1/2)*n*x**2*sqrt(a
/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a*
*(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x*
*2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) + 4*a**(7/2)*a**(n/2 +
1/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a
(9/2)*n2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x
2*gamma(1 - n/2) - 2*a(7/2)*x**2*gamma(1 - n/2)) - 2*a**(7/2)*a**(n/2 +
1/2)*n*x**2*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*ga
mma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(
1 - n/2)) - 2*a**(7/2)*a**(n/2 + 1/2)*x**2*sqrt(a/x**2 + 1)*sinh(n*asinh(x/
sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2)
- 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(
7/2)*x**2*gamma(1 - n/2)) + 2*a**(7/2)*a**(n/2 + 1/2)*x**2*cosh(n*asinh(x/s
qrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2)
- 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7
/2)*x**2*gamma(1 - n/2)) - 2*a**(5/2)*a**(n/2 + 1/2)*n*x**4*sqrt(a/x**2 + 1
)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n*
*2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(
1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**(5/2)*a**(n/2 + 1/2)*n*x*
*4*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n
2*gamma(1 - n/2) - 2*a(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma
(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - 2*a**(5/2)*a**(n/2 + 1/2)*x**
4*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/
2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)
*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**(5/2)*a*
*(n/2 + 1/2)*x**4*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2
)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*
n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - a**4*a**(n/2 +
1/2)*n**2*x*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9
/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*
gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) + a**4*a**(n/2 + 1/2)*n*x*
cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a*
*(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x*
*2*gamma(1 - n/2)) - a**3*a**(n/2 + 1/2)*n**2*x**3*sqrt(a/x**2 + 1)*sinh(n*
asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*
gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamm
a(1 - n/2)) + a**3*a**(n/2 + 1/2)*n*x**3*cosh(n*asinh(x/sqrt(a)))*gamma(-n/
2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)
*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)), Abs(x**2/a) >
1), (-a**(5/2)*a**(n/2 + 1/2)*n**2*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a))
)*gamma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2))
+ 2*a**(5/2)*a**(n/2 + 1/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*g

amma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2))
+ 2*a**(3/2)*a**(n/2 + 1/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a
)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n
/2)) + 2*a**(3/2)*a**(n/2 + 1/2)*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqr
t(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1
- n/2)) - 2*a**2*a**(n/2 + 1/2)*n*x*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)
) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a*
*(5/2)*gamma(1 - n/2)) + a**2*a**(n/2 + 1/2)*n*x*cosh(n*asinh(x/sqrt(a)))*g
amma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)) - 2
*a**2*a**(n/2 + 1/2)*x*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)) + asinh(x/s
qrt(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(
1 - n/2)), True)) + Piecewise((4*a**(71/2)*a**(n/2 + 3/2)*n*cosh(n*asinh(x/
sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n
/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a*
*(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) +
18*a**(69/2)*x**2*gamma(1 - n/2)) - 4*a**(71/2)*a**(n/2 + 3/2)*n*gamma(1 -
n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) +
18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**
(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) + 2*a**
(69/2)*a**(n/2 + 3/2)*n**3*x**2*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)
))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamm
a(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 -
n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 -
n/2)) - 4*a**(69/2)*a**(n/2 + 3/2)*n**2*x**2*sqrt(a/x**2 + 1)*sinh(n*asinh(
x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 -
n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*
a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2)
+ 18*a**(69/2)*x**2*gamma(1 - n/2)) + 12*a**(69/2)*a**(n/2 + 3/2)*n**2*x**2
*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*
n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma
(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2*g
amma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) - 12*a**(69/2)*a**(n/2 +
3/2)*n*x**2*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*
gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1
- n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2
) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2
)) + 22*a**(69/2)*a**(n/2 + 3/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x
/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n
2*gamma(1 - n/2) + 18*a(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*ga
mma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*ga
mma(1 - n/2)) - 4*a**(69/2)*a**(n/2 + 3/2)*n*x**2*gamma(1 - n/2)/(2*a**(71/
2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*ga
mma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**
2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) - 6*a**(67/2)*a**(n/2
+ 3/2)*n**3*x**4*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(

a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*ga
mma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1
- n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1
- n/2)) + 12*a**(67/2)*a**(n/2 + 3/2)*n**3*x**4*cosh(n*asinh(x/sqrt(a)) + 3
*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**
(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4
*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)
*x**2*gamma(1 - n/2)) - 26*a**(67/2)*a**(n/2 + 3/2)*n**2*x**4*sqrt(a/x**2 +
 1)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/
2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*ga
mma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**
2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) + 46*a**(67/2)*a**(n/2
 + 3/2)*n**2*x**4*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n
/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 1
8*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(
69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) - 22*a**
(67/2)*a**(n/2 + 3/2)*n*x**4*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + 3*a
sinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(7
1/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x
2*gamma(1 - n/2) - 20*a(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x
2*gamma(1 - n/2)) + 24*a(67/2)*a**(n/2 + 3/2)*n*x**4*cosh(n*asinh(x/sqr
t(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2)
 - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(6
9/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*
a**(69/2)*x**2*gamma(1 - n/2)) + 6*a**(67/2)*a**(n/2 + 3/2)*x**4*sqrt(a/x**
2 + 1)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(
71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)
*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*
x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) - 18*a**(67/2)*a**(
n/2 + 3/2)*x**4*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2
)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*
a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69
/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) - 14*a**(6
5/2)*a**(n/2 + 3/2)*n**3*x**6*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + 3*
asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(
71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*
x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*
x**2*gamma(1 - n/2)) + 18*a**(65/2)*a**(n/2 + 3/2)*n**3*x**6*cosh(n*asinh(x
/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 -
n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a
(69/2)*n4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) +
 18*a**(69/2)*x**2*gamma(1 - n/2)) - 46*a**(65/2)*a**(n/2 + 3/2)*n**2*x**6*
sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/
2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18
*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(6

9/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) + 58*a**(
65/2)*a**(n/2 + 3/2)*n**2*x**6*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a))
)*gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma
(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n
/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n
/2)) - 2*a**(65/2)*a**(n/2 + 3/2)*n*x**6*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sq
rt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2
) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(
69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18
*a**(69/2)*x**2*gamma(1 - n/2)) - 2*a**(65/2)*a**(n/2 + 3/2)*n*x**6*cosh(n*
asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gam
ma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2
) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 -
 n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) + 30*a**(65/2)*a**(n/2 + 3/2)*x**
6*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 -
n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) +
18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**
(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) - 42*a*
*(65/2)*a**(n/2 + 3/2)*x**6*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*g
amma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1
- n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2)
 - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)
) - 8*a**(63/2)*a**(n/2 + 3/2)*n**3*x**8*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sq
rt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2
) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(
69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18
*a**(69/2)*x**2*gamma(1 - n/2)) + 8*a**(63/2)*a**(n/2 + 3/2)*n**3*x**8*cosh
(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*
gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 -
n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(
1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) - 24*a**(63/2)*a**(n/2 + 3/2)*
n**2*x**8*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*ga
mma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 -
 n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2)
- 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2))
 + 24*a**(63/2)*a**(n/2 + 3/2)*n**2*x**8*cosh(n*asinh(x/sqrt(a)) + 3*asinh(
x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*
n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*g
amma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*g
amma(1 - n/2)) + 8*a**(63/2)*a**(n/2 + 3/2)*n*x**8*sqrt(a/x**2 + 1)*sinh(n*
asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gam
ma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2
) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 -
 n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) - 8*a**(63/2)*a**(n/2 + 3/2)*n*x*
*8*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(71/2

)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gam
ma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2
*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) + 24*a**(63/2)*a**(n/2
+ 3/2)*x**8*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*
gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1
 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2
) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2
)) - 24*a**(63/2)*a**(n/2 + 3/2)*x**8*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x/s
qrt(a)))*gamma(1 - n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**
2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamm
a(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamm
a(1 - n/2)) + 2*a**35*a**(n/2 + 3/2)*n**3*x*cosh(n*asinh(x/sqrt(a)))*gamma(
-n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) +
 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a*
*(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) - 2*a*
*35*a**(n/2 + 3/2)*n**2*x*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)))*gamma(-
n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) +
18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**
(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) - a**34
*a**(n/2 + 3/2)*n**4*x**3*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)))*gamma(-
n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) +
18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**
(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) + 5*a**
34*a**(n/2 + 3/2)*n**3*x**3*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(71/
2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*ga
mma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**
2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) - a**34*a**(n/2 + 3/2)
*n**2*x**3*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(71/
2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*ga
mma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**
2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) - 3*a**34*a**(n/2 + 3/
2)*n*x**3*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(71/2)*n**4*gamma(1 -
n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a
(69/2)*n4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) +
 18*a**(69/2)*x**2*gamma(1 - n/2)) - a**33*a**(n/2 + 3/2)*n**4*x**5*sqrt(a/
x**2 + 1)*sinh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(71/2)*n**4*gamma(1 -
n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a
(69/2)*n4*x**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) +
 18*a**(69/2)*x**2*gamma(1 - n/2)) + 3*a**33*a**(n/2 + 3/2)*n**3*x**5*cosh(
n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(7
1/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x
2*gamma(1 - n/2) - 20*a(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x
2*gamma(1 - n/2)) + a33*a**(n/2 + 3/2)*n**2*x**5*sqrt(a/x**2 + 1)*sinh(
n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(7
1/2)*n**2*gamma(1 - n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x

```
**2*gamma(1 - n/2) - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x
**2*gamma(1 - n/2)) - 3*a**33*a**(n/2 + 3/2)*n*x**5*cosh(n*asinh(x/sqrt(a))
)*gamma(-n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1
- n/2) + 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2)
 - 20*a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)
), Abs(x**2/a) > 1), (-2*a**(73/2)*a**(n/2 + 3/2)*n**2*sqrt(1 + x**2/a)*sin
h(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**
(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4
*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)
*x**2*gamma(1 - n/2)) + 4*a**(73/2)*a**(n/2 + 3/2)*n*cosh(n*asinh(x/sqrt(a)
) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 2
0*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)
*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(
71/2)*x**2*gamma(1 - n/2)) - 4*a**(73/2)*a**(n/2 + 3/2)*n*gamma(1 - n/2)/(2
*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(
73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*
n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) - a**(71/2)*a*
*(n/2 + 3/2)*n**4*x**2*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)))*gamma(-n/2
)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*
a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71
/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) + 2*a**(71
/2)*a**(n/2 + 3/2)*n**3*x**2*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*
gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1
 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2
) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2
)) - a**(71/2)*a**(n/2 + 3/2)*n**2*x**2*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqr
t(a)))*gamma(-n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gam
ma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 -
 n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 -
 n/2)) + 12*a**(71/2)*a**(n/2 + 3/2)*n**2*x**2*cosh(n*asinh(x/sqrt(a)) + 3*
asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(
73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*
x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*
x**2*gamma(1 - n/2)) + 22*a**(71/2)*a**(n/2 + 3/2)*n*x**2*cosh(n*asinh(x/sq
rt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2
) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(
71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18
*a**(71/2)*x**2*gamma(1 - n/2)) - 4*a**(71/2)*a**(n/2 + 3/2)*n*x**2*gamma(1
 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2)
 + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*
a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) - a*
*(69/2)*a**(n/2 + 3/2)*n**4*x**4*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)))*
gamma(-n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 -
n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) -
 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2))
```

+ 12*a**(69/2)*a**(n/2 + 3/2)*n**3*x**4*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x
/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n
2*gamma(1 - n/2) + 18*a(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*ga
mma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*ga
mma(1 - n/2)) + a**(69/2)*a**(n/2 + 3/2)*n**2*x**4*sqrt(1 + x**2/a)*sinh(n*
asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/
2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**
2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**
2*gamma(1 - n/2)) + 46*a**(69/2)*a**(n/2 + 3/2)*n**2*x**4*cosh(n*asinh(x/sq
rt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2
) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(
71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18
*a**(71/2)*x**2*gamma(1 - n/2)) + 24*a**(69/2)*a**(n/2 + 3/2)*n*x**4*cosh(n
*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*ga
mma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/
2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1
 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) - 18*a**(69/2)*a**(n/2 + 3/2)*x*
*4*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2
)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gam
ma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2
*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) + 18*a**(67/2)*a**(n/2
 + 3/2)*n**3*x**6*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/
2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18
*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(7
1/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) + 58*a**(
67/2)*a**(n/2 + 3/2)*n**2*x**6*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a))
)*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma
(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n
/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n
/2)) - 2*a**(67/2)*a**(n/2 + 3/2)*n*x**6*cosh(n*asinh(x/sqrt(a)) + 3*asinh(
x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*
n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*g
amma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*g
amma(1 - n/2)) - 42*a**(67/2)*a**(n/2 + 3/2)*x**6*cosh(n*asinh(x/sqrt(a)) +
 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a
(73/2)*n2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n*
*4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/
2)*x**2*gamma(1 - n/2)) + 8*a**(65/2)*a**(n/2 + 3/2)*n**3*x**8*cosh(n*asinh
(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1
 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2
*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2)
 + 18*a**(71/2)*x**2*gamma(1 - n/2)) + 24*a**(65/2)*a**(n/2 + 3/2)*n**2*x**
8*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)
*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamm
a(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*

gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) - 8*a**(65/2)*a**(n/2 + 3/2)*n*x**8*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) - 24*a**(65/2)*a**(n/2 + 3/2)*x**8*cosh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) + 2*a**36*a**(n/2 + 3/2)*n**3*x*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) - 4*a**36*a**(n/2 + 3/2)*n**2*x*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) - 12*a**36*a**(n/2 + 3/2)*n*x*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) - 6*a**35*a**(n/2 + 3/2)*n**3*x**3*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) + 5*a**35*a**(n/2 + 3/2)*n**3*x**3*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) - 26*a**35*a**(n/2 + 3/2)*n**2*x**3*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) - 22*a**35*a**(n/2 + 3/2)*n*x**3*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) - 3*a**35*a**(n/2 + 3/2)*n*x**3*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) + 6*a**35*a**(n/2 + 3/2)*x**3*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma

(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma
(1 - n/2)) - 14*a**34*a**(n/2 + 3/2)*n**3*x**5*sqrt(1 + x**2/a)*sinh(n*asin
h(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1
 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) +
2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2
) + 18*a**(71/2)*x**2*gamma(1 - n/2)) + 3*a**34*a**(n/2 + 3/2)*n**3*x**5*co
sh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a*
*(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**
4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2
)*x**2*gamma(1 - n/2)) - 46*a**34*a**(n/2 + 3/2)*n**2*x**5*sqrt(1 + x**2/a)
*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*
n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma
(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*g
amma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) - 2*a**34*a**(n/2 + 3/2)*
n*x**5*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma
(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/
2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 2
0*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)) -
3*a**34*a**(n/2 + 3/2)*n*x**5*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(7
3/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*
gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x
2*gamma(1 - n/2) + 18*a(71/2)*x**2*gamma(1 - n/2)) + 30*a**34*a**(n/2 +
 3/2)*x**5*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*g
amma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1
 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2)
 - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2)
) - 8*a**33*a**(n/2 + 3/2)*n**3*x**7*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a
)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) -
20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2
)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**
(71/2)*x**2*gamma(1 - n/2)) - 24*a**33*a**(n/2 + 3/2)*n**2*x**7*sqrt(1 + x*
*2/a)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(7
3/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*
gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x
2*gamma(1 - n/2) + 18*a(71/2)*x**2*gamma(1 - n/2)) + 8*a**33*a**(n/2 +
3/2)*n*x**7*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)) + 3*asinh(x/sqrt(a)))*
gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*a**(73/2)*n**2*gamma(1
 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n**4*x**2*gamma(1 - n/2
) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71/2)*x**2*gamma(1 - n/2
)) + 24*a**33*a**(n/2 + 3/2)*x**7*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a))
+ 3*asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(73/2)*n**4*gamma(1 - n/2) - 20*
a**(73/2)*n**2*gamma(1 - n/2) + 18*a**(73/2)*gamma(1 - n/2) + 2*a**(71/2)*n
4*x2*gamma(1 - n/2) - 20*a**(71/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(71
/2)*x**2*gamma(1 - n/2)), True))

Maxima [F]

$$\int (a + x^2) \left(x + \sqrt{a + x^2}\right)^n \, dx = \int (x^2 + a) \left(x + \sqrt{x^2 + a}\right)^n \, dx$$

```
[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

Giac [F]

$$\int (a + x^2) \left(x + \sqrt{a + x^2}\right)^n \, dx = \int (x^2 + a) \left(x + \sqrt{x^2 + a}\right)^n \, dx$$

```
[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + x^2) \left(x + \sqrt{a + x^2}\right)^n \, dx = \int (x^2 + a) \left(x + \sqrt{x^2 + a}\right)^n \, dx$$

```
[In] int((a + x^2)*(x + (a + x^2)^(1/2))^n,x)
[Out] int((a + x^2)*(x + (a + x^2)^(1/2))^n, x)
```

3.487 $\int \left(x+\sqrt{a+x^2}\right)^n dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \left(x+\sqrt{a+x^2}\right)^n dx=-\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)}+\frac{\left(x+\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}$$

```
[Out] -1/2*a*(x+(x^2+a)^(1/2))^(-1+n)/(1-n)+1/2*(x+(x^2+a)^(1/2))^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2142, 14}

$$\int \left(x+\sqrt{a+x^2}\right)^n dx=\frac{\left(\sqrt{a+x^2}+x\right)^{n+1}}{2(n+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{n-1}}{2(1-n)}$$

```
[In] Int[(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -1/2*(a*(x + Sqrt[a + x^2])^(-1 + n))/(1 - n) + (x + Sqrt[a + x^2])^(1 + n)
/(2*(1 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
```

```
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^{-2+n}\left(a+x^2\right)\,dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+n}+x^n\right)\,dx, x, x+\sqrt{a+x^2}\right)\\ &= -\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)}+\frac{\left(x+\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \left(x+\sqrt{a+x^2}\right)^n\,dx = \frac{\left(x+\sqrt{a+x^2}\right)^{-1+n}\left(an+(-1+n)x\left(x+\sqrt{a+x^2}\right)\right)}{-1+n^2}$$

```
[In] Integrate[(x + Sqrt[a + x^2])^n,x]
[Out] ((x + Sqrt[a + x^2])^(-1 + n)*(a*n + (-1 + n)*x*(x + Sqrt[a + x^2])))/(-1 +
 n^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(44) = 88.

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.31

method	result	size
meijerg	$\frac{a^{\frac{1}{2}+\frac{n}{2}} n \left(\frac{8\sqrt{\pi}\, x^{1+n} a^{-\frac{1}{2}-\frac{n}{2}} \left(\frac{a n}{x^2}+n-1\right)\left(\sqrt{1+\frac{a}{x^2}}+1\right)^{-1+n}}{(1+n)n(-2+2n)}+\frac{4\sqrt{\pi}\, x^{1+n} a^{-\frac{1}{2}-\frac{n}{2}} \sqrt{1+\frac{a}{x^2}}\left(\sqrt{1+\frac{a}{x^2}}+1\right)^{-1+n}}{(1+n)n}\right)}{4\sqrt{\pi}}$	120

```
[In] int((x+(x^2+a)^(1/2))^n,x,method=_RETURNVERBOSE)
[Out] 1/4*a^(1/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(1+n)/n*x^(1+n)*a^(-1/2-1/2*n)*(a/
x^2*n+n-1)/(-2+2*n)*((1+a/x^2)^(1/2)+1)^(-1+n)+4*Pi^(1/2)/(1+n)/n*x^(1+n)*a
^(-1/2-1/2*n)*(1+a/x^2)^(1/2)*((1+a/x^2)^(1/2)+1)^(-1+n))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \left(x+\sqrt{a+x^2}\right)^n \, dx = \frac{\left(\sqrt{x^2+a}n-x\right)\left(x+\sqrt{x^2+a}\right)^n}{n^2-1}$$

```
[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
[Out] (sqrt(x^2 + a)*n - x)*(x + sqrt(x^2 + a))^n/(n^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2236 vs. 2(37) = 74.

Time = 1.71 (sec) , antiderivative size = 2236, normalized size of antiderivative = 43.00

$$\int \left(x+\sqrt{a+x^2}\right)^n \, dx = \text{Too large to display}$$

```
[In] integrate((x+(x**2+a)**(1/2))**n,x)
[Out] Piecewise((2*a**(9/2)*a**(n/2 + 1/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/sq
rt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1
 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n
/2)) - 2*a**(9/2)*a**(n/2 + 1/2)*n*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1
- n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) -
2*a**(7/2)*x**2*gamma(1 - n/2)) - 2*a**(7/2)*a**(n/2 + 1/2)*n*x**2*sqrt(a/x
**2 + 1)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(
9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2
*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) + 4*a**(7/2)*a**(n/2 + 1/
2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**
(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**
2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - 2*a**(7/2)*a**(n/2 + 1
/2)*n*x**2*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamm
a(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1
- n/2)) - 2*a**(7/2)*a**(n/2 + 1/2)*x**2*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sq
rt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) -
 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/
2)*x**2*gamma(1 - n/2)) + 2*a**(7/2)*a**(n/2 + 1/2)*x**2*cosh(n*asinh(x/sqr
t(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) -
2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2
)*x**2*gamma(1 - n/2)) - 2*a**(5/2)*a**(n/2 + 1/2)*n*x**4*sqrt(a/x**2 + 1)*
sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2
```

```
*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1
- n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**(5/2)*a**(n/2 + 1/2)*n*x**4
*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**
2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1
 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - 2*a**(5/2)*a**(n/2 + 1/2)*x**4*
sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)
/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n
**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**(5/2)*a**(
n/2 + 1/2)*x**4*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/
(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n*
*2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - a**4*a**(n/2 + 1
/2)*n**2*x*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2
)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*ga
mma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) + a**4*a**(n/2 + 1/2)*n*x*co
sh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(
9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2
*gamma(1 - n/2)) - a**3*a**(n/2 + 1/2)*n**2*x**3*sqrt(a/x**2 + 1)*sinh(n*as
inh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*ga
mma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(
1 - n/2)) + a**3*a**(n/2 + 1/2)*n*x**3*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)
/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n
**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)), Abs(x**2/a) > 1)
, (-a**(5/2)*a**(n/2 + 1/2)*n**2*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)))*
gamma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)) +
2*a**(5/2)*a**(n/2 + 1/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gam
ma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)) +
2*a**(3/2)*a**(n/2 + 1/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a))
)*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2
)) + 2*a**(3/2)*a**(n/2 + 1/2)*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(
a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 -
n/2)) - 2*a**2*a**(n/2 + 1/2)*n*x*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a))
+ asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(
5/2)*gamma(1 - n/2)) + a**2*a**(n/2 + 1/2)*n*x*cosh(n*asinh(x/sqrt(a)))*gam
ma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)) - 2*a
**2*a**(n/2 + 1/2)*x*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqr
t(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1
- n/2)), True))
```

Maxima [F]

$$\int \left(x+\sqrt{a+x^2}\right)^n dx = \int \left(x+\sqrt{x^2+a}\right)^n dx$$

```
[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + a))^n, x)
```

Giac [F]

$$\int \left(x+\sqrt{a+x^2}\right)^n dx = \int \left(x+\sqrt{x^2+a}\right)^n dx$$

```
[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(x+\sqrt{a+x^2}\right)^n dx = \int \left(x+\sqrt{x^2+a}\right)^n dx$$

```
[In] int((x + (a + x^2)^(1/2))^n,x)
[Out] int((x + (a + x^2)^(1/2))^n, x)
```

3.488 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2} \, dx = \frac{2\left(x+\sqrt{a+x^2}\right)^{1+n} \text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

```
[Out] 2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^(
1/2))^(1+n)/a/(1+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2147, 371}

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2} \, dx = \frac{2\left(\sqrt{a+x^2}+x\right)^{n+1} \text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2),x]
```

```
[Out] (2*(x + Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -
((x + Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^n}{a+x^2}\,dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{2\left(x+\sqrt{a+x^2}\right)^{1+n}\ {}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \frac{2\left(x+\sqrt{a+x^2}\right)^{1+n}\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},1+\frac{1+n}{2},-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

```
[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2),x]
[Out] (2*(x + Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/
2, -((x + Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Maple [F]

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}dx$$

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a),x)
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a),x)
```

Fricas [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="fricas")
[Out] integral((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

Sympy [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$$

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a),x)
[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2), x)
```

Maxima [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

Giac [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2),x)
[Out] int((x + (a + x^2)^(1/2))^n/(a + x^2), x)
```

3.489 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2} dx = \frac{8\left(x+\sqrt{a+x^2}\right)^{3+n} \text{Hypergeometric2F1}\left(3,\frac{3+n}{2},\frac{5+n}{2},-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

```
[Out] 8*hypergeom([3, 3/2+1/2*n],[5/2+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^(
1/2))^(3+n)/a^3/(3+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2147, 371}

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2} dx = \frac{8\left(\sqrt{a+x^2}+x\right)^{n+3} \text{Hypergeometric2F1}\left(3,\frac{n+3}{2},\frac{n+5}{2},-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x + Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5 + n)/2, -
((x + Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\text{integral} = 8\text{Subst}\left(\int \frac{x^{2+n}}{(a+x^2)^3}\,dx, x, x+\sqrt{a+x^2}\right)$$

$$= \frac{8\left(x+\sqrt{a+x^2}\right)^{3+n} {}_2F_1\left(3, \frac{3+n}{2}; \frac{5+n}{2}; -\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^2}\,dx$$

$$= \frac{8\left(x+\sqrt{a+x^2}\right)^{3+n} \text{Hypergeometric2F1}\left(3, \frac{3+n}{2}, 1+\frac{3+n}{2}, -\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

```
[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x + Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, 1 + (3 + n)/
2, -((x + Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Maple [F]

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}dx$$

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x)
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="fricas")
[Out] integral((x + sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx$$

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**2,x)
[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2)**2, x)
```

Maxima [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^2}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^2}\,dx$$

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^2}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^2}\,dx$$

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2)^2,x)
[Out] int((x + (a + x^2)^(1/2))^n/(a + x^2)^2, x)
```

3.490 $\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal result

Integrand size = 23, antiderivative size = 176

$$\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n \, dx = -\frac{a^5\left(x-\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)} - \frac{5a^4\left(x-\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)} - \frac{5a^3\left(x-\sqrt{a+x^2}\right)^{-1+n}}{16(1-n)} + \frac{5a^2\left(x-\sqrt{a+x^2}\right)^{1+n}}{16(1+n)} + \frac{5a\left(x-\sqrt{a+x^2}\right)^{3+n}}{32(3+n)} + \frac{\left(x-\sqrt{a+x^2}\right)^{5+n}}{32(5+n)}$$

```
[Out] -1/32*a^5*(x-(x^2+a)^(1/2))^(-5+n)/(5-n)-5/32*a^4*(x-(x^2+a)^(1/2))^(-3+n)/
(3-n)-5/16*a^3*(x-(x^2+a)^(1/2))^(-1+n)/(1-n)+5/16*a^2*(x-(x^2+a)^(1/2))^(1
+n)/(1+n)+5/32*a*(x-(x^2+a)^(1/2))^(3+n)/(3+n)+1/32*(x-(x^2+a)^(1/2))^(5+n)
/(5+n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2147, 276}

$$\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n \, dx = -\frac{a^5\left(x-\sqrt{a+x^2}\right)^{n-5}}{32(5-n)} - \frac{5a^4\left(x-\sqrt{a+x^2}\right)^{n-3}}{32(3-n)} - \frac{5a^3\left(x-\sqrt{a+x^2}\right)^{n-1}}{16(1-n)} + \frac{5a^2\left(x-\sqrt{a+x^2}\right)^{n+1}}{16(n+1)} + \frac{5a\left(x-\sqrt{a+x^2}\right)^{n+3}}{32(n+3)} + \frac{\left(x-\sqrt{a+x^2}\right)^{n+5}}{32(n+5)}$$

```
[In] Int[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -1/32*(a^5*(x - Sqrt[a + x^2])^(-5 + n))/(5 - n) - (5*a^4*(x - Sqrt[a + x^2
])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x - Sqrt[a + x^2])^(-1 + n))/(16*(1 - n
)) + (5*a^2*(x - Sqrt[a + x^2])^(1 + n))/(16*(1 + n)) + (5*a*(x - Sqrt[a +
x^2])^(3 + n))/(32*(3 + n)) + (x - Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{32}\text{Subst}\left(\int x^{-6+n}\left(a+x^2\right)^5\,dx, x, x-\sqrt{a+x^2}\right)$$
$$= \frac{1}{32}\text{Subst}\left(\int \left(a^5x^{-6+n}+5a^4x^{-4+n}+10a^3x^{-2+n}+10a^2x^n+5ax^{2+n}+x^{4+n}\right)dx, x, x-\sqrt{a+x^2}\right)$$
$$= -\frac{a^5\left(x-\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)}-\frac{5a^4\left(x-\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{-1+n}}{16(1-n)}+\frac{5a^2\left(x-\sqrt{a+x^2}\right)^{1+n}}{16(1+n)}+\frac{5a\left(x-\sqrt{a+x^2}\right)^{3+n}}{32(3+n)}+\frac{\left(x-\sqrt{a+x^2}\right)^{5+n}}{32(5+n)}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.85

$$\int \left(a+x^2\right)^2\left(x-\sqrt{a+x^2}\right)^n dx = \frac{1}{32}\left(x-\sqrt{a+x^2}\right)^{-5+n}\left(\frac{a^5}{-5+n}+\frac{5a^4\left(x-\sqrt{a+x^2}\right)^2}{-3+n}+\frac{10a^3\left(x-\sqrt{a+x^2}\right)^4}{-1+n}+\frac{10a^2\left(x-\sqrt{a+x^2}\right)^6}{1+n}+\frac{5a\left(x-\sqrt{a+x^2}\right)^8}{3+n}+\frac{\left(x-\sqrt{a+x^2}\right)^{10}}{5+n}\right)$$

```
[In] Integrate[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^(-5 + n)*(a^5/(-5 + n) + (5*a^4*(x - Sqrt[a + x^2])^2)
/(-3 + n) + (10*a^3*(x - Sqrt[a + x^2])^4)/(-1 + n) + (10*a^2*(x - Sqrt[a +
 x^2])^6)/(1 + n) + (5*a*(x - Sqrt[a + x^2])^8)/(3 + n) + (x - Sqrt[a + x^2
])^10/(5 + n)))/32
```

Maple [F]

$$\int \left(x^2+a\right)^2 \left(x-\sqrt{x^2+a}\right)^n dx$$

```
[In] int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

$$\int \left(a+x^2\right)^2 \left(x-\sqrt{a+x^2}\right)^n dx =$$

$$-\frac{(5\,(n^4-10\,n^2+9)x^5+10\,(an^4-16\,an^2+15\,a)x^3+5\,(a^2n^4-22\,a^2n^2+45\,a^2)x+(a^2n^5-30\,a^2n^3+(}{n^6-35\,n^4+259\,n^2-}$$

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5*(a^2*n^4
- 22*a^2*n^2 + 45*a^2)*x + (a^2*n^5 - 30*a^2*n^3 + (n^5 - 10*n^3 + 9*n)*x^4
 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*x^2)*sqrt(x^2 + a))*(x - sqrt(
x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2 - 225)
```

Sympy [F]

$$\int \left(a+x^2\right)^2 \left(x-\sqrt{a+x^2}\right)^n dx = \int \left(a+x^2\right)^2 \left(x-\sqrt{a+x^2}\right)^n dx$$

```
[In] integrate((x**2+a)**2*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**2*(x - sqrt(a + x**2))**n, x)
```

Maxima [F]

$$\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n dx = \int (x^2+a)^2 \left(x-\sqrt{x^2+a}\right)^n dx$$

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

Giac [F]

$$\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n dx = \int (x^2+a)^2 \left(x-\sqrt{x^2+a}\right)^n dx$$

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n dx = \int \left(x-\sqrt{x^2+a}\right)^n (x^2+a)^2 dx$$

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^2,x)
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^2, x)
```

3.491 $\int (a+x^2) \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int (a+x^2) \left(x-\sqrt{a+x^2}\right)^n \, dx = -\frac{a^3\left(x-\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)} - \frac{3a^2\left(x-\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)} + \frac{3a\left(x-\sqrt{a+x^2}\right)^{1+n}}{8(1+n)} + \frac{\left(x-\sqrt{a+x^2}\right)^{3+n}}{8(3+n)}$$

```
[Out] -1/8*a^3*(x-(x^2+a)^(1/2))^(-3+n)/(3-n)-3/8*a^2*(x-(x^2+a)^(1/2))^(-1+n)/(1
-n)+3/8*a*(x-(x^2+a)^(1/2))^(1+n)/(1+n)+1/8*(x-(x^2+a)^(1/2))^(3+n)/(3+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2147, 276}

$$\int (a+x^2) \left(x-\sqrt{a+x^2}\right)^n \, dx = -\frac{a^3\left(x-\sqrt{a+x^2}\right)^{n-3}}{8(3-n)} - \frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n-1}}{8(1-n)} + \frac{3a\left(x-\sqrt{a+x^2}\right)^{n+1}}{8(n+1)} + \frac{\left(x-\sqrt{a+x^2}\right)^{n+3}}{8(n+3)}$$

```
[In] Int[(a + x^2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -1/8*(a^3*(x - Sqrt[a + x^2])^(-3 + n))/(3 - n) - (3*a^2*(x - Sqrt[a + x^2]
)^(-1 + n))/(8*(1 - n)) + (3*a*(x - Sqrt[a + x^2])^(1 + n))/(8*(1 + n)) + (
x - Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{8}\text{Subst}\left(\int x^{-4+n}\left(a+x^2\right)^3\,dx, x, x-\sqrt{a+x^2}\right)\\
&= \frac{1}{8}\text{Subst}\left(\int \left(a^3x^{-4+n}+3a^2x^{-2+n}+3ax^n+x^{2+n}\right)\,dx, x, x-\sqrt{a+x^2}\right)\\
&= -\frac{a^3\left(x-\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)}\\
&\quad+\frac{3a\left(x-\sqrt{a+x^2}\right)^{1+n}}{8(1+n)}+\frac{\left(x-\sqrt{a+x^2}\right)^{3+n}}{8(3+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$
\int \left(a+x^2\right)\left(x-\sqrt{a+x^2}\right)^n\,dx = \frac{1}{8}\left(x-\sqrt{a+x^2}\right)^{-3+n}\left(\frac{a^3}{-3+n}+\frac{3a^2\left(x-\sqrt{a+x^2}\right)^2}{-1+n}+\frac{3a\left(x-\sqrt{a+x^2}\right)^4}{1+n}+\frac{\left(x-\sqrt{a+x^2}\right)^6}{3+n}\right)
$$

```
[In] Integrate[(a + x^2)*(x - Sqrt[a + x^2])^n,x]
[Out] ((x - Sqrt[a + x^2])^(-3 + n)*(a^3/(-3 + n) + (3*a^2*(x - Sqrt[a + x^2])^2)
/(-1 + n) + (3*a*(x - Sqrt[a + x^2])^4)/(1 + n) + (x - Sqrt[a + x^2])^6/(3
+ n)))/8
```

Maple [F]

$$\int \left(x^2+a\right)\left(x-\sqrt{x^2+a}\right)^n dx$$

```
[In] int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)
[Out] int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \left(a+x^2\right)\left(x-\sqrt{a+x^2}\right)^n dx = -\frac{\left(3\left(n^2-1\right)x^3+3\left(an^2-3a\right)x+\left(an^3+\left(n^3-n\right)x^2-7an\right)\sqrt{x^2+a}\right)\left(x-\sqrt{x^2+a}\right)^n}{n^4-10n^2+9}$$

```
[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
[Out] -(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x + (a*n^3 + (n^3 - n)*x^2 - 7*a*n)*sqr
t(x^2 + a))*(x - sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)
```

Sympy [F]

$$\int \left(a+x^2\right)\left(x-\sqrt{a+x^2}\right)^n dx = \int \left(a+x^2\right)\left(x-\sqrt{a+x^2}\right)^n dx$$

```
[In] integrate((x**2+a)*(x-(x**2+a)**(1/2))**n,x)
[Out] Integral((a + x**2)*(x - sqrt(a + x**2))**n, x)
```

Maxima [F]

$$\int \left(a+x^2\right)\left(x-\sqrt{a+x^2}\right)^n dx = \int \left(x^2+a\right)\left(x-\sqrt{x^2+a}\right)^n dx$$

```
[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

Giac [F]

$$\int (a+x^2)\left(x-\sqrt{a+x^2}\right)^n\,dx=\int (x^2+a)\left(x-\sqrt{x^2+a}\right)^n\,dx$$

```
[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+x^2)\left(x-\sqrt{a+x^2}\right)^n\,dx=\int \left(x-\sqrt{x^2+a}\right)^n\,(x^2+a)\,dx$$

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2),x)
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2), x)
```

3.492 $\int \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \left(x-\sqrt{a+x^2}\right)^n \, dx = -\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)}+\frac{\left(x-\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}$$

```
[Out] -1/2*a*(x-(x^2+a)^(1/2))^(-1+n)/(1-n)+1/2*(x-(x^2+a)^(1/2))^(1+n)/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2142, 14}

$$\int \left(x-\sqrt{a+x^2}\right)^n \, dx = \frac{\left(x-\sqrt{a+x^2}\right)^{n+1}}{2(n+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n-1}}{2(1-n)}$$

```
[In] Int[(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -1/2*(a*(x - Sqrt[a + x^2])^(-1 + n))/(1 - n) + (x - Sqrt[a + x^2])^(1 + n)
/(2*(1 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
```

```
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^{-2+n}\left(a+x^2\right)\,dx, x, x-\sqrt{a+x^2}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+n}+x^n\right)\,dx, x, x-\sqrt{a+x^2}\right)\\ &= -\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)}+\frac{\left(x-\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \left(x-\sqrt{a+x^2}\right)^n\,dx = \frac{1}{2}\left(x-\sqrt{a+x^2}\right)^{-1+n}\left(\frac{a}{-1+n}+\frac{\left(x-\sqrt{a+x^2}\right)^2}{1+n}\right)$$

```
[In] Integrate[(x - Sqrt[a + x^2])^n,x]
[Out] ((x - Sqrt[a + x^2])^(-1 + n)*(a/(-1 + n) + (x - Sqrt[a + x^2])^2/(1 + n)))
/2
```

Maple [F]

$$\int \left(x-\sqrt{x^2+a}\right)^n dx$$

```
[In] int((x-(x^2+a)^(1/2))^n,x)
[Out] int((x-(x^2+a)^(1/2))^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.59

$$\int \left(x-\sqrt{a+x^2}\right)^n\,dx = -\frac{\left(\sqrt{x^2+a}n+x\right)\left(x-\sqrt{x^2+a}\right)^n}{n^2-1}$$

```
[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
[Out] -(sqrt(x^2 + a)*n + x)*(x - sqrt(x^2 + a))^n/(n^2 - 1)
```

Sympy [F]

$$\int \left(x-\sqrt{a+x^2}\right)^n\,dx=\int \left(x-\sqrt{a+x^2}\right)^n\,dx$$

```
[In] integrate((x-(x**2+a)**(1/2))**n,x)
[Out] Integral((x - sqrt(a + x**2))**n, x)
```

Maxima [F]

$$\int \left(x-\sqrt{a+x^2}\right)^n\,dx=\int \left(x-\sqrt{x^2+a}\right)^n\,dx$$

```
[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^n, x)
```

Giac [F]

$$\int \left(x-\sqrt{a+x^2}\right)^n\,dx=\int \left(x-\sqrt{x^2+a}\right)^n\,dx$$

```
[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(x-\sqrt{a+x^2}\right)^n\,dx=\int \left(x-\sqrt{x^2+a}\right)^n\,dx$$

```
[In] int((x - (a + x^2)^(1/2))^n,x)
[Out] int((x - (a + x^2)^(1/2))^n, x)
```

3.493 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2} \, dx = \frac{2\left(x-\sqrt{a+x^2}\right)^{1+n} \text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

```
[Out] 2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^(
1/2))^(1+n)/a/(1+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2147, 371}

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2} \, dx = \frac{2\left(x-\sqrt{a+x^2}\right)^{n+1} \text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

```
[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2),x]
```

```
[Out] (2*(x - Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -
((x - Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^n}{a+x^2}\,dx, x, x-\sqrt{a+x^2}\right)$$
$$= \frac{2\left(x-\sqrt{a+x^2}\right)^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$$
$$= \frac{2\left(x-\sqrt{a+x^2}\right)^{1+n}\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},1+\frac{1+n}{2},-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2),x]
[Out] (2*(x - Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/
2, -((x - Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Maple [F]

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}dx$$

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a),x)
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a),x)
```

Fricas [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="fricas")
[Out] integral((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

Sympy [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$$

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a),x)
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2), x)
```

Maxima [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

Giac [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2),x)
[Out] int((x - (a + x^2)^(1/2))^n/(a + x^2), x)
```

3.494 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^2}\,dx = \frac{8\left(x-\sqrt{a+x^2}\right)^{3+n}\text{Hypergeometric2F1}\left(3,\frac{3+n}{2},\frac{5+n}{2},-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

```
[Out] 8*hypergeom([3, 3/2+1/2*n],[5/2+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^(
1/2))^(3+n)/a^3/(3+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2147, 371}

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^2}\,dx = \frac{8\left(x-\sqrt{a+x^2}\right)^{n+3}\text{Hypergeometric2F1}\left(3,\frac{n+3}{2},\frac{n+5}{2},-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

```
[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x - Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5 + n)/2, -
((x - Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\text{integral} = 8\text{Subst}\left(\int \frac{x^{2+n}}{(a+x^2)^3}\,dx, x, x-\sqrt{a+x^2}\right)$$

$$= \frac{8\left(x-\sqrt{a+x^2}\right)^{3+n}\ {}_2F_1\left(3,\frac{3+n}{2};\frac{5+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^2}\,dx = \frac{8\left(x-\sqrt{a+x^2}\right)^{3+n}\text{Hypergeometric2F1}\left(3,\frac{3+n}{2},1+\frac{3+n}{2},-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^2,x]
[Out] (8*(x - Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, 1 + (3 + n)/
2, -((x - Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Maple [F]

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}dx$$

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x)
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="fricas")
[Out] integral((x - sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx$$

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**2,x)
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2)**2, x)
```

Maxima [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2)^2,x)
[Out] int((x - (a + x^2)^(1/2))^n/(a + x^2)^2, x)
```

3.495 $\int (a+x^2)^{5/2} \left(x+\sqrt{a+x^2}\right)^n dx$

Optimal result

Integrand size = 23, antiderivative size = 187

$$\int (a+x^2)^{5/2} \left(x+\sqrt{a+x^2}\right)^n dx = -\frac{a^6\left(x+\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)} - \frac{3a^5\left(x+\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)} - \frac{15a^4\left(x+\sqrt{a+x^2}\right)^{-2+n}}{64(2-n)} + \frac{5a^3\left(x+\sqrt{a+x^2}\right)^{n}}{16n} + \frac{15a^2\left(x+\sqrt{a+x^2}\right)^{2+n}}{64(2+n)} + \frac{3a\left(x+\sqrt{a+x^2}\right)^{4+n}}{32(4+n)} + \frac{\left(x+\sqrt{a+x^2}\right)^{6+n}}{64(6+n)}$$

```
[Out] -1/64*a^6*(x+(x^2+a)^(1/2))^(-6+n)/(6-n)-3/32*a^5*(x+(x^2+a)^(1/2))^(-4+n)/
(4-n)-15/64*a^4*(x+(x^2+a)^(1/2))^(-2+n)/(2-n)+5/16*a^3*(x+(x^2+a)^(1/2))^n
/n+15/64*a^2*(x+(x^2+a)^(1/2))^(2+n)/(2+n)+3/32*a*(x+(x^2+a)^(1/2))^(4+n)/(
4+n)+1/64*(x+(x^2+a)^(1/2))^(6+n)/(6+n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2147, 276}

$$\int (a+x^2)^{5/2} \left(x+\sqrt{a+x^2}\right)^n dx = -\frac{a^6\left(\sqrt{a+x^2}+x\right)^{n-6}}{64(6-n)} - \frac{3a^5\left(\sqrt{a+x^2}+x\right)^{n-4}}{32(4-n)} - \frac{15a^4\left(\sqrt{a+x^2}+x\right)^{n-2}}{64(2-n)} + \frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n}}{16n} + \frac{15a^2\left(\sqrt{a+x^2}+x\right)^{n+2}}{64(n+2)} + \frac{3a\left(\sqrt{a+x^2}+x\right)^{n+4}}{32(n+4)} + \frac{\left(\sqrt{a+x^2}+x\right)^{n+6}}{64(n+6)}$$

```
[In] Int[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -1/64*(a^6*(x + Sqrt[a + x^2])^(-6 + n))/(6 - n) - (3*a^5*(x + Sqrt[a + x^2
])^(-4 + n))/(32*(4 - n)) - (15*a^4*(x + Sqrt[a + x^2])^(-2 + n))/(64*(2 -
n)) + (5*a^3*(x + Sqrt[a + x^2])^n)/(16*n) + (15*a^2*(x + Sqrt[a + x^2])^(2
 + n))/(64*(2 + n)) + (3*a*(x + Sqrt[a + x^2])^(4 + n))/(32*(4 + n)) + (x +
 Sqrt[a + x^2])^(6 + n)/(64*(6 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{64}\text{Subst}\left(\int x^{-7+n}\left(a+x^2\right)^6\,dx,x,x+\sqrt{a+x^2}\right)\\
&= \frac{1}{64}\text{Subst}\left(\int \left(a^6x^{-7+n}+6a^5x^{-5+n}+15a^4x^{-3+n}+20a^3x^{-1+n}+15a^2x^{1+n}\right.\right.\\
&\qquad\left.\left.+6ax^{3+n}+x^{5+n}\right)\,dx,x,x+\sqrt{a+x^2}\right)\\
&= -\frac{a^6\left(x+\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)}-\frac{3a^5\left(x+\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)}\\
&\quad-\frac{15a^4\left(x+\sqrt{a+x^2}\right)^{-2+n}}{64(2-n)}+\frac{5a^3\left(x+\sqrt{a+x^2}\right)^{n}}{16n}\\
&\quad+\frac{15a^2\left(x+\sqrt{a+x^2}\right)^{2+n}}{64(2+n)}+\frac{3a\left(x+\sqrt{a+x^2}\right)^{4+n}}{32(4+n)}+\frac{\left(x+\sqrt{a+x^2}\right)^{6+n}}{64(6+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.84

$$\int \left(a+x^2\right)^{5/2} \left(x+\sqrt{a+x^2}\right)^n \, dx = \frac{1}{64}\left(x+\sqrt{a+x^2}\right)^n \left(\frac{20a^3}{n} + \frac{a^6}{(-6+n)\left(x+\sqrt{a+x^2}\right)^6} + \frac{6a^5}{(-4+n)\left(x+\sqrt{a+x^2}\right)^4} + \frac{15a^4}{(-2+n)\left(x+\sqrt{a+x^2}\right)^2} + \frac{15a^2\left(x+\sqrt{a+x^2}\right)^2}{2+n} + \frac{6a\left(x+\sqrt{a+x^2}\right)^4}{4+n} + \frac{\left(x+\sqrt{a+x^2}\right)^6}{6+n}\right)$$

```
[In] Integrate[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^n*((20*a^3)/n + a^6/((-6 + n)*(x + Sqrt[a + x^2])^6) +
 (6*a^5)/((-4 + n)*(x + Sqrt[a + x^2])^4) + (15*a^4)/((-2 + n)*(x + Sqrt[a
+ x^2])^2) + (15*a^2*(x + Sqrt[a + x^2])^2)/(2 + n) + (6*a*(x + Sqrt[a + x^
2])^4)/(4 + n) + (x + Sqrt[a + x^2])^6/(6 + n)))/64
```

Maple [F]

$$\int \left(x^2+a\right)^{\frac{5}{2}} \left(x+\sqrt{x^2+a}\right)^n dx$$

```
[In] int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.07

$$\int \left(a+x^2\right)^{5/2} \left(x+\sqrt{a+x^2}\right)^n \, dx = \frac{\left(a^3n^6 - 50\,a^3n^4 + (n^6 - 20\,n^4 + 64\,n^2)x^6 + 544\,a^3n^2 + 3\,(an^6 - 30\,an^4 + 104\,an^2)x^4 - 7}{}$$

```
[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^2 + 3*(a*n^
6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 - 40*a^2*n^4 + 264*a^2
*n^2)*x^2 - 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2*(a*n^5 - 30*a*n^3 + 104*a*n)*x
^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a^2*n)*x)*sqrt(x^2 + a))*(x + sqrt(x^2 + a
))^n/(n^7 - 56*n^5 + 784*n^3 - 2304*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \left(a+x^2\right)^{5/2}\left(x+\sqrt{a+x^2}\right)^n\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((x**2+a)**(5/2)*(x+(x**2+a)**(1/2))**n,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \left(a+x^2\right)^{5/2}\left(x+\sqrt{a+x^2}\right)^n\,dx=\int \left(x^2+a\right)^{\frac{5}{2}}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

```
[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)
```

Giac [F]

$$\int \left(a+x^2\right)^{5/2}\left(x+\sqrt{a+x^2}\right)^n\,dx=\int \left(x^2+a\right)^{\frac{5}{2}}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

```
[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+x^2\right)^{5/2}\left(x+\sqrt{a+x^2}\right)^n\,dx=\int \left(x^2+a\right)^{5/2}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

```
[In] int((a + x^2)^(5/2)*(x + (a + x^2)^(1/2))^n,x)
[Out] int((a + x^2)^(5/2)*(x + (a + x^2)^(1/2))^n, x)
```

3.496 $\int (a+x^2)^{3/2} \left(x+\sqrt{a+x^2}\right)^n dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int (a+x^2)^{3/2} \left(x+\sqrt{a+x^2}\right)^n dx = -\frac{a^4\left(x+\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)} - \frac{a^3\left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)} + \frac{3a^2\left(x+\sqrt{a+x^2}\right)^n}{8n} + \frac{a\left(x+\sqrt{a+x^2}\right)^{2+n}}{4(2+n)} + \frac{\left(x+\sqrt{a+x^2}\right)^{4+n}}{16(4+n)}$$

```
[Out] -1/16*a^4*(x+(x^2+a)^(1/2))^(-4+n)/(4-n)-1/4*a^3*(x+(x^2+a)^(1/2))^(-2+n)/(
2-n)+3/8*a^2*(x+(x^2+a)^(1/2))^n/n+1/4*a*(x+(x^2+a)^(1/2))^(2+n)/(2+n)+1/16
*(x+(x^2+a)^(1/2))^(4+n)/(4+n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2147, 276}

$$\int (a+x^2)^{3/2} \left(x+\sqrt{a+x^2}\right)^n dx = -\frac{a^4\left(\sqrt{a+x^2}+x\right)^{n-4}}{16(4-n)} - \frac{a^3\left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)} + \frac{3a^2\left(\sqrt{a+x^2}+x\right)^n}{8n} + \frac{a\left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)} + \frac{\left(\sqrt{a+x^2}+x\right)^{n+4}}{16(n+4)}$$

```
[In] Int[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -1/16*(a^4*(x + Sqrt[a + x^2])^(-4 + n))/(4 - n) - (a^3*(x + Sqrt[a + x^2])
^(-2 + n))/(4*(2 - n)) + (3*a^2*(x + Sqrt[a + x^2])^n)/(8*n) + (a*(x + Sqrt
[a + x^2])^(2 + n))/(4*(2 + n)) + (x + Sqrt[a + x^2])^(4 + n)/(16*(4 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{16}\text{Subst}\left(\int x^{-5+n}\left(a+x^2\right)^4\,dx,x,x+\sqrt{a+x^2}\right)\\
&= \frac{1}{16}\text{Subst}\left(\int \left(a^4x^{-5+n}+4a^3x^{-3+n}+6a^2x^{-1+n}+4ax^{1+n}+x^{3+n}\right)\,dx,x,x+\sqrt{a+x^2}\right)\\
&= -\frac{a^4\left(x+\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)}-\frac{a^3\left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}\\
&\quad+\frac{3a^2\left(x+\sqrt{a+x^2}\right)^{n}}{8n}+\frac{a\left(x+\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}+\frac{\left(x+\sqrt{a+x^2}\right)^{4+n}}{16(4+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\begin{aligned}
\int \left(a+x^2\right)^{3/2}\left(x+\sqrt{a+x^2}\right)^n\,dx &= \frac{1}{16}\left(x+\sqrt{a+x^2}\right)^n\left(\frac{6a^2}{n}\right.\\
&\quad+\frac{a^4}{(-4+n)\left(x+\sqrt{a+x^2}\right)^4}+\frac{4a^3}{(-2+n)\left(x+\sqrt{a+x^2}\right)^2}\\
&\quad\left.+\frac{4a\left(x+\sqrt{a+x^2}\right)^2}{2+n}+\frac{\left(x+\sqrt{a+x^2}\right)^4}{4+n}\right)
\end{aligned}$$

```
[In] Integrate[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^n*((6*a^2)/n + a^4/((-4 + n)*(x + Sqrt[a + x^2])^4) +
(4*a^3)/((-2 + n)*(x + Sqrt[a + x^2])^2) + (4*a*(x + Sqrt[a + x^2])^2)/(2 +
 n) + (x + Sqrt[a + x^2])^4/(4 + n)))/16
```

Maple [F]

$$\int \left(x^{2}+a\right)^{\frac{3}{2}} \left(x+\sqrt{x^{2}+a}\right)^{n}dx$$

```
[In] int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)
[Out] int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

$$\int (a+x^2)^{3/2} \left(x+\sqrt{a+x^2}\right)^n dx = \frac{(a^2n^4 + (n^4 - 4\,n^2)x^4 - 16\,a^2n^2 + 2\,(an^4 - 10\,an^2)x^2 + 24\,a^2 - 4\,((n^3 - 4\,n)x^3 + (an^3 -}{n^5 - 20\,n^3 + 64\,n}$$

```
[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
[Out] (a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)*x^2 + 24*a
^2 - 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2 + a))*(x + sqrt(x^2
+ a))^n/(n^5 - 20*n^3 + 64*n)
```

Sympy [F]

$$\int (a+x^2)^{3/2} \left(x+\sqrt{a+x^2}\right)^n dx = \int \left(a+x^{2}\right)^{\frac{3}{2}} \left(x+\sqrt{a+x^{2}}\right)^{n}\, dx$$

```
[In] integrate((x**2+a)**(3/2)*(x+(x**2+a)**(1/2))**n,x)
[Out] Integral((a + x**2)**(3/2)*(x + sqrt(a + x**2))**n, x)
```

Maxima [F]

$$\int (a+x^2)^{3/2} \left(x+\sqrt{a+x^2}\right)^n dx = \int \left(x^{2}+a\right)^{\frac{3}{2}} \left(x+\sqrt{x^{2}+a}\right)^{n}dx$$

```
[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)
```

Giac [F]

$$\int \left(a+x^2\right)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx = \int \left(x^2+a\right)^{\frac{3}{2}} \left(x+\sqrt{x^2+a}\right)^n \, dx$$

```
[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+x^2\right)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx = \int \left(x^2+a\right)^{3/2} \left(x+\sqrt{x^2+a}\right)^n \, dx$$

```
[In] int((a + x^2)^(3/2)*(x + (a + x^2)^(1/2))^n,x)
[Out] int((a + x^2)^(3/2)*(x + (a + x^2)^(1/2))^n, x)
```

3.497 $\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx=-\frac{a^2\left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}+\frac{a\left(x+\sqrt{a+x^2}\right)^n}{2n}+\frac{\left(x+\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}$$

```
[Out] -1/4*a^2*(x+(x^2+a)^(1/2))^(-2+n)/(2-n)+1/2*a*(x+(x^2+a)^(1/2))^n/n+1/4*(x+
(x^2+a)^(1/2))^(2+n)/(2+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2147, 276}

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx=-\frac{a^2\left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{a\left(\sqrt{a+x^2}+x\right)^n}{2n}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}$$

```
[In] Int[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -1/4*(a^2*(x + Sqrt[a + x^2])^(-2 + n))/(2 - n) + (a*(x + Sqrt[a + x^2])^n)
/(2*n) + (x + Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int x^{-3+n}\left(a+x^2\right)^2\,dx, x, x+\sqrt{a+x^2}\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(a^2x^{-3+n}+2ax^{-1+n}+x^{1+n}\right)\,dx, x, x+\sqrt{a+x^2}\right)\\
&= -\frac{a^2\left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}+\frac{a\left(x+\sqrt{a+x^2}\right)^n}{2n}+\frac{\left(x+\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx = \frac{1}{4}\left(x+\sqrt{a+x^2}\right)^n\left(\frac{2a}{n}+\frac{a^2}{(-2+n)\left(x+\sqrt{a+x^2}\right)^2}+\frac{\left(x+\sqrt{a+x^2}\right)^2}{2+n}\right)$$

```
[In] Integrate[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]
[Out] ((x + Sqrt[a + x^2])^n*((2*a)/n + a^2/((-2 + n)*(x + Sqrt[a + x^2])^2) + (x
 + Sqrt[a + x^2])^2/(2 + n)))/4
```

Maple [F]

$$\int \sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n dx$$

[In] int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)

[Out] int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n dx = \frac{\left(n^2x^2+an^2-2\sqrt{x^2+a}nx-2a\right)\left(x+\sqrt{x^2+a}\right)^n}{n^3-4n}$$

[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")

[Out] (n^2*x^2 + a*n^2 - 2*sqrt(x^2 + a)*n*x - 2*a)*(x + sqrt(x^2 + a))^n/(n^3 - 4*n)

Sympy [F]

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n dx = \int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n dx$$

[In] integrate((x**2+a)**(1/2)*(x+(x**2+a)**(1/2))**n,x)

[Out] Integral(sqrt(a + x**2)*(x + sqrt(a + x**2))**n, x)

Maxima [F]

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n dx = \int \sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n dx$$

[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")

[Out] integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)

Giac [F]

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx=\int \sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

```
[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx=\int \sqrt{x^2+a}\,\left(x+\sqrt{x^2+a}\right)^n\,dx$$

```
[In] int((a + x^2)^(1/2)*(x + (a + x^2)^(1/2))^n,x)
[Out] int((a + x^2)^(1/2)*(x + (a + x^2)^(1/2))^n, x)
```

3.498 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 17

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} \, dx=\frac{\left(x+\sqrt{a+x^2}\right)^n}{n}$$

```
[Out] (x+(x^2+a)^(1/2))^n/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2147, 30}

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} \, dx=\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

```
[In] Int[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
```

```
[Out] (x + Sqrt[a + x^2])^n/n
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
```

```
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int x^{-1+n}\,dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{\left(x+\sqrt{a+x^2}\right)^n}{n}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx = \frac{\left(x+\sqrt{a+x^2}\right)^n}{n}$$

```
[In] Integrate[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
[Out] (x + Sqrt[a + x^2])^n/n
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$	16
default	$\frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$	16

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] (x+(x^2+a)^(1/2))^n/n
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx=\frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$$

[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="fricas")

[Out] (x + sqrt(x^2 + a))^n/n

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(12) = 24.

Time = 1.51 (sec) , antiderivative size = 311, normalized size of antiderivative = 18.29

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx = \begin{cases} \frac{\sqrt{a}a^{\frac{n}{2}}\sinh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{nx\sqrt{\frac{a}{x^2}+1}}-\frac{2a^{\frac{n}{2}}\cosh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{n}{2}\right)}{n^2\Gamma\left(-\frac{n}{2}\right)}+\frac{a^{\frac{n}{2}}x\cosh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}n}+\frac{a^{\frac{n}{2}}x\sinh\left(\right.}{} \\ \frac{a^{\frac{n}{2}}\sinh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{n\sqrt{1+\frac{x^2}{a}}}-\frac{2a^{\frac{n}{2}}\cosh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{n}{2}\right)}{n^2\Gamma\left(-\frac{n}{2}\right)}+\frac{a^{\frac{n}{2}}x^2\sinh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{an\sqrt{1+\frac{x^2}{a}}}+\frac{a^{\frac{n}{2}}x\cosh\left(n\right.}{} \end{cases}$$

[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)

[Out] Piecewise((sqrt(a)*a**(n/2)*sinh(n*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(n*
x*sqrt(a/x**2 + 1)) - 2*a**(n/2)*cosh(n*asinh(x/sqrt(a)))*gamma(1 - n/2)/(n
2*gamma(-n/2)) + a(n/2)*x*cosh(n*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(
sqrt(a)*n) + a**(n/2)*x*sinh(n*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(sqrt(a
)*n*sqrt(a/x**2 + 1)), Abs(x**2/a) > 1), (a**(n/2)*sinh(n*asinh(x/sqrt(a))
- asinh(x/sqrt(a)))/(n*sqrt(1 + x**2/a)) - 2*a**(n/2)*cosh(n*asinh(x/sqrt(a
)))*gamma(1 - n/2)/(n**2*gamma(-n/2)) + a**(n/2)*x**2*sinh(n*asinh(x/sqrt(a
)) - asinh(x/sqrt(a)))/(a*n*sqrt(1 + x**2/a)) + a**(n/2)*x*cosh(n*asinh(x/s
qrt(a)) - asinh(x/sqrt(a)))/(sqrt(a)*n), True))

Maxima [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx=\frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$$

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")
[Out] (x + sqrt(x^2 + a))^n/n
```

Mupad [B] (verification not implemented)

Time = 16.97 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx=\frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$$

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(1/2),x)
[Out] (x + (a + x^2)^(1/2))^n/n
```

3.499 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx=\frac{4\left(x+\sqrt{a+x^2}\right)^{2+n}\text{Hypergeometric2F1}\left(2,\frac{2+n}{2},\frac{4+n}{2},-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}$$

```
[Out] 4*hypergeom([2, 1+1/2*n],[2+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^(1/2)
)^(2+n)/a^2/(2+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2147, 371}

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx=\frac{4\left(\sqrt{a+x^2}+x\right)^{n+2}\text{Hypergeometric2F1}\left(2,\frac{n+2}{2},\frac{n+4}{2},-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

```
[Out] (4*(x + Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, -
((x + Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= 4\text{Subst}\left(\int \frac{x^{1+n}}{(a+x^2)^2}\,dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{4\left(x+\sqrt{a+x^2}\right)^{2+n}\,{}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx = \frac{4\left(x+\sqrt{a+x^2}\right)^{2+n}\text{Hypergeometric2F1}\left(2,\frac{2+n}{2},1+\frac{2+n}{2},-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}$$

```
[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
[Out] (4*(x + Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, 1 + (2 + n)/
2, -((x + Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Maple [F]

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx=\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(3/2),x)
[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{3/2}}\,dx$$

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(3/2),x)
[Out] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(3/2), x)
```

3.500 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}} \, dx=\frac{16\left(x+\sqrt{a+x^2}\right)^{4+n} \text{Hypergeometric2F1}\left(4,\frac{4+n}{2},\frac{6+n}{2},-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}$$

```
[Out] 16*hypergeom([4, 2+1/2*n],[3+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^(1/2
))^(4+n)/a^4/(4+n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2147, 371}

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}} \, dx=\frac{16\left(\sqrt{a+x^2}+x\right)^{n+4} \text{Hypergeometric2F1}\left(4,\frac{n+4}{2},\frac{n+6}{2},-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

```
[Out] (16*(x + Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, (6 + n)/2,
-((x + Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= 16\text{Subst}\left(\int \frac{x^{3+n}}{(a+x^2)^4}\,dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{16\left(x+\sqrt{a+x^2}\right)^{4+n}\,{}_2F_1\left(4,\frac{4+n}{2};\frac{6+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx = \frac{16\left(x+\sqrt{a+x^2}\right)^{4+n}\text{Hypergeometric2F1}\left(4,\frac{4+n}{2},1+\frac{4+n}{2},-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}$$

```
[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
[Out] (16*(x + Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, 1 + (4 + n)
/2, -((x + Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Maple [F]

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{5}{2}}}dx$$

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{5}{2}}}\,dx$$

[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n/(x^6 + 3*a*x^4 + 3*a^2*x^2 + a ^3), x)

Sympy [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx = \int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{\frac{5}{2}}}\,dx$$

[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(5/2),x)

[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2)**(5/2), x)

Maxima [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{5}{2}}}\,dx$$

[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)

Giac [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{5}{2}}}\,dx$$

[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{5/2}}\,dx$$

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(5/2),x)
[Out] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(5/2), x)
```

3.501 $\int (a+x^2)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal result

Integrand size = 25, antiderivative size = 201

$$\int (a+x^2)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx = \frac{a^6 \left(x-\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)} + \frac{3a^5 \left(x-\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)} + \frac{15a^4 \left(x-\sqrt{a+x^2}\right)^{-2+n}}{64(2-n)} - \frac{5a^3 \left(x-\sqrt{a+x^2}\right)^{n}}{16n} - \frac{15a^2 \left(x-\sqrt{a+x^2}\right)^{2+n}}{64(2+n)} - \frac{3a \left(x-\sqrt{a+x^2}\right)^{4+n}}{32(4+n)} - \frac{\left(x-\sqrt{a+x^2}\right)^{6+n}}{64(6+n)}$$

```
[Out] 1/64*a^6*(x-(x^2+a)^(1/2))^(-6+n)/(6-n)+3/32*a^5*(x-(x^2+a)^(1/2))^(-4+n)/(
4-n)+15/64*a^4*(x-(x^2+a)^(1/2))^(-2+n)/(2-n)-5/16*a^3*(x-(x^2+a)^(1/2))^n/
n-15/64*a^2*(x-(x^2+a)^(1/2))^(2+n)/(2+n)-3/32*a*(x-(x^2+a)^(1/2))^(4+n)/(4
+n)-1/64*(x-(x^2+a)^(1/2))^(6+n)/(6+n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2147, 276}

$$\int (a+x^2)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx = \frac{a^6 \left(x-\sqrt{a+x^2}\right)^{n-6}}{64(6-n)} + \frac{3a^5 \left(x-\sqrt{a+x^2}\right)^{n-4}}{32(4-n)} + \frac{15a^4 \left(x-\sqrt{a+x^2}\right)^{n-2}}{64(2-n)} - \frac{5a^3 \left(x-\sqrt{a+x^2}\right)^{n}}{16n} - \frac{15a^2 \left(x-\sqrt{a+x^2}\right)^{n+2}}{64(n+2)} - \frac{3a \left(x-\sqrt{a+x^2}\right)^{n+4}}{32(n+4)} - \frac{\left(x-\sqrt{a+x^2}\right)^{n+6}}{64(n+6)}$$

```
[In] Int[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^6*(x - Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) + (3*a^5*(x - Sqrt[a + x^2]
)^(-4 + n))/(32*(4 - n)) + (15*a^4*(x - Sqrt[a + x^2])^(-2 + n))/(64*(2 - n
)) - (5*a^3*(x - Sqrt[a + x^2])^n)/(16*n) - (15*a^2*(x - Sqrt[a + x^2])^(2
+ n))/(64*(2 + n)) - (3*a*(x - Sqrt[a + x^2])^(4 + n))/(32*(4 + n)) - (x -
Sqrt[a + x^2])^(6 + n)/(64*(6 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{64}\text{Subst}\left(\int x^{-7+n}\left(a+x^2\right)^6\,dx, x, x-\sqrt{a+x^2}\right)\right)$$

$$= -\left(\frac{1}{64}\text{Subst}\left(\int\left(a^6x^{-7+n}+6a^5x^{-5+n}+15a^4x^{-3+n}+20a^3x^{-1+n}+15a^2x^{1+n}+6ax^{3+n}+x^{5+n}\right)\,dx, x, x-\sqrt{a+x^2}\right)\right)$$

$$= \frac{a^6\left(x-\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)}+\frac{3a^5\left(x-\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)}+\frac{15a^4\left(x-\sqrt{a+x^2}\right)^{-2+n}}{64(2-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{n}}{16n}-\frac{15a^2\left(x-\sqrt{a+x^2}\right)^{2+n}}{64(2+n)}-\frac{3a\left(x-\sqrt{a+x^2}\right)^{4+n}}{32(4+n)}-\frac{\left(x-\sqrt{a+x^2}\right)^{6+n}}{64(6+n)}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.86

$$\int \left(a+x^2\right)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx = \frac{1}{64}\left(x-\sqrt{a+x^2}\right)^n \left(-\frac{20a^3}{n} - \frac{a^6}{(-6+n)\left(x-\sqrt{a+x^2}\right)^6} - \frac{6a^5}{(-4+n)\left(x-\sqrt{a+x^2}\right)^4} - \frac{15a^4}{(-2+n)\left(x-\sqrt{a+x^2}\right)^2} - \frac{15a^2\left(x-\sqrt{a+x^2}\right)^2}{2+n} - \frac{6a\left(x-\sqrt{a+x^2}\right)^4}{4+n} - \frac{\left(x-\sqrt{a+x^2}\right)^6}{6+n}\right)$$

```
[In] Integrate[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]
[Out] ((x - Sqrt[a + x^2])^n*((-20*a^3)/n - a^6/((-6 + n)*(x - Sqrt[a + x^2])^6)
- (6*a^5)/((-4 + n)*(x - Sqrt[a + x^2])^4) - (15*a^4)/((-2 + n)*(x - Sqrt[a
 + x^2])^2) - (15*a^2*(x - Sqrt[a + x^2])^2)/(2 + n) - (6*a*(x - Sqrt[a + x
^2])^4)/(4 + n) - (x - Sqrt[a + x^2])^6/(6 + n)))/64
```

Maple [F]

$$\int \left(x^2+a\right)^{\frac{5}{2}} \left(x-\sqrt{x^2+a}\right)^n dx$$

```
[In] int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)
[Out] int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \left(a+x^2\right)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx = -\frac{(a^3n^6 - 50\,a^3n^4 + (n^6 - 20\,n^4 + 64\,n^2)x^6 + 544\,a^3n^2 + 3\,(an^6 - 30\,an^4 + 104\,an^2)x^4 - 720\,a^3 + 3\,(a^2n^6 -}{}$$

```
[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
[Out] -(a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^2 + 3*(a*n
^6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 - 40*a^2*n^4 + 264*a^
2*n^2)*x^2 + 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2*(a*n^5 - 30*a*n^3 + 104*a*n)*
x^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a^2*n)*x)*sqrt(x^2 + a))*(x - sqrt(x^2 +
a))^n/(n^7 - 56*n^5 + 784*n^3 - 2304*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \left(a+x^2\right)^{5/2}\left(x-\sqrt{a+x^2}\right)^n\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((x**2+a)**(5/2)*(x-(x**2+a)**(1/2))**n,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \left(a+x^2\right)^{5/2}\left(x-\sqrt{a+x^2}\right)^n\,dx=\int \left(x^2+a\right)^{\frac{5}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

```
[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)
```

Giac [F]

$$\int \left(a+x^2\right)^{5/2}\left(x-\sqrt{a+x^2}\right)^n\,dx=\int \left(x^2+a\right)^{\frac{5}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

```
[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+x^2\right)^{5/2}\left(x-\sqrt{a+x^2}\right)^n\,dx=\int \left(x-\sqrt{x^2+a}\right)^n\left(x^2+a\right)^{5/2}\,dx$$

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(5/2),x)
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(5/2), x)
```

3.502 $\int (a+x^2)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int (a+x^2)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx = \frac{a^4\left(x-\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)} + \frac{a^3\left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)} - \frac{3a^2\left(x-\sqrt{a+x^2}\right)^n}{8n} - \frac{a\left(x-\sqrt{a+x^2}\right)^{2+n}}{4(2+n)} - \frac{\left(x-\sqrt{a+x^2}\right)^{4+n}}{16(4+n)}$$

```
[Out] 1/16*a^4*(x-(x^2+a)^(1/2))^(-4+n)/(4-n)+1/4*a^3*(x-(x^2+a)^(1/2))^(-2+n)/(2
-n)-3/8*a^2*(x-(x^2+a)^(1/2))^n/n-1/4*a*(x-(x^2+a)^(1/2))^(2+n)/(2+n)-1/16*
(x-(x^2+a)^(1/2))^(4+n)/(4+n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2147, 276}

$$\int (a+x^2)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx = \frac{a^4\left(x-\sqrt{a+x^2}\right)^{n-4}}{16(4-n)} + \frac{a^3\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)} - \frac{3a^2\left(x-\sqrt{a+x^2}\right)^n}{8n} - \frac{a\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)} - \frac{\left(x-\sqrt{a+x^2}\right)^{n+4}}{16(n+4)}$$

```
[In] Int[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^4*(x - Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) + (a^3*(x - Sqrt[a + x^2])^
(-2 + n))/(4*(2 - n)) - (3*a^2*(x - Sqrt[a + x^2])^n)/(8*n) - (a*(x - Sqrt[
a + x^2])^(2 + n))/(4*(2 + n)) - (x - Sqrt[a + x^2])^(4 + n)/(16*(4 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{16}\text{Subst}\left(\int x^{-5+n}\left(a+x^2\right)^4\,dx, x, x-\sqrt{a+x^2}\right)\right)\\
&= -\left(\frac{1}{16}\text{Subst}\left(\int \left(a^4x^{-5+n}+4a^3x^{-3+n}+6a^2x^{-1+n}+4ax^{1+n}+x^{3+n}\right)\,dx, x, x-\sqrt{a+x^2}\right)\right)\\
&= \frac{a^4\left(x-\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)}+\frac{a^3\left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}\\
&\quad -\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n}}{8n}-\frac{a\left(x-\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}-\frac{\left(x-\sqrt{a+x^2}\right)^{4+n}}{16(4+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int \left(a+x^2\right)^{3/2}\left(x-\sqrt{a+x^2}\right)^n\,dx = \frac{1}{16}\left(x-\sqrt{a+x^2}\right)^n\left(-\frac{6a^2}{n}-\frac{a^4}{(-4+n)\left(x-\sqrt{a+x^2}\right)^4}-\frac{4a^3}{(-2+n)\left(x-\sqrt{a+x^2}\right)^2}-\frac{4a\left(x-\sqrt{a+x^2}\right)^2}{2+n}-\frac{\left(x-\sqrt{a+x^2}\right)^4}{4+n}\right)$$

```
[In] Integrate[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]
[Out] ((x - Sqrt[a + x^2])^n*((-6*a^2)/n - a^4/((-4 + n)*(x - Sqrt[a + x^2])^4) -
 (4*a^3)/((-2 + n)*(x - Sqrt[a + x^2])^2) - (4*a*(x - Sqrt[a + x^2])^2)/(2
+ n) - (x - Sqrt[a + x^2])^4/(4 + n)))/16
```

Maple [F]

$$\int \left(x^2+a\right)^{\frac{3}{2}} \left(x-\sqrt{x^2+a}\right)^n dx$$

[In] int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)

[Out] int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

$$\int \left(a+x^2\right)^{3/2} \left(x-\sqrt{a+x^2}\right)^n dx = -\frac{\left(a^2n^4+(n^4-4\,n^2)x^4-16\,a^2n^2+2\,(an^4-10\,an^2)x^2+24\,a^2+4\,((n^3-4\,n)x^3+(an^3-10\,an)x)\sqrt{x^2+}\right.}{n^5-20\,n^3+64\,n}$$

[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")

[Out] -(a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)*x^2 + 24*a^2 + 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2 + a))*(x - sqrt(x^2 + a))^n/(n^5 - 20*n^3 + 64*n)

Sympy [F]

$$\int \left(a+x^2\right)^{3/2} \left(x-\sqrt{a+x^2}\right)^n dx = \int \left(a+x^2\right)^{\frac{3}{2}} \left(x-\sqrt{a+x^2}\right)^n dx$$

[In] integrate((x**2+a)**(3/2)*(x-(x**2+a)**(1/2))**n,x)

[Out] Integral((a + x**2)**(3/2)*(x - sqrt(a + x**2))**n, x)

Maxima [F]

$$\int \left(a+x^2\right)^{3/2} \left(x-\sqrt{a+x^2}\right)^n dx = \int \left(x^2+a\right)^{\frac{3}{2}} \left(x-\sqrt{x^2+a}\right)^n dx$$

[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")

[Out] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)

Giac [F]

$$\int \left(a+x^2\right)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx = \int \left(x^2+a\right)^{\frac{3}{2}} \left(x-\sqrt{x^2+a}\right)^n \, dx$$

```
[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+x^2\right)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx = \int \left(x-\sqrt{x^2+a}\right)^n \left(x^2+a\right)^{3/2} \, dx$$

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(3/2),x)
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(3/2), x)
```

3.503 $\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n\,dx$

Optimal result

Integrand size = 25, antiderivative size = 81

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n\,dx=\frac{a^2\left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}-\frac{a\left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}$$

```
[Out] 1/4*a^2*(x-(x^2+a)^(1/2))^(-2+n)/(2-n)-1/2*a*(x-(x^2+a)^(1/2))^n/n-1/4*(x-(
x^2+a)^(1/2))^(2+n)/(2+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2147, 276}

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n\,dx=\frac{a^2\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{a\left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}$$

```
[In] Int[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^2*(x - Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) - (a*(x - Sqrt[a + x^2])^n)/
(2*n) - (x - Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{4}\text{Subst}\left(\int x^{-3+n}\left(a+x^2\right)^2\,dx, x, x-\sqrt{a+x^2}\right)\right)\\ &= -\left(\frac{1}{4}\text{Subst}\left(\int \left(a^2x^{-3+n}+2ax^{-1+n}+x^{1+n}\right)\,dx, x, x-\sqrt{a+x^2}\right)\right)\\ &= \frac{a^2\left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}-\frac{a\left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n\,dx = \frac{1}{4}\left(x-\sqrt{a+x^2}\right)^n\left(-\frac{2a}{n}-\frac{a^2}{(-2+n)\left(x-\sqrt{a+x^2}\right)^2}-\frac{\left(x-\sqrt{a+x^2}\right)^2}{2+n}\right)$$

```
[In] Integrate[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]
[Out] ((x - Sqrt[a + x^2])^n*((-2*a)/n - a^2/((-2 + n)*(x - Sqrt[a + x^2])^2) - (
x - Sqrt[a + x^2])^2/(2 + n)))/4
```

Maple [F]

$$\int \sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n dx$$

```
[In] int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)
[Out] int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.63

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n dx = -\frac{\left(n^2x^2+an^2+2\sqrt{x^2+a}nx-2a\right)\left(x-\sqrt{x^2+a}\right)^n}{n^3-4n}$$

```
[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
[Out] -(n^2*x^2 + a*n^2 + 2*sqrt(x^2 + a)*n*x - 2*a)*(x - sqrt(x^2 + a))^n/(n^3 -
 4*n)
```

Sympy [F]

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n dx = \int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n dx$$

```
[In] integrate((x**2+a)**(1/2)*(x-(x**2+a)**(1/2))**n,x)
[Out] Integral(sqrt(a + x**2)*(x - sqrt(a + x**2))**n, x)
```

Maxima [F]

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n dx = \int \sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n dx$$

```
[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

Giac [F]

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n\,dx=\int \sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

```
[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n\,dx=\int \left(x-\sqrt{x^2+a}\right)^n\sqrt{x^2+a}\,dx$$

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(1/2),x)
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(1/2), x)
```

3.504 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 20

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} \, dx = -\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

```
[Out] -(x-(x^2+a)^(1/2))^n/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2147, 30}

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} \, dx = -\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

```
[In] Int[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
```

```
[Out] -((x - Sqrt[a + x^2])^n/n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
```

```
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int x^{-1+n}\,dx, x, x - \sqrt{a+x^2}\right) \\ &= -\frac{\left(x-\sqrt{a+x^2}\right)^n}{n} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx = -\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

```
[In] Integrate[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
[Out] -((x - Sqrt[a + x^2])^n/n)
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{\left(x-\sqrt{x^2+a}\right)^n}{n}$	19
default	$-\frac{\left(x-\sqrt{x^2+a}\right)^n}{n}$	19

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(x-(x^2+a)^(1/2))^n/n
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx=-\frac{\left(x-\sqrt{x^2+a}\right)^n}{n}$$

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="fricas")
[Out] -(x - sqrt(x^2 + a))^n/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.

Time = 0.95 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx=\begin{cases} -\frac{\left(x-\sqrt{a+x^2}\right)^n}{n} & \text{for}\ n\neq 0 \\ \begin{cases} \log\left(2x+2\sqrt{a+x^2}\right) & \text{for}\ a\neq 0 \\ \frac{x\log(x)}{\sqrt{x^2}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)
[Out] Piecewise((-(x - sqrt(a + x**2))**n/n, Ne(n, 0)), (Piecewise((log(2*x + 2*s
qrt(a + x**2)), Ne(a, 0)), (x*log(x)/sqrt(x**2), True)), True))
```

Maxima [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{\sqrt{a + x^2}} \, dx = -\frac{\left(x - \sqrt{x^2 + a}\right)^n}{n}$$

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")
[Out] -(x - sqrt(x^2 + a))^n/n
```

Mupad [B] (verification not implemented)

Time = 17.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{\sqrt{a + x^2}} \, dx = -\frac{\left(x - \sqrt{x^2 + a}\right)^n}{n}$$

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(1/2),x)
[Out] -(x - (a + x^2)^(1/2))^n/n
```

3.505 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx = -\frac{4\left(x-\sqrt{a+x^2}\right)^{2+n}\text{Hypergeometric2F1}\left(2,\frac{2+n}{2},\frac{4+n}{2},-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}$$

```
[Out] -4*hypergeom([2, 1+1/2*n],[2+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^(1/2
))^(2+n)/a^2/(2+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2147, 371}

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx = -\frac{4\left(x-\sqrt{a+x^2}\right)^{n+2}\text{Hypergeometric2F1}\left(2,\frac{n+2}{2},\frac{n+4}{2},-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

```
[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

```
[Out] (-4*(x - Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2,
-((x - Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\text{integral} = -\left(4\text{Subst}\left(\int \frac{x^{1+n}}{(a+x^2)^2}\,dx, x, x-\sqrt{a+x^2}\right)\right)$$

$$= -\frac{4\left(x-\sqrt{a+x^2}\right)^{2+n}\,{}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx = -\frac{4\left(x-\sqrt{a+x^2}\right)^{2+n}\text{Hypergeometric2F1}\left(2,\frac{2+n}{2},1+\frac{2+n}{2},-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}$$

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
[Out] (-4*(x - Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, 1 + (2 + n)
/2, -((x - Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Maple [F]

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}dx$$

[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)

[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)

Fricas [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx = \int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)

Sympy [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx = \int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{\frac{3}{2}}}\,dx$$

[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(3/2),x)

[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2)**(3/2), x)

Maxima [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx = \int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="maxima")

[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)

Giac [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{3/2}}\,dx$$

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(3/2),x)
[Out] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(3/2), x)
```

3.506 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx = -\frac{16\left(x-\sqrt{a+x^2}\right)^{4+n}\text{Hypergeometric2F1}\left(4,\frac{4+n}{2},\frac{6+n}{2},-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}$$

```
[Out] -16*hypergeom([4, 2+1/2*n],[3+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^(1/
2))^(4+n)/a^4/(4+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2147, 371}

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx = -\frac{16\left(x-\sqrt{a+x^2}\right)^{n+4}\text{Hypergeometric2F1}\left(4,\frac{n+4}{2},\frac{n+6}{2},-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

```
[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

```
[Out] (-16*(x - Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, (6 + n)/2,
 -((x - Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\text{integral} = -\left(16\text{Subst}\left(\int \frac{x^{3+n}}{(a+x^2)^4}\,dx, x, x-\sqrt{a+x^2}\right)\right)$$
$$= -\frac{16\left(x-\sqrt{a+x^2}\right)^{4+n} {}_2F_1\left(4, \frac{4+n}{2}; \frac{6+n}{2}; -\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx = -\frac{16\left(x-\sqrt{a+x^2}\right)^{4+n} \text{Hypergeometric2F1}\left(4, \frac{4+n}{2}, 1+\frac{4+n}{2}, -\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}$$

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

```
[Out] (-16*(x - Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, 1 + (4 + n
)/2, -((x - Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Maple [F]

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{5}{2}}}dx$$

[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)

[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)

Fricas [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{5}{2}}}\,dx$$

[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n/(x^6 + 3*a*x^4 + 3*a^2*x^2 + a^3), x)

Sympy [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx=\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{\frac{5}{2}}}\,dx$$

[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(5/2),x)

[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2)**(5/2), x)

Maxima [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{5}{2}}}\,dx$$

[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="maxima")

[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)

Giac [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{5/2}}\,dx$$

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(5/2),x)
[Out] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(5/2), x)
```

3.507 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n$

Optimal result

Integrand size = 56, antiderivative size = 365

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \frac{(d^2-af^2)^5\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-5+n}}{32ef^4(5-n)} - \frac{5(d^2-af^2)^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-3+n}}{32ef^4(3-n)} + \frac{5(d^2-af^2)^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{16ef^4(1-n)} + \frac{5(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}}{16ef^4(1+n)} - \frac{5(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{3+n}}{32ef^4(3+n)} + \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{5+n}}{32ef^4(5+n)}$$

```
[Out] 1/32*(-a*f^2+d^2)^5*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-5+n)/e/f^
4/(5-n)-5/32*(-a*f^2+d^2)^4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-3
+n)/e/f^4/(3-n)+5/16*(-a*f^2+d^2)^3*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1
/2))^(-1+n)/e/f^4/(1-n)+5/16*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^(1+n)/e/f^4/(1+n)-5/32*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^
2*x^2/f^2)^(1/2))^(3+n)/e/f^4/(3+n)+1/32*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^
2)^(1/2))^(5+n)/e/f^4/(5+n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2146, 12, 276}

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2 \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \frac{(d^2-af^2)^5 \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-5}}{32ef^4(5-n)} - \frac{5(d^2-af^2)^4 \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{32ef^4(3-n)} + \frac{5(d^2-af^2)^3 \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{16ef^4(1-n)} + \frac{5(d^2-af^2)^2 \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{16ef^4(n+1)} - \frac{5(d^2-af^2) \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}}{32ef^4(n+3)} + \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+5}}{32ef^4(n+5)}$$

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f
^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)^5*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-5
+ n))/(32*e*f^4*(5 - n)) - (5*(d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^(-3 + n))/(32*e*f^4*(3 - n)) + (5*(d^2 - a*f^2)^3*
(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(16*e*f^4*(
1 - n)) + (5*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2
)/f^2])^(1 + n))/(16*e*f^4*(1 + n)) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a
+ (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n))/(32*e*f^4*(3 + n)) + (d + e*x +
f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(5 + n)/(32*e*f^4*(5 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{x^{-6+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^5}{64e^6}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{f^4}$$

$$= \frac{\text{Subst}\left(\int x^{-6+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^5\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{32e^6f^4}$$

$$= \frac{\text{Subst}\left(\int \left(-e^5(d^2-af^2)^5x^{-6+n}+5e^5(d^2-af^2)^4x^{-4+n}-10e^5(d^2-af^2)^3x^{-2+n}+10e^5(d^2-a\right.\right.}{32e^6f^4}$$

$$= \frac{(d^2-af^2)^5\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-5+n}}{32ef^4(5-n)} - \frac{5(d^2-af^2)^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-3+n}}{32ef^4(3-n)} + \frac{5(d^2-af^2)^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{16ef^4(1-n)} + \frac{5(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}}{16ef^4(1+n)} - \frac{5(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{3+n}}{32ef^4(3+n)} + \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{5+n}}{32ef^4(5+n)}$$

Mathematica [A] (verified)

Time = 4.20 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.77

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{-5+n}\left(-\frac{(d^2-af^2)^5}{-5+n}+\frac{5(d^2-af^2)^4\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{-3+n}-\frac{10(d^2-af^2)^3\left(d+ex+f\sqrt{a+\frac{ex(\cdot}{}}\right.}{-1+n}\right.}{32ef^4}$$

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*
e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-5 + n)*(-((d^2 - a*f^2)^5/
(-5 + n)) + (5*(d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
)^2)/(-3 + n) - (10*(d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))
/f^2])^4)/(-1 + n) + (10*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (e*x*(2*d +
e*x))/f^2])^6)/(1 + n) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d +
 e*x))/f^2])^8)/(3 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^10/
(5 + n)))/(32*e*f^4)
```

Maple [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

```
[In] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2
))^n,x)
```

```
[Out] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2
))^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.79

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx =$$
$$-\frac{\left(5a^2df^4n^4+225a^2df^4-300ad^3f^2+5(e^5n^4-10e^5n^2+9e^5)x^5+120d^5+25(de^4n^4-10de^4n^2+9d\right.}{}$$

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*a^2*d*f^4*n^4 + 225*a^2*d*f^4 - 300*a*d^3*f^2 + 5*(e^5*n^4 - 10*e^5*n^2
 + 9*e^5)*x^5 + 120*d^5 + 25*(d*e^4*n^4 - 10*d*e^4*n^2 + 9*d*e^4)*x^4 + 10*
(15*a*e^3*f^2 + 30*d^2*e^3 + (a*e^3*f^2 + 4*d^2*e^3)*n^4 - 2*(8*a*e^3*f^2 +
 17*d^2*e^3)*n^2)*x^3 - 10*(11*a^2*d*f^4 - 6*a*d^3*f^2)*n^2 + 10*(45*a*d*e^
2*f^2 + (3*a*d*e^2*f^2 + 2*d^3*e^2)*n^4 - 2*(24*a*d*e^2*f^2 + d^3*e^2)*n^2)
*x^2 + 5*(45*a^2*e*f^4 + (a^2*e*f^4 + 4*a*d^2*e*f^2)*n^4 - 2*(11*a^2*e*f^4
+ 26*a*d^2*e*f^2 - 12*d^4*e)*n^2)*x - (a^2*f^5*n^5 + (e^4*f*n^5 - 10*e^4*f*
n^3 + 9*e^4*f*n)*x^4 - 10*(3*a^2*f^5 - 2*a*d^2*f^3)*n^3 + 4*(d*e^3*f*n^5 -
10*d*e^3*f*n^3 + 9*d*e^3*f*n)*x^3 + 2*((a*e^2*f^3 + 2*d^2*e^2*f)*n^5 - 10*(
2*a*e^2*f^3 + d^2*e^2*f)*n^3 + (19*a*e^2*f^3 + 8*d^2*e^2*f)*n)*x^2 + (149*a
^2*f^5 - 260*a*d^2*f^3 + 120*d^4*f)*n + 4*(a*d*e*f^3*n^5 - 10*(2*a*d*e*f^3
- d^3*e*f)*n^3 + (19*a*d*e*f^3 - 10*d^3*e*f)*n)*x)*sqrt((e^2*x^2 + a*f^2 +
2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*f^4
*n^6 - 35*e*f^4*n^4 + 259*e*f^4*n^2 - 225*e*f^4)
```

Sympy [F(-2)]

Exception generated.

$$\int\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$
$$=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**2*(d+e*x+f*(a+2*d*e*x/f**2+e**2*
x**2/f**2)**(1/2))**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$
$$=\int\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^2\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n\,dx$$

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f^2 + a + 2
*d*e*x/f^2)*f + d)^n, x)
```

Giac [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2 \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^2 \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f^2 + a + 2
*d*e*x/f^2)*f + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2 \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\int \left(d+f\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n \left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)^2 dx$$

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^2,x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^2, x)
```

3.508 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n$

Optimal result

Integrand size = 54, antiderivative size = 239

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \frac{(d^2-af^2)^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-3+n}}{8ef^2(3-n)} - \frac{3(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{8ef^2(1-n)} - \frac{3(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}}{8ef^2(1+n)} + \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{3+n}}{8ef^2(3+n)}$$

```
[Out] 1/8*(-a*f^2+d^2)^3*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-3+n)/e/f^2
/(3-n)-3/8*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-1+n
)/e/f^2/(1-n)-3/8*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^
(1+n)/e/f^2/(1+n)+1/8*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(3+n)/e/f
^2/(3+n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2146, 12, 276}

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\frac{(d^2-af^2)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{8ef^2(3-n)}-\frac{3(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{8ef^2(1-n)}-\frac{3(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{8ef^2(n+1)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}}{8ef^2(n+3)}$$

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-3
+ n))/(8*e*f^2*(3 - n)) - (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x
)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(8*e*f^2*(1 - n)) - (3*(d^2 - a*f^2)*(d +
 e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n))/(8*e*f^2*(1 + n)
) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)/(8*e*f^2*
(3 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
```

```
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{x^{-4+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3}{16e^4}dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{f^2}$$

$$= \frac{\text{Subst}\left(\int x^{-4+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3 dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{8e^4f^2}$$

$$= \frac{\text{Subst}\left(\int \left(-e^3(d^2-af^2)^3x^{-4+n}+3e^3(d^2-af^2)^2x^{-2+n}-3e^3(d^2-af^2)x^n+e^3x^{2+n}\right)dx, x, d+\right.}{8e^4f^2}$$

$$= \frac{(d^2-af^2)^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-3+n}}{8ef^2(3-n)} - \frac{3(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{8ef^2(1-n)} - \frac{3(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}}{8ef^2(1+n)} + \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{3+n}}{8ef^2(3+n)}$$

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.78

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

$$= \frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{-3+n}\left(-\frac{(d^2-af^2)^3}{-3+n}+\frac{3(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{-1+n}-\frac{3(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{ex(\right.}{}}\right.}{1+n}}{8ef^2}$$

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-3 + n)*(-((d^2 - a*f^2)^3/
(-3 + n)) + (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
)^2)/(-1 + n) - (3*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^
2])^4)/(1 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^6/(3 + n)))/
(8*e*f^2)
```

Maple [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

```
[In] int((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))
^n,x)
```

```
[Out] int((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))
^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx =$$
$$-\frac{\left(3\,adf^2n^2-9\,adf^2+3\,(e^3n^2-e^3)x^3+6\,d^3+9\,(de^2n^2-de^2)x^2-3\,(3\,aef^2-(aef^2+2\,d^2e)n^2)x-(af\right.}{ef^2n^4-}$$

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(3*a*d*f^2*n^2 - 9*a*d*f^2 + 3*(e^3*n^2 - e^3)*x^3 + 6*d^3 + 9*(d*e^2*n^2
- d*e^2)*x^2 - 3*(3*a*e*f^2 - (a*e*f^2 + 2*d^2*e)*n^2)*x - (a*f^3*n^3 + (e^
2*f*n^3 - e^2*f*n)*x^2 - (7*a*f^3 - 6*d^2*f)*n + 2*(d*e*f*n^3 - d*e*f*n)*x)
*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*
d*e*x)/f^2) + d)^n/(e*f^2*n^4 - 10*e*f^2*n^2 + 9*e*f^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)*(d+e*x+f*(a+2*d*e*x/f**2+e**2*x**
2/f**2)**(1/2))**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$
$$= \int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n\,dx$$

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d
*e*x/f^2)*f + d)^n, x)
```

Giac [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$
$$= \int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n\,dx$$

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d
*e*x/f^2)*f + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$
$$=\int \left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)\,dx$$

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2), x)
```

3.509 $\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal result

Integrand size = 33, antiderivative size = 107

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)} + \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}$$

```
[Out] 1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-1+n)/e/(1-n)
+1/2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2141, 12, 14}

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)} + \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 +
n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(
1 + n)/(2*e*(1 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)}{4e^2}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$

$$=\frac{\text{Subst}\left(\int x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{2e^2}$$

$$=\frac{\text{Subst}\left(\int \left(-e(d^2-af^2)\,x^{-2+n}+ex^n\right)dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{2e^2}$$

$$=\frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)}+\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.83

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{-1+n}\left(\frac{-d^2+af^2}{-1+n}+\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{1+n}\right)}{2e}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-1 + n)*((-d^2 + a*f^2)/(-1
 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(1 + n)))/(2*e)
```

Maple [F]

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x)
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\left(fn\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}-ex-d\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n}{en^2-e}$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="frica
s")
[Out] (f*n*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) - e*x - d)*(e*x + f*sqrt((e^2*x^
2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*n^2 - e)
```

Sympy [F]

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n,x)
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n, x)
```

Maxima [F]

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxim
a")
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

Giac [F]

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac"
)
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n dx$$

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n,x)
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n, x)
```

3.510 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$

Optimal result

Integrand size = 56, antiderivative size = 122

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = -\frac{2f^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)(1+n)}$$

```
[Out] -2*f^2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(1
+n)/e/(-a*f^2+d^2)/(1+n)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2146, 371}

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = -\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)*Hyper
geometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
&\text{integral}\\
&= (2f^2)\,\text{Subst}\left(\int \frac{x^n}{d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\\
&= -\frac{2f^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = -\frac{2f^2\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{1+n}\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)(1+n)}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d
*e*x)/f^2 + (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(1 + n)*Hypergeometri
c2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Maple [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}dx$$

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2),x)
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2),x)
```

Fricas [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx=\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^2/(e^2*x^2
 + a*f^2 + 2*d*e*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2),x, algorithm="maxima")

[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2 + a + 2*d*e*x/f^2), x)

Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2),x, algorithm="giac")

[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2 + a + 2*d*e*x/f^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = \int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}\,dx$$

[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2),x)

[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2), x)

3.511 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx$

Optimal result

Integrand size = 56, antiderivative size = 122

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx = -\frac{8f^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{3+n}\text{Hypergeometric2F1}\left(3,\frac{3+n}{2},\frac{5+n}{2},\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\,(d^2-af^2)^3\,(3+n)}$$

```
[Out] -8*f^4*hypergeom([3, 3/2+1/2*n],[5/2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(3
+n)/e/(-a*f^2+d^2)^3/(3+n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used

$= \{2146, 12, 371\}$

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx = -\frac{8f^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}\text{Hypergeometric2F1}\left(3,\frac{n+3}{2},\frac{n+5}{2},\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+3)\left(d^2-af^2\right)^3}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2)^2,x]
```

```
[Out] (-8*f^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)*Hyper
geometric2F1[3, (3 + n)/2, (5 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^3*(3 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\text{integral} = \left(2f^4\right)\text{Subst}\left(\int \frac{4e^2x^{2+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$

$$= (8e^2f^4)\,\text{Subst}\left(\int \frac{x^{2+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3}dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$

$$= -\frac{8f^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{3+n} {}_2F_1\left(3,\frac{3+n}{2};\frac{5+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\,(d^2-af^2)^3\,(3+n)}$$

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}dx = -\frac{8f^4\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{3+n}\text{Hypergeometric2F1}\left(3,\frac{3+n}{2},\frac{5+n}{2},\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e\,(d^2-af^2)^3\,(3+n)}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d
*e*x)/f^2 + (e^2*x^2)/f^2)^2,x]
```

```
[Out] (-8*f^4*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(3 + n)*Hypergeometri
c2F1[3, (3 + n)/2, (5 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^3*(3 + n))
```

Maple [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}dx$$

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2)^2,x)
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2)^2,x)
```

Fricas [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx=\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^2}\,dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^4/(e^4*x^4
 + 4*d*e^3*x^3 + a^2*f^4 + 4*a*d*e*f^2*x + 2*(a*e^2*f^2 + 2*d^2*e^2)*x^2),
x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx=\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^2}\,dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2)^2, x)
```

Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx=\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^2}\,dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx=\int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)^2}\,dx$$

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^2,x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^2, x)
```

3.512 $\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx$

Optimal result

Integrand size = 33, antiderivative size = 107

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx = \frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)} + \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}$$

```
[Out] 1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-1+n)/e/(1-n)
+1/2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2143, 2141, 12, 14}

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx = \frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)} + \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 +
n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(
1 + n)/(2*e*(1 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2143

```
Int[((g_.) + (h_.)*((u_) + (f_.)*Sqrt[v_])^(n_))^(p_.), x_Symbol] :> Int[(g
 + h*(ExpandToSum[u, x] + f*Sqrt[ExpandToSum[v, x]])^n)^p, x] /; FreeQ[{f,
g, h, n}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !(LinearMatchQ[u, x]
&& QuadraticMatchQ[v, x]) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x,
 2]*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

$$= 2\text{Subst}\left(\int \frac{x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)}{4e^2}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$

$$= \frac{\text{Subst}\left(\int x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{2e^2}$$

$$= \frac{\text{Subst}\left(\int \left(-e(d^2-af^2)\,x^{-2+n}+ex^n\right)dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{2e^2}$$

$$=\frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)}+\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.83

$$\int\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx$$
$$=\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{-1+n}\left(\frac{-d^2+af^2}{-1+n}+\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{1+n}\right)}{2e}$$

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-1 + n)*((-d^2 + a*f^2)/(-1
 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(1 + n)))/(2*e)
```

Maple [F]

$$\int\left(d+ex+f\sqrt{\frac{a\,f^2+ex\,(ex+2d)}{f^2}}\right)^n dx$$

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx$$
$$=\frac{\left(fn\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}-ex-d\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n}{en^2-e}$$

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="frica
s")
[Out] (f*n*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) - e*x - d)*(e*x + f*sqrt((e^2*x^
2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*n^2 - e)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n\,dx=\text{Timed out}$$

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n\,dx=\int \left(ex+f\left(\frac{\sqrt{af^2+(ex+2\,d)ex}}{f}\right)+d\right)^n\,dx$$

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="maxim
a")
[Out] integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n, x)
```

Giac [F]

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n\,dx=\int \left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n\,dx$$

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="giac"
)
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n\,dx=\int {\left(d+e\,x+f\,\sqrt{\frac{a\,f^2+e\,x\,\left(2\,d+e\,x\right)}{f^2}}\right)}^n\,dx$$

```
[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n,x)
[Out] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n, x)
```

3.513 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$

Optimal result

Integrand size = 56, antiderivative size = 122

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = -\frac{2f^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)(1+n)}$$

```
[Out] -2*f^2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(1
+n)/e/(-a*f^2+d^2)/(1+n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2152, 2146, 371}

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = -\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/(a + (2*d*e*x)/f^2
+ (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)*Hyper
geometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2152

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx \\
&= \left(2f^2\right)\operatorname{Subst}\left(\int \frac{x^n}{d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right) \\
&= -\frac{2f^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = -\frac{2f^2\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{1+n}\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)(1+n)}$$

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/(a + (2*d*e*x
)/f^2 + (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(1 + n)*Hypergeometri
c2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Maple [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{a\,f^2+ex(ex+2d)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}dx$$

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2),x)
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^
2),x)
```

Fricas [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = \int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^2/(e^2*x^2
 + a*f^2 + 2*d*e*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/(a+2*d*e*x/f**2
+e**2*x**2/f**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = \int \frac{\left(ex+d+\sqrt{af^2+(ex+2\,d)ex}\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n/(e^2*x^2/f^2 + a + 2*
d*e*x/f^2), x)
```

Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = \int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="giac")
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d + ex + f\sqrt{\frac{af^2 + ex(2d+ex)}{f^2}}\right)^n}{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\,dx = \int \frac{\left(d + e\,x + f\,\sqrt{\frac{a\,f^2 + e\,x\,(2\,d + e\,x)}{f^2}}\right)^n}{a + \frac{e^2\,x^2}{f^2} + \frac{2\,d\,e\,x}{f^2}}\,dx$$

```
[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/(a + (e^2*x^2)/f^
2 + (2*d*e*x)/f^2),x)
```

```
[Out] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/(a + (e^2*x^2)/f^
2 + (2*d*e*x)/f^2), x)
```

3.514 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right.$

Optimal result

Integrand size = 58, antiderivative size = 297

$$\begin{aligned}
&\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \\
&-\frac{(d^2-af^2)^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-4+n}}{16ef^3(4-n)} \\
&+\frac{(d^2-af^2)^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-2+n}}{4ef^3(2-n)} \\
&+\frac{3(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{n}}{8ef^3n} \\
&-\frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}}{4ef^3(2+n)} \\
&+\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{4+n}}{16ef^3(4+n)}
\end{aligned}$$

```
[Out] -1/16*(-a*f^2+d^2)^4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-4+n)/e/f
^3/(4-n)+1/4*(-a*f^2+d^2)^3*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-2
+n)/e/f^3/(2-n)+3/8*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/
2))^n/e/f^3/n-1/4*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^
(2+n)/e/f^3/(2+n)+1/16*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(4+n)/e/
f^3/(4+n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2146, 12, 276}

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = -\frac{(d^2-af^2)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-4}}{16ef^3(4-n)} + \frac{(d^2-af^2)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef^3(2-n)} + \frac{3(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{8ef^3n} - \frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef^3(n+2)} + \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+4}}{16ef^3(n+4)}$$

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -1/16*((d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]
)^(-4 + n))/(e*f^3*(4 - n)) + ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e
*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4*e*f^3*(2 - n)) + (3*(d^2 - a*f^2)^2*
(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(8*e*f^3*n) - ((d^
2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n))/(
4*e*f^3*(2 + n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(4
 + n)/(16*e*f^3*(4 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{x^{-5+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^4}{32e^5}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{f^3}$$

$$= \frac{\text{Subst}\left(\int x^{-5+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^4\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{16e^5f^3}$$

$$= \frac{\text{Subst}\left(\int \left(e^4(d^2-af^2)^4x^{-5+n}-4e^4(d^2-af^2)^3x^{-3+n}+6e^4(d^2-af^2)^2x^{-1+n}-4e^4(d^2-af^2)\,x\right.\right.}{16e^5f^3}$$

$$= -\frac{(d^2-af^2)^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-4+n}}{16ef^3(4-n)} + \frac{(d^2-af^2)^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-2+n}}{4ef^3(2-n)} + \frac{3(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{n}}{8ef^3n} - \frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}}{4ef^3(2+n)} + \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{4+n}}{16ef^3(4+n)}$$

Mathematica [A] (verified)

Time = 6.54 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.77

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2} \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n \left(\frac{6(d^2-af^2)^2}{n}+\frac{(d^2-af^2)^4}{(-4+n)\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^4}-\frac{}{(-}\right.}{}$$

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (
2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n*((6*(d^2 - a*f^2)^2)/n + (
d^2 - a*f^2)^4/((-4 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4) -
 (4*(d^2 - a*f^2)^3)/((-2 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
)^2) - (4*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2)/(2
 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4/(4 + n)))/(16*e*f^3
)
```

Maple [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}} \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

```
[In] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x)
```

```
[Out] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.27

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2} \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\left(a^2f^4n^4+24\,a^2f^4-48\,ad^2f^2+(e^4n^4-4\,e^4n^2)x^4+24\,d^4+4\,(de^3n^4-4\,de^3\right.}{}$$

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (a^2*f^4*n^4 + 24*a^2*f^4 - 48*a*d^2*f^2 + (e^4*n^4 - 4*e^4*n^2)*x^4 + 24*d
^4 + 4*(d*e^3*n^4 - 4*d*e^3*n^2)*x^3 - 4*(4*a^2*f^4 - 3*a*d^2*f^2)*n^2 + 2*
((a*e^2*f^2 + 2*d^2*e^2)*n^4 - 2*(5*a*e^2*f^2 + d^2*e^2)*n^2)*x^2 + 4*(a*d*
e*f^2*n^4 - 2*(5*a*d*e*f^2 - 3*d^3*e)*n^2)*x - 4*(a*d*f^3*n^3 + (e^3*f*n^3
- 4*e^3*f*n)*x^3 + 3*(d*e^2*f*n^3 - 4*d*e^2*f*n)*x^2 - 2*(5*a*d*f^3 - 3*d^3
*f)*n + ((a*e*f^3 + 2*d^2*e*f)*n^3 - 2*(5*a*e*f^3 + d^2*e*f)*n)*x)*sqrt((e^
2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^
2) + d)^n/(e*f^3*n^5 - 20*e*f^3*n^3 + 64*e*f^3*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(3/2)*(d+e*x+f*(a+2*d*e*x/f**2+e
**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^{\frac{3}{2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x^2/f^2 + a
 + 2*d*e*x/f^2)*f + d)^n, x)
```

Giac [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx=\int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^{\frac{3}{2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x^2/f^2 + a
 + 2*d*e*x/f^2)*f + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx=\int \left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)^{3/2} dx$$

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(3/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(3/2), x)
```

3.515 $\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n d$

Optimal result

Integrand size = 58, antiderivative size = 171

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= -\frac{(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-2+n}}{4ef(2-n)}$$
$$-\frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{2efn}+\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}}{4ef(2+n)}$$

```
[Out] -1/4*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-2+n)/e/f/
(2-n)-1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/e/f/n+
1/4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)/e/f/(2+n)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2146, 12, 276}

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=-\frac{(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)}$$
$$-\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{2efn}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef(n+2)}$$

```
[In] Int[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)
/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -1/4*((d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])
^(-2 + n))/(e*f*(2 - n)) - ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f
^2 + (e^2*x^2)/f^2])^n)/(2*e*f*n) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (
e^2*x^2)/f^2])^(2 + n)/(4*e*f*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{x^{-3+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}{8e^3}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{f}$$

$$= \frac{\text{Subst}\left(\int x^{-3+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2 dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{4e^3f}$$

$$= \frac{\text{Subst}\left(\int \left(e^2(d^2-af^2)^2x^{-3+n}-2e^2(d^2-af^2)x^{-1+n}+e^2x^{1+n}\right) dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{4e^3f}$$

$$= -\frac{(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-2+n}}{4ef(2-n)} - \frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{2efn} + \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}}{4ef(2+n)}$$

Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.79

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n\left(\frac{2(-d^2+af^2)}{n}+\frac{(d^2-af^2)^2}{(-2+n)\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}+\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{2+n}\right)}{4ef}$$

```
[In] Integrate[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]

[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n*((2*(-d^2 + a*f^2))/n + (d
^2 - a*f^2)^2/((-2 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2) +
(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(2 + n)))/(4*e*f)
```

Maple [F]

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

```
[In] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x)
```

```
[Out] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.71

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\frac{\left(e^2n^2x^2+af^2n^2+2\,den^2x-2\,af^2+2\,d^2-2\,(efnx+dfn)\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)}{efn^3-4\,efn}$$

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (e^2*n^2*x^2 + a*f^2*n^2 + 2*d*e*n^2*x - 2*a*f^2 + 2*d^2 - 2*(e*f*n*x + d*f
*n)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 +
 2*d*e*x)/f^2) + d)^n/(e*f*n^3 - 4*e*f*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/f**2+e
**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\int \sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a +
 2*d*e*x/f^2)*f + d)^n, x)
```

Giac [F]

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\int \sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a +
 2*d*e*x/f^2)*f + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\int \left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}\,dx$$

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(1/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(1/2), x)
```

3.516 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$

Optimal result

Integrand size = 58, antiderivative size = 41

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx=\frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}$$

```
[Out] f*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/e/n
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2146, 12, 30}

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx=\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a + (2*d*e
*x)/f^2 + (e^2*x^2)/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= (2f)\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\\ &= \frac{f\text{Subst}\left(\int x^{-1+n}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e}\\ &= \frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx = \frac{f\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n}{en}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a +
(2*d*e*x)/f^2 + (e^2*x^2)/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n)/(e*n)
```

Maple [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}dx$$

[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2),x)

[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx = \frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^nf}{en}$$

[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2),x, algorithm="fricas")

[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f/(e*n)

Sympy [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx = \int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$$

[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2),x)

[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n/sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2), x)

Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx=\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}}\,dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*x^2/
f^2 + a + 2*d*e*x/f^2), x)
```

Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx=\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}}\,dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*x^2/
f^2 + a + 2*d*e*x/f^2), x)
```

Mupad [B] (verification not implemented)

Time = 17.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx=\frac{f\left(d+e\,x+f\,\sqrt{\frac{e^2\,x^2+2\,d\,e\,x+a\,f^2}{f^2}}\right)^n}{e\,n}$$

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(1/2),x)
```

```
[Out] (f*(d + e*x + f*((a*f^2 + e^2*x^2 + 2*d*e*x)/f^2)^(1/2))^n)/(e*n)
```

3.517 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 58, antiderivative size = 122

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx = \frac{4f^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}\text{Hypergeometric2F1}\left(2,\frac{2+n}{2},\right.}{e\left(d^2-af^2\right)^2(2+n)}$$

```
[Out] 4*f^3*hypergeom([2, 1+1/2*n],[2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)
^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)/e
/(-a*f^2+d^2)^2/(2+n)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2146, 12, 371}

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx = \frac{4f^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}\text{Hypergeometric2F1}\left(2,\frac{n+2}{2},\right.}{e(n+2)\left(d^2-af^2\right)^2}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2)^(3/2),x]
```

```
[Out] (4*f^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)*Hyperg
eometric2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
(e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\text{integral} = (2f^3)\,\text{Subst}\left(\int \frac{2ex^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$

$$= (4ef^3)\,\text{Subst}\left(\int \frac{x^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$

$$= \frac{4f^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}{}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)^2(2+n)}$$

Mathematica [A] (verified)

Time = 3.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx=\frac{4f^3\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{2+n}\text{Hypergeometric2F1}\left(2,\frac{2+n}{2},\frac{4+}{2}\right.}{e\left(d^2-af^2\right)^2(2+n)}$$

[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2),x]

[Out] (4*f^3*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(2 + n))

Maple [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}}dx$$

[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2),x)

[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2),x)

Fricas [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx=\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2),x, algorithm="fricas")

[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^4*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^4*x^4 + 4*d*e^3*x^3 + a^2*f^4 + 4*a*d*e*f^2*x + 2*(a*e^2*f^2 + 2*d^2*e^2)*x^2), x)

Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2)**(3/2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2)^(3/2), x)
```

Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
+ a + 2*d*e*x/f^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx=\int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)^{3/2}}\,dx$$

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(3/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(3/2), x)
```

3.518 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx$

Optimal result

Integrand size = 58, antiderivative size = 41

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx=\frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}$$

```
[Out] f*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/e/n
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2152, 2146, 12, 30}

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx=\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 + e*x*(
2*d + e*x))/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2152

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx \\
&= (2f)\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right) \\
&= \frac{f\text{Subst}\left(\int x^{-1+n}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e} \\
&= \frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx = \frac{f\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n}{en}$$

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 +
 e*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n)/(e*n)
```

Maple [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{a\,f^2+ex(ex+2d)}{f^2}}\right)^n}{\sqrt{\frac{a\,f^2+ex(ex+2d)}{f^2}}}dx$$

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*d))/f^
2)^(1/2),x)
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*d))/f^
2)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx = \frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f}{en}$$

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*
d))/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f/(e*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2+e*x*(e
*x+2*d))/f**2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx = \int \frac{\left(ex+d+\sqrt{af^2+(ex+2\,d)ex}\right)^n}{\frac{\sqrt{af^2+(ex+2\,d)ex}}{f}}\,dx$$

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*
d))/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] f*integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n/sqrt(a*f^2 + (e*x +
 2*d)*e*x), x)
```

Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx = \int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}}\,dx$$

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*
d))/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2 +
 (e*x + 2*d)*e*x)/f^2), x)
```

Mupad [B] (verification not implemented)

Time = 16.69 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx=\frac{f\left(d+e\,x+f\,\sqrt{\frac{a\,f^2+e\,x\,(2\,d+e\,x)}{f^2}}\right)^n}{e\,n}$$

```
[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2 + e*x*(2*
d + e*x))/f^2)^(1/2),x)
```

```
[Out] (f*(d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n)/(e*n)
```

3.519 $\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal result

Integrand size = 62, antiderivative size = 327

$$\begin{aligned}
&\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx \\
&= -\frac{(d^2-af^2)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-2+n}}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}} \\
&\quad -\frac{(d^2-af^2)\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{n}}{2efn\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}} \\
&\quad +\frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}}{4ef(2+n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}
\end{aligned}$$

```
[Out] -1/4*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-2+n)*(a*g
+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)/e/f/(2-n)/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(
1/2)-1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n*(a*g+2*
d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)/e/f/n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)+1/
4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)*(a*g+2*d*e*g*x/f^2+e^2*
g*x^2/f^2)^(1/2)/e/f/(2+n)/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2148, 2146, 12, 276}

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= -\frac{(d^2-af^2)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$
$$-\frac{(d^2-af^2)\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{2efn\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$
$$+\frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef(n+2)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

```
[In] Int[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -1/4*((d^2 - a*f^2)^2*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*
x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(e*f*(2 - n)*Sqrt[
a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) - ((d^2 - a*f^2)*Sqrt[a*g + (2*d*e*g*x)
/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2
])^n)/(2*e*f*n*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) + (Sqrt[a*g + (2*d*
e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^
2)/f^2])^(2 + n))/(4*e*f*(2 + n)*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
```

```
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2148

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(i/c)^(m -
 1/2)*(Sqrt[g + h*x + i*x^2]/Sqrt[a + b*x + c*x^2]), Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && IGtQ[m + 1/2, 0] &&  !GtQ[i/c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\int\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

$$=\frac{\left(2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\right)\text{Subst}\left(\int\frac{x^{-3+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}{8e^3}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

$$=\frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\text{Subst}\left(\int x^{-3+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\right.}{4e^3f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

$$=\frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\text{Subst}\left(\int\left(e^2(d^2-af^2)^2x^{-3+n}-2e^2(d^2-af^2)x^{-1+n}+e^2x^{1+n}\right)\,dx,x,d+ex\right.}{4e^3f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

$$=-\frac{(d^2-af^2)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-2+n}}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$
$$-\frac{(d^2-af^2)\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{2efn\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$
$$+\frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}}{4ef(2+n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.54

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\frac{\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n\left(\frac{2(-d^2+af^2)}{n}+\frac{(d^2-af^2)^2}{(-2+n)\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}+\frac{\left(d+ex+f\sqrt{a\right.}}{2}\right.}{4ef\sqrt{a+\frac{ex(2d+ex)}{f^2}}}$$

```
[In] Integrate[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a
 + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] (Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x)
)/f^2])^n*((2*(-d^2 + a*f^2))/n + (d^2 - a*f^2)^2/((-2 + n)*(d + e*x + f*Sq
rt[a + (e*x*(2*d + e*x))/f^2])^2) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))
/f^2])^2/(2 + n)))/(4*e*f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
```

Maple [F]

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2g\,x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

```
[In] int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x)
```

```
[Out] int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.71

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx =$$
$$-\frac{\left(2\,e^3nx^3+6\,de^2nx^2+2\,adf^2n+2\,(aef^2+2\,d^2e)nx-(e^2fn^2x^2+af^3n^2+2\,defn^2x-2\,af^3+2\,d^2f)\right.}{aef^2n^3-4\,aef^2n+(e^3n^3-4\,e^3n)x^2+2\,(de^2n^3}$$

```
[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e
^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(2*e^3*n*x^3 + 6*d*e^2*n*x^2 + 2*a*d*f^2*n + 2*(a*e*f^2 + 2*d^2*e)*n*x - (
e^2*f*n^2*x^2 + a*f^3*n^2 + 2*d*e*f*n^2*x - 2*a*f^3 + 2*d^2*f)*sqrt((e^2*x^
2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) +
 d)^n*sqrt((e^2*g*x^2 + a*f^2*g + 2*d*e*g*x)/f^2)/(a*e*f^2*n^3 - 4*a*e*f^2*
n + (e^3*n^3 - 4*e^3*n)*x^2 + 2*(d*e^2*n^3 - 4*d*e^2*n)*x)
```

Sympy [F(-2)]

Exception generated.

$$\int \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a*g+2*d*e*g*x/f**2+e**2*g*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/
f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \int \sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2\,degx}{f^2}} \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n dx$$

```
[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e
^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e^2*x^2/f^2
 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

Giac [F]

$$\int \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \int \sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2\,degx}{f^2}} \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n dx$$

```
[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e
^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e^2*x^2/f^2
 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$
$$=\int \left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\,\sqrt{a\,g+\frac{e^2\,g\,x^2}{f^2}+\frac{2\,d\,e\,g\,x}{f^2}}\,dx$$

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2), x)
```

3.520 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx$

Optimal result

Integrand size = 62, antiderivative size = 93

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx=\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[Out] f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1
/2))^n/e/n/(a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2150, 2146, 12, 30}

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx=\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g + (2*d
*e*g*x)/f^2 + (e^2*g*x^2)/f^2],x]
```

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2150

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(i/c)^(m +
 1/2)*(Sqrt[a + b*x + c*x^2]/Sqrt[g + h*x + i*x^2]), Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(2f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int\frac{x^{-1+n}}{2e}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int x^{-1+n}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx=\frac{f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n}{en\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g
+ (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2],x]
```

```
[Out] (f*Sqrt[a + (e*x*(2*d + e*x))/f^2]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/
f^2])^n)/(e*n*Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)])
```

Maple [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2g\,x^2}{f^2}}}dx$$

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g*
x^2/f^2)^(1/2),x)
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g*
x^2/f^2)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.26

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx=\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^3\sqrt{\frac{e^2gx^2+af^2g+2\,degx}{f^2}}\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}}{e^3gnx^2+aef^2gn+2\,de^2gnx}$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^3*sqrt((e^2*g*x^2 +
 a*f^2*g + 2*d*e*g*x)/f^2)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^3*g*n*x
^2 + a*e*f^2*g*n + 2*d*e^2*g*n*x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*g*
x/f**2+e**2*g*x**2/f**2)**(1/2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\sqrt{\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}}}\,dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*g*x^
2/f^2 + a*g + 2*d*e*g*x/f^2), x)
```

Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\sqrt{\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}}}\,dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*g*x^
2/f^2 + a*g + 2*d*e*g*x/f^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx=\int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{\sqrt{a\,g+\frac{e^2\,g\,x^2}{f^2}+\frac{2\,d\,e\,g\,x}{f^2}}}\,dx$$

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2), x)
```

3.521 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}} dx$

Optimal result

Integrand size = 62, antiderivative size = 177

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}} dx = \frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n} \text{Hypergeom}}{e\left(d^2-af^2\right)^2 g(2+n)\sqrt{ag+}}$$

```
[Out] 4*f^3*hypergeom([2, 1+1/2*n],[2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)
^(1/2))^2/(-a*f^2+d^2))*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e
*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)/e/(-a*f^2+d^2)^2/g/(2+n)/(a*g+2*d*e*g*x/f^
2+e^2*g*x^2/f^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2150, 2146, 12, 371}

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}} dx = \frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2} \text{Hypergeom}}{eg(n+2)\left(d^2-af^2\right)^2\sqrt{ag+}}$$

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a*g + (2*d*e*g
*x)/f^2 + (e^2*g*x^2)/f^2)^(3/2),x]
```

```
[Out] (4*f^3*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e
*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2
, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(
e*(d^2 - a*f^2)^2*g*(2 + n)*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2150

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(i/c)^(m +
 1/2)*(Sqrt[a + b*x + c*x^2]/Sqrt[g + h*x + i*x^2]), Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx}{g\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(2f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int \frac{2ex^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}\,dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{g\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(4ef^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int\frac{x^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}dx,x,d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{g\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}{}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right.}{e\left(d^2-af^2\right)^2g(2+n)\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.86

$$\int\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}dx = \frac{4f^3\left(a+\frac{ex(2d+ex)}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{2+n}\text{Hypergeom}}{e\left(d^2-af^2\right)^2(2+n)\left(g\left(a+\frac{ex}{}\right.\right.}$$

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a*g + (2
*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2)^(3/2),x]
```

```
[Out] (4*f^3*(a + (e*x*(2*d + e*x))/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (e*x*(2*d +
e*x))/f^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f
*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(2 +
 n)*(g*(a + (e*x*(2*d + e*x))/f^2))^(3/2))
```

Maple [F]

$$\int\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2g\,x^2}{f^2}\right)^{\frac{3}{2}}}dx$$

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g*
x^2/f^2)^(3/2),x)
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g*
x^2/f^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx=\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n}{\left(\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^4*sqrt((e^
2*g*x^2 + a*f^2*g + 2*d*e*g*x)/f^2)/(e^4*g^2*x^4 + 4*d*e^3*g^2*x^3 + a^2*f^
4*g^2 + 4*a*d*e*f^2*g^2*x + 2*(a*e^2*f^2 + 2*d^2*e^2)*g^2*x^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*g*
x/f**2+e**2*g*x**2/f**2)**(3/2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx=\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n}{\left(\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*g*x^2/f^
2 + a*g + 2*d*e*g*x/f^2)^(3/2), x)
```

Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx=\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*g*x^2/f^
2 + a*g + 2*d*e*g*x/f^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx=\int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{\left(a\,g+\frac{e^2\,g\,x^2}{f^2}+\frac{2\,d\,e\,g\,x}{f^2}\right)^{3/2}}\,dx$$

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(3/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(3/2), x)
```

3.522 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx$

Optimal result

Integrand size = 60, antiderivative size = 93

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx=\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[Out] f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1
/2))^n/e/n/(a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2152, 2150, 2146, 12, 30}

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx=\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g + e*g
*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2150

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(i/c)^(m +
 1/2)*(Sqrt[a + b*x + c*x^2]/Sqrt[g + h*x + i*x^2]), Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

Rule 2152

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

Rubi steps

$$\text{integral} = \int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx$$

$$
= \frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}dx}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}
$$

$$
= \frac{\left(2f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int \frac{x^{-1+n}}{2e}dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}
$$

$$
= \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int x^{-1+n}dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}
$$

$$
= \frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82

$$
\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}dx = \frac{f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n}{en\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}}
$$

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g
 + e*g*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*Sqrt[a + (e*x*(2*d + e*x))/f^2]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/
f^2])^n)/(e*n*Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)])
```

Maple [F]

$$
\int \frac{\left(d+ex+f\sqrt{\frac{a\,f^2+ex(ex+2d)}{f^2}}\right)^n}{\sqrt{\frac{a\,f^2g+egx(ex+2d)}{f^2}}}dx
$$

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*x+2*d)
)/f^2)^(1/2),x)
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*x+2*d)
)/f^2)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.26

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx = \frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^3\sqrt{\frac{e^2gx^2+af^2g+2\,degx}{f^2}}\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}}{e^3gnx^2+aef^2gn+2\,de^2gnx}$$

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*
x+2*d))/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^3*sqrt((e^2*g*x^2 +
 a*f^2*g + 2*d*e*g*x)/f^2)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^3*g*n*x
^2 + a*e*f^2*g*n + 2*d*e^2*g*n*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2*g+e*g*
x*(e*x+2*d))/f**2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx = \int \frac{\left(ex+d+\sqrt{af^2+(ex+2\,d)ex}\right)^n}{\frac{\sqrt{af^2g+(ex+2\,d)egx}}{f}}\,dx$$

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*
x+2*d))/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] f*integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n/sqrt(a*f^2*g + (e*x
 + 2*d)*e*g*x), x)
```

Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx = \int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\sqrt{\frac{af^2g+(ex+2\,d)egx}{f^2}}}\,dx$$

[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*x+2*d))/f^2)^(1/2),x, algorithm="giac")

[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2*g + (e*x + 2*d)*e*g*x)/f^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx = \int \frac{\left(d+e\,x+f\,\sqrt{\frac{a\,f^2+e\,x\,(2\,d+e\,x)}{f^2}}\right)^n}{\sqrt{\frac{a\,g\,f^2+e\,g\,x\,(2\,d+e\,x)}{f^2}}}\,dx$$

[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2*g + e*g*x*(2*d + e*x))/f^2)^(1/2),x)

[Out] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2*g + e*g*x*(2*d + e*x))/f^2)^(1/2), x)

3.523 $\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 191

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = -\frac{b\,\text{arctanh}\left(\frac{\sqrt{b^2e+a^2f}\sqrt{c+dx^2}}{\sqrt{b^2c+a^2d}\sqrt{e+fx^2}}\right)}{\sqrt{b^2c+a^2d}\sqrt{b^2e+a^2f}} + \frac{\sqrt{-c}\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(-\frac{b^2c}{a^2d},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[Out] -b*arctanh((a^2*f+b^2*e)^(1/2)*(d*x^2+c)^(1/2)/(a^2*d+b^2*c)^(1/2)/(f*x^2+e
)^(1/2))/(a^2*d+b^2*c)^(1/2)/(a^2*f+b^2*e)^(1/2)+EllipticPi(x*d^(1/2)/(-c)^
(1/2),-b^2*c/a^2/d,(c*f/d/e)^(1/2))*(-c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e
)^(1/2)/a/d^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2138, 552, 551, 585, 95, 214}

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\text{EllipticPi}\left(-\frac{b^2c}{a^2d},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{b\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}\sqrt{a^2f+b^2e}}{\sqrt{e+fx^2}\sqrt{a^2d+b^2c}}\right)}{\sqrt{a^2d+b^2c}\sqrt{a^2f+b^2e}}$$

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[b^2*e + a^2*f]*Sqrt[c + d*x^2])/(Sqrt[b^2*c + a^2*d]*Sqr
t[e + f*x^2])]])/(Sqrt[b^2*c + a^2*d]*Sqrt[b^2*e + a^2*f])) + (Sqrt[-c]*Sqrt
[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b^2*c)/(a^2*d)), ArcSin[(
Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[e + f*x
^2])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
```

```
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= a\int \frac{1}{(a^2-b^2x^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx - b\int \frac{x}{(a^2-b^2x^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx\\
&= -\left(\frac{1}{2}b\text{Subst}\left(\int \frac{1}{(a^2-b^2x)\sqrt{c+dx}\sqrt{e+fx}}\,dx, x, x^2\right)\right)\\
&\quad + \frac{\left(a\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{(a^2-b^2x^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{e+fx^2}}\,dx}{\sqrt{c+dx^2}}\\
&= -\left(b\text{Subst}\left(\int \frac{1}{b^2c+a^2d-(b^2e+a^2f)\,x^2}\,dx, x, \frac{\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\right)\right)\\
&\quad + \frac{\left(a\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\right)\int \frac{1}{(a^2-b^2x^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}}\,dx}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\\
&= -\frac{b\tanh^{-1}\left(\frac{\sqrt{b^2e+a^2f}\sqrt{c+dx^2}}{\sqrt{b^2c+a^2d}\sqrt{e+fx^2}}\right)}{\sqrt{b^2c+a^2d}\sqrt{b^2e+a^2f}} + \frac{\sqrt{-c}\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\Pi\left(-\frac{b^2c}{a^2d};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.19 (sec) , antiderivative size = 772, normalized size of antiderivative = 4.04

$$
\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{2\sqrt{d}\left(\sqrt{c}+i\sqrt{d}x\right)\sqrt{\frac{\left(\sqrt{d}\sqrt{e}-\sqrt{c}\sqrt{f}\right)\left(i\sqrt{c}+\sqrt{d}x\right)}{\left(\sqrt{d}\sqrt{e}+\sqrt{c}\sqrt{f}\right)\left(-i\sqrt{c}+\sqrt{d}x\right)}}\left(\sqrt{e}+i\sqrt{f}x\right)\sqrt{\frac{\sqrt{c}\sqrt{d}(i\sqrt{e}+\sqrt{f}x)}{\left(-\sqrt{d}\sqrt{e}+\sqrt{c}\sqrt{f}\right)\left(-i\sqrt{c}+\sqrt{d}x\right)}}\left(\left(b\sqrt{c}+ia\sqrt{d}\right)\mathrm{E}\right.}{\left(b\sqrt{c}-ia\sqrt{d}\right)\left(b\right.}
$$

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (2*Sqrt[d]*(Sqrt[c] + I*Sqrt[d]*x)*Sqrt[((Sqrt[d]*Sqrt[e] - Sqrt[c]*Sqrt[f]
)*(I*Sqrt[c] + Sqrt[d]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])*((-I)*Sqrt[
c] + Sqrt[d]*x))]*(Sqrt[e] + I*Sqrt[f]*x)*Sqrt[(Sqrt[c]*Sqrt[d]*(I*Sqrt[e]
+ Sqrt[f]*x))/((-(Sqrt[d]*Sqrt[e]) + Sqrt[c]*Sqrt[f])*((-I)*Sqrt[c] + Sqrt[
d]*x))]*((b*Sqrt[c] + I*a*Sqrt[d])*EllipticF[ArcSin[Sqrt[((Sqrt[d]*Sqrt[e]
- Sqrt[c]*Sqrt[f])*(I*Sqrt[c] + Sqrt[d]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqr
```

```
t[f])*((-I)*Sqrt[c] + Sqrt[d]*x))]], (Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])^2/
(Sqrt[d]*Sqrt[e] - Sqrt[c]*Sqrt[f])^2] - 2*b*Sqrt[c]*EllipticPi[((b*Sqrt[c]
 - I*a*Sqrt[d])*(Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f]))/((b*Sqrt[c] + I*a*Sqrt
[d])*(-(Sqrt[d]*Sqrt[e]) + Sqrt[c]*Sqrt[f])), ArcSin[Sqrt[((Sqrt[d]*Sqrt[e]
 - Sqrt[c]*Sqrt[f])*(I*Sqrt[c] + Sqrt[d]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sq
rt[f])*((-I)*Sqrt[c] + Sqrt[d]*x))]], (Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])^2
/(Sqrt[d]*Sqrt[e] - Sqrt[c]*Sqrt[f])^2]))/((b*Sqrt[c] - I*a*Sqrt[d])*(b*Sqr
t[c] + I*a*Sqrt[d])*(-(Sqrt[d]*Sqrt[e]) + Sqrt[c]*Sqrt[f])*Sqrt[(Sqrt[c]*Sq
rt[d]*(Sqrt[e] + I*Sqrt[f]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])*(Sqrt[c
] + I*Sqrt[d]*x))]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.42

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{(cf+de)a^2}{b^2}+2ce+(cf+de)x^2+\frac{2df x^2 a^2}{b^2}}{2\sqrt{\frac{df a^4}{b^4}+\frac{(cf+de)a^2}{b^2}+ce}\sqrt{df x^4+cf x^2+ed x^2+ce}}\right)}{2\sqrt{\frac{df a^4}{b^4}+\frac{(cf+de)a^2}{b^2}+ce}}+\frac{b\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},-\frac{b^2 c}{a^2 d},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)}{\sqrt{-\frac{d}{c}}\,a\sqrt{df x^4+cf x^2+ed x^2+ce}}\right)}{\sqrt{d x^2+c}\sqrt{f x^2+e}\,b}$
default	$\frac{\left(2b\sqrt{\frac{d x^2+c}{c}}\sqrt{\frac{f x^2+e}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},-\frac{b^2 c}{a^2 d},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)\sqrt{\frac{a^4 df+a^2 b^2 cf+a^2 b^2 de+b^4 ec}{b^4}}-\operatorname{arctanh}\left(\frac{2a^2 df x^2+b^2 cf x^2+b^2 de x^2+a^2 cf+a^2 de+2b}{2b^2\sqrt{\frac{a^4 df+a^2 b^2 cf+a^2 b^2 de+b^4 ec}{b^4}}\sqrt{\left(d x^2+c\right)\left(\right.}}\right.\right.}{2ba\sqrt{\frac{a^4 df+a^2 b^2 cf+a^2 b^2 de+b^4 ec}{b^4}}\sqrt{-\frac{d}{c}}\,(df x^4+cf x^2+ed x^2+ce)}$

```
[In] int(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)/b*(-1/2/(d*f*a^
4/b^4+(c*f+d*e)*a^2/b^2+c*e)^(1/2)*arctanh(1/2*((c*f+d*e)*a^2/b^2+2*c*e+(c*
f+d*e)*x^2+2*d*f*x^2*a^2/b^2)/(d*f*a^4/b^4+(c*f+d*e)*a^2/b^2+c*e)^(1/2)/(d*
f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2))+1/(-d/c)^(1/2)/a*b*(1+1/c*d*x^2)^(1/2)*(1
+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/
2),-b^2*c/a^2/d,(-f/e)^(1/2)/(-d/c)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx)\,\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
[Out] Integral(1/((a + b*x)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}(bx+a)}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}(bx+a)}\,dx$$

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,\left(a+b\,x\right)}\,dx$$

```
[In] int(1/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)*(a + b*x)),x)
[Out] int(1/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)*(a + b*x)), x)
```

3.524 $\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 81

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx=-\frac{\log\left(e-2\sqrt{-d}\sqrt{f}x+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}+\frac{\log\left(e+2\sqrt{-d}\sqrt{f}x+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}$$

```
[Out] -1/4*ln(e+2*f*x^2-2*x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+1/4*ln(e+2*f*x
^2+2*x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {6, 1178, 642}

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx=\frac{\log\left(2\sqrt{-d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}-\frac{\log\left(-2\sqrt{-d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}$$

```
[In] Int[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] -1/4*Log[e - 2*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(Sqrt[-d]*Sqrt[f]) + Log[e + 2
*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{e-2fx^2}{e^2+4(d+e)fx^2+4f^2x^4}\,dx \\
&= -\frac{\int \frac{\frac{\sqrt{-d}}{\sqrt{f}}+2x}{-\frac{e}{2f}-\frac{\sqrt{-d}x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{-d}\sqrt{f}}-\frac{\int \frac{\frac{\sqrt{-d}}{\sqrt{f}}-2x}{-\frac{e}{2f}+\frac{\sqrt{-d}x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{-d}\sqrt{f}} \\
&= -\frac{\log\left(e-2\sqrt{-d}\sqrt{f}x+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}+\frac{\log\left(e+2\sqrt{-d}\sqrt{f}x+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 191 vs. 2(81) = 162.

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.36

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx = \frac{-\frac{\left(-d-2e+\sqrt{d}\sqrt{d+2e}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{d+e-\sqrt{d}\sqrt{d+2e}}}\right)}{\sqrt{d+e-\sqrt{d}\sqrt{d+2e}}}-\frac{\left(d+2e+\sqrt{d}\sqrt{d+2e}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{d+e+\sqrt{d}\sqrt{d+2e}}}\right)}{\sqrt{d+e+\sqrt{d}\sqrt{d+2e}}}}{2\sqrt{2}\sqrt{d}\sqrt{d+2e}\sqrt{f}}$$

```
[In] Integrate[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
[Out] (-(((-d - 2*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[d +
e - Sqrt[d]*Sqrt[d + 2*e]]])/Sqrt[d + e - Sqrt[d]*Sqrt[d + 2*e]]) - ((d + 2
*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[d + e + Sqrt[d]
*Sqrt[d + 2*e]]])/Sqrt[d + e + Sqrt[d]*Sqrt[d + 2*e]])/(2*Sqrt[2]*Sqrt[d]*S
qrt[d + 2*e]*Sqrt[f])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\ln\left(-2\sqrt{-df}\, f\,x^{2}-2dfx-\sqrt{-df}\, e\right)}{4\sqrt{-df}}+\frac{\ln\left(-2\sqrt{-df}\, f\,x^{2}+2dfx-\sqrt{-df}\, e\right)}{4\sqrt{-df}}$
default	$f^{2}\left(-\frac{\left(df+2ef-\sqrt{d\,f^{2}(d+2e)}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{fx\sqrt{2}}{\sqrt{-df-ef+\sqrt{d\,f^{2}(d+2e)}}}\right)}{4f^{2}\sqrt{d\,f^{2}(d+2e)}\,\sqrt{-df-ef+\sqrt{d\,f^{2}(d+2e)}}}+\frac{\left(-df-2ef-\sqrt{d\,f^{2}(d+2e)}\right)\sqrt{2}\,\arctan\left(\frac{fx\sqrt{2}}{\sqrt{df+ef+\sqrt{d\,f^{2}}}}\right.}{4f^{2}\sqrt{d\,f^{2}(d+2e)}\,\sqrt{df+ef+\sqrt{d\,f^{2}(d+2e)}}}\right.$

```
[In] int((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(-2*(-d*f)^(1/2)*f*x^2-2*d*f*x-(-d*f)^(1/2)*e)+1/4/(-d*
f)^(1/2)*ln(-2*(-d*f)^(1/2)*f*x^2+2*d*f*x-(-d*f)^(1/2)*e)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.74

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx = \left[-\frac{\sqrt{-df}\log\left(\frac{4\,f^2x^4-4\,(d-e)fx^2+e^2+4\,(2\,fx^3+ex)\sqrt{-df}}{4\,f^2x^4+4\,(d+e)fx^2+e^2}\right)}{4\,df},\ -\frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x}{d}\right)-\sqrt{df}\arctan\left(\frac{(2\,fx^3+(2\,d+e)x)\sqrt{df}}{de}\right)}{2\,df}\right]$$

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^4 - 4*(d - e)*f*x^2 + e^2 + 4*(2*f*x^3 + e*x)
*sqrt(-d*f))/(4*f^2*x^4 + 4*(d + e)*f*x^2 + e^2))/(d*f), -1/2*(sqrt(d*f)*ar
ctan(sqrt(d*f)*x/d) - sqrt(d*f)*arctan((2*f*x^3 + (2*d + e)*x)*sqrt(d*f)/(d
*e)))/(d*f)]
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx = \frac{\sqrt{-\frac{1}{df}}\log\left(-dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4} - \frac{\sqrt{-\frac{1}{df}}\log\left(dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}$$

```
[In] integrate((-2*f*x**2+e)/(4*f**2*x**4+4*d*f*x**2+4*e*f*x**2+e**2),x)
[Out] sqrt(-1/(d*f))*log(-d*x*sqrt(-1/(d*f)) + e/(2*f) + x**2)/4 - sqrt(-1/(d*f))
*log(d*x*sqrt(-1/(d*f)) + e/(2*f) + x**2)/4
```

Maxima [F]

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx = \int -\frac{2\,fx^2-e}{4\,f^2x^4+4\,dfx^2+4\,efx^2+e^2}\,dx$$

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="ma
xima")
[Out] -integrate((2*f*x^2 - e)/(4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2 + e^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(61) = 122.

Time = 0.40 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.28

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx = \frac{\sqrt{2}\left(def|f|-\sqrt{d^2+2\,de}(d+e)f^2+\sqrt{d^2+2\,de}df^2\right)\arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{\frac{4\,df+4\,ef+\sqrt{-16\,e^2f^2+16\,(df+ef)^2}}{f^2}}}\right)}{4\left(d^2+de-\sqrt{d^2+2\,de}d\right)\sqrt{\left(d+e+\sqrt{d^2+2\,de}\right)ff^2}} + \frac{\sqrt{2}\left(def|f|+\sqrt{d^2+2\,de}(d+e)f^2-\sqrt{d^2+2\,de}df^2\right)\arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{\frac{4\,df+4\,ef-\sqrt{-16\,e^2f^2+16\,(df+ef)^2}}{f^2}}}\right)}{4\left(d^2+de+\sqrt{d^2+2\,de}d\right)\sqrt{\left(d+e-\sqrt{d^2+2\,de}\right)ff^2}}$$

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] 1/4*sqrt(2)*(d*e*f*abs(f) - sqrt(d^2 + 2*d*e)*(d + e)*f^2 + sqrt(d^2 + 2*d*
e)*d*f^2)*arctan(4*sqrt(1/2)*x/sqrt((4*d*f + 4*e*f + sqrt(-16*e^2*f^2 + 16*
(d*f + e*f)^2))/f^2))/((d^2 + d*e - sqrt(d^2 + 2*d*e)*d)*sqrt((d + e + sqrt
(d^2 + 2*d*e))*f)*f^2) + 1/4*sqrt(2)*(d*e*f*abs(f) + sqrt(d^2 + 2*d*e)*(d +
 e)*f^2 - sqrt(d^2 + 2*d*e)*d*f^2)*arctan(4*sqrt(1/2)*x/sqrt((4*d*f + 4*e*f
 - sqrt(-16*e^2*f^2 + 16*(d*f + e*f)^2))/f^2))/((d^2 + d*e + sqrt(d^2 + 2*d
*e)*d)*sqrt((d + e - sqrt(d^2 + 2*d*e))*f)*f^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx=\frac{\mathrm{atan}\left(\frac{2\,f^{3/2}\,x^3+2\,d\,\sqrt{f}\,x+e\,\sqrt{f}\,x}{\sqrt{d}\,e}\right)-\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

```
[In] int((e - 2*f*x^2)/(e^2 + 4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2),x)
```

```
[Out] (atan((2*f^(3/2)*x^3 + 2*d*f^(1/2)*x + e*f^(1/2)*x)/(d^(1/2)*e)) - atan((f^
(1/2)*x)/d^(1/2)))/(2*d^(1/2)*f^(1/2))
```

3.525 $\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 73

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx=-\frac{\log\left(e-2\sqrt{d}\sqrt{f}x+2fx^2\right)}{4\sqrt{d}\sqrt{f}}+\frac{\log\left(e+2\sqrt{d}\sqrt{f}x+2fx^2\right)}{4\sqrt{d}\sqrt{f}}$$

```
[Out] -1/4*ln(e+2*f*x^2-2*x*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)+1/4*ln(e+2*f*x^2+2*x
*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {6, 1178, 642}

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx=\frac{\log\left(2\sqrt{d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{d}\sqrt{f}}-\frac{\log\left(-2\sqrt{d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{d}\sqrt{f}}$$

```
[In] Int[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] -1/4*Log[e - 2*Sqrt[d]*Sqrt[f]*x + 2*f*x^2]/(Sqrt[d]*Sqrt[f]) + Log[e + 2*S
qrt[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \int \frac{e-2fx^2}{e^2+(-4d+4e)fx^2+4f^2x^4}\,dx$$

$$= -\frac{\int \frac{\frac{\sqrt{d}}{\sqrt{f}}+2x}{-\frac{e}{2f}-\frac{\sqrt{d}x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{d}\sqrt{f}} - \frac{\int \frac{\frac{\sqrt{d}}{\sqrt{f}}-2x}{-\frac{e}{2f}+\frac{\sqrt{d}x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{d}\sqrt{f}}$$

$$= -\frac{\log\left(e-2\sqrt{d}\sqrt{f}x+2fx^2\right)}{4\sqrt{d}\sqrt{f}} + \frac{\log\left(e+2\sqrt{d}\sqrt{f}x+2fx^2\right)}{4\sqrt{d}\sqrt{f}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.19

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx = \frac{-\frac{\left(-id+2ie+\sqrt{d}\sqrt{-d+2e}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{-d+e-i\sqrt{d}\sqrt{-d+2e}}}\right)}{\sqrt{-d+e-i\sqrt{d}\sqrt{-d+2e}}} - \frac{\left(id-2ie+\sqrt{d}\sqrt{-d+2e}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{-d+e+i\sqrt{d}\sqrt{-d+2e}}}\right)}{\sqrt{-d+e+i\sqrt{d}\sqrt{-d+2e}}}}{2\sqrt{2}\sqrt{d}\sqrt{-d+2e}\sqrt{f}}$$

```
[In] Integrate[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] (-((((-I)*d + (2*I)*e + Sqrt[d]*Sqrt[-d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/
Sqrt[-d + e - I*Sqrt[d]*Sqrt[-d + 2*e]]])/Sqrt[-d + e - I*Sqrt[d]*Sqrt[-d +
```

```
2*e]]) - ((I*d - (2*I)*e + Sqrt[d]*Sqrt[-d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]
*x)/Sqrt[-d + e + I*Sqrt[d]*Sqrt[-d + 2*e]]])/Sqrt[-d + e + I*Sqrt[d]*Sqrt[
-d + 2*e]])/(2*Sqrt[2]*Sqrt[d]*Sqrt[-d + 2*e]*Sqrt[f])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

method	result
risch	$\frac{\ln\left(2\sqrt{df}\, f\, x^{2}+2dfx+\sqrt{df}\, e\right)}{4\sqrt{df}}-\frac{\ln\left(2\sqrt{df}\, f\, x^{2}-2dfx+\sqrt{df}\, e\right)}{4\sqrt{df}}$
default	$f^{2}\left(-\frac{\left(-df+2ef-\sqrt{d\, f^{2}(d-2e)}\right)\sqrt{2}\, \operatorname{arctanh}\left(\frac{fx\sqrt{2}}{\sqrt{df-ef+\sqrt{d\, f^{2}(d-2e)}}}\right)}{4f^{2}\sqrt{d\, f^{2}(d-2e)}\, \sqrt{df-ef+\sqrt{d\, f^{2}(d-2e)}}}+\frac{\left(df-2ef-\sqrt{d\, f^{2}(d-2e)}\right)\sqrt{2}\, \arctan\left(\frac{fx\sqrt{2}}{\sqrt{-df+ef+\sqrt{d\ldots}}}\right.}{4f^{2}\sqrt{d\, f^{2}(d-2e)}\, \sqrt{-df+ef+\sqrt{d\, f^{2}(d-2e\ldots}}}\right.$

```
[In] int((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(2*(d*f)^(1/2)*f*x^2+2*d*f*x+(d*f)^(1/2)*e)-1/4/(d*f)^(1/
2)*ln(2*(d*f)^(1/2)*f*x^2-2*d*f*x+(d*f)^(1/2)*e)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.00

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx = \left[\frac{\sqrt{df}\log\left(\frac{4\,f^2x^4+4\,(d+e)fx^2+e^2+4\,(2\,fx^3+ex)\sqrt{df}}{4\,f^2x^4-4\,(d-e)fx^2+e^2}\right)}{4\,df},\ -\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}x}{d}\right)-\sqrt{-df}\arctan\left(\frac{(2\,fx^3-(2\,d-e)x)\sqrt{-df}}{de}\right)}{2\,df}\right]$$

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*f^2*x^4 + 4*(d + e)*f*x^2 + e^2 + 4*(2*f*x^3 + e*x)*s
qrt(d*f))/(4*f^2*x^4 - 4*(d - e)*f*x^2 + e^2))/(d*f), -1/2*(sqrt(-d*f)*arct
an(sqrt(-d*f)*x/d) - sqrt(-d*f)*arctan((2*f*x^3 - (2*d - e)*x)*sqrt(-d*f)/(
d*e)))/(d*f)]
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx = -\frac{\sqrt{\frac{1}{df}}\log\left(-dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4} + \frac{\sqrt{\frac{1}{df}}\log\left(dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}$$

```
[In] integrate((-2*f*x**2+e)/(4*f**2*x**4-4*d*f*x**2+4*e*f*x**2+e**2),x)
[Out] -sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4 + sqrt(1/(d*f))*l
og(d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4
```

Maxima [F]

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx = \int -\frac{2\,fx^2-e}{4\,f^2x^4-4\,dfx^2+4\,efx^2+e^2}\,dx$$

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="ma
xima")
[Out] -integrate((2*f*x^2 - e)/(4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2 + e^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(53) = 106.

Time = 0.41 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.86

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx = -\frac{\sqrt{2}\left(def|f|+\sqrt{d^2-2\,de}(d-e)f^2-\sqrt{d^2-2\,de}df^2\right)\arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{4\,df-4\,ef+\sqrt{-16\,e^2f^2+16\,(df-ef)^2}}{f^2}}}\right)}{4\left(d^2-de-\sqrt{d^2-2\,de}d\right)\sqrt{-\left(d-e+\sqrt{d^2-2\,de}\right)f}f^2} - \frac{\sqrt{2}\left(def|f|-\sqrt{d^2-2\,de}(d-e)f^2+\sqrt{d^2-2\,de}df^2\right)\arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{4\,df-4\,ef-\sqrt{-16\,e^2f^2+16\,(df-ef)^2}}{f^2}}}\right)}{4\left(d^2-de+\sqrt{d^2-2\,de}d\right)\sqrt{-\left(d-e-\sqrt{d^2-2\,de}\right)f}f^2}$$

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] -1/4*sqrt(2)*(d*e*f*abs(f) + sqrt(d^2 - 2*d*e)*(d - e)*f^2 - sqrt(d^2 - 2*d
*e)*d*f^2)*arctan(4*sqrt(1/2)*x/sqrt(-(4*d*f - 4*e*f + sqrt(-16*e^2*f^2 + 1
6*(d*f - e*f)^2))/f^2))/((d^2 - d*e - sqrt(d^2 - 2*d*e)*d)*sqrt(-(d - e + s
qrt(d^2 - 2*d*e))*f)*f^2) - 1/4*sqrt(2)*(d*e*f*abs(f) - sqrt(d^2 - 2*d*e)*(
d - e)*f^2 + sqrt(d^2 - 2*d*e)*d*f^2)*arctan(4*sqrt(1/2)*x/sqrt(-(4*d*f - 4
*e*f - sqrt(-16*e^2*f^2 + 16*(d*f - e*f)^2))/f^2))/((d^2 - d*e + sqrt(d^2 -
 2*d*e)*d)*sqrt(-(d - e - sqrt(d^2 - 2*d*e))*f)*f^2)
```

Mupad [B] (verification not implemented)

Time = 17.81 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.38

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx=\frac{\operatorname{atanh}\left(\frac{2\,\sqrt{d}\,\sqrt{f}\,x}{2\,f\,x^2+e}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

```
[In] int((e - 2*f*x^2)/(e^2 + 4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2),x)
```

```
[Out] atanh((2*d^(1/2)*f^(1/2)*x)/(e + 2*f*x^2))/(2*d^(1/2)*f^(1/2))
```

3.526 $\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 38

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2118, 211}

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Int[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2118

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
```

```
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\text{integral} = (2e^2)\,\text{Subst}\left(\int \frac{1}{e^2+16de^2fx^2}\,dx, x, \frac{x}{2e+4fx^3}\right)$$
$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.29

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx$$
$$= -\frac{\text{RootSum}\left[e^2+4df\#1^2+4ef\#1^3+4f^2\#1^6\&, \frac{-e\log(x-\#1)+4f\log(x-\#1)\#1^3}{2d\#1+3e\#1^2+6f\#1^5}\&\right]}{4f}$$

```
[In] Integrate[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
[Out] -1/4*RootSum[e^2 + 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + 4*f*Log[x - #1]*#1^3)/(2*d*#1 + 3*e*#1^2 + 6*f*#1^5) & ]/f
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

method	result
default	$\frac{\sum_{\_R=\text{RootOf}\left(4f^2\_Z^6+4ef\_Z^3+4df\_Z^2+e^2\right)}\frac{\left(-4\_R^3f+e\right)\ln\left(x-\_R\right)}{6f\_R^5+3e\_R^2+2d\_R}}{4f}$
risch	$-\frac{\ln\left(\left(32f(-df)^{\frac{3}{2}}d+54e^2f^2d\right)x^3+\left(54e^2(-df)^{\frac{3}{2}}-32d^3f^2\right)x+16e(-df)^{\frac{3}{2}}d+27e^3fd\right)}{4\sqrt{-df}}+\frac{\ln\left(\left(32f(-df)^{\frac{3}{2}}d-54e^2f^2d\right)x^3+\left(54e^2(-\right.\right.}{4\sqrt{}}$

```
[In] int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/f*sum((-4*_R^3*f+e)/(6*_R^5*f+3*_R^2*e+2*_R*d)*ln(x-_R),_R=RootOf(4*_Z^
6*f^2+4*_Z^3*e*f+4*_Z^2*d*f+e^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(28) = 56$.

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 4.03

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx = \left[-\frac{\sqrt{-df}\log\left(\frac{4\,f^2x^6+4\,efx^3-4\,dfx^2+e^2+4\,(2\,fx^4+ex)\sqrt{-df}}{4\,f^2x^6+4\,efx^3+4\,dfx^2+e^2}\right)}{4\,df}, -\frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x^2}{d}\right)-\sqrt{df}\arctan\left(\frac{(2\,fx^5+ex^2+2\,dx)\sqrt{df}}{de}\right)}{2\,df}\right]$$

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2 + 4*(2*f*x^4
+ e*x)*sqrt(-d*f))/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2))/(d*f), -1/2*(
sqrt(d*f)*arctan(sqrt(d*f)*x^2/d) - sqrt(d*f)*arctan((2*f*x^5 + e*x^2 + 2*d
*x)*sqrt(d*f)/(d*e)))/(d*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(34) = 68$.

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx = \frac{\sqrt{-\frac{1}{df}}\log\left(-dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4} - \frac{\sqrt{-\frac{1}{df}}\log\left(dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

```
[In] integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3+4*d*f*x**2+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4 - sqrt(-1/(d*f))
*log(d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4
```

Maxima [F]

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx=\int -\frac{4\,fx^3-e}{4\,f^2x^6+4\,efx^3+4\,dfx^2+e^2}\,dx$$

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(28) = 56$.

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx=-\frac{\sqrt{-df}\log\left(\left|2\,fx^3+2\,\sqrt{-df}x+e\right|\right)}{4\,df}+\frac{\sqrt{-df}\log\left(\left|2\,fx^3-2\,\sqrt{-df}x+e\right|\right)}{4\,df}$$

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] -1/4*sqrt(-d*f)*log(abs(2*f*x^3 + 2*sqrt(-d*f)*x + e))/(d*f) + 1/4*sqrt(-d*
f)*log(abs(2*f*x^3 - 2*sqrt(-d*f)*x + e))/(d*f)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx=\frac{\mathrm{atan}\left(\frac{2\,f^{3/2}\,x^5+2\,d\,\sqrt{f}\,x+e\,\sqrt{f}\,x^2}{\sqrt{d}\,e}\right)-\mathrm{atan}\left(\frac{\sqrt{f}\,x^2}{\sqrt{d}}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

```
[In] int((e - 4*f*x^3)/(e^2 + 4*f^2*x^6 + 4*d*f*x^2 + 4*e*f*x^3),x)
```

```
[Out] (atan((2*f^(3/2)*x^5 + 2*d*f^(1/2)*x + e*f^(1/2)*x^2)/(d^(1/2)*e)) - atan((
f^(1/2)*x^2)/d^(1/2)))/(2*d^(1/2)*f^(1/2))
```

3.527 $\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 38

$$\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx=\frac{\text{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2118, 214}

$$\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx=\frac{\text{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Int[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2118

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
```

```
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\text{integral} = (2e^2)\,\text{Subst}\left(\int \frac{1}{e^2 - 16de^2fx^2}\,dx, x, \frac{x}{2e + 4fx^3}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.29

$$\int \frac{e - 4fx^3}{e^2 - 4dfx^2 + 4efx^3 + 4f^2x^6}\,dx = -\frac{\text{RootSum}\left[e^2 - 4df\#1^2 + 4ef\#1^3 + 4f^2\#1^6\&, \frac{-e\log(x-\#1)+4f\log(x-\#1)\#1^3}{-2d\#1+3e\#1^2+6f\#1^5}\&\right]}{4f}$$

```
[In] Integrate[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
[Out] -1/4*RootSum[e^2 - 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + 4*f*Log[x - #1]*#1^3)/(-2*d*#1 + 3*e*#1^2 + 6*f*#1^5) & ]/f
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

method	result
default	$\frac{\sum_{\_R=\text{RootOf}\left(4f^2\_Z^6+4ef\_Z^3-4df\_Z^2+e^2\right)} \frac{\left(4\_R^3 f-e\right)\ln\left(x-\_R\right)}{-6f\_R^5-3e\_R^2+2d\_R}}{4f}$
risch	$\frac{\ln\left(\left(-32f(df)^{\frac{3}{2}}d+54e^2f^2d\right)x^3+\left(54e^2(df)^{\frac{3}{2}}-32d^3f^2\right)x-16e(df)^{\frac{3}{2}}d+27e^3fd\right)}{4\sqrt{df}} - \frac{\ln\left(\left(-32f(df)^{\frac{3}{2}}d-54e^2f^2d\right)x^3+\left(54e^2(df)^{\frac{3}{2}}+3\right.\right.}{4\sqrt{df}}$

```
[In] int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/f*sum((4*_R^3*f-e)/(-6*_R^5*f-3*_R^2*e+2*_R*d)*ln(x-_R),_R=RootOf(4*_Z^
6*f^2+4*_Z^3*e*f-4*_Z^2*d*f+e^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.

Time = 0.32 (sec) , antiderivative size = 155, normalized size of antiderivative = 4.08

$$\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx = \left[\frac{\sqrt{df}\log\left(\frac{4\,f^2x^6+4\,efx^3+4\,dfx^2+e^2+4\,(2\,fx^4+ex)\sqrt{df}}{4\,f^2x^6+4\,efx^3-4\,dfx^2+e^2}\right)}{4\,df},\ -\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}x^2}{d}\right)-\sqrt{-df}\arctan\left(\frac{(2\,fx^5+ex^2-2\,dx)\sqrt{-df}}{de}\right)}{2\,df}\right]$$

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2 + 4*(2*f*x^4 +
e*x)*sqrt(d*f))/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2))/(d*f), -1/2*(sqr
t(-d*f)*arctan(sqrt(-d*f)*x^2/d) - sqrt(-d*f)*arctan((2*f*x^5 + e*x^2 - 2*d
*x)*sqrt(-d*f)/(d*e)))/(d*f)]
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx = -\frac{\sqrt{\frac{1}{df}}\log\left(-dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

```
[In] integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3-4*d*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4 + sqrt(1/(d*f))*l
og(d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4
```

Maxima [F]

$$\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx = \int -\frac{4\,fx^3-e}{4\,f^2x^6+4\,efx^3-4\,dfx^2+e^2}\,dx$$

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(28) = 56$.

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx = \frac{\sqrt{df}\log\left(\left|2\,fx^3+2\,\sqrt{df}x+e\right|\right)}{4\,df} - \frac{\sqrt{df}\log\left(\left|2\,fx^3-2\,\sqrt{df}x+e\right|\right)}{4\,df}$$

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] 1/4*sqrt(d*f)*log(abs(2*f*x^3 + 2*sqrt(d*f)*x + e))/(d*f) - 1/4*sqrt(d*f)*l
og(abs(2*f*x^3 - 2*sqrt(d*f)*x + e))/(d*f)
```

Mupad [B] (verification not implemented)

Time = 17.55 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx = -\frac{\operatorname{atanh}\left(\frac{-32\,f\,d^2\,x+27\,e^3+54\,f\,e^2\,x^3}{16\,d^{3/2}\,e\,\sqrt{f}+32\,d^{3/2}\,f^{3/2}\,x^3-54\,\sqrt{d}\,e^2\,\sqrt{f}\,x}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

```
[In] int((e - 4*f*x^3)/(e^2 + 4*f^2*x^6 - 4*d*f*x^2 + 4*e*f*x^3),x)
```

```
[Out] -atanh((27*e^3 - 32*d^2*f*x + 54*e^2*f*x^3)/(16*d^(3/2)*e*f^(1/2) + 32*d^(3
/2)*f^(3/2)*x^3 - 54*d^(1/2)*e^2*f^(1/2)*x))/(2*d^(1/2)*f^(1/2))
```

3.528 $\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 38

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2118, 211}

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Int[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2118

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
```

```
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\begin{aligned}&\text{integral}\\&=-\left(\left(e^2(1-n)\right)\text{Subst}\left(\int\frac{1}{e^2+4de^2f(-1+n)^2x^2}\,dx,x,\frac{x}{e(-1+n)+2f(-1+n)x^n}\right)\right)\\&=\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int\frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*
n)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(28) = 56.

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

method	result	size
risch	$-\frac{\ln\left(x^n+\frac{2dfx+\sqrt{-df}\,e}{2\sqrt{-df}\,f}\right)}{4\sqrt{-df}}+\frac{\ln\left(x^n+\frac{-2dfx+\sqrt{-df}\,e}{2\sqrt{-df}\,f}\right)}{4\sqrt{-df}}$	78

```
[In] int((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)+1/4/(
-d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.79

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx = \left[-\frac{\sqrt{-df}\log\left(-\frac{4\,dfx^2-4\,f^2x^{2\,n}-4\,\sqrt{-df}ex-e^2-4\,(2\,\sqrt{-df}\,fx+ef)x^n}{4\,dfx^2+4\,f^2x^{2\,n}+4\,efx^n+e^2}\right)}{4\,df},\ -\frac{\sqrt{df}\arctan\left(\frac{2\,\sqrt{df}\,fx^n+\sqrt{df}e}{2\,dfx}\right)}{2\,df}\right]$$

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*sqrt(-d*f)*e*x - e^2 -
 4*(2*sqrt(-d*f)*f*x + e*f)*x^n)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e
^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*sqrt(d*f)*f*x^n + sqrt(d*f)*e)/(d*
f*x))/(d*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(34) = 68.

Time = 15.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.87

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx = \begin{cases} \frac{x}{e} & \text{for}\: d=0 \wedge f=0 \\ \frac{x}{e+2fx^n} & \text{for}\: d=0 \\ \frac{x}{e} & \text{for}\: f=0 \\ \frac{\log\left(-\frac{e\sqrt{-\frac{1}{df}}}{2}-fx^n\sqrt{-\frac{1}{df}}+x\right)}{4df\sqrt{-\frac{1}{df}}}-\frac{\log\left(\frac{e\sqrt{-\frac{1}{df}}}{2}+fx^n\sqrt{-\frac{1}{df}}+x\right)}{4df\sqrt{-\frac{1}{df}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((e-2*f*(-1+n)*x**n)/(e**2+4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)),
x)
```

```
[Out] Piecewise((x/e, Eq(d, 0) & Eq(f, 0)), (x/(e + 2*f*x**n), Eq(d, 0)), (x/e, E
q(f, 0)), (log(-e*sqrt(-1/(d*f))/2 - f*x**n*sqrt(-1/(d*f)) + x)/(4*d*f*sqrt
(-1/(d*f))) - log(e*sqrt(-1/(d*f))/2 + f*x**n*sqrt(-1/(d*f)) + x)/(4*d*f*sq
rt(-1/(d*f))), True))
```

Maxima [F]

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx = \int -\frac{2\,f(n-1)x^n-e}{4\,dfx^2+4\,f^2x^{2\,n}+4\,efx^n+e^2}\,dx$$

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="maxima")
```

```
[Out] -integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e
^2), x)
```

Giac [F]

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx = \int -\frac{2\,f(n-1)x^n-e}{4\,dfx^2+4\,f^2x^{2\,n}+4\,efx^n+e^2}\,dx$$

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="giac")
```

```
[Out] integrate(-(2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e
^2), x)
```

Mupad [B] (verification not implemented)

Time = 17.51 (sec) , antiderivative size = 196, normalized size of antiderivative = 5.16

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx = \frac{\ln\left(-\frac{e+2\,f\,x^n-2\,f\,n\,x^n}{4\,f^2}-\frac{e^2\,n-4\,d\,f\,x^2+4\,d\,f\,n\,x^2+2\,e\,f\,n\,x^n}{8\,\sqrt{-d}\,f^{5/2}\,x}\right)}{4\,\sqrt{-d}\,\sqrt{f}} - \frac{\mathrm{atan}\left(\frac{x\,(8\,d\,f\,n^2-16\,d\,f\,n+8\,d\,f)}{4\,\sqrt{d}\,\sqrt{f}\,(e\,n-e\,n^2)}\right)}{2\,\sqrt{d}\,\sqrt{f}} - \frac{\ln\left(\frac{e^2\,n-4\,d\,f\,x^2+4\,d\,f\,n\,x^2+2\,e\,f\,n\,x^n}{8\,\sqrt{-d}\,f^{5/2}\,x}-\frac{e+2\,f\,x^n-2\,f\,n\,x^n}{4\,f^2}\right)}{4\,\sqrt{-d}\,\sqrt{f}}$$

```
[In] int((e - 2*f*x^n*(n - 1))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^2 + 4*e*f*x^n),x)
```

```
[Out] log(- (e + 2*f*x^n - 2*f*n*x^n)/(4*f^2) - (e^2*n - 4*d*f*x^2 + 4*d*f*n*x^2
+ 2*e*f*n*x^n)/(8*(-d)^(1/2)*f^(5/2)*x))/(4*(-d)^(1/2)*f^(1/2)) - atan((x*(
8*d*f - 16*d*f*n + 8*d*f*n^2))/(4*d^(1/2)*f^(1/2)*(e*n - e*n^2)))/(2*d^(1/2
)*f^(1/2)) - log((e^2*n - 4*d*f*x^2 + 4*d*f*n*x^2 + 2*e*f*n*x^n)/(8*(-d)^(1
/2)*f^(5/2)*x) - (e + 2*f*x^n - 2*f*n*x^n)/(4*f^2))/(4*(-d)^(1/2)*f^(1/2))
```

3.529 $\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 38

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2118, 214}

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Int[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2118

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
```

```
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\begin{aligned} &\text{integral} \\ &= -\left(\left(e^2(1-n)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(-1+n)^2x^2}\,dx, x, \frac{x}{e(-1+n)+2f(-1+n)x^n}\right)\right) \\ &= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*
n)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(28) = 56.

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

method	result	size
risch	$\frac{\ln\left(x^n+\frac{2dfx+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}-\frac{\ln\left(x^n+\frac{-2dfx+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}$	72

```
[In] int((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)-1/4/(d*f)
^(1/2)*ln(x^n+1/2*(-2*d*f*x+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.79

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx = \left[\frac{\sqrt{df}\log\left(-\frac{4\,dfx^2+4\,f^2x^{2\,n}+4\,\sqrt{df}ex+e^2+4\,(2\,\sqrt{df}fx+ef)x^n}{4\,dfx^2-4\,f^2x^{2\,n}-4\,efx^n-e^2}\right)}{4\,df},\ -\frac{\sqrt{-df}\arctan\left(\frac{2\,\sqrt{-df}fx^n+\sqrt{-df}e}{2\,dfx}\right)}{2\,df}\right]$$

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*sqrt(d*f)*e*x + e^2 + 4*
(2*sqrt(d*f)*f*x + e*f)*x^n)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2))
/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*sqrt(-d*f)*f*x^n + sqrt(-d*f)*e)/(d*f
*x))/(d*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(34) = 68.

Time = 15.59 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.61

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx = \begin{cases} \frac{x}{e} & \text{for } d=0 \wedge f=0 \\ \frac{x}{e+2fx^n} & \text{for } d=0 \\ \frac{x}{e} & \text{for } f=0 \\ -\frac{\log\left(-\frac{e\sqrt{\frac{1}{df}}}{2}-fx^n\sqrt{\frac{1}{df}}+x\right)}{4df\sqrt{\frac{1}{df}}}+\frac{\log\left(\frac{e\sqrt{\frac{1}{df}}}{2}+fx^n\sqrt{\frac{1}{df}}+x\right)}{4df\sqrt{\frac{1}{df}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((e-2*f*(-1+n)*x**n)/(e**2-4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)),
x)
```

```
[Out] Piecewise((x/e, Eq(d, 0) & Eq(f, 0)), (x/(e + 2*f*x**n), Eq(d, 0)), (x/e, E
q(f, 0)), (-log(-e*sqrt(1/(d*f))/2 - f*x**n*sqrt(1/(d*f)) + x)/(4*d*f*sqrt(
1/(d*f))) + log(e*sqrt(1/(d*f))/2 + f*x**n*sqrt(1/(d*f)) + x)/(4*d*f*sqrt(1
/(d*f))), True))
```

Maxima [F]

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\int \frac{2\,f(n-1)x^n-e}{4\,dfx^2-4\,f^2x^{2\,n}-4\,efx^n-e^2}\,dx$$

[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, algorithm="maxima")

[Out] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2), x)

Giac [F]

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\int \frac{2\,f(n-1)x^n-e}{4\,dfx^2-4\,f^2x^{2\,n}-4\,efx^n-e^2}\,dx$$

[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, algorithm="giac")

[Out] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2), x)

Mupad [B] (verification not implemented)

Time = 20.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.66

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\frac{\ln\left(\frac{\left(e+2\,f\,x^n+2\,\sqrt{d}\,\sqrt{f}\,x\right)\,\left(e\,n+2\,\sqrt{d}\,\sqrt{f}\,x-2\,\sqrt{d}\,\sqrt{f}\,n\,x\right)}{x}\right)}{4\,\sqrt{d}\,\sqrt{f}}-\frac{\ln\left(\frac{\left(e+2\,f\,x^n-2\,\sqrt{d}\,\sqrt{f}\,x\right)\,\left(e\,n-2\,\sqrt{d}\,\sqrt{f}\,x+2\,\sqrt{d}\,\sqrt{f}\,n\,x\right)}{x}\right)}{4\,\sqrt{d}\,\sqrt{f}}-\frac{\mathrm{atan}\left(\frac{2\,\sqrt{d}\,\sqrt{f}\,x\,\left(n\,1{}\mathrm{i}-\mathrm{i}\right)}{e\,n}\right)\,1{}\mathrm{i}}{2\,\sqrt{d}\,\sqrt{f}}$$

[In] int((e - 2*f*x^n*(n - 1))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^2 + 4*e*f*x^n),x)

[Out] log(((e + 2*f*x^n + 2*d^(1/2)*f^(1/2)*x)*(e*n + 2*d^(1/2)*f^(1/2)*x - 2*d^(1/2)*f^(1/2)*n*x))/x)/(4*d^(1/2)*f^(1/2)) - log(((e + 2*f*x^n - 2*d^(1/2)*f^(1/2)*x)*(e*n - 2*d^(1/2)*f^(1/2)*x + 2*d^(1/2)*f^(1/2)*n*x))/x)/(4*d^(1/2)*f^(1/2)) - (atan((2*d^(1/2)*f^(1/2)*x*(n*1i - 1i))/(e*n))*1i)/(2*d^(1/2)*f^(1/2))

3.530 $\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 42

$$\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx=\frac{\arctan\left(\frac{\sqrt{f}(e+2(d+f)x^2)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

```
[Out] 1/4*arctan((e+2*(d+f)*x^2)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6, 1121, 632, 210}

$$\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx=\frac{\arctan\left(\frac{\sqrt{f}(2x^2(d+f)+e)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

```
[In] Int[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x}{e^2 + 4efx^2 + 4\left(df + f^2\right)x^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{e^2 + 4efx + 4\left(df + f^2\right)x^2}\,dx, x, x^2\right) \\
&= -\text{Subst}\left(\int \frac{1}{-16de^2f - x^2}\,dx, x, 4f\left(e + 2(d+f)x^2\right)\right) \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{f}\left(e+2(d+f)x^2\right)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$
\int \frac{x}{e^2 + 4efx^2 + 4dfx^4 + 4f^2x^4}\,dx = \frac{\arctan\left(\frac{\sqrt{f}\left(e+2(d+f)x^2\right)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}
$$

```
[In] Integrate[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4),x]
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{2\left(4df+4f^2\right)x^2+4ef}{4\sqrt{df}\,e}\right)}{4\sqrt{df}\,e}$	42
risch	$-\frac{\ln\left(\left(2\sqrt{-df}-2f\right)x^2-e\right)}{8\sqrt{-df}\,e}+\frac{\ln\left(\left(2\sqrt{-df}+2f\right)x^2+e\right)}{8\sqrt{-df}\,e}$	64

```
[In] int(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x,method=_RETURNVERBOSE)
[Out] 1/4/(d*f)^(1/2)/e*arctan(1/4*(2*(4*d*f+4*f^2)*x^2+4*e*f)/(d*f)^(1/2)/e)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.69

$$\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx = \left[-\frac{\sqrt{-df}\log\left(\frac{4\left(d^2f+2\,df^2+f^3\right)x^4-de^2+e^2f+4\left(def+ef^2\right)x^2-2\left(2\left(de+ef\right)x^2+e^2\right)\sqrt{-df}}{4\left(df+f^2\right)x^4+4\,efx^2+e^2}\right)}{8\,def},\ \frac{\sqrt{df}\arctan\left(\frac{\left(2\left(d+f\right)x^2+e\right)\sqrt{df}}{de}\right)}{4\,def}\right]$$

```
[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="fricas")
[Out] [-1/8*sqrt(-d*f)*log((4*(d^2*f + 2*d*f^2 + f^3)*x^4 - d*e^2 + e^2*f + 4*(d*
e*f + e*f^2)*x^2 - 2*(2*(d*e + e*f)*x^2 + e^2)*sqrt(-d*f))/(4*(d*f + f^2)*x
^4 + 4*e*f*x^2 + e^2))/(d*e*f), 1/4*sqrt(d*f)*arctan((2*(d + f)*x^2 + e)*sq
rt(d*f)/(d*e))/(d*e*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

$$\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx = \frac{-\frac{\sqrt{-\frac{1}{df}}\log\left(x^2+\frac{-de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{8}+\frac{\sqrt{-\frac{1}{df}}\log\left(x^2+\frac{de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{8}}{e}$$

```
[In] integrate(x/(4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
[Out] (-sqrt(-1/(d*f))*log(x**2 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/8 + sqrt
(-1/(d*f))*log(x**2 + (d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/8)/e
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{x}{e^2 + 4efx^2 + 4dfx^4 + 4f^2x^4}\,dx = \frac{\arctan\left(\frac{2\,(df+f^2)x^2+ef}{\sqrt{df}e}\right)}{4\,\sqrt{df}e}$$

```
[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")
[Out] 1/4*arctan((2*(d*f + f^2)*x^2 + e*f)/(sqrt(d*f)*e))/(sqrt(d*f)*e)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{x}{e^2 + 4efx^2 + 4dfx^4 + 4f^2x^4}\,dx = \frac{\arctan\left(\frac{2\,dfx^2+2\,f^2x^2+ef}{\sqrt{df}e}\right)}{4\,\sqrt{df}e}$$

```
[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="giac")
[Out] 1/4*arctan((2*d*f*x^2 + 2*f^2*x^2 + e*f)/(sqrt(d*f)*e))/(sqrt(d*f)*e)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x}{e^2 + 4efx^2 + 4dfx^4 + 4f^2x^4}\,dx = \frac{\mathrm{atan}\left(\frac{e\,\sqrt{f}+2\,f^{3/2}\,x^2+2\,d\,\sqrt{f}\,x^2}{\sqrt{d}\,e}\right)}{4\,\sqrt{d}\,e\,\sqrt{f}}$$

```
[In] int(x/(e^2 + 4*f^2*x^4 + 4*d*f*x^4 + 4*e*f*x^2),x)
[Out] atan((e*f^(1/2) + 2*f^(3/2)*x^2 + 2*d*f^(1/2)*x^2)/(d^(1/2)*e))/(4*d^(1/2)*
e*f^(1/2))
```

3.531 $\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4} dx$

Optimal result

Integrand size = 30, antiderivative size = 44

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}(e-2(d-f)x^2)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

```
[Out] -1/4*arctanh((e-2*(d-f)*x^2)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6, 1121, 632, 212}

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}(e-2x^2(d-f))}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

```
[In] Int[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] -1/4*ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^2))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x}{e^2+4efx^2+\left(-4df+4f^2\right)x^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{e^2+4efx+\left(-4df+4f^2\right)x^2}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{16de^2f-x^2}\,dx, x, 4f\left(e-2(d-f)x^2\right)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e+2(-d+f)x^2\right)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx = -\frac{\text{arctanh}\left(\frac{\sqrt{f}\left(e-2dx^2+2fx^2\right)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

```
[In] Integrate[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4),x]
[Out] -1/4*ArcTanh[(Sqrt[f]*(e - 2*d*x^2 + 2*f*x^2))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqrt
[f])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{2\left(4df-4f^2\right)x^2-4ef}{4\sqrt{df}\, e}\right)}{4\sqrt{df}\, e}$	42
risch	$\frac{\ln\left(\left(-2\sqrt{df}-2f\right)x^2-e\right)}{8\sqrt{df}\, e}-\frac{\ln\left(\left(-2\sqrt{df}+2f\right)x^2+e\right)}{8\sqrt{df}\, e}$	60

```
[In] int(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x,method=_RETURNVERBOSE)
[Out] 1/4/(d*f)^(1/2)/e*arctanh(1/4*(2*(4*d*f-4*f^2)*x^2-4*e*f)/(d*f)^(1/2)/e)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.82

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx = \left[\frac{\sqrt{df}\log\left(-\frac{4\,(d^2f-2\,df^2+f^3)x^4+de^2+e^2f-4\,(def-ef^2)x^2+2\,(2\,(de-ef)x^2-e^2)\sqrt{df}}{4\,(df-f^2)x^4-4\,efx^2-e^2}\right)}{8\,def}, \frac{\sqrt{-df}\arctan\left(-\frac{(2\,(d-f)x^2-e)\sqrt{-df}}{de}\right)}{4\,def}\right]$$

```
[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="fricas")
[Out] [1/8*sqrt(d*f)*log(-(4*(d^2*f - 2*d*f^2 + f^3)*x^4 + d*e^2 + e^2*f - 4*(d*e
*f - e*f^2)*x^2 + 2*(2*(d*e - e*f)*x^2 - e^2)*sqrt(d*f))/(4*(d*f - f^2)*x^4
 - 4*e*f*x^2 - e^2))/(d*e*f), 1/4*sqrt(-d*f)*arctan(-(2*(d - f)*x^2 - e)*sq
rt(-d*f)/(d*e))/(d*e*f)]
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx = -\frac{\frac{\sqrt{\frac{1}{df}}\log\left(x^2+\frac{-de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{8}-\frac{\sqrt{\frac{1}{df}}\log\left(x^2+\frac{de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{8}}{e}$$

```
[In] integrate(x/(-4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
[Out] -(sqrt(1/(d*f))*log(x**2 + (-d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/8 - sqrt(1
/(d*f))*log(x**2 + (d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/8)/e
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.52

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx = \frac{\log\left(\frac{2\,(df-f^2)x^2-ef+\sqrt{df}e}{2\,(df-f^2)x^2-ef-\sqrt{df}e}\right)}{8\sqrt{df}e}$$

```
[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")
```

```
[Out] 1/8*log((2*(d*f - f^2)*x^2 - e*f + sqrt(d*f)*e)/(2*(d*f - f^2)*x^2 - e*f -
sqrt(d*f)*e))/(sqrt(d*f)*e)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx = -\frac{\arctan\left(\frac{2\,dfx^2-2\,f^2x^2-ef}{\sqrt{-df}e}\right)}{4\sqrt{-df}e}$$

```
[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="giac")
```

```
[Out] -1/4*arctan((2*d*f*x^2 - 2*f^2*x^2 - e*f)/(sqrt(-d*f)*e))/(sqrt(-d*f)*e)
```

Mupad [B] (verification not implemented)

Time = 17.48 (sec) , antiderivative size = 199, normalized size of antiderivative = 4.52

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx = \frac{\operatorname{atanh}\left(\frac{16\,d^{3/2}\,f^{3/2}\,x^2}{\frac{8\,e\,f^3}{d}-32\,f^3\,x^2-16\,e\,f^2+16\,d\,f^2\,x^2+8\,d\,e\,f+\frac{16\,f^4\,x^2}{d}}-\frac{32\,\sqrt{d}\,f^{5/2}\,x^2}{\frac{8\,e\,f^3}{d}-32\,f^3\,x^2-16\,e\,f^2+16\,d\,f^2\,x^2+8\,d\,e\,f+\frac{16\,f^4\,x^2}{d}}+\frac{\cdots}{\sqrt{d}\left(\frac{8\,e\,f^3}{d}-32\,f^3\cdots\right)}\right)}{4\,\sqrt{d}\,e\,\sqrt{f}}$$

```
[In] int(x/(e^2 + 4*f^2*x^4 - 4*d*f*x^4 + 4*e*f*x^2),x)
```

```
[Out] atanh((16*d^(3/2)*f^(3/2)*x^2)/((8*e*f^3)/d - 32*f^3*x^2 - 16*e*f^2 + 16*d*
f^2*x^2 + 8*d*e*f + (16*f^4*x^2)/d) - (32*d^(1/2)*f^(5/2)*x^2)/((8*e*f^3)/d
 - 32*f^3*x^2 - 16*e*f^2 + 16*d*f^2*x^2 + 8*d*e*f + (16*f^4*x^2)/d) + (16*f
^(7/2)*x^2)/(d^(1/2)*((8*e*f^3)/d - 32*f^3*x^2 - 16*e*f^2 + 16*d*f^2*x^2 +
8*d*e*f + (16*f^4*x^2)/d)))/(4*d^(1/2)*e*f^(1/2))
```

3.532 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 40

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x^3*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2119, 211}

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),x]
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
```

```
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\text{integral} = (3e^2)\,\text{Subst}\left(\int \frac{1}{e^2+36de^2fx^2}\,dx, x, \frac{x^3}{3e+6fx^2}\right)$$
$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.12

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx = \frac{\text{RootSum}\left[e^2+4ef\#1^2+4f^2\#1^4+4df\#1^6\&, \frac{3e\log(x-\#1)\#1+2f\log(x-\#1)\#1^3}{e+2f\#1^2+3d\#1^4}\&\right]}{8f}$$

```
[In] Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),x
]
```

```
[Out] RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 + 4*d*f*#1^6 & , (3*e*Log[x - #1]*#1
+ 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 + 3*d*#1^4) & ]/(8*f)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.85

method	result	size
default	$\frac{\sum_{\_R=\text{RootOf}\left(4df\_Z^6+4f^2\_Z^4+4ef\_Z^2+e^2\right)} \frac{\left(2\_R^4f+3e\_R^2\right)\ln\left(x-\_R\right)}{3d\_R^5+2\_R^3f+e\_R}}{8f}$	74
risch	$-\frac{\ln\left(-2d^2f^2x^3+2(-df)^{\frac{3}{2}}fx^2+(-df)^{\frac{3}{2}}e\right)}{4\sqrt{-df}}+\frac{\ln\left(2d^2f^2x^3+2(-df)^{\frac{3}{2}}fx^2+(-df)^{\frac{3}{2}}e\right)}{4\sqrt{-df}}$	84

```
[In] int(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d+2*_R^3*f+_R*e)*ln(x-_R),_R=RootOf(4
*_Z^6*d*f+4*_Z^4*f^2+4*_Z^2*e*f+e^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(30) = 60.

Time = 0.31 (sec) , antiderivative size = 208, normalized size of antiderivative = 5.20

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx = \left[-\frac{\sqrt{-df}\log\left(\frac{4\,dfx^6-4\,f^2x^4-4\,efx^2-e^2-4\,(2\,fx^5+ex^3)\sqrt{-df}}{4\,dfx^6+4\,f^2x^4+4\,efx^2+e^2}\right)}{4\,df}, \frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x}{f}\right)-\sqrt{df}\arctan\left(\frac{2\,(2\,dfx^5-(de-2\,f^2)x^3}{de^2}\right.}{2\,df}\right.$$

```
[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorith
m="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2 - 4*(2*f*x^5
+ e*x^3)*sqrt(-d*f))/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2))/(d*f), 1/2*
(sqrt(d*f)*arctan(sqrt(d*f)*x/f) - sqrt(d*f)*arctan(2*(2*d*f*x^5 - (d*e - 2
*f^2)*x^3 + e*f*x)*sqrt(d*f)/(d*e^2)) + sqrt(d*f)*arctan((2*d*f*x^3 - (d*e
- 2*f^2)*x)*sqrt(d*f)/(d*e*f)))/(d*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(36) = 72.

Time = 0.65 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx = -\frac{\sqrt{-\frac{1}{df}}\log\left(-\frac{e\sqrt{-\frac{1}{df}}}{2}-fx^2\sqrt{-\frac{1}{df}}+x^3\right)}{4} + \frac{\sqrt{-\frac{1}{df}}\log\left(\frac{e\sqrt{-\frac{1}{df}}}{2}+fx^2\sqrt{-\frac{1}{df}}+x^3\right)}{4}$$

```
[In] integrate(x**2*(2*f*x**2+3*e)/(4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(-1/(d*f))*log(-e*sqrt(-1/(d*f))/2 - f*x**2*sqrt(-1/(d*f)) + x**3)/4 +
 sqrt(-1/(d*f))*log(e*sqrt(-1/(d*f))/2 + f*x**2*sqrt(-1/(d*f)) + x**3)/4
```

Maxima [F]

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx = \int \frac{(2\,fx^2+3\,e)x^2}{4\,dfx^6+4\,f^2x^4+4\,efx^2+e^2}\,dx$$

```
[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorith
m="maxima")
```

```
[Out] integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2), x)
```

Giac [F]

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx = \int \frac{(2\,fx^2+3\,e)x^2}{4\,dfx^6+4\,f^2x^4+4\,efx^2+e^2}\,dx$$

```
[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorith
m="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 17.47 (sec) , antiderivative size = 278, normalized size of antiderivative = 6.95

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx = \frac{\mathrm{atan}\left(\frac{2\,f^2\,x+2\,d\,f\,x^3-d\,e\,x}{\sqrt{d}\,e\,\sqrt{f}}\right)-\mathrm{atan}\left(\frac{1984\,d^{3/2}\,f^{9/2}\,x^3}{432\,d^2\,e^2\,f^2-128\,d\,e\,f^4}+\frac{1728\,d^{5/2}\,f^{7/2}\,x^5}{432\,d^2\,e^2\,f^2-128\,d\,e\,f^4}+\frac{512\,\sqrt{d}\,f^{13/2}\,x^3}{128\,d\,e^2\,f^4-432\,d^2\,e^3\,f^2}+\frac{512\,d^{3/2}\,f^{1}}{128\,d\,e^2\,f^4-43}\right.}{2\,\sqrt{d}\,\sqrt{f}}$$

```
[In] int((x^2*(3*e + 2*f*x^2))/(e^2 + 4*f^2*x^4 + 4*d*f*x^6 + 4*e*f*x^2),x)
```

```
[Out] (atan((2*f^2*x + 2*d*f*x^3 - d*e*x)/(d^(1/2)*e*f^(1/2))) - atan((1984*d^(3/
2)*f^(9/2)*x^3)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (1728*d^(5/2)*f^(7/2)*x^5
)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (512*d^(1/2)*f^(13/2)*x^3)/(128*d*e^2*f
^4 - 432*d^2*e^3*f^2) + (512*d^(3/2)*f^(11/2)*x^5)/(128*d*e^2*f^4 - 432*d^2
*e^3*f^2) - (256*d^(1/2)*f^(11/2)*x)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (864
*d^(3/2)*e*f^(7/2)*x)/(432*d^2*e^2*f^2 - 128*d*e*f^4) - (864*d^(5/2)*e*f^(5
/2)*x^3)/(432*d^2*e^2*f^2 - 128*d*e*f^4)) + atan((d^(1/2)*x)/f^(1/2)))/(2*d
^(1/2)*f^(1/2))
```

3.533 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 40

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx = \frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x^3*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2119, 214}

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx = \frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),x]
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
```

```
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\text{integral} = (3e^2)\,\text{Subst}\left(\int \frac{1}{e^2 - 36de^2fx^2}\,dx, x, \frac{x^3}{3e + 6fx^2}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.12

$$\int \frac{x^2(3e + 2fx^2)}{e^2 + 4efx^2 + 4f^2x^4 - 4dfx^6}\,dx = \frac{\text{RootSum}\left[e^2 + 4ef\#1^2 + 4f^2\#1^4 - 4df\#1^6\&, \frac{3e\log(x-\#1)\#1+2f\log(x-\#1)\#1^3}{e+2f\#1^2-3d\#1^4}\&\right]}{8f}$$

```
[In] Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),x
]
```

```
[Out] RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 - 4*d*f*#1^6 & , (3*e*Log[x - #1]*#1
+ 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 - 3*d*#1^4) & ]/(8*f)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.92

method	result	size
default	$-\frac{\sum_{\_R=\text{RootOf}\left(4df\_Z^6-4f^2\_Z^4-4ef\_Z^2-e^2\right)}\frac{\left(2\_R^4f+3e\_R^2\right)\ln\left(x-\_R\right)}{3d\_R^5-2\_R^3f-e\_R}}{8f}$	77
risch	$\frac{\ln\left(-2d^2f^2x^3-2(df)^{\frac{3}{2}}fx^2-(df)^{\frac{3}{2}}e\right)}{4\sqrt{df}}-\frac{\ln\left(2d^2f^2x^3-2(df)^{\frac{3}{2}}fx^2-(df)^{\frac{3}{2}}e\right)}{4\sqrt{df}}$	80

```
[In] int(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d-2*_R^3*f-_R*e)*ln(x-_R),_R=RootOf(
4*_Z^6*d*f-4*_Z^4*f^2-4*_Z^2*e*f-e^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(30) = 60$.

Time = 0.30 (sec) , antiderivative size = 213, normalized size of antiderivative = 5.32

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx = \left[\frac{\sqrt{df}\log\left(\frac{4\,dfx^6+4\,f^2x^4+4\,efx^2+e^2+4\,(2\,fx^5+ex^3)\sqrt{df}}{4\,dfx^6-4\,f^2x^4-4\,efx^2-e^2}\right)}{4\,df}, -\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}x}{f}\right)-\sqrt{-df}\arctan\left(\frac{2\,(2\,dfx^5-(de+2\,f^2)x^3-efx)\sqrt{-df}}{de^2}\right)+\sqrt{-df}\arctan\left(\frac{(2\,dfx^3-(de+2\,f^2)x)\sqrt{-df}}{def}\right)}{2\,df}\right]$$

```
[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2 + 4*(2*f*x^5 +
e*x^3)*sqrt(d*f))/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2))/(d*f), -1/2*(s
qrt(-d*f)*arctan(sqrt(-d*f)*x/f) - sqrt(-d*f)*arctan(2*(2*d*f*x^5 - (d*e +
2*f^2)*x^3 - e*f*x)*sqrt(-d*f)/(d*e^2)) + sqrt(-d*f)*arctan((2*d*f*x^3 - (d
*e + 2*f^2)*x)*sqrt(-d*f)/(d*e*f)))/(d*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(36) = 72$.

Time = 0.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx = -\frac{\sqrt{\frac{1}{df}}\log\left(-\frac{e\sqrt{\frac{1}{df}}}{2}-fx^2\sqrt{\frac{1}{df}}+x^3\right)}{4} + \frac{\sqrt{\frac{1}{df}}\log\left(\frac{e\sqrt{\frac{1}{df}}}{2}+fx^2\sqrt{\frac{1}{df}}+x^3\right)}{4}$$

```
[In] integrate(x**2*(2*f*x**2+3*e)/(-4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-e*sqrt(1/(d*f))/2 - f*x**2*sqrt(1/(d*f)) + x**3)/4 + sq
rt(1/(d*f))*log(e*sqrt(1/(d*f))/2 + f*x**2*sqrt(1/(d*f)) + x**3)/4
```

Maxima [F]

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx = \int -\frac{(2\,fx^2+3\,e)x^2}{4\,dfx^6-4\,f^2x^4-4\,efx^2-e^2}\,dx$$

```
[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="maxima")
```

```
[Out] -integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2), x
)
```

Giac [F]

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx = \int -\frac{(2\,fx^2+3\,e)x^2}{4\,dfx^6-4\,f^2x^4-4\,efx^2-e^2}\,dx$$

```
[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx = \frac{\mathrm{atanh}\left(\frac{2\,\sqrt{d}\,\sqrt{f}\,x^3}{2\,f\,x^2+e}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

```
[In] int((x^2*(3*e + 2*f*x^2))/(e^2 + 4*f^2*x^4 - 4*d*f*x^6 + 4*e*f*x^2),x)
```

```
[Out] atanh((2*d^(1/2)*f^(1/2)*x^3)/(e + 2*f*x^2))/(2*d^(1/2)*f^(1/2))
```

3.534 $\int \frac{x^m\left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx$

Optimal result

Integrand size = 51, antiderivative size = 42

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2119, 211}

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}(1-m^2)x^{m+1}}{(1-m)(m+1)(e+2fx^2)}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*(1 - m^2)*x^(1 + m))/((1 - m)*(1 + m)*(e + 2*f*x^
2))]/(2*Sqrt[d]*Sqrt[f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\text{integral} = -\left(\left(e^2(1-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(-1+m)^2(1+m)^2x^2}\,dx, x, \frac{x^{1+m}}{e(-1+m)(1+m)+2f(-1+$$

$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}(1-m^2)x^{1+m}}{(1-m)(1+m)(e+2fx^2)}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx = \frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4
 + 4*d*f*x^(2 + 2*m)),x]
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(32) = 64.

Time = 1.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

method	result	size
risch	$-\frac{\ln\left(x^m+\frac{(2fx^2+e)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}+\frac{\ln\left(x^m-\frac{(2fx^2+e)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}$	78

```
[In] int(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+2*m)),
x,method=_RETURNVERBOSE)
[Out] -1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)+1/4/(-d*f)^(1/
2)*ln(x^m-1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.48

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx = \left[-\frac{\sqrt{-df}\log\left(-\frac{4f^2x^4-4dfx^2x^{2m}+4efx^2+4(2fx^3+ex)\sqrt{-df}x^m+e^2}{4f^2x^4+4dfx^2x^{2m}+4efx^2+e^2}\right)}{4df}, -\frac{\sqrt{df}\arctan\left(\frac{(2fx^2+e)\sqrt{df}}{2dfxx^m}\right)}{2df}\right]$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*f^2*x^4 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + 4*(2*f*x
^3 + e*x)*sqrt(-d*f)*x^m + e^2)/(4*f^2*x^4 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^2
+ e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*f*x^2 + e)*sqrt(d*f)/(d*f*x*x^m
))/(d*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(37) = 74.

Time = 98.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.90

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx = \begin{cases} \frac{xx^m}{e} & \text{for } d=0 \wedge f=0 \\ \frac{xx^m}{e+2fx^2} & \text{for } d=0 \\ \frac{xx^m}{e} & \text{for } f=0 \\ \frac{\log\left(-\frac{e\sqrt{-\frac{1}{df}}}{2x}-fx\sqrt{-\frac{1}{df}}+x^m\right)}{4df\sqrt{-\frac{1}{df}}} - \frac{\log\left(\frac{e\sqrt{-\frac{1}{df}}}{2x}+fx\sqrt{-\frac{1}{df}}+x^m\right)}{4df\sqrt{-\frac{1}{df}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4+4*d*f
*x**(2+2*m)),x)
```

```
[Out] Piecewise((x*x**m/e, Eq(d, 0) & Eq(f, 0)), (x*x**m/(e + 2*f*x**2), Eq(d, 0)
), (x*x**m/e, Eq(f, 0)), (log(-e*sqrt(-1/(d*f))/(2*x) - f*x*sqrt(-1/(d*f))
+ x**m)/(4*d*f*sqrt(-1/(d*f))) - log(e*sqrt(-1/(d*f))/(2*x) + f*x*sqrt(-1/(
d*f)) + x**m)/(4*d*f*sqrt(-1/(d*f))), True))
```

Maxima [F]

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-1)x^2+e(m+1))x^m}{4\,f^2x^4+4\,efx^2+4\,dfx^{2\,m+2}+e^2}\,dx$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

Giac [F]

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-1)x^2+e(m+1))x^m}{4\,f^2x^4+4\,efx^2+4\,dfx^{2\,m+2}+e^2}\,dx$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx = \int \frac{x^m\,\left(2\,f\,\left(m-1\right)\,x^2+e\,\left(m+1\right)\right)}{e^2+4\,f^2\,x^4+4\,e\,f\,x^2+4\,d\,f\,x^{2\,m+2}}\,dx$$

```
[In] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 + 4*d*
f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 + 4*d*
f*x^(2*m + 2)), x)
```

3.535 $\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx$

Optimal result

Integrand size = 51, antiderivative size = 42

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2119, 214}

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}(1-m^2)x^{m+1}}{(1-m)(m+1)(e+2fx^2)}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*(1 - m^2)*x^(1 + m))/((1 - m)*(1 + m)*(e + 2*f*x
^2))]/(2*Sqrt[d]*Sqrt[f])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\text{integral} = -\left(\left(e^2(1-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(-1+m)^2(1+m)^2x^2}\,dx, x, \frac{x^{1+m}}{e(-1+m)(1+m)+2f(-1+$$

$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}(1-m^2)x^{1+m}}{(1-m)(1+m)(e+2fx^2)}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4
 - 4*d*f*x^(2 + 2*m)),x]
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(32) = 64.

Time = 1.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.76

method	result	size
risch	$\frac{\ln\left(x^m+\frac{(2fx^2+e)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}-\frac{\ln\left(x^m-\frac{(2fx^2+e)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}$	74

```
[In] int(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+2*m)),
x,method=_RETURNVERBOSE)
[Out] 1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*f)^(1/2)*l
n(x^m-1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.48

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = \left[\frac{\sqrt{df}\log\left(-\frac{4\,f^2x^4+4\,dfx^2x^{2\,m}+4\,efx^2+4\,(2\,fx^3+ex)\sqrt{df}x^m+e^2}{4\,f^2x^4-4\,dfx^2x^{2\,m}+4\,efx^2+e^2}\right)}{4\,df}, -\frac{\sqrt{-df}\arctan\left(\frac{(2\,fx^2+e)\sqrt{-df}}{2\,dfxx^m}\right)}{2\,df}\right]$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*f^2*x^4 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + 4*(2*f*x^3
 + e*x)*sqrt(d*f)*x^m + e^2)/(4*f^2*x^4 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + e
^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*f*x^2 + e)*sqrt(-d*f)/(d*f*x*x^m)
)/(d*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(37) = 74.

Time = 97.94 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.67

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = \begin{cases} \frac{xx^m}{e} & \text{for } d=0 \wedge f=0 \\ \frac{xx^m}{e+2fx^2} & \text{for } d=0 \\ \frac{xx^m}{e} & \text{for } f=0 \\ -\frac{\log\left(-\frac{e\sqrt{\frac{1}{df}}}{2x}-fx\sqrt{\frac{1}{df}}+x^m\right)}{4df\sqrt{\frac{1}{df}}}+\frac{\log\left(\frac{e\sqrt{\frac{1}{df}}}{2x}+fx\sqrt{\frac{1}{df}}+x^m\right)}{4df\sqrt{\frac{1}{df}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4-4*d*f
*x**(2+2*m)),x)
```

```
[Out] Piecewise((x*x**m/e, Eq(d, 0) & Eq(f, 0)), (x*x**m/(e + 2*f*x**2), Eq(d, 0)
), (x*x**m/e, Eq(f, 0)), (-log(-e*sqrt(1/(d*f))/(2*x) - f*x*sqrt(1/(d*f)) +
 x**m)/(4*d*f*sqrt(1/(d*f))) + log(e*sqrt(1/(d*f))/(2*x) + f*x*sqrt(1/(d*f)
) + x**m)/(4*d*f*sqrt(1/(d*f))), True))
```

Maxima [F]

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-1)x^2+e(m+1))x^m}{4\,f^2x^4+4\,efx^2-4\,dfx^{2\,m+2}+e^2}\,dx$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

Giac [F]

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-1)x^2+e(m+1))x^m}{4\,f^2x^4+4\,efx^2-4\,dfx^{2\,m+2}+e^2}\,dx$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = \int \frac{x^m\,\left(2\,f\,\left(m-1\right)\,x^2+e\,\left(m+1\right)\right)}{e^2+4\,f^2\,x^4+4\,e\,f\,x^2-4\,d\,f\,x^{2\,m+2}}\,dx$$

```
[In] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 - 4*d*
f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 - 4*d*
f*x^(2*m + 2)), x)
```

3.536 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6} dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6} dx = \frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x^2*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2119, 211}

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6} dx = \frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
```

```
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\text{integral} = -\left((2e^2)\,\text{Subst}\left(\int \frac{1}{e^2+16de^2fx^2}\,dx, x, \frac{x^2}{-2e-4fx^3}\right)\right)$$
$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.15

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx$$
$$= -\frac{\text{RootSum}\left[e^2+4ef\#1^3+4df\#1^4+4f^2\#1^6\&, \frac{-e\log(x-\#1)+f\log(x-\#1)\#1^3}{3e\#1+4d\#1^2+6f\#1^4}\&\right]}{2f}$$

```
[In] Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]
[Out] -1/2*RootSum[e^2 + 4*e*f*#1^3 + 4*d*f*#1^4 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + f*Log[x - #1]*#1^3)/(3*e*#1 + 4*d*#1^2 + 6*f*#1^4) & ]/f
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.85

method	result
default	$\frac{\sum_{\_R=\text{RootOf}\left(4f^2\_Z^6+4df\_Z^4+4ef\_Z^3+e^2\right)} \frac{\left(-\_R^4 f+e\_R\right)\ln\left(x-\_R\right)}{6f\_R^5+4d\_R^3+3e\_R^2}}{2f}$
risch	$-\frac{\ln\left(\left(16f(-df)^{\frac{3}{2}}d+54d f^3e\right)x^3+\left(54(-df)^{\frac{3}{2}}ef-16d^3f^2\right)x^2+8e(-df)^{\frac{3}{2}}d+27e^2f^2d\right)}{4\sqrt{-df}}+\frac{\ln\left(\left(16f(-df)^{\frac{3}{2}}d-54d f^3e\right)x^3+\left(54(-d\right.\right.}{4\sqrt{}}$

```
[In] int(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/2/f*sum((-_R^4*f+_R*e)/(6*_R^5*f+4*_R^3*d+3*_R^2*e)*ln(x-_R),_R=RootOf(4*
_Z^6*f^2+4*_Z^4*d*f+4*_Z^3*e*f+e^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(30) = 60$.

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.82

$$\int \frac{x(2e - 2fx^3)}{e^2 + 4efx^3 + 4dfx^4 + 4f^2x^6}\,dx = \left[-\frac{\sqrt{-df}\log\left(\frac{4f^2x^6 - 4dfx^4 + 4efx^3 + e^2 + 4(2fx^5 + ex^2)\sqrt{-df}}{4f^2x^6 + 4dfx^4 + 4efx^3 + e^2}\right)}{4df}, -\frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x}{d}\right) - \sqrt{df}\arctan\left(\frac{(2fx^4 + 2dx^2 + ex)\sqrt{df}}{de}\right)}{2df}\right]$$

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2 + 4*(2*f*x^5
+ e*x^2)*sqrt(-d*f))/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2))/(d*f), -1/2
*(sqrt(d*f)*arctan(sqrt(d*f)*x/d) - sqrt(d*f)*arctan((2*f*x^4 + 2*d*x^2 + e
*x)*sqrt(d*f)/(d*e)))/(d*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(36) = 72$.

Time = 0.61 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.82

$$\int \frac{x(2e - 2fx^3)}{e^2 + 4efx^3 + 4dfx^4 + 4f^2x^6}\,dx = \frac{\sqrt{-\frac{1}{df}}\log\left(-dx^2\sqrt{-\frac{1}{df}} + \frac{e}{2f} + x^3\right)}{4} - \frac{\sqrt{-\frac{1}{df}}\log\left(dx^2\sqrt{-\frac{1}{df}} + \frac{e}{2f} + x^3\right)}{4}$$

```
[In] integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6+4*d*f*x**4+4*e*f*x**3+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x**2*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4 - sqrt(-1/(d*
f))*log(d*x**2*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4
```

Maxima [F]

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx = \int -\frac{2\,(fx^3-e)x}{4\,f^2x^6+4\,dfx^4+4\,efx^3+e^2}\,dx$$

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="maxima")
```

```
[Out] -2*integrate((f*x^3 - e)*x/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(30) = 60$.

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx = -\frac{\sqrt{-df}\log\left(\left|2\,fx^3+2\,\sqrt{-df}x^2+e\right|\right)}{4\,df} + \frac{\sqrt{-df}\log\left(\left|2\,fx^3-2\,\sqrt{-df}x^2+e\right|\right)}{4\,df}$$

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="giac")
```

```
[Out] -1/4*sqrt(-d*f)*log(abs(2*f*x^3 + 2*sqrt(-d*f)*x^2 + e))/(d*f) + 1/4*sqrt(-
d*f)*log(abs(2*f*x^3 - 2*sqrt(-d*f)*x^2 + e))/(d*f)
```

Mupad [B] (verification not implemented)

Time = 16.92 (sec) , antiderivative size = 233, normalized size of antiderivative = 5.82

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx = \frac{\operatorname{atan}\left(\frac{128\,d^{7/2}\,\sqrt{f}\,x^2}{64\,d^3\,e+729\,f\,e^3} - \frac{216\,d^{3/2}\,e^2\,\sqrt{f}}{64\,d^3\,e+729\,f\,e^3} + \frac{128\,d^{5/2}\,f^{3/2}\,x^4}{64\,d^3\,e+729\,f\,e^3} + \frac{216\,d^{3/2}\,e\,\sqrt{f}}{64\,d^3+729\,f\,e^2} + \frac{729\,d^{3/2}\,e^2\,f^{3/2}\,x}{64\,d^5+729\,f\,d^2\,e^2} + \frac{1458\,d^{3/2}\,e\,f^{5/2}\,x^4}{64\,d^5+729\,f\,d^2\,e^2} + \frac{64\,d^{5/}}{64\,d^3\,e-}\right.}{2\,\sqrt{d}\,\sqrt{f}}$$

```
[In] int((x*(2*e - 2*f*x^3))/(e^2 + 4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3),x)
```

```
[Out] (atan((128*d^(7/2)*f^(1/2)*x^2)/(64*d^3*e + 729*e^3*f) - (216*d^(3/2)*e^2*f
^(1/2))/(64*d^3*e + 729*e^3*f) + (128*d^(5/2)*f^(3/2)*x^4)/(64*d^3*e + 729*
e^3*f) + (216*d^(3/2)*e*f^(1/2))/(729*e^2*f + 64*d^3) + (729*d^(3/2)*e^2*f^
(3/2)*x)/(64*d^5 + 729*d^2*e^2*f) + (1458*d^(3/2)*e*f^(5/2)*x^4)/(64*d^5 +
729*d^2*e^2*f) + (64*d^(5/2)*e*f^(1/2)*x)/(64*d^3*e + 729*e^3*f) + (1458*d^
(3/2)*e*f^(3/2)*x^2)/(64*d^4 + 729*d*e^2*f)) - atan((f^(1/2)*x)/d^(1/2)))/(
2*d^(1/2)*f^(1/2))
```

3.537 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x^2*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2119, 214}

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
```

```
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left((2e^2)\,\text{Subst}\left(\int \frac{1}{e^2-16de^2fx^2}\,dx, x, \frac{x^2}{-2e-4fx^3}\right)\right)\\ &= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.15

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx = -\frac{\text{RootSum}\left[e^2+4ef\#1^3-4df\#1^4+4f^2\#1^6\&, \frac{-e\log(x-\#1)+f\log(x-\#1)\#1^3}{3e\#1-4d\#1^2+6f\#1^4}\&\right]}{2f}$$

```
[In] Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]
[Out] -1/2*RootSum[e^2 + 4*e*f*#1^3 - 4*d*f*#1^4 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + f*Log[x - #1]*#1^3)/(3*e*#1 - 4*d*#1^2 + 6*f*#1^4) & ]/f
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.85

method	result
default	$\frac{\sum_{\_R=\text{RootOf}\left(4f^2\_Z^6-4df\_Z^4+4ef\_Z^3+e^2\right)}\frac{\left(\_R^4 f-e\_R\right)\ln\left(x-\_R\right)}{-6f\_R^5+4d\_R^3-3e\_R^2}}{2f}$
risch	$\frac{\ln\left(\left(-16f(df)^{\frac{3}{2}}d+54df^3e\right)x^3+\left(54(df)^{\frac{3}{2}}ef-16d^3f^2\right)x^2-8e(df)^{\frac{3}{2}}d+27e^2f^2d\right)}{4\sqrt{df}}-\frac{\ln\left(\left(-16f(df)^{\frac{3}{2}}d-54df^3e\right)x^3+\left(54(df)^{\frac{3}{2}}ef+\right.\right.}{4\sqrt{df}}$

```
[In] int(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/2/f*sum((_R^4*f-_R*e)/(-6*_R^5*f+4*_R^3*d-3*_R^2*e)*ln(x-_R),_R=RootOf(4*
_Z^6*f^2-4*_Z^4*d*f+4*_Z^3*e*f+e^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(30) = 60$.

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.88

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx = \left[\frac{\sqrt{df}\log\left(\frac{4\,f^2x^6+4\,dfx^4+4\,efx^3+e^2+4\,(2\,fx^5+ex^2)\sqrt{df}}{4\,f^2x^6-4\,dfx^4+4\,efx^3+e^2}\right)}{4\,df}, -\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}x}{d}\right)-\sqrt{-df}\arctan\left(\frac{(2\,fx^4-2\,dx^2+ex)\sqrt{-df}}{de}\right)}{2\,df}\right]$$

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2 + 4*(2*f*x^5 +
e*x^2)*sqrt(d*f))/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2))/(d*f), -1/2*(s
qrt(-d*f)*arctan(sqrt(-d*f)*x/d) - sqrt(-d*f)*arctan((2*f*x^4 - 2*d*x^2 + e
*x)*sqrt(-d*f)/(d*e)))/(d*f)]
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx = -\frac{\sqrt{\frac{1}{df}}\log\left(-dx^2\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4} + \frac{\sqrt{\frac{1}{df}}\log\left(dx^2\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

```
[In] integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6-4*d*f*x**4+4*e*f*x**3+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x**2*sqrt(1/(d*f)) + e/(2*f) + x**3)/4 + sqrt(1/(d*f)
)*log(d*x**2*sqrt(1/(d*f)) + e/(2*f) + x**3)/4
```

Maxima [F]

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx = \int -\frac{2\,(fx^3-e)x}{4\,f^2x^6-4\,dfx^4+4\,efx^3+e^2}\,dx$$

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="maxima")
```

```
[Out] -2*integrate((f*x^3 - e)*x/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(30) = 60$.

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.68

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx = \frac{\sqrt{df}\log\left(\left|2\,fx^3+2\,\sqrt{df}x^2+e\right|\right)}{4\,df} - \frac{\sqrt{df}\log\left(\left|2\,fx^3-2\,\sqrt{df}x^2+e\right|\right)}{4\,df}$$

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="giac")
```

```
[Out] 1/4*sqrt(d*f)*log(abs(2*f*x^3 + 2*sqrt(d*f)*x^2 + e))/(d*f) - 1/4*sqrt(d*f)
*log(abs(2*f*x^3 - 2*sqrt(d*f)*x^2 + e))/(d*f)
```

Mupad [B] (verification not implemented)

Time = 17.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.68

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx = -\frac{\operatorname{atanh}\left(\frac{27\,e^2\,\sqrt{f}+54\,e\,f^{3/2}\,x^3-16\,d^2\,\sqrt{f}\,x^2}{8\,d^{3/2}\,e+16\,d^{3/2}\,f\,x^3-54\,\sqrt{d}\,e\,f\,x^2}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

```
[In] int((x*(2*e - 2*f*x^3))/(e^2 + 4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3),x)
```

```
[Out] -atanh((27*e^2*f^(1/2) + 54*e*f^(3/2)*x^3 - 16*d^2*f^(1/2)*x^2)/(8*d^(3/2)*
e + 16*d^(3/2)*f*x^3 - 54*d^(1/2)*e*f*x^2))/(2*d^(1/2)*f^(1/2))
```

3.538 $\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 42

$$\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx=\frac{\arctan\left(\frac{\sqrt{f}(e+2(d+f)x^3)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

```
[Out] 1/6*arctan((e+2*(d+f)*x^3)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6, 1366, 632, 210}

$$\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx=\frac{\arctan\left(\frac{\sqrt{f}(2x^3(d+f)+e)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

```
[In] Int[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2}{e^2+4efx^3+4\left(df+f^2\right)x^6}\,dx \\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{e^2+4efx+4\left(df+f^2\right)x^2}\,dx, x, x^3\right) \\
&= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{-16de^2f-x^2}\,dx, x, 4f\left(e+2(d+f)x^3\right)\right)\right) \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{f}\left(e+2(d+f)x^3\right)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx = \frac{\arctan\left(\frac{\sqrt{f}\left(e+2(d+f)x^3\right)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

```
[In] Integrate[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{2\left(4df+4f^2\right)x^3+4ef}{4\sqrt{df}\,e}\right)}{6\sqrt{df}\,e}$	42
risch	$-\frac{\ln\left(\left(2\sqrt{-df}-2f\right)x^3-e\right)}{12\sqrt{-df}\,e}+\frac{\ln\left(\left(2\sqrt{-df}+2f\right)x^3+e\right)}{12\sqrt{-df}\,e}$	64

[In] int(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x,method=_RETURNVERBOSE)

[Out] 1/6/(d*f)^(1/2)/e*arctan(1/4*(2*(4*d*f+4*f^2)*x^3+4*e*f)/(d*f)^(1/2)/e)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.69

$$\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx = \left[-\frac{\sqrt{-df}\log\left(\frac{4\left(d^2f+2\,df^2+f^3\right)x^6+4\left(def+ef^2\right)x^3-de^2+e^2f-2\left(2\left(de+ef\right)x^3+e^2\right)\sqrt{-df}}{4\left(df+f^2\right)x^6+4\,efx^3+e^2}\right)}{12\,def}, \frac{\sqrt{df}\arctan\left(\frac{\left(2\left(d+f\right)x^3+e\right)\sqrt{df}}{de}\right)}{6\,def}\right]$$

[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="fricas")

[Out] [-1/12*sqrt(-d*f)*log((4*(d^2*f + 2*d*f^2 + f^3)*x^6 + 4*(d*e*f + e*f^2)*x^3 - d*e^2 + e^2*f - 2*(2*(d*e + e*f)*x^3 + e^2)*sqrt(-d*f))/(4*(d*f + f^2)*x^6 + 4*e*f*x^3 + e^2))/(d*e*f), 1/6*sqrt(d*f)*arctan((2*(d + f)*x^3 + e)*sqrt(d*f)/(d*e))/(d*e*f)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.45 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

$$\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx = \frac{-\frac{\sqrt{-\frac{1}{df}}\log\left(x^3+\frac{-de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{12}+\frac{\sqrt{-\frac{1}{df}}\log\left(x^3+\frac{de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{12}}{e}$$

[In] integrate(x**2/(4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)

[Out] (-sqrt(-1/(d*f))*log(x**3 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/12 + sqrt(-1/(d*f))*log(x**3 + (d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/12)/e

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{e^2 + 4efx^3 + 4dfx^6 + 4f^2x^6}\,dx = \frac{\arctan\left(\frac{2\,(df+f^2)x^3+ef}{\sqrt{df}e}\right)}{6\,\sqrt{df}e}$$

```
[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="maxima")
[Out] 1/6*arctan((2*(d*f + f^2)*x^3 + e*f)/(sqrt(d*f)*e))/(sqrt(d*f)*e)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{e^2 + 4efx^3 + 4dfx^6 + 4f^2x^6}\,dx = \frac{\arctan\left(\frac{2\,dfx^3+2\,f^2x^3+ef}{\sqrt{df}e}\right)}{6\,\sqrt{df}e}$$

```
[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="giac")
[Out] 1/6*arctan((2*d*f*x^3 + 2*f^2*x^3 + e*f)/(sqrt(d*f)*e))/(sqrt(d*f)*e)
```

Mupad [B] (verification not implemented)

Time = 17.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{e^2 + 4efx^3 + 4dfx^6 + 4f^2x^6}\,dx = \frac{\mathrm{atan}\left(\frac{e\,\sqrt{f}+2\,f^{3/2}\,x^3+2\,d\,\sqrt{f}\,x^3}{\sqrt{d}\,e}\right)}{6\,\sqrt{d}\,e\,\sqrt{f}}$$

```
[In] int(x^2/(e^2 + 4*f^2*x^6 + 4*d*f*x^6 + 4*e*f*x^3),x)
[Out] atan((e*f^(1/2) + 2*f^(3/2)*x^3 + 2*d*f^(1/2)*x^3)/(d^(1/2)*e))/(6*d^(1/2)*
e*f^(1/2))
```

3.539 $\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 44

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}(e-2(d-f)x^3)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

```
[Out] -1/6*arctanh((e-2*(d-f)*x^3)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6, 1366, 632, 212}

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}(e-2x^3(d-f))}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

```
[In] Int[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] -1/6*ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^3))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2}{e^2+4efx^3+(-4df+4f^2)\,x^6}\,dx \\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{e^2+4efx+(-4df+4f^2)\,x^2}\,dx, x, x^3\right) \\
&= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{16de^2f-x^2}\,dx, x, 4f\left(e-2(d-f)x^3\right)\right)\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{f}\left(e-2(d-f)x^3\right)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}(e-2dx^3+2fx^3)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

```
[In] Integrate[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]
[Out] -1/6*ArcTanh[(Sqrt[f]*(e - 2*d*x^3 + 2*f*x^3))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqrt
[f])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{2\left(4df-4f^{2}\right)x^{3}-4ef}{4\sqrt{df}\,e}\right)}{6\sqrt{df}\,e}$	42
risch	$\frac{\ln\left(\left(-2\sqrt{df}-2f\right)x^{3}-e\right)}{12\sqrt{df}\,e}-\frac{\ln\left(\left(-2\sqrt{df}+2f\right)x^{3}+e\right)}{12\sqrt{df}\,e}$	60

```
[In] int(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x,method=_RETURNVERBOSE)
[Out] 1/6/(d*f)^(1/2)/e*arctanh(1/4*(2*(4*d*f-4*f^2)*x^3-4*e*f)/(d*f)^(1/2)/e)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.82

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx = \left[\frac{\sqrt{df}\log\left(-\frac{4\left(d^2f-2\,df^2+f^3\right)x^6-4\left(def-ef^2\right)x^3+de^2+e^2f+2\left(2\left(de-ef\right)x^3-e^2\right)\sqrt{df}}{4\left(df-f^2\right)x^6-4\,efx^3-e^2}\right)}{12\,def}, \frac{\sqrt{-df}\arctan\left(-\frac{\left(2\left(d-f\right)x^3-e\right)\sqrt{-df}}{de}\right)}{6\,def}\right]$$

```
[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="fricas")
[Out] [1/12*sqrt(d*f)*log(-(4*(d^2*f - 2*d*f^2 + f^3)*x^6 - 4*(d*e*f - e*f^2)*x^3
 + d*e^2 + e^2*f + 2*(2*(d*e - e*f)*x^3 - e^2)*sqrt(d*f))/(4*(d*f - f^2)*x^
6 - 4*e*f*x^3 - e^2))/(d*e*f), 1/6*sqrt(-d*f)*arctan(-(2*(d - f)*x^3 - e)*s
qrt(-d*f)/(d*e))/(d*e*f)]
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx = -\frac{\frac{\sqrt{\frac{1}{df}}\log\left(x^3+\frac{-de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{12}-\frac{\sqrt{\frac{1}{df}}\log\left(x^3+\frac{de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{12}}{e}$$

```
[In] integrate(x**2/(-4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)
[Out] -(sqrt(1/(d*f))*log(x**3 + (-d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/12 - sqrt(
1/(d*f))*log(x**3 + (d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/12)/e
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.52

$$\int \frac{x^2}{e^2 + 4efx^3 - 4dfx^6 + 4f^2x^6}\,dx = \frac{\log\left(\frac{2\,(df-f^2)x^3-ef+\sqrt{df}e}{2\,(df-f^2)x^3-ef-\sqrt{df}e}\right)}{12\,\sqrt{df}e}$$

```
[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="maxima")
[Out] 1/12*log((2*(d*f - f^2)*x^3 - e*f + sqrt(d*f)*e)/(2*(d*f - f^2)*x^3 - e*f -
 sqrt(d*f)*e))/(sqrt(d*f)*e)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{e^2 + 4efx^3 - 4dfx^6 + 4f^2x^6}\,dx = -\frac{\arctan\left(\frac{2\,dfx^3-2\,f^2x^3-ef}{\sqrt{-df}e}\right)}{6\,\sqrt{-df}e}$$

```
[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="giac")
[Out] -1/6*arctan((2*d*f*x^3 - 2*f^2*x^3 - e*f)/(sqrt(-d*f)*e))/(sqrt(-d*f)*e)
```

Mupad [B] (verification not implemented)

Time = 17.36 (sec) , antiderivative size = 923, normalized size of antiderivative = 20.98

$$\int \frac{x^2}{e^2 + 4efx^3 - 4dfx^6 + 4f^2x^6}\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{\frac{\left(x^3\,\left(32\,d^3\,f^3-96\,d^2\,f^4+96\,d\,f^5-32\,f^6\right)+\frac{x^3\,\left(-64\,e\,d^3\,f^4+192\,e\,d^2\,f^5-192\,e\,d\,f^6+64\,e\,f^7\right)+16\,e^2\,f^6-48\,d\,e^2\,f^5+48\,d^2\,e^2\,f^4-16\,d^3\,e^2\,f^3-\frac{x^3\,\left(-384\cdots\right.}{\sqrt{d}\,e\,\sqrt{f}\cdots}}{\sqrt{d}\,e\,\sqrt{f}}\cdots}{\sqrt{d}\,e\,\sqrt{f}}\cdots}{\frac{x^3\,\left(32\,d^3\,f^3-96\,d^2\,f^4+96\,d\,f^5-32\,f^6\right)+\frac{x^3\,\left(-64\,e\,d^3\,f^4+192\,e\,d^2\,f^5-192\,e\,d\,f^6+64\,e\,f^7\right)+16\,e^2\,f^6-48\,d\,e^2\,f^5+48\,d^2\,e^2\,f^4-16\,d^3\,e^2\,f^3-\frac{x^3\,\left(\cdots\right.}{\sqrt{d}\,e\cdots}}{\cdots}}{\sqrt{d}\,e\,\sqrt{f}}\cdots}\right)}{\cdots}$$

```
[In] int(x^2/(e^2 + 4*f^2*x^6 - 4*d*f*x^6 + 4*e*f*x^3),x)
```

```
[Out] (atan((((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) + (x^3*(64*e*f^7
 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5
+ 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 - ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1
152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2
*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)))*1i)/(d
^(1/2)*e*f^(1/2)) + ((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) - (
x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 -
48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 + ((x^3*(384*e^2*f^8 - 1152*
d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3
*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(
1/2)))*1i)/(d^(1/2)*e*f^(1/2)))/((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*
d^3*f^3) + (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 1
6*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 - ((x^3*(384*e^2
*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^
7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d
^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)) - (x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f
^4 + 32*d^3*f^3) - (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*
f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 + ((x^3*
(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 1
6*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1
/2)))/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2))))*1i)/(6*d^(1/2)*e*f^(1/2))
```

3.540 $\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx$

Optimal result

Integrand size = 51, antiderivative size = 42

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2119, 211}

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 + 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
```

```
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\text{integral} = -\left(\left(e^2(2-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(-2+m)^2(1+m)^2x^2}\,dx, x, \frac{x^{1+m}}{e(-2+m)(1+m)+2f(-2+m\right.$$

$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx = \frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6
 + 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(32) = 64.

Time = 2.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

method	result	size
risch	$-\frac{\ln\left(x^m+\frac{\left(2f\,x^3+e\right)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}+\frac{\ln\left(x^m-\frac{\left(2f\,x^3+e\right)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}$	78

```
[In] int(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+2*m)),
x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)+1/4/(-d*f)^(1/
2)*ln(x^m-1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.48

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx = \left[-\frac{\sqrt{-df}\log\left(-\frac{4\,f^2x^6-4\,dfx^2x^{2\,m}+4\,efx^3+4\,(2\,fx^4+ex)\sqrt{-df}x^m+e^2}{4\,f^2x^6+4\,dfx^2x^{2\,m}+4\,efx^3+e^2}\right)}{4\,df}, -\frac{\sqrt{df}\arctan\left(\frac{(2\,fx^3+e)\sqrt{df}}{2\,dfxx^m}\right)}{2\,df}\right]$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*f^2*x^6 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + 4*(2*f*x
^4 + e*x)*sqrt(-d*f)*x^m + e^2)/(4*f^2*x^6 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^3
+ e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*f*x^3 + e)*sqrt(d*f)/(d*f*x*x^m
))/(d*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx = \text{Timed out}$$

```
[In] integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6+4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-2)x^3+e(m+1))x^m}{4\,f^2x^6+4\,efx^3+4\,dfx^{2\,m+2}+e^2}\,dx$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

Giac [F]

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-2)x^3+e(m+1))x^m}{4\,f^2x^6+4\,efx^3+4\,dfx^{2\,m+2}+e^2}\,dx$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*
x^(2*m + 2) + e^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx = \int \frac{x^m\,\left(2\,f\,\left(m-2\right)\,x^3+e\,\left(m+1\right)\right)}{e^2+4\,f^2\,x^6+4\,e\,f\,x^3+4\,d\,f\,x^{2\,m+2}}\,dx$$

```
[In] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 + 4*d*
f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 + 4*d*
f*x^(2*m + 2)), x)
```

3.541 $\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx$

Optimal result

Integrand size = 51, antiderivative size = 42

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2119, 214}

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 - 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
```

```
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\text{integral} = -\left(\left(e^2(2-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(-2+m)^2(1+m)^2x^2}\,dx, x, \frac{x^{1+m}}{e(-2+m)(1+m)+2f(-2+m}\right.\right.$$
$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx = \frac{\text{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6
 - 4*d*f*x^(2 + 2*m)),x]
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(32) = 64.

Time = 2.59 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.76

method	result	size
risch	$\frac{\ln\left(x^m+\frac{\left(2f\,x^3+e\right)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}-\frac{\ln\left(x^m-\frac{\left(2f\,x^3+e\right)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}$	74

```
[In] int(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+2*m)),
x,method=_RETURNVERBOSE)
[Out] 1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*f)^(1/2)*l
n(x^m-1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.48

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx = \left[\frac{\sqrt{df}\log\left(-\frac{4\,f^2x^6+4\,dfx^2x^{2\,m}+4\,efx^3+4\,(2\,fx^4+ex)\sqrt{df}x^m+e^2}{4\,f^2x^6-4\,dfx^2x^{2\,m}+4\,efx^3+e^2}\right)}{4\,df}, -\frac{\sqrt{-df}\arctan\left(\frac{(2\,fx^3+e)\sqrt{-df}}{2\,dfxx^m}\right)}{2\,df}\right]$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*f^2*x^6 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + 4*(2*f*x^4
 + e*x)*sqrt(d*f)*x^m + e^2)/(4*f^2*x^6 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + e
^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*f*x^3 + e)*sqrt(-d*f)/(d*f*x*x^m)
)/(d*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx = \text{Timed out}$$

```
[In] integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6-4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-2)x^3+e(m+1))x^m}{4\,f^2x^6+4\,efx^3-4\,dfx^{2\,m+2}+e^2}\,dx$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

Giac [F]

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-2)x^3+e(m+1))x^m}{4\,f^2x^6+4\,efx^3-4\,dfx^{2\,m+2}+e^2}\,dx$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*
x^(2*m + 2) + e^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx = \int \frac{x^m\,\left(2\,f\,\left(m-2\right)\,x^3+e\,\left(m+1\right)\right)}{e^2+4\,f^2\,x^6+4\,e\,f\,x^3-4\,d\,f\,x^{2\,m+2}}\,dx$$

```
[In] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 - 4*d*
f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 - 4*d*
f*x^(2*m + 2)), x)
```

3.542 $\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$

Optimal result

Integrand size = 56, antiderivative size = 42

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x^(1+m)*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2119, 211}

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m) + 4*e*
f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
```

```
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\text{integral} = (e^2(1+m)(1+m-n))\,\text{Subst}\left(\int \frac{1}{e^2+4de^2f(1+m)^2(1+m-n)^2x^2}\,dx, x, \frac{x^{1+m}}{e(1+m)(1+m-n)+2f(1+m)(1+m-n)}\right)$$
$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(32) = 64.

Time = 3.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.00

method	result	size
risch	$-\frac{\ln\left(x^n+\frac{2x^m dfx+\sqrt{-df}\,e}{2\sqrt{-df}\,f}\right)}{4\sqrt{-df}}+\frac{\ln\left(x^n+\frac{-2x^m dfx+\sqrt{-df}\,e}{2\sqrt{-df}\,f}\right)}{4\sqrt{-df}}$	84

```
[In] int(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2
*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*x^m*d*f*x+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)+1
/4/(-d*f)^(1/2)*ln(x^n+1/2*(-2*x^m*d*f*x+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.93

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \left[-\frac{\sqrt{-df}\log\left(-\frac{4\,dfx^2x^{2\,m}-4\,\sqrt{-df}exx^m-4\,f^2x^{2\,n}-e^2-4\,\left(2\,\sqrt{-df}\,fxx^m+ef\right)x^n}{4\,dfx^2x^{2\,m}+4\,f^2x^{2\,n}+4\,efx^n+e^2}\right)}{4\,df}, -\frac{\sqrt{df}\arctan\left(\frac{2\,\sqrt{df}\,fx^n+\sqrt{df}e}{2\,dfxx^m}\right)}{2\,df}\right]$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*d*f*x^2*x^(2*m) - 4*sqrt(-d*f)*e*x*x^m - 4*f^2*x^(
2*n) - e^2 - 4*(2*sqrt(-d*f)*f*x*x^m + e*f)*x^n)/(4*d*f*x^2*x^(2*m) + 4*f^2
*x^(2*n) + 4*e*f*x^n + e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*sqrt(d*f)*
f*x^n + sqrt(d*f)*e)/(d*f*x*x^m))/(d*f)]
```

Sympy [F]

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \int \frac{x^m(em+e+2fmx^n-2fnx^n+2fx^n)}{4dfx^{2m+2}+e^2+4efx^n+4f^2x^{2n}}\,dx$$

```
[In] integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2+4*d*f*x**(2+2*m)+4*e*f*x**n
+4*f**2*x**(2*n)),x)
```

```
[Out] Integral(x**m*(e*m + e + 2*f*m*x**n - 2*f*n*x**n + 2*f*x**n)/(4*d*f*x**(2*m
 + 2) + e**2 + 4*e*f*x**n + 4*f**2*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \int \frac{(2\,f(m-n+1)x^n+e(m+1))x^m}{4\,dfx^{2\,m+2}+4\,f^2x^{2\,n}+4\,efx^n+e^2}\,dx$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) + 4*f^2*
x^(2*n) + 4*e*f*x^n + e^2), x)
```

Giac [F]

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \int \frac{(2\,f(m-n+1)x^n+e(m+1))x^m}{4\,dfx^{2\,m+2}+4\,f^2x^{2\,n}+4\,efx^n+e^2}\,dx$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) + 4*f^2*
x^(2*n) + 4*e*f*x^n + e^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \int \frac{x^m\,\left(e\,(m+1)+2\,f\,x^n\,(m-n+1)\right)}{e^2+4\,f^2\,x^{2\,n}+4\,d\,f\,x^{2\,m+2}+4\,e\,f\,x^n}\,dx$$

```
[In] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^
(2*m + 2) + 4*e*f*x^n),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^
(2*m + 2) + 4*e*f*x^n), x)
```

3.543 $\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$

Optimal result

Integrand size = 56, antiderivative size = 42

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x^(1+m)*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2119, 214}

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m) + 4*e*
f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
```

```
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\text{integral} = (e^2(1+m)(1+m-n))\,\text{Subst}\left(\int \frac{1}{e^2-4de^2f(1+m)^2(1+m-n)^2x^2}\,dx, x, \frac{x^{1+m}}{e(1+m)(1+m-n)+2f(1+m)(1+m-n)}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)),x]
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(32) = 64.

Time = 3.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

method	result	size
risch	$\frac{\ln\left(x^n+\frac{2x^m dfx+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}-\frac{\ln\left(x^n+\frac{-2x^m dfx+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}$	78

```
[In] int(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2
*n)),x,method=_RETURNVERBOSE)
[Out] 1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*x^m*d*f*x+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)-1/4/(
d*f)^(1/2)*ln(x^n+1/2*(-2*x^m*d*f*x+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.93

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \left[\frac{\sqrt{df}\log\left(-\frac{4\,dfx^2x^{2\,m}+4\,\sqrt{df}exx^m+4\,f^2x^{2\,n}+e^2+4\,(2\,\sqrt{df}fxx^m+ef)x^n}{4\,dfx^2x^{2\,m}-4\,f^2x^{2\,n}-4\,efx^n-e^2}\right)}{4\,df},\ -\frac{\sqrt{-df}\arctan\left(\frac{2\,\sqrt{-df}fx^n+\sqrt{-df}e}{2\,dfxx^m}\right)}{2\,df}\right]$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*d*f*x^2*x^(2*m) + 4*sqrt(d*f)*e*x*x^m + 4*f^2*x^(2*n
) + e^2 + 4*(2*sqrt(d*f)*f*x*x^m + e*f)*x^n)/(4*d*f*x^2*x^(2*m) - 4*f^2*x^(
2*n) - 4*e*f*x^n - e^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*sqrt(-d*f)*f*
x^n + sqrt(-d*f)*e)/(d*f*x*x^m))/(d*f)]
```

Sympy [F]

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = -\int \frac{ex^m}{4dfx^{2m+2}-e^2-4efx^n-4f^2x^{2n}}\,dx - \int \frac{emx^m}{4dfx^{2m+2}-e^2-4efx^n-4f^2x^{2n}}\,dx - \int \frac{2fx^mx^n}{4dfx^{2m+2}-e^2-4efx^n-4f^2x^{2n}}\,dx - \int \frac{2fmx^mx^n}{4dfx^{2m+2}-e^2-4efx^n-4f^2x^{2n}}\,dx - \int \left(-\frac{2fnx^mx^n}{4dfx^{2m+2}-e^2-4efx^n-4f^2x^{2n}}\right)dx$$

```
[In] integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2-4*d*f*x**(2+2*m)+4*e*f*x**n
+4*f**2*x**(2*n)),x)
```

```
[Out] -Integral(e*x**m/(4*d*f*x**(2*m + 2) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n))
, x) - Integral(e*m*x**m/(4*d*f*x**(2*m + 2) - e**2 - 4*e*f*x**n - 4*f**2*x
**(2*n)), x) - Integral(2*f*x**m*x**n/(4*d*f*x**(2*m + 2) - e**2 - 4*e*f*x*
*n - 4*f**2*x**(2*n)), x) - Integral(2*f*m*x**m*x**n/(4*d*f*x**(2*m + 2) -
e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integral(-2*f*n*x**m*x**n/(4*d*f
*x**(2*m + 2) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \int -\frac{(2\,f(m-n+1)x^n+e(m+1))x^m}{4\,dfx^{2\,m+2}-4\,f^2x^{2\,n}-4\,efx^n-e^2}\,dx$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) - 4*f^2
*x^(2*n) - 4*e*f*x^n - e^2), x)
```

Giac [F]

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \int -\frac{(2\,f(m-n+1)x^n+e(m+1))x^m}{4\,dfx^{2\,m+2}-4\,f^2x^{2\,n}-4\,efx^n-e^2}\,dx$$

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(-(2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) - 4*f^2
*x^(2*n) - 4*e*f*x^n - e^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \int \frac{x^m\,\left(e\,(m+1)+2\,f\,x^n\,(m-n+1)\right)}{e^2+4\,f^2\,x^{2\,n}-4\,d\,f\,x^{2\,m+2}+4\,e\,f\,x^n}\,dx$$

```
[In] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^
(2*m + 2) + 4*e*f*x^n),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^
(2*m + 2) + 4*e*f*x^n), x)
```

3.544 $\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}} dx$

Optimal result

Integrand size = 29, antiderivative size = 134

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}} dx = -\frac{(2ac^2-d^2)\,x^2}{2b^2c^3}+\frac{d(2ac^2-d^2)\,\sqrt{a+bx^2}}{b^3c^4}-\frac{d(a+bx^2)^{3/2}}{3b^3c^2}+\frac{(a+bx^2)^2}{4b^3c}+\frac{(ac^2-d^2)^2\log\left(d+c\sqrt{a+bx^2}\right)}{b^3c^5}$$

```
[Out] -1/2*(2*a*c^2-d^2)*x^2/b^2/c^3-1/3*d*(b*x^2+a)^(3/2)/b^3/c^2+1/4*(b*x^2+a)^
2/b^3/c+(a*c^2-d^2)^2*ln(d+c*(b*x^2+a)^(1/2))/b^3/c^5+d*(2*a*c^2-d^2)*(b*x^
2+a)^(1/2)/b^3/c^4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2186, 711}

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}} dx = -\frac{d(a+bx^2)^{3/2}}{3b^3c^2}+\frac{(ac^2-d^2)^2\log\left(c\sqrt{a+bx^2}+d\right)}{b^3c^5}+\frac{d\sqrt{a+bx^2}(2ac^2-d^2)}{b^3c^4}+\frac{(a+bx^2)^2}{4b^3c}-\frac{x^2(2ac^2-d^2)}{2b^2c^3}$$

```
[In] Int[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/2*((2*a*c^2 - d^2)*x^2)/(b^2*c^3) + (d*(2*a*c^2 - d^2)*Sqrt[a + b*x^2])/
(b^3*c^4) - (d*(a + b*x^2)^(3/2))/(3*b^3*c^2) + (a + b*x^2)^2/(4*b^3*c) + (
(a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^2]])/(b^3*c^5)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(a-x^2)^2}{d+cx}\,dx, x, \sqrt{a+bx^2}\right)}{b^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{2ac^2d-d^3}{c^4} - \frac{(2ac^2-d^2)x}{c^3} - \frac{dx^2}{c^2} + \frac{x^3}{c} + \frac{(ac^2-d^2)^2}{c^4(d+cx)}\right)dx, x, \sqrt{a+bx^2}\right)}{b^3}$$

$$= -\frac{(2ac^2-d^2)\,x^2}{2b^2c^3} + \frac{d(2ac^2-d^2)\sqrt{a+bx^2}}{b^3c^4} - \frac{d(a+bx^2)^{3/2}}{3b^3c^2} + \frac{(a+bx^2)^2}{4b^3c} + \frac{(ac^2-d^2)^2\log\left(d+c\sqrt{a+bx^2}\right)}{b^3c^5}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{3c^2(a+bx^2)\left(-3ac^2+2d^2+bc^2x^2\right) - 4cd\sqrt{a+bx^2}(-5ac^2+3d^2+bc^2x^2) + 12(-ac^2+d^2)^2\log\left(d+c\sqrt{a+bx^2}\right)}{12b^3c^5}$$

```
[In] Integrate[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] (3*c^2*(a + b*x^2)*(-3*a*c^2 + 2*d^2 + b*c^2*x^2) - 4*c*d*Sqrt[a + b*x^2]*(
-5*a*c^2 + 3*d^2 + b*c^2*x^2) + 12*(-(a*c^2) + d^2)^2*Log[d + c*Sqrt[a + b*
x^2]])/(12*b^3*c^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1811 vs. 2(122) = 244.

Time = 0.14 (sec) , antiderivative size = 1812, normalized size of antiderivative = 13.52

method	result	size
default	Expression too large to display	1812

```
[In] int(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/3/b^3/c^2*(b*x^2+a)^(3/2)-1/2/b^2*c^2*a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-
d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-1/b
*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/2)*ln
(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*
(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2/b^2*c^2*a^2/((-a*b)^
(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2
)^(1/2))*((b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1
/2)-(-a*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b
*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2*(
-a^2*c^4+2*a*c^2*d^2-d^4)/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))
/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*((b*(x+(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)
^(1/2)/b/c^2)+d^2/c^2)^(1/2)-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln((-(-(a*c^2-d
^2)*b*c^2)^(1/2)/c^2+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x+
(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(
a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^(1
/2)*ln((2*d^2/c^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^
(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*
(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2
)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)))-1/2*(-a^2*c^4+2*a*c^2*d^2-d
^4)/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(
a*c^2-d^2)*b*c^2)^(1/2))/c^2*((b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(
-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)
^(1/2)+(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+(x
-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1
/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)
/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2*(-(
a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2
)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1
/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^2-d^
2)*b*c^2)^(1/2)/b/c^2))))-a*c*(-1/2/b^2/c^2*x^2+1/2*(-a^2*c^4+2*a*c^2*d^2-d
^4)/d^2/c^4/b^3*ln(b*c^2*x^2+a*c^2-d^2)+1/2/b^3*a^2/d^2*ln(b*x^2+a))-b*c*(-
1/2/c^4/b^3*(1/2*b*c^2*x^4-2*a*c^2*x^2+d^2*x^2)+1/2/c^6/b^4*(a^3*c^6-3*a^2*
c^4*d^2+3*a*c^2*d^4-d^6)/d^2*ln(b*c^2*x^2+a*c^2-d^2)-1/2/b^4*a^3/d^2*ln(b*x
^2+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.74

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{3\,b^2c^4x^4 - 6\,(abc^4 - bc^2d^2)x^2 + 6\,(a^2c^4 - 2\,ac^2d^2 + d^4)\log\left(bc^2x^2 + ac^2 - d^2\right) + 3\,(a^2c^4 - 2\,ac^2d^2 + d^4)\log\left(\cdots\right.}{\cdots}$$

```
[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^4*x^4 - 6*(a*b*c^4 - b*c^2*d^2)*x^2 + 6*(a^2*c^4 - 2*a*c^2*d^
2 + d^4)*log(b*c^2*x^2 + a*c^2 - d^2) + 3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log
(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - 3*(a^2*c^4 - 2*a
*c^2*d^2 + d^4)*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2)
 - 4*(b*c^3*d*x^2 - 5*a*c^3*d + 3*c*d^3)*sqrt(b*x^2 + a))/(b^3*c^5)
```

Sympy [A] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{2\left(\frac{(a+bx^2)^2}{8c} - \frac{d(a+bx^2)^{\frac{3}{2}}}{6c^2} + \frac{(a+bx^2)(-2ac^2+d^2)}{4c^3} + \frac{\sqrt{a+bx^2}\cdot(2ac^2d-d^3)}{2c^4} + \frac{(ac^2-d^2)^2\left(\begin{cases}\frac{\sqrt{a+bx^2}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{2c^4}\right)}{b^3} & \text{for } b\neq 0 \\ \frac{x^6}{3\cdot(2\sqrt{a}d+2ac)} & \text{otherwis} \end{cases}$$

```
[In] integrate(x**5/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Piecewise((2*((a + b*x**2)**2/(8*c) - d*(a + b*x**2)**(3/2)/(6*c**2) + (a +
 b*x**2)*(-2*a*c**2 + d**2)/(4*c**3) + sqrt(a + b*x**2)*(2*a*c**2*d - d**3)
/(2*c**4) + (a*c**2 - d**2)**2*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (l
og(c*sqrt(a + b*x**2) + d)/c, True))/(2*c**4))/b**3, Ne(b, 0)), (x**6/(3*(2
*sqrt(a)*d + 2*a*c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{\frac{3\,(bx^2+a)^2c^3-4\,(bx^2+a)^{\frac{3}{2}}c^2d-6\,(2\,ac^3-cd^2)\,(bx^2+a)+12\,(2\,ac^2d-d^3)\,\sqrt{bx^2+a}}{c^4}+\frac{12\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(\sqrt{bx^2+a}c+d\right)}{c^5}}{12\,b^3}$$

```
[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*(b*x^2 + a)^2*c^3 - 4*(b*x^2 + a)^(3/2)*c^2*d - 6*(2*a*c^3 - c*d^2
)*(b*x^2 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^2 + a))/c^4 + 12*(a^2*c^4 - 2
*a*c^2*d^2 + d^4)*log(sqrt(b*x^2 + a)*c + d)/c^5)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{(a^2c^4-2\,ac^2d^2+d^4)\log\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{b^3c^5} + \frac{3\,(bx^2+a)^2b^9c^3-12\,(bx^2+a)ab^9c^3-4\,(bx^2+a)^{\frac{3}{2}}b^9c^2d+24\,\sqrt{bx^2+a}ab^9c^2d+6\,(bx^2+a)b^9cd^2-12}{12\,b^{12}c^4}$$

```
[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] (a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(abs(sqrt(b*x^2 + a)*c + d))/(b^3*c^5) + 1
/12*(3*(b*x^2 + a)^2*b^9*c^3 - 12*(b*x^2 + a)*a*b^9*c^3 - 4*(b*x^2 + a)^(3/
2)*b^9*c^2*d + 24*sqrt(b*x^2 + a)*a*b^9*c^2*d + 6*(b*x^2 + a)*b^9*c*d^2 - 1
2*sqrt(b*x^2 + a)*b^9*d^3)/(b^12*c^4)
```

Mupad [B] (verification not implemented)

Time = 17.48 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.25

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{x^4}{4\,b\,c}-\sqrt{b\,x^2+a}\left(\frac{d^3}{b^3\,c^4}-\frac{2\,a\,d}{b^3\,c^2}\right)-\frac{d\,{(b\,x^2+a)}^{3/2}}{3\,b^3\,c^2} - \frac{x^2\,(a\,c^2-d^2)}{2\,b^2\,c^3}+\frac{\mathrm{atanh}\left(\frac{c\,\sqrt{b\,x^2+a}}{d}\right)\,{(a\,c^2-d^2)}^2}{b^3\,c^5} + \frac{\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)\,(a^2\,c^4-2\,a\,c^2\,d^2+d^4)}{2\,b^3\,c^5}$$

```
[In] int(x^5/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

```
[Out] x^4/(4*b*c) - (a + b*x^2)^(1/2)*(d^3/(b^3*c^4) - (2*a*d)/(b^3*c^2)) - (d*(a
 + b*x^2)^(3/2))/(3*b^3*c^2) - (x^2*(a*c^2 - d^2))/(2*b^2*c^3) + (atanh((c*
(a + b*x^2)^(1/2))/d)*(a*c^2 - d^2)^2)/(b^3*c^5) + (log(a*c^2 - d^2 + b*c^2
*x^2)*(d^4 + a^2*c^4 - 2*a*c^2*d^2))/(2*b^3*c^5)
```

3.545 $\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 69

$$\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\frac{x^2}{2bc}-\frac{d\sqrt{a+bx^2}}{b^2c^2}-\frac{(ac^2-d^2)\log\left(d+c\sqrt{a+bx^2}\right)}{b^2c^3}$$

```
[Out] 1/2*x^2/b/c-(a*c^2-d^2)*ln(d+c*(b*x^2+a)^(1/2))/b^2/c^3-d*(b*x^2+a)^(1/2)/b
^2/c^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2186, 711}

$$\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=-\frac{d\sqrt{a+bx^2}}{b^2c^2}-\frac{(ac^2-d^2)\log\left(c\sqrt{a+bx^2}+d\right)}{b^2c^3}+\frac{x^2}{2bc}$$

```
[In] Int[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] x^2/(2*b*c) - (d*Sqrt[a + b*x^2])/(b^2*c^2) - ((a*c^2 - d^2)*Log[d + c*Sqrt
[a + b*x^2]])/(b^2*c^3)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{-a+x^2}{d+cx}\,dx,x,\sqrt{a+bx^2}\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{d}{c^2}+\frac{x}{c}+\frac{-ac^2+d^2}{c^2(d+cx)}\right)\,dx,x,\sqrt{a+bx^2}\right)}{b^2}\\
&= \frac{x^2}{2bc}-\frac{d\sqrt{a+bx^2}}{b^2c^2}-\frac{(ac^2-d^2)\log\left(d+c\sqrt{a+bx^2}\right)}{b^2c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{c\left(ac+bcx^2-2d\sqrt{a+bx^2}\right)+(-2ac^2+2d^2)\log\left(d+c\sqrt{a+bx^2}\right)}{2b^2c^3}$$

```
[In] Integrate[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
[Out] (c*(a*c + b*c*x^2 - 2*d*Sqrt[a + b*x^2]) + (-2*a*c^2 + 2*d^2)*Log[d + c*Sqr
t[a + b*x^2]])/(2*b^2*c^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1698 vs. 2(63) = 126.

Time = 0.08 (sec) , antiderivative size = 1699, normalized size of antiderivative = 24.62

method	result	size
default	Expression too large to display	1699

```
[In] int(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/2*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*
c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*((b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)
*(x-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)
^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2
)))^(1/2))/b^(1/2))-1/2*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))
/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*((b*(x+1/b*(-a*b)^(1/2))^
2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-(-a*b)^(1/2)*ln(((x+1/b*(-a*b)
^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x
+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))+1/2*(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*
c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*((b
*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x
-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)+(-(a*c^2-d^2)*b*c^2)^(1/2
)/c^2*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^
2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c
^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-
d^2/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(
-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^
2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^
(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)))+1/2*(a*
c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(
-(a*c^2-d^2)*b*c^2)^(1/2))/b*((b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(
-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)
^(1/2)-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln((-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+(
x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(
1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-
(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^
2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(
1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x+(-(a*c^2-d
^2)*b*c^2)^(1/2)/b/c^2)))-a*c*(1/2*(a*c^2-d^2)/b^2/d^2/c^2*ln(b*c^2*x^2+a*
c^2-d^2)-1/2*a/b^2/d^2*ln(b*x^2+a))-b*c*(-1/2/b^2/c^2*x^2+1/2*(-a^2*c^4+2*a
*c^2*d^2-d^4)/d^2/c^4/b^3*ln(b*c^2*x^2+a*c^2-d^2)+1/2/b^3*a^2/d^2*ln(b*x^2+
a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(63) = 126$.

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.33

$$\int \frac{x^3}{ac + bcx^2 + d\sqrt{a + bx^2}} \, dx$$
$$= \frac{2\,bc^2x^2 - 4\,\sqrt{bx^2 + a}cd - 2\,(ac^2 - d^2)\log\left(bc^2x^2 + ac^2 - d^2\right) - (ac^2 - d^2)\log\left(-\frac{bc^2x^2 + ac^2 + 2\,\sqrt{bx^2 + a}cd + d^2}{x^2}\right) + }{4\,b^2c^3}$$

```
[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c^2*x^2 - 4*sqrt(b*x^2 + a)*c*d - 2*(a*c^2 - d^2)*log(b*c^2*x^2 +
a*c^2 - d^2) - (a*c^2 - d^2)*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*
d + d^2)/x^2) + (a*c^2 - d^2)*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c
*d + d^2)/x^2))/(b^2*c^3)
```

Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{2\left(\frac{a+bx^2}{4c}-\frac{d\sqrt{a+bx^2}}{2c^2}-\frac{(ac^2-d^2)\left(\begin{cases}\frac{\sqrt{a+bx^2}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{2c^2}\right)}{b^2} & \text{for } b\neq 0\\ \frac{x^4}{2\cdot\left(2\sqrt{a}d+2ac\right)} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Piecewise((2*((a + b*x**2)/(4*c) - d*sqrt(a + b*x**2)/(2*c**2) - (a*c**2 -
d**2)*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d
)/c, True))/(2*c**2))/b**2, Ne(b, 0)), (x**4/(2*(2*sqrt(a)*d + 2*a*c)), Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{\frac{(bx^2+a)c-2\sqrt{bx^2+a}d}{c^2}-\frac{2\left(ac^2-d^2\right)\log\left(\sqrt{bx^2+a}c+d\right)}{c^3}}{2b^2}$$

```
[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x^2 + a)*c - 2*sqrt(b*x^2 + a)*d)/c^2 - 2*(a*c^2 - d^2)*log(sqrt(b
*x^2 + a)*c + d)/c^3)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = -\frac{\frac{2\,(ac^2-d^2)\log\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{bc^3} - \frac{(bx^2+a)bc-2\sqrt{bx^2+a}bd}{b^2c^2}}{2\,b}$$

```
[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a*c^2 - d^2)*log(abs(sqrt(b*x^2 + a)*c + d))/(b*c^3) - ((b*x^2 + a
)*b*c - 2*sqrt(b*x^2 + a)*b*d)/(b^2*c^2))/b
```

Mupad [B] (verification not implemented)

Time = 17.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.78

$$\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{x^2}{2\,b\,c} - \frac{d\,\sqrt{b\,x^2+a}}{b^2\,c^2} + \frac{\mathrm{atanh}\left(\frac{c\,(a\,c^2-d^2)\,\sqrt{b\,x^2+a}}{d^3-a\,c^2\,d}\right)\,(a\,c^2-d^2)}{b^2\,c^3} - \frac{\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)\,(a\,c^2-d^2)}{2\,b^2\,c^3}$$

```
[In] int(x^3/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

```
[Out] x^2/(2*b*c) - (d*(a + b*x^2)^(1/2))/(b^2*c^2) + (atanh((c*(a*c^2 - d^2)*(a
+ b*x^2)^(1/2))/(d^3 - a*c^2*d))*(a*c^2 - d^2))/(b^2*c^3) - (log(a*c^2 - d^
2 + b*c^2*x^2)*(a*c^2 - d^2))/(2*b^2*c^3)
```

3.546 $\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 23

$$\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{\log\left(d+c\sqrt{a+bx^2}\right)}{bc}$$

```
[Out] ln(d+c*(b*x^2+a)^(1/2))/b/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2186, 31}

$$\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

```
[In] Int[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] Log[d + c*Sqrt[a + b*x^2]]/(b*c)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
```

```
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{d+cx}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\ &= \frac{\log\left(d+c\sqrt{a+bx^2}\right)}{bc}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{\log\left(bd+bc\sqrt{a+bx^2}\right)}{bc}$$

```
[In] Integrate[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
[Out] Log[b*d + b*c*Sqrt[a + b*x^2]]/(b*c)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1627 vs. 2(21) = 42.

Time = 0.07 (sec) , antiderivative size = 1628, normalized size of antiderivative = 70.78

method	result	size
default	Expression too large to display	1628

```
[In] int(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
[Out] d*(1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2
+(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1
/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2
))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(
1/2))/b^(1/2))+1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*
b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b
)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-(-a*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b
-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*
b)^(1/2)))^(1/2))/b^(1/2))-1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(
1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x+(-(a*c^2-d^2)*b
*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^
```

```
2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln((-(-(a*c^2
-d^2)*b*c^2)^(1/2)/c^2+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(
x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(
-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^
(1/2)*ln((2*d^2/c^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2
)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-
2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c
^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)))-1/2*c^2/((-a*b)^(1/2)*c^2
+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))
*((b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^
2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)+(-(a*c^2-d^2)*b*c^2)^
(1/2)/c^2*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/
b/c^2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)
*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1
/2)-d^2/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*
(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*
b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))))+1/
2*a*c/d^2/b*ln(b*c^2*x^2+a*c^2-d^2)-1/2*a*c/d^2/b*ln(b*x^2+a)-b*c*(1/2*(a*c
^2-d^2)/b^2/d^2/c^2*ln(b*c^2*x^2+a*c^2-d^2)-1/2*a/b^2/d^2*ln(b*x^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(21) = 42$.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 4.57

$$\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{2\log\left(bc^2x^2+ac^2-d^2\right)+\log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+a}cd+d^2}{x^2}\right)-\log\left(-\frac{bc^2x^2+ac^2-2\sqrt{bx^2+a}cd+d^2}{x^2}\right)}{4\,bc}$$

```
[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(b*c^2*x^2 + a*c^2 - d^2) + log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^
2 + a)*c*d + d^2)/x^2) - log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d +
d^2)/x^2))/(b*c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.

Time = 1.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \frac{x}{ac + bcx^2 + d\sqrt{a + bx^2}}\, dx = \begin{cases} \frac{\begin{cases} \frac{\sqrt{a+bx^2}}{d} & \text{for}\: c = 0 \\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise} \end{cases}}{b} & \text{for}\: b \neq 0 \\ \frac{x^2}{2\sqrt{a}d+2ac} & \text{otherwise} \end{cases}$$

```
[In] integrate(x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
[Out] Piecewise((Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2
) + d)/c, True))/b, Ne(b, 0)), (x**2/(2*sqrt(a)*d + 2*a*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x}{ac + bcx^2 + d\sqrt{a + bx^2}}\, dx = \frac{\log\left(\sqrt{bx^2 + a}c + d\right)}{bc}$$

```
[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
[Out] log(sqrt(b*x^2 + a)*c + d)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x}{ac + bcx^2 + d\sqrt{a + bx^2}}\, dx = \frac{\log\left(\left|\sqrt{bx^2 + a}c + d\right|\right)}{bc}$$

```
[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
[Out] log(abs(sqrt(b*x^2 + a)*c + d))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 18.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

$$\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\frac{\operatorname{atanh}\left(\frac{c\,\sqrt{b\,x^2+a}}{d}\right)+\frac{\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)}{2}}{b\,c}$$

```
[In] int(x/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
[Out] (atanh((c*(a + b*x^2)^(1/2))/d) + log(a*c^2 - d^2 + b*c^2*x^2)/2)/(b*c)
```

3.547 $\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)} dx$

Optimal result

Integrand size = 29, antiderivative size = 88

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)} dx = \frac{d\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}-\frac{c\log\left(d+c\sqrt{a+bx^2}\right)}{ac^2-d^2}$$

```
[Out] c*ln(x)/(a*c^2-d^2)-c*ln(d+c*(b*x^2+a)^(1/2))/(a*c^2-d^2)+d*arctanh((b*x^2+
a)^(1/2)/a^(1/2))/(a*c^2-d^2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2186, 720, 31, 649, 213, 266}

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)} dx = \frac{d\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\left(ac^2-d^2\right)}-\frac{c\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}+\frac{c\log(x)}{ac^2-d^2}$$

```
[In] Int[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(Sqrt[a]*(a*c^2 - d^2)) + (c*Log[x])/(
a*c^2 - d^2) - (c*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx, x, x^2\right)\\
&= \text{Subst}\left(\int \frac{1}{(d+cx)\left(-a+x^2\right)}\,dx, x, \sqrt{a+bx^2}\right)\\
&= -\frac{c^2\text{Subst}\left(\int \frac{1}{d+cx}\,dx, x, \sqrt{a+bx^2}\right)}{ac^2-d^2}+\frac{\text{Subst}\left(\int \frac{d-cx}{-a+x^2}\,dx, x, \sqrt{a+bx^2}\right)}{-ac^2+d^2}\\
&= -\frac{c\log\left(d+c\sqrt{a+bx^2}\right)}{ac^2-d^2}+\frac{c\text{Subst}\left(\int \frac{x}{-a+x^2}\,dx, x, \sqrt{a+bx^2}\right)}{ac^2-d^2}\\
&\quad -\frac{d\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+bx^2}\right)}{ac^2-d^2}\\
&= \frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}-\frac{c\log\left(d+c\sqrt{a+bx^2}\right)}{ac^2-d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\frac{\frac{2d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}+c\log\left(bx^2\right)-2c\log\left(d+c\sqrt{a+bx^2}\right)}{2ac^2-2d^2}$$

```
[In] Integrate[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] ((2*d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a] + c*Log[b*x^2] - 2*c*Log[d
+ c*Sqrt[a + b*x^2]])/(2*a*c^2 - 2*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1696 vs. 2(80) = 160.

Time = 0.08 (sec) , antiderivative size = 1697, normalized size of antiderivative = 19.28

method	result	size
default	Expression too large to display	1697

```
[In] int(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^2+a*c^2-d^2)+c*ln(x)/(a*c^2-d^2)+1/2*
c/d^2*ln(b*c^2*x^2+a*c^2-d^2)-d*(1/a/(a*c^2-d^2)*((b*x^2+a)^(1/2)-a^(1/2)*l
n((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))-1/2*b*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^
2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-1
/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/2)*
ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+
2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2*b*c^2/a/((-a*b)^(1
/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^
(1/2))*((b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2
)-(-a*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(
-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))+1/2*b*c
^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*
c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+
2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c
^2)^(1/2)+(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2
+(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)
^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1
/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2*
(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/
c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)
^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^2
```

```
-d^2)*b*c^2)^(1/2)/b/c^2)))+1/2*b*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^
2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x+(
-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a
*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2
*ln((-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b
)/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^
(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/
c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*
c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(
1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)+d^2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.59

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$$
$$= \left[-\frac{2\,ac\log\left(bc^2x^2+ac^2-d^2\right)-4\,ac\log\left(x\right)+ac\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}cd+d^2}{x^2}\right)-ac\log\left(-\frac{bc^2x^2+ac^2-2\,\sqrt{bx^2+}}{x^2}\right.}{4\left(a^2c^2-ad^2\right)}\right.$$
$$-\frac{2\,ac\log\left(bc^2x^2+ac^2-d^2\right)-4\,ac\log\left(x\right)+ac\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}cd+d^2}{x^2}\right)-ac\log\left(-\frac{bc^2x^2+ac^2-2\,\sqrt{bx^2+}}{x^2}\right.}{4\left(a^2c^2-ad^2\right)}$$

```
[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c*log(b*c^2*x^2 + a*c^2 - d^2) - 4*a*c*log(x) + a*c*log(-(b*c^2*
x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - a*c*log(-(b*c^2*x^2 + a*c
^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) + 2*sqrt(a)*d*log(-(b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a^2*c^2 - a*d^2), -1/4*(2*a*c*log(b*c^2*x^2
 + a*c^2 - d^2) - 4*a*c*log(x) + a*c*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2
 + a)*c*d + d^2)/x^2) - a*c*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d
 + d^2)/x^2) + 4*sqrt(-a)*d*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/(a^2*c^2 - a*
d^2)]
```

Sympy [A] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.58

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = \begin{cases} \frac{2\left(-\frac{bc^2\left(\begin{cases}\frac{\sqrt{a+bx^2}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{2(ac^2-d^2)}-\frac{b\left(-\frac{c\log\left(-bx^2\right)}{2}+\frac{d\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)}{2(ac^2-d^2)}\right)}{b} & \text{for } b\neq 0\\ \begin{cases}\frac{x^2\log\left(x^2\right)}{2\sqrt{a}dx^2+2acx^2} & \text{for } 2\sqrt{a}d+2ac\neq 0\\ \tilde{\infty}x^2 & \text{otherwise}\end{cases} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Piecewise((2*(-b*c**2*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt
(a + b*x**2) + d)/c, True))/(2*(a*c**2 - d**2)) - b*(-c*log(-b*x**2)/2 + d*
atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a))/(2*(a*c**2 - d**2)))/b, Ne(b, 0))
, (Piecewise((x**2*log(x**2)/(2*sqrt(a)*d*x**2 + 2*a*c*x**2), Ne(2*sqrt(a)*
d + 2*a*c, 0)), (zoo*x**2, True)), True))
```

Maxima [F]

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = \int \frac{1}{\left(bcx^2+ac+\sqrt{bx^2+a}d\right)x}\,dx$$

```
[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = -\frac{c^2\log\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{ac^3-cd^2} + \frac{c\log\left(bx^2\right)}{2\left(ac^2-d^2\right)} - \frac{d\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\left(ac^2-d^2\right)\sqrt{-a}}$$

```
[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -c^2*log(abs(sqrt(b*x^2 + a)*c + d))/(a*c^3 - c*d^2) + 1/2*c*log(b*x^2)/(a*
c^2 - d^2) - d*arctan(sqrt(b*x^2 + a)/sqrt(-a))/((a*c^2 - d^2)*sqrt(-a))
```

Mupad [B] (verification not implemented)

Time = 18.19 (sec) , antiderivative size = 1270, normalized size of antiderivative = 14.43

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = \frac{c\,\ln(x)}{a\,c^2-d^2} - \frac{c\,\mathrm{atan}\left(\frac{\frac{c\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}+\frac{c\left(4\,c^4\,d^5-8\,a\,c^6\,d^3+4\,a^2\,c^8\,d-\frac{c\,\sqrt{b\,x^2+a}\,\left(8\,a^3\,c^{10}-8\,a^2\,c^8\,d^2-8\,a\,c^6\,d^4+8\,c^4\,d^6\right)}{2\,\left(a\,c^2-d^2\right)}\right)}{2\,\left(a\,c^2-d^2\right)}\right)1i}{2\,\left(a\,c^2-d^2\right)}+\frac{c\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}-\frac{c\left(4\,c^4\,d^5 \cdots\right.}{\cdots}\right.}{\cdots}}{\frac{c\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}+\frac{c\left(4\,c^4\,d^5-8\,a\,c^6\,d^3+4\,a^2\,c^8\,d-\frac{c\,\sqrt{b\,x^2+a}\,\left(8\,a^3\,c^{10}-8\,a^2\,c^8\,d^2-8\,a\,c^6\,d^4+8\,c^4\,d^6\right)}{2\,\left(a\,c^2-d^2\right)}\right)}{2\,\left(a\,c^2-d^2\right)}\right)}{2\,\left(a\,c^2-d^2\right)}-\frac{c\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}-\frac{c\left(4\,c^4\,d^5 \cdots\right.}{\cdots}\right.}{\cdots}}\right)}{a\,c^2-d^2} - \frac{c\,\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)}{2\,a\,c^2-2\,d^2} - \frac{d\,\mathrm{atan}\left(\frac{\frac{d\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}+\frac{d\left(4\,c^4\,d^5-8\,a\,c^6\,d^3+4\,a^2\,c^8\,d-\frac{d\,\sqrt{b\,x^2+a}\,\left(8\,a^3\,c^{10}-8\,a^2\,c^8\,d^2-8\,a\,c^6\,d^4+8\,c^4\,d^6\right)}{\sqrt{a}\,\left(2\,a\,c^2-2\,d^2\right)}\right)}{\sqrt{a}\,\left(2\,a\,c^2-2\,d^2\right)}\right)1i}{\sqrt{a}\,\left(2\,a\,c^2-2\,d^2\right)}+\frac{d\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}-\frac{d\left(4\,c^4\,d^5 \cdots\right.}{\cdots}\right.}{\cdots}}{\frac{d\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}+\frac{d\left(4\,c^4\,d^5-8\,a\,c^6\,d^3+4\,a^2\,c^8\,d-\frac{d\,\sqrt{b\,x^2+a}\,\left(8\,a^3\,c^{10}-8\,a^2\,c^8\,d^2-8\,a\,c^6\,d^4+8\,c^4\,d^6\right)}{\sqrt{a}\,\left(2\,a\,c^2-2\,d^2\right)}\right)}{\sqrt{a}\,\left(2\,a\,c^2-2\,d^2\right)}\right)}{\sqrt{a}\,\left(2\,a\,c^2-2\,d^2\right)}-\frac{d\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}-\frac{d\left(4\,c^4\,d^5 \cdots\right.}{\cdots}\right.}{\cdots}}\right)}{\sqrt{a}\,\left(a\,c^2-d^2\right)}$$

```
[In] int(1/(x*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)
```

```
[Out] (c*log(x))/(a*c^2 - d^2) - (c*atan(((c*(4*c^6*d^2*(a + b*x^2)^(1/2) + (c*(4
*c^4*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d - (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8
*c^4*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^
2)))*1i)/(2*(a*c^2 - d^2)) + (c*(4*c^6*d^2*(a + b*x^2)^(1/2) - (c*(4*c^4*d^
5 - 8*a*c^6*d^3 + 4*a^2*c^8*d + (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^
6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2)))*1i
)/(2*(a*c^2 - d^2)))/((c*(4*c^6*d^2*(a + b*x^2)^(1/2) + (c*(4*c^4*d^5 - 8*a
*c^6*d^3 + 4*a^2*c^8*d - (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a
*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2))))/(2*(a*c^
2 - d^2)) - (c*(4*c^6*d^2*(a + b*x^2)^(1/2) - (c*(4*c^4*d^5 - 8*a*c^6*d^3 +
 4*a^2*c^8*d + (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 -
 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2)))
)*1i)/(a*c^2 - d^2) - (c*log(a*c^2 - d^2 + b*c^2*x^2))/(2*a*c^2 - 2*d^2) -
(d*atan(((d*(4*c^6*d^2*(a + b*x^2)^(1/2) + (d*(4*c^4*d^5 - 8*a*c^6*d^3 + 4*
a^2*c^8*d - (d*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 - 8*
a^2*c^8*d^2))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2)))*1i
)/(a^(1/2)*(2*a*c^2 - 2*d^2)) + (d*(4*c^6*d^2*(a + b*x^2)^(1/2) - (d*(4*c^4
*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d + (d*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4
*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)
*(2*a*c^2 - 2*d^2)))*1i)/(a^(1/2)*(2*a*c^2 - 2*d^2)))/((d*(4*c^6*d^2*(a + b
*x^2)^(1/2) + (d*(4*c^4*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d - (d*(a + b*x^2)^(1
/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(a^(1/2)*(2*a*c
^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2)) -
(d*(4*c^6*d^2*(a + b*x^2)^(1/2) - (d*(4*c^4*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d
 + (d*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d
^2))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(
2*a*c^2 - 2*d^2))))*1i)/(a^(1/2)*(a*c^2 - d^2))
```

3.548 $\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 151

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2}-\frac{bd(3ac^2-d^2)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2} - \frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}+\frac{bc^3\log\left(d+c\sqrt{a+bx^2}\right)}{\left(ac^2-d^2\right)^2}$$

```
[Out] -1/2*b*d*(3*a*c^2-d^2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/(a*c^2-d^2)
^2-b*c^3*ln(x)/(a*c^2-d^2)^2+b*c^3*ln(d+c*(b*x^2+a)^(1/2))/(a*c^2-d^2)^2+1/
2*(-a*c+d*(b*x^2+a)^(1/2))/a/(a*c^2-d^2)/x^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2186, 755, 815, 649, 212, 266}

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = -\frac{bd(3ac^2-d^2)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^2}}{2ax^2\left(ac^2-d^2\right)} + \frac{bc^3\log\left(c\sqrt{a+bx^2}+d\right)}{\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

```
[In] Int[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -1/2*(a*c - d*Sqrt[a + b*x^2])/(a*(a*c^2 - d^2)*x^2) - (b*d*(3*a*c^2 - d^2)
*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log
[x])/(a*c^2 - d^2)^2 + (b*c^3*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)^2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx, x, x^2\right)\\ &= b\text{Subst}\left(\int \frac{1}{(d+cx)\left(a-x^2\right)^2}\,dx, x, \sqrt{a+bx^2}\right)\end{aligned}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a(ac^2-d^2)x^2} - \frac{b\text{Subst}\left(\int \frac{-2ac^2+d^2+cdx}{(d+cx)(a-x^2)}\,dx, x, \sqrt{a+bx^2}\right)}{2a(ac^2-d^2)}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a(ac^2-d^2)x^2} - \frac{b\text{Subst}\left(\int \left(-\frac{2ac^4}{(ac^2-d^2)(d+cx)} + \frac{3ac^2d-d^3-2ac^3x}{(ac^2-d^2)(a-x^2)}\right)\,dx, x, \sqrt{a+bx^2}\right)}{2a(ac^2-d^2)}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a(ac^2-d^2)x^2} + \frac{bc^3\log\left(d+c\sqrt{a+bx^2}\right)}{(ac^2-d^2)^2} - \frac{b\text{Subst}\left(\int \frac{3ac^2d-d^3-2ac^3x}{a-x^2}\,dx, x, \sqrt{a+bx^2}\right)}{2a(ac^2-d^2)^2}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a(ac^2-d^2)x^2} + \frac{bc^3\log\left(d+c\sqrt{a+bx^2}\right)}{(ac^2-d^2)^2} + \frac{(bc^3)\,\text{Subst}\left(\int \frac{x}{a-x^2}\,dx, x, \sqrt{a+bx^2}\right)}{(ac^2-d^2)^2} - \frac{(bd(3ac^2-d^2))\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+bx^2}\right)}{2a(ac^2-d^2)^2}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a(ac^2-d^2)x^2} - \frac{bd(3ac^2-d^2)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}(ac^2-d^2)^2} - \frac{bc^3\log(x)}{(ac^2-d^2)^2} + \frac{bc^3\log\left(d+c\sqrt{a+bx^2}\right)}{(ac^2-d^2)^2}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = \frac{bd(-3ac^2+d^2)x^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \sqrt{a}\left(-\left((ac^2-d^2)\left(ac-d\sqrt{a+bx^2}\right)\right) - abc^3x^2\log(bx^2) + 2abc^3x^2\text{lo}\right.}{2a^{3/2}(-ac^2+d^2)^2x^2}$$

```
[In] Integrate[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (b*d*(-3*a*c^2 + d^2)*x^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]] + Sqrt[a]*(-((a*
c^2 - d^2)*(a*c - d*Sqrt[a + b*x^2])) - a*b*c^3*x^2*Log[b*x^2] + 2*a*b*c^3*
x^2*Log[d + c*Sqrt[a + b*x^2]]))/(2*a^(3/2)*(-(a*c^2) + d^2)^2*x^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1909 vs. 2(137) = 274.

Time = 0.10 (sec) , antiderivative size = 1910, normalized size of antiderivative = 12.65

method	result	size
default	Expression too large to display	1910

```
[In] int(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

[Out] a*c*(1/2*b*c^4/(a*c^2-d^2)^2/d^2*ln(b*c^2*x^2+a*c^2-d^2)-1/2/a/(a*c^2-d^2)/x^2-b*(2*a*c^2-d^2)/a^2/(a*c^2-d^2)^2*ln(x)-1/2*b/a^2/d^2*ln(b*x^2+a))+b*c*(-1/2*c^2/(a*c^2-d^2)/d^2*ln(b*c^2*x^2+a*c^2-d^2)+1/a/(a*c^2-d^2)*ln(x)+1/2/a/d^2*ln(b*x^2+a))-d*(1/a/(a*c^2-d^2)*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))-b*(2*a*c^2-d^2)/a^2/(a*c^2-d^2)^2*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))+1/2*b^2*c^2/a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))+1/2*b^2*c^2/a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-(-a*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2*b^2*c^6/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln((-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)))-1/2*b^2*c^6/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)+(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))))

Fricas [A] (verification not implemented)

none

Time = 0.74 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.51

$$\int \frac{1}{x^3 \left(ac + bcx^2 + d\sqrt{a + bx^2}\right)}\, dx$$
$$= \left[\frac{2\, a^2bc^3x^2 \log\left(bc^2x^2 + ac^2 - d^2\right) - 4\, a^2bc^3x^2 \log\left(x\right) + a^2bc^3x^2 \log\left(-\frac{bc^2x^2 + ac^2 + 2\sqrt{bx^2 + a}cd + d^2}{x^2}\right) - a^2bc^3x^2 \log}{4\,(a^4c}\right.$$

```
[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a^2*b*c^3*x^2*log(b*c^2*x^2 + a*c^2 - d^2) - 4*a^2*b*c^3*x^2*log(x)
 + a^2*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2
) - a^2*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^
2) - 2*a^3*c^3 + 2*a^2*c*d^2 - (3*a*b*c^2*d - b*d^3)*sqrt(a)*x^2*log(-(b*x^
2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a^2*c^2*d - a*d^3)*sqrt(b*x^
2 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^2), 1/4*(2*a^2*b*c^3*x^2*log
(b*c^2*x^2 + a*c^2 - d^2) - 4*a^2*b*c^3*x^2*log(x) + a^2*b*c^3*x^2*log(-(b*
c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - a^2*b*c^3*x^2*log(-(b
*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - 2*a^3*c^3 + 2*a^2*c*
d^2 + 2*(3*a*b*c^2*d - b*d^3)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a))
 + 2*(a^2*c^2*d - a*d^3)*sqrt(b*x^2 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d
^4)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3 \left(ac + bcx^2 + d\sqrt{a + bx^2}\right)}\, dx = \int \frac{1}{x^3 \left(ac + bcx^2 + d\sqrt{a + bx^2}\right)}\, dx$$

```
[In] integrate(1/x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**3*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\int \frac{1}{\left(bcx^2+ac+\sqrt{bx^2+a}d\right)x^3}\,dx$$

```
[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\frac{bc^4\log\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{a^2c^5-2\,ac^3d^2+cd^4}-\frac{bc^3\log\left(-bx^2\right)}{2\left(a^2c^4-2\,ac^2d^2+d^4\right)}+\frac{\left(3\,abc^2d-bd^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\left(a^3c^4-2\,a^2c^2d^2+ad^4\right)\sqrt{-a}}-\frac{a^2bc^3-abcd^2-\left(abc^2d-bd^3\right)\sqrt{bx^2+a}}{2\left(ac^2-d^2\right)^2abx^2}$$

```
[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
[Out] b*c^4*log(abs(sqrt(b*x^2 + a)*c + d))/(a^2*c^5 - 2*a*c^3*d^2 + c*d^4) - 1/2
*b*c^3*log(-b*x^2)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + 1/2*(3*a*b*c^2*d - b*d^3
)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4)*sqrt(
-a)) - 1/2*(a^2*b*c^3 - a*b*c*d^2 - (a*b*c^2*d - b*d^3)*sqrt(b*x^2 + a))/((
a*c^2 - d^2)^2*a*b*x^2)
```

Mupad [B] (verification not implemented)

Time = 19.54 (sec) , antiderivative size = 4602, normalized size of antiderivative = 30.48

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\text{Too large to display}$$

```
[In] int(1/(x^3*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)
[Out] (atan((((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 - 18*a^
5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/(16*(a^5*c^6 - a^2*d^6 + 3*
a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - ((a + b*x^2)^(1/2)*(16*(b^2*d^6 - 6*a*b^2*c
^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*
```

d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(512*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) + ((a + b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*1i)/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2) - (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 - 18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/(16*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + ((a + b*x^2)^(1/2)*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(512*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) - ((a + b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*1i)/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))/(((b^3*c^8*d^5)/2 - (3*a*b^3*c^10*d^3)/2)/(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2) + (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 - 18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/(16*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - ((a + b*x^2)^(1/2)*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(512*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) + ((a + b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2) + (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 - 18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/(16*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + ((a + b*x^2)^(1/2)*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^

6*c^12*d^2))/(512*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) - ((a + b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*1i)/(8*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) - c/(2*x^2*(a*c^2 - d^2)) - (b*c^3*log(x))/(d^4 + a^2*c^4 - 2*a*c^2*d^2) + (b*c^3*log(a*c^2 - d^2 + b*c^2*x^2))/(2*d^4 + 2*a^2*c^4 - 4*a*c^2*d^2) - (d*(a + b*x^2)^(1/2))/(2*x^2*(a*d^2 - a^2*c^2)) + (b*c^3*atan(((c^3*(((a + b*x^2)^(1/2)*(c^6*d^6 - 6*a*c^8*d^4 + 13*a^2*c^10*d^2))/(2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)) + (c^3*((8*a*c^4*d^11 - 16*a^6*c^14*d - 48*a^2*c^6*d^9 + 112*a^3*c^8*d^7 - 128*a^4*c^10*d^5 + 72*a^5*c^12*d^3)/(4*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - (c^3*(a + b*x^2)^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(4*(a*c^2 - d^2)^2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2))))/(2*(a*c^2 - d^2)^2))*1i)/(2*(a*c^2 - d^2)^2) + (c^3*(((a + b*x^2)^(1/2)*(c^6*d^6 - 6*a*c^8*d^4 + 13*a^2*c^10*d^2))/(2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)) - (c^3*((8*a*c^4*d^11 - 16*a^6*c^14*d - 48*a^2*c^6*d^9 + 112*a^3*c^8*d^7 - 128*a^4*c^10*d^5 + 72*a^5*c^12*d^3)/(4*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + (c^3*(a + b*x^2)^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(4*(a*c^2 - d^2)^2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2))))/(2*(a*c^2 - d^2)^2))*1i)/(2*(a*c^2 - d^2)^2))/((c^8*d^5 - 3*a*c^10*d^3)/(2*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - (c^3*(((a + b*x^2)^(1/2)*(c^6*d^6 - 6*a*c^8*d^4 + 13*a^2*c^10*d^2))/(2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)) + (c^3*((8*a*c^4*d^11 - 16*a^6*c^14*d - 48*a^2*c^6*d^9 + 112*a^3*c^8*d^7 - 128*a^4*c^10*d^5 + 72*a^5*c^12*d^3)/(4*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - (c^3*(a + b*x^2)^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(4*(a*c^2 - d^2)^2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2))))/(2*(a*c^2 - d^2)^2)))/(2*(a*c^2 - d^2)^2) + (c^3*(((a + b*x^2)^(1/2)*(c^6*d^6 - 6*a*c^8*d^4 + 13*a^2*c^10*d^2))/(2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)) - (c^3*((8*a*c^4*d^11 - 16*a^6*c^14*d - 48*a^2*c^6*d^9 + 112*a^3*c^8*d^7 - 128*a^4*c^10*d^5 + 72*a^5*c^12*d^3)/(4*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + (c^3*(a + b*x^2)^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(4*(a*c^2 - d^2)^2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2))))/(2*(a*c^2 - d^2)^2)))/(2*(a*c^2 - d^2)^2)))*1i)/(a*c^2 - d^2)^2

3.549 $\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 147

$$\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\frac{x}{bc}-\frac{\sqrt{ac^2-d^2}\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}+\frac{\sqrt{ac^2-d^2}\arctan\left(\frac{\sqrt{b}dx}{\sqrt{ac^2-d^2}\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}$$

```
[Out] x/b/c-d*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)/c^2-arctan(c*x*b^(1/2)/(
a*c^2-d^2)^(1/2))*(a*c^2-d^2)^(1/2)/b^(3/2)/c^2+arctan(d*x*b^(1/2)/(a*c^2-d
^2)^(1/2)/(b*x^2+a)^(1/2))*(a*c^2-d^2)^(1/2)/b^(3/2)/c^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2187, 327, 211, 494, 223, 212, 385}

$$\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\frac{\sqrt{ac^2-d^2}\arctan\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{\sqrt{ac^2-d^2}\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}+\frac{x}{bc}$$

```
[In] Int[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] x/(b*c) - (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(b^(3
/2)*c^2) + (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[
```

```
a + b*x^2])])/(b^(3/2)*c^2) - (d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(b^(
3/2)*c^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (ac)\int \frac{x^2}{a^2c^2-ad^2+abc^2x^2}\,dx-(ad)\int \frac{x^2}{\sqrt{a+bx^2}\,(a^2c^2-ad^2+abc^2x^2)}\,dx\\
&= \frac{x}{bc}-\frac{d\int \frac{1}{\sqrt{a+bx^2}}\,dx}{bc^2}-\frac{(a(ac^2-d^2))\int \frac{1}{a^2c^2-ad^2+abc^2x^2}\,dx}{bc}\\
&\quad+\frac{(ad(ac^2-d^2))\int \frac{1}{\sqrt{a+bx^2}(a^2c^2-ad^2+abc^2x^2)}\,dx}{bc^2}\\
&= \frac{x}{bc}-\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{d\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{bc^2}\\
&\quad+\frac{(ad(ac^2-d^2))\,\text{Subst}\left(\int \frac{1}{a^2c^2-ad^2-(-a^2bc^2+b(a^2c^2-ad^2))x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{bc^2}\\
&= \frac{x}{bc}-\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}+\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{ac^2-d^2}\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b}cx+2\sqrt{ac^2-d^2}\arctan\left(\frac{d+c\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{ac^2-d^2}}\right)+d\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}c^2}$$

```
[In] Integrate[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
[Out] (Sqrt[b]*c*x + 2*Sqrt[a*c^2 - d^2]*ArcTan[(d + c*(-(Sqrt[b]*x) + Sqrt[a + b
*x^2]))/Sqrt[a*c^2 - d^2]] + d*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(b^(3/2
)*c^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1779 vs. $2(123) = 246$.

Time = 0.08 (sec) , antiderivative size = 1780, normalized size of antiderivative = 12.11

method	result	size
default	Expression too large to display	1780

```
[In] int(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/2*c^2*a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-
(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-1/b*(-a*b)^(1/2))^2+2*(
-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2
))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*
(-a*b)^(1/2)))^(1/2))/b^(1/2))+1/2*c^2*a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(
a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((b
*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-(-a*b)^(
1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2
))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2*c^2*(a*c^2-d^
2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^
2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((b*(x+(-(a*c^2-d^2)*b*c^2)
^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1
/2)/b/c^2)+d^2/c^2)^(1/2)-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln((-(-(a*c^2-d^2)
*b*c^2)^(1/2)/c^2+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x+(-(
a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c
^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^(1/2)
*ln((2*d^2/c^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/
2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(
a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(
1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)))+1/2*c^2*(a*c^2-d^2)/((-a*b)^(1
/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)
^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)
^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^
2/c^2)^(1/2)+(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/
c^2+(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)
^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2
+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d
^2/c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c
^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*
c^2-d^2)*b*c^2)^(1/2)/b/c^2))))-a*c*((a*c^2-d^2)/b/d^2/c/(b*(a*c^2-d^2))^(1
/2)*arctan(x*b*c/(b*(a*c^2-d^2))^(1/2))-1/d^2*a/b/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2)))-b*c*(-1/b^2/c^2*x+(-a^2*c^4+2*a*c^2*d^2-d^4)/d^2/c^3/b^2/(b*(a*
c^2-d^2))^(1/2)*arctan(x*b*c/(b*(a*c^2-d^2))^(1/2))+1/b^2*a^2/d^2/(a*b)^(1/
2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 1168, normalized size of antiderivative = 7.95

$$\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$$

$$= \left[\frac{4\,bcx + 2\,\sqrt{b}d\log\left(-2\,bx^2 + 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right) + b\sqrt{-\frac{ac^2-d^2}{b}}\log\left(\frac{a^4c^4-2\,a^3c^2d^2+a^2d^4+\left(a^2b^2c^4-8\,ab^2c^2d^2+8\,b^2 \cdots\right.}{\cdots}\right.}{\cdots}\right.$$

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*c*x + 2*sqrt(b)*d*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + b*sqrt(-(a*c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2*b
^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^2
+ 4*a*b*d^4)*x^2 - 4*((a*b^2*c^2*d - 2*b^2*d^3)*x^3 + (a^2*b*c^2*d - a*b*d^
3)*x)*sqrt(b*x^2 + a)*sqrt(-(a*c^2 - d^2)/b))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*
c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*b*sqrt(-(a*c^2 - d^2)/b)*
log((b*c^2*x^2 - 2*b*c*x*sqrt(-(a*c^2 - d^2)/b) - a*c^2 + d^2)/(b*c^2*x^2 +
 a*c^2 - d^2)))/(b^2*c^2), 1/4*(4*b*c*x + 4*sqrt(-b)*d*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) + b*sqrt(-(a*c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^
2*d^4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*
a^2*b*c^2*d^2 + 4*a*b*d^4)*x^2 - 4*((a*b^2*c^2*d - 2*b^2*d^3)*x^3 + (a^2*b*
c^2*d - a*b*d^3)*x)*sqrt(b*x^2 + a)*sqrt(-(a*c^2 - d^2)/b))/(b^2*c^4*x^4 +
a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*b*sqrt(-(a*
c^2 - d^2)/b)*log((b*c^2*x^2 - 2*b*c*x*sqrt(-(a*c^2 - d^2)/b) - a*c^2 + d^2
)/(b*c^2*x^2 + a*c^2 - d^2)))/(b^2*c^2), 1/2*(2*b*c*x + 2*b*sqrt((a*c^2 - d
^2)/b)*arctan(-b*c*x*sqrt((a*c^2 - d^2)/b)/(a*c^2 - d^2)) - b*sqrt((a*c^2 -
 d^2)/b)*arctan(1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(b*x^2
+ a)*sqrt((a*c^2 - d^2)/b)/((a*b*c^2*d - b*d^3)*x^3 + (a^2*c^2*d - a*d^3)*x
)) + sqrt(b)*d*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(b^2*c^2),
1/2*(2*b*c*x + 2*b*sqrt((a*c^2 - d^2)/b)*arctan(-b*c*x*sqrt((a*c^2 - d^2)/b
)/(a*c^2 - d^2)) + 2*sqrt(-b)*d*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - b*sqrt
((a*c^2 - d^2)/b)*arctan(1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sq
rt(b*x^2 + a)*sqrt((a*c^2 - d^2)/b)/((a*b*c^2*d - b*d^3)*x^3 + (a^2*c^2*d -
 a*d^3)*x)))/(b^2*c^2)]
```

Sympy [F]

$$\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$$

```
[In] integrate(x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
[Out] Integral(x**2/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\int \frac{x^2}{bcx^2+ac+\sqrt{bx^2+a}d}\,dx$$

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(x^2/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\int \frac{x^2}{a\,c+d\,\sqrt{b\,x^2+a}+b\,c\,x^2}\,dx$$

```
[In] int(x^2/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
[Out] int(x^2/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2), x)
```

3.550 $\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}-\frac{\arctan\left(\frac{\sqrt{b}dx}{\sqrt{ac^2-d^2}\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

```
[Out] arctan(c*x*b^(1/2)/(a*c^2-d^2)^(1/2))/b^(1/2)/(a*c^2-d^2)^(1/2)-arctan(d*x*
b^(1/2)/(a*c^2-d^2)^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)/(a*c^2-d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2187, 211, 385}

$$\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}-\frac{\arctan\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

```
[In] Int[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]]/(Sqrt[b]*Sqrt[a*c^2 - d^2]) - ArcTa
n[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])]/(Sqrt[b]*Sqrt[a*c^2 -
d^2])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = (ac)\int \frac{1}{a^2c^2 - ad^2 + abc^2x^2}\,dx - (ad)\int \frac{1}{\sqrt{a+bx^2}\,(a^2c^2 - ad^2 + abc^2x^2)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}} - (ad)\text{Subst}\left(\int \frac{1}{a^2c^2 - ad^2 - (-a^2bc^2 + b(a^2c^2 - ad^2))\,x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}} - \frac{\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{ac^2-d^2}\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.61

$$\int \frac{1}{ac + bcx^2 + d\sqrt{a+bx^2}}\,dx = -\frac{2\arctan\left(\frac{d + c\left(-\sqrt{b}x + \sqrt{a+bx^2}\right)}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

```
[In] Integrate[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1),x]
[Out] (-2*ArcTan[(d + c*(-(Sqrt[b]*x) + Sqrt[a + b*x^2]))/Sqrt[a*c^2 - d^2]])/(Sq
rt[b]*Sqrt[a*c^2 - d^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1717 vs. 2(85) = 170.

Time = 0.08 (sec) , antiderivative size = 1718, normalized size of antiderivative = 16.68

method	result	size
default	Expression too large to display	1718

[In] int(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)

[Out] d*(1/2*b*c^2/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2*b*c^2/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-(-a*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))+1/2*b*c^4/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln((-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)))-1/2*b*c^4/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)+(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))))+a*c^2/d^2/(b*(a*c^2-d^2))^(1/2)*arctan(x*b*c/(b*(a*c^2-d^2))^(1/2))-a*c/d^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-b*c*((a*c^2-d^2)/b/d^2/c/(b*(a*c^2-d^2))^(1/2)*arctan(x*b*c/(b*(a*c^2-d^2))^(1/2))-1/d^2*a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(85) = 170$.

Time = 0.31 (sec) , antiderivative size = 510, normalized size of antiderivative = 4.95

$$\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{\sqrt{-abc^2+bd^2}\log\left(\frac{a^4c^4-2\,a^3c^2d^2+a^2d^4+(a^2b^2c^4-8\,ab^2c^2d^2+8\,b^2d^4)x^4+2\,(a^3bc^4-5\,a^2bc^2d^2+4\,abd^4)x^2-4\,\sqrt{-abc^2+bd^2}\,((abc^2d-}{b^2c^4x^4+a^2c^4-2\,ac^2d^2+d^4+2\,(abc^4-bc^2d^2)x^2}\right.}{4\,(abc^2-bd^2)}\right.$$
$$\left.-\frac{2\,\sqrt{abc^2-bd^2}\arctan\left(-\frac{\sqrt{abc^2-bd^2}cx}{ac^2-d^2}\right)-\sqrt{abc^2-bd^2}\arctan\left(\frac{(a^2c^2-ad^2+(abc^2-2\,bd^2)x^2)\sqrt{abc^2-bd^2}\sqrt{bx^2+a}}{2\,((ab^2c^2d-b^2d^3)x^3+(a^2bc^2d-abd^3)x)}\right)}{2\,(abc^2-bd^2)}\right]$$

[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")

[Out] [-1/4*(sqrt(-a*b*c^2 + b*d^2)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^2 + 4*a*b*d^4)*x^2 - 4*sqrt(-a*b*c^2 + b*d^2)*((a*b*c^2*d - 2*b*d^3)*x^3 + (a^2*c^2*d - a*d^3)*x)*sqrt(b*x^2 + a))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*sqrt(-a*b*c^2 + b*d^2)*log((b*c^2*x^2 - a*c^2 - 2*sqrt(-a*b*c^2 + b*d^2)*c*x + d^2)/(b*c^2*x^2 + a*c^2 - d^2)))/(a*b*c^2 - b*d^2), -1/2*(2*sqrt(a*b*c^2 - b*d^2)*arctan(-sqrt(a*b*c^2 - b*d^2)*c*x/(a*c^2 - d^2)) - sqrt(a*b*c^2 - b*d^2)*arctan(1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(a*b*c^2 - b*d^2)*sqrt(b*x^2 + a)/((a*b^2*c^2*d - b^2*d^3)*x^3 + (a^2*b*c^2*d - a*b*d^3)*x)))/(a*b*c^2 - b*d^2)]

Sympy [F]

$$\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$$

[In] integrate(1/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)

[Out] Integral(1/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)

Maxima [F]

$$\int \frac{1}{ac + bcx^2 + d\sqrt{a + bx^2}}\,dx = \int \frac{1}{bcx^2 + ac + \sqrt{bx^2 + a}d}\,dx$$

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04

$$\int \frac{1}{ac + bcx^2 + d\sqrt{a + bx^2}}\,dx = \frac{\arctan\left(\frac{bcx}{\sqrt{abc^2 - bd^2}}\right)}{\sqrt{abc^2 - bd^2}} + \frac{\arctan\left(\frac{\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 c^2 + ac^2 - 2\,d^2}{2\,\sqrt{ac^2 - d^2}d}\right)}{\sqrt{ac^2 - d^2}\sqrt{b}}$$

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
[Out] arctan(b*c*x/sqrt(a*b*c^2 - b*d^2))/sqrt(a*b*c^2 - b*d^2) + arctan(1/2*((sq
rt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 - 2*d^2)/(sqrt(a*c^2 - d^2)*d))/(s
qrt(a*c^2 - d^2)*sqrt(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{ac + bcx^2 + d\sqrt{a + bx^2}}\,dx = \begin{cases} \frac{\mathrm{atan}\left(\frac{b\,c\,x}{\sqrt{a\,b\,c^2 - b\,d^2}}\right)}{\sqrt{a\,b\,c^2 - b\,d^2}} - \frac{d\,x}{\sqrt{a}\,(a\,c^2 - d^2)} & \text{if } b = 0 \lor d = 0 \\ \frac{\mathrm{atan}\left(\frac{b\,c\,x}{\sqrt{a\,b\,c^2 - b\,d^2}}\right)}{\sqrt{a\,b\,c^2 - b\,d^2}} - \frac{d\,\mathrm{atan}\left(\frac{x\,\sqrt{a\,b\,c^2 - b\,(a\,c^2 - d^2)}}{\sqrt{a\,c^2 - d^2}\,\sqrt{b\,x^2 + a}}\right)}{\sqrt{-(a\,c^2 - d^2)\,(b\,(a\,c^2 - d^2) - a\,b\,c^2)}} & \text{if } 0 < b\,d^2 \\ \frac{\mathrm{atan}\left(\frac{b\,c\,x}{\sqrt{a\,b\,c^2 - b\,d^2}}\right)}{\sqrt{a\,b\,c^2 - b\,d^2}} - \frac{d\,\ln\left(\frac{\sqrt{(a\,c^2 - d^2)\,(b\,x^2 + a)} + x\,\sqrt{b\,(a\,c^2 - d^2) - a\,b\,c^2}}{\sqrt{(a\,c^2 - d^2)\,(b\,x^2 + a)} - x\,\sqrt{b\,(a\,c^2 - d^2) - a\,b\,c^2}}\right)}{2\,\sqrt{(a\,c^2 - d^2)\,(b\,(a\,c^2 - d^2) - a\,b\,c^2)}} & \text{if } b\,d^2 < 0 \\ \int \frac{1}{a\,c + d\,\sqrt{b\,x^2 + a} + b\,c\,x^2}\,dx & \text{if } b\,d^2 \notin \mathbb{R} \end{cases}$$

```
[In] int(1/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

```
[Out] piecewise(b == 0 | d == 0, atan((b*c*x)/(- b*d^2 + a*b*c^2)^(1/2))/(- b*d^2
 + a*b*c^2)^(1/2) - (d*x)/(a^(1/2)*(a*c^2 - d^2)), 0 < b*d^2, atan((b*c*x)/
(- b*d^2 + a*b*c^2)^(1/2))/(- b*d^2 + a*b*c^2)^(1/2) - (d*atan((x*(- b*(a*c
^2 - d^2) + a*b*c^2)^(1/2))/((a*c^2 - d^2)^(1/2)*(a + b*x^2)^(1/2))))/(-(a*
c^2 - d^2)*(b*(a*c^2 - d^2) - a*b*c^2))^(1/2), b*d^2 < 0, atan((b*c*x)/(- b
*d^2 + a*b*c^2)^(1/2))/(- b*d^2 + a*b*c^2)^(1/2) - (d*log((((a*c^2 - d^2)*(
a + b*x^2))^(1/2) + x*(b*(a*c^2 - d^2) - a*b*c^2)^(1/2))/(((a*c^2 - d^2)*(a
 + b*x^2))^(1/2) - x*(b*(a*c^2 - d^2) - a*b*c^2)^(1/2))))/(2*((a*c^2 - d^2)
*(b*(a*c^2 - d^2) - a*b*c^2))^(1/2)), ~in(b*d^2, 'real'), int(1/(a*c + d*(a
 + b*x^2)^(1/2) + b*c*x^2), x))
```

3.551 $\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 160

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=-\frac{c}{(ac^2-d^2)\,x}+\frac{d\sqrt{a+bx^2}}{a\,(ac^2-d^2)\,x}-\frac{\sqrt{b}c^2\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}}+\frac{\sqrt{b}c^2\arctan\left(\frac{\sqrt{b}dx}{\sqrt{ac^2-d^2}\sqrt{a+bx^2}}\right)}{(ac^2-d^2)^{3/2}}$$

```
[Out] -c/(a*c^2-d^2)/x-c^2*arctan(c*x*b^(1/2)/(a*c^2-d^2)^(1/2))*b^(1/2)/(a*c^2-d
^2)^(3/2)+c^2*arctan(d*x*b^(1/2)/(a*c^2-d^2)^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)
/(a*c^2-d^2)^(3/2)+d*(b*x^2+a)^(1/2)/a/(a*c^2-d^2)/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2187, 331, 211, 491, 12, 385}

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\frac{\sqrt{b}c^2\arctan\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}}-\frac{\sqrt{b}c^2\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}}+\frac{d\sqrt{a+bx^2}}{ax\,(ac^2-d^2)}-\frac{c}{x\,(ac^2-d^2)}$$

```
[In] Int[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[a + b*x^2])/(a*(a*c^2 - d^2)*x) - (Sqrt[b]
*c^2*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(a*c^2 - d^2)^(3/2) + (Sqrt[b
```

```
]*c^2*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(a*c^2 - d
^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = (ac)\int \frac{1}{x^2\left(a^2c^2-ad^2+abc^2x^2\right)}\,dx-(ad)\int \frac{1}{x^2\sqrt{a+bx^2}\left(a^2c^2-ad^2+abc^2x^2\right)}\,dx$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\left(abc^3\right)\int \frac{1}{a^2c^2-ad^2+abc^2x^2}\,dx}{ac^2-d^2}+\frac{d\int \frac{a^2bc^2}{\sqrt{a+bx^2}\left(a^2c^2-ad^2+abc^2x^2\right)}\,dx}{a\left(ac^2-d^2\right)}$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}+\frac{\left(abc^2d\right)\int \frac{1}{\sqrt{a+bx^2}\left(a^2c^2-ad^2+abc^2x^2\right)}\,dx}{ac^2-d^2}$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}+\frac{\left(abc^2d\right)\text{Subst}\left(\int \frac{1}{a^2c^2-ad^2-\left(-a^2bc^2+b\left(a^2c^2-ad^2\right)\right)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{ac^2-d^2}$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}+\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{ac^2-d^2}\sqrt{a+bx^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=-\frac{ac-d\sqrt{a+bx^2}}{a^2c^2x-ad^2x}+\frac{2\sqrt{b}c^2\arctan\left(\frac{d+c\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}$$

```
[In] Integrate[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
[Out] -((a*c - d*Sqrt[a + b*x^2])/(a^2*c^2*x - a*d^2*x)) + (2*Sqrt[b]*c^2*ArcTan[
(d + c*(-(Sqrt[b]*x) + Sqrt[a + b*x^2]))/Sqrt[a*c^2 - d^2]])/(a*c^2 - d^2)^
(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1810 vs. 2(140) = 280.

Time = 0.09 (sec) , antiderivative size = 1811, normalized size of antiderivative = 11.32

method	result	size
default	Expression too large to display	1811

```
[In] int(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] b*c^2/d^2/(b*(a*c^2-d^2))^(1/2)*arctan(x*b*c/(b*(a*c^2-d^2))^(1/2))-c/(a*c^
2-d^2)/x-a*c^4/(a*c^2-d^2)*b/d^2/(b*(a*c^2-d^2))^(1/2)*arctan(x*b*c/(b*(a*c
^2-d^2))^(1/2))-d*(1/(a*c^2-d^2)/a*(-1/a/x*(b*x^2+a)^(3/2)+2*b/a*(1/2*x*(b*
x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))-1/2*b^2*c^2/a/(-
a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-
(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/
b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2)
)/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1
/2))/b^(1/2))+1/2*b^2*c^2/a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*
c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x+1/b*(-a*b)
^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-(-a*b)^(1/2)*ln(((x+1/
b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^
(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2*b^2*c^6/(a*c^2-d^2)/((-a*b)
^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^
2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^
2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+
d^2/c^2)^(1/2)-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln((-(-(a*c^2-d^2)*b*c^2)^(1/
2)/c^2+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x+(-(a*c^2-d^2)*
b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/
c^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2
*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*
b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x+(-
(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)))+1/2*b^2*c^6/(a*c^2-d^2)/((-a*b)^(1/2)*c^2
+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/
(-(a*c^2-d^2)*b*c^2)^(1/2)*((b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(
a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(
1/2)+(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+(x-(
-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b
/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2*(-(a*
c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^
(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2
)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^2-d^2)
*b*c^2)^(1/2)/b/c^2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 581, normalized size of antiderivative = 3.63

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$$
$$= \left[-\frac{ac^2x\sqrt{-\frac{b}{ac^2-d^2}}\log\left(\frac{a^4c^4-2\,a^3c^2d^2+a^2d^4+(a^2b^2c^4-8\,ab^2c^2d^2+8\,b^2d^4)x^4+2\,(a^3bc^4-5\,a^2bc^2d^2+4\,abd^4)x^2+4\,((a^2bc^4d-3\,abc^2d^3+2\,b\cdots}{b^2c^4x^4+a^2c^4-2\,ac^2d^2+d^4+2\,(abc^4-bc^2d^2)x^2}\cdots\right.}{\cdots}\right.$$
$$-\frac{2\,ac^2x\sqrt{\frac{b}{ac^2-d^2}}\arctan\left(cx\sqrt{\frac{b}{ac^2-d^2}}\right)-ac^2x\sqrt{\frac{b}{ac^2-d^2}}\arctan\left(-\frac{(a^2c^2-ad^2+(abc^2-2\,bd^2)x^2)\sqrt{bx^2+a}\sqrt{\frac{b}{ac^2-d^2}}}{2\,(b^2dx^3+abdx)}\right)+2\cdots}{2\,(a^2c^2-ad^2)x}$$

```
[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*c^2*x*sqrt(-b/(a*c^2 - d^2))*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^
4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*
b*c^2*d^2 + 4*a*b*d^4)*x^2 + 4*((a^2*b*c^4*d - 3*a*b*c^2*d^3 + 2*b*d^5)*x^3
 + (a^3*c^4*d - 2*a^2*c^2*d^3 + a*d^5)*x)*sqrt(b*x^2 + a)*sqrt(-b/(a*c^2 -
d^2)))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)
*x^2)) + 2*a*c^2*x*sqrt(-b/(a*c^2 - d^2))*log((b*c^2*x^2 - a*c^2 + 2*(a*c^3
 - c*d^2)*x*sqrt(-b/(a*c^2 - d^2)) + d^2)/(b*c^2*x^2 + a*c^2 - d^2)) + 4*a*
c - 4*sqrt(b*x^2 + a)*d)/((a^2*c^2 - a*d^2)*x), -1/2*(2*a*c^2*x*sqrt(b/(a*c
^2 - d^2))*arctan(c*x*sqrt(b/(a*c^2 - d^2))) - a*c^2*x*sqrt(b/(a*c^2 - d^2)
)*arctan(-1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(b*x^2 + a)*s
qrt(b/(a*c^2 - d^2))/(b^2*d*x^3 + a*b*d*x)) + 2*a*c - 2*sqrt(b*x^2 + a)*d)/
((a^2*c^2 - a*d^2)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$$

```
[In] integrate(1/x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\int \frac{1}{\left(bcx^2+ac+\sqrt{bx^2+a}d\right)x^2}\,dx$$

```
[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=$$
$$-b^{\frac{3}{2}}d\left(\frac{c^2\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2c^2+ac^2-2\,d^2}{2\,\sqrt{ac^2-d^2}d}\right)}{(abc^2-bd^2)\sqrt{ac^2-d^2}d}+\frac{2}{(abc^2-bd^2)\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)}\right)$$
$$-\frac{bc^2\arctan\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2-bd^2}(ac^2-d^2)}-\frac{c}{(ac^2-d^2)x}$$

```
[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
[Out] -b^(3/2)*d*(c^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 - 2
*d^2)/(sqrt(a*c^2 - d^2)*d))/((a*b*c^2 - b*d^2)*sqrt(a*c^2 - d^2)*d) + 2/((
a*b*c^2 - b*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a))) - b*c^2*arctan(b*c
*x/sqrt(a*b*c^2 - b*d^2))/(sqrt(a*b*c^2 - b*d^2)*(a*c^2 - d^2)) - c/((a*c^2
 - d^2)*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\int \frac{1}{x^2\,\left(a\,c+d\,\sqrt{b\,x^2+a}+b\,c\,x^2\right)}\,dx$$

```
[In] int(1/(x^2*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)
[Out] int(1/(x^2*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)), x)
```

3.552 $\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 140

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=-\frac{(2ac^2-d^2)\,x^3}{3b^2c^3}+\frac{2d(2ac^2-d^2)\,\sqrt{a+bx^3}}{3b^3c^4}-\frac{2d(a+bx^3)^{3/2}}{9b^3c^2}+\frac{(a+bx^3)^2}{6b^3c}+\frac{2(ac^2-d^2)^2\log\left(d+c\sqrt{a+bx^3}\right)}{3b^3c^5}$$

```
[Out] -1/3*(2*a*c^2-d^2)*x^3/b^2/c^3-2/9*d*(b*x^3+a)^(3/2)/b^3/c^2+1/6*(b*x^3+a)^
2/b^3/c+2/3*(a*c^2-d^2)^2*ln(d+c*(b*x^3+a)^(1/2))/b^3/c^5+2/3*d*(2*a*c^2-d^
2)*(b*x^3+a)^(1/2)/b^3/c^4
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2186, 711}

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=-\frac{2d(a+bx^3)^{3/2}}{9b^3c^2}+\frac{2(ac^2-d^2)^2\log\left(c\sqrt{a+bx^3}+d\right)}{3b^3c^5}+\frac{2d\sqrt{a+bx^3}(2ac^2-d^2)}{3b^3c^4}+\frac{(a+bx^3)^2}{6b^3c}-\frac{x^3(2ac^2-d^2)}{3b^2c^3}$$

```
[In] Int[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/3*((2*a*c^2 - d^2)*x^3)/(b^2*c^3) + (2*d*(2*a*c^2 - d^2)*Sqrt[a + b*x^3]
)/(3*b^3*c^4) - (2*d*(a + b*x^3)^(3/2))/(9*b^3*c^2) + (a + b*x^3)^2/(6*b^3*
c) + (2*(a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^3]])/(3*b^3*c^5)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^3\right) \\
&= \frac{2\text{Subst}\left(\int \frac{(a-x^2)^2}{d+cx}\,dx, x, \sqrt{a+bx^3}\right)}{3b^3} \\
&= \frac{2\text{Subst}\left(\int \left(\frac{2ac^2d-d^3}{c^4} - \frac{(2ac^2-d^2)x}{c^3} - \frac{dx^2}{c^2} + \frac{x^3}{c} + \frac{(ac^2-d^2)^2}{c^4(d+cx)}\right)dx, x, \sqrt{a+bx^3}\right)}{3b^3} \\
&= -\frac{(2ac^2-d^2)\,x^3}{3b^2c^3} + \frac{2d(2ac^2-d^2)\sqrt{a+bx^3}}{3b^3c^4} - \frac{2d(a+bx^3)^{3/2}}{9b^3c^2} \\
&\quad + \frac{(a+bx^3)^2}{6b^3c} + \frac{2(ac^2-d^2)^2\log\left(d+c\sqrt{a+bx^3}\right)}{3b^3c^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{3c^2(a+bx^3)\left(-3ac^2+2d^2+bc^2x^3\right) - 4cd\sqrt{a+bx^3}\left(-5ac^2+3d^2+bc^2x^3\right) + 12(-ac^2+d^2)^2\log\left(d+c\sqrt{a\right.}{18b^3c^5}$$

```
[In] Integrate[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (3*c^2*(a + b*x^3)*(-3*a*c^2 + 2*d^2 + b*c^2*x^3) - 4*c*d*Sqrt[a + b*x^3]*(
-5*a*c^2 + 3*d^2 + b*c^2*x^3) + 12*(-(a*c^2) + d^2)^2*Log[d + c*Sqrt[a + b*
x^3]])/(18*b^3*c^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(124) = 248$.

Time = 0.39 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.37

method	result
default	$d\left(-\frac{2(b\,x^3+a)^{\frac{3}{2}}}{9b^3c^2}+\frac{2a^2\sqrt{b\,x^3+a}}{3b^3d^2}-\frac{(a^2c^4-2a\,c^2d^2+d^4)\left(d\ln\left(c\sqrt{b\,x^3+a}-d\right)-d\ln\left(d+c\sqrt{b\,x^3+a}\right)+2c\sqrt{b\,x^3+a}\right)}{3d^2c^5b^3}\right)-ac\left(-\frac{}{3b}\right.$
elliptic	$\frac{\sqrt{b\,x^3+a}\left(d+c\sqrt{b\,x^3+a}\right)\left(c\left(\frac{\frac{1}{2}b\,c^2x^6-c^2x^3a+d^2x^3}{3b^2c^4}+\frac{(a^2c^4-2a\,c^2d^2+d^4)\ln\left(b\,c^2x^3+a\,c^2-d^2\right)}{3b^3c^6}\right)-\frac{2d\,x^3\sqrt{b\,x^3+a}}{9b^2c^2}+\frac{2\left(\frac{(a\,c^2-d^2)d}{b^2c^4}+\frac{2da}{3b^2c^2}\right)}{3b}\right.}{}$

[In] int(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)

[Out] d*(-2/9/b^3/c^2*(b*x^3+a)^(3/2)+2/3*a^2/b^3/d^2*(b*x^3+a)^(1/2)-1/3*(a^2*c^4-2*a*c^2*d^2+d^4)/d^2/c^5/b^3*(d*ln(c*(b*x^3+a)^(1/2)-d)-d*ln(d+c*(b*x^3+a)^(1/2))+2*c*(b*x^3+a)^(1/2)))-a*c*(-1/3/b^2/c^2*x^3+1/3*(-a^2*c^4+2*a*c^2*d^2-d^4)/d^2/c^4/b^3*ln(b*c^2*x^3+a*c^2-d^2)+1/3*a^2/b^3/d^2*ln(b*x^3+a))-b*c*(-1/3/c^4/b^3*(1/2*b*c^2*x^6-2*c^2*x^3*a+d^2*x^3)+1/3/c^6/b^4*(a^3*c^6-3*a^2*c^4*d^2+3*a*c^2*d^4-d^6)/d^2*ln(b*c^2*x^3+a*c^2-d^2)-1/3/b^4*a^3/d^2*ln(b*x^3+a))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.36

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{3\,b^2c^4x^6-6\,(abc^4-bc^2d^2)x^3+6\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(bc^2x^3+ac^2-d^2\right)+6\,(a^2c^4-2\,ac^2d^2+d^4)\log}{18\,b^3}$$

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/18*(3*b^2*c^4*x^6 - 6*(a*b*c^4 - b*c^2*d^2)*x^3 + 6*(a^2*c^4 - 2*a*c^2*d^
2 + d^4)*log(b*c^2*x^3 + a*c^2 - d^2) + 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log
(sqrt(b*x^3 + a)*c + d) - 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(sqrt(b*x^3 +
a)*c - d) - 4*(b*c^3*d*x^3 - 5*a*c^3*d + 3*c*d^3)*sqrt(b*x^3 + a))/(b^3*c^5
)
```

Sympy [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{2\left(\frac{(a+bx^3)^2}{12c}-\frac{d(a+bx^3)^{\frac{3}{2}}}{9c^2}+\frac{(a+bx^3)(-2ac^2+d^2)}{6c^3}+\frac{\sqrt{a+bx^3}\cdot(2ac^2d-d^3)}{3c^4}+\frac{(ac^2-d^2)^2\left(\begin{cases}\frac{\sqrt{a+bx^3}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^3}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{3c^4}\right)}{b^3} & \text{for } b\neq \\ \frac{x^9}{3\cdot(3\sqrt{a}d+3ac)} & \text{otherwi} \end{cases}$$

```
[In] integrate(x**8/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Piecewise((2*((a + b*x**3)**2/(12*c) - d*(a + b*x**3)**(3/2)/(9*c**2) + (a
+ b*x**3)*(-2*a*c**2 + d**2)/(6*c**3) + sqrt(a + b*x**3)*(2*a*c**2*d - d**3
)/(3*c**4) + (a*c**2 - d**2)**2*Piecewise((sqrt(a + b*x**3)/d, Eq(c, 0)), (
log(c*sqrt(a + b*x**3) + d)/c, True))/(3*c**4))/b**3, Ne(b, 0)), (x**9/(3*(
3*sqrt(a)*d + 3*a*c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{\frac{3\,(bx^3+a)^2c^3-4\,(bx^3+a)^{\frac{3}{2}}c^2d-6\,(2\,ac^3-cd^2)\,(bx^3+a)+12\,(2\,ac^2d-d^3)\sqrt{bx^3+a}}{c^4}+\frac{12\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(\sqrt{bx^3+a}c+d\right)}{c^5}}{18\,b^3}$$

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/18*((3*(b*x^3 + a)^2*c^3 - 4*(b*x^3 + a)^(3/2)*c^2*d - 6*(2*a*c^3 - c*d^2
)*(b*x^3 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^3 + a))/c^4 + 12*(a^2*c^4 - 2
*a*c^2*d^2 + d^4)*log(sqrt(b*x^3 + a)*c + d)/c^5)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.11

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{2\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{3\,b^3c^5} + \frac{3\,(bx^3+a)^2b^9c^3-12\,(bx^3+a)ab^9c^3-4\,(bx^3+a)^{\frac{3}{2}}b^9c^2d+24\,\sqrt{bx^3+a}ab^9c^2d+6\,(bx^3+a)b^9cd^2-12\,\sqrt{}}{18\,b^{12}c^4}$$

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(abs(sqrt(b*x^3 + a)*c + d))/(b^3*c^5)
 + 1/18*(3*(b*x^3 + a)^2*b^9*c^3 - 12*(b*x^3 + a)*a*b^9*c^3 - 4*(b*x^3 + a)
^(3/2)*b^9*c^2*d + 24*sqrt(b*x^3 + a)*a*b^9*c^2*d + 6*(b*x^3 + a)*b^9*c*d^2
 - 12*sqrt(b*x^3 + a)*b^9*d^3)/(b^12*c^4)
```

Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.43

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{\left(\frac{2\,d\,(a\,c^2-d^2)}{b^2\,c^4}+\frac{4\,a\,d}{3\,b^2\,c^2}\right)\sqrt{b\,x^3+a}}{3\,b}+\frac{x^6}{6\,b\,c} - \frac{x^3\,(a\,c^2-d^2)}{3\,b^2\,c^3}+\frac{\ln\left(\frac{d+c\,\sqrt{b\,x^3+a}}{d-c\,\sqrt{b\,x^3+a}}\right)\,(a\,c^2-d^2)^2}{3\,b^3\,c^5} + \frac{\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)\,(a^2\,c^4-2\,a\,c^2\,d^2+d^4)}{3\,b^3\,c^5} - \frac{2\,d\,x^3\,\sqrt{b\,x^3+a}}{9\,b^2\,c^2}$$

```
[In] int(x^8/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] (((2*d*(a*c^2 - d^2))/(b^2*c^4) + (4*a*d)/(3*b^2*c^2))*(a + b*x^3)^(1/2))/(
3*b) + x^6/(6*b*c) - (x^3*(a*c^2 - d^2))/(3*b^2*c^3) + (log((d + c*(a + b*x
^3)^(1/2))/(d - c*(a + b*x^3)^(1/2)))*(a*c^2 - d^2)^2)/(3*b^3*c^5) + (log(a
*c^2 - d^2 + b*c^2*x^3)*(d^4 + a^2*c^4 - 2*a*c^2*d^2))/(3*b^3*c^5) - (2*d*x
^3*(a + b*x^3)^(1/2))/(9*b^2*c^2)
```

3.553 $\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 73

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\frac{x^3}{3bc}-\frac{2d\sqrt{a+bx^3}}{3b^2c^2}-\frac{2(ac^2-d^2)\log\left(d+c\sqrt{a+bx^3}\right)}{3b^2c^3}$$

```
[Out] 1/3*x^3/b/c-2/3*(a*c^2-d^2)*ln(d+c*(b*x^3+a)^(1/2))/b^2/c^3-2/3*d*(b*x^3+a)
^(1/2)/b^2/c^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2186, 711}

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=-\frac{2d\sqrt{a+bx^3}}{3b^2c^2}-\frac{2(ac^2-d^2)\log\left(c\sqrt{a+bx^3}+d\right)}{3b^2c^3}+\frac{x^3}{3bc}$$

```
[In] Int[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] x^3/(3*b*c) - (2*d*Sqrt[a + b*x^3])/(3*b^2*c^2) - (2*(a*c^2 - d^2)*Log[d +
c*Sqrt[a + b*x^3]])/(3*b^2*c^3)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^3\right) \\
&= \frac{2\text{Subst}\left(\int \frac{-a+x^2}{d+cx}\,dx, x, \sqrt{a+bx^3}\right)}{3b^2} \\
&= \frac{2\text{Subst}\left(\int \left(-\frac{d}{c^2}+\frac{x}{c}+\frac{-ac^2+d^2}{c^2(d+cx)}\right)\,dx, x, \sqrt{a+bx^3}\right)}{3b^2} \\
&= \frac{x^3}{3bc} - \frac{2d\sqrt{a+bx^3}}{3b^2c^2} - \frac{2(ac^2-d^2)\log\left(d+c\sqrt{a+bx^3}\right)}{3b^2c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{c\left(ac+bcx^3-2d\sqrt{a+bx^3}\right)+(-2ac^2+2d^2)\log\left(d+c\sqrt{a+bx^3}\right)}{3b^2c^3}$$

```
[In] Integrate[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
[Out] (c*(a*c + b*c*x^3 - 2*d*Sqrt[a + b*x^3]) + (-2*a*c^2 + 2*d^2)*Log[d + c*Sqr
t[a + b*x^3]])/(3*b^2*c^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(63) = 126.

Time = 0.16 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.37

method	result
default	$d\left(-\frac{2a\sqrt{b\,x^3+a}}{3b^2d^2}+\frac{(a\,c^2-d^2)\left(d\ln\left(c\sqrt{b\,x^3+a}-d\right)-d\ln\left(d+c\sqrt{b\,x^3+a}\right)+2c\sqrt{b\,x^3+a}\right)}{3b^2d^2c^3}\right)-ac\left(\frac{(a\,c^2-d^2)\ln\left(b\,c^2x^3+a\,c^2-d^2\right)}{3b^2d^2c^2}\right.$
elliptic	$\frac{\sqrt{b\,x^3+a}\left(d+c\sqrt{b\,x^3+a}\right)\left(c\left(\frac{x^3}{3b\,c^2}+\frac{\left(-a\,c^2+d^2\right)\ln\left(b\,c^2x^3+a\,c^2-d^2\right)}{3b^2c^4}\right)-\frac{2d\sqrt{b\,x^3+a}}{3b^2c^2}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\,c^2\_Z^3+a\,c^2-d^2\right)}\frac{(a\,c^2-d^2)(-b}{}\right.}{}\right.}{}$

[In] int(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)

[Out] d*(-2/3*a/b^2/d^2*(b*x^3+a)^(1/2)+1/3*(a*c^2-d^2)/b^2/d^2*(d*ln(c*(b*x^3+a)^(1/2)-d)-d*ln(d+c*(b*x^3+a)^(1/2))+2*c*(b*x^3+a)^(1/2))/c^3)-a*c*(1/3*(a*c^2-d^2)/b^2/d^2/c^2*ln(b*c^2*x^3+a*c^2-d^2)-1/3*a/b^2/d^2*ln(b*x^3+a))-b*c*(-1/3/b^2/c^2*x^3+1/3*(-a^2*c^4+2*a*c^2*d^2-d^4)/d^2/c^4/b^3*ln(b*c^2*x^3+a*c^2-d^2)+1/3*a^2/b^3/d^2*ln(b*x^3+a))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.62

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{bc^2x^3-2\sqrt{bx^3+a}cd-(ac^2-d^2)\log\left(bc^2x^3+ac^2-d^2\right)-(ac^2-d^2)\log\left(\sqrt{bx^3+a}c+d\right)+(ac^2-d^2)\log\left(\sqrt{bx^3+a}c-d\right)}{3\,b^2c^3}$$

```
[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*c^2*x^3 - 2*sqrt(b*x^3 + a)*c*d - (a*c^2 - d^2)*log(b*c^2*x^3 + a*c^
2 - d^2) - (a*c^2 - d^2)*log(sqrt(b*x^3 + a)*c + d) + (a*c^2 - d^2)*log(sqr
t(b*x^3 + a)*c - d))/(b^2*c^3)
```

Sympy [A] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{2\left(\frac{a+bx^3}{6c}-\frac{d\sqrt{a+bx^3}}{3c^2}-\frac{(ac^2-d^2)\left(\begin{cases}\frac{\sqrt{a+bx^3}}{d} & \text{for}\ c=0\\ \frac{\log\left(c\sqrt{a+bx^3}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{3c^2}\right)}{b^2} & \text{for}\ b\neq 0\\ \frac{x^6}{2\cdot\left(3\sqrt{a}d+3ac\right)} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**5/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Piecewise((2*((a + b*x**3)/(6*c) - d*sqrt(a + b*x**3)/(3*c**2) - (a*c**2 -
d**2)*Piecewise((sqrt(a + b*x**3)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**3) + d
)/c, True))/(3*c**2))/b**2, Ne(b, 0)), (x**6/(2*(3*sqrt(a)*d + 3*a*c)), Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{\frac{(bx^3+a)c-2\sqrt{bx^3+a}d}{c^2} - \frac{2\,(ac^2-d^2)\log\left(\sqrt{bx^3+a}c+d\right)}{c^3}}{3\,b^2}$$

```
[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*(((b*x^3 + a)*c - 2*sqrt(b*x^3 + a)*d)/c^2 - 2*(a*c^2 - d^2)*log(sqrt(b
*x^3 + a)*c + d)/c^3)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = -\frac{\frac{2\,(ac^2-d^2)\log\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{bc^3} - \frac{(bx^3+a)bc-2\sqrt{bx^3+a}bd}{b^2c^2}}{3\,b}$$

```
[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/3*(2*(a*c^2 - d^2)*log(abs(sqrt(b*x^3 + a)*c + d))/(b*c^3) - ((b*x^3 + a
)*b*c - 2*sqrt(b*x^3 + a)*b*d)/(b^2*c^2))/b
```

Mupad [B] (verification not implemented)

Time = 18.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.63

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{x^3}{3\,b\,c} - \frac{2\,d\,\sqrt{b\,x^3+a}}{3\,b^2\,c^2} + \frac{\ln\left(\frac{d-c\,\sqrt{b\,x^3+a}}{d+c\,\sqrt{b\,x^3+a}}\right)\,\left(a\,c^2-d^2\right)}{3\,b^2\,c^3} - \frac{\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)\,\left(a\,c^2-d^2\right)}{3\,b^2\,c^3}$$

```
[In] int(x^5/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] x^3/(3*b*c) - (2*d*(a + b*x^3)^(1/2))/(3*b^2*c^2) + (log((d - c*(a + b*x^3)
^(1/2))/(d + c*(a + b*x^3)^(1/2)))*(a*c^2 - d^2))/(3*b^2*c^3) - (log(a*c^2
- d^2 + b*c^2*x^3)*(a*c^2 - d^2))/(3*b^2*c^3)
```

3.554 $\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 26

$$\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}} \, dx = \frac{2\log\left(d+c\sqrt{a+bx^3}\right)}{3bc}$$

```
[Out] 2/3*ln(d+c*(b*x^3+a)^(1/2))/b/c
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2186, 31}

$$\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}} \, dx = \frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

```
[In] Int[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^3]])/(3*b*c)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
```

```
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^3\right)\\
&= \frac{2\text{Subst}\left(\int \frac{1}{d+cx}\,dx,x,\sqrt{a+bx^3}\right)}{3b}\\
&= \frac{2\log\left(d+c\sqrt{a+bx^3}\right)}{3bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{2\log\left(bd+bc\sqrt{a+bx^3}\right)}{3bc}$$

```
[In] Integrate[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
[Out] (2*Log[b*d + b*c*Sqrt[a + b*x^3]])/(3*b*c)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(22) = 44$.

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 6.15

method	result
default	$\frac{\ln\left(d+c\sqrt{b\,x^{3}+a}\right)}{3bc}-\frac{\ln\left(c\sqrt{b\,x^{3}+a}-d\right)}{3cb}+\frac{ac\ln\left(b\,c^{2}x^{3}+a\,c^{2}-d^{2}\right)}{3d^{2}b}-\frac{ac\ln\left(b\,x^{3}+a\right)}{3d^{2}b}-bc\left(\frac{\left(a\,c^{2}-d^{2}\right)\ln\left(b\,c^{2}x^{3}+a\,c^{2}-d^{2}\right)}{3b^{2}d^{2}c^{2}}-\right.$
elliptic	$\frac{\sqrt{b\,x^{3}+a}\left(d+c\sqrt{b\,x^{3}+a}\right)\left(\frac{\ln\left(b\,c^{2}x^{3}+a\,c^{2}-d^{2}\right)}{3bc}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\,c^{2}\_Z^{3}+a\,c^{2}-d^{2}\right)}\frac{\left(-b^{2}a\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-b^{2}a\right)^{\frac{1}{3}}-i\sqrt{3}\left(-b^{2}a\right)^{\frac{1}{3}}}{b}\right)}{\left(-b^{2}a\right)^{\frac{1}{3}}}}}{}\right)}{}\right)}{}$

[In] int(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)

[Out] 1/3*ln(d+c*(b*x^3+a)^(1/2))/b/c-1/3/c/b*ln(c*(b*x^3+a)^(1/2)-d)+1/3*a*c/d^2/b*ln(b*c^2*x^3+a*c^2-d^2)-1/3*a*c/d^2/b*ln(b*x^3+a)-b*c*(1/3*(a*c^2-d^2)/b^2/d^2/c^2*ln(b*c^2*x^3+a*c^2-d^2)-1/3*a/b^2/d^2*ln(b*x^3+a))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(22) = 44$.

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \frac{x^2}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \frac{\log\left(bc^2x^3 + ac^2 - d^2\right) + \log\left(\sqrt{bx^3 + a}c + d\right) - \log\left(\sqrt{bx^3 + a}c - d\right)}{3\,bc}$$

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(log(b*c^2*x^3 + a*c^2 - d^2) + log(sqrt(b*x^3 + a)*c + d) - log(sqrt(b
*x^3 + a)*c - d))/(b*c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(20) = 40$.

Time = 1.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{x^2}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \begin{cases} \frac{2\left(\begin{cases} \frac{\sqrt{a+bx^3}}{d} & \text{for}\ c = 0 \\ \frac{\log\left(c\sqrt{a+bx^3}+d\right)}{c} & \text{otherwise} \end{cases}\right)}{3b} & \text{for}\ b \neq 0 \\ \frac{x^3}{3\sqrt{a}d+3ac} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Piecewise((2*Piecewise((sqrt(a + b*x**3)/d, Eq(c, 0)), (log(c*sqrt(a + b*x*
*3) + d)/c, True))/(3*b), Ne(b, 0)), (x**3/(3*sqrt(a)*d + 3*a*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \frac{2\,\log\left(\sqrt{bx^3 + a}c + d\right)}{3\,bc}$$

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*log(sqrt(b*x^3 + a)*c + d)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\frac{2\log\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{3\,bc}$$

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
[Out] 2/3*log(abs(sqrt(b*x^3 + a)*c + d))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 17.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\frac{\ln\left(\frac{d+c\sqrt{b\,x^3+a}}{d-c\sqrt{b\,x^3+a}}\right)+\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)}{3\,b\,c}$$

```
[In] int(x^2/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
[Out] (log((d + c*(a + b*x^3)^(1/2))/(d - c*(a + b*x^3)^(1/2))) + log(a*c^2 - d^2
 + b*c^2*x^3))/(3*b*c)
```

3.555 $\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)} dx$

Optimal result

Integrand size = 29, antiderivative size = 93

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)} dx = \frac{2d\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)} + \frac{c\log(x)}{ac^2-d^2} - \frac{2c\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}$$

```
[Out] c*ln(x)/(a*c^2-d^2)-2/3*c*ln(d+c*(b*x^3+a)^(1/2))/(a*c^2-d^2)+2/3*d*arctanh
((b*x^3+a)^(1/2)/a^(1/2))/(a*c^2-d^2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2186, 720, 31, 649, 213, 266}

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)} dx = \frac{2d\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)} - \frac{2c\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)} + \frac{c\log(x)}{ac^2-d^2}$$

```
[In] Int[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*(a*c^2 - d^2)) + (c*Log[x
])/(a*c^2 - d^2) - (2*c*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx,x,x^3\right)\\
&= \frac{2}{3}\text{Subst}\left(\int \frac{1}{(d+cx)\left(-a+x^2\right)}\,dx,x,\sqrt{a+bx^3}\right)\\
&= -\frac{2\text{Subst}\left(\int \frac{d-cx}{-a+x^2}\,dx,x,\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}-\frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{d+cx}\,dx,x,\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}\\
&= -\frac{2c\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}+\frac{(2c)\text{Subst}\left(\int \frac{x}{-a+x^2}\,dx,x,\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}\\
&\quad -\frac{(2d)\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx,x,\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}\\
&= \frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}-\frac{2c\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\frac{\frac{2d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}+c\log\left(bx^3\right)-2c\log\left(d+c\sqrt{a+bx^3}\right)}{3ac^2-3d^2}$$

```
[In] Integrate[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] ((2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/Sqrt[a] + c*Log[b*x^3] - 2*c*Log[d
+ c*Sqrt[a + b*x^3]])/(3*a*c^2 - 3*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(81) = 162.

Time = 1.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.40

method	result
default	$\frac{c\ln(x)}{a\,c^2-d^2}-\frac{a\,c^3\ln\left(b\,c^2x^3+a\,c^2-d^2\right)}{3\left(a\,c^2-d^2\right)d^2}+\frac{c\ln\left(b\,c^2x^3+a\,c^2-d^2\right)}{3d^2}-d\left(\frac{\frac{2\sqrt{b\,x^3+a}}{3}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3}}{a\left(a\,c^2-d^2\right)}+\frac{2\sqrt{b\,x^3+a}}{3a\,d^2}-\frac{c\left(d\ln\right.}{}\right.$
elliptic	Expression too large to display

```
[In] int(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] c*ln(x)/(a*c^2-d^2)-1/3*a*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^3+a*c^2-d^2)+1/3*c
/d^2*ln(b*c^2*x^3+a*c^2-d^2)-d*(1/a/(a*c^2-d^2)*(2/3*(b*x^3+a)^(1/2)-2/3*a^
(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+2/3/a/d^2*(b*x^3+a)^(1/2)-1/3*c/(a*
c^2-d^2)/d^2*(d*ln(c*(b*x^3+a)^(1/2)-d)-d*ln(d+c*(b*x^3+a)^(1/2))+2*c*(b*x^
3+a)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.49

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$
$$=\left[-\frac{ac\log\left(bc^2x^3+ac^2-d^2\right)+ac\log\left(\sqrt{bx^3+a}c+d\right)-ac\log\left(\sqrt{bx^3+a}c-d\right)-3\,ac\log\left(x\right)-\sqrt{a}d\log}{3\left(a^2c^2-ad^2\right)}\right.$$
$$-\frac{ac\log\left(bc^2x^3+ac^2-d^2\right)+ac\log\left(\sqrt{bx^3+a}c+d\right)-ac\log\left(\sqrt{bx^3+a}c-d\right)-3\,ac\log\left(x\right)+2\sqrt{-a}d\,a}{3\left(a^2c^2-ad^2\right)}$$

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/3*(a*c*log(b*c^2*x^3 + a*c^2 - d^2) + a*c*log(sqrt(b*x^3 + a)*c + d) -
a*c*log(sqrt(b*x^3 + a)*c - d) - 3*a*c*log(x) - sqrt(a)*d*log((b*x^3 + 2*sq
rt(b*x^3 + a)*sqrt(a) + 2*a)/x^3))/(a^2*c^2 - a*d^2), -1/3*(a*c*log(b*c^2*x
^3 + a*c^2 - d^2) + a*c*log(sqrt(b*x^3 + a)*c + d) - a*c*log(sqrt(b*x^3 + a
)*c - d) - 3*a*c*log(x) + 2*sqrt(-a)*d*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a))/
(a^2*c^2 - a*d^2)]
```

Sympy [A] (verification not implemented)

Time = 2.97 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.61

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$
$$=\begin{cases}\frac{2\left(-\frac{bc^2\left(\begin{cases}\frac{\sqrt{a+bx^3}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^3}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{3\left(ac^2-d^2\right)}-\frac{b\left(-\frac{c\log\left(-bx^3\right)}{2}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)}{3\left(ac^2-d^2\right)}\right)}{b} & \text{for } b\neq 0\\ \begin{cases}\frac{x^3\log\left(x^3\right)}{3\sqrt{a}dx^3+3acx^3} & \text{for } 3\sqrt{a}d+3ac\neq 0\\ \tilde{\infty}x^3 & \text{otherwise}\end{cases} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Piecewise((2*(-b*c**2*Piecewise((sqrt(a + b*x**3)/d, Eq(c, 0)), (log(c*sqrt
(a + b*x**3) + d)/c, True))/(3*(a*c**2 - d**2)) - b*(-c*log(-b*x**3)/2 + d*
atan(sqrt(a + b*x**3)/sqrt(-a))/sqrt(-a))/(3*(a*c**2 - d**2)))/b, Ne(b, 0))
, (Piecewise((x**3*log(x**3)/(3*sqrt(a)*d*x**3 + 3*a*c*x**3), Ne(3*sqrt(a)*
d + 3*a*c, 0)), (zoo*x**3, True)), True))
```

Maxima [F]

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{\left(bcx^3+ac+\sqrt{bx^3+a}d\right)x}\,dx$$

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=-\frac{2\,c^2\log\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{3\,(ac^3-cd^2)}+\frac{c\log\left(bx^3\right)}{3\,(ac^2-d^2)}-\frac{2\,d\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,(ac^2-d^2)\sqrt{-a}}$$

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*c^2*log(abs(sqrt(b*x^3 + a)*c + d))/(a*c^3 - c*d^2) + 1/3*c*log(b*x^3)
/(a*c^2 - d^2) - 2/3*d*arctan(sqrt(b*x^3 + a)/sqrt(-a))/((a*c^2 - d^2)*sqrt
(-a))
```

Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\frac{c\,\ln(x)}{a\,c^2-d^2}+\frac{c\,\ln\left(\frac{d-c\,\sqrt{b\,x^3+a}}{d+c\,\sqrt{b\,x^3+a}}\right)}{3\,\left(a\,c^2-d^2\right)}-\frac{c\,\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)}{3\,a\,c^2-3\,d^2}+\frac{d\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)}{3\,\sqrt{a}\,\left(a\,c^2-d^2\right)}$$

```
[In] int(1/(x*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

```
[Out] (c*log(x))/(a*c^2 - d^2) + (c*log((d - c*(a + b*x^3)^(1/2))/(d + c*(a + b*x
^3)^(1/2))))/(3*(a*c^2 - d^2)) - (c*log(a*c^2 - d^2 + b*c^2*x^3))/(3*a*c^2
- 3*d^2) + (d*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/
2))^3)/x^6))/(3*a^(1/2)*(a*c^2 - d^2))
```

3.556 $\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 154

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=-\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3}-\frac{bd(3ac^2-d^2)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}+\frac{2bc^3\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)^2}$$

```
[Out] -1/3*b*d*(3*a*c^2-d^2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)/(a*c^2-d^2)
^2-b*c^3*ln(x)/(a*c^2-d^2)^2+2/3*b*c^3*ln(d+c*(b*x^3+a)^(1/2))/(a*c^2-d^2)^
2+1/3*(-a*c+d*(b*x^3+a)^(1/2))/a/(a*c^2-d^2)/x^3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2186, 755, 815, 649, 212, 266}

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=-\frac{bd(3ac^2-d^2)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^3}}{3ax^3\left(ac^2-d^2\right)}+\frac{2bc^3\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

```
[In] Int[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -1/3*(a*c - d*Sqrt[a + b*x^3])/(a*(a*c^2 - d^2)*x^3) - (b*d*(3*a*c^2 - d^2)
*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log
```

```
[x])/(a*c^2 - d^2)^2 + (2*b*c^3*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2
)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}(2b)\text{Subst}\left(\int \frac{1}{(d+cx)\,(a-x^2)^2}\,dx, x, \sqrt{a+bx^3}\right)$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\,(ac^2-d^2)\,x^3} - \frac{b\text{Subst}\left(\int \frac{-2ac^2+d^2+cdx}{(d+cx)(a-x^2)}\,dx, x, \sqrt{a+bx^3}\right)}{3a\,(ac^2-d^2)}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\,(ac^2-d^2)\,x^3} - \frac{b\text{Subst}\left(\int \left(-\frac{2ac^4}{(ac^2-d^2)(d+cx)} + \frac{3ac^2d-d^3-2ac^3x}{(ac^2-d^2)(a-x^2)}\right)\,dx, x, \sqrt{a+bx^3}\right)}{3a\,(ac^2-d^2)}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\,(ac^2-d^2)\,x^3} + \frac{2bc^3\log\left(d+c\sqrt{a+bx^3}\right)}{3\,(ac^2-d^2)^2} - \frac{b\text{Subst}\left(\int \frac{3ac^2d-d^3-2ac^3x}{a-x^2}\,dx, x, \sqrt{a+bx^3}\right)}{3a\,(ac^2-d^2)^2}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\,(ac^2-d^2)\,x^3} + \frac{2bc^3\log\left(d+c\sqrt{a+bx^3}\right)}{3\,(ac^2-d^2)^2} + \frac{(2bc^3)\,\text{Subst}\left(\int \frac{x}{a-x^2}\,dx, x, \sqrt{a+bx^3}\right)}{3\,(ac^2-d^2)^2} - \frac{(bd(3ac^2-d^2))\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+bx^3}\right)}{3a\,(ac^2-d^2)^2}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\,(ac^2-d^2)\,x^3} - \frac{bd(3ac^2-d^2)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\,(ac^2-d^2)^2} - \frac{bc^3\log(x)}{(ac^2-d^2)^2} + \frac{2bc^3\log\left(d+c\sqrt{a+bx^3}\right)}{3\,(ac^2-d^2)^2}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx = \frac{bd(-3ac^2+d^2)\,x^3\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) + \sqrt{a}\left(-\left((ac^2-d^2)\left(ac-d\sqrt{a+bx^3}\right)\right) - abc^3x^3\log\left(bx^3\right) + 2abc^3x^3\right.}{3a^{3/2}\left(-ac^2+d^2\right)^2x^3}$$

```
[In] Integrate[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (b*d*(-3*a*c^2 + d^2)*x^3*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + Sqrt[a]*(-((a*
c^2 - d^2)*(a*c - d*Sqrt[a + b*x^3])) - a*b*c^3*x^3*Log[b*x^3] + 2*a*b*c^3*
x^3*Log[d + c*Sqrt[a + b*x^3]]))/(3*a^(3/2)*(-(a*c^2) + d^2)^2*x^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. 2(138) = 276.

Time = 0.97 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.63

method	result
default	$ac\left(-\frac{b\ln(b\,x^3+a)}{3a^2d^2}-\frac{1}{3a(a\,c^2-d^2)x^3}-\frac{b(2a\,c^2-d^2)\ln(x)}{a^2(a\,c^2-d^2)^2}+\frac{b\,c^4\ln(b\,c^2x^3+a\,c^2-d^2)}{3(a\,c^2-d^2)^2d^2}\right)+bc\left(\frac{\ln(b\,x^3+a)}{3a\,d^2}+\frac{\ln(x)}{a(a\,c^2-d^2)}-\right.$
elliptic	Expression too large to display

```
[In] int(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] a*c*(-1/3*b/a^2/d^2*ln(b*x^3+a)-1/3/a/(a*c^2-d^2)/x^3-b*(2*a*c^2-d^2)/a^2/(
a*c^2-d^2)^2*ln(x)+1/3*b*c^4/(a*c^2-d^2)^2/d^2*ln(b*c^2*x^3+a*c^2-d^2))+b*c
*(1/3/a/d^2*ln(b*x^3+a)+1/a/(a*c^2-d^2)*ln(x)-1/3*c^2/(a*c^2-d^2)/d^2*ln(b*
c^2*x^3+a*c^2-d^2))-d*(1/a/(a*c^2-d^2)*(-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arct
anh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))-2/3*b/a^2/d^2*(b*x^3+a)^(1/2)-b*(2*a*
c^2-d^2)/a^2/(a*c^2-d^2)^2*(2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arctanh((b*x^3+
a)^(1/2)/a^(1/2)))+1/3*b*c^3/(a*c^2-d^2)^2/d^2*(d*ln(c*(b*x^3+a)^(1/2)-d)-d
*ln(d+c*(b*x^3+a)^(1/2))+2*c*(b*x^3+a)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.89

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$
$$=\left[\frac{2\,a^2bc^3x^3\log\left(bc^2x^3+ac^2-d^2\right)+2\,a^2bc^3x^3\log\left(\sqrt{bx^3+a}c+d\right)-2\,a^2bc^3x^3\log\left(\sqrt{bx^3+a}c-d\right)-6\,a^2bc}{6\left(a^4c^4-2\,a^3c\right.}\right.$$

```
[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^2*b*c^3*x^3*log(b*c^2*x^3 + a*c^2 - d^2) + 2*a^2*b*c^3*x^3*log(sq
rt(b*x^3 + a)*c + d) - 2*a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c - d) - 6*a^2*b
*c^3*x^3*log(x) - 2*a^3*c^3 - (3*a*b*c^2*d - b*d^3)*sqrt(a)*x^3*log((b*x^3
+ 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*a^2*c*d^2 + 2*(a^2*c^2*d - a*d^
3)*sqrt(b*x^3 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^3), 1/3*(a^2*b*c
^3*x^3*log(b*c^2*x^3 + a*c^2 - d^2) + a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c +
 d) - a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c - d) - 3*a^2*b*c^3*x^3*log(x) - a
^3*c^3 + (3*a*b*c^2*d - b*d^3)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)
```

```
/a) + a^2*c*d^2 + (a^2*c^2*d - a*d^3)*sqrt(b*x^3 + a))/((a^4*c^4 - 2*a^3*c^
2*d^2 + a^2*d^4)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$

```
[In] integrate(1/x**4/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
[Out] Integral(1/(x**4*(a*c + b*c*x**3 + d*sqrt(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{\left(bcx^3+ac+\sqrt{bx^3+a}d\right)x^4}\,dx$$

```
[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\frac{2\,bc^4\log\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{3\,(a^2c^5-2\,ac^3d^2+cd^4)}-\frac{bc^3\log\left(-bx^3\right)}{3\,(a^2c^4-2\,ac^2d^2+d^4)}+\frac{(3\,abc^2d-bd^3)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,(a^3c^4-2\,a^2c^2d^2+ad^4)\sqrt{-a}}-\frac{a^2bc^3-abcd^2-(abc^2d-bd^3)\sqrt{bx^3+a}}{3\,(ac^2-d^2)^2abx^3}$$

```
[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
[Out] 2/3*b*c^4*log(abs(sqrt(b*x^3 + a)*c + d))/(a^2*c^5 - 2*a*c^3*d^2 + c*d^4) -
 1/3*b*c^3*log(-b*x^3)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + 1/3*(3*a*b*c^2*d - b
*d^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4)*s
qrt(-a)) - 1/3*(a^2*b*c^3 - a*b*c*d^2 - (a*b*c^2*d - b*d^3)*sqrt(b*x^3 + a)
)/((a*c^2 - d^2)^2*a*b*x^3)
```

Mupad [B] (verification not implemented)

Time = 18.90 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^4 \left(ac + bcx^3 + d\sqrt{a + bx^3}\right)}\,dx = \frac{b\,c^3\,\ln\left(b\,c^2\,x^3 + a\,c^2 - d^2\right)}{3\,a^2\,c^4 - 6\,a\,c^2\,d^2 + 3\,d^4} - \frac{b\,c^3\,\ln\left(x\right)}{a^2\,c^4 - 2\,a\,c^2\,d^2 + d^4} - \frac{c}{3\,x^3\,\left(a\,c^2 - d^2\right)} + \frac{b\,c^3\,\ln\left(\frac{d + c\,\sqrt{b\,x^3 + a}}{d - c\,\sqrt{b\,x^3 + a}}\right)}{3\,\left(a\,c^2 - d^2\right)^2} + \frac{d\,\sqrt{b\,x^3 + a}}{3\,a\,x^3\,\left(a\,c^2 - d^2\right)} + \frac{b\,d\,\ln\left(\frac{\left(\sqrt{b\,x^3 + a} - \sqrt{a}\right)^3\,\left(\sqrt{b\,x^3 + a} + \sqrt{a}\right)}{x^6}\right)\,\left(3\,a\,c^2 - d^2\right)}{6\,a^{3/2}\,\left(a\,c^2 - d^2\right)^2}$$

```
[In] int(1/(x^4*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

```
[Out] (b*c^3*log(a*c^2 - d^2 + b*c^2*x^3))/(3*d^4 + 3*a^2*c^4 - 6*a*c^2*d^2) - (b
*c^3*log(x))/(d^4 + a^2*c^4 - 2*a*c^2*d^2) - c/(3*x^3*(a*c^2 - d^2)) + (b*c
^3*log((d + c*(a + b*x^3)^(1/2))/(d - c*(a + b*x^3)^(1/2))))/(3*(a*c^2 - d^
2)^2) + (d*(a + b*x^3)^(1/2))/(3*a*x^3*(a*c^2 - d^2)) + (b*d*log((((a + b*x
^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*(3*a*c^2 - d^2))
/(6*a^(3/2)*(a*c^2 - d^2)^2)
```

3.557 $\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 311

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{x}{bc}-\frac{dx^4\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4(ac^2-d^2)\sqrt{a+bx^3}}+\frac{\sqrt[3]{ac^2-d^2}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}c^{5/3}}-\frac{\sqrt[3]{ac^2-d^2}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{4/3}c^{5/3}}+\frac{\sqrt[3]{ac^2-d^2}\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6b^{4/3}c^{5/3}}$$

```
[Out] x/b/c-1/3*(a*c^2-d^2)^(1/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*x)/b^(4/3)
/c^(5/3)+1/6*(a*c^2-d^2)^(1/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2/3)*(a*c^2-
d^2)^(1/3)*x+b^(2/3)*c^(4/3)*x^2)/b^(4/3)/c^(5/3)+1/3*(a*c^2-d^2)^(1/3)*arc
tan(1/3*(1-2*b^(1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/b^(4/3)/c^(5/3)*
3^(1/2)-1/4*d*x^4*AppellF1(4/3,1/2,1,7/3,-b*x^3/a,-b*c^2*x^3/(a*c^2-d^2))*(
1+b*x^3/a)^(1/2)/(a*c^2-d^2)/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2187, 327, 206, 31, 648, 631, 210, 642, 525, 524}

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$$
$$=-\frac{dx^4\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\sqrt{a+bx^3}\,(ac^2-d^2)}$$
$$+\frac{\sqrt[3]{ac^2-d^2}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}c^{5/3}}$$
$$+\frac{\sqrt[3]{ac^2-d^2}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+(ac^2-d^2)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{4/3}c^{5/3}}$$
$$-\frac{\sqrt[3]{ac^2-d^2}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{4/3}c^{5/3}}+\frac{x}{bc}$$

```
[In] Int[x^3/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] x/(b*c) - (d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a
), -((b*c^2*x^3)/(a*c^2 - d^2))])/(4*(a*c^2 - d^2)*Sqrt[a + b*x^3]) + ((a*c
^2 - d^2)^(1/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt
[3]])/(Sqrt[3]*b^(4/3)*c^(5/3)) - ((a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(1
/3) + b^(1/3)*c^(2/3)*x])/(3*b^(4/3)*c^(5/3)) + ((a*c^2 - d^2)^(1/3)*Log[(a
*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)
*x^2])/(6*b^(4/3)*c^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = (ac)\int \frac{x^3}{a^2c^2 - ad^2 + abc^2x^3}\,dx - (ad)\int \frac{x^3}{\sqrt{a+bx^3}\,(a^2c^2 - ad^2 + abc^2x^3)}\,dx$$

$$= \frac{x}{bc} - \frac{(a(ac^2-d^2))\int \frac{1}{a^2c^2-ad^2+abc^2x^3}\,dx}{bc} - \frac{\left(ad\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{x^3}{\sqrt{1+\frac{bx^3}{a}}\,(a^2c^2-ad^2+abc^2x^3)}\,dx}{\sqrt{a+bx^3}}$$

$$= \frac{x}{bc} - \frac{dx^4\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\,(ac^2-d^2)\sqrt{a+bx^3}} - \frac{\left(\sqrt[3]{a}\sqrt[3]{ac^2-d^2}\right)\int \frac{1}{\sqrt[3]{a}\,\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\,\sqrt[3]{b}c^{2/3}x}\,dx}{3bc} - \frac{\left(\sqrt[3]{a}\sqrt[3]{ac^2-d^2}\right)\int \frac{2\sqrt[3]{a}\,\sqrt[3]{ac^2-d^2}-\sqrt[3]{a}\,\sqrt[3]{b}c^{2/3}x}{a^{2/3}(ac^2-d^2)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,dx}{3bc}$$

$$= \frac{x}{bc} - \frac{dx^4\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\,(ac^2-d^2)\sqrt{a+bx^3}} - \frac{\sqrt[3]{ac^2-d^2}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{4/3}c^{5/3}} + \frac{\sqrt[3]{ac^2-d^2}\int \frac{-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}+2a^{2/3}b^{2/3}c^{4/3}x}{a^{2/3}(ac^2-d^2)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,dx}{6b^{4/3}c^{5/3}} - \frac{\left(a^{2/3}(ac^2-d^2)^{2/3}\right)\int \frac{1}{a^{2/3}(ac^2-d^2)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,dx}{2bc}$$

$$=\frac{x}{bc}-\frac{dx^4\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4(ac^2-d^2)\sqrt{a+bx^3}}-\frac{\sqrt[3]{ac^2-d^2}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{4/3}c^{5/3}}+\frac{\sqrt[3]{ac^2-d^2}\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6b^{4/3}c^{5/3}}-\frac{\sqrt[3]{ac^2-d^2}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}\right)}{b^{4/3}c^{5/3}}$$

$$=\frac{x}{bc}-\frac{dx^4\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4(ac^2-d^2)\sqrt{a+bx^3}}+\frac{\sqrt[3]{ac^2-d^2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}c^{5/3}}-\frac{\sqrt[3]{ac^2-d^2}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{4/3}c^{5/3}}+\frac{\sqrt[3]{ac^2-d^2}\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6b^{4/3}c^{5/3}}$$

Mathematica [A] (verified)

Time = 10.43 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.95

$$\int\frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=-\frac{dx^4\sqrt{1+\frac{bx^3}{a}}\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(4ac^2-4d^2)\sqrt{a+bx^3}}+\frac{6\sqrt[3]{b}c^{2/3}x-2\sqrt{3}\sqrt[3]{ac^2-d^2}\arctan\left(\frac{-1+\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)-2\sqrt[3]{ac^2-d^2}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)+\sqrt[3]{a}}{6b^{4/3}c^{5/3}}$$

```
[In] Integrate[x^3/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] -((d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -((b*
c^2*x^3)/(a*c^2 - d^2))])/((4*a*c^2 - 4*d^2)*Sqrt[a + b*x^3])) + (6*b^(1/3)
*c^(2/3)*x - 2*Sqrt[3]*(a*c^2 - d^2)^(1/3)*ArcTan[(-1 + (2*b^(1/3)*c^(2/3)*
x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*(a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^
(1/3) + b^(1/3)*c^(2/3)*x] + (a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(2/3) -
b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*b^(4/3)*c^
(5/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 0.99 (sec) , antiderivative size = 994, normalized size of antiderivative = 3.20

method	result	size
elliptic	Expression too large to display	994
default	Expression too large to display	1672

```
[In] int(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^3+a)^(1/2)*(d+c*(b*x^3+a)^(1/2))/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2))*(c*(1
/b/c^2*x+(1/3/b/c^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x+((a*c^2-d^2)/b/c^2)^(1/3
))-1/6/b/c^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(1/3)*x+(
(a*c^2-d^2)/b/c^2)^(2/3))+1/3/b/c^2/((a*c^2-d^2)/b/c^2)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1)))*(-a*c^2+d^2)/b/c^2)+2/3*I
*d/b^2/c^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(
-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2
*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/
3*I/d/b^4/c^2*2^(1/2)*sum((a*c^2-d^2)/_alpha^2*(-b^2*a)^(1/3)*(1/2*I*b*(2*x
+1/b*((-b^2*a)^(1/3)-I*3^(1/2)*(-b^2*a)^(1/3)))/(-b^2*a)^(1/3))^(1/2)*(b*(x
-1/b*(-b^2*a)^(1/3))/(-3*(-b^2*a)^(1/3)+I*3^(1/2)*(-b^2*a)^(1/3)))^(1/2)*(-
1/2*I*b*(2*x+1/b*((-b^2*a)^(1/3)+I*3^(1/2)*(-b^2*a)^(1/3)))/(-b^2*a)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*(I*(-b^2*a)^(1/3)*3^(1/2)*_alpha*b-I*(-b^2*a)^(2/3)*
3^(1/2)+2*_alpha^2*b^2-(-b^2*a)^(1/3)*_alpha*b-(-b^2*a)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2),-1/2/b*c^2*(2*I*(-b^2*a)^(1/3)*3^(1/2)*_alpha^2*
b-I*(-b^2*a)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-b^2*a)^(2/3)*_alpha-3*a
*b)/d^2,(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$$

```
[In] integrate(x**3/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
[Out] Integral(x**3/(a*c + b*c*x**3 + d*sqrt(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \int \frac{x^3}{bcx^3+ac+\sqrt{bx^3+a}d}\,dx$$

```
[In] integrate(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(x^3/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Giac [F]

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \int \frac{x^3}{bcx^3+ac+\sqrt{bx^3+a}d}\,dx$$

```
[In] integrate(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
[Out] integrate(x^3/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \int \frac{x^3}{a\,c+d\,\sqrt{b\,x^3+a}+b\,c\,x^3}\,dx$$

```
[In] int(x^3/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
[Out] int(x^3/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3), x)
```

3.558 $\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 304

$$\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = -\frac{dx^2\sqrt{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)\sqrt{a+bx^3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}} - \frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}} + \frac{\log\left(\left(ac^2-d^2\right)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}$$

```
[Out] -1/3*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*x)/b^(2/3)/c^(1/3)/(a*c^2-d^2)^(1
/3)+1/6*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2/3)*(a*c^2-d^2)^(1/3)*x+b^(2/3)*c^
(4/3)*x^2)/b^(2/3)/c^(1/3)/(a*c^2-d^2)^(1/3)-1/3*arctan(1/3*(1-2*b^(1/3)*c^
(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/b^(2/3)/c^(1/3)/(a*c^2-d^2)^(1/3)*3^(1/
2)-1/2*d*x^2*AppellF1(2/3,1/2,1,5/3,-b*x^3/a,-b*c^2*x^3/(a*c^2-d^2))*(1+b*x
^3/a)^(1/2)/(a*c^2-d^2)/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2187, 298, 31, 648, 631, 210, 642, 525, 524}

$$\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = -\frac{dx^2\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\sqrt{a+bx^3}\,(ac^2-d^2)} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}} + \frac{\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+(ac^2-d^2)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}} - \frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}$$

```
[In] Int[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/2*(d*x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -(
(b*c^2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^3]) - ArcTan[(1 -
(2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)*c^(1/3
)*(a*c^2 - d^2)^(1/3)) - Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x]/(3*b^
(2/3)*c^(1/3)*(a*c^2 - d^2)^(1/3)) + Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2
/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]/(6*b^(2/3)*c^(1/3)*(a*c^2
- d^2)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = (ac)\int \frac{x}{a^2c^2-ad^2+abc^2x^3}\,dx-(ad)\int \frac{x}{\sqrt{a+bx^3}\,(a^2c^2-ad^2+abc^2x^3)}\,dx$$

$$=-\frac{\left(\sqrt[3]{a}\sqrt[3]{c}\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x}\,dx}{3\sqrt[3]{b}\sqrt[3]{ac^2-d^2}}+\frac{\left(\sqrt[3]{a}\sqrt[3]{c}\right)\int \frac{\sqrt[3]{a}\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x}{a^{2/3}(ac^2-d^2)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,dx}{3\sqrt[3]{b}\sqrt[3]{ac^2-d^2}}-\frac{\left(ad\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{x}{\sqrt{1+\frac{bx^3}{a}}\,(a^2c^2-ad^2+abc^2x^3)}\,dx}{\sqrt{a+bx^3}}$$

$$=-\frac{dx^2\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\,(ac^2-d^2)\sqrt{a+bx^3}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}+\frac{\left(a^{2/3}\sqrt[3]{c}\right)\int \frac{1}{a^{2/3}(ac^2-d^2)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,dx}{2\sqrt[3]{b}}+\frac{\int \frac{-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}+2a^{2/3}b^{2/3}c^{4/3}x}{a^{2/3}(ac^2-d^2)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,dx}{6b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}$$

$$=-\frac{dx^2\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\,(ac^2-d^2)\sqrt{a+bx^3}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}+\frac{\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}\right)}{b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}$$

$$=-\frac{dx^2\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\,(ac^2-d^2)\sqrt{a+bx^3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}+\frac{\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}$$

Mathematica [A] (verified)

Time = 10.21 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.83

$$\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = -\frac{dx^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(2ac^2-2d^2)\sqrt{a+bx^3}} + \frac{2\sqrt{3}\arctan\left(\frac{-1+\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)+\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}\right.}{6b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}$$

```
[In] Integrate[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] -((d*x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*
c^2*x^3)/(a*c^2 - d^2))])/((2*a*c^2 - 2*d^2)*Sqrt[a + b*x^3])) + (2*Sqrt[3]
*ArcTan[(-1 + (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*Log[(
a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x] + Log[(a*c^2 - d^2)^(2/3) - b^(1/3)
*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*b^(2/3)*c^(1/3)*(
a*c^2 - d^2)^(1/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 0.88 (sec) , antiderivative size = 665, normalized size of antiderivative = 2.19

method	result
elliptic	$\dfrac{\sqrt{b x^3+a}\left(d+c\sqrt{b x^3+a}\right)\left(-\frac{\ln\left(x+\left(\frac{a c^2-d^2}{b c^2}\right)^{\frac{1}{3}}\right)}{3bc\left(\frac{a c^2-d^2}{b c^2}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a c^2-d^2}{b c^2}\right)^{\frac{1}{3}}x+\left(\frac{a c^2-d^2}{b c^2}\right)^{\frac{2}{3}}\right)}{6bc\left(\frac{a c^2-d^2}{b c^2}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a c^2-d^2}{b c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3bc\left(\frac{a c^2-d^2}{b c^2}\right)^{\frac{1}{3}}}-\cdots i\sqrt{\cdots}\right.}{\cdots}$
default	Expression too large to display

```
[In] int(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^3+a)^(1/2)*(d+c*(b*x^3+a)^(1/2))/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2))*(-1/3
/b/c/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x+((a*c^2-d^2)/b/c^2)^(1/3))+1/6/b/c/((a*
c^2-d^2)/b/c^2)^(1/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(1/3)*x+((a*c^2-d^2)/b/c^2
)^(2/3))+1/3/b/c*3^(1/2)/((a*c^2-d^2)/b/c^2)^(1/3)*arctan(1/3*3^(1/2)*(2/((
a*c^2-d^2)/b/c^2)^(1/3)*x-1))-1/3*I/d/b^3*2^(1/2)*sum(1/_alpha*(-b^2*a)^(1/
3)*(1/2*I*b*(2*x+1/b*((-b^2*a)^(1/3)-I*3^(1/2)*(-b^2*a)^(1/3)))/(-b^2*a)^(1
/3))^(1/2)*(b*(x-1/b*(-b^2*a)^(1/3))/(-3*(-b^2*a)^(1/3)+I*3^(1/2)*(-b^2*a)^
(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-b^2*a)^(1/3)+I*3^(1/2)*(-b^2*a)^(1/3)))
/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-b^2*a)^(1/3)*3^(1/2)*_alpha*b-I
*(-b^2*a)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-b^2*a)^(1/3)*_alpha*b-(-b^2*a)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),-1/2/b*c^2*(2*I*(-b^2*a)^(1/3)*3
^(1/2)*_alpha^2*b-I*(-b^2*a)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-b^2*a)^
```

```
(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2)
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \int \frac{x}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx$$

```
[In] integrate(x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
[Out] Integral(x/(a*c + b*c*x**3 + d*sqrt(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \int \frac{x}{bcx^3 + ac + \sqrt{bx^3 + a}d}\,dx$$

```
[In] integrate(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(x/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Giac [F]

$$\int \frac{x}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \int \frac{x}{bcx^3 + ac + \sqrt{bx^3 + a}d}\,dx$$

```
[In] integrate(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
[Out] integrate(x/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\int \frac{x}{a\,c+d\,\sqrt{b\,x^3+a}+b\,c\,x^3}\,dx$$

```
[In] int(x/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
[Out] int(x/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3), x)
```

3.559 $\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 300

$$\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = -\frac{dx\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^3}} - \frac{\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\,(ac^2-d^2)^{2/3}} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}\,(ac^2-d^2)^{2/3}} - \frac{\sqrt[3]{c}\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}$$

```
[Out] 1/3*c^(1/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*x)/b^(1/3)/(a*c^2-d^2)^(2/
3)-1/6*c^(1/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2/3)*(a*c^2-d^2)^(1/3)*x+b^(
2/3)*c^(4/3)*x^2)/b^(1/3)/(a*c^2-d^2)^(2/3)-1/3*c^(1/3)*arctan(1/3*(1-2*b^(
1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/b^(1/3)/(a*c^2-d^2)^(2/3)*3^(1/2
)-d*x*AppellF1(1/3,1/2,1,4/3,-b*x^3/a,-b*c^2*x^3/(a*c^2-d^2))*(1+b*x^3/a)^(
1/2)/(a*c^2-d^2)/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2187, 206, 31, 648, 631, 210, 642, 441, 440}

$$\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = -\frac{dx\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\,(ac^2-d^2)} - \frac{\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\,(ac^2-d^2)^{2/3}} - \frac{\sqrt[3]{c}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+(ac^2-d^2)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\sqrt[3]{b}\,(ac^2-d^2)^{2/3}} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}$$

```
[In] Int[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1),x]
```

```
[Out] -((d*x*Sqrt[1 + (b*x^3)/a]*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^
2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^3])) - (c^(1/3)*ArcTan[
(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)*
(a*c^2 - d^2)^(2/3)) + (c^(1/3)*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x
])/(3*b^(1/3)*(a*c^2 - d^2)^(2/3)) - (c^(1/3)*Log[(a*c^2 - d^2)^(2/3) - b^(
1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*b^(1/3)*(a*c^
2 - d^2)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = (ac)\int \frac{1}{a^2c^2-ad^2+abc^2x^3}\,dx-(ad)\int \frac{1}{\sqrt{a+bx^3}\,(a^2c^2-ad^2+abc^2x^3)}\,dx$$

$$= \frac{\left(\sqrt[3]{a}c\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x}\,dx}{3\left(ac^2-d^2\right)^{2/3}}+\frac{\left(\sqrt[3]{a}c\right)\int \frac{2\sqrt[3]{a}\sqrt[3]{ac^2-d^2}-\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x}{a^{2/3}(ac^2-d^2)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,dx}{3\left(ac^2-d^2\right)^{2/3}}-\frac{\left(ad\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{1}{\sqrt{1+\frac{bx^3}{a}}(a^2c^2-ad^2+abc^2x^3)}\,dx}{\sqrt{a+bx^3}}$$

$$= -\frac{dx\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^3}}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}}-\frac{\sqrt[3]{c}\int \frac{-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}+2a^{2/3}b^{2/3}c^{4/3}x}{a^{2/3}(ac^2-d^2)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,dx}{6\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}}+\frac{\left(a^{2/3}c\right)\int \frac{1}{a^{2/3}(ac^2-d^2)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,dx}{2\sqrt[3]{ac^2-d^2}}$$

$$= -\frac{dx\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^3}}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}}-\frac{\sqrt[3]{c}\log\left(\left(ac^2-d^2\right)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}}+\frac{\sqrt[3]{c}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}\right)}{\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}}$$

$$= -\frac{dx\sqrt{1+\frac{bx^3}{a}}F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^3}}-\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}}-\frac{\sqrt[3]{c}\log\left(\left(ac^2-d^2\right)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6\sqrt[3]{b}\left(ac^2-d^2\right)^{2/3}}$$

Mathematica [A] (warning: unable to verify)

Time = 10.46 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.42

$$\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = -\frac{8ad(ac^2-d^2)\,x\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},\cdots\right)}{\sqrt{a+bx^3}\,(ac^2-d^2+bc^2x^3)\left(8a\,(ac^2-d^2)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)-3bx^3\left(2ac^2\,\text{AppellF1}\left(\frac{4}{3},\cdots\right)\cdots\right)\right)} + \frac{\sqrt[3]{c}\left(2\sqrt{3}\arctan\left(\frac{-1+\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)-\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}\,\cdots\right)\right)}{6\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}$$

```
[In] Integrate[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1),x]
```

```
[Out] (-8*a*d*(a*c^2 - d^2)*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^2*x
^3)/(a*c^2 - d^2))])/(Sqrt[a + b*x^3]*(a*c^2 - d^2 + b*c^2*x^3)*(8*a*(a*c^2
 - d^2)*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2
))] - 3*b*x^3*(2*a*c^2*AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*c^2*x^
3)/(a*c^2 - d^2))] + (a*c^2 - d^2)*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a),
 -((b*c^2*x^3)/(a*c^2 - d^2))]))) + (c^(1/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*b^(
1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] + 2*Log[(a*c^2 - d^2)^(1/3) +
 b^(1/3)*c^(2/3)*x] - Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^
2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]))/(6*b^(1/3)*(a*c^2 - d^2)^(2/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 0.86 (sec) , antiderivative size = 665, normalized size of antiderivative = 2.22

method	result
elliptic	$\frac{\sqrt{b\,x^{3}+a}\,\left(d+c\sqrt{b\,x^{3}+a}\right)\left(\frac{\ln\left(x+\left(\frac{a\,c^{2}-d^{2}}{b\,c^{2}}\right)^{\frac{1}{3}}\right)}{3bc\left(\frac{a\,c^{2}-d^{2}}{b\,c^{2}}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a\,c^{2}-d^{2}}{b\,c^{2}}\right)^{\frac{1}{3}}x+\left(\frac{a\,c^{2}-d^{2}}{b\,c^{2}}\right)^{\frac{2}{3}}\right)}{6bc\left(\frac{a\,c^{2}-d^{2}}{b\,c^{2}}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a\,c^{2}-d^{2}}{b\,c^{2}}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3bc\left(\frac{a\,c^{2}-d^{2}}{b\,c^{2}}\right)^{\frac{2}{3}}}-\frac{i\sqrt{2}\cdots}{\cdots}\right)}{\cdots}$
default	Expression too large to display

```
[In] int(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^3+a)^(1/2)*(d+c*(b*x^3+a)^(1/2))/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2))*(1/3/
b/c/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x+((a*c^2-d^2)/b/c^2)^(1/3))-1/6/b/c/((a*c
^2-d^2)/b/c^2)^(2/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(1/3)*x+((a*c^2-d^2)/b/c^2)
^(2/3))+1/3/b/c/((a*c^2-d^2)/b/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/((a
*c^2-d^2)/b/c^2)^(1/3)*x-1))-1/3*I/d/b^3*2^(1/2)*sum(1/_alpha^2*(-b^2*a)^(1
/3)*(1/2*I*b*(2*x+1/b*((-b^2*a)^(1/3)-I*3^(1/2)*(-b^2*a)^(1/3)))/(-b^2*a)^(
1/3))^(1/2)*(b*(x-1/b*(-b^2*a)^(1/3))/(-3*(-b^2*a)^(1/3)+I*3^(1/2)*(-b^2*a)
^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-b^2*a)^(1/3)+I*3^(1/2)*(-b^2*a)^(1/3))
)/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-b^2*a)^(1/3)*3^(1/2)*_alpha*b-
I*(-b^2*a)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-b^2*a)^(1/3)*_alpha*b-(-b^2*a)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),-1/2/b*c^2*(2*I*(-b^2*a)^(1/3)*
3^(1/2)*_alpha^2*b-I*(-b^2*a)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-b^2*a)
```

```
^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)
+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2
)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

Sympy [F]

$$\int \frac{1}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \int \frac{1}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx$$

```
[In] integrate(1/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
[Out] Integral(1/(a*c + b*c*x**3 + d*sqrt(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \int \frac{1}{bcx^3 + ac + \sqrt{bx^3 + a}d}\,dx$$

```
[In] integrate(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Giac [F]

$$\int \frac{1}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \int \frac{1}{bcx^3 + ac + \sqrt{bx^3 + a}d}\,dx$$

```
[In] integrate(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
[Out] integrate(1/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\int \frac{1}{a\,c+d\,\sqrt{b\,x^3+a}+b\,c\,x^3}\,dx$$

```
[In] int(1/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
[Out] int(1/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3), x)
```

3.560 $\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 319

$$\begin{aligned}\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx &= -\frac{c}{(ac^2-d^2)\,x}+\frac{d\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(ac^2-d^2)\,x\sqrt{a+bx^3}}\\&\quad+\frac{\sqrt[3]{b}c^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,(ac^2-d^2)^{4/3}}+\frac{\sqrt[3]{b}c^{5/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\,(ac^2-d^2)^{4/3}}\\&\quad-\frac{\sqrt[3]{b}c^{5/3}\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}\,x+b^{2/3}c^{4/3}x^2\right)}{6\,(ac^2-d^2)^{4/3}}\end{aligned}$$

```
[Out] -c/(a*c^2-d^2)/x+1/3*b^(1/3)*c^(5/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*x
)/(a*c^2-d^2)^(4/3)-1/6*b^(1/3)*c^(5/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2/3
)*(a*c^2-d^2)^(1/3)*x+b^(2/3)*c^(4/3)*x^2)/(a*c^2-d^2)^(4/3)+1/3*b^(1/3)*c^
(5/3)*arctan(1/3*(1-2*b^(1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/(a*c^2-
d^2)^(4/3)*3^(1/2)+d*AppellF1(-1/3,1/2,1,2/3,-b*x^3/a,-b*c^2*x^3/(a*c^2-d^2
))*(1+b*x^3/a)^(1/2)/(a*c^2-d^2)/x/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2187, 331, 298, 31, 648, 631, 210, 642, 525, 524}

$$\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$
$$=\frac{d\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{x\sqrt{a+bx^3}\,(ac^2-d^2)}+\frac{\sqrt[3]{b}c^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,(ac^2-d^2)^{4/3}}$$
$$-\frac{\sqrt[3]{b}c^{5/3}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+(ac^2-d^2)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\,(ac^2-d^2)^{4/3}}$$
$$+\frac{\sqrt[3]{b}c^{5/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\,(ac^2-d^2)^{4/3}}-\frac{c}{x\,(ac^2-d^2)}$$

```
[In] Int[1/(x^2*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[1 + (b*x^3)/a]*AppellF1[-1/3, 1/2, 1, 2/3,
 -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*x*Sqrt[a + b*x
^3]) + (b^(1/3)*c^(5/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/
3))/Sqrt[3]])/(Sqrt[3]*(a*c^2 - d^2)^(4/3)) + (b^(1/3)*c^(5/3)*Log[(a*c^2 -
 d^2)^(1/3) + b^(1/3)*c^(2/3)*x])/(3*(a*c^2 - d^2)^(4/3)) - (b^(1/3)*c^(5/3
)*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)
*c^(4/3)*x^2])/(6*(a*c^2 - d^2)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = (ac)\int \frac{1}{x^2\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx-(ad)\int \frac{1}{x^2\sqrt{a+bx^3}\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}-\frac{\left(abc^3\right)\int \frac{x}{a^2c^2-ad^2+abc^2x^3}\,dx}{ac^2-d^2}-\frac{\left(ad\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{1}{x^2\sqrt{1+\frac{bx^3}{a}}\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx}{\sqrt{a+bx^3}}$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)x\sqrt{a+bx^3}}+\frac{\left(\sqrt[3]{a}b^{2/3}c^{7/3}\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x}\,dx}{3\left(ac^2-d^2\right)^{4/3}}-\frac{\left(\sqrt[3]{a}b^{2/3}c^{7/3}\right)\int \frac{\sqrt[3]{a}\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x}{a^{2/3}\left(ac^2-d^2\right)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,dx}{3\left(ac^2-d^2\right)^{4/3}}$$

$$=-\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)x\sqrt{a+bx^3}}+\frac{\sqrt[3]{b}c^{5/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\left(ac^2-d^2\right)^{4/3}}-\frac{\left(\sqrt[3]{b}c^{5/3}\right)\int \frac{-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}+2a^{2/3}b^{2/3}c^{4/3}x}{a^{2/3}\left(ac^2-d^2\right)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,dx}{6\left(ac^2-d^2\right)^{4/3}}-\frac{\left(a^{2/3}b^{2/3}c^{7/3}\right)\int \frac{1}{a^{2/3}\left(ac^2-d^2\right)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,dx}{2\left(ac^2-d^2\right)}$$

$$= -\frac{c}{(ac^2-d^2)x} + \frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(ac^2-d^2)x\sqrt{a+bx^3}} + \frac{\sqrt[3]{b}c^{5/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3(ac^2-d^2)^{4/3}} - \frac{\sqrt[3]{b}c^{5/3}\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6(ac^2-d^2)^{4/3}} - \frac{\left(\sqrt[3]{b}c^{5/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}\right)}{(ac^2-d^2)^{4/3}}$$

$$= -\frac{c}{(ac^2-d^2)x} + \frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(ac^2-d^2)x\sqrt{a+bx^3}} + \frac{\sqrt[3]{b}c^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}(ac^2-d^2)^{4/3}} + \frac{\sqrt[3]{b}c^{5/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3(ac^2-d^2)^{4/3}} - \frac{\sqrt[3]{b}c^{5/3}\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6(ac^2-d^2)^{4/3}}$$

Mathematica [A] (verified)

Time = 10.52 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.55

$$\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$

$$= \frac{15bd\sqrt[3]{ac^2-d^2}(ac^2+d^2)x^3\sqrt{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)-6b^2c^2d\sqrt[3]{ac^2-d^2}x^6\sqrt{1+\frac{bx^3}{a}}\,A}{}$$

```
[In] Integrate[1/(x^2*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (15*b*d*(a*c^2 - d^2)^(1/3)*(a*c^2 + d^2)*x^3*Sqrt[1 + (b*x^3)/a]*AppellF1[
2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 6*b^2*c^2*d
*(a*c^2 - d^2)^(1/3)*x^6*Sqrt[1 + (b*x^3)/a]*AppellF1[5/3, 1/2, 1, 8/3, -((
b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 10*(a*c^2 - d^2)*(-6*a*d*(a*c^2
- d^2)^(1/3) - 6*b*d*(a*c^2 - d^2)^(1/3)*x^3 + 6*a*c*(a*c^2 - d^2)^(1/3)*Sq
rt[a + b*x^3] + 2*Sqrt[3]*a*b^(1/3)*c^(5/3)*x*Sqrt[a + b*x^3]*ArcTan[(-1 +
```

```
(2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*a*b^(1/3)*c^(5/3)*x
*Sqrt[a + b*x^3]*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x] + a*b^(1/3)*c
^(5/3)*x*Sqrt[a + b*x^3]*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 -
 d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]))/(60*a*(a*c^2 - d^2)^(7/3)*x*Sqrt[a +
 b*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 0.83 (sec) , antiderivative size = 1200, normalized size of antiderivative = 3.76

method	result	size
elliptic	Expression too large to display	1200
default	Expression too large to display	2404

```
[In] int(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^3+a)^(1/2)*(d+c*(b*x^3+a)^(1/2))/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2))*(c*(-
1/(a*c^2-d^2)/x-(-1/3/b/c^2/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x+((a*c^2-d^2)/b/c
^2)^(1/3))+1/6/b/c^2/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(
1/3)*x+((a*c^2-d^2)/b/c^2)^(2/3))+1/3*3^(1/2)/b/c^2/((a*c^2-d^2)/b/c^2)^(1/
3)*arctan(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1)))*b*c^2/(a*c^2-d^2)
)+d/a/(a*c^2-d^2)*(b*x^3+a)^(1/2)/x+1/3*I*d/a/(a*c^2-d^2)*3^(1/2)*(-b^2*a)^
(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*
(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3)))^(1/2)))+1/3*I/d/b^2*c^2*2^(1/2)*sum(1/(a*c^2-d^2)/_alpha*(
-b^2*a)^(1/3)*(1/2*I*b*(2*x+1/b*((-b^2*a)^(1/3)-I*3^(1/2)*(-b^2*a)^(1/3)))/
(-b^2*a)^(1/3))^(1/2)*(b*(x-1/b*(-b^2*a)^(1/3))/(-3*(-b^2*a)^(1/3)+I*3^(1/2
)*(-b^2*a)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-b^2*a)^(1/3)+I*3^(1/2)*(-b^2
*a)^(1/3)))/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-b^2*a)^(1/3)*3^(1/2)
*_alpha*b-I*(-b^2*a)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-b^2*a)^(1/3)*_alpha*b-(
-b^2*a)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),-1/2/b*c^2*(2*I*(-b^2
*a)^(1/3)*3^(1/2)*_alpha^2*b-I*(-b^2*a)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-
3*(-b^2*a)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^
2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2
+a*c^2-d^2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$

```
[In] integrate(1/x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
[Out] Integral(1/(x**2*(a*c + b*c*x**3 + d*sqrt(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{\left(bcx^3+ac+\sqrt{bx^3+a}d\right)x^2}\,dx$$

```
[In] integrate(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{\left(bcx^3+ac+\sqrt{bx^3+a}d\right)x^2}\,dx$$

```
[In] integrate(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{x^2\,\left(a\,c+d\,\sqrt{b\,x^3+a}+b\,c\,x^3\right)}\,dx$$

```
[In] int(1/(x^2*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
[Out] int(1/(x^2*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)), x)
```

3.561 $\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 324

$$\begin{aligned}
&\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx \\
&= -\frac{c}{2(ac^2-d^2)x^2}+\frac{d\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2(ac^2-d^2)x^2\sqrt{a+bx^3}} \\
&\quad+\frac{b^{2/3}c^{7/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}(ac^2-d^2)^{5/3}}-\frac{b^{2/3}c^{7/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3(ac^2-d^2)^{5/3}} \\
&\quad+\frac{b^{2/3}c^{7/3}\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6(ac^2-d^2)^{5/3}}
\end{aligned}$$

```
[Out] -1/2*c/(a*c^2-d^2)/x^2-1/3*b^(2/3)*c^(7/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(
2/3)*x)/(a*c^2-d^2)^(5/3)+1/6*b^(2/3)*c^(7/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3)*
c^(2/3)*(a*c^2-d^2)^(1/3)*x+b^(2/3)*c^(4/3)*x^2)/(a*c^2-d^2)^(5/3)+1/3*b^(2
/3)*c^(7/3)*arctan(1/3*(1-2*b^(1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/(
a*c^2-d^2)^(5/3)*3^(1/2)+1/2*d*AppellF1(-2/3,1/2,1,1/3,-b*x^3/a,-b*c^2*x^3/
(a*c^2-d^2))*(1+b*x^3/a)^(1/2)/(a*c^2-d^2)/x^2/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2187, 331, 206, 31, 648, 631, 210, 642, 525, 524}

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$

$$=\frac{d\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2x^2\sqrt{a+bx^3}\,(ac^2-d^2)}+\frac{b^{2/3}c^{7/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,(ac^2-d^2)^{5/3}}$$
$$+\frac{b^{2/3}c^{7/3}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+(ac^2-d^2)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\,(ac^2-d^2)^{5/3}}$$
$$-\frac{b^{2/3}c^{7/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\,(ac^2-d^2)^{5/3}}-\frac{c}{2x^2\,(ac^2-d^2)}$$

```
[In] Int[1/(x^3*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -1/2*c/((a*c^2 - d^2)*x^2) + (d*Sqrt[1 + (b*x^3)/a]*AppellF1[-2/3, 1/2, 1,
1/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(2*(a*c^2 - d^2)*x^2*Sqrt
[a + b*x^3]) + (b^(2/3)*c^(7/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 -
d^2)^(1/3))/Sqrt[3]])/(Sqrt[3]*(a*c^2 - d^2)^(5/3)) - (b^(2/3)*c^(7/3)*Log[
(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x])/(3*(a*c^2 - d^2)^(5/3)) + (b^(2/3
)*c^(7/3)*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x +
 b^(2/3)*c^(4/3)*x^2])/(6*(a*c^2 - d^2)^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = (ac)\int \frac{1}{x^3\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx-(ad)\int \frac{1}{x^3\sqrt{a+bx^3}\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx$$

$$=-\frac{c}{2\left(ac^2-d^2\right)x^2}-\frac{\left(abc^3\right)\int \frac{1}{a^2c^2-ad^2+abc^2x^3}\,dx}{ac^2-d^2}-\frac{\left(ad\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{1}{x^3\sqrt{1+\frac{bx^3}{a}}\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx}{\sqrt{a+bx^3}}$$

$$=-\frac{c}{2\left(ac^2-d^2\right)x^2}+\frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}}-\frac{\left(\sqrt[3]{a}bc^3\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x}\,dx}{3\left(ac^2-d^2\right)^{5/3}}-\frac{\left(\sqrt[3]{a}bc^3\right)\int \frac{2\sqrt[3]{a}\sqrt[3]{ac^2-d^2}-\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x}{a^{2/3}\left(ac^2-d^2\right)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,dx}{3\left(ac^2-d^2\right)^{5/3}}$$

$$=-\frac{c}{2\left(ac^2-d^2\right)x^2}+\frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}}-\frac{b^{2/3}c^{7/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\left(ac^2-d^2\right)^{5/3}}+\frac{\left(b^{2/3}c^{7/3}\right)\int \frac{-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}+2a^{2/3}b^{2/3}c^{4/3}x}{a^{2/3}\left(ac^2-d^2\right)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,dx}{6\left(ac^2-d^2\right)^{5/3}}-\frac{\left(a^{2/3}bc^3\right)\int \frac{1}{a^{2/3}\left(ac^2-d^2\right)^{2/3}-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}b^{2/3}c^{4/3}x^2}\,dx}{2\left(ac^2-d^2\right)^{4/3}}$$

$$= -\frac{c}{2(ac^2-d^2)x^2} + \frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2(ac^2-d^2)x^2\sqrt{a+bx^3}} - \frac{b^{2/3}c^{7/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3(ac^2-d^2)^{5/3}} + \frac{b^{2/3}c^{7/3}\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6(ac^2-d^2)^{5/3}} - \frac{\left(b^{2/3}c^{7/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}dx,x,1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}\right)}{(ac^2-d^2)^{5/3}}$$

$$= -\frac{c}{2(ac^2-d^2)x^2} + \frac{d\sqrt{1+\frac{bx^3}{a}}F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2(ac^2-d^2)x^2\sqrt{a+bx^3}} + \frac{b^{2/3}c^{7/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}(ac^2-d^2)^{5/3}} - \frac{b^{2/3}c^{7/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3(ac^2-d^2)^{5/3}} + \frac{b^{2/3}c^{7/3}\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6(ac^2-d^2)^{5/3}}$$

Mathematica [A] (warning: unable to verify)

Time = 13.07 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.86

$$\int\frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}dx = \frac{b^2c^2dx^4\sqrt{1+\frac{bx^3}{a}}\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{16a(-ac^2+d^2)^2\sqrt{a+bx^3}} + \frac{2bd(-5ac^2+d^2)x\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\right.}{\sqrt{a+bx^3}(ac^2-d^2+bc^2x^3)\left(8a(-ac^2+d^2)\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)+3bx^3\left(2ac^2\text{AppellF1}\right.\right.} + \frac{-3ac(ac^2-d^2)^{2/3}+3d(ac^2-d^2)^{2/3}\sqrt{a+bx^3}-2\sqrt{3}ab^{2/3}c^{7/3}x^2\arctan\left(\frac{-1+\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)-2ab^{2/3}c^{7/3}}{6a(ac^2-d^2)}$$

```
[In] Integrate[1/(x^3*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (b^2*c^2*d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a),
 -((b*c^2*x^3)/(a*c^2 - d^2))])/(16*a*(-(a*c^2) + d^2)^2*Sqrt[a + b*x^3]) +
```

```
(2*b*d*(-5*a*c^2 + d^2)*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^
2*x^3)/(a*c^2 - d^2))])/(Sqrt[a + b*x^3]*(a*c^2 - d^2 + b*c^2*x^3)*(8*a*(-(
a*c^2) + d^2)*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2
 - d^2))] + 3*b*x^3*(2*a*c^2*AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*
c^2*x^3)/(a*c^2 - d^2))] + (a*c^2 - d^2)*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^
3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))]))) + (-3*a*c*(a*c^2 - d^2)^(2/3) + 3*d
*(a*c^2 - d^2)^(2/3)*Sqrt[a + b*x^3] - 2*Sqrt[3]*a*b^(2/3)*c^(7/3)*x^2*ArcT
an[(-1 + (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*a*b^(2/3)*
c^(7/3)*x^2*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x] + a*b^(2/3)*c^(7/3
)*x^2*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(
2/3)*c^(4/3)*x^2])/(6*a*(a*c^2 - d^2)^(5/3)*x^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 0.91 (sec) , antiderivative size = 1049, normalized size of antiderivative = 3.24

method	result	size
elliptic	Expression too large to display	1049
default	Expression too large to display	1948

```
[In] int(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^3+a)^(1/2)*(d+c*(b*x^3+a)^(1/2))/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2))*(c*(-
1/2/(a*c^2-d^2)/x^2-(1/3/b/c^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x+((a*c^2-d^2)/
b/c^2)^(1/3))-1/6/b/c^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x^2-((a*c^2-d^2)/b/c^2
)^(1/3)*x+((a*c^2-d^2)/b/c^2)^(2/3))+1/3/b/c^2/((a*c^2-d^2)/b/c^2)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1)))*b*c^2/(a*c^2-d
^2))+1/2*d/a/(a*c^2-d^2)*(b*x^3+a)^(1/2)/x^2-1/6*I*d/a/(a*c^2-d^2)*3^(1/2)*
(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3
^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/3*I/d/b^2*c^2*2^(1/
2)*sum(1/_alpha^2/(a*c^2-d^2)*(-b^2*a)^(1/3)*(1/2*I*b*(2*x+1/b*((-b^2*a)^(1
/3)-I*3^(1/2)*(-b^2*a)^(1/3)))/(-b^2*a)^(1/3))^(1/2)*(b*(x-1/b*(-b^2*a)^(1/
3))/(-3*(-b^2*a)^(1/3)+I*3^(1/2)*(-b^2*a)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*
((-b^2*a)^(1/3)+I*3^(1/2)*(-b^2*a)^(1/3)))/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^
(1/2)*(I*(-b^2*a)^(1/3)*3^(1/2)*_alpha*b-I*(-b^2*a)^(2/3)*3^(1/2)+2*_alpha^
2*b^2-(-b^2*a)^(1/3)*_alpha*b-(-b^2*a)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2),-1/2/b*c^2*(2*I*(-b^2*a)^(1/3)*3^(1/2)*_alpha^2*b-I*(-b^2*a)^(2/3
```

```
)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-b^2*a)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2
)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$

```
[In] integrate(1/x**3/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
[Out] Integral(1/(x**3*(a*c + b*c*x**3 + d*sqrt(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{\left(bcx^3+ac+\sqrt{bx^3+a}d\right)x^3}\,dx$$

```
[In] integrate(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{\left(bcx^3+ac+\sqrt{bx^3+a}d\right)x^3}\,dx$$

```
[In] integrate(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{x^3\,\left(a\,c+d\,\sqrt{b\,x^3+a}+b\,c\,x^3\right)}\,dx$$

```
[In] int(1/(x^3*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
[Out] int(1/(x^3*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)), x)
```

3.562 $\int \frac{1}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{1}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = -\frac{dx\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^n}} + \frac{cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}$$

```
[Out] c*x*hypergeom([1, 1/n],[1+1/n],-b*c^2*x^n/(a*c^2-d^2))/(a*c^2-d^2)-d*x*Appe
llF1(1/n,1/2,1,1+1/n,-b*x^n/a,-b*c^2*x^n/(a*c^2-d^2))*(1+b*x^n/a)^(1/2)/(a*
c^2-d^2)/(a+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2187, 251, 441, 440}

$$\int \frac{1}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = \frac{cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2} - \frac{dx\sqrt{\frac{bx^n}{a}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^n}}$$

```
[In] Int[(a*c + b*c*x^n + d*Sqrt[a + b*x^n])^(-1),x]
```

```
[Out] -((d*x*Sqrt[1 + (b*x^n)/a]*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a
), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^n])) + (c*x*H
```

```
ypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*c^2*x^n)/(a*c^2 - d^2))])/(a*c
^2 - d^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (ac)\int \frac{1}{a^2c^2-ad^2+abc^2x^n}\,dx-(ad)\int \frac{1}{\sqrt{a+bx^n}\,(a^2c^2-ad^2+abc^2x^n)}\,dx\\
&= \frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{\left(ad\sqrt{1+\frac{bx^n}{a}}\right)\int \frac{1}{\sqrt{1+\frac{bx^n}{a}}\,(a^2c^2-ad^2+abc^2x^n)}\,dx}{\sqrt{a+bx^n}}\\
&= -\frac{dx\sqrt{1+\frac{bx^n}{a}}\,F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^n}}+\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 320 vs. 2(135) = 270.

Time = 0.55 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.37

$$\int \frac{1}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx =$$

$$-\frac{2ad(ac^2-d^2)(1+n)x\,\text{AppellF1}}{\sqrt{a+bx^n}\,(ac^2-d^2+bc^2x^n)\left(-2abc^2nx^n\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)+(ac^2-d^2)\left(-bn\right.\right.}$$
$$+\frac{cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}$$

```
[In] Integrate[(a*c + b*c*x^n + d*Sqrt[a + b*x^n])^(-1),x]
```

```
[Out] (-2*a*d*(a*c^2 - d^2)*(1 + n)*x*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x
^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/(Sqrt[a + b*x^n]*(a*c^2 - d^2 + b*c^
2*x^n)*(-2*a*b*c^2*n*x^n*AppellF1[1 + n^(-1), 1/2, 2, 2 + n^(-1), -((b*x^n)
/a), -((b*c^2*x^n)/(a*c^2 - d^2))] + (a*c^2 - d^2)*(-(b*n*x^n*AppellF1[1 +
n^(-1), 3/2, 1, 2 + n^(-1), -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))]) +
2*a*(1 + n)*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((b*c^2*x^n
)/(a*c^2 - d^2))]))) + (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*c
^2*x^n)/(a*c^2 - d^2))])/(a*c^2 - d^2)
```

Maple [F]

$$\int \frac{1}{ac+bc\,x^n+d\sqrt{a+b\,x^n}}dx$$

```
[In] int(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

```
[Out] int(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

Fricas [F]

$$\int \frac{1}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = \int \frac{1}{bcx^n+ac+\sqrt{bx^n+a}d}\,dx$$

```
[In] integrate(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((b*c*x^n + a*c - sqrt(b*x^n + a)*d)/(b^2*c^2*x^(2*n) + a^2*c^2 - a
*d^2 + (2*a*b*c^2 - b*d^2)*x^n), x)
```

Sympy [F]

$$\int \frac{1}{ac + bcx^n + d\sqrt{a + bx^n}}\,dx = \int \frac{1}{ac + bcx^n + d\sqrt{a + bx^n}}\,dx$$

```
[In] integrate(1/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
[Out] Integral(1/(a*c + b*c*x**n + d*sqrt(a + b*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{ac + bcx^n + d\sqrt{a + bx^n}}\,dx = \int \frac{1}{bcx^n + ac + \sqrt{bx^n + a}d}\,dx$$

```
[In] integrate(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

Giac [F]

$$\int \frac{1}{ac + bcx^n + d\sqrt{a + bx^n}}\,dx = \int \frac{1}{bcx^n + ac + \sqrt{bx^n + a}d}\,dx$$

```
[In] integrate(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
[Out] integrate(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{ac + bcx^n + d\sqrt{a + bx^n}}\,dx = \int \frac{1}{a\,c + d\,\sqrt{a + b\,x^n} + b\,c\,x^n}\,dx$$

```
[In] int(1/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n),x)
[Out] int(1/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n), x)
```

3.563 $\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 167

$$\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = -\frac{dx^{1+m}\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1+m}{n},\frac{1}{2},1,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\,(1+m)\sqrt{a+bx^n}} + \frac{cx^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\,(1+m)}$$

```
[Out] c*x^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*c^2*x^n/(a*c^2-d^2))/(a*c^2
-d^2)/(1+m)-d*x^(1+m)*AppellF1((1+m)/n,1/2,1,(1+m+n)/n,-b*x^n/a,-b*c^2*x^n/
(a*c^2-d^2))*(1+b*x^n/a)^(1/2)/(a*c^2-d^2)/(1+m)/(a+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2187, 371, 525, 524}

$$\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = \frac{cx^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\,(ac^2-d^2)} - \frac{dx^{m+1}\sqrt{\frac{bx^n}{a}+1}\,\text{AppellF1}\left(\frac{m+1}{n},\frac{1}{2},1,\frac{m+n+1}{n},-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\,(ac^2-d^2)\,\sqrt{a+bx^n}}$$

```
[In] Int[x^m/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] -((d*x^(1 + m)*Sqrt[1 + (b*x^n)/a]*AppellF1[(1 + m)/n, 1/2, 1, (1 + m + n)/
n, -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*(1 + m)*Sqrt
[a + b*x^n])) + (c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
 -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = (ac)\int \frac{x^m}{a^2c^2 - ad^2 + abc^2x^n}\,dx - (ad)\int \frac{x^m}{\sqrt{a+bx^n}\,(a^2c^2 - ad^2 + abc^2x^n)}\,dx$$

$$= \frac{cx^{1+m}\,{}_2F_1\left(1, \frac{1+m}{n}; \frac{1+m+n}{n}; -\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\,(1+m)} - \frac{\left(ad\sqrt{1+\frac{bx^n}{a}}\right)\int \frac{x^m}{\sqrt{1+\frac{bx^n}{a}}\,(a^2c^2-ad^2+abc^2x^n)}\,dx}{\sqrt{a+bx^n}}$$

$$= -\frac{dx^{1+m}\sqrt{1+\frac{bx^n}{a}}\,F_1\left(\frac{1+m}{n}; \frac{1}{2}, 1; \frac{1+m+n}{n}; -\frac{bx^n}{a}, -\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\,(1+m)\sqrt{a+bx^n}} + \frac{cx^{1+m}\,{}_2F_1\left(1, \frac{1+m}{n}; \frac{1+m+n}{n}; -\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\,(1+m)}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = \frac{x^{1+m}\left(-d\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1+m}{n},\frac{1}{2},1,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)+c\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n}\right.\right.}{(ac^2-d^2)\,(1+m)\sqrt{a+bx^n}}$$

```
[In] Integrate[x^m/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] (x^(1 + m)*(-(d*Sqrt[1 + (b*x^n)/a]*AppellF1[(1 + m)/n, 1/2, 1, (1 + m + n)
/n, -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))]) + c*Sqrt[a + b*x^n]*Hyperg
eometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*c^2*x^n)/(a*c^2 - d^2))]))/((
a*c^2 - d^2)*(1 + m)*Sqrt[a + b*x^n])
```

Maple [F]

$$\int \frac{x^m}{ac+bc\,x^n+d\sqrt{a+b\,x^n}}dx$$

```
[In] int(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

```
[Out] int(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

Fricas [F]

$$\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = \int \frac{x^m}{bcx^n+ac+\sqrt{bx^n+a}d}\,dx$$

```
[In] integrate(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((b*c*x^m*x^n + a*c*x^m - sqrt(b*x^n + a)*d*x^m)/(b^2*c^2*x^(2*n) +
 a^2*c^2 - a*d^2 + (2*a*b*c^2 - b*d^2)*x^n), x)
```

Sympy [F]

$$\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = \int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$$

```
[In] integrate(x**m/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

```
[Out] Integral(x**m/(a*c + b*c*x**n + d*sqrt(a + b*x**n)), x)
```

Maxima [F]

$$\int \frac{x^m}{ac + bcx^n + d\sqrt{a + bx^n}}\, dx = \int \frac{x^m}{bcx^n + ac + \sqrt{bx^n + a}d}\, dx$$

```
[In] integrate(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
[Out] integrate(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

Giac [F]

$$\int \frac{x^m}{ac + bcx^n + d\sqrt{a + bx^n}}\, dx = \int \frac{x^m}{bcx^n + ac + \sqrt{bx^n + a}d}\, dx$$

```
[In] integrate(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
[Out] integrate(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{ac + bcx^n + d\sqrt{a + bx^n}}\, dx = \int \frac{x^m}{a\,c + d\,\sqrt{a + b\,x^n} + b\,c\,x^n}\, dx$$

```
[In] int(x^m/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n),x)
[Out] int(x^m/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n), x)
```

3.564 $\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 27

$$\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx=\frac{2\log\left(d+c\sqrt{a+bx^n}\right)}{bcn}$$

```
[Out] 2*ln(d+c*(a+b*x^n)^(1/2))/b/c/n
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2186, 31}

$$\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx=\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

```
[In] Int[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^n]])/(b*c*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
```

```
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^n\right)}{n}\\
&= \frac{2\text{Subst}\left(\int \frac{1}{d+cx}\,dx, x, \sqrt{a+bx^n}\right)}{bn}\\
&= \frac{2\log\left(d+c\sqrt{a+bx^n}\right)}{bcn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = \frac{2\log\left(bdn+bcn\sqrt{a+bx^n}\right)}{bcn}$$

```
[In] Integrate[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
[Out] (2*Log[b*d*n + b*c*n*Sqrt[a + b*x^n]])/(b*c*n)
```

Maple [F]

$$\int \frac{x^{-1+n}}{ac+bc\,x^n+d\sqrt{a+b\,x^n}}dx$$

```
[In] int(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
[Out] int(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = \frac{2\,\log\left(\sqrt{bx^n+a}c+d\right)}{bcn}$$

```
[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
[Out] 2*log(sqrt(b*x^n + a)*c + d)/(b*c*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(20) = 40$.

Time = 6.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = \begin{cases} \frac{2\left(\begin{cases} \frac{\sqrt{a+bx^n}}{d} & \text{for}\ c=0 \\ \frac{\log\left(c\sqrt{a+bx^n}+d\right)}{c} & \text{otherwise} \end{cases}\right)}{bn} & \text{for}\ b\neq 0 \\ \frac{x^n}{\sqrt{a}dn+acn} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(-1+n)/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

```
[Out] Piecewise((2*Piecewise((sqrt(a + b*x**n)/d, Eq(c, 0)), (log(c*sqrt(a + b*x*
*n) + d)/c, True))/(b*n), Ne(b, 0)), (x**n/(sqrt(a)*d*n + a*c*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(25) = 50$.

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

$$\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = -\frac{\log\left(\frac{bx^n+a}{b}\right)}{bcn}+\frac{2\log\left(\frac{bcx^n+ac+\sqrt{bx^n+a}d}{d}\right)}{bcn}$$

```
[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
```

```
[Out] -log((b*x^n + a)/b)/(b*c*n) + 2*log((b*c*x^n + a*c + sqrt(b*x^n + a)*d)/d)/
(b*c*n)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = \frac{2\log\left(\left|\sqrt{bx^n+a}c+d\right|\right)}{bcn}$$

```
[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*log(abs(sqrt(b*x^n + a)*c + d))/(b*c*n)
```

Mupad [B] (verification not implemented)

Time = 16.92 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx=\frac{2\ \ln\left(d+c\,\sqrt{a+b\,x^n}\right)}{b\,c\,n}$$

```
[In] int(x^(n - 1)/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n),x)
[Out] (2*log(d + c*(a + b*x^n)^(1/2)))/(b*c*n)
```

3.565 $\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 8

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx = \arctan\left(2\sqrt{x}\right)$$

```
[Out] arctan(2*x^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1607, 65, 209}

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx = \arctan\left(2\sqrt{x}\right)$$

```
[In] Int[(Sqrt[x] + 4*x^(3/2))^(-1),x]
```

```
[Out] ArcTan[2*Sqrt[x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{x}(1+4x)}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \sqrt{x}\right)\\ &= \tan^{-1}\left(2\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx = \arctan\left(2\sqrt{x}\right)$$

```
[In] Integrate[(Sqrt[x] + 4*x^(3/2))^(-1),x]
[Out] ArcTan[2*Sqrt[x]]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\arctan\left(2\sqrt{x}\right)$	7
default	$\arctan\left(2\sqrt{x}\right)$	7
meijerg	$\arctan\left(2\sqrt{x}\right)$	7
trager	$\frac{\text{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-4\,\text{RootOf}\left(\_Z^2+1\right)x+4\sqrt{x}+\text{RootOf}\left(\_Z^2+1\right)}{1+4x}\right)}{2}$	39

```
[In] int(1/(4*x^(3/2)+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] arctan(2*x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx = \arctan\left(2\sqrt{x}\right)$$

```
[In] integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="fricas")
[Out] arctan(2*sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx = \operatorname{atan}\left(2\sqrt{x}\right)$$

```
[In] integrate(1/(4*x**(3/2)+x**(1/2)),x)
[Out] atan(2*sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx = \arctan\left(2\sqrt{x}\right)$$

```
[In] integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="maxima")
[Out] arctan(2*sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx = \arctan\left(2\sqrt{x}\right)$$

```
[In] integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="giac")
[Out] arctan(2*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx=\operatorname{atan}\left(2\sqrt{x}\right)$$

```
[In] int(1/(x^(1/2) + 4*x^(3/2)),x)
[Out] atan(2*x^(1/2))
```

3.566 $\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx = \arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1607, 335, 218, 212, 209}

$$\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx = \arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$$

```
[In] Int[(Sqrt[x] - x^(5/2))^(-1),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right)\\ &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\ &= \tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx = \arctan\left(\sqrt{x}\right) + \operatorname{arctanh}\left(\sqrt{x}\right)$$

```
[In] Integrate[(Sqrt[x] - x^(5/2))^(-1),x]
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
default	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
meijerg	$-\frac{\sqrt{x}\left(\ln\left(1-\left(x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(x^{2}\right)^{\frac{1}{4}}\right)\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}$	40
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)x+2\sqrt{x}-\operatorname{RootOf}\left(\_Z^{2}+1\right)}{x+1}\right)}{2}+\frac{\ln\left(\frac{2\sqrt{x}+1+x}{x-1}\right)}{2}$	56

```
[In] int(1/(-x^(5/2)+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx = \arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

```
[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="fricas")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx = -\frac{\log\left(\sqrt{x}-1\right)}{2}+\frac{\log\left(\sqrt{x}+1\right)}{2}+\operatorname{atan}\left(\sqrt{x}\right)$$

```
[In] integrate(1/(-x**(5/2)+x**(1/2)),x)
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx = \arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

```
[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="maxima")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(9) = 18$.

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx = \arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

```
[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="giac")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 16.49 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx = \mathrm{atan}\left(\sqrt{x}\right) + \mathrm{atanh}\left(\sqrt{x}\right)$$

```
[In] int(1/(x^(1/2) - x^(5/2)),x)
[Out] atan(x^(1/2)) + atanh(x^(1/2))
```

3.567 $\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}} dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}} dx = 4\sqrt[4]{x} + 2\sqrt{x} + 4\log\left(1-\sqrt[4]{x}\right)$$

```
[Out] 4*x^(1/4)+4*ln(1-x^(1/4))+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1607, 272, 45}

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}} dx = 2\sqrt{x} + 4\sqrt[4]{x} + 4\log\left(1-\sqrt[4]{x}\right)$$

```
[In] Int[(-x^(1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 - x^(1/4)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\left(-1+\sqrt[4]{x}\right)\sqrt[4]{x}}\,dx \\ &= 4\text{Subst}\left(\int \frac{x^2}{-1+x}\,dx, x, \sqrt[4]{x}\right) \\ &= 4\text{Subst}\left(\int \left(1+\frac{1}{-1+x}+x\right)\,dx, x, \sqrt[4]{x}\right) \\ &= 4\sqrt[4]{x}+2\sqrt{x}+4\log\left(1-\sqrt[4]{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx = 2\left(2+\sqrt[4]{x}\right)\sqrt[4]{x}+4\log\left(-1+\sqrt[4]{x}\right)$$

```
[In] Integrate[(-x^(1/4) + Sqrt[x])^(-1),x]
[Out] 2*(2 + x^(1/4))*x^(1/4) + 4*Log[-1 + x^(1/4)]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$2\sqrt{x}+4x^{\frac{1}{4}}+4\ln\left(x^{\frac{1}{4}}-1\right)$	20
default	$2\sqrt{x}+4x^{\frac{1}{4}}+4\ln\left(x^{\frac{1}{4}}-1\right)$	20
meijerg	$\frac{2x^{\frac{1}{4}}\left(6+3x^{\frac{1}{4}}\right)}{3}+4\ln\left(1-x^{\frac{1}{4}}\right)$	24

```
[In] int(1/(-x^(1/4)+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*x^(1/2)+4*x^(1/4)+4*ln(x^(1/4)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx = 2\,\sqrt{x}+4\,x^{\frac{1}{4}}+4\,\log\left(x^{\frac{1}{4}}-1\right)$$

```
[In] integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx = 4\sqrt[4]{x}+2\sqrt{x}+4\log\left(\sqrt[4]{x}-1\right)$$

```
[In] integrate(1/(-x**(1/4)+x**(1/2)),x)
[Out] 4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx = 2\,\sqrt{x}+4\,x^{\frac{1}{4}}+4\,\log\left(x^{\frac{1}{4}}-1\right)$$

```
[In] integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx = 2\,\sqrt{x}+4\,x^{\frac{1}{4}}+4\,\log\left(\left|x^{\frac{1}{4}}-1\right|\right)$$

```
[In] integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(abs(x^(1/4) - 1))
```

Mupad [B] (verification not implemented)

Time = 16.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx = 4\,\ln\left(x^{1/4}-1\right)+2\,\sqrt{x}+4\,x^{1/4}$$

```
[In] int(1/(x^(1/2) - x^(1/4)),x)
[Out] 4*log(x^(1/4) - 1) + 2*x^(1/2) + 4*x^(1/4)
```

3.568 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)$$

```
[Out] 6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1607, 272, 45}

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

```
[In] Int[(x^(1/3) + Sqrt[x])^(-1),x]
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\left(1+\sqrt[6]{x}\right)\sqrt[3]{x}}\,dx\\ &= 6\text{Subst}\left(\int \frac{x^3}{1+x}\,dx, x, \sqrt[6]{x}\right)\\ &= 6\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2\right)\,dx, x, \sqrt[6]{x}\right)\\ &= 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = \left(6-3\sqrt[6]{x}+2\sqrt[3]{x}\right)\sqrt[6]{x}-6\log\left(1+\sqrt[6]{x}\right)$$

```
[In] Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
[Out] (6 - 3*x^(1/6) + 2*x^(1/3))*x^(1/6) - 6*Log[1 + x^(1/6)]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result
derivativedivides	$6x^{\frac{1}{6}}-3x^{\frac{1}{3}}-6\ln\left(1+x^{\frac{1}{6}}\right)+2\sqrt{x}$
meijerg	$\frac{x^{\frac{1}{6}}\left(4x^{\frac{1}{3}}-6x^{\frac{1}{6}}+12\right)}{2}-6\ln\left(1+x^{\frac{1}{6}}\right)$
default	$2\ln\left(x^{\frac{1}{6}}-1\right)-\ln\left(x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+\ln\left(1-x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)-2\ln\left(1+x^{\frac{1}{6}}\right)+2\sqrt{x}+\ln\left(-\right.$

```
[In] int(1/(x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = \int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$$

```
[In] integrate(1/(x**(1/3)+x**(1/2)),x)
[Out] Integral(1/(x**(1/3) + sqrt(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\,\sqrt{x}-6\,\ln\left(x^{1/6}+1\right)-3\,x^{1/3}+6\,x^{1/6}$$

```
[In] int(1/(x^(1/2) + x^(1/3)),x)
[Out] 2*x^(1/2) - 6*log(x^(1/6) + 1) - 3*x^(1/3) + 6*x^(1/6)
```

3.569 $\int \frac{1}{\sqrt[4]{x}+\sqrt{x}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}} \, dx = -4\sqrt[4]{x}+2\sqrt{x}+4\log\left(1+\sqrt[4]{x}\right)$$

```
[Out] -4*x^(1/4)+4*ln(1+x^(1/4))+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1607, 272, 45}

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}} \, dx = 2\sqrt{x}-4\sqrt[4]{x}+4\log\left(\sqrt[4]{x}+1\right)$$

```
[In] Int[(x^(1/4) + Sqrt[x])^(-1),x]
```

```
[Out] -4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 + x^(1/4)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\left(1+\sqrt[4]{x}\right)\sqrt[4]{x}}\,dx \\
&= 4\text{Subst}\left(\int \frac{x^2}{1+x}\,dx, x, \sqrt[4]{x}\right) \\
&= 4\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x}\right)\,dx, x, \sqrt[4]{x}\right) \\
&= -4\sqrt[4]{x}+2\sqrt{x}+4\log\left(1+\sqrt[4]{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx = 2\left(-2+\sqrt[4]{x}\right)\sqrt[4]{x}+4\log\left(1+\sqrt[4]{x}\right)$$

```
[In] Integrate[(x^(1/4) + Sqrt[x])^(-1),x]
[Out] 2*(-2 + x^(1/4))*x^(1/4) + 4*Log[1 + x^(1/4)]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-4x^{\frac{1}{4}}+4\ln\left(1+x^{\frac{1}{4}}\right)+2\sqrt{x}$	20
default	$-4x^{\frac{1}{4}}+4\ln\left(1+x^{\frac{1}{4}}\right)+2\sqrt{x}$	20
meijerg	$-\frac{2x^{\frac{1}{4}}\left(-3x^{\frac{1}{4}}+6\right)}{3}+4\ln\left(1+x^{\frac{1}{4}}\right)$	22

```
[In] int(1/(x^(1/4)+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -4*x^(1/4)+4*ln(1+x^(1/4))+2*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 4\,x^{\frac{1}{4}} + 4\,\log\left(x^{\frac{1}{4}}+1\right)$$

```
[In] integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx = -4\sqrt[4]{x} + 2\sqrt{x} + 4\log\left(\sqrt[4]{x}+1\right)$$

```
[In] integrate(1/(x**(1/4)+x**(1/2)),x)
[Out] -4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 4\,x^{\frac{1}{4}} + 4\,\log\left(x^{\frac{1}{4}}+1\right)$$

```
[In] integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 4\,x^{\frac{1}{4}} + 4\,\log\left(x^{\frac{1}{4}}+1\right)$$

```
[In] integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx=4\,\ln\left(x^{1/4}+1\right)+2\,\sqrt{x}-4\,x^{1/4}$$

```
[In] int(1/(x^(1/2) + x^(1/4)),x)
[Out] 4*log(x^(1/4) + 1) + 2*x^(1/2) - 4*x^(1/4)
```

3.570 $\int \frac{1}{-\sqrt[3]{x}+x^{2/3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}} \, dx=3 \sqrt[3]{x}+3 \log \left(1-\sqrt[3]{x}\right)$$

```
[Out] 3*x^(1/3)+3*ln(1-x^(1/3))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1607, 272, 45}

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}} \, dx=3 \sqrt[3]{x}+3 \log \left(1-\sqrt[3]{x}\right)$$

```
[In] Int[(-x^(1/3) + x^(2/3))^(-1),x]
```

```
[Out] 3*x^(1/3) + 3*Log[1 - x^(1/3)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\left(-1+\sqrt[3]{x}\right)\sqrt[3]{x}}\,dx\\ &= 3\text{Subst}\left(\int \frac{x}{-1+x}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\text{Subst}\left(\int \left(1+\frac{1}{-1+x}\right)\,dx, x, \sqrt[3]{x}\right)\\ &= 3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx = 3\sqrt[3]{x}+3\log\left(-1+\sqrt[3]{x}\right)$$

```
[In] Integrate[(-x^(1/3) + x^(2/3))^(-1),x]
[Out] 3*x^(1/3) + 3*Log[-1 + x^(1/3)]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$3x^{\frac{1}{3}}+3\ln\left(x^{\frac{1}{3}}-1\right)$	15
default	$3x^{\frac{1}{3}}+3\ln\left(x^{\frac{1}{3}}-1\right)$	15
meijerg	$3x^{\frac{1}{3}}+3\ln\left(1-x^{\frac{1}{3}}\right)$	17
trager	$3x^{\frac{1}{3}}+\ln\left(-3x^{\frac{2}{3}}+3x^{\frac{1}{3}}+x-1\right)$	21

```
[In] int(1/(-x^(1/3)+x^(2/3)),x,method=_RETURNVERBOSE)
[Out] 3*x^(1/3)+3*ln(x^(1/3)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx = 3\,x^{\frac{1}{3}} + 3\,\log\left(x^{\frac{1}{3}} - 1\right)$$

```
[In] integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="fricas")
[Out] 3*x^(1/3) + 3*log(x^(1/3) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx = 3\sqrt[3]{x} + 3\log\left(\sqrt[3]{x} - 1\right)$$

```
[In] integrate(1/(-x**(1/3)+x**(2/3)),x)
[Out] 3*x**(1/3) + 3*log(x**(1/3) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx = 3\,x^{\frac{1}{3}} + 3\,\log\left(x^{\frac{1}{3}} - 1\right)$$

```
[In] integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="maxima")
[Out] 3*x^(1/3) + 3*log(x^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx = 3\,x^{\frac{1}{3}} + 3\,\log\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

```
[In] integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="giac")
[Out] 3*x^(1/3) + 3*log(abs(x^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx = 3\,\ln\left(x^{1/3} - 1\right) + 3\,x^{1/3}$$

```
[In] int(-1/(x^(1/3) - x^(2/3)),x)
[Out] 3*log(x^(1/3) - 1) + 3*x^(1/3)
```

3.571 $\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx=2\sqrt{x}+\frac{4\arctan\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{4}{3}\log\left(1+\sqrt[4]{x}\right)-\frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)$$

```
[Out] 4/3*ln(1+x^(1/4))-2/3*ln(1-x^(1/4)+x^(1/2))+4/3*arctan(1/3*(1-2*x^(1/4))*3^
(1/2))*3^(1/2)+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {1607, 348, 327, 298, 31, 648, 632, 210, 642}

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx=\frac{4\arctan\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}+2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)$$

```
[In] Int[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (4*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[1 + x^(1/4
)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{x}}{1+x^{3/4}}\,dx\\
&= 4\text{Subst}\left(\int \frac{x^4}{1+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x} - 4\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x} + \frac{4}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[4]{x}\right) - \frac{4}{3}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)\\
&\quad - 2\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)\\
&= 2\sqrt{x}+\frac{4}{3}\log\left(1+\sqrt[4]{x}\right)-\frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)+4\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[4]{x}\right)\\
&= 2\sqrt{x} + \frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = \frac{2}{3}\left(3\sqrt{x}+2\sqrt{3}\arctan\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)+2\log\left(1+\sqrt[4]{x}\right)-\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)\right)$$

```
[In] Integrate[(x^(-1/4) + Sqrt[x])^(-1),x]
[Out] (2*(3*Sqrt[x] + 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]] + 2*Log[1 + x^(1/
4)] - Log[1 - x^(1/4) + Sqrt[x]]))/3
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$2\sqrt{x}+\frac{4\ln\left(1+x^{\frac{1}{4}}\right)}{3}-\frac{2\ln\left(1-x^{\frac{1}{4}}+\sqrt{x}\right)}{3}-\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{4}}-1\right)\sqrt{3}}{3}\right)}{3}$	46
default	$2\sqrt{x}+\frac{4\ln\left(1+x^{\frac{1}{4}}\right)}{3}-\frac{2\ln\left(1-x^{\frac{1}{4}}+\sqrt{x}\right)}{3}-\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{4}}-1\right)\sqrt{3}}{3}\right)}{3}$	46
meijerg	$2\sqrt{x}-\frac{4\sqrt{x}\left(-\frac{\ln\left(1+x^{\frac{1}{4}}\right)}{\sqrt{x}}+\frac{\ln\left(1-x^{\frac{1}{4}}+\sqrt{x}\right)}{2\sqrt{x}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^{\frac{1}{4}}}{2-x^{\frac{1}{4}}}\right)}{\sqrt{x}}\right)}{3}$	65

```
[In] int(1/(1/x^(1/4)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)+4/3*ln(1+x^(1/4))-2/3*ln(1-x^(1/4)+x^(1/2))-4/3*3^(1/2)*arctan(1/
3*(2*x^(1/4)-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = -\frac{4}{3}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}x^{\frac{1}{4}}-\frac{1}{3}\,\sqrt{3}\right)+2\,\sqrt{x}-\frac{2}{3}\,\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\,\log\left(x^{\frac{1}{4}}+1\right)$$

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/4) - 1/3*sqrt(3)) + 2*sqrt(x) - 2/3*lo
g(sqrt(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = 2\sqrt{x}+\frac{4\log\left(\sqrt[4]{x}+1\right)}{3}-\frac{2\log\left(-4\sqrt[4]{x}+4\sqrt{x}+4\right)}{3} - \frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[4]{x}}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate(1/(1/x**(1/4)+x**(1/2)),x)
[Out] 2*sqrt(x) + 4*log(x**(1/4) + 1)/3 - 2*log(-4*x**(1/4) + 4*sqrt(x) + 4)/3 -
4*sqrt(3)*atan(2*sqrt(3)*x**(1/4)/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = -\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x} - \frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="maxima")
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = -\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x} - \frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="giac")
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx=\frac{4\,\ln\left(16\,x^{1/4}+16\right)}{3}+\ln\left(9\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)^2+16\,x^{1/4}\right)\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)-\ln\left(9\,\left(\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)^2+16\,x^{1/4}\right)\,\left(\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)+2\,\sqrt{x}$$

```
[In] int(1/(x^(1/2) + 1/x^(1/4)),x)
[Out] (4*log(16*x^(1/4) + 16))/3 + log(9*((3^(1/2)*2i)/3 - 2/3)^2 + 16*x^(1/4))*(
(3^(1/2)*2i)/3 - 2/3) - log(9*((3^(1/2)*2i)/3 + 2/3)^2 + 16*x^(1/4))*((3^(1
/2)*2i)/3 + 2/3) + 2*x^(1/2)
```

3.572 $\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = -12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5}+2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2}+12\log\left(1+\sqrt[12]{x}\right)$$

```
[Out] -12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2*
x^(2/3)+12*ln(1+x^(1/12))+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1607, 272, 45}

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

```
[In] Int[(x^(1/4) + x^(1/3))^(-1),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 + 12*Log[1 + x^(1/12)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\left(1+\sqrt[12]{x}\right)\sqrt[4]{x}}\,dx \\
&= 12\text{Subst}\left(\int \frac{x^8}{1+x}\,dx, x, \sqrt[12]{x}\right) \\
&= 12\text{Subst}\left(\int \left(-1+x-x^2+x^3-x^4+x^5-x^6+x^7+\frac{1}{1+x}\right)dx, x, \sqrt[12]{x}\right) \\
&= -12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5}+2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2}+12\log\left(1+\sqrt[12]{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \frac{1}{70}\left(-840\sqrt[12]{x}+420\sqrt[6]{x}-280\sqrt[4]{x}+210\sqrt[3]{x}-168x^{5/12}+140\sqrt{x} - 120x^{7/12}+105x^{2/3}\right)+12\log\left(1+\sqrt[12]{x}\right)$$

```
[In] Integrate[(x^(1/4) + x^(1/3))^(-1),x]
[Out] (-840*x^(1/12) + 420*x^(1/6) - 280*x^(1/4) + 210*x^(1/3) - 168*x^(5/12) + 1
40*Sqrt[x] - 120*x^(7/12) + 105*x^(2/3))/70 + 12*Log[1 + x^(1/12)]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-12x^{\frac{1}{12}}+6x^{\frac{1}{6}}-4x^{\frac{1}{4}}+3x^{\frac{1}{3}}-\frac{12x^{\frac{5}{12}}}{5}-\frac{12x^{\frac{7}{12}}}{7}+\frac{3x^{\frac{2}{3}}}{2}+12\ln\left(1+x^{\frac{1}{12}}\right)+2\sqrt{x}$
meijerg	$-\frac{x^{\frac{1}{12}}\left(-315x^{\frac{7}{12}}+360\sqrt{x}-420x^{\frac{5}{12}}+504x^{\frac{1}{3}}-630x^{\frac{1}{4}}+840x^{\frac{1}{6}}-1260x^{\frac{1}{12}}+2520\right)}{210}+12\ln\left(1+x^{\frac{1}{12}}\right)$
default	$2\sqrt{x}-4x^{\frac{1}{4}}+3x^{\frac{1}{3}}+6x^{\frac{1}{6}}+\frac{3x^{\frac{2}{3}}}{2}+\ln(x-1)+2\ln\left(x^{\frac{1}{6}}-1\right)-\ln\left(x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)-2\ln$

```
[In] int(1/(x^(1/4)+x^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] -12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2*
x^(2/3)+12*ln(1+x^(1/12))+2*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx=\frac{3}{2}\,x^{\frac{2}{3}}-\frac{12}{7}\,x^{\frac{7}{12}}+2\,\sqrt{x}-\frac{12}{5}\,x^{\frac{5}{12}}+3\,x^{\frac{1}{3}}-4\,x^{\frac{1}{4}}+6\,x^{\frac{1}{6}}-12\,x^{\frac{1}{12}}+12\,\log\left(x^{\frac{1}{12}}+1\right)$$

```
[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(
1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx=\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$$

```
[In] integrate(1/(x**(1/4)+x**(1/3)),x)
```

```
[Out] Integral(1/(x**(1/4) + x**(1/3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}} - \frac{12}{7}\,x^{\frac{7}{12}} + 2\,\sqrt{x} - \frac{12}{5}\,x^{\frac{5}{12}} + 3\,x^{\frac{1}{3}} - 4\,x^{\frac{1}{4}} + 6\,x^{\frac{1}{6}} - 12\,x^{\frac{1}{12}} + 12\,\log\left(x^{\frac{1}{12}} + 1\right)$$

```
[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(
1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}} - \frac{12}{7}\,x^{\frac{7}{12}} + 2\,\sqrt{x} - \frac{12}{5}\,x^{\frac{5}{12}} + 3\,x^{\frac{1}{3}} - 4\,x^{\frac{1}{4}} + 6\,x^{\frac{1}{6}} - 12\,x^{\frac{1}{12}} + 12\,\log\left(x^{\frac{1}{12}} + 1\right)$$

```
[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(
1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = 12\,\ln\left(x^{1/12}+1\right) + 2\,\sqrt{x} + 3\,x^{1/3} - 4\,x^{1/4} + \frac{3\,x^{2/3}}{2} + 6\,x^{1/6} - 12\,x^{1/12} - \frac{12\,x^{5/12}}{5} - \frac{12\,x^{7/12}}{7}$$

```
[In] int(1/(x^(1/3) + x^(1/4)),x)
```

```
[Out] 12*log(x^(1/12) + 1) + 2*x^(1/2) + 3*x^(1/3) - 4*x^(1/4) + (3*x^(2/3))/2 +
6*x^(1/6) - 12*x^(1/12) - (12*x^(5/12))/5 - (12*x^(7/12))/7
```

3.573 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}} dx$

Optimal result

Integrand size = 13, antiderivative size = 130

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}} dx = 12\sqrt[12]{x}-6\sqrt[6]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}+\frac{12x^{5/12}}{5}-2\sqrt{x}+\frac{12x^{7/12}}{7}-\frac{3x^{2/3}}{2}+\frac{4x^{3/4}}{3} -\frac{6x^{5/6}}{5}+\frac{12x^{11/12}}{11}-x+\frac{12x^{13/12}}{13}-\frac{6x^{7/6}}{7}+\frac{4x^{5/4}}{5}-12\log\left(1+\sqrt[12]{x}\right)$$

```
[Out] 12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7/12)-3/2*x
^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*x^(13/12)-6/7*x^(7/6
)+4/5*x^(5/4)-12*ln(1+x^(1/12))-2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1607, 272, 45}

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}} dx = \frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7} +\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-12\log\left(\sqrt[12]{x}+1\right)$$

```
[In] Int[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*Sqrt[
x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*
```

```
x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*
Log[1 + x^(1/12)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt[3]{x}}{1+\sqrt[12]{x}}\,dx \\
&= 12\text{Subst}\left(\int \frac{x^{15}}{1+x}\,dx, x, \sqrt[12]{x}\right) \\
&= 12\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2-x^3+x^4-x^5+x^6-x^7+x^8-x^9+x^{10}-x^{11}\right.\right. \\
&\qquad \left.\left. +x^{12}-x^{13}+x^{14}\right)dx, x, \sqrt[12]{x}\right) \\
&= 12\sqrt[12]{x}-6\sqrt[6]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}+\frac{12x^{5/12}}{5}-2\sqrt{x}+\frac{12x^{7/12}}{7}-\frac{3x^{2/3}}{2}+\frac{4x^{3/4}}{3} \\
&\quad -\frac{6x^{5/6}}{5}+\frac{12x^{11/12}}{11}-x+\frac{12x^{13/12}}{13}-\frac{6x^{7/6}}{7}+\frac{4x^{5/4}}{5}-12\log\left(1+\sqrt[12]{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx$$
$$= \frac{360360\sqrt[12]{x}-180180\sqrt[6]{x}+120120\sqrt[4]{x}-90090\sqrt[3]{x}+72072x^{5/12}-60060\sqrt{x}+51480x^{7/12}-45045x^{2/3}+4}{30030}$$
$$-12\log\left(1+\sqrt[12]{x}\right)$$

```
[In] Integrate[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] (360360*x^(1/12) - 180180*x^(1/6) + 120120*x^(1/4) - 90090*x^(1/3) + 72072*
x^(5/12) - 60060*Sqrt[x] + 51480*x^(7/12) - 45045*x^(2/3) + 40040*x^(3/4) -
 36036*x^(5/6) + 32760*x^(11/12) - 30030*x + 27720*x^(13/12) - 25740*x^(7/6
) + 24024*x^(5/4))/30030 - 12*Log[1 + x^(1/12)]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.64

method	result
derivativedivides	$12x^{\frac{1}{12}}-6x^{\frac{1}{6}}+4x^{\frac{1}{4}}-3x^{\frac{1}{3}}+\frac{12x^{\frac{5}{12}}}{5}+\frac{12x^{\frac{7}{12}}}{7}-\frac{3x^{\frac{2}{3}}}{2}+\frac{4x^{\frac{3}{4}}}{3}-\frac{6x^{\frac{5}{6}}}{5}+\frac{12x^{\frac{11}{12}}}{11}-x+\frac{12x^{\frac{13}{12}}}{13}-\frac{6x}{7}$
default	$12x^{\frac{1}{12}}-6x^{\frac{1}{6}}+4x^{\frac{1}{4}}-3x^{\frac{1}{3}}+\frac{12x^{\frac{5}{12}}}{5}+\frac{12x^{\frac{7}{12}}}{7}-\frac{3x^{\frac{2}{3}}}{2}+\frac{4x^{\frac{3}{4}}}{3}-\frac{6x^{\frac{5}{6}}}{5}+\frac{12x^{\frac{11}{12}}}{11}-x+\frac{12x^{\frac{13}{12}}}{13}-\frac{6x}{7}$
meijerg	$\frac{x^{\frac{1}{12}}\left(48048x^{\frac{7}{6}}-51480x^{\frac{13}{12}}+55440x-60060x^{\frac{11}{12}}+65520x^{\frac{5}{6}}-72072x^{\frac{3}{4}}+80080x^{\frac{2}{3}}-90090x^{\frac{7}{12}}+102960\sqrt{x}-120120x^{\frac{5}{12}}+14\right.}{60060}$

```
[In] int(1/(1/x^(1/3)+1/x^(1/4)),x,method=_RETURNVERBOSE)
```

```
[Out] 12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7/12)-3/2*x
^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*x^(13/12)-6/7*x^(7/6
)+4/5*x^(5/4)-12*ln(1+x^(1/12))-2*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx=\frac{4}{5}\,(x+5)x^{\frac{1}{4}}-\frac{6}{7}\,(x+7)x^{\frac{1}{6}}+\frac{12}{13}\,(x+13)x^{\frac{1}{12}}-x+\frac{12}{11}\,x^{\frac{11}{12}}-\frac{6}{5}\,x^{\frac{5}{6}}+\frac{4}{3}\,x^{\frac{3}{4}}-\frac{3}{2}\,x^{\frac{2}{3}}+\frac{12}{7}\,x^{\frac{7}{12}}-2\,\sqrt{x}+\frac{12}{5}\,x^{\frac{5}{12}}-3\,x^{\frac{1}{3}}-12\,\log\left(x^{\frac{1}{12}}+1\right)$$

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="fricas")
```

```
[Out] 4/5*(x + 5)*x^(1/4) - 6/7*(x + 7)*x^(1/6) + 12/13*(x + 13)*x^(1/12) - x + 1
2/11*x^(11/12) - 6/5*x^(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) -
2*sqrt(x) + 12/5*x^(5/12) - 3*x^(1/3) - 12*log(x^(1/12) + 1)
```

Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.93

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx=\frac{12x^{\frac{13}{12}}}{13}+\frac{12x^{\frac{11}{12}}}{11}+\frac{12x^{\frac{7}{12}}}{7}+\frac{12x^{\frac{5}{12}}}{5}+12\sqrt[12]{x}-\frac{6x^{\frac{7}{6}}}{7}-\frac{6x^{\frac{5}{6}}}{5}-6\sqrt[6]{x}+\frac{4x^{\frac{5}{4}}}{5}+\frac{4x^{\frac{3}{4}}}{3}+4\sqrt[4]{x}-\frac{3x^{\frac{2}{3}}}{2}-3\sqrt[3]{x}-2\sqrt{x}-x-12\log\left(\sqrt[12]{x}+1\right)$$

```
[In] integrate(1/(1/x**(1/3)+1/x**(1/4)),x)
```

```
[Out] 12*x**(13/12)/13 + 12*x**(11/12)/11 + 12*x**(7/12)/7 + 12*x**(5/12)/5 + 12*
x**(1/12) - 6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 4*x**(5/4)/5 + 4*x**
(3/4)/3 + 4*x**(1/4) - 3*x**(2/3)/2 - 3*x**(1/3) - 2*sqrt(x) - x - 12*log(x
**(1/12) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx=\frac{4}{5}\,x^{\frac{5}{4}}-\frac{6}{7}\,x^{\frac{7}{6}}+\frac{12}{13}\,x^{\frac{13}{12}}-x+\frac{12}{11}\,x^{\frac{11}{12}}-\frac{6}{5}\,x^{\frac{5}{6}}+\frac{4}{3}\,x^{\frac{3}{4}}-\frac{3}{2}\,x^{\frac{2}{3}}+\frac{12}{7}\,x^{\frac{7}{12}}-2\,\sqrt{x}+\frac{12}{5}\,x^{\frac{5}{12}}-3\,x^{\frac{1}{3}}+4\,x^{\frac{1}{4}}-6\,x^{\frac{1}{6}}+12\,x^{\frac{1}{12}}-12\,\log\left(x^{\frac{1}{12}}+1\right)$$

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="maxima")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx = \frac{4}{5}\,x^{\frac{5}{4}} - \frac{6}{7}\,x^{\frac{7}{6}} + \frac{12}{13}\,x^{\frac{13}{12}} - x + \frac{12}{11}\,x^{\frac{11}{12}} - \frac{6}{5}\,x^{\frac{5}{6}} + \frac{4}{3}\,x^{\frac{3}{4}} - \frac{3}{2}\,x^{\frac{2}{3}} + \frac{12}{7}\,x^{\frac{7}{12}} - 2\,\sqrt{x} + \frac{12}{5}\,x^{\frac{5}{12}} - 3\,x^{\frac{1}{3}} + 4\,x^{\frac{1}{4}} - 6\,x^{\frac{1}{6}} + 12\,x^{\frac{1}{12}} - 12\,\log\left(x^{\frac{1}{12}} + 1\right)$$

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="giac")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx = 4\,x^{1/4} - 12\,\ln\left(x^{1/12}+1\right) - 2\,\sqrt{x} - 3\,x^{1/3} - x - \frac{3\,x^{2/3}}{2} - 6\,x^{1/6} + \frac{4\,x^{3/4}}{3} + \frac{4\,x^{5/4}}{5} - \frac{6\,x^{5/6}}{5} + 12\,x^{1/12} - \frac{6\,x^{7/6}}{7} + \frac{12\,x^{5/12}}{5} + \frac{12\,x^{7/12}}{7} + \frac{12\,x^{11/12}}{11} + \frac{12\,x^{13/12}}{13}$$

```
[In] int(1/(1/x^(1/3) + 1/x^(1/4)),x)
```

```
[Out] 4*x^(1/4) - 12*log(x^(1/12) + 1) - 2*x^(1/2) - 3*x^(1/3) - x - (3*x^(2/3))/
2 - 6*x^(1/6) + (4*x^(3/4))/3 + (4*x^(5/4))/5 - (6*x^(5/6))/5 + 12*x^(1/12)
 - (6*x^(7/6))/7 + (12*x^(5/12))/5 + (12*x^(7/12))/7 + (12*x^(11/12))/11 +
(12*x^(13/12))/13
```

3.574 $\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 200

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = 2\sqrt{x}+\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\arctan\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}+4\sqrt[6]{x}\right)\right) + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right) - \frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)$$

```
[Out] 6/5*ln(1-x^(1/6))-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*(-5^(1/2)+1)
-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*(5^(1/2)+1)+2*x^(1/2)+3/5*arc
tan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-3/5*ar
ctan(1/20*(1+4*x^(1/6)+5^(1/2))*(50+10*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used

$= \{1607, 348, 327, 300, 648, 632, 210, 642, 31\}$

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = \frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\arctan\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right) + 2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right) - \frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)$$

```
[In] Int[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/Sqrt[
2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[5])/
10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 + Sqr
t[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])
*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 300

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*
k - 1)*m*(Pi/n)] + s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k
 - 1)*(Pi/n)]*x + s^2*x^2), x]; (r^(m + 1)/(a*n*s^m))*Int[1/(r - s*x), x] -
 Dist[2*((-r)^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fre
eQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && NegQ[
a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx \\
&= 6\text{Subst}\left(\int \frac{x^7}{-1+x^5}\,dx, x, \sqrt[6]{x}\right) \\
&= 2\sqrt{x}+6\text{Subst}\left(\int \frac{x^2}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)
\end{aligned}$$

$$=2\sqrt{x}-\frac{6}{5}\text{Subst}\left(\int\frac{1}{1-x}\,dx,x,\sqrt[6]{x}\right)$$
$$-\frac{12}{5}\text{Subst}\left(\int\frac{\frac{1}{4}\left(-1-\sqrt{5}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx,x,\sqrt[6]{x}\right)$$
$$-\frac{12}{5}\text{Subst}\left(\int\frac{\frac{1}{4}\left(-1+\sqrt{5}\right)+\frac{1}{4}\left(1-\sqrt{5}\right)x}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx,x,\sqrt[6]{x}\right)$$

$$=2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)+\frac{3\text{Subst}\left(\int\frac{1}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx,x,\sqrt[6]{x}\right)}{\sqrt{5}}$$
$$-\frac{3\text{Subst}\left(\int\frac{1}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx,x,\sqrt[6]{x}\right)}{\sqrt{5}}$$
$$-\frac{1}{10}\left(3\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int\frac{\frac{1}{2}\left(1+\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx,x,\sqrt[6]{x}\right)$$
$$-\frac{1}{10}\left(3\left(1+\sqrt{5}\right)\right)\text{Subst}\left(\int\frac{\frac{1}{2}\left(1-\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx,x,\sqrt[6]{x}\right)$$

$$=2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)$$
$$-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)$$
$$-\frac{6\text{Subst}\left(\int\frac{1}{\frac{1}{2}\left(-5-\sqrt{5}\right)-x^2}\,dx,x,\frac{1}{2}\left(1-\sqrt{5}\right)+2\sqrt[6]{x}\right)}{\sqrt{5}}$$
$$+\frac{6\text{Subst}\left(\int\frac{1}{\frac{1}{2}\left(-5+\sqrt{5}\right)-x^2}\,dx,x,\frac{1}{2}\left(1+\sqrt{5}\right)+2\sqrt[6]{x}\right)}{\sqrt{5}}$$

$$=2\sqrt{x}+6\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)$$
$$-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}+4\sqrt[6]{x}\right)\right)$$
$$+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)$$
$$-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.63

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = 2\sqrt{x}+\frac{6}{5}\log\left(-1+\sqrt[6]{x}\right)-\frac{6}{5}\text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\&,\frac{-\log\left(\sqrt[6]{x}-\#1\right)-2\log\left(\sqrt[6]{x}-\#1\right)\#1+2\log\left(\sqrt[6]{x}-\#1\right)\#1^2+\log\left(\sqrt[6]{x}-\#1\right)\#1^3}{1+2\#1+3\#1^2+4\#1^3}\&\right]$$

```
[In] Integrate[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (6*Log[-1 + x^(1/6)])/5 - (6*RootSum[1 + #1 + #1^2 + #1^3 + #1^
4 & , (-Log[x^(1/6) - #1] - 2*Log[x^(1/6) - #1]*#1 + 2*Log[x^(1/6) - #1]*#1
^2 + Log[x^(1/6) - #1]*#1^3)/(1 + 2*#1 + 3*#1^2 + 4*#1^3) & ])/5
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.62

method	result
meijerg	$-\frac{6(-1)^{\frac{2}{5}}\left(\frac{5\sqrt{x}\,(-1)^{\frac{3}{5}}}{3}+(-1)^{\frac{3}{5}}\left(\ln\left(1-x^{\frac{1}{6}}\right)-\cos\left(\frac{\pi}{5}\right)\ln\left(1-2\cos\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)+2\sin\left(\frac{\pi}{5}\right)\arctan\left(\frac{\sin\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}}{1-\cos\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}}\right)+\mathrm{c}\right.\right.}{5}$
derivativedivides	$2\sqrt{x}+\frac{3\left(\sqrt{5}-1\right)\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{5}\right)}{10}+\frac{12\left(-\sqrt{5}+1-\frac{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}{4}\right)\arctan\left(\frac{1+4x^{\frac{1}{6}}+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{3\ln\left(2+x\right.}{}$
default	$2\sqrt{x}+\frac{3\left(\sqrt{5}-1\right)\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{5}\right)}{10}+\frac{12\left(-\sqrt{5}+1-\frac{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}{4}\right)\arctan\left(\frac{1+4x^{\frac{1}{6}}+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{3\ln\left(2+x\right.}{}$

```
[In] int(1/(-1/x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -6/5*(-1)^(2/5)*(5/3*x^(1/2)*(-1)^(3/5)+(-1)^(3/5)*(ln(1-x^(1/6))-cos(1/5*P
i)*ln(1-2*cos(2/5*Pi)*x^(1/6)+x^(1/3))+2*sin(1/5*Pi)*arctan(sin(2/5*Pi)*x^(
1/6)/(1-cos(2/5*Pi)*x^(1/6)))+cos(2/5*Pi)*ln(1+2*cos(1/5*Pi)*x^(1/6)+x^(1/3
))-2*sin(2/5*Pi)*arctan(sin(1/5*Pi)*x^(1/6)/(1+cos(1/5*Pi)*x^(1/6)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(133) = 266$.

Time = 0.98 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.19

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}} + \sqrt{x}}\,dx$$

$$= \frac{1}{10}\left(3\sqrt{5} - \sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5} + \sqrt{5} + 1\right)^2 + \frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5} + \sqrt{5} - 3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5} - \sqrt{5} - 1\right) + \frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5} - \sqrt{5} - 1\right)^2}\right.$$

$$+3\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5} + \sqrt{5} + 1\right)^2 + \frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5} + \sqrt{5} - 3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5} - \sqrt{5} - 1\right) - \frac{27}{4}\left(\cdots\right.}$$

$$\left. + 72\,x^{\frac{1}{6}} + 36\right)$$

$$+\frac{1}{10}\left(3\sqrt{5} + \sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5} + \sqrt{5} + 1\right)^2 + \frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5} + \sqrt{5} - 3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5} - \sqrt{5} - 1\right) + \frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5} - \sqrt{5} - 1\right)^2}\right.$$

$$-3\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5} + \sqrt{5} + 1\right)^2 + \frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5} + \sqrt{5} - 3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5} - \sqrt{5} - 1\right) - \frac{27}{4}\left(\cdots\right.}$$

$$\left. + 72\,x^{\frac{1}{6}} + 36\right)$$

$$- \frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5} + \sqrt{5} + 1\right)\log\left(-\frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5} + \sqrt{5} + 1\right)^2 + 36\,x^{\frac{1}{6}}\right)$$

$$+ \frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5} - \sqrt{5} - 1\right)\log\left(-\frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5} - \sqrt{5} - 1\right)^2 + 36\,x^{\frac{1}{6}}\right)$$

$$+ 2\sqrt{x} + \frac{6}{5}\log\left(x^{\frac{1}{6}} - 1\right)$$

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 +
9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) -
sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2
)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) -
```

```
5) + sqrt(5) + 1)^2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 3*
sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(
sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/
4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5
) + 18*sqrt(5) - 90)*(sqrt(5) - 1) + 72*x^(1/6) + 36) + 1/10*(3*sqrt(5) + s
qrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(s
qrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4
*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5)
 + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 - 3*sqrt(-27/4*(sqrt(2)
*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqr
t(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90
)*(sqrt(5) - 1) + 72*x^(1/6) + 36) - 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) + 1)*log(-9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 36*x^(1/6))
 + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(-9/4*(sqrt(2)*sqrt(sq
rt(5) - 5) - sqrt(5) - 1)^2 + 36*x^(1/6)) + 2*sqrt(x) + 6/5*log(x^(1/6) - 1
)
```

Sympy [F]

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}} + \sqrt{x}}\, dx = \int \frac{\sqrt[3]{x}}{\left(\sqrt[6]{x} - 1\right)\left(\sqrt[6]{x} + x^{\frac{2}{3}} + \sqrt[3]{x} + \sqrt{x} + 1\right)}\, dx$$

```
[In] integrate(1/(-1/x**(1/3)+x**(1/2)),x)
[Out] Integral(x**(1/3)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3) + sqrt(x)
 + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(133) = 266$.

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.36

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = -\frac{6}{5}(-1)^{\frac{3}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right) - \frac{6\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}} + \frac{6\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right)}{5\sqrt{-2\sqrt{5}-10}} + 2\sqrt{x} + \frac{6\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{2}{5}}+(-1)^{\frac{2}{5}}\right)} - \frac{6\log\left(x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{2}{5}}-(-1)^{\frac{2}{5}}\right)}$$

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -6/5*(-1)^(3/5)*log((-1)^(1/5) + x^(1/6)) - 6/5*sqrt(5)*(-1)^(3/5)*log((sqr
t(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))
/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(
1/6)))/sqrt(2*sqrt(5) - 10) + 6/5*sqrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)^(1/5
) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1
)^(1/5) + (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(
-2*sqrt(5) - 10) + 2*sqrt(x) + 6/5*log(-x^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(2/5) + (-1)^(2/5)) - 6/5*
log(x^(1/6)*(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(
sqrt(5)*(-1)^(2/5) - (-1)^(2/5))
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.70

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = \frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right) - \frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right) + \frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+1\right) - \frac{3}{10}\sqrt{5}\log\left(-\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+x^{\frac{1}{3}}+1\right) + 2\sqrt{x} - \frac{3}{10}\log\left(x^{\frac{2}{3}}+\sqrt{x}+x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right) + \frac{6}{5}\log\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 3/5*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2*
sqrt(5) + 10)) + 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
- 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + 2*sqrt(x) -
3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1/6) -
 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.12

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = \frac{6\ln\left(1296x^{1/6}-1296\right)}{5} - \ln\left(-750x^{1/6}\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)^3 - 1296\right)\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right) + \ln\left(750x^{1/6}\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}+\frac{3\sqrt{5}}{10}-\frac{3}{10}\right)^3 - 1296\right)\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}+\frac{3\sqrt{5}}{10}-\frac{3}{10}\right) - \ln\left(-750\right.$$

```
[In] int(1/(x^(1/2) - 1/x^(1/3)),x)
```

```
[Out] (6*log(1296*x^(1/6) - 1296))/5 - log(- 750*x^(1/6)*((3*2^(1/2)*(- 5^(1/2) -
 5)^(1/2))/10 - (3*5^(1/2))/10 + 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2) - 5
```

```
)^(1/2))/10 - (3*5^(1/2))/10 + 3/10) + log(750*x^(1/6)*((3*2^(1/2)*(- 5^(1/
2) - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2)
 - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10) - log(- 750*x^(1/6)*((3*5^(1/2))/1
0 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))/10 -
(3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) - log(- 750*x^(1/6)*((3*5^(1/2))
/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))/10
+ (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) + 2*x^(1/2)
```

3.575 $\int \frac{\sqrt{x}}{x+x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{\sqrt{x}}{x+x^2} dx = 2\arctan\left(\sqrt{x}\right)$$

```
[Out] 2*arctan(x^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {661, 65, 209}

$$\int \frac{\sqrt{x}}{x+x^2} dx = 2\arctan\left(\sqrt{x}\right)$$

```
[In] Int[Sqrt[x]/(x + x^2),x]
```

```
[Out] 2*ArcTan[Sqrt[x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 661

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{x}(1+x)}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\ &= 2\tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{x+x^2}\,dx = 2\arctan\left(\sqrt{x}\right)$$

```
[In] Integrate[Sqrt[x]/(x + x^2),x]
[Out] 2*ArcTan[Sqrt[x]]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$2\arctan\left(\sqrt{x}\right)$	7
default	$2\arctan\left(\sqrt{x}\right)$	7
meijerg	$2\arctan\left(\sqrt{x}\right)$	7
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{x}+x-1}{x+1}\right)$	29

```
[In] int(x^(1/2)/(x^2+x),x,method=_RETURNVERBOSE)
[Out] 2*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{x+x^2} \, dx = 2 \arctan\left(\sqrt{x}\right)$$

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="fricas")
[Out] 2*arctan(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{x}}{x+x^2} \, dx = 2 \operatorname{atan}\left(\sqrt{x}\right)$$

```
[In] integrate(x**(1/2)/(x**2+x),x)
[Out] 2*atan(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{x+x^2} \, dx = 2 \arctan\left(\sqrt{x}\right)$$

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="maxima")
[Out] 2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{x+x^2} \, dx = 2 \arctan\left(\sqrt{x}\right)$$

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="giac")
[Out] 2*arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{x + x^2} \, dx = 2 \operatorname{atan}\left(\sqrt{x}\right)$$

```
[In] int(x^(1/2)/(x + x^2),x)
[Out] 2*atan(x^(1/2))
```

3.576 $\int \frac{x}{4\sqrt{x}+x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{x}{4\sqrt{x}+x} \, dx = -8\sqrt{x}+x+32\log\left(4+\sqrt{x}\right)$$

```
[Out] x+32*ln(4+x^(1/2))-8*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1598, 272, 45}

$$\int \frac{x}{4\sqrt{x}+x} \, dx = x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

```
[In] Int[x/(4*Sqrt[x] + x),x]
```

```
[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{x}}{4+\sqrt{x}}\,dx \\
&= 2\text{Subst}\left(\int \frac{x^2}{4+x}\,dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \left(-4+x+\frac{16}{4+x}\right)\,dx, x, \sqrt{x}\right) \\
&= -8\sqrt{x}+x+32\log\left(4+\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x}{4\sqrt{x}+x}\,dx = -8\sqrt{x}+x+32\log\left(4+\sqrt{x}\right)$$

```
[In] Integrate[x/(4*Sqrt[x] + x),x]
[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$x+32\ln\left(4+\sqrt{x}\right)-8\sqrt{x}$	16
default	$x+32\ln\left(4+\sqrt{x}\right)-8\sqrt{x}$	16
trager	$x-1-8\sqrt{x}+16\ln\left(8\sqrt{x}+16+x\right)$	20
meijerg	$-\frac{4\sqrt{x}\left(-\frac{3\sqrt{x}}{4}+6\right)}{3}+32\ln\left(1+\frac{\sqrt{x}}{4}\right)$	24

```
[In] int(x/(x+4*x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x+32*ln(4+x^(1/2))-8*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x}{4\sqrt{x}+x}\,dx = x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

```
[In] integrate(x/(x+4*x^(1/2)),x, algorithm="fricas")
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x}{4\sqrt{x}+x}\,dx = -8\sqrt{x}+x+32\log\left(\sqrt{x}+4\right)$$

```
[In] integrate(x/(x+4*x**(1/2)),x)
[Out] -8*sqrt(x) + x + 32*log(sqrt(x) + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x}{4\sqrt{x}+x}\,dx = x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

```
[In] integrate(x/(x+4*x^(1/2)),x, algorithm="maxima")
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x}{4\sqrt{x}+x}\,dx = x - 8\sqrt{x} + 32\,\log\left(\sqrt{x}+4\right)$$

```
[In] integrate(x/(x+4*x^(1/2)),x, algorithm="giac")
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x}{4\sqrt{x}+x}\,dx = x + 32\,\ln\left(\sqrt{x}+4\right) - 8\sqrt{x}$$

```
[In] int(x/(x + 4*x^(1/2)),x)
[Out] x + 32*log(x^(1/2) + 4) - 8*x^(1/2)
```

3.577 $\int \frac{\sqrt{x}}{\sqrt[3]{x}+x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x} \, dx = 2\sqrt{x}+\frac{3\arctan\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}-\frac{3\arctan\left(1+\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}-\frac{3\log\left(1-\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}}+\frac{3\log\left(1+\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}}$$

```
[Out] -3/2*arctan(-1+x^(1/6)*2^(1/2))*2^(1/2)-3/2*arctan(1+x^(1/6)*2^(1/2))*2^(1/
2)-3/4*ln(1+x^(1/3)-x^(1/6)*2^(1/2))*2^(1/2)+3/4*ln(1+x^(1/3)+x^(1/6)*2^(1/
2))*2^(1/2)+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1598, 348, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x} \, dx = \frac{3\arctan\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}-\frac{3\arctan\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}+2\sqrt{x}-\frac{3\log\left(\sqrt[3]{x}-\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\log\left(\sqrt[3]{x}+\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}$$

```
[In] Int[Sqrt[x]/(x^(1/3) + x),x]
```

```
[Out] 2*Sqrt[x] + (3*ArcTan[1 - Sqrt[2]*x^(1/6)])/Sqrt[2] - (3*ArcTan[1 + Sqrt[2]
*x^(1/6)])/Sqrt[2] - (3*Log[1 - Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt[2]) + (
3*Log[1 + Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt[6]{x}}{1+x^{2/3}}\,dx \\
&= 3\text{Subst}\left(\int \frac{x^{5/2}}{1+x^2}\,dx, x, \sqrt[3]{x}\right) \\
&= 2\sqrt{x} - 3\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \sqrt[3]{x}\right) \\
&= 2\sqrt{x} - 6\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right) \\
&= 2\sqrt{x} + 3\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right) - 3\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right) \\
&= 2\sqrt{x} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&\quad - \frac{3}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&\quad - \frac{3\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt[6]{x}\right)}{2\sqrt{2}} - \frac{3\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt[6]{x}\right)}{2\sqrt{2}}
\end{aligned}
$$

$$= 2\sqrt{x} - \frac{3\log\left(1-\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} - \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}} + \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}$$

$$= 2\sqrt{x} + \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}} - \frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx = 2\sqrt{x} - \frac{3\arctan\left(\frac{-1+\sqrt[3]{x}}{\sqrt{2}\sqrt[6]{x}}\right)}{\sqrt{2}} + \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[6]{x}}{1+\sqrt[3]{x}}\right)}{\sqrt{2}}$$

```
[In] Integrate[Sqrt[x]/(x^(1/3) + x),x]
```

```
[Out] 2*Sqrt[x] - (3*ArcTan[(-1 + x^(1/3))/(Sqrt[2]*x^(1/6))])/Sqrt[2] + (3*ArcTa
nh[(Sqrt[2]*x^(1/6))/(1 + x^(1/3))])/Sqrt[2]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

method	result	size
derivativedivides	$2\sqrt{x}-\frac{3\sqrt{2}\left(\ln\left(\frac{1+x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{2}}{1+x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{2}}\right)+2\arctan\left(1+x^{\frac{1}{6}}\sqrt{2}\right)+2\arctan\left(-1+x^{\frac{1}{6}}\sqrt{2}\right)\right)}{4}$	66
default	$2\sqrt{x}-\frac{3\sqrt{2}\left(\ln\left(\frac{1+x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{2}}{1+x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{2}}\right)+2\arctan\left(1+x^{\frac{1}{6}}\sqrt{2}\right)+2\arctan\left(-1+x^{\frac{1}{6}}\sqrt{2}\right)\right)}{4}$	66
meijerg	$2\sqrt{x}-\frac{3\sqrt{x}\left(\frac{\sqrt{2}\,\ln\left(1+x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{2}\right)}{2\sqrt{x}}+\frac{\sqrt{2}\,\arctan\left(\frac{x^{\frac{1}{6}}\sqrt{2}}{2-x^{\frac{1}{6}}\sqrt{2}}\right)}{\sqrt{x}}-\frac{\sqrt{2}\,\ln\left(1+x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{2}\right)}{2\sqrt{x}}+\frac{\sqrt{2}\,\arctan\left(\frac{x^{\frac{1}{6}}\sqrt{2}}{2+x^{\frac{1}{6}}\sqrt{2}}\right)}{\sqrt{x}}\right)}{2}$	112

```
[In] int(x^(1/2)/(x^(1/3)+x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)-3/4*2^(1/2)*(ln((1+x^(1/3)-x^(1/6)*2^(1/2))/(1+x^(1/3)+x^(1/6)*2^
(1/2)))+2*arctan(1+x^(1/6)*2^(1/2))+2*arctan(-1+x^(1/6)*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx = -\left(\frac{3}{4}i-\frac{3}{4}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2x^{\frac{1}{6}}\right) + \left(\frac{3}{4}i+\frac{3}{4}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2x^{\frac{1}{6}}\right) - \left(\frac{3}{4}i+\frac{3}{4}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2x^{\frac{1}{6}}\right) + \left(\frac{3}{4}i-\frac{3}{4}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2x^{\frac{1}{6}}\right) + 2\sqrt{x}$$

```
[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="fricas")
[Out] -(3/4*I - 3/4)*sqrt(2)*log((I + 1)*sqrt(2) + 2*x^(1/6)) + (3/4*I + 3/4)*sqr
t(2)*log(-(I - 1)*sqrt(2) + 2*x^(1/6)) - (3/4*I + 3/4)*sqrt(2)*log((I - 1)*
sqrt(2) + 2*x^(1/6)) + (3/4*I - 3/4)*sqrt(2)*log(-(I + 1)*sqrt(2) + 2*x^(1/
6)) + 2*sqrt(x)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx = \int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx$$

```
[In] integrate(x**(1/2)/(x**(1/3)+x),x)
[Out] Integral(sqrt(x)/(x**(1/3) + x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx = -\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2x^{\frac{1}{6}}\right)\right) - \frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2x^{\frac{1}{6}}\right)\right) + \frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right) - \frac{3}{4}\sqrt{2}\log\left(-\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right) + 2\sqrt{x}$$

```
[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="maxima")
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x^
(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 2*sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx = -\frac{3}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,x^{\frac{1}{6}}\right)\right) - \frac{3}{2}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,x^{\frac{1}{6}}\right)\right) + \frac{3}{4}\,\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right) - \frac{3}{4}\,\sqrt{2}\log\left(-\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right) + 2\,\sqrt{x}$$

```
[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x^
(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx = 2\,\sqrt{x} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^{1/6}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{3}{2}+\frac{3}{2}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^{1/6}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)$$

```
[In] int(x^(1/2)/(x + x^(1/3)),x)
```

```
[Out] 2*x^(1/2) - 2^(1/2)*atan(2^(1/2)*x^(1/6)*(1/2 + 1i/2))*(3/2 + 3i/2) - 2^(1/
2)*atan(2^(1/2)*x^(1/6)*(1/2 - 1i/2))*(3/2 - 3i/2)
```

3.578 $\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}} dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}} dx = -12\sqrt[12]{x}+3\sqrt[3]{x}-\frac{12x^{7/12}}{7}+\frac{6x^{5/6}}{5} - 4\sqrt{3}\arctan\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)+6\log\left(1+\sqrt[12]{x}\right)-2\log\left(1+\sqrt[4]{x}\right)$$

```
[Out] -12*x^(1/12)+3*x^(1/3)-12/7*x^(7/12)+6/5*x^(5/6)+6*ln(1+x^(1/12))-2*ln(1+x^
(1/4))-4*arctan(1/3*(1-2*x^(1/12))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1598, 348, 52, 60, 632, 210, 31}

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}} dx = -4\sqrt{3}\arctan\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)+\frac{6x^{5/6}}{5}-\frac{12x^{7/12}}{7} +3\sqrt[3]{x}-12\sqrt[12]{x}+6\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[4]{x}+1\right)$$

```
[In] Int[x^(1/3)/(x^(1/4) + Sqrt[x]),x]
```

```
[Out] -12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 - 4*Sqrt[3]*ArcT
an[(1 - 2*x^(1/12))/Sqrt[3]] + 6*Log[1 + x^(1/12)] - 2*Log[1 + x^(1/4)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt[12]{x}}{1+\sqrt[4]{x}}\,dx$$

$$= 4\text{Subst}\left(\int \frac{x^{10/3}}{1+x}\,dx, x, \sqrt[4]{x}\right)$$

$$= \frac{6x^{5/6}}{5} - 4\text{Subst}\left(\int \frac{x^{7/3}}{1+x}\,dx, x, \sqrt[4]{x}\right)$$

$$= -\frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 4\text{Subst}\left(\int \frac{x^{4/3}}{1+x}\,dx, x, \sqrt[4]{x}\right)$$

$$= 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 4\text{Subst}\left(\int \frac{\sqrt[3]{x}}{1+x}\,dx, x, \sqrt[4]{x}\right)$$

$$= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 4\text{Subst}\left(\int \frac{1}{x^{2/3}(1+x)}\,dx, x, \sqrt[4]{x}\right)$$

$$= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 2\log\left(1+\sqrt[4]{x}\right) + 6\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[12]{x}\right) + 6\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[12]{x}\right)$$

$$= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 6\log\left(1+\sqrt[12]{x}\right) - 2\log\left(1+\sqrt[4]{x}\right) - 12\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[12]{x}\right)$$

$$= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right) + 6\log\left(1+\sqrt[12]{x}\right) - 2\log\left(1+\sqrt[4]{x}\right)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx = -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 4\sqrt{3}\arctan\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right) + 4\log\left(1+\sqrt[12]{x}\right) - 2\log\left(1-\sqrt[12]{x}+\sqrt[6]{x}\right)$$

```
[In] Integrate[x^(1/3)/(x^(1/4) + Sqrt[x]),x]
[Out] -12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 - 4*Sqrt[3]*ArcT
an[(1 - 2*x^(1/12))/Sqrt[3]] + 4*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12) + x
^(1/6)]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{6x^{\frac{5}{6}}}{5}-\frac{12x^{\frac{7}{12}}}{7}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}+4\ln\left(1+x^{\frac{1}{12}}\right)-2\ln\left(1-x^{\frac{1}{12}}+x^{\frac{1}{6}}\right)+4\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{12}}\right.}{}\right.$
default	$\frac{6x^{\frac{5}{6}}}{5}-\frac{12x^{\frac{7}{12}}}{7}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}+4\ln\left(1+x^{\frac{1}{12}}\right)-2\ln\left(1-x^{\frac{1}{12}}+x^{\frac{1}{6}}\right)+4\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{12}}\right.}{}\right.$
meijerg	$-\frac{3x^{\frac{1}{12}}\left(-182x^{\frac{3}{4}}+260\sqrt{x}-455x^{\frac{1}{4}}+1820\right)}{455}+4x^{\frac{1}{12}}\left(\frac{\ln\left(1+x^{\frac{1}{12}}\right)}{x^{\frac{1}{12}}}-\frac{\ln\left(1-x^{\frac{1}{12}}+x^{\frac{1}{6}}\right)}{2x^{\frac{1}{12}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^{\frac{1}{12}}}{2-x^{\frac{1}{12}}}\right)}{x^{\frac{1}{12}}}\right)$

```
[In] int(x^(1/3)/(x^(1/4)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 6/5*x^(5/6)-12/7*x^(7/12)+3*x^(1/3)-12*x^(1/12)+4*ln(1+x^(1/12))-2*ln(1-x^(
1/12)+x^(1/6))+4*3^(1/2)*arctan(1/3*(2*x^(1/12)-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx = 4\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}x^{\frac{1}{12}}-\frac{1}{3}\,\sqrt{3}\right)+\frac{6}{5}\,x^{\frac{5}{6}}-\frac{12}{7}\,x^{\frac{7}{12}}+3\,x^{\frac{1}{3}}-12\,x^{\frac{1}{12}}-2\,\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\,\log\left(x^{\frac{1}{12}}+1\right)$$

```
[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 4*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/12) - 1/3*sqrt(3)) + 6/5*x^(5/6) - 12/7*x
^(7/12) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x
^(1/12) + 1)
```

Sympy [F]

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx = \int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx$$

```
[In] integrate(x**(1/3)/(x**(1/4)+x**(1/2)),x)
```

```
[Out] Integral(x**(1/3)/(x**(1/4) + sqrt(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx = 4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{\frac{1}{12}}-1\right)\right)+\frac{6}{5}\,x^{\frac{5}{6}}-\frac{12}{7}\,x^{\frac{7}{12}}+3\,x^{\frac{1}{3}} - 12\,x^{\frac{1}{12}}-2\,\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\,\log\left(x^{\frac{1}{12}}+1\right)$$

```
[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12/7*x^(7/12
) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x^(1/12
) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx = 4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{\frac{1}{12}}-1\right)\right)+\frac{6}{5}\,x^{\frac{5}{6}}-\frac{12}{7}\,x^{\frac{7}{12}}+3\,x^{\frac{1}{3}} - 12\,x^{\frac{1}{12}}-2\,\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\,\log\left(x^{\frac{1}{12}}+1\right)$$

```
[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12/7*x^(7/12
) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x^(1/12
) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx = 4\,\ln\left(144\,x^{1/12}+144\right)-\ln\left(18-36\,x^{1/12}+\sqrt{3}\,18\mathrm{i}\right)\,\left(2+\sqrt{3}\,2\mathrm{i}\right) + \ln\left(36\,x^{1/12}-18+\sqrt{3}\,18\mathrm{i}\right)\,\left(-2+\sqrt{3}\,2\mathrm{i}\right)+3\,x^{1/3}+\frac{6\,x^{5/6}}{5}-12\,x^{1/12}-\frac{12\,x^{7/12}}{7}$$

```
[In] int(x^(1/3)/(x^(1/2) + x^(1/4)),x)
```

```
[Out] 4*log(144*x^(1/12) + 144) - log(3^(1/2)*18i - 36*x^(1/12) + 18)*(3^(1/2)*2i
 + 2) + log(3^(1/2)*18i + 36*x^(1/12) - 18)*(3^(1/2)*2i - 2) + 3*x^(1/3) +
(6*x^(5/6))/5 - 12*x^(1/12) - (12*x^(7/12))/7
```

3.579 $\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 119

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}} \, dx = -12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5}+2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2}-\frac{4x^{3/4}}{3}+\frac{6x^{5/6}}{5}-\frac{12x^{11/12}}{11}+x-\frac{12x^{13/12}}{13}+\frac{6x^{7/6}}{7}+12\log\left(1+\sqrt[12]{x}\right)$$

```
[Out] -12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2*
x^(2/3)-4/3*x^(3/4)+6/5*x^(5/6)-12/11*x^(11/12)+x-12/13*x^(13/12)+6/7*x^(7/
6)+12*ln(1+x^(1/12))+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1598, 272, 45}

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}} \, dx = \frac{6x^{7/6}}{7}-\frac{12x^{13/12}}{13}-\frac{12x^{11/12}}{11}+\frac{6x^{5/6}}{5}-\frac{4x^{3/4}}{3}+\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+x+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

```
[In] Int[Sqrt[x]/(x^(1/4) + x^(1/3)),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 - (4*x^(3/4))/3 + (6*x^(5/6))/5 - (12
*x^(11/12))/11 + x - (12*x^(13/12))/13 + (6*x^(7/6))/7 + 12*Log[1 + x^(1/12
)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt[4]{x}}{1+\sqrt[12]{x}}\,dx \\
&= 12\text{Subst}\left(\int \frac{x^{14}}{1+x}\,dx, x, \sqrt[12]{x}\right) \\
&= 12\text{Subst}\left(\int \left(-1+x-x^2+x^3-x^4+x^5-x^6+x^7-x^8+x^9-x^{10}+x^{11}-x^{12}\right.\right. \\
&\qquad \left.\left.+x^{13}+\frac{1}{1+x}\right)dx, x, \sqrt[12]{x}\right) \\
&= -12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5}+2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2} \\
&\quad -\frac{4x^{3/4}}{3}+\frac{6x^{5/6}}{5}-\frac{12x^{11/12}}{11}+x-\frac{12x^{13/12}}{13}+\frac{6x^{7/6}}{7}+12\log\left(1+\sqrt[12]{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$$
$$= \frac{-360360\sqrt[12]{x}+180180\sqrt[6]{x}-120120\sqrt[4]{x}+90090\sqrt[3]{x}-72072x^{5/12}+60060\sqrt{x}-51480x^{7/12}+45045x^{2/3}}{30030}$$
$$+12\log\left(1+\sqrt[12]{x}\right)$$

```
[In] Integrate[Sqrt[x]/(x^(1/4) + x^(1/3)),x]
```

```
[Out] (-360360*x^(1/12) + 180180*x^(1/6) - 120120*x^(1/4) + 90090*x^(1/3) - 72072
*x^(5/12) + 60060*Sqrt[x] - 51480*x^(7/12) + 45045*x^(2/3) - 40040*x^(3/4)
+ 36036*x^(5/6) - 32760*x^(11/12) + 30030*x - 27720*x^(13/12) + 25740*x^(7/
6))/30030 + 12*Log[1 + x^(1/12)]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.64

method	result
derivativedivides	$-12x^{\frac{1}{12}}+6x^{\frac{1}{6}}-4x^{\frac{1}{4}}+3x^{\frac{1}{3}}-\frac{12x^{\frac{5}{12}}}{5}-\frac{12x^{\frac{7}{12}}}{7}+\frac{3x^{\frac{2}{3}}}{2}-\frac{4x^{\frac{3}{4}}}{3}+\frac{6x^{\frac{5}{6}}}{5}-\frac{12x^{\frac{11}{12}}}{11}+x-\frac{12x^{\frac{13}{12}}}{13}+$
default	$-12x^{\frac{1}{12}}+6x^{\frac{1}{6}}-4x^{\frac{1}{4}}+3x^{\frac{1}{3}}-\frac{12x^{\frac{5}{12}}}{5}-\frac{12x^{\frac{7}{12}}}{7}+\frac{3x^{\frac{2}{3}}}{2}-\frac{4x^{\frac{3}{4}}}{3}+\frac{6x^{\frac{5}{6}}}{5}-\frac{12x^{\frac{11}{12}}}{11}+x-\frac{12x^{\frac{13}{12}}}{13}+$
meijerg	$-\frac{x^{\frac{1}{12}}\left(-25740x^{\frac{13}{12}}+27720x-30030x^{\frac{11}{12}}+32760x^{\frac{5}{6}}-36036x^{\frac{3}{4}}+40040x^{\frac{2}{3}}-45045x^{\frac{7}{12}}+51480\sqrt{x}-60060x^{\frac{5}{12}}+72072x^{\frac{1}{3}}-\right.}{30030}$

```
[In] int(x^(1/2)/(x^(1/4)+x^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] -12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2*
x^(2/3)-4/3*x^(3/4)+6/5*x^(5/6)-12/11*x^(11/12)+x-12/13*x^(13/12)+6/7*x^(7/
6)+12*ln(1+x^(1/12))+2*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx=\frac{6}{7}\,(x+7)x^{\frac{1}{6}}-\frac{12}{13}\,(x+13)x^{\frac{1}{12}}+x-\frac{12}{11}\,x^{\frac{11}{12}}+\frac{6}{5}\,x^{\frac{5}{6}}-\frac{4}{3}\,x^{\frac{3}{4}}+\frac{3}{2}\,x^{\frac{2}{3}}-\frac{12}{7}\,x^{\frac{7}{12}}+2\,\sqrt{x}-\frac{12}{5}\,x^{\frac{5}{12}}+3\,x^{\frac{1}{3}}-4\,x^{\frac{1}{4}}+12\,\log\left(x^{\frac{1}{12}}+1\right)$$

```
[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="fricas")
```

```
[Out] 6/7*(x + 7)*x^(1/6) - 12/13*(x + 13)*x^(1/12) + x - 12/11*x^(11/12) + 6/5*x
^(5/6) - 4/3*x^(3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/
12) + 3*x^(1/3) - 4*x^(1/4) + 12*log(x^(1/12) + 1)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$$

```
[In] integrate(x**(1/2)/(x**(1/4)+x**(1/3)),x)
[Out] Integral(sqrt(x)/(x**(1/4) + x**(1/3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \frac{6}{7}x^{\frac{7}{6}} - \frac{12}{13}x^{\frac{13}{12}} + x - \frac{12}{11}x^{\frac{11}{12}} + \frac{6}{5}x^{\frac{5}{6}} - \frac{4}{3}x^{\frac{3}{4}} + \frac{3}{2}x^{\frac{2}{3}} - \frac{12}{7}x^{\frac{7}{12}} + 2\sqrt{x} - \frac{12}{5}x^{\frac{5}{12}} + 3x^{\frac{1}{3}} - 4x^{\frac{1}{4}} + 6x^{\frac{1}{6}} - 12x^{\frac{1}{12}} + 12\log\left(x^{\frac{1}{12}}+1\right)$$

```
[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="maxima")
[Out] 6/7*x^(7/6) - 12/13*x^(13/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6) - 4/3*x^(
3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3)
- 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \frac{6}{7}x^{\frac{7}{6}} - \frac{12}{13}x^{\frac{13}{12}} + x - \frac{12}{11}x^{\frac{11}{12}} + \frac{6}{5}x^{\frac{5}{6}} - \frac{4}{3}x^{\frac{3}{4}} + \frac{3}{2}x^{\frac{2}{3}} - \frac{12}{7}x^{\frac{7}{12}} + 2\sqrt{x} - \frac{12}{5}x^{\frac{5}{12}} + 3x^{\frac{1}{3}} - 4x^{\frac{1}{4}} + 6x^{\frac{1}{6}} - 12x^{\frac{1}{12}} + 12\log\left(x^{\frac{1}{12}}+1\right)$$

```
[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="giac")
[Out] 6/7*x^(7/6) - 12/13*x^(13/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6) - 4/3*x^(
3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3)
- 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = x+12\,\ln\left(x^{1/12}+1\right)+2\sqrt{x}+3\,x^{1/3}-4\,x^{1/4}+\frac{3\,x^{2/3}}{2}+6\,x^{1/6}-\frac{4\,x^{3/4}}{3}+\frac{6\,x^{5/6}}{5}-12\,x^{1/12}+\frac{6\,x^{7/6}}{7}-\frac{12\,x^{5/12}}{5}-\frac{12\,x^{7/12}}{7}-\frac{12\,x^{11/12}}{11}-\frac{12\,x^{13/12}}{13}$$

```
[In] int(x^(1/2)/(x^(1/3) + x^(1/4)),x)
```

```
[Out] x + 12*log(x^(1/12) + 1) + 2*x^(1/2) + 3*x^(1/3) - 4*x^(1/4) + (3*x^(2/3))/
2 + 6*x^(1/6) - (4*x^(3/4))/3 + (6*x^(5/6))/5 - 12*x^(1/12) + (6*x^(7/6))/7
 - (12*x^(5/12))/5 - (12*x^(7/12))/7 - (12*x^(11/12))/11 - (12*x^(13/12))/1
3
```

3.580 $\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 201

$$\begin{aligned}\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}} \, dx = {} & 6\sqrt[6]{x}+x-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) \\ & -\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}+4\sqrt[6]{x}\right)\right) \\ & +\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right) \\ & -\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)\end{aligned}$$

```
[Out] 6*x^(1/6)+x+6/5*ln(1-x^(1/6))-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*
(-5^(1/2)+1)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*(5^(1/2)+1)-3/5*a
rctan(1/20*(1+4*x^(1/6)+5^(1/2))*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2
)-3/5*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/
2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1598, 348, 308, 208, 648, 632, 210, 642, 31}

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx=-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right)+x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)$$

```
[In] Int[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/S
qrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[
5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 -
 Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 + Sqrt
[5])*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r + s*Cos[(2*k
- 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; (r/(a*
n))*Int[1/(r - s*x), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 1)/2}], x],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^{5/6}}{-1+x^{5/6}}\,dx \\
&= 6\text{Subst}\left(\int \frac{x^{10}}{-1+x^5}\,dx, x, \sqrt[6]{x}\right) \\
&= 6\text{Subst}\left(\int \left(1+x^5+\frac{1}{-1+x^5}\right)\,dx, x, \sqrt[6]{x}\right) \\
&= 6\sqrt[6]{x}+x+6\text{Subst}\left(\int \frac{1}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)
\end{aligned}$$

$$= 6\sqrt[6]{x} + x - \frac{6}{5}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[6]{x}\right) - \frac{12}{5}\text{Subst}\left(\int \frac{1+\frac{1}{4}\left(1-\sqrt{5}\right)x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right) - \frac{12}{5}\text{Subst}\left(\int \frac{1+\frac{1}{4}\left(1+\sqrt{5}\right)x}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)$$

$$= 6\sqrt[6]{x} + x + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) - \frac{1}{10}\left(3\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(1-\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right) - \frac{1}{10}\left(3\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right) - \frac{1}{10}\left(3\left(1+\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(1+\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right) - \frac{1}{10}\left(3\left(5+\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)$$

$$= 6\sqrt[6]{x} + x + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) - \frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right) - \frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right) + \frac{1}{5}\left(3\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\left(-5+\sqrt{5}\right)-x^2}\,dx, x, \frac{1}{2}\left(1+\sqrt{5}\right)+2\sqrt[6]{x}\right) + \frac{1}{5}\left(3\left(5+\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\left(-5-\sqrt{5}\right)-x^2}\,dx, x, \frac{1}{2}\left(1-\sqrt{5}\right)+2\sqrt[6]{x}\right)$$

$$= 6\sqrt[6]{x} + x - \frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}+4\sqrt[6]{x}\right)\right) + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) - \frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right) - \frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = 6\sqrt[6]{x}+x+\frac{6}{5}\log\left(-1+\sqrt[6]{x}\right)-\frac{6}{5}\text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\&,\frac{4\log\left(\sqrt[6]{x}-\#1\right)+3\log\left(\sqrt[6]{x}-\#1\right)\#1+2\log\left(\sqrt[6]{x}-\#1\right)\#1^2+\log\left(\sqrt[6]{x}-\#1\right)\#1^3}{1+2\#1+3\#1^2+4\#1^3}\&\right]$$

```
[In] Integrate[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x + (6*Log[-1 + x^(1/6)])/5 - (6*RootSum[1 + #1 + #1^2 + #1^3 +
 #1^4 & , (4*Log[x^(1/6) - #1] + 3*Log[x^(1/6) - #1]*#1 + 2*Log[x^(1/6) - #
1]*#1^2 + Log[x^(1/6) - #1]*#1^3)/(1 + 2*#1 + 3*#1^2 + 4*#1^3) & ])/5
```

Maple [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.66

method	result
meijerg	$\frac{6(-1)^{\frac{4}{5}}\left(-\frac{5x^{\frac{1}{6}}(-1)^{\frac{1}{5}}\left(11x^{\frac{5}{6}}+66\right)}{66}-(-1)^{\frac{1}{5}}\left(\ln\left(1-x^{\frac{1}{6}}\right)+\cos\left(\frac{2\pi}{5}\right)\ln\left(1-2\cos\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)-2\sin\left(\frac{2\pi}{5}\right)\arctan\left(\frac{\sin(}{1-\cos(}\right.\right.\right.}{5}$
derivativedivides	$x+6x^{\frac{1}{6}}+\frac{6\ln\left(x^{\frac{1}{6}}-1\right)}{5}-\frac{3\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{5}\right)\left(-\sqrt{5}+1\right)}{10}-\frac{12\left(4-\frac{\left(-\sqrt{5}+1\right)^2}{4}\right)\arctan\left(\frac{1+4x^{\frac{1}{6}}-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}$
default	$x+6x^{\frac{1}{6}}+\frac{6\ln\left(x^{\frac{1}{6}}-1\right)}{5}-\frac{3\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{5}\right)\left(-\sqrt{5}+1\right)}{10}-\frac{12\left(4-\frac{\left(-\sqrt{5}+1\right)^2}{4}\right)\arctan\left(\frac{1+4x^{\frac{1}{6}}-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}$

```
[In] int(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 6/5*(-1)^(4/5)*(-5/66*x^(1/6)*(-1)^(1/5)*(11*x^(5/6)+66)-(-1)^(1/5)*(ln(1-x
^(1/6))+cos(2/5*Pi)*ln(1-2*cos(2/5*Pi)*x^(1/6)+x^(1/3))-2*sin(2/5*Pi)*arcta
n(sin(2/5*Pi)*x^(1/6)/(1-cos(2/5*Pi)*x^(1/6)))-cos(1/5*Pi)*ln(1+2*cos(1/5*P
i)*x^(1/6)+x^(1/3))-2*sin(1/5*Pi)*arctan(sin(1/5*Pi)*x^(1/6)/(1+cos(1/5*Pi)
*x^(1/6)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(134) = 268.

Time = 0.94 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.72

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = -\frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)\log\left(\frac{3}{2}\sqrt{2}\sqrt{\sqrt{5}-5}+\frac{3}{2}\sqrt{5}+6x^{\frac{1}{6}}+\frac{3}{2}\right) + \frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)\log\left(-\frac{3}{2}\sqrt{2}\sqrt{\sqrt{5}-5}+\frac{3}{2}\sqrt{5}+6x^{\frac{1}{6}}+\frac{3}{2}\right) + \frac{1}{10}\left(3\sqrt{5}-\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1 \cdots\right.}\right. + \sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{2}\cdots\right.} \left. + 12x^{\frac{1}{6}}+3\right) + \frac{1}{10}\left(3\sqrt{5}+\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1 \cdots\right.}\right. - \sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{2}\cdots\right.} \left. + 12x^{\frac{1}{6}}+3\right) + x + 6x^{\frac{1}{6}} + \frac{6}{5}\log\left(x^{\frac{1}{6}}-1\right)$$

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(3/2*sqrt(2)*sqrt(sqrt(5
) - 5) + 3/2*sqrt(5) + 6*x^(1/6) + 3/2) + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) -
 sqrt(5) - 1)*log(-3/2*sqrt(2)*sqrt(sqrt(5) - 5) + 3/2*sqrt(5) + 6*x^(1/6)
+ 3/2) + 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5)
+ 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5
) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 +
18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(-3*sqrt(5) + sqrt(
-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(
5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sq
rt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 1
8*sqrt(5) - 90) + 12*x^(1/6) + 3) + 1/10*(3*sqrt(5) + sqrt(-27/4*(sqrt(2)*s
qrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5
) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90)
```

```
- 3)*log(-3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5
) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sq
rt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) + 12*x^(1/6) + 3) + x + 6*x^(1/6
) + 6/5*log(x^(1/6) - 1)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx=\int \frac{x^{\frac{5}{6}}}{\left(\sqrt[6]{x}-1\right)\left(\sqrt[6]{x}+x^{\frac{2}{3}}+\sqrt[3]{x}+\sqrt{x}+1\right)}\,dx$$

```
[In] integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(x**(5/6)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3) + sqrt(x)
 + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(134) = 268$.

Time = 0.28 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.46

$$\begin{aligned}\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = &-\frac{3\sqrt{5}(-1)^{\frac{1}{5}}\left(\sqrt{5}-1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}\\ &-\frac{3\sqrt{5}(-1)^{\frac{1}{5}}\left(\sqrt{5}+1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right)}{5\sqrt{-2\sqrt{5}-10}}\\ &-\frac{6}{5}(-1)^{\frac{1}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right)+x\\ &-\frac{3\left(\sqrt{5}+3\right)\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{4}{5}}+(-1)^{\frac{4}{5}}\right)}\\ &-\frac{3\left(\sqrt{5}-3\right)\log\left(x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{4}{5}}-(-1)^{\frac{4}{5}}\right)}+6x^{\frac{1}{6}}\end{aligned}$$

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) - 1)*log((sqrt(5)*(-1)^(1/5) + (-1)^(1/5)*
sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) - (-1)^(
1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6)))/sqrt(2*sqrt(5) - 10) -
```

```
 3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) + 1)*log((sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*
sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(-2*sqrt(5) - 10
) - 6/5*(-1)^(1/5)*log((-1)^(1/5) + x^(1/6)) + x - 3/5*(sqrt(5) + 3)*log(-x
^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(
5)*(-1)^(4/5) + (-1)^(4/5)) - 3/5*(sqrt(5) - 3)*log(x^(1/6)*(sqrt(5)*(-1)^(
1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(4/5) - (-1)^(
4/5)) + 6*x^(1/6)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = -\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right) - \frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right) - \frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+1\right) + \frac{3}{10}\sqrt{5}\log\left(-\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+x^{\frac{1}{3}}+1\right) + x + 6x^{\frac{1}{6}} - \frac{3}{10}\log\left(x^{\frac{2}{3}}+\sqrt{x}+x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right) + \frac{6}{5}\log\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -3/5*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2
*sqrt(5) + 10)) - 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
 + 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + x + 6*x^(1/
6) - 3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1
/6) - 1))
```

Mupad [B] (verification not implemented)

Time = 17.04 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = x+\frac{6\,\ln\left(1296\,x^{1/6}-1296\right)}{5}$$
$$-\ln\left(270\,\sqrt{2}\,\sqrt{-\sqrt{5}-5}-270\,\sqrt{5}+1080\,x^{1/6}+270\right)\,\left(\frac{3\,\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}-\frac{3\,\sqrt{5}}{10}+\frac{3}{10}\right)$$
$$+\ln\left(270\,\sqrt{2}\,\sqrt{-\sqrt{5}-5}+270\,\sqrt{5}-1080\,x^{1/6}-270\right)\,\left(\frac{3\,\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}+\frac{3\,\sqrt{5}}{10}-\frac{3}{10}\right)+6\,x^{1/6}-\ln\left(270\right.$$

```
[In] int(x^(1/2)/(x^(1/2) - 1/x^(1/3)),x)
```

```
[Out] x + (6*log(1296*x^(1/6) - 1296))/5 - log(270*2^(1/2)*(- 5^(1/2) - 5)^(1/2)
- 270*5^(1/2) + 1080*x^(1/6) + 270)*((3*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/10 -
 (3*5^(1/2))/10 + 3/10) + log(270*2^(1/2)*(- 5^(1/2) - 5)^(1/2) + 270*5^(1/
2) - 1080*x^(1/6) - 270)*((3*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/10 + (3*5^(1/2)
)/10 - 3/10) + 6*x^(1/6) - log(270*5^(1/2) + 1080*x^(1/6) - 270*2^(1/2)*(5^
(1/2) - 5)^(1/2) + 270)*((3*5^(1/2))/10 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/1
0 + 3/10) - log(270*5^(1/2) + 1080*x^(1/6) + 270*2^(1/2)*(5^(1/2) - 5)^(1/2
) + 270)*((3*5^(1/2))/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)
```

3.581 $\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 36

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx=\frac{2\sqrt{b-\frac{a}{x}}x^{1+m}}{(1+2m)\sqrt{a-bx}}$$

```
[Out] 2*x^(1+m)*(b-a/x)^(1/2)/(1+2*m)/(-b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {529, 23, 30}

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx=\frac{2x^{m+1}\sqrt{b-\frac{a}{x}}}{(2m+1)\sqrt{a-bx}}$$

```
[In] Int[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^(1 + m))/((1 + 2*m)*Sqrt[a - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int\frac{x^{-\frac{1}{2}+m}\sqrt{-a+bx}}{\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int x^{-\frac{1}{2}+m}\,dx}{\sqrt{a-bx}}$$
$$= \frac{2\sqrt{b-\frac{a}{x}}\,x^{1+m}}{(1+2m)\sqrt{a-bx}}$$

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int\frac{\sqrt{b-\frac{a}{x}}\,x^m}{\sqrt{a-bx}}\,dx = \frac{\sqrt{b-\frac{a}{x}}\,x^{1+m}}{\left(\frac{1}{2}+m\right)\sqrt{a-bx}}$$

```
[In] Integrate[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
[Out] (Sqrt[b - a/x]*x^(1 + m))/((1/2 + m)*Sqrt[a - b*x])
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{2x^{1+m}\sqrt{-\frac{-bx+a}{x}}}{(1+2m)\sqrt{-bx+a}}$	36

```
[In] int(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*x^(1+m)/(1+2*m)/(-b*x+a)^(1/2)*(-(-b*x+a)/x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx = \frac{2\sqrt{-bx+a}xx^m\sqrt{\frac{bx-a}{x}}}{2\,am-(2\,bm+b)x+a}$$

```
[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(-b*x + a)*x*x^m*sqrt((b*x - a)/x)/(2*a*m - (2*b*m + b)*x + a)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx = \int \frac{x^m\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

```
[In] integrate(x**m*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
[Out] Integral(x**m*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx = \frac{2\sqrt{x}x^m}{2i\,m+i}$$

```
[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x)*x^m/(2*I*m + I)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 17.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx=\frac{2\,x^{m+1}\,\sqrt{b-\frac{a}{x}}}{(2\,m+1)\,\sqrt{a-b\,x}}$$

```
[In] int((x^m*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
[Out] (2*x^(m + 1)*(b - a/x)^(1/2))/((2*m + 1)*(a - b*x)^(1/2))
```

3.582 $\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = \frac{2\sqrt{b-\frac{a}{x}}x^3}{5\sqrt{a-bx}}$$

```
[Out] 2/5*x^3*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {529, 23, 30}

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = \frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

```
[In] Int[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
[Out] (2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int\frac{x^{3/2}\sqrt{-a+bx}}{\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}\\ &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int x^{3/2}\,dx}{\sqrt{a-bx}}\\ &= \frac{2\sqrt{b-\frac{a}{x}}x^3}{5\sqrt{a-bx}}\end{aligned}$$

Mathematica [A] (verified)

Time = 8.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int\frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = -\frac{2x^2\sqrt{a-bx}}{5\sqrt{b-\frac{a}{x}}}$$

```
[In] Integrate[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
[Out] (-2*x^2*Sqrt[a - b*x])/(5*Sqrt[b - a/x])
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{2x^3\sqrt{-\frac{-bx+a}{x}}}{5\sqrt{-bx+a}}$	27
default	$\frac{2x^3\sqrt{-\frac{-bx+a}{x}}}{5\sqrt{-bx+a}}$	27
risch	$-\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-(-bx+a)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)x^3}{5(-bx+a)^{\frac{3}{2}}\sqrt{(bx-a)x}\,\sqrt{-x}}$	80

```
[In] int(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*x^3*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = -\frac{2\sqrt{-bx+a}x^3\sqrt{\frac{bx-a}{x}}}{5\,(bx-a)}$$

```
[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2/5*sqrt(-b*x + a)*x^3*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = \int \frac{x^2\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

```
[In] integrate(x**2*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
[Out] Integral(x**2*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = -\frac{2}{5}i\,x^{\frac{5}{2}}$$

```
[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] -2/5*I*x^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(23) = 46.

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.62

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = \frac{2\sqrt{-ab}a^2|b|\mathrm{sgn}(x)}{5\,b^4} - \frac{2\left(\sqrt{-ab}a^2 - \frac{((bx-a)b+ab)^2\sqrt{-(bx-a)b-ab}}{b^2}\right)|b|\mathrm{sgn}(x)}{5\,b^4}$$

```
[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] 2/5*sqrt(-a*b)*a^2*abs(b)*sgn(x)/b^4 - 2/5*(sqrt(-a*b)*a^2 - ((b*x - a)*b +
 a*b)^2*sqrt(-(b*x - a)*b - a*b)/b^2)*abs(b)*sgn(x)/b^4
```

Mupad [B] (verification not implemented)

Time = 16.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx=\frac{2\,x^3\,\sqrt{b-\frac{a}{x}}}{5\,\sqrt{a-b\,x}}$$

```
[In] int((x^2*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
[Out] (2*x^3*(b - a/x)^(1/2))/(5*(a - b*x)^(1/2))
```

3.583 $\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 29

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx=\frac{2\sqrt{b-\frac{a}{x}}x^2}{3\sqrt{a-bx}}$$

```
[Out] 2/3*x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {529, 23, 30}

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx=\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

```
[In] Int[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{\sqrt{x}\sqrt{-a+bx}}{\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \sqrt{x}\,dx}{\sqrt{a-bx}}$$
$$= \frac{2\sqrt{b-\frac{a}{x}}x^2}{3\sqrt{a-bx}}$$

Mathematica [A] (verified)

Time = 5.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx = -\frac{2x\sqrt{a-bx}}{3\sqrt{b-\frac{a}{x}}}$$

```
[In] Integrate[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
[Out] (-2*x*Sqrt[a - b*x])/(3*Sqrt[b - a/x])
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{2x^2\sqrt{-\frac{-bx+a}{x}}}{3\sqrt{-bx+a}}$	27
default	$\frac{2x^2\sqrt{-\frac{-bx+a}{x}}}{3\sqrt{-bx+a}}$	27
risch	$-\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-(-bx+a)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)x^2}{3(-bx+a)^{\frac{3}{2}}\,\sqrt{(bx-a)x}\,\sqrt{-x}}$	80

```
[In] int(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*x^2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx = -\frac{2\sqrt{-bx+a}x^2\sqrt{\frac{bx-a}{x}}}{3(bx-a)}$$

```
[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(-b*x + a)*x^2*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx = \int \frac{x\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

```
[In] integrate(x*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
[Out] Integral(x*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx = -\frac{2}{3}i\,x^{\frac{3}{2}}$$

```
[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] -2/3*I*x^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(23) = 46.

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.93

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx = \frac{2\sqrt{-ab}a|b|\mathrm{sgn}(x)}{3b^3} - \frac{2\left(\sqrt{-ab}a + \frac{(-(bx-a)b-ab)^{\frac{3}{2}}}{b}\right)|b|\mathrm{sgn}(x)}{3b^3}$$

```
[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(-a*b)*a*abs(b)*sgn(x)/b^3 - 2/3*(sqrt(-a*b)*a + (-(b*x - a)*b - a*
b)^(3/2)/b)*abs(b)*sgn(x)/b^3
```

Mupad [B] (verification not implemented)

Time = 17.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx=\frac{2\,x^2\,\sqrt{b-\frac{a}{x}}}{3\,\sqrt{a-b\,x}}$$

```
[In] int((x*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
[Out] (2*x^2*(b - a/x)^(1/2))/(3*(a - b*x)^(1/2))
```

3.584 $\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}} dx$

Optimal result

Integrand size = 23, antiderivative size = 25

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}} dx = \frac{2\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}$$

```
[Out] 2*x*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {446, 23, 30}

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}} dx = \frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

```
[In] Int[Sqrt[b - a/x]/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 446

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart[
q]), Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int\frac{\sqrt{-a+bx}}{\sqrt{x}\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}\\ &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int\frac{1}{\sqrt{x}}\,dx}{\sqrt{a-bx}}\\ &= \frac{2\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\end{aligned}$$

Mathematica [A] (verified)

Time = 4.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int\frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx = -\frac{2\sqrt{a-bx}}{\sqrt{b-\frac{a}{x}}}$$

```
[In] Integrate[Sqrt[b - a/x]/Sqrt[a - b*x],x]
[Out] (-2*Sqrt[a - b*x])/Sqrt[b - a/x]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{2x\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$	25
default	$\frac{2x\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$	25
risch	$-\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-(-bx+a)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)x}{(-bx+a)^{\frac{3}{2}}\sqrt{(bx-a)x}\,\sqrt{-x}}$	78

```
[In] int((b-a/x)^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*x*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx = -\frac{2\sqrt{-bx+a}\,x\sqrt{\frac{bx-a}{x}}}{bx-a}$$

```
[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-b*x + a)*x*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx = \int \frac{\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

```
[In] integrate((b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
[Out] Integral(sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx = -2i\sqrt{x}$$

```
[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] -2*I*sqrt(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(21) = 42$.

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx = \frac{2\left(\sqrt{-(bx-a)b-ab}-\sqrt{-ab}\right)|b|\mathrm{sgn}(x)}{b^2} + \frac{2\sqrt{-ab}|b|\mathrm{sgn}(x)}{b^2}$$

```
[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] 2*(sqrt(-(b*x - a)*b - a*b) - sqrt(-a*b))*abs(b)*sgn(x)/b^2 + 2*sqrt(-a*b)*
abs(b)*sgn(x)/b^2
```

Mupad [B] (verification not implemented)

Time = 17.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx=\frac{2\,x\,\sqrt{b-\frac{a}{x}}}{\sqrt{a-b\,x}}$$

```
[In] int((b - a/x)^(1/2)/(a - b*x)^(1/2),x)
[Out] (2*x*(b - a/x)^(1/2))/(a - b*x)^(1/2)
```

3.585 $\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 24

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx=-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

```
[Out] -2*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {529, 23, 30}

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx=-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

```
[In] Int[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int\frac{\sqrt{-a+bx}}{x^{3/2}\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}\\&=\frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int\frac{1}{x^{3/2}}\,dx}{\sqrt{a-bx}}\\&=-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int\frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx=-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

```
[In] Integrate[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]
[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$	24
default	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$	24
risch	$\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-(-bx+a)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)}{(-bx+a)^{\frac{3}{2}}\sqrt{(bx-a)x}\,\sqrt{-x}}$	77

```
[In] int((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = \frac{2\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{bx-a}$$

```
[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = \int \frac{\sqrt{-\frac{a}{x}+b}}{x\sqrt{a-bx}}\,dx$$

```
[In] integrate((b-a/x)**(1/2)/x/(-b*x+a)**(1/2),x)
[Out] Integral(sqrt(-a/x + b)/(x*sqrt(a - b*x)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = \frac{2i}{\sqrt{x}}$$

```
[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2*I/sqrt(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = \frac{2\left(\frac{b^3}{\sqrt{-(bx-a)b-ab}}-\frac{b^3}{\sqrt{-ab}}\right)|b|\mathrm{sgn}(x)}{b^3}$$

```
[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] 2*(b^3/sqrt(-(b*x - a)*b - a*b) - b^3/sqrt(-a*b))*abs(b)*sgn(x)/b^3
```

Mupad [B] (verification not implemented)

Time = 16.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = -\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-b\,x}}$$

```
[In] int((b - a/x)^(1/2)/(x*(a - b*x)^(1/2)),x)
[Out] -(2*(b - a/x)^(1/2))/(a - b*x)^(1/2)
```

3.586 $\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx=-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

```
[Out] -2/3*(b-a/x)^(1/2)/x/(-b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {529, 23, 30}

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx=-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

```
[In] Int[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]
[Out] (-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int\frac{\sqrt{-a+bx}}{x^{5/2}\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}\\ &= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int\frac{1}{x^{5/2}}\,dx}{\sqrt{a-bx}}\\ &= -\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}\end{aligned}$$

Mathematica [A] (verified)

Time = 3.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int\frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \frac{2\left(b-\frac{a}{x}\right)^{3/2}}{3(a-bx)^{3/2}}$$

```
[In] Integrate[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]
[Out] (2*(b - a/x)^(3/2))/(3*(a - b*x)^(3/2))
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{3x\sqrt{-bx+a}}$	27
default	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{3x\sqrt{-bx+a}}$	27
risch	$\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-(-bx+a)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)}{3(-bx+a)^{\frac{3}{2}}\sqrt{(bx-a)x}\,\sqrt{-x}\,x}$	80

```
[In] int((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(-(-b*x+a)/x)^(1/2)/x/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \frac{2\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{3\,(bx^2-ax)}$$

```
[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x^2 - a*x)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \int \frac{\sqrt{-\frac{a}{x}+b}}{x^2\sqrt{a-bx}}\,dx$$

```
[In] integrate((b-a/x)**(1/2)/x**2/(-b*x+a)**(1/2),x)
[Out] Integral(sqrt(-a/x + b)/(x**2*sqrt(a - b*x)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \frac{2i}{3\,x^{\frac{3}{2}}}$$

```
[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/3*I/x^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(23) = 46.

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \frac{2\left(\frac{b^5}{((bx-a)b+ab)\sqrt{-(bx-a)b-ab}}-\frac{b^4}{\sqrt{-ab}a}\right)|b|\mathrm{sgn}(x)}{3\,b^3}$$

```
[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] 2/3*(b^5/(((b*x - a)*b + a*b)*sqrt(-(b*x - a)*b - a*b)) - b^4/(sqrt(-a*b)*a
))*abs(b)*sgn(x)/b^3
```

Mupad [B] (verification not implemented)

Time = 16.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = -\frac{2\,\sqrt{b-\frac{a}{x}}}{3\,x\,\sqrt{a-b\,x}}$$

```
[In] int((b - a/x)^(1/2)/(x^2*(a - b*x)^(1/2)),x)
[Out] -(2*(b - a/x)^(1/2))/(3*x*(a - b*x)^(1/2))
```

3.587 $\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx = \frac{\left(a+\frac{b}{x}\right)^m x\left(1+\frac{ax}{b}\right)^{-m} (c+dx)^n \left(1+\frac{dx}{c}\right)^{-n} \text{AppellF1}\left(1-m,-m,-n,2-m,-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}$$

```
[Out] (a+b/x)^m*x*(d*x+c)^n*AppellF1(1-m,-m,-n,2-m,-a*x/b,-d*x/c)/(1-m)/((1+a*x/b
)^m)/((1+d*x/c)^n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {446, 140, 138}

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx = \frac{x\left(a+\frac{b}{x}\right)^m \left(\frac{ax}{b}+1\right)^{-m} (c+dx)^n \left(\frac{dx}{c}+1\right)^{-n} \text{AppellF1}\left(1-m,-m,-n,2-m,-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}$$

```
[In] Int[(a + b/x)^m*(c + d*x)^n,x]
```

```
[Out] ((a + b/x)^m*x*(c + d*x)^n*AppellF1[1 - m, -m, -n, 2 - m, -((a*x)/b), -((d*
x)/c)])/((1 - m)*(1 + (a*x)/b)^m*(1 + (d*x)/c)^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
```

```
m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 446

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart[
q]), Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\left(a+\frac{b}{x}\right)^m x^m (b+ax)^{-m}\right)\int x^{-m}(b+ax)^m (c+dx)^n\,dx\\
&= \left(\left(a+\frac{b}{x}\right)^m x^m \left(1+\frac{ax}{b}\right)^{-m}\right)\int x^{-m}\left(1+\frac{ax}{b}\right)^m (c+dx)^n\,dx\\
&= \left(\left(a+\frac{b}{x}\right)^m x^m \left(1+\frac{ax}{b}\right)^{-m}(c+dx)^n\left(1+\frac{dx}{c}\right)^{-n}\right)\int x^{-m}\left(1+\frac{ax}{b}\right)^m\left(1+\frac{dx}{c}\right)^n dx\\
&= \frac{\left(a+\frac{b}{x}\right)^m x\left(1+\frac{ax}{b}\right)^{-m}(c+dx)^n\left(1+\frac{dx}{c}\right)^{-n}F_1\left(1-m;-m,-n;2-m;-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}
\end{aligned}$$

Mathematica [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx = \int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$$

```
[In] Integrate[(a + b/x)^m*(c + d*x)^n,x]
[Out] Integrate[(a + b/x)^m*(c + d*x)^n, x]
```

Maple [F]

$$\int \left(a+\frac{b}{x}\right)^m (dx+c)^n\,dx$$

```
[In] int((a+b/x)^m*(d*x+c)^n,x)
[Out] int((a+b/x)^m*(d*x+c)^n,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx=\int (dx+c)^n\left(a+\frac{b}{x}\right)^m\,dx$$

```
[In] integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="fricas")
[Out] integral((d*x + c)^n*((a*x + b)/x)^m, x)
```

Sympy [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx=\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$$

```
[In] integrate((a+b/x)**m*(d*x+c)**n,x)
[Out] Integral((a + b/x)**m*(c + d*x)**n, x)
```

Maxima [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx=\int (dx+c)^n\left(a+\frac{b}{x}\right)^m\,dx$$

```
[In] integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="maxima")
[Out] integrate((d*x + c)^n*(a + b/x)^m, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx=\int (dx+c)^n\left(a+\frac{b}{x}\right)^m\,dx$$

```
[In] integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="giac")
[Out] integrate((d*x + c)^n*(a + b/x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx=\int \left(a+\frac{b}{x}\right)^m (c+d\,x)^n\,dx$$

```
[In] int((a + b/x)^m*(c + d*x)^n,x)
[Out] int((a + b/x)^m*(c + d*x)^n, x)
```

3.588 $\int \left(a+\frac{b}{x}\right)^m (c+dx)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 138

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2 \, dx = \frac{d(6ac-bd(2-m))\left(a+\frac{b}{x}\right)^{1+m} x^2}{6a^2} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+m} x^3}{3a} - \frac{b(6a^2c^2-6abcd(1-m)+b^2d^2(2-3m+m^2))\left(a+\frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left(2,1+m,2+m,1+\frac{b}{ax}\right)}{6a^4(1+m)}$$

```
[Out] 1/6*d*(6*a*c-b*d*(2-m))*(a+b/x)^(1+m)*x^2/a^2+1/3*d^2*(a+b/x)^(1+m)*x^3/a-1
/6*b*(6*a^2*c^2-6*a*b*c*d*(1-m)+b^2*d^2*(m^2-3*m+2))*(a+b/x)^(1+m)*hypergeo
m([2, 1+m],[2+m],1+b/a/x)/a^4/(1+m)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {445, 457, 91, 79, 67}

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2 \, dx = \frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(2-m))}{6a^2} - \frac{b\left(a+\frac{b}{x}\right)^{m+1}(6a^2c^2-6abcd(1-m)+b^2d^2(m^2-3m+2)) \text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{b}{ax}+1\right)}{6a^4(m+1)} + \frac{d^2x^3\left(a+\frac{b}{x}\right)^{m+1}}{3a}$$

```
[In] Int[(a + b/x)^m*(c + d*x)^2,x]
```

```
[Out] (d*(6*a*c - b*d*(2 - m))*(a + b/x)^(1 + m)*x^2)/(6*a^2) + (d^2*(a + b/x)^(1
 + m)*x^3)/(3*a) - (b*(6*a^2*c^2 - 6*a*b*c*d*(1 - m) + b^2*d^2*(2 - 3*m + m
```

^2))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(6*
a^4*(1 + m))

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a+\frac{b}{x}\right)^m \left(d+\frac{c}{x}\right)^2 x^2\,dx \\
&= -\text{Subst}\left(\int \frac{(a+bx)^m(d+cx)^2}{x^4}\,dx, x, \frac{1}{x}\right) \\
&= \frac{d^2\left(a+\frac{b}{x}\right)^{1+m} x^3}{3a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^m(d(6ac-bd(2-m))+3ac^2x)}{x^3}\,dx, x, \frac{1}{x}\right)}{3a} \\
&= \frac{d(6ac-bd(2-m))\left(a+\frac{b}{x}\right)^{1+m} x^2}{6a^2} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+m} x^3}{3a} \\
&\quad - \frac{1}{6}\left(6c^2 - \frac{bd(6ac-bd(2-m))(1-m)}{a^2}\right)\text{Subst}\left(\int \frac{(a+bx)^m}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{d(6ac-bd(2-m))\left(a+\frac{b}{x}\right)^{1+m} x^2}{6a^2} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+m} x^3}{3a} \\
&\quad - \frac{b(6a^2c^2-6abcd(1-m)+b^2d^2(2-3m+m^2))\left(a+\frac{b}{x}\right)^{1+m} {}_2F_1\left(2,1+m;2+m;1+\frac{b}{ax}\right)}{6a^4(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.81

$$
\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx
$$

$$
= \frac{\left(a+\frac{b}{x}\right)^m (b+ax)\left(a^2d(1+m)x^2(bd(-2+m)+2a(3c+dx)) - b(6a^2c^2+6abcd(-1+m)+b^2d^2(2-3m\right.}{6a^4(1+m)x}
$$

```
[In] Integrate[(a + b/x)^m*(c + d*x)^2,x]
```

```
[Out] ((a + b/x)^m*(b + a*x)*(a^2*d*(1 + m)*x^2*(b*d*(-2 + m) + 2*a*(3*c + d*x))
- b*(6*a^2*c^2 + 6*a*b*c*d*(-1 + m) + b^2*d^2*(2 - 3*m + m^2))*Hypergeometr
ic2F1[2, 1 + m, 2 + m, 1 + b/(a*x)]))/(6*a^4*(1 + m)*x)
```

Maple [F]

$$\int \left(a+\frac{b}{x}\right)^m (dx+c)^2\,dx$$

```
[In] int((a+b/x)^m*(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^m*(d*x+c)^2,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx = \int (dx+c)^2\left(a+\frac{b}{x}\right)^m\,dx$$

```
[In] integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*((a*x + b)/x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.74 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx = \frac{b^m c^2 x^{1-m}\Gamma(1-m)\,{}_2F_1\left(\begin{matrix}-m,1-m\\2-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)} + \frac{2b^m cdx^{2-m}\Gamma(2-m)\,{}_2F_1\left(\begin{matrix}-m,2-m\\3-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(3-m)} + \frac{b^m d^2 x^{3-m}\Gamma(3-m)\,{}_2F_1\left(\begin{matrix}-m,3-m\\4-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(4-m)}$$

```
[In] integrate((a+b/x)**m*(d*x+c)**2,x)
```

```
[Out] b**m*c**2*x**(1 - m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_pola
r(I*pi)/b)/gamma(2 - m) + 2*b**m*c*d*x**(2 - m)*gamma(2 - m)*hyper((-m, 2 -
 m), (3 - m,), a*x*exp_polar(I*pi)/b)/gamma(3 - m) + b**m*d**2*x**(3 - m)*g
amma(3 - m)*hyper((-m, 3 - m), (4 - m,), a*x*exp_polar(I*pi)/b)/gamma(4 - m
)
```

Maxima [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2 \, dx = \int (dx+c)^2 \left(a+\frac{b}{x}\right)^m \, dx$$

```
[In] integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^2*(a + b/x)^m, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2 \, dx = \int (dx+c)^2 \left(a+\frac{b}{x}\right)^m \, dx$$

```
[In] integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^2*(a + b/x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2 \, dx = \int \left(a+\frac{b}{x}\right)^m (c+d\,x)^2 \, dx$$

```
[In] int((a + b/x)^m*(c + d*x)^2,x)
[Out] int((a + b/x)^m*(c + d*x)^2, x)
```

3.589 $\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx = \frac{d\left(a+\frac{b}{x}\right)^{1+m} x^2}{2a} - \frac{b(2ac-bd(1-m))\left(a+\frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left(2,1+m,2+m,1+\frac{b}{ax}\right)}{2a^3(1+m)}$$

```
[Out] 1/2*d*(a+b/x)^(1+m)*x^2/a-1/2*b*(2*a*c-b*d*(1-m))*(a+b/x)^(1+m)*hypergeom([
2, 1+m],[2+m],1+b/a/x)/a^3/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {445, 457, 79, 67}

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx = \frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}}{2a} - \frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(1-m))\,\text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{b}{ax}+1\right)}{2a^3(m+1)}$$

```
[In] Int[(a + b/x)^m*(c + d*x),x]
```

```
[Out] (d*(a + b/x)^(1 + m)*x^2)/(2*a) - (b*(2*a*c - b*d*(1 - m))*(a + b/x)^(1 + m
)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(2*a^3*(1 + m))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a+\frac{b}{x}\right)^m \left(d+\frac{c}{x}\right) x\,dx \\
&= -\text{Subst}\left(\int \frac{(a+bx)^m(d+cx)}{x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{d\left(a+\frac{b}{x}\right)^{1+m} x^2}{2a} - \frac{(2ac+bd(-1+m))\text{Subst}\left(\int \frac{(a+bx)^m}{x^2}\,dx, x, \frac{1}{x}\right)}{2a} \\
&= \frac{d\left(a+\frac{b}{x}\right)^{1+m} x^2}{2a} - \frac{b(2ac-bd(1-m))\left(a+\frac{b}{x}\right)^{1+m} {}_2F_1\left(2,1+m;2+m;1+\frac{b}{ax}\right)}{2a^3(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx = \frac{\left(a+\frac{b}{x}\right)^m (b+ax)\left(a^2d(1+m)x^2+b(-2ac-bd(-1+m))\,\text{Hypergeometric2F1}\left(2,1+m,2+m,1+\frac{b}{ax}\right)\right)}{2a^3(1+m)x}$$

```
[In] Integrate[(a + b/x)^m*(c + d*x),x]
```

```
[Out] ((a + b/x)^m*(b + a*x)*(a^2*d*(1 + m)*x^2 + b*(-2*a*c - b*d*(-1 + m))*Hyper
geometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)]))/(2*a^3*(1 + m)*x)
```

Maple [F]

$$\int \left(a+\frac{b}{x}\right)^m (dx+c)\,dx$$

```
[In] int((a+b/x)^m*(d*x+c),x)
```

```
[Out] int((a+b/x)^m*(d*x+c),x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx = \int (dx+c)\left(a+\frac{b}{x}\right)^m\,dx$$

```
[In] integrate((a+b/x)^m*(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*((a*x + b)/x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx = \frac{b^m c x^{1-m}\Gamma(1-m)\,{}_2F_1\left(\begin{matrix}-m,1-m\\2-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)} + \frac{b^m d x^{2-m}\Gamma(2-m)\,{}_2F_1\left(\begin{matrix}-m,2-m\\3-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(3-m)}$$

```
[In] integrate((a+b/x)**m*(d*x+c),x)
```

```
[Out] b**m*c*x**(1 - m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_polar(I
*pi)/b)/gamma(2 - m) + b**m*d*x**(2 - m)*gamma(2 - m)*hyper((-m, 2 - m), (3
 - m,), a*x*exp_polar(I*pi)/b)/gamma(3 - m)
```

Maxima [F]

$$\int \left(a + \frac{b}{x}\right)^m (c + dx)\, dx = \int (dx + c)\left(a + \frac{b}{x}\right)^m\, dx$$

```
[In] integrate((a+b/x)^m*(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*(a + b/x)^m, x)
```

Giac [F]

$$\int \left(a + \frac{b}{x}\right)^m (c + dx)\, dx = \int (dx + c)\left(a + \frac{b}{x}\right)^m\, dx$$

```
[In] integrate((a+b/x)^m*(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(a + b/x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a + \frac{b}{x}\right)^m (c + dx)\, dx = \int \left(a + \frac{b}{x}\right)^m (c + d\,x)\, dx$$

```
[In] int((a + b/x)^m*(c + d*x),x)
```

```
[Out] int((a + b/x)^m*(c + d*x), x)
```

3.590 $\int \left(a+\frac{b}{x}\right)^m \, dx$

Optimal result

Integrand size = 9, antiderivative size = 40

$$\int \left(a+\frac{b}{x}\right)^m \, dx = -\frac{b\left(a+\frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left(2,1+m,2+m,1+\frac{b}{ax}\right)}{a^2(1+m)}$$

```
[Out] -b*(a+b/x)^(1+m)*hypergeom([2, 1+m],[2+m],1+b/a/x)/a^2/(1+m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {248, 67}

$$\int \left(a+\frac{b}{x}\right)^m \, dx = -\frac{b\left(a+\frac{b}{x}\right)^{m+1} \text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{b}{ax}+1\right)}{a^2(m+1)}$$

```
[In] Int[(a + b/x)^m,x]
```

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(a^
2*(1 + m)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{(a+bx)^m}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{b\left(a+\frac{b}{x}\right)^{1+m}\,{}_2F_1\left(2,1+m;2+m;1+\frac{b}{ax}\right)}{a^2(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \left(a+\frac{b}{x}\right)^m dx = -\frac{\left(a+\frac{b}{x}\right)^m x\left(1+\frac{ax}{b}\right)^{-m} \text{Hypergeometric2F1}\left(1-m,-m,2-m,-\frac{ax}{b}\right)}{-1+m}$$

```
[In] Integrate[(a + b/x)^m,x]
[Out] -(((a + b/x)^m*x*Hypergeometric2F1[1 - m, -m, 2 - m, -((a*x)/b)])/((-1 + m)
*(1 + (a*x)/b)^m))
```

Maple [F]

$$\int \left(a+\frac{b}{x}\right)^m dx$$

```
[In] int((a+b/x)^m,x)
[Out] int((a+b/x)^m,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^m dx = \int \left(a+\frac{b}{x}\right)^m dx$$

```
[In] integrate((a+b/x)^m,x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^m dx = \frac{b^m x^{1-m}\Gamma(1-m)\, {}_2F_1\left(\begin{matrix} -m, 1-m \\ 2-m \end{matrix} \middle| \frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)}$$

```
[In] integrate((a+b/x)**m,x)
```

```
[Out] b**m*x**(1 - m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_polar(I*p
i)/b)/gamma(2 - m)
```

Maxima [F]

$$\int \left(a+\frac{b}{x}\right)^m dx = \int \left(a+\frac{b}{x}\right)^m dx$$

```
[In] integrate((a+b/x)^m,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x}\right)^m dx = \int \left(a+\frac{b}{x}\right)^m dx$$

```
[In] integrate((a+b/x)^m,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m, x)
```

Mupad [B] (verification not implemented)

Time = 16.89 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \left(a+\frac{b}{x}\right)^m dx = -\frac{x\left(a+\frac{b}{x}\right)^m {}_2\mathrm{F}_1\left(1-m,-m;\ 2-m;\ -\frac{a\,x}{b}\right)}{\left(\frac{a\,x}{b}+1\right)^m (m-1)}$$

```
[In] int((a + b/x)^m,x)
```

```
[Out] -(x*(a + b/x)^m*hypergeom([1 - m, -m], 2 - m, -(a*x)/b))/(((a*x)/b + 1)^m*(
m - 1))
```

3.591 $\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx} \, dx = -\frac{c\left(a+\frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(ac-bd)(1+m)} + \frac{\left(a+\frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,1+\frac{b}{ax}\right)}{ad(1+m)}$$

```
[Out] -c*(a+b/x)^(1+m)*hypergeom([1, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/d/(a*c-b*d)/
(1+m)+(a+b/x)^(1+m)*hypergeom([1, 1+m],[2+m],1+b/a/x)/a/d/(1+m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {445, 457, 88, 67, 70}

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx} \, dx = \frac{\left(a+\frac{b}{x}\right)^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{b}{ax}+1\right)}{ad(m+1)} - \frac{c\left(a+\frac{b}{x}\right)^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(m+1)(ac-bd)}$$

```
[In] Int[(a + b/x)^m/(c + d*x),x]
```

```
[Out] -((c*(a + b/x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(a + b/x))/(a*
c - b*d)])/(d*(a*c - b*d)*(1 + m))) + ((a + b/x)^(1 + m)*Hypergeometric2F1[
1, 1 + m, 2 + m, 1 + b/(a*x)])/(a*d*(1 + m))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)x}\,dx \\
&= -\text{Subst}\left(\int \frac{(a+bx)^m}{x(d+cx)}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\text{Subst}\left(\int \frac{(a+bx)^m}{x}\,dx, x, \frac{1}{x}\right)}{d} + \frac{c\text{Subst}\left(\int \frac{(a+bx)^m}{d+cx}\,dx, x, \frac{1}{x}\right)}{d} \\
&= -\frac{c\left(a+\frac{b}{x}\right)^{1+m} {}_2F_1\left(1,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(ac-bd)(1+m)} + \frac{\left(a+\frac{b}{x}\right)^{1+m} {}_2F_1\left(1,1+m;2+m;1+\frac{b}{ax}\right)}{ad(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx = \frac{\left(a+\frac{b}{x}\right)^m(b+ax)\left(ac\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(-ac+bd)\,\text{Hypergeometric2F1}\left(\right.\right.}{ad(-ac+bd)(1+m)x}$$

```
[In] Integrate[(a + b/x)^m/(c + d*x),x]
```

```
[Out] ((a + b/x)^m*(b + a*x)*(a*c*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(a + b/x)
)/(a*c - b*d)] + (-(a*c) + b*d)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + b/(a
*x)]))/(a*d*(-(a*c) + b*d)*(1 + m)*x)
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{dx+c}dx$$

```
[In] int((a+b/x)^m/(d*x+c),x)
```

```
[Out] int((a+b/x)^m/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{dx+c}\,dx$$

```
[In] integrate((a+b/x)^m/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx$$

```
[In] integrate((a+b/x)**m/(d*x+c),x)
```

```
[Out] Integral((a + b/x)**m/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{dx+c}\,dx$$

```
[In] integrate((a+b/x)^m/(d*x+c),x, algorithm="maxima")
[Out] integrate((a + b/x)^m/(d*x + c), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{dx+c}\,dx$$

```
[In] integrate((a+b/x)^m/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^m/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{c+d\,x}\,dx$$

```
[In] int((a + b/x)^m/(c + d*x),x)
[Out] int((a + b/x)^m/(c + d*x), x)
```

3.592 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2} \, dx = -\frac{b\left(a+\frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left(2,1+m,2+m,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+m)}$$

```
[Out] -b*(a+b/x)^(1+m)*hypergeom([2, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/(a*c-b*d)^2/
(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {445, 455, 70}

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2} \, dx = -\frac{b\left(a+\frac{b}{x}\right)^{m+1} \text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(m+1)(ac-bd)^2}$$

```
[In] Int[(a + b/x)^m/(c + d*x)^2,x]
```

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a*
c - b*d)])/((a*c - b*d)^2*(1 + m)))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)^2 x^2}\,dx\\ &= -\text{Subst}\left(\int \frac{(a+bx)^m}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{b\left(a+\frac{b}{x}\right)^{1+m}\,{}_2F_1\left(2,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx = -\frac{b\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{c\left(a+\frac{b}{x}\right)}{-ac+bd}\right)}{(-ac+bd)^2(1+m)}$$

```
[In] Integrate[(a + b/x)^m/(c + d*x)^2,x]
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((c*(a + b/x))/(
-(a*c) + b*d))])/((-(a*c) + b*d)^2*(1 + m)))
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(dx+c\right)^2}dx$$

[In] int((a+b/x)^m/(d*x+c)^2,x)

[Out] int((a+b/x)^m/(d*x+c)^2,x)

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(dx+c\right)^2}\,dx$$

[In] integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="fricas")

[Out] integral(((a*x + b)/x)^m/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(c+dx\right)^2}\,dx$$

[In] integrate((a+b/x)**m/(d*x+c)**2,x)

[Out] Integral((a + b/x)**m/(c + d*x)**2, x)

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(dx+c\right)^2}\,dx$$

[In] integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((a + b/x)^m/(d*x + c)^2, x)

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^2}\,dx$$

```
[In] integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^m/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{(c+d\,x)^2}\,dx$$

```
[In] int((a + b/x)^m/(c + d*x)^2,x)
[Out] int((a + b/x)^m/(c + d*x)^2, x)
```

3.593 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 112

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3} dx = -\frac{d\left(a+\frac{b}{x}\right)^{1+m}}{2c(ac-bd)\left(d+\frac{c}{x}\right)^2} - \frac{b(2ac-bd(1+m))\left(a+\frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left(2,1+m,2+m,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(ac-bd)^3(1+m)}$$

```
[Out] -1/2*d*(a+b/x)^(1+m)/c/(a*c-b*d)/(d+c/x)^2-1/2*b*(2*a*c-b*d*(1+m))*(a+b/x)^
(1+m)*hypergeom([2, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/c/(a*c-b*d)^3/(1+m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {445, 457, 79, 70}

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3} dx = -\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(m+1)) \text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(m+1)(ac-bd)^3} - \frac{d\left(a+\frac{b}{x}\right)^{m+1}}{2c\left(\frac{c}{x}+d\right)^2(ac-bd)}$$

```
[In] Int[(a + b/x)^m/(c + d*x)^3,x]
```

```
[Out] -1/2*(d*(a + b/x)^(1 + m))/(c*(a*c - b*d)*(d + c/x)^2) - (b*(2*a*c - b*d*(1
 + m))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(
a*c - b*d)])/(2*c*(a*c - b*d)^3*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)^3 x^3}\,dx$$

$$= -\text{Subst}\left(\int \frac{x(a+bx)^m}{(d+cx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d\left(a+\frac{b}{x}\right)^{1+m}}{2c(ac-bd)\left(d+\frac{c}{x}\right)^2} - \frac{(2ac-bd(1+m))\text{Subst}\left(\int \frac{(a+bx)^m}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)}{2c(ac-bd)}$$

$$= -\frac{d\left(a+\frac{b}{x}\right)^{1+m}}{2c(ac-bd)\left(d+\frac{c}{x}\right)^2} - \frac{b(2ac-bd(1+m))\left(a+\frac{b}{x}\right)^{1+m}{}_2F_1\left(2,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(ac-bd)^3(1+m)}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx = \frac{\left(a+\frac{b}{x}\right)^{1+m}\left(-\frac{dx^2}{(c+dx)^2} + \frac{b(-2ac+bd(1+m))\,\text{Hypergeometric2F1}\left(2,1+m,2+m,\frac{bc+acx}{acx-bdx}\right)}{(ac-bd)^2(1+m)}\right)}{2c(ac-bd)}$$

```
[In] Integrate[(a + b/x)^m/(c + d*x)^3,x]
```

```
[Out] ((a + b/x)^(1 + m)*(-((d*x^2)/(c + d*x)^2) + (b*(-2*a*c + b*d*(1 + m))*Hype
rgeometric2F1[2, 1 + m, 2 + m, (b*c + a*c*x)/(a*c*x - b*d*x)])/((a*c - b*d)
^2*(1 + m))))/(2*c*(a*c - b*d))
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(dx+c\right)^3}dx$$

```
[In] int((a+b/x)^m/(d*x+c)^3,x)
```

```
[Out] int((a+b/x)^m/(d*x+c)^3,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(dx+c\right)^3}\,dx$$

```
[In] integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(c+dx\right)^3}\,dx$$

```
[In] integrate((a+b/x)**m/(d*x+c)**3,x)
[Out] Integral((a + b/x)**m/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(dx+c\right)^3}\,dx$$

```
[In] integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="maxima")
[Out] integrate((a + b/x)^m/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(dx+c\right)^3}\,dx$$

```
[In] integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="giac")
[Out] integrate((a + b/x)^m/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(c+d\,x\right)^3}\,dx$$

```
[In] int((a + b/x)^m/(c + d*x)^3,x)
[Out] int((a + b/x)^m/(c + d*x)^3, x)
```

3.594 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 185

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4} dx = \frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3} - \frac{d(6ac-bd(4+m))\left(a+\frac{b}{x}\right)^{1+m}}{6c^2(ac-bd)^2\left(d+\frac{c}{x}\right)^2} - \frac{b(6a^2c^2-6abcd(1+m)+b^2d^2(2+3m+m^2))\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(2,1+m,2+m,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{6c^2(ac-bd)^4(1+m)}$$

```
[Out] 1/3*d^2*(a+b/x)^(1+m)/c^2/(a*c-b*d)/(d+c/x)^3-1/6*d*(6*a*c-b*d*(4+m))*(a+b/
x)^(1+m)/c^2/(a*c-b*d)^2/(d+c/x)^2-1/6*b*(6*a^2*c^2-6*a*b*c*d*(1+m)+b^2*d^2
*(m^2+3*m+2))*(a+b/x)^(1+m)*hypergeom([2, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/c
^2/(a*c-b*d)^4/(1+m)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {445, 457, 91, 79, 70}

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4} dx = -\frac{b\left(a+\frac{b}{x}\right)^{m+1}(6a^2c^2-6abcd(m+1)+b^2d^2(m^2+3m+2))\text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{6c^2(m+1)(ac-bd)^4} + \frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-bd)} - \frac{d\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(m+4))}{6c^2\left(\frac{c}{x}+d\right)^2(ac-bd)^2}$$

```
[In] Int[(a + b/x)^m/(c + d*x)^4,x]
```

```
[Out] (d^2*(a + b/x)^(1 + m))/(3*c^2*(a*c - b*d)*(d + c/x)^3) - (d*(6*a*c - b*d*(
4 + m))*(a + b/x)^(1 + m))/(6*c^2*(a*c - b*d)^2*(d + c/x)^2) - (b*(6*a^2*c^
2 - 6*a*b*c*d*(1 + m) + b^2*d^2*(2 + 3*m + m^2))*(a + b/x)^(1 + m)*Hypergeo
metric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a*c - b*d)])/(6*c^2*(a*c - b*d)^4
*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)^4 x^4} \, dx \\
&= -\text{Subst}\left(\int \frac{x^2(a+bx)^m}{(d+cx)^4} \, dx, x, \frac{1}{x}\right) \\
&= \frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3} - \frac{\text{Subst}\left(\int \frac{(a+bx)^m(-d(3ac-bd(1+m))+3c(ac-bd)x)}{(d+cx)^3} \, dx, x, \frac{1}{x}\right)}{3c^2(ac-bd)} \\
&= \frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3} - \frac{d(6ac-bd(4+m))\left(a+\frac{b}{x}\right)^{1+m}}{6c^2(ac-bd)^2\left(d+\frac{c}{x}\right)^2} \\
&\quad - \frac{\left(6a^2c^2-6abcd(1+m)+b^2d^2(2+3m+m^2)\right)\text{Subst}\left(\int \frac{(a+bx)^m}{(d+cx)^2} \, dx, x, \frac{1}{x}\right)}{6c^2(ac-bd)^2} \\
&= \frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3} - \frac{d(6ac-bd(4+m))\left(a+\frac{b}{x}\right)^{1+m}}{6c^2(ac-bd)^2\left(d+\frac{c}{x}\right)^2} \\
&\quad - \frac{b\left(6a^2c^2-6abcd(1+m)+b^2d^2(2+3m+m^2)\right)\left(a+\frac{b}{x}\right)^{1+m} {}_2F_1\left(2,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{6c^2(ac-bd)^4(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

$$
\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4} \, dx = \frac{\left(a+\frac{b}{x}\right)^{1+m}\left(\frac{2d^2(ac-bd)x^3}{(c+dx)^3} + \frac{d(-6ac+bd(4+m))x^2}{(c+dx)^2} - \frac{b(6a^2c^2-6abcd(1+m)+b^2d^2(2+3m+m^2))\,\text{Hypergeometric2F1}\left(2,1+m,2+m,\frac{bc+}{acx}\right.}{(ac-bd)^2(1+m)}\right.}{6c^2(ac-bd)^2}
$$

```
[In] Integrate[(a + b/x)^m/(c + d*x)^4,x]
```

```
[Out] ((a + b/x)^(1 + m)*((2*d^2*(a*c - b*d)*x^3)/(c + d*x)^3 + (d*(-6*a*c + b*d*
(4 + m))*x^2)/(c + d*x)^2 - (b*(6*a^2*c^2 - 6*a*b*c*d*(1 + m) + b^2*d^2*(2
+ 3*m + m^2))*Hypergeometric2F1[2, 1 + m, 2 + m, (b*c + a*c*x)/(a*c*x - b*d
*x)])/((a*c - b*d)^2*(1 + m))))/(6*c^2*(a*c - b*d)^2)
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^4}dx$$

```
[In] int((a+b/x)^m/(d*x+c)^4,x)
[Out] int((a+b/x)^m/(d*x+c)^4,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^4}\,dx$$

```
[In] integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^m/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x
 + c^4), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx$$

```
[In] integrate((a+b/x)**m/(d*x+c)**4,x)
[Out] Integral((a + b/x)**m/(c + d*x)**4, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^4}\,dx$$

```
[In] integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="maxima")
[Out] integrate((a + b/x)^m/(d*x + c)^4, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^4}\,dx$$

```
[In] integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="giac")
[Out] integrate((a + b/x)^m/(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+d\,x)^4}\,dx$$

```
[In] int((a + b/x)^m/(c + d*x)^4,x)
[Out] int((a + b/x)^m/(c + d*x)^4, x)
```

3.595 $\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 33

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{b-\frac{a}{x^2}}x^{1+m}}{m\sqrt{a-bx^2}}$$

```
[Out] x^(1+m)*(b-a/x^2)^(1/2)/m/(-b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {529, 23, 30}

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx=\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

```
[In] Int[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int\frac{x^{-1+m}\sqrt{-a+bx^2}}{\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}\\
&= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int x^{-1+m}\,dx}{\sqrt{a-bx^2}}\\
&= \frac{\sqrt{b-\frac{a}{x^2}}x^{1+m}}{m\sqrt{a-bx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int\frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{b-\frac{a}{x^2}}x^{1+m}}{m\sqrt{a-bx^2}}$$

```
[In] Integrate[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]
[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x^{1+m}\sqrt{-\frac{-b\,x^2+a}{x^2}}}{m\sqrt{-b\,x^2+a}}$	35
risch	$\frac{i\sqrt{-\frac{-b\,x^2+a}{x^2}}\,(b\,x^2-a)x\sqrt{\frac{-b\,x^2+a}{b\,x^2-a}}\,x^m}{(-b\,x^2+a)^{\frac{3}{2}}m}$	67

```
[In] int(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1+m)/m/(-b*x^2+a)^(1/2)*(-(-b*x^2+a)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{-bx^2+a}\,xx^m\sqrt{\frac{bx^2-a}{x^2}}}{bmx^2-am}$$

```
[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-b*x^2 + a)*x*x^m*sqrt((b*x^2 - a)/x^2)/(b*m*x^2 - a*m)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx = \int \frac{x^m\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

```
[In] integrate(x**m*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
[Out] Integral(x**m*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx = -\frac{i\,x^m}{m}$$

```
[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -I*x^m/m
```

Giac [F]

$$\int \frac{\sqrt{b - \frac{a}{x^2}}x^m}{\sqrt{a - bx^2}}\,dx = \int \frac{\sqrt{b - \frac{a}{x^2}}x^m}{\sqrt{-bx^2 + a}}\,dx$$

```
[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b - a/x^2)*x^m/sqrt(-b*x^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 16.81 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{b - \frac{a}{x^2}}x^m}{\sqrt{a - bx^2}}\,dx = \frac{x^{m+1}\sqrt{b - \frac{a}{x^2}}}{m\sqrt{a - b\,x^2}}$$

```
[In] int((x^m*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
[Out] (x^(m + 1)*(b - a/x^2)^(1/2))/(m*(a - b*x^2)^(1/2))
```

3.596 $\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 31

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{b-\frac{a}{x^2}}x^3}{2\sqrt{a-bx^2}}$$

```
[Out] 1/2*x^3*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {529, 23, 30}

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx=\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

```
[In] Int[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int\frac{x\sqrt{-a+bx^2}}{\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}} \\ &= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int x\,dx}{\sqrt{a-bx^2}} \\ &= \frac{\sqrt{b-\frac{a}{x^2}}x^3}{2\sqrt{a-bx^2}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int\frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{b-\frac{a}{x^2}}x\sqrt{a-bx^2}}{2b}$$

```
[In] Integrate[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]
[Out] -1/2*(Sqrt[b - a/x^2]*x*Sqrt[a - b*x^2])/b
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{x^3\sqrt{-\frac{-b\,x^2+a}{x^2}}}{2\sqrt{-b\,x^2+a}}$	31
default	$\frac{x^3\sqrt{-\frac{-b\,x^2+a}{x^2}}}{2\sqrt{-b\,x^2+a}}$	31
risch	$\frac{ix^3\sqrt{-\frac{-b\,x^2+a}{x^2}}\,(b\,x^2-a)\sqrt{\frac{-b\,x^2+a}{b\,x^2-a}}}{2(-b\,x^2+a)^{\frac{3}{2}}}$	63

```
[In] int(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^3*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{-bx^2+a}x^3\sqrt{\frac{bx^2-a}{x^2}}}{2\,(bx^2-a)}$$

```
[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-b*x^2 + a)*x^3*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx = \int \frac{x^2\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

```
[In] integrate(x**2*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
[Out] Integral(x**2*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx = -\frac{1}{2}i\,x^2$$

```
[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -1/2*I*x^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx = -\frac{i\,bx^2 - i\,a}{2\,b}$$

```
[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] -1/2*(I*b*x^2 - I*a)/b
```

Mupad [B] (verification not implemented)

Time = 16.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx = \frac{x^3\,\sqrt{b-\frac{a}{x^2}}}{2\,\sqrt{a-b\,x^2}}$$

```
[In] int((x^2*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
[Out] (x^3*(b - a/x^2)^(1/2))/(2*(a - b*x^2)^(1/2))
```

3.597 $\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 28

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}$$

```
[Out] x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {529, 23, 8}

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx = \frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

```
[In] Int[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
```

```
GtQ[b/d, 0])
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int\frac{\sqrt{-a+bx^2}}{\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}\\&=\frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int 1\,dx}{\sqrt{a-bx^2}}\\&=\frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int\frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}$$

```
[In] Integrate[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{x^2\sqrt{-\frac{-b\,x^2+a}{x^2}}}{\sqrt{-b\,x^2+a}}$	30
risch	$\frac{i x^2\sqrt{-\frac{-b\,x^2+a}{x^2}}\,(b\,x^2-a)\sqrt{\frac{-b\,x^2+a}{b\,x^2-a}}}{(-b\,x^2+a)^{\frac{3}{2}}}$	63

```
[In] int(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] x^2*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{-bx^2+a}x^2\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

```
[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-b*x^2 + a)*x^2*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx = \int \frac{x\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

```
[In] integrate(x*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
[Out] Integral(x*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx = -i\sqrt{x^2}$$

```
[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -I*sqrt(x^2)
```

Giac [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{-bx^2+a}}\,dx$$

```
[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b - a/x^2)*x/sqrt(-b*x^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 17.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{b\,x^2-a}\,\sqrt{x^2}}{\sqrt{a-b\,x^2}}$$

```
[In] int((x*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
[Out] ((b*x^2 - a)^(1/2)*(x^2)^(1/2))/(a - b*x^2)^(1/2)
```

3.598 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 28

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{b-\frac{a}{x^2}}x\log(x)}{\sqrt{a-bx^2}}$$

```
[Out] x*ln(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {446, 23, 29}

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx=\frac{x\log(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

```
[In] Int[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]
[Out] (Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 446

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart[
q]), Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int\frac{\sqrt{-a+bx^2}}{x\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}\\ &= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int\frac{1}{x}\,dx}{\sqrt{a-bx^2}}\\ &= \frac{\sqrt{b-\frac{a}{x^2}}\,x\log(x)}{\sqrt{a-bx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{b-\frac{a}{x^2}}\,x\log(x)}{\sqrt{a-bx^2}}$$

```
[In] Integrate[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]
[Out] (Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,x\ln\left(x\right)}{\sqrt{-b\,x^{2}+a}}$	30
risch	$\frac{i\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,\left(b\,x^{2}-a\right)x\sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}\,\ln\left(x\right)}{\left(-b\,x^{2}+a\right)^{\frac{3}{2}}}$	63

```
[In] int((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-(-b*x^2+a)/x^2)^(1/2)*x/(-b*x^2+a)^(1/2)*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(24) = 48$.

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx = -\arctan\left(\frac{\sqrt{-bx^2+a}(x^3+x)\sqrt{\frac{bx^2-a}{x^2}}}{bx^4-(a+b)x^2+a}\right)$$

```
[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-b*x^2 + a)*(x^3 + x)*sqrt((b*x^2 - a)/x^2)/(b*x^4 - (a + b)*x
^2 + a))
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

```
[In] integrate((b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx = -i\log(x)$$

```
[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -I*log(x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-b\,x^2}}\,dx$$

```
[In] int((b - a/x^2)^(1/2)/(a - b*x^2)^(1/2),x)
```

```
[Out] int((b - a/x^2)^(1/2)/(a - b*x^2)^(1/2), x)
```

3.599 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 26

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx=-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

```
[Out] -(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {529, 23, 30}

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx=-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

```
[In] Int[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]
```

```
[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int\frac{\sqrt{-a+bx^2}}{x^2\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}\\ &= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int\frac{1}{x^2}\,dx}{\sqrt{a-bx^2}}\\ &= -\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int\frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

```
[In] Integrate[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]
[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}}{\sqrt{-b\,x^2+a}}$	28
default	$-\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}}{\sqrt{-b\,x^2+a}}$	28
risch	$-\frac{i\sqrt{-\frac{-b\,x^2+a}{x^2}}\,(b\,x^2-a)\sqrt{\frac{-b\,x^2+a}{b\,x^2-a}}}{(-b\,x^2+a)^{\frac{3}{2}}}$	60

```
[In] int((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{-bx^2+a}(x-1)\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

```
[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-b*x^2 + a)*(x - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-\frac{a}{x^2}+b}}{x\sqrt{a-bx^2}}\,dx$$

```
[In] integrate((b-a/x**2)**(1/2)/x/(-b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(-a/x**2 + b)/(x*sqrt(a - b*x**2)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = \frac{i}{\sqrt{x^2}}$$

```
[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] I/sqrt(x^2)
```

Giac [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx=\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{-bx^2+ax}}\,dx$$

```
[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b - a/x^2)/(sqrt(-b*x^2 + a)*x), x)
```

Mupad [B] (verification not implemented)

Time = 16.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx=-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-b\,x^2}}$$

```
[In] int((b - a/x^2)^(1/2)/(x*(a - b*x^2)^(1/2)),x)
[Out] -(b - a/x^2)^(1/2)/(a - b*x^2)^(1/2)
```

3.600 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 31

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx=-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

```
[Out] -1/2*(b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {529, 23, 30}

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx=-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

```
[In] Int[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]
```

```
[Out] -1/2*Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int\frac{\sqrt{-a+bx^2}}{x^3\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}} \\ &= \frac{\left(\sqrt{b-\frac{a}{x^2}}x\right)\int\frac{1}{x^3}\,dx}{\sqrt{a-bx^2}} \\ &= -\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int\frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

```
[In] Integrate[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]
[Out] -1/2*Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2])
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}}{2x\sqrt{-b\,x^2+a}}$	31
default	$-\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}}{2x\sqrt{-b\,x^2+a}}$	31
risch	$-\frac{i\sqrt{-\frac{-b\,x^2+a}{x^2}}\,(b\,x^2-a)\sqrt{\frac{-b\,x^2+a}{b\,x^2-a}}}{2x(-b\,x^2+a)^{\frac{3}{2}}}$	63

```
[In] int((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(-(-b*x^2+a)/x^2)^(1/2)/x/(-b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{-bx^2+a}(x^2-1)\sqrt{\frac{bx^2-a}{x^2}}}{2\,(bx^3-ax)}$$

```
[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-b*x^2 + a)*(x^2 - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^3 - a*x)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-\frac{a}{x^2}+b}}{x^2\sqrt{a-bx^2}}\,dx$$

```
[In] integrate((b-a/x**2)**(1/2)/x**2/(-b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(-a/x**2 + b)/(x**2*sqrt(a - b*x**2)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = \frac{i}{2\,x^2}$$

```
[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] 1/2*I/x^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = \frac{i}{2\,x^2}$$

```
[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/2*I/x^2
```

Mupad [B] (verification not implemented)

Time = 17.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{b-\frac{a}{x^2}}}{2\,x\,\sqrt{a-b\,x^2}}$$

```
[In] int((b - a/x^2)^(1/2)/(x^2*(a - b*x^2)^(1/2)),x)
[Out] -(b - a/x^2)^(1/2)/(2*x*(a - b*x^2)^(1/2))
```

3.601 $\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 406

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{2c\sqrt{c+dx}(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}x} + \frac{2(c+dx)^{3/2}(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}x} + \frac{2\sqrt{b}(ac^2-3bd^2)\sqrt{c+dx}\sqrt{1+\frac{ax^2}{b}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}d\sqrt{a+\frac{b}{x^2}}x\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}} - \frac{2\sqrt{b}c(ac^2+bd^2)\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}\sqrt{1+\frac{ax^2}{b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right),-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}d\sqrt{a+\frac{b}{x^2}}x\sqrt{c+dx}}$$

```
[Out] 2/5*(d*x+c)^(3/2)*(a*x^2+b)/a/x/(a+b/x^2)^(1/2)+2/5*c*(a*x^2+b)*(d*x+c)^(1/
2)/a/x/(a+b/x^2)^(1/2)+2/5*(a*c^2-3*b*d^2)*EllipticE(1/2*(1-x*(-a)^(1/2)/b^
(1/2))^(1/2)*2^(1/2),(-2*d*(-a)^(1/2)*b^(1/2)/(a*c-d*(-a)^(1/2)*b^(1/2)))^(
1/2))*b^(1/2)*(d*x+c)^(1/2)*(1+a*x^2/b)^(1/2)/(-a)^(3/2)/d/x/(a+b/x^2)^(1/2
)/(a*(d*x+c)/(a*c-d*(-a)^(1/2)*b^(1/2)))^(1/2)-2/5*c*(a*c^2+b*d^2)*Elliptic
F(1/2*(1-x*(-a)^(1/2)/b^(1/2))^(1/2)*2^(1/2),(-2*d*(-a)^(1/2)*b^(1/2)/(a*c-
d*(-a)^(1/2)*b^(1/2)))^(1/2))*b^(1/2)*(1+a*x^2/b)^(1/2)*(a*(d*x+c)/(a*c-d*(
-a)^(1/2)*b^(1/2)))^(1/2)/(-a)^(3/2)/d/x/(a+b/x^2)^(1/2)/(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1464, 847, 858, 733, 435, 430}

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = -\frac{2\sqrt{b}c\sqrt{\frac{ax^2}{b}+1}(ac^2+bd^2)\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right),-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}dx\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx}} + \frac{2\sqrt{b}\sqrt{\frac{ax^2}{b}+1}\sqrt{c+dx}(ac^2-3bd^2)\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}dx\sqrt{a+\frac{b}{x^2}}\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}} + \frac{2(ax^2+b)(c+dx)^{3/2}}{5ax\sqrt{a+\frac{b}{x^2}}} + \frac{2c(ax^2+b)\sqrt{c+dx}}{5ax\sqrt{a+\frac{b}{x^2}}}$$

```
[In] Int[(c + d*x)^(3/2)/Sqrt[a + b/x^2],x]
```

```
[Out] (2*c*Sqrt[c + d*x]*(b + a*x^2))/(5*a*Sqrt[a + b/x^2]*x) + (2*(c + d*x)^(3/2
)*(b + a*x^2))/(5*a*Sqrt[a + b/x^2]*x) + (2*Sqrt[b]*(a*c^2 - 3*b*d^2)*Sqrt[
c + d*x]*Sqrt[1 + (a*x^2)/b]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[-a]*x)/Sqrt[b]
]/Sqrt[2]], (-2*Sqrt[-a]*Sqrt[b]*d)/(a*c - Sqrt[-a]*Sqrt[b]*d)])/(5*(-a)^(3
/2)*d*Sqrt[a + b/x^2]*x*Sqrt[(a*(c + d*x))/(a*c - Sqrt[-a]*Sqrt[b]*d)]) - (
2*Sqrt[b]*c*(a*c^2 + b*d^2)*Sqrt[(a*(c + d*x))/(a*c - Sqrt[-a]*Sqrt[b]*d)]*
Sqrt[1 + (a*x^2)/b]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[-a]*x)/Sqrt[b]]/Sqrt[2]
], (-2*Sqrt[-a]*Sqrt[b]*d)/(a*c - Sqrt[-a]*Sqrt[b]*d)])/(5*(-a)^(3/2)*d*Sqr
t[a + b/x^2]*x*Sqrt[c + d*x])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1464

```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Sy
mbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(c + a*x^(2*
n))^FracPart[p]), Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x], x] /
; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[mn2, -2*n] &&  !IntegerQ[p] &&  !I
ntegerQ[q] && PosQ[n]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{b+ax^2}\int \frac{x(c+dx)^{3/2}}{\sqrt{b+ax^2}}\,dx}{\sqrt{a+\frac{b}{x^2}}x}$$

$$= \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}x} + \frac{\left(2\sqrt{b+ax^2}\right)\int \frac{\left(-\frac{3bd}{2}+\frac{3acx}{2}\right)\sqrt{c+dx}}{\sqrt{b+ax^2}}\,dx}{5a\sqrt{a+\frac{b}{x^2}}x}$$

$$= \frac{2c\sqrt{c+dx}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}x} + \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}x} + \frac{\left(4\sqrt{b+ax^2}\right)\int \frac{-3abcd+\frac{3}{4}a\left(ac^2-3bd^2\right)x}{\sqrt{c+dx}\sqrt{b+ax^2}}\,dx}{15a^2\sqrt{a+\frac{b}{x^2}}x}$$

$$= \frac{2c\sqrt{c+dx}(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}x} + \frac{2(c+dx)^{3/2}(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}x} + \frac{\left((ac^2-3bd^2)\sqrt{b+ax^2}\right)\int \frac{\sqrt{c+dx}}{\sqrt{b+ax^2}}\,dx}{5ad\sqrt{a+\frac{b}{x^2}}x} - \frac{\left(c(ac^2+bd^2)\sqrt{b+ax^2}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{b+ax^2}}\,dx}{5ad\sqrt{a+\frac{b}{x^2}}x}$$

$$= \frac{2c\sqrt{c+dx}(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}x} + \frac{2(c+dx)^{3/2}(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}x} + \frac{\left(2\sqrt{-a}\sqrt{b}(ac^2-3bd^2)\sqrt{c+dx}\sqrt{1+\frac{ax^2}{b}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{-a}\sqrt{b}dx^2}{ac-\sqrt{-a}\sqrt{b}d}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right)}{5a^2d\sqrt{a+\frac{b}{x^2}}x\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}} - \frac{\left(2\sqrt{-a}\sqrt{b}c(ac^2+bd^2)\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}\sqrt{1+\frac{ax^2}{b}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{-a}\sqrt{b}dx^2}{ac-\sqrt{-a}\sqrt{b}d}}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right)}{5a^2d\sqrt{a+\frac{b}{x^2}}x\sqrt{c+dx}}$$

$$= \frac{2c\sqrt{c+dx}(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}x} + \frac{2(c+dx)^{3/2}(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}x} + \frac{2\sqrt{b}(ac^2-3bd^2)\sqrt{c+dx}\sqrt{1+\frac{ax^2}{b}}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}d\sqrt{a+\frac{b}{x^2}}x\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}} - \frac{2\sqrt{b}c(ac^2+bd^2)\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}\sqrt{1+\frac{ax^2}{b}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}d\sqrt{a+\frac{b}{x^2}}x\sqrt{c+dx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.58 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\sqrt{c+dx}\left(\frac{2(2c+dx)(b+ax^2)}{a} + \frac{2\left(d^2\sqrt{-c-\frac{i\sqrt{b}d}{\sqrt{a}}}(ac^2-3bd^2)(b+ax^2)+\sqrt{a}\left(-ia^{3/2}c^3+a\sqrt{b}c^2d+3i\sqrt{a}bcd^2-3b^{3/2}d^3\right.\right.}{\cdots}\right.}{\cdots}$$

[In] Integrate[(c + d*x)^(3/2)/Sqrt[a + b/x^2],x]

[Out] (Sqrt[c + d*x]*((2*(2*c + d*x)*(b + a*x^2))/a + (2*(d^2*Sqrt[-c - (I*Sqrt[b]*d)/Sqrt[a]]*(a*c^2 - 3*b*d^2)*(b + a*x^2) + Sqrt[a]*((-I)*a^(3/2)*c^3 + a*Sqrt[b]*c^2*d + (3*I)*Sqrt[a]*b*c*d^2 - 3*b^(3/2)*d^3)*Sqrt[(d*((I*Sqrt[b])/Sqrt[a] + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[b]*d)/Sqrt[a] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c - (I*Sqrt[b]*d)/Sqrt[a]]/Sqrt[c + d*x]], (Sqrt[a]*c - I*Sqrt[b]*d)/(Sqrt[a]*c + I*Sqrt[b]*d)] - Sqrt[a]*Sqrt[b]*d*(a*c^2 + (4*I)*Sqrt[a]*Sqrt[b]*c*d - 3*b*d^2)*Sqrt[(d*((I*Sqrt[b])/Sqrt[a] + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[b]*d)/Sqrt[a] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c - (I*Sqrt[b]*d)/Sqrt[a]]/Sqrt[c + d*x]], (Sqrt[a]*c - I*Sqrt[b]*d)/(Sqrt[a]*c + I*Sqrt[b]*d)]))/(a^2*d^2*Sqrt[-c - (I*Sqrt[b]*d)/Sqrt[a]]*(c + d*x))))/(5*Sqrt[a + b/x^2]*x)

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.53

method	result
risch	$\frac{2(dx+2c)(a\,x^2+b)\sqrt{dx+c}}{5a\sqrt{\frac{a\,x^2+b}{x^2}}\,x} + \frac{\left(-\frac{8bcd\left(\frac{c}{d}-\frac{\sqrt{-ab}}{a}\right)\sqrt{\frac{\frac{c}{d}+x}{\frac{c}{d}-\frac{\sqrt{-ab}}{a}}}\sqrt{\frac{x-\frac{\sqrt{-ab}}{a}}{-\frac{c}{d}-\frac{\sqrt{-ab}}{a}}}\sqrt{\frac{x+\frac{\sqrt{-ab}}{a}}{-\frac{c}{d}+\frac{\sqrt{-ab}}{a}}}\,F\left(\sqrt{\frac{\frac{c}{d}+x}{\frac{c}{d}-\frac{\sqrt{-ab}}{a}}},\sqrt{\frac{-\frac{c}{d}+\frac{\sqrt{-ab}}{a}}{-\frac{c}{d}-\frac{\sqrt{-ab}}{a}}}\right)}{\sqrt{ad\,x^3+ac\,x^2+bdx+bc}}+\frac{2\left(a\,c^2-\cdots\right.}{\cdots}\right.}{\cdots}$
default	Expression too large to display

[In] int((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] 2/5*(d*x+2*c)*(a*x^2+b)*(d*x+c)^(1/2)/a/((a*x^2+b)/x^2)^(1/2)/x+1/5/a*(-8*b
*c*d*(c/d-1/a*(-a*b)^(1/2))*((c/d+x)/(c/d-1/a*(-a*b)^(1/2)))^(1/2)*((x-1/a*
(-a*b)^(1/2))/(-c/d-1/a*(-a*b)^(1/2)))^(1/2)*((x+1/a*(-a*b)^(1/2))/(-c/d+1/
a*(-a*b)^(1/2)))^(1/2)/(a*d*x^3+a*c*x^2+b*d*x+b*c)^(1/2)*EllipticF(((c/d+x)
/(c/d-1/a*(-a*b)^(1/2)))^(1/2),((-c/d+1/a*(-a*b)^(1/2))/(-c/d-1/a*(-a*b)^(1
/2)))^(1/2))+2*(a*c^2-3*b*d^2)*(c/d-1/a*(-a*b)^(1/2))*((c/d+x)/(c/d-1/a*(-a
*b)^(1/2)))^(1/2)*((x-1/a*(-a*b)^(1/2))/(-c/d-1/a*(-a*b)^(1/2)))^(1/2)*((x+
1/a*(-a*b)^(1/2))/(-c/d+1/a*(-a*b)^(1/2)))^(1/2)/(a*d*x^3+a*c*x^2+b*d*x+b*c
)^(1/2)*((-c/d-1/a*(-a*b)^(1/2))*EllipticE(((c/d+x)/(c/d-1/a*(-a*b)^(1/2)))
^(1/2),((-c/d+1/a*(-a*b)^(1/2))/(-c/d-1/a*(-a*b)^(1/2)))^(1/2))+1/a*(-a*b)^
(1/2)*EllipticF(((c/d+x)/(c/d-1/a*(-a*b)^(1/2)))^(1/2),((-c/d+1/a*(-a*b)^(1
/2))/(-c/d-1/a*(-a*b)^(1/2)))^(1/2))))/((a*x^2+b)/x^2)^(1/2)/x*((a*x^2+b)*(
d*x+c))^(1/2)/(d*x+c)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.58

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx =$$

$$-\frac{2\left((ac^3+9\,bcd^2)\sqrt{ad}\text{weierstrassPInverse}\left(\frac{4\,(ac^2-3\,bd^2)}{3\,ad^2},-\frac{8\,(ac^3+9\,bcd^2)}{27\,ad^3},\frac{3\,dx+c}{3\,d}\right)+3\,(ac^2d-3\,bd^3)\sqrt{ad}\text{weierstr}\right.}{}$$

```
[In] integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*((a*c^3 + 9*b*c*d^2)*sqrt(a*d)*weierstrassPInverse(4/3*(a*c^2 - 3*b*d
^2)/(a*d^2), -8/27*(a*c^3 + 9*b*c*d^2)/(a*d^3), 1/3*(3*d*x + c)/d) + 3*(a*c
^2*d - 3*b*d^3)*sqrt(a*d)*weierstrassZeta(4/3*(a*c^2 - 3*b*d^2)/(a*d^2), -8
/27*(a*c^3 + 9*b*c*d^2)/(a*d^3), weierstrassPInverse(4/3*(a*c^2 - 3*b*d^2)/
(a*d^2), -8/27*(a*c^3 + 9*b*c*d^2)/(a*d^3), 1/3*(3*d*x + c)/d)) - 3*(a*d^3*
x^2 + 2*a*c*d^2*x)*sqrt(d*x + c)*sqrt((a*x^2 + b)/x^2))/(a^2*d^2)
```

Sympy [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \int \frac{(c+dx)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

```
[In] integrate((d*x+c)**(3/2)/(a+b/x**2)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/sqrt(a + b/x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

[In] integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="maxima")

[Out] integrate((d*x + c)^(3/2)/sqrt(a + b/x^2), x)

Giac [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

[In] integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="giac")

[Out] integrate((d*x + c)^(3/2)/sqrt(a + b/x^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \int \frac{(c+d\,x)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

[In] int((c + d*x)^(3/2)/(a + b/x^2)^(1/2),x)

[Out] int((c + d*x)^(3/2)/(a + b/x^2)^(1/2), x)

3.602 $\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx=\frac{3}{4}\sqrt[3]{-4x+x^4}$$

```
[Out] 3/4*(x^4-4*x)^(1/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1602}

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx=\frac{3}{4}\sqrt[3]{x^4-4x}$$

```
[In] Int[(-1 + x^3)/(-4*x + x^4)^(2/3),x]
```

```
[Out] (3*(-4*x + x^4)^(1/3))/4
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{3}{4}\sqrt[3]{-4x+x^4}$$

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx = \frac{3}{4}\sqrt[3]{x\,(-4+x^3)}$$

```
[In] Integrate[(-1 + x^3)/(-4*x + x^4)^(2/3),x]
[Out] (3*(x*(-4 + x^3))^(1/3))/4
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{3(x^4-4x)^{\frac{1}{3}}}{4}$	12
trager	$\frac{3(x^4-4x)^{\frac{1}{3}}}{4}$	12
pseudoelliptic	$\frac{3(x(x^3-4))^{\frac{1}{3}}}{4}$	12
gosper	$\frac{3x(x^3-4)}{4(x^4-4x)^{\frac{2}{3}}}$	18
risch	$\frac{3x(x^3-4)}{4(x(x^3-4))^{\frac{2}{3}}}$	18
meijerg	$-\frac{3\,2^{\frac{2}{3}}\left(-\operatorname{signum}\left(-1+\frac{x^3}{4}\right)\right)^{\frac{2}{3}} x^{\frac{1}{3}}\,{}_2F_1\left(\frac{1}{9},\frac{2}{3};\frac{10}{9};\frac{x^3}{4}\right)}{4\operatorname{signum}\left(-1+\frac{x^3}{4}\right)^{\frac{2}{3}}}+\frac{3\,2^{\frac{2}{3}}\left(-\operatorname{signum}\left(-1+\frac{x^3}{4}\right)\right)^{\frac{2}{3}} x^{\frac{10}{3}}\,{}_2F_1\left(\frac{2}{3},\frac{10}{9};\frac{19}{9};\frac{x^3}{4}\right)}{40\operatorname{signum}\left(-1+\frac{x^3}{4}\right)^{\frac{2}{3}}}$	84

```
[In] int((x^3-1)/(x^4-4*x)^(2/3),x,method=_RETURNVERBOSE)
[Out] 3/4*(x^4-4*x)^(1/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx = \frac{3}{4}\left(x^4-4\,x\right)^{\frac{1}{3}}$$

```
[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="fricas")
[Out] 3/4*(x^4 - 4*x)^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx = \frac{3\sqrt[3]{x^4-4x}}{4}$$

```
[In] integrate((x**3-1)/(x**4-4*x)**(2/3),x)
[Out] 3*(x**4 - 4*x)**(1/3)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx = \frac{3}{4}\left(x^4-4\,x\right)^{\frac{1}{3}}$$

```
[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="maxima")
[Out] 3/4*(x^4 - 4*x)^(1/3)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx = \frac{3}{4}\left(x^4-4\,x\right)^{\frac{1}{3}}$$

```
[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="giac")
[Out] 3/4*(x^4 - 4*x)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx = \frac{3\left(x^4-4\,x\right)^{1/3}}{4}$$

```
[In] int((x^3 - 1)/(x^4 - 4*x)^(2/3),x)
[Out] (3*(x^4 - 4*x)^(1/3))/4
```

3.603 $\int (2-x^2) \sqrt[4]{6x-x^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 17

$$\int (2-x^2) \sqrt[4]{6x-x^3} \, dx = \frac{4}{15}(6x-x^3)^{5/4}$$

```
[Out] 4/15*(-x^3+6*x)^(5/4)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1602}

$$\int (2-x^2) \sqrt[4]{6x-x^3} \, dx = \frac{4}{15}(6x-x^3)^{5/4}$$

```
[In] Int[(2 - x^2)*(6*x - x^3)^(1/4),x]
```

```
[Out] (4*(6*x - x^3)^(5/4))/15
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{4}{15}(6x-x^3)^{5/4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 4.24

$$\int (2-x^2)\sqrt[4]{6x-x^3}\,dx = -\frac{4\sqrt[4]{-x(-6+x^2)}\left(-26x\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{8},\frac{13}{8},\frac{x^2}{6}\right)+5x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{13}{8},\frac{21}{8},\frac{x^2}{6}\right)\right)}{65\sqrt[4]{1-\frac{x^2}{6}}}$$

```
[In] Integrate[(2 - x^2)*(6*x - x^3)^(1/4),x]
[Out] (-4*(-(x*(-6 + x^2)))^(1/4)*(-26*x*Hypergeometric2F1[-1/4, 5/8, 13/8, x^2/6
] + 5*x^3*Hypergeometric2F1[-1/4, 13/8, 21/8, x^2/6]))/(65*(1 - x^2/6)^(1/4
))
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{4(-x^3+6x)^{\frac{5}{4}}}{15}$	14
risch	$-\frac{4(-x(x^2-6))^{\frac{1}{4}}x(x^2-6)}{15}$	19
pseudoelliptic	$-\frac{4(-x(x^2-6))^{\frac{1}{4}}x(x^2-6)}{15}$	19
gosper	$-\frac{4(-x^3+6x)^{\frac{1}{4}}x(x^2-6)}{15}$	20
trager	$-\frac{4(-x^3+6x)^{\frac{1}{4}}x(x^2-6)}{15}$	20
meijerg	$\frac{8\,6^{\frac{1}{4}}x^{\frac{5}{4}}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{8};\frac{13}{8};\frac{x^2}{6}\right)}{5}-\frac{4\,6^{\frac{1}{4}}x^{\frac{13}{4}}\,{}_2F_1\left(-\frac{1}{4},\frac{13}{8};\frac{21}{8};\frac{x^2}{6}\right)}{13}$	40

```
[In] int((-x^2+2)*(-x^3+6*x)^(1/4),x,method=_RETURNVERBOSE)
[Out] 4/15*(-x^3+6*x)^(5/4)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \left(2-x^2\right) \sqrt[4]{6x-x^3}\,dx = -\frac{4}{15}\left(x^3-6\,x\right)\left(-x^3+6\,x\right)^{\frac{1}{4}}$$

```
[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="fricas")
[Out] -4/15*(x^3 - 6*x)*(-x^3 + 6*x)^(1/4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \left(2-x^2\right) \sqrt[4]{6x-x^3}\,dx = -\frac{4x^3\sqrt[4]{-x^3+6x}}{15}+\frac{8x\sqrt[4]{-x^3+6x}}{5}$$

```
[In] integrate((-x**2+2)*(-x**3+6*x)**(1/4),x)
[Out] -4*x**3*(-x**3 + 6*x)**(1/4)/15 + 8*x*(-x**3 + 6*x)**(1/4)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(2-x^2\right) \sqrt[4]{6x-x^3}\,dx = \frac{4}{15}\left(-x^3+6\,x\right)^{\frac{5}{4}}$$

```
[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="maxima")
[Out] 4/15*(-x^3 + 6*x)^(5/4)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(2-x^2\right) \sqrt[4]{6x-x^3}\, dx = \frac{4}{15} \left(-x^3+6\,x\right)^{\frac{5}{4}}$$

```
[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="giac")
[Out] 4/15*(-x^3 + 6*x)^(5/4)
```

Mupad [B] (verification not implemented)

Time = 17.77 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \left(2-x^2\right) \sqrt[4]{6x-x^3}\, dx = -\frac{4\,x\,\left(x^2-6\right)\,{\left(6\,x-x^3\right)}^{1/4}}{15}$$

```
[In] int(-(x^2 - 2)*(6*x - x^3)^(1/4),x)
[Out] -(4*x*(x^2 - 6)*(6*x - x^3)^(1/4))/15
```

3.604 $\int (1+x^4) \sqrt{5x+x^5} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int (1+x^4) \sqrt{5x+x^5} \, dx = \frac{2}{15} \left(5x+x^5\right)^{3/2}$$

```
[Out] 2/15*(x^5+5*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1602}

$$\int (1+x^4) \sqrt{5x+x^5} \, dx = \frac{2}{15} \left(x^5+5x\right)^{3/2}$$

```
[In] Int[(1 + x^4)*Sqrt[5*x + x^5],x]
```

```
[Out] (2*(5*x + x^5)^(3/2))/15
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2}{15} \left(5x+x^5\right)^{3/2}$$

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1+x^4) \sqrt{5x+x^5}\, dx = \frac{2}{15}\left(x\left(5+x^4\right)\right)^{3/2}$$

```
[In] Integrate[(1 + x^4)*Sqrt[5*x + x^5],x]
[Out] (2*(x*(5 + x^4))^(3/2))/15
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{2\left(x^5+5x\right)^{\frac{3}{2}}}{15}$	12
gosper	$\frac{2x\left(x^4+5\right)\sqrt{x^5+5x}}{15}$	18
trager	$\frac{2x\left(x^4+5\right)\sqrt{x^5+5x}}{15}$	18
pseudoelliptic	$\frac{2x\left(x^4+5\right)\sqrt{x\left(x^4+5\right)}}{15}$	18
risch	$\frac{2x^2\left(x^4+5\right)^2}{15\sqrt{x\left(x^4+5\right)}}$	22
meijerg	$\frac{2\sqrt{5}\, x^{\frac{3}{2}}\, {}_2F_1\left(-\frac{1}{2},\frac{3}{8};\frac{11}{8};-\frac{x^4}{5}\right)}{3}+\frac{2\sqrt{5}\, x^{\frac{11}{2}}\, {}_2F_1\left(-\frac{1}{2},\frac{11}{8};\frac{19}{8};-\frac{x^4}{5}\right)}{11}$	40

```
[In] int((x^4+1)*(x^5+5*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(x^5+5*x)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (1+x^4) \sqrt{5x+x^5}\, dx = \frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

```
[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="fricas")
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int (1+x^4)\sqrt{5x+x^5}\,dx = \frac{2x^5\sqrt{x^5+5x}}{15} + \frac{2x\sqrt{x^5+5x}}{3}$$

```
[In] integrate((x**4+1)*(x**5+5*x)**(1/2),x)
[Out] 2*x**5*sqrt(x**5 + 5*x)/15 + 2*x*sqrt(x**5 + 5*x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (1+x^4)\sqrt{5x+x^5}\,dx = \frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

```
[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="maxima")
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (1+x^4)\sqrt{5x+x^5}\,dx = \frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

```
[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="giac")
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 17.50 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \left(1+x^4\right) \sqrt{5x+x^5}\, dx = \frac{2\left(x^5+5\,x\right)^{3/2}}{15}$$

```
[In] int((5*x + x^5)^(1/2)*(x^4 + 1),x)
[Out] (2*(5*x + x^5)^(3/2))/15
```

3.605 $\int (2+5x^4) \sqrt{2x+x^5} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 15

$$\int (2+5x^4) \sqrt{2x+x^5} \, dx = \frac{2}{3}(2x+x^5)^{3/2}$$

```
[Out] 2/3*(x^5+2*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1602}

$$\int (2+5x^4) \sqrt{2x+x^5} \, dx = \frac{2}{3}(x^5+2x)^{3/2}$$

```
[In] Int[(2 + 5*x^4)*Sqrt[2*x + x^5],x]
```

```
[Out] (2*(2*x + x^5)^(3/2))/3
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2}{3}(2x+x^5)^{3/2}$$

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (2+5x^4)\sqrt{2x+x^5}\,dx = \frac{2}{3}\left(x\left(2+x^4\right)\right)^{3/2}$$

```
[In] Integrate[(2 + 5*x^4)*Sqrt[2*x + x^5],x]
[Out] (2*(x*(2 + x^4))^(3/2))/3
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{2(x^5+2x)^{\frac{3}{2}}}{3}$	12
default	$\frac{2(x^5+2x)^{\frac{3}{2}}}{3}$	12
gosper	$\frac{2x(x^4+2)\sqrt{x^5+2x}}{3}$	18
trager	$\frac{2x(x^4+2)\sqrt{x^5+2x}}{3}$	18
pseudoelliptic	$\frac{2x(x^4+2)\sqrt{x(x^4+2)}}{3}$	18
risch	$\frac{2x^2(x^4+2)^2}{3\sqrt{x(x^4+2)}}$	22
meijerg	$\frac{4\sqrt{2}\,x^{\frac{3}{2}}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{8};\frac{11}{8};-\frac{x^4}{2}\right)}{3}+\frac{10\sqrt{2}\,x^{\frac{11}{2}}\,{}_2F_1\left(-\frac{1}{2},\frac{11}{8};\frac{19}{8};-\frac{x^4}{2}\right)}{11}$	40

```
[In] int((5*x^4+2)*(x^5+2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(x^5+2*x)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (2+5x^4)\sqrt{2x+x^5}\,dx = \frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

```
[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \left(2+5x^4\right)\sqrt{2x+x^5}\,dx = \frac{2x^5\sqrt{x^5+2x}}{3}+\frac{4x\sqrt{x^5+2x}}{3}$$

```
[In] integrate((5*x**4+2)*(x**5+2*x)**(1/2),x)
[Out] 2*x**5*sqrt(x**5 + 2*x)/3 + 4*x*sqrt(x**5 + 2*x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \left(2+5x^4\right)\sqrt{2x+x^5}\,dx = \frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

```
[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \left(2+5x^4\right)\sqrt{2x+x^5}\,dx = \frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

```
[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="giac")
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 17.40 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (2+5x^4)\sqrt{2x+x^5}\,dx=\frac{2\left(x^5+2\,x\right)^{3/2}}{3}$$

```
[In] int((2*x + x^5)^(1/2)*(5*x^4 + 2),x)
[Out] (2*(2*x + x^5)^(3/2))/3
```

3.606 $\int \frac{x+3x^2}{\sqrt{x^2+2x^3}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 13

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}} \, dx = \sqrt{x^2+2x^3}$$

```
[Out] (2*x^3+x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1602}

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}} \, dx = \sqrt{2x^3+x^2}$$

```
[In] Int[(x + 3*x^2)/Sqrt[x^2 + 2*x^3],x]
```

```
[Out] Sqrt[x^2 + 2*x^3]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \sqrt{x^2+2x^3}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx=\sqrt{x^2(1+2x)}$$

```
[In] Integrate[(x + 3*x^2)/Sqrt[x^2 + 2*x^3],x]
[Out] Sqrt[x^2*(1 + 2*x)]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
trager	$\sqrt{2x^3+x^2}$	12
pseudoelliptic	$\sqrt{x^2\left(1+2x\right)}$	12
gosper	$\frac{x^2(1+2x)}{\sqrt{2x^3+x^2}}$	21
default	$\frac{x^2(1+2x)}{\sqrt{2x^3+x^2}}$	21
risch	$\frac{x^2(1+2x)}{\sqrt{x^2(1+2x)}}$	21
meijerg	$\frac{\sqrt{\pi}-\frac{\sqrt{\pi}\,(-8x+8)\sqrt{1+2x}}{8}}{\sqrt{\pi}}+\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1+2x}}{2\sqrt{\pi}}$	53

```
[In] int((3*x^2+x)/(2*x^3+x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (2*x^3+x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx=\sqrt{2\,x^3+x^2}$$

```
[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(2*x^3 + x^2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx=\sqrt{2x^3+x^2}$$

```
[In] integrate((3*x**2+x)/(2*x**3+x**2)**(1/2),x)
[Out] sqrt(2*x**3 + x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx=\sqrt{2\,x^3+x^2}$$

```
[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(2*x^3 + x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx=\frac{(2\,x+1)^{\frac{3}{2}}-\sqrt{2\,x+1}}{2\,\mathrm{sgn}\left(x\right)}$$

```
[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="giac")
[Out] 1/2*((2*x + 1)^(3/2) - sqrt(2*x + 1))/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 17.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx=|x|\,\sqrt{2\,x+1}$$

```
[In] int((x + 3*x^2)/(x^2 + 2*x^3)^(1/2),x)
[Out] abs(x)*(2*x + 1)^(1/2)
```

3.607 $\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 44

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}} \, dx = -\frac{9}{5}\sqrt[3]{1-5x}+\frac{3}{10}(1-5x)^{2/3}+x+\frac{27}{5}\log \left(3+\sqrt[3]{1-5x}\right)$$

```
[Out] -9/5*(1-5*x)^(1/3)+3/10*(1-5*x)^(2/3)+x+27/5*ln(3+(1-5*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {442, 383, 78}

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}} \, dx = x+\frac{3}{10}(1-5x)^{2/3}-\frac{9}{5}\sqrt[3]{1-5x}+\frac{27}{5}\log \left(\sqrt[3]{1-5x}+3\right)$$

```
[In] Int[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]
```

```
[Out] (-9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^(2/3))/10 + x + (27*Log[3 + (1 - 5*x)
^(1/3)])/5
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 442

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{5}\text{Subst}\left(\int \frac{2+\sqrt[3]{x}}{3+\sqrt[3]{x}}\,dx, x, 1-5x\right)\right)\\
&= -\left(\frac{3}{5}\text{Subst}\left(\int \frac{x^2(2+x)}{3+x}\,dx, x, \sqrt[3]{1-5x}\right)\right)\\
&= -\left(\frac{3}{5}\text{Subst}\left(\int \left(3-x+x^2-\frac{9}{3+x}\right)\,dx, x, \sqrt[3]{1-5x}\right)\right)\\
&= -\frac{9}{5}\sqrt[3]{1-5x}+\frac{3}{10}(1-5x)^{2/3}+x+\frac{27}{5}\log\left(3+\sqrt[3]{1-5x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx = \frac{1}{10}\left(-2-18\sqrt[3]{1-5x}+3(1-5x)^{2/3}+10x+54\log\left(3+\sqrt[3]{1-5x}\right)\right)$$

```
[In] Integrate[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]
```

```
[Out] (-2 - 18*(1 - 5*x)^(1/3) + 3*(1 - 5*x)^(2/3) + 10*x + 54*Log[3 + (1 - 5*x)^
(1/3)])/10
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{1}{5}+x+\frac{3(1-5x)^{\frac{2}{3}}}{10}-\frac{9(1-5x)^{\frac{1}{3}}}{5}+\frac{27\ln\left(3+(1-5x)^{\frac{1}{3}}\right)}{5}$	34
default	$-\frac{1}{5}+x+\frac{3(1-5x)^{\frac{2}{3}}}{10}-\frac{9(1-5x)^{\frac{1}{3}}}{5}+\frac{27\ln\left(3+(1-5x)^{\frac{1}{3}}\right)}{5}$	34
trager	$x-\frac{9(1-5x)^{\frac{1}{3}}}{5}+\frac{3(1-5x)^{\frac{2}{3}}}{10}+\frac{9\ln\left(9(1-5x)^{\frac{2}{3}}+27(1-5x)^{\frac{1}{3}}-5x+28\right)}{5}$	47

```
[In] int((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x,method=_RETURNVERBOSE)
[Out] -1/5+x+3/10*(1-5*x)^(2/3)-9/5*(1-5*x)^(1/3)+27/5*ln(3+(1-5*x)^(1/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx = x+\frac{3}{10}\,(-5\,x+1)^{\frac{2}{3}}-\frac{9}{5}\,(-5\,x+1)^{\frac{1}{3}}+\frac{27}{5}\,\log\left((-5\,x+1)^{\frac{1}{3}}+3\right)$$

```
[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="fricas")
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3
) + 3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx = x+\frac{3(1-5x)^{\frac{2}{3}}}{10}-\frac{9\sqrt[3]{1-5x}}{5}+\frac{27\log\left(\sqrt[3]{1-5x}+3\right)}{5}$$

```
[In] integrate((2+(1-5*x)**(1/3))/(3+(1-5*x)**(1/3)),x)
[Out] x + 3*(1 - 5*x)**(2/3)/10 - 9*(1 - 5*x)**(1/3)/5 + 27*log((1 - 5*x)**(1/3)
+ 3)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx = x+\frac{3}{10}\,(-5\,x+1)^{\frac{2}{3}}-\frac{9}{5}\,(-5\,x+1)^{\frac{1}{3}}+\frac{27}{5}\,\log\left((-5\,x+1)^{\frac{1}{3}}+3\right)-\frac{1}{5}$$

```
[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="maxima")
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3
) + 3) - 1/5
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx = x+\frac{3}{10}\,(-5\,x+1)^{\frac{2}{3}}-\frac{9}{5}\,(-5\,x+1)^{\frac{1}{3}}+\frac{27}{5}\,\log\left((-5\,x+1)^{\frac{1}{3}}+3\right)-\frac{1}{5}$$

```
[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="giac")
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3
) + 3) - 1/5
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx = x+\frac{27\,\ln\left({(1-5\,x)}^{1/3}+3\right)}{5}-\frac{9\,{(1-5\,x)}^{1/3}}{5}+\frac{3\,{(1-5\,x)}^{2/3}}{10}$$

```
[In] int(((1 - 5*x)^(1/3) + 2)/((1 - 5*x)^(1/3) + 3),x)
[Out] x + (27*log((1 - 5*x)^(1/3) + 3))/5 - (9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^
(2/3))/10
```

3.608 $\int \frac{1+\sqrt{x}}{-1+\sqrt{x}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}} \, dx = 4\sqrt{x}+x+4\log\left(1-\sqrt{x}\right)$$

```
[Out] x+4*ln(1-x^(1/2))+4*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {383, 78}

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}} \, dx = x+4\sqrt{x}+4\log\left(1-\sqrt{x}\right)$$

```
[In] Int[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
[Out] 4*Sqrt[x] + x + 4*Log[1 - Sqrt[x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{x(1+x)}{-1+x}\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int \left(2+\frac{2}{-1+x}+x\right)\,dx, x, \sqrt{x}\right)\\ &= 4\sqrt{x}+x+4\log\left(1-\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = 4\sqrt{x}+x+4\log\left(-1+\sqrt{x}\right)$$

```
[In] Integrate[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
[Out] 4*Sqrt[x] + x + 4*Log[-1 + Sqrt[x]]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$x+4\sqrt{x}+4\ln\left(-1+\sqrt{x}\right)$	16
default	$x+4\sqrt{x}+4\ln\left(-1+\sqrt{x}\right)$	16
trager	$x-2+4\sqrt{x}+2\ln\left(2\sqrt{x}-1-x\right)$	22
meijerg	$2\sqrt{x}+4\ln\left(1-\sqrt{x}\right)+\frac{\sqrt{x}\,(6+3\sqrt{x})}{3}$	29

```
[In] int((1+x^(1/2))/(-1+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] x+4*x^(1/2)+4*ln(-1+x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\, dx = x + 4\,\sqrt{x} + 4\,\log\left(\sqrt{x} - 1\right)$$

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="fricas")
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\, dx = 4\sqrt{x} + x + 4\log\left(\sqrt{x} - 1\right)$$

```
[In] integrate((1+x**(1/2))/(-1+x**(1/2)),x)
[Out] 4*sqrt(x) + x + 4*log(sqrt(x) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\, dx = x + 4\,\sqrt{x} + 4\,\log\left(\sqrt{x} - 1\right)$$

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="maxima")
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\, dx = x + 4\,\sqrt{x} + 4\,\log\left(\left|\sqrt{x} - 1\right|\right)$$

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="giac")
[Out] x + 4*sqrt(x) + 4*log(abs(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 17.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = x + 4\,\ln\left(\sqrt{x}-1\right) + 4\,\sqrt{x}$$

```
[In] int((x^(1/2) + 1)/(x^(1/2) - 1),x)
[Out] x + 4*log(x^(1/2) - 1) + 4*x^(1/2)
```

3.609 $\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 33

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx = -x+\frac{4}{3}\sqrt{2+3x}-\frac{4}{3}\log\left(1+\sqrt{2+3x}\right)$$

```
[Out] -x-4/3*ln(1+(2+3*x)^(1/2))+4/3*(2+3*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {442, 383, 78}

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx = -x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

```
[In] Int[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]),x]
```

```
[Out] -x + (4*Sqrt[2 + 3*x])/3 - (4*Log[1 + Sqrt[2 + 3*x]])/3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 442

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1-\sqrt{x}}{1+\sqrt{x}}\,dx, x, 2+3x\right)\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{(1-x)x}{1+x}\,dx, x, \sqrt{2+3x}\right)\\ &= \frac{2}{3}\text{Subst}\left(\int \left(2-x-\frac{2}{1+x}\right)\,dx, x, \sqrt{2+3x}\right)\\ &= -x+\frac{4}{3}\sqrt{2+3x}-\frac{4}{3}\log\left(1+\sqrt{2+3x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx = \frac{1}{3}\left(-2-3x+4\sqrt{2+3x}-4\log\left(1+\sqrt{2+3x}\right)\right)$$

```
[In] Integrate[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]),x]
[Out] (-2 - 3*x + 4*Sqrt[2 + 3*x] - 4*Log[1 + Sqrt[2 + 3*x]])/3
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-x-\frac{2}{3}+\frac{4\sqrt{3x+2}}{3}-\frac{4\ln\left(1+\sqrt{3x+2}\right)}{3}$	27
default	$-x-\frac{2}{3}+\frac{4\sqrt{3x+2}}{3}-\frac{4\ln\left(1+\sqrt{3x+2}\right)}{3}$	27
trager	$-x+\frac{4\sqrt{3x+2}}{3}-\frac{2\ln\left(2\sqrt{3x+2}+3+3x\right)}{3}$	31

```
[In] int((1-(3*x+2)^(1/2))/(1+(3*x+2)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -x-2/3+4/3*(3*x+2)^(1/2)-4/3*ln(1+(3*x+2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx = -x+\frac{4}{3}\sqrt{3\,x+2}-\frac{4}{3}\,\log\left(\sqrt{3\,x+2}+1\right)$$

```
[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="fricas")
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx = -x+\frac{4\sqrt{3x+2}}{3}-\frac{4\log\left(\sqrt{3x+2}+1\right)}{3}$$

```
[In] integrate((1-(2+3*x)**(1/2))/(1+(2+3*x)**(1/2)),x)
[Out] -x + 4*sqrt(3*x + 2)/3 - 4*log(sqrt(3*x + 2) + 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx = -x+\frac{4}{3}\sqrt{3\,x+2}-\frac{4}{3}\,\log\left(\sqrt{3\,x+2}+1\right)-\frac{2}{3}$$

```
[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="maxima")
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1) - 2/3
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx = -x + \frac{4}{3}\sqrt{3\,x+2} - \frac{4}{3}\log\left(\sqrt{3\,x+2}+1\right) - \frac{2}{3}$$

```
[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="giac")
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1) - 2/3
```

Mupad [B] (verification not implemented)

Time = 17.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx = \frac{4\sqrt{3\,x+2}}{3} - \frac{4\,\ln\left(\sqrt{3\,x+2}+1\right)}{3} - x$$

```
[In] int(-((3*x + 2)^(1/2) - 1)/((3*x + 2)^(1/2) + 1),x)
[Out] (4*(3*x + 2)^(1/2))/3 - (4*log((3*x + 2)^(1/2) + 1))/3 - x
```

3.610 $\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 33

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = x-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(1+\sqrt{a+bx}\right)}{b}$$

```
[Out] x+4*ln(1+(b*x+a)^(1/2))/b-4*(b*x+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {442, 383, 78}

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = -\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(\sqrt{a+bx}+1\right)}{b}+x$$

```
[In] Int[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]
```

```
[Out] x - (4*Sqrt[a + b*x])/b + (4*Log[1 + Sqrt[a + b*x]])/b
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 442

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \frac{-1+\sqrt{x}}{1+\sqrt{x}}\,dx, x, a+bx\right)}{b} \\ &= \frac{2\text{Subst}\left(\int \frac{(-1+x)x}{1+x}\,dx, x, \sqrt{a+bx}\right)}{b} \\ &= \frac{2\text{Subst}\left(\int \left(-2+x+\frac{2}{1+x}\right)\,dx, x, \sqrt{a+bx}\right)}{b} \\ &= x-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(1+\sqrt{a+bx}\right)}{b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = \frac{a+bx-4\sqrt{a+bx}+4\log\left(b\left(1+\sqrt{a+bx}\right)\right)}{b}$$

```
[In] Integrate[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]
[Out] (a + b*x - 4*Sqrt[a + b*x] + 4*Log[b*(1 + Sqrt[a + b*x])])/b
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{bx+a-4\sqrt{bx+a}+4\ln\left(1+\sqrt{bx+a}\right)}{b}$	35
default	$\frac{bx+a-4\sqrt{bx+a}+4\ln\left(1+\sqrt{bx+a}\right)}{b}$	35

```
[In] int((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/b*(1/2*b*x+1/2*a-2*(b*x+a)^(1/2)+2*ln(1+(b*x+a)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = \frac{bx-4\sqrt{bx+a}+4\log\left(\sqrt{bx+a}+1\right)}{b}$$

```
[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="fricas")
[Out] (b*x - 4*sqrt(b*x + a) + 4*log(sqrt(b*x + a) + 1))/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = \begin{cases} x-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(\sqrt{a+bx}+1\right)}{b} & \text{for } b\neq 0 \\ \frac{x(\sqrt{a}-1)}{\sqrt{a}+1} & \text{otherwise} \end{cases}$$

```
[In] integrate((-1+(b*x+a)**(1/2))/(1+(b*x+a)**(1/2)),x)
[Out] Piecewise((x - 4*sqrt(a + b*x)/b + 4*log(sqrt(a + b*x) + 1)/b, Ne(b, 0)), (
x*(sqrt(a) - 1)/(sqrt(a) + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = \frac{bx+a-4\sqrt{bx+a}+4\,\log\left(\sqrt{bx+a}+1\right)}{b}$$

```
[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="maxima")
[Out] (b*x + a - 4*sqrt(b*x + a) + 4*log(sqrt(b*x + a) + 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = \frac{4\,\log\left(\sqrt{bx+a}+1\right)}{b} + \frac{(bx+a)b-4\sqrt{bx+a}b}{b^2}$$

```
[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="giac")
[Out] 4*log(sqrt(b*x + a) + 1)/b + ((b*x + a)*b - 4*sqrt(b*x + a)*b)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = x + \frac{4\,\ln\left(\sqrt{a+b\,x}+1\right)}{b} - \frac{4\,\sqrt{a+b\,x}}{b}$$

```
[In] int(((a + b*x)^(1/2) - 1)/((a + b*x)^(1/2) + 1),x)
[Out] x + (4*log((a + b*x)^(1/2) + 1))/b - (4*(a + b*x)^(1/2))/b
```

3.611 $\int \frac{a+bnx^{-1+n}}{ax+bx^n} dx$

Optimal result

Integrand size = 22, antiderivative size = 10

$$\int \frac{a+bnx^{-1+n}}{ax+bx^n} dx = \log(ax+bx^n)$$

```
[Out] ln(a*x+b*x^n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 528, 457, 78}

$$\int \frac{a+bnx^{-1+n}}{ax+bx^n} dx = \log\left(ax^{1-n}+b\right)+n\log(x)$$

```
[In] Int[(a + b*n*x^(-1 + n))/(a*x + b*x^n),x]
```

```
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx \\
&= \int \frac{bn+ax^{1-n}}{x\left(b+ax^{1-n}\right)}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{bn+ax}{x(b+ax)}\,dx,x,x^{1-n}\right)}{1-n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{n}{x}+\frac{a-an}{b+ax}\right)\,dx,x,x^{1-n}\right)}{1-n} \\
&= n\log(x)+\log\left(b+ax^{1-n}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$
\int \frac{a+bnx^{-1+n}}{ax+bx^n}\,dx = \log\left(ax+bx^n\right)
$$

```
[In] Integrate[(a + b*n*x^(-1 + n))/(a*x + b*x^n),x]
[Out] Log[a*x + b*x^n]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

method	result	size
risch	$\ln\left(x^{n}+\frac{ax}{b}\right)$	12
norman	$\ln\left(ax+b\,{\mathrm e}^{n\ln\left(x\right)}\right)$	13

```
[In] int((a+b*n*x^(-1+n))/(a*x+b*x^n),x,method=_RETURNVERBOSE)
[Out] ln(x^n+a*x/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+bnx^{-1+n}}{ax+bx^n}\,dx = \log\left(ax+bx^n\right)$$

```
[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="fricas")
[Out] log(a*x + b*x^n)
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{a+bnx^{-1+n}}{ax+bx^n}\,dx = \begin{cases} \log\left(x+\frac{bx^n}{a}\right) & \text{for } a\neq 0 \\ n\log\left(x\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*n*x**(-1+n))/(a*x+b*x**n),x)
[Out] Piecewise((log(x + b*x**n/a), Ne(a, 0)), (n*log(x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a + bnx^{-1+n}}{ax + bx^n} \, dx = \log \left(ax + bx^n \right)$$

```
[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="maxima")
[Out] log(a*x + b*x^n)
```

Giac [F]

$$\int \frac{a + bnx^{-1+n}}{ax + bx^n} \, dx = \int \frac{bnx^{n-1} + a}{ax + bx^n} \, dx$$

```
[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="giac")
[Out] integrate((b*n*x^(n - 1) + a)/(a*x + b*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 17.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a + bnx^{-1+n}}{ax + bx^n} \, dx = \ln \left(b\, x^n + a\, x \right)$$

```
[In] int((a + b*n*x^(n - 1))/(b*x^n + a*x),x)
[Out] log(b*x^n + a*x)
```

3.612 $\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}} dx$

Optimal result

Integrand size = 29, antiderivative size = 17

$$\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}} dx = n\log(x) + \log\left(b+ax^{1-n}\right)$$

```
[Out] n*ln(x)+ln(b+a*x^(1-n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {528, 457, 78}

$$\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}} dx = \log\left(ax^{1-n}+b\right) + n\log(x)$$

```
[In] Int[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]
```

```
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{bn+ax^{1-n}}{x\left(b+ax^{1-n}\right)}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{bn+ax}{x(b+ax)}\,dx,x,x^{1-n}\right)}{1-n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{n}{x}+\frac{a-an}{b+ax}\right)\,dx,x,x^{1-n}\right)}{1-n} \\
&= n\log(x)+\log\left(b+ax^{1-n}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{x^{-n}\left(a+bnx^{-1+n}\right)}{b+ax^{1-n}}\,dx = \log\left(ax+bx^n\right)$$

```
[In] Integrate[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]
[Out] Log[a*x + b*x^n]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
risch	$\ln\left(x^n+\frac{ax}{b}\right)$	12
norman	$\ln\left(ax+b\,{\mathrm e}^{n\ln(x)}\right)$	13

```
[In] int((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x^n+a*x/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx = \log\left(ax+bx^n\right)$$

```
[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="fricas")
[Out] log(a*x + b*x^n)
```

Sympy [A] (verification not implemented)

Time = 12.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx = \begin{cases} n\log(x)+\log(x)+\log\left(x^{-n}+\frac{b}{ax}\right) & \text{for } a\neq 0 \\ n\log(x) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*n*x**(-1+n))/(x**n)/(b+a*x**(1-n)),x)
[Out] Piecewise((n*log(x) + log(x) + log(x**(-n) + b/(a*x)), Ne(a, 0)), (n*log(x)
, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(17) = 34.

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 5.06

$$\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx = bn\left(\frac{\log(x)}{b}-\frac{n\log(x)}{b(n-1)}+\frac{\log\left(\frac{ax+bx^n}{b}\right)}{b(n-1)}\right) + a\left(\frac{n\log(x)}{a(n-1)}-\frac{\log\left(\frac{ax+bx^n}{b}\right)}{a(n-1)}\right)$$

```
[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="maxima")
[Out] b*n*(log(x)/b - n*log(x)/(b*(n - 1)) + log((a*x + b*x^n)/b)/(b*(n - 1))) +
a*(n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n - 1)))
```

Giac [F]

$$\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx=\int \frac{bnx^{n-1}+a}{(ax^{-n+1}+b)x^n}\,dx$$

```
[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="giac")
[Out] integrate((b*n*x^(n - 1) + a)/((a*x^(-n + 1) + b)*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 17.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx=-\frac{\ln\left(b+a\,x^{1-n}\right)-2\,n\,\mathrm{atanh}\left(\frac{2\,a\,x^{1-n}}{b}+1\right)}{n-1}$$

```
[In] int((a + b*n*x^(n - 1))/(x^n*(b + a*x^(1 - n))),x)
[Out] -(log(b + a*x^(1 - n)) - 2*n*atanh((2*a*x^(1 - n))/b + 1))/(n - 1)
```

3.613 $\int x(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (2ad+(3b$

Optimal result

Integrand size = 176, antiderivative size = 37

$$\int x(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (2ad+(3bd+3ae+bdm+aen)x \\ +(4cd+4be+4af+2cdm+bem+ben+2afn)x^2 \\ +(5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3 \\ +(6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5)\,dx = x^2(a+bx \\ +cx^2)^{1+m}(d+ex+fx^2+gx^3)^{1+n}$$

```
[Out] x^2*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$, Rules used = {1604}

$$\int x(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (2ad+(3bd+3ae+bdm+aen)x \\ +(4cd+4be+4af+2cdm+bem+ben+2afn)x^2 \\ +(5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3 \\ +(6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5)\,dx = x^2(a+bx \\ +cx^2)^{m+1}(d+ex+fx^2+gx^3)^{n+1}$$

```
[In] Int[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*d + 3*a
*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b*e*m + b*e*n +
2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n +
 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 +
 c*g*(7 + 2*m + 3*n)*x^5),x]
```

```
[Out] x^2*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = x^2(a+bx+cx^2)^{1+m}\left(d+ex+fx^2+gx^3\right)^{1+n}$$

Mathematica [A] (verified)

Time = 3.89 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x(a+bx+cx^2)^m\left(d+ex+fx^2+gx^3\right)^n\left(2ad+(3bd+3ae+bdm+aen)x+(4cd+4be+4af+2cdm+bem+ben+2afn)x^2+(5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3+(6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5\right)dx = x^2(a+x(b+cx))^{1+m}(d+x(e+x(f+gx)))^{1+n}$$

```
[In] Integrate[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*d
 + 3*a*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b*e*m + b*
e*n + 2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b
*f*n + 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)
*x^4 + c*g*(7 + 2*m + 3*n)*x^5),x]
```

```
[Out] x^2*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

Maple [A] (verified)

Time = 56.81 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result
gosper	$x^2(c\,x^2+bx+a)^{1+m}\left(g\,x^3+f\,x^2+ex+d\right)^{1+n}$
risch	$x^2(cg\,x^5+bg\,x^4+cf\,x^4+ag\,x^3+bf\,x^3+c\,x^3e+af\,x^2+e\,x^2b+x^2cd+aex+bdx+ad)$ (
parallelrisch	$\frac{x^7(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+d)^n c^2g^2+x^6(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+d)^n bc\,g^2+x^6(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+}{}$

```
[In] int(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e+3*b*d
)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*m+2*b*
f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+6*b
*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x,method=_RETURNVERBOSE)
```

```
[Out] x^2*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Fricas [F(-1)]

Timed out.

$$\int x(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(2ad+(3bd+3ae+bdm+aen)x+(4cd+4be+4af+2cdm+bem+ben+2afn)x^2+(5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3+(6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5)\,dx=\text{Timed out}$$

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int x(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(2ad+(3bd+3ae+bdm+aen)x+(4cd+4be+4af+2cdm+bem+ben+2afn)x^2+(5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3+(6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5)\,dx=\text{Timed out}$$

```
[In] integrate(x*(c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(2*a*d+(a*e*n+b*d*m+
3*a*e+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x**2+(3*a*g*
n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*
m+2*c*f*n+6*b*g+6*c*f)*x**4+c*g*(7+2*m+3*n)*x**5),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(37) = 74.

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.62

$$\int x(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (2ad+(3bd+3ae+bdm+aen)x + (4cd+4be+4af+2cdm+bem+ben+2afn)x^2 + (5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3 + (6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5)\,dx = (cgx^7+(cf+bg)x^6+(ce+bf+ag)x^5+(cd+be+af)x^4+adx^2+(bd+ae)x^3)e^{(n\log(gx^3+fx^2+ex+d)+m}$$

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="maxima")
```

```
[Out] (c*g*x^7 + (c*f + b*g)*x^6 + (c*e + b*f + a*g)*x^5 + (c*d + b*e + a*f)*x^4
+ a*d*x^2 + (b*d + a*e)*x^3)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^
2 + b*x + a))
```

Giac [F(-1)]

Timed out.

$$\int x(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (2ad+(3bd+3ae+bdm+aen)x + (4cd+4be+4af+2cdm+bem+ben+2afn)x^2 + (5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3 + (6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5)\,dx = \text{Timed out}$$

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n \left(2ad+(3bd+3ae+bdm+aen)x + (4cd+4be+4af+2cdm+bem+ben+2afn)x^2 + (5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3 + (6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5\right)\,dx = \text{Hanged}$$

```
[In] int(x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + x^4*(6*b*g +
 6*c*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(4*a*f + 4*b*e + 4*c*d
+ b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x*(3*a*e + 3*b*d + b*d*m + a*e*n) +
x^3*(5*a*g + 5*b*f + 5*c*e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) +
 c*g*x^5*(2*m + 3*n + 7)),x)
```

```
[Out] \text{Hanged}
```

3.614 $\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (ad+(2bd$

Optimal result

Integrand size = 174, antiderivative size = 35

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n \left(ad+(2bd+2ae+bdm+aen)x + (3cd+3be+3af+2cdm+bem+ben+2afn)x^2 + (4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3 + (5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5\right)\,dx = x(a+bx+cx^2)^{1+m}(d+ex+fx^2+gx^3)^{1+n}$$

```
[Out] x*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.006, Rules used = {1604}

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n \left(ad+(2bd+2ae+bdm+aen)x + (3cd+3be+3af+2cdm+bem+ben+2afn)x^2 + (4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3 + (5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5\right)\,dx = x(a+bx+cx^2)^{m+1}(d+ex+fx^2+gx^3)^{n+1}$$

```
[In] Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d + 2*a*e +
 b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*
f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a
*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g
*(6 + 2*m + 3*n)*x^5),x]
```

```
[Out] x*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = x\left(a+bx+cx^2\right)^{1+m}\left(d+ex+fx^2+gx^3\right)^{1+n}$$

Mathematica [A] (verified)

Time = 10.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(ad+(2bd+2ae+bdm+aen)x+(3cd+3be+3af+2cdm+bem+ben+2afn)x^2+(4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3+(5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5\right)dx=x(a+x(b+cx))^{1+m}(d+x(e+x(f+gx)))^{1+n}$$

```
[In] Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d + 2
*a*e + b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m + b*e*n
+ 2*a*f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n
 + 3*a*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4
 + c*g*(6 + 2*m + 3*n)*x^5),x]
```

```
[Out] x*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

Maple [A] (verified)

Time = 43.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result
gosper	$x(c\,x^2+bx+a)^{1+m}\left(g\,x^3+f\,x^2+ex+d\right)^{1+n}$
risch	$x(cg\,x^5+bg\,x^4+cf\,x^4+ag\,x^3+bf\,x^3+c\,x^3e+af\,x^2+e\,x^2b+x^2cd+aex+bdx+ad)\,(c$
parallelrisch	$\frac{x^6(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+d)^n c^2g^2+x^5(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+d)^n bc\,g^2+x^5(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+d)}{}$

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b*d)*x+
(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+
2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5*b*g+5
*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x,method=_RETURNVERBOSE)
```

```
[Out] x*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Fricas [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(ad+(2bd+2ae+bdm+aen)x\right.$$
$$+(3cd+3be+3af+2cdm+bem+ben+2afn)x^2$$
$$+(4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3$$
$$\left.+(5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5\right)\,dx=\text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(ad+(2bd+2ae+bdm+aen)x\right.$$
$$+(3cd+3be+3af+2cdm+bem+ben+2afn)x^2$$
$$+(4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3$$
$$\left.+(5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5\right)\,dx=\text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(a*d+(a*e*n+b*d*m+2*a*
e+2*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x**2+(3*a*g*n+b*
f*m+2*b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*
c*f*n+5*b*g+5*c*f)*x**4+c*g*(6+2*m+3*n)*x**5),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(35) = 70.

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.71

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (ad+(2bd+2ae+bdm+aen)x + (3cd+3be+3af+2cdm+bem+ben+2afn)x^2 + (4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3 + (5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5)\,dx = (cgx^6+(cf+bg)x^5+(ce+bf+ag)x^4+(cd+be+af)x^3+adx+(bd+ae)x^2)e^{(n\log(gx^3+fx^2+ex+d)+m\log}$$

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="maxima")
```

```
[Out] (c*g*x^6 + (c*f + b*g)*x^5 + (c*e + b*f + a*g)*x^4 + (c*d + b*e + a*f)*x^3
+ a*d*x + (b*d + a*e)*x^2)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2
+ b*x + a))
```

Giac [F(-1)]

Timed out.

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (ad+(2bd+2ae+bdm+aen)x + (3cd+3be+3af+2cdm+bem+ben+2afn)x^2 + (4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3 + (5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5)\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(ad+(2bd+2ae+bdm+aen)x + (3cd+3be+3af+2cdm+bem+ben+2afn)x^2 + (4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3 + (5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5\right)\,dx = \text{Hanged}$$

```
[In] int((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + x^4*(5*b*g + 5*c
*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(3*a*f + 3*b*e + 3*c*d + b*
e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x*(2*a*e + 2*b*d + b*d*m + a*e*n) + x^3*
(4*a*g + 4*b*f + 4*c*e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g
*x^5*(2*m + 3*n + 6)),x)
```

```
[Out] \text{Hanged}
```

3.615 $\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+$

Optimal result

Integrand size = 164, antiderivative size = 34

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4)\,dx=(a+bx+cx^2)^{1+m}(d+ex+fx^2+gx^3)^{1+n}$$

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$, Rules used = {1604}

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4)\,dx=(a+bx+cx^2)^{m+1}(d+ex+fx^2+gx^3)^{n+1}$$

```
[In] Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d*m + a*
e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x + (3*c*
e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^2 + (4*c
*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*(5 + 2*m + 3*n)
*x^4),x]
```

```
[Out] (a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\text{integral} = \left(a+bx+cx^2\right)^{1+m}\left(d+ex+fx^2+gx^3\right)^{1+n}$$

Mathematica [A] (verified)

Time = 6.96 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\big(bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4\big)\,dx=(a+x(b+cx))^{1+m}(d+x(e+x(f+gx)))^{1+n}$$

```
[In] Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d*
m + a*e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x +
 (3*c*e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^2
+ (4*c*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*(5 + 2*m
+ 3*n)*x^4),x]
```

```
[Out] (a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

Maple [A] (verified)

Time = 32.99 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result
gosper	$\left(c\,x^2+bx+a\right)^{1+m}\left(g\,x^3+f\,x^2+ex+d\right)^{1+n}$
risch	$\left(cg\,x^5+bg\,x^4+cf\,x^4+ag\,x^3+bf\,x^3+c\,x^3e+af\,x^2+e\,x^2b+x^2cd+aex+bdx+ad\right)(c$
parallelrisch	$\frac{x^5(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+d)^n c^2g^2+x^4(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+d)^n bc\,g^2+x^4(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+}{}$

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f*n+b*e
*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+
3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^3+c*g*
(5+2*m+3*n)*x^4),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Fricas [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4\right)\,dx=\text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4\right)\,dx=\text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(b*d+a*e+b*d*m+a*e*n+(
2*a*f*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c
*e*m+c*e*n+3*a*g+3*b*f+3*c*e)*x**2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c
*f)*x**3+c*g*(5+2*m+3*n)*x**4),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(34) = 68$.

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.71

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4)\,dx = (cgx^5+(cf+bg)x^4+(ce+bf+ag)x^3+(cd+be+af)x^2+ad+(bd+ae)x)e^{(n\log(gx^3+fx^2+ex+d)+m\log($$

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 + b*
x + a))
```

Giac [F(-1)]

Timed out.

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4)\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 27.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.35

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4)\,dx = (g\,x^3+f\,x^2+e\,x+d)^n\,\left(x^4\,(b\,g+c\,f)\,(c\,x^2+b\,x+a)^m+x^2\,(c\,x^2+b\,x+a)^m\,(a\,f+b\,e+c\,d)+x^3\,(c\,x^2+b\,x+a)^m\,(a\,g+b\,f+c\,e)+a\,d\,(c\,x^2+b\,x+a)^m+x\,(a\,e+b\,d)\,(c\,x^2+b\,x+a)^m+c\,g\,x^5\,(c\,x^2+b\,x+a)^m\right)$$

```
[In] int((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*e + b*d + x^3*(4*b*g
 + 4*c*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(3*a*g + 3*b*f + 3*c*
e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + x*(2*a*f + 2*b*e + 2*c*d
 + b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + b*d*m + a*e*n + c*g*x^4*(2*m + 3*n
+ 5)),x)
```

```
[Out] (d + e*x + f*x^2 + g*x^3)^n*(x^4*(b*g + c*f)*(a + b*x + c*x^2)^m + x^2*(a +
 b*x + c*x^2)^m*(a*f + b*e + c*d) + x^3*(a + b*x + c*x^2)^m*(a*g + b*f + c*
e) + a*d*(a + b*x + c*x^2)^m + x*(a*e + b*d)*(a + b*x + c*x^2)^m + c*g*x^5*
(a + b*x + c*x^2)^m)
```

3.616 $\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+$

Optimal result

Integrand size = 167, antiderivative size = 37

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2$$

$$=\frac{(a+bx+cx^2)^{1+m}(d+ex+fx^2+gx^3)^{1+n}}{x}$$

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x
```

Rubi [F]

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2$$

$$=\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+$$

```
[In] Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d) + (b*d*m + a*e
*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (2*c*e
+ 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (3*c*f
 + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4 + 2*m + 3*n)*x
^5))/x^2,x]
```

```
[Out] (c*(d + 2*d*m) + b*e*(1 + m + n) + a*f*(1 + 2*n))*Defer[Int][(a + b*x + c*x
^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] - a*d*Defer[Int][((a + b*x + c*x^2)^m
*(d + e*x + f*x^2 + g*x^3)^n)/x^2, x] + (b*d*m + a*e*n)*Defer[Int][((a + b*
x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x, x] + (c*e*(2 + 2*m + n) + b*f*
(2 + m + 2*n) + a*g*(2 + 3*n))*Defer[Int][x*(a + b*x + c*x^2)^m*(d + e*x +
f*x^2 + g*x^3)^n, x] + (c*f*(3 + 2*m + 2*n) + b*g*(3 + m + 3*n))*Defer[Int]
```

```
[x^2*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + c*g*(4 + 2*m + 3
*n)*Defer[Int][x^3*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x]
```

Rubi steps

$$\text{integral} = \int \left(cd\left(1+\frac{2cdm+be(1+m+n)+a(f+2fn)}{cd}\right)(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n - \frac{ad(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n}{x^2} + \frac{(bdm+aen)(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n}{x} + (ce(2+2m+n)+bf(2+m+2n)+ag(2+3n))x(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n + (cf(3+2m+2n)+bg(3+m+3n))x^2(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n + cg(4+2m+3n)x^3(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n\right)dx$$

$$= -\left((ad)\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n}{x^2}dx\right) + (cg(4+2m+3n))\int x^3(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n dx + (bdm+aen)\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n}{x}dx + (c(d+2dm)+be(1+m+n)+af(1+2n))\int (a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n dx + (ce(2+2m+n)+bf(2+m+2n)+ag(2+3n))\int x(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n dx + (cf(3+2m+2n)+bg(3+m+3n))\int x^2(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n dx$$

Mathematica [A] (verified)

Time = 8.77 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2a$$

$$= \frac{(a+x(b+cx))^{1+m}(d+x(e+x(f+gx)))^{1+n}}{x}$$

```
[In] Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d) + (b*d*m
 + a*e*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (
2*c*e + 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 +
(3*c*f + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4 + 2*m +
3*n)*x^5))/x^2,x]
```

```
[Out] ((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x
```

Maple [A] (verified)

Time = 84.55 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result
gosper	$\frac{(c x^2+bx+a)^{1+m} (g x^3+f x^2+ex+d)^{1+n}}{x}$
risch	$\frac{(cg x^5+bg x^4+cf x^4+ag x^3+bf x^3+c x^3e+af x^2+e x^2b+x^2cd+aex+bdx+ad)(c x^2+bx+a)^m (g x^3+f x^2+ex+d)^n}{x}$
parallelrisch	$\frac{x^5 (c x^2+bx+a)^m (g x^3+f x^2+ex+d)^n c^2g^2+x^4 (c x^2+bx+a)^m (g x^3+f x^2+ex+d)^n bc g^2+x^4 (c x^2+bx+a)^m (g x^3+f x^2+ex+}{}$

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c*g*(4+2*m+3*n)*x^5)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2}{}$$

= Timed out

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2}{}$$

= Timed out

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-a*d+(a*e*n+b*d*m)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+2*a*g+2*b*f+2*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x**4+c*g*(4+2*m+3*n)*x**5)/x**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.57

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2a}{}$$

$$= \frac{(cgx^5+(cf+bg)x^4+(ce+bf+ag)x^3+(cd+be+af)x^2+ad+(bd+ae)x)e^{(n\log(gx^3+fx^2+ex+d)+m\log(cx}}{x}$$

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 + b*
x + a))/x
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2a}{}$$

$$= \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 25.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2a}{}$$

$$= \frac{(c\,x^2+b\,x+a)^{m+1}\,(g\,x^3+f\,x^2+e\,x+d)^{n+1}}{x}$$

```
[In] int(((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(x^4*(3*b*g + 3*c*f +
b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) - a*d + x^2*(a*f + b*e + c*d + b*e*m +
```

```
 2*c*d*m + 2*a*f*n + b*e*n) + x*(b*d*m + a*e*n) + x^3*(2*a*g + 2*b*f + 2*c*
e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g*x^5*(2*m + 3*n + 4))
)/x^2,x)
```

```
[Out] ((a + b*x + c*x^2)^(m + 1)*(d + e*x + f*x^2 + g*x^3)^(n + 1))/x
```

3.617 $\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-2ad+(-bd-ae+bdm+aen)x+(2c$

Optimal result

Integrand size = 163, antiderivative size = 37

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn$$

$$=\frac{(a+bx+cx^2)^{1+m}(d+ex+fx^2+gx^3)^{1+n}}{x^2}$$

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x^2
```

Rubi [F]

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn$$

$$=\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2a$$

```
[In] Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d + (-(b*d) - a*
e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c*e + b*f
 + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (2*c*f + 2*b*g
+ 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m + 3*n)*x^5))/x^3,
x]
```

```
[Out] (c*e*(1 + 2*m + n) + b*f*(1 + m + 2*n) + a*g*(1 + 3*n))*Defer[Int][(a + b*x
 + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] - 2*a*d*Defer[Int][((a + b*x +
c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x^3, x] - (b*d*(1 - m) + a*e*(1 - n))
*Defer[Int][((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x^2, x] + (2*
c*d*m + 2*a*f*n + b*e*(m + n))*Defer[Int][((a + b*x + c*x^2)^m*(d + e*x + f
*x^2 + g*x^3)^n)/x, x] + (2*c*f*(1 + m + n) + b*g*(2 + m + 3*n))*Defer[Int]
```

```
[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + c*g*(3 + 2*m + 3*n
)*Defer[Int][x^2*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x]
```

Rubi steps

$$\text{integral} = \int \left(ce\left(1+\frac{ce(2m+n)+bf(1+m+2n)+a(g+3gn)}{ce}\right)(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n - \frac{2ad(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n}{x^3} + \frac{(-bd(1-m)-ae(1-n))(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n}{x^2} + \frac{(2cdm+2afn+be(m+n))(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n}{x} + (2cf(1+m+n)+bg(2+m+3n))x(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n + cg(3+2m+3n)x^2(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n\right) dx$$

$$= -\left((2ad)\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n}{x^3}\,dx\right) + (-bd(1-m)-ae(1-n))\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n}{x^2}\,dx + (cg(3+2m+3n))\int x^2(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n\,dx + (2cdm+2afn+be(m+n))\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n}{x}\,dx + (ce(1+2m+n)+bf(1+m+2n)+ag(1+3n))\int (a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n\,dx + (2cf(1+m+n)+bg(2+m+3n))\int x(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n\,dx$$

Mathematica [A] (verified)

Time = 9.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2af$$

$$= \frac{(a+x(b+cx))^{1+m}(d+x(e+x(f+gx)))^{1+n}}{x^2}$$

```
[In] Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d + (-(b*d
) - a*e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c*e
 + b*f + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (2*c*f +
2*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m + 3*n)*x^5)
)/x^3,x]
```

```
[Out] ((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x^2
```

Maple [A] (verified)

Time = 82.80 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result
gosper	$\frac{(c x^2+bx+a)^{1+m} (g x^3+f x^2+ex+d)^{1+n}}{x^2}$
risch	$\frac{(cg x^5+bg x^4+cf x^4+ag x^3+bf x^3+c x^3 e+af x^2+e x^2 b+x^2 cd+aex+bdx+ad) (c x^2+bx+a)^m (g x^3+f x^2+ex+d)^n}{x^2}$
parallelrisch	$x^5 (c x^2+bx+a)^m (g x^3+f x^2+ex+d)^n c^2 g^2+x^4 (c x^2+bx+a)^m (g x^3+f x^2+ex+d)^n bc g^2+x^4 (c x^2+bx+a)^m (g x^3+f x^2+ex+d)$

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*d)*x+(
2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+a*g+b
*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*m+3*n)
*x^5)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn}{}$$

= Timed out

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn}{}$$

= Timed out

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-2*a*d+(a*e*n+b*d*m-a
*e-b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m
+c*e*n+a*g+b*f+c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x**4+c
*g*(3+2*m+3*n)*x**5)/x**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(37) = 74.

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.57

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2af}{}$$

$$=\frac{(cgx^5+(cf+bg)x^4+(ce+bf+ag)x^3+(cd+be+af)x^2+ad+(bd+ae)x)e^{(n\log(gx^3+fx^2+ex+d)+m\log(}}{x^2}$$

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 + b*
x + a))/x^2
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2af}{}$$

$$= \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 26.02 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.95

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2af}{}$$

$$= (c\,x^2+b\,x+a)^m\,(g\,x^3+f\,x^2+e\,x+d)^n\,(a\,f+b\,e+c\,d+c\,g\,x^3+a\,g\,x+b\,f\,x+c\,e\,x + b\,g\,x^2+c\,f\,x^2) + \frac{(a\,e+b\,d)\,(c\,x^2+b\,x+a)^m\,(g\,x^3+f\,x^2+e\,x+d)^n}{x} + \frac{a\,d\,(c\,x^2+b\,x+a)^m\,(g\,x^3+f\,x^2+e\,x+d)^n}{x^2}$$

```
[In] int(((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(x^4*(2*b*g + 2*c*f +
b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) - 2*a*d - x*(a*e + b*d - b*d*m - a*e*n
) + x^2*(b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x^3*(a*g + b*f + c*e + b*f*m
+ 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g*x^5*(2*m + 3*n + 3)))/x^3,x)
```

```
[Out] (a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*f + b*e + c*d + c*g*x^3
+ a*g*x + b*f*x + c*e*x + b*g*x^2 + c*f*x^2) + ((a*e + b*d)*(a + b*x + c*x^
2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x + (a*d*(a + b*x + c*x^2)^m*(d + e*x + f
*x^2 + g*x^3)^n)/x^2
```

3.618 $\int x^3 \left(a+b\sqrt{c+dx}\right)^2 dx$

Optimal result

Integrand size = 19, antiderivative size = 185

$$\int x^3 \left(a+b\sqrt{c+dx}\right)^2 dx = -\frac{a^2c^3x}{d^3} - \frac{4abc^3(c+dx)^{3/2}}{3d^4} + \frac{c^2(3a^2-b^2c)(c+dx)^2}{2d^4} + \frac{12abc^2(c+dx)^{5/2}}{5d^4} - \frac{c(a^2-b^2c)(c+dx)^3}{d^4} - \frac{12abc(c+dx)^{7/2}}{7d^4} + \frac{(a^2-3b^2c)(c+dx)^4}{4d^4} + \frac{4ab(c+dx)^{9/2}}{9d^4} + \frac{b^2(c+dx)^5}{5d^4}$$

```
[Out] -a^2*c^3*x/d^3-4/3*a*b*c^3*(d*x+c)^(3/2)/d^4+1/2*c^2*(-b^2*c+3*a^2)*(d*x+c)
^2/d^4+12/5*a*b*c^2*(d*x+c)^(5/2)/d^4-c*(-b^2*c+a^2)*(d*x+c)^3/d^4-12/7*a*b
*c*(d*x+c)^(7/2)/d^4+1/4*(-3*b^2*c+a^2)*(d*x+c)^4/d^4+4/9*a*b*(d*x+c)^(9/2)
/d^4+1/5*b^2*(d*x+c)^5/d^4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {378, 1412, 786}

$$\int x^3 \left(a+b\sqrt{c+dx}\right)^2 dx = \frac{c^2(3a^2-b^2c)(c+dx)^2}{2d^4} + \frac{(a^2-3b^2c)(c+dx)^4}{4d^4} - \frac{c(a^2-b^2c)(c+dx)^3}{d^4} - \frac{a^2c^3x}{d^3} - \frac{4abc^3(c+dx)^{3/2}}{3d^4} + \frac{12abc^2(c+dx)^{5/2}}{5d^4} + \frac{4ab(c+dx)^{9/2}}{9d^4} - \frac{12abc(c+dx)^{7/2}}{7d^4} + \frac{b^2(c+dx)^5}{5d^4}$$

```
[In] Int[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] -((a^2*c^3*x)/d^3) - (4*a*b*c^3*(c + d*x)^(3/2))/(3*d^4) + (c^2*(3*a^2 - b^
2*c)*(c + d*x)^2)/(2*d^4) + (12*a*b*c^2*(c + d*x)^(5/2))/(5*d^4) - (c*(a^2
- b^2*c)*(c + d*x)^3)/d^4 - (12*a*b*c*(c + d*x)^(7/2))/(7*d^4) + ((a^2 - 3*
b^2*c)*(c + d*x)^4)/(4*d^4) + (4*a*b*(c + d*x)^(9/2))/(9*d^4) + (b^2*(c + d
*x)^5)/(5*d^4)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^2(-c+x)^3\,dx,x,c+dx\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int x(a+bx)^2\left(-c+x^2\right)^3\,dx,x,\sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int \left(-a^2c^3x-2abc^3x^2-c^2\left(-3a^2+b^2c\right)x^3+6abc^2x^4+3c\left(-a^2+b^2c\right)x^5-6abcx^6+\left(a^2-\right.\right.}{d^4}$$

$$= -\frac{a^2c^3x}{d^3}-\frac{4abc^3(c+dx)^{3/2}}{3d^4}+\frac{c^2\left(3a^2-b^2c\right)(c+dx)^2}{2d^4} + \frac{12abc^2(c+dx)^{5/2}}{5d^4}-\frac{c\left(a^2-b^2c\right)(c+dx)^3}{d^4}-\frac{12abc(c+dx)^{7/2}}{7d^4} + \frac{\left(a^2-3b^2c\right)(c+dx)^4}{4d^4}+\frac{4ab(c+dx)^{9/2}}{9d^4}+\frac{b^2(c+dx)^5}{5d^4}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.57

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx = -\frac{16ab\sqrt{c+dx}(16c^4-8c^3dx+6c^2d^2x^2-5cd^3x^3-35d^4x^4)+315a^2(c^4-d^4x^4)+63b^2(c^5-5cd^4x^4-4d}{1260d^4}$$

```
[In] Integrate[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] -1/1260*(16*a*b*Sqrt[c + d*x]*(16*c^4 - 8*c^3*d*x + 6*c^2*d^2*x^2 - 5*c*d^3
*x^3 - 35*d^4*x^4) + 315*a^2*(c^4 - d^4*x^4) + 63*b^2*(c^5 - 5*c*d^4*x^4 -
4*d^5*x^5))/d^4
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.42

method	result
default	$b^2\left(\frac{1}{5}x^5d+\frac{1}{4}c\,x^4\right)+\frac{4ab\left(\frac{(dx+c)^{\frac{9}{2}}}{9}-\frac{3c(dx+c)^{\frac{7}{2}}}{7}+\frac{3c^2(dx+c)^{\frac{5}{2}}}{5}-\frac{c^3(dx+c)^{\frac{3}{2}}}{3}\right)}{d^4}+\frac{a^2x^4}{4}$
trager	$\frac{(4b^2dx+5b^2c+5a^2)x^4}{20}-\frac{4ab(-35d^4x^4-5d^3c\,x^3+6c^2x^2d^2-8c^3dx+16c^4)\sqrt{dx+c}}{315d^4}$
derivativedivides	$\frac{\frac{b^2(dx+c)^5}{5}+\frac{4ab(dx+c)^{\frac{9}{2}}}{9}+\frac{\left(-3b^2c+a^2\right)(dx+c)^4}{4}-\frac{12cab(dx+c)^{\frac{7}{2}}}{7}+\frac{\left(3b^2c^2-3c\,a^2\right)(dx+c)^3}{3}+\frac{12c^2ab(dx+c)^{\frac{5}{2}}}{5}+\frac{\left(-b^2c^3+3a^2c^2\right)}{2}}{d^4}$

```
[In] int(x^3*(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/5*x^5*d+1/4*c*x^4)+4*a*b/d^4*(1/9*(d*x+c)^(9/2)-3/7*c*(d*x+c)^(7/2)+
3/5*c^2*(d*x+c)^(5/2)-1/3*c^3*(d*x+c)^(3/2))+1/4*a^2*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.51

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx = \frac{252\,b^2d^5x^5+315\,(b^2c+a^2)d^4x^4+16\,(35\,abd^4x^4+5\,abcd^3x^3-6\,abc^2d^2x^2+8\,abc^3dx-16\,abc^4)\sqrt{dx+c}}{1260\,d^4}$$

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

[Out] 1/1260*(252*b^2*d^5*x^5 + 315*(b^2*c + a^2)*d^4*x^4 + 16*(35*a*b*d^4*x^4 + 5*a*b*c*d^3*x^3 - 6*a*b*c^2*d^2*x^2 + 8*a*b*c^3*d*x - 16*a*b*c^4)*sqrt(d*x + c))/d^4

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.96

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx = \begin{cases} \frac{2\left(-\frac{a^2c^3(c+dx)}{2}-\frac{2abc^3(c+dx)^{\frac{3}{2}}}{3}+\frac{6abc^2(c+dx)^{\frac{5}{2}}}{5}-\frac{6abc(c+dx)^{\frac{7}{2}}}{7}+\frac{2ab(c+dx)^{\frac{9}{2}}}{9}+\frac{b^2(c+dx)^5}{10}+\frac{(a^2-3b^2c)(c+dx)^4}{8}+\frac{(c+dx)^3(-3a^2c+3b^2c^2)}{6}+\frac{(c+dx)^2\cdot(}{} \right.}{d^4} \\ \frac{x^4(a+b\sqrt{c})^2}{4} \end{cases}$$

[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**2,x)

[Out] Piecewise((2*(-a**2*c**3*(c + d*x)/2 - 2*a*b*c**3*(c + d*x)**(3/2)/3 + 6*a*b*c**2*(c + d*x)**(5/2)/5 - 6*a*b*c*(c + d*x)**(7/2)/7 + 2*a*b*(c + d*x)**(9/2)/9 + b**2*(c + d*x)**5/10 + (a**2 - 3*b**2*c)*(c + d*x)**4/8 + (c + d*x)**3*(-3*a**2*c + 3*b**2*c**2)/6 + (c + d*x)**2*(3*a**2*c**2 - b**2*c**3)/4)/d**4, Ne(d, 0)), (x**4*(a + b*sqrt(c))**2/4, True))

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.82

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx = \frac{252\,(dx+c)^5b^2+560\,(dx+c)^{\frac{9}{2}}ab-2160\,(dx+c)^{\frac{7}{2}}abc+3024\,(dx+c)^{\frac{5}{2}}abc^2-1680\,(dx+c)^{\frac{3}{2}}abc^3-1260}{1260\,d^{}}$$

[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] 1/1260*(252*(d*x + c)^5*b^2 + 560*(d*x + c)^(9/2)*a*b - 2160*(d*x + c)^(7/2)*a*b*c + 3024*(d*x + c)^(5/2)*a*b*c^2 - 1680*(d*x + c)^(3/2)*a*b*c^3 - 1260*(d*x + c)*a^2*c^3 - 315*(3*b^2*c - a^2)*(d*x + c)^4 + 1260*(b^2*c^2 - a^2*c)*(d*x + c)^3 - 630*(b^2*c^3 - 3*a^2*c^2)*(d*x + c)^2)/d^4

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.82

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{252\,b^2d^2x^5+315\,b^2cdx^4+315\,a^2dx^4+\frac{144\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}c^3\right)abc}{d^3}+\frac{16\left(35\,(dx+c)^{\frac{9}{2}}-180\right.}{}}{1260\,d}$$

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/1260*(252*b^2*d^2*x^5 + 315*b^2*c*d*x^4 + 315*a^2*d*x^4 + 144*(5*(d*x + c
)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*
c^3)*a*b*c/d^3 + 16*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x
+ c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b/d^3)/
d
```

Mupad [B] (verification not implemented)

Time = 17.65 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{b^2\,(c+d\,x)^5}{5\,d^4}-\frac{(6\,b^2\,c-2\,a^2)\,(c+d\,x)^4}{8\,d^4}+\frac{(6\,a^2\,c^2-2\,b^2\,c^3)\,(c+d\,x)^2}{4\,d^4}-\frac{a^2\,c^3\,x}{d^3}+\frac{4\,a\,b\,(c+d\,x)^{9/2}}{9\,d^4}+\frac{c\,(b^2\,c-a^2)\,(c+d\,x)^3}{d^4}-\frac{4\,a\,b\,c^3\,(c+d\,x)^{3/2}}{3\,d^4}+\frac{12\,a\,b\,c^2\,(c+d\,x)^{5/2}}{5\,d^4}-\frac{12\,a\,b\,c\,(c+d\,x)^{7/2}}{7\,d^4}$$

```
[In] int(x^3*(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (b^2*(c + d*x)^5)/(5*d^4) - ((6*b^2*c - 2*a^2)*(c + d*x)^4)/(8*d^4) + ((6*a
^2*c^2 - 2*b^2*c^3)*(c + d*x)^2)/(4*d^4) - (a^2*c^3*x)/d^3 + (4*a*b*(c + d*
x)^(9/2))/(9*d^4) + (c*(b^2*c - a^2)*(c + d*x)^3)/d^4 - (4*a*b*c^3*(c + d*x
)^(3/2))/(3*d^4) + (12*a*b*c^2*(c + d*x)^(5/2))/(5*d^4) - (12*a*b*c*(c + d*
x)^(7/2))/(7*d^4)
```

3.619 $\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 138

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}-\frac{c(2a^2-b^2c)\,(c+dx)^2}{2d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{(a^2-2b^2c)\,(c+dx)^3}{3d^3}+\frac{4ab(c+dx)^{7/2}}{7d^3}+\frac{b^2(c+dx)^4}{4d^3}$$

```
[Out] a^2*c^2*x/d^2+4/3*a*b*c^2*(d*x+c)^(3/2)/d^3-1/2*c*(-b^2*c+2*a^2)*(d*x+c)^2/
d^3-8/5*a*b*c*(d*x+c)^(5/2)/d^3+1/3*(-2*b^2*c+a^2)*(d*x+c)^3/d^3+4/7*a*b*(d
*x+c)^(7/2)/d^3+1/4*b^2*(d*x+c)^4/d^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {378, 1412, 786}

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{(a^2-2b^2c)\,(c+dx)^3}{3d^3}-\frac{c(2a^2-b^2c)\,(c+dx)^2}{2d^3}+\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}+\frac{4ab(c+dx)^{7/2}}{7d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{b^2(c+dx)^4}{4d^3}$$

```
[In] Int[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (a^2*c^2*x)/d^2 + (4*a*b*c^2*(c + d*x)^(3/2))/(3*d^3) - (c*(2*a^2 - b^2*c)*
(c + d*x)^2)/(2*d^3) - (8*a*b*c*(c + d*x)^(5/2))/(5*d^3) + ((a^2 - 2*b^2*c)
```

```
*(c + d*x)^3)/(3*d^3) + (4*a*b*(c + d*x)^(7/2))/(7*d^3) + (b^2*(c + d*x)^4)
/(4*d^3)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^2 (-c+x)^2\,dx, x, c+dx\right)}{d^3}\\
&= \frac{2\text{Subst}\left(\int x(a+bx)^2 \left(-c+x^2\right)^2\,dx, x, \sqrt{c+dx}\right)}{d^3}\\
&= \frac{2\text{Subst}\left(\int \left(a^2c^2x+2abc^2x^2+c\left(-2a^2+b^2c\right)x^3-4abcx^4+\left(a^2-2b^2c\right)x^5+2abx^6+b^2x^7\right)\,dx, x, \cdot\right.}{d^3}\\
&= \frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}-\frac{c\left(2a^2-b^2c\right)(c+dx)^2}{2d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}\\
&\quad+\frac{\left(a^2-2b^2c\right)(c+dx)^3}{3d^3}+\frac{4ab(c+dx)^{7/2}}{7d^3}+\frac{b^2(c+dx)^4}{4d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx = \frac{140a^2(c^3+d^3x^3)+16ab\sqrt{c+dx}(8c^3-4c^2dx+3cd^2x^2+15d^3x^3)+35b^2(c^4+4cd^3x^3+3d^4x^4)}{420d^3}$$

```
[In] Integrate[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (140*a^2*(c^3 + d^3*x^3) + 16*a*b*Sqrt[c + d*x]*(8*c^3 - 4*c^2*d*x + 3*c*d^
2*x^2 + 15*d^3*x^3) + 35*b^2*(c^4 + 4*c*d^3*x^3 + 3*d^4*x^4))/(420*d^3)
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.48

method	result	size
default	$b^2\left(\frac{1}{4}d\,x^4+\frac{1}{3}c\,x^3\right)+\frac{4ab\left(\frac{(dx+c)^{\frac{7}{2}}}{7}-\frac{2c(dx+c)^{\frac{5}{2}}}{5}+\frac{c^2(dx+c)^{\frac{3}{2}}}{3}\right)}{d^3}+\frac{a^2x^3}{3}$	66
trager	$\frac{(3b^2dx+4b^2c+4a^2)x^3}{12}+\frac{4ab(15d^3x^3+3c\,d^2x^2-4c^2dx+8c^3)\sqrt{dx+c}}{105d^3}$	70
derivativedivides	$\frac{\frac{b^2(dx+c)^4}{4}+\frac{4ab(dx+c)^{\frac{7}{2}}}{7}+\frac{\left(-2b^2c+a^2\right)(dx+c)^3}{3}-\frac{8cab(dx+c)^{\frac{5}{2}}}{5}+\frac{\left(b^2c^2-2c\,a^2\right)(dx+c)^2}{2}+\frac{4c^2ab(dx+c)^{\frac{3}{2}}}{3}+c^2a^2(dx+c)}{d^3}$	111

```
[In] int(x^2*(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/4*d*x^4+1/3*c*x^3)+4*a*b/d^3*(1/7*(d*x+c)^(7/2)-2/5*c*(d*x+c)^(5/2)+
1/3*c^2*(d*x+c)^(3/2))+1/3*a^2*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.59

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx = \frac{105\,b^2d^4x^4+140\,(b^2c+a^2)d^3x^3+16\,(15\,abd^3x^3+3\,abcd^2x^2-4\,abc^2dx+8\,abc^3)\sqrt{dx+c}}{420\,d^3}$$

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/420*(105*b^2*d^4*x^4 + 140*(b^2*c + a^2)*d^3*x^3 + 16*(15*a*b*d^3*x^3 + 3
*a*b*c*d^2*x^2 - 4*a*b*c^2*d*x + 8*a*b*c^3)*sqrt(d*x + c))/d^3
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\begin{cases}\frac{2\left(\frac{a^2c^2(c+dx)}{2}+\frac{2abc^2(c+dx)^{\frac{3}{2}}}{3}-\frac{4abc(c+dx)^{\frac{5}{2}}}{5}+\frac{2ab(c+dx)^{\frac{7}{2}}}{7}+\frac{b^2(c+dx)^4}{8}+\frac{(a^2-2b^2c)(c+dx)^3}{6}+\frac{(c+dx)^2(-2a^2c+b^2c^2)}{4}\right)}{d^3} & \text{for } d\neq 0\\ \frac{x^3\left(a+b\sqrt{c}\right)^2}{3} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((2*(a**2*c**2*(c + d*x)/2 + 2*a*b*c**2*(c + d*x)**(3/2)/3 - 4*a*b
*c*(c + d*x)**(5/2)/5 + 2*a*b*(c + d*x)**(7/2)/7 + b**2*(c + d*x)**4/8 + (a
**2 - 2*b**2*c)*(c + d*x)**3/6 + (c + d*x)**2*(-2*a**2*c + b**2*c**2)/4)/d*
*3, Ne(d, 0)), (x**3*(a + b*sqrt(c))**2/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.81

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{105\,(dx+c)^4b^2+240\,(dx+c)^{\frac{7}{2}}ab-672\,(dx+c)^{\frac{5}{2}}abc+560\,(dx+c)^{\frac{3}{2}}abc^2+420\,(dx+c)a^2c^2-140\,(2\,b^2}{420\,d^3}$$

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/420*(105*(d*x + c)^4*b^2 + 240*(d*x + c)^(7/2)*a*b - 672*(d*x + c)^(5/2)*
a*b*c + 560*(d*x + c)^(3/2)*a*b*c^2 + 420*(d*x + c)*a^2*c^2 - 140*(2*b^2*c
- a^2)*(d*x + c)^3 + 210*(b^2*c^2 - 2*a^2*c)*(d*x + c)^2)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{105\,b^2d^2x^4+140\,b^2cdx^3+140\,a^2dx^3+\frac{112\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}c^2\right)abc}{d^2}+\frac{48\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(d}{d^2}}{420\,d}$$

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/420*(105*b^2*d^2*x^4 + 140*b^2*c*d*x^3 + 140*a^2*d*x^3 + 112*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b*c/d^2 + 48*(5*(d*
x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x
+ c)*c^3)*a*b/d^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{b^2\,{\left(c+d\,x\right)}^4}{4\,d^3}-\frac{\left(4\,a^2\,c-2\,b^2\,c^2\right)\,{\left(c+d\,x\right)}^2}{4\,d^3}-\frac{\left(4\,b^2\,c-2\,a^2\right)\,{\left(c+d\,x\right)}^3}{6\,d^3}+\frac{a^2\,c^2\,x}{d^2}+\frac{4\,a\,b\,{\left(c+d\,x\right)}^{7/2}}{7\,d^3}+\frac{4\,a\,b\,c^2\,{\left(c+d\,x\right)}^{3/2}}{3\,d^3}-\frac{8\,a\,b\,c\,{\left(c+d\,x\right)}^{5/2}}{5\,d^3}$$

```
[In] int(x^2*(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (b^2*(c + d*x)^4)/(4*d^3) - ((4*a^2*c - 2*b^2*c^2)*(c + d*x)^2)/(4*d^3) - (
(4*b^2*c - 2*a^2)*(c + d*x)^3)/(6*d^3) + (a^2*c^2*x)/d^2 + (4*a*b*(c + d*x)
^(7/2))/(7*d^3) + (4*a*b*c^2*(c + d*x)^(3/2))/(3*d^3) - (8*a*b*c*(c + d*x)^
(5/2))/(5*d^3)
```

3.620 $\int x\left(a+b\sqrt{c+dx}\right)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx=-\frac{a^2cx}{d}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{(a^2-b^2c)\,(c+dx)^2}{2d^2}+\frac{4ab(c+dx)^{5/2}}{5d^2}+\frac{b^2(c+dx)^3}{3d^2}$$

```
[Out] -a^2*c*x/d-4/3*a*b*c*(d*x+c)^(3/2)/d^2+1/2*(-b^2*c+a^2)*(d*x+c)^2/d^2+4/5*a
*b*(d*x+c)^(5/2)/d^2+1/3*b^2*(d*x+c)^3/d^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {378, 1412, 786}

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{(a^2-b^2c)\,(c+dx)^2}{2d^2}-\frac{a^2cx}{d}+\frac{4ab(c+dx)^{5/2}}{5d^2}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{b^2(c+dx)^3}{3d^2}$$

```
[In] Int[x*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] -((a^2*c*x)/d) - (4*a*b*c*(c + d*x)^(3/2))/(3*d^2) + ((a^2 - b^2*c)*(c + d*
x)^2)/(2*d^2) + (4*a*b*(c + d*x)^(5/2))/(5*d^2) + (b^2*(c + d*x)^3)/(3*d^2)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
```

```
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^2 (-c+x)\,dx, x, c+dx\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int x(a+bx)^2\left(-c+x^2\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int \left(-a^2cx-2abcx^2+\left(a^2-b^2c\right)x^3+2abx^4+b^2x^5\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= -\frac{a^2cx}{d}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{\left(a^2-b^2c\right)(c+dx)^2}{2d^2}+\frac{4ab(c+dx)^{5/2}}{5d^2}+\frac{b^2(c+dx)^3}{3d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx = \frac{(c+dx)\left(-15a^2(c-dx)-8ab(2c-3dx)\sqrt{c+dx}+5b^2\left(-c^2+cdx+2d^2x^2\right)\right)}{30d^2}$$

```
[In] Integrate[x*(a + b*Sqrt[c + d*x])^2,x]
[Out] ((c + d*x)*(-15*a^2*(c - d*x) - 8*a*b*(2*c - 3*d*x)*Sqrt[c + d*x] + 5*b^2*(
-c^2 + c*d*x + 2*d^2*x^2)))/(30*d^2)
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

method	result	size
default	$b^{2}\left(\frac{1}{3}x^{3}d+\frac{1}{2}c\,x^{2}\right)+\frac{4ab\left(\frac{(dx+c)^{\frac{5}{2}}}{5}-\frac{c(dx+c)^{\frac{3}{2}}}{3}\right)}{d^{2}}+\frac{a^{2}x^{2}}{2}$	54
trager	$\frac{(2b^{2}dx+3b^{2}c+3a^{2})x^{2}}{6}-\frac{4ab(-3d^{2}x^{2}-cdx+2c^{2})\sqrt{dx+c}}{15d^{2}}$	59
derivativedivides	$\frac{\frac{b^{2}(dx+c)^{3}}{3}+\frac{4ab(dx+c)^{\frac{5}{2}}}{5}+\frac{\left(-b^{2}c+a^{2}\right)(dx+c)^{2}}{2}-\frac{4cab(dx+c)^{\frac{3}{2}}}{3}-c\,a^{2}(dx+c)}{d^{2}}$	72

```
[In] int(x*(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/3*x^3*d+1/2*c*x^2)+4*a*b/d^2*(1/5*(d*x+c)^(5/2)-1/3*c*(d*x+c)^(3/2))
+1/2*a^2*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.75

$$\int x\left(a+b\sqrt{c+dx}\right)^{2}\,dx$$
$$=\frac{10\,b^{2}d^{3}x^{3}+15\,(b^{2}c+a^{2})d^{2}x^{2}+8\,(3\,abd^{2}x^{2}+abcdx-2\,abc^{2})\sqrt{dx+c}}{30\,d^{2}}$$

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^2*d^3*x^3 + 15*(b^2*c + a^2)*d^2*x^2 + 8*(3*a*b*d^2*x^2 + a*b*c*
d*x - 2*a*b*c^2)*sqrt(d*x + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int x\left(a+b\sqrt{c+dx}\right)^{2}\,dx$$
$$=\begin{cases}\frac{2\left(-\frac{a^{2}c(c+dx)}{2}-\frac{2abc(c+dx)^{\frac{3}{2}}}{3}+\frac{2ab(c+dx)^{\frac{5}{2}}}{5}+\frac{b^{2}(c+dx)^{3}}{6}+\frac{\left(a^{2}-b^{2}c\right)(c+dx)^{2}}{4}\right)}{d^{2}} & \text{for } d\neq 0\\ \frac{x^{2}\left(a+b\sqrt{c}\right)^{2}}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((2*(-a**2*c*(c + d*x)/2 - 2*a*b*c*(c + d*x)**(3/2)/3 + 2*a*b*(c +
 d*x)**(5/2)/5 + b**2*(c + d*x)**3/6 + (a**2 - b**2*c)*(c + d*x)**2/4)/d**2
, Ne(d, 0)), (x**2*(a + b*sqrt(c))**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx = \frac{10\,(dx+c)^3b^2+24\,(dx+c)^{\frac{5}{2}}ab-40\,(dx+c)^{\frac{3}{2}}abc-30\,(dx+c)a^2c-15\,(b^2c-a^2)(dx+c)^2}{30\,d^2}$$

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(10*(d*x + c)^3*b^2 + 24*(d*x + c)^(5/2)*a*b - 40*(d*x + c)^(3/2)*a*b*
c - 30*(d*x + c)*a^2*c - 15*(b^2*c - a^2)*(d*x + c)^2)/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.47

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx = \frac{10\,b^2d^2x^3+\frac{40\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}c\right)abc}{d}+\frac{15\left((dx+c)^2-2\,(dx+c)c\right)b^2c}{d}+\frac{15\left((dx+c)^2-2\,(dx+c)c\right)a^2}{d}+\frac{8\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+\right.}{d}}{30\,d}$$

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/30*(10*b^2*d^2*x^3 + 40*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c/d + 1
5*((d*x + c)^2 - 2*(d*x + c)*c)*b^2*c/d + 15*((d*x + c)^2 - 2*(d*x + c)*c)*
a^2/d + 8*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)
*a*b/d)/d
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{b^2\,{\left(c+d\,x\right)}^3}{3\,d^2}-\frac{\left(2\,b^2\,c-2\,a^2\right)\,{\left(c+d\,x\right)}^2}{4\,d^2}+\frac{4\,a\,b\,{\left(c+d\,x\right)}^{5/2}}{5\,d^2}-\frac{a^2\,c\,x}{d}-\frac{4\,a\,b\,c\,{\left(c+d\,x\right)}^{3/2}}{3\,d^2}$$

```
[In] int(x*(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (b^2*(c + d*x)^3)/(3*d^2) - ((2*b^2*c - 2*a^2)*(c + d*x)^2)/(4*d^2) + (4*a*
b*(c + d*x)^(5/2))/(5*d^2) - (a^2*c*x)/d - (4*a*b*c*(c + d*x)^(3/2))/(3*d^2
)
```

3.621 $\int \left(a+b\sqrt{c+dx}\right)^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \left(a+b\sqrt{c+dx}\right)^2\,dx = a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}$$

```
[Out] a^2*x+4/3*a*b*(d*x+c)^(3/2)/d+1/2*b^2*(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {253, 196, 45}

$$\int \left(a+b\sqrt{c+dx}\right)^2\,dx = a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}$$

```
[In] Int[(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] a^2*x + (4*a*b*(c + d*x)^(3/2))/(3*d) + (b^2*(c + d*x)^2)/(2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
```

```
IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^2 \,dx, x, c+dx\right)}{d} \\
&= \frac{2\text{Subst}\left(\int x(a+bx)^2 \,dx, x, \sqrt{c+dx}\right)}{d} \\
&= \frac{2\text{Subst}\left(\int \left(a^2x+2abx^2+b^2x^3\right) \,dx, x, \sqrt{c+dx}\right)}{d} \\
&= a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt{c+dx}\right)^2 \,dx = \frac{(c+dx)\left(6a^2+8ab\sqrt{c+dx}+3b^2(c+dx)\right)}{6d}$$

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2,x]
[Out] ((c + d*x)*(6*a^2 + 8*a*b*Sqrt[c + d*x] + 3*b^2*(c + d*x)))/(6*d)
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

method	result	size
default	$b^2\left(cx+\frac{1}{2}d\,x^2\right)+\frac{4ab(dx+c)^{\frac{3}{2}}}{3d}+a^2x$	35
trager	$\frac{\left(b^2dx+2b^2c+2a^2\right)x}{2}+\frac{4ab(dx+c)^{\frac{3}{2}}}{3d}$	37
derivativedivides	$\frac{\frac{b^2(dx+c)^2}{2}+\frac{4ab(dx+c)^{\frac{3}{2}}}{3}+a^2(dx+c)}{d}$	40

```
[In] int((a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(c*x+1/2*d*x^2)+4/3*a*b*(d*x+c)^(3/2)/d+a^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{3\,b^2d^2x^2+6\,(b^2c+a^2)dx+8\,(abdx+abc)\sqrt{dx+c}}{6\,d}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*d^2*x^2 + 6*(b^2*c + a^2)*d*x + 8*(a*b*d*x + a*b*c)*sqrt(d*x + c
))/d
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.66

$$\int \left(a+b\sqrt{c+dx}\right)^2\,dx=\begin{cases} a^2x+\frac{4abc\sqrt{c+dx}}{3d}+\frac{4abx\sqrt{c+dx}}{3}+b^2cx+\frac{b^2dx^2}{2} & \text{for } d\neq 0\\ x\left(a+b\sqrt{c}\right)^2 & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((a**2*x + 4*a*b*c*sqrt(c + d*x)/(3*d) + 4*a*b*x*sqrt(c + d*x)/3 +
 b**2*c*x + b**2*d*x**2/2, Ne(d, 0)), (x*(a + b*sqrt(c))**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{1}{2}\,(dx^2+2\,cx)b^2+a^2x+\frac{4\,(dx+c)^{\frac{3}{2}}ab}{3\,d}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x^2 + 2*c*x)*b^2 + a^2*x + 4/3*(d*x + c)^(3/2)*a*b/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(35) = 70$.

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \left(a+b\sqrt{c+dx}\right)^2 dx$$
$$= \frac{6\,(dx+c)b^2c+24\,\sqrt{dx+c}abc+6\,(dx+c)a^2+8\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}c\right)ab+3\left((dx+c)^2-2\,(dx+c\right.}{6\,d}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
[Out] 1/6*(6*(d*x + c)*b^2*c + 24*sqrt(d*x + c)*a*b*c + 6*(d*x + c)*a^2 + 8*((d*x
 + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b + 3*((d*x + c)^2 - 2*(d*x + c)*c)*b^2)
/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \left(a+b\sqrt{c+dx}\right)^2 dx = \frac{3\,b^2\,(c+d\,x)^2+8\,a\,b\,(c+d\,x)^{3/2}+6\,a^2\,d\,x}{6\,d}$$

```
[In] int((a + b*(c + d*x)^(1/2))^2,x)
[Out] (3*b^2*(c + d*x)^2 + 8*a*b*(c + d*x)^(3/2) + 6*a^2*d*x)/(6*d)
```

3.622 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx=b^2dx+4ab\sqrt{c+dx}-4ab\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\left(a^2+b^2c\right)\log(x)$$

```
[Out] b^2*d*x+(b^2*c+a^2)*ln(x)-4*a*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a*
b*(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {378, 1412, 815, 649, 213, 266}

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx=\log(x)\left(a^2+b^2c\right)-4ab\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+4ab\sqrt{c+dx}+b^2dx$$

```
[In] Int[(a + b*Sqrt[c + d*x])^2/x,x]
```

```
[Out] b^2*d*x + 4*a*b*Sqrt[c + d*x] - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]
] + (a^2 + b^2*c)*Log[x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2}{-c+x}\,dx, x, c+dx\right)\\
&= 2\text{Subst}\left(\int \frac{x(a+bx)^2}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\\
&= 2\text{Subst}\left(\int \left(2ab+b^2x+\frac{2abc+\left(a^2+b^2c\right)x}{-c+x^2}\right)\,dx, x, \sqrt{c+dx}\right)\\
&= b^2dx+4ab\sqrt{c+dx}+2\text{Subst}\left(\int \frac{2abc+\left(a^2+b^2c\right)x}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\\
&= b^2dx+4ab\sqrt{c+dx}+(4abc)\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\\
&\quad+\left(2\left(a^2+b^2c\right)\right)\text{Subst}\left(\int \frac{x}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)
\end{aligned}$$

$$= b^2dx + 4ab\sqrt{c+dx} - 4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + (a^2+b^2c)\log(x)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = b\left(bc+bdx+4a\sqrt{c+dx}\right) - 4ab\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + (a^2+b^2c)\log(-dx)$$

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x,x]
[Out] b*(b*c + b*d*x + 4*a*Sqrt[c + d*x]) - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/S
qrt[c]] + (a^2 + b^2*c)*Log[-(d*x)]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

method	result	size
default	$b^2(dx+c\ln(x)) + 2ab\left(2\sqrt{dx+c} - 2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\right) + a^2\ln(x)$	51
derivativedivides	$(dx+c)\,b^2 + 4ab\sqrt{dx+c} - (-b^2c-a^2)\ln(-dx) - 4ab\,\text{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\sqrt{c}$	60

```
[In] int((a+b*(d*x+c)^(1/2))^2/x,x,method=_RETURNVERBOSE)
[Out] b^2*(d*x+c*ln(x))+2*a*b*(2*(d*x+c)^(1/2)-2*c^(1/2)*arctanh((d*x+c)^(1/2)/c^
(1/2)))+a^2*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.07

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = \left[b^2dx + 2\,ab\sqrt{c}\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right) + 4\sqrt{dx+c}ab + (b^2c+a^2)\log(x),\, b^2dx + 4\,ab\sqrt{-c}\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right) + 4\sqrt{dx+c}ab + (b^2c+a^2)\log(x)\right]$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="fricas")
```

```
[Out] [b^2*d*x + 2*a*b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 4*s
qrt(d*x + c)*a*b + (b^2*c + a^2)*log(x), b^2*d*x + 4*a*b*sqrt(-c)*arctan(sq
rt(d*x + c)*sqrt(-c)/c) + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(x)]
```

Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.53

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = \begin{cases} \frac{4abc\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 4ab\sqrt{c+dx} + b^2(c+dx) - 2\left(-\frac{a^2}{2} - \frac{b^2c}{2}\right)\log\left(-dx\right) & \text{for}\ d \neq 0 \\ \left(a+b\sqrt{c}\right)^2\log\left(x\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x,x)
```

```
[Out] Piecewise((4*a*b*c*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 4*a*b*sqrt(c + d
*x) + b**2*(c + d*x) - 2*(-a**2/2 - b**2*c/2)*log(-d*x), Ne(d, 0)), ((a + b
*sqrt(c))**2*log(x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = 2\,ab\sqrt{c}\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right) + (dx+c)b^2 + 4\,\sqrt{dx+c}ab + \left(b^2c+a^2\right)\log\left(dx\right)$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="maxima")
```

```
[Out] 2*a*b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + (d
*x + c)*b^2 + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(d*x)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = \frac{4\,abc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + (dx+c)b^2 + 4\,\sqrt{dx+c}ab + \left(b^2c+a^2\right)\log\left(dx\right)$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] 4*a*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) + (d*x + c)*b^2 + 4*sqrt(d*
x + c)*a*b + (b^2*c + a^2)*log(d*x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.28

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = \ln\left(\left(2\,a^2+2\,c\,b^2\right)\,\sqrt{c+d\,x} - 2\,\left(a+b\,\sqrt{c}\right)^2\,\sqrt{c+d\,x} + 4\,a\,b\,c\right)\,\left(a+b\,\sqrt{c}\right)^2 + \ln\left(\left(2\,a^2+2\,c\,b^2\right)\,\sqrt{c+d\,x} - 2\,\left(a-b\,\sqrt{c}\right)^2\,\sqrt{c+d\,x} + 4\,a\,b\,c\right)\,\left(a-b\,\sqrt{c}\right)^2 + 4\,a\,b\,\sqrt{c+d\,x} + b^2\,d\,x$$

```
[In] int((a + b*(c + d*x)^(1/2))^2/x,x)
```

```
[Out] log((2*b^2*c + 2*a^2)*(c + d*x)^(1/2) - 2*(a + b*c^(1/2))^2*(c + d*x)^(1/2)
 + 4*a*b*c)*(a + b*c^(1/2))^2 + log((2*b^2*c + 2*a^2)*(c + d*x)^(1/2) - 2*(
a - b*c^(1/2))^2*(c + d*x)^(1/2) + 4*a*b*c)*(a - b*c^(1/2))^2 + 4*a*b*(c +
d*x)^(1/2) + b^2*d*x
```

3.623 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2} dx = -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} - \frac{2abd\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} + b^2 d\log(x)$$

```
[Out] b^2*d*ln(x)-2*a*b*d*arctanh((d*x+c)^(1/2)/c^(1/2))/c^(1/2)-(a+b*(d*x+c)^(1/
2))^2/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {378, 1412, 833, 649, 213, 266}

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2} dx = -\frac{2abd\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{\left(a+b\sqrt{c+dx}\right)^2}{x} + b^2 d\log(x)$$

```
[In] Int[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

```
[Out] -((a + b*Sqrt[c + d*x])^2/x) - (2*a*b*d*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/Sqr
t[c] + b^2*d*Log[x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\text{integral} = d\text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2}{(-c+x)^2}\,dx, x, c+dx\right)$$

$$= (2d)\text{Subst}\left(\int \frac{x(a+bx)^2}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} - \frac{d\text{Subst}\left(\int \frac{-2abc-2b^2cx}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)}{c}$$

$$= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} + (2abd)\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, \sqrt{c+dx}\right) + \left(2b^2d\right)\text{Subst}\left(\int \frac{x}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} - \frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} + b^2d\log(x)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx = -\frac{a^2+b^2c+2ab\sqrt{c+dx}}{x} - \frac{2abd\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} + b^2d\log(-dx)$$

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

```
[Out] -((a^2 + b^2*c + 2*a*b*Sqrt[c + d*x])/x) - (2*a*b*d*ArcTanh[Sqrt[c + d*x]/S
qrt[c]])/Sqrt[c] + b^2*d*Log[-(d*x)]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

method	result	size
default	$b^2\left(d\ln(x)-\frac{c}{x}\right)+4abd\left(-\frac{\sqrt{dx+c}}{2dx}-\frac{\text{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)-\frac{a^2}{x}$	63
derivativedivides	$2d\left(-\frac{ab\sqrt{dx+c}+\frac{b^2c}{2}+\frac{a^2}{2}}{dx}+b\left(\frac{b\ln(-dx)}{2}-\frac{a\ \text{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)\right)$	64

```
[In] int((a+b*(d*x+c)^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(d*ln(x)-1/x*c)+4*a*b*d*(-1/2*(d*x+c)^(1/2)/d/x-1/2/c^(1/2)*arctanh((d*
x+c)^(1/2)/c^(1/2)))-a^2/x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.72

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx$$
$$=\left[\frac{b^2cdx\log(x)+ab\sqrt{c}dx\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)-b^2c^2-2\sqrt{dx+c}abc-a^2c}{cx},\frac{b^2cdx\log(x)+2ab\sqrt{-c}dx\,\mathrm{a}}{}\right.$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="fricas")
```

```
[Out] [(b^2*c*d*x*log(x) + a*b*sqrt(c)*d*x*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2
*c)/x) - b^2*c^2 - 2*sqrt(d*x + c)*a*b*c - a^2*c)/(c*x), (b^2*c*d*x*log(x)
+ 2*a*b*sqrt(-c)*d*x*arctan(sqrt(d*x + c)*sqrt(-c)/c) - b^2*c^2 - 2*sqrt(d*
x + c)*a*b*c - a^2*c)/(c*x)]
```

Sympy [A] (verification not implemented)

Time = 22.67 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx=-\frac{a^2}{x}-\frac{2ab\sqrt{d}\sqrt{\frac{c}{dx}+1}}{\sqrt{x}}-\frac{2abd\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\sqrt{x}}\right)}{\sqrt{c}}-\frac{b^2c}{x}+b^2d\log(x)$$

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x**2,x)
```

```
[Out] -a**2/x - 2*a*b*sqrt(d)*sqrt(c/(d*x) + 1)/sqrt(x) - 2*a*b*d*asinh(sqrt(c)/(
sqrt(d)*sqrt(x)))/sqrt(c) - b**2*c/x + b**2*d*log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx=\left(b^2\log(dx)+\frac{ab\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{b^2c+2\sqrt{dx+c}ab+a^2}{dx}\right)d$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="maxima")
```

```
[Out] (b^2*log(d*x) + a*b*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))
)/sqrt(c) - (b^2*c + 2*sqrt(d*x + c)*a*b + a^2)/(d*x))*d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.48

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx = \frac{b^2d^2\log\left(dx\right) + \frac{2\,abd^2\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{b^2cd^2+2\,\sqrt{dx+c}abd^2+a^2d^2}{dx}}{d}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="giac")
```

```
[Out] (b^2*d^2*log(d*x) + 2*a*b*d^2*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) - (b^
2*c*d^2 + 2*sqrt(d*x + c)*a*b*d^2 + a^2*d^2)/(d*x))/d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.43

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx = b\,d\,\ln\left(2\,b\,d\left(b+\frac{a}{\sqrt{c}}\right)\sqrt{c+d\,x}-2\,b^2\,d\,\sqrt{c+d\,x}-2\,a\,b\,d\right)\left(b+\frac{a}{\sqrt{c}}\right)-\frac{a^2\,d+b^2\,c\,d+2\,a\,b\,d\,\sqrt{c+d\,x}}{d\,x}+b\,d\,\ln\left(2\,b\,d\left(b-\frac{a}{\sqrt{c}}\right)\sqrt{c+d\,x}-2\,b^2\,d\,\sqrt{c+d\,x}-2\,a\,b\,d\right)\left(b-\frac{a}{\sqrt{c}}\right)$$

```
[In] int((a + b*(c + d*x)^(1/2))^2/x^2,x)
```

```
[Out] b*d*log(2*b*d*(b + a/c^(1/2))*(c + d*x)^(1/2) - 2*b^2*d*(c + d*x)^(1/2) - 2
*a*b*d)*(b + a/c^(1/2)) - (a^2*d + b^2*c*d + 2*a*b*d*(c + d*x)^(1/2))/(d*x)
 + b*d*log(2*b*d*(b - a/c^(1/2))*(c + d*x)^(1/2) - 2*b^2*d*(c + d*x)^(1/2)
- 2*a*b*d)*(b - a/c^(1/2))
```

3.624 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx=-\frac{bd\left(bc+a\sqrt{c+dx}\right)}{2cx}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2}+\frac{abd^2\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}$$

```
[Out] 1/2*a*b*d^2*arctanh((d*x+c)^(1/2)/c^(1/2))/c^(3/2)-1/2*b*d*(b*c+a*(d*x+c)^(
1/2))/c/x-1/2*(a+b*(d*x+c)^(1/2))^2/x^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {378, 1412, 835, 12, 653, 213}

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx=\frac{abd^2\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2}-\frac{bd\left(a\sqrt{c+dx}+bc\right)}{2cx}$$

```
[In] Int[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

```
[Out] -1/2*(b*d*(b*c + a*Sqrt[c + d*x]))/(c*x) - (a + b*Sqrt[c + d*x])^2/(2*x^2)
+ (a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\text{integral} = d^2\text{Subst}\left(\int \frac{(a+b\sqrt{x})^2}{(-c+x)^3}\,dx, x, c+dx\right)$$

$$= (2d^2)\,\text{Subst}\left(\int \frac{x(a+bx)^2}{(-c+x^2)^3}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{(a+b\sqrt{c+dx})^2}{2x^2} - \frac{d^2\text{Subst}\left(\int -\frac{2bc(a+bx)}{(-c+x^2)^2}\,dx, x, \sqrt{c+dx}\right)}{2c}$$

$$= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} + \left(bd^2\right)\text{Subst}\left(\int \frac{a+bx}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{bd\left(bc+a\sqrt{c+dx}\right)}{2cx} - \frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} - \frac{\left(abd^2\right)\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)}{2c}$$

$$= -\frac{bd\left(bc+a\sqrt{c+dx}\right)}{2cx} - \frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} + \frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = -\frac{a^2c+ab\sqrt{c+dx}(2c+dx)+b^2c(c+2dx)}{2cx^2} + \frac{abd^2\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}$$

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

```
[Out] -1/2*(a^2*c + a*b*Sqrt[c + d*x]*(2*c + d*x) + b^2*c*(c + 2*d*x))/(c*x^2) +
(a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

method	result	size
derivativedivides	$2d^2\left(-\frac{-\frac{ab(dx+c)^{\frac{3}{2}}}{4c}+\frac{(dx+c)b^2}{2}+\frac{ab\sqrt{dx+c}}{4}-\frac{b^2c}{4}+\frac{a^2}{4}}{d^2x^2}+\frac{ab\,\text{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{4c^{\frac{3}{2}}}\right)$	81
default	$b^2\left(-\frac{c}{2x^2}-\frac{d}{x}\right)+4ab\,d^2\left(-\frac{-\frac{(dx+c)^{\frac{3}{2}}}{8c}+\frac{\sqrt{dx+c}}{8}}{d^2x^2}+\frac{\text{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)-\frac{a^2}{2x^2}$	82

```
[In] int((a+b*(d*x+c)^(1/2))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*(-(1/4*a*b/c*(d*x+c)^(3/2)+1/2*(d*x+c)*b^2+1/4*a*b*(d*x+c)^(1/2)-1/4*
b^2*c+1/4*a^2)/d^2/x^2+1/4*a*b/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.26

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = \left[\frac{ab\sqrt{c}d^2x^2\log\left(\frac{dx+2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)-4b^2c^2dx-2b^2c^3-2a^2c^2-2\left(abcdx+2abc^2\right)\sqrt{dx+c}}{4c^2x^2}, -\frac{ab\sqrt{-c}d^2x^2\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+2b^2c^2dx+b^2c^3+a^2c^2+\left(abcdx+2abc^2\right)\sqrt{dx+c}}{2c^2x^2}\right]$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(a*b*sqrt(c)*d^2*x^2*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 4*
b^2*c^2*d*x - 2*b^2*c^3 - 2*a^2*c^2 - 2*(a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x +
c))/(c^2*x^2), -1/2*(a*b*sqrt(-c)*d^2*x^2*arctan(sqrt(d*x + c)*sqrt(-c)/c)
+ 2*b^2*c^2*d*x + b^2*c^3 + a^2*c^2 + (a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x + c)
)/(c^2*x^2)]
```

Sympy [A] (verification not implemented)

Time = 90.95 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.68

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = -\frac{a^2}{2x^2}-\frac{abc}{\sqrt{d}x^{\frac{5}{2}}\sqrt{\frac{c}{dx}+1}}-\frac{3ab\sqrt{d}}{2x^{\frac{3}{2}}\sqrt{\frac{c}{dx}+1}}-\frac{abd^{\frac{3}{2}}}{2c\sqrt{x}\sqrt{\frac{c}{dx}+1}}+\frac{abd^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\sqrt{x}}\right)}{2c^{\frac{3}{2}}}-\frac{b^2c}{2x^2}-\frac{b^2d}{x}$$

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) - a*b*c/(sqrt(d)*x**(5/2)*sqrt(c/(d*x) + 1)) - 3*a*b*sqrt(d)
/(2*x**(3/2)*sqrt(c/(d*x) + 1)) - a*b*d**(3/2)/(2*c*sqrt(x)*sqrt(c/(d*x) +
1)) + a*b*d**2*asinh(sqrt(c)/(sqrt(d)*sqrt(x)))/(2*c**(3/2)) - b**2*c/(2*x*
*2) - b**2*d/x
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = -\frac{1}{4}\left(\frac{ab\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(2\,(dx+c)b^2c-b^2c^2+(dx+c)^{\frac{3}{2}}ab+\sqrt{dx+c}abc+a^2c\right)}{(dx+c)^2c-2\,(dx+c)c^2+c^3}\right)d^2$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a*b*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/c^(3/2)
+ 2*(2*(d*x + c)*b^2*c - b^2*c^2 + (d*x + c)^(3/2)*a*b + sqrt(d*x + c)*a*b*
c + a^2*c)/((d*x + c)^2*c - 2*(d*x + c)*c^2 + c^3))*d^2
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = -\frac{\frac{abd^3\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}+\frac{2\,(dx+c)b^2cd^3-b^2c^2d^3+(dx+c)^{\frac{3}{2}}abd^3+\sqrt{dx+c}abcd^3+a^2cd^3}{cd^2x^2}}{2\,d}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(a*b*d^3*arctan(sqrt(d*x + c)/sqrt(-c))/(sqrt(-c)*c) + (2*(d*x + c)*b^
2*c*d^3 - b^2*c^2*d^3 + (d*x + c)^(3/2)*a*b*d^3 + sqrt(d*x + c)*a*b*c*d^3 +
 a^2*c*d^3)/(c*d^2*x^2))/d
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = \frac{a\,b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}}{\sqrt{c}}\right)}{2\,c^{3/2}}-\frac{b^2\,c}{2\,x^2}-\frac{b^2\,d}{x}-\frac{a\,b\,\sqrt{c+d\,x}}{2\,x^2}-\frac{a\,b\,{\left(c+d\,x\right)}^{3/2}}{2\,c\,x^2}-\frac{a^2}{2\,x^2}$$

```
[In] int((a + b*(c + d*x)^(1/2))^2/x^3,x)
```

```
[Out] (a*b*d^2*atanh((c + d*x)^(1/2)/c^(1/2)))/(2*c^(3/2)) - (b^2*c)/(2*x^2) - (b
^2*d)/x - (a*b*(c + d*x)^(1/2))/(2*x^2) - (a*b*(c + d*x)^(3/2))/(2*c*x^2) -
 a^2/(2*x^2)
```

3.625 $\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 326

$$\begin{aligned}\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx = & -\frac{4a(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4} \\ & +\frac{4(a^2-b^2c)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4} \\ & -\frac{12a(7a^2-3b^2c)\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4} \\ & +\frac{4(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4} \\ & -\frac{20a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4} \\ & +\frac{12(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4} \\ & -\frac{28a\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{17/2}}{17b^8d^4}\end{aligned}$$

```
[Out] -4/3*a*(-b^2*c+a^2)^3*(a+b*(d*x+c)^(1/2))^(3/2)/b^8/d^4+4/5*(-b^2*c+a^2)^2*
(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(5/2)/b^8/d^4-12/7*a*(-3*b^2*c+7*a^2)*(-
b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(7/2)/b^8/d^4+4/9*(3*b^4*c^2-30*a^2*b^2*c+35
*a^4)*(a+b*(d*x+c)^(1/2))^(9/2)/b^8/d^4-20/11*a*(-3*b^2*c+7*a^2)*(a+b*(d*x+
c)^(1/2))^(11/2)/b^8/d^4+12/13*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(13/2)/b^
8/d^4-28/15*a*(a+b*(d*x+c)^(1/2))^(15/2)/b^8/d^4+4/17*(a+b*(d*x+c)^(1/2))^(
17/2)/b^8/d^4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {378, 1412, 786}

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{12(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4} - \frac{20a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4} - \frac{12a(7a^2-3b^2c)(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4} + \frac{4(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4} - \frac{4a(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4} + \frac{4(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4} + \frac{4\left(a+b\sqrt{c+dx}\right)^{17/2}}{17b^8d^4} - \frac{28a\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}$$

```
[In] Int[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8*d^4) + (4*(a^2 -
b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8*d^4) - (12*a*(
7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8*d^4) + (
4*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d
^4) - (20*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) +
(12*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^8*d^4) - (28*a*(a +
 b*Sqrt[c + d*x])^(15/2))/(15*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(17/2))/(
17*b^8*d^4)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}(-c+x)^3\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int x\sqrt{a+bx}\left(-c+x^2\right)^3\,dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{a(a^2-b^2c)^3\sqrt{a+bx}}{b^7} - \frac{(-7a^2+b^2c)(-a^2+b^2c)^2(a+bx)^{3/2}}{b^7} - \frac{3(7a^5-10a^3b^2c+3ab^4c^2)(a+bx)^{5/2}}{b^7} + \frac{(35a^4-30\right.\right.}{}$$

$$= -\frac{4a(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4} + \frac{4(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4} - \frac{12a(7a^2-3b^2c)(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4} + \frac{4(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4} - \frac{20a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4} + \frac{12(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4} - \frac{28a\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4} + \frac{4\left(a+b\sqrt{c+dx}\right)^{17/2}}{17b^8d^4}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$

$$= \frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-14336a^7+3840a^5b^2(10c-7dx)+21504a^6b\sqrt{c+dx}-640a^4b^3(104c-49dx)\sqrt{c+\right.}{}$$

```
[In] Integrate[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-14336*a^7 + 3840*a^5*b^2*(10*c - 7*d*x) +
21504*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(104*c - 49*d*x)*Sqrt[c + d*x] - 48
*a^3*b^4*(616*c^2 - 1080*c*d*x + 735*d^2*x^2) + 24*a^2*b^5*Sqrt[c + d*x]*(2
960*c^2 - 2716*c*d*x + 1617*d^2*x^2) + 6*a*b^6*(320*c^3 - 3936*c^2*d*x + 57
54*c*d^2*x^2 - 7007*d^3*x^3) - 231*b^7*Sqrt[c + d*x]*(128*c^3 - 160*c^2*d*x
 + 180*c*d^2*x^2 - 195*d^3*x^3)))/(765765*b^8*d^4)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.18

method	result
derivativedivides	$-\frac{4\left(-\frac{(a+b\sqrt{dx+c})^{\frac{17}{2}}}{17}+\frac{7a(a+b\sqrt{dx+c})^{\frac{15}{2}}}{15}+\frac{(3b^2c-21a^2)(a+b\sqrt{dx+c})^{\frac{13}{2}}}{13}+\frac{(8(-b^2c+a^2)a+2a(-2b^2c+6a^2)+(-3b^2c+15a^2)a)}{11}\right.}{}$
default	$-\frac{4\left(-\frac{(a+b\sqrt{dx+c})^{\frac{17}{2}}}{17}+\frac{7a(a+b\sqrt{dx+c})^{\frac{15}{2}}}{15}+\frac{(3b^2c-21a^2)(a+b\sqrt{dx+c})^{\frac{13}{2}}}{13}+\frac{(8(-b^2c+a^2)a+2a(-2b^2c+6a^2)+(-3b^2c+15a^2)a)}{11}\right.}{}$

```
[In] int(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/d^4/b^8*(-1/17*(a+b*(d*x+c)^(1/2))^(17/2)+7/15*a*(a+b*(d*x+c)^(1/2))^(15
/2)+1/13*(3*b^2*c-21*a^2)*(a+b*(d*x+c)^(1/2))^(13/2)+1/11*(8*(-b^2*c+a^2)*a
+2*a*(-2*b^2*c+6*a^2)+(-3*b^2*c+15*a^2)*a)*(a+b*(d*x+c)^(1/2))^(11/2)+1/9*(
-(-b^2*c+a^2)*(-2*b^2*c+6*a^2)-8*a^2*(-b^2*c+a^2)-(-b^2*c+a^2)^2+(-8*(-b^2*
c+a^2)*a-2*a*(-2*b^2*c+6*a^2))*a)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*(6*(-b^2*c+
a^2)^2*a+((-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2)*
a)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*(-(-b^2*c+a^2)^3-6*(-b^2*c+a^2)^2*a^2)*(a+
b*(d*x+c)^(1/2))^(5/2)+1/3*(-b^2*c+a^2)^3*a*(a+b*(d*x+c)^(1/2))^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.88

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\frac{4\left(45045\,b^8d^4x^4-29568\,b^8c^4+72960\,a^2b^6c^3-96128\,a^4b^4c^2+59904\,a^6b^2c-14336\,a^8+231\left(15\,b^8c-14\,a^2\right.\right.}{}$$

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/765765*(45045*b^8*d^4*x^4 - 29568*b^8*c^4 + 72960*a^2*b^6*c^3 - 96128*a^4
*b^4*c^2 + 59904*a^6*b^2*c - 14336*a^8 + 231*(15*b^8*c - 14*a^2*b^6)*d^3*x^
3 - 28*(165*b^8*c^2 - 291*a^2*b^6*c + 140*a^4*b^4)*d^2*x^2 + 32*(231*b^8*c^
3 - 555*a^2*b^6*c^2 + 520*a^4*b^4*c - 168*a^6*b^2)*d*x + (3003*a*b^7*d^3*x^
3 - 27648*a*b^7*c^3 + 41472*a^3*b^5*c^2 - 28160*a^5*b^3*c + 7168*a^7*b - 35
28*(2*a*b^7*c - a^3*b^5)*d^2*x^2 + 32*(417*a*b^7*c^2 - 417*a^3*b^5*c + 140*
a^5*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.10

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$

$$= \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{7a(a+b\sqrt{c+dx})^{\frac{15}{2}}}{15b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{17}{2}}}{17b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{13}{2}}\cdot(21a^2-3b^2c)}{13b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{11}{2}}(-35a^3+15ab^2c)}{11b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{9}{2}}\cdot(35a^4-30a^2b^2c+3b\ldots}{9b^6}\right.}{\ldots} \\ \frac{\sqrt{a}d^4x^4}{8} \end{cases}\right.}{\ldots} \\ \frac{x^4\sqrt{a+b\sqrt{c}}}{4} \end{cases}$$

```
[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((2*(-7*a*(a + b*sqrt(c + d*x))**(15/2)/(15*b**6) + (
a + b*sqrt(c + d*x))**(17/2)/(17*b**6) + (a + b*sqrt(c + d*x))**(13/2)*(21*
a**2 - 3*b**2*c)/(13*b**6) + (a + b*sqrt(c + d*x))**(11/2)*(-35*a**3 + 15*a
*b**2*c)/(11*b**6) + (a + b*sqrt(c + d*x))**(9/2)*(35*a**4 - 30*a**2*b**2*c
 + 3*b**4*c**2)/(9*b**6) + (a + b*sqrt(c + d*x))**(7/2)*(-21*a**5 + 30*a**3
*b**2*c - 9*a*b**4*c**2)/(7*b**6) + (a + b*sqrt(c + d*x))**(5/2)*(7*a**6 -
15*a**4*b**2*c + 9*a**2*b**4*c**2 - b**6*c**3)/(5*b**6) + (a + b*sqrt(c + d
*x))**(3/2)*(-a**7 + 3*a**5*b**2*c - 3*a**3*b**4*c**2 + a*b**6*c**3)/(3*b**
6))/b**2, Ne(b, 0)), (sqrt(a)*d**4*x**4/8, True))/d**4, Ne(d, 0)), (x**4*sq
rt(a + b*sqrt(c))/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.82

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$

$$= \frac{4\left(45045\left(\sqrt{dx+c}b+a\right)^{\frac{17}{2}}-357357\left(\sqrt{dx+c}b+a\right)^{\frac{15}{2}}a-176715\left(b^2c-7a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{13}{2}}+34807\ldots\right.}{\ldots}$$

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/765765*(45045*(sqrt(d*x + c)*b + a)^(17/2) - 357357*(sqrt(d*x + c)*b + a)
^(15/2)*a - 176715*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(13/2) + 348075*(3
*a*b^2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(11/2) + 85085*(3*b^4*c^2 - 30*a^2*
b^2*c + 35*a^4)*(sqrt(d*x + c)*b + a)^(9/2) - 328185*(3*a*b^4*c^2 - 10*a^3*
b^2*c + 7*a^5)*(sqrt(d*x + c)*b + a)^(7/2) - 153153*(b^6*c^3 - 9*a^2*b^4*c^
2 + 15*a^4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(5/2) + 255255*(a*b^6*c^3 -
 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*(sqrt(d*x + c)*b + a)^(3/2))/(b^8*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. 2(279) = 558.

Time = 0.42 (sec) , antiderivative size = 915, normalized size of antiderivative = 2.81

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/765765*(17*(15015*(sqrt(d*x + c)*b + a)^(3/2)*b^6*c^3 - 45045*sqrt(sqrt(
d*x + c)*b + a)*a*b^6*c^3 - 19305*(sqrt(d*x + c)*b + a)^(7/2)*b^4*c^2 + 810
81*(sqrt(d*x + c)*b + a)^(5/2)*a*b^4*c^2 - 135135*(sqrt(d*x + c)*b + a)^(3/
2)*a^2*b^4*c^2 + 135135*sqrt(sqrt(d*x + c)*b + a)*a^3*b^4*c^2 + 12285*(sqrt
(d*x + c)*b + a)^(11/2)*b^2*c - 75075*(sqrt(d*x + c)*b + a)^(9/2)*a*b^2*c +
 193050*(sqrt(d*x + c)*b + a)^(7/2)*a^2*b^2*c - 270270*(sqrt(d*x + c)*b + a
)^(5/2)*a^3*b^2*c + 225225*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^2*c - 135135*s
qrt(sqrt(d*x + c)*b + a)*a^5*b^2*c - 3003*(sqrt(d*x + c)*b + a)^(15/2) + 24
255*(sqrt(d*x + c)*b + a)^(13/2)*a - 85995*(sqrt(d*x + c)*b + a)^(11/2)*a^2
 + 175175*(sqrt(d*x + c)*b + a)^(9/2)*a^3 - 225225*(sqrt(d*x + c)*b + a)^(7
/2)*a^4 + 189189*(sqrt(d*x + c)*b + a)^(5/2)*a^5 - 105105*(sqrt(d*x + c)*b
+ a)^(3/2)*a^6 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^7)*a/(b^7*d^3) + (153153
*(sqrt(d*x + c)*b + a)^(5/2)*b^6*c^3 - 510510*(sqrt(d*x + c)*b + a)^(3/2)*a
*b^6*c^3 + 765765*sqrt(sqrt(d*x + c)*b + a)*a^2*b^6*c^3 - 255255*(sqrt(d*x
+ c)*b + a)^(9/2)*b^4*c^2 + 1312740*(sqrt(d*x + c)*b + a)^(7/2)*a*b^4*c^2 -
 2756754*(sqrt(d*x + c)*b + a)^(5/2)*a^2*b^4*c^2 + 3063060*(sqrt(d*x + c)*b
 + a)^(3/2)*a^3*b^4*c^2 - 2297295*sqrt(sqrt(d*x + c)*b + a)*a^4*b^4*c^2 + 1
76715*(sqrt(d*x + c)*b + a)^(13/2)*b^2*c - 1253070*(sqrt(d*x + c)*b + a)^(1
1/2)*a*b^2*c + 3828825*(sqrt(d*x + c)*b + a)^(9/2)*a^2*b^2*c - 6563700*(sqr
t(d*x + c)*b + a)^(7/2)*a^3*b^2*c + 6891885*(sqrt(d*x + c)*b + a)^(5/2)*a^4
*b^2*c - 4594590*(sqrt(d*x + c)*b + a)^(3/2)*a^5*b^2*c + 2297295*sqrt(sqrt(
d*x + c)*b + a)*a^6*b^2*c - 45045*(sqrt(d*x + c)*b + a)^(17/2) + 408408*(sq
rt(d*x + c)*b + a)^(15/2)*a - 1649340*(sqrt(d*x + c)*b + a)^(13/2)*a^2 + 38
98440*(sqrt(d*x + c)*b + a)^(11/2)*a^3 - 5955950*(sqrt(d*x + c)*b + a)^(9/2
)*a^4 + 6126120*(sqrt(d*x + c)*b + a)^(7/2)*a^5 - 4288284*(sqrt(d*x + c)*b
+ a)^(5/2)*a^6 + 2042040*(sqrt(d*x + c)*b + a)^(3/2)*a^7 - 765765*sqrt(sqrt
(d*x + c)*b + a)*a^8)/(b^7*d^3))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx=\int x^3\,\sqrt{a+b\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(x^3*(a + b*(c + d*x)^(1/2))^(1/2),x)
[Out] int(x^3*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.626 $\int x^2 \sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 224

$$\int x^2 \sqrt{a+b\sqrt{c+dx}}\,dx = -\frac{4a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3} + \frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3} - \frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3} + \frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3} - \frac{20a\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3} + \frac{4\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^6d^3}$$

```
[Out] -4/3*a*(-b^2*c+a^2)^2*(a+b*(d*x+c)^(1/2))^(3/2)/b^6/d^3+4/5*(b^4*c^2-6*a^2*
b^2*c+5*a^4)*(a+b*(d*x+c)^(1/2))^(5/2)/b^6/d^3-8/7*a*(-3*b^2*c+5*a^2)*(a+b*
(d*x+c)^(1/2))^(7/2)/b^6/d^3+8/9*(-b^2*c+5*a^2)*(a+b*(d*x+c)^(1/2))^(9/2)/b
^6/d^3-20/11*a*(a+b*(d*x+c)^(1/2))^(11/2)/b^6/d^3+4/13*(a+b*(d*x+c)^(1/2))^
(13/2)/b^6/d^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {378, 1412, 786}

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3} - \frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3} - \frac{4a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3} + \frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3} + \frac{4\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^6d^3} - \frac{20a\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3}$$

```
[In] Int[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6*d^3) + (4*(5*a^4
- 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6*d^3) - (8*a*(5
*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^6*d^3) + (8*(5*a^2 - b^2*
c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^6*d^3) - (20*a*(a + b*Sqrt[c + d*x])^(
11/2))/(11*b^6*d^3) + (4*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^6*d^3)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}(-c+x)^2\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\text{Subst}\left(\int x\sqrt{a+bx}\left(-c+x^2\right)^2\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{a(a^2-b^2c)^2\sqrt{a+bx}}{b^5} + \frac{(5a^4-6a^2b^2c+b^4c^2)(a+bx)^{3/2}}{b^5} - \frac{2(5a^3-3ab^2c)(a+bx)^{5/2}}{b^5} - \frac{2(-5a^2+b^2c)(a+bx)^{7/2}}{b^5} - \right.\right.}{d^3}$$

$$= -\frac{4a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3} + \frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3} - \frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3} + \frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3} - \frac{20a\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3} + \frac{4\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^6d^3}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.66

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$

$$= \frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-1280a^5+32a^3b^2(68c-75dx)+1920a^4b\sqrt{c+dx}+16a^2b^3\sqrt{c+dx}(-254c+175dx)\right.}{45045b^6d^3}$$

```
[In] Integrate[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-1280*a^5 + 32*a^3*b^2*(68*c - 75*d*x) + 19
20*a^4*b*Sqrt[c + d*x] + 16*a^2*b^3*Sqrt[c + d*x]*(-254*c + 175*d*x) + 77*b
^5*Sqrt[c + d*x]*(32*c^2 - 40*c*d*x + 45*d^2*x^2) - 6*a*b^4*(96*c^2 - 380*c
*d*x + 525*d^2*x^2)))/(45045*b^6*d^3)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{4(a+b\sqrt{dx+c})^{\frac{13}{2}}}{13}-\frac{20a(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}-\frac{4(2b^2c-10a^2)(a+b\sqrt{dx+c})^{\frac{9}{2}}}{9}-\frac{4(4(-b^2c+a^2)a+a(-2b^2c+6a^2))(a+b\sqrt{dx+c})^{\frac{7}{2}}}{7}-\frac{4(-}{}}{d^3b^6}$
default	$\frac{\frac{4(a+b\sqrt{dx+c})^{\frac{13}{2}}}{13}-\frac{20a(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}-\frac{4(2b^2c-10a^2)(a+b\sqrt{dx+c})^{\frac{9}{2}}}{9}-\frac{4(4(-b^2c+a^2)a+a(-2b^2c+6a^2))(a+b\sqrt{dx+c})^{\frac{7}{2}}}{7}-\frac{4(-}{}}{d^3b^6}$

```
[In] int(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 4/d^3/b^6*(1/13*(a+b*(d*x+c)^(1/2))^(13/2)-5/11*a*(a+b*(d*x+c)^(1/2))^(11/2
)-1/9*(2*b^2*c-10*a^2)*(a+b*(d*x+c)^(1/2))^(9/2)-1/7*(4*(-b^2*c+a^2)*a+a*(-
2*b^2*c+6*a^2))*(a+b*(d*x+c)^(1/2))^(7/2)-1/5*(-(-b^2*c+a^2)^2-4*a^2*(-b^2*
c+a^2))*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*(-b^2*c+a^2)^2*a*(a+b*(d*x+c)^(1/2))^
(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.82

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\frac{4\left(3465\,b^6d^3x^3+2464\,b^6c^3-4640\,a^2b^4c^2+4096\,a^4b^2c-1280\,a^6+35\left(11\,b^6c-10\,a^2b^4\right)d^2x^2-8\left(77\,b^6c^2-\right.}{}$$

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 4/45045*(3465*b^6*d^3*x^3 + 2464*b^6*c^3 - 4640*a^2*b^4*c^2 + 4096*a^4*b^2*
c - 1280*a^6 + 35*(11*b^6*c - 10*a^2*b^4)*d^2*x^2 - 8*(77*b^6*c^2 - 127*a^2
*b^4*c + 60*a^4*b^2)*d*x + (315*a*b^5*d^2*x^2 + 1888*a*b^5*c^2 - 1888*a^3*b
^3*c + 640*a^5*b - 400*(2*a*b^5*c - a^3*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(
d*x + c)*b + a)/(b^6*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.08

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\begin{cases}\frac{2\left(\begin{cases}\frac{2\left(-\frac{5a(a+b\sqrt{c+dx})^{\frac{11}{2}}}{11b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{13}{2}}}{13b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{9}{2}}\cdot(10a^2-2b^2c)}{9b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{7}{2}}(-10a^3+6ab^2c)}{7b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{5}{2}}\cdot(5a^4-6a^2b^2c+b^4c^2)}{5b^4}\right.}{b^2} \\ \frac{\sqrt{a}d^3x^3}{6}\end{cases}\right.}{d^3} \\ \frac{x^3\sqrt{a+b\sqrt{c}}}{3}\end{cases}$$

```
[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((2*(-5*a*(a + b*sqrt(c + d*x))**(11/2)/(11*b**4) + (
a + b*sqrt(c + d*x))**(13/2)/(13*b**4) + (a + b*sqrt(c + d*x))**(9/2)*(10*a
**2 - 2*b**2*c)/(9*b**4) + (a + b*sqrt(c + d*x))**(7/2)*(-10*a**3 + 6*a*b**
2*c)/(7*b**4) + (a + b*sqrt(c + d*x))**(5/2)*(5*a**4 - 6*a**2*b**2*c + b**4
*c**2)/(5*b**4) + (a + b*sqrt(c + d*x))**(3/2)*(-a**5 + 2*a**3*b**2*c - a*b
**4*c**2)/(3*b**4))/b**2, Ne(b, 0)), (sqrt(a)*d**3*x**3/6, True))/d**3, Ne(
d, 0)), (x**3*sqrt(a + b*sqrt(c))/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.75

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$= \frac{4\left(3465\left(\sqrt{dx+c}b+a\right)^{\frac{13}{2}}-20475\left(\sqrt{dx+c}b+a\right)^{\frac{11}{2}}a-10010\left(b^2c-5a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{9}{2}}+12870\left(3ab\right.}{}$$

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/45045*(3465*(sqrt(d*x + c)*b + a)^(13/2) - 20475*(sqrt(d*x + c)*b + a)^(1
1/2)*a - 10010*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(9/2) + 12870*(3*a*b^2
*c - 5*a^3)*(sqrt(d*x + c)*b + a)^(7/2) + 9009*(b^4*c^2 - 6*a^2*b^2*c + 5*a
^4)*(sqrt(d*x + c)*b + a)^(5/2) - 15015*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*(sq
rt(d*x + c)*b + a)^(3/2))/(b^6*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(188) = 376.

Time = 0.39 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.45

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$= \frac{4\left(\frac{13\left(1155\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}b^4c^2-3465\sqrt{\sqrt{dx+c}b+a}\,ab^4c^2-990\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}b^2c+4158\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}ab^2c-6930\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}a^2b^2c+6930\right.}{}\right.}{}$$

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/45045*(13*(1155*(sqrt(d*x + c)*b + a)^(3/2)*b^4*c^2 - 3465*sqrt(sqrt(d*x
+ c)*b + a)*a*b^4*c^2 - 990*(sqrt(d*x + c)*b + a)^(7/2)*b^2*c + 4158*(sqrt(
```

```
d*x + c)*b + a)^(5/2)*a*b^2*c - 6930*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^2*c
+ 6930*sqrt(sqrt(d*x + c)*b + a)*a^3*b^2*c + 315*(sqrt(d*x + c)*b + a)^(11/
2) - 1925*(sqrt(d*x + c)*b + a)^(9/2)*a + 4950*(sqrt(d*x + c)*b + a)^(7/2)*
a^2 - 6930*(sqrt(d*x + c)*b + a)^(5/2)*a^3 + 5775*(sqrt(d*x + c)*b + a)^(3/
2)*a^4 - 3465*sqrt(sqrt(d*x + c)*b + a)*a^5)*a/(b^5*d^2) + (9009*(sqrt(d*x
+ c)*b + a)^(5/2)*b^4*c^2 - 30030*(sqrt(d*x + c)*b + a)^(3/2)*a*b^4*c^2 + 4
5045*sqrt(sqrt(d*x + c)*b + a)*a^2*b^4*c^2 - 10010*(sqrt(d*x + c)*b + a)^(9
/2)*b^2*c + 51480*(sqrt(d*x + c)*b + a)^(7/2)*a*b^2*c - 108108*(sqrt(d*x +
c)*b + a)^(5/2)*a^2*b^2*c + 120120*(sqrt(d*x + c)*b + a)^(3/2)*a^3*b^2*c -
90090*sqrt(sqrt(d*x + c)*b + a)*a^4*b^2*c + 3465*(sqrt(d*x + c)*b + a)^(13/
2) - 24570*(sqrt(d*x + c)*b + a)^(11/2)*a + 75075*(sqrt(d*x + c)*b + a)^(9/
2)*a^2 - 128700*(sqrt(d*x + c)*b + a)^(7/2)*a^3 + 135135*(sqrt(d*x + c)*b +
 a)^(5/2)*a^4 - 90090*(sqrt(d*x + c)*b + a)^(3/2)*a^5 + 45045*sqrt(sqrt(d*x
 + c)*b + a)*a^6)/(b^5*d^2))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx=\int x^2\,\sqrt{a+b\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(x^2*(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x^2*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.627 $\int x\sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx=-\frac{4a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4d^2}$$

```
[Out] -4/3*a*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)/b^4/d^2+4/5*(-b^2*c+3*a^2)*(a
+b*(d*x+c)^(1/2))^(5/2)/b^4/d^2-12/7*a*(a+b*(d*x+c)^(1/2))^(7/2)/b^4/d^2+4/
9*(a+b*(d*x+c)^(1/2))^(9/2)/b^4/d^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {378, 1412, 786}

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx=\frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{4a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}$$

```
[In] Int[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) + (4*(3*a^2 -
b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^4*d^2) - (12*a*(a + b*Sqrt[c + d*x
])^(7/2))/(7*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^4*d^2)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}(-c+x)\,dx, x, c+dx\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int x\sqrt{a+bx}\left(-c+x^2\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{(-a^3+ab^2c)\sqrt{a+bx}}{b^3}+\frac{(3a^2-b^2c)(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= -\frac{4a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}\\
&\quad -\frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.63

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-16a^3+6ab^2(2c-5dx)+24a^2b\sqrt{c+dx}+7b^3\sqrt{c+dx}(-4c+5dx)\right)}{315b^4d^2}$$

[In] Integrate[x*Sqrt[a + b*Sqrt[c + d*x]],x]

[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-16*a^3 + 6*a*b^2*(2*c - 5*d*x) + 24*a^2*b*
Sqrt[c + d*x] + 7*b^3*Sqrt[c + d*x]*(-4*c + 5*d*x)))/(315*b^4*d^2)

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{9}{2}}}{9}+\frac{3a\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}+\frac{\left(b^{2}c-3a^{2}\right)\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}+\frac{\left(-b^{2}c+a^{2}\right)a\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}\right)}{d^{2}b^{4}}$	93
default	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{9}{2}}}{9}+\frac{3a\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}+\frac{\left(b^{2}c-3a^{2}\right)\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}+\frac{\left(-b^{2}c+a^{2}\right)a\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}\right)}{d^{2}b^{4}}$	93

[In] int(x*(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)

[Out] -4/d^2/b^4*(-1/9*(a+b*(d*x+c)^(1/2))^(9/2)+3/7*a*(a+b*(d*x+c)^(1/2))^(7/2)+
1/5*(b^2*c-3*a^2)*(a+b*(d*x+c)^(1/2))^(5/2)+1/3*(-b^2*c+a^2)*a*(a+b*(d*x+c)
^(1/2))^(3/2))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\frac{4\left(35\,b^{4}d^{2}x^{2}-28\,b^{4}c^{2}+36\,a^{2}b^{2}c-16\,a^{4}+(7\,b^{4}c-6\,a^{2}b^{2})dx+(5\,ab^{3}dx-16\,ab^{3}c+8\,a^{3}b)\sqrt{dx+c}\right)\sqrt{\sqrt{dx}}}{315\,b^{4}d^{2}}$$

[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 4/315*(35*b^4*d^2*x^2 - 28*b^4*c^2 + 36*a^2*b^2*c - 16*a^4 + (7*b^4*c - 6*a
^2*b^2)*d*x + (5*a*b^3*d*x - 16*a*b^3*c + 8*a^3*b)*sqrt(d*x + c))*sqrt(sqrt
(d*x + c)*b + a)/(b^4*d^2)

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.17

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\begin{cases}\frac{2\left(\begin{cases}\frac{2\left(-\frac{3a(a+b\sqrt{c+dx})^{\frac{7}{2}}}{7b^2}+\frac{(a+b\sqrt{c+dx})^{\frac{9}{2}}}{9b^2}+\frac{(a+b\sqrt{c+dx})^{\frac{5}{2}}\cdot(3a^2-b^2c)}{5b^2}+\frac{(a+b\sqrt{c+dx})^{\frac{3}{2}}(-a^3+ab^2c)}{3b^2}\right)}{b^2} & \text{for } b\neq 0\\ \sqrt{a}\left(-\frac{c(c+dx)}{2}+\frac{(c+dx)^2}{4}\right) & \text{otherwise}\end{cases}\right)}{d^2} & \text{for } d\neq 0\\ \frac{x^2\sqrt{a+b\sqrt{c}}}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((2*(-3*a*(a + b*sqrt(c + d*x))**(7/2)/(7*b**2) + (a
+ b*sqrt(c + d*x))**(9/2)/(9*b**2) + (a + b*sqrt(c + d*x))**(5/2)*(3*a**2 -
 b**2*c)/(5*b**2) + (a + b*sqrt(c + d*x))**(3/2)*(-a**3 + a*b**2*c)/(3*b**2
))/b**2, Ne(b, 0)), (sqrt(a)*(-c*(c + d*x)/2 + (c + d*x)**2/4), True))/d**2
, Ne(d, 0)), (x**2*sqrt(a + b*sqrt(c))/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.70

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\frac{4\left(35\left(\sqrt{dx+c}b+a\right)^{\frac{9}{2}}-135\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}a-63\left(b^2c-3a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}+105\left(ab^2c-a^3\right)\left(\sqrt{d\right.}}{315\,b^4d^2}$$

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/315*(35*(sqrt(d*x + c)*b + a)^(9/2) - 135*(sqrt(d*x + c)*b + a)^(7/2)*a -
 63*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(5/2) + 105*(a*b^2*c - a^3)*(sqrt
(d*x + c)*b + a)^(3/2))/(b^4*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(109) = 218.

Time = 0.39 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.10

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx =$$

$$-\frac{4\left(\frac{3\left(35\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}b^2c-105\sqrt{\sqrt{dx+c}b+a}ab^2c-15\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}+63\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}a-105\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}a^2+105\sqrt{\sqrt{dx+c}b+a}a^3\right)a}{b^3d}\right.}{}$$

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/315*(3*(35*(sqrt(d*x + c)*b + a)^(3/2)*b^2*c - 105*sqrt(sqrt(d*x + c)*b
+ a)*a*b^2*c - 15*(sqrt(d*x + c)*b + a)^(7/2) + 63*(sqrt(d*x + c)*b + a)^(5
/2)*a - 105*(sqrt(d*x + c)*b + a)^(3/2)*a^2 + 105*sqrt(sqrt(d*x + c)*b + a)
*a^3)*a/(b^3*d) + (63*(sqrt(d*x + c)*b + a)^(5/2)*b^2*c - 210*(sqrt(d*x + c
)*b + a)^(3/2)*a*b^2*c + 315*sqrt(sqrt(d*x + c)*b + a)*a^2*b^2*c - 35*(sqrt
(d*x + c)*b + a)^(9/2) + 180*(sqrt(d*x + c)*b + a)^(7/2)*a - 378*(sqrt(d*x
+ c)*b + a)^(5/2)*a^2 + 420*(sqrt(d*x + c)*b + a)^(3/2)*a^3 - 315*sqrt(sqrt
(d*x + c)*b + a)*a^4)/(b^3*d))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx = \int x\,\sqrt{a+b\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(x*(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.628 $\int \sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx=-\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}$$

```
[Out] -4/3*a*(a+b*(d*x+c)^(1/2))^(3/2)/b^2/d+4/5*(a+b*(d*x+c)^(1/2))^(5/2)/b^2/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {253, 196, 45}

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx=\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}-\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^2*d) + (4*(a + b*Sqrt[c + d*x])^(5/
2))/(5*b^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d} \\
&= \frac{2\text{Subst}\left(\int x\sqrt{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d} \\
&= \frac{2\text{Subst}\left(\int \left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)\,dx, x, \sqrt{c+dx}\right)}{d} \\
&= -\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\sqrt{a+b\sqrt{c+dx}}\left(-2a^2+ab\sqrt{c+dx}+3b^2(c+dx)\right)}{15b^2d}$$

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]],x]
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-2*a^2 + a*b*Sqrt[c + d*x] + 3*b^2*(c + d*x))
)/(15*b^2*d)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}-\frac{4a\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}}{b^{2}d}$	41
default	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}-\frac{4a\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}}{b^{2}d}$	41

[In] int((a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)

[Out] 4/d/b^2*(1/5*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*a*(a+b*(d*x+c)^(1/2))^(3/2))

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int\sqrt{a+b\sqrt{c+dx}}\,dx=\frac{4\left(3\,b^{2}dx+3\,b^{2}c+\sqrt{dx+c}ab-2\,a^{2}\right)\sqrt{\sqrt{dx+c}b+a}}{15\,b^{2}d}$$

[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 4/15*(3*b^2*d*x + 3*b^2*c + sqrt(d*x + c)*a*b - 2*a^2)*sqrt(sqrt(d*x + c)*b + a)/(b^2*d)

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25

$$\int\sqrt{a+b\sqrt{c+dx}}\,dx=\begin{cases}\frac{2\left(\begin{cases}\frac{2\left(-\frac{a\left(a+b\sqrt{c+dx}\right)^{\frac{3}{2}}}{3}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{5}{2}}}{5}\right)}{b^{2}} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}(c+dx)}{2} & \text{otherwise}\end{cases}\right)}{d} & \text{for}\ d\neq 0\\ x\sqrt{a+b\sqrt{c}} & \text{otherwise}\end{cases}$$

[In] integrate((a+b*(d*x+c)**(1/2))**(1/2),x)

[Out] Piecewise((2*Piecewise((2*(-a*(a + b*sqrt(c + d*x))**(3/2)/3 + (a + b*sqrt(c + d*x))**(5/2)/5)/b**2, Ne(b, 0)), (sqrt(a)*(c + d*x)/2, True))/d, Ne(d, 0)), (x*sqrt(a + b*sqrt(c)), True))

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\left(\frac{3\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}}{b^2} - \frac{5\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}a}{b^2}\right)}{15\,d}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/15*(3*(sqrt(d*x + c)*b + a)^(5/2)/b^2 - 5*(sqrt(d*x + c)*b + a)^(3/2)*a/b
^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(44) = 88$.

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.77

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\left(\frac{5\left(\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}-3\sqrt{\sqrt{dx+c}b+a}a\right)a}{b} + \frac{3\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}-10\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}a+15\sqrt{\sqrt{dx+c}b+a}a^2}{b}\right)}{15\,bd}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(5*((sqrt(d*x + c)*b + a)^(3/2) - 3*sqrt(sqrt(d*x + c)*b + a)*a)*a/b +
 (3*(sqrt(d*x + c)*b + a)^(5/2) - 10*(sqrt(d*x + c)*b + a)^(3/2)*a + 15*sqr
t(sqrt(d*x + c)*b + a)*a^2)/b)/(b*d)
```

Mupad [B] (verification not implemented)

Time = 17.90 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\left(a+b\sqrt{c+d\,x}\right)^{5/2}}{5\,b^2\,d} - \frac{4\,a\left(a+b\sqrt{c+d\,x}\right)^{3/2}}{3\,b^2\,d}$$

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] (4*(a + b*(c + d*x)^(1/2))^(5/2))/(5*b^2*d) - (4*a*(a + b*(c + d*x)^(1/2))^
(3/2))/(3*b^2*d)
```

3.629 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x} \, dx = 4\sqrt{a+b\sqrt{c+dx}} - 2\sqrt{a-b\sqrt{c}}\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right) - 2\sqrt{a+b\sqrt{c}}\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

```
[Out] -2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))*(a-b*c^(1/2))^(1/
2)-2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1/2))*(a+b*c^(1/2))^(
1/2)+4*(a+b*(d*x+c)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {378, 1412, 839, 841, 1180, 213}

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x} \, dx = -2\sqrt{a-b\sqrt{c}}\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right) - 2\sqrt{a+b\sqrt{c}}\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right) + 4\sqrt{a+b\sqrt{c+dx}}$$

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[a - b*Sqrt[c]]*ArcTanh[Sqrt[a + b*Sqrt
[c + d*x]]/Sqrt[a - b*Sqrt[c]]] - 2*Sqrt[a + b*Sqrt[c]]*ArcTanh[Sqrt[a + b*
Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 839

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[(d + e*x)^(m -
 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}}}{-c+x}\,dx, x, c+dx\right)$$

$$= 2\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= 4\sqrt{a+b\sqrt{c+dx}} + 2\text{Subst}\left(\int \frac{bc+ax}{\sqrt{a+bx}\,(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)$$

$$= 4\sqrt{a+b\sqrt{c+dx}} + 4\text{Subst}\left(\int \frac{-a^2+b^2c+ax^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)$$

$$= 4\sqrt{a+b\sqrt{c+dx}} + \left(2\left(a-b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right) + \left(2\left(a+b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{1}{-a-b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)$$

$$= 4\sqrt{a+b\sqrt{c+dx}} - 2\sqrt{a-b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right) - 2\sqrt{a+b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = 4\sqrt{a+b\sqrt{c+dx}} - 2\sqrt{-a-b\sqrt{c}}\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right) - 2\sqrt{-a+b\sqrt{c}}\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)$$

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[-a - b*Sqrt[c]]*ArcTan[Sqrt[a + b*Sqrt
[c + d*x]]/Sqrt[-a - b*Sqrt[c]]] - 2*Sqrt[-a + b*Sqrt[c]]*ArcTan[Sqrt[a + b
*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$4\sqrt{a+b\sqrt{dx+c}}-\frac{2\left(-b^2c-a\sqrt{b^2c}\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{\sqrt{b^2c}\,\sqrt{-\sqrt{b^2c}-a}}-\frac{2\left(b^2c-a\sqrt{b^2c}\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\sqrt{b^2c}\,\sqrt{\sqrt{b^2c}-a}}$	154
default	$4\sqrt{a+b\sqrt{dx+c}}-\frac{2\left(-b^2c-a\sqrt{b^2c}\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{\sqrt{b^2c}\,\sqrt{-\sqrt{b^2c}-a}}-\frac{2\left(b^2c-a\sqrt{b^2c}\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\sqrt{b^2c}\,\sqrt{\sqrt{b^2c}-a}}$	154

```
[In] int((a+b*(d*x+c)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 4*(a+b*(d*x+c)^(1/2))^(1/2)-2*(-b^2*c-a*(b^2*c)^(1/2))/(b^2*c)^(1/2)/(-(b^2
*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2
))-2*(b^2*c-a*(b^2*c)^(1/2))/(b^2*c)^(1/2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((
a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(88) = 176.

Time = 0.35 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = -\sqrt{a+\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}+2\sqrt{a+\sqrt{b^2c}}\right) + \sqrt{a+\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}-2\sqrt{a+\sqrt{b^2c}}\right) - \sqrt{a-\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}+2\sqrt{a-\sqrt{b^2c}}\right) + \sqrt{a-\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}-2\sqrt{a-\sqrt{b^2c}}\right) + 4\sqrt{\sqrt{dx+c}b+a}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) + 2*sqrt(a + sqrt(b^
2*c))) + sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) - 2*sqrt(a +
 sqrt(b^2*c))) - sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) + 2*
sqrt(a - sqrt(b^2*c))) + sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b +
 a) - 2*sqrt(a - sqrt(b^2*c))) + 4*sqrt(sqrt(d*x + c)*b + a)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = \int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$$

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x,x)
[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = \int \frac{\sqrt{\sqrt{dx+c}b+a}}{x}\,dx$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x, x)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = \frac{2\left(2\sqrt{\sqrt{dx+c}b+a}b - \frac{(b^3c-a^2b)\sqrt{b\sqrt{c}-a}\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{b^2c-a^2} - \frac{(b^3c-a^2b)\sqrt{-b\sqrt{c}-a}\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{b^2c-a^2}\right)}{b}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="giac")
[Out] 2*(2*sqrt(sqrt(d*x + c)*b + a)*b - (b^3*c - a^2*b)*sqrt(b*sqrt(c) - a)*arct
an(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a + sqrt(b^2*c)))/(b^2*c - a^2) - (b^3*c
 - a^2*b)*sqrt(-b*sqrt(c) - a)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - s
qrt(b^2*c)))/(b^2*c - a^2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = \int \frac{\sqrt{a+b\,\sqrt{c+d\,x}}}{x}\,dx$$

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2)/x,x)
[Out] int((a + b*(c + d*x)^(1/2))^(1/2)/x, x)
```

3.630 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx = -\frac{\sqrt{a+b\sqrt{c+dx}}}{x}+\frac{bd\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{a-b\sqrt{c}}\sqrt{c}}-\frac{bd\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{a+b\sqrt{c}}\sqrt{c}}$$

```
[Out] 1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/c^(1/2)/(a-b
*c^(1/2))^(1/2)-1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1/
2))/c^(1/2)/(a+b*c^(1/2))^(1/2)-(a+b*(d*x+c)^(1/2))^(1/2)/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {378, 1412, 835, 12, 722, 1107, 213}

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx = \frac{bd\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}}-\frac{bd\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}-\frac{\sqrt{a+b\sqrt{c+dx}}}{x}$$

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] -(Sqrt[a + b*Sqrt[c + d*x]]/x) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqr
t[a - b*Sqrt[c]]])/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[c]) - (b*d*ArcTanh[Sqrt[a +
b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(2*Sqrt[a + b*Sqrt[c]]*Sqrt[c])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}}}{(-c+x)^2}\,dx, x, c+dx\right)\\
&= (2d)\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} - \frac{d\text{Subst}\left(\int -\frac{bc}{2\sqrt{a+bx}(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)}{c}\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{1}{2}(bd)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \left(b^2d\right)\text{Subst}\left(\int \frac{1}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{(bd)\text{Subst}\left(\int \frac{1}{-a-b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{2\sqrt{c}}\\
&\quad - \frac{(bd)\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{2\sqrt{c}}\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{a-b\sqrt{c}}\sqrt{c}} - \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{a+b\sqrt{c}}\sqrt{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx = \frac{1}{2}\left(-\frac{2\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{bd\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\sqrt{-a-b\sqrt{c}}\sqrt{c}} - \frac{bd\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\sqrt{-a+b\sqrt{c}}\sqrt{c}}\right)$$

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
[Out] ((-2*Sqrt[a + b*Sqrt[c + d*x]])/x + (b*d*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/S
qrt[-a - b*Sqrt[c]]])/(Sqrt[-a - b*Sqrt[c]]*Sqrt[c]) - (b*d*ArcTan[Sqrt[a +
 b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]])/(Sqrt[-a + b*Sqrt[c]]*Sqrt[c]))/2
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.21

method	result	s
derivativedivides	$4d\,b^{2}\left(-\frac{\sqrt{a+b\sqrt{dx+c}}}{4\left(\left(a+b\sqrt{dx+c}\right)^{2}-2a\left(a+b\sqrt{dx+c}\right)-b^{2}c+a^{2}\right)}-\frac{\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^{2}c}-a}}\right)}{8\sqrt{b^{2}c}\,\sqrt{\sqrt{b^{2}c}-a}}+\frac{\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^{2}c}-a}}\right)}{8\sqrt{b^{2}c}\,\sqrt{-\sqrt{b^{2}c}-a}}\right)$	1
default	$4d\,b^{2}\left(-\frac{\sqrt{a+b\sqrt{dx+c}}}{4\left(\left(a+b\sqrt{dx+c}\right)^{2}-2a\left(a+b\sqrt{dx+c}\right)-b^{2}c+a^{2}\right)}-\frac{\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^{2}c}-a}}\right)}{8\sqrt{b^{2}c}\,\sqrt{\sqrt{b^{2}c}-a}}+\frac{\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^{2}c}-a}}\right)}{8\sqrt{b^{2}c}\,\sqrt{-\sqrt{b^{2}c}-a}}\right)$	1

```
[In] int((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4*d*b^2*(-1/4*(a+b*(d*x+c)^(1/2))^(1/2)/((a+b*(d*x+c)^(1/2))^2-2*a*(a+b*(d*
x+c)^(1/2))-b^2*c+a^2)-1/8/(b^2*c)^(1/2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+
b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))+1/8/(b^2*c)^(1/2)/(-(b^2*c)
^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. 2(101) = 202.

Time = 0.35 (sec) , antiderivative size = 1003, normalized size of antiderivative = 7.32

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx =$$

$$-\frac{x\sqrt{-\frac{ab^2d^2+\sqrt{\frac{b^6d^4}{b^4c^3-2\,a^2b^2c^2+a^4c}}(b^2c^2-a^2c)}{b^2c^2-a^2c}}\log\left(\sqrt{\sqrt{dx+c}b+a}b^4d^3+\left(b^4cd^2-\sqrt{\frac{b^6d^4}{b^4c^3-2\,a^2b^2c^2+a^4c}}(ab^2c^2-a^3c)\right.\right.}{}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*
(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3
 + (b^4*c*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2
- a^3*c))*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)
)*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^
4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*
log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 - sqrt(b^6*d^4/(b^4*c^3
- 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 + sqrt(b^6*
d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c)
)) + x*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(
```

```
b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3
+ (b^4*c*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 -
 a^3*c))*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))
*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4
/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*l
og(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 + sqrt(b^6*d^4/(b^4*c^3 -
 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 - sqrt(b^6*d
^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))
) + 4*sqrt(sqrt(d*x + c)*b + a))/x
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx = \int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx$$

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**2,x)
[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx = \int \frac{\sqrt{\sqrt{dx+c}b+a}}{x^2}\,dx$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(101) = 202$.

Time = 0.41 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx = \frac{\frac{2\sqrt{\sqrt{dx+c}b+a}\,b^3d^2}{b^2c-\left(\sqrt{dx+c}b+a\right)^2+2\left(\sqrt{dx+c}b+a\right)a-a^2} - \frac{\left(b^3cd^2|b|+ab^3\sqrt{c}d^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(bc^{\frac{3}{2}}+ac\right)\sqrt{b\sqrt{c}-a}\,|b|} + \frac{\left(b^3cd^2|b|-ab^3\sqrt{c}d^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\left(bc^{\frac{3}{2}}-ac\right)\sqrt{-b\sqrt{c}-a}\,|b|}}{2\,bd}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(sqrt(d*x + c)*b + a)*b^3*d^2/(b^2*c - (sqrt(d*x + c)*b + a)^2 +
 2*(sqrt(d*x + c)*b + a)*a - a^2) - (b^3*c*d^2*abs(b) + a*b^3*sqrt(c)*d^2)*
arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a + sqrt(b^2*c)))/((b*c^(3/2) + a*c)
*sqrt(b*sqrt(c) - a)*abs(b)) + (b^3*c*d^2*abs(b) - a*b^3*sqrt(c)*d^2)*arcta
n(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*c^(3/2) - a*c)*sqrt
(-b*sqrt(c) - a)*abs(b)))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx=\int \frac{\sqrt{a+b\,\sqrt{c+d\,x}}}{x^2} \,dx$$

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int((a + b*(c + d*x)^(1/2))^(1/2)/x^2, x)
```

3.631 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 224

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2}+\frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} - \frac{b\left(2a-3b\sqrt{c}\right)d^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16\left(a-b\sqrt{c}\right)^{3/2}c^{3/2}} + \frac{b\left(2a+3b\sqrt{c}\right)d^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16\left(a+b\sqrt{c}\right)^{3/2}c^{3/2}}$$

```
[Out] -1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))*(2*a-3*b
*c^(1/2))/c^(3/2)/(a-b*c^(1/2))^(3/2)+1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/2)
)^(1/2)/(a+b*c^(1/2))^(1/2))*(2*a+3*b*c^(1/2))/c^(3/2)/(a+b*c^(1/2))^(3/2)-
1/2*(a+b*(d*x+c)^(1/2))^(1/2)/x^2+1/8*b*d*(b*c-a*(d*x+c)^(1/2))*(a+b*(d*x+c
)^(1/2))^(1/2)/c/(-b^2*c+a^2)/x
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used

$= \{378, 1412, 835, 12, 755, 841, 1180, 213\}$

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8cx\,(a^2-b^2c)} - \frac{bd^2\left(2a-3b\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{3/2}} + \frac{bd^2\left(2a+3b\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}\left(a+b\sqrt{c}\right)^{3/2}} - \frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2}$$

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + b*Sqrt[c + d*x]]/x^2 + (b*d*(b*c - a*Sqrt[c + d*x])*Sqrt[a +
b*Sqrt[c + d*x]])/(8*c*(a^2 - b^2*c)*x) - (b*(2*a - 3*b*Sqrt[c])*d^2*ArcTan
h[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(16*(a - b*Sqrt[c])^(3/2)
*c^(3/2)) + (b*(2*a + 3*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sq
rt[a + b*Sqrt[c]]])/(16*(a + b*Sqrt[c])^(3/2)*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d^2\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}}}{(-c+x)^3}\,dx, x, c+dx\right)\\
&= \left(2d^2\right)\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} - \frac{d^2\text{Subst}\left(\int -\frac{bc}{2\sqrt{a+bx}(-c+x^2)^2}\,dx, x, \sqrt{c+dx}\right)}{2c}\\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{1}{4}\left(bd^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)
\end{aligned}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} + \frac{(bd^2)\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-2a^2+3b^2c\right)-\frac{abx}{2}}{\sqrt{a+bx}\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)}{8c\left(a^2-b^2c\right)}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} + \frac{(bd^2)\,\text{Subst}\left(\int \frac{\frac{a^2b}{2}+\frac{1}{2}b\left(-2a^2+3b^2c\right)-\frac{1}{2}abx^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{4c\left(a^2-b^2c\right)}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} + \frac{\left(b\left(2a-3b\sqrt{c}\right)d^2\right)\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{16\left(a-b\sqrt{c}\right)c^{3/2}} - \frac{\left(b\left(2a+3b\sqrt{c}\right)d^2\right)\text{Subst}\left(\int \frac{1}{-a-b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{16\left(a+b\sqrt{c}\right)c^{3/2}}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} - \frac{b\left(2a-3b\sqrt{c}\right)d^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16\left(a-b\sqrt{c}\right)^{3/2}c^{3/2}} + \frac{b\left(2a+3b\sqrt{c}\right)d^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16\left(a+b\sqrt{c}\right)^{3/2}c^{3/2}}$$

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \frac{-\frac{2\sqrt{c}\sqrt{a+b\sqrt{c+dx}}\left(4a^2c+abdx\sqrt{c+dx}-b^2c(4c+dx)\right)}{(a^2-b^2c)x^2} + \frac{b\left(2a+3b\sqrt{c}\right)d^2\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\left(-a-b\sqrt{c}\right)^{3/2}} + \frac{b\left(-2a+3b\sqrt{c}\right)d^2\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\left(-a+b\sqrt{c}\right)^{3/2}}}{16c^{3/2}}$$

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

```
[Out] ((-2*Sqrt[c]*Sqrt[a + b*Sqrt[c + d*x]]*(4*a^2*c + a*b*d*x*Sqrt[c + d*x] - b
^2*c*(4*c + d*x)))/((a^2 - b^2*c)*x^2) + (b*(2*a + 3*b*Sqrt[c])*d^2*ArcTan[
Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a - b*Sqrt[c]]])/(-a - b*Sqrt[c])^(3/2) + (
b*(-2*a + 3*b*Sqrt[c])*d^2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqr
t[c]]])/(-a + b*Sqrt[c])^(3/2))/(16*c^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(174) = 348$.

Time = 0.30 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.66

method	result
derivativedivides	$-4d^2b^4\left(\frac{\frac{a(a+b\sqrt{dx+c})^{\frac{7}{2}}}{32b^2c(-b^2c+a^2)}-\frac{(b^2c+3a^2)(a+b\sqrt{dx+c})^{\frac{5}{2}}}{32b^2c(-b^2c+a^2)}+\frac{a(b^2c+3a^2)(a+b\sqrt{dx+c})^{\frac{3}{2}}}{32b^2c(-b^2c+a^2)}-\frac{(-3b^2c+a^2)\sqrt{a+b\sqrt{dx+c}}}{32b^2c}}{\left((a+b\sqrt{dx+c})^2-2a(a+b\sqrt{dx+c})-b^2c+a^2\right)^2}+\frac{(3b^2c}{}\right.$
default	$-4d^2b^4\left(\frac{\frac{a(a+b\sqrt{dx+c})^{\frac{7}{2}}}{32b^2c(-b^2c+a^2)}-\frac{(b^2c+3a^2)(a+b\sqrt{dx+c})^{\frac{5}{2}}}{32b^2c(-b^2c+a^2)}+\frac{a(b^2c+3a^2)(a+b\sqrt{dx+c})^{\frac{3}{2}}}{32b^2c(-b^2c+a^2)}-\frac{(-3b^2c+a^2)\sqrt{a+b\sqrt{dx+c}}}{32b^2c}}{\left((a+b\sqrt{dx+c})^2-2a(a+b\sqrt{dx+c})-b^2c+a^2\right)^2}+\frac{(3b^2c}{}\right.$

```
[In] int((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4*d^2*b^4*((1/32*a/b^2/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(7/2)-1/32*(b^2*
c+3*a^2)/b^2/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(5/2)+1/32*a*(b^2*c+3*a^2)/
b^2/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)-1/32*(-3*b^2*c+a^2)/b^2/c*(a+b
*(d*x+c)^(1/2))^(1/2))/((a+b*(d*x+c)^(1/2))^2-2*a*(a+b*(d*x+c)^(1/2))-b^2*c
+a^2)^2+1/32/b^2/c/(-b^2*c+a^2)*(1/2*(3*b^2*c+a*(b^2*c)^(1/2)-2*a^2)/(b^2*c
)^(1/2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(
1/2)-a)^(1/2))+1/2*(-3*b^2*c+a*(b^2*c)^(1/2)+2*a^2)/(b^2*c)^(1/2)/(-(b^2*c)
^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2856 vs. $2(175) = 350$.

Time = 0.56 (sec) , antiderivative size = 2856, normalized size of antiderivative = 12.75

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/32*((b^2*c^2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2
)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c
^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b
^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^
6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2
```

*b^8*c + 20*a^4*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 - 24*a^2
*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*
c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b
^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*
a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^
4*c + 4*a^5*b^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*
sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10
*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 +
a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) - (b^2*c^
2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 + (b^6*
c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b
^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a
^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2
*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a
^4*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*
a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b
^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^1
2*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 -
 6*a^10*b^2*c^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b
^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14
*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4
*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(
b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) + (b^2*c^2 - a^2*c)*x^
2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2*b
^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^
4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 +
15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*
a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*sqrt(
sqrt(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*
d^4 + 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*
c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2
*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c
^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^
6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2
*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20
*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a
^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) - (b^2*c^2 - a^2*c)*x^2*sqrt(-((15*
a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^
4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(
b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^
5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 -
 a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*sqrt(sqrt(d*x + c)
*b + a)*d^6 - ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 + 2*(2*a*
b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81
*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 1

```
5*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3
)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2
*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*
a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6
+ 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 +
3*a^4*b^2*c^4 - a^6*c^3))) - 4*(b^2*c*d*x - sqrt(d*x + c)*a*b*d*x + 4*b^2*c
^2 - 4*a^2*c)*sqrt(sqrt(d*x + c)*b + a))/((b^2*c^2 - a^2*c)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \int \frac{\sqrt{\sqrt{dx+c}b+a}}{x^3}\,dx$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(175) = 350$.

Time = 0.61 (sec) , antiderivative size = 895, normalized size of antiderivative = 4.00

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \frac{\frac{\left(\left(b^3c^2-a^2bc\right)^2ab^3\sqrt{c}d^3-\left(3\,b^7c^3-4\,a^2b^5c^2+a^4b^3c\right)d^3\left|b^3c^2-a^2bc\right|+\left(3\,ab^9c^{\frac{9}{2}}-8\,a^3b^7c^{\frac{7}{2}}+7\,a^5b^5c^{\frac{5}{2}}-2\,a^7b^3c^{\frac{3}{2}}\right)d^3\right)\arctan\left(\frac{}{\sqrt{-\frac{ab^2c^2-a^3c+\sqrt{(ab}}{}}}\right.}{\left(b^5c^{\frac{9}{2}}-ab^4c^4-2\,a^2b^3c^{\frac{7}{2}}+2\,a^3b^2c^3+a^4bc^{\frac{5}{2}}-a^5c^2\right)\sqrt{-b\sqrt{c}-a}\left|b^3c^2-a^2bc\right|}}{}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(((b^3*c^2 - a^2*b*c)^2*a*b^3*sqrt(c)*d^3 - (3*b^7*c^3 - 4*a^2*b^5*c^2
 + a^4*b^3*c)*d^3*abs(b^3*c^2 - a^2*b*c) + (3*a*b^9*c^(9/2) - 8*a^3*b^7*c^(
7/2) + 7*a^5*b^5*c^(5/2) - 2*a^7*b^3*c^(3/2))*d^3)*arctan(sqrt(sqrt(d*x + c
)*b + a)/sqrt(-(a*b^2*c^2 - a^3*c + sqrt((a*b^2*c^2 - a^3*c)^2 + (b^4*c^3 -
 2*a^2*b^2*c^2 + a^4*c)*(b^2*c^2 - a^2*c)))/(b^2*c^2 - a^2*c)))/((b^5*c^(9/
2) - a*b^4*c^4 - 2*a^2*b^3*c^(7/2) + 2*a^3*b^2*c^3 + a^4*b*c^(5/2) - a^5*c^
2)*sqrt(-b*sqrt(c) - a)*abs(b^3*c^2 - a^2*b*c)) + ((b^3*c^2 - a^2*b*c)^2*a*
b^3*d^3 + (3*b^7*c^(5/2) - 4*a^2*b^5*c^(3/2) + a^4*b^3*sqrt(c))*d^3*abs(b^3
*c^2 - a^2*b*c) + (3*a*b^9*c^4 - 8*a^3*b^7*c^3 + 7*a^5*b^5*c^2 - 2*a^7*b^3*
c)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c^2 - a^3*c - sqrt((a
*b^2*c^2 - a^3*c)^2 + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(b^2*c^2 - a^2*c)))
/(b^2*c^2 - a^2*c)))/((b^5*c^4 + a*b^4*c^(7/2) - 2*a^2*b^3*c^3 - 2*a^3*b^2*
c^(5/2) + a^4*b*c^2 + a^5*c^(3/2))*sqrt(b*sqrt(c) - a)*abs(b^3*c^2 - a^2*b*
c)) - 2*(3*sqrt(sqrt(d*x + c)*b + a)*b^7*c^2*d^3 + (sqrt(d*x + c)*b + a)^(5
/2)*b^5*c*d^3 - (sqrt(d*x + c)*b + a)^(3/2)*a*b^5*c*d^3 - 4*sqrt(sqrt(d*x +
 c)*b + a)*a^2*b^5*c*d^3 - (sqrt(d*x + c)*b + a)^(7/2)*a*b^3*d^3 + 3*(sqrt(
d*x + c)*b + a)^(5/2)*a^2*b^3*d^3 - 3*(sqrt(d*x + c)*b + a)^(3/2)*a^3*b^3*d
^3 + sqrt(sqrt(d*x + c)*b + a)*a^4*b^3*d^3)/((b^2*c^2 - a^2*c)*(b^2*c - (sq
rt(d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a)*a - a^2)^2))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \int \frac{\sqrt{a+b\,\sqrt{c+d\,x}}}{x^3}\,dx$$

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int((a + b*(c + d*x)^(1/2))^(1/2)/x^3, x)
```

3.632 $\int \frac{x^3}{a+b\sqrt{c+dx}} dx$

Optimal result

Integrand size = 19, antiderivative size = 230

$$\begin{aligned}\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx &= -\frac{a(a^4-3a^2b^2c+3b^4c^2)\,x}{b^6d^3}+\frac{2(a^2-b^2c)^3\sqrt{c+dx}}{b^7d^4} \\ &\quad+\frac{2(a^4-3a^2b^2c+3b^4c^2)\,(c+dx)^{3/2}}{3b^5d^4}-\frac{a(a^2-3b^2c)\,(c+dx)^2}{2b^4d^4} \\ &\quad+\frac{2(a^2-3b^2c)\,(c+dx)^{5/2}}{5b^3d^4}-\frac{a(c+dx)^3}{3b^2d^4} \\ &\quad+\frac{2(c+dx)^{7/2}}{7bd^4}-\frac{2a(a^2-b^2c)^3\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}\end{aligned}$$

```
[Out] -a*(3*b^4*c^2-3*a^2*b^2*c+a^4)*x/b^6/d^3+2/3*(3*b^4*c^2-3*a^2*b^2*c+a^4)*(d
*x+c)^(3/2)/b^5/d^4-1/2*a*(-3*b^2*c+a^2)*(d*x+c)^2/b^4/d^4+2/5*(-3*b^2*c+a^
2)*(d*x+c)^(5/2)/b^3/d^4-1/3*a*(d*x+c)^3/b^2/d^4+2/7*(d*x+c)^(7/2)/b/d^4-2*
a*(-b^2*c+a^2)^3*ln(a+b*(d*x+c)^(1/2))/b^8/d^4+2*(-b^2*c+a^2)^3*(d*x+c)^(1/
2)/b^7/d^4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used

$= \{378, 1412, 786\}$

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = -\frac{2a(a^2-b^2c)^3\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4} + \frac{2(a^2-b^2c)^3\sqrt{c+dx}}{b^7d^4} - \frac{a(a^2-3b^2c)(c+dx)^2}{2b^4d^4} + \frac{2(a^2-3b^2c)(c+dx)^{5/2}}{5b^3d^4} - \frac{ax(a^4-3a^2b^2c+3b^4c^2)}{b^6d^3} + \frac{2(a^4-3a^2b^2c+3b^4c^2)(c+dx)^{3/2}}{3b^5d^4} - \frac{a(c+dx)^3}{3b^2d^4} + \frac{2(c+dx)^{7/2}}{7bd^4}$$

```
[In] Int[x^3/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3)) + (2*(a^2 - b^2*c)^3*Sqr
t[c + d*x])/(b^7*d^4) + (2*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*(c + d*x)^(3/2))
/(3*b^5*d^4) - (a*(a^2 - 3*b^2*c)*(c + d*x)^2)/(2*b^4*d^4) + (2*(a^2 - 3*b^
2*c)*(c + d*x)^(5/2))/(5*b^3*d^4) - (a*(c + d*x)^3)/(3*b^2*d^4) + (2*(c + d
*x)^(7/2))/(7*b*d^4) - (2*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(b^8*
d^4)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int \frac{x(-c+x^2)^3}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{(-a^2+b^2c)^3}{b^7} - \frac{a(a^4-3a^2b^2c+3b^4c^2)x}{b^6} + \frac{(a^4-3a^2b^2c+3b^4c^2)x^2}{b^5} - \frac{a(a^2-3b^2c)x^3}{b^4} - \frac{(-a^2+3b^2c)x^4}{b^3} - \frac{ax^5}{b^2}\right.\right.}{d^4}$$

$$= -\frac{a(a^4 - 3a^2b^2c + 3b^4c^2)\,x}{b^6d^3} + \frac{2(a^2-b^2c)^3\sqrt{c+dx}}{b^7d^4} + \frac{2(a^4-3a^2b^2c+3b^4c^2)\,(c+dx)^{3/2}}{3b^5d^4} - \frac{a(a^2-3b^2c)\,(c+dx)^2}{2b^4d^4} + \frac{2(a^2-3b^2c)\,(c+dx)^{5/2}}{5b^3d^4} - \frac{a(c+dx)^3}{3b^2d^4} + \frac{2(c+dx)^{7/2}}{7bd^4} - \frac{2a(a^2-b^2c)^3\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{b\left(420a^6\sqrt{c+dx} - 140a^4b^2(8c-dx)\sqrt{c+dx} - 210a^5b(c+dx) + 105a^3b^3(5c^2+4cdx-d^2x^2) + 84a^2b^4\sqrt{\right.}{}$$

```
[In] Integrate[x^3/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(420*a^6*Sqrt[c + d*x] - 140*a^4*b^2*(8*c - d*x)*Sqrt[c + d*x] - 210*a^5
*b*(c + d*x) + 105*a^3*b^3*(5*c^2 + 4*c*d*x - d^2*x^2) + 84*a^2*b^4*Sqrt[c
+ d*x]*(11*c^2 - 3*c*d*x + d^2*x^2) - 35*a*b^5*(11*c^3 + 6*c^2*d*x - 3*c*d^
2*x^2 + 2*d^3*x^3) + 12*b^6*Sqrt[c + d*x]*(-16*c^3 + 8*c^2*d*x - 6*c*d^2*x^
2 + 5*d^3*x^3)) - 420*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(210*b^8*
d^4)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{2\left(\frac{(dx+c)^{\frac{7}{2}}b^6}{7} - \frac{a(dx+c)^3b^5}{6} - \frac{3b^6c(dx+c)^{\frac{5}{2}}}{5} + \frac{a^2b^4(dx+c)^{\frac{5}{2}}}{5} + \frac{3a\,b^5c(dx+c)^2}{4} + b^6c^2(dx+c)^{\frac{3}{2}} - \frac{a^3b^3(dx+c)^2}{4} - a^2b^4c(dx+c)^{\frac{3}{2}} - \frac{3a\,b}{}\right.}{b^7}$
default	$\frac{2\left(\frac{(dx+c)^{\frac{7}{2}}b^6}{7} - \frac{a(dx+c)^3b^5}{6} - \frac{3b^6c(dx+c)^{\frac{5}{2}}}{5} + \frac{a^2b^4(dx+c)^{\frac{5}{2}}}{5} + \frac{3a\,b^5c(dx+c)^2}{4} + b^6c^2(dx+c)^{\frac{3}{2}} - \frac{a^3b^3(dx+c)^2}{4} - a^2b^4c(dx+c)^{\frac{3}{2}} - \frac{3a\,b}{}\right.}{b^7}$

```
[In] int(x^3/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^4*(1/b^7*(1/7*(d*x+c)^(7/2)*b^6-1/6*a*(d*x+c)^3*b^5-3/5*b^6*c*(d*x+c)^(
5/2)+1/5*a^2*b^4*(d*x+c)^(5/2)+3/4*a*b^5*c*(d*x+c)^2+b^6*c^2*(d*x+c)^(3/2)-
1/4*a^3*b^3*(d*x+c)^2-a^2*b^4*c*(d*x+c)^(3/2)-3/2*a*b^5*c^2*(d*x+c)-b^6*c^3
*(d*x+c)^(1/2)+1/3*a^4*b^2*(d*x+c)^(3/2)+3/2*a^3*b^3*c*(d*x+c)+3*a^2*b^4*c^
2*(d*x+c)^(1/2)-1/2*a^5*b*(d*x+c)-3*a^4*b^2*c*(d*x+c)^(1/2)+a^6*(d*x+c)^(1/
2))-a*(-b^6*c^3+3*a^2*b^4*c^2-3*a^4*b^2*c+a^6)/b^8*ln(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = -\frac{70\,ab^6d^3x^3-105\,(ab^6c-a^3b^4)d^2x^2+210\,(ab^6c^2-2\,a^3b^4c+a^5b^2)dx-420\,(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-$$

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/210*(70*a*b^6*d^3*x^3 - 105*(a*b^6*c - a^3*b^4)*d^2*x^2 + 210*(a*b^6*c^2
 - 2*a^3*b^4*c + a^5*b^2)*d*x - 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*
c - a^7)*log(sqrt(d*x + c)*b + a) - 4*(15*b^7*d^3*x^3 - 48*b^7*c^3 + 231*a^
2*b^5*c^2 - 280*a^4*b^3*c + 105*a^6*b - 3*(6*b^7*c - 7*a^2*b^5)*d^2*x^2 + (
24*b^7*c^2 - 63*a^2*b^5*c + 35*a^4*b^3)*d*x)*sqrt(d*x + c))/(b^8*d^4)
```

Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.10

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \begin{cases} \frac{2\left(-\frac{a(c+dx)^3}{6b^2}-\frac{a\left(a^2-b^2c\right)^3\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^7}+\frac{(c+dx)^{\frac{7}{2}}}{7b}+\frac{\left(a^2-3b^2c\right)(c+dx)^{\frac{5}{2}}}{5b^3}+\frac{\left(-a^3+3ab^2c\right)(c+dx)^2}{4b^4}+\frac{(c+dx)^{\frac{3}{2}}\left(a^4-3a^2b^2c\right.}{3b^5}\right.}{d^4} \\ \frac{x^4}{4(a+b\sqrt{c})} \end{cases}$$

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((2*(-a*(c + d*x)**3/(6*b**2) - a*(a**2 - b**2*c)**3*Piecewise((sq
rt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/b**7 + (c + d
```

```
*x)**(7/2)/(7*b) + (a**2 - 3*b**2*c)*(c + d*x)**(5/2)/(5*b**3) + (-a**3 + 3
*a*b**2*c)*(c + d*x)**2/(4*b**4) + (c + d*x)**(3/2)*(a**4 - 3*a**2*b**2*c +
 3*b**4*c**2)/(3*b**5) + (c + d*x)*(-a**5 + 3*a**3*b**2*c - 3*a*b**4*c**2)/
(2*b**6) + sqrt(c + d*x)*(a**6 - 3*a**4*b**2*c + 3*a**2*b**4*c**2 - b**6*c*
*3)/b**7)/d**4, Ne(d, 0)), (x**4/(4*(a + b*sqrt(c))), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{\frac{60\,(dx+c)^{\frac{7}{2}}b^6-70\,(dx+c)^3ab^5-84\,(3\,b^6c-a^2b^4)(dx+c)^{\frac{5}{2}}+105\,(3\,ab^5c-a^3b^3)(dx+c)^2+140\,(3\,b^6c^2-3\,a^2b^4c+a^4b^2)(dx+c)^{\frac{3}{2}}-210\,(3\,ab^5c^2-3\,a^3b}{b^7}}{210\,d^4}$$

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/210*((60*(d*x + c)^(7/2)*b^6 - 70*(d*x + c)^3*a*b^5 - 84*(3*b^6*c - a^2*b
^4)*(d*x + c)^(5/2) + 105*(3*a*b^5*c - a^3*b^3)*(d*x + c)^2 + 140*(3*b^6*c^
2 - 3*a^2*b^4*c + a^4*b^2)*(d*x + c)^(3/2) - 210*(3*a*b^5*c^2 - 3*a^3*b^3*c
 + a^5*b)*(d*x + c) - 420*(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*sqr
t(d*x + c))/b^7 + 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(s
qrt(d*x + c)*b + a)/b^8)/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.48

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{2\,(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^8d^4} + \frac{60\,(dx+c)^{\frac{7}{2}}b^6d^{24}-252\,(dx+c)^{\frac{5}{2}}b^6cd^{24}+420\,(dx+c)^{\frac{3}{2}}b^6c^2d^{24}-420\,\sqrt{dx+c}b^6c^3d^{24}-70\,(dx+c)^3ab}{}$$

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(abs(sqrt(d*x + c)*b +
 a))/(b^8*d^4) + 1/210*(60*(d*x + c)^(7/2)*b^6*d^24 - 252*(d*x + c)^(5/2)*b
^6*c*d^24 + 420*(d*x + c)^(3/2)*b^6*c^2*d^24 - 420*sqrt(d*x + c)*b^6*c^3*d^
24 - 70*(d*x + c)^3*a*b^5*d^24 + 315*(d*x + c)^2*a*b^5*c*d^24 - 630*(d*x +
c)*a*b^5*c^2*d^24 + 84*(d*x + c)^(5/2)*a^2*b^4*d^24 - 420*(d*x + c)^(3/2)*a
^2*b^4*c*d^24 + 1260*sqrt(d*x + c)*a^2*b^4*c^2*d^24 - 105*(d*x + c)^2*a^3*b
^3*d^24 + 630*(d*x + c)*a^3*b^3*c*d^24 + 140*(d*x + c)^(3/2)*a^4*b^2*d^24 -
 1260*sqrt(d*x + c)*a^4*b^2*c*d^24 - 210*(d*x + c)*a^5*b*d^24 + 420*sqrt(d*
x + c)*a^6*d^24)/(b^7*d^28)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{2\,(c+d\,x)^{7/2}}{7\,b\,d^4} - \left(\frac{a^2\,\left(\frac{a^2\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{b^2}-\frac{6\,c^2}{b\,d^4}\right)}{b^2}+\frac{2\,c^3}{b\,d^4}\right)\sqrt{c+d\,x} - \left(\frac{a^2\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{3\,b^2}-\frac{2\,c^2}{b\,d^4}\right)(c+d\,x)^{3/2} - \left(\frac{6\,c}{5\,b\,d^4}-\frac{2\,a^2}{5\,b^3\,d^4}\right)(c+d\,x)^{5/2} + \frac{a\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)(c+d\,x)^2}{4\,b} - \frac{a\,(c+d\,x)^3}{3\,b^2\,d^4} - \frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,a^7-6\,a^5\,b^2\,c+6\,a^3\,b^4\,c^2-2\,a\,b^6\,c^3\right)}{b^8\,d^4} + \frac{a\,d\,x\,\left(\frac{a^2\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{b^2}-\frac{6\,c^2}{b\,d^4}\right)}{2\,b}$$

```
[In] int(x^3/(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(c + d*x)^(7/2))/(7*b*d^4) - ((a^2*((a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d
^4)))/b^2 - (6*c^2)/(b*d^4)))/b^2 + (2*c^3)/(b*d^4))*(c + d*x)^(1/2) - ((a^
2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d^4)))/(3*b^2) - (2*c^2)/(b*d^4))*(c + d*x)
^(3/2) - ((6*c)/(5*b*d^4) - (2*a^2)/(5*b^3*d^4))*(c + d*x)^(5/2) + (a*((6*c
)/(b*d^4) - (2*a^2)/(b^3*d^4))*(c + d*x)^2)/(4*b) - (a*(c + d*x)^3)/(3*b^2*
d^4) - (log(a + b*(c + d*x)^(1/2))*(2*a^7 - 6*a^5*b^2*c - 2*a*b^6*c^3 + 6*a
^3*b^4*c^2))/(b^8*d^4) + (a*d*x*((a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d^4)))/
b^2 - (6*c^2)/(b*d^4)))/(2*b)
```

3.633 $\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 151

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = -\frac{a(a^2-2b^2c)\,x}{b^4d^2}+\frac{2(a^2-b^2c)^2\sqrt{c+dx}}{b^5d^3}+\frac{2(a^2-2b^2c)\,(c+dx)^{3/2}}{3b^3d^3} - \frac{a(c+dx)^2}{2b^2d^3}+\frac{2(c+dx)^{5/2}}{5bd^3}-\frac{2a(a^2-b^2c)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}$$

```
[Out] -a*(-2*b^2*c+a^2)*x/b^4/d^2+2/3*(-2*b^2*c+a^2)*(d*x+c)^(3/2)/b^3/d^3-1/2*a*
(d*x+c)^2/b^2/d^3+2/5*(d*x+c)^(5/2)/b/d^3-2*a*(-b^2*c+a^2)^2*ln(a+b*(d*x+c)
^(1/2))/b^6/d^3+2*(-b^2*c+a^2)^2*(d*x+c)^(1/2)/b^5/d^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {378, 1412, 786}

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = -\frac{2a(a^2-b^2c)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3} + \frac{2(a^2-b^2c)^2\sqrt{c+dx}}{b^5d^3}-\frac{ax(a^2-2b^2c)}{b^4d^2} + \frac{2(a^2-2b^2c)\,(c+dx)^{3/2}}{3b^3d^3}-\frac{a(c+dx)^2}{2b^2d^3}+\frac{2(c+dx)^{5/2}}{5bd^3}$$

```
[In] Int[x^2/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*(a^2 - 2*b^2*c)*x)/(b^4*d^2)) + (2*(a^2 - b^2*c)^2*Sqrt[c + d*x])/(b^5
*d^3) + (2*(a^2 - 2*b^2*c)*(c + d*x)^(3/2))/(3*b^3*d^3) - (a*(c + d*x)^2)/(
```

```
2*b^2*d^3) + (2*(c + d*x)^(5/2))/(5*b*d^3) - (2*a*(a^2 - b^2*c)^2*Log[a + b
*Sqrt[c + d*x]])/(b^6*d^3)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d^3}\\
&= \frac{2\text{Subst}\left(\int \frac{x(-c+x^2)^2}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d^3}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{(-a^2+b^2c)^2}{b^5} - \frac{a(a^2-2b^2c)x}{b^4} - \frac{(-a^2+2b^2c)x^2}{b^3} - \frac{ax^3}{b^2} + \frac{x^4}{b} - \frac{a(a^2-b^2c)^2}{b^5(a+bx)}\right)dx, x, \sqrt{c+dx}\right)}{d^3}\\
&= -\frac{a(a^2-2b^2c)\,x}{b^4d^2} + \frac{2(a^2-b^2c)^2\sqrt{c+dx}}{b^5d^3} + \frac{2(a^2-2b^2c)\,(c+dx)^{3/2}}{3b^3d^3}\\
&\quad - \frac{a(c+dx)^2}{2b^2d^3} + \frac{2(c+dx)^{5/2}}{5bd^3} - \frac{2a(a^2-b^2c)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx$$
$$= \frac{b\left(60a^4\sqrt{c+dx}-20a^2b^2(5c-dx)\sqrt{c+dx}-30a^3b(c+dx)+15ab^3(3c^2+2cdx-d^2x^2)+4b^4\sqrt{c+dx}(8c\right.}{30b^6d^3}$$

```
[In] Integrate[x^2/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(60*a^4*Sqrt[c + d*x] - 20*a^2*b^2*(5*c - d*x)*Sqrt[c + d*x] - 30*a^3*b*
(c + d*x) + 15*a*b^3*(3*c^2 + 2*c*d*x - d^2*x^2) + 4*b^4*Sqrt[c + d*x]*(8*c
^2 - 4*c*d*x + 3*d^2*x^2)) - 60*a*(a^2 - b^2*c)^2*Log[a + b*Sqrt[c + d*x]])
/(30*b^6*d^3)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{2\left(\frac{(dx+c)^{\frac{5}{2}}b^4}{5}-\frac{a(dx+c)^2b^3}{4}-\frac{2b^4c(dx+c)^{\frac{3}{2}}}{3}+\frac{a^2b^2(dx+c)^{\frac{3}{2}}}{3}+a\,b^3c(dx+c)+b^4c^2\sqrt{dx+c}-\frac{a^3b(dx+c)}{2}-2a^2b^2c\sqrt{dx+c}+a^4\sqrt{dx+c}\right)}{b^5}}{d^3}$
default	$\frac{\frac{2\left(\frac{(dx+c)^{\frac{5}{2}}b^4}{5}-\frac{a(dx+c)^2b^3}{4}-\frac{2b^4c(dx+c)^{\frac{3}{2}}}{3}+\frac{a^2b^2(dx+c)^{\frac{3}{2}}}{3}+a\,b^3c(dx+c)+b^4c^2\sqrt{dx+c}-\frac{a^3b(dx+c)}{2}-2a^2b^2c\sqrt{dx+c}+a^4\sqrt{dx+c}\right)}{b^5}}{d^3}$

```
[In] int(x^2/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(1/b^5*(1/5*(d*x+c)^(5/2)*b^4-1/4*a*(d*x+c)^2*b^3-2/3*b^4*c*(d*x+c)^(
3/2)+1/3*a^2*b^2*(d*x+c)^(3/2)+a*b^3*c*(d*x+c)+b^4*c^2*(d*x+c)^(1/2)-1/2*a^
3*b*(d*x+c)-2*a^2*b^2*c*(d*x+c)^(1/2)+a^4*(d*x+c)^(1/2))-a*(b^4*c^2-2*a^2*b
^2*c+a^4)/b^6*ln(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx =$$
$$-\frac{15\,ab^4d^2x^2-30\,(ab^4c-a^3b^2)dx+60\,(ab^4c^2-2\,a^3b^2c+a^5)\log\left(\sqrt{dx+c}b+a\right)-4\,(3\,b^5d^2x^2+8\,b^5c^2-}{30\,b^6d^3}$$

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/30*(15*a*b^4*d^2*x^2 - 30*(a*b^4*c - a^3*b^2)*d*x + 60*(a*b^4*c^2 - 2*a^
3*b^2*c + a^5)*log(sqrt(d*x + c)*b + a) - 4*(3*b^5*d^2*x^2 + 8*b^5*c^2 - 25
*a^2*b^3*c + 15*a^4*b - (4*b^5*c - 5*a^2*b^3)*d*x)*sqrt(d*x + c))/(b^6*d^3)
```

Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = \begin{cases} \frac{2\left(-\frac{a(c+dx)^2}{4b^2} - \frac{a\left(a^2-b^2c\right)^2\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^5} + \frac{(c+dx)^{\frac{5}{2}}}{5b} + \frac{\left(a^2-2b^2c\right)(c+dx)^{\frac{3}{2}}}{3b^3} + \frac{\left(-a^3+2ab^2c\right)(c+dx)}{2b^4} + \frac{\sqrt{c+dx}\left(a^4-2a^2b^2c+b^4\right.}{b^5} \right.}{d^3} \\ \frac{x^3}{3(a+b\sqrt{c})} \end{cases}$$

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((2*(-a*(c + d*x)**2/(4*b**2) - a*(a**2 - b**2*c)**2*Piecewise((sq
rt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/b**5 + (c + d
*x)**(5/2)/(5*b) + (a**2 - 2*b**2*c)*(c + d*x)**(3/2)/(3*b**3) + (-a**3 + 2
*a*b**2*c)*(c + d*x)/(2*b**4) + sqrt(c + d*x)*(a**4 - 2*a**2*b**2*c + b**4*
c**2)/b**5)/d**3, Ne(d, 0)), (x**3/(3*(a + b*sqrt(c))), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = \frac{\frac{12\,(dx+c)^{\frac{5}{2}}b^4-15\,(dx+c)^2ab^3-20\,(2\,b^4c-a^2b^2)(dx+c)^{\frac{3}{2}}+30\,(2\,ab^3c-a^3b)(dx+c)+60\,(b^4c^2-2\,a^2b^2c+a^4)\sqrt{dx+c}}{b^5} - \frac{60\,(ab^4c^2-2\,a^3b^2c+a^5)\log(}{b^6}}{30\,d^3}$$

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/30*((12*(d*x + c)^(5/2)*b^4 - 15*(d*x + c)^2*a*b^3 - 20*(2*b^4*c - a^2*b^
2)*(d*x + c)^(3/2) + 30*(2*a*b^3*c - a^3*b)*(d*x + c) + 60*(b^4*c^2 - 2*a^2
*b^2*c + a^4)*sqrt(d*x + c))/b^5 - 60*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*log(s
qrt(d*x + c)*b + a)/b^6)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.31

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = -\frac{2\left(ab^4c^2-2\,a^3b^2c+a^5\right)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^6d^3} + \frac{12\,(dx+c)^{\frac{5}{2}}b^4d^{12}-40\,(dx+c)^{\frac{3}{2}}b^4cd^{12}+60\,\sqrt{dx+c}b^4c^2d^{12}-15\,(dx+c)^2ab^3d^{12}+60\,(dx+c)ab^3cd^{12}}{30\,b^5d^{15}}$$

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*log(abs(sqrt(d*x + c)*b + a))/(b^6*d^3)
+ 1/30*(12*(d*x + c)^(5/2)*b^4*d^12 - 40*(d*x + c)^(3/2)*b^4*c*d^12 + 60*sq
rt(d*x + c)*b^4*c^2*d^12 - 15*(d*x + c)^2*a*b^3*d^12 + 60*(d*x + c)*a*b^3*c
*d^12 + 20*(d*x + c)^(3/2)*a^2*b^2*d^12 - 120*sqrt(d*x + c)*a^2*b^2*c*d^12
- 30*(d*x + c)*a^3*b*d^12 + 60*sqrt(d*x + c)*a^4*d^12)/(b^5*d^15)
```

Mupad [B] (verification not implemented)

Time = 17.68 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = \frac{2\,{(c+d\,x)}^{5/2}}{5\,b\,d^3} - \left(\frac{a^2\,\left(\frac{4\,c}{b\,d^3}-\frac{2\,a^2}{b^3\,d^3}\right)}{b^2} - \frac{2\,c^2}{b\,d^3}\right)\,\sqrt{c+d\,x} - \left(\frac{4\,c}{3\,b\,d^3}-\frac{2\,a^2}{3\,b^3\,d^3}\right)\,{(c+d\,x)}^{3/2} - \frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,a^5-4\,a^3\,b^2\,c+2\,a\,b^4\,c^2\right)}{b^6\,d^3} - \frac{a\,{(c+d\,x)}^2}{2\,b^2\,d^3} + \frac{a\,d\,x\,\left(\frac{4\,c}{b\,d^3}-\frac{2\,a^2}{b^3\,d^3}\right)}{2\,b}$$

```
[In] int(x^2/(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(c + d*x)^(5/2))/(5*b*d^3) - ((a^2*((4*c)/(b*d^3) - (2*a^2)/(b^3*d^3)))/
b^2 - (2*c^2)/(b*d^3))*(c + d*x)^(1/2) - ((4*c)/(3*b*d^3) - (2*a^2)/(3*b^3*
d^3))*(c + d*x)^(3/2) - (log(a + b*(c + d*x)^(1/2))*(2*a^5 - 4*a^3*b^2*c +
2*a*b^4*c^2))/(b^6*d^3) - (a*(c + d*x)^2)/(2*b^2*d^3) + (a*d*x*((4*c)/(b*d^
3) - (2*a^2)/(b^3*d^3)))/(2*b)
```

3.634 $\int \frac{x}{a+b\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx=-\frac{ax}{b^2d}+\frac{2(a^2-b^2c)\sqrt{c+dx}}{b^3d^2}+\frac{2(c+dx)^{3/2}}{3bd^2}-\frac{2a(a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

```
[Out] -a*x/b^2/d+2/3*(d*x+c)^(3/2)/b/d^2-2*a*(-b^2*c+a^2)*ln(a+b*(d*x+c)^(1/2))/b
^4/d^2+2*(-b^2*c+a^2)*(d*x+c)^(1/2)/b^3/d^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {378, 1412, 786}

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx=-\frac{2a(a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{2(a^2-b^2c)\sqrt{c+dx}}{b^3d^2}-\frac{ax}{b^2d}+\frac{2(c+dx)^{3/2}}{3bd^2}$$

```
[In] Int[x/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*x)/(b^2*d)) + (2*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^3*d^2) + (2*(c + d*x)
^(3/2))/(3*b*d^2) - (2*a*(a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
```

```
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-c+x}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int \frac{x(-c+x^2)}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{a^2-b^2c}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}+\frac{-a^3+ab^2c}{b^3(a+bx)}\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= -\frac{ax}{b^2d}+\frac{2(a^2-b^2c)\sqrt{c+dx}}{b^3d^2}+\frac{2(c+dx)^{3/2}}{3bd^2}-\frac{2a(a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$
\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \frac{b\left(6a^2\sqrt{c+dx}+2b^2(-2c+dx)\sqrt{c+dx}-3ab(c+dx)\right)-6(a^3-ab^2c)\log\left(a+b\sqrt{c+dx}\right)}{3b^4d^2}
$$

```
[In] Integrate[x/(a + b*Sqrt[c + d*x]),x]
[Out] (b*(6*a^2*Sqrt[c + d*x] + 2*b^2*(-2*c + d*x)*Sqrt[c + d*x] - 3*a*b*(c + d*x
)) - 6*(a^3 - a*b^2*c)*Log[a + b*Sqrt[c + d*x]])/(3*b^4*d^2)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\frac{2\left(\frac{(dx+c)^{\frac{3}{2}}b^2}{3}-\frac{a(dx+c)b}{2}-b^2c\sqrt{dx+c}+a^2\sqrt{dx+c}\right)}{b^3}-\frac{2a\left(-b^2c+a^2\right)\ln\left(a+b\sqrt{dx+c}\right)}{b^4}}{d^2}$	85
default	$\frac{\frac{2\left(\frac{(dx+c)^{\frac{3}{2}}b^2}{3}-\frac{a(dx+c)b}{2}-b^2c\sqrt{dx+c}+a^2\sqrt{dx+c}\right)}{b^3}-\frac{2a\left(-b^2c+a^2\right)\ln\left(a+b\sqrt{dx+c}\right)}{b^4}}{d^2}$	85

```
[In] int(x/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/d^2*(1/b^3*(1/3*(d*x+c)^(3/2)*b^2-1/2*a*(d*x+c)*b-b^2*c*(d*x+c)^(1/2)+a^2
*(d*x+c)^(1/2))-a*(-b^2*c+a^2)/b^4*ln(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.79

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = -\frac{3\,ab^2dx-6\left(ab^2c-a^3\right)\log\left(\sqrt{dx+c}b+a\right)-2\left(b^3dx-2\,b^3c+3\,a^2b\right)\sqrt{dx+c}}{3\,b^4d^2}$$

```
[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
[Out] -1/3*(3*a*b^2*d*x - 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a) - 2*(b^3*d*x
 - 2*b^3*c + 3*a^2*b)*sqrt(d*x + c))/(b^4*d^2)
```

Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \begin{cases} \frac{2\left(-\frac{a(c+dx)}{2b^2}-\frac{a\left(a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for}\ b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^3}+\frac{(c+dx)^{\frac{3}{2}}}{3b}+\frac{\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3}\right)}{d^2} & \text{for}\ d\neq 0\\ \frac{x^2}{2\left(a+b\sqrt{c}\right)} & \text{otherwise}\end{cases}$$

```
[In] integrate(x/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((2*(-a*(c + d*x)/(2*b**2) - a*(a**2 - b**2*c)*Piecewise((sqrt(c +
 d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/b**3 + (c + d*x)**(
3/2)/(3*b) + (a**2 - b**2*c)*sqrt(c + d*x)/b**3)/d**2, Ne(d, 0)), (x**2/(2*
(a + b*sqrt(c))), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx=\frac{\frac{2\,(dx+c)^{\frac{3}{2}}b^2-3\,(dx+c)ab-6\,(b^2c-a^2)\sqrt{dx+c}}{b^3}+\frac{6\,(ab^2c-a^3)\log\left(\sqrt{dx+c}b+a\right)}{b^4}}{3\,d^2}$$

```
[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*((2*(d*x + c)^(3/2)*b^2 - 3*(d*x + c)*a*b - 6*(b^2*c - a^2)*sqrt(d*x +
c))/b^3 + 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a)/b^4)/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx =\frac{\frac{6\,(ab^2c-a^3)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^4d}+\frac{2\,(dx+c)^{\frac{3}{2}}b^2d^2-6\,\sqrt{dx+c}b^2cd^2-3\,(dx+c)abd^2+6\,\sqrt{dx+c}a^2d^2}{b^3d^3}}{3\,d}$$

```
[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*(6*(a*b^2*c - a^3)*log(abs(sqrt(d*x + c)*b + a))/(b^4*d) + (2*(d*x + c)
^(3/2)*b^2*d^2 - 6*sqrt(d*x + c)*b^2*c*d^2 - 3*(d*x + c)*a*b*d^2 + 6*sqrt(d
*x + c)*a^2*d^2)/(b^3*d^3))/d
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \frac{2\,{\left(c+d\,x\right)}^{3/2}}{3\,b\,d^2}-\left(\frac{2\,c}{b\,d^2}-\frac{2\,a^2}{b^3\,d^2}\right)\,\sqrt{c+d\,x}-\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,a^3-2\,a\,b^2\,c\right)}{b^4\,d^2}-\frac{a\,x}{b^2\,d}$$

```
[In] int(x/(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(c + d*x)^(3/2))/(3*b*d^2) - ((2*c)/(b*d^2) - (2*a^2)/(b^3*d^2))*(c + d*
x)^(1/2) - (log(a + b*(c + d*x)^(1/2))*(2*a^3 - 2*a*b^2*c))/(b^4*d^2) - (a*
x)/(b^2*d)
```

3.635 $\int \frac{1}{a+b\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx=\frac{2\sqrt{c+dx}}{bd}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[Out] -2*a*ln(a+b*(d*x+c)^(1/2))/b^2/d+2*(d*x+c)^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {253, 196, 45}

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx=\frac{2\sqrt{c+dx}}{bd}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[In] Int[(a + b*Sqrt[c + d*x])^(-1),x]
```

```
[Out] (2*Sqrt[c + d*x])/(b*d) - (2*a*Log[a + b*Sqrt[c + d*x]])/(b^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
```

```
IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d}\\ &= \frac{2\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d}\\ &= \frac{2\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)\,dx, x, \sqrt{c+dx}\right)}{d}\\ &= \frac{2\sqrt{c+dx}}{bd}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = \frac{2b\sqrt{c+dx}-2a\log\left(bd\left(a+b\sqrt{c+dx}\right)\right)}{b^2d}$$

```
[In] Integrate[(a + b*Sqrt[c + d*x])^(-1),x]
[Out] (2*b*Sqrt[c + d*x] - 2*a*Log[b*d*(a + b*Sqrt[c + d*x])])/(b^2*d)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\frac{2\sqrt{dx+c}}{b}-\frac{2a\ln\left(a+b\sqrt{dx+c}\right)}{b^2}}{d}$	36
default	$\frac{2\sqrt{dx+c}}{bd}+\frac{a\ln\left(-a+b\sqrt{dx+c}\right)}{b^2d}-\frac{a\ln\left(a+b\sqrt{dx+c}\right)}{b^2d}-\frac{a\ln\left(b^2dx+b^2c-a^2\right)}{b^2d}$	87

```
[In] int(1/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/d*(1/b*(d*x+c)^(1/2)-a/b^2*ln(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = -\frac{2\left(a\log\left(\sqrt{dx+c}b+a\right)-\sqrt{dx+c}b\right)}{b^2d}$$

```
[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
[Out] -2*(a*log(sqrt(d*x + c)*b + a) - sqrt(d*x + c)*b)/(b^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = \begin{cases} \frac{x}{a} & \text{for } b=0\wedge(b=0\vee d=0) \\ \frac{x}{a+b\sqrt{c}} & \text{for } d=0 \\ -\frac{2a\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{b^2d}+\frac{2\sqrt{c+dx}}{bd} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+b*(d*x+c)**(1/2)),x)
[Out] Piecewise((x/a, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c)), Eq(d
, 0)), (-2*a*log(a/b + sqrt(c + d*x))/(b**2*d) + 2*sqrt(c + d*x)/(b*d), Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = -\frac{2\left(\frac{a\log\left(\sqrt{dx+c}b+a\right)}{b^2}-\frac{\sqrt{dx+c}}{b}\right)}{d}$$

```
[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
[Out] -2*(a*log(sqrt(d*x + c)*b + a)/b^2 - sqrt(d*x + c)/b)/d
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = -\frac{2\,a\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^2d} + \frac{2\,\sqrt{dx+c}}{bd}$$

```
[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
[Out] -2*a*log(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*sqrt(d*x + c)/(b*d)
```

Mupad [B] (verification not implemented)

Time = 17.45 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = -\frac{2\left(a\,\ln\left(a+b\,\sqrt{c+d\,x}\right)-b\,\sqrt{c+d\,x}\right)}{b^2\,d}$$

```
[In] int(1/(a + b*(c + d*x)^(1/2)),x)
[Out] -(2*(a*log(a + b*(c + d*x)^(1/2)) - b*(c + d*x)^(1/2)))/(b^2*d)
```

3.636 $\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx=\frac{2b\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}$$

```
[Out] a*ln(x)/(-b^2*c+a^2)-2*a*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)+2*b*arctanh((d*
x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c+a^2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {378, 1412, 815, 649, 212, 266}

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx=\frac{2b\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}$$

```
[In] Int[1/(x*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(a^2 - b^2*c) + (a*Log[x])/(a^
2 - b^2*c) - (2*a*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)(-c+x)}\,dx, x, c+dx\right)\\
&= 2\text{Subst}\left(\int \frac{x}{(a+bx)\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)\\
&= 2\text{Subst}\left(\int \left(-\frac{ab}{\left(a^2-b^2c\right)(a+bx)}+\frac{bc-ax}{\left(a^2-b^2c\right)\left(c-x^2\right)}\right)\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{2\text{Subst}\left(\int \frac{bc-ax}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{a^2-b^2c}
\end{aligned}
$$

$$= -\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c} - \frac{(2a)\mathrm{Subst}\left(\int \frac{x}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{a^2-b^2c} + \frac{(2bc)\mathrm{Subst}\left(\int \frac{1}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{a^2-b^2c}$$

$$= \frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c} + \frac{a\log(x)}{a^2-b^2c} - \frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{2b\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + a\log(-dx) - 2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}$$

```
[In] Integrate[1/(x*(a + b*Sqrt[c + d*x])),x]
[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + a*Log[-(d*x)] - 2*a*Log[a + b
*Sqrt[c + d*x]])/(a^2 - b^2*c)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{a\ln(-dx)+2\sqrt{c}\,b\ \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{-b^2c+a^2} - \frac{2a\ln\left(a+b\sqrt{dx+c}\right)}{-b^2c+a^2}$	69
default	$\frac{a\ln(-dx)+2\sqrt{c}\,b\ \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{-b^2c+a^2} - \frac{2a\ln\left(a+b\sqrt{dx+c}\right)}{-b^2c+a^2}$	69

```
[In] int(1/x/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/(-b^2*c+a^2)*(1/2*a*ln(-d*x)+c^(1/2)*b*arctanh((d*x+c)^(1/2)/c^(1/2)))-2*
a*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.52

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx = \left[\frac{b\sqrt{c}\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2a\log\left(\sqrt{dx+c}b+a\right)-a\log\left(x\right)}{b^2c-a^2}, \frac{2b\sqrt{-c}\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+2a\log\left(\sqrt{d\right.}}{b^2c-a^2}\right.$$

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [(b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*a*log(sqrt(d*x
 + c)*b + a) - a*log(x))/(b^2*c - a^2), (2*b*sqrt(-c)*arctan(sqrt(d*x + c)*
sqrt(-c)/c) + 2*a*log(sqrt(d*x + c)*b + a) - a*log(x))/(b^2*c - a^2)]
```

Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx = \begin{cases} -\frac{2ab\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for}\ b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{a^2-b^2c} - \frac{2\left(-\frac{a\log\left(-dx\right)}{2}+\frac{bc\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}\right)}{a^2-b^2c} & \text{for}\ d\neq 0\\ \frac{\log\left(x\right)}{a+b\sqrt{c}} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((-2*a*b*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c
+ d*x))/b, True))/(a**2 - b**2*c) - 2*(-a*log(-d*x)/2 + b*c*atan(sqrt(c + d
*x)/sqrt(-c))/sqrt(-c))/(a**2 - b**2*c), Ne(d, 0)), (log(x)/(a + b*sqrt(c))
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{b\sqrt{c}\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{b^2c-a^2} - \frac{a\log\left(dx\right)}{b^2c-a^2} + \frac{2\,a\log\left(\sqrt{dx+c}b+a\right)}{b^2c-a^2}$$

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/(b^2*c -
 a^2) - a*log(d*x)/(b^2*c - a^2) + 2*a*log(sqrt(d*x + c)*b + a)/(b^2*c - a^
2)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{2\,ab\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^3c-a^2b} + \frac{2\,bc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{(b^2c-a^2)\sqrt{-c}} - \frac{a\log\left(dx\right)}{b^2c-a^2}$$

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*a*b*log(abs(sqrt(d*x + c)*b + a))/(b^3*c - a^2*b) + 2*b*c*arctan(sqrt(d*x
 + c)/sqrt(-c))/((b^2*c - a^2)*sqrt(-c)) - a*log(d*x)/(b^2*c - a^2)
```

Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.21

$$\begin{aligned}\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx &= \frac{\ln\left(2\,b^3\,c^{3/2}-2\,b^3\,c\,\sqrt{c+d\,x}-6\,a\,b^2\,c+6\,a\,b^2\,\sqrt{c}\,\sqrt{c+d\,x}\right)}{a+b\,\sqrt{c}}\\ &\quad+\frac{\ln\left(-2\,b^3\,c^{3/2}-2\,b^3\,c\,\sqrt{c+d\,x}-6\,a\,b^2\,c-6\,a\,b^2\,\sqrt{c}\,\sqrt{c+d\,x}\right)}{a-b\,\sqrt{c}}\\ &\quad+\frac{2\,a\,\ln\left(4\,b^5\,c^2\,\sqrt{c+d\,x}-36\,a^3\,b^2\,c+4\,a\,b^4\,c^2-36\,a^2\,b^3\,c\,\sqrt{c+d\,x}\right)}{b^2\,c-a^2}\end{aligned}$$

```
[In] int(1/(x*(a + b*(c + d*x)^(1/2))),x)
```

```
[Out] log(2*b^3*c^(3/2) - 2*b^3*c*(c + d*x)^(1/2) - 6*a*b^2*c + 6*a*b^2*c^(1/2)*(
c + d*x)^(1/2))/(a + b*c^(1/2)) + log(- 2*b^3*c^(3/2) - 2*b^3*c*(c + d*x)^(
1/2) - 6*a*b^2*c - 6*a*b^2*c^(1/2)*(c + d*x)^(1/2))/(a - b*c^(1/2)) + (2*a*
log(4*b^5*c^2*(c + d*x)^(1/2) - 36*a^3*b^2*c + 4*a*b^4*c^2 - 36*a^2*b^3*c*(
c + d*x)^(1/2)))/(b^2*c - a^2)
```

3.637 $\int \frac{1}{x^2(a+b\sqrt{c+dx})} dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx = -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x}+\frac{b(a^2+b^2c)\,d\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2-b^2c)^2}+\frac{ab^2d\log(x)}{(a^2-b^2c)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

```
[Out] a*b^2*d*ln(x)/(-b^2*c+a^2)^2-2*a*b^2*d*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^2
+b*(b^2*c+a^2)*d*arctanh((d*x+c)^(1/2)/c^(1/2))/(-b^2*c+a^2)^2/c^(1/2)+(-a+
b*(d*x+c)^(1/2))/(-b^2*c+a^2)/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {378, 1412, 837, 815, 649, 212, 266}

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{bd(a^2+b^2c)\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2-b^2c)^2}-\frac{a-b\sqrt{c+dx}}{x\,(a^2-b^2c)}+\frac{ab^2d\log(x)}{(a^2-b^2c)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

```
[In] Int[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] -((a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*x)) + (b*(a^2 + b^2*c)*d*ArcTanh[Sqr
t[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^2) + (a*b^2*d*Log[x])/(a^2 - b^
2*c)^2 - (2*a*b^2*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
```

```
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\text{integral} = d\text{Subst}\left(\int \frac{1}{(a+b\sqrt{x})(-c+x)^2}\,dx, x, c+dx\right)$$
$$= (2d)\text{Subst}\left(\int \frac{x}{(a+bx)(-c+x^2)^2}\,dx, x, \sqrt{c+dx}\right)$$
$$= -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x} + \frac{d\text{Subst}\left(\int \frac{-abc+b^2cx}{(a+bx)(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)}{c\,(a^2-b^2c)}$$
$$= -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x} + \frac{d\text{Subst}\left(\int \left(-\frac{2ab^3c}{(a^2-b^2c)(a+bx)} - \frac{bc(a^2+b^2c-2abx)}{(-a^2+b^2c)(c-x^2)}\right)dx, x, \sqrt{c+dx}\right)}{c\,(a^2-b^2c)}$$
$$= -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x} - \frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2} + \frac{(bd)\text{Subst}\left(\int \frac{a^2+b^2c-2abx}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$
$$= -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x} - \frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2} - \frac{(2ab^2d)\,\text{Subst}\left(\int \frac{x}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{(a^2-b^2c)^2} + \frac{(b(a^2+b^2c)\,d)\,\text{Subst}\left(\int \frac{1}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$
$$= -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x} + \frac{b(a^2+b^2c)\,d\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2-b^2c)^2} + \frac{ab^2d\log(x)}{(a^2-b^2c)^2} - \frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{b(a^2+b^2c)\,dx\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + \sqrt{c}\left(-\left((a^2-b^2c)\left(a-b\sqrt{c+dx}\right)\right) + ab^2dx\log(-dx) - 2ab^2dx\log\left(a+\right.\right.}{\sqrt{c}\,(a^2-b^2c)^2\,x}$$

```
[In] Integrate[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (b*(a^2 + b^2*c)*d*x*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + Sqrt[c]*(-((a^2 - b^2
*c)*(a - b*Sqrt[c + d*x])) + a*b^2*d*x*Log[-(d*x)] - 2*a*b^2*d*x*Log[a + b*
Sqrt[c + d*x]]))/(Sqrt[c]*(a^2 - b^2*c)^2*x)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

method	result	siz
derivativedivides	$2d\left(-\frac{a\,b^{2}\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^{2}c+a^{2}\right)^{2}}+\frac{-\frac{\left(\frac{1}{2}b^{3}c-\frac{1}{2}a^{2}b\right)\sqrt{dx+c}-\frac{a\,b^{2}c}{2}+\frac{a^{3}}{2}}{dx}+\frac{b\left(ab\ln\left(-dx\right)+\frac{\left(b^{2}c+a^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{2}}{\left(-b^{2}c+a^{2}\right)^{2}}\right)$	12
default	$2d\left(-\frac{a\,b^{2}\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^{2}c+a^{2}\right)^{2}}+\frac{-\frac{\left(\frac{1}{2}b^{3}c-\frac{1}{2}a^{2}b\right)\sqrt{dx+c}-\frac{a\,b^{2}c}{2}+\frac{a^{3}}{2}}{dx}+\frac{b\left(ab\ln\left(-dx\right)+\frac{\left(b^{2}c+a^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{2}}{\left(-b^{2}c+a^{2}\right)^{2}}\right)$	12

```
[In] int(1/x^2/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*d*(-a*b^2/(-b^2*c+a^2)^2*ln(a+b*(d*x+c)^(1/2))+1/(-b^2*c+a^2)^2*(-((1/2*b
^3*c-1/2*a^2*b)*(d*x+c)^(1/2)-1/2*a*b^2*c+1/2*a^3)/d/x+1/2*b*(a*b*ln(-d*x)+
(b^2*c+a^2)/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$$
$$=\left[-\frac{4\,ab^2cdx\log\left(\sqrt{dx+c}b+a\right)-2\,ab^2cdx\log\left(x\right)-2\,ab^2c^2-(b^3c+a^2b)\sqrt{c}dx\log\left(\frac{dx+2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)+2\,a}{2\,(b^4c^3-2\,a^2b^2c^2+a^4c)x}\right.$$
$$-\frac{2\,ab^2cdx\log\left(\sqrt{dx+c}b+a\right)-ab^2cdx\log\left(x\right)-ab^2c^2+(b^3c+a^2b)\sqrt{-c}dx\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+a^3c+(}{(b^4c^3-2\,a^2b^2c^2+a^4c)x}$$

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
[Out] [-1/2*(4*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - 2*a*b^2*c*d*x*log(x) - 2*a*
b^2*c^2 - (b^3*c + a^2*b)*sqrt(c)*d*x*log((d*x + 2*sqrt(d*x + c)*sqrt(c) +
2*c)/x) + 2*a^3*c + 2*(b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a^2*
b^2*c^2 + a^4*c)*x), -(2*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - a*b^2*c*d*x
*log(x) - a*b^2*c^2 + (b^3*c + a^2*b)*sqrt(-c)*d*x*arctan(sqrt(d*x + c)*sqr
t(-c)/c) + a^3*c + (b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a^2*b^2
*c^2 + a^4*c)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx = \int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$$

```
[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2)),x)
[Out] Integral(1/(x**2*(a + b*sqrt(c + d*x))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{1}{2}\left(\frac{2\,ab^2\log\left(dx\right)}{b^4c^2-2\,a^2b^2c+a^4} - \frac{4\,ab^2\log\left(\sqrt{dx+c}b+a\right)}{b^4c^2-2\,a^2b^2c+a^4} - \frac{\left(b^3c+a^2b\right)\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{c}} + \frac{2\left(\sqrt{dx+c}b-\right.}{b^2c^2-a^2c-\left(b^2c-\right.}\right.$$

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
[Out] 1/2*(2*a*b^2*log(d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - 4*a*b^2*log(sqrt(d*x
+ c)*b + a)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - (b^3*c + a^2*b)*log((sqrt(d*x +
 c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sq
rt(c)) + 2*(sqrt(d*x + c)*b - a)/(b^2*c^2 - a^2*c - (b^2*c - a^2)*(d*x + c)
))*d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.47

$$\begin{aligned}\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx = &-\frac{2\,ab^3d\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^5c^2-2\,a^2b^3c+a^4b} + \frac{ab^2d\log\left(-dx\right)}{b^4c^2-2\,a^2b^2c+a^4} \\ &- \frac{\left(b^3cd+a^2bd\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{-c}} \\ &+ \frac{ab^2cd-a^3d-\left(b^3cd-a^2bd\right)\sqrt{dx+c}}{\left(b^2c-a^2\right)^2dx}\end{aligned}$$

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*a*b^3*d*log(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c + a^4*b) +
a*b^2*d*log(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - (b^3*c*d + a^2*b*d)*arcta
n(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt(-c)) + (a*b^2
*c*d - a^3*d - (b^3*c*d - a^2*b*d)*sqrt(d*x + c))/((b^2*c - a^2)^2*d*x)
```

Mupad [B] (verification not implemented)

Time = 18.52 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)\,\left(4\,a\,b^2\,c\,d-b\,\sqrt{c}\,d\,(2\,a^2+2\,c\,b^2)\right)}{4\,a^4\,c-8\,a^2\,b^2\,c^2+4\,b^4\,c^3} + \frac{\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)\,\left(4\,a\,b^2\,c\,d+b\,\sqrt{c}\,d\,(2\,a^2+2\,c\,b^2)\right)}{4\,a^4\,c-8\,a^2\,b^2\,c^2+4\,b^4\,c^3} + \frac{\frac{a\,d}{b^2\,c-a^2}-\frac{b\,d\,\sqrt{c+d\,x}}{b^2\,c-a^2}}{d\,x} - \frac{2\,a\,b^2\,d\,\ln\left(a+b\,\sqrt{c+d\,x}\right)}{\left(b^2\,c-a^2\right)^2}$$

```
[In] int(1/(x^2*(a + b*(c + d*x)^(1/2))),x)
```

```
[Out] (log((c + d*x)^(1/2) - c^(1/2))*(4*a*b^2*c*d - b*c^(1/2)*d*(2*b^2*c + 2*a^2
)))/(4*a^4*c + 4*b^4*c^3 - 8*a^2*b^2*c^2) + (log((c + d*x)^(1/2) + c^(1/2))
*(4*a*b^2*c*d + b*c^(1/2)*d*(2*b^2*c + 2*a^2)))/(4*a^4*c + 4*b^4*c^3 - 8*a^
2*b^2*c^2) + ((a*d)/(b^2*c - a^2) - (b*d*(c + d*x)^(1/2))/(b^2*c - a^2))/(d
*x) - (2*a*b^2*d*log(a + b*(c + d*x)^(1/2)))/(b^2*c - a^2)^2
```

3.638 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 204

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2}-\frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} - \frac{b\left(a^4-6a^2b^2c-3b^4c^2\right)d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}\left(a^2-b^2c\right)^3} + \frac{ab^4d^2\log(x)}{\left(a^2-b^2c\right)^3}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}$$

```
[Out] -1/4*b*(-3*b^4*c^2-6*a^2*b^2*c+a^4)*d^2*arctanh((d*x+c)^(1/2)/c^(1/2))/c^(3
/2)/(-b^2*c+a^2)^3+a*b^4*d^2*ln(x)/(-b^2*c+a^2)^3-2*a*b^4*d^2*ln(a+b*(d*x+c
)^(1/2))/(-b^2*c+a^2)^3+1/2*(-a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)/x^2-1/4*b*d*(
4*a*b*c-(3*b^2*c+a^2)*(d*x+c)^(1/2))/c/(-b^2*c+a^2)^2/x
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {378, 1412, 837, 815, 649, 212, 266}

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = -\frac{a-b\sqrt{c+dx}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4cx\left(a^2-b^2c\right)^2} + \frac{ab^4d^2\log(x)}{\left(a^2-b^2c\right)^3}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3} - \frac{bd^2\left(a^4-6a^2b^2c-3b^4c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}\left(a^2-b^2c\right)^3}$$

```
[In] Int[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] -1/2*(a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*x^2) - (b*d*(4*a*b*c - (a^2 + 3*b
^2*c)*Sqrt[c + d*x]))/(4*c*(a^2 - b^2*c)^2*x) - (b*(a^4 - 6*a^2*b^2*c - 3*b
^4*c^2)*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(4*c^(3/2)*(a^2 - b^2*c)^3) + (
a*b^4*d^2*Log[x])/(a^2 - b^2*c)^3 - (2*a*b^4*d^2*Log[a + b*Sqrt[c + d*x]])/
(a^2 - b^2*c)^3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
```

```
[2*m, 2*p])
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^2\text{Subst}\left(\int \frac{1}{(a+b\sqrt{x})\,(-c+x)^3}\,dx, x, c+dx\right)\\
&= (2d^2)\,\text{Subst}\left(\int \frac{x}{(a+bx)\,(-c+x^2)^3}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{a-b\sqrt{c+dx}}{2\,(a^2-b^2c)\,x^2}+\frac{d^2\text{Subst}\left(\int \frac{-abc+3b^2cx}{(a+bx)(-c+x^2)^2}\,dx, x, \sqrt{c+dx}\right)}{2c\,(a^2-b^2c)}\\
&= -\frac{a-b\sqrt{c+dx}}{2\,(a^2-b^2c)\,x^2}-\frac{bd\left(4abc-(a^2+3b^2c)\sqrt{c+dx}\right)}{4c\,(a^2-b^2c)^2\,x}\\
&\quad+\frac{d^2\text{Subst}\left(\int \frac{abc(a^2-5b^2c)+b^2c(a^2+3b^2c)x}{(a+bx)(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)}{4c^2\,(a^2-b^2c)^2}\\
&= -\frac{a-b\sqrt{c+dx}}{2\,(a^2-b^2c)\,x^2}-\frac{bd\left(4abc-(a^2+3b^2c)\sqrt{c+dx}\right)}{4c\,(a^2-b^2c)^2\,x}\\
&\quad+\frac{d^2\text{Subst}\left(\int \left(-\frac{8ab^5c^2}{(a^2-b^2c)(a+bx)}-\frac{bc(-a^4+6a^2b^2c+3b^4c^2-8ab^3cx)}{(-a^2+b^2c)(c-x^2)}\right)dx, x, \sqrt{c+dx}\right)}{4c^2\,(a^2-b^2c)^2}\\
&= -\frac{a-b\sqrt{c+dx}}{2\,(a^2-b^2c)\,x^2}-\frac{bd\left(4abc-(a^2+3b^2c)\sqrt{c+dx}\right)}{4c\,(a^2-b^2c)^2\,x}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^3}\\
&\quad+\frac{(bd^2)\,\text{Subst}\left(\int \frac{-a^4+6a^2b^2c+3b^4c^2-8ab^3cx}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{4c\,(a^2-b^2c)^3}\\
&= -\frac{a-b\sqrt{c+dx}}{2\,(a^2-b^2c)\,x^2}-\frac{bd\left(4abc-(a^2+3b^2c)\sqrt{c+dx}\right)}{4c\,(a^2-b^2c)^2\,x}\\
&\quad-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^3}-\frac{(2ab^4d^2)\,\text{Subst}\left(\int \frac{x}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{(a^2-b^2c)^3}\\
&\quad-\frac{(b(a^4-6a^2b^2c-3b^4c^2)\,d^2)\,\text{Subst}\left(\int \frac{1}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{4c\,(a^2-b^2c)^3}
\end{aligned}
$$

$$= -\frac{a-b\sqrt{c+dx}}{2(a^2-b^2c)x^2} - \frac{bd(4abc-(a^2+3b^2c)\sqrt{c+dx})}{4c(a^2-b^2c)^2x} - \frac{b(a^4-6a^2b^2c-3b^4c^2)d^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}(a^2-b^2c)^3} + \frac{ab^4d^2\log(x)}{(a^2-b^2c)^3} - \frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^3}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$$
$$= \frac{b(a^4-6a^2b^2c-3b^4c^2)\,d^2x^2\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\sqrt{c}\left((a^2-b^2c)\left(2a^3c-2ab^2c(c-2dx)+b^3c(2c-3dx)\sqrt{c+\right.\right.}{4c^{3/2}\left(-a^2+b^2c\right)^3x^2}$$

```
[In] Integrate[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (b*(a^4 - 6*a^2*b^2*c - 3*b^4*c^2)*d^2*x^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] +
 Sqrt[c]*((a^2 - b^2*c)*(2*a^3*c - 2*a*b^2*c*(c - 2*d*x) + b^3*c*(2*c - 3*d
*x)*Sqrt[c + d*x] - a^2*b*Sqrt[c + d*x]*(2*c + d*x)) - 4*a*b^4*c*d^2*x^2*Lo
g[-(d*x)] + 8*a*b^4*c*d^2*x^2*Log[a + b*Sqrt[c + d*x]]))/(4*c^(3/2)*(-a^2 +
 b^2*c)^3*x^2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.11

method	result
derivativedivides	$2d^2\left(-\frac{a\,b^4\ln\left(a+b\sqrt{dx+c}\right)}{(-b^2c+a^2)^3}-\frac{-\frac{b\left(-3b^4c^2+2a^2b^2c+a^4\right)(dx+c)^{\frac{3}{2}}}{8c}+\left(-\frac{1}{2}a\,b^4c+\frac{1}{2}a^3b^2\right)(dx+c)+\left(\frac{3}{4}a^2b^3c-\frac{1}{8}b\,a^4-\frac{5}{8}c^2b^5\right)\sqrt{dx}}{d^2x^2}}{(-b^2c+}\right.$
default	$2d^2\left(-\frac{a\,b^4\ln\left(a+b\sqrt{dx+c}\right)}{(-b^2c+a^2)^3}-\frac{-\frac{b\left(-3b^4c^2+2a^2b^2c+a^4\right)(dx+c)^{\frac{3}{2}}}{8c}+\left(-\frac{1}{2}a\,b^4c+\frac{1}{2}a^3b^2\right)(dx+c)+\left(\frac{3}{4}a^2b^3c-\frac{1}{8}b\,a^4-\frac{5}{8}c^2b^5\right)\sqrt{dx}}{d^2x^2}}{(-b^2c+}\right.$

```
[In] int(1/x^3/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*(-a*b^4/(-b^2*c+a^2)^3*ln(a+b*(d*x+c)^(1/2))-1/(-b^2*c+a^2)^3*((-1/8*
b*(-3*b^4*c^2+2*a^2*b^2*c+a^4)/c*(d*x+c)^(3/2)+(-1/2*a*b^4*c+1/2*a^3*b^2)*(
d*x+c)+(3/4*a^2*b^3*c-1/8*b*a^4-5/8*c^2*b^5)*(d*x+c)^(1/2)+3/4*a*b^4*c^2-a^
3*b^2*c+1/4*a^5)/d^2/x^2+1/8*b/c*(-4*a*b^3*c*ln(-d*x)+(-3*b^4*c^2-6*a^2*b^2
*c+a^4)/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.68 (sec) , antiderivative size = 534, normalized size of antiderivative = 2.62

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$$
$$= \left[\frac{16\,ab^4c^2d^2x^2\log\left(\sqrt{dx+c}b+a\right)-8\,ab^4c^2d^2x^2\log\left(x\right)+4\,ab^4c^4-8\,a^3b^2c^3+4\,a^5c^2+\left(3\,b^5c^2+6\,a^2b^3c-\right.}{8\,(b^6c^5}\right.$$

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/8*(16*a*b^4*c^2*d^2*x^2*log(sqrt(d*x + c)*b + a) - 8*a*b^4*c^2*d^2*x^2*l
og(x) + 4*a*b^4*c^4 - 8*a^3*b^2*c^3 + 4*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*c
- a^4*b)*sqrt(c)*d^2*x^2*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 8*(
a*b^4*c^3 - a^3*b^2*c^2)*d*x - 2*(2*b^5*c^4 - 4*a^2*b^3*c^3 + 2*a^4*b*c^2 -
 (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c)*d*x)*sqrt(d*x + c))/((b^6*c^5 - 3*a^
2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)*x^2), 1/4*(8*a*b^4*c^2*d^2*x^2*log(sqr
t(d*x + c)*b + a) - 4*a*b^4*c^2*d^2*x^2*log(x) + 2*a*b^4*c^4 - 4*a^3*b^2*c^
3 + 2*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*sqrt(-c)*d^2*x^2*arctan(s
qrt(d*x + c)*sqrt(-c)/c) - 4*(a*b^4*c^3 - a^3*b^2*c^2)*d*x - (2*b^5*c^4 - 4
*a^2*b^3*c^3 + 2*a^4*b*c^2 - (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c)*d*x)*sqr
t(d*x + c))/((b^6*c^5 - 3*a^2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx=\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$$

```
[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x**3*(a + b*sqrt(c + d*x))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.80

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = -\frac{1}{8}\left(\frac{8\,ab^4\log\left(dx\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{16\,ab^4\log\left(\sqrt{dx+c}b+a\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{\left(3\,b^5c^2+6\,a^2b^3c-a^4b\right)\log\left(\frac{\sqrt{dx+c}}{\sqrt{dx+c}}\right.}{\left(b^6c^4-3\,a^2b^4c^3+3\,a^4b^2c^2-a^6c\right)}\right.$$

[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")

[Out] -1/8*(8*a*b^4*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6) - 16*a*b^4*log(sqrt(d*x + c)*b + a)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6) - (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt(c)) + 2*(4*(d*x + c)*a*b^2*c - 6*a*b^2*c^2 + 2*a^3*c - (3*b^3*c + a^2*b)*(d*x + c)^(3/2) + (5*b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/(b^4*c^5 - 2*a^2*b^2*c^4 + a^4*c^3 + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(d*x + c)^2 - 2*(b^4*c^4 - 2*a^2*b^2*c^3 + a^4*c^2)*(d*x + c)))*d^2

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.84

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{2\,ab^5d^2\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^7c^3-3\,a^2b^5c^2+3\,a^4b^3c-a^6b} - \frac{ab^4d^2\log\left(dx\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6} + \frac{\left(3\,b^5c^2d^2+6\,a^2b^3cd^2-a^4bd^2\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{4\left(b^6c^4-3\,a^2b^4c^3+3\,a^4b^2c^2-a^6c\right)\sqrt{-c}} + \frac{6\,ab^4c^3d^2-8\,a^3b^2c^2d^2+2\,a^5cd^2+\left(3\,b^5c^2d^2-2\,a^2b^3cd^2-a^4bd^2\right)(dx+c)^{\frac{3}{2}}-4\left(ab^4c^2d^2-a^3b^2cd^2\right)(dx+}{4\left(b^2c-a^2\right)^3cd^2x^2}$$

[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")

[Out] 2*a*b^5*d^2*log(abs(sqrt(d*x + c)*b + a))/(b^7*c^3 - 3*a^2*b^5*c^2 + 3*a^4*b^3*c - a^6*b) - a*b^4*d^2*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6) + 1/4*(3*b^5*c^2*d^2 + 6*a^2*b^3*c*d^2 - a^4*b*d^2)*arctan(sqrt(d*x + c)/sqrt(-c))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt(-c)) + 1/4*(6*a*b^4*c^3*d^2 - 8*a^3*b^2*c^2*d^2 + 2*a^5*c*d^2 + (3*b^5*c^2*d^2 - 2*a^2*b^3*c*d^2 - a^4*b*d^2)*(d*x + c)^(3/2) - 4*(a*b^4*c^2*d^2 - a^3*b^2*c*d^2)*(d*x + c) - (5*b^5*c^3*d^2 - 6*a^2*b^3*c^2*d^2 + a^4*b*c*d^2)*sqrt(d*x + c))/((b^2*c - a^2)^3*c*d^2*x^2)

Mupad [B] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 1094, normalized size of antiderivative = 5.36

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$$

$$= \frac{\ln\left(\frac{b^5\,d^4\,\left(a^2+3\,c\,b^2\right)^2\,\sqrt{c+d\,x}}{16\,c^2\,\left(b^2\,c-a^2\right)^4}-\frac{a\,b^4\,d^4\,\left(-a^4+2\,a^2\,b^2\,c+15\,b^4\,c^2\right)}{16\,c^2\,\left(b^2\,c-a^2\right)^4}-\frac{b\,d^2\,\sqrt{c^3}\,\left(\frac{b^2\,d^2\,\left(3\,b^2\,c-a^2\right)}{4\,c\,\left(b^2\,c-a^2\right)}+\frac{b^2\,d^2\,\sqrt{c^3}\,\left(a^2\,\sqrt{c+d\,x}+4\,a\,b\,c+3\,b^2\,c\,\sqrt{c+d\,x}\right)}{4\,c^3\,\left(b^2\,c-a^2\right.}\right.}{}\right.}{8\,\left(a^6\right.}$$

$$-\frac{\frac{a^3\,d^2-3\,a\,b^2\,c\,d^2}{2\,\left(a^4-2\,a^2\,b^2\,c+b^4\,c^2\right)}-\frac{\left(a^2\,b\,d^2+3\,c\,b^3\,d^2\right)\,{\left(c+d\,x\right)}^{3/2}}{4\,c\,\left(a^4-2\,a^2\,b^2\,c+b^4\,c^2\right)}+\frac{b\,d^2\,\left(5\,b^2\,c-a^2\right)\,\sqrt{c+d\,x}}{4\,\left(a^4-2\,a^2\,b^2\,c+b^4\,c^2\right)}+\frac{a\,b^2\,d^2\,\left(c+d\,x\right)}{a^4-2\,a^2\,b^2\,c+b^4\,c^2}}{{\left(c+d\,x\right)}^2-2\,c\,\left(c+d\,x\right)+c^2}$$

$$+\frac{\ln\left(\frac{b^5\,d^4\,\left(a^2+3\,c\,b^2\right)^2\,\sqrt{c+d\,x}}{16\,c^2\,\left(b^2\,c-a^2\right)^4}-\frac{a\,b^4\,d^4\,\left(-a^4+2\,a^2\,b^2\,c+15\,b^4\,c^2\right)}{16\,c^2\,\left(b^2\,c-a^2\right)^4}-\frac{b\,d^2\,\sqrt{c^3}\,\left(\frac{b^2\,d^2\,\left(3\,b^2\,c-a^2\right)}{4\,c\,\left(b^2\,c-a^2\right)}+\frac{b^2\,d^2\,\sqrt{c^3}\,\left(a^2\,\sqrt{c+d\,x}+4\,a\,b\,c+3\,b^2\,c\,\sqrt{c+c\right.}{4\,c^3\,\left(b^2\,c\right.}\right.}{}\right.}{8\,\left(\right.}$$

$$+\frac{2\,a\,b^4\,d^2\,\ln\left(a+b\,\sqrt{c+d\,x}\right)}{{\left(b^2\,c-a^2\right)}^3}$$

```
[In] int(1/(x^3*(a + b*(c + d*x)^(1/2))),x)
```

```
[Out] (log((b^5*d^4*(3*b^2*c + a^2)^2*(c + d*x)^(1/2))/(16*c^2*(b^2*c - a^2)^4) -
 (a*b^4*d^4*(15*b^4*c^2 - a^4 + 2*a^2*b^2*c))/(16*c^2*(b^2*c - a^2)^4) - (b
*d^2*(c^3)^(1/2)*((b^2*d^2*(3*b^2*c - a^2))/(4*c*(b^2*c - a^2)) + (b^2*d^2*
(c^3)^(1/2)*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*(3*b^
4*c^2 - a^4 + 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(4*c^3*(b^2*c - a^2)^3) -
 (a*b^3*d^2*(9*b^2*c - a^2)*(c + d*x)^(1/2))/(2*c*(b^2*c - a^2)^2))*(3*b^4*
c^2 - a^4 + 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(8*c^3*(b^2*c - a^2)^3))*(8
*a*b^4*c^3*d^2 - a^4*b*d^2*(c^3)^(1/2) + 3*b^5*c^2*d^2*(c^3)^(1/2) + 6*a^2*
b^3*c*d^2*(c^3)^(1/2)))/(8*(a^6*c^3 - b^6*c^6 - 3*a^4*b^2*c^4 + 3*a^2*b^4*c
^5)) - ((a^3*d^2 - 3*a*b^2*c*d^2)/(2*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) - ((a^2
*b*d^2 + 3*b^3*c*d^2)*(c + d*x)^(3/2))/(4*c*(a^4 + b^4*c^2 - 2*a^2*b^2*c))
+ (b*d^2*(5*b^2*c - a^2)*(c + d*x)^(1/2))/(4*(a^4 + b^4*c^2 - 2*a^2*b^2*c))
 + (a*b^2*d^2*(c + d*x))/(a^4 + b^4*c^2 - 2*a^2*b^2*c))/((c + d*x)^2 - 2*c*
(c + d*x) + c^2) + (log((b^5*d^4*(3*b^2*c + a^2)^2*(c + d*x)^(1/2))/(16*c^2
*(b^2*c - a^2)^4) - (a*b^4*d^4*(15*b^4*c^2 - a^4 + 2*a^2*b^2*c))/(16*c^2*(b
^2*c - a^2)^4) - (b*d^2*(c^3)^(1/2)*((b^2*d^2*(3*b^2*c - a^2))/(4*c*(b^2*c
- a^2)) + (b^2*d^2*(c^3)^(1/2)*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c
+ d*x)^(1/2))*(a^4 - 3*b^4*c^2 - 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(4*c^3
*(b^2*c - a^2)^3) - (a*b^3*d^2*(9*b^2*c - a^2)*(c + d*x)^(1/2))/(2*c*(b^2*c
 - a^2)^2))*(a^4 - 3*b^4*c^2 - 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(8*c^3*(
b^2*c - a^2)^3))*(8*a*b^4*c^3*d^2 + a^4*b*d^2*(c^3)^(1/2) - 3*b^5*c^2*d^2*(
```

```
c^3)^(1/2) - 6*a^2*b^3*c*d^2*(c^3)^(1/2)))/(8*(a^6*c^3 - b^6*c^6 - 3*a^4*b^
2*c^4 + 3*a^2*b^4*c^5)) + (2*a*b^4*d^2*log(a + b*(c + d*x)^(1/2)))/(b^2*c -
 a^2)^3
```

3.639 $\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 240

$$\begin{aligned}\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx =& \frac{(5a^4-9a^2b^2c+3b^4c^2)\,x}{b^6d^3}-\frac{12a(a^2-b^2c)^2\sqrt{c+dx}}{b^7d^4}\\&-\frac{4a(2a^2-3b^2c)\,(c+dx)^{3/2}}{3b^5d^4}+\frac{3(a^2-b^2c)\,(c+dx)^2}{2b^4d^4}\\&-\frac{4a(c+dx)^{5/2}}{5b^3d^4}+\frac{(c+dx)^3}{3b^2d^4}+\frac{2a(a^2-b^2c)^3}{b^8d^4\left(a+b\sqrt{c+dx}\right)}\\&+\frac{2(a^2-b^2c)^2\,(7a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}\end{aligned}$$

```
[Out] (3*b^4*c^2-9*a^2*b^2*c+5*a^4)*x/b^6/d^3-4/3*a*(-3*b^2*c+2*a^2)*(d*x+c)^(3/2
)/b^5/d^4+3/2*(-b^2*c+a^2)*(d*x+c)^2/b^4/d^4-4/5*a*(d*x+c)^(5/2)/b^3/d^4+1/
3*(d*x+c)^3/b^2/d^4+2*(-b^2*c+a^2)^2*(-b^2*c+7*a^2)*ln(a+b*(d*x+c)^(1/2))/b
^8/d^4-12*a*(-b^2*c+a^2)^2*(d*x+c)^(1/2)/b^7/d^4+2*a*(-b^2*c+a^2)^3/b^8/d^4
/(a+b*(d*x+c)^(1/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used

$= \{378, 1412, 786\}$

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2a(a^2-b^2c)^3}{b^8d^4\left(a+b\sqrt{c+dx}\right)} + \frac{2(7a^2-b^2c)\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4} - \frac{12a(a^2-b^2c)^2\sqrt{c+dx}}{b^7d^4} - \frac{4a(2a^2-3b^2c)\left(c+dx\right)^{3/2}}{3b^5d^4} + \frac{3(a^2-b^2c)\left(c+dx\right)^2}{2b^4d^4} + \frac{x(5a^4-9a^2b^2c+3b^4c^2)}{b^6d^3} - \frac{4a(c+dx)^{5/2}}{5b^3d^4} + \frac{(c+dx)^3}{3b^2d^4}$$

```
[In] Int[x^3/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] ((5*a^4 - 9*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3) - (12*a*(a^2 - b^2*c)^2*Sqr
t[c + d*x])/(b^7*d^4) - (4*a*(2*a^2 - 3*b^2*c)*(c + d*x)^(3/2))/(3*b^5*d^4)
 + (3*(a^2 - b^2*c)*(c + d*x)^2)/(2*b^4*d^4) - (4*a*(c + d*x)^(5/2))/(5*b^3
*d^4) + (c + d*x)^3/(3*b^2*d^4) + (2*a*(a^2 - b^2*c)^3)/(b^8*d^4*(a + b*Sqr
t[c + d*x])) + (2*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])
/(b^8*d^4)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int \frac{x(-c+x^2)^3}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{6a(a^2-b^2c)^2}{b^7} + \frac{(5a^4-9a^2b^2c+3b^4c^2)x}{b^6} - \frac{2a(2a^2-3b^2c)x^2}{b^5} - \frac{3(-a^2+b^2c)x^3}{b^4} - \frac{2ax^4}{b^3} + \frac{x^5}{b^2} - \frac{a(a^2-b^2c)}{b^7(a+bx)^2}\right.\right.}{d^4}$$

$$= \frac{(5a^4-9a^2b^2c+3b^4c^2)x}{b^6d^3} - \frac{12a(a^2-b^2c)^2\sqrt{c+dx}}{b^7d^4} - \frac{4a(2a^2-3b^2c)(c+dx)^{3/2}}{3b^5d^4} + \frac{3(a^2-b^2c)(c+dx)^2}{2b^4d^4} - \frac{4a(c+dx)^{5/2}}{5b^3d^4} + \frac{(c+dx)^3}{3b^2d^4} + \frac{2a(a^2-b^2c)^3}{b^8d^4(a+b\sqrt{c+dx})} + \frac{2(a^2-b^2c)^2(7a^2-b^2c)\log(a+b\sqrt{c+dx})}{b^8d^4}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{(a+b\sqrt{c+dx})^2}\,dx = \frac{60a^7 - 360a^6b\sqrt{c+dx} - 30a^5b^2(13c+7dx) + 10a^4b^3\sqrt{c+dx}(79c+7dx) - 3a^2b^5\sqrt{c+dx}(163c^2+36cdx\ldots}{}$$

```
[In] Integrate[x^3/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (60*a^7 - 360*a^6*b*Sqrt[c + d*x] - 30*a^5*b^2*(13*c + 7*d*x) + 10*a^4*b^3*
Sqrt[c + d*x]*(79*c + 7*d*x) - 3*a^2*b^5*Sqrt[c + d*x]*(163*c^2 + 36*c*d*x
- 7*d^2*x^2) + 5*a^3*b^4*(119*c^2 + 76*c*d*x - 7*d^2*x^2) + 5*b^7*Sqrt[c +
d*x]*(11*c^3 + 6*c^2*d*x - 3*c*d^2*x^2 + 2*d^3*x^3) - a*b^6*(269*c^3 + 162*
c^2*d*x - 33*c*d^2*x^2 + 14*d^3*x^3) + 60*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(
a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(30*b^8*d^4*(a + b*Sqrt[c +
d*x]))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.15

method	result
derivativedivides	$-\frac{2\left(-\frac{(dx+c)^3 b^5}{6}+\frac{2a(dx+c)^{\frac{5}{2}} b^4}{5}+\frac{3b^5 c(dx+c)^2}{4}-\frac{3a^2 b^3 (dx+c)^2}{4}-2a\,b^4 c(dx+c)^{\frac{3}{2}}-\frac{3b^5 c^2 (dx+c)}{2}+\frac{4a^3 b^2 (dx+c)^{\frac{3}{2}}}{3}+\frac{9a^2 b^3 c(dx+c)}{2}+\right.}{b^7}$
default	$-\frac{2\left(-\frac{(dx+c)^3 b^5}{6}+\frac{2a(dx+c)^{\frac{5}{2}} b^4}{5}+\frac{3b^5 c(dx+c)^2}{4}-\frac{3a^2 b^3 (dx+c)^2}{4}-2a\,b^4 c(dx+c)^{\frac{3}{2}}-\frac{3b^5 c^2 (dx+c)}{2}+\frac{4a^3 b^2 (dx+c)^{\frac{3}{2}}}{3}+\frac{9a^2 b^3 c(dx+c)}{2}+\right.}{b^7}$

```
[In] int(x^3/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^4*(-1/b^7*(-1/6*(d*x+c)^3*b^5+2/5*a*(d*x+c)^(5/2)*b^4+3/4*b^5*c*(d*x+c)
^2-3/4*a^2*b^3*(d*x+c)^2-2*a*b^4*c*(d*x+c)^(3/2)-3/2*b^5*c^2*(d*x+c)+4/3*a^
3*b^2*(d*x+c)^(3/2)+9/2*a^2*b^3*c*(d*x+c)+6*a*c^2*b^4*(d*x+c)^(1/2)-5/2*a^4
*b*(d*x+c)-12*a^3*c*b^2*(d*x+c)^(1/2)+6*a^5*(d*x+c)^(1/2))+1/b^8*(-b^6*c^3+
9*a^2*b^4*c^2-15*a^4*b^2*c+7*a^6)*ln(a+b*(d*x+c)^(1/2))+a*(-b^6*c^3+3*a^2*b
^4*c^2-3*a^4*b^2*c+a^6)/b^8/(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.63

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$=\frac{10\,b^8 d^4 x^4+55\,b^8 c^4-220\,a^2 b^6 c^3+195\,a^4 b^4 c^2+30\,a^6 b^2 c-60\,a^8-5\,(b^8 c-7\,a^2 b^6)d^3 x^3+15\,(b^8 c^2-8\,a^2 b^6 c}{}$$

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^8*d^4*x^4 + 55*b^8*c^4 - 220*a^2*b^6*c^3 + 195*a^4*b^4*c^2 + 30*
a^6*b^2*c - 60*a^8 - 5*(b^8*c - 7*a^2*b^6)*d^3*x^3 + 15*(b^8*c^2 - 8*a^2*b^
6*c + 7*a^4*b^4)*d^2*x^2 + 5*(17*b^8*c^3 - 87*a^2*b^6*c^2 + 96*a^4*b^4*c -
30*a^6*b^2)*d*x - 60*(b^8*c^4 - 10*a^2*b^6*c^3 + 24*a^4*b^4*c^2 - 22*a^6*b^
2*c + 7*a^8 + (b^8*c^3 - 9*a^2*b^6*c^2 + 15*a^4*b^4*c - 7*a^6*b^2)*d*x)*log
(sqrt(d*x + c)*b + a) - 4*(6*a*b^7*d^3*x^3 + 81*a*b^7*c^3 - 271*a^3*b^5*c^2
 + 295*a^5*b^3*c - 105*a^7*b - 2*(6*a*b^7*c - 7*a^3*b^5)*d^2*x^2 + 2*(24*a*
b^7*c^2 - 61*a^3*b^5*c + 35*a^5*b^3)*d*x)*sqrt(d*x + c))/(b^10*d^5*x + (b^1
0*c - a^2*b^8)*d^4)
```

Sympy [A] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

$$= \begin{cases} \frac{2\left(-\frac{2a(c+dx)^{\frac{5}{2}}}{5b^3} - \frac{a\left(a^2-b^2c\right)^3\left(\begin{cases}\frac{\sqrt{c+dx}}{a^2} & \text{for } b=0 \\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise}\end{cases}\right)}{b^7} + \frac{(c+dx)^3}{6b^2} + \frac{\left(3a^2-3b^2c\right)(c+dx)^2}{4b^4} + \frac{\left(-4a^3+6ab^2c\right)(c+dx)^{\frac{3}{2}}}{3b^5} + \frac{(c+dx)\left(5a^4-9a\right.}{2b^6}\right.}{d^4} \\ \frac{x^4}{4\left(a+b\sqrt{c}\right)^2} \end{cases}$$

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((2*(-2*a*(c + d*x)**(5/2)/(5*b**3) - a*(a**2 - b**2*c)**3*Piecewi
se((sqrt(c + d*x)/a**2, Eq(b, 0)), (-1/(b*(a + b*sqrt(c + d*x))), True))/b*
*7 + (c + d*x)**3/(6*b**2) + (3*a**2 - 3*b**2*c)*(c + d*x)**2/(4*b**4) + (-
4*a**3 + 6*a*b**2*c)*(c + d*x)**(3/2)/(3*b**5) + (c + d*x)*(5*a**4 - 9*a**2
*b**2*c + 3*b**4*c**2)/(2*b**6) + (a**2 - b**2*c)**2*(7*a**2 - b**2*c)*Piec
ewise((sqrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/b**7
 + sqrt(c + d*x)*(-6*a**5 + 12*a**3*b**2*c - 6*a*b**4*c**2)/b**7)/d**4, Ne(
d, 0)), (x**4/(4*(a + b*sqrt(c))**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx =$$

$$-\frac{\frac{60\,\left(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7\right)}{\sqrt{dx+c}b^9+ab^8} - \frac{10\,(dx+c)^3b^5-24\,(dx+c)^{\frac{5}{2}}ab^4-45\,\left(b^5c-a^2b^3\right)(dx+c)^2+40\,\left(3\,ab^4c-2\,a^3b^2\right)(dx+c)^{\frac{3}{2}}+30\,\left(3\,b^5c^2-9\,a^2\right.}{b^7}}{30\,d^4}$$

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(60*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/(sqrt(d*x + c)*b^
9 + a*b^8) - (10*(d*x + c)^3*b^5 - 24*(d*x + c)^(5/2)*a*b^4 - 45*(b^5*c - a
^2*b^3)*(d*x + c)^2 + 40*(3*a*b^4*c - 2*a^3*b^2)*(d*x + c)^(3/2) + 30*(3*b^
5*c^2 - 9*a^2*b^3*c + 5*a^4*b)*(d*x + c) - 360*(a*b^4*c^2 - 2*a^3*b^2*c + a
^5)*sqrt(d*x + c))/b^7 + 60*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6
)*log(sqrt(d*x + c)*b + a)/b^8)/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.35

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = -\frac{2\left(b^6c^3-9\,a^2b^4c^2+15\,a^4b^2c-7\,a^6\right)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^8d^4} - \frac{2\left(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7\right)}{\left(\sqrt{dx+c}b+a\right)b^8d^4} + \frac{10\,(dx+c)^3b^{10}d^{20}-45\,(dx+c)^2b^{10}cd^{20}+90\,(dx+c)b^{10}c^2d^{20}-24\,(dx+c)^{\frac{5}{2}}ab^9d^{20}+120\,(dx+c)^{\frac{3}{2}}ab^9c}{}$$

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -2*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6)*log(abs(sqrt(d*x + c)*b
 + a))/(b^8*d^4) - 2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/((sqrt
(d*x + c)*b + a)*b^8*d^4) + 1/30*(10*(d*x + c)^3*b^10*d^20 - 45*(d*x + c)^2
*b^10*c*d^20 + 90*(d*x + c)*b^10*c^2*d^20 - 24*(d*x + c)^(5/2)*a*b^9*d^20 +
 120*(d*x + c)^(3/2)*a*b^9*c*d^20 - 360*sqrt(d*x + c)*a*b^9*c^2*d^20 + 45*(
d*x + c)^2*a^2*b^8*d^20 - 270*(d*x + c)*a^2*b^8*c*d^20 - 80*(d*x + c)^(3/2)
*a^3*b^7*d^20 + 720*sqrt(d*x + c)*a^3*b^7*c*d^20 + 150*(d*x + c)*a^4*b^6*d^
20 - 360*sqrt(d*x + c)*a^5*b^5*d^20)/(b^12*d^24)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.92

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \left(\frac{4\,a^3}{3\,b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{3\,b}\right)\,{\left(c+d\,x\right)}^{3/2} - \left(\frac{3\,c}{2\,b^2\,d^4}-\frac{3\,a^2}{2\,b^4\,d^4}\right)\,{\left(c+d\,x\right)}^2 - \left(\frac{2\,a\,\left(\frac{a^2\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b^2}-\frac{2\,a\,\left(\frac{4\,a^3}{b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b}\right)}{b}+\frac{6\,c^2}{b^2\,d^4}\right)}{b} + \frac{a^2\,\left(\frac{4\,a^3}{b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b}\right)}{b^2}\right)\,\sqrt{c+d\,x} + \frac{{\left(c+d\,x\right)}^3}{3\,b^2\,d^4}+\frac{2\,\left(a^7-3\,a^5\,b^2\,c+3\,a^3\,b^4\,c^2-a\,b^6\,c^3\right)}{b\,\left(b^8\,d^4\,\sqrt{c+d\,x}+a\,b^7\,d^4\right)} + d\,x\,\left(\frac{a^2\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{2\,b^2}-\frac{a\,\left(\frac{4\,a^3}{b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b}\right)}{b}+\frac{3\,c^2}{b^2\,d^4}\right) + \frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(14\,a^6-30\,a^4\,b^2\,c+18\,a^2\,b^4\,c^2-2\,b^6\,c^3\right)}{b^8\,d^4} - \frac{4\,a\,{\left(c+d\,x\right)}^{5/2}}{5\,b^3\,d^4}$$

```
[In] int(x^3/(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] ((4*a^3)/(3*b^5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/(3*b))*(
c + d*x)^(3/2) - ((3*c)/(2*b^2*d^4) - (3*a^2)/(2*b^4*d^4))*(c + d*x)^2 - ((
2*a*((a^2*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b^2 - (2*a*((4*a^3)/(b^5*d
^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b))/b + (6*c^2)/(b^2*d^4)
```

))/b + (a^2*((4*a^3)/(b^5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b))/b^2)*(c + d*x)^(1/2) + (c + d*x)^3/(3*b^2*d^4) + (2*(a^7 - 3*a^5*b^2*c - a*b^6*c^3 + 3*a^3*b^4*c^2))/(b*(b^8*d^4*(c + d*x)^(1/2) + a*b^7*d^4)) + d*x*((a^2*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/(2*b^2) - (a*((4*a^3)/(b^5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b))/b + (3*c^2)/(b^2*d^4)) + (log(a + b*(c + d*x)^(1/2))*(14*a^6 - 2*b^6*c^3 - 30*a^4*b^2*c + 18*a^2*b^4*c^2))/(b^8*d^4) - (4*a*(c + d*x)^(5/2))/(5*b^3*d^4)

3.640 $\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 166

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2} dx = \frac{(3a^2-2b^2c)\,x}{b^4d^2} - \frac{8a(a^2-b^2c)\sqrt{c+dx}}{b^5d^3} - \frac{4a(c+dx)^{3/2}}{3b^3d^3} + \frac{(c+dx)^2}{2b^2d^3} + \frac{2a(a^2-b^2c)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)} + \frac{2(5a^4-6a^2b^2c+b^4c^2)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}$$

```
[Out] (-2*b^2*c+3*a^2)*x/b^4/d^2-4/3*a*(d*x+c)^(3/2)/b^3/d^3+1/2*(d*x+c)^2/b^2/d^
3+2*(b^4*c^2-6*a^2*b^2*c+5*a^4)*ln(a+b*(d*x+c)^(1/2))/b^6/d^3-8*a*(-b^2*c+a
^2)*(d*x+c)^(1/2)/b^5/d^3+2*a*(-b^2*c+a^2)^2/b^6/d^3/(a+b*(d*x+c)^(1/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {378, 1412, 786}

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2} dx = \frac{2a(a^2-b^2c)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)} - \frac{8a(a^2-b^2c)\sqrt{c+dx}}{b^5d^3} + \frac{x(3a^2-2b^2c)}{b^4d^2} + \frac{2(5a^4-6a^2b^2c+b^4c^2)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3} - \frac{4a(c+dx)^{3/2}}{3b^3d^3} + \frac{(c+dx)^2}{2b^2d^3}$$

```
[In] Int[x^2/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] ((3*a^2 - 2*b^2*c)*x)/(b^4*d^2) - (8*a*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^5*d^
3) - (4*a*(c + d*x)^(3/2))/(3*b^3*d^3) + (c + d*x)^2/(2*b^2*d^3) + (2*a*(a^
2 - b^2*c)^2)/(b^6*d^3*(a + b*Sqrt[c + d*x])) + (2*(5*a^4 - 6*a^2*b^2*c + b
^4*c^2)*Log[a + b*Sqrt[c + d*x]])/(b^6*d^3)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d^3}\\
&= \frac{2\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^2}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d^3}\\
&= \frac{2\text{Subst}\left(\int \left(-\frac{4a(a^2-b^2c)}{b^5}-\frac{(-3a^2+2b^2c)x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}-\frac{a\left(a^2-b^2c\right)^2}{b^5(a+bx)^2}+\frac{5a^4-6a^2b^2c+b^4c^2}{b^5(a+bx)}\right)\,dx, x, \sqrt{c+dx}\right)}{d^3}\\
&= \frac{(3a^2-2b^2c)\,x}{b^4d^2}-\frac{8a(a^2-b^2c)\sqrt{c+dx}}{b^5d^3}-\frac{4a(c+dx)^{3/2}}{3b^3d^3}+\frac{(c+dx)^2}{2b^2d^3}\\
&\quad+\frac{2a(a^2-b^2c)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)}+\frac{2(5a^4-6a^2b^2c+b^4c^2)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{12a^5-48a^4b\sqrt{c+dx}-6a^3b^2(9c+5dx)+2a^2b^3\sqrt{c+dx}(29c+5dx)+ab^4(43c^2+26cdx-5d^2x^2)+3b^5}{6b^6d^3\left(a+b\sqrt{c+\right.}}$$

[In] Integrate[x^2/(a + b*Sqrt[c + d*x])^2,x]

[Out] (12*a^5 - 48*a^4*b*Sqrt[c + d*x] - 6*a^3*b^2*(9*c + 5*d*x) + 2*a^2*b^3*Sqrt[c + d*x]*(29*c + 5*d*x) + a*b^4*(43*c^2 + 26*c*d*x - 5*d^2*x^2) + 3*b^5*Sqrt[c + d*x]*(-3*c^2 - 2*c*d*x + d^2*x^2) + 12*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(6*b^6*d^3*(a + b*Sqrt[c + d*x]))

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(dx+c)^2b^3}{4}+\frac{2a(dx+c)^{\frac{3}{2}}b^2}{3}+b^3c(dx+c)-\frac{3a^2b(dx+c)}{2}-4acb^2\sqrt{dx+c}+4a^3\sqrt{dx+c}\right)}{b^5}+\frac{2a\left(b^4c^2-2a^2b^2c+a^4\right)}{b^6\left(a+b\sqrt{dx+c}\right)}+\frac{2\left(b^4c^2-6a^2b^2\right.}{}}{d^3}$
default	$\frac{-\frac{2\left(-\frac{(dx+c)^2b^3}{4}+\frac{2a(dx+c)^{\frac{3}{2}}b^2}{3}+b^3c(dx+c)-\frac{3a^2b(dx+c)}{2}-4acb^2\sqrt{dx+c}+4a^3\sqrt{dx+c}\right)}{b^5}+\frac{2a\left(b^4c^2-2a^2b^2c+a^4\right)}{b^6\left(a+b\sqrt{dx+c}\right)}+\frac{2\left(b^4c^2-6a^2b^2\right.}{}}{d^3}$

[In] int(x^2/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)

[Out] 2/d^3*(-1/b^5*(-1/4*(d*x+c)^2*b^3+2/3*a*(d*x+c)^(3/2)*b^2+b^3*c*(d*x+c)-3/2*a^2*b*(d*x+c)-4*a*c*b^2*(d*x+c)^(1/2)+4*a^3*(d*x+c)^(1/2))+a*(b^4*c^2-2*a^2*b^2*c+a^4)/b^6/(a+b*(d*x+c)^(1/2))+1/b^6*(b^4*c^2-6*a^2*b^2*c+5*a^4)*ln(a+b*(d*x+c)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.62

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{3\,b^6d^3x^3 - 9\,b^6c^3 + 15\,a^2b^4c^2 + 6\,a^4b^2c - 12\,a^6 - 3\,(b^6c - 5\,a^2b^4)d^2x^2 - 3\,(5\,b^6c^2 - 14\,a^2b^4c + 6\,a^4b^2)dx + 1}{}$$

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^6*d^3*x^3 - 9*b^6*c^3 + 15*a^2*b^4*c^2 + 6*a^4*b^2*c - 12*a^6 - 3*
(b^6*c - 5*a^2*b^4)*d^2*x^2 - 3*(5*b^6*c^2 - 14*a^2*b^4*c + 6*a^4*b^2)*d*x
+ 12*(b^6*c^3 - 7*a^2*b^4*c^2 + 11*a^4*b^2*c - 5*a^6 + (b^6*c^2 - 6*a^2*b^4
*c + 5*a^4*b^2)*d*x)*log(sqrt(d*x + c)*b + a) - 4*(2*a*b^5*d^2*x^2 - 13*a*b
^5*c^2 + 28*a^3*b^3*c - 15*a^5*b - 2*(4*a*b^5*c - 5*a^3*b^3)*d*x)*sqrt(d*x
+ c))/(b^8*d^4*x + (b^8*c - a^2*b^6)*d^3)
```

Sympy [A] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \begin{cases} \frac{2\left(-\frac{2a(c+dx)^{\frac{3}{2}}}{3b^3} - \frac{a\left(a^2-b^2c\right)^2\left(\begin{cases}\frac{\sqrt{c+dx}}{a^2} & \text{for } b=0\\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise}\end{cases}\right)}{b^5} + \frac{(c+dx)^2}{4b^2} + \frac{\left(3a^2-2b^2c\right)(c+dx)}{2b^4} + \frac{\left(a^2-b^2c\right)\left(5a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} \\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b}\end{cases}\right.}{b^5}\right.}{d^3} \\ \frac{x^3}{3\left(a+b\sqrt{c}\right)^2} \end{cases}$$

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((2*(-2*a*(c + d*x)**(3/2)/(3*b**3) - a*(a**2 - b**2*c)**2*Piecewi
se((sqrt(c + d*x)/a**2, Eq(b, 0)), (-1/(b*(a + b*sqrt(c + d*x))), True))/b*
*5 + (c + d*x)**2/(4*b**2) + (3*a**2 - 2*b**2*c)*(c + d*x)/(2*b**4) + (a**2
 - b**2*c)*(5*a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (log(a
+ b*sqrt(c + d*x))/b, True))/b**5 + (-4*a**3 + 4*a*b**2*c)*sqrt(c + d*x)/b*
*5)/d**3, Ne(d, 0)), (x**3/(3*(a + b*sqrt(c))**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\frac{12\,(ab^4c^2-2\,a^3b^2c+a^5)}{\sqrt{dx+c}b^7+ab^6} + \frac{3\,(dx+c)^2b^3-8\,(dx+c)^{\frac{3}{2}}ab^2-6\,(2\,b^3c-3\,a^2b)(dx+c)+48\,(ab^2c-a^3)\sqrt{dx+c}}{b^5} + \frac{12\,(b^4c^2-6\,a^2b^2c+5\,a^4)\log(\sqrt{dx+c}}{b^6}}{6\,d^3}$$

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(12*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/(sqrt(d*x + c)*b^7 + a*b^6) + (3*(d
*x + c)^2*b^3 - 8*(d*x + c)^(3/2)*a*b^2 - 6*(2*b^3*c - 3*a^2*b)*(d*x + c) +
 48*(a*b^2*c - a^3)*sqrt(d*x + c))/b^5 + 12*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)
*log(sqrt(d*x + c)*b + a)/b^6)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2\,(b^4c^2-6\,a^2b^2c+5\,a^4)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^6d^3} + \frac{2\,(ab^4c^2-2\,a^3b^2c+a^5)}{\left(\sqrt{dx+c}b+a\right)b^6d^3} + \frac{3\,(dx+c)^2b^6d^9-12\,(dx+c)b^6cd^9-8\,(dx+c)^{\frac{3}{2}}ab^5d^9+48\,\sqrt{dx+c}ab^5cd^9+18\,(dx+c)a^2b^4d^9-48\,\sqrt{}}{6\,b^8d^{12}}$$

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*log(abs(sqrt(d*x + c)*b + a))/(b^6*d^3) +
 2*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/((sqrt(d*x + c)*b + a)*b^6*d^3) + 1/6*(3
*(d*x + c)^2*b^6*d^9 - 12*(d*x + c)*b^6*c*d^9 - 8*(d*x + c)^(3/2)*a*b^5*d^9
 + 48*sqrt(d*x + c)*a*b^5*c*d^9 + 18*(d*x + c)*a^2*b^4*d^9 - 48*sqrt(d*x +
c)*a^3*b^3*d^9)/(b^8*d^12)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \left(\frac{4\,a^3}{b^5\,d^3}+\frac{2\,a\,\left(\frac{4\,c}{b^2\,d^3}-\frac{6\,a^2}{b^4\,d^3}\right)}{b}\right)\,\sqrt{c+d\,x} + \frac{2\,\left(a^5-2\,a^3\,b^2\,c+a\,b^4\,c^2\right)}{b\,\left(b^6\,d^3\,\sqrt{c+d\,x}+a\,b^5\,d^3\right)}+\frac{{\left(c+d\,x\right)}^2}{2\,b^2\,d^3} - d\,x\,\left(\frac{2\,c}{b^2\,d^3}-\frac{3\,a^2}{b^4\,d^3}\right)-\frac{4\,a\,{\left(c+d\,x\right)}^{3/2}}{3\,b^3\,d^3} + \frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(10\,a^4-12\,a^2\,b^2\,c+2\,b^4\,c^2\right)}{b^6\,d^3}$$

```
[In] int(x^2/(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] ((4*a^3)/(b^5*d^3) + (2*a*((4*c)/(b^2*d^3) - (6*a^2)/(b^4*d^3)))/b)*(c + d*
x)^(1/2) + (2*(a^5 - 2*a^3*b^2*c + a*b^4*c^2))/(b*(b^6*d^3*(c + d*x)^(1/2)
+ a*b^5*d^3)) + (c + d*x)^2/(2*b^2*d^3) - d*x*((2*c)/(b^2*d^3) - (3*a^2)/(b
^4*d^3)) - (4*a*(c + d*x)^(3/2))/(3*b^3*d^3) + (log(a + b*(c + d*x)^(1/2))*
(10*a^4 + 2*b^4*c^2 - 12*a^2*b^2*c))/(b^6*d^3)
```

3.641 $\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{x}{b^2d}-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{2a(a^2-b^2c)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2(3a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

```
[Out] x/b^2/d+2*(-b^2*c+3*a^2)*ln(a+b*(d*x+c)^(1/2))/b^4/d^2-4*a*(d*x+c)^(1/2)/b^
3/d^2+2*a*(-b^2*c+a^2)/b^4/d^2/(a+b*(d*x+c)^(1/2))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {378, 1412, 786}

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2a(a^2-b^2c)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2(3a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{x}{b^2d}$$

```
[In] Int[x/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] x/(b^2*d) - (4*a*Sqrt[c + d*x])/(b^3*d^2) + (2*a*(a^2 - b^2*c))/(b^4*d^2*(a
 + b*Sqrt[c + d*x])) + (2*(3*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^
2)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-c+x}{\left(a+b\sqrt{x}\right)^2}\,dx,x,c+dx\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int \frac{x(-c+x^2)}{(a+bx)^2}\,dx,x,\sqrt{c+dx}\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}+\frac{-a^3+ab^2c}{b^3(a+bx)^2}+\frac{3a^2-b^2c}{b^3(a+bx)}\right)\,dx,x,\sqrt{c+dx}\right)}{d^2}\\
&= \frac{x}{b^2d}-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{2a(a^2-b^2c)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2(3a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$
\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2a^3-2ab^2c-4a^2b\sqrt{c+dx}-3ab^2(c+dx)+b^3(c+dx)^{3/2}}{b^4d^2\left(a+b\sqrt{c+dx}\right)} - \frac{2(-3a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}
$$

```
[In] Integrate[x/(a + b*Sqrt[c + d*x])^2,x]
[Out] (2*a^3 - 2*a*b^2*c - 4*a^2*b*Sqrt[c + d*x] - 3*a*b^2*(c + d*x) + b^3*(c + d
*x)^(3/2))/(b^4*d^2*(a + b*Sqrt[c + d*x])) - (2*(-3*a^2 + b^2*c)*Log[a + b*
Sqrt[c + d*x]])/(b^4*d^2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{-\frac{2\left(-\frac{(dx+c)b}{2}+2a\sqrt{dx+c}\right)}{b^{3}}+\frac{2\left(-b^{2}c+3a^{2}\right)\ln\left(a+b\sqrt{dx+c}\right)}{b^{4}}+\frac{2a\left(-b^{2}c+a^{2}\right)}{b^{4}\left(a+b\sqrt{dx+c}\right)}}{d^{2}}$	87
default	$\frac{-\frac{2\left(-\frac{(dx+c)b}{2}+2a\sqrt{dx+c}\right)}{b^{3}}+\frac{2\left(-b^{2}c+3a^{2}\right)\ln\left(a+b\sqrt{dx+c}\right)}{b^{4}}+\frac{2a\left(-b^{2}c+a^{2}\right)}{b^{4}\left(a+b\sqrt{dx+c}\right)}}{d^{2}}$	87

```
[In] int(x/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^2*(-1/b^3*(-1/2*(d*x+c)*b+2*a*(d*x+c)^(1/2))+1/b^4*(-b^2*c+3*a^2)*ln(a+
b*(d*x+c)^(1/2))+a*(-b^2*c+a^2)/b^4/(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.72

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{b^4d^2x^2+b^4c^2+a^2b^2c-2\,a^4+(2\,b^4c-a^2b^2)dx-2\,(b^4c^2-4\,a^2b^2c+3\,a^4+(b^4c-3\,a^2b^2)dx)\log\left(\sqrt{dx+\right.}{b^6d^3x+(b^6c-a^2b^4)d^2}$$

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] (b^4*d^2*x^2 + b^4*c^2 + a^2*b^2*c - 2*a^4 + (2*b^4*c - a^2*b^2)*d*x - 2*(b
^4*c^2 - 4*a^2*b^2*c + 3*a^4 + (b^4*c - 3*a^2*b^2)*d*x)*log(sqrt(d*x + c)*b
 + a) - 2*(2*a*b^3*d*x + 3*a*b^3*c - 3*a^3*b)*sqrt(d*x + c))/(b^6*d^3*x + (
b^6*c - a^2*b^4)*d^2)
```

Sympy [A] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.33

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \begin{cases} \frac{2\left(-\frac{a\left(a^2-b^2c\right)\left(\begin{cases} \frac{\sqrt{c+dx}}{a^2} & \text{for } b=0 \\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise} \end{cases}\right)}{b^3} - \frac{2a\sqrt{c+dx}}{b^3} + \frac{c+dx}{2b^2} + \frac{\left(3a^2-b^2c\right)\left(\begin{cases} \frac{\sqrt{c+dx}}{a} & \text{for } b=0 \\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise} \end{cases}\right)}{b^3}\right)}{d^2} & \text{for } d \neq 0 \\ \frac{x^2}{2\left(a+b\sqrt{c}\right)^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((2*(-a*(a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a**2, Eq(b, 0)),
(-1/(b*(a + b*sqrt(c + d*x))), True))/b**3 - 2*a*sqrt(c + d*x)/b**3 + (c +
d*x)/(2*b**2) + (3*a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (l
og(a + b*sqrt(c + d*x))/b, True))/b**3)/d**2, Ne(d, 0)), (x**2/(2*(a + b*sq
rt(c))**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = -\frac{\frac{2\left(ab^2c-a^3\right)}{\sqrt{dx+c}b^5+ab^4} - \frac{(dx+c)b-4\sqrt{dx+c}a}{b^3} + \frac{2\left(b^2c-3a^2\right)\log\left(\sqrt{dx+c}b+a\right)}{b^4}}{d^2}$$

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a*b^2*c - a^3)/(sqrt(d*x + c)*b^5 + a*b^4) - ((d*x + c)*b - 4*sqrt(d*x
 + c)*a)/b^3 + 2*(b^2*c - 3*a^2)*log(sqrt(d*x + c)*b + a)/b^4)/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx=-\frac{\frac{2\,\left(b^2c-3\,a^2\right)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^4d}-\frac{(dx+c)b^2d-4\,\sqrt{dx+c}abd}{b^4d^2}+\frac{2\,\left(ab^2c-a^3\right)}{\left(\sqrt{dx+c}b+a\right)b^4d}}{d}$$

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -(2*(b^2*c - 3*a^2)*log(abs(sqrt(d*x + c)*b + a))/(b^4*d) - ((d*x + c)*b^2*
d - 4*sqrt(d*x + c)*a*b*d)/(b^4*d^2) + 2*(a*b^2*c - a^3)/((sqrt(d*x + c)*b
+ a)*b^4*d))/d
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx=\frac{x}{b^2\,d}+\frac{2\,\left(a^3-a\,b^2\,c\right)}{b\,\left(b^4\,d^2\,\sqrt{c+d\,x}+a\,b^3\,d^2\right)}-\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,b^2\,c-6\,a^2\right)}{b^4\,d^2}-\frac{4\,a\,\sqrt{c+d\,x}}{b^3\,d^2}$$

```
[In] int(x/(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] x/(b^2*d) + (2*(a^3 - a*b^2*c))/(b*(b^4*d^2*(c + d*x)^(1/2) + a*b^3*d^2)) -
 (log(a + b*(c + d*x)^(1/2))*(2*b^2*c - 6*a^2))/(b^4*d^2) - (4*a*(c + d*x)^
(1/2))/(b^3*d^2)
```

3.642 $\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2} dx = \frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)} + \frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[Out] 2*ln(a+b*(d*x+c)^(1/2))/b^2/d+2*a/b^2/d/(a+b*(d*x+c)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {253, 196, 45}

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2} dx = \frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)} + \frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[In] Int[(a + b*Sqrt[c + d*x])^(-2),x]
```

```
[Out] (2*a)/(b^2*d*(a + b*Sqrt[c + d*x])) + (2*Log[a + b*Sqrt[c + d*x]])/(b^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d}\\
&= \frac{2\text{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d}\\
&= \frac{2\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)\,dx, x, \sqrt{c+dx}\right)}{d}\\
&= \frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2\left(\frac{a}{a+b\sqrt{c+dx}}+\log\left(bd\left(a+b\sqrt{c+dx}\right)\right)\right)}{b^2d}$$

```
[In] Integrate[(a + b*Sqrt[c + d*x])^(-2),x]
[Out] (2*(a/(a + b*Sqrt[c + d*x]) + Log[b*d*(a + b*Sqrt[c + d*x])]))/(b^2*d)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{2\ln\left(a+b\sqrt{dx+c}\right)}{b^2}+\frac{2a}{b^2\left(a+b\sqrt{dx+c}\right)}}{d}$
default	$\frac{a^2}{(-b^2dx-b^2c+a^2)b^2d}+\frac{c}{(-b^2dx-b^2c+a^2)d}+b^2d\left(-\frac{-b^2c+a^2}{b^4d^2(b^2dx+b^2c-a^2)}+\frac{\ln(b^2dx+b^2c-a^2)}{b^4d^2}\right)+\frac{a}{b^2d(-a+b\sqrt{c}}$

[In] int(1/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)

[Out] 2/d*(1/b^2*ln(a+b*(d*x+c)^(1/2))+a/b^2/(a+b*(d*x+c)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx=\frac{2\left(\sqrt{dx+c}ab-a^2+(b^2dx+b^2c-a^2)\log\left(\sqrt{dx+c}b+a\right)\right)}{b^4d^2x+(b^4c-a^2b^2)d}$$

[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] 2*(sqrt(d*x + c)*a*b - a^2 + (b^2*d*x + b^2*c - a^2)*log(sqrt(d*x + c)*b + a))/(b^4*d^2*x + (b^4*c - a^2*b^2)*d)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(39) = 78.

Time = 0.56 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.64

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \begin{cases} \frac{x}{a^2} & \text{for } b=0\wedge(b=0\vee d=0) \\ \frac{x}{\left(a+b\sqrt{c}\right)^2} & \text{for } d=0 \\ \frac{2a\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}}+\frac{2a}{ab^2d+b^3d\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}} & \text{otherwise} \end{cases}$$

[In] integrate(1/(a+b*(d*x+c)**(1/2))**2,x)

[Out] Piecewise((x/a**2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c))**2, Eq(d, 0)), (2*a*log(a/b + sqrt(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*x)) + 2*a/(a*b**2*d + b**3*d*sqrt(c + d*x)) + 2*b*sqrt(c + d*x)*log(a/b + sqrt(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*x)), True))

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2\left(\frac{a}{\sqrt{dx+c}b^3+ab^2}+\frac{\log\left(\sqrt{dx+c}b+a\right)}{b^2}\right)}{d}$$

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
[Out] 2*(a/(sqrt(d*x + c)*b^3 + a*b^2) + log(sqrt(d*x + c)*b + a)/b^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2\,\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^2d}+\frac{2\,a}{\left(\sqrt{dx+c}b+a\right)b^2d}$$

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
[Out] 2*log(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*a/((sqrt(d*x + c)*b + a)*b^2*d)
```

Mupad [B] (verification not implemented)

Time = 17.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2\,\ln\left(a+b\,\sqrt{c+d\,x}\right)}{b^2\,d}+\frac{2\,a}{b^2\,\left(a\,d+b\,d\,\sqrt{c+d\,x}\right)}$$

```
[In] int(1/(a + b*(c + d*x)^(1/2))^2,x)
[Out] (2*log(a + b*(c + d*x)^(1/2)))/(b^2*d) + (2*a)/(b^2*(a*d + b*d*(c + d*x)^(1
/2)))
```

3.643 $\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 129

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx=\frac{2a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}+\frac{4ab\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{(a^2-b^2c)^2}+\frac{(a^2+b^2c)\log(x)}{(a^2-b^2c)^2}-\frac{2(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

```
[Out] (b^2*c+a^2)*ln(x)/(-b^2*c+a^2)^2-2*(b^2*c+a^2)*ln(a+b*(d*x+c)^(1/2))/(-b^2*
c+a^2)^2+4*a*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c+a^2)^2+2*a/(-
b^2*c+a^2)/(a+b*(d*x+c)^(1/2))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {378, 1412, 815, 649, 212, 266}

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx=\frac{4ab\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{(a^2-b^2c)^2}+\frac{2a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}-\frac{2(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}+\frac{\log(x)(a^2+b^2c)}{(a^2-b^2c)^2}$$

```
[In] Int[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (2*a)/((a^2 - b^2*c)*(a + b*Sqrt[c + d*x])) + (4*a*b*Sqrt[c]*ArcTanh[Sqrt[c
 + d*x]/Sqrt[c]])/(a^2 - b^2*c)^2 + ((a^2 + b^2*c)*Log[x])/(a^2 - b^2*c)^2
- (2*(a^2 + b^2*c)*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(-c+x)}\,dx, x, c+dx\right)\\ &= 2\text{Subst}\left(\int \frac{x}{(a+bx)^2\,(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)\end{aligned}$$

$$=2\text{Subst}\left(\int\left(-\frac{ab}{(a^2-b^2c)\,(a+bx)^2}-\frac{b(a^2+b^2c)}{(a^2-b^2c)^2\,(a+bx)}+\frac{2abc-(a^2+b^2c)\,x}{(a^2-b^2c)^2\,(c-x^2)}\right)dx,x,\sqrt{c+dx}\right)$$

$$=\frac{2a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}-\frac{2(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}+\frac{2\text{Subst}\left(\int\frac{2abc-(a^2+b^2c)x}{c-x^2}\,dx,x,\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

$$=\frac{2a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}-\frac{2(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}+\frac{(4abc)\text{Subst}\left(\int\frac{1}{c-x^2}\,dx,x,\sqrt{c+dx}\right)}{(a^2-b^2c)^2}-\frac{(2(a^2+b^2c))\,\text{Subst}\left(\int\frac{x}{c-x^2}\,dx,x,\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

$$=\frac{2a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}+\frac{4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{(a^2-b^2c)^2}+\frac{(a^2+b^2c)\log(x)}{(a^2-b^2c)^2}-\frac{2(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.83

$$\int\frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx=\frac{\frac{2a(a^2-b^2c)}{a+b\sqrt{c+dx}}+4ab\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+(a^2+b^2c)\log(-dx)-2(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

```
[In] Integrate[1/(x*(a + b*Sqrt[c + d*x])^2),x]
[Out] ((2*a*(a^2 - b^2*c))/(a + b*Sqrt[c + d*x]) + 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c +
 d*x]/Sqrt[c]] + (a^2 + b^2*c)*Log[-(d*x)] - 2*(a^2 + b^2*c)*Log[a + b*Sqrt
[c + d*x]])/(a^2 - b^2*c)^2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{2a}{(-b^2c+a^2)(a+b\sqrt{dx+c})}-\frac{2(b^2c+a^2)\ln(a+b\sqrt{dx+c})}{(-b^2c+a^2)^2}+\frac{-(-b^2c-a^2)\ln(-dx)+4ab\ \mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\sqrt{c}}{(-b^2c+a^2)^2}$	118
default	$\frac{2a}{(-b^2c+a^2)(a+b\sqrt{dx+c})}-\frac{2(b^2c+a^2)\ln(a+b\sqrt{dx+c})}{(-b^2c+a^2)^2}+\frac{-(-b^2c-a^2)\ln(-dx)+4ab\ \mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\sqrt{c}}{(-b^2c+a^2)^2}$	118

```
[In] int(1/x/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a/(-b^2*c+a^2)/(a+b*(d*x+c)^(1/2))-2*(b^2*c+a^2)*ln(a+b*(d*x+c)^(1/2))/(-
b^2*c+a^2)^2+2/(-b^2*c+a^2)^2*(-1/2*(-b^2*c-a^2)*ln(-d*x)+2*a*b*arctanh((d*
x+c)^(1/2)/c^(1/2))*c^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.44

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \left[\frac{2\,a^2b^2c-2\,a^4+2\,(ab^3dx+ab^3c-a^3b)\sqrt{c}\log\left(\frac{dx+2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)-2\,(b^4c^2-a^4+(b^4c+a^2b^2)dx)\log\left(\sqrt{\right.}\right.}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6+(b^6c^2-2\,a}\right.$$

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*a^2*b^2*c - 2*a^4 + 2*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(c)*log((d*x +
2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*
x)*log(sqrt(d*x + c)*b + a) + (b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(x
) - 2*(a*b^3*c - a^3*b)*sqrt(d*x + c))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2
*c - a^6 + (b^6*c^2 - 2*a^2*b^4*c + a^4*b^2)*d*x), (2*a^2*b^2*c - 2*a^4 - 4
*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) -
2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(sqrt(d*x + c)*b + a) + (b^4*c
^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(x) - 2*(a*b^3*c - a^3*b)*sqrt(d*x + c
))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6 + (b^6*c^2 - 2*a^2*b^4*c +
a^4*b^2)*d*x)]
```

Sympy [A] (verification not implemented)

Time = 4.64 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \begin{cases} -\frac{2ab\left(\begin{cases}\frac{\sqrt{c+dx}}{a^2} & \text{for } b=0\\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise}\end{cases}\right)}{a^2-b^2c} - \frac{2b\left(a^2+b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{\left(a^2-b^2c\right)^2} - \frac{2\cdot\left(\frac{2abc\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\left(-\frac{a^2}{2}-\frac{b^2 \ldots}{\ldots}\right.}{\left(a^2-b^2c\right)^2} \\ \frac{\log(x)}{\left(a+b\sqrt{c}\right)^2} \end{cases}$$

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((-2*a*b*Piecewise((sqrt(c + d*x)/a**2, Eq(b, 0)), (-1/(b*(a + b*s
qrt(c + d*x))), True))/(a**2 - b**2*c) - 2*b*(a**2 + b**2*c)*Piecewise((sqr
t(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(a**2 - b**2*c
)**2 - 2*(2*a*b*c*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + (-a**2/2 - b**2*c
/2)*log(-d*x))/(a**2 - b**2*c)**2, Ne(d, 0)), (log(x)/(a + b*sqrt(c))**2, T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = -\frac{2\,ab\sqrt{c}\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{b^4c^2-2\,a^2b^2c+a^4} + \frac{\left(b^2c+a^2\right)\log\left(dx\right)}{b^4c^2-2\,a^2b^2c+a^4} - \frac{2\left(b^2c+a^2\right)\log\left(\sqrt{dx+c}b+a\right)}{b^4c^2-2\,a^2b^2c+a^4} - \frac{2\,a}{ab^2c-a^3+\left(b^3c-a^2b\right)\sqrt{dx+c}}$$

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/(b^
4*c^2 - 2*a^2*b^2*c + a^4) + (b^2*c + a^2)*log(d*x)/(b^4*c^2 - 2*a^2*b^2*c
+ a^4) - 2*(b^2*c + a^2)*log(sqrt(d*x + c)*b + a)/(b^4*c^2 - 2*a^2*b^2*c +
a^4) - 2*a/(a*b^2*c - a^3 + (b^3*c - a^2*b)*sqrt(d*x + c))
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.35

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = -\frac{4\,abc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{-c}} + \frac{\left(b^2c+a^2\right)\log\left(-dx\right)}{b^4c^2-2\,a^2b^2c+a^4} - \frac{2\left(b^3c+a^2b\right)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^5c^2-2\,a^2b^3c+a^4b} - \frac{2\left(ab^2c-a^3\right)}{\left(b^2c-a^2\right)^2\left(\sqrt{dx+c}b+a\right)}$$

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -4*a*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt
(-c)) + (b^2*c + a^2)*log(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - 2*(b^3*c +
a^2*b)*log(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c + a^4*b) - 2*(a
*b^2*c - a^3)/((b^2*c - a^2)^2*(sqrt(d*x + c)*b + a))
```

Mupad [B] (verification not implemented)

Time = 17.95 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\left(a+b\,\sqrt{c}\right)^2} + \ln\left(a+b\,\sqrt{c+d\,x}\right)\left(\frac{2}{b^2\,c-a^2}-\frac{4\,b^2\,c}{\left(b^2\,c-a^2\right)^2}\right) + \frac{\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)}{\left(a-b\,\sqrt{c}\right)^2} - \frac{2\,a}{\left(b^2\,c-a^2\right)\,\left(a+b\,\sqrt{c+d\,x}\right)}$$

```
[In] int(1/(x*(a + b*(c + d*x)^(1/2))^2),x)
```

```
[Out] log((c + d*x)^(1/2) - c^(1/2))/(a + b*c^(1/2))^2 + log(a + b*(c + d*x)^(1/2
))*(2/(b^2*c - a^2) - (4*b^2*c)/(b^2*c - a^2)^2) + log((c + d*x)^(1/2) + c^
(1/2))/(a - b*c^(1/2))^2 - (2*a)/((b^2*c - a^2)*(a + b*(c + d*x)^(1/2)))
```

3.644 $\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 202

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{4ab^2d}{(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x\left(a+b\sqrt{c+dx}\right)} + \frac{2ab(a^2+3b^2c)\,d\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2-b^2c)^3} + \frac{b^2(3a^2+b^2c)\,d\log(x)}{(a^2-b^2c)^3} - \frac{2b^2(3a^2+b^2c)\,d\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^3}$$

```
[Out] b^2*(b^2*c+3*a^2)*d*ln(x)/(-b^2*c+a^2)^3-2*b^2*(b^2*c+3*a^2)*d*ln(a+b*(d*x+
c)^(1/2))/(-b^2*c+a^2)^3+2*a*b*(3*b^2*c+a^2)*d*arctanh((d*x+c)^(1/2)/c^(1/2
))/(-b^2*c+a^2)^3/c^(1/2)+4*a*b^2*d/(-b^2*c+a^2)^2/(a+b*(d*x+c)^(1/2))+(-a+
b*(d*x+c)^(1/2))/(-b^2*c+a^2)/x/(a+b*(d*x+c)^(1/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {378, 1412, 837, 815, 649, 212, 266}

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2abd(a^2+3b^2c)\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2-b^2c)^3} + \frac{4ab^2d}{(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{x\,(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)} + \frac{b^2d\log(x)\,(3a^2+b^2c)}{(a^2-b^2c)^3} - \frac{2b^2d(3a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^3}$$

```
[In] Int[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (4*a*b^2*d)/((a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])) - (a - b*Sqrt[c + d*x])
/((a^2 - b^2*c)*x*(a + b*Sqrt[c + d*x])) + (2*a*b*(a^2 + 3*b^2*c)*d*ArcTanh
[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^3) + (b^2*(3*a^2 + b^2*c)*d
*Log[x])/(a^2 - b^2*c)^3 - (2*b^2*(3*a^2 + b^2*c)*d*Log[a + b*Sqrt[c + d*x]
])/(a^2 - b^2*c)^3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
```

```
[2*m, 2*p])
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(-c+x)^2}\,dx, x, c+dx\right)\\
&= (2d)\text{Subst}\left(\int \frac{x}{(a+bx)^2\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x\left(a+b\sqrt{c+dx}\right)}+\frac{d\text{Subst}\left(\int \frac{-2abc+2b^2cx}{(a+bx)^2(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)}{c\,(a^2-b^2c)}\\
&= -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x\left(a+b\sqrt{c+dx}\right)}\\
&\quad+\frac{d\text{Subst}\left(\int \left(-\frac{4ab^3c}{(a^2-b^2c)(a+bx)^2}-\frac{2b^3c(3a^2+b^2c)}{(-a^2+b^2c)^2(a+bx)}+\frac{2bc\left(a\left(a^2+3b^2c\right)-b\left(3a^2+b^2c\right)x\right)}{(a^2-b^2c)^2(c-x^2)}\right)dx, x, \sqrt{c+dx}\right)}{c\,(a^2-b^2c)}\\
&= \frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x\left(a+b\sqrt{c+dx}\right)}\\
&\quad-\frac{2b^2(3a^2+b^2c)\,d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}\\
&\quad+\frac{(2bd)\text{Subst}\left(\int \frac{a\left(a^2+3b^2c\right)-b\left(3a^2+b^2c\right)x}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}\\
&= \frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x\left(a+b\sqrt{c+dx}\right)}\\
&\quad-\frac{2b^2(3a^2+b^2c)\,d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}\\
&\quad-\frac{\left(2b^2(3a^2+b^2c)\,d\right)\text{Subst}\left(\int \frac{x}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}\\
&\quad+\frac{\left(2ab(a^2+3b^2c)\,d\right)\text{Subst}\left(\int \frac{1}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}
\end{aligned}$$

$$= \frac{4ab^2d}{(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x\left(a+b\sqrt{c+dx}\right)} + \frac{2ab(a^2+3b^2c)\,d\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2-b^2c)^3} + \frac{b^2(3a^2+b^2c)\,d\log(x)}{(a^2-b^2c)^3} - \frac{2b^2(3a^2+b^2c)\,d\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^3}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\frac{(a^2-b^2c)\left(-a^3+a^2b\sqrt{c+dx}-b^3c\sqrt{c+dx}+ab^2(c+4dx)\right)}{x\left(a+b\sqrt{c+dx}\right)} + \frac{2ab(a^2+3b^2c)d\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} + b^2(3a^2+b^2c)\,d\log(-dx) - 2b^2(3}{(a^2-b^2c)^3}$$

```
[In] Integrate[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (((a^2 - b^2*c)*(-a^3 + a^2*b*Sqrt[c + d*x] - b^3*c*Sqrt[c + d*x] + a*b^2*(
c + 4*d*x)))/(x*(a + b*Sqrt[c + d*x])) + (2*a*b*(a^2 + 3*b^2*c)*d*ArcTanh[S
qrt[c + d*x]/Sqrt[c]])/Sqrt[c] + b^2*(3*a^2 + b^2*c)*d*Log[-(d*x)] - 2*b^2*
(3*a^2 + b^2*c)*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^3
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.90

method	result
derivativedivides	$2d\left(\frac{a\,b^2}{(-b^2c+a^2)^2\left(a+b\sqrt{dx+c}\right)} - \frac{b^2\left(b^2c+3a^2\right)\ln\left(a+b\sqrt{dx+c}\right)}{(-b^2c+a^2)^3} + \frac{-\frac{\left(a\,b^3c-a^3b\right)\sqrt{dx+c}-\frac{b^4c^2}{2}+\frac{a^4}{2}}{dx}+b\left(-\frac{\left(-b^3c-3a^2\right.}{2}\right.}{(-b^2c+a}\right.$
default	$2d\left(\frac{a\,b^2}{(-b^2c+a^2)^2\left(a+b\sqrt{dx+c}\right)} - \frac{b^2\left(b^2c+3a^2\right)\ln\left(a+b\sqrt{dx+c}\right)}{(-b^2c+a^2)^3} + \frac{-\frac{\left(a\,b^3c-a^3b\right)\sqrt{dx+c}-\frac{b^4c^2}{2}+\frac{a^4}{2}}{dx}+b\left(-\frac{\left(-b^3c-3a^2\right.}{2}\right.}{(-b^2c+a}\right.$

```
[In] int(1/x^2/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(a*b^2/(-b^2*c+a^2)^2/(a+b*(d*x+c)^(1/2))-b^2*(b^2*c+3*a^2)/(-b^2*c+a^2
)^3*ln(a+b*(d*x+c)^(1/2))+1/(-b^2*c+a^2)^3*(-((a*b^3*c-a^3*b)*(d*x+c)^(1/2)
```

```
-1/2*b^4*c^2+1/2*a^4)/d/x+b*(-1/2*(-b^3*c-3*a^2*b)*ln(-d*x)+(3*a*b^2*c+a^3)
/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(195) = 390.

Time = 0.53 (sec) , antiderivative size = 854, normalized size of antiderivative = 4.23

$$\int \frac{1}{x^2 \left(a+b\sqrt{c+dx}\right)^2} dx$$
$$= \left[-\frac{b^6c^4 - a^2b^4c^3 - a^4b^2c^2 + a^6c + (b^6c^3 + 2\,a^2b^4c^2 - 3\,a^4b^2c)dx - ((3\,ab^5c + a^3b^3)d^2x^2 + (3\,ab^5c^2 - 2\,a^3b^3c}{} \right.$$
$$-\frac{b^6c^4 - a^2b^4c^3 - a^4b^2c^2 + a^6c + (b^6c^3 + 2\,a^2b^4c^2 - 3\,a^4b^2c)dx - 2\,((3\,ab^5c + a^3b^3)d^2x^2 + (3\,ab^5c^2 - 2\,a^3b^3}{}$$

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [-(b^6*c^4 - a^2*b^4*c^3 - a^4*b^2*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2 -
 3*a^4*b^2*c)*d*x - ((3*a*b^5*c + a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3*b
^3*c - a^5*b)*d*x)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2
*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)
*d*x)*log(sqrt(d*x + c)*b + a) + ((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^
3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(x) - 2*(a*b^5*c^3 - 2*a^3*b^3*c^2
 + a^5*b*c + 2*(a*b^5*c^2 - a^3*b^3*c)*d*x)*sqrt(d*x + c))/((b^8*c^4 - 3*a^
2*b^6*c^3 + 3*a^4*b^4*c^2 - a^6*b^2*c)*d*x^2 + (b^8*c^5 - 4*a^2*b^6*c^4 + 6
*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*x), -(b^6*c^4 - a^2*b^4*c^3 - a^4*b^2
*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x - 2*((3*a*b^5*c
+ a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3*b^3*c - a^5*b)*d*x)*sqrt(-c)*arct
an(sqrt(d*x + c)*sqrt(-c)/c) - 2*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^
3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(sqrt(d*x + c)*b + a) + ((b^6*c^2
+ 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(x
) - 2*(a*b^5*c^3 - 2*a^3*b^3*c^2 + a^5*b*c + 2*(a*b^5*c^2 - a^3*b^3*c)*d*x)
*sqrt(d*x + c))/((b^8*c^4 - 3*a^2*b^6*c^3 + 3*a^4*b^4*c^2 - a^6*b^2*c)*d*x^
2 + (b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

```
[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2))**2,x)
[Out] Integral(1/(x**2*(a + b*sqrt(c + d*x))**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx =$$
$$-d\left(\frac{(b^4c+3\,a^2b^2)\log{(dx)}}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{2\,(b^4c+3\,a^2b^2)\log{\left(\sqrt{dx+c}b+a\right)}}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{(3\,ab^3c+a^3b)\log{\left(\frac{\sqrt{dx+c}-}{\sqrt{dx+c}+}\right.}}{(b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-}\right.$$

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
[Out] -d*((b^4*c + 3*a^2*b^2)*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a
^6) - 2*(b^4*c + 3*a^2*b^2)*log(sqrt(d*x + c)*b + a)/(b^6*c^3 - 3*a^2*b^4*c
^2 + 3*a^4*b^2*c - a^6) - (3*a*b^3*c + a^3*b)*log((sqrt(d*x + c) - sqrt(c))
/(sqrt(d*x + c) + sqrt(c)))/((b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*
sqrt(c)) + (4*(d*x + c)*a*b^2 - 3*a*b^2*c - a^3 - (b^3*c - a^2*b)*sqrt(d*x
+ c))/(a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c - (b^5*c^2 - 2*a^2*b^3*c + a^4*b)*
(d*x + c)^(3/2) - (a*b^4*c^2 - 2*a^3*b^2*c + a^5)*(d*x + c) + (b^5*c^3 - 2*
a^2*b^3*c^2 + a^4*b*c)*sqrt(d*x + c)))
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.54

$$\begin{aligned}\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx &= -\frac{(b^4cd+3\,a^2b^2d)\log{(-dx)}}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}+\frac{2\,(b^5cd+3\,a^2b^3d)\log{\left(\left|-\sqrt{dx+c}b-a\right|\right)}}{b^7c^3-3\,a^2b^5c^2+3\,a^4b^3c-a^6b}\\&\quad+\frac{2\,(3\,ab^3cd+a^3bd)\arctan{\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}}{(b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6)\sqrt{-c}}\\&\quad-\frac{\sqrt{dx+c}b^3cd-4\,(dx+c)ab^2d+3\,ab^2cd-\sqrt{dx+c}a^2bd+a^3d}{(b^4c^2-2\,a^2b^2c+a^4)\left((dx+c)^{\frac{3}{2}}b-\sqrt{dx+c}bc+(dx+c)a-ac\right)}\end{aligned}$$

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -(b^4*c*d + 3*a^2*b^2*d)*log(-d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c -
 a^6) + 2*(b^5*c*d + 3*a^2*b^3*d)*log(abs(-sqrt(d*x + c)*b - a))/(b^7*c^3 -
 3*a^2*b^5*c^2 + 3*a^4*b^3*c - a^6*b) + 2*(3*a*b^3*c*d + a^3*b*d)*arctan(sq
rt(d*x + c)/sqrt(-c))/((b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*sqrt(-
c)) - (sqrt(d*x + c)*b^3*c*d - 4*(d*x + c)*a*b^2*d + 3*a*b^2*c*d - sqrt(d*x
 + c)*a^2*b*d + a^3*d)/((b^4*c^2 - 2*a^2*b^2*c + a^4)*((d*x + c)^(3/2)*b -
sqrt(d*x + c)*b*c + (d*x + c)*a - a*c))
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^2 \left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{b\,d\,\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)}{a^3\,\sqrt{c}-b^3\,c^2+3\,a\,b^2\,c^{3/2}-3\,a^2\,b\,c} - \frac{\frac{a\,d\,\left(a^2+3\,c\,b^2\right)}{{\left(b^2\,c-a^2\right)}^2}+\frac{b\,d\,\sqrt{c+d\,x}}{b^2\,c-a^2}-\frac{4\,a\,b^2\,d\,\left(c+d\,x\right)}{a^4-2\,a^2\,b^2\,c+b^4\,c^2}}{b\,{\left(c+d\,x\right)}^{3/2}-a\,c+a\,\left(c+d\,x\right)-b\,c\,\sqrt{c+d\,x}} - \frac{b\,d\,\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{a^3\,\sqrt{c}+b^3\,c^2+3\,a\,b^2\,c^{3/2}+3\,a^2\,b\,c} - \ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(\frac{6\,b^2\,d}{{\left(b^2\,c-a^2\right)}^2}-\frac{8\,b^4\,c\,d}{{\left(b^2\,c-a^2\right)}^3}\right)$$

```
[In] int(1/(x^2*(a + b*(c + d*x)^(1/2))^2),x)
```

```
[Out] (b*d*log((c + d*x)^(1/2) + c^(1/2)))/(a^3*c^(1/2) - b^3*c^2 + 3*a*b^2*c^(3/
2) - 3*a^2*b*c) - ((a*d*(3*b^2*c + a^2))/(b^2*c - a^2)^2 + (b*d*(c + d*x)^(
1/2))/(b^2*c - a^2) - (4*a*b^2*d*(c + d*x))/(a^4 + b^4*c^2 - 2*a^2*b^2*c))/
(b*(c + d*x)^(3/2) - a*c + a*(c + d*x) - b*c*(c + d*x)^(1/2)) - (b*d*log((c
 + d*x)^(1/2) - c^(1/2)))/(a^3*c^(1/2) + b^3*c^2 + 3*a*b^2*c^(3/2) + 3*a^2*
b*c) - log(a + b*(c + d*x)^(1/2))*((6*b^2*d)/(b^2*c - a^2)^2 - (8*b^4*c*d)/
(b^2*c - a^2)^3)
```

3.645 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 306

$$\begin{aligned}\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx &= \frac{ab^2(a^2+11b^2c)\,d^2}{2c\,(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\,(a^2-b^2c)\,x^2\left(a+b\sqrt{c+dx}\right)} \\ &\quad - \frac{bd\left(3abc-(a^2+2b^2c)\sqrt{c+dx}\right)}{2c\,(a^2-b^2c)^2\,x\left(a+b\sqrt{c+dx}\right)} \\ &\quad - \frac{ab(a^4-10a^2b^2c-15b^4c^2)\,d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}\,(a^2-b^2c)^4} \\ &\quad + \frac{b^4(5a^2+b^2c)\,d^2\log(x)}{(a^2-b^2c)^4} - \frac{2b^4(5a^2+b^2c)\,d^2\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^4}\end{aligned}$$

```
[Out] -1/2*a*b*(-15*b^4*c^2-10*a^2*b^2*c+a^4)*d^2*arctanh((d*x+c)^(1/2)/c^(1/2))/
c^(3/2)/(-b^2*c+a^2)^4+b^4*(b^2*c+5*a^2)*d^2*ln(x)/(-b^2*c+a^2)^4-2*b^4*(b^
2*c+5*a^2)*d^2*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^4+1/2*a*b^2*(11*b^2*c+a^2
)*d^2/c/(-b^2*c+a^2)^3/(a+b*(d*x+c)^(1/2))+1/2*(-a+b*(d*x+c)^(1/2))/(-b^2*c
+a^2)/x^2/(a+b*(d*x+c)^(1/2))-1/2*b*d*(3*a*b*c-(2*b^2*c+a^2)*(d*x+c)^(1/2))
/c/(-b^2*c+a^2)^2/x/(a+b*(d*x+c)^(1/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {378, 1412, 837, 815, 649, 212, 266}

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{ab^2d^2(a^2+11b^2c)}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2x^2\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2cx\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)} + \frac{b^4d^2\log(x)\left(5a^2+b^2c\right)}{\left(a^2-b^2c\right)^4} - \frac{2b^4d^2(5a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4} - \frac{abd^2(a^4-10a^2b^2c-15b^4c^2)\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}\left(a^2-b^2c\right)^4}$$

```
[In] Int[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (a*b^2*(a^2 + 11*b^2*c)*d^2)/(2*c*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])) -
(a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2*(a + b*Sqrt[c + d*x])) - (b*d*(3
*a*b*c - (a^2 + 2*b^2*c)*Sqrt[c + d*x]))/(2*c*(a^2 - b^2*c)^2*x*(a + b*Sqrt
[c + d*x])) - (a*b*(a^4 - 10*a^2*b^2*c - 15*b^4*c^2)*d^2*ArcTanh[Sqrt[c + d
*x]/Sqrt[c]])/(2*c^(3/2)*(a^2 - b^2*c)^4) + (b^4*(5*a^2 + b^2*c)*d^2*Log[x]
)/(a^2 - b^2*c)^4 - (2*b^4*(5*a^2 + b^2*c)*d^2*Log[a + b*Sqrt[c + d*x]])/(a
^2 - b^2*c)^4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^2\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(-c+x)^3}\,dx, x, c+dx\right)\\
&= \left(2d^2\right)\text{Subst}\left(\int \frac{x}{(a+bx)^2\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)}+\frac{d^2\text{Subst}\left(\int \frac{-2abc+4b^2cx}{(a+bx)^2\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)}\\
&= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)}-\frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^2x\left(a+b\sqrt{c+dx}\right)}\\
&\quad+\frac{d^2\text{Subst}\left(\int \frac{2abc\left(a^2-7b^2c\right)+4b^2c\left(a^2+2b^2c\right)x}{(a+bx)^2\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)}{4c^2\left(a^2-b^2c\right)^2}
\end{aligned}
$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^2x\left(a+b\sqrt{c+dx}\right)} + \frac{d^2\text{Subst}\left(\int\left(-\frac{2ab^3c(a^2+11b^2c)}{(a^2-b^2c)(a+bx)^2}-\frac{8b^5c^2(5a^2+b^2c)}{(-a^2+b^2c)^2(a+bx)}+\frac{2bc(-a(a^4-10a^2b^2c-15b^4c^2)-4b^3c(5a^2+b^2c)x)}{(a^2-b^2c)^2(c-x^2)}\right)dx,x,\sqrt{c}\right.}{4c^2\left(a^2-b^2c\right)^2}$$

$$= \frac{ab^2(a^2+11b^2c)d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^2x\left(a+b\sqrt{c+dx}\right)} - \frac{2b^4(5a^2+b^2c)d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4} + \frac{(bd^2)\,\text{Subst}\left(\int\frac{-a(a^4-10a^2b^2c-15b^4c^2)-4b^3c(5a^2+b^2c)x}{c-x^2}dx,x,\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^4}$$

$$= \frac{ab^2(a^2+11b^2c)d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^2x\left(a+b\sqrt{c+dx}\right)} - \frac{2b^4(5a^2+b^2c)d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4} - \frac{(2b^4(5a^2+b^2c)d^2)\,\text{Subst}\left(\int\frac{x}{c-x^2}dx,x,\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4} - \frac{(ab(a^4-10a^2b^2c-15b^4c^2)d^2)\,\text{Subst}\left(\int\frac{1}{c-x^2}dx,x,\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^4}$$

$$= \frac{ab^2(a^2+11b^2c)d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^2x\left(a+b\sqrt{c+dx}\right)} - \frac{ab(a^4-10a^2b^2c-15b^4c^2)d^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}\left(a^2-b^2c\right)^4} + \frac{b^4(5a^2+b^2c)d^2\log(x)}{\left(a^2-b^2c\right)^4} - \frac{2b^4(5a^2+b^2c)d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$=\frac{1}{2}\left(\frac{a^5c-b^5c^2(c-2dx)\sqrt{c+dx}+a^2b^3c(2c-dx)\sqrt{c+dx}-a^4b(c+dx)^{3/2}+ab^4c(c^2-3cdx-11d^2x^2)-}{c\left(-a^2+b^2c\right)^3x^2\left(a+b\sqrt{c+dx}\right)}\right.$$
$$+\frac{\left(-a^5b+10a^3b^3c+15ab^5c^2\right)d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{c^{3/2}\left(a^2-b^2c\right)^4}+\frac{2b^4(5a^2+b^2c)\,d^2\log(-dx)}{\left(a^2-b^2c\right)^4}$$
$$\left.-\frac{4b^4(5a^2+b^2c)\,d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4}\right)$$

```
[In] Integrate[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] ((a^5*c - b^5*c^2*(c - 2*d*x)*Sqrt[c + d*x] + a^2*b^3*c*(2*c - d*x)*Sqrt[c
+ d*x] - a^4*b*(c + d*x)^(3/2) + a*b^4*c*(c^2 - 3*c*d*x - 11*d^2*x^2) - a^3
*b^2*(2*c^2 - 3*c*d*x + d^2*x^2))/(c*(-a^2 + b^2*c)^3*x^2*(a + b*Sqrt[c + d
*x])) + ((-(a^5*b) + 10*a^3*b^3*c + 15*a*b^5*c^2)*d^2*ArcTanh[Sqrt[c + d*x]
/Sqrt[c]])/(c^(3/2)*(a^2 - b^2*c)^4) + (2*b^4*(5*a^2 + b^2*c)*d^2*Log[-(d*x
)])/(a^2 - b^2*c)^4 - (4*b^4*(5*a^2 + b^2*c)*d^2*Log[a + b*Sqrt[c + d*x]])/
(a^2 - b^2*c)^4)/2
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.99

method	result
derivativedivides	$2d^2\left(-\frac{\frac{-\frac{ab\left(-7b^4c^2+6a^2b^2c+a^4\right)(dx+c)^{\frac{3}{2}}}{4c}+\left(-\frac{1}{2}c^2b^6-a^2b^4c+\frac{3}{2}b^2a^4\right)(dx+c)+\left(-\frac{9}{4}ab^5c^2+\frac{5}{2}a^3b^3c-\frac{1}{4}a^5b\right)\sqrt{dx+c}+\frac{3b^6c^3}{4}+}{d^2x^2}}{(-b^2c+a^2)^4}\right.$
default	$2d^2\left(-\frac{\frac{-\frac{ab\left(-7b^4c^2+6a^2b^2c+a^4\right)(dx+c)^{\frac{3}{2}}}{4c}+\left(-\frac{1}{2}c^2b^6-a^2b^4c+\frac{3}{2}b^2a^4\right)(dx+c)+\left(-\frac{9}{4}ab^5c^2+\frac{5}{2}a^3b^3c-\frac{1}{4}a^5b\right)\sqrt{dx+c}+\frac{3b^6c^3}{4}+}{d^2x^2}}{(-b^2c+a^2)^4}\right.$

```
[In] int(1/x^3/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*(-1/(-b^2*c+a^2)^4*((-1/4*a*b*(-7*b^4*c^2+6*a^2*b^2*c+a^4)/c*(d*x+c)^
(3/2)+(-1/2*c^2*b^6-a^2*b^4*c+3/2*b^2*a^4)*(d*x+c)+(-9/4*a*b^5*c^2+5/2*a^3*
b^3*c-1/4*a^5*b)*(d*x+c)^(1/2)+3/4*b^6*c^3+3/4*a^2*b^4*c^2-7/4*a^4*b^2*c+1/
4*a^6)/d^2/x^2+1/4*b/c*(-1/2*(4*b^5*c^2+20*a^2*b^3*c)*ln(-d*x)+(-15*a*b^4*c
^2-10*a^3*b^2*c+a^5)/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))))+b^4/(-b^2*c+a
^2)^3*a/(a+b*(d*x+c)^(1/2))-b^4*(b^2*c+5*a^2)/(-b^2*c+a^2)^4*ln(a+b*(d*x+c)
^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(289) = 578$.

Time = 1.25 (sec) , antiderivative size = 1252, normalized size of antiderivative = 4.09

$$\int \frac{1}{x^3 \left(a+b\sqrt{c+dx}\right)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*b^8*c^6 - 4*a^2*b^6*c^5 + 4*a^6*b^2*c^3 - 2*a^8*c^2 - 4*(b^8*c^4 +
 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2 - 2*(b^8*c^5 + 3*a^2*b^6*c^4 - 9*a^
4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x - ((15*a*b^7*c^2 + 10*a^3*b^5*c - a^5*b^3)*d
^3*x^3 + (15*a*b^7*c^3 - 5*a^3*b^5*c^2 - 11*a^5*b^3*c + a^7*b)*d^2*x^2)*sqr
t(c)*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 8*((b^8*c^3 + 5*a^2*b^6
*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(sqrt
(d*x + c)*b + a) - 4*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*
b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(x) - 2*(2*a*b^7*c^5 - 6*a^3*b^5*c^4 +
 6*a^5*b^3*c^3 - 2*a^7*b*c^2 - (11*a*b^7*c^3 - 10*a^3*b^5*c^2 - a^5*b^3*c)*
d^2*x^2 - (5*a*b^7*c^4 - 9*a^3*b^5*c^3 + 3*a^5*b^3*c^2 + a^7*b*c)*d*x)*sqrt
(d*x + c))/((b^10*c^6 - 4*a^2*b^8*c^5 + 6*a^4*b^6*c^4 - 4*a^6*b^4*c^3 + a^8
*b^2*c^2)*d*x^3 + (b^10*c^7 - 5*a^2*b^8*c^6 + 10*a^4*b^6*c^5 - 10*a^6*b^4*c
^4 + 5*a^8*b^2*c^3 - a^10*c^2)*x^2), -1/2*(b^8*c^6 - 2*a^2*b^6*c^5 + 2*a^6*
b^2*c^3 - a^8*c^2 - 2*(b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2 - (
b^8*c^5 + 3*a^2*b^6*c^4 - 9*a^4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x + ((15*a*b^7*c
^2 + 10*a^3*b^5*c - a^5*b^3)*d^3*x^3 + (15*a*b^7*c^3 - 5*a^3*b^5*c^2 - 11*a
^5*b^3*c + a^7*b)*d^2*x^2)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) + 4*((
b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2
)*d^2*x^2)*log(sqrt(d*x + c)*b + a) - 2*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3
+ (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(x) - (2*a*b^7*c^5
- 6*a^3*b^5*c^4 + 6*a^5*b^3*c^3 - 2*a^7*b*c^2 - (11*a*b^7*c^3 - 10*a^3*b^5*
c^2 - a^5*b^3*c)*d^2*x^2 - (5*a*b^7*c^4 - 9*a^3*b^5*c^3 + 3*a^5*b^3*c^2 + a
^7*b*c)*d*x)*sqrt(d*x + c))/((b^10*c^6 - 4*a^2*b^8*c^5 + 6*a^4*b^6*c^4 - 4*
a^6*b^4*c^3 + a^8*b^2*c^2)*d*x^3 + (b^10*c^7 - 5*a^2*b^8*c^6 + 10*a^4*b^6*c
^5 - 10*a^6*b^4*c^4 + 5*a^8*b^2*c^3 - a^10*c^2)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx = \int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

```
[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**3*(a + b*sqrt(c + d*x))**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. 2(289) = 578.

Time = 0.29 (sec) , antiderivative size = 659, normalized size of antiderivative = 2.15

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{1}{4}d^2\left(\frac{4\,(b^6c+5\,a^2b^4)\log{(dx)}}{b^8c^4-4\,a^2b^6c^3+6\,a^4b^4c^2-4\,a^6b^2c+a^8} - \frac{8\,(b^6c+5\,a^2b^4)\log{\left(\sqrt{dx+c}b+a\right)}}{b^8c^4-4\,a^2b^6c^3+6\,a^4b^4c^2-4\,a^6b^2c+a^8} - \frac{(15\,ab^5c^2}{(b^8c^5-4\,a^2}\right.$$

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*d^2*(4*(b^6*c + 5*a^2*b^4)*log(d*x)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^
4*c^2 - 4*a^6*b^2*c + a^8) - 8*(b^6*c + 5*a^2*b^4)*log(sqrt(d*x + c)*b + a)
/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^4*c^2 - 4*a^6*b^2*c + a^8) - (15*a*b^5*
c^2 + 10*a^3*b^3*c - a^5*b)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) +
sqrt(c)))/((b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c
)*sqrt(c)) - 2*(7*a*b^4*c^3 + 6*a^3*b^2*c^2 - a^5*c + (11*a*b^4*c + a^3*b^2
)*(d*x + c)^2 - (2*b^5*c^2 - a^2*b^3*c - a^4*b)*(d*x + c)^(3/2) - (19*a*b^4
*c^2 + 5*a^3*b^2*c)*(d*x + c) + 3*(b^5*c^3 - a^2*b^3*c^2)*sqrt(d*x + c))/(a
*b^6*c^6 - 3*a^3*b^4*c^5 + 3*a^5*b^2*c^4 - a^7*c^3 + (b^7*c^4 - 3*a^2*b^5*c
^3 + 3*a^4*b^3*c^2 - a^6*b*c)*(d*x + c)^(5/2) + (a*b^6*c^4 - 3*a^3*b^4*c^3
+ 3*a^5*b^2*c^2 - a^7*c)*(d*x + c)^2 - 2*(b^7*c^5 - 3*a^2*b^5*c^4 + 3*a^4*b
^3*c^3 - a^6*b*c^2)*(d*x + c)^(3/2) - 2*(a*b^6*c^5 - 3*a^3*b^4*c^4 + 3*a^5*
b^2*c^3 - a^7*c^2)*(d*x + c) + (b^7*c^6 - 3*a^2*b^5*c^5 + 3*a^4*b^3*c^4 - a
^6*b*c^3)*sqrt(d*x + c)))
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{(b^6cd^2+5\,a^2b^4d^2)\log\left(-dx\right)}{b^8c^4-4\,a^2b^6c^3+6\,a^4b^4c^2-4\,a^6b^2c+a^8} - \frac{2\,(b^7cd^2+5\,a^2b^5d^2)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^9c^4-4\,a^2b^7c^3+6\,a^4b^5c^2-4\,a^6b^3c+a^8b}$$
$$- \frac{(15\,ab^5c^2d^2+10\,a^3b^3cd^2-a^5bd^2)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{2\,(b^8c^5-4\,a^2b^6c^4+6\,a^4b^4c^3-4\,a^6b^2c^2+a^8c)\sqrt{-c}}$$
$$- \frac{7\,ab^6c^4d^2-a^3b^4c^3d^2-7\,a^5b^2c^2d^2+a^7cd^2+(11\,ab^6c^2d^2-10\,a^3b^4cd^2-a^5b^2d^2)(dx+c)^2-(2\,b^7c^3d^2-3\,\cdots}{2\,(b^2c-\cdots}$$

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] (b^6*c*d^2 + 5*a^2*b^4*d^2)*log(-d*x)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^4*
c^2 - 4*a^6*b^2*c + a^8) - 2*(b^7*c*d^2 + 5*a^2*b^5*d^2)*log(abs(sqrt(d*x +
 c)*b + a))/(b^9*c^4 - 4*a^2*b^7*c^3 + 6*a^4*b^5*c^2 - 4*a^6*b^3*c + a^8*b)
 - 1/2*(15*a*b^5*c^2*d^2 + 10*a^3*b^3*c*d^2 - a^5*b*d^2)*arctan(sqrt(d*x +
c)/sqrt(-c))/((b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^
8*c)*sqrt(-c)) - 1/2*(7*a*b^6*c^4*d^2 - a^3*b^4*c^3*d^2 - 7*a^5*b^2*c^2*d^2
 + a^7*c*d^2 + (11*a*b^6*c^2*d^2 - 10*a^3*b^4*c*d^2 - a^5*b^2*d^2)*(d*x + c
)^2 - (2*b^7*c^3*d^2 - 3*a^2*b^5*c^2*d^2 + a^6*b*d^2)*(d*x + c)^(3/2) - (19
*a*b^6*c^3*d^2 - 14*a^3*b^4*c^2*d^2 - 5*a^5*b^2*c*d^2)*(d*x + c) + 3*(b^7*c
^4*d^2 - 2*a^2*b^5*c^3*d^2 + a^4*b^3*c^2*d^2)*sqrt(d*x + c))/((b^2*c - a^2)
^4*(sqrt(d*x + c)*b + a)*c*d^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 21.42 (sec) , antiderivative size = 1441, normalized size of antiderivative = 4.71

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^3*(a + b*(c + d*x)^(1/2))^2),x)
```

```
[Out] (((5*a^3*b^2*d^2 + 19*a*b^4*c*d^2)*(c + d*x))/(2*(b^2*c - a^2)*(a^4 + b^4*c
^2 - 2*a^2*b^2*c)) + ((a^3*b^2*d^2 + 11*a*b^4*c*d^2)*(c + d*x)^2)/(2*c*(a^6
 - b^6*c^3 - 3*a^4*b^2*c + 3*a^2*b^4*c^2)) - (a*(7*b^4*c^2*d^2 - a^4*d^2 +
6*a^2*b^2*c*d^2))/(2*(b^2*c - a^2)*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) + (b*(a^2
*d^2 + 2*b^2*c*d^2)*(c + d*x)^(3/2))/(2*c*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) -
(3*b^3*c*d^2*(c + d*x)^(1/2))/(2*(a^4 + b^4*c^2 - 2*a^2*b^2*c)))/(a*(c + d*
```

x)^2 + b*(c + d*x)^(5/2) + a*c^2 - 2*a*c*(c + d*x) - 2*b*c*(c + d*x)^(3/2)
+ b*c^2*(c + d*x)^(1/2)) + log(a + b*(c + d*x)^(1/2))*((10*b^4*d^2)/(b^2*c
- a^2)^3 - (12*b^6*c*d^2)/(b^2*c - a^2)^4) + (log((a*b^4*d^4*(a^6 - 44*b^6*
c^3 + 2*a^4*b^2*c - 103*a^2*b^4*c^2))/(4*c^2*(b^2*c - a^2)^6) - (b*d^2*((b^
2*d^2*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*(a^5*(c^3)^
(1/2) + 4*b^5*c^4 + 20*a^2*b^3*c^3 - 10*a^3*b^2*c*(c^3)^(1/2) - 15*a*b^4*c^
2*(c^3)^(1/2)))/(2*c^3*(b^2*c - a^2)^4) - (b^3*d^2*(c + d*x)^(1/2)*(6*b^4*c
^2 - a^4 + 19*a^2*b^2*c))/(c*(b^2*c - a^2)^3) + (a*b^2*d^2*(7*b^2*c - a^2))
/(2*c*(b^2*c - a^2)^2))*(a^5*(c^3)^(1/2) + 4*b^5*c^4 + 20*a^2*b^3*c^3 - 10*
a^3*b^2*c*(c^3)^(1/2) - 15*a*b^4*c^2*(c^3)^(1/2)))/(4*c^3*(b^2*c - a^2)^4)
+ (a^2*b^5*d^4*(11*b^2*c + a^2)^2*(c + d*x)^(1/2))/(4*c^2*(b^2*c - a^2)^6))
*(4*b^6*c^4*d^2 + 20*a^2*b^4*c^3*d^2 + a^5*b*d^2*(c^3)^(1/2) - 10*a^3*b^3*c
d^2(c^3)^(1/2) - 15*a*b^5*c^2*d^2*(c^3)^(1/2)))/(4*(a^8*c^3 + b^8*c^7 - 4
*a^6*b^2*c^4 + 6*a^4*b^4*c^5 - 4*a^2*b^6*c^6)) + (log((a*b^4*d^4*(a^6 - 44*
b^6*c^3 + 2*a^4*b^2*c - 103*a^2*b^4*c^2))/(4*c^2*(b^2*c - a^2)^6) - (b*d^2*
((b^2*d^2*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*(4*b^5*
c^4 - a^5*(c^3)^(1/2) + 20*a^2*b^3*c^3 + 10*a^3*b^2*c*(c^3)^(1/2) + 15*a*b^
4*c^2*(c^3)^(1/2)))/(2*c^3*(b^2*c - a^2)^4) - (b^3*d^2*(c + d*x)^(1/2)*(6*b
^4*c^2 - a^4 + 19*a^2*b^2*c))/(c*(b^2*c - a^2)^3) + (a*b^2*d^2*(7*b^2*c - a
^2))/(2*c*(b^2*c - a^2)^2))*(4*b^5*c^4 - a^5*(c^3)^(1/2) + 20*a^2*b^3*c^3 +
 10*a^3*b^2*c*(c^3)^(1/2) + 15*a*b^4*c^2*(c^3)^(1/2)))/(4*c^3*(b^2*c - a^2)
^4) + (a^2*b^5*d^4*(11*b^2*c + a^2)^2*(c + d*x)^(1/2))/(4*c^2*(b^2*c - a^2)
^6))*(4*b^6*c^4*d^2 + 20*a^2*b^4*c^3*d^2 - a^5*b*d^2*(c^3)^(1/2) + 10*a^3*b
^3*c*d^2*(c^3)^(1/2) + 15*a*b^5*c^2*d^2*(c^3)^(1/2)))/(4*(a^8*c^3 + b^8*c^7
 - 4*a^6*b^2*c^4 + 6*a^4*b^4*c^5 - 4*a^2*b^6*c^6))

3.646 $\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 324

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = & -\frac{4a(a^2-b^2c)^3\sqrt{a+b\sqrt{c+dx}}}{b^8d^4} \\ & +\frac{4(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4} \\ & -\frac{12a(7a^2-3b^2c)(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4} \\ & +\frac{4(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4} \\ & -\frac{20a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4} \\ & +\frac{12(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4} \\ & -\frac{28a\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}\end{aligned}$$

```
[Out] 4/3*(-b^2*c+a^2)^2*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(3/2)/b^8/d^4-12/5*a*
(-3*b^2*c+7*a^2)*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(5/2)/b^8/d^4+4/7*(3*b^4*
c^2-30*a^2*b^2*c+35*a^4)*(a+b*(d*x+c)^(1/2))^(7/2)/b^8/d^4-20/9*a*(-3*b^2*c
+7*a^2)*(a+b*(d*x+c)^(1/2))^(9/2)/b^8/d^4+12/11*(-b^2*c+7*a^2)*(a+b*(d*x+c)
^(1/2))^(11/2)/b^8/d^4-28/13*a*(a+b*(d*x+c)^(1/2))^(13/2)/b^8/d^4+4/15*(a+b
*(d*x+c)^(1/2))^(15/2)/b^8/d^4-4*a*(-b^2*c+a^2)^3*(a+b*(d*x+c)^(1/2))^(1/2)
/b^8/d^4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {378, 1412, 786}

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{12(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4} - \frac{20a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4} - \frac{12a(7a^2-3b^2c)\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4} + \frac{4(a^2-b^2c)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4} - \frac{4a(a^2-b^2c)^3\sqrt{a+b\sqrt{c+dx}}}{b^8d^4} + \frac{4(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4} + \frac{4\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4} - \frac{28a\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}$$

```
[In] Int[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^3*Sqrt[a + b*Sqrt[c + d*x]])/(b^8*d^4) + (4*(a^2 - b^2*
c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8*d^4) - (12*a*(7*a^
2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8*d^4) + (4*(3
5*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8*d^4)
- (20*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d^4) + (12*(7
*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) - (28*a*(a + b*Sqr
t[c + d*x])^(13/2))/(13*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(15/2))/(15*b^8
*d^4)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d^4}$$
$$= \frac{2\text{Subst}\left(\int \frac{x(-c+x^2)^3}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d^4}$$
$$= \frac{2\text{Subst}\left(\int \left(-\frac{a(a^2-b^2c)^3}{b^7\sqrt{a+bx}} - \frac{(-7a^2+b^2c)(-a^2+b^2c)^2\sqrt{a+bx}}{b^7} - \frac{3(7a^5-10a^3b^2c+3ab^4c^2)(a+bx)^{3/2}}{b^7} + \frac{(35a^4-30a^2b^2c+3b^4c^2}{b^7}\right.\right.}{d^4}$$
$$= -\frac{4a(a^2-b^2c)^3\sqrt{a+b\sqrt{c+dx}}}{b^8d^4} + \frac{4(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4} - \frac{12a(7a^2-3b^2c)(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4} + \frac{4(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4} - \frac{20a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4} + \frac{12(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4} - \frac{28a\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4} + \frac{4\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\sqrt{a+b\sqrt{c+dx}}\left(-14336a^7+768a^5b^2(58c-7dx)+7168a^6b\sqrt{c+dx}-640a^4b^3(32c-7dx)\sqrt{c+dx}+24\right.}{}$$

```
[In] Integrate[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-14336*a^7 + 768*a^5*b^2*(58*c - 7*d*x) + 716
8*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(32*c - 7*d*x)*Sqrt[c + d*x] + 24*a^2*b
^5*Sqrt[c + d*x]*(784*c^2 - 356*c*d*x + 147*d^2*x^2) - 16*a^3*b^4*(2936*c^2
```

```
 - 680*c*d*x + 245*d^2*x^2) + 6*a*b^6*(2880*c^3 - 928*c^2*d*x + 658*c*d^2*x
^2 - 539*d^3*x^3) - 39*b^7*Sqrt[c + d*x]*(128*c^3 - 96*c^2*d*x + 84*c*d^2*x
^2 - 77*d^3*x^3)))/(45045*b^8*d^4)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.19

method	result
derivativedivides	$-\frac{4\left(-\frac{(a+b\sqrt{dx+c})^{\frac{15}{2}}}{15}+\frac{7a(a+b\sqrt{dx+c})^{\frac{13}{2}}}{13}+\frac{(3b^2c-21a^2)(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}+\frac{(8(-b^2c+a^2)a+2a(-2b^2c+6a^2)+(-3b^2c+15a^2)\cdots}{9}\cdots\right.}{\cdots}$
default	$-\frac{4\left(-\frac{(a+b\sqrt{dx+c})^{\frac{15}{2}}}{15}+\frac{7a(a+b\sqrt{dx+c})^{\frac{13}{2}}}{13}+\frac{(3b^2c-21a^2)(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}+\frac{(8(-b^2c+a^2)a+2a(-2b^2c+6a^2)+(-3b^2c+15a^2)\cdots}{9}\cdots\right.}{\cdots}$

```
[In] int(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/d^4/b^8*(-1/15*(a+b*(d*x+c)^(1/2))^(15/2)+7/13*a*(a+b*(d*x+c)^(1/2))^(13
/2)+1/11*(3*b^2*c-21*a^2)*(a+b*(d*x+c)^(1/2))^(11/2)+1/9*(8*(-b^2*c+a^2)*a+
2*a*(-2*b^2*c+6*a^2)+(-3*b^2*c+15*a^2)*a)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*(-(
-b^2*c+a^2)*(-2*b^2*c+6*a^2)-8*a^2*(-b^2*c+a^2)-(-b^2*c+a^2)^2+(-8*(-b^2*c+
a^2)*a-2*a*(-2*b^2*c+6*a^2))*a)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*(6*(-b^2*c+a^
2)^2*a+((-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2)*a)
*(a+b*(d*x+c)^(1/2))^(5/2)+1/3*(-(-b^2*c+a^2)^3-6*(-b^2*c+a^2)^2*a^2)*(a+b*
(d*x+c)^(1/2))^(3/2)+(-b^2*c+a^2)^3*a*(a+b*(d*x+c)^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{4\left(3234\,ab^6d^3x^3-17280\,ab^6c^3+46976\,a^3b^4c^2-44544\,a^5b^2c+14336\,a^7-28\,(141\,ab^6c-140\,a^3b^4)d^2x^2\cdots\right.}{\cdots}$$

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/45045*(3234*a*b^6*d^3*x^3 - 17280*a*b^6*c^3 + 46976*a^3*b^4*c^2 - 44544*
a^5*b^2*c + 14336*a^7 - 28*(141*a*b^6*c - 140*a^3*b^4)*d^2*x^2 + 64*(87*a*b
^6*c^2 - 170*a^3*b^4*c + 84*a^5*b^2)*d*x - (3003*b^7*d^3*x^3 - 4992*b^7*c^3
 + 18816*a^2*b^5*c^2 - 20480*a^4*b^3*c + 7168*a^6*b - 252*(13*b^7*c - 14*a^
2*b^5)*d^2*x^2 + 32*(117*b^7*c^2 - 267*a^2*b^5*c + 140*a^4*b^3)*d*x)*sqrt(d
*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.10

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$= \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{7a(a+b\sqrt{c+dx})^{\frac{13}{2}}}{13b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{15}{2}}}{15b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{11}{2}}\cdot(21a^2-3b^2c)}{11b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{9}{2}}(-35a^3+15ab^2c)}{9b^6}+\frac{(a+b\sqrt{c+dx})^{\frac{7}{2}}\cdot(35a^4-30a^2b^2c+3b^4c^2 \ldots}{7b^6}\right.}{b^2} \\ \frac{d^4x^4}{8\sqrt{a}} \end{cases}\right.}{} \\ \frac{x^4}{4\sqrt{a+b\sqrt{c}}} \end{cases}$$

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((2*(-7*a*(a + b*sqrt(c + d*x))**(13/2)/(13*b**6) + (
a + b*sqrt(c + d*x))**(15/2)/(15*b**6) + (a + b*sqrt(c + d*x))**(11/2)*(21*
a**2 - 3*b**2*c)/(11*b**6) + (a + b*sqrt(c + d*x))**(9/2)*(-35*a**3 + 15*a*
b**2*c)/(9*b**6) + (a + b*sqrt(c + d*x))**(7/2)*(35*a**4 - 30*a**2*b**2*c +
 3*b**4*c**2)/(7*b**6) + (a + b*sqrt(c + d*x))**(5/2)*(-21*a**5 + 30*a**3*b
**2*c - 9*a*b**4*c**2)/(5*b**6) + (a + b*sqrt(c + d*x))**(3/2)*(7*a**6 - 15
*a**4*b**2*c + 9*a**2*b**4*c**2 - b**6*c**3)/(3*b**6) + sqrt(a + b*sqrt(c +
 d*x))*(-a**7 + 3*a**5*b**2*c - 3*a**3*b**4*c**2 + a*b**6*c**3)/b**6)/b**2,
 Ne(b, 0)), (d**4*x**4/(8*sqrt(a)), True))/d**4, Ne(d, 0)), (x**4/(4*sqrt(a
 + b*sqrt(c))), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$= \frac{4\left(3003\left(\sqrt{dx+c}b+a\right)^{\frac{15}{2}}-24255\left(\sqrt{dx+c}b+a\right)^{\frac{13}{2}}a-12285\left(b^2c-7a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{11}{2}}+25025\left(3ab \ldots\right.}{}$$

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/45045*(3003*(sqrt(d*x + c)*b + a)^(15/2) - 24255*(sqrt(d*x + c)*b + a)^(1
3/2)*a - 12285*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(11/2) + 25025*(3*a*b^
2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(9/2) + 6435*(3*b^4*c^2 - 30*a^2*b^2*c +
 35*a^4)*(sqrt(d*x + c)*b + a)^(7/2) - 27027*(3*a*b^4*c^2 - 10*a^3*b^2*c +
7*a^5)*(sqrt(d*x + c)*b + a)^(5/2) - 15015*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^
4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(3/2) + 45045*(a*b^6*c^3 - 3*a^3*b^4
*c^2 + 3*a^5*b^2*c - a^7)*sqrt(sqrt(d*x + c)*b + a))/(b^8*d^4)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.26

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx =$$
$$-\frac{4\left(15015\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}b^6c^3-45045\sqrt{\sqrt{dx+c}b+a}ab^6c^3-19305\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}b^4c^2+81081\left(\sqrt{\right.\right.}{}$$

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/45045*(15015*(sqrt(d*x + c)*b + a)^(3/2)*b^6*c^3 - 45045*sqrt(sqrt(d*x +
 c)*b + a)*a*b^6*c^3 - 19305*(sqrt(d*x + c)*b + a)^(7/2)*b^4*c^2 + 81081*(s
qrt(d*x + c)*b + a)^(5/2)*a*b^4*c^2 - 135135*(sqrt(d*x + c)*b + a)^(3/2)*a^
2*b^4*c^2 + 135135*sqrt(sqrt(d*x + c)*b + a)*a^3*b^4*c^2 + 12285*(sqrt(d*x
+ c)*b + a)^(11/2)*b^2*c - 75075*(sqrt(d*x + c)*b + a)^(9/2)*a*b^2*c + 1930
50*(sqrt(d*x + c)*b + a)^(7/2)*a^2*b^2*c - 270270*(sqrt(d*x + c)*b + a)^(5/
2)*a^3*b^2*c + 225225*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^2*c - 135135*sqrt(s
qrt(d*x + c)*b + a)*a^5*b^2*c - 3003*(sqrt(d*x + c)*b + a)^(15/2) + 24255*(
sqrt(d*x + c)*b + a)^(13/2)*a - 85995*(sqrt(d*x + c)*b + a)^(11/2)*a^2 + 17
5175*(sqrt(d*x + c)*b + a)^(9/2)*a^3 - 225225*(sqrt(d*x + c)*b + a)^(7/2)*a
^4 + 189189*(sqrt(d*x + c)*b + a)^(5/2)*a^5 - 105105*(sqrt(d*x + c)*b + a)^
(3/2)*a^6 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^7)/(b^8*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{x^3}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

```
[In] int(x^3/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x^3/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.647 $\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 222

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx &= -\frac{4a(a^2-b^2c)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3} \\ &+ \frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3} \\ &- \frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3} \\ &+ \frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3} \\ &- \frac{20a\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3} + \frac{4\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3}\end{aligned}$$

```
[Out] 4/3*(b^4*c^2-6*a^2*b^2*c+5*a^4)*(a+b*(d*x+c)^(1/2))^(3/2)/b^6/d^3-8/5*a*(-3
*b^2*c+5*a^2)*(a+b*(d*x+c)^(1/2))^(5/2)/b^6/d^3+8/7*(-b^2*c+5*a^2)*(a+b*(d*
x+c)^(1/2))^(7/2)/b^6/d^3-20/9*a*(a+b*(d*x+c)^(1/2))^(9/2)/b^6/d^3+4/11*(a+
b*(d*x+c)^(1/2))^(11/2)/b^6/d^3-4*a*(-b^2*c+a^2)^2*(a+b*(d*x+c)^(1/2))^(1/2
)/b^6/d^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {378, 1412, 786}

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3} - \frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3} - \frac{4a(a^2-b^2c)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3} + \frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3} + \frac{4\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3} - \frac{20a\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}$$

```
[In] Int[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^2*Sqrt[a + b*Sqrt[c + d*x]])/(b^6*d^3) + (4*(5*a^4 - 6*
a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6*d^3) - (8*a*(5*a^2
 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6*d^3) + (8*(5*a^2 - b^2*c)*(
a + b*Sqrt[c + d*x])^(7/2))/(7*b^6*d^3) - (20*a*(a + b*Sqrt[c + d*x])^(9/2)
)/(9*b^6*d^3) + (4*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^6*d^3)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\text{Subst}\left(\int \frac{x(-c+x^2)^2}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{a(a^2-b^2c)^2}{b^5\sqrt{a+bx}} + \frac{(5a^4-6a^2b^2c+b^4c^2)\sqrt{a+bx}}{b^5} - \frac{2(5a^3-3ab^2c)(a+bx)^{3/2}}{b^5} - \frac{2(-5a^2+b^2c)(a+bx)^{5/2}}{b^5} - \frac{5a(a+bx}{b^5}\right.\right.}{d^3}$$

$$= -\frac{4a(a^2-b^2c)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3} + \frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3} - \frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3} + \frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3} - \frac{20a\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3} + \frac{4\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\sqrt{a+b\sqrt{c+dx}}\left(-1280a^5+96a^3b^2(28c-5dx)+640a^4b\sqrt{c+dx}-16a^2b^3(74c-25dx)\sqrt{c+dx}+15b^5\sqrt{}\right.}{3465b^6d^3}$$

```
[In] Integrate[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-1280*a^5 + 96*a^3*b^2*(28*c - 5*d*x) + 640*a
^4*b*Sqrt[c + d*x] - 16*a^2*b^3*(74*c - 25*d*x)*Sqrt[c + d*x] + 15*b^5*Sqrt
[c + d*x]*(32*c^2 - 24*c*d*x + 21*d^2*x^2) - 2*a*b^4*(736*c^2 - 244*c*d*x +
 175*d^2*x^2)))/(3465*b^6*d^3)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{4(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}-\frac{20a(a+b\sqrt{dx+c})^{\frac{9}{2}}}{9}-\frac{4(2b^2c-10a^2)(a+b\sqrt{dx+c})^{\frac{7}{2}}}{7}-\frac{4(4(-b^2c+a^2)a+a(-2b^2c+6a^2))(a+b\sqrt{dx+c})^{\frac{5}{2}}}{5}-\frac{4(-}{}}{d^3b^6}$
default	$\frac{\frac{4(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}-\frac{20a(a+b\sqrt{dx+c})^{\frac{9}{2}}}{9}-\frac{4(2b^2c-10a^2)(a+b\sqrt{dx+c})^{\frac{7}{2}}}{7}-\frac{4(4(-b^2c+a^2)a+a(-2b^2c+6a^2))(a+b\sqrt{dx+c})^{\frac{5}{2}}}{5}-\frac{4(-}{}}{d^3b^6}$

```
[In] int(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d^3/b^6*(1/11*(a+b*(d*x+c)^(1/2))^(11/2)-5/9*a*(a+b*(d*x+c)^(1/2))^(9/2)-
1/7*(2*b^2*c-10*a^2)*(a+b*(d*x+c)^(1/2))^(7/2)-1/5*(4*(-b^2*c+a^2)*a+a*(-2*
b^2*c+6*a^2))*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*(-(-b^2*c+a^2)^2-4*a^2*(-b^2*c+
a^2))*(a+b*(d*x+c)^(1/2))^(3/2)-(-b^2*c+a^2)^2*a*(a+b*(d*x+c)^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{4\left(350\,ab^4d^2x^2+1472\,ab^4c^2-2688\,a^3b^2c+1280\,a^5-8\,(61\,ab^4c-60\,a^3b^2)dx-(315\,b^5d^2x^2+480\,b^5c^2\right.}{3465\,b^6d^3}$$

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/3465*(350*a*b^4*d^2*x^2 + 1472*a*b^4*c^2 - 2688*a^3*b^2*c + 1280*a^5 - 8
*(61*a*b^4*c - 60*a^3*b^2)*d*x - (315*b^5*d^2*x^2 + 480*b^5*c^2 - 1184*a^2*
b^3*c + 640*a^4*b - 40*(9*b^5*c - 10*a^2*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt
(d*x + c)*b + a)/(b^6*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{5a(a+b\sqrt{c+dx})^{\frac{9}{2}}}{9b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{11}{2}}}{11b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{7}{2}}\cdot(10a^2-2b^2c)}{7b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{5}{2}}(-10a^3+6ab^2c)}{5b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{3}{2}}\cdot(5a^4-6a^2b^2c+b^4c^2)}{3b^4}+\right.}{b^2} \\ \frac{d^3x^3}{6\sqrt{a}} \end{cases}\right.}{d^3} \\ \frac{x^3}{3\sqrt{a+b\sqrt{c}}} \end{cases}$$

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((2*(-5*a*(a + b*sqrt(c + d*x))**(9/2)/(9*b**4) + (a
+ b*sqrt(c + d*x))**(11/2)/(11*b**4) + (a + b*sqrt(c + d*x))**(7/2)*(10*a**
2 - 2*b**2*c)/(7*b**4) + (a + b*sqrt(c + d*x))**(5/2)*(-10*a**3 + 6*a*b**2*
c)/(5*b**4) + (a + b*sqrt(c + d*x))**(3/2)*(5*a**4 - 6*a**2*b**2*c + b**4*c
**2)/(3*b**4) + sqrt(a + b*sqrt(c + d*x))*(-a**5 + 2*a**3*b**2*c - a*b**4*c
**2)/b**4)/b**2, Ne(b, 0)), (d**3*x**3/(6*sqrt(a)), True))/d**3, Ne(d, 0)),
 (x**3/(3*sqrt(a + b*sqrt(c))), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\left(315\left(\sqrt{dx+c}b+a\right)^{\frac{11}{2}} - 1925\left(\sqrt{dx+c}b+a\right)^{\frac{9}{2}}a - 990\left(b^2c-5a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}} + 1386\left(3ab^2c-5\right.\right.}{3}$$

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3465*(315*(sqrt(d*x + c)*b + a)^(11/2) - 1925*(sqrt(d*x + c)*b + a)^(9/2)
*a - 990*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(7/2) + 1386*(3*a*b^2*c - 5*
a^3)*(sqrt(d*x + c)*b + a)^(5/2) + 1155*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*(sq
rt(d*x + c)*b + a)^(3/2) - 3465*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*sqrt(sqrt(d
*x + c)*b + a))/(b^6*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\left(1155\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}b^4c^2 - 3465\sqrt{\sqrt{dx+c}b+a}\,ab^4c^2 - 990\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}b^2c + 4158\left(\sqrt{dx+c}b+\right.\right.}{}$$

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/3465*(1155*(sqrt(d*x + c)*b + a)^(3/2)*b^4*c^2 - 3465*sqrt(sqrt(d*x + c)*
b + a)*a*b^4*c^2 - 990*(sqrt(d*x + c)*b + a)^(7/2)*b^2*c + 4158*(sqrt(d*x +
 c)*b + a)^(5/2)*a*b^2*c - 6930*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^2*c + 693
```

```
0*sqrt(sqrt(d*x + c)*b + a)*a^3*b^2*c + 315*(sqrt(d*x + c)*b + a)^(11/2) -
1925*(sqrt(d*x + c)*b + a)^(9/2)*a + 4950*(sqrt(d*x + c)*b + a)^(7/2)*a^2 -
 6930*(sqrt(d*x + c)*b + a)^(5/2)*a^3 + 5775*(sqrt(d*x + c)*b + a)^(3/2)*a^
4 - 3465*sqrt(sqrt(d*x + c)*b + a)*a^5)/(b^6*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{x^2}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

```
[In] int(x^2/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x^2/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.648 $\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{4a(a^2-b^2c)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2}+\frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2} - \frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}$$

```
[Out] 4/3*(-b^2*c+3*a^2)*(a+b*(d*x+c)^(1/2))^(3/2)/b^4/d^2-12/5*a*(a+b*(d*x+c)^(1
/2))^(5/2)/b^4/d^2+4/7*(a+b*(d*x+c)^(1/2))^(7/2)/b^4/d^2-4*a*(-b^2*c+a^2)*(
a+b*(d*x+c)^(1/2))^(1/2)/b^4/d^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {378, 1412, 786}

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}-\frac{4a(a^2-b^2c)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2} + \frac{4\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}$$

```
[In] Int[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)*Sqrt[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (4*(3*a^2 - b^2*
c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) - (12*a*(a + b*Sqrt[c + d*x])^(
5/2))/(5*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^4*d^2)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-c+x}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int \frac{x(-c+x^2)}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{-a^3+ab^2c}{b^3\sqrt{a+bx}}+\frac{(3a^2-b^2c)\sqrt{a+bx}}{b^3}-\frac{3a(a+bx)^{3/2}}{b^3}+\frac{(a+bx)^{5/2}}{b^3}\right)dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= -\frac{4a(a^2-b^2c)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2}+\frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}\\
&\quad -\frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.64

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\sqrt{a+b\sqrt{c+dx}}\left(-48a^3+2ab^2(26c-9dx)+24a^2b\sqrt{c+dx}+5b^3\sqrt{c+dx}(-4c+3dx)\right)}{105b^4d^2}$$

[In] Integrate[x/Sqrt[a + b*Sqrt[c + d*x]],x]

[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-48*a^3 + 2*a*b^2*(26*c - 9*d*x) + 24*a^2*b*Sqrt[c + d*x] + 5*b^3*Sqrt[c + d*x]*(-4*c + 3*d*x)))/(105*b^4*d^2)

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}+\frac{3a\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}+\frac{\left(b^2c-3a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}+\left(-b^2c+a^2\right)a\sqrt{a+b\sqrt{dx+c}}\right)}{d^2b^4}$	92
default	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}+\frac{3a\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}+\frac{\left(b^2c-3a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}+\left(-b^2c+a^2\right)a\sqrt{a+b\sqrt{dx+c}}\right)}{d^2b^4}$	92

[In] int(x/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)

[Out] -4/d^2/b^4*(-1/7*(a+b*(d*x+c)^(1/2))^(7/2)+3/5*a*(a+b*(d*x+c)^(1/2))^(5/2)+1/3*(b^2*c-3*a^2)*(a+b*(d*x+c)^(1/2))^(3/2)+(-b^2*c+a^2)*a*(a+b*(d*x+c)^(1/2))^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.54

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{4\left(18\,ab^2dx-52\,ab^2c+48\,a^3-\left(15\,b^3dx-20\,b^3c+24\,a^2b\right)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}b+a}}{105\,b^4d^2}$$

[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")

[Out] -4/105*(18*a*b^2*d*x - 52*a*b^2*c + 48*a^3 - (15*b^3*d*x - 20*b^3*c + 24*a^2*b)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^4*d^2)

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.18

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{3a\left(a+b\sqrt{c+dx}\right)^{\frac{5}{2}}}{5b^2}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{7}{2}}}{7b^2}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{3}{2}}\cdot\left(3a^2-b^2c\right)}{3b^2}+\frac{\sqrt{a+b\sqrt{c+dx}}\left(-a^3+ab^2c\right)}{b^2}\right)}{b^2} & \text{for } b\neq 0 \\ \frac{-\frac{c(c+dx)}{2}+\frac{(c+dx)^2}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}\right)}{d^2} & \text{for } d\neq 0 \\ \frac{x^2}{2\sqrt{a+b\sqrt{c}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((2*(-3*a*(a + b*sqrt(c + d*x))**(5/2)/(5*b**2) + (a
+ b*sqrt(c + d*x))**(7/2)/(7*b**2) + (a + b*sqrt(c + d*x))**(3/2)*(3*a**2 -
 b**2*c)/(3*b**2) + sqrt(a + b*sqrt(c + d*x))*(-a**3 + a*b**2*c)/b**2)/b**2
, Ne(b, 0)), ((-c*(c + d*x)/2 + (c + d*x)**2/4)/sqrt(a), True))/d**2, Ne(d,
 0)), (x**2/(2*sqrt(a + b*sqrt(c))), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.71

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\left(15\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}-63\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}a-35\left(b^2c-3a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}+105\left(ab^2c-a^3\right)\sqrt{\sqrt{d}}\right.}{105\,b^4d^2}$$

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/105*(15*(sqrt(d*x + c)*b + a)^(7/2) - 63*(sqrt(d*x + c)*b + a)^(5/2)*a -
35*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(3/2) + 105*(a*b^2*c - a^3)*sqrt(s
qrt(d*x + c)*b + a))/(b^4*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.88

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{4\left(35\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}b^2c-105\sqrt{\sqrt{dx+c}b+a}ab^2c-15\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}+63\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}a-1\right.}{105\,b^4d^2}$$

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/105*(35*(sqrt(d*x + c)*b + a)^(3/2)*b^2*c - 105*sqrt(sqrt(d*x + c)*b + a
)*a*b^2*c - 15*(sqrt(d*x + c)*b + a)^(7/2) + 63*(sqrt(d*x + c)*b + a)^(5/2)
*a - 105*(sqrt(d*x + c)*b + a)^(3/2)*a^2 + 105*sqrt(sqrt(d*x + c)*b + a)*a^
3)/(b^4*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{x}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

```
[In] int(x/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.649 $\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx=-\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

```
[Out] 4/3*(a+b*(d*x+c)^(1/2))^(3/2)/b^2/d-4*a*(a+b*(d*x+c)^(1/2))^(1/2)/b^2/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {253, 196, 45}

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx=\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}-\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}$$

```
[In] Int[1/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*Sqrt[a + b*Sqrt[c + d*x]])/(b^2*d) + (4*(a + b*Sqrt[c + d*x])^(3/2))/
(3*b^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d}\\
&= \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d}\\
&= \frac{2\text{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)\,dx, x, \sqrt{c+dx}\right)}{d}\\
&= -\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx=\frac{4\left(-2a+b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{3b^2d}$$

```
[In] Integrate[1/Sqrt[a + b*Sqrt[c + d*x]],x]
[Out] (4*(-2*a + b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(3*b^2*d)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}-4a\sqrt{a+b\sqrt{dx+c}}}{b^2d}$	41
default	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}-4a\sqrt{a+b\sqrt{dx+c}}}{b^2d}$	41

```
[In] int(1/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 4/d/b^2*(1/3*(a+b*(d*x+c)^(1/2))^(3/2)-a*(a+b*(d*x+c)^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\,\sqrt{\sqrt{dx+c}b+a}\left(\sqrt{dx+c}b-2\,a\right)}{3\,b^2d}$$

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 4/3*sqrt(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b - 2*a)/(b^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-a\sqrt{a+b\sqrt{c+dx}}+\frac{(a+b\sqrt{c+dx})^{\frac{3}{2}}}{3}\right)}{b^2} & \text{for } b\neq 0 \\ \frac{c+dx}{2\sqrt{a}} & \text{otherwise} \end{cases}\right)}{d} & \text{for } d\neq 0 \\ \frac{x}{\sqrt{a+b\sqrt{c}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+b*(d*x+c)**(1/2))**(1/2),x)
[Out] Piecewise((2*Piecewise((2*(-a*sqrt(a + b*sqrt(c + d*x)) + (a + b*sqrt(c + d
*x))**(3/2)/3)/b**2, Ne(b, 0)), ((c + d*x)/(2*sqrt(a)), True))/d, Ne(d, 0))
, (x/sqrt(a + b*sqrt(c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\left(\frac{\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}}{b^2}-\frac{3\,\sqrt{\sqrt{dx+c}b+a}a}{b^2}\right)}{3\,d}$$

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
[Out] 4/3*((sqrt(d*x + c)*b + a)^(3/2)/b^2 - 3*sqrt(sqrt(d*x + c)*b + a)*a/b^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\left(\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}-3\sqrt{\sqrt{dx+c}b+a}a\right)}{3b^2d}$$

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
[Out] 4/3*((sqrt(d*x + c)*b + a)^(3/2) - 3*sqrt(sqrt(d*x + c)*b + a)*a)/(b^2*d)
```

Mupad [B] (verification not implemented)

Time = 17.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\left(a+b\sqrt{c+d\,x}\right)^{3/2}}{3\,b^2\,d}-\frac{4\,a\,\sqrt{a+b\sqrt{c+d\,x}}}{b^2\,d}$$

```
[In] int(1/(a + b*(c + d*x)^(1/2))^(1/2),x)
[Out] (4*(a + b*(c + d*x)^(1/2))^(3/2))/(3*b^2*d) - (4*a*(a + b*(c + d*x)^(1/2))^
(1/2))/(b^2*d)
```

3.650 $\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

```
[Out] -2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/(a-b*c^(1/2))^(1/
2)-2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1/2))/(a+b*c^(1/2))^(
1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {378, 1412, 841, 1180, 213}

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

```
[In] Int[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/Sqrt[a - b*Sqrt
[c]] - (2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/Sqrt[a +
b*Sqrt[c]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}(-c+x)}\,dx, x, c+dx\right)\\
&= 2\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\,(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)\\
&= 4\text{Subst}\left(\int \frac{-a+x^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)\\
&= 2\text{Subst}\left(\int \frac{1}{-a-b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)\\
&\quad + 2\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)
\end{aligned}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\sqrt{-a-b\sqrt{c}}} + \frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\sqrt{-a+b\sqrt{c}}}$$

```
[In] Integrate[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a - b*Sqrt[c]]])/Sqrt[-a - b*Sqrt
[c]] + (2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]])/Sqrt[-a +
 b*Sqrt[c]]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{\sqrt{-\sqrt{b^2c}-a}}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\sqrt{\sqrt{b^2c}-a}}$	92
default	$\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{\sqrt{-\sqrt{b^2c}-a}}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\sqrt{\sqrt{b^2c}-a}}$	92

```
[In] int(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)
-a)^(1/2))+2/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2
*c)^(1/2)-a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(73) = 146$.

Time = 0.33 (sec) , antiderivative size = 743, normalized size of antiderivative = 7.66

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$= \sqrt{-\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2a^2b^2c+a^4}}+a}{b^2c-a^2}}\log\left(4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2a^2b^2c+a^4}}-a\right)\sqrt{-\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2a^2}}}{b^2c-a^2}}\right.$$
$$\left.+4\sqrt{\sqrt{dx+c}b+a}\right)$$
$$-\sqrt{-\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2a^2b^2c+a^4}}+a}{b^2c-a^2}}\log\left(-4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2a^2b^2c+a^4}}-a\right)\sqrt{-\frac{(b^2c-a^2)\sqrt{\frac{}{b^4c^2}}}{b^2c-}}\right.$$
$$\left.+4\sqrt{\sqrt{dx+c}b+a}\right)$$
$$-\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2a^2b^2c+a^4}}-a}{b^2c-a^2}}\log\left(4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2a^2b^2c+a^4}}+a\right)\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2a^2b^2}}}{b^2c-a^2}}\right.$$
$$\left.+4\sqrt{\sqrt{dx+c}b+a}\right)$$
$$+\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2a^2b^2c+a^4}}-a}{b^2c-a^2}}\log\left(-4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2a^2b^2c+a^4}}+a\right)\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{b^2}{b^4c^2-2a^2}}}{b^2c-a^2}}\right.$$
$$\left.+4\sqrt{\sqrt{dx+c}b+a}\right)$$

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c
- a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)
```

```
*sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c
 - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) - sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(
b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2))*log(-4*((b^2*c - a^2)*sqr
t(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)*sqrt(-((b^2*c - a^2)*sqrt(b^2*c
/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*
b + a)) - sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a
)/(b^2*c - a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a
^4)) + a)*sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a
)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) + sqrt(((b^2*c - a^2)*sqrt(
b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2))*log(-4*((b^2*c - a
^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)*sqrt(((b^2*c - a^2)*sqrt
(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x
 + c)*b + a))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx=\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$$

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2))**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*sqrt(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx=\int \frac{1}{\sqrt{\sqrt{dx+c}b+a}\,x}\,dx$$

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.44

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx=\frac{2\left(\frac{\left(b^2\sqrt{c}|b|+ab^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(b\sqrt{c}+a\right)\sqrt{b\sqrt{c}-a}}+\frac{\left(b^2\sqrt{c}|b|-ab^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\left(b\sqrt{c}-a\right)\sqrt{-b\sqrt{c}-a}}\right)}{b^2}$$

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2*((b^2*sqrt(c)*abs(b) + a*b^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a +
sqrt(b^2*c)))/((b*sqrt(c) + a)*sqrt(b*sqrt(c) - a)) + (b^2*sqrt(c)*abs(b) -
 a*b^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*sqrt(c
) - a)*sqrt(-b*sqrt(c) - a)))/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{x\,\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

```
[In] int(1/(x*(a + b*(c + d*x)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

3.651 $\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 163

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x} - \frac{bd\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\left(a-b\sqrt{c}\right)^{3/2}\sqrt{c}} + \frac{bd\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\left(a+b\sqrt{c}\right)^{3/2}\sqrt{c}}$$

```
[Out] -1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/c^(1/2)/(a-
b*c^(1/2))^(3/2)+1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1
/2))/c^(1/2)/(a+b*c^(1/2))^(3/2)-(a-b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))^(1
/2)/(-b^2*c+a^2)/x
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {378, 1412, 837, 841, 1180, 213}

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{\sqrt{a+b\sqrt{c+dx}}\left(a-b\sqrt{c+dx}\right)}{x\,(a^2-b^2c)} - \frac{bd\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\left(a-b\sqrt{c}\right)^{3/2}} + \frac{bd\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\left(a+b\sqrt{c}\right)^{3/2}}$$

```
[In] Int[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -(((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c)*x)) - (b
*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(2*(a - b*Sqrt[c
```

```
])^(3/2)*Sqrt[c]) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[
c]]])/(2*(a + b*Sqrt[c])^(3/2)*Sqrt[c])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
```

```
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}(-c+x)^2}\,dx, x, c+dx\right)\\
&= (2d)\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\,(-c+x^2)^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x}+\frac{d\text{Subst}\left(\int \frac{-\frac{1}{2}abc+\frac{1}{2}b^2cx}{\sqrt{a+bx}(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)}{c\,(a^2-b^2c)}\\
&= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x}+\frac{(2d)\text{Subst}\left(\int \frac{-ab^2c+\frac{1}{2}b^2cx^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{c\,(a^2-b^2c)}\\
&= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x}\\
&\quad+\frac{(bd)\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{2\left(a-b\sqrt{c}\right)\sqrt{c}}\\
&\quad-\frac{(bd)\text{Subst}\left(\int \frac{1}{-a-b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{2\left(a+b\sqrt{c}\right)\sqrt{c}}\\
&= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\left(a-b\sqrt{c}\right)^{3/2}\sqrt{c}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\left(a+b\sqrt{c}\right)^{3/2}\sqrt{c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.04

$$
\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{1}{2}\left(-\frac{2\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x}+\frac{bd\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\left(-a-b\sqrt{c}\right)^{3/2}\sqrt{c}}-\frac{bd\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\left(-a+b\sqrt{c}\right)^{3/2}\sqrt{c}}\right)
$$

```
[In] Integrate[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
[Out] ((-2*(a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c)*x) + (
b*d*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a - b*Sqrt[c]]])/((-a - b*Sqrt[c
])^(3/2)*Sqrt[c]) - (b*d*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[
c]]])/((-a + b*Sqrt[c])^(3/2)*Sqrt[c]))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(125) = 250$.

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.59

method	result
derivativedivides	$4d\,b^2\left(-\frac{\sqrt{b^2c}\left(\frac{2\sqrt{a+b\sqrt{dx+c}}}{\left(4\sqrt{b^2c}-4a\right)\left(b\sqrt{dx+c}+\sqrt{b^2c}\right)}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\left(4\sqrt{b^2c}-4a\right)\sqrt{\sqrt{b^2c}-a}}\right)}{4b^2c}+\frac{\sqrt{b^2c}\left(-\frac{2\sqrt{a+b\sqrt{dx+c}}}{\left(-4\sqrt{b^2c}-4a\right)\left(-b\sqrt{dx+c}+\sqrt{b^2c}\right)}\right.}{4b^2c}\right.$
default	$4d\,b^2\left(-\frac{\sqrt{b^2c}\left(\frac{2\sqrt{a+b\sqrt{dx+c}}}{\left(4\sqrt{b^2c}-4a\right)\left(b\sqrt{dx+c}+\sqrt{b^2c}\right)}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\left(4\sqrt{b^2c}-4a\right)\sqrt{\sqrt{b^2c}-a}}\right)}{4b^2c}+\frac{\sqrt{b^2c}\left(-\frac{2\sqrt{a+b\sqrt{dx+c}}}{\left(-4\sqrt{b^2c}-4a\right)\left(-b\sqrt{dx+c}+\sqrt{b^2c}\right)}\right.}{4b^2c}\right.$

```
[In] int(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4*d*b^2*(-1/4*(b^2*c)^(1/2)/b^2/c*(2*(a+b*(d*x+c)^(1/2))^(1/2)/(4*(b^2*c)^(
1/2)-4*a)/(b*(d*x+c)^(1/2)+(b^2*c)^(1/2))+2/(4*(b^2*c)^(1/2)-4*a)/((b^2*c)^
(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2)))+1
/4*(b^2*c)^(1/2)/b^2/c*(-2*(a+b*(d*x+c)^(1/2))^(1/2)/(-4*(b^2*c)^(1/2)-4*a)
/(-b*(d*x+c)^(1/2)+(b^2*c)^(1/2))+2/(-4*(b^2*c)^(1/2)-4*a)/(-(b^2*c)^(1/2)-
a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2493 vs. $2(127) = 254$.

Time = 0.41 (sec) , antiderivative size = 2493, normalized size of antiderivative = 15.29

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^
4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^
4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4
*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2
- a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 + (2*(a*b^6
*c^2 + 3*a^3*b^4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)
```

*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - (b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 - (2*(a*b^6*c^2 + 3*a^3*b^4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) + (b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 + (2*(a*b^6*c^2 + 3*a^3*b^4*c)*d^2 + (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - (b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 - (2*(a*b^6*c^2 + 3*a^3*b^4*c)*d^2 + (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - 4*sqrt(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b - a))/((b^2*c - a^2)*x)

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$$

[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)

[Out] Integral(1/(x**2*sqrt(a + b*sqrt(c + d*x))), x)

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{\sqrt{\sqrt{dx+c}b+ax^2}}\,dx$$

[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^2), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. 2(127) = 254.

Time = 0.62 (sec) , antiderivative size = 644, normalized size of antiderivative = 3.95

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$$

$$= \frac{\frac{\left((b^3c-a^2b)^2b^4cd^2-2\left(ab^6c^{\frac{3}{2}}-a^3b^4\sqrt{c}\right)d^2|-b^3c+a^2b|+(a^2b^8c^2-2a^4b^6c+a^6b^4)d^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-\frac{ab^2c-a^3+\sqrt{\left(ab^2c-a^3\right)^2+\left(b^4c^2-2a^2b^2c+a^4\right)\left(b^2c-\right.}}{b^2c-a^2}}}\right.}{\left(b^5c^3+ab^4c^{\frac{5}{2}}-2a^2b^3c^2-2a^3b^2c^{\frac{3}{2}}+a^4bc+a^5\sqrt{c}\right)\sqrt{b\sqrt{c}-a}|-b^3c+a^2b|}}{}$$

[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")

[Out] 1/2*(((b^3*c - a^2*b)^2*b^4*c*d^2 - 2*(a*b^6*c^(3/2) - a^3*b^4*sqrt(c))*d^2 *abs(-b^3*c + a^2*b) + (a^2*b^8*c^2 - 2*a^4*b^6*c + a^6*b^4)*d^2)*arctan(sq rt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c - a^3 + sqrt((a*b^2*c - a^3)^2 + (b^ 4*c^2 - 2*a^2*b^2*c + a^4)*(b^2*c - a^2)))/(b^2*c - a^2)))/((b^5*c^3 + a*b^ 4*c^(5/2) - 2*a^2*b^3*c^2 - 2*a^3*b^2*c^(3/2) + a^4*b*c + a^5*sqrt(c))*sqrt (b*sqrt(c) - a)*abs(-b^3*c + a^2*b)) + ((b^3*c - a^2*b)^2*b^4*c*d^2 + 2*(a* b^6*c^(3/2) - a^3*b^4*sqrt(c))*d^2*abs(-b^3*c + a^2*b) + (a^2*b^8*c^2 - 2*a ^4*b^6*c + a^6*b^4)*d^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c - a^3 - sqrt((a*b^2*c - a^3)^2 + (b^4*c^2 - 2*a^2*b^2*c + a^4)*(b^2*c - a^2))

```
)/(b^2*c - a^2)))/((b^5*c^3 - a*b^4*c^(5/2) - 2*a^2*b^3*c^2 + 2*a^3*b^2*c^(
3/2) + a^4*b*c - a^5*sqrt(c))*sqrt(-b*sqrt(c) - a)*abs(-b^3*c + a^2*b)) + 2
*((sqrt(d*x + c)*b + a)^(3/2)*b^4*d^2 - 2*sqrt(sqrt(d*x + c)*b + a)*a*b^4*d
^2)/((b^2*c - (sqrt(d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a)*a - a^2)*(b
^2*c - a^2)))/(b^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{x^2\,\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

```
[In] int(1/(x^2*(a + b*(c + d*x)^(1/2))^(1/2)),x)
[Out] int(1/(x^2*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

3.652 $\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 261

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8c\left(a^2-b^2c\right)^2x} + \frac{b\left(2a-5b\sqrt{c}\right)d^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16\left(a-b\sqrt{c}\right)^{5/2}c^{3/2}} - \frac{b\left(2a+5b\sqrt{c}\right)d^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16\left(a+b\sqrt{c}\right)^{5/2}c^{3/2}}$$

```
[Out] 1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))*(2*a-5*b*
c^(1/2))/c^(3/2)/(a-b*c^(1/2))^(5/2)-1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/2))
^(1/2)/(a+b*c^(1/2))^(1/2))*(2*a+5*b*c^(1/2))/c^(3/2)/(a+b*c^(1/2))^(5/2)-1
/2*(a-b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))^(1/2)/(-b^2*c+a^2)/x^2-1/8*b*d*(
6*a*b*c-(5*b^2*c+a^2)*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))^(1/2)/c/(-b^2*c+a^
2)^2/x
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {378, 1412, 837, 841, 1180, 213}

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2x^2\left(a^2-b^2c\right)} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8cx\left(a^2-b^2c\right)^2} + \frac{bd^2\left(2a-5b\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{5/2}} - \frac{bd^2\left(2a+5b\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}\left(a+b\sqrt{c}\right)^{5/2}}$$

```
[In] Int[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -1/2*((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c)*x^2)
- (b*d*Sqrt[a + b*Sqrt[c + d*x]]*(6*a*b*c - (a^2 + 5*b^2*c)*Sqrt[c + d*x]))
/(8*c*(a^2 - b^2*c)^2*x) + (b*(2*a - 5*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sq
rt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(16*(a - b*Sqrt[c])^(5/2)*c^(3/2)) - (b*
(2*a + 5*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c
]]])/(16*(a + b*Sqrt[c])^(5/2)*c^(3/2))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
```

```
a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^2\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}(-c+x)^3}\,dx, x, c+dx\right)\\
&= \left(2d^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2}+\frac{d^2\text{Subst}\left(\int \frac{-\frac{1}{2}abc+\frac{5}{2}b^2cx}{\sqrt{a+bx}(-c+x^2)^2}\,dx, x, \sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)}\\
&= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2}\\
&\quad-\frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8c\left(a^2-b^2c\right)^2x}\\
&\quad+\frac{d^2\text{Subst}\left(\int \frac{\frac{1}{2}abc(a^2-4b^2c)+\frac{1}{4}b^2c(a^2+5b^2c)x}{\sqrt{a+bx}(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)}{4c^2\left(a^2-b^2c\right)^2}
\end{aligned}
$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8c\left(a^2-b^2c\right)^2 x} + \frac{d^2\text{Subst}\left(\int \frac{\frac{1}{2}ab^2c(a^2-4b^2c)-\frac{1}{4}ab^2c(a^2+5b^2c)+\frac{1}{4}b^2c(a^2+5b^2c)x^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{2c^2\left(a^2-b^2c\right)^2}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8c\left(a^2-b^2c\right)^2 x} - \frac{\left(b\left(2a-5b\sqrt{c}\right)d^2\right)\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{16\left(a-b\sqrt{c}\right)^2 c^{3/2}} + \frac{\left(b\left(2a+5b\sqrt{c}\right)d^2\right)\text{Subst}\left(\int \frac{1}{-a-b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{16\left(a+b\sqrt{c}\right)^2 c^{3/2}}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8c\left(a^2-b^2c\right)^2 x} + \frac{b\left(2a-5b\sqrt{c}\right)d^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16\left(a-b\sqrt{c}\right)^{5/2}c^{3/2}} - \frac{b\left(2a+5b\sqrt{c}\right)d^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16\left(a+b\sqrt{c}\right)^{5/2}c^{3/2}}$$

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$$

$$= \frac{-\frac{2\sqrt{c}\sqrt{a+b\sqrt{c+dx}}\left(4a^3c+b^3c(4c-5dx)\sqrt{c+dx}-a^2b\sqrt{c+dx}(4c+dx)+2ab^2c(-2c+3dx)\right)}{\left(a^2-b^2c\right)^2x^2} + \frac{b\left(2a+5b\sqrt{c}\right)d^2\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\left(-a-b\sqrt{c}\right)^{5/2}} + \frac{b(-2a+}{}}{16c^{3/2}}$$

```
[In] Integrate[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] ((-2*Sqrt[c]*Sqrt[a + b*Sqrt[c + d*x]]*(4*a^3*c + b^3*c*(4*c - 5*d*x)*Sqrt[
c + d*x] - a^2*b*Sqrt[c + d*x]*(4*c + d*x) + 2*a*b^2*c*(-2*c + 3*d*x)))/((a
^2 - b^2*c)^2*x^2) + (b*(2*a + 5*b*Sqrt[c])*d^2*ArcTan[Sqrt[a + b*Sqrt[c +
d*x]]/Sqrt[-a - b*Sqrt[c]]])/(-a - b*Sqrt[c])^(5/2) + (b*(-2*a + 5*b*Sqrt[c
])*d^2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]])/(-a + b*Sqrt
[c])^(5/2))/(16*c^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(209) = 418$.

Time = 0.42 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.64

method	result
derivativedivides	$-4d^2b^4\left(\frac{\frac{-\frac{\left(5\sqrt{b^2c}+2a\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{4\left(b^2c+2a\sqrt{b^2c}+a^2\right)}+\frac{\left(7\sqrt{b^2c}+2a\right)\sqrt{a+b\sqrt{dx+c}}}{4\sqrt{b^2c}+4a}}{\left(-b\sqrt{dx+c}+\sqrt{b^2c}\right)^2}-\frac{\left(5\sqrt{b^2c}+2a\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{4\left(b^2c+2a\sqrt{b^2c}+a^2\right)\sqrt{-\sqrt{b^2c}-a}}}{16b^2c\sqrt{b^2c}}-\frac{-\frac{\left(-5\sqrt{b^2c}+\right.}{4\left(b^2c-\right.}}{}\right.$
default	$-4d^2b^4\left(\frac{\frac{-\frac{\left(5\sqrt{b^2c}+2a\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{4\left(b^2c+2a\sqrt{b^2c}+a^2\right)}+\frac{\left(7\sqrt{b^2c}+2a\right)\sqrt{a+b\sqrt{dx+c}}}{4\sqrt{b^2c}+4a}}{\left(-b\sqrt{dx+c}+\sqrt{b^2c}\right)^2}-\frac{\left(5\sqrt{b^2c}+2a\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{4\left(b^2c+2a\sqrt{b^2c}+a^2\right)\sqrt{-\sqrt{b^2c}-a}}}{16b^2c\sqrt{b^2c}}-\frac{-\frac{\left(-5\sqrt{b^2c}+\right.}{4\left(b^2c-\right.}}{}\right.$

```
[In] int(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*d^2*b^4*(1/16/b^2/c/(b^2*c)^(1/2)*((-1/4*(5*(b^2*c)^(1/2)+2*a)/(b^2*c+2*
a*(b^2*c)^(1/2)+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)+1/4*(7*(b^2*c)^(1/2)+2*a)/((
b^2*c)^(1/2)+a)*(a+b*(d*x+c)^(1/2))^(1/2))/(-b*(d*x+c)^(1/2)+(b^2*c)^(1/2))
^2-1/4*(5*(b^2*c)^(1/2)+2*a)/(b^2*c+2*a*(b^2*c)^(1/2)+a^2)/(-(b^2*c)^(1/2)-
a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)))-1/16/b
^2/c/(b^2*c)^(1/2)*((-1/4*(-5*(b^2*c)^(1/2)+2*a)/(b^2*c-2*a*(b^2*c)^(1/2)+a
^2)*(a+b*(d*x+c)^(1/2))^(3/2)+1/4*(-7*(b^2*c)^(1/2)+2*a)/(-(b^2*c)^(1/2)+a)
*(a+b*(d*x+c)^(1/2))^(1/2))/(-b*(d*x+c)^(1/2)-(b^2*c)^(1/2))^2-1/4*(5*(b^2*
c)^(1/2)-2*a)/(-b^2*c+2*a*(b^2*c)^(1/2)-a^2)/((b^2*c)^(1/2)-a)^(1/2)*arctan
((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4390 vs. $2(212) = 424$.

Time = 1.22 (sec) , antiderivative size = 4390, normalized size of antiderivative = 16.82

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3
*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a
^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4
```

+ 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10
)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10
+ 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^
6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c
^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*log((625*
b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x
+ c)*b + a)*d^6 + ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 +
35*a^7*b^6*c^2)*d^4 - (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*
a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c
^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6
*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^
11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^
8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*
sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b
^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 -
a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10
780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*
b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*
a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*
c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*
b^2*c^4 - a^10*c^3))) - (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a
*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a
^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sq
rt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*
c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 1
20*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7
- 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*
c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^1
0*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^
6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 6
09*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 - (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a
^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b
^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^1
4*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12
+ 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*
c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c
^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*
a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5
+ 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*
a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^
18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^1
0*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^1
8*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6
*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*
x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4

- (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c
^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2
- 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*
a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 +
210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a
^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*
a^8*b^2*c^4 - a^10*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b
^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 + ((325*a*b^12*c^5 + 1977
*a^3*b^10*c^4 - 609*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 + (5*b^14*c^10 - 16*a
^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^
4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^
3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 -
10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9
- 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^
5 - 10*a^18*b^2*c^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 -
35*a^5*b^4*c + 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6
- 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*
b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20
*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b
^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16
*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4
*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) - (b^4*c^3 - 2*a^2*
b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c
+ 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*
c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21
966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^
2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252
*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10
*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10
*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10
*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((325*
a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 + (5
*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6
*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 +
7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)
*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 +
210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6
+ 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 +
70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7
+ 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^1
8*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^
8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14
*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*
b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2
*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) +

```
4*(6*a*b^2*c*d*x - 4*a*b^2*c^2 + 4*a^3*c + (4*b^3*c^2 - 4*a^2*b*c - (5*b^3*
c + a^2*b)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a))/((b^4*c^3 - 2*a^2
*b^2*c^2 + a^4*c)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{\sqrt{\sqrt{dx+c}b+a}x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1303 vs. $2(212) = 424$.

Time = 0.44 (sec) , antiderivative size = 1303, normalized size of antiderivative = 4.99

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
[Out] 1/16*(((b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)^2*(5*b^6*c + a^2*b^4)*d^3 - (13*
a*b^10*c^(7/2) - 27*a^3*b^8*c^(5/2) + 15*a^5*b^6*c^(3/2) - a^7*b^4*sqrt(c))
*d^3*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c) + 2*(4*a^2*b^14*c^6 - 17*a^4*b^
12*c^5 + 28*a^6*b^10*c^4 - 22*a^8*b^8*c^3 + 8*a^10*b^6*c^2 - a^12*b^4*c)*d^
3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*
c + sqrt((a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c)^2 + (b^6*c^4 - 3*a^2*b^4*c^3 +
 3*a^4*b^2*c^2 - a^6*c)*(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/(b^4*c^3 - 2*a^
2*b^2*c^2 + a^4*c)))/((b^9*c^6 - a*b^8*c^(11/2) - 4*a^2*b^7*c^5 + 4*a^3*b^6
*c^(9/2) + 6*a^4*b^5*c^4 - 6*a^5*b^4*c^(7/2) - 4*a^6*b^3*c^3 + 4*a^7*b^2*c^
(5/2) + a^8*b*c^2 - a^9*c^(3/2))*sqrt(-b*sqrt(c) - a)*abs(b^5*c^3 - 2*a^2*b
```

```
^3*c^2 + a^4*b*c)) + ((b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)^2*(5*b^6*c + a^2*
b^4)*d^3 + (13*a*b^10*c^(7/2) - 27*a^3*b^8*c^(5/2) + 15*a^5*b^6*c^(3/2) - a
^7*b^4*sqrt(c))*d^3*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c) + 2*(4*a^2*b^14*
c^6 - 17*a^4*b^12*c^5 + 28*a^6*b^10*c^4 - 22*a^8*b^8*c^3 + 8*a^10*b^6*c^2 -
 a^12*b^4*c)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^4*c^3 - 2*a^3
*b^2*c^2 + a^5*c - sqrt((a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c)^2 + (b^6*c^4 -
3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/
(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/((b^9*c^6 + a*b^8*c^(11/2) - 4*a^2*b^7*
c^5 - 4*a^3*b^6*c^(9/2) + 6*a^4*b^5*c^4 + 6*a^5*b^4*c^(7/2) - 4*a^6*b^3*c^3
 - 4*a^7*b^2*c^(5/2) + a^8*b*c^2 + a^9*c^(3/2))*sqrt(b*sqrt(c) - a)*abs(b^5
*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)) - 2*(9*(sqrt(d*x + c)*b + a)^(3/2)*b^8*c^2
*d^3 - 19*sqrt(sqrt(d*x + c)*b + a)*a*b^8*c^2*d^3 - 5*(sqrt(d*x + c)*b + a)
^(7/2)*b^6*c*d^3 + 21*(sqrt(d*x + c)*b + a)^(5/2)*a*b^6*c*d^3 - 30*(sqrt(d*
x + c)*b + a)^(3/2)*a^2*b^6*c*d^3 + 18*sqrt(sqrt(d*x + c)*b + a)*a^3*b^6*c*
d^3 - (sqrt(d*x + c)*b + a)^(7/2)*a^2*b^4*d^3 + 3*(sqrt(d*x + c)*b + a)^(5/
2)*a^3*b^4*d^3 - 3*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^4*d^3 + sqrt(sqrt(d*x
+ c)*b + a)*a^5*b^4*d^3)/((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(b^2*c - (sqrt(
d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a)*a - a^2)^2))/(b^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{x^3\,\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

```
[In] int(1/(x^3*(a + b*(c + d*x)^(1/2))^(1/2)),x)
[Out] int(1/(x^3*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

3.653 $\int x^3 \left(a+b\sqrt{c+dx}\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 350

$$\begin{aligned}\int x^3 \left(a+b\sqrt{c+dx}\right)^p \, dx =& -\frac{2a(a^2-b^2c)^3 \left(a+b\sqrt{c+dx}\right)^{1+p}}{b^8d^4(1+p)} \\ &+ \frac{2(a^2-b^2c)^2 (7a^2-b^2c) \left(a+b\sqrt{c+dx}\right)^{2+p}}{b^8d^4(2+p)} \\ &- \frac{6a(7a^2-3b^2c)(a^2-b^2c) \left(a+b\sqrt{c+dx}\right)^{3+p}}{b^8d^4(3+p)} \\ &+ \frac{2(35a^4-30a^2b^2c+3b^4c^2) \left(a+b\sqrt{c+dx}\right)^{4+p}}{b^8d^4(4+p)} \\ &- \frac{10a(7a^2-3b^2c) \left(a+b\sqrt{c+dx}\right)^{5+p}}{b^8d^4(5+p)} \\ &+ \frac{6(7a^2-b^2c) \left(a+b\sqrt{c+dx}\right)^{6+p}}{b^8d^4(6+p)} \\ &- \frac{14a\left(a+b\sqrt{c+dx}\right)^{7+p}}{b^8d^4(7+p)} + \frac{2\left(a+b\sqrt{c+dx}\right)^{8+p}}{b^8d^4(8+p)}\end{aligned}$$

```
[Out] -2*a*(-b^2*c+a^2)^3*(a+b*(d*x+c)^(1/2))^(p+1)/b^8/d^4/(p+1)+2*(-b^2*c+a^2)^
2*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(2+p)/b^8/d^4/(2+p)-6*a*(-3*b^2*c+7*a^
2)*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3+p)/b^8/d^4/(3+p)+2*(3*b^4*c^2-30*a^2
*b^2*c+35*a^4)*(a+b*(d*x+c)^(1/2))^(4+p)/b^8/d^4/(4+p)-10*a*(-3*b^2*c+7*a^2
)*(a+b*(d*x+c)^(1/2))^(5+p)/b^8/d^4/(5+p)+6*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/
2))^(6+p)/b^8/d^4/(6+p)-14*a*(a+b*(d*x+c)^(1/2))^(7+p)/b^8/d^4/(7+p)+2*(a+b
*(d*x+c)^(1/2))^(8+p)/b^8/d^4/(8+p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {378, 1412, 786}

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx=-\frac{2a(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^8d^4(p+1)}+\frac{2(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^8d^4(p+2)}-\frac{6a(7a^2-3b^2c)(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^8d^4(p+3)}-\frac{10a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{p+5}}{b^8d^4(p+5)}+\frac{6(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+6}}{b^8d^4(p+6)}+\frac{2(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^8d^4(p+4)}-\frac{14a\left(a+b\sqrt{c+dx}\right)^{p+7}}{b^8d^4(p+7)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+8}}{b^8d^4(p+8)}$$

```
[In] Int[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(1 + p))/(b^8*d^4*(1 + p)) + (2
*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^8*d^4*(2
 + p)) - (6*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p)
)/(b^8*d^4*(3 + p)) + (2*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c
+ d*x])^(4 + p))/(b^8*d^4*(4 + p)) - (10*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c
+ d*x])^(5 + p))/(b^8*d^4*(5 + p)) + (6*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x
])^(6 + p))/(b^8*d^4*(6 + p)) - (14*a*(a + b*Sqrt[c + d*x])^(7 + p))/(b^8*d
^4*(7 + p)) + (2*(a + b*Sqrt[c + d*x])^(8 + p))/(b^8*d^4*(8 + p))
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
```

```
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^p (-c+x)^3 \, dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int x(a+bx)^p \left(-c+x^2\right)^3 \, dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{a(a^2-b^2c)^3(a+bx)^p}{b^7} - \frac{(-7a^2+b^2c)\left(-a^2+b^2c\right)^2(a+bx)^{1+p}}{b^7} - \frac{3(7a^5-10a^3b^2c+3ab^4c^2)(a+bx)^{2+p}}{b^7} + \frac{(35a^4-}{}\right.\right.}{}$$

$$= -\frac{2a(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^8d^4(1+p)} + \frac{2(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^8d^4(2+p)}$$
$$- \frac{6a(7a^2-3b^2c)(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3+p}}{b^8d^4(3+p)}$$
$$+ \frac{2(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{4+p}}{b^8d^4(4+p)}$$
$$- \frac{10a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{5+p}}{b^8d^4(5+p)} + \frac{6(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{6+p}}{b^8d^4(6+p)}$$
$$- \frac{14a\left(a+b\sqrt{c+dx}\right)^{7+p}}{b^8d^4(7+p)} + \frac{2\left(a+b\sqrt{c+dx}\right)^{8+p}}{b^8d^4(8+p)}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.59

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p dx =$$
$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(5040a^7-5040a^6b(1+p)\sqrt{c+dx}+360a^5b^2(6c(-7+p+p^2)+7d(2+3p+p^2)x\right.}{}$$

```
[In] Integrate[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(5040*a^7 - 5040*a^6*b*(1 + p)*Sqrt[c + d
*x] + 360*a^5*b^2*(6*c*(-7 + p + p^2) + 7*d*(2 + 3*p + p^2)*x) - 120*a^4*b^
3*(1 + p)*Sqrt[c + d*x]*(2*c*(-63 - 5*p + 2*p^2) + 7*d*(6 + 5*p + p^2)*x) +
 6*a^3*b^4*(8*c^2*(315 - 124*p - 139*p^2 - 14*p^3 + p^4) + 40*c*d*(-42 - 61
*p - 16*p^2 + 4*p^3 + p^4)*x + 35*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p^4)*x
^2) - 6*a^2*b^5*(1 + p)*Sqrt[c + d*x]*(-24*c^2*(-105 - 24*p + 5*p^2 + p^3)
+ 4*c*d*(-420 - 386*p - 94*p^2 - p^3 + p^4)*x + 7*d^2*(120 + 154*p + 71*p^2
 + 14*p^3 + p^4)*x^2) - b^7*(105 + 176*p + 86*p^2 + 16*p^3 + p^4)*Sqrt[c +
d*x]*(-48*c^3 + 24*c^2*d*(2 + p)*x - 6*c*d^2*(8 + 6*p + p^2)*x^2 + d^3*(48
+ 44*p + 12*p^2 + p^3)*x^3) + a*b^6*(48*c^3*(-105 + 103*p + 138*p^2 + 38*p^
3 + 3*p^4) - 24*c^2*d*(-210 - 283*p - 21*p^2 + 74*p^3 + 24*p^4 + 2*p^5)*x +
 6*c*d^2*(-840 - 1726*p - 1151*p^2 - 265*p^3 + 10*p^4 + 11*p^5 + p^6)*x^2 +
 7*d^3*(720 + 1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6)*x^3)))/
(b^8*d^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p)*(7 + p)*(8 + p))
```

Maple [F]

$$\int x^3 \left(a + b\sqrt{dx + c}\right)^p dx$$

```
[In] int(x^3*(a+b*(d*x+c)^(1/2))^p,x)
```

```
[Out] int(x^3*(a+b*(d*x+c)^(1/2))^p,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(335) = 670$.

Time = 0.41 (sec) , antiderivative size = 1416, normalized size of antiderivative = 4.05

$$\int x^3 \left(a + b\sqrt{c + dx}\right)^p dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] -2*(5040*b^8*c^4 - 20160*a^2*b^6*c^3 + 30240*a^4*b^4*c^2 - 20160*a^6*b^2*c
+ 5040*a^8 + 48*(b^8*c^4 + 6*a^2*b^6*c^3 + a^4*b^4*c^2)*p^4 - (b^8*d^4*p^7
+ 28*b^8*d^4*p^6 + 322*b^8*d^4*p^5 + 1960*b^8*d^4*p^4 + 6769*b^8*d^4*p^3 +
13132*b^8*d^4*p^2 + 13068*b^8*d^4*p + 5040*b^8*d^4)*x^4 + 384*(2*b^8*c^4 +
7*a^2*b^6*c^3 - 3*a^4*b^4*c^2)*p^3 - (b^8*c*d^3*p^7 + (22*b^8*c - 7*a^2*b^6
)*d^3*p^6 + 5*(38*b^8*c - 21*a^2*b^6)*d^3*p^5 + 5*(164*b^8*c - 119*a^2*b^6)
*d^3*p^4 + (1849*b^8*c - 1575*a^2*b^6)*d^3*p^3 + 2*(1019*b^8*c - 959*a^2*b^
6)*d^3*p^2 + 840*(b^8*c - a^2*b^6)*d^3*p)*x^3 + 48*(86*b^8*c^4 + 81*a^2*b^6
*c^3 - 124*a^4*b^4*c^2 + 45*a^6*b^2*c)*p^2 + 6*(18*b^8*c^2*d^2*p^5 + (b^8*c
^2 + a^2*b^6*c)*d^2*p^6 + (118*b^8*c^2 - 95*a^2*b^6*c + 35*a^4*b^4)*d^2*p^4
 + 6*(58*b^8*c^2 - 80*a^2*b^6*c + 35*a^4*b^4)*d^2*p^3 + (457*b^8*c^2 - 806*
```

```
a^2*b^6*c + 385*a^4*b^4)*d^2*p^2 + 210*(b^8*c^2 - 2*a^2*b^6*c + a^4*b^4)*d^
2*p)*x^2 + 192*(44*b^8*c^4 - 71*a^2*b^6*c^3 + 54*a^4*b^4*c^2 - 15*a^6*b^2*c
)*p - 24*((b^8*c^3 + 3*a^2*b^6*c^2)*d*p^5 + 2*(8*b^8*c^3 + 9*a^2*b^6*c^2 -
5*a^4*b^4*c)*d*p^4 + (86*b^8*c^3 - 57*a^2*b^6*c^2 + 15*a^4*b^4*c)*d*p^3 + (
176*b^8*c^3 - 387*a^2*b^6*c^2 + 340*a^4*b^4*c - 105*a^6*b^2)*d*p^2 + 105*(b
^8*c^3 - 3*a^2*b^6*c^2 + 3*a^4*b^4*c - a^6*b^2)*d*p)*x + (192*(a*b^7*c^3 +
a^3*b^5*c^2)*p^4 + 96*(27*a*b^7*c^3 + 2*a^3*b^5*c^2 - 5*a^5*b^3*c)*p^3 - (a
*b^7*d^3*p^7 + 21*a*b^7*d^3*p^6 + 175*a*b^7*d^3*p^5 + 735*a*b^7*d^3*p^4 + 1
624*a*b^7*d^3*p^3 + 1764*a*b^7*d^3*p^2 + 720*a*b^7*d^3*p)*x^3 + 192*(56*a*b
^7*c^3 - 49*a^3*b^5*c^2 + 15*a^5*b^3*c)*p^2 + 6*(2*a*b^7*c*d^2*p^6 + (33*a*
b^7*c - 7*a^3*b^5)*d^2*p^5 + 10*(20*a*b^7*c - 7*a^3*b^5)*d^2*p^4 + 5*(111*a
*b^7*c - 49*a^3*b^5)*d^2*p^3 + 2*(349*a*b^7*c - 175*a^3*b^5)*d^2*p^2 + 24*(
13*a*b^7*c - 7*a^3*b^5)*d^2*p)*x^2 + 48*(279*a*b^7*c^3 - 511*a^3*b^5*c^2 +
385*a^5*b^3*c - 105*a^7*b)*p - 24*((3*a*b^7*c^2 + a^3*b^5*c)*d*p^5 + 2*(21*
a*b^7*c^2 - 5*a^3*b^5*c)*d*p^4 + (192*a*b^7*c^2 - 135*a^3*b^5*c + 35*a^5*b^
3)*d*p^3 + (327*a*b^7*c^2 - 320*a^3*b^5*c + 105*a^5*b^3)*d*p^2 + 2*(87*a*b^
7*c^2 - 98*a^3*b^5*c + 35*a^5*b^3)*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x + c)*b
+ a)^p/(b^8*d^4*p^8 + 36*b^8*d^4*p^7 + 546*b^8*d^4*p^6 + 4536*b^8*d^4*p^5 +
 22449*b^8*d^4*p^4 + 67284*b^8*d^4*p^3 + 118124*b^8*d^4*p^2 + 109584*b^8*d^
4*p + 40320*b^8*d^4)
```

Sympy [F]

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx=\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx$$

```
[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**p,x)
[Out] Integral(x**3*(a + b*sqrt(c + d*x))**p, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(335)=670$.

Time = 0.22 (sec) , antiderivative size = 728, normalized size of antiderivative = 2.08

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx=$$

$$-\frac{2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}\,abp-a^2\right)\left(\sqrt{dx+c}\,b+a\right)^p c^3}{(p^2+3p+2)b^2}-\frac{3\left((p^3+6p^2+11p+6)(dx+c)^2b^4+(p^3+3p^2+2p)(dx+c)^{\frac{3}{2}}ab^3-3(p^2+p)(dx+c)c\cdots\right.}{(p^4+10p^3+35p^2+50p+24)b^4}\cdots\right.}{\cdots}$$

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] -2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b +
a)^p*c^3/((p^2 + 3*p + 2)*b^2) - 3*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^
4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b
^2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p*c^2/((p^4 + 1
0*p^3 + 35*p^2 + 50*p + 24)*b^4) + 3*((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 27
4*p + 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x +
 c)^(5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 + 20*(
p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x + c)*a^4*b^2
 + 120*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*b + a)^p*c/((p^6 + 2
1*p^5 + 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6) - ((p^7 + 28*p^6
+ 322*p^5 + 1960*p^4 + 6769*p^3 + 13132*p^2 + 13068*p + 5040)*(d*x + c)^4*b
^8 + (p^7 + 21*p^6 + 175*p^5 + 735*p^4 + 1624*p^3 + 1764*p^2 + 720*p)*(d*x
+ c)^(7/2)*a*b^7 - 7*(p^6 + 15*p^5 + 85*p^4 + 225*p^3 + 274*p^2 + 120*p)*(d
*x + c)^3*a^2*b^6 + 42*(p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x + c)^(5
/2)*a^3*b^5 - 210*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^4*b^4 + 840*(p
^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^5*b^3 - 2520*(p^2 + p)*(d*x + c)*a^6*b^
2 + 5040*sqrt(d*x + c)*a^7*b*p - 5040*a^8)*(sqrt(d*x + c)*b + a)^p/((p^8 +
36*p^7 + 546*p^6 + 4536*p^5 + 22449*p^4 + 67284*p^3 + 118124*p^2 + 109584*p
 + 40320)*b^8))/d^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5699 vs. $2(335) = 670$.

Time = 0.51 (sec) , antiderivative size = 5699, normalized size of antiderivative = 16.28

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] -2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^7 - (sqrt(d*x
 + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^7 + 34*(sqrt(d*x + c)*b +
a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^6 - 35*(sqrt(d*x + c)*b + a)*(sqrt(d
*x + c)*b + a)^p*a*b^6*c^3*p^6 - 3*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b
 + a)^p*b^4*c^2*p^7 + 9*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b
^4*c^2*p^7 - 9*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2*
p^7 + 3*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p^7 + 478
*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^5 - 511*(sqrt(d*
x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^5 - 96*(sqrt(d*x + c)*b +
 a)^4*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^6 + 297*(sqrt(d*x + c)*b + a)^3*(sq
rt(d*x + c)*b + a)^p*a*b^4*c^2*p^6 - 306*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x
+ c)*b + a)^p*a^2*b^4*c^2*p^6 + 105*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b
+ a)^p*a^3*b^4*c^2*p^6 + 3*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*
b^2*c*p^7 - 15*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^7
+ 30*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^7 - 30*(sq
```

rt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^7 + 15*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^7 - 3*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*b^2*c*p^7 + 3580*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^4 - 4025*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^4 - 1254*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^5 + 4023*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^5 - 4302*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2*p^5 + 1533*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p^5 + 90*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p^6 - 465*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^6 + 960*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^6 - 990*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^6 + 510*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^6 - 105*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*b^2*c*p^6 - (sqrt(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*p^7 + 7*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p*a*p^7 - 21*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^7 + 35*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3*p^7 - 35*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^4*p^7 + 21*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^5*p^7 - 7*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^6*p^7 + (sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p^7 + 15289*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^3 - 18424*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^3 - 8592*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^4 + 28755*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^4 - 32220*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2*p^4 + 12075*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p^4 + 1098*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p^5 - 5865*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^5 + 12540*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^5 - 13410*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^5 + 7170*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^5 - 1533*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*b^2*c*p^5 - 28*(sqrt(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*p^6 + 203*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p*a*p^6 - 630*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^6 + 1085*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3*p^6 - 1120*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^4*p^6 + 693*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^5*p^6 - 238*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^6*p^6 + 35*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p^6 + 36706*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^2 - 48860*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^2 - 32979*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^3 + 115776*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^3 - 137601*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2*p^3 + 55272*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p^3 + 7020*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p^4 - 38715*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^4 + 85920*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x

+ c)*b + a)^p*a^2*b^2*c*p^4 - 95850*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*
b + a)^p*a^3*b^2*c*p^4 + 53700*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a
)^p*a^4*b^2*c*p^4 - 12075*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5
*b^2*c*p^4 - 322*(sqrt(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*p^5 + 2401
*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p*a*p^5 - 7686*(sqrt(d*x + c
)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^5 + 13685*(sqrt(d*x + c)*b + a)^5*
(sqrt(d*x + c)*b + a)^p*a^3*p^5 - 14630*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x +
 c)*b + a)^p*a^4*p^5 + 9387*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p
*a^5*p^5 - 3346*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^6*p^5 + 5
11*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p^5 + 44712*(sqrt(d*x
+ c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p - 69264*(sqrt(d*x + c)*b +
a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p - 69936*(sqrt(d*x + c)*b + a)^4*(sqr
t(d*x + c)*b + a)^p*b^4*c^2*p^2 + 258228*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x
+ c)*b + a)^p*a*b^4*c^2*p^2 - 330354*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)
*b + a)^p*a^2*b^4*c^2*p^2 + 146580*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b +
 a)^p*a^3*b^4*c^2*p^2 + 25227*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)
^p*b^2*c*p^3 - 143160*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2
*c*p^3 + 329790*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p
^3 - 385920*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^3 +
 229335*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^3 - 552
72*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*b^2*c*p^3 - 1960*(sqrt
(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*p^4 + 14945*(sqrt(d*x + c)*b + a
)^7*(sqrt(d*x + c)*b + a)^p*a*p^4 - 49140*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x
 + c)*b + a)^p*a^2*p^4 + 90335*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a
)^p*a^3*p^4 - 100240*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^4*p^
4 + 67095*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^5*p^4 - 25060*(
sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^6*p^4 + 4025*(sqrt(d*x + c
)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p^4 + 20160*(sqrt(d*x + c)*b + a)^2*(s
qrt(d*x + c)*b + a)^p*b^6*c^3 - 40320*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*
b + a)^p*a*b^6*c^3 - 74628*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*
b^4*c^2*p + 288432*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^4*c^
2*p - 402408*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2*p
+ 207792*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p + 5049
0*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p^2 - 293460*(sqrt(
d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^2 + 699360*(sqrt(d*x +
c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^2 - 860760*(sqrt(d*x + c)*b
 + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^2 + 550590*(sqrt(d*x + c)*b + a
)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^2 - 146580*(sqrt(d*x + c)*b + a)*(s
qrt(d*x + c)*b + a)^p*a^5*b^2*c*p^2 - 6769*(sqrt(d*x + c)*b + a)^8*(sqrt(d*
x + c)*b + a)^p*p^3 + 52528*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p
*a*p^3 - 176589*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^3 + 3
34040*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3*p^3 - 384755*(sqr
t(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^4*p^3 + 270144*(sqrt(d*x + c)
b + a)^3(sqrt(d*x + c)*b + a)^p*a^5*p^3 - 107023*(sqrt(d*x + c)*b + a)^2*
(sqrt(d*x + c)*b + a)^p*a^6*p^3 + 18424*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c

)*b + a)^p*a^7*p^3 - 30240*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*
b^4*c^2 + 120960*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2
- 181440*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2 + 1209
60*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2 + 51432*(sqrt(
d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p - 304560*(sqrt(d*x + c)*b
 + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p + 746280*(sqrt(d*x + c)*b + a)^4*
(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p - 961440*(sqrt(d*x + c)*b + a)^3*(sqrt(
d*x + c)*b + a)^p*a^3*b^2*c*p + 670680*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x +
c)*b + a)^p*a^4*b^2*c*p - 207792*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a
)^p*a^5*b^2*c*p - 13132*(sqrt(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*p^2
 + 103292*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p*a*p^2 - 353430*(s
qrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^2 + 684740*(sqrt(d*x +
c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3*p^2 - 815920*(sqrt(d*x + c)*b + a)^
4*(sqrt(d*x + c)*b + a)^p*a^4*p^2 + 602532*(sqrt(d*x + c)*b + a)^3*(sqrt(d*
x + c)*b + a)^p*a^5*p^2 - 256942*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b +
 a)^p*a^6*p^2 + 48860*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p^2
 + 20160*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c - 120960*(sq
rt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c + 302400*(sqrt(d*x + c
)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c - 403200*(sqrt(d*x + c)*b + a)
^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c + 302400*(sqrt(d*x + c)*b + a)^2*(sqrt
(d*x + c)*b + a)^p*a^4*b^2*c - 120960*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*
b + a)^p*a^5*b^2*c - 13068*(sqrt(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*
p + 103824*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p*a*p - 360024*(sq
rt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p + 710640*(sqrt(d*x + c)*
b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3*p - 870660*(sqrt(d*x + c)*b + a)^4*(sq
rt(d*x + c)*b + a)^p*a^4*p + 673008*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*
b + a)^p*a^5*p - 312984*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^6
p + 69264(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p - 5040*(sqrt
(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p + 40320*(sqrt(d*x + c)*b + a)^7*
(sqrt(d*x + c)*b + a)^p*a - 141120*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b
 + a)^p*a^2 + 282240*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3 -
352800*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^4 + 282240*(sqrt(d
*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^5 - 141120*(sqrt(d*x + c)*b + a)
^2*(sqrt(d*x + c)*b + a)^p*a^6 + 40320*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)
*b + a)^p*a^7)/((b^6*d^3*p^8 + 36*b^6*d^3*p^7 + 546*b^6*d^3*p^6 + 4536*b^6*
d^3*p^5 + 22449*b^6*d^3*p^4 + 67284*b^6*d^3*p^3 + 118124*b^6*d^3*p^2 + 1095
84*b^6*d^3*p + 40320*b^6*d^3)*b^2*d)

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx=\int x^3\,\left(a+b\,\sqrt{c+d\,x}\right)^p\,dx$$

```
[In] int(x^3*(a + b*(c + d*x)^(1/2))^p,x)
[Out] int(x^3*(a + b*(c + d*x)^(1/2))^p, x)
```

3.654 $\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 242

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx=-\frac{2a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^6d^3(1+p)}+\frac{2(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^6d^3(2+p)}-\frac{4a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{3+p}}{b^6d^3(3+p)}+\frac{4(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{4+p}}{b^6d^3(4+p)}-\frac{10a\left(a+b\sqrt{c+dx}\right)^{5+p}}{b^6d^3(5+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{6+p}}{b^6d^3(6+p)}$$

```
[Out] -2*a*(-b^2*c+a^2)^2*(a+b*(d*x+c)^(1/2))^(p+1)/b^6/d^3/(p+1)+2*(b^4*c^2-6*a^
2*b^2*c+5*a^4)*(a+b*(d*x+c)^(1/2))^(2+p)/b^6/d^3/(2+p)-4*a*(-3*b^2*c+5*a^2)
*(a+b*(d*x+c)^(1/2))^(3+p)/b^6/d^3/(3+p)+4*(-b^2*c+5*a^2)*(a+b*(d*x+c)^(1/2
))^(4+p)/b^6/d^3/(4+p)-10*a*(a+b*(d*x+c)^(1/2))^(5+p)/b^6/d^3/(5+p)+2*(a+b*
(d*x+c)^(1/2))^(6+p)/b^6/d^3/(6+p)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {378, 1412, 786}

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx=-\frac{2a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^6d^3(p+1)}-\frac{4a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^6d^3(p+3)}+\frac{4(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^6d^3(p+4)}+\frac{2(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^6d^3(p+2)}-\frac{10a\left(a+b\sqrt{c+dx}\right)^{p+5}}{b^6d^3(p+5)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+6}}{b^6d^3(p+6)}$$

```
[In] Int[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(1 + p))/(b^6*d^3*(1 + p)) + (2
*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^6*d^3*(2
 + p)) - (4*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p))/(b^6*d^3*(3
+ p)) + (4*(5*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(4 + p))/(b^6*d^3*(4 + p))
 - (10*a*(a + b*Sqrt[c + d*x])^(5 + p))/(b^6*d^3*(5 + p)) + (2*(a + b*Sqrt[
c + d*x])^(6 + p))/(b^6*d^3*(6 + p))
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
```

```
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^p (-c+x)^2\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\text{Subst}\left(\int x(a+bx)^p\left(-c+x^2\right)^2\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{a(a^2-b^2c)^2(a+bx)^p}{b^5}+\frac{(5a^4-6a^2b^2c+b^4c^2)(a+bx)^{1+p}}{b^5}-\frac{2(5a^3-3ab^2c)(a+bx)^{2+p}}{b^5}-\frac{2(-5a^2+b^2c)(a+bx)^{3+}}{b^5}\right.\right.}{d^3}$$

$$= -\frac{2a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^6d^3(1+p)}+\frac{2(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^6d^3(2+p)} - \frac{4a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{3+p}}{b^6d^3(3+p)}+\frac{4(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{4+p}}{b^6d^3(4+p)} - \frac{10a\left(a+b\sqrt{c+dx}\right)^{5+p}}{b^6d^3(5+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{6+p}}{b^6d^3(6+p)}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.17

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx = -\frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(120a^5-120a^4b(1+p)\sqrt{c+dx}+12a^3b^2\left(4c(-5+p+p^2)+5d(2+3p+p^2)\,x\right)-\right.}{}$$

```
[In] Integrate[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(120*a^5 - 120*a^4*b*(1 + p)*Sqrt[c + d*x
] + 12*a^3*b^2*(4*c*(-5 + p + p^2) + 5*d*(2 + 3*p + p^2)*x) - 4*a^2*b^3*(1
+ p)*Sqrt[c + d*x]*(2*c*(-30 - 4*p + p^2) + 5*d*(6 + 5*p + p^2)*x) - b^5*(1
5 + 23*p + 9*p^2 + p^3)*Sqrt[c + d*x]*(8*c^2 - 4*c*d*(2 + p)*x + d^2*(8 + 6
*p + p^2)*x^2) + a*b^4*(-8*c^2*(-15 + 10*p + 12*p^2 + 2*p^3) + 4*c*d*(-30 -
 43*p - 10*p^2 + 4*p^3 + p^4)*x + 5*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p^4)
*x^2)))/(b^6*d^3*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p))
```

Maple [F]

$$\int x^2 \left(a + b\sqrt{dx + c}\right)^p dx$$

[In] int(x^2*(a+b*(d*x+c)^(1/2))^p,x)

[Out] int(x^2*(a+b*(d*x+c)^(1/2))^p,x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(230) = 460$.

Time = 0.33 (sec) , antiderivative size = 712, normalized size of antiderivative = 2.94

$$\int x^2 \left(a + b\sqrt{c + dx}\right)^p dx$$
$$= \frac{2\left(120\,b^6c^3 - 360\,a^2b^4c^2 + 360\,a^4b^2c - 120\,a^6 + 8\,(b^6c^3 + 3\,a^2b^4c^2)p^3 + (b^6d^3p^5 + 15\,b^6d^3p^4 + 85\,b^6d^3p^3 + 22\right.}{}$$

[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")

[Out] 2*(120*b^6*c^3 - 360*a^2*b^4*c^2 + 360*a^4*b^2*c - 120*a^6 + 8*(b^6*c^3 + 3 *a^2*b^4*c^2)*p^3 + (b^6*d^3*p^5 + 15*b^6*d^3*p^4 + 85*b^6*d^3*p^3 + 225*b^ 6*d^3*p^2 + 274*b^6*d^3*p + 120*b^6*d^3)*x^3 + 24*(3*b^6*c^3 + 3*a^2*b^4*c^ 2 - 2*a^4*b^2*c)*p^2 + (b^6*c*d^2*p^5 + (11*b^6*c - 5*a^2*b^4)*d^2*p^4 + (4 1*b^6*c - 30*a^2*b^4)*d^2*p^3 + (61*b^6*c - 55*a^2*b^4)*d^2*p^2 + 30*(b^6*c - a^2*b^4)*d^2*p)*x^2 + 8*(23*b^6*c^3 - 24*a^2*b^4*c^2 + 9*a^4*b^2*c)*p - 4*((b^6*c^2 + a^2*b^4*c)*d*p^4 + 3*(3*b^6*c^2 - a^2*b^4*c)*d*p^3 + (23*b^6* c^2 - 34*a^2*b^4*c + 15*a^4*b^2)*d*p^2 + 15*(b^6*c^2 - 2*a^2*b^4*c + a^4*b^ 2)*d*p)*x + (8*(3*a*b^5*c^2 + a^3*b^3*c)*p^3 + 24*(7*a*b^5*c^2 - 3*a^3*b^3* c)*p^2 + (a*b^5*d^2*p^5 + 10*a*b^5*d^2*p^4 + 35*a*b^5*d^2*p^3 + 50*a*b^5*d^ 2*p^2 + 24*a*b^5*d^2*p)*x^2 + 8*(33*a*b^5*c^2 - 40*a^3*b^3*c + 15*a^5*b)*p - 4*(2*a*b^5*c*d*p^4 + 5*(3*a*b^5*c - a^3*b^3)*d*p^3 + (31*a*b^5*c - 15*a^3 *b^3)*d*p^2 + 2*(9*a*b^5*c - 5*a^3*b^3)*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x + c)*b + a)^p/(b^6*d^3*p^6 + 21*b^6*d^3*p^5 + 175*b^6*d^3*p^4 + 735*b^6*d^3*p ^3 + 1624*b^6*d^3*p^2 + 1764*b^6*d^3*p + 720*b^6*d^3)

Sympy [F]

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx=\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx$$

```
[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**p,x)
[Out] Integral(x**2*(a + b*sqrt(c + d*x))**p, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.66

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx = \frac{2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}abp-a^2\right)\left(\sqrt{dx+c}b+a\right)^pc^2}{(p^2+3p+2)b^2}-\frac{2\left((p^3+6p^2+11p+6)(dx+c)^2b^4+(p^3+3p^2+2p)(dx+c)^{\frac{3}{2}}ab^3-3(p^2+p)(dx+c)a^2b^2\right.}{(p^4+10p^3+35p^2+50p+24)b^4}\right.}{}$$

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
[Out] 2*((((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b + a
)^p*c^2/((p^2 + 3*p + 2)*b^2) - 2*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^4
 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b^
2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p*c/((p^4 + 10*p
^3 + 35*p^2 + 50*p + 24)*b^4) + ((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 274*p +
 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x + c)^(
5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 + 20*(p^3 +
 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x + c)*a^4*b^2 + 12
0*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*b + a)^p/((p^6 + 21*p^5 +
 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6))/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2511 vs. 2(230) = 460.

Time = 0.36 (sec) , antiderivative size = 2511, normalized size of antiderivative = 10.38

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx=\text{Too large to display}$$

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

[Out] 2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^5 - (sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^5 + 19*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^4 - 20*(sqrt(d*x + c)*b + a)*(sqrt(d* x + c)*b + a)^p*a*b^4*c^2*p^4 - 2*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^2*c*p^5 + 6*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^2* c*p^5 - 6*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^5 + 2 *(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^5 + 137*(sqrt(d* x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^3 - 155*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^3 - 34*(sqrt(d*x + c)*b + a)^4*(sq rt(d*x + c)*b + a)^p*b^2*c*p^4 + 108*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c) *b + a)^p*a*b^2*c*p^4 - 114*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p *a^2*b^2*c*p^4 + 40*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c *p^4 + (sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^5 - 5*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^5 + 10*(sqrt(d*x + c)*b + a)^4*(sqr t(d*x + c)*b + a)^p*a^2*p^5 - 10*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*p^5 + 5*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*p^5 - (sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^5 + 461*(sqrt(d*x + c) *b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^2 - 580*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^2 - 214*(sqrt(d*x + c)*b + a)^4*(sqrt(d* x + c)*b + a)^p*b^2*c*p^3 + 726*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^3 - 822*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2* b^2*c*p^3 + 310*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^3 + 15*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^4 - 80*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^4 + 170*(sqrt(d*x + c)*b + a)^4*(sq rt(d*x + c)*b + a)^p*a^2*p^4 - 180*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*p^4 + 95*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*p^ 4 - 20*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^4 + 702*(sqrt(d* x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p - 1044*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p - 614*(sqrt(d*x + c)*b + a)^4*(sqrt (d*x + c)*b + a)^p*b^2*c*p^2 + 2232*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)* b + a)^p*a*b^2*c*p^2 - 2766*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p *a^2*b^2*c*p^2 + 1160*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2 *c*p^2 + 85*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^3 - 475*(sqrt (d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^3 + 1070*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*p^3 - 1210*(sqrt(d*x + c)*b + a)^3*(sqrt(d *x + c)*b + a)^p*a^3*p^3 + 685*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*p^3 - 155*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^3 + 3 60*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2 - 720*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2 - 792*(sqrt(d*x + c)*b + a)^4 *(sqrt(d*x + c)*b + a)^p*b^2*c*p + 3048*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p - 4212*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a) ^p*a^2*b^2*c*p + 2088*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2 *c*p + 225*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^2 - 1300*(sqrt (d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^2 + 3070*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*p^2 - 3720*(sqrt(d*x + c)*b + a)^3*(sqrt(d

```
*x + c)*b + a)^p*a^3*p^2 + 2305*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b +
a)^p*a^4*p^2 - 580*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^2 -
360*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^2*c + 1440*(sqrt(d*x
+ c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^2*c - 2160*(sqrt(d*x + c)*b + a)^
2*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c + 1440*(sqrt(d*x + c)*b + a)*(sqrt(d*x
+ c)*b + a)^p*a^3*b^2*c + 274*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)
^p*p - 1620*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p + 3960*(sqr
t(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*p - 5080*(sqrt(d*x + c)*b +
 a)^3*(sqrt(d*x + c)*b + a)^p*a^3*p + 3510*(sqrt(d*x + c)*b + a)^2*(sqrt(d*
x + c)*b + a)^p*a^4*p - 1044*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*
a^5*p + 120*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p - 720*(sqrt(d*x
 + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a + 1800*(sqrt(d*x + c)*b + a)^4*(sq
rt(d*x + c)*b + a)^p*a^2 - 2400*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
a)^p*a^3 + 1800*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4 - 720*(
sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5)/((b^4*d^2*p^6 + 21*b^4*d^
2*p^5 + 175*b^4*d^2*p^4 + 735*b^4*d^2*p^3 + 1624*b^4*d^2*p^2 + 1764*b^4*d^2
*p + 720*b^4*d^2)*b^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx=\int x^2\,\left(a+b\,\sqrt{c+d\,x}\right)^p\,dx$$

```
[In] int(x^2*(a + b*(c + d*x)^(1/2))^p,x)
[Out] int(x^2*(a + b*(c + d*x)^(1/2))^p, x)
```

3.655 $\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 145

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=-\frac{2a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^4d^2(1+p)}+\frac{2(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^4d^2(2+p)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{3+p}}{b^4d^2(3+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{4+p}}{b^4d^2(4+p)}$$

```
[Out] -2*a*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(p+1)/b^4/d^2/(p+1)+2*(-b^2*c+3*a^2)*
(a+b*(d*x+c)^(1/2))^(2+p)/b^4/d^2/(2+p)-6*a*(a+b*(d*x+c)^(1/2))^(3+p)/b^4/d
^2/(3+p)+2*(a+b*(d*x+c)^(1/2))^(4+p)/b^4/d^2/(4+p)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {378, 1412, 786}

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=-\frac{2a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^4d^2(p+1)}+\frac{2(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^4d^2(p+2)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^4d^2(p+3)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^4d^2(p+4)}$$

```
[In] Int[x*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(1 + p))/(b^4*d^2*(1 + p)) + (2*(
3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^4*d^2*(2 + p)) - (6*a*(a +
 b*Sqrt[c + d*x])^(3 + p))/(b^4*d^2*(3 + p)) + (2*(a + b*Sqrt[c + d*x])^(4
+ p))/(b^4*d^2*(4 + p))
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^p (-c+x)\,dx, x, c+dx\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int x(a+bx)^p \left(-c+x^2\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{(-a^3+ab^2c)(a+bx)^p}{b^3}+\frac{(3a^2-b^2c)(a+bx)^{1+p}}{b^3}-\frac{3a(a+bx)^{2+p}}{b^3}+\frac{(a+bx)^{3+p}}{b^3}\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}\\
&= -\frac{2a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^4d^2(1+p)}+\frac{2(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^4d^2(2+p)}\\
&\quad-\frac{6a\left(a+b\sqrt{c+dx}\right)^{3+p}}{b^4d^2(3+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{4+p}}{b^4d^2(4+p)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx =$$

$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(6a^3-6a^2b(1+p)\sqrt{c+dx}-b^3(3+4p+p^2)\sqrt{c+dx}(-2c+d(2+p)x)+ab^2(2c(\right.}{b^4d^2(1+p)(2+p)(3+p)(4+p)}$$

```
[In] Integrate[x*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(6*a^3 - 6*a^2*b*(1 + p)*Sqrt[c + d*x] -
b^3*(3 + 4*p + p^2)*Sqrt[c + d*x]*(-2*c + d*(2 + p)*x) + a*b^2*(2*c*(-3 + p
 + p^2) + 3*d*(2 + 3*p + p^2)*x)))/(b^4*d^2*(1 + p)*(2 + p)*(3 + p)*(4 + p)
)
```

Maple [F]

$$\int x\left(a+b\sqrt{dx+c}\right)^p\,dx$$

```
[In] int(x*(a+b*(d*x+c)^(1/2))^p,x)
```

```
[Out] int(x*(a+b*(d*x+c)^(1/2))^p,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(137) = 274.

Time = 0.31 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.03

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx =$$

$$-\frac{2\left(6\,b^4c^2-12\,a^2b^2c+6\,a^4+2\,(b^4c^2+a^2b^2c)p^2-(b^4d^2p^3+6\,b^4d^2p^2+11\,b^4d^2p+6\,b^4d^2)x^2+4\,(2\,b^4c^2-\right.}{b}$$

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] -2*(6*b^4*c^2 - 12*a^2*b^2*c + 6*a^4 + 2*(b^4*c^2 + a^2*b^2*c)*p^2 - (b^4*d
^2*p^3 + 6*b^4*d^2*p^2 + 11*b^4*d^2*p + 6*b^4*d^2)*x^2 + 4*(2*b^4*c^2 - a^2
*b^2*c)*p - (b^4*c*d*p^3 + (4*b^4*c - 3*a^2*b^2)*d*p^2 + 3*(b^4*c - a^2*b^2
)*d*p)*x + (4*a*b^3*c*p^2 + 2*(5*a*b^3*c - 3*a^3*b)*p - (a*b^3*d*p^3 + 3*a*
b^3*d*p^2 + 2*a*b^3*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x + c)*b + a)^p/(b^4*d^2
*p^4 + 10*b^4*d^2*p^3 + 35*b^4*d^2*p^2 + 50*b^4*d^2*p + 24*b^4*d^2)
```

Sympy [F]

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$$

[In] integrate(x*(a+b*(d*x+c)**(1/2))**p,x)

[Out] Integral(x*(a + b*sqrt(c + d*x))**p, x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.29

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=-\frac{2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}abp-a^2\right)\left(\sqrt{dx+c}b+a\right)^pc}{(p^2+3p+2)b^2}-\frac{\left((p^3+6p^2+11p+6)(dx+c)^2b^4+(p^3+3p^2+2p)(dx+c)^{\frac{3}{2}}ab^3-3(p^2+p)(dx+c)a^2b\right.}{(p^4+10p^3+35p^2+50p+24)b^4}\right.}{d^2}$$

[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")

[Out] -2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b + a)^p*c/((p^2 + 3*p + 2)*b^2) - ((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b^2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p/((p^4 + 10*p^3 + 35*p^2 + 50*p + 24)*b^4))/d^2

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. 2(137) = 274.

Time = 0.35 (sec) , antiderivative size = 806, normalized size of antiderivative = 5.56

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=-\frac{2\left(\left(\sqrt{dx+c}b+a\right)^2\left(\sqrt{dx+c}b+a\right)^pb^2cp^3-\left(\sqrt{dx+c}b+a\right)\left(\sqrt{dx+c}b+a\right)^pab^2cp^3+8\left(\sqrt{dx+c}b+a\right.\right.}{}$$

[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")

[Out] -2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c*p^3 - (sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^3 + 8*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c*p^2 - 9*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*

```
b + a)^p*a*b^2*c*p^2 - (sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*p^3
+ 3*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*p^3 - 3*(sqrt(d*x + c
)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*p^3 + (sqrt(d*x + c)*b + a)*(sqrt(d*
x + c)*b + a)^p*a^3*p^3 + 19*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^
p*b^2*c*p - 26*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p - 6*
(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*p^2 + 21*(sqrt(d*x + c)*b +
 a)^3*(sqrt(d*x + c)*b + a)^p*a*p^2 - 24*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x
+ c)*b + a)^p*a^2*p^2 + 9*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3
*p^2 + 12*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c - 24*(sqrt(
d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c - 11*(sqrt(d*x + c)*b + a)^
4*(sqrt(d*x + c)*b + a)^p*p + 42*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
 a)^p*a*p - 57*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*p + 26*(
sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*p - 6*(sqrt(d*x + c)*b + a
)^4*(sqrt(d*x + c)*b + a)^p + 24*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
 a)^p*a - 36*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2 + 24*(sqrt
(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3)/((b^2*p^4 + 10*b^2*p^3 + 35*b
^2*p^2 + 50*b^2*p + 24*b^2)*b^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=\int x\,\left(a+b\,\sqrt{c+d\,x}\right)^p\,dx$$

```
[In] int(x*(a + b*(c + d*x)^(1/2))^p,x)
[Out] int(x*(a + b*(c + d*x)^(1/2))^p, x)
```

3.656 $\int \left(a+b\sqrt{c+dx}\right)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx=-\frac{2a\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^2d(1+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^2d(2+p)}$$

```
[Out] -2*a*(a+b*(d*x+c)^(1/2))^(p+1)/b^2/d/(p+1)+2*(a+b*(d*x+c)^(1/2))^(2+p)/b^2/
d/(2+p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {253, 196, 45}

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx=\frac{2\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^2d(p+2)}-\frac{2a\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^2d(p+1)}$$

```
[In] Int[(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a + b*Sqrt[c + d*x])^(1 + p))/(b^2*d*(1 + p)) + (2*(a + b*Sqrt[c + d
*x])^(2 + p))/(b^2*d*(2 + p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^p \, dx, x, c+dx\right)}{d} \\ &= \frac{2\text{Subst}\left(\int x(a+bx)^p \, dx, x, \sqrt{c+dx}\right)}{d} \\ &= \frac{2\text{Subst}\left(\int \left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right) \, dx, x, \sqrt{c+dx}\right)}{d} \\ &= -\frac{2a\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^2d(1+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^2d(2+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \left(a+b\sqrt{c+dx}\right)^p \, dx = \frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(-a+b(1+p)\sqrt{c+dx}\right)}{b^2d(1+p)(2+p)}$$

```
[In] Integrate[(a + b*Sqrt[c + d*x])^p,x]
[Out] (2*(a + b*Sqrt[c + d*x])^(1 + p)*(-a + b*(1 + p)*Sqrt[c + d*x]))/(b^2*d*(1
+ p)*(2 + p))
```

Maple [F]

$$\int \left(a+b\sqrt{dx+c}\right)^p dx$$

```
[In] int((a+b*(d*x+c)^(1/2))^p,x)
[Out] int((a+b*(d*x+c)^(1/2))^p,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.31

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx = \frac{2\left(b^2cp+\sqrt{dx+c}abp+b^2c-a^2+(b^2dp+b^2d)x\right)\left(\sqrt{dx+c}b+a\right)^p}{b^2dp^2+3\,b^2dp+2\,b^2d}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
[Out] 2*(b^2*c*p + sqrt(d*x + c)*a*b*p + b^2*c - a^2 + (b^2*d*p + b^2*d)*x)*(sqrt
(d*x + c)*b + a)^p/(b^2*d*p^2 + 3*b^2*d*p + 2*b^2*d)
```

Sympy [F]

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx = \int \left(a+b\sqrt{c+dx}\right)^p\,dx$$

```
[In] integrate((a+b*(d*x+c)**(1/2))**p,x)
[Out] Integral((a + b*sqrt(c + d*x))**p, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx = \frac{2\left((dx+c)b^2(p+1)+\sqrt{dx+c}abp-a^2\right)\left(\sqrt{dx+c}b+a\right)^p}{(p^2+3\,p+2)b^2d}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
[Out] 2*((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b + a)
^p/((p^2 + 3*p + 2)*b^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(58) = 116.

Time = 0.57 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.08

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx = \frac{2\left(\left(\sqrt{dx+c}b+a\right)^2\left(\sqrt{dx+c}b+a\right)^p p-\left(\sqrt{dx+c}b+a\right)\left(\sqrt{dx+c}b+a\right)^p ap+\left(\sqrt{dx+c}b+a\right)^2\left(\sqrt{dx+c}b\right.\right.}{(p^2+3p+2)b^2d}$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*p - (sqrt(d*x + c)*b + a
)*(sqrt(d*x + c)*b + a)^p*a*p + (sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b +
a)^p - 2*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a)/((p^2 + 3*p + 2)*
b^2*d)
```

Mupad [B] (verification not implemented)

Time = 17.52 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.35

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx = \begin{cases} -\frac{2\,a\,\ln\left(a+b\,\sqrt{c+d\,x}\right)-2\,b\,\sqrt{c+d\,x}}{b^2\,d} & \text{if } p=-1 \\ \frac{2\,\left(\ln\left(a+b\,\sqrt{c+d\,x}\right)+\frac{a}{a+b\,\sqrt{c+d\,x}}\right)}{b^2\,d} & \text{if } p=-2 \\ \frac{4\,\left(a+b\,\sqrt{c+d\,x}\right)^{p+2}}{b^2\,d\,(2\,p+4)}-\frac{4\,a\,\left(a+b\,\sqrt{c+d\,x}\right)^{p+1}}{b^2\,d\,(2\,p+2)} & \text{if } p\neq -1\land p\neq -2 \end{cases}$$

```
[In] int((a + b*(c + d*x)^(1/2))^p,x)
```

```
[Out] piecewise(p == -1, -(2*a*log(a + b*(c + d*x)^(1/2)) - 2*b*(c + d*x)^(1/2))/
(b^2*d), p == -2, (2*(log(a + b*(c + d*x)^(1/2)) + a/(a + b*(c + d*x)^(1/2)
)))/(b^2*d), p ~= -1 & p ~= -2, (4*(a + b*(c + d*x)^(1/2))^(p + 2))/(b^2*d*
(2*p + 4)) - (4*a*(a + b*(c + d*x)^(1/2))^(p + 1))/(b^2*d*(2*p + 2)))
```

3.657 $\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 139

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx = -\frac{\left(a+b\sqrt{c+dx}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{\left(a-b\sqrt{c}\right)(1+p)} - \frac{\left(a+b\sqrt{c+dx}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)}{\left(a+b\sqrt{c}\right)(1+p)}$$

```
[Out] -hypergeom([1, p+1],[2+p],(a+b*(d*x+c)^(1/2))/(a-b*c^(1/2)))*(a+b*(d*x+c)^(
1/2))^(p+1)/(p+1)/(a-b*c^(1/2))-hypergeom([1, p+1],[2+p],(a+b*(d*x+c)^(1/2)
)/(a+b*c^(1/2)))*(a+b*(d*x+c)^(1/2))^(p+1)/(p+1)/(a+b*c^(1/2))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {378, 1412, 845, 70}

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx = -\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{(p+1)\left(a-b\sqrt{c}\right)} - \frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)}{(p+1)\left(a+b\sqrt{c}\right)}$$

```
[In] Int[(a + b*Sqrt[c + d*x])^p/x,x]
```

```
[Out] -(((a + b*Sqrt[c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*
Sqrt[c + d*x])/(a - b*Sqrt[c])])/((a - b*Sqrt[c])*(1 + p))) - ((a + b*Sqrt[
c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d*x])/
(a + b*Sqrt[c])])/((a + b*Sqrt[c])*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^p}{-c+x}\,dx, x, c+dx\right)\\
&= 2\text{Subst}\left(\int \frac{x(a+bx)^p}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\\
&= 2\text{Subst}\left(\int \left(-\frac{(a+bx)^p}{2\left(\sqrt{c}-x\right)}+\frac{(a+bx)^p}{2\left(\sqrt{c}+x\right)}\right)\,dx, x, \sqrt{c+dx}\right)\\
&= -\text{Subst}\left(\int \frac{(a+bx)^p}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)+\text{Subst}\left(\int \frac{(a+bx)^p}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)
\end{aligned}$$

$$= -\frac{\left(a+b\sqrt{c+dx}\right)^{1+p}\ {}_2F_1\left(1,1+p;2+p;\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{\left(a-b\sqrt{c}\right)(1+p)} - \frac{\left(a+b\sqrt{c+dx}\right)^{1+p}\ {}_2F_1\left(1,1+p;2+p;\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)}{\left(a+b\sqrt{c}\right)(1+p)}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx = -\frac{\left(a+b\sqrt{c+dx}\right)^{1+p}\left(\left(a+b\sqrt{c}\right)\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)+\left(a-b\sqrt{c}\right)\text{Hypergeo}\right.}{\left(a-b\sqrt{c}\right)\left(a+b\sqrt{c}\right)(1+p)}$$

```
[In] Integrate[(a + b*Sqrt[c + d*x])^p/x,x]
```

```
[Out] -(((a + b*Sqrt[c + d*x])^(1 + p)*((a + b*Sqrt[c])*Hypergeometric2F1[1, 1 +
p, 2 + p, (a + b*Sqrt[c + d*x])/(a - b*Sqrt[c])] + (a - b*Sqrt[c])*Hypergeo
metric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d*x])/(a + b*Sqrt[c])]))/((a - b
*Sqrt[c])*(a + b*Sqrt[c])*(1 + p)))
```

Maple [F]

$$\int \frac{\left(a+b\sqrt{dx+c}\right)^p}{x}dx$$

```
[In] int((a+b*(d*x+c)^(1/2))^p/x,x)
```

```
[Out] int((a+b*(d*x+c)^(1/2))^p/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx = \int \frac{\left(\sqrt{dx+c}b+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="fricas")
```

```
[Out] integral((sqrt(d*x + c)*b + a)^p/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx=\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx$$

```
[In] integrate((a+b*(d*x+c)**(1/2))**p/x,x)
[Out] Integral((a + b*sqrt(c + d*x))**p/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx=\int \frac{\left(\sqrt{dx+c}b+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="maxima")
[Out] integrate((sqrt(d*x + c)*b + a)^p/x, x)
```

Giac [F]

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx=\int \frac{\left(\sqrt{dx+c}b+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="giac")
[Out] integrate((sqrt(d*x + c)*b + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx=\int \frac{\left(a+b\,\sqrt{c+d\,x}\right)^p}{x}\,dx$$

```
[In] int((a + b*(c + d*x)^(1/2))^p/x,x)
[Out] int((a + b*(c + d*x)^(1/2))^p/x, x)
```

3.658 $\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] 2/3*a*(a+b*(c*x)^n)^(3/2)/n+2/5*(a+b*(c*x)^n)^(5/2)/n-2*a^(5/2)*arctanh((a+
b*(c*x)^n)^(1/2)/a^(1/2))/n+2*a^2*(a+b*(c*x)^n)^(1/2)/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {374, 12, 272, 52, 65, 214}

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = -\frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n} + \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n}$$

```
[In] Int[(a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*a^2*Sqrt[a + b*(c*x)^n])/n + (2*a*(a + b*(c*x)^n)^(3/2))/(3*n) + (2*(a +
 b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])
/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^{5/2}}{x}\,dx, x, cx\right)}{c}$$

$$= \text{Subst}\left(\int \frac{(a+bx^n)^{5/2}}{x}\,dx, x, cx\right)$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx,x,(cx)^n\right)}{n}\\
&= \frac{2(a+b(cx)^n)^{5/2}}{5n}+\frac{a\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,(cx)^n\right)}{n}\\
&= \frac{2a(a+b(cx)^n)^{3/2}}{3n}+\frac{2(a+b(cx)^n)^{5/2}}{5n}+\frac{a^2\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,(cx)^n\right)}{n}\\
&= \frac{2a^2\sqrt{a+b(cx)^n}}{n}+\frac{2a(a+b(cx)^n)^{3/2}}{3n}+\frac{2(a+b(cx)^n)^{5/2}}{5n}+\frac{a^3\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,(cx)^n\right)}{n}\\
&= \frac{2a^2\sqrt{a+b(cx)^n}}{n}+\frac{2a(a+b(cx)^n)^{3/2}}{3n}+\frac{2(a+b(cx)^n)^{5/2}}{5n}\\
&\quad+\frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b(cx)^n}\right)}{bn}\\
&= \frac{2a^2\sqrt{a+b(cx)^n}}{n}+\frac{2a(a+b(cx)^n)^{3/2}}{3n}+\frac{2(a+b(cx)^n)^{5/2}}{5n}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$
\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx=\frac{2\sqrt{a+b(cx)^n}(23a^2+11ab(cx)^n+3b^2(cx)^{2n})-30a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{15n}
$$

```
[In] Integrate[(a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n]*(23*a^2 + 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30*a
^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\frac{2(a+b(cx)^n)^{\frac{5}{2}}}{5}+\frac{2a(a+b(cx)^n)^{\frac{3}{2}}}{3}+2a^2\sqrt{a+b(cx)^n}-2a^{\frac{5}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	70
default	$\frac{\frac{2(a+b(cx)^n)^{\frac{5}{2}}}{5}+\frac{2a(a+b(cx)^n)^{\frac{3}{2}}}{3}+2a^2\sqrt{a+b(cx)^n}-2a^{\frac{5}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	70
risch	$\frac{2(3b^2\mathrm{e}^{2n\ln(cx)}+11a\,\mathrm{e}^{n\ln(cx)}b+23a^2)\sqrt{a+b\,\mathrm{e}^{n\ln(cx)}}}{15n}-\frac{2a^{\frac{5}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{e}^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	77

```
[In] int((a+b*(c*x)^n)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2/5*(a+b*(c*x)^n)^(5/2)+2/3*a*(a+b*(c*x)^n)^(3/2)+2*a^2*(a+b*(c*x)^n)^
(1/2)-2*a^(5/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.76

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \left[\frac{15\,a^{\frac{5}{2}}\log\left(\frac{(cx)^n b-2\,\sqrt{(cx)^n b+a}\sqrt{a}+2\,a}{(cx)^n}\right)+2\left(11\,(cx)^n\,ab+3\,(cx)^{2\,n}\,b^2+23\,a^2\right)\sqrt{(cx)^n\,b-}}{15\,n}\right.$$

```
[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^(5/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*
x)^n) + 2*(11*(c*x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b + a)
)/n, 2/15*(15*sqrt(-a)*a^2*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + (11*(c*
x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b + a))/n]
```

Sympy [A] (verification not implemented)

Time = 18.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.44

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \begin{cases} \frac{\begin{cases} \frac{2a^3\,\mathrm{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2a^2\sqrt{a+b\,(cx)^n}+\frac{2a(a+b(cx)^n)^{\frac{3}{2}}}{3}+\frac{2(a+b(cx)^n)^{\frac{5}{2}}}{5} & \text{for } b\neq 0 \\ a^{\frac{5}{2}}\log\left((cx)^n\right) & \text{otherwise} \end{cases}}{n} & \mathrm{f} \\ -\left(-a^2\sqrt{a+b}-2ab\sqrt{a+b}-b^2\sqrt{a+b}\right)\log(cx) & \mathrm{e} \end{cases}$$

```
[In] integrate((a+b*(c*x)**n)**(5/2)/x,x)
```

```
[Out] Piecewise((Piecewise((2*a**3*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a) +
 2*a**2*sqrt(a + b*(c*x)**n) + 2*a*(a + b*(c*x)**n)**(3/2)/3 + 2*(a + b*(c*
x)**n)**(5/2)/5, Ne(b, 0)), (a**(5/2)*log((c*x)**n), True))/n, Ne(n, 0)), (
-(-a**2*sqrt(a + b) - 2*a*b*sqrt(a + b) - b**2*sqrt(a + b))*log(c*x), True)
)
```

Maxima [F]

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx=\int \frac{((cx)^n b+a)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")
[Out] integrate(((c*x)^n*b + a)^(5/2)/x, x)
```

Giac [F]

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx=\int \frac{((cx)^n b+a)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")
[Out] integrate(((c*x)^n*b + a)^(5/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx=\int \frac{(a+b\,(c\,x)^n)^{5/2}}{x}\,dx$$

```
[In] int((a + b*(c*x)^n)^(5/2)/x,x)
[Out] int((a + b*(c*x)^n)^(5/2)/x, x)
```

3.659 $\int \frac{(a+b(cx)^n)^{3/2}}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{(a+b(cx)^n)^{3/2}}{x} dx = \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} - \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] 2/3*(a+b*(c*x)^n)^(3/2)/n-2*a^(3/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/n+
2*a*(a+b*(c*x)^n)^(1/2)/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {374, 12, 272, 52, 65, 214}

$$\int \frac{(a+b(cx)^n)^{3/2}}{x} dx = -\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n} + \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n}$$

```
[In] Int[(a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (2*a*Sqrt[a + b*(c*x)^n])/n + (2*(a + b*(c*x)^n)^(3/2))/(3*n) - (2*a^(3/2)*
ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^{3/2}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{(a+bx^n)^{3/2}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2(a+b(cx)^n)^{3/2}}{3n} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, (cx)^n\right)}{n}
\end{aligned}$$

$$= \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} + \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{n}$$

$$= \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{bn}$$

$$= \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \frac{2\sqrt{a+b(cx)^n}(4a+b(cx)^n) - 6a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{3n}$$

```
[In] Integrate[(a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n]*(4*a + b*(c*x)^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*(c*
x)^n]/Sqrt[a]])/(3*n)
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\frac{2(a+b(cx)^n)^{\frac{3}{2}}}{3}+2a\sqrt{a+b(cx)^n}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	54
default	$\frac{\frac{2(a+b(cx)^n)^{\frac{3}{2}}}{3}+2a\sqrt{a+b(cx)^n}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	54
risch	$\frac{2\left(b\,e^{n\ln(cx)}+4a\right)\sqrt{a+b\,e^{n\ln(cx)}}}{3n}-\frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,e^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	59

```
[In] int((a+b*(c*x)^n)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2/3*(a+b*(c*x)^n)^(3/2)+2*a*(a+b*(c*x)^n)^(1/2)-2*a^(3/2)*arctanh((a+b
*(c*x)^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.86

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{(cx)^n b-2\,\sqrt{(cx)^n b+a}\sqrt{a}+2\,a}{(cx)^n}\right)+2\left((cx)^n\,b+4\,a\right)\sqrt{(cx)^n\,b+a}}{3\,n}, \frac{2\left(3\,\sqrt{-a}a\,\mathrm{arct}\right.}{}\right.$$

```
[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)
^n) + 2*((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n, 2/3*(3*sqrt(-a)*a*arctan(
sqrt((c*x)^n*b + a)*sqrt(-a)/a) + ((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n]
```

Sympy [A] (verification not implemented)

Time = 12.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \begin{cases} \frac{\begin{cases} \frac{2a^2\,\mathrm{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2a\sqrt{a+b\,(cx)^n}+\frac{2(a+b(cx)^n)^{\frac{3}{2}}}{3} & \text{for } b\neq 0 \\ a^{\frac{3}{2}}\log\left((cx)^n\right) & \text{otherwise} \end{cases}}{n} & \text{for } n\neq 0 \\ \left(a\sqrt{a+b}+b\sqrt{a+b}\right)\log\left(x\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] Piecewise((Piecewise((2*a**2*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a) +
 2*a*sqrt(a + b*(c*x)**n) + 2*(a + b*(c*x)**n)**(3/2)/3, Ne(b, 0)), (a**(3/
2)*log((c*x)**n), True))/n, Ne(n, 0)), ((a*sqrt(a + b) + b*sqrt(a + b))*log
(x), True))
```

Maxima [F]

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{\left((cx)^n\,b+a\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b + a)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{((cx)^n b+a)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")
[Out] integrate(((c*x)^n*b + a)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{(a+b\,(c\,x)^n)^{3/2}}{x}\,dx$$

```
[In] int((a + b*(c*x)^n)^(3/2)/x,x)
[Out] int((a + b*(c*x)^n)^(3/2)/x, x)
```

3.660 $\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx = \frac{2\sqrt{a+b(cx)^n}}{n} - \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] -2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(a+b*(c*x)^n)^(1/2)/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {374, 12, 272, 52, 65, 214}

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx = \frac{2\sqrt{a+b(cx)^n}}{n} - \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[In] Int[Sqrt[a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]]
)/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{c\sqrt{a+bx^n}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{\sqrt{a+bx^n}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2\sqrt{a+b(cx)^n}}{n} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2\sqrt{a+b(cx)^n}}{n} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{bn}
\end{aligned}
$$

$$= \frac{2\sqrt{a+b(cx)^n}}{n} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx = \frac{2\left(\sqrt{a+b(cx)^n} - \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)\right)}{n}$$

```
[In] Integrate[Sqrt[a + b*(c*x)^n]/x,x]
[Out] (2*(Sqrt[a + b*(c*x)^n] - Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]]))/n
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2\sqrt{a+b(cx)^n}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	40
default	$\frac{2\sqrt{a+b(cx)^n}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	40
risch	$\frac{2\sqrt{a+b\,\mathrm{e}^{n\ln(cx)}}}{n} - \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{e}^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	46

```
[In] int((a+b*(c*x)^n)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/n*(2*(a+b*(c*x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx = \left[\frac{\sqrt{a}\log\left(\frac{(cx)^n b - 2\sqrt{(cx)^n b+a}\sqrt{a}+2a}{(cx)^n}\right) + 2\sqrt{(cx)^n b + a}}{n}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{(cx)^n b+a}\sqrt{-a}}{a}\right) + \sqrt{(cx)^n b + a}\right)}{n}\right]$$

```
[In] integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) +
2*sqrt((c*x)^n*b + a))/n, 2*(sqrt(-a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a
) + sqrt((c*x)^n*b + a))/n]
```

Sympy [F]

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx=\int \frac{\sqrt{a+b\,(cx)^n}}{x}\,dx$$

```
[In] integrate((a+b*(c*x)**n)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*(c*x)**n)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx=\int \frac{\sqrt{(cx)^n\,b+a}}{x}\,dx$$

```
[In] integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((c*x)^n*b + a)/x, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx=\int \frac{\sqrt{(cx)^n\,b+a}}{x}\,dx$$

```
[In] integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x)^n*b + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx=\int \frac{\sqrt{a+b\,(c\,x)^n}}{x}\,dx$$

```
[In] int((a + b*(c*x)^n)^(1/2)/x,x)
```

```
[Out] int((a + b*(c*x)^n)^(1/2)/x, x)
```

3.661 $\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

```
[Out] -2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {374, 12, 272, 65, 214}

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

```
[In] Int[1/(x*Sqrt[a + b*(c*x)^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^n}}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{bn}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

```
[In] Integrate[1/(x*Sqrt[a + b*(c*x)^n]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n\sqrt{a}}$	25
default	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n\sqrt{a}}$	25

```
[In] int(1/x/(a+b*(c*x)^n)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.60

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \left[\frac{\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\sqrt{a}+2\,a}{(cx)^n}\right)}{\sqrt{a}n}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{(cx)^n b+a}\sqrt{-a}}{a}\right)}{an}\right]$$

```
[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="fricas")
[Out] [log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n)/(sqrt(a)*n)
, 2*sqrt(-a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a)/(a*n)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \int \frac{1}{x\sqrt{a+b\,(cx)^n}}\,dx$$

```
[In] integrate(1/x/(a+b*(c*x)**n)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*(c*x)**n)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \int \frac{1}{\sqrt{(cx)^n\,b+a}\,x}\,dx$$

```
[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt((c*x)^n*b + a)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \int \frac{1}{\sqrt{(cx)^n\,b+a}\,x}\,dx$$

```
[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt((c*x)^n*b + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \int \frac{1}{x\,\sqrt{a+b\,(c\,x)^n}}\,dx$$

```
[In] int(1/(x*(a + b*(c*x)^n)^(1/2)),x)
[Out] int(1/(x*(a + b*(c*x)^n)^(1/2)), x)
```

3.662 $\int \frac{1}{x(a+b(cx)^n)^{3/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 52

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}} dx = \frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

```
[Out] -2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(3/2)/n+2/a/n/(a+b*(c*x)^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {374, 12, 272, 53, 65, 214}

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}} dx = \frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

```
[In] Int[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] 2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(
3/2)*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{c}{x(a+bx^n)^{3/2}}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\left(a+bx^n\right)^{3/2}}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2}{an\sqrt{a+b(cx)^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{an}
\end{aligned}$$

$$= \frac{2}{an\sqrt{a+b(cx)^n}} + \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{abn}$$

$$= \frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx = \frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

```
[In] Integrate[1/(x*(a + b*(c*x)^n)^(3/2)),x]
[Out] 2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(
3/2)*n)
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\frac{2}{a\sqrt{a+b(cx)^n}}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{n}$	43
default	$\frac{\frac{2}{a\sqrt{a+b(cx)^n}}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{n}$	43

```
[In] int(1/x/(a+b*(c*x)^n)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/n*(2/a/(a+b*(c*x)^n)^(1/2)-2/a^(3/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.15

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx = \left[\frac{\left((cx)^n\sqrt{a}b+a^{\frac{3}{2}}\right)\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\sqrt{a}+2a}{(cx)^n}\right)+2\sqrt{(cx)^n b+a}a}{(cx)^n a^2bn+a^3n}, \frac{2\left(\left((cx)^n\sqrt{-\right.\right.}{}\right.$$

```
[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [((((c*x)^n*sqrt(a)*b + a^(3/2))*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt
(a) + 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n), 2
*(((c*x)^n*sqrt(-a)*b + sqrt(-a)*a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a)
+ sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n)]
```

Sympy [A] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{b}{an\sqrt{a+b(cx)^n}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{an\sqrt{-a}}\right)}{b} & \text{for } b\neq 0 \\ \frac{\log\left((cx)^n\right)}{a^{\frac{3}{2}}n} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/(a+b*(c*x)**n)**(3/2),x)
```

```
[Out] Piecewise((2*(b/(a*n*sqrt(a + b*(c*x)**n)) + b*atan(sqrt(a + b*(c*x)**n)/sq
rt(-a))/(a*n*sqrt(-a)))/b, Ne(b, 0)), (log((c*x)**n)/(a**(3/2)*n), True))
```

Maxima [F]

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx = \int \frac{1}{\left((cx)^n b+a\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx = \int \frac{1}{\left((cx)^n b+a\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/((((c*x)^n*b + a)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx = \int \frac{1}{x\left(a+b\left(c\,x\right)^n\right)^{3/2}}\,dx$$

```
[In] int(1/(x*(a + b*(c*x)^n)^(3/2)),x)
[Out] int(1/(x*(a + b*(c*x)^n)^(3/2)), x)
```

3.663 $\int \frac{1}{x(a+b(cx)^n)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 75

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx=\frac{2}{3an\left(a+b(cx)^n\right)^{3/2}}+\frac{2}{a^2n\sqrt{a+b(cx)^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

```
[Out] 2/3/a/n/(a+b*(c*x)^n)^(3/2)-2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(5/2)/
n+2/a^2/n/(a+b*(c*x)^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {374, 12, 272, 53, 65, 214}

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{a^2n\sqrt{a+b(cx)^n}}+\frac{2}{3an\left(a+b(cx)^n\right)^{3/2}}$$

```
[In] Int[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] 2/(3*a*n*(a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTan
h[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{c}{x(a+bx^n)^{5/2}}\,dx, x, cx\right)}{c} \\
&= \text{Subst}\left(\int \frac{1}{x\,(a+bx^n)^{5/2}}\,dx, x, cx\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, (cx)^n\right)}{n} \\
&= \frac{2}{3an\,(a+b(cx)^n)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, (cx)^n\right)}{an}
\end{aligned}$$

$$= \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}}+\frac{2}{a^2n\sqrt{a+b(cx)^n}}+\frac{\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,(cx)^n\right)}{a^2n}$$

$$= \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}}+\frac{2}{a^2n\sqrt{a+b(cx)^n}}+\frac{2\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b(cx)^n}\right)}{a^2bn}$$

$$= \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}}+\frac{2}{a^2n\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int\frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx=\frac{2(a+3(a+b(cx)^n))}{3a^2n\left(a+b(cx)^n\right)^{3/2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

```
[In] Integrate[1/(x*(a + b*(c*x)^n)^(5/2)),x]
[Out] (2*(a + 3*(a + b*(c*x)^n)))/(3*a^2*n*(a + b*(c*x)^n)^(3/2)) - (2*ArcTanh[Sq
rt[a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{2}{a^2\sqrt{a+b(cx)^n}}+\frac{2}{3a(a+b(cx)^n)^{\frac{3}{2}}}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}}{n}$	59
default	$\frac{\frac{2}{a^2\sqrt{a+b(cx)^n}}+\frac{2}{3a(a+b(cx)^n)^{\frac{3}{2}}}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}}{n}$	59

```
[In] int(1/x/(a+b*(c*x)^n)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/n*(2/a^2/(a+b*(c*x)^n)^(1/2)+2/3/a/(a+b*(c*x)^n)^(3/2)-2/a^(5/2)*arctanh(
(a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(63) = 126.

Time = 0.31 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.49

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx = \left[\frac{3\left(2\,(cx)^n a^{\frac{3}{2}}b+(cx)^{2\,n}\sqrt{a}b^2+a^{\frac{5}{2}}\right)\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\sqrt{a}+2\,a}{(cx)^n}\right)+2\left(3\,(cx)^n ab\right.}{3\left(2\,(cx)^n a^4bn+(cx)^{2\,n}a^3b^2n+a^5n\right)}\right.$$

```
[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(2*(c*x)^n*a^(3/2)*b + (c*x)^(2*n)*sqrt(a)*b^2 + a^(5/2))*log(((c*x
)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) + 2*(3*(c*x)^n*a*b +
4*a^2)*sqrt((c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n + a^
5*n), 2/3*(3*(2*(c*x)^n*sqrt(-a)*a*b + (c*x)^(2*n)*sqrt(-a)*b^2 + sqrt(-a)*
a^2)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + (3*(c*x)^n*a*b + 4*a^2)*sqrt(
(c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n + a^5*n)]
```

Sympy [A] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{b}{3an\left(a+b(cx)^n\right)^{\frac{3}{2}}}+\frac{b}{a^2n\sqrt{a+b(cx)^n}}+\frac{b\operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{a^2n\sqrt{-a}}\right)}{b} & \text{for}\ b\neq 0 \\ \frac{\log\left((cx)^n\right)}{a^{\frac{5}{2}}n} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/(a+b*(c*x)**n)**(5/2),x)
```

```
[Out] Piecewise((2*(b/(3*a*n*(a + b*(c*x)**n)**(3/2)) + b/(a**2*n*sqrt(a + b*(c*x
)**n)) + b*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/(a**2*n*sqrt(-a)))/b, Ne(b,
0)), (log((c*x)**n)/(a**(5/2)*n), True))
```

Maxima [F]

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx = \int \frac{1}{\left((cx)^n b+a\right)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx=\int \frac{1}{\left((cx)^n b+a\right)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="giac")
[Out] integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx=\int \frac{1}{x\left(a+b\left(c\,x\right)^n\right)^{5/2}}\,dx$$

```
[In] int(1/(x*(a + b*(c*x)^n)^(5/2)),x)
[Out] int(1/(x*(a + b*(c*x)^n)^(5/2)), x)
```

3.664 $\int \frac{(-a+b(cx)^n)^{5/2}}{x} dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x} dx = \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] -2/3*a*(-a+b*(c*x)^n)^(3/2)/n+2/5*(-a+b*(c*x)^n)^(5/2)/n-2*a^(5/2)*arctan((
-a+b*(c*x)^n)^(1/2)/a^(1/2))/n+2*a^2*(-a+b*(c*x)^n)^(1/2)/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {374, 12, 272, 52, 65, 211}

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x} dx = -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n} + \frac{2a^2\sqrt{b(cx)^n-a}}{n} - \frac{2a(b(cx)^n-a)^{3/2}}{3n} + \frac{2(b(cx)^n-a)^{5/2}}{5n}$$

```
[In] Int[(-a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*a^2*Sqrt[-a + b*(c*x)^n])/n - (2*a*(-a + b*(c*x)^n)^(3/2))/(3*n) + (2*(-
a + b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a
]])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{c(-a+bx^n)^{5/2}}{x}\,dx, x, cx\right)}{c}\\ &= \text{Subst}\left(\int \frac{(-a+bx^n)^{5/2}}{x}\,dx, x, cx\right)\end{aligned}$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int \frac{(-a+bx)^{5/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{a\text{Subst}\left(\int \frac{(-a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= -\frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n}\\
&\quad + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{a^3\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n}\\
&\quad - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{bn}\\
&= \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$
\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx = \frac{2\sqrt{-a+b(cx)^n}(23a^2-11ab(cx)^n+3b^2(cx)^{2n}) - 30a^{5/2}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{15n}
$$

```
[In] Integrate[(-a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[-a + b*(c*x)^n]*(23*a^2 - 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30*
a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\frac{2(-a+b(cx)^n)^{\frac{5}{2}}}{5}-\frac{2a(-a+b(cx)^n)^{\frac{3}{2}}}{3}+2a^2\sqrt{-a+b(cx)^n}-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	78
default	$\frac{\frac{2(-a+b(cx)^n)^{\frac{5}{2}}}{5}-\frac{2a(-a+b(cx)^n)^{\frac{3}{2}}}{3}+2a^2\sqrt{-a+b(cx)^n}-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	78
risch	$-\frac{2(3b^2e^{2n\ln(cx)}-11a\,e^{n\ln(cx)}b+23a^2)(a-b\,e^{n\ln(cx)})}{15n\sqrt{-a+b\,e^{n\ln(cx)}}}-\frac{2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a+b\,e^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	93

```
[In] int((-a+b*(c*x)^n)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2/5*(-a+b*(c*x)^n)^(5/2)-2/3*a*(-a+b*(c*x)^n)^(3/2)+2*a^2*(-a+b*(c*x)^
n)^(1/2)-2*a^(5/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.67

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx = \left[\frac{15\sqrt{-a}a^2\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)-2\left(11\,(cx)^n\,ab-3\,(cx)^{2n}\,b^2-23\,a^2\right)\sqrt{}}{15\,n}\right.$$
$$\left.-\frac{2\left(15\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)+\left(11\,(cx)^n\,ab-3\,(cx)^{2n}\,b^2-23\,a^2\right)\sqrt{(cx)^n\,b-a}\right)}{15\,n}\right]$$

```
[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*sqrt(-a)*a^2*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*
a)/(c*x)^n) - 2*(11*(c*x)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^n*
b - a))/n, -2/15*(15*a^(5/2)*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) + (11*(c*x
)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^n*b - a))/n]
```

Sympy [A] (verification not implemented)

Time = 17.82 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.30

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx = \begin{cases} \frac{\begin{cases} -2a^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)+2a^2\sqrt{-a+b\,(cx)^n}-\frac{2a(-a+b(cx)^n)^{\frac{3}{2}}}{3}+\frac{2(-a+b(cx)^n)^{\frac{5}{2}}}{5} \\ a^2\sqrt{-a}\log\left((cx)^n\right) \end{cases}}{n} \\ -\left(-a^2\sqrt{-a+b}+2ab\sqrt{-a+b}-b^2\sqrt{-a+b}\right)\log\left(cx\right) \end{cases}$$

```
[In] integrate((-a+b*(c*x)**n)**(5/2)/x,x)
```

```
[Out] Piecewise((Piecewise((-2*a**(5/2)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) + 2*a
**2*sqrt(-a + b*(c*x)**n) - 2*a*(-a + b*(c*x)**n)**(3/2)/3 + 2*(-a + b*(c*x
)**n)**(5/2)/5, Ne(b, 0)), (a**2*sqrt(-a)*log((c*x)**n), True))/n, Ne(n, 0)
), (-(-a**2*sqrt(-a + b) + 2*a*b*sqrt(-a + b) - b**2*sqrt(-a + b))*log(c*x)
, True))
```

Maxima [F]

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx=\int \frac{((cx)^n b-a)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b - a)^(5/2)/x, x)
```

Giac [F]

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx=\int \frac{((cx)^n b-a)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b - a)^(5/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx=\int \frac{(b\,(c\,x)^n-a)^{5/2}}{x}\,dx$$

```
[In] int((b*(c*x)^n - a)^(5/2)/x,x)
```

```
[Out] int((b*(c*x)^n - a)^(5/2)/x, x)
```

3.665 $\int \frac{(-a+b(cx)^n)^{3/2}}{x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x} \, dx = -\frac{2a\sqrt{-a+b(cx)^n}}{n}+\frac{2(-a+b(cx)^n)^{3/2}}{3n}+\frac{2a^{3/2}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] 2/3*(-a+b*(c*x)^n)^(3/2)/n+2*a^(3/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n
-2*a*(-a+b*(c*x)^n)^(1/2)/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {374, 12, 272, 52, 65, 211}

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x} \, dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2a\sqrt{b(cx)^n-a}}{n}+\frac{2(b(cx)^n-a)^{3/2}}{3n}$$

```
[In] Int[(-a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (-2*a*Sqrt[-a + b*(c*x)^n])/n + (2*(-a + b*(c*x)^n)^(3/2))/(3*n) + (2*a^(3/
2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{c(-a+bx^n)^{3/2}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{(-a+bx^n)^{3/2}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{(-a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2(-a+b(cx)^n)^{3/2}}{3n} - \frac{a\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x}\,dx, x, (cx)^n\right)}{n}
\end{aligned}$$

$$= -\frac{2a\sqrt{-a+b(cx)^n}}{n} + \frac{2(-a+b(cx)^n)^{3/2}}{3n} + \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{n}$$

$$= -\frac{2a\sqrt{-a+b(cx)^n}}{n} + \frac{2(-a+b(cx)^n)^{3/2}}{3n} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{bn}$$

$$= -\frac{2a\sqrt{-a+b(cx)^n}}{n} + \frac{2(-a+b(cx)^n)^{3/2}}{3n} + \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx = \frac{-2(4a-b(cx)^n)\sqrt{-a+b(cx)^n} + 6a^{3/2}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{3n}$$

```
[In] Integrate[(-a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (-2*(4*a - b*(c*x)^n)*Sqrt[-a + b*(c*x)^n] + 6*a^(3/2)*ArcTan[Sqrt[-a + b*(
c*x)^n]/Sqrt[a]])/(3*n)
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{2(-a+b(cx)^n)^{\frac{3}{2}}}{3}-2a\sqrt{-a+b(cx)^n}+2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	60
default	$\frac{\frac{2(-a+b(cx)^n)^{\frac{3}{2}}}{3}-2a\sqrt{-a+b(cx)^n}+2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	60
risch	$\frac{2(-b\,\mathrm{e}^{n\ln(cx)}+4a)(a-b\,\mathrm{e}^{n\ln(cx)})}{3n\sqrt{-a+b\,\mathrm{e}^{n\ln(cx)}}}+\frac{2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a+b\,\mathrm{e}^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	76

```
[In] int((-a+b*(c*x)^n)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2/3*(-a+b*(c*x)^n)^(3/2)-2*a*(-a+b*(c*x)^n)^(1/2)+2*a^(3/2)*arctan((-a
+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.78

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx = \left[\frac{3\sqrt{-a}a\log\left(\frac{(cx)^n b+2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^n b-a}((cx)^n b-4a)}{3n}, \frac{2\left(3a^{\frac{3}{2}}\right.}{}\right.$$

```
[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(-a)*a*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(
c*x)^n) + 2*sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n, 2/3*(3*a^(3/2)*arctan
(sqrt((c*x)^n*b - a)/sqrt(a)) + sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n]
```

Sympy [A] (verification not implemented)

Time = 11.99 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.24

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx = \begin{cases} \frac{\begin{cases} 2a^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)-2a\sqrt{-a+b\left(cx\right)^n}+\frac{2(-a+b(cx)^n)^{\frac{3}{2}}}{3} & \text{for}\ b\neq 0 \\ -a\sqrt{-a}\log\left(\left(cx\right)^n\right) & \text{otherwise} \end{cases}}{n} & \text{for}\ n \\ \left(-a\sqrt{-a+b}+b\sqrt{-a+b}\right)\log\left(x\right) & \text{othe} \end{cases}$$

```
[In] integrate((-a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] Piecewise((Piecewise((2*a**(3/2)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) - 2*a*
sqrt(-a + b*(c*x)**n) + 2*(-a + b*(c*x)**n)**(3/2)/3, Ne(b, 0)), (-a*sqrt(-
a)*log((c*x)**n), True))/n, Ne(n, 0)), ((-a*sqrt(-a + b) + b*sqrt(-a + b))*
log(x), True))
```

Maxima [F]

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{\left((cx)^n b-a\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b - a)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx=\int \frac{\left((cx)^n b-a\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")
[Out] integrate(((c*x)^n*b - a)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx=\int \frac{\left(b\,(c\,x)^n-a\right)^{3/2}}{x}\,dx$$

```
[In] int((b*(c*x)^n - a)^(3/2)/x,x)
[Out] int((b*(c*x)^n - a)^(3/2)/x, x)
```

3.666 $\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx=\frac{2\sqrt{-a+b(cx)^n}}{n}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] -2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(-a+b*(c*x)^n)^(1/2)/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {374, 12, 272, 52, 65, 211}

$$\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx=\frac{2\sqrt{b(cx)^n-a}}{n}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

```
[In] Int[Sqrt[-a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[-a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]
])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{c\sqrt{-a+bx^n}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{\sqrt{-a+bx^n}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{(2a)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{bn}
\end{aligned}$$

$$=\frac{2\sqrt{-a+b(cx)^n}}{n}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int\frac{\sqrt{-a+b(cx)^n}}{x}\,dx=\frac{2\left(\sqrt{-a+b(cx)^n}-\sqrt{a}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)\right)}{n}$$

```
[In] Integrate[Sqrt[-a + b*(c*x)^n]/x,x]
[Out] (2*(Sqrt[-a + b*(c*x)^n] - Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]]))/n
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2\sqrt{-a+b(cx)^n}-2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	44
default	$\frac{2\sqrt{-a+b(cx)^n}-2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	44
risch	$-\frac{2\left(a-b\,\mathrm{e}^{n\ln(cx)}\right)}{n\sqrt{-a+b\,\mathrm{e}^{n\ln(cx)}}}-\frac{2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b\,\mathrm{e}^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	62

```
[In] int((-a+b*(c*x)^n)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/n*(2*(-a+b*(c*x)^n)^(1/2)-2*a^(1/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.08

$$\int\frac{\sqrt{-a+b(cx)^n}}{x}\,dx=\left[\frac{\sqrt{-a}\log\left(\frac{(cx)^nb-2\sqrt{(cx)^nb-a}\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^nb-a}}{n},\right.$$
$$\left.-\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{(cx)^nb-a}}{\sqrt{a}}\right)-\sqrt{(cx)^nb-a}\right)}{n}\right]$$

```
[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="fricas")
[Out] [(sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n)
+ 2*sqrt((c*x)^n*b - a))/n, -2*(sqrt(a)*arctan(sqrt((c*x)^n*b - a)/sqrt(a))
 - sqrt((c*x)^n*b - a))/n]
```

Sympy [F]

$$\int \frac{\sqrt{-a + b(cx)^n}}{x}\,dx = \int \frac{\sqrt{-a + b\,(cx)^n}}{x}\,dx$$

```
[In] integrate((-a+b*(c*x)**n)**(1/2)/x,x)
[Out] Integral(sqrt(-a + b*(c*x)**n)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{-a + b(cx)^n}}{x}\,dx = \int \frac{\sqrt{(cx)^n\,b - a}}{x}\,dx$$

```
[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt((c*x)^n*b - a)/x, x)
```

Giac [F]

$$\int \frac{\sqrt{-a + b(cx)^n}}{x}\,dx = \int \frac{\sqrt{(cx)^n\,b - a}}{x}\,dx$$

```
[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt((c*x)^n*b - a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a + b(cx)^n}}{x}\,dx = \int \frac{\sqrt{b\,(c\,x)^n - a}}{x}\,dx$$

```
[In] int((b*(c*x)^n - a)^(1/2)/x,x)
[Out] int((b*(c*x)^n - a)^(1/2)/x, x)
```

3.667 $\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

```
[Out] 2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {374, 12, 272, 65, 211}

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

```
[In] Int[1/(x*Sqrt[-a + b*(c*x)^n]),x]
```

```
[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{-a+bx^n}}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx^n}}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{bn}\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

```
[In] Integrate[1/(x*Sqrt[-a + b*(c*x)^n]),x]
[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n\sqrt{a}}$	27
default	$\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n\sqrt{a}}$	27

```
[In] int(1/x/(-a+b*(c*x)^n)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \left[-\frac{\sqrt{-a}\log\left(\frac{(cx)^nb-2\sqrt{(cx)^nb-a}\sqrt{-a}-2a}{(cx)^n}\right)}{an}, \frac{2\arctan\left(\frac{\sqrt{(cx)^nb-a}}{\sqrt{a}}\right)}{\sqrt{a}n}\right]$$

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="fricas")
[Out] [-sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n)/
(a*n), 2*arctan(sqrt((c*x)^n*b - a)/sqrt(a))/(sqrt(a)*n)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \int \frac{1}{x\sqrt{-a+b\,(cx)^n}}\,dx$$

```
[In] integrate(1/x/(-a+b*(c*x)**n)**(1/2),x)
[Out] Integral(1/(x*sqrt(-a + b*(c*x)**n)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \int \frac{1}{\sqrt{(cx)^n\,b-a}x}\,dx$$

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt((c*x)^n*b - a)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \int \frac{1}{\sqrt{(cx)^n\,b-a}x}\,dx$$

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt((c*x)^n*b - a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \int \frac{1}{x\,\sqrt{b\,(c\,x)^n-a}}\,dx$$

```
[In] int(1/(x*(b*(c*x)^n - a)^(1/2)),x)
[Out] int(1/(x*(b*(c*x)^n - a)^(1/2)), x)
```

3.668 $\int \frac{1}{x(-a+b(cx)^n)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx=-\frac{2}{an\sqrt{-a+b(cx)^n}}-\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

```
[Out] -2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(3/2)/n-2/a/n/(-a+b*(c*x)^n)^(1/2
)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {374, 12, 272, 53, 65, 211}

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}n}-\frac{2}{an\sqrt{b(cx)^n-a}}$$

```
[In] Int[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] -2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a
^(3/2)*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{c}{x(-a+bx^n)^{3/2}}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\left(-a+bx^n\right)^{3/2}}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{3/2}}\,dx, x, (cx)^n\right)}{n}\\
&= -\frac{2}{an\sqrt{-a+b(cx)^n}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{an}
\end{aligned}$$

$$= -\frac{2}{an\sqrt{-a+b(cx)^n}} - \frac{2\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{abn}$$

$$= -\frac{2}{an\sqrt{-a+b(cx)^n}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(-a+b(cx)^n)^{3/2}}\,dx = -\frac{2}{an\sqrt{-a+b(cx)^n}} - \frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

```
[In] Integrate[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
[Out] -2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a
^(3/2)*n)
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{-\frac{2}{a\sqrt{-a+b(cx)^n}}-\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{n}$	47
default	$\frac{-\frac{2}{a\sqrt{-a+b(cx)^n}}-\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{n}$	47

```
[In] int(1/x/(-a+b*(c*x)^n)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/n*(-2/a/(-a+b*(c*x)^n)^(1/2)-2/a^(3/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2
)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.12

$$\int \frac{1}{x(-a+b(cx)^n)^{3/2}}\,dx = \left[-\frac{\left((cx)^n\sqrt{-a}b-\sqrt{-a}a\right)\log\left(\frac{(cx)^n b+2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^n b-a}a}{(cx)^n a^2bn-a^3n}, -\frac{2\left(\left((cx)^n\sqrt{a}b-a^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)+\sqrt{(cx)^n b-a}a\right)}{(cx)^n a^2bn-a^3n}\right]$$

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(((c*x)^n*sqrt(-a)*b - sqrt(-a)*a)*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a)
*sqrt(-a) - 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b - a)*a)/((c*x)^n*a^2*b*n - a^3
*n), -2*(((c*x)^n*sqrt(a)*b - a^(3/2))*arctan(sqrt((c*x)^n*b - a)/sqrt(a))
+ sqrt((c*x)^n*b - a)*a)/((c*x)^n*a^2*b*n - a^3*n)]
```

Sympy [A] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(-a+b(cx)^n)^{3/2}}\,dx = -\begin{cases} \frac{2\left(\frac{b}{an\sqrt{-a+b(cx)^n}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}n}\right)}{b} & \text{for } b\neq 0 \\ \frac{\log\left((cx)^n\right)}{an\sqrt{-a}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/(-a+b*(c*x)**n)**(3/2),x)
```

```
[Out] -Piecewise((2*(b/(a*n*sqrt(-a + b*(c*x)**n)) + b*atan(sqrt(-a + b*(c*x)**n)
/sqrt(a))/(a**(3/2)*n))/b, Ne(b, 0)), (log((c*x)**n)/(a*n*sqrt(-a)), True))
```

Maxima [F]

$$\int \frac{1}{x(-a+b(cx)^n)^{3/2}}\,dx = \int \frac{1}{\left((cx)^n b-a\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b - a)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx=\int \frac{1}{\left((cx)^n\,b-a\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/(((c*x)^n*b - a)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx=\int \frac{1}{x\left(b\,(c\,x)^n-a\right)^{3/2}}\,dx$$

```
[In] int(1/(x*(b*(c*x)^n - a)^(3/2)),x)
[Out] int(1/(x*(b*(c*x)^n - a)^(3/2)), x)
```

3.669 $\int \frac{1}{x(-a+b(cx)^n)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx = -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{-a+b(cx)^n}} + \frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

```
[Out] -2/3/a/n/(-a+b*(c*x)^n)^(3/2)+2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(5/2
)/n+2/a^2/n/(-a+b*(c*x)^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {374, 12, 272, 53, 65, 211}

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{5/2}n} + \frac{2}{a^2n\sqrt{b(cx)^n-a}} - \frac{2}{3an\left(b(cx)^n-a\right)^{3/2}}$$

```
[In] Int[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] -2/(3*a*n*(-a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[-a + b*(c*x)^n]) + (2*Arc
Tan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{c}{x(-a+bx^n)^{5/2}}\,dx, x, cx\right)}{c}\\ &= \text{Subst}\left(\int \frac{1}{x\,(-a+bx^n)^{5/2}}\,dx, x, cx\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{5/2}}\,dx, x, (cx)^n\right)}{n}\end{aligned}$$

$$= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{3/2}}\,dx, x, (cx)^n\right)}{an}$$

$$= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{-a+b(cx)^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{a^2n}$$

$$= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{-a+b(cx)^n}} + \frac{2\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{a^2bn}$$

$$= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{-a+b(cx)^n}} + \frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx = \frac{2\left(\frac{\sqrt{a}(-4a+3b(cx)^n)}{(-a+b(cx)^n)^{3/2}} + 3\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)\right)}{3a^{5/2}n}$$

```
[In] Integrate[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] (2*((Sqrt[a]*(-4*a + 3*b*(c*x)^n))/(-a + b*(c*x)^n)^(3/2) + 3*ArcTan[Sqrt[-
a + b*(c*x)^n]/Sqrt[a]]))/(3*a^(5/2)*n)
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{-\frac{2}{3a(-a+b(cx)^n)^{\frac{3}{2}}}+\frac{2}{a^2\sqrt{-a+b(cx)^n}}+\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}}{n}$	65
default	$\frac{-\frac{2}{3a(-a+b(cx)^n)^{\frac{3}{2}}}+\frac{2}{a^2\sqrt{-a+b(cx)^n}}+\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}}{n}$	65

```
[In] int(1/x/(-a+b*(c*x)^n)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-2/3/a/(-a+b*(c*x)^n)^(3/2)+2/a^2/(-a+b*(c*x)^n)^(1/2)+2/a^(5/2)*arcta
n((-a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.42

$$\int \frac{1}{x(-a+b(cx)^n)^{5/2}}\,dx = \left[-\frac{3\left(2\,(cx)^n\sqrt{-a}ab-(cx)^{2\,n}\sqrt{-a}b^2-\sqrt{-a}a^2\right)\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2\,a}{(cx)^n}\right)}{3\left(2\,(cx)^n a^4bn-(cx)^{2\,n}a^3b^2n-a^5n\right)}\right.$$

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(2*(c*x)^n*sqrt(-a)*a*b - (c*x)^(2*n)*sqrt(-a)*b^2 - sqrt(-a)*a^2)
*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n) + 2*(3*(c*
x)^n*a*b - 4*a^2)*sqrt((c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*a^3
*b^2*n - a^5*n), 2/3*(3*(2*(c*x)^n*a^(3/2)*b - (c*x)^(2*n)*sqrt(a)*b^2 - a^
(5/2))*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) - (3*(c*x)^n*a*b - 4*a^2)*sqrt((
c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*a^3*b^2*n - a^5*n)]
```

Sympy [A] (verification not implemented)

Time = 4.69 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(-a+b(cx)^n)^{5/2}}\,dx = \begin{cases} \frac{2\left(-\frac{b}{3an(-a+b(cx)^n)^{\frac{3}{2}}}+\frac{b}{a^2n\sqrt{-a+b(cx)^n}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}n}\right)}{b} & \text{for } b\neq 0 \\ \frac{\log\left((cx)^n\right)}{a^2n\sqrt{-a}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/(-a+b*(c*x)**n)**(5/2),x)
```

```
[Out] Piecewise((2*(-b/(3*a*n*(-a + b*(c*x)**n)**(3/2)) + b/(a**2*n*sqrt(-a + b*(
c*x)**n)) + b*atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/(a**(5/2)*n))/b, Ne(b, 0)
), (log((c*x)**n)/(a**2*n*sqrt(-a)), True))
```

Maxima [F]

$$\int \frac{1}{x(-a+b(cx)^n)^{5/2}}\,dx = \int \frac{1}{\left((cx)^n b-a\right)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b - a)^(5/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx=\int \frac{1}{\left(\left(cx\right)^n b-a\right)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="giac")
[Out] integrate(1/((((c*x)^n*b - a)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx=\int \frac{1}{x\left(b\left(c\,x\right)^n-a\right)^{5/2}}\,dx$$

```
[In] int(1/(x*(b*(c*x)^n - a)^(5/2)),x)
[Out] int(1/(x*(b*(c*x)^n - a)^(5/2)), x)
```

3.670 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {65, 214}

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Int[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Integrate[1/(x*Sqrt[a + b*x]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
pseudoelliptic	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18

```
[In] int(1/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.43

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{2\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 16.70 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

3.671 $\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

```
[Out] -2*arctanh((a+b*(c*x)^m)^(1/2)/a^(1/2))/m/a^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {374, 12, 272, 65, 214}

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

```
[In] Int[1/(x*Sqrt[a + b*(c*x)^m]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^m}}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^m}}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^m\right)}{m}\\
&= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^m}\right)}{bm}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

```
[In] Integrate[1/(x*Sqrt[a + b*(c*x)^m]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{m\sqrt{a}}$	25
default	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{m\sqrt{a}}$	25

```
[In] int(1/x/(a+b*(c*x)^m)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((a+b*(c*x)^m)^(1/2)/a^(1/2))/m/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.60

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = \left[\frac{\log\left(\frac{(cx)^m b-2\sqrt{(cx)^m b+a}\sqrt{a}+2a}{(cx)^m}\right)}{\sqrt{a}m}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{(cx)^m b+a}\sqrt{-a}}{a}\right)}{am}\right]$$

```
[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="fricas")
[Out] [log(((c*x)^m*b - 2*sqrt((c*x)^m*b + a)*sqrt(a) + 2*a)/(c*x)^m)/(sqrt(a)*m)
, 2*sqrt(-a)*arctan(sqrt((c*x)^m*b + a)*sqrt(-a)/a)/(a*m)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = \int \frac{1}{x\sqrt{a+b\,(cx)^m}}\,dx$$

```
[In] integrate(1/x/(a+b*(c*x)**m)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*(c*x)**m)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = \int \frac{1}{\sqrt{(cx)^m\,b+a}\,x}\,dx$$

```
[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt((c*x)^m*b + a)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = \int \frac{1}{\sqrt{(cx)^m\,b+a}\,x}\,dx$$

```
[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt((c*x)^m*b + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = \int \frac{1}{x\,\sqrt{a+b\,{(c\,x)}^m}}\,dx$$

```
[In] int(1/(x*(a + b*(c*x)^m)^(1/2)),x)
[Out] int(1/(x*(a + b*(c*x)^m)^(1/2)), x)
```

3.672 $\int \frac{1}{x\sqrt{a+b(c(dx)^m)^n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

```
[Out] -2*arctanh((a+b*(c*(d*x)^m)^n)^(1/2)/a^(1/2))/m/n/a^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {374, 12, 272, 65, 214}

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

```
[In] Int[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx, x, (dx)^m\right)}{m}\\
&= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^n}}\,dx, x, c(dx)^m\right)}{cm}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, c(dx)^m\right)}{m}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (c(dx)^m)^n\right)}{mn}\\
&= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\,(c(dx)^m)^n}\right)}{bmn}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

```
[In] Integrate[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{mn\sqrt{a}}$	32
default	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{mn\sqrt{a}}$	32

```
[In] int(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((a+b*(c*(d*x)^m)^n)^(1/2)/a^(1/2))/m/n/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.14

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx$$
$$=\left[\frac{\log\left(\left(be^{(mn\log(dx)+n\log(c))}-2\sqrt{be^{(mn\log(dx)+n\log(c))}+a}\sqrt{a}+2\,a\right)e^{(-mn\log(dx)-n\log(c))}\right)}{\sqrt{a}mn},\frac{2\sqrt{-a}\arctan\left(\right.}{}\right.$$

```
[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="fricas")
[Out] [log((b*e^(m*n*log(d*x) + n*log(c)) - 2*sqrt(b*e^(m*n*log(d*x) + n*log(c))
+ a)*sqrt(a) + 2*a)*e^(-m*n*log(d*x) - n*log(c)))/(sqrt(a)*m*n), 2*sqrt(-a)
*arctan(sqrt(b*e^(m*n*log(d*x) + n*log(c)) + a)*sqrt(-a)/a)/(a*m*n)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx = \int \frac{1}{x\sqrt{a+b\left(c\left(dx\right)^m\right)^n}}\,dx$$

```
[In] integrate(1/x/(a+b*(c*(d*x)**m)**n)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*(c*(d*x)**m)**n)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx = \int \frac{1}{\sqrt{\left((dx)^m\,c\right)^n b + a}\,x}\,dx$$

```
[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx = \int \frac{1}{\sqrt{\left((dx)^m\,c\right)^n b + a}\,x}\,dx$$

```
[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx = \int \frac{1}{x\,\sqrt{a+b\left(c\left(d\,x\right)^m\right)^n}}\,dx$$

```
[In] int(1/(x*(a + b*(c*(d*x)^m)^n)^(1/2)),x)
[Out] int(1/(x*(a + b*(c*(d*x)^m)^n)^(1/2)), x)
```

3.673 $\int \frac{1}{x\sqrt{a+b(c(d(ex)^m)^n)^p}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 44

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}$$

```
[Out] -2*arctanh((a+b*(c*(d*(e*x)^m)^n)^p)^(1/2)/a^(1/2))/m/n/p/a^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {374, 12, 272, 65, 214}

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}$$

```
[In] Int[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(c(dx)^n)^p}}\,dx, x, (ex)^m\right)}{m} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^p}}\,dx, x, (d(ex)^m)^n\right)}{mn} \\
&= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^p}}\,dx, x, c(d(ex)^m)^n\right)}{cmn} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^p}}\,dx, x, c(d(ex)^m)^n\right)}{mn} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (c(d(ex)^m)^n)^p\right)}{mnp} \\
&= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}\right)}{bmnp} \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}$$

```
[In] Integrate[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{mnp\sqrt{a}}$	39
default	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{mnp\sqrt{a}}$	39

```
[In] int(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((a+b*(c*(d*(e*x)^m)^n)^p)^(1/2)/a^(1/2))/m/n/p/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.34

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx = \left[\frac{\log\left(\left(be^{(mnp\log(ex)+np\log(d)+p\log(c))} - 2\sqrt{be^{(mnp\log(ex)+np\log(d)+p\log(c))}+a}\sqrt{a} + 2a\right)e^{(-mnp\log(ex)-np\log(d)-p}\right.}{\sqrt{a}mnp}\right.$$

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="fricas")
[Out] [log((b*e^(m*n*p*log(e*x) + n*p*log(d) + p*log(c)) - 2*sqrt(b*e^(m*n*p*log(
e*x) + n*p*log(d) + p*log(c)) + a)*sqrt(a) + 2*a)*e^(-m*n*p*log(e*x) - n*p*
log(d) - p*log(c)))/(sqrt(a)*m*n*p), 2*sqrt(-a)*arctan(sqrt(b*e^(m*n*p*log(
e*x) + n*p*log(d) + p*log(c)) + a)*sqrt(-a)/a)/(a*m*n*p)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx = \int \frac{1}{x\sqrt{a+b\left(c\left(d\left(ex\right)^m\right)^n\right)^p}}\,dx$$

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)**m)**n)**p)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*(c*(d*(e*x)**m)**n)**p)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx = \int \frac{1}{\sqrt{\left(\left((ex)^m d\right)^n c\right)^p b+a}x}\,dx$$

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx = \int \frac{1}{\sqrt{\left(\left((ex)^m d\right)^n c\right)^p b+a}x}\,dx$$

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx = \int \frac{1}{x\,\sqrt{a+b\left(c\left(d\left(e\,x\right)^m\right)^n\right)^p}}\,dx$$

```
[In] int(1/(x*(a + b*(c*(d*(e*x)^m)^n)^p)^(1/2)),x)
[Out] int(1/(x*(a + b*(c*(d*(e*x)^m)^n)^p)^(1/2)), x)
```

3.674 $\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 51

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}mnpq}$$

```
[Out] -2*arctanh((a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)/a^(1/2))/m/n/p/q/a^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {374, 12, 272, 65, 214}

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}mnpq}$$

```
[In] Int[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt[a]*m*n*p*
q)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(c(d(ex)^n)^p)^q}}\,dx, x, (fx)^m\right)}{m}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(c(dx)^p)^q}}\,dx, x, (e(fx)^m)^n\right)}{mn}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^q}}\,dx, x, (d(e(fx)^m)^n)^p\right)}{mnp}$$

$$= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^q}}\,dx, x, c(d(e(fx)^m)^n)^p\right)}{cmnp}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^q}}\,dx, x, c(d(e(fx)^m)^n)^p\right)}{mnp}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (c(d(e(fx)^m)^n)^p)^q\right)}{mnpq}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}\right)}{bmnpq}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(d(e(fx)^m)^n)^p)^q}}{\sqrt{a}}\right)}{\sqrt{a}mnpq}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(d(e(fx)^m)^n)^p)^q}}{\sqrt{a}}\right)}{\sqrt{a}mnpq}$$

```
[In] Integrate[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt[a]*m*n*p*
q)
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(c(d(e(fx)^m)^n)^p)^q}}{\sqrt{a}}\right)}{mnpq\sqrt{a}}$	46
default	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(c(d(e(fx)^m)^n)^p)^q}}{\sqrt{a}}\right)}{mnpq\sqrt{a}}$	46

```
[In] int(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)/a^(1/2))/m/n/p/q/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.57

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx$$
$$= \left[\frac{\log\left(\left(be^{(mnpq\log(fx)+npq\log(e)+pq\log(d)+q\log(c))} - 2\sqrt{be^{(mnpq\log(fx)+npq\log(e)+pq\log(d)+q\log(c))}+a}\sqrt{a}+2a\right)e^{(-}\right.}{\sqrt{a}mnpq}\right.$$

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="fricas")
```

[Out] [log((b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*log(d) + q*log(c)) - 2*sqr
t(b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*log(d) + q*log(c)) + a)*sqrt(a
) + 2*a)*e^(-m*n*p*q*log(f*x) - n*p*q*log(e) - p*q*log(d) - q*log(c)))/(sqr
t(a)*m*n*p*q), 2*sqrt(-a)*arctan(sqrt(b*e^(m*n*p*q*log(f*x) + n*p*q*log(e)
+ p*q*log(d) + q*log(c)) + a)*sqrt(-a)/a)/(a*m*n*p*q)]

Sympy [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx=\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e\left(fx\right)^m\right)^n\right)^p\right)^q}}\,dx$$

[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)**m)**n)**p)**q)**(1/2),x)

[Out] Integral(1/(x*sqrt(a + b*(c*(d*(e*(f*x)**m)**n)**p)**q)), x)

Maxima [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx=\int \frac{1}{\sqrt{\left(\left(\left((fx)^m e\right)^n d\right)^p c\right)^q b+a}x}\,dx$$

[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(((((f*x)^m*e)^n*d)^p*c)^q*b + a)*x), x)

Giac [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx=\int \frac{1}{\sqrt{\left(\left(\left((fx)^m e\right)^n d\right)^p c\right)^q b+a}x}\,dx$$

[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(((((f*x)^m*e)^n*d)^p*c)^q*b + a)*x), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx=\int \frac{1}{x\,\sqrt{a+b\,{\left(c\,{\left(d\,{\left(e\,{\left(f\,x\right)}^m\right)}^n\right)}^p\right)}^q}} \,d x$$

```
[In] int(1/(x*(a + b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)),x)
[Out] int(1/(x*(a + b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)), x)
```

3.675 $\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x} dx = \frac{35}{16}\sqrt{-1+\frac{1}{x^2}} - \frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 - \frac{35}{16}\arctan\left(\sqrt{-1+\frac{1}{x^2}}\right)$$

```
[Out] -35/48*(-1+1/x^2)^(3/2)*x^2-7/24*(-1+1/x^2)^(5/2)*x^4-1/6*(-1+1/x^2)^(7/2)*
x^6-35/16*arctan((-1+1/x^2)^(1/2))+35/16*(-1+1/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {25, 272, 43, 52, 65, 209}

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x} dx = -\frac{35}{16}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right) - \frac{35}{48}\left(\frac{1}{x^2}-1\right)^{3/2} x^2 + \frac{35}{16}\sqrt{\frac{1}{x^2}-1} - \frac{1}{6}\left(\frac{1}{x^2}-1\right)^{7/2} x^6 - \frac{7}{24}\left(\frac{1}{x^2}-1\right)^{5/2} x^4$$

```
[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

```
[Out] (35*Sqrt[-1 + x^(-2)])/16 - (35*(-1 + x^(-2))^(3/2)*x^2)/48 - (7*(-1 + x^(-
2))^(5/2)*x^4)/24 - ((-1 + x^(-2))^(7/2)*x^6)/6 - (35*ArcTan[Sqrt[-1 + x^(-
2)]])/16
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\int \left(-1+\frac{1}{x^2}\right)^{7/2} x^5\, dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^{7/2}}{x^4}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2}x^6 + \frac{7}{12}\text{Subst}\left(\int \frac{(-1+x)^{5/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2}x^6 + \frac{35}{48}\text{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2}x^6 + \frac{35}{32}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, \frac{1}{x^2}\right)$$

$$= \frac{35}{16}\sqrt{-1+\frac{1}{x^2}} - \frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2}x^6 - \frac{35}{32}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x}\,dx, x, \frac{1}{x^2}\right)$$

$$= \frac{35}{16}\sqrt{-1+\frac{1}{x^2}} - \frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2}x^6 - \frac{35}{16}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\frac{1}{x^2}}\right)$$

$$= \frac{35}{16}\sqrt{-1+\frac{1}{x^2}} - \frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2}x^6 - \frac{35}{16}\tan^{-1}\left(\sqrt{-1+\frac{1}{x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x}\,dx = \frac{1}{48}\sqrt{-1+\frac{1}{x^2}}(48+87x^2-38x^4+8x^6) - \frac{35\sqrt{-1+\frac{1}{x^2}}x\text{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)}{8\sqrt{-1+x^2}}$$

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(48 + 87*x^2 - 38*x^4 + 8*x^6))/48 - (35*Sqrt[-1 + x^(-2
)]*x*ArcTanh[Sqrt[-1 + x^2]/(-1 + x)])/(8*Sqrt[-1 + x^2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

method	result	size
trager	$2\left(\frac{1}{12}x^{6}-\frac{19}{48}x^{4}+\frac{29}{32}x^{2}+\frac{1}{2}\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}-\frac{35\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\left(\sqrt{-\frac{x^{2}-1}{x^{2}}}+\mathrm{RootOf}\left(\_Z^{2}+1\right)\right)x\right)}{16}$	63
risch	$\frac{\left(8x^{8}-46x^{6}+125x^{4}-39x^{2}-48\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}}{48x^{2}-48}-\frac{35\arcsin\left(x\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}\,x\sqrt{-x^{2}+1}}{16\left(x^{2}-1\right)}$	78
default	$\frac{\sqrt{-\frac{x^{2}-1}{x^{2}}}\left(-8x^{4}\left(-x^{2}+1\right)^{\frac{3}{2}}+30x^{2}\left(-x^{2}+1\right)^{\frac{3}{2}}+48\left(-x^{2}+1\right)^{\frac{3}{2}}+105x^{2}\sqrt{-x^{2}+1}+105\arcsin\left(x\right)x\right)}{48\sqrt{-x^{2}+1}}$	83

```
[In] int((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/12*x^6-19/48*x^4+29/32*x^2+1/2)*(-(x^2-1)/x^2)^(1/2)-35/16*RootOf(_Z^2
+1)*ln(((-(x^2-1)/x^2)^(1/2)+RootOf(_Z^2+1))*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^3}{x}\,dx=\frac{1}{48}\left(8\,x^{6}-38\,x^{4}+87\,x^{2}+48\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}-\frac{35}{8}\arctan\left(\frac{x\sqrt{-\frac{x^{2}-1}{x^{2}}}-1}{x}\right)$$

```
[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/48*(8*x^6 - 38*x^4 + 87*x^2 + 48)*sqrt(-(x^2 - 1)/x^2) - 35/8*arctan((x*s
qrt(-(x^2 - 1)/x^2) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 51.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x}\,dx = -\frac{x^6\left(-1+\frac{1}{x^2}\right)^{\frac{3}{2}}}{6} - \frac{5x^4\sqrt{-1+\frac{1}{x^2}}\cdot\left(2-\frac{1}{x^2}\right)}{16} + \frac{3x^2\sqrt{-1+\frac{1}{x^2}}}{2} + \sqrt{-1+\frac{1}{x^2}} - \frac{35\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{16}$$

```
[In] integrate((x**2-1)**3*(-1+1/x**2)**(1/2)/x,x)
[Out] -x**6*(-1 + x**(-2))**(3/2)/6 - 5*x**4*sqrt(-1 + x**(-2))*(2 - 1/x**2)/16 +
 3*x**2*sqrt(-1 + x**(-2))/2 + sqrt(-1 + x**(-2)) - 35*atan(sqrt(-1 + x**(-
2)))/16
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(56) = 112.

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x}\,dx = \frac{3}{2}x^2\sqrt{\frac{1}{x^2}-1} + \sqrt{\frac{1}{x^2}-1} - \frac{3\left(\frac{1}{x^2}-1\right)^{\frac{5}{2}} + 8\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}} - 3\sqrt{\frac{1}{x^2}-1}}{48\left(\left(\frac{1}{x^2}-1\right)^3 + 3\left(\frac{1}{x^2}-1\right)^2 + \frac{3}{x^2} - 2\right)} + \frac{3\left(\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}} - \sqrt{\frac{1}{x^2}-1}\right)}{8\left(\left(\frac{1}{x^2}-1\right)^2 + \frac{2}{x^2} - 1\right)} - \frac{35}{16}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

```
[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
[Out] 3/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 1/48*(3*(1/x^2 - 1)^(5/2) + 8*(
1/x^2 - 1)^(3/2) - 3*sqrt(1/x^2 - 1))/((1/x^2 - 1)^3 + 3*(1/x^2 - 1)^2 + 3/
x^2 - 2) + 3/8*((1/x^2 - 1)^(3/2) - sqrt(1/x^2 - 1))/((1/x^2 - 1)^2 + 2/x^2
 - 1) - 35/16*arctan(sqrt(1/x^2 - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x}\,dx = \frac{1}{48}\left(2\left(4\,x^2\mathrm{sgn}(x)-19\,\mathrm{sgn}(x)\right)x^2+87\,\mathrm{sgn}(x)\right)\sqrt{-x^2+1}x + \frac{35}{16}\arcsin\left(x\right)\mathrm{sgn}(x) - \frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} + \frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

```
[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2*sgn(x) - 19*sgn(x))*x^2 + 87*sgn(x))*sqrt(-x^2 + 1)*x + 35/1
6*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1
) - 1)*sgn(x)/x
```

Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x}\,dx = \sqrt{\frac{1}{x^2}-1} - \frac{35\,\mathrm{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{16} + \frac{19\,x^6\,\sqrt{\frac{1}{x^2}-1}}{16} + \frac{17\,x^6\,\left(\frac{1}{x^2}-1\right)^{3/2}}{6} + \frac{29\,x^6\,\left(\frac{1}{x^2}-1\right)^{5/2}}{16}$$

```
[In] int(((1/x^2 - 1)^(1/2)*(x^2 - 1)^3)/x,x)
```

```
[Out] (1/x^2 - 1)^(1/2) - (35*atan((1/x^2 - 1)^(1/2)))/16 + (19*x^6*(1/x^2 - 1)^(
1/2))/16 + (17*x^6*(1/x^2 - 1)^(3/2))/6 + (29*x^6*(1/x^2 - 1)^(5/2))/16
```

3.676 $\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x} dx = -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}}+\frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4+\frac{15}{8}\arctan\left(\sqrt{-1+\frac{1}{x^2}}\right)$$

```
[Out] 5/8*(-1+1/x^2)^(3/2)*x^2+1/4*(-1+1/x^2)^(5/2)*x^4+15/8*arctan((-1+1/x^2)^(1
/2))-15/8*(-1+1/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {25, 272, 43, 52, 65, 209}

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x} dx = \frac{15}{8}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)+\frac{5}{8}\left(\frac{1}{x^2}-1\right)^{3/2}x^2 - \frac{15}{8}\sqrt{\frac{1}{x^2}-1}+\frac{1}{4}\left(\frac{1}{x^2}-1\right)^{5/2}x^4$$

```
[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

```
[Out] (-15*Sqrt[-1 + x^(-2)])/8 + (5*(-1 + x^(-2))^(3/2)*x^2)/8 + ((-1 + x^(-2))^
(5/2)*x^4)/4 + (15*ArcTan[Sqrt[-1 + x^(-2)]])/8
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \int \left(-1 + \frac{1}{x^2}\right)^{5/2} x^3 \, dx$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^{5/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4 - \frac{5}{8}\text{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$

$$= \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4 - \frac{15}{16}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}} + \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4 + \frac{15}{16}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}} + \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4 + \frac{15}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\frac{1}{x^2}}\right)$$

$$= -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}} + \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4 + \frac{15}{8}\tan^{-1}\left(\sqrt{-1+\frac{1}{x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x}\,dx = \frac{1}{8}\sqrt{-1+\frac{1}{x^2}}(-8-9x^2+2x^4) + \frac{15\sqrt{-1+\frac{1}{x^2}}x\,\text{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)}{4\sqrt{-1+x^2}}$$

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(-8 - 9*x^2 + 2*x^4))/8 + (15*Sqrt[-1 + x^(-2)]*x*ArcTan
h[Sqrt[-1 + x^2]/(-1 + x)])/(4*Sqrt[-1 + x^2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.99 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

method	result	size
trager	$2\left(\frac{1}{8}x^4-\frac{9}{16}x^2-\frac{1}{2}\right)\sqrt{-\frac{x^2-1}{x^2}}+\frac{15\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\left(\sqrt{-\frac{x^2-1}{x^2}}+\mathrm{RootOf}\left(\_Z^2+1\right)\right)x\right)}{8}$	58
default	$-\frac{\sqrt{-\frac{x^2-1}{x^2}}\left(2x^2(-x^2+1)^{\frac{3}{2}}+8(-x^2+1)^{\frac{3}{2}}+15x^2\sqrt{-x^2+1}+15\arcsin(x)x\right)}{8\sqrt{-x^2+1}}$	69
risch	$\frac{(2x^6-11x^4+x^2+8)\sqrt{-\frac{x^2-1}{x^2}}}{8x^2-8}+\frac{15\arcsin(x)\sqrt{-\frac{x^2-1}{x^2}}\,x\sqrt{-x^2+1}}{8(x^2-1)}$	71

```
[In] int((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 2*(1/8*x^4-9/16*x^2-1/2)*(-(x^2-1)/x^2)^(1/2)+15/8*RootOf(_Z^2+1)*ln(((-(x^
2-1)/x^2)^(1/2)+RootOf(_Z^2+1))*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x}\,dx=\frac{1}{8}\left(2\,x^4-9\,x^2-8\right)\sqrt{-\frac{x^2-1}{x^2}}+\frac{15}{4}\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

```
[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
[Out] 1/8*(2*x^4 - 9*x^2 - 8)*sqrt(-(x^2 - 1)/x^2) + 15/4*arctan((x*sqrt(-(x^2 -
1)/x^2) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 23.56 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x}\,dx=\frac{x^4\sqrt{-1+\frac{1}{x^2}}\cdot\left(2-\frac{1}{x^2}\right)}{8}-x^2\sqrt{-1+\frac{1}{x^2}}-\sqrt{-1+\frac{1}{x^2}}+\frac{15\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{8}$$

```
[In] integrate((x**2-1)**2*(-1+1/x**2)**(1/2)/x,x)
[Out] x**4*sqrt(-1 + x**(-2))*(2 - 1/x**2)/8 - x**2*sqrt(-1 + x**(-2)) - sqrt(-1
+ x**(-2)) + 15*atan(sqrt(-1 + x**(-2)))/8
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x}\,dx = -x^2\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}-1}-\frac{\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-\sqrt{\frac{1}{x^2}-1}}{8\left(\left(\frac{1}{x^2}-1\right)^2+\frac{2}{x^2}-1\right)} + \frac{15}{8}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

```
[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -x^2*sqrt(1/x^2 - 1) - sqrt(1/x^2 - 1) - 1/8*((1/x^2 - 1)^(3/2) - sqrt(1/x^
2 - 1))/((1/x^2 - 1)^2 + 2/x^2 - 1) + 15/8*arctan(sqrt(1/x^2 - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x}\,dx = \frac{1}{8}\left(2x^2\mathrm{sgn}(x)-9\,\mathrm{sgn}(x)\right)\sqrt{-x^2+1}x-\frac{15}{8}\arcsin(x)\,\mathrm{sgn}(x) + \frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}-\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2x}$$

```
[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*x^2*sgn(x) - 9*sgn(x))*sqrt(-x^2 + 1)*x - 15/8*arcsin(x)*sgn(x) + 1/
2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

Mupad [B] (verification not implemented)

Time = 17.65 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x}\,dx = \frac{15\,\mathrm{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{8}-\sqrt{\frac{1}{x^2}-1} - \frac{7\,x^4\,\sqrt{\frac{1}{x^2}-1}}{8}-\frac{9\,x^4\,\left(\frac{1}{x^2}-1\right)^{3/2}}{8}$$

```
[In] int(((1/x^2 - 1)^(1/2)*(x^2 - 1)^2)/x,x)
```

```
[Out] (15*atan((1/x^2 - 1)^(1/2)))/8 - (1/x^2 - 1)^(1/2) - (7*x^4*(1/x^2 - 1)^(1/
2))/8 - (9*x^4*(1/x^2 - 1)^(3/2))/8
```

3.677 $\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 44

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x} dx = \frac{3}{2}\sqrt{-1+\frac{1}{x^2}} - \frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 - \frac{3}{2}\arctan\left(\sqrt{-1+\frac{1}{x^2}}\right)$$

```
[Out] -1/2*(-1+1/x^2)^(3/2)*x^2-3/2*arctan((-1+1/x^2)^(1/2))+3/2*(-1+1/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {25, 272, 43, 52, 65, 209}

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x} dx = -\frac{3}{2}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right) - \frac{1}{2}\left(\frac{1}{x^2}-1\right)^{3/2} x^2 + \frac{3}{2}\sqrt{\frac{1}{x^2}-1}$$

```
[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
```

```
[Out] (3*Sqrt[-1 + x^(-2)])/2 - ((-1 + x^(-2))^(3/2)*x^2)/2 - (3*ArcTan[Sqrt[-1 +
 x^(-2)]])/2
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \left(-1+\frac{1}{x^2}\right)^{3/2} x\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 + \frac{3}{4}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, \frac{1}{x^2}\right)
\end{aligned}$$

$$= \frac{3}{2}\sqrt{-1+\frac{1}{x^2}} - \frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2 - \frac{3}{4}\text{Subst}\left(\int \frac{1}{\sqrt{-1+xx}}\,dx, x, \frac{1}{x^2}\right)$$

$$= \frac{3}{2}\sqrt{-1+\frac{1}{x^2}} - \frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2 - \frac{3}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\frac{1}{x^2}}\right)$$

$$= \frac{3}{2}\sqrt{-1+\frac{1}{x^2}} - \frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2 - \frac{3}{2}\tan^{-1}\left(\sqrt{-1+\frac{1}{x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \frac{1}{2}\sqrt{-1+\frac{1}{x^2}}\left(2+x^2-\frac{6x\text{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)}{\sqrt{-1+x^2}}\right)$$

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
[Out] (Sqrt[-1 + x^(-2)]*(2 + x^2 - (6*x*ArcTanh[Sqrt[-1 + x^2]/(-1 + x)])/Sqrt[-
1 + x^2]))/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

method	result	size
trager	$2\left(\frac{x^2}{4}+\frac{1}{2}\right)\sqrt{-\frac{x^2-1}{x^2}} - \frac{3\,\text{RootOf}\left(\_Z^2+1\right)\ln\left(\left(\sqrt{-\frac{x^2-1}{x^2}}+\text{RootOf}\left(\_Z^2+1\right)\right)x\right)}{2}$	53
default	$\frac{\sqrt{-\frac{x^2-1}{x^2}}\left(2\left(-x^2+1\right)^{\frac{3}{2}}+3x^2\sqrt{-x^2+1}+3\arcsin\left(x\right)x\right)}{2\sqrt{-x^2+1}}$	55
risch	$\frac{\left(x^4+x^2-2\right)\sqrt{-\frac{x^2-1}{x^2}}}{2x^2-2} - \frac{3\arcsin\left(x\right)\sqrt{-\frac{x^2-1}{x^2}}\,x\sqrt{-x^2+1}}{2\left(x^2-1\right)}$	64

```
[In] int((x^2-1)*(-1+1/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 2*(1/4*x^2+1/2)*(-(x^2-1)/x^2)^(1/2)-3/2*RootOf(_Z^2+1)*ln(((-(x^2-1)/x^2)^
(1/2)+RootOf(_Z^2+1))*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \frac{1}{2}\left(x^2+2\right)\sqrt{-\frac{x^2-1}{x^2}} - 3\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

```
[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
[Out] 1/2*(x^2 + 2)*sqrt(-(x^2 - 1)/x^2) - 3*arctan((x*sqrt(-(x^2 - 1)/x^2) - 1)/
x)
```

Sympy [A] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \frac{x^2\sqrt{-1+\frac{1}{x^2}}}{2} + \sqrt{-1+\frac{1}{x^2}} - \frac{3\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{2}$$

```
[In] integrate((x**2-1)*(-1+1/x**2)**(1/2)/x,x)
[Out] x**2*sqrt(-1 + x**(-2))/2 + sqrt(-1 + x**(-2)) - 3*atan(sqrt(-1 + x**(-2)))
/2
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \frac{1}{2}\,x^2\sqrt{\frac{1}{x^2}-1} + \sqrt{\frac{1}{x^2}-1} - \frac{3}{2}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

```
[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
[Out] 1/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 3/2*arctan(sqrt(1/x^2 - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \frac{1}{2}\sqrt{-x^2+1}x\text{sgn}(x) + \frac{3}{2}\arcsin(x)\,\text{sgn}(x) - \frac{x\text{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} + \frac{\left(\sqrt{-x^2+1}-1\right)\text{sgn}(x)}{2\,x}$$

```
[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*sgn(x) + 3/2*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x^
2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

Mupad [B] (verification not implemented)

Time = 18.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \sqrt{\frac{1}{x^2}-1} - \frac{3\,\text{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{2} + \frac{x^2\sqrt{\frac{1}{x^2}-1}}{2}$$

```
[In] int(((1/x^2 - 1)^(1/2)*(x^2 - 1))/x,x)
```

```
[Out] (1/x^2 - 1)^(1/2) - (3*atan((1/x^2 - 1)^(1/2)))/2 + (x^2*(1/x^2 - 1)^(1/2))
/2
```

3.678 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 9

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx=\sqrt{-1+\frac{1}{x^2}}$$

```
[Out] (-1+1/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {25, 267}

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx=\sqrt{\frac{1}{x^2}-1}$$

```
[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]
```

```
[Out] Sqrt[-1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.), 
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d, 
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\int \frac{1}{\sqrt{-1+\frac{1}{x^2}}x^3}\,dx$$
$$= \sqrt{-1+\frac{1}{x^2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)}\,dx = \sqrt{-1+\frac{1}{x^2}}$$

```
[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]
[Out] Sqrt[-1 + x^(-2)]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

method	result	size
gosper	$\sqrt{-\frac{x^2-1}{x^2}}$	13
default	$\sqrt{-\frac{x^2-1}{x^2}}$	13
trager	$\sqrt{-\frac{x^2-1}{x^2}}$	13
risch	$\sqrt{-\frac{x^2-1}{x^2}}$	13

```
[In] int((-1+1/x^2)^(1/2)/x/(x^2-1),x,method=_RETURNVERBOSE)
[Out] (-(x^2-1)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx=\sqrt{-\frac{x^2-1}{x^2}}$$

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="fricas")
[Out] sqrt(-(x^2 - 1)/x^2)
```

Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx=\sqrt{-1+\frac{1}{x^2}}$$

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1),x)
[Out] sqrt(-1 + x**(-2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(7) = 14$.

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx=\frac{\sqrt{x+1}\sqrt{-x+1}}{x}$$

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="maxima")
[Out] sqrt(x + 1)*sqrt(-x + 1)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(7) = 14$.

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 4.11

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx = -\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}+\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="giac")
[Out] -1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx = \frac{\sqrt{1-x^2}}{|x|}$$

```
[In] int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)),x)
[Out] (1 - x^2)^(1/2)/abs(x)
```

3.679 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 21

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx=\frac{1}{\sqrt{-1+\frac{1}{x^2}}}-\sqrt{-1+\frac{1}{x^2}}$$

```
[Out] 1/(-1+1/x^2)^(1/2)-(-1+1/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {25, 272, 45}

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx=\frac{1}{\sqrt{\frac{1}{x^2}-1}}-\sqrt{\frac{1}{x^2}-1}$$

```
[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]
```

```
[Out] 1/Sqrt[-1 + x^(-2)] - Sqrt[-1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\left(-1+\frac{1}{x^2}\right)^{3/2} x^5}\,dx \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x}{(-1+x)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1+x)^{3/2}}+\frac{1}{\sqrt{-1+x}}\right)\,dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{1}{\sqrt{-1+\frac{1}{x^2}}}-\sqrt{-1+\frac{1}{x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx = \frac{\sqrt{-1+\frac{1}{x^2}}(1-2x^2)}{-1+x^2}$$

```
[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]
[Out] (Sqrt[-1 + x^(-2)]*(1 - 2*x^2))/(-1 + x^2)
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

method	result	size
gosper	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$	29
trager	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$	29
risch	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$	29
default	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{(x-1)(x+1)}$	32

```
[In] int((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x,method=_RETURNVERBOSE)
[Out] -(2*x^2-1)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\,(-1+x^2)^2}\,dx = -\frac{(2\,x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$$

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="fricas")
[Out] -(2*x^2 - 1)*sqrt(-(x^2 - 1)/x^2)/(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\,(-1+x^2)^2}\,dx = -\sqrt{-1+\frac{1}{x^2}} + \frac{1}{\sqrt{-1+\frac{1}{x^2}}}$$

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**2,x)
[Out] -sqrt(-1 + x**(-2)) + 1/sqrt(-1 + x**(-2))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\,(-1+x^2)^2}\,dx = -\frac{(2\,x^2-1)\sqrt{x+1}\sqrt{-x+1}}{x^3-x}$$

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -(2*x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)/(x^3 - x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(17) = 34.

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.76

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\,(-1+x^2)^2}\,dx = -\frac{\sqrt{-x^2+1}x\mathrm{sgn}(x)}{x^2-1} + \frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} - \frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)*x*sgn(x)/(x^2 - 1) + 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1/
2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

Mupad [B] (verification not implemented)

Time = 18.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\,(-1+x^2)^2}\,dx = \frac{x\,\sqrt{\frac{1}{x^2}-1}\,(2\,x^2-1)}{x-x^3}$$

```
[In] int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)^2),x)
```

```
[Out] (x*(1/x^2 - 1)^(1/2)*(2*x^2 - 1))/(x - x^3)
```

3.680 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3}\,dx = -\frac{1}{3\left(-1+\frac{1}{x^2}\right)^{3/2}} - \frac{2}{\sqrt{-1+\frac{1}{x^2}}} + \sqrt{-1+\frac{1}{x^2}}$$

```
[Out] -1/3/(-1+1/x^2)^(3/2)-2/(-1+1/x^2)^(1/2)+(-1+1/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {25, 272, 45}

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3}\,dx = \sqrt{\frac{1}{x^2}-1} - \frac{2}{\sqrt{\frac{1}{x^2}-1}} - \frac{1}{3\left(\frac{1}{x^2}-1\right)^{3/2}}$$

```
[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
```

```
[Out] -1/3*1/(-1 + x^(-2))^(3/2) - 2/Sqrt[-1 + x^(-2)] + Sqrt[-1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{1}{\left(-1+\frac{1}{x^2}\right)^{5/2} x^7}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(-1+x)^{5/2}}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1+x)^{5/2}}+\frac{2}{(-1+x)^{3/2}}+\frac{1}{\sqrt{-1+x}}\right)dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{3\left(-1+\frac{1}{x^2}\right)^{3/2}}-\frac{2}{\sqrt{-1+\frac{1}{x^2}}}+\sqrt{-1+\frac{1}{x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx = \frac{\sqrt{-1+\frac{1}{x^2}}\left(3-12x^2+8x^4\right)}{3\left(-1+x^2\right)^2}$$

```
[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
[Out] (Sqrt[-1 + x^(-2)]*(3 - 12*x^2 + 8*x^4))/(3*(-1 + x^2)^2)
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{(8x^4-12x^2+3)\sqrt{-\frac{x^2-1}{x^2}}}{3(x^2-1)^2}$	34
trager	$\frac{(8x^4-12x^2+3)\sqrt{-\frac{x^2-1}{x^2}}}{3(x^2-1)^2}$	34
risch	$\frac{(8x^4-12x^2+3)\sqrt{-\frac{x^2-1}{x^2}}}{3(x^2-1)^2}$	34
default	$\frac{(8x^4-12x^2+3)\sqrt{-\frac{x^2-1}{x^2}}}{3(x-1)^2(x+1)^2}$	37

```
[In] int((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x,method=_RETURNVERBOSE)
[Out] 1/3*(8*x^4-12*x^2+3)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3}\,dx = \frac{(8\,x^4-12\,x^2+3)\sqrt{-\frac{x^2-1}{x^2}}}{3\,(x^4-2\,x^2+1)}$$

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="fricas")
[Out] 1/3*(8*x^4 - 12*x^2 + 3)*sqrt(-(x^2 - 1)/x^2)/(x^4 - 2*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3}\,dx = \sqrt{-1+\frac{1}{x^2}} - \frac{2}{\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{3\left(-1+\frac{1}{x^2}\right)^{\frac{3}{2}}}$$

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**3,x)
[Out] sqrt(-1 + x**(-2)) - 2/sqrt(-1 + x**(-2)) - 1/(3*(-1 + x**(-2))**(3/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx=\frac{\left(8\,x^4-12\,x^2+3\right)\sqrt{x+1}\sqrt{-x+1}}{3\,\left(x^5-2\,x^3+x\right)}$$

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="maxima")
[Out] 1/3*(8*x^4 - 12*x^2 + 3)*sqrt(x + 1)*sqrt(-x + 1)/(x^5 - 2*x^3 + x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(26) = 52.

Time = 0.49 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx=-\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}+\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}-\frac{\left(5\,x^2\mathrm{sgn}(x)-6\,\mathrm{sgn}(x)\right)x}{3\,\left(x^2-1\right)\sqrt{-x^2+1}}$$

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="giac")
[Out] -1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x - 1/
3*(5*x^2*sgn(x) - 6*sgn(x))*x/((x^2 - 1)*sqrt(-x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 18.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx=\frac{\sqrt{\frac{1}{x^2}-1}\,\left(8\,x^4-12\,x^2+3\right)}{3\,{\left(x^2-1\right)}^2}$$

```
[In] int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)^3),x)
[Out] ((1/x^2 - 1)^(1/2)*(8*x^4 - 12*x^2 + 3))/(3*(x^2 - 1)^2)
```

3.681 $\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 9

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

```
[Out] 1/(1+1/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {25, 267}

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

```
[In] Int[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
```

```
[Out] 1/Sqrt[1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\left(1+\frac{1}{x^2}\right)^{3/2} x^3}\, dx\\ &= \frac{1}{\sqrt{1+\frac{1}{x^2}}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 20 vs. $2(9) = 18$.

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\, dx = \frac{\sqrt{1+\frac{1}{x^2}}x^2}{1+x^2}$$

```
[In] Integrate[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
[Out] (Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(7) = 14$.

Time = 0.98 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.56

method	result	size
gosper	$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}}{x^2+1}$	23
default	$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}}{x^2+1}$	23
risch	$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}}{x^2+1}$	23
trager	$\frac{x^2\sqrt{-\frac{-x^2-1}{x^2}}}{x^2+1}$	26

```
[In] int(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/(x^2+1)*x^2*((x^2+1)/x^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(7) = 14$.

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 3.11

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{x^2\sqrt{\frac{x^2+1}{x^2}}+x^2+1}{x^2+1}$$

```
[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="fricas")
[Out] (x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{x}{\sqrt{x^2+1}}$$

```
[In] integrate(x*(1+1/x**2)**(1/2)/(x**2+1)**2,x)
[Out] x/sqrt(x**2 + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$$

```
[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/sqrt((x^2 + 1)/x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{x\mathrm{sgn}(x)}{\sqrt{x^2+1}}$$

```
[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="giac")
[Out] x*sgn(x)/sqrt(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 18.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{x^2\sqrt{\frac{1}{x^2}+1}}{x^2+1}$$

```
[In] int((x*(1/x^2 + 1)^(1/2))/(x^2 + 1)^2,x)
[Out] (x^2*(1/x^2 + 1)^(1/2))/(x^2 + 1)
```

3.682 $\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x(1+x^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 9

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x(1+x^2)}\,dx = \frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

```
[Out] 1/(1+1/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {25, 267}

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x(1+x^2)}\,dx = \frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

```
[In] Int[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]
```

```
[Out] 1/Sqrt[1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\left(1+\frac{1}{x^2}\right)^{3/2} x^3}\,dx\\ &= \frac{1}{\sqrt{1+\frac{1}{x^2}}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\left(1+x^2\right)}\,dx = \frac{\sqrt{1+\frac{1}{x^2}}x^2}{1+x^2}$$

```
[In] Integrate[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]
[Out] (Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

method	result	size
gosper	$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$	12
default	$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$	12
risch	$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$	12
trager	$\frac{x^2\sqrt{-\frac{-x^2-1}{x^2}}}{x^2+1}$	26

```
[In] int(1/x/(x^2+1)/(1+1/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/((x^2+1)/x^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(7) = 14$.

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 3.11

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\left(1+x^2\right)}\,dx = \frac{x^2\sqrt{\frac{x^2+1}{x^2}}+x^2+1}{x^2+1}$$

```
[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="fricas")
[Out] (x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\left(1+x^2\right)}\,dx = \frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

```
[In] integrate(1/x/(x**2+1)/(1+1/x**2)**(1/2),x)
[Out] 1/sqrt(1 + x**(-2))
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\left(1+x^2\right)}\,dx = \int \frac{1}{(x^2+1)x\sqrt{\frac{1}{x^2}+1}}\,dx$$

```
[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 1)*x*sqrt(1/x^2 + 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\left(1+x^2\right)}\,dx = \frac{x}{\sqrt{x^2+1}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="giac")
[Out] x/(sqrt(x^2 + 1)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 18.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\left(1+x^2\right)}\,dx=\frac{x^2\,\sqrt{\frac{1}{x^2}+1}}{x^2+1}$$

```
[In] int(1/(x*(1/x^2 + 1)^(1/2)*(x^2 + 1)),x)
[Out] (x^2*(1/x^2 + 1)^(1/2))/(x^2 + 1)
```

3.683 $\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx = \frac{\log\left(1+\sqrt{a+bx^2}\right)}{b}$$

```
[Out] ln(1+(b*x^2+a)^(1/2))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2186, 31}

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx = \frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

```
[In] Int[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]
```

```
[Out] Log[1 + Sqrt[a + b*x^2]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
```

```
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{1}{2}\text{Subst}\left(\int\frac{1}{a+bx+\sqrt{a+bx}}\,dx,x,x^2\right)\\&=\frac{\text{Subst}\left(\int\frac{1}{1+x}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\&=\frac{\log\left(1+\sqrt{a+bx^2}\right)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int\frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx=\frac{\log\left(b+b\sqrt{a+bx^2}\right)}{b}$$

```
[In] Integrate[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]
[Out] Log[b + b*Sqrt[a + b*x^2]]/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 894 vs. 2(16) = 32.

Time = 0.09 (sec) , antiderivative size = 895, normalized size of antiderivative = 49.72

method	result
default	$-\frac{\sqrt{b\left(x-\frac{\sqrt{-ab}}{b}\right)^2+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}+\frac{\sqrt{-ab}\,\ln\left(\frac{\left(x-\frac{\sqrt{-ab}}{b}\right)b+\sqrt{-ab}}{\sqrt{b}}+\sqrt{b\left(x-\frac{\sqrt{-ab}}{b}\right)^2+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}\right)}{\sqrt{b}}}{2\left(\sqrt{-(a-1)b}+\sqrt{-ab}\right)\left(-\sqrt{-(a-1)b}+\sqrt{-ab}\right)}-\frac{\sqrt{b\left(x+\frac{\sqrt{-ab}}{b}\right)^2}}{}$

```
[In] int(x/(a+b*x^2+(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*((b*(
x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/
2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))
^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2/((-(a-1)*b)^(1/
2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*((b*(x+1/b*(-a*b)^(1/2))^
2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-(-a*b)^(1/2)*ln(((x+1/b*(-a*b)
^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x
```

```
+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))+1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(
-(a-1)*b)^(1/2)+(-a*b)^(1/2))*((b*(x+(-(a-1)*b)^(1/2)/b)^2-2*(-(a-1)*b)^(1/
2)*(x+(-(a-1)*b)^(1/2)/b)+1)^(1/2)-(-(a-1)*b)^(1/2)*ln(((x+(-(a-1)*b)^(1/2)
/b)*b-(-(a-1)*b)^(1/2))/b^(1/2)+(b*(x+(-(a-1)*b)^(1/2)/b)^2-2*(-(a-1)*b)^(1
/2)*(x+(-(a-1)*b)^(1/2)/b)+1)^(1/2))/b^(1/2)-arctanh(1/2*(2-2*(-(a-1)*b)^(1
/2)*(x+(-(a-1)*b)^(1/2)/b))/(b*(x+(-(a-1)*b)^(1/2)/b)^2-2*(-(a-1)*b)^(1/2)*
(x+(-(a-1)*b)^(1/2)/b)+1)^(1/2)))+1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(
a-1)*b)^(1/2)+(-a*b)^(1/2))*((b*(x-(-(a-1)*b)^(1/2)/b)^2+2*(-(a-1)*b)^(1/2)
*(x-(-(a-1)*b)^(1/2)/b)+1)^(1/2)+(-(a-1)*b)^(1/2)*ln(((x-(-(a-1)*b)^(1/2)/b
)*b+(-(a-1)*b)^(1/2))/b^(1/2)+(b*(x-(-(a-1)*b)^(1/2)/b)^2+2*(-(a-1)*b)^(1/2
)*(x-(-(a-1)*b)^(1/2)/b)+1)^(1/2))/b^(1/2)-arctanh(1/2*(2+2*(-(a-1)*b)^(1/2
)*(x-(-(a-1)*b)^(1/2)/b))/(b*(x-(-(a-1)*b)^(1/2)/b)^2+2*(-(a-1)*b)^(1/2)*(x
-(-(a-1)*b)^(1/2)/b)+1)^(1/2)))+1/2*a/b*ln(b*x^2+a-1)-1/2*a/b*ln(b*x^2+a)-b
*(1/2*(a-1)/b^2*ln(b*x^2+a-1)-1/2*a/b^2*ln(b*x^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.72

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx = \frac{2\log\left(bx^2+a-1\right)+\log\left(\frac{bx^2+a+2\sqrt{bx^2+a}+1}{x^2}\right)-\log\left(\frac{bx^2+a-2\sqrt{bx^2+a}+1}{x^2}\right)}{4b}$$

```
[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(b*x^2 + a - 1) + log((b*x^2 + a + 2*sqrt(b*x^2 + a) + 1)/x^2) -
log((b*x^2 + a - 2*sqrt(b*x^2 + a) + 1)/x^2))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(14) = 28$.

Time = 0.74 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.94

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{2\left(-\frac{\log\left(2\sqrt{a+bx^2}\right)}{4}+\frac{\log\left(2\sqrt{a+bx^2}+2\right)}{4}+\frac{\log\left(2a+2bx^2+2\sqrt{a+bx^2}\right)}{4}\right)}{b} & \text{for } b\neq 0 \\ \frac{x^2}{2\sqrt{a}+2a} & \text{otherwise} \end{cases}$$

```
[In] integrate(x/(a+b*x**2+(b*x**2+a)**(1/2)),x)
```

```
[Out] Piecewise((2*(-log(2*sqrt(a + b*x**2))/4 + log(2*sqrt(a + b*x**2) + 2)/4 +
log(2*a + 2*b*x**2 + 2*sqrt(a + b*x**2))/4)/b, Ne(b, 0)), (x**2/(2*sqrt(a)
+ 2*a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx=\frac{\log\left(\sqrt{bx^2+a}+1\right)}{b}$$

```
[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="maxima")
[Out] log(sqrt(b*x^2 + a) + 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx=\frac{\log\left(\sqrt{bx^2+a}+1\right)}{b}$$

```
[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="giac")
[Out] log(sqrt(b*x^2 + a) + 1)/b
```

Mupad [B] (verification not implemented)

Time = 19.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx=\frac{\mathrm{atanh}\left(\sqrt{b\,x^2+a}\right)+\frac{\ln\left(b\,x^2+a-1\right)}{2}}{b}$$

```
[In] int(x/(a + b*x^2 + (a + b*x^2)^(1/2)),x)
[Out] (atanh((a + b*x^2)^(1/2)) + log(a + b*x^2 - 1)/2)/b
```

3.684 $\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx=\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

```
[Out] 3/4*ln(1-(x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6847, 1607, 266}

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx=\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

```
[In] Int[x/(x^2 - (x^2)^(1/3)),x]
```

```
[Out] (3*Log[1 - (x^2)^(2/3)])/4
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-\sqrt[3]{x}+x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x^{2/3})\sqrt[3]{x}}\,dx, x, x^2\right)\\ &= \frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx = \frac{3}{4}\log\left(-1+\sqrt[3]{x^2}\right)+\frac{3}{4}\log\left(1+\sqrt[3]{x^2}\right)$$

```
[In] Integrate[x/(x^2 - (x^2)^(1/3)),x]
[Out] (3*Log[-1 + (x^2)^(1/3)])/4 + (3*Log[1 + (x^2)^(1/3)])/4
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result
meijerg	$\frac{3\ln\left(1-\frac{x^2}{\left(x^2\right)^{\frac{1}{3}}}\right)}{4}$
derivativedivides	$\frac{\ln(x^2-1)}{4}+\frac{\ln(x^2+1)}{4}-\frac{\ln\left(\left(x^2\right)^{\frac{2}{3}}+\left(x^2\right)^{\frac{1}{3}}+1\right)}{4}+\frac{\ln\left(\left(x^2\right)^{\frac{1}{3}}-1\right)}{2}-\frac{\ln\left(\left(x^2\right)^{\frac{2}{3}}-\left(x^2\right)^{\frac{1}{3}}+1\right)}{4}+\frac{\ln\left(\left(x^2\right)^{\frac{1}{3}}+1\right)}{2}$
default	$\frac{\ln(x^2-1)}{4}+\frac{\ln(x^2+1)}{4}-\frac{\ln\left(\left(x^2\right)^{\frac{2}{3}}+\left(x^2\right)^{\frac{1}{3}}+1\right)}{4}+\frac{\ln\left(\left(x^2\right)^{\frac{1}{3}}-1\right)}{2}-\frac{\ln\left(\left(x^2\right)^{\frac{2}{3}}-\left(x^2\right)^{\frac{1}{3}}+1\right)}{4}+\frac{\ln\left(\left(x^2\right)^{\frac{1}{3}}+1\right)}{2}$
trager	$-\frac{\ln\left(-\frac{x^8+3\left(x^2\right)^{\frac{1}{3}}x^6+6\left(x^2\right)^{\frac{2}{3}}x^4+7x^4+6x^2\left(x^2\right)^{\frac{1}{3}}+3\left(x^2\right)^{\frac{2}{3}}+1}{(x-1)^3(x+1)^3\left(x^2+1\right)^3}\right)}{4}$

```
[In] int(x/(x^2-(x^2)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*ln(1-x^2/(x^2)^(1/3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{x}{x^2 - \sqrt[3]{x^2}}\,dx = -3\log\left(\frac{(x^2)^{\frac{1}{3}}}{x}\right) + \frac{3}{4}\log\left(-\frac{x^2 - (x^2)^{\frac{1}{3}}}{x^2}\right)$$

```
[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="fricas")
[Out] -3*log((x^2)^(1/3)/x) + 3/4*log(-(x^2 - (x^2)^(1/3))/x^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{x}{x^2 - \sqrt[3]{x^2}}\,dx = -\frac{\log(x)}{2} + \frac{3\log\left(-x^2 + \sqrt[3]{x^2}\right)}{4}$$

```
[In] integrate(x/(x**2-(x**2)**(1/3)),x)
[Out] -log(x)/2 + 3*log(-x**2 + (x**2)**(1/3))/4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{x}{x^2 - \sqrt[3]{x^2}}\,dx = \frac{3}{4}\log\left((x^2)^{\frac{1}{3}} + 1\right) + \frac{3}{4}\log\left((x^2)^{\frac{1}{3}} - 1\right)$$

```
[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="maxima")
[Out] 3/4*log((x^2)^(1/3) + 1) + 3/4*log((x^2)^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{x^2 - \sqrt[3]{x^2}}\,dx = \frac{3}{4}\log\left(\left|(x\mathrm{sgn}(x))^{\frac{1}{3}}\,x\mathrm{sgn}(x) - 1\right|\right)$$

```
[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="giac")
[Out] 3/4*log(abs((x*sgn(x))^(1/3)*x*sgn(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 18.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x}{x^2 - \sqrt[3]{x^2}}\,dx = \frac{3\,\ln\left(\left(x^2\right)^{2/3} - 1\right)}{4}$$

```
[In] int(-x/((x^2)^(1/3) - x^2),x)
[Out] (3*log((x^2)^(2/3) - 1))/4
```

3.685 $\int x(1+x^2)^3\sqrt{2+2x^2+x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 44

$$\int x\left(1+x^2\right)^3\sqrt{2+2x^2+x^4}\,dx=-\frac{1}{15}\left(2+2x^2+x^4\right)^{3/2}+\frac{1}{10}\left(1+x^2\right)^2\left(2+2x^2+x^4\right)^{3/2}$$

```
[Out] -1/15*(x^4+2*x^2+2)^(3/2)+1/10*(x^2+1)^2*(x^4+2*x^2+2)^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1261, 706, 643}

$$\int x\left(1+x^2\right)^3\sqrt{2+2x^2+x^4}\,dx=\frac{1}{10}\left(x^2+1\right)^2\left(x^4+2x^2+2\right)^{3/2}-\frac{1}{15}\left(x^4+2x^2+2\right)^{3/2}$$

```
[In] Int[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4],x]
[Out] -1/15*(2 + 2*x^2 + x^4)^(3/2) + ((1 + x^2)^2*(2 + 2*x^2 + x^4)^(3/2))/10
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 706

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[2*d*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(b*(m + 2*p
 + 1))), x] + Dist[d^2*(m - 1)*((b^2 - 4*a*c)/(b^2*(m + 2*p + 1))), Int[(d
```

```
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (1+x)^3\sqrt{2+2x+x^2}\,dx, x, x^2\right)\\ &= \frac{1}{10}\left(1+x^2\right)^2\left(2+2x^2+x^4\right)^{3/2}-\frac{1}{5}\text{Subst}\left(\int (1+x)\sqrt{2+2x+x^2}\,dx, x, x^2\right)\\ &= -\frac{1}{15}\left(2+2x^2+x^4\right)^{3/2}+\frac{1}{10}\left(1+x^2\right)^2\left(2+2x^2+x^4\right)^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int x\left(1+x^2\right)^3\sqrt{2+2x^2+x^4}\,dx = \frac{1}{30}\sqrt{2+2x^2+x^4}\left(2+14x^2+19x^4+12x^6+3x^8\right)$$

```
[In] Integrate[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4],x]
[Out] (Sqrt[2 + 2*x^2 + x^4]*(2 + 14*x^2 + 19*x^4 + 12*x^6 + 3*x^8))/30
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(x^4+2x^2+2)^{\frac{3}{2}}(3x^4+6x^2+1)}{30}$	27
elliptic	$\frac{(x^4+2x^2+2)^{\frac{3}{2}}(3x^4+6x^2+1)}{30}$	27
pseudoelliptic	$\frac{(x^4+2x^2+2)^{\frac{3}{2}}(3x^4+6x^2+1)}{30}$	27
trager	$\left(\frac{1}{10}x^8+\frac{2}{5}x^6+\frac{19}{30}x^4+\frac{7}{15}x^2+\frac{1}{15}\right)\sqrt{x^4+2x^2+2}$	36
risch	$\frac{(3x^8+12x^6+19x^4+14x^2+2)\sqrt{x^4+2x^2+2}}{30}$	37
default	$\frac{x^4(x^4+2x^2+2)^{\frac{3}{2}}}{10}+\frac{x^2(x^4+2x^2+2)^{\frac{3}{2}}}{5}+\frac{(x^4+2x^2+2)^{\frac{3}{2}}}{30}$	50

```
[In] int(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/30*(x^4+2*x^2+2)^(3/2)*(3*x^4+6*x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int x(1+x^2)^3\sqrt{2+2x^2+x^4}\,dx=\frac{1}{30}\left(3x^8+12x^6+19x^4+14x^2+2\right)\sqrt{x^4+2x^2+2}$$

```
[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/30*(3*x^8 + 12*x^6 + 19*x^4 + 14*x^2 + 2)*sqrt(x^4 + 2*x^2 + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(36) = 72.

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.14

$$\int x(1+x^2)^3\sqrt{2+2x^2+x^4}\,dx=\frac{x^8\sqrt{x^4+2x^2+2}}{10}+\frac{2x^6\sqrt{x^4+2x^2+2}}{5}+\frac{19x^4\sqrt{x^4+2x^2+2}}{30}+\frac{7x^2\sqrt{x^4+2x^2+2}}{15}+\frac{\sqrt{x^4+2x^2+2}}{15}$$

```
[In] integrate(x*(x**2+1)**3*(x**4+2*x**2+2)**(1/2),x)
[Out] x**8*sqrt(x**4 + 2*x**2 + 2)/10 + 2*x**6*sqrt(x**4 + 2*x**2 + 2)/5 + 19*x**
4*sqrt(x**4 + 2*x**2 + 2)/30 + 7*x**2*sqrt(x**4 + 2*x**2 + 2)/15 + sqrt(x**
4 + 2*x**2 + 2)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int x(1+x^2)^3\sqrt{2+2x^2+x^4}\,dx = \frac{1}{10}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}x^4 + \frac{1}{5}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}x^2 + \frac{1}{30}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="maxima")
[Out] 1/10*(x^4 + 2*x^2 + 2)^(3/2)*x^4 + 1/5*(x^4 + 2*x^2 + 2)^(3/2)*x^2 + 1/30*(
x^4 + 2*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int x(1+x^2)^3\sqrt{2+2x^2+x^4}\,dx = \frac{1}{10}\left(x^4+2\,x^2+2\right)^{\frac{5}{2}} - \frac{1}{6}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="giac")
[Out] 1/10*(x^4 + 2*x^2 + 2)^(5/2) - 1/6*(x^4 + 2*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int x(1+x^2)^3\sqrt{2+2x^2+x^4}\,dx = \frac{\left(x^4+2\,x^2+2\right)^{3/2}\left(3\,x^4+6\,x^2+1\right)}{30}$$

```
[In] int(x*(x^2 + 1)^3*(2*x^2 + x^4 + 2)^(1/2),x)
[Out] ((2*x^2 + x^4 + 2)^(3/2)*(6*x^2 + 3*x^4 + 1))/30
```

3.686 $\int x^5\sqrt{1-x^3}(1+x^9)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int x^5\sqrt{1-x^3}(1+x^9)^2 \, dx = -\frac{8}{9}(1-x^3)^{3/2}+\frac{32}{15}(1-x^3)^{5/2}-\frac{22}{7}(1-x^3)^{7/2}+\frac{86}{27}(1-x^3)^{9/2}-\frac{74}{33}(1-x^3)^{11/2}+\frac{14}{13}(1-x^3)^{13/2}-\frac{14}{45}(1-x^3)^{15/2}+\frac{2}{51}(1-x^3)^{17/2}$$

```
[Out] -8/9*(-x^3+1)^(3/2)+32/15*(-x^3+1)^(5/2)-22/7*(-x^3+1)^(7/2)+86/27*(-x^3+1)
^(9/2)-74/33*(-x^3+1)^(11/2)+14/13*(-x^3+1)^(13/2)-14/45*(-x^3+1)^(15/2)+2/
51*(-x^3+1)^(17/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1835, 1634}

$$\int x^5\sqrt{1-x^3}(1+x^9)^2 \, dx = \frac{2}{51}(1-x^3)^{17/2}-\frac{14}{45}(1-x^3)^{15/2}+\frac{14}{13}(1-x^3)^{13/2}-\frac{74}{33}(1-x^3)^{11/2}+\frac{86}{27}(1-x^3)^{9/2}-\frac{22}{7}(1-x^3)^{7/2}+\frac{32}{15}(1-x^3)^{5/2}-\frac{8}{9}(1-x^3)^{3/2}$$

```
[In] Int[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

```
[Out] (-8*(1 - x^3)^(3/2))/9 + (32*(1 - x^3)^(5/2))/15 - (22*(1 - x^3)^(7/2))/7 +
 (86*(1 - x^3)^(9/2))/27 - (74*(1 - x^3)^(11/2))/33 + (14*(1 - x^3)^(13/2))
/13 - (14*(1 - x^3)^(15/2))/45 + (2*(1 - x^3)^(17/2))/51
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \sqrt{1-x}x\left(1+x^3\right)^2\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(4\sqrt{1-x}-16(1-x)^{3/2}\right.\right.\\ &\quad \left.\left.+33(1-x)^{5/2}-43(1-x)^{7/2}+37(1-x)^{9/2}-21(1-x)^{11/2}+7(1-x)^{13/2}-(1-x)^{15/2}\right)\,dx,x,x^3\right)\\ &= -\frac{8}{9}\left(1-x^3\right)^{3/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}\\ &\quad +\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{74}{33}\left(1-x^3\right)^{11/2}+\frac{14}{13}\left(1-x^3\right)^{13/2}-\frac{14}{45}\left(1-x^3\right)^{15/2}+\frac{2}{51}\left(1-x^3\right)^{17/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.47

$$\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx$$
$$=\frac{2\sqrt{1-x^3}(-173014-86507x^3+126561x^6-22160x^9-19390x^{12}+135702x^{15}-3234x^{18}-3003x^{21}+4}{2297295}$$

```
[In] Integrate[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

```
[Out] (2*Sqrt[1 - x^3]*(-173014 - 86507*x^3 + 126561*x^6 - 22160*x^9 - 19390*x^12
 + 135702*x^15 - 3234*x^18 - 3003*x^21 + 45045*x^24))/2297295
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.40

method	result
pseudoelliptic	$-\frac{2\left(-x^{3}+1\right)^{\frac{3}{2}}\left(45045 x^{21}+42042 x^{18}+38808 x^{15}+174510 x^{12}+155120 x^{9}+132960 x^{6}+259521 x^{3}+173014\right)}{2297295}$
trager	$\left(\frac{2}{51} x^{24}-\frac{2}{765} x^{21}-\frac{28}{9945} x^{18}+\frac{1436}{12155} x^{15}-\frac{1108}{65637} x^{12}-\frac{8864}{459459} x^{9}+\frac{84374}{765765} x^{6}-\frac{173014}{2297295} x^{3}-\frac{346028}{2297295}\right)$
gosper	$\frac{2\sqrt{-x^{3}+1}\,\left(45045 x^{21}+42042 x^{18}+38808 x^{15}+174510 x^{12}+155120 x^{9}+132960 x^{6}+259521 x^{3}+173014\right)(x-1)\left(x^{2}+x+1\right)}{2297295}$
risch	$-\frac{2\left(45045 x^{24}-3003 x^{21}-3234 x^{18}+135702 x^{15}-19390 x^{12}-22160 x^{9}+126561 x^{6}-86507 x^{3}-173014\right)\left(x^{3}-1\right)}{2297295\sqrt{-x^{3}+1}}$
default	$\frac{84374 x^{6}\sqrt{-x^{3}+1}}{765765}-\frac{173014 x^{3}\sqrt{-x^{3}+1}}{2297295}-\frac{346028\sqrt{-x^{3}+1}}{2297295}+\frac{2 x^{24}\sqrt{-x^{3}+1}}{51}-\frac{2 x^{21}\sqrt{-x^{3}+1}}{765}-\frac{28 x^{18}\sqrt{-x^{3}+1}}{9945}+1$
elliptic	$\frac{84374 x^{6}\sqrt{-x^{3}+1}}{765765}-\frac{173014 x^{3}\sqrt{-x^{3}+1}}{2297295}-\frac{346028\sqrt{-x^{3}+1}}{2297295}+\frac{2 x^{24}\sqrt{-x^{3}+1}}{51}-\frac{2 x^{21}\sqrt{-x^{3}+1}}{765}-\frac{28 x^{18}\sqrt{-x^{3}+1}}{9945}+1$
meijerg	$-\frac{-\frac{8192\sqrt{\pi}}{109395}+\frac{4\sqrt{\pi}\,\left(-x^{3}+1\right)^{\frac{3}{2}}\left(6435 x^{21}+6006 x^{18}+5544 x^{15}+5040 x^{12}+4480 x^{9}+3840 x^{6}+3072 x^{3}+2048\right)}{109395}}{6\sqrt{\pi}}+\frac{\frac{512\sqrt{\pi}}{3465}-\frac{4\sqrt{\pi}\,\left(-x^{3}+1\right)^{\frac{3}{2}}(}{}}{}$

```
[In] int(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/2297295*(-x^3+1)^(3/2)*(45045*x^21+42042*x^18+38808*x^15+174510*x^12+155
120*x^9+132960*x^6+259521*x^3+173014)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

$$\int x^{5}\sqrt{1-x^{3}}\left(1+x^{9}\right)^{2}\,dx$$
$$=\frac{2}{2297295}\left(45045\,x^{24}-3003\,x^{21}-3234\,x^{18}+135702\,x^{15}-19390\,x^{12}-22160\,x^{9}+126561\,x^{6}-86507\,x^{3}-\right.$$

```
[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/2297295*(45045*x^24 - 3003*x^21 - 3234*x^18 + 135702*x^15 - 19390*x^12 -
22160*x^9 + 126561*x^6 - 86507*x^3 - 173014)*sqrt(-x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10

$$\int x^5\sqrt{1-x^3}(1+x^9)^2\,dx=\frac{2x^{24}\sqrt{1-x^3}}{51}-\frac{2x^{21}\sqrt{1-x^3}}{765}-\frac{28x^{18}\sqrt{1-x^3}}{9945}+\frac{1436x^{15}\sqrt{1-x^3}}{12155}-\frac{1108x^{12}\sqrt{1-x^3}}{65637}-\frac{8864x^{9}\sqrt{1-x^3}}{459459}+\frac{84374x^{6}\sqrt{1-x^3}}{765765}-\frac{173014x^{3}\sqrt{1-x^3}}{2297295}-\frac{346028\sqrt{1-x^3}}{2297295}$$

```
[In] integrate(x**5*(x**9+1)**2*(-x**3+1)**(1/2),x)
```

```
[Out] 2*x**24*sqrt(1 - x**3)/51 - 2*x**21*sqrt(1 - x**3)/765 - 28*x**18*sqrt(1 -
x**3)/9945 + 1436*x**15*sqrt(1 - x**3)/12155 - 1108*x**12*sqrt(1 - x**3)/65
637 - 8864*x**9*sqrt(1 - x**3)/459459 + 84374*x**6*sqrt(1 - x**3)/765765 -
173014*x**3*sqrt(1 - x**3)/2297295 - 346028*sqrt(1 - x**3)/2297295
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int x^5\sqrt{1-x^3}(1+x^9)^2\,dx=\frac{2}{51}\left(-x^3+1\right)^{\frac{17}{2}}-\frac{14}{45}\left(-x^3+1\right)^{\frac{15}{2}}+\frac{14}{13}\left(-x^3+1\right)^{\frac{13}{2}}-\frac{74}{33}\left(-x^3+1\right)^{\frac{11}{2}}+\frac{86}{27}\left(-x^3+1\right)^{\frac{9}{2}}-\frac{22}{7}\left(-x^3+1\right)^{\frac{7}{2}}+\frac{32}{15}\left(-x^3+1\right)^{\frac{5}{2}}-\frac{8}{9}\left(-x^3+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/51*(-x^3 + 1)^(17/2) - 14/45*(-x^3 + 1)^(15/2) + 14/13*(-x^3 + 1)^(13/2)
- 74/33*(-x^3 + 1)^(11/2) + 86/27*(-x^3 + 1)^(9/2) - 22/7*(-x^3 + 1)^(7/2)
+ 32/15*(-x^3 + 1)^(5/2) - 8/9*(-x^3 + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14

$$\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx = \frac{2}{51}\left(x^3-1\right)^8\sqrt{-x^3+1}+\frac{14}{45}\left(x^3-1\right)^7\sqrt{-x^3+1}+\frac{14}{13}\left(x^3-1\right)^6\sqrt{-x^3+1}+\frac{74}{33}\left(x^3-1\right)^5\sqrt{-x^3+1}+\frac{86}{27}\left(x^3-1\right)^4\sqrt{-x^3+1}+\frac{22}{7}\left(x^3-1\right)^3\sqrt{-x^3+1}+\frac{32}{15}\left(x^3-1\right)^2\sqrt{-x^3+1}-\frac{8}{9}\left(-x^3+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/51*(x^3 - 1)^8*sqrt(-x^3 + 1) + 14/45*(x^3 - 1)^7*sqrt(-x^3 + 1) + 14/13*
(x^3 - 1)^6*sqrt(-x^3 + 1) + 74/33*(x^3 - 1)^5*sqrt(-x^3 + 1) + 86/27*(x^3
- 1)^4*sqrt(-x^3 + 1) + 22/7*(x^3 - 1)^3*sqrt(-x^3 + 1) + 32/15*(x^3 - 1)^2
*sqrt(-x^3 + 1) - 8/9*(-x^3 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx = \frac{84374\,x^6\,\sqrt{1-x^3}}{765765}-\frac{173014\,x^3\,\sqrt{1-x^3}}{2297295}-\frac{8864\,x^9\,\sqrt{1-x^3}}{459459}-\frac{1108\,x^{12}\,\sqrt{1-x^3}}{65637}+\frac{1436\,x^{15}\,\sqrt{1-x^3}}{12155}-\frac{28\,x^{18}\,\sqrt{1-x^3}}{9945}-\frac{2\,x^{21}\,\sqrt{1-x^3}}{765}+\frac{2\,x^{24}\,\sqrt{1-x^3}}{51}-\frac{346028\,\sqrt{1-x^3}}{2297295}$$

```
[In] int(x^5*(1 - x^3)^(1/2)*(x^9 + 1)^2,x)
```

```
[Out] (84374*x^6*(1 - x^3)^(1/2))/765765 - (173014*x^3*(1 - x^3)^(1/2))/2297295 -
 (8864*x^9*(1 - x^3)^(1/2))/459459 - (1108*x^12*(1 - x^3)^(1/2))/65637 + (1
436*x^15*(1 - x^3)^(1/2))/12155 - (28*x^18*(1 - x^3)^(1/2))/9945 - (2*x^21*
(1 - x^3)^(1/2))/765 + (2*x^24*(1 - x^3)^(1/2))/51 - (346028*(1 - x^3)^(1/2
))/2297295
```

3.687 $\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx$

Optimal result

Integrand size = 34, antiderivative size = 50

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx = -\frac{1}{b\sqrt{a+bx^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/b/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {267, 455, 65, 214}

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\sqrt{a+bx^2}}$$

```
[In] Int[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x}{(a+bx^2)^{3/2}}\,dx + \int \frac{x}{(1+x^2)\sqrt{a+bx^2}}\,dx \\
&= -\frac{1}{b\sqrt{a+bx^2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= -\frac{1}{b\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b} \\
&= -\frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \left(\frac{x}{(a+bx^2)^{3/2}} + \frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx = -\frac{1}{b\sqrt{a+bx^2}} + \frac{\arctan\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

```
[In] Integrate[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
[Out] -(1/(b*Sqrt[a + b*x^2])) + ArcTan[Sqrt[a + b*x^2]/Sqrt[-a + b]]/Sqrt[-a + b
]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{1}{b\sqrt{b\,x^{2}+a}}+\frac{\arctan\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	42

```
[In] int(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/b/(b*x^2+a)^(1/2)+1/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(42) = 84.

Time = 0.32 (sec) , antiderivative size = 268, normalized size of antiderivative = 5.36

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)\,dx=\left[\frac{(b^2x^2+ab)\sqrt{a-b}\log\left(\frac{b^2x^4+2\,(4\,ab-3\,b^2)x^2-4\,(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right)-4\,\sqrt{bx^2+a}(a-b)}{4\,(a^2b-ab^2+(ab^2-b^3)x^2)},\ -\frac{(b^2x^2+ab)\sqrt{-a+b}\arctan\left(-\frac{(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{-a+b}}{2\,((ab-b^2)x^2+a^2-ab)}\right)+2\,\sqrt{bx^2+a}(a-b)}{2\,(a^2b-ab^2+(ab^2-b^3)x^2)}\right]$$

```
[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="fricas"
)
[Out] [1/4*((b^2*x^2 + a*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4*
(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4 +
 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^
2), -1/2*((b^2*x^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sqrt(b
*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*sqrt(b*x^2 + a)*(
a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]
```

Sympy [A] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.66

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx = \begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} + \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} & \text{for } b\neq 0 \\ \begin{cases} \tilde{\infty}x^2 & \text{for } \sqrt{a}=0 \\ \frac{\log(2\sqrt{a}x^2+2\sqrt{a})}{2\sqrt{a}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

```
[In] integrate(x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) +
 Piecewise((atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b), Ne(b, 0)), (P
iecewise((zoo*x**2, Eq(sqrt(a), 0)), (log(2*sqrt(a)*x**2 + 2*sqrt(a))/(2*sq
rt(a)), True)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+a}b}$$

```
[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 18.79 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\,\sqrt{b\,x^2+a}}$$

```
[In] int(x/(a + b*x^2)^(3/2) + x/((x^2 + 1)*(a + b*x^2)^(1/2)),x)
[Out] - atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - 1/(b*(a + b*x^2)^(
1/2))
```

3.688 $\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 50

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = -\frac{1}{b\sqrt{a+bx^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/b/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {6, 585, 79, 65, 214}

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{1}{b\sqrt{a+bx^2}}$$

```
[In] Int[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(1+a+(1+b)x^2)}{(1+x^2)\,(a+bx^2)^{3/2}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+a+(1+b)x}{(1+x)(a+bx)^{3/2}}\,dx, x, x^2\right) \\
&= -\frac{1}{b\sqrt{a+bx^2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= -\frac{1}{b\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b} \\
&= -\frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)\,(a+bx^2)^{3/2}}\,dx = -\frac{1}{b\sqrt{a+bx^2}}+\frac{\arctan\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

```
[In] Integrate[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
[Out] -(1/(b*Sqrt[a + b*x^2])) + ArcTan[Sqrt[a + b*x^2]/Sqrt[-a + b]]/Sqrt[-a + b
]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$-\frac{1}{b\sqrt{b\,x^2+a}}+\frac{\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	42
default	$-\frac{b+1}{b\sqrt{b\,x^2+a}}+(a-b)\left(\frac{1}{(a-b)\sqrt{b\,x^2+a}}+\frac{\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}\right)$	76

```
[In] int(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/b/(b*x^2+a)^(1/2)+1/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(42) = 84.

Time = 0.31 (sec) , antiderivative size = 268, normalized size of antiderivative = 5.36

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)\,(a+bx^2)^{3/2}}\,dx = \left[\frac{(b^2x^2+ab)\sqrt{a-b}\log\left(\frac{b^2x^4+2\,(4\,ab-3\,b^2)x^2-4\,(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right)-}{4\,(a^2b-ab^2+(ab^2-b^3)x^2)}\right.$$
$$\left.-\frac{(b^2x^2+ab)\sqrt{-a+b}\arctan\left(-\frac{(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{-a+b}}{2\,((ab-b^2)x^2+a^2-ab)}\right)+2\sqrt{bx^2+a}(a-b)}{2\,(a^2b-ab^2+(ab^2-b^3)x^2)}\right]$$

```
[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] [1/4*((b^2*x^2 + a*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4*
(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4 +
```

```
 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^
2), -1/2*((b^2*x^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sqrt(b
*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*sqrt(b*x^2 + a)*(
a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]
```

Sympy [F]

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = \int \frac{x(a+bx^2+x^2+1)}{(a+bx^2)^{\frac{3}{2}}(x^2+1)}\,dx$$

```
[In] integrate(x*(b*x**2+x**2+a+1)/(x**2+1)/(b*x**2+a)**(3/2),x)
[Out] Integral(x*(a + b*x**2 + x**2 + 1)/((a + b*x**2)**(3/2)*(x**2 + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{1}{\sqrt{bx^2+a}b}$$

```
[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 19.35 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)\,(a+bx^2)^{3/2}}\,dx = \frac{1}{\sqrt{b\,x^2+a}\,\,(a-b)} - \frac{a}{\sqrt{b\,x^2+a}\,\,(a\,b-b^2)} - \frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}$$

```
[In] int((x*(a + b*x^2 + x^2 + 1))/((x^2 + 1)*(a + b*x^2)^(3/2)),x)
```

```
[Out] 1/((a + b*x^2)^(1/2)*(a - b)) - a/((a + b*x^2)^(1/2)*(a*b - b^2)) - (a*atan
h((a + b*x^2)^(1/2)/(a - b)^(1/2)))/(a - b)^(3/2) + (b*atanh((a + b*x^2)^(1
/2)/(a - b)^(1/2)))/(a - b)^(3/2)
```

3.689 $\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx$

Optimal result

Integrand size = 47, antiderivative size = 68

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx=-\frac{1}{3b(a+bx^2)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -1/3/b/(b*x^2+a)^(3/2)-arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/b
/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.085, Rules used = {267, 455, 65, 214}

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b(a+bx^2)^{3/2}}$$

```
[In] Int[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2
]),x]
```

```
[Out] -1/3*1/(b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x}{(a+bx^2)^{5/2}}\,dx + \int \frac{x}{(a+bx^2)^{3/2}}\,dx + \int \frac{x}{(1+x^2)\sqrt{a+bx^2}}\,dx \\
&= -\frac{1}{3b\,(a+bx^2)^{3/2}} - \frac{1}{b\sqrt{a+bx^2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= -\frac{1}{3b\,(a+bx^2)^{3/2}} - \frac{1}{b\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b} \\
&= -\frac{1}{3b\,(a+bx^2)^{3/2}} - \frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=\frac{-1-3a-3bx^2}{3b(a+bx^2)^{3/2}}+\frac{\arctan\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

```
[In] Integrate[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a +
 b*x^2]),x]
```

```
[Out] (-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) + ArcTan[Sqrt[a + b*x^2]/Sqrt[
-a + b]]/Sqrt[-a + b]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{1}{3b(b\,x^2+a)^{\frac{3}{2}}}-\frac{1}{b\sqrt{b\,x^2+a}}+\frac{\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	56

```
[In] int(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -1/3/b/(b*x^2+a)^(3/2)-1/b/(b*x^2+a)^(1/2)+1/(-a+b)^(1/2)*arctan((b*x^2+a)^
(1/2)/(-a+b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(56) = 112.

Time = 0.33 (sec) , antiderivative size = 382, normalized size of antiderivative = 5.62

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=\left[\frac{3\,(b^3x^4+2\,ab^2x^2+a^2b)\sqrt{a-b}\log\left(\frac{b^2x^4+2\,(4\,ab-3\,b^2)x^2-4\,(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{a-b}+8}{x^4+2\,x^2+1}\right.}{12\,((ab^3-b^4)x^4+a^3b-a^2b^2+}\right.$$

```
[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*
b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 -
8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*
b + a)*sqrt(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 -
 a*b^3)*x^2), -1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b)*arctan(-
1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 -
 a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2 + a))/(
(a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]
```

Sympy [A] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.93

$$\int\left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}+\begin{cases}\frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} & \text{for } b\neq 0\\ \begin{cases}\tilde{\infty}x^2 & \text{for } \sqrt{a}=0\\ \frac{\log\left(2\sqrt{a}x^2+2\sqrt{a}\right)}{2\sqrt{a}} & \text{otherwise}\end{cases} & \text{otherwise}\end{cases}+\begin{cases}-\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}$$

```
[In] integrate(x/(b*x**2+a)**(5/2)+x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1
/2),x)
```

```
[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) +
 Piecewise((atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b), Ne(b, 0)), (P
iecewise((zoo*x**2, Eq(sqrt(a), 0)), (log(2*sqrt(a)*x**2 + 2*sqrt(a))/(2*sq
rt(a)), True)), True)) + Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**
2*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int\left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)\,dx=\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+ab}}-\frac{1}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

```
[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b) -
 1/3/((b*x^2 + a)^(3/2)*b)
```

Mupad [B] (verification not implemented)

Time = 18.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)\,dx=-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\,\sqrt{b\,x^2+a}}-\frac{1}{3\,b\,(b\,x^2+a)^{3/2}}$$

```
[In] int(x/(a + b*x^2)^(3/2) + x/(a + b*x^2)^(5/2) + x/((x^2 + 1)*(a + b*x^2)^(1
/2)),x)
```

```
[Out] - atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - 1/(b*(a + b*x^2)^(
1/2)) - 1/(3*b*(a + b*x^2)^(3/2))
```

3.690 $\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 58, antiderivative size = 68

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}}\,dx = -\frac{1}{3b(a+bx^2)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -1/3/b/(b*x^2+a)^(3/2)-arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/b
/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {6, 6847, 911, 1275, 213}

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b(a+bx^2)^{3/2}}$$

```
[In] Int[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*x^4))/(
(1 + x^2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] -1/3*1/(b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x(1+a+a^2+(1+a)x^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx \\
&= \int \frac{x(1+a+a^2+2abx^2+(1+a+b)x^2+bx^4+b^2x^4)}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx \\
&= \int \frac{x(1+a+a^2+(1+a+b+2ab)x^2+bx^4+b^2x^4)}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx \\
&= \int \frac{x(1+a+a^2+(1+a+b+2ab)x^2+(b+b^2)\,x^4)}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx
\end{aligned}
$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1+a+a^2+(1+a+b+2ab)x+(b+b^2)\,x^2}{(1+x)(a+bx)^{5/2}}\,dx,x,x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\frac{(1+a+a^2)b^2-ab(1+a+b+2ab)+a^2(b+b^2)}{b^2}-\frac{\left(-b(1+a+b+2ab)+2a(b+b^2)\right)x^2}{b^2}+\frac{(b+b^2)x^4}{b^2}}{x^4\left(\frac{-a+b}{b}+\frac{x^2}{b}\right)}\,dx,x,\sqrt{a+bx^2}\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x^4}+\frac{1}{x^2}+\frac{b}{-a+b+x^2}\right)\,dx,x,\sqrt{a+bx^2}\right)}{b}$$

$$= -\frac{1}{3b\,(a+bx^2)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}+\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx,x,\sqrt{a+bx^2}\right)$$

$$= -\frac{1}{3b\,(a+bx^2)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\,(a+bx^2)^{5/2}}\,dx = \frac{-1-3a-3bx^2}{3b\,(a+bx^2)^{3/2}} + \frac{\arctan\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

```
[In] Integrate[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*x
^4))/((1 + x^2)*(a + b*x^2)^(5/2)),x]
[Out] (-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) + ArcTan[Sqrt[a + b*x^2]/Sqrt[
-a + b]]/Sqrt[-a + b]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)b\left(b\,x^2+a\right)^{\frac{3}{2}}-\sqrt{-a+b}\,\left(b\,x^2+a+\frac{1}{3}\right)}{\sqrt{-a+b}\,\left(b\,x^2+a\right)^{\frac{3}{2}}b}$
default	$(b^2+b)\left(-\frac{x^2}{b(b\,x^2+a)^{\frac{3}{2}}}-\frac{2a}{3b^2(b\,x^2+a)^{\frac{3}{2}}}\right)-\frac{2ab-b^2+a+1}{3b(b\,x^2+a)^{\frac{3}{2}}}+(a^2-2ab+b^2)\left(\frac{1}{(a-b)^2\sqrt{b\,x^2+a}}+\frac{\arctan\left(\right.}{(a-b)^{2}}\right.$

```
[In] int(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(
5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-a+b)^(1/2)*(arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))*b*(b*x^2+a)^(3/2)-(-a+
b)^(1/2)*(b*x^2+a+1/3))/(b*x^2+a)^(3/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(56) = 112$.

Time = 0.32 (sec) , antiderivative size = 382, normalized size of antiderivative = 5.62

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx = \left[\frac{3\left(b^3x^4+2\,ab^2x^2+a^2b\right)\sqrt{a-b}\log\left(\frac{b^2x^4+2\,(\cdots}{\cdots}\right)\cdots}{\cdots}\right.$$

$$-\frac{3\left(b^3x^4+2\,ab^2x^2+a^2b\right)\sqrt{-a+b}\arctan\left(-\frac{(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{-a+b}}{2\left((ab-b^2)x^2+a^2-ab\right)}\right)+2\left(3\,(ab-b^2)x^2+3\,a^2-(3\,a+1)b\cdots\right.}{6\left((ab^3-b^4)x^4+a^3b-a^2b^2+2\left(a^2b^2-ab^3\right)x^2\right)}$$

```
[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^
2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*
b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 -
8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*
b + a)*sqrt(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 -
 a*b^3)*x^2), -1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b)*arctan(-
1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 -
 a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2 + a))/(
(a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]
```

Sympy [F]

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx = \int \frac{x(a^2+2abx^2+ax^2+a+b^2x^4+bx^4+bx\cdots}{\left(a+bx^2\right)^{\frac{5}{2}}\left(x^2+1\right)}$$

```
[In] integrate(x*(b**2*x**4+b*x**4+2*a*b*x**2+a*x**2+b*x**2+a**2+x**2+a+1)/(x**2
+1)/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral(x*(a**2 + 2*a*b*x**2 + a*x**2 + a + b**2*x**4 + b*x**4 + b*x**2 +
x**2 + 1)/((a + b*x**2)**(5/2)*(x**2 + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^
2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{3\,bx^2+3\,a+1}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

```
[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^
2+a)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/3*(3*b*x^2 + 3*a + 1)
/((b*x^2 + a)^(3/2)*b)
```

Mupad [B] (verification not implemented)

Time = 18.90 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{b\,x^2+a+\frac{1}{3}}{b\,(b\,x^2+a)^{3/2}}$$

```
[In] int((x*(a + a*x^2 + b*x^2 + b*x^4 + a^2 + x^2 + b^2*x^4 + 2*a*b*x^2 + 1))/(
(x^2 + 1)*(a + b*x^2)^(5/2)),x)
```

```
[Out] - atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - (a + b*x^2 + 1/3)/
(b*(a + b*x^2)^(3/2))
```

3.691 $\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx = 2\sqrt{\sqrt{x}+x} - 2\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$

```
[Out] -2*arctanh(x^(1/2)/(x+x^(1/2))^(1/2))+2*(x+x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2035, 2038, 634, 212}

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx = 2\sqrt{x+\sqrt{x}} - 2\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

```
[In] Int[1/Sqrt[Sqrt[x] + x],x]
```

```
[Out] 2*Sqrt[Sqrt[x] + x] - 2*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2035

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[-2*(Sqrt
[a*x^j + b*x^n]/(b*(n - 2)*x^(n - 1))), x] - Dist[a*((2*n - j - 2)/(b*(n -
2))), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\sqrt{\sqrt{x}+x}-\frac{1}{2}\int\frac{1}{\sqrt{x}\sqrt{\sqrt{x}+x}}\,dx \\
&= 2\sqrt{\sqrt{x}+x}-\text{Subst}\left(\int\frac{1}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right) \\
&= 2\sqrt{\sqrt{x}+x}-2\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right) \\
&= 2\sqrt{\sqrt{x}+x}-2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int\frac{1}{\sqrt{\sqrt{x}+x}}\,dx=2\sqrt{\sqrt{x}+x}+\log\left(-1-2\sqrt{x}+2\sqrt{\sqrt{x}+x}\right)$$

```
[In] Integrate[1/Sqrt[Sqrt[x] + x],x]
[Out] 2*Sqrt[Sqrt[x] + x] + Log[-1 - 2*Sqrt[x] + 2*Sqrt[Sqrt[x] + x]]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$2\sqrt{x+\sqrt{x}}-\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)$	26
meijerg	$\frac{2\sqrt{\pi}\, x^{\frac{1}{4}}\sqrt{1+\sqrt{x}}-2\sqrt{\pi}\, \operatorname{arcsinh}\left(x^{\frac{1}{4}}\right)}{\sqrt{\pi}}$	30
default	$\frac{\sqrt{x+\sqrt{x}}\left(2\sqrt{x+\sqrt{x}}-\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)\right)}{\sqrt{\sqrt{x}\,(1+\sqrt{x})}}$	45

```
[In] int(1/(x+x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(x+x^(1/2))^(1/2)-ln(1/2+x^(1/2)+(x+x^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx = 2\sqrt{x+\sqrt{x}}+\frac{1}{2}\log\left(4\sqrt{x+\sqrt{x}}(2\sqrt{x}+1)-8x-8\sqrt{x}-1\right)$$

```
[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x + sqrt(x)) + 1/2*log(4*sqrt(x + sqrt(x))*(2*sqrt(x) + 1) - 8*x - 8
*sqrt(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx = 2\sqrt{\sqrt{x}+x}-\log\left(2\sqrt{x}+2\sqrt{\sqrt{x}+x}+1\right)$$

```
[In] integrate(1/(x+x**(1/2))**(1/2),x)
[Out] 2*sqrt(sqrt(x) + x) - log(2*sqrt(x) + 2*sqrt(sqrt(x) + x) + 1)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx = \int \frac{1}{\sqrt{x+\sqrt{x}}}\,dx$$

```
[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x + sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx = 2\sqrt{x+\sqrt{x}} + \log\left(-2\sqrt{x+\sqrt{x}} + 2\sqrt{x} + 1\right)$$

```
[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x + sqrt(x)) + log(-2*sqrt(x + sqrt(x)) + 2*sqrt(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 18.75 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx = \frac{2\sqrt{x}\left(\sqrt{x}+1\right) + x^{1/4}\,\mathrm{asin}\left(x^{1/4}\,1\mathrm{i}\right)\,\sqrt{\sqrt{x}+1}\,2\mathrm{i}}{\sqrt{x+\sqrt{x}}}$$

```
[In] int(1/(x + x^(1/2))^(1/2),x)
[Out] (2*x^(1/2)*(x^(1/2) + 1) + x^(1/4)*asin(x^(1/4)*1i)*(x^(1/2) + 1)^(1/2)*2i)
/(x + x^(1/2))^(1/2)
```

3.692 $\int \sqrt{\sqrt{x}+x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 74

$$\int \sqrt{\sqrt{x}+x}\,dx=-\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{4}\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$

```
[Out] 1/4*arctanh(x^(1/2)/(x+x^(1/2))^(1/2))-1/4*(x+x^(1/2))^(1/2)+2/3*x*(x+x^(1/
2))^(1/2)+1/6*x^(1/2)*(x+x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {2029, 2043, 684, 654, 634, 212}

$$\int \sqrt{\sqrt{x}+x}\,dx=\frac{1}{4}\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)+\frac{2}{3}\sqrt{x+\sqrt{x}}x+\frac{1}{6}\sqrt{x+\sqrt{x}}\sqrt{x}-\frac{\sqrt{x+\sqrt{x}}}{4}$$

```
[In] Int[Sqrt[Sqrt[x] + x],x]
```

```
[Out] -1/4*Sqrt[Sqrt[x] + x] + (Sqrt[x]*Sqrt[Sqrt[x] + x])/6 + (2*x*Sqrt[Sqrt[x]
+ x])/3 + ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{6}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\,dx \\
&= \frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right) \\
&= \frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}-\frac{1}{4}\text{Subst}\left(\int \frac{x}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right) \\
&= -\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right)
\end{aligned}$$

$$= -\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{4}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$

$$= -\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int\sqrt{\sqrt{x}+x}\,dx = \frac{1}{12}\sqrt{\sqrt{x}+x}\left(-3+2\sqrt{x}+8x\right)+\frac{1}{4}\text{arctanh}\left(\frac{\sqrt{\sqrt{x}+x}}{\sqrt{x}}\right)$$

```
[In] Integrate[Sqrt[Sqrt[x] + x],x]
[Out] (Sqrt[Sqrt[x] + x]*(-3 + 2*Sqrt[x] + 8*x))/12 + ArcTanh[Sqrt[Sqrt[x] + x]/S
qrt[x]]/4
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
meijerg	$-\frac{\frac{\sqrt{\pi}\,x^{\frac{1}{4}}\left(-40x-10\sqrt{x}+15\right)\sqrt{1+\sqrt{x}}}{60}-\frac{\sqrt{\pi}\,\text{arcsinh}\left(x^{\frac{1}{4}}\right)}{4}}{\sqrt{\pi}}$	41
derivativedivides	$\frac{2\left(x+\sqrt{x}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x}\right)\sqrt{x+\sqrt{x}}}{4}+\frac{\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)}{8}$	42
default	$\frac{2\left(x+\sqrt{x}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x}\right)\sqrt{x+\sqrt{x}}}{4}+\frac{\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)}{8}$	42

```
[In] int((x+x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/Pi^(1/2)*(1/60*Pi^(1/2)*x^(1/4)*(-40*x-10*x^(1/2)+15)*(1+x^(1/2))^(1/2)-
1/4*Pi^(1/2)*arcsinh(x^(1/4)))
```

Fricas [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.66

$$\int \sqrt{\sqrt{x}+x}\,dx = \frac{1}{12}\left(8\,x+2\,\sqrt{x}-3\right)\sqrt{x+\sqrt{x}} + \frac{1}{16}\,\log\left(4\,\sqrt{x+\sqrt{x}}\left(2\,\sqrt{x}+1\right)+8\,x+8\,\sqrt{x}+1\right)$$

```
[In] integrate((x+x^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/12*(8*x + 2*sqrt(x) - 3)*sqrt(x + sqrt(x)) + 1/16*log(4*sqrt(x + sqrt(x))
*(2*sqrt(x) + 1) + 8*x + 8*sqrt(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int \sqrt{\sqrt{x}+x}\,dx = 2\sqrt{\sqrt{x}+x}\left(\frac{\sqrt{x}}{12}+\frac{x}{3}-\frac{1}{8}\right)+\frac{\log\left(2\sqrt{x}+2\sqrt{\sqrt{x}+x}+1\right)}{8}$$

```
[In] integrate((x+x**(1/2))**(1/2),x)
[Out] 2*sqrt(sqrt(x) + x)*(sqrt(x)/12 + x/3 - 1/8) + log(2*sqrt(x) + 2*sqrt(sqrt(
x) + x) + 1)/8
```

Maxima [F]

$$\int \sqrt{\sqrt{x}+x}\,dx = \int \sqrt{x+\sqrt{x}}\,dx$$

```
[In] integrate((x+x^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.58

$$\int \sqrt{\sqrt{x}+x}\,dx = \frac{1}{12}\left(2\sqrt{x}(4\sqrt{x}+1)-3\right)\sqrt{x+\sqrt{x}}-\frac{1}{8}\log\left(-2\sqrt{x+\sqrt{x}}+2\sqrt{x}+1\right)$$

```
[In] integrate((x+x^(1/2))^(1/2),x, algorithm="giac")
[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) - 3)*sqrt(x + sqrt(x)) - 1/8*log(-2*sqrt(x
+ sqrt(x)) + 2*sqrt(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 18.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.36

$$\int \sqrt{\sqrt{x}+x}\,dx = \frac{4\,x\,\sqrt{x+\sqrt{x}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{5}{2};\ \frac{7}{2};\ -\sqrt{x}\right)}{5\,\sqrt{\sqrt{x}+1}}$$

```
[In] int((x + x^(1/2))^(1/2),x)
[Out] (4*x*(x + x^(1/2))^(1/2)*hypergeom([-1/2, 5/2], 7/2, -x^(1/2)))/(5*(x^(1/2)
 + 1)^(1/2))
```

3.693 $\int \sqrt{-x}(\sqrt{-x}+x)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \sqrt{-x}(\sqrt{-x}+x)\,dx=\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}$$

```
[Out] 2/5*(-x)^(5/2)-1/2*x^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\int \sqrt{-x}(\sqrt{-x}+x)\,dx=\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}$$

```
[In] Int[Sqrt[-x]*(Sqrt[-x] + x),x]
[Out] (2*(-x)^(5/2))/5 - x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(-(-x)^{3/2}-x\right)\,dx\\&=\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \sqrt{-x}\left(\sqrt{-x}+x\right)\,dx=\frac{1}{10}\left(-5+4\sqrt{-x}\right)x^2$$

```
[In] Integrate[Sqrt[-x]*(Sqrt[-x] + x),x]
[Out] ((-5 + 4*Sqrt[-x])*x^2)/10
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{2(-x)^{\frac{5}{2}}}{5}-\frac{x^2}{2}$	14
default	$\frac{2(-x)^{\frac{5}{2}}}{5}-\frac{x^2}{2}$	14
trager	$-\frac{(x-1)(x+1)}{2}+\frac{2x^2\sqrt{-x}}{5}$	20

```
[In] int((-x)^(1/2)*(x+(-x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/5*(-x)^(5/2)-1/2*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \sqrt{-x}\left(\sqrt{-x}+x\right)\,dx=\frac{2}{5}\sqrt{-x}x^2-\frac{1}{2}x^2$$

```
[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="fricas")
[Out] 2/5*sqrt(-x)*x^2 - 1/2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \sqrt{-x}\left(\sqrt{-x}+x\right)\,dx = \frac{2x^2\sqrt{-x}}{5} - \frac{x^2}{2}$$

```
[In] integrate((-x)**(1/2)*(x+(-x)**(1/2)),x)
[Out] 2*x**2*sqrt(-x)/5 - x**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \sqrt{-x}\left(\sqrt{-x}+x\right)\,dx = \frac{2}{5}\,(-x)^{\frac{5}{2}} - \frac{1}{2}\,x^2$$

```
[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="maxima")
[Out] 2/5*(-x)^(5/2) - 1/2*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \sqrt{-x}\left(\sqrt{-x}+x\right)\,dx = \frac{2}{5}\,\sqrt{-x}x^2 - \frac{1}{2}\,x^2$$

```
[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="giac")
[Out] 2/5*sqrt(-x)*x^2 - 1/2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \sqrt{-x}\left(\sqrt{-x}+x\right)\,dx = \frac{2\,(-x)^{5/2}}{5} - \frac{x^2}{2}$$

```
[In] int((-x)^(1/2)*(x + (-x)^(1/2)),x)
[Out] (2*(-x)^(5/2))/5 - x^2/2
```

3.694 $\int \frac{5+\sqrt[4]{x}}{-6+x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{5+\sqrt[4]{x}}{-6+x} \, dx = 4\sqrt[4]{x}-2\sqrt[4]{6}\arctan\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\text{arctanh}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)+5\log(6-x)$$

```
[Out] 4*x^(1/4)-2*6^(1/4)*arctan(1/6*x^(1/4)*6^(3/4))-2*6^(1/4)*arctanh(1/6*x^(1/
4)*6^(3/4))+5*ln(6-x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1845, 266, 327, 218, 212, 209}

$$\int \frac{5+\sqrt[4]{x}}{-6+x} \, dx = -2\sqrt[4]{6}\arctan\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\text{arctanh}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)+4\sqrt[4]{x}+5\log(6-x)$$

```
[In] Int[(5 + x^(1/4))/(-6 + x),x]
```

```
[Out] 4*x^(1/4) - 2*6^(1/4)*ArcTan[x^(1/4)/6^(1/4)] - 2*6^(1/4)*ArcTanh[x^(1/4)/6
^(1/4)] + 5*Log[6 - x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 4\text{Subst}\left(\int \frac{x^3(5+x)}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\text{Subst}\left(\int \left(\frac{5x^3}{-6+x^4}+\frac{x^4}{-6+x^4}\right)\,dx, x, \sqrt[4]{x}\right)\\
&= 4\text{Subst}\left(\int \frac{x^4}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)+20\text{Subst}\left(\int \frac{x^3}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\sqrt[4]{x}+5\log(6-x)+24\text{Subst}\left(\int \frac{1}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)
\end{aligned}$$

$$= 4\sqrt[4]{x} + 5\log(6-x) - \left(2\sqrt{6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{6}-x^2}\,dx, x, \sqrt[4]{x}\right) - \left(2\sqrt{6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{6}+x^2}\,dx, x, \sqrt[4]{x}\right)$$

$$= 4\sqrt[4]{x} - 2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right) - 2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right) + 5\log(6-x)$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx = 4\sqrt[4]{x} - 2\sqrt[4]{6}\arctan\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right) - 2\sqrt[4]{6}\text{arctanh}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right) + 5\log(-6+x)$$

```
[In] Integrate[(5 + x^(1/4))/(-6 + x),x]
[Out] 4*x^(1/4) - 2*6^(1/4)*ArcTan[x^(1/4)/6^(1/4)] - 2*6^(1/4)*ArcTanh[x^(1/4)/6
^(1/4)] + 5*Log[-6 + x]
```

Maple [A] (verified)

Time = 9.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result
derivativedivides	$4x^{\frac{1}{4}} - 6^{\frac{1}{4}}\left(\ln\left(\frac{x^{\frac{1}{4}}+6^{\frac{1}{4}}}{x^{\frac{1}{4}}-6^{\frac{1}{4}}}\right) + 2\arctan\left(\frac{x^{\frac{1}{4}}6^{\frac{3}{4}}}{6}\right)\right) + 5\ln(-6+x)$
default	$4x^{\frac{1}{4}} - 6^{\frac{1}{4}}\left(\ln\left(\frac{x^{\frac{1}{4}}+6^{\frac{1}{4}}}{x^{\frac{1}{4}}-6^{\frac{1}{4}}}\right) + 2\arctan\left(\frac{x^{\frac{1}{4}}6^{\frac{3}{4}}}{6}\right)\right) + 5\ln(-6+x)$
meijerg	$5\ln\left(1-\frac{x}{6}\right) - 6^{\frac{1}{4}}(-1)^{\frac{3}{4}}\left(\frac{2\,8^{\frac{1}{4}}3^{\frac{3}{4}}x^{\frac{1}{4}}(-1)^{\frac{1}{4}}}{3} + (-1)^{\frac{1}{4}}\left(\ln\left(1-\frac{x^{\frac{1}{4}}6^{\frac{3}{4}}}{6}\right) - \ln\left(1+\frac{x^{\frac{1}{4}}6^{\frac{3}{4}}}{6}\right) - 2\,a\right.\right.$
trager	Expression too large to display

```
[In] int((5+x^(1/4))/(-6+x),x,method=_RETURNVERBOSE)
[Out] 4*x^(1/4)-6^(1/4)*(ln((x^(1/4)+6^(1/4))/(x^(1/4)-6^(1/4)))+2*arctan(1/6*x^(
1/4)*6^(3/4)))+5*ln(-6+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(42) = 84$.

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.78

$$\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx = -\left(\sqrt{-\sqrt{6}}-5\right)\log\left(2\sqrt{-\sqrt{6}}+2x^{\frac{1}{4}}\right) + \left(\sqrt{-\sqrt{6}}+5\right)\log\left(-2\sqrt{-\sqrt{6}}+2x^{\frac{1}{4}}\right) - \left(6^{\frac{1}{4}}-5\right)\log\left(2\cdot 6^{\frac{1}{4}}+2x^{\frac{1}{4}}\right) + \left(6^{\frac{1}{4}}+5\right)\log\left(-2\cdot 6^{\frac{1}{4}}+2x^{\frac{1}{4}}\right) + 4x^{\frac{1}{4}}$$

```
[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="fricas")
```

```
[Out] -(sqrt(-sqrt(6)) - 5)*log(2*sqrt(-sqrt(6)) + 2*x^(1/4)) + (sqrt(-sqrt(6)) +
 5)*log(-2*sqrt(-sqrt(6)) + 2*x^(1/4)) - (6^(1/4) - 5)*log(2*6^(1/4) + 2*x^
(1/4)) + (6^(1/4) + 5)*log(-2*6^(1/4) + 2*x^(1/4)) + 4*x^(1/4)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.85

$$\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx = 4\sqrt[4]{x} + \sqrt[4]{6}\log\left(\sqrt[4]{x}-\sqrt[4]{6}\right) + 5\log\left(\sqrt[4]{x}-\sqrt[4]{6}\right) - \sqrt[4]{6}\log\left(\sqrt[4]{x}+\sqrt[4]{6}\right) + 5\log\left(\sqrt[4]{x}+\sqrt[4]{6}\right) + 5\log\left(\sqrt{x}+\sqrt{6}\right) - 2\cdot\sqrt[4]{6}\,\mathrm{atan}\left(\frac{6^{\frac{3}{4}}\sqrt[4]{x}}{6}\right)$$

```
[In] integrate((5+x**(1/4))/(-6+x),x)
```

```
[Out] 4*x**(1/4) + 6**(1/4)*log(x**(1/4) - 6**(1/4)) + 5*log(x**(1/4) - 6**(1/4))
 - 6**(1/4)*log(x**(1/4) + 6**(1/4)) + 5*log(x**(1/4) + 6**(1/4)) + 5*log(s
qrt(x) + sqrt(6)) - 2*6**(1/4)*atan(6**(3/4)*x**(1/4)/6)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx = -2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right) + 6^{\frac{1}{4}}\log\left(-\frac{6^{\frac{1}{4}}-x^{\frac{1}{4}}}{6^{\frac{1}{4}}+x^{\frac{1}{4}}}\right) + 4x^{\frac{1}{4}} + 5\log\left(\sqrt{6}+\sqrt{x}\right) + 5\log\left(-\sqrt{6}+\sqrt{x}\right)$$

```
[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="maxima")
```

```
[Out] -2*6^(1/4)*arctan(1/6*6^(3/4)*x^(1/4)) + 6^(1/4)*log(-(6^(1/4) - x^(1/4))/(
6^(1/4) + x^(1/4))) + 4*x^(1/4) + 5*log(sqrt(6) + sqrt(x)) + 5*log(-sqrt(6)
 + sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx = -2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right) - 6^{\frac{1}{4}}\log\left(6^{\frac{1}{4}}+x^{\frac{1}{4}}\right) + 6^{\frac{1}{4}}\log\left(\left|-6^{\frac{1}{4}}+x^{\frac{1}{4}}\right|\right) + 4\,x^{\frac{1}{4}} + 5\,\log\left(|x-6|\right)$$

```
[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="giac")
```

```
[Out] -2*6^(1/4)*arctan(1/6*6^(3/4)*x^(1/4)) - 6^(1/4)*log(6^(1/4) + x^(1/4)) + 6
^(1/4)*log(abs(-6^(1/4) + x^(1/4))) + 4*x^(1/4) + 5*log(abs(x - 6))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.00

$$\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx = \ln\left(11520\,x^{1/4} - \left(6^{1/4}+5\right)\,\left(2304\,x^{1/4}-2304\,6^{1/4}+11520\right)+57600\right)\,\left(6^{1/4}+5\right)-\ln\left(\left(6^{1/4}-5\right)\,\left(2304\,6^{1/4}+2304\,x^{1/4}+1152\right.\right.$$

```
[In] int((x^(1/4) + 5)/(x - 6),x)
```

```
[Out] log(11520*x^(1/4) - (6^(1/4) + 5)*(2304*x^(1/4) - 2304*6^(1/4) + 11520) + 5
7600)*(6^(1/4) + 5) - log((6^(1/4) - 5)*(2304*6^(1/4) + 2304*x^(1/4) + 1152
0) + 11520*x^(1/4) + 57600)*(6^(1/4) - 5) - log(11520*x^(1/4) + ((-6^(1/2))
^(1/2) - 5)*(2304*(-6^(1/2))^(1/2) + 2304*x^(1/4) + 11520) + 57600)*((-6^(1
/2))^(1/2) - 5) + log(11520*x^(1/4) - ((-6^(1/2))^(1/2) + 5)*(2304*x^(1/4)
- 2304*(-6^(1/2))^(1/2) + 11520) + 57600)*((-6^(1/2))^(1/2) + 5) + 4*x^(1/4
)
```

3.695 $\int \frac{1}{4+\sqrt{4-x}-x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \frac{1}{4+\sqrt{4-x}-x} \, dx = -2\log\left(1+\sqrt{4-x}\right)$$

```
[Out] -2*ln(1+(4-x)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {31}

$$\int \frac{1}{4+\sqrt{4-x}-x} \, dx = -2\log\left(\sqrt{4-x}+1\right)$$

```
[In] Int[(4 + Sqrt[4 - x] - x)^(-1),x]
```

```
[Out] -2*Log[1 + Sqrt[4 - x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\left(2\text{Subst}\left(\int \frac{1}{1+x} \, dx, x, \sqrt{4-x}\right)\right) \\ &= -2\log\left(1+\sqrt{4-x}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{4+\sqrt{4-x}-x}\,dx = -2\log\left(1+\sqrt{4-x}\right)$$

```
[In] Integrate[(4 + Sqrt[4 - x] - x)^(-1),x]
[Out] -2*Log[1 + Sqrt[4 - x]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-2\ln\left(1+\sqrt{4-x}\right)$	13
default	$-\ln\left(-3+x\right)-2\,\mathrm{arctanh}\left(\sqrt{4-x}\right)$	18
trager	$-\ln\left(2\sqrt{4-x}+5-x\right)$	18

```
[In] int(1/(4-x+(4-x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -2*ln(1+(4-x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{4+\sqrt{4-x}-x}\,dx = -2\,\log\left(\sqrt{-x+4}+1\right)$$

```
[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="fricas")
[Out] -2*log(sqrt(-x + 4) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(12) = 24.

Time = 0.65 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \frac{1}{4+\sqrt{4-x}-x}\,dx = \log\left(2\sqrt{4-x}\right) - \log\left(2\sqrt{4-x}+2\right) - \log\left(-x+\sqrt{4-x}+4\right)$$

```
[In] integrate(1/(4-x+(4-x)**(1/2)),x)
[Out] log(2*sqrt(4 - x)) - log(2*sqrt(4 - x) + 2) - log(-x + sqrt(4 - x) + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{4+\sqrt{4-x}-x}\,dx = -2\,\log\left(\sqrt{-x+4}+1\right)$$

```
[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="maxima")
[Out] -2*log(sqrt(-x + 4) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{4+\sqrt{4-x}-x}\,dx = -2\,\log\left(\sqrt{-x+4}+1\right)$$

```
[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="giac")
[Out] -2*log(sqrt(-x + 4) + 1)
```

Mupad [B] (verification not implemented)

Time = 17.96 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{4+\sqrt{4-x}-x}\,dx = -2\,\ln\left(\sqrt{4-x}+1\right)$$

```
[In] int(1/((4 - x)^(1/2) - x + 4),x)
[Out] -2*log((4 - x)^(1/2) + 1)
```

3.696 $\int \frac{1}{1+x-\sqrt{2+x}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \frac{1}{1+x-\sqrt{2+x}} \, dx = \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{2+x}\right) + \frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{2+x}\right)$$

```
[Out] 1/5*ln(1-5^(1/2)-2*(2+x)^(1/2))*(5-5^(1/2))+1/5*ln(1+5^(1/2)-2*(2+x)^(1/2))
*(5+5^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {646, 31}

$$\int \frac{1}{1+x-\sqrt{2+x}} \, dx = \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+2}-\sqrt{5}+1\right) + \frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+2}+\sqrt{5}+1\right)$$

```
[In] Int[(1 + x - Sqrt[2 + x])^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[2 + x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[2 + x]])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{x}{-1-x+x^2}\,dx, x, \sqrt{2+x}\right)\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, \sqrt{2+x}\right)\\ &\quad+\frac{1}{5}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, \sqrt{2+x}\right)\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{2+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{2+x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{1}{1+x-\sqrt{2+x}}\,dx = \frac{1}{5}\left(\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{2+x}\right) - \left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2\sqrt{2+x}\right)\right)$$

```
[In] Integrate[(1 + x - Sqrt[2 + x])^(-1),x]
[Out] ((5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[2 + x]] - (-5 + Sqrt[5])*Log[-1 + S
qrt[5] + 2*Sqrt[2 + x]])/5
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.54

method	result
derivativedivides	$\ln\left(1+x-\sqrt{x+2}\right)-\frac{2\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2\sqrt{x+2}-1\right)\sqrt{5}}{5}\right)}{5}$
default	$-\frac{\operatorname{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)\sqrt{5}}{5}+\frac{\ln(x^2+x-1)}{2}+\frac{\ln(1+x-\sqrt{x+2})}{2}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2\sqrt{x+2}-1)\sqrt{5}}{5}\right)}{5}-\frac{\ln(x+1+\sqrt{x+2})}{2}$
trager	$-\ln\left(-1-x+\sqrt{x+2}\right)\operatorname{RootOf}\left(5\_Z^2-10\_Z+4\right)+\ln\Big(150\operatorname{RootOf}\left(5\_Z^2-10\_Z\right.$

```
[In] int(1/(1+x-(x+2)^(1/2)),x,method=_RETURNVERBOSE)
[Out] ln(1+x-(x+2)^(1/2))-2/5*5^(1/2)*arctanh(1/5*(2*(x+2)^(1/2)-1)*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \frac{1}{1+x-\sqrt{2+x}}\,dx = \frac{1}{5}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(x+3)-\left(\sqrt{5}(2x+1)-5\right)\sqrt{x+2}+7x+3}{x^2+x-1}\right) + \log\left(x-\sqrt{x+2}+1\right)$$

```
[In] integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="fricas")
[Out] 1/5*sqrt(5)*log((2*x^2 - sqrt(5)*(x + 3) - (sqrt(5)*(2*x + 1) - 5)*sqrt(x +
 2) + 7*x + 3)/(x^2 + x - 1)) + log(x - sqrt(x + 2) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{1}{1+x-\sqrt{2+x}}\,dx = \frac{\sqrt{5}\left(-\log\left(\sqrt{x+2}-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+\log\left(\sqrt{x+2}-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\right)}{5} + \log\left(x-\sqrt{x+2}+1\right)$$

```
[In] integrate(1/(1+x-(2+x)**(1/2)),x)
[Out] sqrt(5)*(-log(sqrt(x + 2) - 1/2 + sqrt(5)/2) + log(sqrt(x + 2) - sqrt(5)/2
- 1/2))/5 + log(x - sqrt(x + 2) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+x-\sqrt{2+x}}\,dx = \frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{x+2}+1}{\sqrt{5}+2\sqrt{x+2}-1}\right)+\log\left(x-\sqrt{x+2}+1\right)$$

```
[In] integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 2) + 1)/(sqrt(5) + 2*sqrt(x + 2) - 1
)) + log(x - sqrt(x + 2) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+x-\sqrt{2+x}}\,dx = \frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{x+2}-1\right|}{\left|\sqrt{5}+2\sqrt{x+2}-1\right|}\right)+\log\left(\left|x-\sqrt{x+2}+1\right|\right)$$

```
[In] integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(x + 2) - 1)/abs(sqrt(5) + 2*sqrt(x +
2) - 1)) + log(abs(x - sqrt(x + 2) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

$$\int \frac{1}{1+x-\sqrt{2+x}}\,dx = \ln\left(2\sqrt{x+2}-\left(\frac{\sqrt{5}}{5}+1\right)\left(2\sqrt{x+2}-1\right)\right)\left(\frac{\sqrt{5}}{5}+1\right) - \ln\left(2\sqrt{x+2}+\left(\frac{\sqrt{5}}{5}-1\right)\left(2\sqrt{x+2}-1\right)\right)\left(\frac{\sqrt{5}}{5}-1\right)$$

```
[In] int(1/(x - (x + 2)^(1/2) + 1),x)
```

```
[Out] log(2*(x + 2)^(1/2) - (5^(1/2)/5 + 1)*(2*(x + 2)^(1/2) - 1))*(5^(1/2)/5 + 1
) - log(2*(x + 2)^(1/2) + (5^(1/2)/5 - 1)*(2*(x + 2)^(1/2) - 1))*(5^(1/2)/5
 - 1)
```

3.697 $\int \frac{1}{4+x+\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx=-\frac{2\arctan\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}}+\log\left(4+x+\sqrt{1+x}\right)$$

```
[Out] ln(4+x+(1+x)^(1/2))-2/11*arctan(1/11*(1+2*(1+x)^(1/2))*11^(1/2))*11^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {648, 632, 210, 642}

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx=\log\left(x+\sqrt{x+1}+4\right)-\frac{2\arctan\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

```
[In] Int[(4 + x + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= -\text{Subst}\left(\int \frac{1}{3+x+x^2}\,dx, x, \sqrt{1+x}\right) + \text{Subst}\left(\int \frac{1+2x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= \log\left(4+x+\sqrt{1+x}\right) + 2\text{Subst}\left(\int \frac{1}{-11-x^2}\,dx, x, 1+2\sqrt{1+x}\right)\\
&= -\frac{2\tan^{-1}\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}} + \log\left(4+x+\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = -\frac{2\arctan\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}} + \log\left(4+x+\sqrt{1+x}\right)$$

```
[In] Integrate[(4 + x + Sqrt[1 + x])^(-1),x]
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\ln \left(4+x+\sqrt{x+1}\right)-\frac{2 \arctan \left(\frac{\left(1+2 \sqrt{x+1}\right) \sqrt{11}}{11}\right) \sqrt{11}}{11}$
default	$\frac{\ln \left(4+x+\sqrt{x+1}\right)}{2}-\frac{\arctan \left(\frac{\left(1+2 \sqrt{x+1}\right) \sqrt{11}}{11}\right) \sqrt{11}}{11}-\frac{\ln \left(x+4-\sqrt{x+1}\right)}{2}-\frac{\sqrt{11}\, \arctan \left(\frac{\left(2 \sqrt{x+1}-1\right) \sqrt{11}}{11}\right)}{11}+\frac{\sqrt{11}\, \mathrm{arct}}{}$
trager	$-\ln \left(4+x+\sqrt{x+1}\right) \operatorname{RootOf}\left(11\_Z^{2}-22\_Z+12\right)+\ln \left(-847 \operatorname{RootOf}\left(11\_Z^{2}-22\_\right.\right.$

```
[In] int(1/(4+x+(x+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] ln(4+x+(x+1)^(1/2))-2/11*arctan(1/11*(1+2*(x+1)^(1/2))*11^(1/2))*11^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx=-\frac{2}{11}\,\sqrt{11}\arctan\left(\frac{2}{11}\,\sqrt{11}\sqrt{x+1}+\frac{1}{11}\,\sqrt{11}\right)+\log\left(x+\sqrt{x+1}+4\right)$$

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="fricas")
[Out] -2/11*sqrt(11)*arctan(2/11*sqrt(11)*sqrt(x + 1) + 1/11*sqrt(11)) + log(x +
sqrt(x + 1) + 4)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx=\log\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\operatorname{atan}\left(\frac{2\sqrt{11}\left(\sqrt{x+1}+\frac{1}{2}\right)}{11}\right)}{11}$$

```
[In] integrate(1/(4+x+(1+x)**(1/2)),x)
[Out] log(x + sqrt(x + 1) + 4) - 2*sqrt(11)*atan(2*sqrt(11)*(sqrt(x + 1) + 1/2)/1
1)/11
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = -\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="maxima")
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = -\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="giac")
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

Mupad [B] (verification not implemented)

Time = 18.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = \ln\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\,\mathrm{atan}\left(\frac{\sqrt{11}}{11}+\frac{2\sqrt{11}\sqrt{x+1}}{11}\right)}{11}$$

```
[In] int(1/(x + (x + 1)^(1/2) + 4),x)
[Out] log(x + (x + 1)^(1/2) + 4) - (2*11^(1/2)*atan(11^(1/2)/11 + (2*11^(1/2)*(x
+ 1)^(1/2))/11))/11
```

3.698 $\int \frac{1}{x-\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{1}{x-\sqrt{1+x}}\,dx=\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{1+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{1+x}\right)$$

```
[Out] 1/5*ln(1-5^(1/2)-2*(1+x)^(1/2))*(5-5^(1/2))+1/5*ln(1+5^(1/2)-2*(1+x)^(1/2))
*(5+5^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {646, 31}

$$\int \frac{1}{x-\sqrt{1+x}}\,dx=\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+1}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+1}+\sqrt{5}+1\right)$$

```
[In] Int[(x - Sqrt[1 + x])^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[1 + x]])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{x}{-1-x+x^2}\,dx, x, \sqrt{1+x}\right)\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, \sqrt{1+x}\right)\\ &\quad+\frac{1}{5}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, \sqrt{1+x}\right)\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{1+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{1}{x-\sqrt{1+x}}\,dx = \frac{1}{5}\left(\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{1+x}\right) - \left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2\sqrt{1+x}\right)\right)$$

```
[In] Integrate[(x - Sqrt[1 + x])^(-1),x]
[Out] ((5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[1 + x]] - (-5 + Sqrt[5])*Log[-1 + S
qrt[5] + 2*Sqrt[1 + x]])/5
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

method	result
derivativedivides	$\ln\left(x-\sqrt{x+1}\right)-\frac{2\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2\sqrt{x+1}-1)\sqrt{5}}{5}\right)}{5}$
default	$\frac{\ln(x^2-x-1)}{2}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{5}+\frac{\ln(x-\sqrt{x+1})}{2}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2\sqrt{x+1}-1)\sqrt{5}}{5}\right)}{5}-\frac{\ln(x+\sqrt{x+1})}{2}-\frac{\sqrt{5}}{}$
trager	$\operatorname{RootOf}\left(5\_Z^2-10\_Z+4\right)\ln\left(x-\sqrt{x+1}\right)-\ln\left(5\operatorname{RootOf}\left(5\_Z^2-10\_Z+4\right)^2x-5\right.$

[In] int(1/(x-(x+1)^(1/2)),x,method=_RETURNVERBOSE)

[Out] ln(x-(x+1)^(1/2))-2/5*5^(1/2)*arctanh(1/5*(2*(x+1)^(1/2)-1)*5^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{1}{x-\sqrt{1+x}}\,dx=\frac{1}{5}\sqrt{5}\log\left(\frac{2\,x^2-\sqrt{5}(x+2)-\left(\sqrt{5}(2\,x-1)-5\right)\sqrt{x+1}+3\,x-2}{x^2-x-1}\right)+\log\left(x-\sqrt{x+1}\right)$$

[In] integrate(1/(x-(1+x)^(1/2)),x, algorithm="fricas")

[Out] 1/5*sqrt(5)*log((2*x^2 - sqrt(5)*(x + 2) - (sqrt(5)*(2*x - 1) - 5)*sqrt(x + 1) + 3*x - 2)/(x^2 - x - 1)) + log(x - sqrt(x + 1))

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{1}{x-\sqrt{1+x}}\,dx=\frac{\sqrt{5}\left(-\log\left(\sqrt{x+1}-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+\log\left(\sqrt{x+1}-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\right)}{5}+\log\left(x-\sqrt{x+1}\right)$$

[In] integrate(1/(x-(1+x)**(1/2)),x)

[Out] sqrt(5)*(-log(sqrt(x + 1) - 1/2 + sqrt(5)/2) + log(sqrt(x + 1) - sqrt(5)/2 - 1/2))/5 + log(x - sqrt(x + 1))

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \frac{1}{x-\sqrt{1+x}}\,dx = \frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{x+1}+1}{\sqrt{5}+2\sqrt{x+1}-1}\right)+\log\left(x-\sqrt{x+1}\right)$$

```
[In] integrate(1/(x-(1+x)^(1/2)),x, algorithm="maxima")
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 1) + 1)/(sqrt(5) + 2*sqrt(x + 1) - 1
)) + log(x - sqrt(x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{1}{x-\sqrt{1+x}}\,dx = \frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{x+1}-1\right|}{\left|\sqrt{5}+2\sqrt{x+1}-1\right|}\right)+\log\left(\left|x-\sqrt{x+1}\right|\right)$$

```
[In] integrate(1/(x-(1+x)^(1/2)),x, algorithm="giac")
[Out] 1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(x + 1) - 1)/abs(sqrt(5) + 2*sqrt(x +
1) - 1)) + log(abs(x - sqrt(x + 1)))
```

Mupad [B] (verification not implemented)

Time = 17.77 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

$$\int \frac{1}{x-\sqrt{1+x}}\,dx = \ln\left(2\sqrt{x+1}-\left(\frac{\sqrt{5}}{5}+1\right)\left(2\sqrt{x+1}-1\right)\right)\left(\frac{\sqrt{5}}{5}+1\right) - \ln\left(2\sqrt{x+1}+\left(\frac{\sqrt{5}}{5}-1\right)\left(2\sqrt{x+1}-1\right)\right)\left(\frac{\sqrt{5}}{5}-1\right)$$

```
[In] int(1/(x - (x + 1)^(1/2)),x)
[Out] log(2*(x + 1)^(1/2) - (5^(1/2)/5 + 1)*(2*(x + 1)^(1/2) - 1))*(5^(1/2)/5 + 1
) - log(2*(x + 1)^(1/2) + (5^(1/2)/5 - 1)*(2*(x + 1)^(1/2) - 1))*(5^(1/2)/5
 - 1)
```

3.699 $\int \frac{1}{x-\sqrt{2+x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{x-\sqrt{2+x}}\,dx=\frac{4}{3}\log\left(2-\sqrt{2+x}\right)+\frac{2}{3}\log\left(1+\sqrt{2+x}\right)$$

```
[Out] 4/3*ln(2-(2+x)^(1/2))+2/3*ln(1+(2+x)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {646, 31}

$$\int \frac{1}{x-\sqrt{2+x}}\,dx=\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

```
[In] Int[(x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \text{integral} &= 2\text{Subst}\left(\int \frac{x}{-2-x+x^2}\,dx, x, \sqrt{2+x}\right) \\ &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{2+x}\right) + \frac{4}{3}\text{Subst}\left(\int \frac{1}{-2+x}\,dx, x, \sqrt{2+x}\right) \\ &= \frac{4}{3}\log\left(2-\sqrt{2+x}\right) + \frac{2}{3}\log\left(1+\sqrt{2+x}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \frac{4}{3}\log\left(-2+\sqrt{2+x}\right) + \frac{2}{3}\log\left(1+\sqrt{2+x}\right)$$

```
[In] Integrate[(x - Sqrt[2 + x])^(-1),x]
[Out] (4*Log[-2 + Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2\ln\left(1+\sqrt{x+2}\right)}{3}+\frac{4\ln\left(\sqrt{x+2}-2\right)}{3}$	22
trager	$\frac{\ln\left(6\sqrt{x+2}\,x^2-x^3+16\sqrt{x+2}\,x-15x^2+8\sqrt{x+2}-24x-12\right)}{3}$	44
default	$\frac{\ln(x+1)}{3}+\frac{2\ln(x-2)}{3}+\frac{\ln\left(1+\sqrt{x+2}\right)}{3}-\frac{2\ln\left(\sqrt{x+2}+2\right)}{3}-\frac{\ln\left(\sqrt{x+2}-1\right)}{3}+\frac{2\ln\left(\sqrt{x+2}-2\right)}{3}$	54

```
[In] int(1/(x-(x+2)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/3*ln(1+(x+2)^(1/2))+4/3*ln((x+2)^(1/2)-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="fricas")
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \log\left(x-\sqrt{x+2}\right)+\frac{\log\left(2\sqrt{x+2}-4\right)}{3}-\frac{\log\left(2\sqrt{x+2}+2\right)}{3}$$

```
[In] integrate(1/(x-(2+x)**(1/2)),x)
[Out] log(x - sqrt(x + 2)) + log(2*sqrt(x + 2) - 4)/3 - log(2*sqrt(x + 2) + 2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="maxima")
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\left|\sqrt{x+2}-2\right|\right)$$

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="giac")
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(abs(sqrt(x + 2) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \frac{2\ln\left(\frac{2\sqrt{x+2}}{3}+\frac{2}{3}\right)}{3}+\frac{4\ln\left(\frac{4}{3}-\frac{2\sqrt{x+2}}{3}\right)}{3}$$

```
[In] int(1/(x - (x + 2)^(1/2)),x)
[Out] (2*log((2*(x + 2)^(1/2))/3 + 2/3))/3 + (4*log(4/3 - (2*(x + 2)^(1/2))/3))/3
```

3.700 $\int \frac{1}{-\sqrt{1-x}+x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{1}{-\sqrt{1-x}+x} \, dx = \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}+2\sqrt{1-x}\right) + \frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2\sqrt{1-x}\right)$$

```
[Out] 1/5*ln(1-5^(1/2)+2*(1-x)^(1/2))*(5-5^(1/2))+1/5*ln(1+5^(1/2)+2*(1-x)^(1/2))
*(5+5^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {646, 31}

$$\int \frac{1}{-\sqrt{1-x}+x} \, dx = \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(2\sqrt{1-x}-\sqrt{5}+1\right) + \frac{1}{5}\left(5+\sqrt{5}\right)\log\left(2\sqrt{1-x}+\sqrt{5}+1\right)$$

```
[In] Int[(-Sqrt[1 - x] + x)^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 - x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] + 2*Sqrt[1 - x]])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{x}{-1+x+x^2}\,dx, x, \sqrt{1-x}\right)\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, \sqrt{1-x}\right)\\ &\quad+\frac{1}{5}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, \sqrt{1-x}\right)\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}+2\sqrt{1-x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2\sqrt{1-x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{1}{-\sqrt{1-x}+x}\,dx = \frac{1}{5}\left(-\left(\left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2\sqrt{1-x}\right)\right)+\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2\sqrt{1-x}\right)\right)$$

```
[In] Integrate[(-Sqrt[1 - x] + x)^(-1),x]
[Out] (-((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*Sqrt[1 - x]]) + (5 + Sqrt[5])*Log[1
+ Sqrt[5] + 2*Sqrt[1 - x]])/5
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

method	result
derivativedivides	$\ln\left(-x+\sqrt{1-x}\right)+\frac{2\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2\sqrt{1-x}+1)\sqrt{5}}{5}\right)}{5}$
default	$\frac{\ln(x^2+x-1)}{2}+\frac{\operatorname{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)\sqrt{5}}{5}-\frac{\ln(-x-\sqrt{1-x})}{2}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2\sqrt{1-x}-1)\sqrt{5}}{5}\right)}{5}+\frac{\ln(-x+\sqrt{1-x})}{2}+$
trager	$-\ln\left(x-\sqrt{1-x}\right)\operatorname{RootOf}\left(5\_Z^2-10\_Z+4\right)+\ln\left(5\operatorname{RootOf}\left(5\_Z^2-10\_Z+4\right)^2x\right.$

```
[In] int(1/(x-(1-x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] ln(-x+(1-x)^(1/2))+2/5*5^(1/2)*arctanh(1/5*(2*(1-x)^(1/2)+1)*5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{1}{-\sqrt{1-x}+x}\,dx = \frac{1}{5}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(x-2)-\left(\sqrt{5}(2x+1)+5\right)\sqrt{-x+1}-3x-2}{x^2+x-1}\right) + \log\left(-x+\sqrt{-x+1}\right)$$

```
[In] integrate(1/(x-(1-x)^(1/2)),x, algorithm="fricas")
[Out] 1/5*sqrt(5)*log((2*x^2 + sqrt(5)*(x - 2) - (sqrt(5)*(2*x + 1) + 5)*sqrt(-x
+ 1) - 3*x - 2)/(x^2 + x - 1)) + log(-x + sqrt(-x + 1))
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{1}{-\sqrt{1-x}+x}\,dx = -\frac{\sqrt{5}\left(-\log\left(\sqrt{1-x}+\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+\log\left(\sqrt{1-x}-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\right)}{5} + \log\left(x-\sqrt{1-x}\right)$$

```
[In] integrate(1/(x-(1-x)**(1/2)),x)
[Out] -sqrt(5)*(-log(sqrt(1 - x) + 1/2 + sqrt(5)/2) + log(sqrt(1 - x) - sqrt(5)/2
 + 1/2))/5 + log(x - sqrt(1 - x))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{1}{-\sqrt{1-x}+x}\,dx = -\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{-x+1}-1}{\sqrt{5}+2\sqrt{-x+1}+1}\right)+\log\left(-x+\sqrt{-x+1}\right)$$

```
[In] integrate(1/(x-(1-x)^(1/2)),x, algorithm="maxima")
[Out] -1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(-x + 1) - 1)/(sqrt(5) + 2*sqrt(-x + 1)
+ 1)) + log(-x + sqrt(-x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{1}{-\sqrt{1-x}+x}\,dx = -\frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{-x+1}+1\right|}{\sqrt{5}+2\sqrt{-x+1}+1}\right)+\log\left(\left|-x+\sqrt{-x+1}\right|\right)$$

```
[In] integrate(1/(x-(1-x)^(1/2)),x, algorithm="giac")
[Out] -1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(-x + 1) + 1)/(sqrt(5) + 2*sqrt(-x +
1) + 1)) + log(abs(-x + sqrt(-x + 1)))
```

Mupad [B] (verification not implemented)

Time = 18.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \frac{1}{-\sqrt{1-x}+x}\,dx = \ln\left(2\sqrt{1-x}-\left(\frac{\sqrt{5}}{5}+1\right)\left(2\sqrt{1-x}+1\right)\right)\left(\frac{\sqrt{5}}{5}+1\right) - \ln\left(2\sqrt{1-x}+\left(\frac{\sqrt{5}}{5}-1\right)\left(2\sqrt{1-x}+1\right)\right)\left(\frac{\sqrt{5}}{5}-1\right)$$

```
[In] int(1/(x - (1 - x)^(1/2)),x)
[Out] log(2*(1 - x)^(1/2) - (5^(1/2)/5 + 1)*(2*(1 - x)^(1/2) + 1))*(5^(1/2)/5 + 1
) - log(2*(1 - x)^(1/2) + (5^(1/2)/5 - 1)*(2*(1 - x)^(1/2) + 1))*(5^(1/2)/5
 - 1)
```

3.701 $\int \sqrt{1+\sqrt{x}+x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 62

$$\int \sqrt{1+\sqrt{x}+x}\,dx=-\frac{1}{4}(1+2\sqrt{x})\sqrt{1+\sqrt{x}+x}+\frac{2}{3}(1+\sqrt{x}+x)^{3/2}-\frac{3}{8}\text{arcsinh}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)$$

```
[Out] -3/8*arcsinh(1/3*(1+2*x^(1/2))*3^(1/2))+2/3*(1+x+x^(1/2))^(3/2)-1/4*(1+2*x^
(1/2))*(1+x+x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1355, 654, 626, 633, 221}

$$\int \sqrt{1+\sqrt{x}+x}\,dx=-\frac{3}{8}\text{arcsinh}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)+\frac{2}{3}(x+\sqrt{x}+1)^{3/2}-\frac{1}{4}(2\sqrt{x}+1)\sqrt{x+\sqrt{x}+1}$$

```
[In] Int[Sqrt[1 + Sqrt[x] + x],x]
```

```
[Out] -1/4*((1 + 2*Sqrt[x])*Sqrt[1 + Sqrt[x] + x]) + (2*(1 + Sqrt[x] + x)^(3/2))/
3 - (3*ArcSinh[(1 + 2*Sqrt[x])/Sqrt[3]])/8
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
```

```
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2} - \text{Subst}\left(\int \sqrt{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+\sqrt{x}+x}+\frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2}-\frac{3}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+\sqrt{x}+x}+\frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2}-\frac{1}{8}\sqrt{3}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, 1+2\sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+\sqrt{x}+x}+\frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2}-\frac{3}{8}\sinh^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \sqrt{1+\sqrt{x}+x}\,dx = \frac{1}{12}\sqrt{1+\sqrt{x}+x}\left(5+2\sqrt{x}+8x\right)+\frac{3}{8}\log\left(-1-2\sqrt{x}+2\sqrt{1+\sqrt{x}+x}\right)$$

```
[In] Integrate[Sqrt[1 + Sqrt[x] + x],x]
[Out] (Sqrt[1 + Sqrt[x] + x]*(5 + 2*Sqrt[x] + 8*x))/12 + (3*Log[-1 - 2*Sqrt[x] +
2*Sqrt[1 + Sqrt[x] + x]])/8
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{2(1+x+\sqrt{x})^{\frac{3}{2}}}{3}-\frac{(1+2\sqrt{x})\sqrt{1+x+\sqrt{x}}}{4}-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x}+\frac{1}{2}\right)}{3}\right)}{8}$	42
default	$\frac{2(1+x+\sqrt{x})^{\frac{3}{2}}}{3}-\frac{(1+2\sqrt{x})\sqrt{1+x+\sqrt{x}}}{4}-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x}+\frac{1}{2}\right)}{3}\right)}{8}$	42

```
[In] int((1+x+x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(1+x+x^(1/2))^(3/2)-1/4*(1+2*x^(1/2))*(1+x+x^(1/2))^(1/2)-3/8*arcsinh(2
/3*3^(1/2)*(x^(1/2)+1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \sqrt{1+\sqrt{x}+x}\,dx = \frac{1}{12}\left(8\,x+2\,\sqrt{x}+5\right)\sqrt{x+\sqrt{x}+1} + \frac{3}{16}\log\left(4\,\sqrt{x+\sqrt{x}+1}(2\,\sqrt{x}+1)-8\,x-8\,\sqrt{x}-5\right)$$

```
[In] integrate((1+x+x^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/12*(8*x + 2*sqrt(x) + 5)*sqrt(x + sqrt(x) + 1) + 3/16*log(4*sqrt(x + sqrt
(x) + 1)*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 5)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \sqrt{1+\sqrt{x}+x}\,dx = 2\left(\frac{\sqrt{x}}{12}+\frac{x}{3}+\frac{5}{24}\right)\sqrt{\sqrt{x}+x+1}-\frac{3\operatorname{asinh}\left(\frac{2\sqrt{3}\left(\sqrt{x}+\frac{1}{2}\right)}{3}\right)}{8}$$

```
[In] integrate((1+x+x**(1/2))**(1/2),x)
[Out] 2*(sqrt(x)/12 + x/3 + 5/24)*sqrt(sqrt(x) + x + 1) - 3*asinh(2*sqrt(3)*(sqrt
(x) + 1/2)/3)/8
```

Maxima [F]

$$\int \sqrt{1+\sqrt{x}+x}\,dx = \int \sqrt{x+\sqrt{x}+1}\,dx$$

```
[In] integrate((1+x+x^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(x) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \sqrt{1+\sqrt{x}+x}\,dx = \frac{1}{12}\left(2\sqrt{x}(4\sqrt{x}+1)+5\right)\sqrt{x+\sqrt{x}+1} + \frac{3}{8}\log\left(2\sqrt{x+\sqrt{x}+1}-2\sqrt{x}-1\right)$$

```
[In] integrate((1+x+x^(1/2))^(1/2),x, algorithm="giac")
[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) + 5)*sqrt(x + sqrt(x) + 1) + 3/8*log(2*sqrt
(x + sqrt(x) + 1) - 2*sqrt(x) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sqrt{x}+x}\,dx = \int \sqrt{x+\sqrt{x}+1}\,dx$$

```
[In] int((x + x^(1/2) + 1)^(1/2),x)
[Out] int((x + x^(1/2) + 1)^(1/2), x)
```

3.702 $\int \sqrt{1+x+\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \sqrt{1+x+\sqrt{1+x}}\,dx=\frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2}-\frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right)+\frac{1}{4}\text{arctanh}\left(\frac{\sqrt{1+x}}{\sqrt{1+x+\sqrt{1+x}}}\right)$$

```
[Out] 1/4*arctanh((1+x)^(1/2)/(1+x+(1+x)^(1/2))^(1/2))+2/3*(1+x+(1+x)^(1/2))^(3/2
)-1/4*(1+2*(1+x)^(1/2))*(1+x+(1+x)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1976, 654, 626, 634, 212}

$$\int \sqrt{1+x+\sqrt{1+x}}\,dx=\frac{1}{4}\text{arctanh}\left(\frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}+1}}\right)+\frac{2}{3}\left(x+\sqrt{x+1}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}+1}$$

```
[In] Int[Sqrt[1 + x + Sqrt[1 + x]],x]
```

```
[Out] (2*(1 + x + Sqrt[1 + x])^(3/2))/3 - (Sqrt[1 + x + Sqrt[1 + x]]*(1 + 2*Sqrt[
1 + x]))/4 + ArcTanh[Sqrt[1 + x]/Sqrt[1 + x + Sqrt[1 + x]]]/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x\sqrt{x(1+x)}\,dx, x, \sqrt{1+x}\right) \\
&= 2\text{Subst}\left(\int x\sqrt{x+x^2}\,dx, x, \sqrt{1+x}\right) \\
&= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \text{Subst}\left(\int \sqrt{x+x^2}\,dx, x, \sqrt{1+x}\right) \\
&= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} \\
&\quad - \frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,dx, x, \sqrt{1+x}\right) \\
&= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) \\
&\quad + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{1+x}}{\sqrt{1+x+\sqrt{1+x}}}\right)
\end{aligned}
$$

$$= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1+x+\sqrt{1+x}}}\right)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \sqrt{1+x+\sqrt{1+x}}\,dx = \frac{1}{12}\left(\sqrt{1+x+\sqrt{1+x}}\left(5+8x+2\sqrt{1+x}\right) + 3\text{arctanh}\left(\frac{\sqrt{1+x+\sqrt{1+x}}}{\sqrt{1+x}}\right)\right)$$

```
[In] Integrate[Sqrt[1 + x + Sqrt[1 + x]],x]
```

```
[Out] (Sqrt[1 + x + Sqrt[1 + x]]*(5 + 8*x + 2*Sqrt[1 + x]) + 3*ArcTanh[Sqrt[1 + x
 + Sqrt[1 + x]]/Sqrt[1 + x]])/12
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{2\left(1+x+\sqrt{x+1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x+1}\right)\sqrt{1+x+\sqrt{x+1}}}{4}+\frac{\ln\left(\frac{1}{2}+\sqrt{x+1}+\sqrt{1+x+\sqrt{x+1}}\right)}{8}$	55
default	$\frac{2\left(1+x+\sqrt{x+1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x+1}\right)\sqrt{1+x+\sqrt{x+1}}}{4}+\frac{\ln\left(\frac{1}{2}+\sqrt{x+1}+\sqrt{1+x+\sqrt{x+1}}\right)}{8}$	55

```
[In] int((1+x+(x+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1+x+(x+1)^(1/2))^(3/2)-1/4*(1+2*(x+1)^(1/2))*(1+x+(x+1)^(1/2))^(1/2)+1
/8*ln(1/2+(x+1)^(1/2)+(1+x+(x+1)^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \sqrt{1+x+\sqrt{1+x}}\,dx = \frac{1}{12}\left(8\,x+2\,\sqrt{x+1}+5\right)\sqrt{x+\sqrt{x+1}+1} + \frac{1}{16}\,\log\left(-4\,\sqrt{x+\sqrt{x+1}+1}\left(2\,\sqrt{x+1}+1\right)-8\,x - 8\,\sqrt{x+1}-9\right)$$

```
[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x + 1) + 5)*sqrt(x + sqrt(x + 1) + 1) + 1/16*log(-4*sqrt
(x + sqrt(x + 1) + 1)*(2*sqrt(x + 1) + 1) - 8*x - 8*sqrt(x + 1) - 9)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

$$\int \sqrt{1+x+\sqrt{1+x}}\,dx = 2\left(\frac{x}{3}+\frac{\sqrt{x+1}}{12}+\frac{5}{24}\right)\sqrt{x+\sqrt{x+1}+1} + \frac{\log\left(2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}+1}+1\right)}{8}$$

```
[In] integrate((1+x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] 2*(x/3 + sqrt(x + 1)/12 + 5/24)*sqrt(x + sqrt(x + 1) + 1) + log(2*sqrt(x +
1) + 2*sqrt(x + sqrt(x + 1) + 1) + 1)/8
```

Maxima [F]

$$\int \sqrt{1+x+\sqrt{1+x}}\,dx = \int \sqrt{x+\sqrt{x+1}+1}\,dx$$

```
[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.73

$$\int \sqrt{1+x+\sqrt{1+x}}\,dx = \frac{1}{12}\left(2\sqrt{x+1}\left(4\sqrt{x+1}+1\right)-3\right)\sqrt{x+\sqrt{x+1}+1} - \frac{1}{8}\log\left(-2\sqrt{x+\sqrt{x+1}+1}+2\sqrt{x+1}+1\right)$$

```
[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="giac")
[Out] 1/12*(2*sqrt(x + 1)*(4*sqrt(x + 1) + 1) - 3)*sqrt(x + sqrt(x + 1) + 1) - 1/
8*log(-2*sqrt(x + sqrt(x + 1) + 1) + 2*sqrt(x + 1) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+x+\sqrt{1+x}}\,dx = \int \sqrt{x+\sqrt{x+1}+1}\,dx$$

```
[In] int((x + (x + 1)^(1/2) + 1)^(1/2),x)
[Out] int((x + (x + 1)^(1/2) + 1)^(1/2), x)
```

3.703 $\int \sqrt{\sqrt{-1+x}+x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \frac{3}{8}\operatorname{arcsinh}\left(\frac{1+2\sqrt{-1+x}}{\sqrt{3}}\right)$$

```
[Out] -3/8*arcsinh(1/3*(1+2*(-1+x)^(1/2))*3^(1/2))+2/3*(x+(-1+x)^(1/2))^(3/2)-1/4
*(1+2*(-1+x)^(1/2))*(x+(-1+x)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {654, 626, 633, 221}

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = -\frac{3}{8}\operatorname{arcsinh}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right) + \frac{2}{3}\left(x+\sqrt{x-1}\right)^{3/2} - \frac{1}{4}\left(2\sqrt{x-1}+1\right)\sqrt{x+\sqrt{x-1}}$$

```
[In] Int[Sqrt[Sqrt[-1 + x] + x],x]
```

```
[Out] -1/4*((1 + 2*Sqrt[-1 + x])*Sqrt[Sqrt[-1 + x] + x]) + (2*(Sqrt[-1 + x] + x)^
(3/2))/3 - (3*ArcSinh[(1 + 2*Sqrt[-1 + x])/Sqrt[3]])/8
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx, x, \sqrt{-1+x}\right) \\
&= \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \text{Subst}\left(\int \sqrt{1+x+x^2}\,dx, x, \sqrt{-1+x}\right) \\
&= -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} \\
&\quad - \frac{3}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\,dx, x, \sqrt{-1+x}\right) \\
&= -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} \\
&\quad - \frac{1}{8}\sqrt{3}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, 1+2\sqrt{-1+x}\right) \\
&= -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \frac{3}{8}\sinh^{-1}\left(\frac{1+2\sqrt{-1+x}}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = \frac{1}{12}\left(5+2\sqrt{-1+x}+8(-1+x)\right)\sqrt{\sqrt{-1+x}+x} + \frac{3}{8}\log\left(-1-2\sqrt{-1+x}+2\sqrt{\sqrt{-1+x}+x}\right)$$

```
[In] Integrate[Sqrt[Sqrt[-1 + x] + x],x]
[Out] ((5 + 2*Sqrt[-1 + x] + 8*(-1 + x))*Sqrt[Sqrt[-1 + x] + x])/12 + (3*Log[-1 -
 2*Sqrt[-1 + x] + 2*Sqrt[Sqrt[-1 + x] + x]])/8
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2\left(x+\sqrt{x-1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x-1}\right)\sqrt{x+\sqrt{x-1}}}{4}-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}+\frac{1}{2}\right)}{3}\right)}{8}$	48
default	$\frac{2\left(x+\sqrt{x-1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x-1}\right)\sqrt{x+\sqrt{x-1}}}{4}-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}+\frac{1}{2}\right)}{3}\right)}{8}$	48

```
[In] int((x+(x-1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(x+(x-1)^(1/2))^(3/2)-1/4*(1+2*(x-1)^(1/2))*(x+(x-1)^(1/2))^(1/2)-3/8*a
rcsinh(2/3*3^(1/2)*((x-1)^(1/2)+1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = \frac{1}{12}\left(8\,x+2\,\sqrt{x-1}-3\right)\sqrt{x+\sqrt{x-1}} + \frac{3}{16}\log\left(-4\,\sqrt{x+\sqrt{x-1}}\left(2\,\sqrt{x-1}+1\right)+8\,x+8\,\sqrt{x-1}-3\right)$$

```
[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/12*(8*x + 2*sqrt(x - 1) - 3)*sqrt(x + sqrt(x - 1)) + 3/16*log(-4*sqrt(x +
 sqrt(x - 1))*(2*sqrt(x - 1) + 1) + 8*x + 8*sqrt(x - 1) - 3)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = 2\sqrt{x+\sqrt{x-1}}\left(\frac{x}{3}+\frac{\sqrt{x-1}}{12}-\frac{1}{8}\right)-\frac{3\operatorname{asinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}+\frac{1}{2}\right)}{3}\right)}{8}$$

```
[In] integrate((x+(-1+x)**(1/2))**(1/2),x)
[Out] 2*sqrt(x + sqrt(x - 1))*(x/3 + sqrt(x - 1)/12 - 1/8) - 3*asinh(2*sqrt(3)*(s
qrt(x - 1) + 1/2)/3)/8
```

Maxima [F]

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = \int \sqrt{x+\sqrt{x-1}}\,dx$$

```
[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(x - 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = \frac{1}{12}\left(2\sqrt{x-1}\left(4\sqrt{x-1}+1\right)+5\right)\sqrt{x+\sqrt{x-1}} + \frac{3}{8}\log\left(2\sqrt{x+\sqrt{x-1}}-2\sqrt{x-1}-1\right)$$

```
[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="giac")
[Out] 1/12*(2*sqrt(x - 1)*(4*sqrt(x - 1) + 1) + 5)*sqrt(x + sqrt(x - 1)) + 3/8*lo
g(2*sqrt(x + sqrt(x - 1)) - 2*sqrt(x - 1) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sqrt{-1+x}+x}\,dx=\int \sqrt{x+\sqrt{x-1}}\,dx$$

```
[In] int((x + (x - 1)^(1/2))^(1/2),x)
[Out] int((x + (x - 1)^(1/2))^(1/2), x)
```

3.704 $\int \sqrt{2x+\sqrt{-1+2x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx=\frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2}-\frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right)-\frac{3}{16}\text{arcsinh}\left(\frac{1+2\sqrt{-1+2x}}{\sqrt{3}}\right)$$

```
[Out] -3/16*arcsinh(1/3*(1+2*(-1+2*x)^(1/2))*3^(1/2))+1/3*(2*x+(-1+2*x)^(1/2))^(3
/2)-1/8*(1+2*(-1+2*x)^(1/2))*(2*x+(-1+2*x)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {654, 626, 633, 221}

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx=-\frac{3}{16}\text{arcsinh}\left(\frac{2\sqrt{2x-1}+1}{\sqrt{3}}\right)+\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{3/2}-\frac{1}{8}\left(2\sqrt{2x-1}+1\right)\sqrt{2x+\sqrt{2x-1}}$$

```
[In] Int[Sqrt[2*x + Sqrt[-1 + 2*x]],x]
```

```
[Out] (2*x + Sqrt[-1 + 2*x])^(3/2)/3 - (Sqrt[2*x + Sqrt[-1 + 2*x]]*(1 + 2*Sqrt[-1
 + 2*x]))/8 - (3*ArcSinh[(1 + 2*Sqrt[-1 + 2*x])/Sqrt[3]])/16
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx, x, \sqrt{-1+2x}\right)\\
&= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{2}\text{Subst}\left(\int \sqrt{1+x+x^2}\,dx, x, \sqrt{-1+2x}\right)\\
&= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right)\\
&\quad - \frac{3}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\,dx, x, \sqrt{-1+2x}\right)\\
&= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right)\\
&\quad - \frac{1}{16}\sqrt{3}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, 1+2\sqrt{-1+2x}\right)\\
&= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2}\\
&\quad - \frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right) - \frac{3}{16}\sinh^{-1}\left(\frac{1+2\sqrt{-1+2x}}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx = \frac{1}{48}\left(2\sqrt{2x+\sqrt{-1+2x}}\left(-3+16x+2\sqrt{-1+2x}\right)+9\log\left(-1-2\sqrt{-1+2x}+2\sqrt{2x+\sqrt{-1+2x}}\right)\right)$$

```
[In] Integrate[Sqrt[2*x + Sqrt[-1 + 2*x]],x]
[Out] (2*Sqrt[2*x + Sqrt[-1 + 2*x]]*(-3 + 16*x + 2*Sqrt[-1 + 2*x]) + 9*Log[-1 - 2
*Sqrt[-1 + 2*x] + 2*Sqrt[2*x + Sqrt[-1 + 2*x]]])/48
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\left(2x+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{2x-1}\right)\sqrt{2x+\sqrt{2x-1}}}{8}-\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{2x-1}+\frac{1}{2}\right)}{3}\right)}{16}$	60
default	$\frac{\left(2x+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{2x-1}\right)\sqrt{2x+\sqrt{2x-1}}}{8}-\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{2x-1}+\frac{1}{2}\right)}{3}\right)}{16}$	60

```
[In] int((2*x+(2*x-1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(2*x+(2*x-1)^(1/2))^(3/2)-1/8*(1+2*(2*x-1)^(1/2))*(2*x+(2*x-1)^(1/2))^(
1/2)-3/16*arcsinh(2/3*3^(1/2)*((2*x-1)^(1/2)+1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx = \frac{1}{24}\left(16\,x+2\,\sqrt{2\,x-1}-3\right)\sqrt{2\,x+\sqrt{2\,x-1}}+\frac{3}{32}\,\log\left(-4\,\sqrt{2\,x+\sqrt{2\,x-1}}\left(2\,\sqrt{2\,x-1}+1\right)+16\,x+8\,\sqrt{2\,x-1}-3\right)$$

```
[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
```

[Out] 1/24*(16*x + 2*sqrt(2*x - 1) - 3)*sqrt(2*x + sqrt(2*x - 1)) + 3/32*log(-4*s
qrt(2*x + sqrt(2*x - 1))*(2*sqrt(2*x - 1) + 1) + 16*x + 8*sqrt(2*x - 1) - 3
)

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx = \sqrt{2x+\sqrt{2x-1}}\cdot\left(\frac{2x}{3}+\frac{\sqrt{2x-1}}{12}-\frac{1}{8}\right)-\frac{3\operatorname{asinh}\left(\frac{2\sqrt{3}\left(\sqrt{2x-1}+\frac{1}{2}\right)}{3}\right)}{16}$$

[In] integrate((2*x+(-1+2*x)**(1/2))**(1/2),x)

[Out] sqrt(2*x + sqrt(2*x - 1))*(2*x/3 + sqrt(2*x - 1)/12 - 1/8) - 3*asinh(2*sqrt
(3)*(sqrt(2*x - 1) + 1/2)/3)/16

Maxima [F]

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx = \int \sqrt{2\,x+\sqrt{2\,x-1}}\,dx$$

[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(2*x + sqrt(2*x - 1)), x)

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx = \frac{1}{24}\left(2\sqrt{2\,x-1}\left(4\sqrt{2\,x-1}+1\right)+5\right)\sqrt{2\,x+\sqrt{2\,x-1}} + \frac{3}{16}\log\left(2\sqrt{2\,x+\sqrt{2\,x-1}}-2\sqrt{2\,x-1}-1\right)$$

[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")

[Out] 1/24*(2*sqrt(2*x - 1)*(4*sqrt(2*x - 1) + 1) + 5)*sqrt(2*x + sqrt(2*x - 1))
+ 3/16*log(2*sqrt(2*x + sqrt(2*x - 1)) - 2*sqrt(2*x - 1) - 1)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx=\int \sqrt{2\,x+\sqrt{2\,x-1}}\,dx$$

```
[In] int((2*x + (2*x - 1)^(1/2))^(1/2),x)
[Out] int((2*x + (2*x - 1)^(1/2))^(1/2), x)
```

3.705 $\int \sqrt{3x+\sqrt{-7+8x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 109

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx=-\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}}+\frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3/2}}{72\sqrt{2}}-\frac{47\operatorname{arcsinh}\left(\frac{4+3\sqrt{-7+8x}}{\sqrt{47}}\right)}{36\sqrt{6}}$$

```
[Out] -47/216*arcsinh(1/47*(4+3*(-7+8*x)^(1/2))*47^(1/2))*6^(1/2)+1/144*(24*x+8*(
-7+8*x)^(1/2))^(3/2)*2^(1/2)-1/36*(4+3*(-7+8*x)^(1/2))*(6*x+2*(-7+8*x)^(1/2
))^(1/2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {654, 626, 633, 221}

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx=-\frac{47\operatorname{arcsinh}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)}{36\sqrt{6}}+\frac{\left(-3(7-8x)+8\sqrt{8x-7}+21\right)^{3/2}}{72\sqrt{2}}-\frac{\left(3\sqrt{8x-7}+4\right)\sqrt{-3(7-8x)+8\sqrt{8x-7}+21}}{36\sqrt{2}}$$

```
[In] Int[Sqrt[3*x + Sqrt[-7 + 8*x]],x]
```

```
[Out] -1/36*((4 + 3*Sqrt[-7 + 8*x])*Sqrt[21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x]])/Sq
rt[2] + (21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x])^(3/2)/(72*Sqrt[2]) - (47*ArcS
inh[(4 + 3*Sqrt[-7 + 8*x])/Sqrt[47]])/(36*Sqrt[6])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int x\sqrt{\frac{21}{8}+x+\frac{3x^2}{8}}\,dx, x, \sqrt{-7+8x}\right)\\
&= \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3/2}}{72\sqrt{2}} - \frac{1}{3}\text{Subst}\left(\int \sqrt{\frac{21}{8}+x+\frac{3x^2}{8}}\,dx, x, \sqrt{-7+8x}\right)\\
&= -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}}\\
&\quad + \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3/2}}{72\sqrt{2}}\\
&\quad - \frac{47}{144}\text{Subst}\left(\int \frac{1}{\sqrt{\frac{21}{8}+x+\frac{3x^2}{8}}}\,dx, x, \sqrt{-7+8x}\right)
\end{aligned}$$

$$= -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}} + \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3/2}}{72\sqrt{2}} - \frac{1}{9}\sqrt{\frac{47}{6}}\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{16x^2}{47}}}\,dx,x,1+\frac{3}{4}\sqrt{-7+8x}\right)$$

$$= -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}} + \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3/2}}{72\sqrt{2}} - \frac{47\sinh^{-1}\left(\frac{4+3\sqrt{-7+8x}}{\sqrt{47}}\right)}{36\sqrt{6}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.76

$$\int\sqrt{3x+\sqrt{-7+8x}}\,dx = \frac{1}{18}\sqrt{3x+\sqrt{-7+8x}}\left(-4+12x+\sqrt{-7+8x}\right) + \frac{47\log\left(-4-3\sqrt{-7+8x}+2\sqrt{6}\sqrt{3x+\sqrt{-7+8x}}\right)}{36\sqrt{6}}$$

```
[In] Integrate[Sqrt[3*x + Sqrt[-7 + 8*x]],x]
[Out] (Sqrt[3*x + Sqrt[-7 + 8*x]]*(-4 + 12*x + Sqrt[-7 + 8*x]))/18 + (47*Log[-4 -
 3*Sqrt[-7 + 8*x] + 2*Sqrt[6]*Sqrt[3*x + Sqrt[-7 + 8*x]]])/(36*Sqrt[6])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.61

method	result	size
derivativedivides	$\frac{\left(48x+16\sqrt{-7+8x}\right)^{\frac{3}{2}}}{288}-\frac{\left(12\sqrt{-7+8x}+16\right)\sqrt{48x+16\sqrt{-7+8x}}}{288}-\frac{47\sqrt{6}\,\operatorname{arcsinh}\left(\frac{3\sqrt{47}\left(\sqrt{-7+8x}+\frac{4}{3}\right)}{47}\right)}{216}$	67
default	$\frac{\left(48x+16\sqrt{-7+8x}\right)^{\frac{3}{2}}}{288}-\frac{\left(12\sqrt{-7+8x}+16\right)\sqrt{48x+16\sqrt{-7+8x}}}{288}-\frac{47\sqrt{6}\,\operatorname{arcsinh}\left(\frac{3\sqrt{47}\left(\sqrt{-7+8x}+\frac{4}{3}\right)}{47}\right)}{216}$	67

```
[In] int((3*x+(-7+8*x)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/288*(48*x+16*(-7+8*x)^(1/2))^(3/2)-1/288*(12*(-7+8*x)^(1/2)+16)*(48*x+16*
(-7+8*x)^(1/2))^(1/2)-47/216*6^(1/2)*arcsinh(3/47*47^(1/2)*((-7+8*x)^(1/2)+
4/3))
```

Fricas [A] (verification not implemented)

none

Time = 1.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx = \frac{1}{18}\,(12\,x+\sqrt{8\,x-7}-4)\sqrt{3\,x+\sqrt{8\,x-7}} + \frac{47}{864}\,\sqrt{6}\log\left(-41472\,x^2-192\,(144\,x-47)\sqrt{8\,x-7} + 8\left(3\,\sqrt{6}(144\,x+17)\sqrt{8\,x-7}+4\,\sqrt{6}(432\,x-299)\right)\sqrt{3\,x+\sqrt{8\,x-7}}-9792\,x + 30047\right)$$

```
[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*(12*x + sqrt(8*x - 7) - 4)*sqrt(3*x + sqrt(8*x - 7)) + 47/864*sqrt(6)*
log(-41472*x^2 - 192*(144*x - 47)*sqrt(8*x - 7) + 8*(3*sqrt(6)*(144*x + 17)
*sqrt(8*x - 7) + 4*sqrt(6)*(432*x - 299))*sqrt(3*x + sqrt(8*x - 7)) - 9792*
x + 30047)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx = \frac{\sqrt{3x+\sqrt{8x-7}}\cdot\left(\frac{8x}{3}+\frac{2\sqrt{8x-7}}{9}-\frac{8}{9}\right)}{4} - \frac{47\sqrt{6}\operatorname{asinh}\left(\frac{3\sqrt{47}\left(\sqrt{8x-7}+\frac{4}{3}\right)}{47}\right)}{216}$$

```
[In] integrate((3*x+(-7+8*x)**(1/2))**(1/2),x)
```

```
[Out] sqrt(3*x + sqrt(8*x - 7))*(8*x/3 + 2*sqrt(8*x - 7)/9 - 8/9)/4 - 47*sqrt(6)*
asinh(3*sqrt(47)*(sqrt(8*x - 7) + 4/3)/47)/216
```

Maxima [F]

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx=\int \sqrt{3\,x+\sqrt{8\,x-7}}\,dx$$

```
[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(3*x + sqrt(8*x - 7)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx$$
$$=\frac{1}{216}\,\sqrt{2}\Big(3\,\sqrt{2}\big(\sqrt{8\,x-7}\big(3\,\sqrt{8\,x-7}+2\big)+13\big)\sqrt{3\,x+\sqrt{8\,x-7}}+47\,\sqrt{3}\log\Big(-\sqrt{3}\Big(\sqrt{3}\sqrt{8\,x-7}-2\,\sqrt{2}$$

```
[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="giac")
[Out] 1/216*sqrt(2)*(3*sqrt(2)*(sqrt(8*x - 7)*(3*sqrt(8*x - 7) + 2) + 13)*sqrt(3*
x + sqrt(8*x - 7)) + 47*sqrt(3)*log(-sqrt(3)*(sqrt(3)*sqrt(8*x - 7) - 2*sqr
t(2)*sqrt(3*x + sqrt(8*x - 7))) - 4))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx=\int \sqrt{3\,x+\sqrt{8\,x-7}}\,dx$$

```
[In] int((3*x + (8*x - 7)^(1/2))^(1/2),x)
[Out] int((3*x + (8*x - 7)^(1/2))^(1/2), x)
```

3.706 $\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = 2\sqrt{x+\sqrt{1+x}} - \operatorname{arctanh}\left(\frac{1+2\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)$$

```
[Out] -arctanh(1/2*(1+2*(1+x)^(1/2))/(x+(1+x)^(1/2))^(1/2))+2*(x+(1+x)^(1/2))^(1/
2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {654, 635, 212}

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = 2\sqrt{x+\sqrt{x+1}} - \operatorname{arctanh}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

```
[In] Int[1/Sqrt[x + Sqrt[1 + x]],x]
[Out] 2*Sqrt[x + Sqrt[1 + x]] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 +
x]])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)\\
&= 2\sqrt{x+\sqrt{1+x}} - \text{Subst}\left(\int \frac{1}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)\\
&= 2\sqrt{x+\sqrt{1+x}} - 2\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{1+2\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right)\\
&= 2\sqrt{x+\sqrt{1+x}} - \tanh^{-1}\left(\frac{1+2\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = 2\sqrt{x+\sqrt{1+x}} + \log\left(-1-2\sqrt{1+x}+2\sqrt{x+\sqrt{1+x}}\right)$$

```
[In] Integrate[1/Sqrt[x + Sqrt[1 + x]],x]
[Out] 2*Sqrt[x + Sqrt[1 + x]] + Log[-1 - 2*Sqrt[1 + x] + 2*Sqrt[x + Sqrt[1 + x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$2\sqrt{x+\sqrt{x+1}}-\ln\left(\frac{1}{2}+\sqrt{x+1}+\sqrt{x+\sqrt{x+1}}\right)$	32
default	$2\sqrt{x+\sqrt{x+1}}-\ln\left(\frac{1}{2}+\sqrt{x+1}+\sqrt{x+\sqrt{x+1}}\right)$	32

```
[In] int(1/(x+(x+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(x+(x+1)^(1/2))^(1/2)-ln(1/2+(x+1)^(1/2)+(x+(x+1)^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = 2\,\sqrt{x+\sqrt{x+1}} + \frac{1}{2}\,\log\left(4\,\sqrt{x+\sqrt{x+1}}\left(2\,\sqrt{x+1}+1\right) - 8\,x - 8\,\sqrt{x+1} - 5\right)$$

```
[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + sqrt(x + 1)) + 1/2*log(4*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) +
1) - 8*x - 8*sqrt(x + 1) - 5)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = 2\sqrt{x+\sqrt{x+1}} - \log\left(2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}}+1\right)$$

```
[In] integrate(1/(x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] 2*sqrt(x + sqrt(x + 1)) - log(2*sqrt(x + 1) + 2*sqrt(x + sqrt(x + 1)) + 1)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = \int \frac{1}{\sqrt{x+\sqrt{x+1}}}\,dx$$

```
[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x + sqrt(x + 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = 2\sqrt{x+\sqrt{x+1}} + \log\left(-2\sqrt{x+\sqrt{x+1}} + 2\sqrt{x+1} + 1\right)$$

```
[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x + sqrt(x + 1)) + log(-2*sqrt(x + sqrt(x + 1)) + 2*sqrt(x + 1) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = \int \frac{1}{\sqrt{x+\sqrt{x+1}}}\,dx$$

```
[In] int(1/(x + (x + 1)^(1/2))^(1/2),x)
[Out] int(1/(x + (x + 1)^(1/2))^(1/2), x)
```

3.707 $\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx = x-2\sqrt{3}\sqrt{-3+2x}+4\sqrt{6}\arctan\left(\frac{3+\sqrt{-9+6x}}{2\sqrt{6}}\right)+3\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)$$

```
[Out] x+3*ln(4+x+(-3+2*x)^(1/2)*3^(1/2))+4*arctan(1/12*(3+(-9+6*x)^(1/2))*6^(1/2)
)*6^(1/2)-2*(-3+2*x)^(1/2)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1642, 648, 632, 210, 642}

$$\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx = 4\sqrt{6}\arctan\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)+x-2\sqrt{3}\sqrt{2x-3}+3\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)$$

```
[In] Int[(1 + x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] x - 2*Sqrt[3]*Sqrt[-3 + 2*x] + 4*Sqrt[6]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqr
t[6])] + 3*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x(15+x^2)}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(-6+x+\frac{18(11+x)}{33+6x+x^2}\right)\,dx, x, \sqrt{-9+6x}\right) \\
&= x-2\sqrt{3}\sqrt{-3+2x}+6\text{Subst}\left(\int \frac{11+x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right) \\
&= x-2\sqrt{3}\sqrt{-3+2x}+3\text{Subst}\left(\int \frac{6+2x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right) \\
&\quad +48\text{Subst}\left(\int \frac{1}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right) \\
&= x-2\sqrt{3}\sqrt{-3+2x}+3\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right) \\
&\quad -96\text{Subst}\left(\int \frac{1}{-96-x^2}\,dx, x, 6+2\sqrt{-9+6x}\right)
\end{aligned}$$

$$=x-2\sqrt{3}\sqrt{-3+2x}+4\sqrt{6}\tan^{-1}\left(\frac{3+\sqrt{3}\sqrt{-3+2x}}{2\sqrt{6}}\right)+3\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int\frac{1+x}{4+x+\sqrt{-9+6x}}\,dx=-\frac{3}{2}+x-2\sqrt{-9+6x}+4\sqrt{6}\arctan\left(\frac{\sqrt{3}+\sqrt{-3+2x}}{2\sqrt{2}}\right)+3\log\left(4+x+\sqrt{-9+6x}\right)$$

```
[In] Integrate[(1 + x)/(4 + x + Sqrt[-9 + 6*x]),x]
[Out] -3/2 + x - 2*Sqrt[-9 + 6*x] + 4*Sqrt[6]*ArcTan[(Sqrt[3] + Sqrt[-3 + 2*x])/(
2*Sqrt[2])] + 3*Log[4 + x + Sqrt[-9 + 6*x]]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result
derivativedivides	$-\frac{3}{2}+x-2\sqrt{-9+6x}+3\ln\left(24+6x+6\sqrt{-9+6x}\right)+4\sqrt{6}\arctan\left(\frac{(2\sqrt{-9+6x}+6)\sqrt{6}}{24}\right)$
default	$-\frac{3}{2}+x-2\sqrt{-9+6x}+3\ln\left(24+6x+6\sqrt{-9+6x}\right)+4\sqrt{6}\arctan\left(\frac{(2\sqrt{-9+6x}+6)\sqrt{6}}{24}\right)$
trager	$x-2\sqrt{-9+6x}+\mathrm{RootOf}\left(\_Z^2-6\_Z+33\right)\ln\left(4+x+\sqrt{-9+6x}\right)-\ln\left(161\,\mathrm{Root}\right.$

```
[In] int((x+1)/(4+x+(-9+6*x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -3/2+x-2*(-9+6*x)^(1/2)+3*ln(24+6*x+6*(-9+6*x)^(1/2))+4*6^(1/2)*arctan(1/24
*(2*(-9+6*x)^(1/2)+6)*6^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int\frac{1+x}{4+x+\sqrt{-9+6x}}\,dx=4\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\sqrt{6x-9}+\frac{1}{4}\sqrt{6}\right)+x-2\sqrt{6x-9}+3\log\left(x+\sqrt{6x-9}+4\right)$$

```
[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*sqrt(6*x - 9) + 1/4*sqrt(6)) + x - 2*sqrt(6*x
 - 9) + 3*log(x + sqrt(6*x - 9) + 4)
```

Sympy [A] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx = x-2\sqrt{6x-9}+3\log\left(6x+6\sqrt{6x-9}+24\right) + 4\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\left(\sqrt{6x-9}+3\right)}{12}\right)-\frac{3}{2}$$

```
[In] integrate((1+x)/(4+x+(-9+6*x)**(1/2)),x)
```

```
[Out] x - 2*sqrt(6*x - 9) + 3*log(6*x + 6*sqrt(6*x - 9) + 24) + 4*sqrt(6)*atan(sq
rt(6)*(sqrt(6*x - 9) + 3)/12) - 3/2
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx = 4\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)+x - 2\sqrt{6x-9}+3\log\left(6x+6\sqrt{6x-9}+24\right)-\frac{3}{2}$$

```
[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) + x - 2*sqrt(6*x - 9) +
3*log(6*x + 6*sqrt(6*x - 9) + 24) - 3/2
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx = 4\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(\sqrt{3}+\sqrt{2x-3}\right)\right)-2\sqrt{3}\sqrt{2x-3} + x+3\log\left(2\sqrt{3}\sqrt{2x-3}+2x+8\right)-\frac{3}{2}$$

```
[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 4*sqrt(3)*sqrt(2)*arctan(1/4*sqrt(2)*(sqrt(3) + sqrt(2*x - 3))) - 2*sqrt(3)
*sqrt(2*x - 3) + x + 3*log(2*sqrt(3)*sqrt(2*x - 3) + 2*x + 8) - 3/2
```

Mupad [B] (verification not implemented)

Time = 17.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

$$\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx = x+3\,\ln\left(\left(6\,\sqrt{6\,x-9}+\left(-3+\sqrt{6}\,2\mathrm{i}\right)\,\left(2\,\sqrt{6\,x-9}+6\right)+66\right)\,\left(6\,\sqrt{6\,x-9}-\left(3+\sqrt{6}\,2\mathrm{i}\right)\,\left(2\,\sqrt{6\,x-9}+6\right)+66\right)\right)+4\,\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,\sqrt{6\,x-9}}{12}+\frac{\sqrt{6}}{4}\right)-2\,\sqrt{6\,x-9}$$

```
[In] int((x + 1)/(x + (6*x - 9)^(1/2) + 4),x)
```

```
[Out] x + 3*log((6*(6*x - 9)^(1/2) + (6^(1/2)*2i - 3)*(2*(6*x - 9)^(1/2) + 6) + 6
6)*(6*(6*x - 9)^(1/2) - (6^(1/2)*2i + 3)*(2*(6*x - 9)^(1/2) + 6) + 66)) + 4
*6^(1/2)*atan((6^(1/2)*(6*x - 9)^(1/2))/12 + 6^(1/2)/4) - 2*(6*x - 9)^(1/2)
```

3.708 $\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 71

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = -x+2\sqrt{3}\sqrt{-3+2x}-21\sqrt{\frac{3}{2}}\arctan\left(\frac{3+\sqrt{-9+6x}}{2\sqrt{6}}\right) + 10\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)$$

```
[Out] -x+10*ln(4+x+(-3+2*x)^(1/2)*3^(1/2))-21/2*arctan(1/12*(3+(-9+6*x)^(1/2))*6^
(1/2))*6^(1/2)+2*(-3+2*x)^(1/2)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1642, 648, 632, 210, 642}

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = -21\sqrt{\frac{3}{2}}\arctan\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)-x + 2\sqrt{3}\sqrt{2x-3}+10\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)$$

```
[In] Int[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] -x + 2*Sqrt[3]*Sqrt[-3 + 2*x] - 21*Sqrt[3/2]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2
*Sqrt[6])] + 10*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{x(-63+x^2)}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)\right)\\
&= -\left(\frac{1}{3}\text{Subst}\left(\int \left(-6+x+\frac{6(33-10x)}{33+6x+x^2}\right)\,dx, x, \sqrt{-9+6x}\right)\right)\\
&= -x+2\sqrt{3}\sqrt{-3+2x}-2\text{Subst}\left(\int \frac{33-10x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)\\
&= -x+2\sqrt{3}\sqrt{-3+2x}+10\text{Subst}\left(\int \frac{6+2x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)\\
&\quad -126\text{Subst}\left(\int \frac{1}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)\\
&= -x+2\sqrt{3}\sqrt{-3+2x}+10\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)\\
&\quad +252\text{Subst}\left(\int \frac{1}{-96-x^2}\,dx, x, 6+2\sqrt{-9+6x}\right)
\end{aligned}
$$

$$= -x+2\sqrt{3}\sqrt{-3+2x}-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{3+\sqrt{3}\sqrt{-3+2x}}{2\sqrt{6}}\right)+10\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = \frac{1}{2}\left(3-2x+4\sqrt{-9+6x}-21\sqrt{6}\arctan\left(\frac{\sqrt{3}+\sqrt{-3+2x}}{2\sqrt{2}}\right)+20\log\left(4+x+\sqrt{-9+6x}\right)\right)$$

```
[In] Integrate[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]
[Out] (3 - 2*x + 4*Sqrt[-9 + 6*x] - 21*Sqrt[6]*ArcTan[(Sqrt[3] + Sqrt[-3 + 2*x])/
(2*Sqrt[2])] + 20*Log[4 + x + Sqrt[-9 + 6*x]])/2
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{3}{2}-x+2\sqrt{-9+6x}+10\ln\left(24+6x+6\sqrt{-9+6x}\right)-\frac{21\sqrt{6}\,\arctan\left(\frac{\left(2\sqrt{-9+6x}+6\right)\sqrt{6}}{24}\right)}{2}$
default	$\frac{3}{2}-x+2\sqrt{-9+6x}+10\ln\left(24+6x+6\sqrt{-9+6x}\right)-\frac{21\sqrt{6}\,\arctan\left(\frac{\left(2\sqrt{-9+6x}+6\right)\sqrt{6}}{24}\right)}{2}$
trager	$-x+2\sqrt{-9+6x}-\ln\left(4+x+\sqrt{-9+6x}\right)\operatorname{RootOf}\left(8\_Z^2-160\_Z+2123\right)+\ln\Big(184$

```
[In] int((12-x)/(4+x+(-9+6*x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 3/2-x+2*(-9+6*x)^(1/2)+10*ln(24+6*x+6*(-9+6*x)^(1/2))-21/2*6^(1/2)*arctan(1
/24*(2*(-9+6*x)^(1/2)+6)*6^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = -\frac{21}{2}\,\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{12}\,\sqrt{3}\sqrt{2}\sqrt{6\,x-9}+\frac{1}{4}\,\sqrt{3}\sqrt{2}\right) - x + 2\,\sqrt{6\,x-9} + 10\,\log\left(x+\sqrt{6\,x-9}+4\right)$$

```
[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="fricas")
[Out] -21/2*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3)*sqrt(2)*sqrt(6*x - 9) + 1/4*sqrt(
3)*sqrt(2)) - x + 2*sqrt(6*x - 9) + 10*log(x + sqrt(6*x - 9) + 4)
```

Sympy [A] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = -x+2\sqrt{6x-9}+10\log\left(6x+6\sqrt{6x-9}+24\right) - \frac{21\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\left(\sqrt{6x-9}+3\right)}{12}\right)}{2} + \frac{3}{2}$$

```
[In] integrate((12-x)/(4+x+(-9+6*x)**(1/2)),x)
[Out] -x + 2*sqrt(6*x - 9) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) - 21*sqrt(6)*atan
(sqrt(6)*(sqrt(6*x - 9) + 3)/12)/2 + 3/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = -\frac{21}{2}\,\sqrt{6}\arctan\left(\frac{1}{12}\,\sqrt{6}\left(\sqrt{6\,x-9}+3\right)\right) - x + 2\,\sqrt{6\,x-9} + 10\,\log\left(6\,x+6\,\sqrt{6\,x-9}+24\right) + \frac{3}{2}$$

```
[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="maxima")
[Out] -21/2*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) - x + 2*sqrt(6*x - 9
) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) + 3/2
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = -\frac{21}{2}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(\sqrt{3}+\sqrt{2x-3}\right)\right) + 2\sqrt{3}\sqrt{2x-3} - x + 10\log\left(2\sqrt{3}\sqrt{2x-3}+2x+8\right) + \frac{3}{2}$$

```
[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="giac")
```

```
[Out] -21/2*sqrt(3)*sqrt(2)*arctan(1/4*sqrt(2)*(sqrt(3) + sqrt(2*x - 3))) + 2*sqr
t(3)*sqrt(2*x - 3) - x + 10*log(2*sqrt(3)*sqrt(2*x - 3) + 2*x + 8) + 3/2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.66

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = 2\sqrt{6x-9} + 10\ln\left(\left(\left(2\sqrt{6x-9}+6\right)\left(-10+\frac{\sqrt{2}\sqrt{3}\,21i}{4}\right) + 20\sqrt{6x-9}-66\right)\left(\left(2\sqrt{6x-9}+6\right)\left(10+\frac{\sqrt{2}\sqrt{3}\,21i}{4}\right) - 20\sqrt{6x-9}+66\right)\right) - x - \frac{21\sqrt{2}\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{3}}{4}+\frac{\sqrt{2}\sqrt{3}\sqrt{6x-9}}{12}\right)}{2}$$

```
[In] int(-(x - 12)/(x + (6*x - 9)^(1/2) + 4),x)
```

```
[Out] 10*log(((2*(6*x - 9)^(1/2) + 6)*((2^(1/2)*3^(1/2)*21i)/4 - 10) + 20*(6*x -
9)^(1/2) - 66)*((2*(6*x - 9)^(1/2) + 6)*((2^(1/2)*3^(1/2)*21i)/4 + 10) - 20
*(6*x - 9)^(1/2) + 66)) - x + 2*(6*x - 9)^(1/2) - (21*2^(1/2)*3^(1/2)*atan(
(2^(1/2)*3^(1/2))/4 + (2^(1/2)*3^(1/2)*(6*x - 9)^(1/2))/12))/2
```

3.709 $\int \frac{-1+x^3}{\sqrt{x}(1+x^2)} dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx = \frac{2x^{3/2}}{3} + \sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{x}\right) - \sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right)$$

```
[Out] 2/3*x^(3/2)-arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-arctan(1+2^(1/2)*x^(1/2))*2^
(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1816, 841, 1176, 631, 210}

$$\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx = \sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{x}\right) - \sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right) + \frac{2x^{3/2}}{3}$$

```
[In] Int[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] (2*x^(3/2))/3 + Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] - Sqrt[2]*ArcTan[1 + Sq
rt[2]*Sqrt[x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\sqrt{x}-\frac{1+x}{\sqrt{x}\,(1+x^2)}\right)\,dx\\
&= \frac{2x^{3/2}}{3}-\int \frac{1+x}{\sqrt{x}\,(1+x^2)}\,dx\\
&= \frac{2x^{3/2}}{3}-2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= \frac{2x^{3/2}}{3}-\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)-\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)\\
&= \frac{2x^{3/2}}{3}-\sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)+\sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{x}\right)\\
&= \frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \frac{-1+x^3}{\sqrt{x}(1+x^2)} dx = \frac{2x^{3/2}}{3} - \sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)$$

```
[In] Integrate[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]
[Out] (2*x^(3/2))/3 - Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

method	result
trager	$\frac{2x^{\frac{3}{2}}}{3} - \frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)x^2-4\operatorname{RootOf}\left(\_Z^2+2\right)x-4x^{\frac{3}{2}}+\operatorname{RootOf}\left(\_Z^2+2\right)+4\sqrt{x}}{x^2+1}\right)}{2}$
risch	$\frac{2x^{\frac{3}{2}}}{3} - \arctan\left(1+\sqrt{2}\sqrt{x}\right)\sqrt{2} - \arctan\left(-1+\sqrt{2}\sqrt{x}\right)\sqrt{2} - \frac{\sqrt{2}\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)}{4} - \frac{\sqrt{2}\ln\left(\frac{x}{x}\right.}{}$
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3} - \frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4} - \frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\right.\right.}{4}$
default	$\frac{2x^{\frac{3}{2}}}{3} - \frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4} - \frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\right.\right.}{4}$
meijerg	$\frac{2x^{\frac{3}{2}}}{3} - \frac{x^{\frac{3}{2}}\left(\frac{\sqrt{2}\ln\left(1-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2\left(x^2\right)^{\frac{3}{4}}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}\right)}{\left(x^2\right)^{\frac{3}{4}}} - \frac{\sqrt{2}\ln\left(1+\sqrt{2}\left(x^2\right)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2\left(x^2\right)^{\frac{3}{4}}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2\right.}{2+\sqrt{2}\left(x^2\right.}\right.}{\left(x^2\right)^{\frac{3}{4}}}\right.}{2}$

```
[In] int((x^3-1)/(x^2+1)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*x^(3/2)-1/2*RootOf(_Z^2+2)*ln(-(RootOf(_Z^2+2)*x^2-4*RootOf(_Z^2+2)*x-4
*x^(3/2)+RootOf(_Z^2+2)+4*x^(1/2))/(x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.44

$$\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx=\frac{2}{3}\,x^{\frac{3}{2}}-\sqrt{2}\arctan\left(\frac{\sqrt{2}(x-1)}{2\,\sqrt{x}}\right)$$

```
[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="fricas")
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(x - 1)/sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx=\frac{2x^{\frac{3}{2}}}{3}-\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)$$

```
[In] integrate((x**3-1)/(x**2+1)/x**(1/2),x)
[Out] 2*x**(3/2)/3 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1) - sqrt(2)*atan(sqrt(2)*sqr
t(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx=\frac{2}{3}\,x^{\frac{3}{2}}-\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right)-\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right)$$

```
[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="maxima")
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x)))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx = \frac{2}{3}\,x^{\frac{3}{2}} - \sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right) - \sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right)$$

```
[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="giac")
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x)))
```

Mupad [B] (verification not implemented)

Time = 18.59 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx = \frac{2\,x^{3/2}}{3} - \frac{\sqrt{2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{x}}{2}+\frac{\sqrt{2}\,x^{3/2}}{2}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)\right)}{2}$$

```
[In] int((x^3 - 1)/(x^(1/2)*(x^2 + 1)),x)
[Out] (2*x^(3/2))/3 - (2^(1/2)*(2*atan((2^(1/2)*x^(1/2))/2 + (2^(1/2)*x^(3/2))/2)
 + 2*atan((2^(1/2)*x^(1/2))/2)))/2
```

3.710 $\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 20

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = -\text{arcsinh}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)$$

```
[Out] -arcsinh(1/3*(1-2*(-1+x)^(1/2))*3^(1/2))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {12, 633, 221}

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = -\text{arcsinh}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

```
[In] Int[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

```
[Out] -ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx \\
&= \text{Subst}\left(\int \frac{1}{\sqrt{1-x+x^2}}\,dx, x, \sqrt{-1+x}\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, -1+2\sqrt{-1+x}\right)}{\sqrt{3}} \\
&= -\sinh^{-1}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = -\log\left(1-2\sqrt{-1+x}+2\sqrt{-\sqrt{-1+x}+x}\right)$$

```
[In] Integrate[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
[Out] -Log[1 - 2*Sqrt[-1 + x] + 2*Sqrt[-Sqrt[-1 + x] + x]]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$	14
default	$\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$	14

```
[In] int(1/2/(x-1)^(1/2)/(x-(x-1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(2/3*3^(1/2)*((x-1)^(1/2)-1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = \frac{1}{2}\log\left(4\sqrt{x-\sqrt{x-1}}\left(2\sqrt{x-1}-1\right)+8x-8\sqrt{x-1}-3\right)$$

```
[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/2*log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqrt(x - 1) -
 3)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = \operatorname{asinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$$

```
[In] integrate(1/2/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)
[Out] asinh(2*sqrt(3)*(sqrt(x - 1) - 1/2)/3)
```

Maxima [F]

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = \int \frac{1}{2\sqrt{x-\sqrt{x-1}}\sqrt{x-1}}\,dx$$

```
[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] 1/2*integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = -\log\left(2\sqrt{x-\sqrt{x-1}}-2\sqrt{x-1}+1\right)$$

```
[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="giac")
[Out] -log(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = \int \frac{1}{2\sqrt{x-\sqrt{x-1}}\sqrt{x-1}}\,dx$$

```
[In] int(1/(2*(x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)),x)
[Out] int(1/(2*(x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)), x)
```

3.711 $\int \frac{1+x^{7/2}}{1-x^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{1+x^{7/2}}{1-x^2} dx = -2\sqrt{x} - \frac{2x^{5/2}}{5} + \arctan\left(\sqrt{x}\right) - \log\left(1-\sqrt{x}\right) + \frac{1}{2}\log(1+x)$$

```
[Out] -2/5*x^(5/2)+arctan(x^(1/2))+1/2*ln(1+x)-ln(1-x^(1/2))-2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1847, 281, 212, 308, 218, 209}

$$\int \frac{1+x^{7/2}}{1-x^2} dx = \arctan\left(\sqrt{x}\right) + \text{arctanh}\left(\sqrt{x}\right) + \text{arctanh}(x) - \frac{2x^{5/2}}{5} - 2\sqrt{x}$$

```
[In] Int[(1 + x^(7/2))/(1 - x^2),x]
```

```
[Out] -2*Sqrt[x] - (2*x^(5/2))/5 + ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]] + ArcTanh[x
]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x(1+x^7)}{1-x^4}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x}{1-x^4}+\frac{x^8}{1-x^4}\right)\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \frac{x}{1-x^4}\,dx, x, \sqrt{x}\right)+2\text{Subst}\left(\int \frac{x^8}{1-x^4}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(-1-x^4+\frac{1}{1-x^4}\right)\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x\right)\\
&= -2\sqrt{x}-\frac{2x^{5/2}}{5}+\tanh^{-1}(x)+2\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right)\\
&= -2\sqrt{x}-\frac{2x^{5/2}}{5}+\tanh^{-1}(x)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)
\end{aligned}
$$

$$= -2\sqrt{x} - \frac{2x^{5/2}}{5} + \tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right) + \tanh^{-1}(x)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1+x^{7/2}}{1-x^2}\,dx = -\frac{2}{5}\sqrt{x}(5+x^2) + \arctan\left(\sqrt{x}\right) - \log\left(-1+\sqrt{x}\right) + \frac{1}{2}\log(1+x)$$

```
[In] Integrate[(1 + x^(7/2))/(1 - x^2),x]
[Out] (-2*Sqrt[x]*(5 + x^2))/5 + ArcTan[Sqrt[x]] - Log[-1 + Sqrt[x]] + Log[1 + x]
/2
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x} + \frac{\ln(x+1)}{2} + \arctan\left(\sqrt{x}\right) - \ln\left(-1+\sqrt{x}\right)$
default	$-\frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x} - \frac{\ln(-1+\sqrt{x})}{2} + \frac{\ln(1+\sqrt{x})}{2} + \arctan\left(\sqrt{x}\right) + \operatorname{arctanh}(x)$
meijerg	$\operatorname{arctanh}(x) - \frac{(-1)^{\frac{3}{4}}\left(-\frac{4\sqrt{x}\,(-1)^{\frac{1}{4}}(9x^2+45)}{45} - \frac{\sqrt{x}\,(-1)^{\frac{1}{4}}\left(\ln\left(1-(x^2)^{\frac{1}{4}}\right) - \ln\left(1+(x^2)^{\frac{1}{4}}\right) - 2\arctan\left((x^2)^{\frac{1}{4}}\right)\right)}{(x^2)^{\frac{1}{4}}}\right)}{2}$
trager	$\left(-\frac{2x^2}{5} - 2\right)\sqrt{x} - \ln\Big(-\frac{-24\,\mathrm{RootOf}\left(8\_Z^2-4\_Z+1\right)^2 x+16\,\mathrm{RootOf}\left(8\_Z^2-4\_Z+1\right)\sqrt{x}+48\,\mathrm{RootOf}\big(8\_$

```
[In] int((1+x^(7/2))/(-x^2+1),x,method=_RETURNVERBOSE)
[Out] -2/5*x^(5/2)-2*x^(1/2)+1/2*ln(x+1)+arctan(x^(1/2))-ln(-1+x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1+x^{7/2}}{1-x^2}\,dx = -\frac{2}{5}\left(x^2+5\right)\sqrt{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(x+1\right)-\log\left(\sqrt{x}-1\right)$$

```
[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="fricas")
[Out] -2/5*(x^2 + 5)*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(sqrt(x) - 1
)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1+x^{7/2}}{1-x^2}\,dx = -\frac{2x^{\frac{5}{2}}}{5}-2\sqrt{x}-\log\left(\sqrt{x}-1\right)+\frac{\log\left(x+1\right)}{2}+\operatorname{atan}\left(\sqrt{x}\right)$$

```
[In] integrate((1+x**(7/2))/(-x**2+1),x)
[Out] -2*x**(5/2)/5 - 2*sqrt(x) - log(sqrt(x) - 1) + log(x + 1)/2 + atan(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1+x^{7/2}}{1-x^2}\,dx = -\frac{2}{5}\,x^{\frac{5}{2}}-2\,\sqrt{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(x+1\right)-\log\left(\sqrt{x}-1\right)$$

```
[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="maxima")
[Out] -2/5*x^(5/2) - 2*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(sqrt(x) -
 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{1+x^{7/2}}{1-x^2}\,dx = -\frac{2}{5}\,x^{\frac{5}{2}} - 2\,\sqrt{x} + \arctan\left(\sqrt{x}\right) + \frac{1}{2}\,\log\left(x+1\right) - \log\left(\left|\sqrt{x}-1\right|\right)$$

```
[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="giac")
[Out] -2/5*x^(5/2) - 2*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(abs(sqrt(
x) - 1))
```

Mupad [B] (verification not implemented)

Time = 18.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \frac{1+x^{7/2}}{1-x^2}\,dx = -\ln\left(10\,\sqrt{x}-10\right) - 2\,\sqrt{x} - \frac{2\,x^{5/2}}{5} + \ln\left(1+\sqrt{x}\,\left(-3-\mathrm{i}\right)-3\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right) + \ln\left(1+\sqrt{x}\,\left(-3+1\mathrm{i}\right)+3\mathrm{i}\right)\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

```
[In] int(-(x^(7/2) + 1)/(x^2 - 1),x)
[Out] log((1 - 3i) - x^(1/2)*(3 + 1i))*(1/2 + 1i/2) - log(10*x^(1/2) - 10) + log(
(1 + 3i) - x^(1/2)*(3 - 1i))*(1/2 - 1i/2) - 2*x^(1/2) - (2*x^(5/2))/5
```

3.712 $\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 116

$$\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx = -x+18\sqrt[6]{-1+2x}-9\sqrt[3]{-1+2x}+6\sqrt{-1+2x} -\frac{3}{4}(-1+2x)^{2/3}+\frac{3}{5}(-1+2x)^{5/6}+\frac{3}{7}(-1+2x)^{7/6} -\frac{3}{8}(-1+2x)^{4/3}+\frac{1}{3}(-1+2x)^{3/2}-18\log\left(1+\sqrt[6]{-1+2x}\right)$$

```
[Out] -x+18*(-1+2*x)^(1/6)-9*(-1+2*x)^(1/3)-3/4*(-1+2*x)^(2/3)+3/5*(-1+2*x)^(5/6)
+3/7*(-1+2*x)^(7/6)-3/8*(-1+2*x)^(4/3)+1/3*(-1+2*x)^(3/2)-18*ln(1+(-1+2*x)^
(1/6))+6*(-1+2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1634}

$$\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx = \frac{1}{3}(2x-1)^{3/2}-\frac{3}{8}(2x-1)^{4/3}+\frac{3}{7}(2x-1)^{7/6} +\frac{3}{5}(2x-1)^{5/6}-\frac{3}{4}(2x-1)^{2/3}+6\sqrt{2x-1}-9\sqrt[3]{2x-1}+18\sqrt[6]{2x-1}-x-18\log\left(\sqrt[6]{2x-1}+1\right)$$

```
[In] Int[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]
```

```
[Out] -x + 18*(-1 + 2*x)^(1/6) - 9*(-1 + 2*x)^(1/3) + 6*Sqrt[-1 + 2*x] - (3*(-1 +
 2*x)^(2/3))/4 + (3*(-1 + 2*x)^(5/6))/5 + (3*(-1 + 2*x)^(7/6))/7 - (3*(-1 +
 2*x)^(4/3))/8 + (-1 + 2*x)^(3/2)/3 - 18*Log[1 + (-1 + 2*x)^(1/6)]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{x^3(5+x^6)}{1+x}\,dx, x, \sqrt[6]{-1+2x}\right)\\
&= 3\text{Subst}\left(\int \left(6-6x+6x^2-x^3+x^4-x^5+x^6-x^7+x^8-\frac{6}{1+x}\right)\,dx, x, \sqrt[6]{-1+2x}\right)\\
&= -x+18\sqrt[6]{-1+2x}-9\sqrt[3]{-1+2x}+6\sqrt{-1+2x}-\frac{3}{4}(-1+2x)^{2/3}+\frac{3}{5}(-1+2x)^{5/6}\\
&\quad+\frac{3}{7}(-1+2x)^{7/6}-\frac{3}{8}(-1+2x)^{4/3}+\frac{1}{3}(-1+2x)^{3/2}-18\log\left(1+\sqrt[6]{-1+2x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\begin{aligned}
&\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx\\
&=2\left(\frac{1}{4}+\frac{123}{14}\sqrt[6]{-1+2x}-\frac{69}{16}\sqrt[3]{-1+2x}+\frac{17}{6}\sqrt{-1+2x}-\frac{3}{8}(-1+2x)^{2/3}+\frac{3}{10}(-1+2x)^{5/6}\right.\\
&\quad\left.+x\left(-\frac{1}{2}+\frac{3}{7}\sqrt[6]{-1+2x}-\frac{3}{8}\sqrt[3]{-1+2x}+\frac{1}{3}\sqrt{-1+2x}\right)-9\log\left(1+\sqrt[6]{-1+2x}\right)\right)
\end{aligned}$$

```
[In] Integrate[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]
```

```
[Out] 2*(1/4 + (123*(-1 + 2*x)^(1/6))/14 - (69*(-1 + 2*x)^(1/3))/16 + (17*Sqrt[-1
 + 2*x])/6 - (3*(-1 + 2*x)^(2/3))/8 + (3*(-1 + 2*x)^(5/6))/10 + x*(-1/2 + (
3*(-1 + 2*x)^(1/6))/7 - (3*(-1 + 2*x)^(1/3))/8 + Sqrt[-1 + 2*x]/3) - 9*Log[
1 + (-1 + 2*x)^(1/6)])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{(2x-1)^{\frac{3}{2}}}{3}-\frac{3(2x-1)^{\frac{4}{3}}}{8}+\frac{3(2x-1)^{\frac{7}{6}}}{7}-x+\frac{1}{2}+\frac{3(2x-1)^{\frac{5}{6}}}{5}-\frac{3(2x-1)^{\frac{2}{3}}}{4}+6\sqrt{2x-1}-9(2x-1)^{\frac{1}{3}}$
default	$\frac{(2x-1)^{\frac{3}{2}}}{3}-\frac{3(2x-1)^{\frac{4}{3}}}{8}+\frac{3(2x-1)^{\frac{7}{6}}}{7}-x+\frac{1}{2}+\frac{3(2x-1)^{\frac{5}{6}}}{5}-\frac{3(2x-1)^{\frac{2}{3}}}{4}+6\sqrt{2x-1}-9(2x-1)^{\frac{1}{3}}$

```
[In] int((4+2*x)/((2*x-1)^(1/3)+(2*x-1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*x-1)^(3/2)-3/8*(2*x-1)^(4/3)+3/7*(2*x-1)^(7/6)-x+1/2+3/5*(2*x-1)^(5/
6)-3/4*(2*x-1)^(2/3)+6*(2*x-1)^(1/2)-9*(2*x-1)^(1/3)+18*(2*x-1)^(1/6)-18*ln
(1+(2*x-1)^(1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx = \frac{1}{3}\,(2\,x+17)\sqrt{2\,x-1}-\frac{3}{8}\,(2\,x+23)(2\,x-1)^{\frac{1}{3}} + \frac{3}{7}\,(2\,x+41)(2\,x-1)^{\frac{1}{6}}-x+\frac{3}{5}\,(2\,x-1)^{\frac{5}{6}} - \frac{3}{4}\,(2\,x-1)^{\frac{2}{3}}-18\,\log\left((2\,x-1)^{\frac{1}{6}}+1\right)$$

```
[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x + 17)*sqrt(2*x - 1) - 3/8*(2*x + 23)*(2*x - 1)^(1/3) + 3/7*(2*x +
41)*(2*x - 1)^(1/6) - x + 3/5*(2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) - 18*lo
g((2*x - 1)^(1/6) + 1)
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx = -x+\frac{3(2x-1)^{\frac{7}{6}}}{7}+\frac{3(2x-1)^{\frac{5}{6}}}{5}+18\sqrt[6]{2x-1} - \frac{3(2x-1)^{\frac{4}{3}}}{8}-\frac{3(2x-1)^{\frac{2}{3}}}{4}-9\sqrt[3]{2x-1} + \frac{(2x-1)^{\frac{3}{2}}}{3}+6\sqrt{2x-1}-18\log\left(\sqrt[6]{2x-1}+1\right)+\frac{1}{2}$$

```
[In] integrate((4+2*x)/((-1+2*x)**(1/3)+(-1+2*x)**(1/2)),x)
```

```
[Out] -x + 3*(2*x - 1)**(7/6)/7 + 3*(2*x - 1)**(5/6)/5 + 18*(2*x - 1)**(1/6) - 3*
(2*x - 1)**(4/3)/8 - 3*(2*x - 1)**(2/3)/4 - 9*(2*x - 1)**(1/3) + (2*x - 1)*
*(3/2)/3 + 6*sqrt(2*x - 1) - 18*log((2*x - 1)**(1/6) + 1) + 1/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx = \frac{1}{3}\,(2\,x-1)^{\frac{3}{2}} - \frac{3}{8}\,(2\,x-1)^{\frac{4}{3}} + \frac{3}{7}\,(2\,x-1)^{\frac{7}{6}} - x + \frac{3}{5}\,(2\,x-1)^{\frac{5}{6}} - \frac{3}{4}\,(2\,x-1)^{\frac{2}{3}} + 6\,\sqrt{2\,x-1} - 9\,(2\,x-1)^{\frac{1}{3}} + 18\,(2\,x-1)^{\frac{1}{6}} - 18\,\log\left((2\,x-1)^{\frac{1}{6}}+1\right) + \frac{1}{2}$$

```
[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x - 1)^(3/2) - 3/8*(2*x - 1)^(4/3) + 3/7*(2*x - 1)^(7/6) - x + 3/5*(
2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) + 6*sqrt(2*x - 1) - 9*(2*x - 1)^(1/3)
+ 18*(2*x - 1)^(1/6) - 18*log((2*x - 1)^(1/6) + 1) + 1/2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx = \frac{1}{3}\,(2\,x-1)^{\frac{3}{2}} - \frac{3}{8}\,(2\,x-1)^{\frac{4}{3}} + \frac{3}{7}\,(2\,x-1)^{\frac{7}{6}} - x + \frac{3}{5}\,(2\,x-1)^{\frac{5}{6}} - \frac{3}{4}\,(2\,x-1)^{\frac{2}{3}} + 6\,\sqrt{2\,x-1} - 9\,(2\,x-1)^{\frac{1}{3}} + 18\,(2\,x-1)^{\frac{1}{6}} - 18\,\log\left((2\,x-1)^{\frac{1}{6}}+1\right) + \frac{1}{2}$$

```
[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*(2*x - 1)^(3/2) - 3/8*(2*x - 1)^(4/3) + 3/7*(2*x - 1)^(7/6) - x + 3/5*(
2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) + 6*sqrt(2*x - 1) - 9*(2*x - 1)^(1/3)
+ 18*(2*x - 1)^(1/6) - 18*log((2*x - 1)^(1/6) + 1) + 1/2
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

$$\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx = 6\sqrt{2x-1}-18\ln\left((2x-1)^{1/6}+1\right)-x-9\,(2x-1)^{1/3}-\frac{3\,(2x-1)^{2/3}}{4}+\frac{(2x-1)^{3/2}}{3}+18\,(2x-1)^{1/6}-\frac{3\,(2x-1)^{4/3}}{8}+\frac{3\,(2x-1)^{5/6}}{5}+\frac{3\,(2x-1)^{7/6}}{7}$$

```
[In] int((2*x + 4)/((2*x - 1)^(1/2) + (2*x - 1)^(1/3)),x)
```

```
[Out] 6*(2*x - 1)^(1/2) - 18*log((2*x - 1)^(1/6) + 1) - x - 9*(2*x - 1)^(1/3) - (
3*(2*x - 1)^(2/3))/4 + (2*x - 1)^(3/2)/3 + 18*(2*x - 1)^(1/6) - (3*(2*x - 1
)^(4/3))/8 + (3*(2*x - 1)^(5/6))/5 + (3*(2*x - 1)^(7/6))/7
```

3.713 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 83

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = -48\sqrt{2+\sqrt{1+\sqrt{x}}}+\frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2} -\frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2}+\frac{8}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{7/2}$$

```
[Out] 88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(1
+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {378, 1412, 786}

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2} +\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

```
[In] Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/2))/3 - (4
8*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + Sqrt[x]])^(7/2))/7
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x}{\sqrt{2+\sqrt{1+x}}}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \frac{-1+x}{\sqrt{2+\sqrt{x}}}\,dx, x, 1+\sqrt{x}\right)\\
&= 4\text{Subst}\left(\int \frac{x(-1+x^2)}{\sqrt{2+x}}\,dx, x, \sqrt{1+\sqrt{x}}\right)\\
&= 4\text{Subst}\left(\int \left(-\frac{6}{\sqrt{2+x}}+11\sqrt{2+x}-6(2+x)^{3/2}+(2+x)^{5/2}\right)\,dx, x, \sqrt{1+\sqrt{x}}\right)\\
&= -48\sqrt{2+\sqrt{1+\sqrt{x}}}+\frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2}\\
&\quad-\frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2}+\frac{8}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{7/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8}{105}\sqrt{2+\sqrt{1+\sqrt{x}}}\left(-280+76\sqrt{1+\sqrt{x}}+3\left(-12+5\sqrt{1+\sqrt{x}}\right)\sqrt{x}\right)$$

```
[In] Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(-12 + 5*Sq
rt[1 + Sqrt[x]])*Sqrt[x]))/105
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{88\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{1+\sqrt{x}}}$	54
default	$\frac{88\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{1+\sqrt{x}}}$	54

```
[In] int(1/(2+(1+x^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(1
+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8}{105}\left((15\sqrt{x}+76)\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)\sqrt{\sqrt{\sqrt{x}+1}+2}$$

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sqrt(sqrt(sq
rt(x) + 1) + 2)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}}{7} - \frac{48\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}}{5} + \frac{88\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}}{3} - 48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

```
[In] integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
[Out] 8*(sqrt(sqrt(x) + 1) + 2)**(7/2)/7 - 48*(sqrt(sqrt(x) + 1) + 2)**(5/2)/5 +
88*(sqrt(sqrt(x) + 1) + 2)**(3/2)/3 - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}} - \frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}} + \frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}} - 48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
[Out] 8/7*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 48/5*(sqrt(sqrt(x) + 1) + 2)^(5/2) + 88
/3*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8\left(15\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}} - 126\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}} + 385\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}} - 630\sqrt{\sqrt{\sqrt{x}+1}+2}\right)}{105\,\mathrm{sgn}\left(4\left(\sqrt{x}+1\right)^{2} - 8\sqrt{x} - 7\right)\mathrm{sgn}\left(4x-3\right)}$$

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/105*(15*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 126*(sqrt(sqrt(x) + 1) + 2)^(5/2)
 + 385*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 630*sqrt(sqrt(sqrt(x) + 1) + 2))/(sg
n(4*(sqrt(x) + 1)^2 - 8*sqrt(x) - 7)*sgn(4*x - 3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

```
[In] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2),x)
```

```
[Out] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2), x)
```

3.714 $\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = \frac{64}{5}\left(2+\sqrt{4+\sqrt{x}}\right)^{5/2} - \frac{48}{7}\left(2+\sqrt{4+\sqrt{x}}\right)^{7/2} + \frac{8}{9}\left(2+\sqrt{4+\sqrt{x}}\right)^{9/2}$$

```
[Out] 64/5*(2+(4+x^(1/2))^(1/2))^(5/2)-48/7*(2+(4+x^(1/2))^(1/2))^(7/2)+8/9*(2+(4
+x^(1/2))^(1/2))^(9/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {378, 1412, 786}

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = \frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{9/2} - \frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{7/2} + \frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{5/2}$$

```
[In] Int[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
```

```
[Out] (64*(2 + Sqrt[4 + Sqrt[x]])^(5/2))/5 - (48*(2 + Sqrt[4 + Sqrt[x]])^(7/2))/7
 + (8*(2 + Sqrt[4 + Sqrt[x]])^(9/2))/9
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x\sqrt{2+\sqrt{4+x}}\,dx,x,\sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \sqrt{2+\sqrt{x}}(-4+x)\,dx,x,4+\sqrt{x}\right)\\
&= 4\text{Subst}\left(\int x\sqrt{2+x}\left(-4+x^2\right)\,dx,x,\sqrt{4+\sqrt{x}}\right)\\
&= 4\text{Subst}\left(\int \left(8(2+x)^{3/2}-6(2+x)^{5/2}+(2+x)^{7/2}\right)\,dx,x,\sqrt{4+\sqrt{x}}\right)\\
&= \frac{64}{5}\left(2+\sqrt{4+\sqrt{x}}\right)^{5/2}-\frac{48}{7}\left(2+\sqrt{4+\sqrt{x}}\right)^{7/2}+\frac{8}{9}\left(2+\sqrt{4+\sqrt{x}}\right)^{9/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int\sqrt{2+\sqrt{4+\sqrt{x}}}\,dx=\frac{8}{315}\sqrt{2+\sqrt{4+\sqrt{x}}}\left(-64\left(2+\sqrt{4+\sqrt{x}}\right)+2\left(2+5\sqrt{4+\sqrt{x}}\right)\sqrt{x}+35x\right)$$

```
[In] Integrate[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
[Out] (8*Sqrt[2 + Sqrt[4 + Sqrt[x]]]*(-64*(2 + Sqrt[4 + Sqrt[x]]) + 2*(2 + 5*Sqrt
[4 + Sqrt[x]])*Sqrt[x] + 35*x))/315
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.27

method	result	size
meijerg	$2x\,{}_3F_2\left(-\frac{1}{4},\frac{1}{4},2;\frac{1}{2},3;-\frac{\sqrt{x}}{4}\right)$	17
derivativedivides	$\frac{64\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{5}{2}}}{5}-\frac{48\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{7}{2}}}{7}+\frac{8\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{9}{2}}}{9}$	41
default	$\frac{64\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{5}{2}}}{5}-\frac{48\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{7}{2}}}{7}+\frac{8\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{9}{2}}}{9}$	41

```
[In] int((2+(4+x^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*x*hypergeom([-1/4,1/4,2],[1/2,3],-1/4*x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int\sqrt{2+\sqrt{4+\sqrt{x}}}\,dx=\frac{8}{315}\left(2\left(5\sqrt{x}-32\right)\sqrt{\sqrt{x}+4}+35\,x+4\sqrt{x}-128\right)\sqrt{\sqrt{\sqrt{x}+4}+2}$$

```
[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
[Out] 8/315*(2*(5*sqrt(x) - 32)*sqrt(sqrt(x) + 4) + 35*x + 4*sqrt(x) - 128)*sqrt(
sqrt(sqrt(x) + 4) + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(54) = 108.

Time = 1.40 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.38

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = -\frac{2\sqrt{2}\sqrt{x}\sqrt{\sqrt{x}+4}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{63\pi} - \frac{4\sqrt{2}\sqrt{x}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi} - \frac{\sqrt{2}x\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{9\pi} + \frac{64\sqrt{2}\sqrt{\sqrt{x}+4}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi} + \frac{128\sqrt{2}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}$$

```
[In] integrate((2+(4+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] -2*sqrt(2)*sqrt(x)*sqrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(-1/4
)*gamma(1/4)/(63*pi) - 4*sqrt(2)*sqrt(x)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(
-1/4)*gamma(1/4)/(315*pi) - sqrt(2)*x*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(-1/
4)*gamma(1/4)/(9*pi) + 64*sqrt(2)*sqrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) + 4)
+ 2)*gamma(-1/4)*gamma(1/4)/(315*pi) + 128*sqrt(2)*sqrt(sqrt(sqrt(x) + 4) +
 2)*gamma(-1/4)*gamma(1/4)/(315*pi)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = \frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{9}{2}} - \frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}} + \frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}$$

```
[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*(sqrt(sqrt(x) + 4) + 2)^(9/2) - 48/7*(sqrt(sqrt(x) + 4) + 2)^(7/2) + 64
/5*(sqrt(sqrt(x) + 4) + 2)^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(40) = 80.

Time = 0.39 (sec) , antiderivative size = 268, normalized size of antiderivative = 4.19

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx$$
$$= \frac{8}{315}\left(\left(35\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{9}{2}} - 360\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}} + 1512\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}} - 3360\left(\sqrt{\sqrt{x}+4}+2 \right.\right.\right.$$
$$- 15)$$

```
[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/315*((35*(sqrt(sqrt(x) + 4) + 2)^(9/2) - 360*(sqrt(sqrt(x) + 4) + 2)^(7/2
) + 1512*(sqrt(sqrt(x) + 4) + 2)^(5/2) - 3360*(sqrt(sqrt(x) + 4) + 2)^(3/2)
 + 5040*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 7
9) + 18*(5*(sqrt(sqrt(x) + 4) + 2)^(7/2) - 42*(sqrt(sqrt(x) + 4) + 2)^(5/2)
 + 140*(sqrt(sqrt(x) + 4) + 2)^(3/2) - 280*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn
(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79) - 84*(3*(sqrt(sqrt(x) + 4) + 2)^(5/2)
 - 20*(sqrt(sqrt(x) + 4) + 2)^(3/2) + 60*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4
*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79) - 840*((sqrt(sqrt(x) + 4) + 2)^(3/2) -
6*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79))*sg
n(4*x - 15)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = \int \sqrt{\sqrt{\sqrt{x}+4}+2}\,dx$$

```
[In] int(((x^(1/2) + 4)^(1/2) + 2)^(1/2),x)
```

```
[Out] int(((x^(1/2) + 4)^(1/2) + 2)^(1/2), x)
```

3.715 $\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx=\frac{64}{25}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{5/2}-\frac{48}{35}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{7/2}+\frac{8}{45}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{9/2}$$

```
[Out] 64/25*(2-(4+(-9+5*x)^(1/2))^(1/2))^(5/2)-48/35*(2-(4+(-9+5*x)^(1/2))^(1/2))
^(7/2)+8/45*(2-(4+(-9+5*x)^(1/2))^(1/2))^(9/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {378, 1412, 786}

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx=\frac{8}{45}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{9/2}-\frac{48}{35}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{7/2}+\frac{64}{25}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{5/2}$$

```
[In] Int[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

```
[Out] (64*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(5/2))/25 - (48*(2 - Sqrt[4 + Sqrt[-9 +
5*x]])^(7/2))/35 + (8*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(9/2))/45
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{5}\text{Subst}\left(\int x\sqrt{2-\sqrt{4+x}}\,dx,x,\sqrt{-9+5x}\right)\\
&= \frac{2}{5}\text{Subst}\left(\int \sqrt{2-\sqrt{x}}(-4+x)\,dx,x,4+\sqrt{-9+5x}\right)\\
&= \frac{4}{5}\text{Subst}\left(\int \sqrt{2-x}x\left(-4+x^2\right)\,dx,x,\sqrt{4+\sqrt{-9+5x}}\right)\\
&= \frac{4}{5}\text{Subst}\left(\int \left(-8(2-x)^{3/2}+6(2-x)^{5/2}-(2-x)^{7/2}\right)\,dx,x,\sqrt{4+\sqrt{-9+5x}}\right)\\
&= \frac{64}{25}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{5/2}\\
&\quad -\frac{48}{35}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{7/2}+\frac{8}{45}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{9/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = -\frac{8\sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\left(443-175x-4\sqrt{-9+5x}-64\sqrt{4+\sqrt{-9+5x}}+10\sqrt{-9+5x}\sqrt{4+\sqrt{-9+5x}}\right)}{1575}$$

```
[In] Integrate[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

```
[Out] (-8*Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]]*(443 - 175*x - 4*Sqrt[-9 + 5*x] - 64
*Sqrt[4 + Sqrt[-9 + 5*x]] + 10*Sqrt[-9 + 5*x]*Sqrt[4 + Sqrt[-9 + 5*x]]))/15
75
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{64\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{5}{2}}}{25}-\frac{48\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{7}{2}}}{35}+\frac{8\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{9}{2}}}{45}$	59
default	$\frac{64\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{5}{2}}}{25}-\frac{48\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{7}{2}}}{35}+\frac{8\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{9}{2}}}{45}$	59

```
[In] int((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 64/25*(2-(4+(-9+5*x)^(1/2))^(1/2))^(5/2)-48/35*(2-(4+(-9+5*x)^(1/2))^(1/2))
^(7/2)+8/45*(2-(4+(-9+5*x)^(1/2))^(1/2))^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = -\frac{8}{1575}\left(2\left(5\sqrt{5x-9}-32\right)\sqrt{\sqrt{5x-9}+4}-175x-4\sqrt{5x-9}+443\right)\sqrt{-\sqrt{\sqrt{5x-9}+4}+2}$$

```
[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -8/1575*(2*(5*sqrt(5*x - 9) - 32)*sqrt(sqrt(5*x - 9) + 4) - 175*x - 4*sqrt(
5*x - 9) + 443)*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = \frac{8\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{\frac{9}{2}}}{45} - \frac{48\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{\frac{7}{2}}}{35} + \frac{64\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{\frac{5}{2}}}{25}$$

```
[In] integrate((2-(4+(-9+5*x)**(1/2))**(1/2))**(1/2),x)
[Out] 8*(2 - sqrt(sqrt(5*x - 9) + 4))**(9/2)/45 - 48*(2 - sqrt(sqrt(5*x - 9) + 4)
)**(7/2)/35 + 64*(2 - sqrt(sqrt(5*x - 9) + 4))**(5/2)/25
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.71

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = \frac{8}{45}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{9}{2}} - \frac{48}{35}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{7}{2}} + \frac{64}{25}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{5}{2}}$$

```
[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
[Out] 8/45*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(9/2) - 48/35*(-sqrt(sqrt(5*x - 9) + 4)
 + 2)^(7/2) + 64/25*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(58) = 116.

Time = 0.41 (sec) , antiderivative size = 474, normalized size of antiderivative = 5.78

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx =$$

$$-\frac{8}{1575}\left(\left(35\left(\sqrt{\sqrt{5\,x-9}+4}-2\right)^{4}\sqrt{-\sqrt{\sqrt{5\,x-9}+4}+2}+360\left(\sqrt{\sqrt{5\,x-9}+4}-2\right)^{3}\sqrt{-\sqrt{\sqrt{5\,x}}}\right.\right.$$

$$-\,51)$$

```
[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -8/1575*((35*(sqrt(sqrt(5*x - 9) + 4) - 2)^4*sqrt(-sqrt(sqrt(5*x - 9) + 4)
+ 2) + 360*(sqrt(sqrt(5*x - 9) + 4) - 2)^3*sqrt(-sqrt(sqrt(5*x - 9) + 4) +
2) + 1512*(sqrt(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2
) - 3360*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(3/2) + 5040*sqrt(-sqrt(sqrt(5*x -
9) + 4) + 2))*sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 18*(5
*(sqrt(sqrt(5*x - 9) + 4) - 2)^3*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) + 42*(s
qrt(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) - 140*(-sq
rt(sqrt(5*x - 9) + 4) + 2)^(3/2) + 280*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*
sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 84*(3*(sqrt(sqrt(5*
x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) - 20*(-sqrt(sqrt(5*x
- 9) + 4) + 2)^(3/2) + 60*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sgn(-4*(sqrt(
5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 840*((-sqrt(sqrt(5*x - 9) + 4) +
 2)^(3/2) - 6*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sgn(-4*(sqrt(5*x - 9) + 4
)^2 + 32*sqrt(5*x - 9) + 79))*sgn(20*x - 51)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx=\int \sqrt{2-\sqrt{\sqrt{5\,x-9}+4}}\,dx$$

```
[In] int((2 - ((5*x - 9)^(1/2) + 4)^(1/2))^(1/2),x)
```

```
[Out] int((2 - ((5*x - 9)^(1/2) + 4)^(1/2))^(1/2), x)
```

3.716 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}} dx$

Optimal result

Integrand size = 17, antiderivative size = 83

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}} dx = -48\sqrt{2+\sqrt{1+\sqrt{x}}}+\frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2} -\frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2}+\frac{8}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{7/2}$$

```
[Out] 88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(1
+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {378, 1412, 786}

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}} dx = \frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2} +\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

```
[In] Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/2))/3 - (4
8*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + Sqrt[x]])^(7/2))/7
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x}{\sqrt{2+\sqrt{1+x}}}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \frac{-1+x}{\sqrt{2+\sqrt{x}}}\,dx, x, 1+\sqrt{x}\right)\\
&= 4\text{Subst}\left(\int \frac{x(-1+x^2)}{\sqrt{2+x}}\,dx, x, \sqrt{1+\sqrt{x}}\right)\\
&= 4\text{Subst}\left(\int \left(-\frac{6}{\sqrt{2+x}}+11\sqrt{2+x}-6(2+x)^{3/2}+(2+x)^{5/2}\right)\,dx, x, \sqrt{1+\sqrt{x}}\right)\\
&= -48\sqrt{2+\sqrt{1+\sqrt{x}}}+\frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2}\\
&\quad -\frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2}+\frac{8}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{7/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx=\frac{8}{105}\sqrt{2+\sqrt{1+\sqrt{x}}}\left(-280+76\sqrt{1+\sqrt{x}}+3\left(-12+5\sqrt{1+\sqrt{x}}\right)\sqrt{x}\right)$$

```
[In] Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(-12 + 5*Sq
rt[1 + Sqrt[x]])*Sqrt[x]))/105
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{88\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{1+\sqrt{x}}}$	54
default	$\frac{88\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{1+\sqrt{x}}}$	54

```
[In] int(1/(2+(1+x^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(1
+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx=\frac{8}{105}\left(\left(15\sqrt{x}+76\right)\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)\sqrt{\sqrt{\sqrt{x}+1}+2}$$

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sqrt(sqrt(sq
rt(x) + 1) + 2)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}}{7} - \frac{48\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}}{5} + \frac{88\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}}{3} - 48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

```
[In] integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] 8*(sqrt(sqrt(x) + 1) + 2)**(7/2)/7 - 48*(sqrt(sqrt(x) + 1) + 2)**(5/2)/5 +
88*(sqrt(sqrt(x) + 1) + 2)**(3/2)/3 - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}} - \frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}} + \frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}} - 48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/7*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 48/5*(sqrt(sqrt(x) + 1) + 2)^(5/2) + 88
/3*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8\left(15\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}} - 126\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}} + 385\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}} - 630\sqrt{\sqrt{\sqrt{x}+1}+2}\right)}{105\,\mathrm{sgn}\left(4\left(\sqrt{x}+1\right)^{2} - 8\sqrt{x} - 7\right)\mathrm{sgn}\left(4\,x-3\right)}$$

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/105*(15*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 126*(sqrt(sqrt(x) + 1) + 2)^(5/2)
 + 385*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 630*sqrt(sqrt(sqrt(x) + 1) + 2))/(sg
n(4*(sqrt(x) + 1)^2 - 8*sqrt(x) - 7)*sgn(4*x - 3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

```
[In] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2),x)
```

```
[Out] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2), x)
```

3.717 $\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 190

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = -\frac{32}{5}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{5/2} + \frac{48}{7}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{7/2} + \frac{112}{9}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{9/2} - \frac{320}{11}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{11/2} + \frac{288}{13}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{13/2} - \frac{112}{15}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{15/2} + \frac{16}{17}\Big(1$$

```
[Out] -32/5*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(5/2)+48/7*(1+(1+(1+x^(1/2))^(1/2))^(
1/2))^(7/2)+112/9*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(9/2)-320/11*(1+(1+(1+x^(
1/2))^(1/2))^(1/2))^(11/2)+288/13*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(13/2)-11
2/15*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(15/2)+16/17*(1+(1+(1+x^(1/2))^(1/2))^
(1/2))^(17/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used

$= \{1632, 1634\}$

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = \frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{17/2} - \frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{15/2} + \frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{13/2} - \frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{11/2} + \frac{112}{9}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{9/2} + \frac{48}{7}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{7/2} - \frac{32}{5}\left(\sqrt{\right.$$

```
[In] Int[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

```
[Out] (-32*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2))/5 + (48*(1 + Sqrt[1 + Sqrt[1
+ Sqrt[x]]])^(7/2))/7 + (112*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(9/2))/9 - (
320*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(11/2))/11 + (288*(1 + Sqrt[1 + Sqrt[
1 + Sqrt[x]]])^(13/2))/13 - (112*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(15/2))/
15 + (16*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(17/2))/17
```

Rule 1632

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*x)^n, x]
 /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && EqQ[PolynomialRemainder
[Px, a + b*x, x], 0]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x\sqrt{1+\sqrt{1+\sqrt{1+x}}}\,dx, x, \sqrt{x}\right) \\
&= 4\text{Subst}\left(\int x\left(-1+x^2\right)\sqrt{1+\sqrt{1+x}}\,dx, x, \sqrt{1+\sqrt{x}}\right) \\
&= 8\text{Subst}\left(\int x^3\sqrt{1+x}\left(-2+x^2\right)\left(-1+x^2\right)\,dx, x, \sqrt{1+\sqrt{1+\sqrt{x}}}\right) \\
&= 8\text{Subst}\left(\int x^3(1+x)^{3/2}\left(2-2x-x^2+x^3\right)\,dx, x, \sqrt{1+\sqrt{1+\sqrt{x}}}\right)
\end{aligned}$$

$$= 8\text{Subst}\left(\int (-2(1+x)^{3/2}$$

$$+3(1+x)^{5/2}+7(1+x)^{7/2}-20(1+x)^{9/2}+18(1+x)^{11/2}-7(1+x)^{13/2}+(1+x)^{15/2})\ dx, x, \sqrt{1+\sqrt{1+$$

$$= -\frac{32}{5}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{5/2}$$

$$+\frac{48}{7}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{7/2}+\frac{112}{9}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{9/2}$$

$$-\frac{320}{11}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{11/2}+\frac{288}{13}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{13/2}-\frac{112}{15}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{15/2}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.88

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$$

$$= \frac{16\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\left(-8\left(3519-1094\sqrt{1+\sqrt{1+\sqrt{x}}}+163\sqrt{1+\sqrt{x}}+584\sqrt{1+\sqrt{1+\sqrt{x}}}\sqrt{1+\right.\right.}{76576}$$

```
[In] Integrate[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

```
[Out] (16*Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]]*(-8*(3519 - 1094*Sqrt[1 + Sqrt[1
+ Sqrt[x]]] + 163*Sqrt[1 + Sqrt[x]] + 584*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*Sqrt[
1 + Sqrt[x]]) + 7*(659 - 504*Sqrt[1 + Sqrt[1 + Sqrt[x]]] + 33*Sqrt[1 + Sqrt
[x]] + 429*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*Sqrt[1 + Sqrt[x]])*Sqrt[x] + 45045*x
))/765765
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.64

method	result
derivativedivides	$-\frac{32\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{5}{2}}}{5}+\frac{48\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{7}{2}}}{7}+\frac{112\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{9}{2}}}{9}-\frac{320\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{11}{2}}}{11}-$
default	$-\frac{32\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{5}{2}}}{5}+\frac{48\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{7}{2}}}{7}+\frac{112\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{9}{2}}}{9}-\frac{320\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{11}{2}}}{11}-$

```
[In] int((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] -32/5*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(5/2)+48/7*(1+(1+(1+x^(1/2))^(1/2))^(
1/2))^(7/2)+112/9*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(9/2)-320/11*(1+(1+(1+x^(
1/2))^(1/2))^(1/2))^(11/2)+288/13*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(13/2)-11
2/15*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(15/2)+16/17*(1+(1+(1+x^(1/2))^(1/2))^
(1/2))^(17/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.40

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$$
$$= \frac{16}{765765}\left(\left(231\sqrt{x}-1304\right)\sqrt{\sqrt{x}+1}+\left(\left(3003\sqrt{x}-4672\right)\sqrt{\sqrt{x}+1}-3528\sqrt{x}+8752\right)\sqrt{\sqrt{\sqrt{x}+1}+\right.$$

```
[In] integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
[Out] 16/765765*((231*sqrt(x) - 1304)*sqrt(sqrt(x) + 1) + ((3003*sqrt(x) - 4672)*
sqrt(sqrt(x) + 1) - 3528*sqrt(x) + 8752)*sqrt(sqrt(sqrt(x) + 1) + 1) + 4504
5*x + 4613*sqrt(x) - 28152)*sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = \frac{16\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{17}{2}}}{17} - \frac{112\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{15}{2}}}{15} + \frac{288\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{13}{2}}}{13} - \frac{320\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{11}{2}}}{11} + \frac{112\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{9}{2}}}{9} + \frac{48\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{7}{2}}}{7} - \frac{32\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{5}{2}}}{5}$$

```
[In] integrate((1+(1+(1+x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

```
[Out] 16*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(17/2)/17 - 112*(sqrt(sqrt(sqrt(x) +
1) + 1) + 1)**(15/2)/15 + 288*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(13/2)/13
- 320*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(11/2)/11 + 112*(sqrt(sqrt(sqrt(x)
 + 1) + 1) + 1)**(9/2)/9 + 48*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(7/2)/7 -
32*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(5/2)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.63

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = \frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{17}{2}} - \frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{15}{2}} + \frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{13}{2}} - \frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{11}{2}} + \frac{112}{9}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{9}{2}} + \frac{48}{7}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{7}{2}} - \frac{32}{5}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{5}{2}}$$

```
[In] integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 16/17*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(17/2) - 112/15*(sqrt(sqrt(sqrt(x)
+ 1) + 1) + 1)^(15/2) + 288/13*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(13/2) - 3
20/11*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(11/2) + 112/9*(sqrt(sqrt(sqrt(x) +
 1) + 1) + 1)^(9/2) + 48/7*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(7/2) - 32/5*(
sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7916 vs. 2(120) = 240.

Time = 43.80 (sec) , antiderivative size = 7916, normalized size of antiderivative = 41.66

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = \text{Too large to display}$$

```
[In] integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 16/765765*(7*(6435*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(17/2) - 58344*(sqrt(s
qrt(sqrt(x) + 1) + 1) + 1)^(15/2) + 235620*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1
)^(13/2) - 556920*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(11/2) + 850850*(sqrt(s
qrt(sqrt(x) + 1) + 1) + 1)^(9/2) - 875160*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)
^(7/2) + 612612*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2) - 291720*(sqrt(sqrt
(sqrt(x) + 1) + 1) + 1)^(3/2) + 109395*sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1
))*sgn(70368744177664*(sqrt(sqrt(x) + 1) + 1)^92 - 6473924464345088*(sqrt(s
qrt(x) + 1) + 1)^91 + 291326600895528960*(sqrt(sqrt(x) + 1) + 1)^90 - 85455
80292935516160*(sqrt(sqrt(x) + 1) + 1)^89 + 183728762437276532736*(sqrt(sqr
t(x) + 1) + 1)^88 - 3086556782054646743040*(sqrt(sqrt(x) + 1) + 1)^87 + 421
79809308639429132288*(sqrt(sqrt(x) + 1) + 1)^86 - 481978846822841400164352*
(sqrt(sqrt(x) + 1) + 1)^85 + 4697911198078384159588352*(sqrt(sqrt(x) + 1) +
 1)^84 - 39651330432185076620984320*(sqrt(sqrt(x) + 1) + 1)^83 + 2931836397
16003233721745408*(sqrt(sqrt(x) + 1) + 1)^82 - 1916656336440269370174734336
*(sqrt(sqrt(x) + 1) + 1)^81 + 11160164453620451334571425792*(sqrt(sqrt(x) +
 1) + 1)^80 - 58223902019906429347317153792*(sqrt(sqrt(x) + 1) + 1)^79 + 27
3479024956137655533112918016*(sqrt(sqrt(x) + 1) + 1)^78 - 11609566078829931
55309408616448*(sqrt(sqrt(x) + 1) + 1)^77 + 4467886822469532994953426239488
*(sqrt(sqrt(x) + 1) + 1)^76 - 15624039803063454614788052615168*(sqrt(sqrt(x
) + 1) + 1)^75 + 49728771914087708805425247813632*(sqrt(sqrt(x) + 1) + 1)^7
4 - 144204022361387642459669217148928*(sqrt(sqrt(x) + 1) + 1)^73 + 38109938
4933784007520636056371200*(sqrt(sqrt(x) + 1) + 1)^72 - 9174887252144158139
57123995336704*(sqrt(sqrt(x) + 1) + 1)^71 + 2009521130818998104990097239703
552*(sqrt(sqrt(x) + 1) + 1)^70 - 3994471142582563999654557691936768*(sqrt(sq
rt(x) + 1) + 1)^69 + 7177812996901911023337169833951232*(sqrt(sqrt(x) + 1)
+ 1)^68 - 11588332903437268712897290291904512*(sqrt(sqrt(x) + 1) + 1)^67 +
16646690083818302450699356048719872*(sqrt(sqrt(x) + 1) + 1)^66 - 2093668615
18988043128935803571404 80*(sqrt(sqrt(x) + 1) + 1)^65 + 2238278803888389909
915245434686668 8*(sqrt(sqrt(x) + 1) + 1)^64 - 1905635422748711967745134244659
2000*(sqrt(sqrt(x) + 1) + 1)^63 + 10446792239109173739071175649132544*(sqr
t(sqrt(x) + 1) + 1)^62 + 15117537962174503606803037851484 16*(sqrt(sqrt(x) +
 1) + 1)^61 - 126157040901937139880885371902361 60*(sqrt(sqrt(x) + 1) + 1)^6
0 + 182107690102765240544353331697418 24*(sqrt(sqrt(x) + 1) + 1)^59 - 158886
182399254786350503282218106 88*(sqrt(sqrt(x) + 1) + 1)^58 + 726498029835240 3
```

064896955164393472*(sqrt(sqrt(x) + 1) + 1)^57 + 27171592356346826247012374З
9758336*(sqrt(sqrt(x) + 1) + 1)^56 - 8806173737385529153533018462224384*(sq
rt(sqrt(x) + 1) + 1)^55 + 8704589509518681571761496954765312*(sqrt(sqrt(x)
+ 1) + 1)^54 - 4141051044270206270604188407824384*(sqrt(sqrt(x) + 1) + 1)^5
3 - 944047265435153343329682904317952*(sqrt(sqrt(x) + 1) + 1)^52 + 34414217
59241742702311709805117440*(sqrt(sqrt(x) + 1) + 1)^51 - 2875820730830681791
678590352359424*(sqrt(sqrt(x) + 1) + 1)^50 + 881068299799276284483428560142
336*(sqrt(sqrt(x) + 1) + 1)^49 + 656876670010853235917344051560448*(sqrt(sq
rt(x) + 1) + 1)^48 - 985314730141923394087336160526336*(sqrt(sqrt(x) + 1) +
 1)^47 + 512961170622589184570169885720576*(sqrt(sqrt(x) + 1) + 1)^46 + 178
399963186035530484128698859552*(sqrt(sqrt(x) + 1) + 1)^45 - 2210745729060236
19230346738925568*(sqrt(sqrt(x) + 1) + 1)^44 + 1533206437006286733306258668
91264*(sqrt(sqrt(x) + 1) + 1)^43 - 26652891419311593866038343630848*(sqrt(s
qrt(x) + 1) + 1)^42 - 34964525177019636722858108911616*(sqrt(sqrt(x) + 1) +
 1)^41 + 31682113944794289518835974275072*(sqrt(sqrt(x) + 1) + 1)^40 - 9233
374080713604069270669492224*(sqrt(sqrt(x) + 1) + 1)^39 - 383353858045854843
1139339501568*(sqrt(sqrt(x) + 1) + 1)^38 + 5085184419428714337736452997120*
(sqrt(sqrt(x) + 1) + 1)^37 - 1982823679057600833030660816896*(sqrt(sqrt(x)
+ 1) + 1)^36 - 262480534359793423136287883264*(sqrt(sqrt(x) + 1) + 1)^35 +
711320861924448823343914680320*(sqrt(sqrt(x) + 1) + 1)^34 - 328697875402249
596865599242240*(sqrt(sqrt(x) + 1) + 1)^33 - 16461430004162620889537183744*
(sqrt(sqrt(x) + 1) + 1)^32 + 95428601176521400977360683008*(sqrt(sqrt(x) +
1) + 1)^31 - 41349359848761873167586164736*(sqrt(sqrt(x) + 1) + 1)^30 - 486
3977456557543561269084160*(sqrt(sqrt(x) + 1) + 1)^29 + 11544647980057943904
629669888*(sqrt(sqrt(x) + 1) + 1)^28 - 3333524342970261558455762944*(sqrt(s
qrt(x) + 1) + 1)^27 - 1191572683417725493401812992*(sqrt(sqrt(x) + 1) + 1)^
26 + 1026232209353398029110476800*(sqrt(sqrt(x) + 1) + 1)^25 - 934813837556
79278573023232*(sqrt(sqrt(x) + 1) + 1)^24 - 146062154264152631122427904*(sq
rt(sqrt(x) + 1) + 1)^23 + 51788232298428869952700416*(sqrt(sqrt(x) + 1) + 1
)^22 + 905329857951631397525913б*(sqrt(sqrt(x) + 1) + 1)^21 - 8934958448492
427163846656*(sqrt(sqrt(x) + 1) + 1)^20 + 641389659530470477504512*(sqrt(sq
rt(x) + 1) + 1)^19 + 915449581849135293882368*(sqrt(sqrt(x) + 1) + 1)^18 -
220733028492743248314368*(sqrt(sqrt(x) + 1) + 1)^17 - 562395365146609338639
36*(sqrt(sqrt(x) + 1) + 1)^16 + 27627111260485487730688*(sqrt(sqrt(x) + 1)
+ 1)^15 + 1165421058926413062144*(sqrt(sqrt(x) + 1) + 1)^14 - 2200556830148
466212864*(sqrt(sqrt(x) + 1) + 1)^13 + 144065426934920400768*(sqrt(sqrt(x)
+ 1) + 1)^12 + 121664054013852993024*(sqrt(sqrt(x) + 1) + 1)^11 - 176530742
66194861568*(sqrt(sqrt(x) + 1) + 1)^10 - 4695714500414910464*(sqrt(sqrt(x)
+ 1) + 1)^9 + 1046901206612471360*(sqrt(sqrt(x) + 1) + 1)^8 + 1215281202812
14464*(sqrt(sqrt(x) + 1) + 1)^7 - 39242096244066816*(sqrt(sqrt(x) + 1) + 1)
^6 - 1900915774507008*(sqrt(sqrt(x) + 1) + 1)^5 + 947064578357268*(sqrt(sqr
t(x) + 1) + 1)^4 + 13573235584944*(sqrt(sqrt(x) + 1) + 1)^3 - 1357323558494
4*(sqrt(sqrt(x) + 1) + 1)^2 + 88253338509) + 119*(429*(sqrt(sqrt(sqrt(x) +
1) + 1) + 1)^(15/2) - 3465*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(13/2) + 12285
(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(11/2) - 25025(sqrt(sqrt(sqrt(x) + 1) +

1) + 1)^(9/2) + 32175*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(7/2) - 27027*(sqr
t(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2) + 15015*(sqrt(sqrt(sqrt(x) + 1) + 1) +
1)^(3/2) - 6435*sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1))*sgn(70368744177664*(
sqrt(sqrt(x) + 1) + 1)^92 - 6473924464345088*(sqrt(sqrt(x) + 1) + 1)^91 + 2
91326600895528960*(sqrt(sqrt(x) + 1) + 1)^90 - 8545580292935516160*(sqrt(sq
rt(x) + 1) + 1)^89 + 183728762437276532736*(sqrt(sqrt(x) + 1) + 1)^88 - 308
6556782054646743040*(sqrt(sqrt(x) + 1) + 1)^87 + 42179809308639429132288*(s
qrt(sqrt(x) + 1) + 1)^86 - 481978846822841400164352*(sqrt(sqrt(x) + 1) + 1)
^85 + 4697911198078384159588352*(sqrt(sqrt(x) + 1) + 1)^84 - 39651330432185
076620984320*(sqrt(sqrt(x) + 1) + 1)^83 + 293183639716003233721745408*(sqrt
(sqrt(x) + 1) + 1)^82 - 1916656336440269370174734336*(sqrt(sqrt(x) + 1) + 1
)^81 + 11160164453620451334571425792*(sqrt(sqrt(x) + 1) + 1)^80 - 582239020
19906429347317153792*(sqrt(sqrt(x) + 1) + 1)^79 + 273479024956137655533112
918016*(sqrt(sqrt(x) + 1) + 1)^78 - 1160956607882993155309408616448*(sqrt(sq
rt(x) + 1) + 1)^77 + 4467886822469532994953426239488*(sqrt(sqrt(x) + 1) + 1
)^76 - 15624039803063454614788052615168*(sqrt(sqrt(x) + 1) + 1)^75 + 497287
71914087708805425247813632*(sqrt(sqrt(x) + 1) + 1)^74 - 1442040223613876424
59669217148928*(sqrt(sqrt(x) + 1) + 1)^73 + 381099384933784007520636056371
200*(sqrt(sqrt(x) + 1) + 1)^72 - 917488725214415813957123995336704*(sqrt(sqr
t(x) + 1) + 1)^71 + 2009521130818998104990097239703552*(sqrt(sqrt(x) + 1) +
1)^70 - 3994471142582563999654557691936768*(sqrt(sqrt(x) + 1) + 1)^69 + 71
77812996901911023337169833951232*(sqrt(sqrt(x) + 1) + 1)^68 - 1158833290343
7268712897290291904512*(sqrt(sqrt(x) + 1) + 1)^67 + 166466900838183024506993
56048719872*(sqrt(sqrt(x) + 1) + 1)^66 - 2093668615189880431289358035714048
0*(sqrt(sqrt(x) + 1) + 1)^65 + 22382788038883899099152454346866688*(sqrt(s
qrt(x) + 1) + 1)^64 - 19056354227487119677451342446592000*(sqrt(sqrt(x) + 1
) + 1)^63 + 10446792239109173739071175649132544*(sqrt(sqrt(x) + 1) + 1)^62
+ 15117537962174503606803037851484 16*(sqrt(sqrt(x) + 1) + 1)^61 - 126157040
9019371398808853719023616 0*(sqrt(sqrt(x) + 1) + 1)^60 + 1821076901027652405
4435333169741824*(sqrt(sqrt(x) + 1) + 1)^59 - 15888618239925478635050328221
810688*(sqrt(sqrt(x) + 1) + 1)^58 + 7264980298352403064896955164393472*(sqr
t(sqrt(x) + 1) + 1)^57 + 2717159235634682624701237439758336*(sqrt(sqrt(x) +
1) + 1)^56 - 8806173737385529153533018462224384*(sqrt(sqrt(x) + 1) + 1)^55
+ 8704589509518681571761496954765312*(sqrt(sqrt(x) + 1) + 1)^54 - 41410510
44270206270604188407824384*(sqrt(sqrt(x) + 1) + 1)^53 - 9440472654351533433
29682904317952*(sqrt(sqrt(x) + 1) + 1)^52 + 3441421759241742702311709805117
440*(sqrt(sqrt(x) + 1) + 1)^51 - 2875820730830681791678590352359424*(sqrt(s
qrt(x) + 1) + 1)^50 + 881068299799276284483428560142336*(sqrt(sqrt(x) + 1)
+ 1)^49 + 656876670010853235917344051560448*(sqrt(sqrt(x) + 1) + 1)^48 - 98
5314730141923394087336160526336*(sqrt(sqrt(x) + 1) + 1)^47 + 51296117062258
9184570169885720576*(sqrt(sqrt(x) + 1) + 1)^46 + 178399963186035530484128869
885952*(sqrt(sqrt(x) + 1) + 1)^45 - 221074572906023619230346738925568*(sqrt
(sqrt(x) + 1) + 1)^44 + 153320643700628673330625866891264*(sqrt(sqrt(x) + 1
) + 1)^43 - 26652891419311593866038343630848*(sqrt(sqrt(x) + 1) + 1)^42 - 3
4964525177019636722858108911616*(sqrt(sqrt(x) + 1) + 1)^41 + 31682113944794

289518835974275072*(sqrt(sqrt(x) + 1) + 1)^40 - 92333740807136040692706694922224*(sqrt(sqrt(x) + 1) + 1)^39 - 383353858045854843113933950156 8*(sqrt(sqrt(x) + 1) + 1)^38 + 508518441942871433773645299712 0*(sqrt(sqrt(x) + 1) + 1)^37 - 198282367905760083303066081689 6*(sqrt(sqrt(x) + 1) + 1)^36 - 26248053435979342313628788326 4*(sqrt(sqrt(x) + 1) + 1)^35 + 71132086192444882334391468032 0*(sqrt(sqrt(x) + 1) + 1)^34 - 32869787540224959686559924224 0*(sqrt(sqrt(x) + 1) + 1)^33 - 16461430004162620889537183744*(sqrt(sqrt(x) + 1) + 1)^32 + 95428601176521400977360683008*(sqrt(sqrt(x) + 1) + 1)^31 - 4134935984876187316758616473 6*(sqrt(sqrt(x) + 1) + 1)^30 - 4863977456557543561269084160*(sqrt(sqrt(x) + 1) + 1)^29 + 1154464798005794390462966988 8*(sqrt(sqrt(x) + 1) + 1)^28 - 3333524342970261558455762944*(sqrt(sqrt(x) + 1) + 1)^27 - 1191572683417725493401812992*(sqrt(sqrt(x) + 1) + 1)^26 + 1026232209353398029110476800*(sqrt(sqrt(x) + 1) + 1)^25 - 93481383755679278573023232*(sqrt(sqrt(x) + 1) + 1)^24 - 146062154264152631122427904*(sqrt(sqrt(x) + 1) + 1)^23 + 51788232298428869952700416*(sqrt(sqrt(x) + 1) + 1)^22 + 9053298579516313975259136*(sqrt(sqrt(x) + 1) + 1)^21 - 8934958448492427163846656*(sqrt(sqrt(x) + 1) + 1)^20 + 641389659530470477504512*(sqrt(sqrt(x) + 1) + 1)^19 + 91544958184913529388236 8*(sqrt(sqrt(x) + 1) + 1)^18 - 2207330284927432483143 68*(sqrt(sqrt(x) + 1) + 1)^17 - 56239536514660933863936*(sqrt(sqrt(x) + 1) + 1)^16 + 27627111260485487730688*(sqrt(sqrt(x) + 1) + 1)^15 + 1165421058926413062144*(sqrt(sqrt(x) + 1) + 1)^14 - 2200556830148466212864*(sqrt(sqrt(x) + 1) + 1)^13 + 144065426934920400768*(sqrt(sqrt(x) + 1) + 1)^12 + 121664054013852993024*(sqrt(sqrt(x) + 1) + 1)^11 - 17653074266194861568*(sqrt(sqrt(x) + 1) + 1)^10 - 4695714500414910464*(sqrt(sqrt(x) + 1) + 1)^9 + 1046901206612471360*(sqrt(sqrt(x) + 1) + 1)^8 + 121528120281214464*(sqrt(sqrt(x) + 1) + 1)^7 - 39242096244066816*(sqrt(sqrt(x) + 1) + 1)^6 - 1900915774507008*(sqrt(sqrt(x) + 1) + 1)^5 + 947064578357268*(sqrt(sqrt(x) + 1) + 1)^4 + 13573235584944*(sqrt(sqrt(x) + 1) + 1)^3 - 13573235584944*(sqrt(sqrt(x) + 1) + 1)^2 + 88253338509) - 765*(231*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(13/2) - 1638*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(11/2) + 5005*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(9/2) - 8580*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(7/2) + 9009*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2) - 6006*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(3/2) + 3003*sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1))*sgn(70368744177664*(sqrt(sqrt(x) + 1) + 1)^92 - 6473924464345088*(sqrt(sqrt(x) + 1) + 1)^91 + 29132600895528960*(sqrt(sqrt(x) + 1) + 1)^90 - 854558029293551616 0*(sqrt(sqrt(x) + 1) + 1)^89 + 18372876243727653273 6*(sqrt(sqrt(x) + 1) + 1)^88 - 3086556782054646743040*(sqrt(sqrt(x) + 1) + 1)^87 + 4217980930863942913228 8*(sqrt(sqrt(x) + 1) + 1)^86 - 4819788468228414001643 52*(sqrt(sqrt(x) + 1) + 1)^85 + 4697911198078384159588352*(sqrt(sqrt(x) + 1) + 1)^84 - 39651330432185076620984320*(sqrt(sqrt(x) + 1) + 1)^83 + 293183639716003233721745408*(sqrt(sqrt(x) + 1) + 1)^82 - 1916656336440269370174734336*(sqrt(sqrt(x) + 1) + 1)^81 + 11160164453620451334571425792*(sqrt(sqrt(x) + 1) + 1)^80 - 58223902019906429347317153792*(sqrt(sqrt(x) + 1) + 1)^79 + 2734790249561376555331129 18016*(sqrt(sqrt(x) + 1) + 1)^78 - 1160956607882993155309408616448*(sqrt(sqrt(x) + 1) + 1)^77 + 4467886822469532994953426239488*(sqrt(sqrt(x) + 1) + 1)

^76 - 156240398030634546147880526151 68*(sqrt(sqrt(x) + 1) + 1)^75 + 4972877
191408770880542524781363 2*(sqrt(sqrt(x) + 1) + 1)^74 - 14420402236138764245
9669217148928*(sqrt(sqrt(x) + 1) + 1)^73 + 381099384933784007520636056371 20
0*(sqrt(sqrt(x) + 1) + 1)^72 - 917488725214415813957123995336704*(sqrt(sqrt
(x) + 1) + 1)^71 + 200952113081899810499009723970 3552*(sqrt(sqrt(x) + 1) +
1)^70 - 399447114258256399965455769193 6768*(sqrt(sqrt(x) + 1) + 1)^69 + 717
781299690191102333716983395123 2*(sqrt(sqrt(x) + 1) + 1)^68 - 11588332903437
268712897290291904512*(sqrt(sqrt(x) + 1) + 1)^67 + 166466900838183024506993
56048719872*(sqrt(sqrt(x) + 1) + 1)^66 - 209366861518988043128935803571404 8
0*(sqrt(sqrt(x) + 1) + 1)^65 + 223827880388838990991524543468666 88*(sqrt(sq
rt(x) + 1) + 1)^64 - 190563542274871196774513424465920 00*(sqrt(sqrt(x) + 1)
 + 1)^63 + 104467922391091737390711756491325 44*(sqrt(sqrt(x) + 1) + 1)^62 +
 151175379621745036068030378514841 6*(sqrt(sqrt(x) + 1) + 1)^61 - 1261570409
019371398808853719023616 0*(sqrt(sqrt(x) + 1) + 1)^60 + 18210769010276524054
435333169741824*(sqrt(sqrt(x) + 1) + 1)^59 - 158886182399254786350503282218
10688*(sqrt(sqrt(x) + 1) + 1)^58 + 726498029835240306489695516439347 2*(sqrt
(sqrt(x) + 1) + 1)^57 + 271715923563468262470123743975833 6*(sqrt(sqrt(x) +
1) + 1)^56 - 880617373738552915353301846222438 4*(sqrt(sqrt(x) + 1) + 1)^55
+ 870458950951868157176149695476531 2*(sqrt(sqrt(x) + 1) + 1)^54 - 414105104
427020627060418840782438 4*(sqrt(sqrt(x) + 1) + 1)^53 - 94404726543515334332
9682904317952*(sqrt(sqrt(x) + 1) + 1)^52 + 344142175924174270231170980511 74
40*(sqrt(sqrt(x) + 1) + 1)^51 - 287582073083068179167859035235942 4*(sqrt(sq
rt(x) + 1) + 1)^50 + 881068299799276284483428560142336*(sqrt(sqrt(x) + 1) +
 1)^49 + 656876670010853235917344051560448*(sqrt(sqrt(x) + 1) + 1)^48 - 985
314730141923394087336160526336*(sqrt(sqrt(x) + 1) + 1)^47 + 512961170622589
184570169885720576*(sqrt(sqrt(x) + 1) + 1)^46 + 178399963186035530484128698
85952*(sqrt(sqrt(x) + 1) + 1)^45 - 221074572906023619230346738925568*(sqrt(
sqrt(x) + 1) + 1)^44 + 153320643700628673330625866891264*(sqrt(sqrt(x) + 1)
 + 1)^43 - 266528914193115938660383436308 48*(sqrt(sqrt(x) + 1) + 1)^42 - 34
964525177019636722858108911616*(sqrt(sqrt(x) + 1) + 1)^41 + 316821139447942
89518835974275072*(sqrt(sqrt(x) + 1) + 1)^40 - 923337408071360406927066949 2
224*(sqrt(sqrt(x) + 1) + 1)^39 - 383353858045854843113933950156 8*(sqrt(sqrt
(x) + 1) + 1)^38 + 508518441942871433773645299712 0*(sqrt(sqrt(x) + 1) + 1)^
37 - 198282367905760083303066081689 6*(sqrt(sqrt(x) + 1) + 1)^36 - 262480534
359793423136287883264*(sqrt(sqrt(x) + 1) + 1)^35 + 711320861924448823343914
680320*(sqrt(sqrt(x) + 1) + 1)^34 - 328697875402249596865599242240*(sqrt(sq
rt(x) + 1) + 1)^33 - 164614300041626208895371837 44*(sqrt(sqrt(x) + 1) + 1)^
32 + 954286011765214009773606830 08*(sqrt(sqrt(x) + 1) + 1)^31 - 41349359848
761873167586164736*(sqrt(sqrt(x) + 1) + 1)^30 - 486397745655754356126908416
0*(sqrt(sqrt(x) + 1) + 1)^29 + 115446479800579439046296698 88*(sqrt(sqrt(x)
+ 1) + 1)^28 - 333352434297026155845576294 4*(sqrt(sqrt(x) + 1) + 1)^27 - 11
915726834177254934018129 92*(sqrt(sqrt(x) + 1) + 1)^26 + 1026232209353398029
110476800*(sqrt(sqrt(x) + 1) + 1)^25 - 934813837556792785730232 32*(sqrt(sqr
t(x) + 1) + 1)^24 - 146062154264152631122427904*(sqrt(sqrt(x) + 1) + 1)^23
+ 517882322984288699527004 16*(sqrt(sqrt(x) + 1) + 1)^22 + 90532985795163139

75259136*(sqrt(sqrt(x) + 1) + 1)^21 - 8934958448492427163846656*(sqrt(sqrt(
x) + 1) + 1)^20 + 6413896595304704775045 12*(sqrt(sqrt(x) + 1) + 1)^19 + 915
449581849135293882368*(sqrt(sqrt(x) + 1) + 1)^18 - 2207330284927432483143 68
(sqrt(sqrt(x) + 1) + 1)^17 - 562395365146609338 63936(sqrt(sqrt(x) + 1) +
1)^16 + 276271112604854877306 88*(sqrt(sqrt(x) + 1) + 1)^15 + 11654210589264
13062144*(sqrt(sqrt(x) + 1) + 1)^14 - 220055683014846621 2864*(sqrt(sqrt(x)
+ 1) + 1)^13 + 14406542693492040 0768*(sqrt(sqrt(x) + 1) + 1)^12 + 121664054
013852993024*(sqrt(sqrt(x) + 1) + 1)^11 - 1765307426619486 1568*(sqrt(sqrt(x
) + 1) + 1)^10 - 4695714500414910464*(sqrt(sqrt(x) + 1) + 1)^9 + 1046901206
612471360*(sqrt(sqrt(x) + 1) + 1)^8 + 121528120281214464*(sqrt(sqrt(x) + 1)
+ 1)^7 - 39242096244066816*(sqrt(sqrt(x) + 1) + 1)^6 - 1900915774507008*(s
qrt(sqrt(x) + 1) + 1)^5 + 947064578357268*(sqrt(sqrt(x) + 1) + 1)^4 + 13573
235584944*(sqrt(sqrt(x) + 1) + 1)^3 - 13573235584944*(sqrt(sqrt(x) + 1) + 1
)^2 + 88253338509) - 3315*(63*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(11/2) - 38
5*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(9/2) + 990*(sqrt(sqrt(sqrt(x) + 1) + 1
) + 1)^(7/2) - 1386*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2) + 1155*(sqrt(sq
rt(sqrt(x) + 1) + 1) + 1)^(3/2) - 693*sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)
)*sgn(70368744177664*(sqrt(sqrt(x) + 1) + 1)^92 - 6473924464345088*(sqrt(sq
rt(x) + 1) + 1)^91 + 291326600895528960*(sqrt(sqrt(x) + 1) + 1)^90 - 854558
0292935516160*(sqrt(sqrt(x) + 1) + 1)^89 + 183728762437276532736*(sqrt(sqrt
(x) + 1) + 1)^88 - 3086556782054646743040*(sqrt(sqrt(x) + 1) + 1)^87 + 4217
9809308639429132288*(sqrt(sqrt(x) + 1) + 1)^86 - 481978846822841400164352*(
sqrt(sqrt(x) + 1) + 1)^85 + 4697911198078384159588352*(sqrt(sqrt(x) + 1) +
1)^84 - 39651330432185076620984320*(sqrt(sqrt(x) + 1) + 1)^83 + 29318363971
6003233721745408*(sqrt(sqrt(x) + 1) + 1)^82 - 1916656336440269370174734336*
(sqrt(sqrt(x) + 1) + 1)^81 + 11160164453620451334571425792*(sqrt(sqrt(x) +
1) + 1)^80 - 58223902019906429347317153792*(sqrt(sqrt(x) + 1) + 1)^79 + 273
479024956137655533112918016*(sqrt(sqrt(x) + 1) + 1)^78 - 116095660788299315
5309408616448*(sqrt(sqrt(x) + 1) + 1)^77 + 4467886822469532994953426239488*
(sqrt(sqrt(x) + 1) + 1)^76 - 15624039803063454614788052615168*(sqrt(sqrt(x)
+ 1) + 1)^75 + 49728771914087708805425247813632*(sqrt(sqrt(x) + 1) + 1)^74
- 144204022361387642459669217148928*(sqrt(sqrt(x) + 1) + 1)^73 + 381099384
933784007520636056371200*(sqrt(sqrt(x) + 1) + 1)^72 - 917488725214415813957
123995336704*(sqrt(sqrt(x) + 1) + 1)^71 + 200952113081899810499009723970355
2*(sqrt(sqrt(x) + 1) + 1)^70 - 3994471142582563999654557691936768*(sqrt(sqr
t(x) + 1) + 1)^69 + 7177812996901911023337169833951232*(sqrt(sqrt(x) + 1) +
1)^68 - 11588332903437268712897290291904512*(sqrt(sqrt(x) + 1) + 1)^67 + 1
6646690083818302450699356048719872*(sqrt(sqrt(x) + 1) + 1)^66 - 20936686151
898804312893580357140480*(sqrt(sqrt(x) + 1) + 1)^65 + 22382788038883899099 1
52454346866688*(sqrt(sqrt(x) + 1) + 1)^64 - 1905635422748711967745134244659
2000*(sqrt(sqrt(x) + 1) + 1)^63 + 10446792239109173739071175649132544*(sqrt
(sqrt(x) + 1) + 1)^62 + 1511753796217450360680303785148416*(sqrt(sqrt(x) +
1) + 1)^61 - 12615704090193713988088537190236160*(sqrt(sqrt(x) + 1) + 1)^60
+ 18210769010276524054435333169741824*(sqrt(sqrt(x) + 1) + 1)^59 - 1588861
823992547863505032822181068 8*(sqrt(sqrt(x) + 1) + 1)^58 + 72649802983524030

```
64896955164393472*(sqrt(sqrt(x) + 1) + 1)^57 + 27171592356346826247012374
39758336*(sqrt(sqrt(x) + 1) + 1)^56 - 8806173737385529153533018462224384*(sqr
t(sqrt(x) + 1) + 1)^55 + 8704589509518681571761496954765312*(sqrt(sqrt(x) +
 1) + 1)^54 - 4141051044270206270604188407824384*(sqrt(sqrt(x) + 1) + 1)^53
 - 944047265435153343329682904317952*(sqrt(sqrt(x) + 1) + 1)^52 + 344142175
92417427023117098051174440*(sqrt(sqrt(x) + 1) + 1)^51 - 28758207308306817916
78590352359424*(sqrt(sqrt(x) + 1) + 1)^50 + 881068299799276284483428560142
336*(sqrt(sqrt(x) + 1) + 1)^49 + 6568766700108532359173440515604448*(sqrt(sqr
t(x) + 1) + 1)^48 - 9853147301419233940873361605263336*(sqrt(sqrt(x) + 1) +
1)^47 + 5129611706225891845701698857205576*(sqrt(sqrt(x) + 1) + 1)^46 + 1783
999631860355304841286988595952*(sqrt(sqrt(x) + 1) + 1)^45 - 22107457290602361
9230346738925568*(sqrt(sqrt(x) + 1) + 1)^44 + 15332064370062867333062586689
1264*(sqrt(sqrt(x) + 1) + 1)^43 - 2665289141931159386603834363084 8*(sqrt(sq
rt(x) + 1) + 1)^42 - 349645251770196367228581089116 16*(sqrt(sqrt(x) + 1) +
1)^41 + 316821139447942895188359742750 72*(sqrt(sqrt(x) + 1) + 1)^40 - 92333
740807136040692706694922224*(sqrt(sqrt(x) + 1) + 1)^39 - 383353858045854 8431
139339501568*(sqrt(sqrt(x) + 1) + 1)^38 + 5085184419428714337736452997120*(
sqrt(sqrt(x) + 1) + 1)^37 - 1982823679057600833030660816896*(sqrt(sqrt(x) +
 1) + 1)^36 - 262480534359793423136287883264*(sqrt(sqrt(x) + 1) + 1)^35 + 7
1132086192444882334391468 0320*(sqrt(sqrt(x) + 1) + 1)^34 - 3286978754022495
96865599242240*(sqrt(sqrt(x) + 1) + 1)^33 - 164614300041626208895371837 44*(
sqrt(sqrt(x) + 1) + 1)^32 + 95428601176521400977360683008*(sqrt(sqrt(x) + 1
) + 1)^31 - 41349359848761873167586164736*(sqrt(sqrt(x) + 1) + 1)^30 - 4863
977456557543561269084160*(sqrt(sqrt(x) + 1) + 1)^29 + 115446479800579439046
29669888*(sqrt(sqrt(x) + 1) + 1)^28 - 3333524342970261558455762944*(sqrt(sq
rt(x) + 1) + 1)^27 - 1191572683417725493401812992*(sqrt(sqrt(x) + 1) + 1)^2
6 + 1026232209353398029110476800*(sqrt(sqrt(x) + 1) + 1)^25 - 9348138375567
9278573023232*(sqrt(sqrt(x) + 1) + 1)^24 - 146062154264152631122427904*(sqr
t(sqrt(x) + 1) + 1)^23 + 51788232298428869952700416*(sqrt(sqrt(x) + 1) + 1)
^22 + 9053298579516313975259136*(sqrt(sqrt(x) + 1) + 1)^21 - 89349584484924
27163846656*(sqrt(sqrt(x) + 1) + 1)^20 + 641389659530470477504512*(sqrt(sqr
t(x) + 1) + 1)^19 + 915449581849135293882368*(sqrt(sqrt(x) + 1) + 1)^18 - 2
20733028492743248314368*(sqrt(sqrt(x) + 1) + 1)^17 - 562395365146609338639 3
6*(sqrt(sqrt(x) + 1) + 1)^16 + 27627111260485487730688*(sqrt(sqrt(x) + 1) +
 1)^15 + 1165421058926413062144*(sqrt(sqrt(x) + 1) + 1)^14 - 22005568301484
66212864*(sqrt(sqrt(x) + 1) + 1)^13 + 144065426934920400768*(sqrt(sqrt(x) +
 1) + 1)^12 + 121664054013852993024*(sqrt(sqrt(x) + 1) + 1)^11 - 1765307426
6194861568*(sqrt(sqrt(x) + 1) + 1)^10 - 4695714500414910464*(sqrt(sqrt(x) +
 1) + 1)^9 + 1046901206612471360*(sqrt(sqrt(x) + 1) + 1)^8 + 12152812028121
4464*(sqrt(sqrt(x) + 1) + 1)^7 - 39242096244066816*(sqrt(sqrt(x) + 1) + 1)^
6 - 1900915774507008*(sqrt(sqrt(x) + 1) + 1)^5 + 947064578357268*(sqrt(sqrt
(x) + 1) + 1)^4 + 13573235584944*(sqrt(sqrt(x) + 1) + 1)^3 - 13573235584944
*(sqrt(sqrt(x) + 1) + 1)^2 + 88253338509) + 4862*(35*(sqrt(sqrt(sqrt(x) + 1
) + 1) + 1)^(9/2) - 180*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(7/2) + 378*(sqrt
(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2) - 420*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^
```

(3/2) + 315*sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1))*sgn(70368744177664*(sqrt
(sqrt(x) + 1) + 1)^92 - 6473924464345088*(sqrt(sqrt(x) + 1) + 1)^91 + 29132
6600895528960*(sqrt(sqrt(x) + 1) + 1)^90 - 8545580292935516160*(sqrt(sqrt(x
) + 1) + 1)^89 + 183728762437276532736*(sqrt(sqrt(x) + 1) + 1)^88 - 3086556
782054646743040*(sqrt(sqrt(x) + 1) + 1)^87 + 42179809308639429132288*(sqrt(
sqrt(x) + 1) + 1)^86 - 481978846822841400164352*(sqrt(sqrt(x) + 1) + 1)^85
+ 4697911198078384159588352*(sqrt(sqrt(x) + 1) + 1)^84 - 396513304321850766
20984320*(sqrt(sqrt(x) + 1) + 1)^83 + 293183639716003233721745408*(sqrt(sqr
t(x) + 1) + 1)^82 - 1916656336440269370174734336*(sqrt(sqrt(x) + 1) + 1)^81
 + 11160164453620451334571425792*(sqrt(sqrt(x) + 1) + 1)^80 - 5822390201990
6429347317153792*(sqrt(sqrt(x) + 1) + 1)^79 + 273479024956137655533112918016*(sqrt(sqrt(x) + 1) + 1)^78 - 1160956607882993155309408616448*(sqrt(sqrt(x
) + 1) + 1)^77 + 4467886822469532994953426239488*(sqrt(sqrt(x) + 1) + 1)^76
 - 15624039803063454614788052615168*(sqrt(sqrt(x) + 1) + 1)^75 + 4972877191
4087708805425247813632*(sqrt(sqrt(x) + 1) + 1)^74 - 14420402236138764245966
9217148928*(sqrt(sqrt(x) + 1) + 1)^73 + 381099384933784007520636056371200*(
sqrt(sqrt(x) + 1) + 1)^72 - 917488725214415813957123995336704*(sqrt(sqrt(x)
 + 1) + 1)^71 + 2009521130818998104990097239703552*(sqrt(sqrt(x) + 1) + 1)^
70 - 3994471142582563999654557691936768*(sqrt(sqrt(x) + 1) + 1)^69 + 717781
2996901911023337169833951232*(sqrt(sqrt(x) + 1) + 1)^68 - 1158833290343726
8712897290291904512*(sqrt(sqrt(x) + 1) + 1)^67 + 166466900838183024506993560
48719872*(sqrt(sqrt(x) + 1) + 1)^66 - 20936686151898804312893580357140480*(
sqrt(sqrt(x) + 1) + 1)^65 + 22382788038883899099152454346866688*(sqrt(sqrt(
x) + 1) + 1)^64 - 19056354227487119677451342446592000*(sqrt(sqrt(x) + 1) +
1)^63 + 10446792239109173739071175649132544*(sqrt(sqrt(x) + 1) + 1)^62 + 15
11753796217450360680303785148416*(sqrt(sqrt(x) + 1) + 1)^61 - 1261570409019
3713988088537190236160*(sqrt(sqrt(x) + 1) + 1)^60 + 182107690102765240544435
333169741824*(sqrt(sqrt(x) + 1) + 1)^59 - 158886182399254786350503282218106
88*(sqrt(sqrt(x) + 1) + 1)^58 + 7264980298352403064896955164393472*(sqrt(sq
rt(x) + 1) + 1)^57 + 2717159235634682624701237439758336*(sqrt(sqrt(x) + 1)
+ 1)^56 - 8806173737385529153533018462224384*(sqrt(sqrt(x) + 1) + 1)^55 + 8
704589509518681571761496954765312*(sqrt(sqrt(x) + 1) + 1)^54 - 414105104427
0206270604188407824384*(sqrt(sqrt(x) + 1) + 1)^53 - 94404726543515334332968
2904317952*(sqrt(sqrt(x) + 1) + 1)^52 + 3441421759241742702311709805117440*
(sqrt(sqrt(x) + 1) + 1)^51 - 2875820730830681791678590352359424*(sqrt(sqrt(
x) + 1) + 1)^50 + 881068299799276284483428560142336*(sqrt(sqrt(x) + 1) + 1)
^49 + 656876670010853235917344051560448*(sqrt(sqrt(x) + 1) + 1)^48 - 985314
730141923394087336160526336*(sqrt(sqrt(x) + 1) + 1)^47 + 512961170622589184
570169885720576*(sqrt(sqrt(x) + 1) + 1)^46 + 1783999631860355304841286988859
52*(sqrt(sqrt(x) + 1) + 1)^45 - 221074572906023619230346738925568*(sqrt(sqr
t(x) + 1) + 1)^44 + 153320643700628673330625866891264*(sqrt(sqrt(x) + 1) +
1)^43 - 26652891419311593866038343630848*(sqrt(sqrt(x) + 1) + 1)^42 - 34964
525177019636722858108911616*(sqrt(sqrt(x) + 1) + 1)^41 + 316821139447942895
18835974275072*(sqrt(sqrt(x) + 1) + 1)^40 - 9233374080713604069270669492224
(sqrt(sqrt(x) + 1) + 1)^39 - 3833538580458548431139339501568(sqrt(sqrt(x)

+ 1) + 1)^38 + 508518441942871433773645299712O*(sqrt(sqrt(x) + 1) + 1)^37
- 198282367905760083303066081689 6*(sqrt(sqrt(x) + 1) + 1)^36 - 262480534359
793423136287883264*(sqrt(sqrt(x) + 1) + 1)^35 + 711320861924448823343914680
320*(sqrt(sqrt(x) + 1) + 1)^34 - 328697875402249596865599242240*(sqrt(sqrt(
x) + 1) + 1)^33 - 164614300041626208895371837 44*(sqrt(sqrt(x) + 1) + 1)^32
+ 954286011765214009773606830 08*(sqrt(sqrt(x) + 1) + 1)^31 - 41349359848761
873167586164736*(sqrt(sqrt(x) + 1) + 1)^30 - 486397745655754356126908416 0*(
sqrt(sqrt(x) + 1) + 1)^29 + 115446479800579439046296698 88*(sqrt(sqrt(x) + 1
) + 1)^28 - 333352434297026155845576294 4*(sqrt(sqrt(x) + 1) + 1)^27 - 11915
726834177254934018129 92*(sqrt(sqrt(x) + 1) + 1)^26 + 1026232209353398029110
476800*(sqrt(sqrt(x) + 1) + 1)^25 - 934813837556792785730232 32*(sqrt(sqrt(x
) + 1) + 1)^24 - 146062154264152631122427904*(sqrt(sqrt(x) + 1) + 1)^23 + 5
178823229842886995270041 6*(sqrt(sqrt(x) + 1) + 1)^22 + 905329857951631397 52
59136*(sqrt(sqrt(x) + 1) + 1)^21 - 893495844849242716384665 6*(sqrt(sqrt(x)
+ 1) + 1)^20 + 641389659530470477504512*(sqrt(sqrt(x) + 1) + 1)^19 + 915449
581849135293882368*(sqrt(sqrt(x) + 1) + 1)^18 - 220733028492743248314368*(s
qrt(sqrt(x) + 1) + 1)^17 - 562395365146609338639 36*(sqrt(sqrt(x) + 1) + 1)^
16 + 27627111260485487730688*(sqrt(sqrt(x) + 1) + 1)^15 + 11654210589264130
62144*(sqrt(sqrt(x) + 1) + 1)^14 - 220055683014846621286 4*(sqrt(sqrt(x) + 1
) + 1)^13 + 144065426934920400768*(sqrt(sqrt(x) + 1) + 1)^12 + 121664054013
852993024*(sqrt(sqrt(x) + 1) + 1)^11 - 17653074266194861568*(sqrt(sqrt(x) +
 1) + 1)^10 - 4695714500414910464*(sqrt(sqrt(x) + 1) + 1)^9 + 1046901206612
471360*(sqrt(sqrt(x) + 1) + 1)^8 + 121528120281214464*(sqrt(sqrt(x) + 1) +
1)^7 - 39242096244066816*(sqrt(sqrt(x) + 1) + 1)^6 - 1900915774507008*(sqrt
(sqrt(x) + 1) + 1)^5 + 947064578357268*(sqrt(sqrt(x) + 1) + 1)^4 + 13573235
584944*(sqrt(sqrt(x) + 1) + 1)^3 - 13573235584944*(sqrt(sqrt(x) + 1) + 1)^2
 + 88253338509) + 43758*(5*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(7/2) - 21*(sq
rt(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2) + 35*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)
^(3/2) - 35*sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1))*sgn(70368744177664*(sqrt
(sqrt(x) + 1) + 1)^92 - 6473924464345088*(sqrt(sqrt(x) + 1) + 1)^91 + 29132
6600895528960*(sqrt(sqrt(x) + 1) + 1)^90 - 8545580292935516160*(sqrt(sqrt(x
) + 1) + 1)^89 + 183728762437276532736*(sqrt(sqrt(x) + 1) + 1)^88 - 3086556
782054646743040*(sqrt(sqrt(x) + 1) + 1)^87 + 42179809308639429132288*(sqrt(
sqrt(x) + 1) + 1)^86 - 481978846822841400164352*(sqrt(sqrt(x) + 1) + 1)^85
+ 4697911198078384159588352*(sqrt(sqrt(x) + 1) + 1)^84 - 396513304321850766
20984320*(sqrt(sqrt(x) + 1) + 1)^83 + 293183639716003233721745408*(sqrt(sqr
t(x) + 1) + 1)^82 - 1916656336440269370174734336*(sqrt(sqrt(x) + 1) + 1)^81
 + 11160164453620451334571425792*(sqrt(sqrt(x) + 1) + 1)^80 - 5822390201990
6429347317153792*(sqrt(sqrt(x) + 1) + 1)^79 + 273479024956137655533112918O1
6*(sqrt(sqrt(x) + 1) + 1)^78 - 1160956607882993155309408616448*(sqrt(sqrt(x
) + 1) + 1)^77 + 4467886822469532994953426239488*(sqrt(sqrt(x) + 1) + 1)^76
 - 15624039803063454614788052615168*(sqrt(sqrt(x) + 1) + 1)^75 + 4972877191
4087708805425247813632*(sqrt(sqrt(x) + 1) + 1)^74 - 1442040223613876424596 6
9217148928*(sqrt(sqrt(x) + 1) + 1)^73 + 381099384933784007520636056371200*(
sqrt(sqrt(x) + 1) + 1)^72 - 917488725214415813957123995336704*(sqrt(sqrt(x)

+ 1) + 1)^71 + 2009521130818998104990097239703552*(sqrt(sqrt(x) + 1) + 1)^
70 - 3994471142582563999654557691936768*(sqrt(sqrt(x) + 1) + 1)^69 + 717781
2996901911023337169833951232*(sqrt(sqrt(x) + 1) + 1)^68 - 11588332903437268
712897290291904512*(sqrt(sqrt(x) + 1) + 1)^67 + 1664669008381830245069935560
48719872*(sqrt(sqrt(x) + 1) + 1)^66 - 2093668615189880431289358035714048O*(
sqrt(sqrt(x) + 1) + 1)^65 + 2238278803888389909915245434686688*(sqrt(sqrt(
x) + 1) + 1)^64 - 1905635422748711967745134244659200O*(sqrt(sqrt(x) + 1) +
1)^63 + 1044679223910917373907117564913254 4*(sqrt(sqrt(x) + 1) + 1)^62 + 15
117537962174503606803037851484 16*(sqrt(sqrt(x) + 1) + 1)^61 - 1261570409019
371398808853719023616O*(sqrt(sqrt(x) + 1) + 1)^60 + 1821076901027652405443 5
333169741824*(sqrt(sqrt(x) + 1) + 1)^59 - 1588861823992547863505032822181 06
88*(sqrt(sqrt(x) + 1) + 1)^58 + 7264980298352403064896955164393472*(sqrt(sq
rt(x) + 1) + 1)^57 + 2717159235634682624701237439758336*(sqrt(sqrt(x) + 1)
+ 1)^56 - 8806173737385529153533018462224384*(sqrt(sqrt(x) + 1) + 1)^55 + 8
704589509518681571761496954765312*(sqrt(sqrt(x) + 1) + 1)^54 - 414105104427
020627060418840782438 4*(sqrt(sqrt(x) + 1) + 1)^53 - 9440472654351533433296 8
2904317952*(sqrt(sqrt(x) + 1) + 1)^52 + 3441421759241742702311709805117440*
(sqrt(sqrt(x) + 1) + 1)^51 - 2875820730830681791678590352359424*(sqrt(sqrt(
x) + 1) + 1)^50 + 881068299799276284483428560142336*(sqrt(sqrt(x) + 1) + 1)
^49 + 656876670010853235917344051560448*(sqrt(sqrt(x) + 1) + 1)^48 - 985314
730141923394087336160526336*(sqrt(sqrt(x) + 1) + 1)^47 + 512961170622589184
570169885720576*(sqrt(sqrt(x) + 1) + 1)^46 + 1783996318603553048412869885 9
52*(sqrt(sqrt(x) + 1) + 1)^45 - 221074572906023619230346738925568*(sqrt(sqr
t(x) + 1) + 1)^44 + 153320643700628673330625866891264*(sqrt(sqrt(x) + 1) +
1)^43 - 26652891419311593866038343630848*(sqrt(sqrt(x) + 1) + 1)^42 - 34964
525177019636722858108911616*(sqrt(sqrt(x) + 1) + 1)^41 + 316821139447942895
18835974275072*(sqrt(sqrt(x) + 1) + 1)^40 - 923337408071360406927066949222 4
(sqrt(sqrt(x) + 1) + 1)^39 - 3833538580458548431139339501568(sqrt(sqrt(x)
 + 1) + 1)^38 + 5085184419428714337736452997120*(sqrt(sqrt(x) + 1) + 1)^37
- 1982823679057600833030660816896*(sqrt(sqrt(x) + 1) + 1)^36 - 262480534359
793423136287883264*(sqrt(sqrt(x) + 1) + 1)^35 + 7113208619244488233439146 80
320*(sqrt(sqrt(x) + 1) + 1)^34 - 328697875402249596865599242240*(sqrt(sqrt(
x) + 1) + 1)^33 - 16461430004162620889537183744*(sqrt(sqrt(x) + 1) + 1)^32
+ 95428601176521400977360683008*(sqrt(sqrt(x) + 1) + 1)^31 - 41349359848761
873167586164736*(sqrt(sqrt(x) + 1) + 1)^30 - 48639774565575435612690841 60*(
sqrt(sqrt(x) + 1) + 1)^29 + 11544647980057943904629669888*(sqrt(sqrt(x) + 1
) + 1)^28 - 3333524342970261558455762944*(sqrt(sqrt(x) + 1) + 1)^27 - 11915
72683417725493401812992*(sqrt(sqrt(x) + 1) + 1)^26 + 102623220935339802911 0
476800*(sqrt(sqrt(x) + 1) + 1)^25 - 934813837556792785730232 32*(sqrt(sqrt(x
) + 1) + 1)^24 - 146062154264152631122427904*(sqrt(sqrt(x) + 1) + 1)^23 + 5
1788232298428869952700416*(sqrt(sqrt(x) + 1) + 1)^22 + 905329857951631397 52
59136*(sqrt(sqrt(x) + 1) + 1)^21 - 893495844849242716384665 6*(sqrt(sqrt(x)
+ 1) + 1)^20 + 641389659530470477504512*(sqrt(sqrt(x) + 1) + 1)^19 + 915449
581849135293882368*(sqrt(sqrt(x) + 1) + 1)^18 - 220733028492743248314368*(s
qrt(sqrt(x) + 1) + 1)^17 - 56239536514660933863936*(sqrt(sqrt(x) + 1) + 1)^

```
16 + 276271112604854877306888*(sqrt(sqrt(x) + 1) + 1)^15 + 11654210589264130
62144*(sqrt(sqrt(x) + 1) + 1)^14 - 2200556830148466212864*(sqrt(sqrt(x) + 1
) + 1)^13 + 1440654269349204007688*(sqrt(sqrt(x) + 1) + 1)^12 + 121664054013
852993024*(sqrt(sqrt(x) + 1) + 1)^11 - 176530742661948615688*(sqrt(sqrt(x) +
 1) + 1)^10 - 46957145004149104644*(sqrt(sqrt(x) + 1) + 1)^9 + 1046901206612
471360*(sqrt(sqrt(x) + 1) + 1)^8 + 121528120281214464*(sqrt(sqrt(x) + 1) +
1)^7 - 39242096244066816*(sqrt(sqrt(x) + 1) + 1)^6 - 1900915774507008*(sqrt
(sqrt(x) + 1) + 1)^5 + 947064578357268*(sqrt(sqrt(x) + 1) + 1)^4 + 13573235
584944*(sqrt(sqrt(x) + 1) + 1)^3 - 13573235584944*(sqrt(sqrt(x) + 1) + 1)^2
 + 88253338509))*sgn(70368744177664*x^23 - 1213860837064704*x^22 + 96273238
12806656*x^21 - 46162995692175360*x^20 + 147222133058043904*x^19 - 31873749
4495461376*x^18 + 433452446019223552*x^17 - 184515450053328896*x^16 - 72952
2204785508352*x^15 + 2240383466879320064*x^14 - 3757915392534642688*x^13 +
4526529501076652032*x^12 - 4210103957592211456*x^11 + 3112008713686220800*x
^10 - 1850368000801112064*x^9 + 887763592420786176*x^8 - 342554000588423168
*x^7 + 105330954055192576*x^6 - 25400752095938560*x^5 + 4684945271584256*x^
4 - 635434110546048*x^3 + 59403617953344*x^2 - 3393308896236*x + 8825333850
9)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = \int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\,dx$$

```
[In] int((((x^(1/2) + 1)^(1/2) + 1)^(1/2) + 1)^(1/2),x)
[Out] int((((x^(1/2) + 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)
```

3.718 $\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 233

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = -\frac{16}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{3/2} + \frac{136}{5}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{5/2} - \frac{480}{7}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{7/2} + \frac{304}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{9/2} - \frac{760}{11}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{11/2} + \frac{300}{13}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{13/}$$

```
[Out] -16/3*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(3/2)+136/5*(2+(3+(-1+2*x^(1/2))^(
1/2))^(1/2))^(5/2)-480/7*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(7/2)+304/3*(2+
(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(9/2)-760/11*(2+(3+(-1+2*x^(1/2))^(1/2))^(1
/2))^(11/2)+300/13*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(13/2)-56/15*(2+(3+(-
1+2*x^(1/2))^(1/2))^(1/2))^(15/2)+4/17*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(
17/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1634}

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = \frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{17/2} - \frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{15/2} + \frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{13/2} - \frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{11/2} + \frac{304}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{9/2} - \frac{480}{7}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{7/2} + \frac{13}{5}$$

```
[In] Int[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
[Out] (-16*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(3/2))/3 + (136*(2 + Sqrt[3 + Sqr
t[-1 + 2*Sqrt[x]]])^(5/2))/5 - (480*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(7
/2))/7 + (304*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(9/2))/3 - (760*(2 + Sqr
t[3 + Sqrt[-1 + 2*Sqrt[x]]])^(11/2))/11 + (300*(2 + Sqrt[3 + Sqrt[-1 + 2*Sq
rt[x]]])^(13/2))/13 - (56*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(15/2))/15 +
 (4*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(17/2))/17
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x\sqrt{2+\sqrt{3+\sqrt{-1+2x}}}\,dx, x, \sqrt{x}\right) \\
&= \text{Subst}\left(\int x(1+x^2)\sqrt{2+\sqrt{3+x}}\,dx, x, \sqrt{-1+2\sqrt{x}}\right) \\
&= 2\text{Subst}\left(\int x\sqrt{2+x}(-3+x^2)\left(1+(-3+x^2)^2\right)\,dx, x, \sqrt{3+\sqrt{-1+2\sqrt{x}}}\right) \\
&= 2\text{Subst}\Bigg(\int \Big(-4\sqrt{2+x}+34(2+x)^{3/2} \\
&\quad -120(2+x)^{5/2}+228(2+x)^{7/2}-190(2+x)^{9/2}+75(2+x)^{11/2}-14(2+x)^{13/2}+(2+x)^{15/2}\Big)\,dx, x, \sqrt{3
\end{aligned}$$

$$= -\frac{16}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{3/2} + \frac{136}{5}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{5/2} - \frac{480}{7}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{7/2} + \frac{304}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{9/2} - \frac{760}{11}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{11/2} + \frac{300}{13}\left(2+\sqrt{3+\sqrt{-1+2\right.$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.80

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = \frac{8\sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\left(8\left(-15510-7428\sqrt{3+\sqrt{-1+2\sqrt{x}}}+211\sqrt{-1+2\sqrt{x}}+1700\sqrt{3+\sqrt{-1}\right.\right.}{}$$

```
[In] Integrate[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

```
[Out] (8*Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]]*(8*(-15510 - 7428*Sqrt[3 + Sqrt
[-1 + 2*Sqrt[x]]] + 211*Sqrt[-1 + 2*Sqrt[x]] + 1700*Sqrt[3 + Sqrt[-1 + 2*Sq
rt[x]]]*Sqrt[-1 + 2*Sqrt[x]]) + 7*(-549 - 672*Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]
] - 121*Sqrt[-1 + 2*Sqrt[x]] + 286*Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]*Sqrt[-1 +
 2*Sqrt[x]])*Sqrt[x] + 30030*x))/255255
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.66

method	result
derivativedivides	$-\frac{16\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{3}{2}}}{3}+\frac{136\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{5}{2}}}{5}-\frac{480\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{7}{2}}}{7}+\frac{304\left(2+\sqrt{3+\sqrt{-1}\right.}{3}$
default	$-\frac{16\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{3}{2}}}{3}+\frac{136\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{5}{2}}}{5}-\frac{480\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{7}{2}}}{7}+\frac{304\left(2+\sqrt{3+\sqrt{-1}\right.}{3}$

```
[In] int((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/3*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(3/2)+136/5*(2+(3+(-1+2*x^(1/2))^(
1/2))^(1/2))^(5/2)-480/7*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(7/2)+304/3*(2+
(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(9/2)-760/11*(2+(3+(-1+2*x^(1/2))^(1/2))^(1
/2))^(11/2)+300/13*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(13/2)-56/15*(2+(3+(-
```

```
1+2*x^(1/2))^(1/2))^(1/2))^(15/2)+4/17*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(
17/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.36

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx =$$

$$-\frac{8}{255255}\left(\left(847\sqrt{x}-1688\right)\sqrt{2\sqrt{x}-1}-2\left(\left(1001\sqrt{x}+6800\right)\sqrt{2\sqrt{x}-1}-2352\sqrt{x}-29712\right)\sqrt{\sqrt{2}\right.$$

```
[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -8/255255*((847*sqrt(x) - 1688)*sqrt(2*sqrt(x) - 1) - 2*((1001*sqrt(x) + 68
00)*sqrt(2*sqrt(x) - 1) - 2352*sqrt(x) - 29712)*sqrt(sqrt(2*sqrt(x) - 1) +
3) - 30030*x + 3843*sqrt(x) + 124080)*sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3) +
2)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.87

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = \frac{4\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}}}{17} - \frac{56\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}}}{15} + \frac{300\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{13}{2}}}{13} - \frac{760\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{11}{2}}}{11} + \frac{304\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{9}{2}}}{3} - \frac{480\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{7}{2}}}{7} + \frac{136\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{5}{2}}}{5} - \frac{16\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{3}{2}}}{3}$$

```
[In] integrate((2+(3+(-1+2*x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

```
[Out] 4*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)**(17/2)/17 - 56*(sqrt(sqrt(2*sqrt(x)
- 1) + 3) + 2)**(15/2)/15 + 300*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)**(13/2)
/13 - 760*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)**(11/2)/11 + 304*(sqrt(sqrt(2
*sqrt(x) - 1) + 3) + 2)**(9/2)/3 - 480*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)*
*(7/2)/7 + 136*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)**(5/2)/5 - 16*(sqrt(sqrt
(2*sqrt(x) - 1) + 3) + 2)**(3/2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.66

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = \frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}} - \frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}} + \frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{13}{2}} - \frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{11}{2}} + \frac{304}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{9}{2}} - \frac{480}{7}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{7}{2}} + \frac{136}{5}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{5}{2}} - \frac{16}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{3}{2}}$$

```
[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/17*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(17/2) - 56/15*(sqrt(sqrt(2*sqrt(x
) - 1) + 3) + 2)^(15/2) + 300/13*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(13/2)
 - 760/11*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(11/2) + 304/3*(sqrt(sqrt(2*s
qrt(x) - 1) + 3) + 2)^(9/2) - 480/7*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(7/
2) + 136/5*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(5/2) - 16/3*(sqrt(sqrt(2*sq
rt(x) - 1) + 3) + 2)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 4.88 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.16

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx$$
$$= \frac{4}{255255}\left(15015\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}} - 238238\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}} + 1472625\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{13}{2}} \ldots\right.$$
$$+ 376832\,x^{22} + 8224768\,x^{21} + 113971200\,x^{20} + 1130782720\,x^{19} + 8582063104\,x^{18}$$
$$+ 51933387264\,x^{17} + 257575619584\,x^{16} + 1066188686592\,x^{15} + 3723204389632\,x^{14}$$
$$+ 11019822890016\,x^{13} + 27631512444352\,x^{12} + 58424530490176\,x^{11}$$
$$+ 103336828749760\,x^{10} + 151203890043312\,x^{9} + 180411181747936\,x^{8}$$
$$+ 172287199292960\,x^{7} + 128457231939048\,x^{6} + 72257964298210\,x^{5} + 29175203228012\,x^{4}$$
$$\left. + 7830371130072\,x^{3} + 1228114804752\,x^{2} + 87490886400\,x + 933120000\right)$$

```
[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/255255*(15015*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(17/2) - 238238*(sqrt(s
qrt(2*sqrt(x) - 1) + 3) + 2)^(15/2) + 1472625*(sqrt(sqrt(2*sqrt(x) - 1) + 3
) + 2)^(13/2) - 4408950*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(11/2) + 646646
0*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(9/2) - 4375800*(sqrt(sqrt(2*sqrt(x)
- 1) + 3) + 2)^(7/2) + 1735734*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(5/2) -
340340*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(3/2))*sgn(8192*x^23 + 376832*x^
22 + 8224768*x^21 + 113971200*x^20 + 1130782720*x^19 + 8582063104*x^18 + 51
933387264*x^17 + 257575619584*x^16 + 1066188686592*x^15 + 3723204389632*x^1
4 + 11019822890016*x^13 + 27631512444352*x^12 + 58424530490176*x^11 + 10333
6828749760*x^10 + 151203890043312*x^9 + 180411181747936*x^8 + 1722871992929
60*x^7 + 128457231939048*x^6 + 72257964298210*x^5 + 29175203228012*x^4 + 78
30371130072*x^3 + 1228114804752*x^2 + 87490886400*x + 933120000)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = \int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\,dx$$

```
[In] int((((2*x^(1/2) - 1)^(1/2) + 3)^(1/2) + 2)^(1/2),x)
```

```
[Out] int((((2*x^(1/2) - 1)^(1/2) + 3)^(1/2) + 2)^(1/2), x)
```

3.719 $\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 160

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}} \, dx = \frac{16}{5}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{5/2} - \frac{24}{7}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{7/2} + 8\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{9/2} - \frac{160}{11}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{11/2} + \frac{144}{13}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{13/2} - \frac{56}{15}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{15/2} + \frac{8}{17}\left(1+\right.$$

```
[Out] 16/5*(1+(1+(-1+x)^(1/2))^(1/2))^(5/2)-24/7*(1+(1+(-1+x)^(1/2))^(1/2))^(7/2)
+8*(1+(1+(-1+x)^(1/2))^(1/2))^(9/2)-160/11*(1+(1+(-1+x)^(1/2))^(1/2))^(11/2
)+144/13*(1+(1+(-1+x)^(1/2))^(1/2))^(13/2)-56/15*(1+(1+(-1+x)^(1/2))^(1/2))
^(15/2)+8/17*(1+(1+(-1+x)^(1/2))^(1/2))^(17/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1632, 1634}

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}} \, dx = \frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{17/2} - \frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{15/2} + \frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{13/2} - \frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{11/2} + 8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{9/2} - \frac{24}{7}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{7/2} + \frac{16}{5}\left(\sqrt{\sqrt{x-1}+\right.$$

```
[In] Int[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

```
[Out] (16*(1 + Sqrt[1 + Sqrt[-1 + x]])^(5/2))/5 - (24*(1 + Sqrt[1 + Sqrt[-1 + x]]
)^(7/2))/7 + 8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(9/2) - (160*(1 + Sqrt[1 + Sqrt
[-1 + x]])^(11/2))/11 + (144*(1 + Sqrt[1 + Sqrt[-1 + x]])^(13/2))/13 - (56*
(1 + Sqrt[1 + Sqrt[-1 + x]])^(15/2))/15 + (8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(
17/2))/17
```

Rule 1632

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*x)^n, x]
 /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && EqQ[PolynomialRemainder
[Px, a + b*x, x], 0]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x(1+x^2)\sqrt{1+\sqrt{1+x}}\,dx, x, \sqrt{-1+x}\right)\\
&= 4\text{Subst}\left(\int x\sqrt{1+x}(-1+x^2)\left(1+(-1+x^2)^2\right)\,dx, x, \sqrt{1+\sqrt{-1+x}}\right)\\
&= 4\text{Subst}\left(\int x(1+x)^{3/2}\left(-2+2x+2x^2-2x^3-x^4+x^5\right)\,dx, x, \sqrt{1+\sqrt{-1+x}}\right)\\
&= 4\text{Subst}\left(\int \left(2(1+x)^{3/2}\right.\right.\\
&\quad \left.\left.-3(1+x)^{5/2}+9(1+x)^{7/2}-20(1+x)^{9/2}+18(1+x)^{11/2}-7(1+x)^{13/2}+(1+x)^{15/2}\right)\,dx, x, \sqrt{1+\sqrt{-1+x}}\right)\\
&= \frac{16}{5}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{5/2}-\frac{24}{7}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{7/2}+8\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{9/2}\\
&\quad -\frac{160}{11}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{11/2}+\frac{144}{13}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{13/2}-\frac{56}{15}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{15/2}+\frac{8}{17}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{17/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.59

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx$$
$$= \frac{8\sqrt{1+\sqrt{1+\sqrt{-1+x}}}\left(-8872+1109\sqrt{-1+x}+28231(-1+x)+77(-1+x)^{3/2}+15015(-1+x)^2+\right.}{255255}$$

```
[In] Integrate[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

```
[Out] (8*Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*(-8872 + 1109*Sqrt[-1 + x] + 28231*(-1
+ x) + 77*(-1 + x)^(3/2) + 15015*(-1 + x)^2 + Sqrt[1 + Sqrt[-1 + x]]*(-7696
 + 4544*Sqrt[-1 + x] + 7*(-168 + 143*Sqrt[-1 + x])*x)))/255255
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{16\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{5}{2}}}{5}-\frac{24\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{7}{2}}}{7}+8\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{9}{2}}-\frac{160\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{11}{2}}}{11}+$
default	$\frac{16\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{5}{2}}}{5}-\frac{24\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{7}{2}}}{7}+8\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{9}{2}}-\frac{160\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{11}{2}}}{11}+$

```
[In] int(x*(1+(1+(x-1)^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 16/5*(1+(1+(x-1)^(1/2))^(1/2))^(5/2)-24/7*(1+(1+(x-1)^(1/2))^(1/2))^(7/2)+8
*(1+(1+(x-1)^(1/2))^(1/2))^(9/2)-160/11*(1+(1+(x-1)^(1/2))^(1/2))^(11/2)+14
4/13*(1+(1+(x-1)^(1/2))^(1/2))^(13/2)-56/15*(1+(1+(x-1)^(1/2))^(1/2))^(15/2
)+8/17*(1+(1+(x-1)^(1/2))^(1/2))^(17/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.39

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx$$
$$= \frac{8}{255255}\left(15015\,x^2+(77\,x+1032)\sqrt{x-1}+\left((1001\,x+4544)\sqrt{x-1}-1176\,x-7696\right)\sqrt{\sqrt{x-1}+1}-\right.$$

```
[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/255255*(15015*x^2 + (77*x + 1032)*sqrt(x - 1) + ((1001*x + 4544)*sqrt(x -
 1) - 1176*x - 7696)*sqrt(sqrt(x - 1) + 1) - 1799*x - 22088)*sqrt(sqrt(sqrt
(x - 1) + 1) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx = \frac{8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{17}{2}}}{17} - \frac{56\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{15}{2}}}{15} + \frac{144\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{13}{2}}}{13} - \frac{160\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{11}{2}}}{11} + 8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{9}{2}} - \frac{24\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{7}{2}}}{7} + \frac{16\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{5}{2}}}{5}$$

```
[In] integrate(x*(1+(1+(-1+x)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] 8*(sqrt(sqrt(x - 1) + 1) + 1)**(17/2)/17 - 56*(sqrt(sqrt(x - 1) + 1) + 1)**
(15/2)/15 + 144*(sqrt(sqrt(x - 1) + 1) + 1)**(13/2)/13 - 160*(sqrt(sqrt(x -
 1) + 1) + 1)**(11/2)/11 + 8*(sqrt(sqrt(x - 1) + 1) + 1)**(9/2) - 24*(sqrt(
sqrt(x - 1) + 1) + 1)**(7/2)/7 + 16*(sqrt(sqrt(x - 1) + 1) + 1)**(5/2)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.66

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx = \frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{17}{2}} - \frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{15}{2}} + \frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{13}{2}} - \frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{11}{2}} + 8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{9}{2}} - \frac{24}{7}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{7}{2}} + \frac{16}{5}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{5}{2}}$$

```
[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/17*(sqrt(sqrt(x - 1) + 1) + 1)^(17/2) - 56/15*(sqrt(sqrt(x - 1) + 1) + 1)
^(15/2) + 144/13*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 160/11*(sqrt(sqrt(x -
 1) + 1) + 1)^(11/2) + 8*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) - 24/7*(sqrt(sqr
t(x - 1) + 1) + 1)^(7/2) + 16/5*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(106) = 212$.

Time = 0.43 (sec) , antiderivative size = 859, normalized size of antiderivative = 5.37

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx = \text{Too large to display}$$

```
[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/765765*(7*(6435*(sqrt(sqrt(x - 1) + 1) + 1)^(17/2) - 58344*(sqrt(sqrt(x -
 1) + 1) + 1)^(15/2) + 235620*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 556920*(
sqrt(sqrt(x - 1) + 1) + 1)^(11/2) + 850850*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2
) - 875160*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) + 612612*(sqrt(sqrt(x - 1) + 1
) + 1)^(5/2) - 291720*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 109395*sqrt(sqrt(
sqrt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) + 119
*(429*(sqrt(sqrt(x - 1) + 1) + 1)^(15/2) - 3465*(sqrt(sqrt(x - 1) + 1) + 1)
^(13/2) + 12285*(sqrt(sqrt(x - 1) + 1) + 1)^(11/2) - 25025*(sqrt(sqrt(x - 1
) + 1) + 1)^(9/2) + 32175*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) - 27027*(sqrt(s
qrt(x - 1) + 1) + 1)^(5/2) + 15015*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) - 6435
*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1)
 - 7) - 765*(231*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 1638*(sqrt(sqrt(x - 1
) + 1) + 1)^(11/2) + 5005*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) - 8580*(sqrt(sq
rt(x - 1) + 1) + 1)^(7/2) + 9009*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2) - 6006*(
sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 3003*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*sg
n(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) - 3315*(63*(sqrt(sqrt(x - 1) +
 1) + 1)^(11/2) - 385*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) + 990*(sqrt(sqrt(x
- 1) + 1) + 1)^(7/2) - 1386*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2) + 1155*(sqrt(
sqrt(x - 1) + 1) + 1)^(3/2) - 693*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*sgn(4*(s
qrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) + 9724*(35*(sqrt(sqrt(x - 1) + 1) +
1)^(9/2) - 180*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) + 378*(sqrt(sqrt(x - 1) +
1) + 1)^(5/2) - 420*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 315*sqrt(sqrt(sqrt(
x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) + 87516*(5
*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) - 21*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2) +
 35*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) - 35*sqrt(sqrt(sqrt(x - 1) + 1) + 1))
*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) - 102102*(3*(sqrt(sqrt(x -
1) + 1) + 1)^(5/2) - 10*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 15*sqrt(sqrt(sq
rt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) - 51051
```

```
0*((sqrt(sqrt(x - 1) + 1) + 1)^(3/2) - 3*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*s
gn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7))*sgn(4*x - 7)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx = \int x\sqrt{\sqrt{\sqrt{x-1}+1}+1}\,dx$$

```
[In] int(x*(((x - 1)^(1/2) + 1)^(1/2) + 1)^(1/2),x)
[Out] int(x*(((x - 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)
```

3.720 $\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 20

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = -2\operatorname{arcsinh}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)$$

```
[Out] -2*arcsinh(1/3*(1-2*(-1+x)^(1/2))*3^(1/2))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {633, 221}

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = -2\operatorname{arcsinh}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

```
[In] Int[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

```
[Out] -2*ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{\sqrt{1-x+x^2}}\,dx, x, \sqrt{-1+x}\right)\\ &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, -1+2\sqrt{-1+x}\right)}{\sqrt{3}}\\ &= -2\sinh^{-1}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = -2\log\left(1-2\sqrt{-1+x}+2\sqrt{-\sqrt{-1+x}+x}\right)$$

```
[In] Integrate[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
[Out] -2*Log[1 - 2*Sqrt[-1 + x] + 2*Sqrt[-Sqrt[-1 + x] + x]]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$2\ \text{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$	16
default	$2\ \text{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$	16

```
[In] int(1/(x-1)^(1/2)/(x-(x-1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arcsinh(2/3*3^(1/2)*((x-1)^(1/2)-1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = \log\left(4\sqrt{x-\sqrt{x-1}}(2\sqrt{x-1}-1)+8x-8\sqrt{x-1}-3\right)$$

```
[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqrt(x - 1) - 3)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = 2\,\mathrm{asinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$$

```
[In] integrate(1/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)
[Out] 2*asinh(2*sqrt(3)*(sqrt(x - 1) - 1/2)/3)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = \int \frac{1}{\sqrt{x-\sqrt{x-1}}\sqrt{x-1}}\,dx$$

```
[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = -2\,\log\left(2\sqrt{x-\sqrt{x-1}}-2\sqrt{x-1}+1\right)$$

```
[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="giac")
[Out] -2*log(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = \int \frac{1}{\sqrt{x-\sqrt{x-1}}\,\sqrt{x-1}}\,dx$$

```
[In] int(1/((x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)),x)
[Out] int(1/((x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)), x)
```

3.721 $\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}} dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}} dx = 2\sqrt{1+x+\sqrt{-1+2x}} - \sqrt{2}\text{arcsinh}\left(\frac{1+\sqrt{-1+2x}}{\sqrt{2}}\right)$$

```
[Out] -arcsinh(1/2*(1+(-1+2*x)^(1/2))*2^(1/2))*2^(1/2)+2*(1+x+(-1+2*x)^(1/2))^(1/
2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {654, 633, 221}

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}} dx = \sqrt{2}\sqrt{2x+2\sqrt{2x-1}+2} - \sqrt{2}\text{arcsinh}\left(\frac{\sqrt{2x-1}+1}{\sqrt{2}}\right)$$

```
[In] Int[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]
```

```
[Out] Sqrt[2]*Sqrt[2 + 2*x + 2*Sqrt[-1 + 2*x]] - Sqrt[2]*ArcSinh[(1 + Sqrt[-1 + 2
*x])/Sqrt[2]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x}{\sqrt{\frac{3}{2}+x+\frac{x^2}{2}}}\,dx, x, \sqrt{-1+2x}\right)\\
&= \sqrt{2}\sqrt{2+2x+2\sqrt{-1+2x}} - \text{Subst}\left(\int \frac{1}{\sqrt{\frac{3}{2}+x+\frac{x^2}{2}}}\,dx, x, \sqrt{-1+2x}\right)\\
&= \sqrt{2}\sqrt{2+2x+2\sqrt{-1+2x}} - \text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}}\,dx, x, 1+\sqrt{-1+2x}\right)\\
&= \sqrt{2}\sqrt{2+2x+2\sqrt{-1+2x}} - \sqrt{2}\sinh^{-1}\left(\frac{1+\sqrt{-1+2x}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = 2\sqrt{1+x+\sqrt{-1+2x}} + \sqrt{2}\log\left(-1-\sqrt{-1+2x}+\sqrt{2+2x+2\sqrt{-1+2x}}\right)$$

```
[In] Integrate[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]
[Out] 2*Sqrt[1 + x + Sqrt[-1 + 2*x]] + Sqrt[2]*Log[-1 - Sqrt[-1 + 2*x] + Sqrt[2 +
 2*x + 2*Sqrt[-1 + 2*x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\sqrt{4x+4+4\sqrt{2x-1}}-\operatorname{arcsinh}\left(\frac{(1+\sqrt{2x-1})\sqrt{2}}{2}\right)\sqrt{2}$	38
default	$\sqrt{4x+4+4\sqrt{2x-1}}-\operatorname{arcsinh}\left(\frac{(1+\sqrt{2x-1})\sqrt{2}}{2}\right)\sqrt{2}$	38

```
[In] int(1/(1+x+(2*x-1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] (4*x+4+4*(2*x-1)^(1/2))^(1/2)-arcsinh(1/2*(1+(2*x-1)^(1/2))*2^(1/2))*2^(1/2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(35) = 70.

Time = 0.65 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.93

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = \frac{1}{4}\sqrt{2}\log\Big(-8x^2-8(2x+1)\sqrt{2x-1} +2\left(\sqrt{2}(2x+3)\sqrt{2x-1}+\sqrt{2}(6x-1)\right)\sqrt{x+\sqrt{2x-1}+1} -24x+7\Big)+2\sqrt{x+\sqrt{2x-1}+1}$$

```
[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log(-8*x^2 - 8*(2*x + 1)*sqrt(2*x - 1) + 2*(sqrt(2)*(2*x + 3)*s
qrt(2*x - 1) + sqrt(2)*(6*x - 1))*sqrt(x + sqrt(2*x - 1) + 1) - 24*x + 7) +
 2*sqrt(x + sqrt(2*x - 1) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = 2\sqrt{x+\sqrt{2x-1}+1}-\sqrt{2}\operatorname{asinh}\left(\frac{\sqrt{2}\left(\sqrt{2x-1}+1\right)}{2}\right)$$

```
[In] integrate(1/(1+x+(-1+2*x)**(1/2))**(1/2),x)
[Out] 2*sqrt(x + sqrt(2*x - 1) + 1) - sqrt(2)*asinh(sqrt(2)*(sqrt(2*x - 1) + 1)/2
)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = \int \frac{1}{\sqrt{x+\sqrt{2\,x-1}+1}}\,dx$$

```
[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x + sqrt(2*x - 1) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = \sqrt{2}\left(\sqrt{2\,x+2\,\sqrt{2\,x-1}+2}+\log\left(\sqrt{2\,x+2\,\sqrt{2\,x-1}+2}-\sqrt{2\,x-1}-1\right)\right)$$

```
[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
[Out] sqrt(2)*(sqrt(2*x + 2*sqrt(2*x - 1) + 2) + log(sqrt(2*x + 2*sqrt(2*x - 1) +
 2) - sqrt(2*x - 1) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = \int \frac{1}{\sqrt{x+\sqrt{2\,x-1}+1}}\,dx$$

```
[In] int(1/(x + (2*x - 1)^(1/2) + 1)^(1/2),x)
[Out] int(1/(x + (2*x - 1)^(1/2) + 1)^(1/2), x)
```

3.722 $\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 54

$$\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)}\,dx=\frac{px}{a}-\frac{2fp\sqrt{b+ax}}{a^2}-\frac{2(bp-f^2p-aq)\log\left(f+\sqrt{b+ax}\right)}{a^2}$$

```
[Out] p*x/a-2*(-f^2*p-a*q+b*p)*ln(f+(a*x+b)^(1/2))/a^2-2*f*p*(a*x+b)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {711}

$$\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)}\,dx=-\frac{2(-aq+bp+f^2(-p))\log\left(\sqrt{ax+b}+f\right)}{a^2}-\frac{2fp\sqrt{ax+b}}{a^2}+\frac{px}{a}$$

```
[In] Int[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]
```

```
[Out] (p*x)/a - (2*f*p*Sqrt[b + a*x])/a^2 - (2*(b*p - f^2*p - a*q)*Log[f + Sqrt[b
 + a*x]])/a^2
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\text{Subst}\left(\int \frac{-bp+aq+px^2}{f+x}\,dx, x, \sqrt{b+ax}\right)}{a^2}\\ &= \frac{2\text{Subst}\left(\int \left(-fp+px+\frac{-bp+f^2p+aq}{f+x}\right)\,dx, x, \sqrt{b+ax}\right)}{a^2}\\ &= \frac{px}{a}-\frac{2fp\sqrt{b+ax}}{a^2}-\frac{2(bp-f^2p-aq)\log\left(f+\sqrt{b+ax}\right)}{a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)}\,dx = \frac{p\left(b+ax-2f\sqrt{b+ax}\right)+2(-bp+f^2p+aq)\log\left(f+\sqrt{b+ax}\right)}{a^2}$$

```
[In] Integrate[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]
[Out] (p*(b + a*x - 2*f*Sqrt[b + a*x]) + 2*(-(b*p) + f^2*p + a*q)*Log[f + Sqrt[b
+ a*x]])/a^2
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{-2fp\sqrt{ax+b}+p(ax+b)+2\left(f^2p+aq-bp\right)\ln\left(f+\sqrt{ax+b}\right)}{a^2}$	50
default	$\frac{-2fp\sqrt{ax+b}+p(ax+b)+2\left(f^2p+aq-bp\right)\ln\left(f+\sqrt{ax+b}\right)}{a^2}$	50

```
[In] int((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/a^2*(-f*p*(a*x+b)^(1/2)+1/2*p*(a*x+b)+(f^2*p+a*q-b*p)*ln(f+(a*x+b)^(1/2))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{q+px}{\sqrt{b+ax}\,(f+\sqrt{b+ax})}\,dx = \frac{apx-2\sqrt{ax+b}fp+2\left((f^2-b)p+aq\right)\log\left(f+\sqrt{ax+b}\right)}{a^2}$$

```
[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="fricas")
[Out] (a*p*x - 2*sqrt(a*x + b)*f*p + 2*((f^2 - b)*p + a*q)*log(f + sqrt(a*x + b))
)/a^2
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int \frac{q+px}{\sqrt{b+ax}\,(f+\sqrt{b+ax})}\,dx = \begin{cases} \frac{2\left(-\frac{fp\sqrt{ax+b}}{a}+\frac{p(ax+b)}{2a}-\frac{(-aq+bp-f^2p)\log(f+\sqrt{ax+b})}{a}\right)}{a} & \text{for } a\neq 0 \\ \frac{\frac{px^2}{2}+qx}{\sqrt{b}(\sqrt{b}+f)} & \text{otherwise} \end{cases}$$

```
[In] integrate((p*x+q)/(a*x+b)**(1/2)/(f+(a*x+b)**(1/2)),x)
[Out] Piecewise((2*(-f*p*sqrt(a*x + b)/a + p*(a*x + b)/(2*a) - (-a*q + b*p - f**2
*p)*log(f + sqrt(a*x + b))/a)/a, Ne(a, 0)), ((p*x**2/2 + q*x)/(sqrt(b)*(sqr
t(b) + f)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{q+px}{\sqrt{b+ax}\,(f+\sqrt{b+ax})}\,dx = \frac{\frac{2\left((f^2-b)p+aq\right)\log(f+\sqrt{ax+b})}{a}-\frac{2\sqrt{ax+b}fp-(ax+b)p}{a}}{a}$$

```
[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="maxima")
[Out] (2*((f^2 - b)*p + a*q)*log(f + sqrt(a*x + b))/a - (2*sqrt(a*x + b)*f*p - (a
*x + b)*p)/a)/a
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \frac{q+px}{\sqrt{b+ax}\,\left(f+\sqrt{b+ax}\right)}\,dx = \frac{2\,(f^2p-bp+aq)\log\left(\left|f+\sqrt{ax+b}\right|\right)}{a^2} - \frac{2\,\sqrt{ax+b}a^2fp-(ax+b)a^2p}{a^4}$$

```
[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="giac")
[Out] 2*(f^2*p - b*p + a*q)*log(abs(f + sqrt(a*x + b)))/a^2 - (2*sqrt(a*x + b)*a^
2*f*p - (a*x + b)*a^2*p)/a^4
```

Mupad [B] (verification not implemented)

Time = 18.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{q+px}{\sqrt{b+ax}\,\left(f+\sqrt{b+ax}\right)}\,dx = \frac{\ln\left(f+\sqrt{b+a\,x}\right)\,\left(2\,p\,f^2+2\,a\,q-2\,b\,p\right)}{a^2} + \frac{p\,x}{a} - \frac{2\,f\,p\,\sqrt{b+a\,x}}{a^2}$$

```
[In] int((q + p*x)/((f + (b + a*x)^(1/2))*(b + a*x)^(1/2)),x)
[Out] (log(f + (b + a*x)^(1/2))*(2*a*q - 2*b*p + 2*f^2*p))/a^2 + (p*x)/a - (2*f*p
*(b + a*x)^(1/2))/a^2
```

3.723 $\int \sqrt{1-\sqrt{x}-x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 70

$$\int \sqrt{1-\sqrt{x}-x}\,dx=-\frac{1}{4}(1+2\sqrt{x})\sqrt{1-\sqrt{x}-x}-\frac{2}{3}(1-\sqrt{x}-x)^{3/2}-\frac{5}{8}\arcsin\left(\frac{1+2\sqrt{x}}{\sqrt{5}}\right)$$

```
[Out] -5/8*arcsin(1/5*(1+2*x^(1/2))*5^(1/2))-2/3*(1-x-x^(1/2))^(3/2)-1/4*(1+2*x^(
1/2))*(1-x-x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1355, 654, 626, 633, 222}

$$\int \sqrt{1-\sqrt{x}-x}\,dx=-\frac{5}{8}\arcsin\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)-\frac{2}{3}\left(-x-\sqrt{x}+1\right)^{3/2}-\frac{1}{4}(2\sqrt{x}+1)\sqrt{-x-\sqrt{x}+1}$$

```
[In] Int[Sqrt[1 - Sqrt[x] - x],x]
```

```
[Out] -1/4*((1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x]) - (2*(1 - Sqrt[x] - x)^(3/2))/
3 - (5*ArcSin[(1 + 2*Sqrt[x])/Sqrt[5]])/8
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x\sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} - \text{Subst}\left(\int \sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} - \frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} - \frac{5}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-x-x^2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} - \frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2}\\
&\quad + \frac{1}{8}\sqrt{5}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{5}}}\,dx, x, -1-2\sqrt{x}\right)\\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} - \frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} - \frac{5}{8}\sin^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{5}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int \sqrt{1-\sqrt{x}-x}\,dx = \frac{1}{12}\sqrt{1-\sqrt{x}-x}(-11+2\sqrt{x}+8x)-\frac{5}{4}\arctan\left(\frac{\sqrt{x}}{-1+\sqrt{1-\sqrt{x}-x}}\right)$$

```
[In] Integrate[Sqrt[1 - Sqrt[x] - x],x]
[Out] (Sqrt[1 - Sqrt[x] - x]*(-11 + 2*Sqrt[x] + 8*x))/12 - (5*ArcTan[Sqrt[x]/(-1
+ Sqrt[1 - Sqrt[x] - x])])/4
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{2(1-x-\sqrt{x})^{\frac{3}{2}}}{3}+\frac{(-2\sqrt{x}-1)\sqrt{1-x-\sqrt{x}}}{4}-\frac{5\arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{8}$	50
default	$-\frac{2(1-x-\sqrt{x})^{\frac{3}{2}}}{3}+\frac{(-2\sqrt{x}-1)\sqrt{1-x-\sqrt{x}}}{4}-\frac{5\arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{8}$	50

```
[In] int((1-x-x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(1-x-x^(1/2))^(3/2)+1/4*(-2*x^(1/2)-1)*(1-x-x^(1/2))^(1/2)-5/8*arcsin(
2/5*5^(1/2)*(x^(1/2)+1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.84 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.20

$$\int \sqrt{1-\sqrt{x}-x}\,dx = \frac{1}{12}\left(8x+2\sqrt{x}-11\right)\sqrt{-x-\sqrt{x}+1} + \frac{5}{16}\arctan\left(-\frac{\left(8x^2-(16x^2-38x+11)\sqrt{x}-9x+3\right)\sqrt{-x-\sqrt{x}+1}}{4\left(4x^3-13x^2+7x-1\right)}\right)$$

```
[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/12*(8*x + 2*sqrt(x) - 11)*sqrt(-x - sqrt(x) + 1) + 5/16*arctan(-1/4*(8*x^
2 - (16*x^2 - 38*x + 11)*sqrt(x) - 9*x + 3)*sqrt(-x - sqrt(x) + 1)/(4*x^3 -
 13*x^2 + 7*x - 1))
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int \sqrt{1-\sqrt{x}-x}\,dx = 2\sqrt{-\sqrt{x}-x+1}\left(\frac{\sqrt{x}}{12}+\frac{x}{3}-\frac{11}{24}\right)-\frac{5\,\mathrm{asin}\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{8}$$

```
[In] integrate((1-x-x**(1/2))**(1/2),x)
[Out] 2*sqrt(-sqrt(x) - x + 1)*(sqrt(x)/12 + x/3 - 11/24) - 5*asin(2*sqrt(5)*(sqr
t(x) + 1/2)/5)/8
```

Maxima [F]

$$\int \sqrt{1-\sqrt{x}-x}\,dx = \int \sqrt{-x-\sqrt{x}+1}\,dx$$

```
[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x - sqrt(x) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int \sqrt{1-\sqrt{x}-x}\,dx = \frac{1}{12}\left(2\sqrt{x}(4\sqrt{x}+1)-11\right)\sqrt{-x-\sqrt{x}+1} - \frac{5}{8}\arcsin\left(\frac{1}{5}\sqrt{5}(2\sqrt{x}+1)\right)$$

```
[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="giac")
[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) - 11)*sqrt(-x - sqrt(x) + 1) - 5/8*arcsin(1
/5*sqrt(5)*(2*sqrt(x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-\sqrt{x}-x}\,dx = \int \sqrt{1-\sqrt{x}-x}\,dx$$

```
[In] int((1 - x^(1/2) - x)^(1/2),x)
[Out] int((1 - x^(1/2) - x)^(1/2), x)
```

3.724 $\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 19

$$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x} \, dx = 4\sqrt{x}+x+2\log\left(4+\sqrt{x}\right)$$

```
[Out] x+2*ln(4+x^(1/2))+4*x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {28, 1411, 785}

$$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x} \, dx = x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

```
[In] Int[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]
[Out] 4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
```

```
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1411

```
Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g
 - 1)*(d + e*x^(g*n))^q*(a + b*x^(g*n) + c*x^(2*g*n))^p, x], x, x^(1/g)], x
]] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\left(3+\sqrt{x}\right)^2}{4\sqrt{x}+x}\,dx\\ &= 2\text{Subst}\left(\int \frac{x(3+x)^2}{4x+x^2}\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int \left(2+x+\frac{1}{4+x}\right)\,dx, x, \sqrt{x}\right)\\ &= 4\sqrt{x}+x+2\log\left(4+\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx = 4\sqrt{x}+x+2\log\left(4+\sqrt{x}\right)$$

```
[In] Integrate[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]
[Out] 4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$x+2\ln\left(4+\sqrt{x}\right)+4\sqrt{x}$	16
default	$x+2\ln\left(4+\sqrt{x}\right)+4\sqrt{x}$	16
trager	$x-1+4\sqrt{x}+\ln\left(8\sqrt{x}+16+x\right)$	18
meijerg	$2\ln\left(1+\frac{\sqrt{x}}{4}\right)-\frac{4\sqrt{x}\left(-\frac{3\sqrt{x}}{4}+6\right)}{3}+12\sqrt{x}$	29

```
[In] int((9+x+6*x^(1/2))/(x+4*x^(1/2)),x,method=_RETURNVERBOSE)
[Out] x+2*ln(4+x^(1/2))+4*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx = x+4\,\sqrt{x}+2\,\log\left(\sqrt{x}+4\right)$$

```
[In] integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="fricas")
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx = 4\sqrt{x}+x+2\log\left(\sqrt{x}+4\right)$$

```
[In] integrate((9+x+6*x**(1/2))/(x+4*x**(1/2)),x)
[Out] 4*sqrt(x) + x + 2*log(sqrt(x) + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx = x+4\,\sqrt{x}+2\,\log\left(\sqrt{x}+4\right)$$

```
[In] integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="maxima")
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx = x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

```
[In] integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="giac")
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

Mupad [B] (verification not implemented)

Time = 18.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx = x+2\ln\left(\sqrt{x}+4\right)+4\sqrt{x}$$

```
[In] int((x + 6*x^(1/2) + 9)/(x + 4*x^(1/2)),x)
[Out] x + 2*log(x^(1/2) + 4) + 4*x^(1/2)
```

3.725 $\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = -\frac{56145628\sqrt{x}}{43046721}+\frac{125000x}{4782969}+\frac{50000x^{3/2}}{1594323}+\frac{2500x^2}{59049}+\frac{400x^{5/2}}{6561}+\frac{200x^3}{2187}+\frac{80x^{7/2}}{567}+\frac{2x^4}{9}-\frac{280728140\log\left(5-9\sqrt{x}\right)}{387420489}$$

```
[Out] 125000/4782969*x+50000/1594323*x^(3/2)+2500/59049*x^2+400/6561*x^(5/2)+200/
2187*x^3+80/567*x^(7/2)+2/9*x^4-280728140/387420489*ln(5-9*x^(1/2))-5614562
8/43046721*x^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1907, 196, 45, 272}

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = \frac{80x^{7/2}}{567}+\frac{400x^{5/2}}{6561}+\frac{50000x^{3/2}}{1594323}+\frac{2x^4}{9}+\frac{200x^3}{2187}+\frac{2500x^2}{59049}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\log\left(5-9\sqrt{x}\right)}{387420489}$$

```
[In] Int[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]),x]
```

```
[Out] (-56145628*Sqrt[x])/43046721 + (125000*x)/4782969 + (50000*x^(3/2))/1594323
 + (2500*x^2)/59049 + (400*x^(5/2))/6561 + (200*x^3)/2187 + (80*x^(7/2))/56
7 + (2*x^4)/9 - (280728140*Log[5 - 9*Sqrt[x]])/387420489
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{6}{-5+9\sqrt{x}}+\frac{8x^{7/2}}{-5+9\sqrt{x}}\right)dx \\
&= -\left(6\int \frac{1}{-5+9\sqrt{x}}\,dx\right)+8\int \frac{x^{7/2}}{-5+9\sqrt{x}}\,dx \\
&= -\left(12\text{Subst}\left(\int \frac{x}{-5+9x}\,dx,x,\sqrt{x}\right)\right)+16\text{Subst}\left(\int \frac{x^8}{-5+9x}\,dx,x,\sqrt{x}\right) \\
&= -\left(12\text{Subst}\left(\int \left(\frac{1}{9}+\frac{5}{9(-5+9x)}\right)dx,x,\sqrt{x}\right)\right)+16\text{Subst}\left(\int \left(\frac{78125}{43046721}+\frac{15625x}{4782969}\right.\right. \\
&\quad \left.\left.+\frac{3125x^2}{531441}+\frac{625x^3}{59049}+\frac{125x^4}{6561}+\frac{25x^5}{729}+\frac{5x^6}{81}+\frac{x^7}{9}+\frac{390625}{43046721(-5+9x)}\right)dx,x,\sqrt{x}\right) \\
&= -\frac{56145628\sqrt{x}}{43046721}+\frac{125000x}{4782969}+\frac{50000x^{3/2}}{1594323}+\frac{2500x^2}{59049}+\frac{400x^{5/2}}{6561} \\
&\quad +\frac{200x^3}{2187}+\frac{80x^{7/2}}{567}+\frac{2x^4}{9}-\frac{280728140\log\left(5-9\sqrt{x}\right)}{387420489}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = \frac{2\sqrt{x}\left(-196509698+3937500\sqrt{x}+4725000x+6378750x^{3/2}+9185400x^2+13778100x^{5/2}\right.}{301327047}$$
$$-\frac{280728140\log\left(-5+9\sqrt{x}\right)}{387420489}$$

```
[In] Integrate[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]),x]
```

```
[Out] (2*Sqrt[x]*(-196509698 + 3937500*Sqrt[x] + 4725000*x + 6378750*x^(3/2) + 91
85400*x^2 + 13778100*x^(5/2) + 21257640*x^3 + 33480783*x^(7/2)))/301327047
- (280728140*Log[-5 + 9*Sqrt[x]])/387420489
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{2x^4}{9}+\frac{80x^{\frac{7}{2}}}{567}+\frac{200x^3}{2187}+\frac{400x^{\frac{5}{2}}}{6561}+\frac{2500x^2}{59049}+\frac{50000x^{\frac{3}{2}}}{1594323}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\ln(-5+9\sqrt{x})}{387420489}$
default	$\frac{2x^4}{9}+\frac{80x^{\frac{7}{2}}}{567}+\frac{200x^3}{2187}+\frac{400x^{\frac{5}{2}}}{6561}+\frac{2500x^2}{59049}+\frac{50000x^{\frac{3}{2}}}{1594323}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\ln(-5+9\sqrt{x})}{387420489}$
trager	$\frac{2(531441x^3+750141x^2+851391x+913891)(x-1)}{4782969}+2\left(\frac{40}{567}x^3+\frac{200}{6561}x^2+\frac{25000}{1594323}x-\frac{28072814}{43046721}\right)\sqrt{x}-1403$
meijerg	$-\frac{4\sqrt{x}}{3}-\frac{280728140\ln\left(1-\frac{9\sqrt{x}}{5}\right)}{387420489}+\frac{31250\sqrt{x}\left(\frac{301327047x^{\frac{7}{2}}}{15625}+\frac{38263752x^3}{3125}+\frac{4960116x^{\frac{5}{2}}}{625}+\frac{3306744x^2}{625}+\frac{91854x^{\frac{3}{2}}}{25}+\frac{13608x}{5}\right.}{2711943423}$

```
[In] int((6-8*x^(7/2))/(5-9*x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*x^4+80/567*x^(7/2)+200/2187*x^3+400/6561*x^(5/2)+2500/59049*x^2+50000/1
594323*x^(3/2)+125000/4782969*x-56145628/43046721*x^(1/2)-280728140/3874204
89*ln(-5+9*x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = \frac{2}{9}x^4+\frac{200}{2187}x^3+\frac{2500}{59049}x^2$$
$$+\frac{4}{301327047}\left(10628820x^3+4592700x^2+2362500x-98254849\right)\sqrt{x}$$
$$+\frac{125000}{4782969}x-\frac{280728140}{387420489}\log\left(9\sqrt{x}-5\right)$$

```
[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/9*x^4 + 200/2187*x^3 + 2500/59049*x^2 + 4/301327047*(10628820*x^3 + 45927
00*x^2 + 2362500*x - 98254849)*sqrt(x) + 125000/4782969*x - 280728140/38742
0489*log(9*sqrt(x) - 5)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = \frac{80x^{\frac{7}{2}}}{567} + \frac{400x^{\frac{5}{2}}}{6561} + \frac{50000x^{\frac{3}{2}}}{1594323} - \frac{56145628\sqrt{x}}{43046721} + \frac{2x^4}{9} + \frac{200x^3}{2187} + \frac{2500x^2}{59049} + \frac{125000x}{4782969} - \frac{280728140\log\left(9\sqrt{x}-5\right)}{387420489}$$

```
[In] integrate((6-8*x**(7/2))/(5-9*x**(1/2)),x)
```

```
[Out] 80*x**(7/2)/567 + 400*x**(5/2)/6561 + 50000*x**(3/2)/1594323 - 56145628*sqr
t(x)/43046721 + 2*x**4/9 + 200*x**3/2187 + 2500*x**2/59049 + 125000*x/47829
69 - 280728140*log(9*sqrt(x) - 5)/387420489
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = \frac{2}{9}x^4 + \frac{80}{567}x^{\frac{7}{2}} + \frac{200}{2187}x^3 + \frac{400}{6561}x^{\frac{5}{2}} + \frac{2500}{59049}x^2 + \frac{50000}{1594323}x^{\frac{3}{2}} + \frac{125000}{4782969}x - \frac{56145628}{43046721}\sqrt{x} - \frac{280728140}{387420489}\log\left(9\sqrt{x}-5\right)$$

```
[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500/59049*x^2
 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/43046721*sqrt(x) - 2
80728140/387420489*log(9*sqrt(x) - 5)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.65

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = \frac{2}{9}\,x^4 + \frac{80}{567}\,x^{\frac{7}{2}} + \frac{200}{2187}\,x^3 + \frac{400}{6561}\,x^{\frac{5}{2}} + \frac{2500}{59049}\,x^2 + \frac{50000}{1594323}\,x^{\frac{3}{2}} + \frac{125000}{4782969}\,x - \frac{56145628}{43046721}\,\sqrt{x} - \frac{280728140}{387420489}\,\log\left(\left|9\,\sqrt{x}-5\right|\right)$$

```
[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500/59049*x^2
 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/43046721*sqrt(x) - 2
80728140/387420489*log(abs(9*sqrt(x) - 5))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = \frac{125000\,x}{4782969} - \frac{280728140\,\ln\left(\sqrt{x}-\frac{5}{9}\right)}{387420489} + \frac{2500\,x^2}{59049} - \frac{56145628\,\sqrt{x}}{43046721} + \frac{200\,x^3}{2187} + \frac{2\,x^4}{9} + \frac{50000\,x^{3/2}}{1594323} + \frac{400\,x^{5/2}}{6561} + \frac{80\,x^{7/2}}{567}$$

```
[In] int((8*x^(7/2) - 6)/(9*x^(1/2) - 5),x)
```

```
[Out] (125000*x)/4782969 - (280728140*log(x^(1/2) - 5/9))/387420489 + (2500*x^2)/
59049 - (56145628*x^(1/2))/43046721 + (200*x^3)/2187 + (2*x^4)/9 + (50000*x
^(3/2))/1594323 + (400*x^(5/2))/6561 + (80*x^(7/2))/567
```

3.726 $\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 80

$$\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2} dx = -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + (1-i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+x}}{\sqrt{1-i}}\right) + (1+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+x}}{\sqrt{1+i}}\right)$$

```
[Out] -2/3*(1+x)^(3/2)+2/5*(1+x)^(5/2)+(1-I)^(3/2)*arctanh((1+x)^(1/2)/(1-I)^(1/2
))+(1+I)^(3/2)*arctanh((1+x)^(1/2)/(1+I)^(1/2))-2*(1+x)^(1/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 224 vs. 2(80) = 160.

Time = 0.21 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.80, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules

used = {1639, 1643, 839, 12, 722, 1108, 648, 632, 210, 642}

$$\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx = -\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right) + \sqrt{1+\sqrt{2}}\arctan\left(\frac{2\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right) + \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1} - \frac{\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}} + \frac{\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}$$

```
[In] Int[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2),x]
```

```
[Out] -2*Sqrt[1 + x] - (2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5 - Sqrt[1 + Sqrt[
2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]]
+ Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + x])/Sqrt[2*(
-1 + Sqrt[2])]] - Log[1 + Sqrt[2] + x - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]]/
(2*Sqrt[1 + Sqrt[2]]) + Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1
+ x]]/(2*Sqrt[1 + Sqrt[2]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 839

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[(d + e*x)^(m -
 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*(a + c*x^2)^p,
x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[PolynomialRemaind
er[Pq, d + e*x, x], 0]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \frac{(1+x)^{3/2}(1-x+x^2)}{1+x^2}\,dx$$
$$= \int \left((1+x)^{3/2} - \frac{x(1+x)^{3/2}}{1+x^2}\right)dx$$
$$= \frac{2}{5}(1+x)^{5/2} - \int \frac{x(1+x)^{3/2}}{1+x^2}\,dx$$
$$= -\frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \int \frac{(-1+x)\sqrt{1+x}}{1+x^2}\,dx$$
$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \int -\frac{2}{\sqrt{1+x}\,(1+x^2)}\,dx$$
$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + 2\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx$$
$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + 4\text{Subst}\left(\int \frac{1}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right)$$
$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+x}\right)}{\sqrt{1+\sqrt{2}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+x}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+x}\right)}{\sqrt{1+\sqrt{2}}}$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+x}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+x}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{-\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx, x, \sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}}$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \frac{\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}} + \frac{\log\left(1+\sqrt{2}+x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}} - \sqrt{2}\text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\,dx, x, -\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}\right) - \sqrt{2}\text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)-x^2}\,dx, x, \sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}\right)$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{-1+\sqrt{2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{-1+\sqrt{2}}} - \frac{\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}} + \frac{\log\left(1+\sqrt{2}+x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx = \frac{2}{15}\sqrt{1+x}(-17+x+3x^2)+\sqrt{2+2i}\arctan\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\sqrt{1+x}\right)+\sqrt{2-2i}\arctan\left(\sqrt{-\frac{1}{2}+\frac{i}{2}}\sqrt{1+x}\right)$$

```
[In] Integrate[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2),x]
```

```
[Out] (2*Sqrt[1 + x]*(-17 + x + 3*x^2))/15 + Sqrt[2 + 2*I]*ArcTan[Sqrt[-1/2 - I/2
]*Sqrt[1 + x]] + Sqrt[2 - 2*I]*ArcTan[Sqrt[-1/2 + I/2]*Sqrt[1 + x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(58) = 116.

Time = 1.37 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.61

method	result
derivativedivides	$\frac{2(x+1)^{\frac{5}{2}}}{5}-\frac{2(x+1)^{\frac{3}{2}}}{3}-2\sqrt{x+1}-\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\ln\left(x+1-\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)}{4}-\frac{\left(-2\sqrt{2}+\right.}{}$
default	$\frac{2(x+1)^{\frac{5}{2}}}{5}-\frac{2(x+1)^{\frac{3}{2}}}{3}-2\sqrt{x+1}-\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\ln\left(x+1-\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)}{4}-\frac{\left(-2\sqrt{2}+\right.}{}$
trager	$\left(\frac{2}{5}x^2+\frac{2}{15}x-\frac{34}{15}\right)\sqrt{x+1}-\mathrm{RootOf}\left(\_Z^2+16\,\mathrm{RootOf}\left(512\_Z^4+32\_Z^2+1\right)^2+1\right)\ln$
risch	$\frac{2(3x^2+x-17)\sqrt{x+1}}{15}-\frac{\ln\left(x+1+\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)\sqrt{2+2\sqrt{2}}\,\sqrt{2}}{4}+\frac{\ln\left(x+1+\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)\sqrt{2+2\sqrt{2}}}{2}+$

```
[In] int((x^3+1)*(x+1)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(x+1)^(5/2)-2/3*(x+1)^(3/2)-2*(x+1)^(1/2)-1/4*(-(2+2*2^(1/2))^(1/2)*2^(
1/2)+2*(2+2*2^(1/2))^(1/2))*ln(x+1-(x+1)^(1/2)*(2+2*2^(1/2))^(1/2)+2^(1/2))
-(-2*2^(1/2)+1/2*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2))*(2+2*
2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)-(2+2*2^(1/2))^(1
/2))/(-2+2*2^(1/2))^(1/2))+1/4*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2)
)^(1/2))*ln(x+1+(x+1)^(1/2)*(2+2*2^(1/2))^(1/2)+2^(1/2))+(2*2^(1/2)-1/2*(-(
2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2))/(-2+
2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx = \frac{2}{15}\left(3x^2+x-17\right)\sqrt{x+1} + \frac{1}{2}\sqrt{2i-2}\log\left(-(i-1)\sqrt{2i-2}+2\sqrt{x+1}\right) - \frac{1}{2}\sqrt{2i-2}\log\left((i-1)\sqrt{2i-2}+2\sqrt{x+1}\right) + \frac{1}{2}\sqrt{-2i-2}\log\left((i+1)\sqrt{-2i-2}+2\sqrt{x+1}\right) - \frac{1}{2}\sqrt{-2i-2}\log\left(-(i+1)\sqrt{-2i-2}+2\sqrt{x+1}\right)$$

```
[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] 2/15*(3*x^2 + x - 17)*sqrt(x + 1) + 1/2*sqrt(2*I - 2)*log(-(I - 1)*sqrt(2*I
 - 2) + 2*sqrt(x + 1)) - 1/2*sqrt(2*I - 2)*log((I - 1)*sqrt(2*I - 2) + 2*sq
rt(x + 1)) + 1/2*sqrt(-2*I - 2)*log((I + 1)*sqrt(-2*I - 2) + 2*sqrt(x + 1))
 - 1/2*sqrt(-2*I - 2)*log(-(I + 1)*sqrt(-2*I - 2) + 2*sqrt(x + 1))
```

Sympy [F]

$$\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx = \int \frac{(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)}{x^2+1}\,dx$$

```
[In] integrate((x**3+1)*(1+x)**(1/2)/(x**2+1),x)
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx = \int \frac{(x^3+1)\sqrt{x+1}}{x^2+1}\,dx$$

```
[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate((x^3 + 1)*sqrt(x + 1)/(x^2 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(50) = 100$.

Time = 0.83 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.14

$$\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx = \frac{2}{5}\,(x+1)^{\frac{5}{2}} - \frac{2}{3}\,(x+1)^{\frac{3}{2}} + \sqrt{\sqrt{2}+1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right) + \sqrt{\sqrt{2}+1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right) + \frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right) - \frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(-2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right) - 2\sqrt{x+1}$$

```
[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2) + sqrt(sqrt(2) + 1)*arctan(1/2*2^(3/4
)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) + sqrt(sq
rt(2) + 1)*arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(x + 1))/
sqrt(-sqrt(2) + 2)) + 1/2*sqrt(sqrt(2) - 1)*log(2^(1/4)*sqrt(x + 1)*sqrt(sq
rt(2) + 2) + x + sqrt(2) + 1) - 1/2*sqrt(sqrt(2) - 1)*log(-2^(1/4)*sqrt(x +
 1)*sqrt(sqrt(2) + 2) + x + sqrt(2) + 1) - 2*sqrt(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.19

$$\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx = \frac{2\,(x+1)^{5/2}}{5} - \frac{2\,(x+1)^{3/2}}{3} - 2\sqrt{x+1} - \mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{x+1}\,64\mathrm{i}}{256\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64} - \frac{\sqrt{2}\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{x+1}\,64\mathrm{i}}{256\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64}\right)\left(\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}+\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\right. + \mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{x+1}\,64\mathrm{i}}{256\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64} + \frac{\sqrt{2}\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{x+1}\,64\mathrm{i}}{256\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64}\right)\left(\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}-\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\right.$$

```
[In] int(((x^3 + 1)*(x + 1)^(1/2))/(x^2 + 1),x)
```

```
[Out] (2*(x + 1)^(5/2))/5 - (2*(x + 1)^(3/2))/3 - 2*(x + 1)^(1/2) - atan((2^(1/2)
*(- 2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*
(- 2^(1/2)/4 - 1/4)^(1/2) - 64) - (2^(1/2)*(2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^
(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) - 64))*((
- 2^(1/2)/4 - 1/4)^(1/2)*2i + (2^(1/2)/4 - 1/4)^(1/2)*2i) + atan((2^(1/2)*(
- 2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(-
 2^(1/2)/4 - 1/4)^(1/2) + 64) + (2^(1/2)*(2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1
/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) + 64))*((-
2^(1/2)/4 - 1/4)^(1/2)*2i - (2^(1/2)/4 - 1/4)^(1/2)*2i)
```

3.727 $\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx = \arctan\left(\frac{3-\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) - 2\text{arctanh}\left(\frac{1-2\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) - \text{arctanh}\left(\frac{1+3\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right)$$

```
[Out] arctan(1/2*(3-x^(1/2))/(-1+x-x^(1/2))^(1/2))-2*arctanh(1/2*(1-2*x^(1/2))/(-
1+x-x^(1/2))^(1/2))-arctanh(1/2*(1+3*x^(1/2))/(-1+x-x^(1/2))^(1/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1004, 635, 212, 1047, 738, 210}

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx = \arctan\left(\frac{3-\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right) - 2\text{arctanh}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right) - \text{arctanh}\left(\frac{3\sqrt{x}+1}{2\sqrt{x-\sqrt{x}-1}}\right)$$

```
[In] Int[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]),x]
```

```
[Out] ArcTan[(3 - Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - 2*ArcTanh[(1 - 2*Sqrt[x]
)/(2*Sqrt[-1 - Sqrt[x] + x])] - ArcTanh[(1 + 3*Sqrt[x])/(2*Sqrt[-1 - Sqrt[x
] + x])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1004

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{\sqrt{-1-x+x^2}}{-1+x^2}\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int \frac{1}{\sqrt{-1-x+x^2}}\,dx, x, \sqrt{x}\right) - 2\text{Subst}\left(\int \frac{x}{(-1+x^2)\sqrt{-1-x+x^2}}\,dx, x, \sqrt{x}\right)\end{aligned}$$

$$
\begin{aligned}
&= 4\mathrm{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1+2\sqrt{x}}{\sqrt{-1-\sqrt{x}+x}}\right) \\
&\quad - \mathrm{Subst}\left(\int \frac{1}{(-1+x)\sqrt{-1-x+x^2}}\,dx, x, \sqrt{x}\right) \\
&\quad - \mathrm{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1-x+x^2}}\,dx, x, \sqrt{x}\right) \\
&= -2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) + 2\mathrm{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3+\sqrt{x}}{\sqrt{-1-\sqrt{x}+x}}\right) \\
&\quad + 2\mathrm{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1-3\sqrt{x}}{\sqrt{-1-\sqrt{x}+x}}\right) \\
&= \tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) - 2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) - \tanh^{-1}\left(\frac{1+3\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$
\begin{aligned}
\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx &= -2\arctan\left(1-\sqrt{x}+\sqrt{-1-\sqrt{x}+x}\right) \\
&\quad - 2\mathrm{arctanh}\left(1+\sqrt{x}-\sqrt{-1-\sqrt{x}+x}\right) \\
&\quad - 2\log\left(1-2\sqrt{x}+2\sqrt{-1-\sqrt{x}+x}\right)
\end{aligned}
$$

```
[In] Integrate[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]),x]
[Out] -2*ArcTan[1 - Sqrt[x] + Sqrt[-1 - Sqrt[x] + x]] - 2*ArcTanh[1 + Sqrt[x] - S
qrt[-1 - Sqrt[x] + x]] - 2*Log[1 - 2*Sqrt[x] + 2*Sqrt[-1 - Sqrt[x] + x]]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.46

method	result
derivativedivides	$-\sqrt{\left(1+\sqrt{x}\right)^2-3\sqrt{x}-2}+\frac{3\ln\left(-\frac{1}{2}+\sqrt{x}+\sqrt{\left(1+\sqrt{x}\right)^2-3\sqrt{x}-2}\right)}{2}+\mathrm{arctanh}\left(\frac{-1-3\sqrt{x}}{2\sqrt{\left(1+\sqrt{x}\right)^2-3\sqrt{x}-2}}\right.$
default	$-\sqrt{\left(1+\sqrt{x}\right)^2-3\sqrt{x}-2}+\frac{3\ln\left(-\frac{1}{2}+\sqrt{x}+\sqrt{\left(1+\sqrt{x}\right)^2-3\sqrt{x}-2}\right)}{2}+\mathrm{arctanh}\left(\frac{-1-3\sqrt{x}}{2\sqrt{\left(1+\sqrt{x}\right)^2-3\sqrt{x}-2}}\right.$

```
[In] int((-1+x-x^(1/2))^(1/2)/(x-1)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((1+x^(1/2))^2-3*x^(1/2)-2)^(1/2)+3/2*ln(-1/2+x^(1/2)+((1+x^(1/2))^2-3*x^(
1/2)-2)^(1/2))+arctanh(1/2*(-1-3*x^(1/2))/((1+x^(1/2))^2-3*x^(1/2)-2)^(1/2)
)+((-1+x^(1/2))^2+x^(1/2)-2)^(1/2)+1/2*ln(-1/2+x^(1/2)+((-1+x^(1/2))^2+x^(1
/2)-2)^(1/2))-arctan(1/2*(-3+x^(1/2))/((-1+x^(1/2))^2+x^(1/2)-2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 2.55 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx = -\arctan\left(\frac{\left((x-4)\sqrt{x}-2\,x+3\right)\sqrt{x-\sqrt{x}-1}}{2\,(x^2-3\,x+1)}\right) + \log\left(-\frac{8\,x^2+2\,\left((4\,x-5)\sqrt{x}+2\,x-1\right)\sqrt{x-\sqrt{x}-1}-17\,x-2\,\sqrt{x}+11}{x-1}\right)$$

```
[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*((x - 4)*sqrt(x) - 2*x + 3)*sqrt(x - sqrt(x) - 1)/(x^2 - 3*x +
1)) + log(-(8*x^2 + 2*((4*x - 5)*sqrt(x) + 2*x - 1)*sqrt(x - sqrt(x) - 1) -
 17*x - 2*sqrt(x) + 11)/(x - 1))
```

Sympy [F]

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx = \int \frac{\sqrt{-\sqrt{x}+x-1}}{\sqrt{x}\,(x-1)}\,dx$$

```
[In] integrate((-1+x-x**(1/2))**(1/2)/(-1+x)/x**(1/2),x)
```

```
[Out] Integral(sqrt(-sqrt(x) + x - 1)/(sqrt(x)*(x - 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx = \int \frac{\sqrt{x-\sqrt{x}-1}}{(x-1)\sqrt{x}}\,dx$$

```
[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x - sqrt(x) - 1)/((x - 1)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx = -2\arctan\left(\sqrt{x-\sqrt{x}-1}-\sqrt{x}+1\right) - \log\left(-\sqrt{x-\sqrt{x}-1}+\sqrt{x}+2\right) + \log\left(-\sqrt{x-\sqrt{x}-1}+\sqrt{x}\right) - 2\log\left(\left|2\sqrt{x-\sqrt{x}-1}-2\sqrt{x}+1\right|\right)$$

```
[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="giac")
[Out] -2*arctan(sqrt(x - sqrt(x) - 1) - sqrt(x) + 1) - log(-sqrt(x - sqrt(x) - 1)
 + sqrt(x) + 2) + log(-sqrt(x - sqrt(x) - 1) + sqrt(x)) - 2*log(abs(2*sqrt(
x - sqrt(x) - 1) - 2*sqrt(x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx = \int \frac{\sqrt{x-\sqrt{x}-1}}{\sqrt{x}\,(x-1)}\,dx$$

```
[In] int((x - x^(1/2) - 1)^(1/2)/(x^(1/2)*(x - 1)),x)
[Out] int((x - x^(1/2) - 1)^(1/2)/(x^(1/2)*(x - 1)), x)
```

3.728 $\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 61

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = -\arctan\left(\frac{3+\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)+3\text{arctanh}\left(\frac{1-3\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)$$

```
[Out] -arctan(1/2*(3+(1+x)^(1/2))/(x+(1+x)^(1/2))^(1/2))+3*arctanh(1/2*(1-3*(1+x)
^(1/2))/(x+(1+x)^(1/2))^(1/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1047, 738, 212, 210}

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = 3\text{arctanh}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)-\arctan\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)$$

```
[In] Int[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]),x]
```

```
[Out] -ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + 3*ArcTanh[(1 - 3*Sqr
t[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)\\
&= 3\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)\\
&\quad + \text{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)\\
&= -\left(2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3-\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right)\right) - 6\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1+3\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right)\\
&= -\tan^{-1}\left(\frac{3+\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right) + 3\tanh^{-1}\left(\frac{1-3\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = -2\arctan\left(1+\sqrt{1+x}-\sqrt{x+\sqrt{1+x}}\right) - 6\text{arctanh}\left(1-\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}\right)$$

```
[In] Integrate[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]),x]
[Out] -2*ArcTan[1 + Sqrt[1 + x] - Sqrt[x + Sqrt[1 + x]]] - 6*ArcTanh[1 - Sqrt[1 +
 x] + Sqrt[x + Sqrt[1 + x]]]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\arctan\left(\frac{-3-\sqrt{x+1}}{2\sqrt{\left(1+\sqrt{x+1}\right)^{2}-\sqrt{x+1}-2}}\right)-3\,\operatorname{arctanh}\left(\frac{-1+3\sqrt{x+1}}{2\sqrt{\left(\sqrt{x+1}-1\right)^{2}+3\sqrt{x+1}-2}}\right)$	68
default	$\arctan\left(\frac{-3-\sqrt{x+1}}{2\sqrt{\left(1+\sqrt{x+1}\right)^{2}-\sqrt{x+1}-2}}\right)-3\,\operatorname{arctanh}\left(\frac{-1+3\sqrt{x+1}}{2\sqrt{\left(\sqrt{x+1}-1\right)^{2}+3\sqrt{x+1}-2}}\right)$	68

```
[In] int((1+2*(x+1)^(1/2))/x/(x+1)^(1/2)/(x+(x+1)^(1/2))^(1/2),x,method=_RETURNV
ERBOSE)
[Out] arctan(1/2*(-3-(x+1)^(1/2))/((1+(x+1)^(1/2))^2-(x+1)^(1/2)-2)^(1/2))-3*arct
anh(1/2*(-1+3*(x+1)^(1/2))/(((x+1)^(1/2)-1)^2+3*(x+1)^(1/2)-2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 1.61 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = \arctan\left(\frac{2\,\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}-3\right)}{x-8}\right) + 3\,\log\left(\frac{2\,\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}+1\right)-3\,x-2\,\sqrt{x+1}-2}{x}\right)$$

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorith
m="fricas")
[Out] arctan(2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) - 3)/(x - 8)) + 3*log((2*sqrt(x
 + sqrt(x + 1))*(sqrt(x + 1) + 1) - 3*x - 2*sqrt(x + 1) - 2)/x)
```

Sympy [F]

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx=\int \frac{2\sqrt{x+1}+1}{x\sqrt{x+1}\sqrt{x+\sqrt{x+1}}}\,dx$$

```
[In] integrate((1+2*(1+x)**(1/2))/x/(1+x)**(1/2)/(x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral((2*sqrt(x + 1) + 1)/(x*sqrt(x + 1)*sqrt(x + sqrt(x + 1))), x)
```

Maxima [F]

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx=\int \frac{2\sqrt{x+1}+1}{\sqrt{x+\sqrt{x+1}}\sqrt{x+1}x}\,dx$$

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((2*sqrt(x + 1) + 1)/(sqrt(x + sqrt(x + 1))*sqrt(x + 1)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx=2\arctan\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}-1\right)-3\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}+2\right|\right)+3\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right|\right)$$

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorith
m="giac")
```

```
[Out] 2*arctan(sqrt(x + sqrt(x + 1)) - sqrt(x + 1) - 1) - 3*log(abs(sqrt(x + sqrt
(x + 1)) - sqrt(x + 1) + 2)) + 3*log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1
)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = \int \frac{2\,\sqrt{x+1}+1}{x\,\sqrt{x+\sqrt{x+1}}\,\sqrt{x+1}}\,dx$$

```
[In] int((2*(x + 1)^(1/2) + 1)/(x*(x + (x + 1)^(1/2))^(1/2)*(x + 1)^(1/2)),x)
[Out] int((2*(x + 1)^(1/2) + 1)/(x*(x + (x + 1)^(1/2))^(1/2)*(x + 1)^(1/2)), x)
```

3.729 $\int \frac{1}{\sqrt{x}\sqrt{1+x}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}} \, dx = 2\text{arcsinh}\left(\sqrt{x}\right)$$

```
[Out] 2*arcsinh(x^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {56, 221}

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}} \, dx = 2\text{arcsinh}\left(\sqrt{x}\right)$$

```
[In] Int[1/(Sqrt[x]*Sqrt[1 + x]),x]
```

```
[Out] 2*ArcSinh[Sqrt[x]]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)\\ &= 2\sinh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 18 vs. 2(8) = 16.

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = -2\log\left(-\sqrt{x}+\sqrt{1+x}\right)$$

```
[In] Integrate[1/(Sqrt[x]*Sqrt[1 + x]),x]
[Out] -2*Log[-Sqrt[x] + Sqrt[1 + x]]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
meijerg	$2 \operatorname{arcsinh}\left(\sqrt{x}\right)$	7
default	$\frac{\sqrt{(x+1)x}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{\sqrt{x}\,\sqrt{x+1}}$	28

```
[In] int(1/x^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arcsinh(x^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(6) = 12.

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = -\log\left(2\sqrt{x+1}\sqrt{x}-2x-1\right)$$

```
[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] -log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 3.25

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = \begin{cases} 2\operatorname{acosh}\left(\sqrt{x+1}\right) & \text{for } |x+1| > 1 \\ -2i\operatorname{asin}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x**(1/2)/(1+x)**(1/2),x)
[Out] Piecewise((2*acosh(sqrt(x + 1)), Abs(x + 1) > 1), (-2*I*asin(sqrt(x + 1)),
True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = \log\left(\frac{\sqrt{x+1}}{\sqrt{x}} + 1\right) - \log\left(\frac{\sqrt{x+1}}{\sqrt{x}} - 1\right)$$

```
[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] log(sqrt(x + 1)/sqrt(x) + 1) - log(sqrt(x + 1)/sqrt(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = -2\log\left(\sqrt{x+1} - \sqrt{x}\right)$$

```
[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(x + 1) - sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = 4\,\mathrm{atanh}\left(\frac{\sqrt{x+1}-1}{\sqrt{x}}\right)$$

```
[In] int(1/(x^(1/2)*(x + 1)^(1/2)),x)
[Out] 4*atanh(((x + 1)^(1/2) - 1)/x^(1/2))
```

3.730 $\int \frac{\sqrt{\frac{x}{1+x}}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 8

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x} dx = 2\text{arcsinh}\left(\sqrt{x}\right)$$

```
[Out] 2*arcsinh(x^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1978, 56, 221}

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x} dx = 2\text{arcsinh}\left(\sqrt{x}\right)$$

```
[In] Int[Sqrt[x/(1 + x)]/x,x]
```

```
[Out] 2*ArcSinh[Sqrt[x]]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx \\ &= 2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right) \\ &= 2\sinh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 18 vs. 2(8) = 16.

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = -2\log\left(-\sqrt{x}+\sqrt{1+x}\right)$$

```
[In] Integrate[Sqrt[x/(1 + x)]/x,x]
[Out] -2*Log[-Sqrt[x] + Sqrt[1 + x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(6) = 12.

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 4.00

method	result	size
default	$\frac{\sqrt{\frac{x}{x+1}}\,(x+1)\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{\sqrt{(x+1)x}}$	32
trager	$-\ln\left(2\sqrt{\frac{x}{x+1}}\,x+2\sqrt{\frac{x}{x+1}}-2x-1\right)$	32

```
[In] int((x/(x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] (x/(x+1))^(1/2)*(x+1)/((x+1)*x)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = \log\left(\sqrt{\frac{x}{x+1}}+1\right) - \log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

```
[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="fricas")
[Out] log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = \int \frac{\sqrt{\frac{x}{x+1}}}{x}\,dx$$

```
[In] integrate((x/(1+x))**(1/2)/x,x)
[Out] Integral(sqrt(x/(x + 1))/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = \log\left(\sqrt{\frac{x}{x+1}}+1\right) - \log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

```
[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="maxima")
[Out] log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(6) = 12$.

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.75

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = -\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1)$$

```
[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="giac")
[Out] -log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{x}{x+1}}\right)$$

```
[In] int((x/(x + 1))^(1/2)/x,x)
[Out] 2*atanh((x/(x + 1))^(1/2))
```

3.731 $\int \frac{\sqrt{x}}{\sqrt{1+x}} dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{\sqrt{x}}{\sqrt{1+x}} dx = \sqrt{x}\sqrt{1+x} - \operatorname{arcsinh}\left(\sqrt{x}\right)$$

```
[Out] -arcsinh(x^(1/2))+x^(1/2)*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 56, 221}

$$\int \frac{\sqrt{x}}{\sqrt{1+x}} dx = \sqrt{x}\sqrt{x+1} - \operatorname{arcsinh}\left(\sqrt{x}\right)$$

```
[In] Int[Sqrt[x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \sqrt{x}\sqrt{1+x} - \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx \\ &= \sqrt{x}\sqrt{1+x} - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right) \\ &= \sqrt{x}\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. $2(22) = 44$.

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \frac{\sqrt{\frac{x}{1+x}}\left(\sqrt{x}(1+x) + \sqrt{1+x}\log\left(-\sqrt{x} + \sqrt{1+x}\right)\right)}{\sqrt{x}}$$

```
[In] Integrate[Sqrt[x]/Sqrt[1 + x],x]
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) + Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]]
))/Sqrt[x]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

method	result	size
meijerg	$\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{x+1}-\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{x}\right)}{\sqrt{\pi}}$	27
default	$\sqrt{x}\,\sqrt{x+1}-\frac{\sqrt{(x+1)x}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{2\sqrt{x}\,\sqrt{x+1}}$	39
risch	$\sqrt{x}\,\sqrt{x+1}-\frac{\sqrt{(x+1)x}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{2\sqrt{x}\,\sqrt{x+1}}$	39

```
[In] int(x^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/Pi^(1/2)*(Pi^(1/2)*x^(1/2)*(x+1)^(1/2)-Pi^(1/2)*arcsinh(x^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{x} + \frac{1}{2}\,\log\left(2\,\sqrt{x+1}\sqrt{x} - 2\,x - 1\right)$$

```
[In] integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] sqrt(x + 1)*sqrt(x) + 1/2*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \begin{cases} \sqrt{x}\sqrt{x+1} - \operatorname{acosh}\left(\sqrt{x+1}\right) & \text{for}\ |x+1| > 1 \\ i\operatorname{asin}\left(\sqrt{x+1}\right) - \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{-x}} + \frac{i\sqrt{x+1}}{\sqrt{-x}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(1/2)/(1+x)**(1/2),x)
[Out] Piecewise((sqrt(x)*sqrt(x + 1) - acosh(sqrt(x + 1)), Abs(x + 1) > 1), (I*as
in(sqrt(x + 1)) - I*(x + 1)**(3/2)/sqrt(-x) + I*sqrt(x + 1)/sqrt(-x), True)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(16) = 32.

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \frac{\sqrt{x+1}}{\sqrt{x}\left(\frac{x+1}{x} - 1\right)} - \frac{1}{2}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}} + 1\right) + \frac{1}{2}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}} - 1\right)$$

```
[In] integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] sqrt(x + 1)/(sqrt(x)*((x + 1)/x - 1)) - 1/2*log(sqrt(x + 1)/sqrt(x) + 1) +
1/2*log(sqrt(x + 1)/sqrt(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{x} + \log\left(\sqrt{x+1} - \sqrt{x}\right)$$

```
[In] integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] sqrt(x + 1)*sqrt(x) + log(sqrt(x + 1) - sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 19.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \sqrt{x}\,\sqrt{x+1} - 2\,\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)$$

```
[In] int(x^(1/2)/(x + 1)^(1/2),x)
[Out] x^(1/2)*(x + 1)^(1/2) - 2*atanh(x^(1/2)/((x + 1)^(1/2) - 1))
```

3.732 $\int \sqrt{\frac{x}{1+x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \sqrt{\frac{x}{1+x}}\,dx = \sqrt{x}\sqrt{1+x} - \operatorname{arcsinh}\left(\sqrt{x}\right)$$

```
[Out] -arcsinh(x^(1/2))+x^(1/2)*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1978, 52, 56, 221}

$$\int \sqrt{\frac{x}{1+x}}\,dx = \sqrt{x}\sqrt{x+1} - \operatorname{arcsinh}\left(\sqrt{x}\right)$$

```
[In] Int[Sqrt[x/(1 + x)],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\
&= \sqrt{x}\sqrt{1+x} - \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\
&= \sqrt{x}\sqrt{1+x} - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)\\
&= \sqrt{x}\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(22) = 44.

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \sqrt{\frac{x}{1+x}}\,dx = \frac{\sqrt{\frac{x}{1+x}}\left(\sqrt{x}(1+x) + \sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)\right)}{\sqrt{x}}$$

```
[In] Integrate[Sqrt[x/(1 + x)],x]
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) + Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]]
))/Sqrt[x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(16) = 32$.

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

method	result	size
default	$\frac{\sqrt{\frac{x}{x+1}}\,(x+1)\left(2\sqrt{x^2+x}-\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\right)}{2\sqrt{(x+1)x}}$	45
risch	$(x+1)\sqrt{\frac{x}{x+1}}-\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\sqrt{\frac{x}{x+1}}\,\sqrt{(x+1)x}}{2x}$	47
trager	$2\left(\frac{x}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{x+1}}-\frac{\ln\left(2\sqrt{\frac{x}{x+1}}\,x+2\sqrt{\frac{x}{x+1}}+2x+1\right)}{2}$	49

```
[In] int((x/(x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(x/(x+1))^(1/2)*(x+1)*(2*(x^2+x)^(1/2)-ln(x+1/2+(x^2+x)^(1/2)))/((x+1)*
x)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(16) = 32$.

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int\sqrt{\frac{x}{1+x}}\,dx=(x+1)\sqrt{\frac{x}{x+1}}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="fricas")
[Out] (x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x
+ 1)) - 1)
```

Sympy [F]

$$\int\sqrt{\frac{x}{1+x}}\,dx=\int\sqrt{\frac{x}{x+1}}\,dx$$

```
[In] integrate((x/(1+x))**(1/2),x)
[Out] Integral(sqrt(x/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(16) = 32$.

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \sqrt{\frac{x}{1+x}}\,dx = -\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1} - \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="maxima")
[Out] -sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(s
qrt(x/(x + 1)) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{x}{1+x}}\,dx = \frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1) + \sqrt{x^2+x}\operatorname{sgn}(x+1)$$

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="giac")
[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x +
 1)
```

Mupad [B] (verification not implemented)

Time = 18.84 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{x}{1+x}}\,dx = -\operatorname{atanh}\left(\sqrt{\frac{x}{x+1}}\right) - \frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

```
[In] int((x/(x + 1))^(1/2),x)
[Out] - atanh((x/(x + 1))^(1/2)) - (x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

3.733 $\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 36

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \arctan\left(\sqrt{-1+x}\sqrt{1+x}\right)$$

```
[Out] arctan((-1+x)^(1/2)*(1+x)^(1/2))-(-1+x)^(1/2)*(1+x)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {96, 94, 209}

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = \arctan\left(\sqrt{x-1}\sqrt{x+1}\right) - \frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

```
[In] Int[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \int \frac{1}{\sqrt{-1+x}x\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\sqrt{1+x}\right) \\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \tan^{-1}\left(\sqrt{-1+x}\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}(1+x) + 2x\sqrt{1+x}\arctan\left(x-\sqrt{-1+x}\sqrt{1+x}\right)\right)}{\sqrt{-1+x}x}$$

```
[In] Integrate[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
[Out] -((Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(1 + x) + 2*x*Sqrt[1 + x]*ArcTan[x
- Sqrt[-1 + x]*Sqrt[1 + x]]))/(Sqrt[-1 + x]*x))
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{\left(-\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)x-\sqrt{x^2-1}\right)\sqrt{x-1}\,\sqrt{x+1}}{x\sqrt{x^2-1}}$	43
risch	$-\frac{\sqrt{x-1}\,\sqrt{x+1}}{x}-\frac{\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)\sqrt{(x-1)(x+1)}}{\sqrt{x-1}\,\sqrt{x+1}}$	46

```
[In] int((x-1)^(1/2)/x^2/(x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-arctan(1/(x^2-1)^(1/2))*x-(x^2-1)^(1/2))*(x-1)^(1/2)*(x+1)^(1/2)/x/(x^2-1
)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = \frac{2\,x\arctan\left(\sqrt{x+1}\sqrt{x-1}-x\right)-\sqrt{x+1}\sqrt{x-1}-x}{x}$$

```
[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="fricas")
[Out] (2*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

Sympy [F]

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = \int \frac{\sqrt{x-1}}{x^2\sqrt{x+1}}\,dx$$

```
[In] integrate((-1+x)**(1/2)/x**2/(1+x)**(1/2),x)
[Out] Integral(sqrt(x - 1)/(x**2*sqrt(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{x^2-1}}{x}-\arcsin\left(\frac{1}{|x|}\right)$$

```
[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(x^2 - 1)/x - arcsin(1/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^4+4} - 2\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right)$$

```
[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="giac")
[Out] -8/((sqrt(x + 1) - sqrt(x - 1))^4 + 4) - 2*arctan(1/2*(sqrt(x + 1) - sqrt(x
 - 1))^2)
```

Mupad [B] (verification not implemented)

Time = 19.79 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.83

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\ln\left(\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1\right)1\mathrm{i} + \ln\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)1\mathrm{i} - \frac{\sqrt{x-1}-\mathrm{i}}{4\left(\sqrt{x+1}-1\right)} - \frac{\frac{5\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1}{\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)}{\sqrt{x+1}-1}}$$

```
[In] int((x - 1)^(1/2)/(x^2*(x + 1)^(1/2)),x)
[Out] log(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1))*1i - log(((x - 1)^(1/2) - 1i)
^2/((x + 1)^(1/2) - 1)^2 + 1)*1i - ((x - 1)^(1/2) - 1i)/(4*((x + 1)^(1/2) -
 1)) - ((5*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 + 1)/((4*((x - 1)^
(1/2) - 1i)^3)/((x + 1)^(1/2) - 1)^3 + (4*((x - 1)^(1/2) - 1i))/((x + 1)^(1
/2) - 1))
```

3.734 $\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} dx = -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \arctan\left(\sqrt{-1+x}\sqrt{1+x}\right)$$

```
[Out] arctan((-1+x)^(1/2)*(1+x)^(1/2))-(-1+x)^(1/2)*(1+x)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1978, 96, 94, 209}

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} dx = \arctan\left(\sqrt{x-1}\sqrt{x+1}\right) - \frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

```
[In] Int[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \int \frac{1}{\sqrt{-1+x}x\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\sqrt{1+x}\right) \\
&= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \tan^{-1}\left(\sqrt{-1+x}\sqrt{1+x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$
\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = -\frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}(1+x)+2x\sqrt{1+x}\arctan\left(x-\sqrt{-1+x}\sqrt{1+x}\right)\right)}{\sqrt{-1+x}x}
$$

```
[In] Integrate[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
```

```
[Out] -((Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(1 + x) + 2*x*Sqrt[1 + x]*ArcTan[x
 - Sqrt[-1 + x]*Sqrt[1 + x]]))/(Sqrt[-1 + x]*x))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

method	result	size
risch	$-\frac{(x+1)\sqrt{\frac{x-1}{x+1}}}{x}-\frac{\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)\sqrt{\frac{x-1}{x+1}}\,\sqrt{(x-1)(x+1)}}{x-1}$	56
default	$\frac{\sqrt{\frac{x-1}{x+1}}\,(x+1)\left((x^2-1)^{\frac{3}{2}}-x^2\sqrt{x^2-1}-\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)x\right)}{\sqrt{(x-1)(x+1)}\,x}$	59
trager	$-\frac{(x+1)\sqrt{-\frac{1-x}{x+1}}}{x}+\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{1-x}{x+1}}\,x+\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{1-x}{x+1}}-1}{x}\right)$	82

```
[In] int(((x-1)/(x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -(x+1)/x*((x-1)/(x+1))^(1/2)-arctan(1/(x^2-1)^(1/2))*((x-1)/(x+1))^(1/2)*((
x-1)*(x+1))^(1/2)/(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = \frac{2\,x\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)-(x+1)\sqrt{\frac{x-1}{x+1}}}{x}$$

```
[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="fricas")
[Out] (2*x*arctan(sqrt((x - 1)/(x + 1))) - (x + 1)*sqrt((x - 1)/(x + 1)))/x
```

Sympy [F]

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = \int \frac{\sqrt{\frac{x-1}{x+1}}}{x^2}\,dx$$

```
[In] integrate(((-1+x)/(1+x))**(1/2)/x**2,x)
[Out] Integral(sqrt((x - 1)/(x + 1))/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}+1} + 2\,\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)$$

```
[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -2*sqrt((x - 1)/(x + 1))/((x - 1)/(x + 1) + 1) + 2*arctan(sqrt((x - 1)/(x +
 1)))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = -\frac{1}{2}\,(\pi-2)\mathrm{sgn}(x+1) + 2\,\arctan\left(-x+\sqrt{x^2-1}\right)\mathrm{sgn}(x+1) - \frac{2\,\mathrm{sgn}(x+1)}{\left(x-\sqrt{x^2-1}\right)^2+1}$$

```
[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(pi - 2)*sgn(x + 1) + 2*arctan(-x + sqrt(x^2 - 1))*sgn(x + 1) - 2*sgn(
x + 1)/((x - sqrt(x^2 - 1))^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = 2\,\mathrm{atan}\left(\sqrt{\frac{x-1}{x+1}}\right) - \frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}+1}$$

```
[In] int(((x - 1)/(x + 1))^(1/2)/x^2,x)
```

```
[Out] 2*atan(((x - 1)/(x + 1))^(1/2)) - (2*((x - 1)/(x + 1))^(1/2))/((x - 1)/(x +
 1) + 1)
```

3.735 $\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3\operatorname{arccosh}(x)}{8}$$

```
[Out] 3/8*arccosh(x)+1/24*(7-2*x)*(-1+x)^(3/2)*(1+x)^(1/2)+1/4*(-1+x)^(3/2)*x^2*(
1+x)^(1/2)-3/8*(-1+x)^(1/2)*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {102, 152, 52, 54}

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{3\operatorname{arccosh}(x)}{8}+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2 + \frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\sqrt{x+1}$$

```
[In] Int[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x])/24
 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/8
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{1}{4}\int\frac{(2-x)\sqrt{-1+x}x}{\sqrt{1+x}}\,dx\\
&= \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}-\frac{3}{8}\int\frac{\sqrt{-1+x}}{\sqrt{1+x}}\,dx\\
&= -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}\\
&\quad+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3}{8}\int\frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx\\
&= -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3}{8}\cosh^{-1}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}(-16-7x+x^2-2x^3+6x^4)-18\sqrt{1+x}\log\left(\sqrt{-1+x}-\sqrt{1+x}\right)\right)}{24\sqrt{-1+x}}$$

```
[In] Integrate[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(-16 - 7*x + x^2 - 2*x^3 + 6*x^4) - 1
8*Sqrt[1 + x]*Log[Sqrt[-1 + x] - Sqrt[1 + x]]))/(24*Sqrt[-1 + x])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

method	result	size
risch	$\frac{(6x^3-8x^2+9x-16)\sqrt{x-1}\,\sqrt{x+1}}{24}+\frac{3\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(x+1)}}{8\sqrt{x-1}\,\sqrt{x+1}}$	60
default	$\frac{\sqrt{x-1}\,\sqrt{x+1}\left(6x^3\sqrt{x^2-1}-8x^2\sqrt{x^2-1}+9x\sqrt{x^2-1}+9\ln\left(x+\sqrt{x^2-1}\right)-16\sqrt{x^2-1}\right)}{24\sqrt{x^2-1}}$	76

```
[In] int(x^3*(x-1)^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(6*x^3-8*x^2+9*x-16)*(x-1)^(1/2)*(x+1)^(1/2)+3/8*ln(x+(x^2-1)^(1/2))*(
(x-1)*(x+1))^(1/2)/(x-1)^(1/2)/(x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{1}{24}\left(6\,x^3-8\,x^2+9\,x-16\right)\sqrt{x+1}\sqrt{x-1}-\frac{3}{8}\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)$$

```
[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^3 - 8*x^2 + 9*x - 16)*sqrt(x + 1)*sqrt(x - 1) - 3/8*log(sqrt(x +
1)*sqrt(x - 1) - x)
```

Sympy [F]

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \int \frac{x^3\sqrt{x-1}}{\sqrt{x+1}}\,dx$$

```
[In] integrate(x**3*(-1+x)**(1/2)/(1+x)**(1/2),x)
[Out] Integral(x**3*sqrt(x - 1)/sqrt(x + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{1}{4}\left(x^2-1\right)^{\frac{3}{2}}x - \frac{1}{3}\left(x^2-1\right)^{\frac{3}{2}} + \frac{5}{8}\sqrt{x^2-1}x - \sqrt{x^2-1} + \frac{3}{8}\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

```
[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] 1/4*(x^2 - 1)^(3/2)*x - 1/3*(x^2 - 1)^(3/2) + 5/8*sqrt(x^2 - 1)*x - sqrt(x^
2 - 1) + 3/8*log(2*x + 2*sqrt(x^2 - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{1}{24}\left((2\,(3\,x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{x-1} - \frac{3}{4}\log\left(\sqrt{x+1}-\sqrt{x-1}\right)$$

```
[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] 1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(x - 1) - 3
/4*log(sqrt(x + 1) - sqrt(x - 1))
```

Mupad [B] (verification not implemented)

Time = 32.74 (sec) , antiderivative size = 473, normalized size of antiderivative = 6.86

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{3\,\mathrm{atanh}\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)}{2} + \frac{\frac{23\left(\sqrt{x-1}-\mathrm{i}\right)^3}{2\left(\sqrt{x+1}-1\right)^3} - \frac{\left(\sqrt{x-1}-\mathrm{i}\right)^4 64\mathrm{i}}{\left(\sqrt{x+1}-1\right)^4} + \frac{333\left(\sqrt{x-1}-\mathrm{i}\right)^5}{2\left(\sqrt{x+1}-1\right)^5} + \frac{\left(\sqrt{x-1}-\mathrm{i}\right)^6 256\mathrm{i}}{3\left(\sqrt{x+1}-1\right)^6} + \frac{671\left(\sqrt{x-1}-\mathrm{i}\right)^7}{2\left(\sqrt{x+1}-1\right)^7} - \frac{\left(\sqrt{x-1}-\mathrm{i}\right)^8 128\mathrm{i}}{3\left(\sqrt{x+1}-1\right)^8} + \frac{671\left(\sqrt{x-1}-\mathrm{i}\right)^9}{2\left(\sqrt{x+1}-1\right)^9} + \cdots}{1 + \frac{28\left(\sqrt{x-1}-\mathrm{i}\right)^4}{\left(\sqrt{x+1}-1\right)^4} - \frac{56\left(\sqrt{x-1}-\mathrm{i}\right)^6}{\left(\sqrt{x+1}-1\right)^6} + \frac{70\left(\sqrt{x-1}-\mathrm{i}\right)^8}{\left(\sqrt{x+1}-1\right)^8} - \frac{56\left(\sqrt{x-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{x+1}-1\right)^{10}} + \cdots}$$

```
[In] int((x^3*(x - 1)^(1/2))/(x + 1)^(1/2),x)
```

```
[Out] (3*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)))/2 + ((23*((x - 1)^(1/2)
 - 1i)^3)/(2*((x + 1)^(1/2) - 1)^3) - (((x - 1)^(1/2) - 1i)^4*64i)/((x + 1)
^(1/2) - 1)^4 + (333*((x - 1)^(1/2) - 1i)^5)/(2*((x + 1)^(1/2) - 1)^5) + ((
(x - 1)^(1/2) - 1i)^6*256i)/(3*((x + 1)^(1/2) - 1)^6) + (671*((x - 1)^(1/2)
 - 1i)^7)/(2*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^8*128i)/(3*((x
+ 1)^(1/2) - 1)^8) + (671*((x - 1)^(1/2) - 1i)^9)/(2*((x + 1)^(1/2) - 1)^9)
 + (((x - 1)^(1/2) - 1i)^10*256i)/(3*((x + 1)^(1/2) - 1)^10) + (333*((x - 1
)^(1/2) - 1i)^11)/(2*((x + 1)^(1/2) - 1)^11) - (((x - 1)^(1/2) - 1i)^12*64i
)/((x + 1)^(1/2) - 1)^12 + (23*((x - 1)^(1/2) - 1i)^13)/(2*((x + 1)^(1/2) -
 1)^13) - (3*((x - 1)^(1/2) - 1i)^15)/(2*((x + 1)^(1/2) - 1)^15) - (3*((x -
 1)^(1/2) - 1i))/(2*((x + 1)^(1/2) - 1)))/((28*((x - 1)^(1/2) - 1i)^4)/((x
+ 1)^(1/2) - 1)^4 - (8*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 - (56*
((x - 1)^(1/2) - 1i)^6)/((x + 1)^(1/2) - 1)^6 + (70*((x - 1)^(1/2) - 1i)^8)
/((x + 1)^(1/2) - 1)^8 - (56*((x - 1)^(1/2) - 1i)^10)/((x + 1)^(1/2) - 1)^1
0 + (28*((x - 1)^(1/2) - 1i)^12)/((x + 1)^(1/2) - 1)^12 - (8*((x - 1)^(1/2)
 - 1i)^14)/((x + 1)^(1/2) - 1)^14 + ((x - 1)^(1/2) - 1i)^16/((x + 1)^(1/2)
- 1)^16 + 1)
```

3.736 $\int x^3 \sqrt{\frac{-1+x}{1+x}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int x^3 \sqrt{\frac{-1+x}{1+x}} \, dx = -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x} + \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} + \frac{3\text{arccosh}(x)}{8}$$

```
[Out] 3/8*arccosh(x)+1/24*(7-2*x)*(-1+x)^(3/2)*(1+x)^(1/2)+1/4*(-1+x)^(3/2)*x^2*(
1+x)^(1/2)-3/8*(-1+x)^(1/2)*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1978, 102, 152, 52, 54}

$$\int x^3 \sqrt{\frac{-1+x}{1+x}} \, dx = \frac{3\text{arccosh}(x)}{8} + \frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2 + \frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1} - \frac{3}{8}\sqrt{x-1}\sqrt{x+1}$$

```
[In] Int[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x])/24
 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/8
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx$$

$$= \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} + \frac{1}{4}\int \frac{(2-x)\sqrt{-1+x}x}{\sqrt{1+x}}\,dx$$

$$= \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} - \frac{3}{8}\int \frac{\sqrt{-1+x}}{\sqrt{1+x}}\,dx$$

$$= -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x} + \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} + \frac{3}{8}\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx$$

$$= -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x} + \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} + \frac{3}{8}\cosh^{-1}(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx = \frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}(-16-7x+x^2-2x^3+6x^4) - 18\sqrt{1+x}\log\left(\sqrt{-1+x}-\sqrt{1+x}\right)\right)}{24\sqrt{-1+x}}$$

```
[In] Integrate[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

```
[Out] (Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(-16 - 7*x + x^2 - 2*x^3 + 6*x^4) - 1
8*Sqrt[1 + x]*Log[Sqrt[-1 + x] - Sqrt[1 + x]]))/(24*Sqrt[-1 + x])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

method	result	size
risch	$\frac{(6x^3-8x^2+9x-16)(x+1)\sqrt{\frac{x-1}{x+1}}}{24}+\frac{3\ln\left(x+\sqrt{x^2-1}\right)\sqrt{\frac{x-1}{x+1}}\,\sqrt{(x-1)(x+1)}}{8(x-1)}$	70
trager	$\frac{(x+1)(6x^3-8x^2+9x-16)\sqrt{-\frac{1-x}{x+1}}}{24}-\frac{3\ln\left(-\sqrt{-\frac{1-x}{x+1}}\,x-\sqrt{-\frac{1-x}{x+1}}+x\right)}{8}$	74
default	$\frac{\sqrt{\frac{x-1}{x+1}}\,(x+1)\left(6x(x^2-1)^{\frac{3}{2}}-8((x-1)(x+1))^{\frac{3}{2}}+15x\sqrt{x^2-1}-24\sqrt{x^2-1}+9\ln\left(x+\sqrt{x^2-1}\right)\right)}{24\sqrt{(x-1)(x+1)}}$	79

```
[In] int(x^3*((x-1)/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(6*x^3-8*x^2+9*x-16)*(x+1)*((x-1)/(x+1))^(1/2)+3/8*ln(x+(x^2-1)^(1/2))
*((x-1)/(x+1))^(1/2)*((x-1)*(x+1))^(1/2)/(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx = \frac{1}{24}\left(6\,x^4-2\,x^3+x^2-7\,x-16\right)\sqrt{\frac{x-1}{x+1}} + \frac{3}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{3}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="fricas")
[Out] 1/24*(6*x^4 - 2*x^3 + x^2 - 7*x - 16)*sqrt((x - 1)/(x + 1)) + 3/8*log(sqrt(
(x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx = \int x^{3}\sqrt{\frac{x-1}{x+1}}\,dx$$

```
[In] integrate(x**3*((-1+x)/(1+x))**(1/2),x)
[Out] Integral(x**3*sqrt((x - 1)/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(49) = 98.

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.00

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx = -\frac{39\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}}-31\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+49\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-9\sqrt{\frac{x-1}{x+1}}}{12\left(\frac{4\,(x-1)}{x+1}-\frac{6\,(x-1)^2}{(x+1)^2}+\frac{4\,(x-1)^3}{(x+1)^3}-\frac{(x-1)^4}{(x+1)^4}-1\right)} + \frac{3}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{3}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

```
[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="maxima")
[Out] -1/12*(39*((x - 1)/(x + 1))^(7/2) - 31*((x - 1)/(x + 1))^(5/2) + 49*((x - 1
)/(x + 1))^(3/2) - 9*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)^
2/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) + 3/8*log(sq
rt((x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx = -\frac{3}{8}\log\left(\left|-x+\sqrt{x^2-1}\right|\right)\operatorname{sgn}(x+1) + \frac{1}{24}\left((2\,(3\,x\operatorname{sgn}(x+1)-4\operatorname{sgn}(x+1))x+9\operatorname{sgn}(x+1))x-16\operatorname{sgn}(x+1)\right)\sqrt{x^2-1}$$

```
[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] -3/8*log(abs(-x + sqrt(x^2 - 1)))*sgn(x + 1) + 1/24*((2*(3*x*sgn(x + 1) - 4
*sgn(x + 1))*x + 9*sgn(x + 1))*x - 16*sgn(x + 1))*sqrt(x^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.72

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx = \frac{3\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4} - \frac{\frac{3\sqrt{\frac{x-1}{x+1}}}{4} - \frac{49\left(\frac{x-1}{x+1}\right)^{3/2}}{12} + \frac{31\left(\frac{x-1}{x+1}\right)^{5/2}}{12} - \frac{13\left(\frac{x-1}{x+1}\right)^{7/2}}{4}}{\frac{6\,(x-1)^2}{(x+1)^2} - \frac{4\,(x-1)}{x+1} - \frac{4\,(x-1)^3}{(x+1)^3} + \frac{(x-1)^4}{(x+1)^4} + 1}$$

```
[In] int(x^3*((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] (3*atanh(((x - 1)/(x + 1))^(1/2)))/4 - ((3*((x - 1)/(x + 1))^(1/2))/4 - (49
*((x - 1)/(x + 1))^(3/2))/12 + (31*((x - 1)/(x + 1))^(5/2))/12 - (13*((x -
1)/(x + 1))^(7/2))/4)/((6*(x - 1)^2)/(x + 1)^2 - (4*(x - 1))/(x + 1) - (4*(
x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1)
```

3.737 $\int \frac{\sqrt{-\frac{x}{1+x}}}{x} dx$

Optimal result

Integrand size = 16, antiderivative size = 15

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x} dx = 2\arctan\left(\sqrt{-\frac{x}{1+x}}\right)$$

```
[Out] 2*arctan((-x/(1+x))^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1980, 210}

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x} dx = 2\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

```
[In] Int[Sqrt[-(x/(1 + x))]/x,x]
```

```
[Out] 2*ArcTan[Sqrt[-(x/(1 + x))]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
```

```
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{-\frac{x}{1+x}}\right)\right)\\ &= 2\tan^{-1}\left(\sqrt{-\frac{x}{1+x}}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(15) = 30.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.80

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = -\frac{2\sqrt{-\frac{x}{1+x}}\sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)}{\sqrt{x}}$$

```
[In] Integrate[Sqrt[-(x/(1 + x))]/x,x]
[Out] (-2*Sqrt[-(x/(1 + x))]*Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]])/Sqrt[x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

method	result
default	$\frac{\sqrt{-\frac{x}{x+1}}\,(x+1)\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{\sqrt{(x+1)x}}$
trager	$-\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(2\sqrt{-\frac{x}{x+1}}\,x-2\operatorname{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-\frac{x}{x+1}}-\operatorname{RootOf}\left(\_Z^2+1\right)\right)$

```
[In] int((-x/(x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] (-x/(x+1))^(1/2)*(x+1)/((x+1)*x)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = 2\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

```
[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="fricas")
[Out] 2*arctan(sqrt(-x/(x + 1)))
```

Sympy [F]

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = \int \frac{\sqrt{-\frac{x}{x+1}}}{x}\,dx$$

```
[In] integrate((-x/(1+x))**(1/2)/x,x)
[Out] Integral(sqrt(-x/(x + 1))/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = 2\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

```
[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="maxima")
[Out] 2*arctan(sqrt(-x/(x + 1)))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = -\frac{1}{2}\pi\mathrm{sgn}(x+1) - \arcsin\left(2\,x+1\right)\mathrm{sgn}(x+1)$$

```
[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="giac")
[Out] -1/2*pi*sgn(x + 1) - arcsin(2*x + 1)*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x} \, dx = 2 \operatorname{atan}\left(\sqrt{-\frac{x}{x+1}}\right)$$

```
[In] int((-x/(x + 1))^(1/2)/x,x)
[Out] 2*atan((-x/(x + 1))^(1/2))
```

3.738 $\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = 2\arctan\left(\sqrt{\frac{1-x}{1+x}}\right)$$

```
[Out] 2*arctan(((1-x)/(1+x))^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1983, 210}

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = 2\arctan\left(\sqrt{\frac{1-x}{x+1}}\right)$$

```
[In] Int[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]
```

```
[Out] 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1983

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n),
Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(
b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1/
n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rubi steps

$$\begin{aligned}\text{integral}&=-\left(4\text{Subst}\left(\int\frac{1}{-2-2x^2}\,dx,x,\sqrt{\frac{1-x}{1+x}}\right)\right)\\&=2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(18) = 36.

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int\frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx=-\frac{2\sqrt{\frac{1-x}{1+x}}\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)}{-1+x}$$

```
[In] Integrate[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]
[Out] (-2*Sqrt[(1 - x)/(1 + x)]*Sqrt[1 - x^2]*ArcTan[Sqrt[1 - x^2]/(-1 + x)])/(-1
 + x)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

method	result	size
default	$-\frac{\sqrt{-\frac{x-1}{x+1}}\,(x+1)\arcsin(x)}{\sqrt{-(x-1)(x+1)}}$	30
trager	$\text{RootOf}\left(\_Z^2+1\right)\ln\left(-\text{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{x-1}{x+1}}\,x-\text{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{x-1}{x+1}}+x\right)$	52

```
[In] int(((1-x)/(x+1))^(1/2)/(x-1),x,method=_RETURNVERBOSE)
[Out] -(-(x-1)/(x+1))^(1/2)*(x+1)/(-(x-1)*(x+1))^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = 2\,\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

```
[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="fricas")
[Out] 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Sympy [F]

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = \int \frac{\sqrt{-\frac{x-1}{x+1}}}{x-1}\,dx$$

```
[In] integrate(((1-x)/(1+x))**(1/2)/(-1+x),x)
[Out] Integral(sqrt(-(x - 1)/(x + 1))/(x - 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = 2\,\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

```
[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="maxima")
[Out] 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = -\frac{1}{2}\,\pi\mathrm{sgn}(x+1) - \arcsin{(x)}\,\mathrm{sgn}(x+1)$$

```
[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="giac")
[Out] -1/2*pi*sgn(x + 1) - arcsin(x)*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 17.87 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = 2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

```
[In] int((-(x - 1)/(x + 1))^(1/2)/(x - 1),x)
[Out] 2*atan((-(x - 1)/(x + 1))^(1/2))
```

3.739 $\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 24

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx=\frac{2\arctan\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

```
[Out] 2*arctan(((b*x+a)/(-b*x+c))^(1/2))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1983, 12, 209}

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx=\frac{2\arctan\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

```
[In] Int[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]
[Out] (2*ArcTan[Sqrt[(a + b*x)/(c - b*x)]])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1983

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n),
Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(
b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1/
n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rubi steps

$$\text{integral} = (2b(a+c))\text{Subst}\left(\int \frac{1}{b^2(a+c)\,(1+x^2)}\,dx, x, \sqrt{\frac{a+bx}{c-bx}}\right)$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$
$$= \frac{2\tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 63 vs. 2(24) = 48.

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = \frac{2\sqrt{c-bx}\sqrt{\frac{a+bx}{c-bx}}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{c-bx}}\right)}{b\sqrt{a+bx}}$$

```
[In] Integrate[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]
[Out] (2*Sqrt[c - b*x]*Sqrt[(a + b*x)/(c - b*x)]*ArcTan[Sqrt[a + b*x]/Sqrt[c - b*
x]])/(b*Sqrt[a + b*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(22) = 44$.

Time = 1.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.54

method	result	size
default	$-\frac{\arctan\left(\frac{\sqrt{b^2}\,(2bx+a-c)}{2b\sqrt{-(bx+a)(bx-c)}}\right)(bx-c)\sqrt{-\frac{bx+a}{bx-c}}}{\sqrt{b^2}\,\sqrt{-(bx+a)(bx-c)}}$	85

```
[In] int(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -arctan(1/2*(b^2)^(1/2)/b*(2*b*x+a-c)/(-(b*x+a)*(b*x-c))^(1/2))*(b*x-c)*(-(
b*x+a)/(b*x-c))^(1/2)/(b^2)^(1/2)/(-(b*x+a)*(b*x-c))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = \frac{2\arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

```
[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="fricas")
[Out] 2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = \int \frac{\sqrt{\frac{a+bx}{-bx+c}}}{a+bx}\,dx$$

```
[In] integrate(((b*x+a)/(-b*x+c))**(1/2)/(b*x+a),x)
[Out] Integral(sqrt((a + b*x)/(-b*x + c))/(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = \frac{2\arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

```
[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")
[Out] 2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = -\frac{\arcsin\left(-\frac{2\,bx+a-c}{a+c}\right)\operatorname{sgn}(-ab-bc)\operatorname{sgn}(bx-c)}{|b|}$$

```
[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="giac")
[Out] -arcsin(-(2*b*x + a - c)/(a + c))*sgn(-a*b - b*c)*sgn(b*x - c)/abs(b)
```

Mupad [B] (verification not implemented)

Time = 18.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = -\frac{2\sqrt{-b}\,\mathrm{atanh}\left(\frac{\sqrt{-b}\,\sqrt{\frac{a+b\,x}{c-b\,x}}}{\sqrt{b}}\right)}{b^{3/2}}$$

```
[In] int(((a + b*x)/(c - b*x))^(1/2)/(a + b*x),x)
[Out] -(2*(-b)^(1/2)*atanh(((-b)^(1/2)*((a + b*x)/(c - b*x))^(1/2))/b^(1/2)))/b^(
3/2)
```

3.740 $\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 41

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} \, dx = \frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] 2*arctanh(d^(1/2)*((b*x+a)/(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1983, 12, 214}

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} \, dx = \frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[In] Int[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1983

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n),
Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(
b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1/
n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (2(bc-ad))\text{Subst}\left(\int \frac{1}{(bc-ad)\left(b-dx^2\right)}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)\\ &= 2\text{Subst}\left(\int \frac{1}{b-dx^2}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = \frac{2\sqrt{\frac{a+bx}{c+dx}}\sqrt{c+dx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{d}\sqrt{a+bx}}$$

```
[In] Integrate[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
[Out] (2*Sqrt[(a + b*x)/(c + d*x)]*Sqrt[c + d*x]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/
(Sqrt[d]*Sqrt[a + b*x])])/(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(31) = 62$.

Time = 1.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{\ln\left(\frac{2bdx+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}}$	80

```
[In] int(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))
*(d*x+c)*((b*x+a)/(d*x+c))^(1/2)/((b*x+a)*(d*x+c))^(1/2)/(b*d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = \left[\frac{\sqrt{bd}\log\left(2\,bdx+bc+ad+2\,\sqrt{bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}\right)}{bd}, -\frac{2\,\sqrt{-bd}\arctan\left(\frac{\sqrt{-bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{bdx+ad}\right)}{bd}\right]$$

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="fricas")
[Out] [sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x + c)*sqrt((b*x + a)/(
d*x + c)))/(b*d), -2*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*x + a)/
(d*x + c))/(b*d*x + a*d))/(b*d)]
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = \int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx$$

```
[In] integrate(((b*x+a)/(d*x+c))**(1/2)/(b*x+a),x)
[Out] Integral(sqrt((a + b*x)/(c + d*x))/(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = -\frac{\log\left(\frac{d\sqrt{\frac{bx+a}{dx+c}}-\sqrt{bd}}{d\sqrt{\frac{bx+a}{dx+c}}+\sqrt{bd}}\right)}{\sqrt{bd}}$$

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")
[Out] -log((d*sqrt((b*x + a)/(d*x + c)) - sqrt(b*d))/(d*sqrt((b*x + a)/(d*x + c))
 + sqrt(b*d)))/sqrt(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = -\frac{\log\left(\left|-bc-ad-2\sqrt{bd}\left(\sqrt{bd}x-\sqrt{bdx^2+bcx+adx+ac}\right)\right|\right)\operatorname{sgn}(dx+c)}{\sqrt{bd}}$$

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="giac")
[Out] -log(abs(-b*c - a*d - 2*sqrt(b*d)*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d
*x + a*c))))*sgn(d*x + c)/sqrt(b*d)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = \frac{2\,\operatorname{atanh}\left(\frac{\sqrt{d}\,\sqrt{\frac{a+b\,x}{c+d\,x}}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

```
[In] int(((a + b*x)/(c + d*x))^(1/2)/(a + b*x),x)
[Out] (2*atanh((d^(1/2)*((a + b*x)/(c + d*x))^(1/2))/b^(1/2)))/(b^(1/2)*d^(1/2))
```

3.741 $\int \sqrt{-\frac{x}{1+x}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \sqrt{-\frac{x}{1+x}}\,dx = \sqrt{-\frac{x}{1+x}}(1+x) - \arctan\left(\sqrt{-\frac{x}{1+x}}\right)$$

```
[Out] -arctan((-x/(1+x))^(1/2))+(1+x)*(-x/(1+x))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1979, 294, 210}

$$\int \sqrt{-\frac{x}{1+x}}\,dx = \sqrt{-\frac{x}{x+1}}(x+1) - \arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

```
[In] Int[Sqrt[-(x/(1 + x))],x]
```

```
[Out] Sqrt[-(x/(1 + x))]*(1 + x) - ArcTan[Sqrt[-(x/(1 + x))]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int \frac{x^2}{(-1-x^2)^2}\,dx, x, \sqrt{-\frac{x}{1+x}}\right)\right)\\
&= \sqrt{-\frac{x}{1+x}}(1+x) + \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{-\frac{x}{1+x}}\right)\\
&= \sqrt{-\frac{x}{1+x}}(1+x) - \tan^{-1}\left(\sqrt{-\frac{x}{1+x}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \sqrt{-\frac{x}{1+x}}\,dx = \frac{\sqrt{-\frac{x}{1+x}}\left(\sqrt{x}(1+x)+\sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)\right)}{\sqrt{x}}$$

```
[In] Integrate[Sqrt[-(x/(1 + x))],x]
[Out] (Sqrt[-(x/(1 + x))]*(Sqrt[x]*(1 + x) + Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 +
x]]))/Sqrt[x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

method	result	size
risch	$(x+1)\sqrt{-\frac{x}{x+1}}-\frac{\arcsin(1+2x)\sqrt{-\frac{x}{x+1}}\,\sqrt{-(x+1)x}}{2x}$	45
default	$\frac{\sqrt{-\frac{x}{x+1}}\,(x+1)\left(2\sqrt{x^2+x}-\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\right)}{2\sqrt{(x+1)x}}$	46
trager	$2\left(\frac{x}{2}+\frac{1}{2}\right)\sqrt{-\frac{x}{x+1}}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(2\sqrt{-\frac{x}{x+1}}\,x-2\,\mathrm{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-\frac{x}{x+1}}-\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{2}$	71

```
[In] int((-x/(x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] (x+1)*(-x/(x+1))^(1/2)-1/2*arcsin(1+2*x)*(-x/(x+1))^(1/2)/x*(-(x+1)*x)^(1/2
)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int\sqrt{-\frac{x}{1+x}}\,dx=(x+1)\sqrt{-\frac{x}{x+1}}-\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

```
[In] integrate((-x/(1+x))^(1/2),x, algorithm="fricas")
[Out] (x + 1)*sqrt(-x/(x + 1)) - arctan(sqrt(-x/(x + 1)))
```

Sympy [F]

$$\int\sqrt{-\frac{x}{1+x}}\,dx=\int\sqrt{-\frac{x}{x+1}}\,dx$$

```
[In] integrate((-x/(1+x))**(1/2),x)
[Out] Integral(sqrt(-x/(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int\sqrt{-\frac{x}{1+x}}\,dx=-\frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1}-1}-\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

```
[In] integrate((-x/(1+x))^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x/(x + 1))/(x/(x + 1) - 1) - arctan(sqrt(-x/(x + 1)))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \sqrt{-\frac{x}{1+x}}\,dx = \frac{1}{4}\,\pi\mathrm{sgn}(x+1) + \frac{1}{2}\,\arcsin\left(2\,x+1\right)\mathrm{sgn}(x+1) + \sqrt{-x^2-x}\mathrm{sgn}(x+1)$$

```
[In] integrate((-x/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x + 1) + 1/2*arcsin(2*x + 1)*sgn(x + 1) + sqrt(-x^2 - x)*sgn(x +
 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \sqrt{-\frac{x}{1+x}}\,dx = -\mathrm{atan}\left(\sqrt{-\frac{x}{x+1}}\right) - \frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

```
[In] int((-x/(x + 1))^(1/2),x)
```

```
[Out] - atan((-x/(x + 1))^(1/2)) - (-x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

3.742 $\int \sqrt{\frac{1-x}{1+x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \sqrt{\frac{1-x}{1+x}}(1+x) - 2\arctan\left(\sqrt{\frac{1-x}{1+x}}\right)$$

```
[Out] -2*arctan(((1-x)/(1+x))^(1/2))+(1+x)*((1-x)/(1+x))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 294, 210}

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \sqrt{\frac{1-x}{x+1}}(x+1) - 2\arctan\left(\sqrt{\frac{1-x}{x+1}}\right)$$

```
[In] Int[Sqrt[(1 - x)/(1 + x)],x]
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(4\text{Subst}\left(\int \frac{x^2}{(-1-x^2)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)\\ &= \sqrt{\frac{1-x}{1+x}}(1+x) + 2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\\ &= \sqrt{\frac{1-x}{1+x}}(1+x) - 2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.68

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \frac{\sqrt{\frac{1-x}{1+x}}\sqrt{1+x}\left(\sqrt{1-x^2} - 2\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)\right)}{\sqrt{1-x}}$$

```
[In] Integrate[Sqrt[(1 - x)/(1 + x)],x]
[Out] (Sqrt[(1 - x)/(1 + x)]*Sqrt[1 + x]*(Sqrt[1 - x^2] - 2*ArcTan[Sqrt[1 - x^2]/
(-1 + x)]))/Sqrt[1 - x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result
default	$\frac{\sqrt{-\frac{x-1}{x+1}}\,(x+1)\left(\sqrt{-x^{2}+1}+\arcsin\left(x\right)\right)}{\sqrt{-\left(x-1\right)\left(x+1\right)}}$
risch	$\sqrt{-\frac{x-1}{x+1}}\,(x+1)-\frac{\arcsin\left(x\right)\sqrt{-\frac{x-1}{x+1}}\,\sqrt{-\left(x-1\right)\left(x+1\right)}}{x-1}$
trager	$\sqrt{-\frac{x-1}{x+1}}\,(x+1)+\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{x-1}{x+1}}\,x+\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{x}{x}}\right.$

```
[In] int(((1-x)/(x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] (-(x-1)/(x+1))^(1/2)*(x+1)/(-(x-1)*(x+1))^(1/2)*((-x^2+1)^(1/2)+arcsin(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \sqrt{\frac{1-x}{1+x}}\,dx=(x+1)\sqrt{-\frac{x-1}{x+1}}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="fricas")
[Out] (x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Sympy [F]

$$\int \sqrt{\frac{1-x}{1+x}}\,dx=\int \sqrt{\frac{1-x}{x+1}}\,dx$$

```
[In] integrate(((1-x)/(1+x))**(1/2),x)
[Out] Integral(sqrt((1 - x)/(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} - 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-(x - 1)/(x
 + 1)))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \frac{1}{2}\pi\mathrm{sgn}(x+1) + \arcsin(x)\,\mathrm{sgn}(x+1) + \sqrt{-x^2+1}\mathrm{sgn}(x+1)$$

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="giac")
[Out] 1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right) - \frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

```
[In] int((-(x - 1)/(x + 1))^(1/2),x)
[Out] - 2*atan((-(x - 1)/(x + 1))^(1/2)) - (2*(-(x - 1)/(x + 1))^(1/2))/((x - 1)/
(x + 1) - 1)
```

3.743 $\int \sqrt{\frac{a+x}{a-x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = -\left((a-x)\sqrt{\frac{a+x}{a-x}}\right)+2a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)$$

```
[Out] 2*a*arctan(((a+x)/(a-x))^(1/2))-(a-x)*((a+x)/(a-x))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 294, 209}

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = 2a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

```
[In] Int[Sqrt[(a + x)/(a - x)],x]
[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x)]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= (4a)\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right) \\ &= -\left((a-x)\sqrt{\frac{a+x}{a-x}}\right) + (2a)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right) \\ &= -\left((a-x)\sqrt{\frac{a+x}{a-x}}\right) + 2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.60

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = \frac{\sqrt{\frac{a+x}{a-x}}\left((-a+x)\sqrt{a+x} + 2a\sqrt{a-x}\arctan\left(\frac{\sqrt{a+x}}{\sqrt{a-x}}\right)\right)}{\sqrt{a+x}}$$

```
[In] Integrate[Sqrt[(a + x)/(a - x)],x]
[Out] (Sqrt[(a + x)/(a - x)]*((-a + x)*Sqrt[a + x] + 2*a*Sqrt[a - x]*ArcTan[Sqrt[
a + x]/Sqrt[a - x]]))/Sqrt[a + x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{\sqrt{\frac{a+x}{a-x}}\,(a-x)\left(a\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)-\sqrt{a^2-x^2}\right)}{\sqrt{(a-x)(a+x)}}$	61
risch	$-\frac{(a-x)\sqrt{\frac{a+x}{a-x}}\,\sqrt{(a-x)(a+x)}}{\sqrt{-(-a+x)(a+x)}}+\frac{a\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)\sqrt{\frac{a+x}{a-x}}\,\sqrt{(a-x)(a+x)}}{a+x}$	90

```
[In] int(((a+x)/(a-x))^(1/2),x,method=_RETURNVERBOSE)
[Out] ((a+x)/(a-x))^(1/2)*(a-x)*(a*arctan(x/(a^2-x^2)^(1/2))-(a^2-x^2)^(1/2))/((a
-x)*(a+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int\sqrt{\frac{a+x}{a-x}}\,dx=2\,a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="fricas")
[Out] 2*a*arctan(sqrt((a + x)/(a - x))) - (a - x)*sqrt((a + x)/(a - x))
```

Sympy [F]

$$\int\sqrt{\frac{a+x}{a-x}}\,dx=\int\sqrt{\frac{a+x}{a-x}}\,dx$$

```
[In] integrate(((a+x)/(a-x))**(1/2),x)
[Out] Integral(sqrt((a + x)/(a - x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int\sqrt{\frac{a+x}{a-x}}\,dx=-2\,a\left(\frac{\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}-\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)\right)$$

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="maxima")
[Out] -2*a*(sqrt((a + x)/(a - x))/((a + x)/(a - x) + 1) - arctan(sqrt((a + x)/(a
- x))))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = a\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a-x)\operatorname{sgn}(a) - \sqrt{a^2-x^2}\operatorname{sgn}(a-x)$$

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="giac")
[Out] a*arcsin(x/a)*sgn(a - x)*sgn(a) - sqrt(a^2 - x^2)*sgn(a - x)
```

Mupad [B] (verification not implemented)

Time = 17.92 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = 2\,a\,\mathrm{atan}\left(\sqrt{\frac{a+x}{a-x}}\right) - \frac{2\,a\,\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}$$

```
[In] int(((a + x)/(a - x))^(1/2),x)
[Out] 2*a*atan(((a + x)/(a - x))^(1/2)) - (2*a*((a + x)/(a - x))^(1/2))/((a + x)/
(a - x) + 1)
```

3.744 $\int \sqrt{\frac{-a+x}{a+x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = \sqrt{-\frac{a-x}{a+x}}(a+x) - 2a\operatorname{arctanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

```
[Out] -2*a*arctanh(((-a+x)/(a+x))^(1/2))+(a+x)*((-a+x)/(a+x))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 294, 212}

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = \sqrt{-\frac{a-x}{a+x}}(a+x) - 2a\operatorname{arctanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

```
[In] Int[Sqrt[(-a + x)/(a + x)],x]
[Out] Sqrt[-((a - x)/(a + x))]*(a + x) - 2*a*ArcTanh[Sqrt[-((a - x)/(a + x))]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (4a)\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2}\,dx, x, \sqrt{\frac{-a+x}{a+x}}\right)\\ &= \sqrt{-\frac{a-x}{a+x}}(a+x) - (2a)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\frac{-a+x}{a+x}}\right)\\ &= \sqrt{-\frac{a-x}{a+x}}(a+x) - 2a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.63

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = \frac{\sqrt{\frac{-a+x}{a+x}}\left(\sqrt{-a+x}(a+x) - 2a\sqrt{a+x}\,\text{arctanh}\left(\frac{\sqrt{a+x}}{\sqrt{-a+x}}\right)\right)}{\sqrt{-a+x}}$$

```
[In] Integrate[Sqrt[(-a + x)/(a + x)],x]
[Out] (Sqrt[(-a + x)/(a + x)]*(Sqrt[-a + x]*(a + x) - 2*a*Sqrt[a + x]*ArcTanh[Sqr
t[a + x]/Sqrt[-a + x]]))/Sqrt[-a + x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.51

method	result	size
default	$-\frac{\sqrt{-\frac{a-x}{a+x}}\,(a+x)\left(a\ln\left(x+\sqrt{-a^{2}+x^{2}}\right)-\sqrt{-a^{2}+x^{2}}\right)}{\sqrt{-(a-x)(a+x)}}$	62
risch	$\frac{(a+x)\sqrt{-\frac{a-x}{a+x}}\,\sqrt{-(a-x)(a+x)}}{\sqrt{(-a+x)(a+x)}}+\frac{a\ln\left(x+\sqrt{-a^{2}+x^{2}}\right)\sqrt{-\frac{a-x}{a+x}}\,\sqrt{-(a-x)(a+x)}}{a-x}$	92

```
[In] int(((-a+x)/(a+x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-(a-x)/(a+x))^(1/2)*(a+x)*(a*ln(x+(-a^2+x^2)^(1/2))-(-a^2+x^2)^(1/2))/(-(
a-x)*(a+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int\sqrt{\frac{-a+x}{a+x}}\,dx=-a\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+a\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)+(a+x)\sqrt{-\frac{a-x}{a+x}}$$

```
[In] integrate(((-a+x)/(a+x))^(1/2),x, algorithm="fricas")
[Out] -a*log(sqrt(-(a - x)/(a + x)) + 1) + a*log(sqrt(-(a - x)/(a + x)) - 1) + (a
 + x)*sqrt(-(a - x)/(a + x))
```

Sympy [F]

$$\int\sqrt{\frac{-a+x}{a+x}}\,dx=\int\sqrt{\frac{-a+x}{a+x}}\,dx$$

```
[In] integrate(((-a+x)/(a+x))**(1/2),x)
[Out] Integral(sqrt((-a + x)/(a + x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.71

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = a\left(\frac{2\sqrt{-\frac{a-x}{a+x}}}{\frac{a-x}{a+x}+1} - \log\left(\sqrt{-\frac{a-x}{a+x}}+1\right) + \log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)\right)$$

```
[In] integrate(((-a+x)/(a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] a*(2*sqrt(-(a - x)/(a + x))/((a - x)/(a + x) + 1) - log(sqrt(-(a - x)/(a +
x)) + 1) + log(sqrt(-(a - x)/(a + x)) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = a\log\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)\mathrm{sgn}(a+x) + \sqrt{-a^2+x^2}\,\mathrm{sgn}(a+x)$$

```
[In] integrate(((-a+x)/(a+x))^(1/2),x, algorithm="giac")
```

```
[Out] a*log(abs(-x + sqrt(-a^2 + x^2)))*sgn(a + x) + sqrt(-a^2 + x^2)*sgn(a + x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = \frac{2\,a\,\sqrt{-\frac{a-x}{a+x}}}{\frac{a-x}{a+x}+1} - 2\,a\,\mathrm{atanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

```
[In] int((-(a - x)/(a + x))^(1/2),x)
```

```
[Out] (2*a*(-(a - x)/(a + x))^(1/2))/((a - x)/(a + x) + 1) - 2*a*atanh((-(a - x)/
(a + x))^(1/2))
```

3.745 $\int \sqrt{\frac{a+bx}{c+dx}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx=\frac{\sqrt{\frac{a+bx}{c+dx}}(c+dx)}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}d^{3/2}}$$

```
[Out] -(-a*d+b*c)*arctanh(d^(1/2)*((b*x+a)/(d*x+c))^(1/2)/b^(1/2))/d^(3/2)/b^(1/2
)+(d*x+c)*((b*x+a)/(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1979, 294, 214}

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx=\frac{(c+dx)\sqrt{\frac{a+bx}{c+dx}}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}d^{3/2}}$$

```
[In] Int[Sqrt[(a + b*x)/(c + d*x)],x]
```

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*(c + d*x))/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqr
t[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*d^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (2(bc-ad))\text{Subst}\left(\int \frac{x^2}{(b-dx^2)^2}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)\\ &= \frac{\sqrt{\frac{a+bx}{c+dx}}(c+dx)}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{b-dx^2}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)}{d}\\ &= \frac{\sqrt{\frac{a+bx}{c+dx}}(c+dx)}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}d^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = \frac{\sqrt{\frac{a+bx}{c+dx}}\left(\sqrt{d}(c+dx) + \frac{(-bc+ad)\sqrt{c+dx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{a+bx}}\right)}{d^{3/2}}$$

```
[In] Integrate[Sqrt[(a + b*x)/(c + d*x)],x]
```

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*(Sqrt[d]*(c + d*x) + ((-(b*c) + a*d)*Sqrt[c + d*
x]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(Sqrt[b]*Sqrt[
a + b*x])))/d^(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(64) = 128$.

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.00

method	result	siz
default	$\frac{\sqrt{\frac{bx+a}{dx+c}}\,(dx+c)\left(\ln\left(\frac{2bdx+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)ad-\ln\left(\frac{2bdx+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)bc+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}\right)}{2\sqrt{(bx+a)(dx+c)}\,d\sqrt{bd}}$	15

```
[In] int(((b*x+a)/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((b*x+a)/(d*x+c))^(1/2)*(d*x+c)*(ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1
/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*d-ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c
))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b*c+2*((b*x+a)*(d*x+c))^(1/2)*(b
*d)^(1/2))/((b*x+a)*(d*x+c))^(1/2)/d/(b*d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.37

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = \left[-\frac{(bc-ad)\sqrt{bd}\log\left(2\,bdx+bc+ad+2\,\sqrt{bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}\right)-2\,(bd^2x+bcd)\sqrt{\frac{bx+a}{dx+c}}}{2\,bd^2}, \frac{(bc-ad)\sqrt{-bd}\,\mathrm{ar}}{}\right.$$

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c - a*d)*sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x + c
)*sqrt((b*x + a)/(d*x + c))) - 2*(b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x + c)
))/(b*d^2), ((b*c - a*d)*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*x +
 a)/(d*x + c))/(b*d*x + a*d)) + (b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x + c))
)/(b*d^2)]
```

Sympy [F]

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = \int \sqrt{\frac{a+bx}{c+dx}}\,dx$$

```
[In] integrate(((b*x+a)/(d*x+c))**(1/2),x)
[Out] Integral(sqrt((a + b*x)/(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.55

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = \frac{(bc-ad)\sqrt{\frac{bx+a}{dx+c}}}{bd-\frac{(bx+a)d^2}{dx+c}} + \frac{(bc-ad)\log\left(\frac{d\sqrt{\frac{bx+a}{dx+c}}-\sqrt{bd}}{d\sqrt{\frac{bx+a}{dx+c}}+\sqrt{bd}}\right)}{2\sqrt{bd}d}$$

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="maxima")
[Out] (b*c - a*d)*sqrt((b*x + a)/(d*x + c))/(b*d - (b*x + a)*d^2/(d*x + c)) + 1/2
*(b*c - a*d)*log((d*sqrt((b*x + a)/(d*x + c)) - sqrt(b*d))/(d*sqrt((b*x + a
)/(d*x + c)) + sqrt(b*d)))/(sqrt(b*d)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.43

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = \frac{(bc\,\mathrm{sgn}(dx+c)-ad\,\mathrm{sgn}(dx+c))\log\left(\left|-bc-ad-2\sqrt{bd}\left(\sqrt{bd}x-\sqrt{bdx^2+bcx+adx+ac}\right)\right|\right)}{2\sqrt{bd}d} + \frac{\sqrt{bdx^2+bcx+adx+ac}\,\mathrm{sgn}(dx+c)}{d}$$

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="giac")
[Out] 1/2*(b*c*sgn(d*x + c) - a*d*sgn(d*x + c))*log(abs(-b*c - a*d - 2*sqrt(b*d)*
(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))))/(sqrt(b*d)*d) + sqrt(
b*d*x^2 + b*c*x + a*d*x + a*c)*sgn(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.18

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx=\frac{\mathrm{atanh}\left(\frac{\sqrt{d}\,\sqrt{\frac{a+b\,x}{c+d\,x}}}{\sqrt{b}}\right)\,\left(a\,d-b\,c\right)}{\sqrt{b}\,d^{3/2}}+\frac{\left(a\,d-b\,c\right)\,\sqrt{\frac{a+b\,x}{c+d\,x}}}{b\,d\,\left(\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}-1\right)}$$

```
[In] int(((a + b*x)/(c + d*x))^(1/2),x)
```

```
[Out] (atanh((d^(1/2)*((a + b*x)/(c + d*x))^(1/2))/b^(1/2))*(a*d - b*c))/(b^(1/2)
*d^(3/2)) + ((a*d - b*c)*((a + b*x)/(c + d*x))^(1/2))/(b*d*((d*(a + b*x))/(
b*(c + d*x)) - 1))
```

3.746 $\int \sqrt{\frac{-1+x}{5+3x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = \frac{1}{3}\sqrt{-1+x}\sqrt{5+3x}-\frac{8\text{arcsinh}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{-1+x}\right)}{3\sqrt{3}}$$

```
[Out] -8/9*arcsinh(1/4*6^(1/2)*(-1+x)^(1/2))*3^(1/2)+1/3*(-1+x)^(1/2)*(5+3*x)^(1/
2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1978, 52, 56, 221}

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = \frac{1}{3}\sqrt{x-1}\sqrt{3x+5}-\frac{8\text{arcsinh}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{x-1}\right)}{3\sqrt{3}}$$

```
[In] Int[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

```
[Out] (Sqrt[-1 + x]*Sqrt[5 + 3*x])/3 - (8*ArcSinh[(Sqrt[3/2]*Sqrt[-1 + x])/2])/(3
*Sqrt[3])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt{-1+x}}{\sqrt{5+3x}}\,dx\\ &= \frac{1}{3}\sqrt{-1+x}\sqrt{5+3x} - \frac{4}{3}\int \frac{1}{\sqrt{-1+x}\sqrt{5+3x}}\,dx\\ &= \frac{1}{3}\sqrt{-1+x}\sqrt{5+3x} - \frac{8}{3}\text{Subst}\left(\int \frac{1}{\sqrt{8+3x^2}}\,dx, x, \sqrt{-1+x}\right)\\ &= \frac{1}{3}\sqrt{-1+x}\sqrt{5+3x} - \frac{8\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{-1+x}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = \frac{\sqrt{\frac{-1+x}{5+3x}}\left(3\sqrt{-1+x}(5+3x) - 8\sqrt{15+9x}\operatorname{arctanh}\left(\frac{\sqrt{5+3x}}{\sqrt{3}\sqrt{-1+x}}\right)\right)}{9\sqrt{-1+x}}$$

```
[In] Integrate[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

```
[Out] (Sqrt[(-1 + x)/(5 + 3*x)]*(3*Sqrt[-1 + x]*(5 + 3*x) - 8*Sqrt[15 + 9*x]*ArcT
anh[Sqrt[5 + 3*x]/(Sqrt[3]*Sqrt[-1 + x])]))/(9*Sqrt[-1 + x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(31) = 62$.

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.55

method	result	si
default	$-\frac{\sqrt{\frac{x-1}{5+3x}}\,(5+3x)\left(4\ln\left(x\sqrt{3}+\frac{\sqrt{3}}{3}+\sqrt{3x^{2}+2x-5}\right)\sqrt{3}-3\sqrt{3x^{2}+2x-5}\right)}{9\sqrt{(5+3x)(x-1)}}$	7
risch	$\frac{(5+3x)\sqrt{\frac{x-1}{5+3x}}}{3}-\frac{4\ln\left(\frac{(1+3x)\sqrt{3}}{3}+\sqrt{3x^{2}+2x-5}\right)\sqrt{3}\,\sqrt{\frac{x-1}{5+3x}}\,\sqrt{(5+3x)(x-1)}}{9(x-1)}$	8
trager	$5\left(\frac{1}{3}+\frac{x}{5}\right)\sqrt{-\frac{1-x}{5+3x}}+\frac{4\,\mathrm{RootOf}\left(\_Z^{2}-3\right)\ln\left(9\sqrt{-\frac{1-x}{5+3x}}\,x-3\,\mathrm{RootOf}\left(\_Z^{2}-3\right)x+15\sqrt{-\frac{1-x}{5+3x}}-\mathrm{RootOf}\left(\_Z^{2}-3\right)\right)}{9}$	8

```
[In] int(((x-1)/(5+3*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*((x-1)/(5+3*x))^(1/2)*(5+3*x)*(4*ln(x*3^(1/2)+1/3*3^(1/2)+(3*x^2+2*x-5
)^(1/2))*3^(1/2)-3*(3*x^2+2*x-5)^(1/2))/((5+3*x)*(x-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int\sqrt{\frac{-1+x}{5+3x}}\,dx=\frac{1}{3}\,(3\,x+5)\sqrt{\frac{x-1}{3\,x+5}}+\frac{4}{9}\,\sqrt{3}\log\left(\sqrt{3}(3\,x+5)\sqrt{\frac{x-1}{3\,x+5}}-3\,x-1\right)$$

```
[In] integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x + 5)*sqrt((x - 1)/(3*x + 5)) + 4/9*sqrt(3)*log(sqrt(3)*(3*x + 5)*s
qrt((x - 1)/(3*x + 5)) - 3*x - 1)
```

Sympy [F]

$$\int\sqrt{\frac{-1+x}{5+3x}}\,dx=\int\sqrt{\frac{x-1}{3x+5}}\,dx$$

```
[In] integrate(((-1+x)/(5+3*x))**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)/(3*x + 5)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(31) = 62$.

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = \frac{4}{9}\sqrt{3}\log\left(-\frac{\sqrt{3}-3\sqrt{\frac{x-1}{3x+5}}}{\sqrt{3}+3\sqrt{\frac{x-1}{3x+5}}}\right) - \frac{8\sqrt{\frac{x-1}{3x+5}}}{3\left(\frac{3(x-1)}{3x+5}-1\right)}$$

```
[In] integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="maxima")
[Out] 4/9*sqrt(3)*log(-(sqrt(3) - 3*sqrt((x - 1)/(3*x + 5)))/(sqrt(3) + 3*sqrt((x
 - 1)/(3*x + 5)))) - 8/3*sqrt((x - 1)/(3*x + 5))/(3*(x - 1)/(3*x + 5) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.

Time = 0.39 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\begin{aligned}\int \sqrt{\frac{-1+x}{5+3x}}\,dx = & -\frac{8}{9}\sqrt{3}\log(2)\operatorname{sgn}(3x+5) \\ & + \frac{4}{9}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2x-5}\right)-1\right|\right)\operatorname{sgn}(3x+5) \\ & + \frac{1}{3}\sqrt{3x^2+2x-5}\operatorname{sgn}(3x+5)\end{aligned}$$

```
[In] integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="giac")
[Out] -8/9*sqrt(3)*log(2)*sgn(3*x + 5) + 4/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)*x
- sqrt(3*x^2 + 2*x - 5)) - 1))*sgn(3*x + 5) + 1/3*sqrt(3*x^2 + 2*x - 5)*sgn
(3*x + 5)
```

Mupad [B] (verification not implemented)

Time = 18.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = -\frac{8\sqrt{3}\operatorname{atanh}\left(\sqrt{3}\sqrt{\frac{x-1}{3x+5}}\right)}{9} - \frac{8\sqrt{\frac{x-1}{3x+5}}}{3\left(\frac{3x-3}{3x+5}-1\right)}$$

```
[In] int(((x - 1)/(3*x + 5))^(1/2),x)
[Out] - (8*3^(1/2)*atanh(3^(1/2)*((x - 1)/(3*x + 5))^(1/2)))/9 - (8*((x - 1)/(3*x
 + 5))^(1/2))/(3*((3*x - 3)/(3*x + 5) - 1))
```

3.747 $\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2} dx = -\frac{\sqrt{-1+5x}\sqrt{1+7x}}{x} - 12\arctan\left(\frac{\sqrt{1+7x}}{\sqrt{-1+5x}}\right)$$

```
[Out] -12*arctan((1+7*x)^(1/2)/(-1+5*x)^(1/2))-(-1+5*x)^(1/2)*(1+7*x)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1978, 96, 95, 210}

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2} dx = -12\arctan\left(\frac{\sqrt{7x+1}}{\sqrt{5x-1}}\right) - \frac{\sqrt{5x-1}\sqrt{7x+1}}{x}$$

```
[In] Int[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

```
[Out] -((Sqrt[-1 + 5*x]*Sqrt[1 + 7*x])/x) - 12*ArcTan[Sqrt[1 + 7*x]/Sqrt[-1 + 5*x
]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt{-1+5x}}{x^2\sqrt{1+7x}}\,dx\\ &= -\frac{\sqrt{-1+5x}\sqrt{1+7x}}{x}+6\int \frac{1}{x\sqrt{-1+5x}\sqrt{1+7x}}\,dx\\ &= -\frac{\sqrt{-1+5x}\sqrt{1+7x}}{x}+12\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\frac{\sqrt{1+7x}}{\sqrt{-1+5x}}\right)\\ &= -\frac{\sqrt{-1+5x}\sqrt{1+7x}}{x}-12\tan^{-1}\left(\frac{\sqrt{1+7x}}{\sqrt{-1+5x}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = -\frac{\sqrt{\frac{-1+5x}{1+7x}}\left(\sqrt{-1+5x}(1+7x)+12x\sqrt{1+7x}\arctan\left(\sqrt{35}x-\sqrt{-1+5x}\sqrt{1+7x}\right)\right)}{x\sqrt{-1+5x}}$$

```
[In] Integrate[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
[Out] -((Sqrt[(-1 + 5*x)/(1 + 7*x)]*(Sqrt[-1 + 5*x]*(1 + 7*x) + 12*x*Sqrt[1 + 7*x
]*ArcTan[Sqrt[35]*x - Sqrt[-1 + 5*x]*Sqrt[1 + 7*x]]))/(x*Sqrt[-1 + 5*x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(38) = 76$.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

method	result
risch	$-\frac{(1+7x)\sqrt{\frac{-1+5x}{1+7x}}}{x}+\frac{6\arctan\left(\frac{-2x-2}{2\sqrt{35x^2-2x-1}}\right)\sqrt{\frac{-1+5x}{1+7x}}\,\sqrt{(-1+5x)(1+7x)}}{-1+5x}$
trager	$-\frac{(1+7x)\sqrt{-\frac{1-5x}{1+7x}}}{x}+6\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{7\sqrt{-\frac{1-5x}{1+7x}}\,x+\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-\frac{1-5x}{1+7x}}+\mathrm{RootOf}\left(\_Z^2+1\right)}{x}\right)$
default	$-\frac{\sqrt{\frac{-1+5x}{1+7x}}\,(1+7x)\left(-(35x^2-2x-1)^{\frac{3}{2}}+35\sqrt{35x^2-2x-1}\,x^2+6\arctan\left(\frac{x+1}{\sqrt{35x^2-2x-1}}\right)x-2\sqrt{35x^2-2x-1}\,x\right)}{\sqrt{(-1+5x)(1+7x)}\,x}$

```
[In] int(((-1+5*x)/(1+7*x))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(1+7*x)/x*((-1+5*x)/(1+7*x))^(1/2)+6*arctan(1/2*(-2*x-2)/(35*x^2-2*x-1)^(1
/2))*((-1+5*x)/(1+7*x))^(1/2)*((-1+5*x)*(1+7*x))^(1/2)/(-1+5*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx=\frac{12\,x\arctan\left(\sqrt{\frac{5\,x-1}{7\,x+1}}\right)-(7\,x+1)\sqrt{\frac{5\,x-1}{7\,x+1}}}{x}$$

```
[In] integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (12*x*arctan(sqrt((5*x - 1)/(7*x + 1))) - (7*x + 1)*sqrt((5*x - 1)/(7*x + 1
)))/x
```

Sympy [F]

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx=\int \frac{\sqrt{\frac{5x-1}{7x+1}}}{x^2}\,dx$$

```
[In] integrate(((-1+5*x)/(1+7*x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((5*x - 1)/(7*x + 1))/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = -\frac{12\sqrt{\frac{5x-1}{7x+1}}}{\frac{5x-1}{7x+1}+1} + 12\arctan\left(\sqrt{\frac{5x-1}{7x+1}}\right)$$

```
[In] integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -12*sqrt((5*x - 1)/(7*x + 1))/((5*x - 1)/(7*x + 1) + 1) + 12*arctan(sqrt((5
*x - 1)/(7*x + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(38) = 76.

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.48

$$\begin{aligned}\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx &= \left(\sqrt{35} - 12\arctan\left(\frac{1}{7}\sqrt{35}\right)\right)\mathrm{sgn}(7x+1) \\ &\quad + 12\arctan\left(-\sqrt{35}x + \sqrt{35x^2-2x-1}\right)\mathrm{sgn}(7x+1) \\ &\quad - \frac{2\left(\left(\sqrt{35}x - \sqrt{35x^2-2x-1}\right)\mathrm{sgn}(7x+1) + \sqrt{35}\,\mathrm{sgn}(7x+1)\right)}{\left(\sqrt{35}x - \sqrt{35x^2-2x-1}\right)^2+1}\end{aligned}$$

```
[In] integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (sqrt(35) - 12*arctan(1/7*sqrt(35)))*sgn(7*x + 1) + 12*arctan(-sqrt(35)*x +
 sqrt(35*x^2 - 2*x - 1))*sgn(7*x + 1) - 2*((sqrt(35)*x - sqrt(35*x^2 - 2*x
- 1))*sgn(7*x + 1) + sqrt(35)*sgn(7*x + 1))/((sqrt(35)*x - sqrt(35*x^2 - 2*
x - 1))^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = 12\,\mathrm{atan}\left(\frac{\sqrt{5}\sqrt{7}\sqrt{35}\sqrt{\frac{5x-1}{7x+1}}}{35}\right) - \frac{12\sqrt{5}\sqrt{7}\sqrt{35}\sqrt{\frac{5x-1}{7x+1}}}{25\left(\frac{7x-\frac{7}{5}}{7x+1}+\frac{7}{5}\right)}$$

```
[In] int(((5*x - 1)/(7*x + 1))^(1/2)/x^2,x)
```

```
[Out] 12*atan((5^(1/2)*7^(1/2)*35^(1/2)*((5*x - 1)/(7*x + 1))^(1/2))/35) - (12*5^
(1/2)*7^(1/2)*35^(1/2)*((5*x - 1)/(7*x + 1))^(1/2))/(25*((7*x - 7/5)/(7*x +
 1) + 7/5))
```

3.748 $\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 20

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = -\sqrt{\frac{1-x}{1+x}}(1+x)$$

```
[Out] -(1+x)*((1-x)/(1+x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1984, 12, 391}

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = -\sqrt{\frac{1-x}{x+1}}(x+1)$$

```
[In] Int[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]
```

```
[Out] -(Sqrt[(1 - x)/(1 + x)]*(1 + x))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 1984

```
Int[(u_)^(r_.)*(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)
*(x_)^(n_.)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c
- a*d)/n), Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^
((m + 1)/n - 1)/(b*e - d*x^q)^((m + 1)/n + 1))*(u /. x -> ((-a)*e + c*x^q)^
(1/n)/(b*e - d*x^q)^(1/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q
)], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] &
& IntegerQ[1/n] && IntegersQ[m, r]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(4\text{Subst}\left(\int \frac{1-x^2}{2(1+x^2)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)\\ &= -\left(2\text{Subst}\left(\int \frac{1-x^2}{(1+x^2)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)\\ &= -\sqrt{\frac{1-x}{1+x}}(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = \frac{-1+x}{\sqrt{\frac{1-x}{1+x}}}$$

```
[In] Integrate[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]
[Out] (-1 + x)/Sqrt[(1 - x)/(1 + x)]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$	17
risch	$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$	17
trager	$(-x-1)\sqrt{-\frac{x-1}{x+1}}$	19
default	$\frac{(x-1)\sqrt{-x^2+1}}{\sqrt{-\frac{x-1}{x+1}}\,\sqrt{-(x-1)(x+1)}}$	36

```
[In] int(x/(x+1)/((1-x)/(x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] (x-1)/(-(x-1)/(x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = -(x+1)\sqrt{-\frac{x-1}{x+1}}$$

```
[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="fricas")
[Out] -(x + 1)*sqrt(-(x - 1)/(x + 1))
```

Sympy [F]

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = \int \frac{x}{\sqrt{-\frac{x-1}{x+1}}\,(x+1)}\,dx$$

```
[In] integrate(x/(1+x)/((1-x)/(1+x))**(1/2),x)
[Out] Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = \frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

```
[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = -\frac{\sqrt{-x^2+1}}{\operatorname{sgn}(x+1)}$$

```
[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 1)/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = -\sqrt{-\frac{x-1}{x+1}}\,(x+1)$$

```
[In] int(x/((-(x - 1)/(x + 1))^(1/2)*(x + 1)),x)
[Out] -(-(x - 1)/(x + 1))^(1/2)*(x + 1)
```

3.749 $\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 18

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = -\left((1+x)\sqrt{-1+\frac{2}{1+x}}\right)$$

```
[Out] -(1+x)*(-1+2/(1+x))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {526, 528, 382, 75}

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = -\left((x+1)\sqrt{\frac{2}{x+1}-1}\right)$$

```
[In] Int[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]
```

```
[Out] -((1 + x)*Sqrt[-1 + 2/(1 + x)])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 526

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_.)*((c_.) + (d_.)*(v_)^(n_))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/Coefficient[v, x, 1]^(m + 1), Subst[Int[SimplifyIn
tegrand[(x - Coefficient[v, x, 0])^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x], x
, v], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[v, x] && IntegerQ[m]
 && NeQ[v, x]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{-1+x}{\sqrt{-1+\frac{2}{x}}x}\,dx, x, 1+x\right)\\
&= \text{Subst}\left(\int \frac{1-\frac{1}{x}}{\sqrt{-1+\frac{2}{x}}}\,dx, x, 1+x\right)\\
&= -\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{-1+2x}}\,dx, x, \frac{1}{1+x}\right)\\
&= -\left((1+x)\sqrt{-1+\frac{2}{1+x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$
\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = \frac{-1+x}{\sqrt{\frac{1-x}{1+x}}}
$$

```
[In] Integrate[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]
[Out] (-1 + x)/Sqrt[(1 - x)/(1 + x)]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$	17
risch	$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$	17
trager	$(-x-1)\sqrt{-\frac{x-1}{x+1}}$	19
default	$-\frac{\sqrt{-\frac{x-1}{x+1}}\,(x+1)\sqrt{-x^{2}+1}}{\sqrt{-(x-1)(x+1)}}$	37

```
[In] int(x/(x+1)/(-1+2/(x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] (x-1)/(-(x-1)/(x+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = -(x+1)\sqrt{-\frac{x-1}{x+1}}$$

```
[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="fricas")
[Out] -(x + 1)*sqrt(-(x - 1)/(x + 1))
```

Sympy [F]

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = \int \frac{x}{\sqrt{-\frac{x-1}{x+1}}\,(x+1)}\,dx$$

```
[In] integrate(x/(1+x)/(-1+2/(1+x))**(1/2),x)
[Out] Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = \frac{\sqrt{x+1}(x-1)}{\sqrt{-x+1}}$$

```
[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="maxima")
[Out] sqrt(x + 1)*(x - 1)/sqrt(-x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = -\frac{\sqrt{-x^2+1}}{\operatorname{sgn}(x+1)}$$

```
[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 1)/sgn(x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = \int \frac{x}{(x+1)\,\sqrt{\frac{2}{x+1}-1}}\,dx$$

```
[In] int(x/((x + 1)*(2/(x + 1) - 1)^(1/2)),x)
[Out] int(x/((x + 1)*(2/(x + 1) - 1)^(1/2)), x)
```

3.750 $\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}} dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}} dx = \sqrt{2+x}\sqrt{3+x} - \operatorname{arcsinh}\left(\sqrt{2+x}\right) + 2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{2+x}}{\sqrt{3+x}}\right)$$

```
[Out] -arcsinh((2+x)^(1/2))+2*arctanh(2^(1/2)*(2+x)^(1/2)/(3+x)^(1/2))*2^(1/2)+(2
+x)^(1/2)*(3+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1978, 159, 163, 56, 221, 95, 213}

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}} dx = -\operatorname{arcsinh}\left(\sqrt{x+2}\right) + 2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x+2}}{\sqrt{x+3}}\right) + \sqrt{x+2}\sqrt{x+3}$$

```
[In] Int[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

```
[Out] Sqrt[2 + x]*Sqrt[3 + x] - ArcSinh[Sqrt[2 + x]] + 2*Sqrt[2]*ArcTanh[(Sqrt[2]
*Sqrt[2 + x])/Sqrt[3 + x]]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\text{integral} = \int \frac{x\sqrt{3+x}}{(1+x)\sqrt{2+x}}\,dx$$

$$= \sqrt{2+x}\sqrt{3+x} + \int \frac{-\frac{5}{2}-\frac{x}{2}}{(1+x)\sqrt{2+x}\sqrt{3+x}}\,dx$$

$$= \sqrt{2+x}\sqrt{3+x} - \frac{1}{2}\int \frac{1}{\sqrt{2+x}\sqrt{3+x}}\,dx - 2\int \frac{1}{(1+x)\sqrt{2+x}\sqrt{3+x}}\,dx$$

$$= \sqrt{2+x}\sqrt{3+x} - 4\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \frac{\sqrt{2+x}}{\sqrt{3+x}}\right) - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2+x}\right)$$

$$= \sqrt{2+x}\sqrt{3+x} - \sinh^{-1}\left(\sqrt{2+x}\right) + 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2+x}}{\sqrt{3+x}}\right)$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = \sqrt{2+x}\sqrt{3+x} + 2\sqrt{2}\text{arctanh}\left(\frac{-1-x+\sqrt{2+x}\sqrt{3+x}}{\sqrt{2}}\right) + \log\left(\sqrt{2+x}-\sqrt{3+x}\right)$$

```
[In] Integrate[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
[Out] Sqrt[2 + x]*Sqrt[3 + x] + 2*Sqrt[2]*ArcTanh[(-1 - x + Sqrt[2 + x]*Sqrt[3 +
x])/Sqrt[2]] + Log[Sqrt[2 + x] - Sqrt[3 + x]]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.46

method	result
default	$-\frac{(x+2)\left(-2\sqrt{2}\,\text{arctanh}\left(\frac{(7+3x)\sqrt{2}}{4\sqrt{x^2+5x+6}}\right)+\ln\left(\frac{5}{2}+x+\sqrt{x^2+5x+6}\right)-2\sqrt{x^2+5x+6}\right)}{2\sqrt{\frac{x+2}{3+x}}\,\sqrt{(3+x)(x+2)}}$
risch	$\frac{x+2}{\sqrt{\frac{x+2}{3+x}}}+\frac{\left(-\frac{\ln\left(\frac{5}{2}+x+\sqrt{x^2+5x+6}\right)}{2}+\sqrt{2}\,\text{arctanh}\left(\frac{(7+3x)\sqrt{2}}{4\sqrt{(x+1)^2+3x+5}}\right)\right)\sqrt{(3+x)(x+2)}}{\sqrt{\frac{x+2}{3+x}}\,(3+x)}$
trager	$3\left(1+\frac{x}{3}\right)\sqrt{-\frac{-x-2}{3+x}}-\frac{\ln\left(2\sqrt{-\frac{-x-2}{3+x}}\,x+6\sqrt{-\frac{-x-2}{3+x}}+2x+5\right)}{2}+\text{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{4\sqrt{-\frac{-x-2}{3+x}}\,x+3\,\text{RootOf}\left(\_\right.}{}\right.$

```
[In] int(x/(x+1)/((x+2)/(3+x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(x+2)*(-2*2^(1/2)*arctanh(1/4*(7+3*x)*2^(1/2)/(x^2+5*x+6)^(1/2))+ln(5/
2+x+(x^2+5*x+6)^(1/2))-2*(x^2+5*x+6)^(1/2))/((x+2)/(3+x))^(1/2)/((3+x)*(x+2
))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(40) = 80$.

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = (x+3)\sqrt{\frac{x+2}{x+3}} + \sqrt{2}\log\left(\frac{2\sqrt{2}(x+3)\sqrt{\frac{x+2}{x+3}} + 3x + 7}{x+1}\right) - \frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}} + 1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}} - 1\right)$$

```
[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="fricas")
[Out] (x + 3)*sqrt((x + 2)/(x + 3)) + sqrt(2)*log((2*sqrt(2)*(x + 3)*sqrt((x + 2)
/(x + 3)) + 3*x + 7)/(x + 1)) - 1/2*log(sqrt((x + 2)/(x + 3)) + 1) + 1/2*lo
g(sqrt((x + 2)/(x + 3)) - 1)
```

Sympy [F]

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = \int \frac{x}{\sqrt{\frac{x+2}{x+3}}\,(x+1)}\,dx$$

```
[In] integrate(x/(1+x)/((2+x)/(3+x))**(1/2),x)
[Out] Integral(x/(sqrt((x + 2)/(x + 3))*(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.91

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = -\sqrt{2}\log\left(-\frac{\sqrt{2} - 2\sqrt{\frac{x+2}{x+3}}}{\sqrt{2} + 2\sqrt{\frac{x+2}{x+3}}}\right) - \frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3} - 1} - \frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}} + 1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}} - 1\right)$$

```
[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="maxima")
[Out] -sqrt(2)*log(-(sqrt(2) - 2*sqrt((x + 2)/(x + 3)))/(sqrt(2) + 2*sqrt((x + 2)
/(x + 3)))) - sqrt((x + 2)/(x + 3))/((x + 2)/(x + 3) - 1) - 1/2*log(sqrt((x
 + 2)/(x + 3)) + 1) + 1/2*log(sqrt((x + 2)/(x + 3)) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(40) = 80$.

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.39

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = \sqrt{2}\log\left(-\frac{\sqrt{2}-2}{\sqrt{2}+2}\right)\operatorname{sgn}(x+3) - \frac{\sqrt{2}\log\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+5\,x+6}-2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+5\,x+6}-2\right|}\right)}{\operatorname{sgn}(x+3)} + \frac{\log\left(\left|-2\,x+2\,\sqrt{x^2+5\,x+6}-5\right|\right)}{2\operatorname{sgn}(x+3)} + \frac{\sqrt{x^2+5\,x+6}}{\operatorname{sgn}(x+3)}$$

```
[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="giac")
[Out] sqrt(2)*log(-(sqrt(2) - 2)/(sqrt(2) + 2))*sgn(x + 3) - sqrt(2)*log(abs(-2*x
 - 2*sqrt(2) + 2*sqrt(x^2 + 5*x + 6) - 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2
 + 5*x + 6) - 2))/sgn(x + 3) + 1/2*log(abs(-2*x + 2*sqrt(x^2 + 5*x + 6) - 5
))/sgn(x + 3) + sqrt(x^2 + 5*x + 6)/sgn(x + 3)
```

Mupad [B] (verification not implemented)

Time = 18.00 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = 2\,\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sqrt{\frac{x+2}{x+3}}\right) - \frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1} - \mathrm{atanh}\left(\sqrt{\frac{x+2}{x+3}}\right)$$

```
[In] int(x/(((x + 2)/(x + 3))^(1/2)*(x + 1)),x)
[Out] 2*2^(1/2)*atanh(2^(1/2)*((x + 2)/(x + 3))^(1/2)) - ((x + 2)/(x + 3))^(1/2)/
((x + 2)/(x + 3) - 1) - atanh(((x + 2)/(x + 3))^(1/2))
```

3.751 $\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} \, dx = \frac{2}{\sqrt{1+\frac{1}{x}}}$$

```
[Out] 2/(1+1/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {25, 267}

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} \, dx = \frac{2}{\sqrt{\frac{1}{x}+1}}$$

```
[In] Int[Sqrt[1 + x^(-1)]/(1 + x)^2,x]
```

```
[Out] 2/Sqrt[1 + x^(-1)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\left(1+\frac{1}{x}\right)^{3/2} x^2}\,dx$$
$$= \frac{2}{\sqrt{1+\frac{1}{x}}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \frac{2x\sqrt{\frac{1+x}{x}}}{1+x}$$

```
[In] Integrate[Sqrt[1 + x^(-1)]/(1 + x)^2,x]
[Out] (2*x*Sqrt[(1 + x)/x])/(1 + x)
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

method	result	size
gosper	$\frac{2x\sqrt{\frac{x+1}{x}}}{x+1}$	18
risch	$\frac{2x\sqrt{\frac{x+1}{x}}}{x+1}$	18
trager	$\frac{2x\sqrt{-\frac{-x-1}{x}}}{x+1}$	21
default	$\frac{2\sqrt{x^2+x}\,x\sqrt{\frac{x+1}{x}}}{(x+1)\sqrt{(x+1)x}}$	32

```
[In] int((1+1/x)^(1/2)/(x+1)^2,x,method=_RETURNVERBOSE)
[Out] 2/(x+1)*x*((x+1)/x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \frac{2\,x\sqrt{\frac{x+1}{x}}}{x+1}$$

```
[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="fricas")
[Out] 2*x*sqrt((x + 1)/x)/(x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \int \frac{\sqrt{1+\frac{1}{x}}}{(x+1)^2}\,dx$$

```
[In] integrate((1+1/x)**(1/2)/(1+x)**2,x)
[Out] Integral(sqrt(1 + 1/x)/(x + 1)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \int \frac{\sqrt{\frac{1}{x}+1}}{(x+1)^2}\,dx$$

```
[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="maxima")
[Out] integrate(sqrt(1/x + 1)/(x + 1)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(9) = 18.

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \frac{2\,\mathrm{sgn}(x)}{x-\sqrt{x^2+x}+1} - 2\,\mathrm{sgn}(x)$$

```
[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="giac")
[Out] 2*sgn(x)/(x - sqrt(x^2 + x) + 1) - 2*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 18.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \frac{2\,x\,\sqrt{\frac{1}{x}+1}}{x+1}$$

```
[In] int((1/x + 1)^(1/2)/(x + 1)^2,x)
[Out] (2*x*(1/x + 1)^(1/2))/(x + 1)
```

3.752 $\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx=-\frac{\sqrt{1+\frac{1}{x}}\sqrt{x}\arcsin(1-2x)}{\sqrt{1+x}}$$

[Out] arcsin(-1+2*x)*(1+1/x)^(1/2)*x^(1/2)/(1+x)^(1/2)

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1462, 26, 55, 633, 222}

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx=-\frac{\sqrt{\frac{1}{x}+1}\sqrt{x}\arcsin(1-2x)}{\sqrt{x+1}}$$

[In] Int[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]

[Out] -((Sqrt[1 + x^(-1)]*Sqrt[x]*ArcSin[1 - 2*x])/Sqrt[1 + x])

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1462

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sym
bol] :> Dist[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(x^mn*e)))^F
racPart[q]))/x^(mn*FracPart[q]), Int[x^(mn*q)*(1 + d*(1/(x^mn*e)))^q*(a + c
*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, mn, p, q}, x] && EqQ[n2, -2*mn] &&
!IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int\frac{\sqrt{1+x}}{\sqrt{x}\sqrt{1-x^2}}\,dx}{\sqrt{1+x}}\\
&= \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int\frac{1}{\sqrt{1-x}\sqrt{x}}\,dx}{\sqrt{1+x}}\\
&= \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int\frac{1}{\sqrt{x-x^2}}\,dx}{\sqrt{1+x}}\\
&= -\frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)}{\sqrt{1+x}}\\
&= -\frac{\sqrt{1+\frac{1}{x}}\sqrt{x}\sin^{-1}(1-2x)}{\sqrt{1+x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = -\arctan\left(\frac{\sqrt{\frac{1+x}{x}}(-1+2x)\sqrt{1-x^2}}{2\,(-1+x^2)}\right)$$

```
[In] Integrate[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]
[Out] -ArcTan[(Sqrt[(1 + x)/x]*(-1 + 2*x)*Sqrt[1 - x^2])/(2*(-1 + x^2))]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{\sqrt{\frac{x+1}{x}}\,x\sqrt{-x^2+1}\,\arcsin(2x-1)}{(x+1)\sqrt{-(x-1)x}}$	40

```
[In] int((1+1/x)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((x+1)/x)^(1/2)*x*(-x^2+1)^(1/2)/(x+1)/(-(x-1)*x)^(1/2)*arcsin(2*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = -\arctan\left(\frac{2\sqrt{-x^2+1}x\sqrt{\frac{x+1}{x}}}{2\,x^2+x-1}\right)$$

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -arctan(2*sqrt(-x^2 + 1)*x*sqrt((x + 1)/x)/(2*x^2 + x - 1))
```

Sympy [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{-(x-1)(x+1)}}\,dx$$

```
[In] integrate((1+1/x)**(1/2)/(-x**2+1)**(1/2),x)
[Out] Integral(sqrt(1 + 1/x)/sqrt(-(x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{-x^2+1}}\,dx$$

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{-x^2+1}}\,dx$$

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{1-x^2}}\,dx$$

```
[In] int((1/x + 1)^(1/2)/(1 - x^2)^(1/2),x)
[Out] int((1/x + 1)^(1/2)/(1 - x^2)^(1/2), x)
```

3.753 $\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 180

$$\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx = \arctan\left(\frac{\sqrt{3}-\sqrt{3-2x-x^2}}{x}\right) - \frac{1}{2}\log\left(-\frac{3-x-\sqrt{3}\sqrt{3-2x-x^2}}{x^2}\right) + \frac{1}{14}\left(7+\sqrt{7}\right)\log\left(1+\sqrt{3}-\sqrt{7}-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right) + \frac{1}{14}\left(7-\sqrt{7}\right)\log\left(1+\sqrt{3}+\sqrt{7}-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)$$

```
[Out] arctan((3^(1/2)-(-x^2-2*x+3)^(1/2))/x)-1/2*ln((-3+x+3^(1/2)*(-x^2-2*x+3)^(1
/2))/x^2)+1/14*ln(1+3^(1/2)+7^(1/2)-3^(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x)
*(7-7^(1/2))+1/14*ln(1+3^(1/2)-7^(1/2)-3^(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/2))
/x)*(7+7^(1/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{1088, 646, 31, 649, 209, 266\}$

$$\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx = \arctan\left(\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right) - \frac{1}{2}\log\left(-\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-x+3}{x^2}\right) + \frac{1}{14}\left(7 + \sqrt{7}\right)\log\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x} - \sqrt{7}+\sqrt{3}+1\right) + \frac{1}{14}\left(7-\sqrt{7}\right)\log\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x} + \sqrt{7}+\sqrt{3} + 1\right)$$

```
[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-1),x]
```

```
[Out] ArcTan[(Sqrt[3] - Sqrt[3 - 2*x - x^2])/x] - Log[-((3 - x - Sqrt[3]*Sqrt[3 -
 2*x - x^2])/x^2)]/2 + ((7 + Sqrt[7])*Log[1 + Sqrt[3] - Sqrt[7] - (Sqrt[3]*
(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14 + ((7 - Sqrt[7])*Log[1 + Sqrt[3] +
Sqrt[7] - (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1088

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d
*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*b*B*f - a*A*c
*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x],
x] + Dist[1/q, Int[(c*C*d^2 + b*B*d*f - A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*
c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[
{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

integral

$$= 2\text{Subst}\left(\int \frac{\sqrt{3}-2x-\sqrt{3}x^2}{(1+x^2)\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}x^2\right)}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$= \frac{1}{16}\text{Subst}\left(\int \frac{-6+2\sqrt{3}(2-\sqrt{3})-4(1+\sqrt{3})-\left(-2\sqrt{3}+2(2-\sqrt{3})+4\sqrt{3}(1+\sqrt{3})\right)x}{1+x^2}\,dx, x, \frac{-\sqrt{3}+\sqrt{}}{}\right.$$

$$+\frac{1}{16}\text{Subst}\left(\int \frac{3\sqrt{3}-\sqrt{3}(2-\sqrt{3})^2+4(2-\sqrt{3})\left(1+\sqrt{3}\right)+4\sqrt{3}(1+\sqrt{3})^2+\sqrt{3}(-2\sqrt{3}+2(2-\sqrt{3})+}{2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}x^2}\right.$$

$$= -\left(\frac{1}{2}\left(\sqrt{\frac{3}{7}}\left(1-\sqrt{7}\right)\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{3}+\sqrt{7}+\sqrt{3}x}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)\right)$$

$$+\frac{1}{2}\left(\sqrt{\frac{3}{7}}\left(1+\sqrt{7}\right)\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{3}-\sqrt{7}+\sqrt{3}x}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$-\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$-\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$
= -\tan^{-1}\left(\frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)-\frac{1}{2}\log\left(\frac{-3+x+\sqrt{3}\sqrt{3-2x-x^2}}{x^2}\right)
+\frac{1}{14}\left(7+\sqrt{7}\right)\log\left(1+\sqrt{3}-\sqrt{7}-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)
+\frac{1}{14}\left(7-\sqrt{7}\right)\log\left(1+\sqrt{3}+\sqrt{7}-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.62

$$
\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx = \frac{1}{14}\left(-14\arctan\left(\frac{\sqrt{3-2x-x^2}}{3+x}\right)-7\log(-1+x)
-\left(-7+\sqrt{7}\right)\log\left(-2+\sqrt{7}(-1+x)+2x-\sqrt{3-2x-x^2}\right)
+\left(7+\sqrt{7}\right)\log\left(2+\sqrt{7}(-1+x)-2x+\sqrt{3-2x-x^2}\right)\right)
$$

```
[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-1),x]
[Out] (-14*ArcTan[Sqrt[3 - 2*x - x^2]/(3 + x)] - 7*Log[-1 + x] - (-7 + Sqrt[7])*L
og[-2 + Sqrt[7]*(-1 + x) + 2*x - Sqrt[3 - 2*x - x^2]] + (7 + Sqrt[7])*Log[2
 + Sqrt[7]*(-1 + x) - 2*x + Sqrt[3 - 2*x - x^2]])/14
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(138) = 276$.

Time = 1.26 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.99

method	result
default	$-\frac{\sqrt{7}\left(\frac{\sqrt{-4\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)^2+4\left(-1-\sqrt{7}\right)\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)+8-2\sqrt{7}}}{4}+\frac{\left(-1-\sqrt{7}\right)\arcsin\left(\frac{x+1}{\sqrt{2-\frac{\sqrt{7}}{2}+\frac{\left(-1-\sqrt{7}\right)^2}{4}}}\right)}{4}-\frac{\left(2-\frac{\sqrt{7}}{2}\right)\operatorname{arctanh}\left(\frac{}{\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)\cdots}\right.}{}\right.}{7}$
trager	Expression too large to display

```
[In] int(1/(x+(-x^2-2*x+3)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/7*7^(1/2)*(1/4*(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/
2))+8-2*7^(1/2))^(1/2)+1/4*(-1-7^(1/2))*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(
```

```
1/2))^2)^(1/2)*(x+1))-1/2*(2-1/2*7^(1/2))/(-1/2+1/2*7^(1/2))*arctanh((4-7^(
1/2)+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1/2-1/2*7^
(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2))+1/7*7^(1/2
)*(1/4*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(
1/2))^(1/2)+1/4*(-1+7^(1/2))*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1
/2)*(x+1))-1/2*(2+1/2*7^(1/2))/(1/2+1/2*7^(1/2))*arctanh((4+7^(1/2)+(-1+7^(
1/2))*(x+1/2+1/2*7^(1/2)))/(1/2+1/2*7^(1/2))/(-4*(x+1/2+1/2*7^(1/2))^2+4*(-
1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)))+1/4*ln(2*x^2+2*x-3)+1/1
4*7^(1/2)*arctanh(1/14*(4*x+2)*7^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(136) = 272$.

Time = 0.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.07

$$\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx$$
$$= \frac{1}{56}\sqrt{7}\log\left(\frac{24x^4+62x^3-153x^2+2\sqrt{7}(3x^4+x^3-45x^2+45x)-\left(14x^3-84x^2+\sqrt{7}(8x^3-30x^2+\right.}{4x^4+8x^3-8x^2-12x+9}\right.$$
$$+\frac{1}{56}\sqrt{7}\log\left(\frac{24x^4+62x^3-153x^2-2\sqrt{7}(3x^4+x^3-45x^2+45x)+\left(14x^3-84x^2-\sqrt{7}(8x^3-30x^2\right.}{4x^4+8x^3-8x^2-12x+9}\right.$$
$$+\frac{1}{28}\sqrt{7}\log\left(\frac{2x^2+\sqrt{7}(2x+1)+2x+4}{2x^2+2x-3}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-2x+3}(x+1)}{x^2+2x-3}\right)$$
$$+\frac{1}{4}\log\left(2x^2+2x-3\right)-\frac{1}{8}\log\left(\frac{2\sqrt{-x^2-2x+3}x+2x-3}{x^2}\right)$$
$$+\frac{1}{8}\log\left(-\frac{2\sqrt{-x^2-2x+3}x-2x+3}{x^2}\right)$$

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/56*sqrt(7)*log((24*x^4 + 62*x^3 - 153*x^2 + 2*sqrt(7)*(3*x^4 + x^3 - 45*x
^2 + 45*x) - (14*x^3 - 84*x^2 + sqrt(7)*(8*x^3 - 30*x^2 + 27*x - 27) + 126*
x)*sqrt(-x^2 - 2*x + 3) + 180*x - 135)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9))
+ 1/56*sqrt(7)*log((24*x^4 + 62*x^3 - 153*x^2 - 2*sqrt(7)*(3*x^4 + x^3 - 45
*x^2 + 45*x) + (14*x^3 - 84*x^2 - sqrt(7)*(8*x^3 - 30*x^2 + 27*x - 27) + 12
6*x)*sqrt(-x^2 - 2*x + 3) + 180*x - 135)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)
) + 1/28*sqrt(7)*log((2*x^2 + sqrt(7)*(2*x + 1) + 2*x + 4)/(2*x^2 + 2*x - 3
)) - 1/2*arctan(sqrt(-x^2 - 2*x + 3)*(x + 1)/(x^2 + 2*x - 3)) + 1/4*log(2*x
^2 + 2*x - 3) - 1/8*log((2*sqrt(-x^2 - 2*x + 3)*x + 2*x - 3)/x^2) + 1/8*log
(-(2*sqrt(-x^2 - 2*x + 3)*x - 2*x + 3)/x^2)
```

Sympy [F]

$$\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx=\int \frac{1}{x+\sqrt{-x^2-2x+3}}\,dx$$

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2)),x)
[Out] Integral(1/(x + sqrt(-x**2 - 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx=\int \frac{1}{x+\sqrt{-x^2-2\,x+3}}\,dx$$

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(x + sqrt(-x^2 - 2*x + 3)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(136) = 272.

Time = 0.39 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.59

$$\begin{aligned}
&\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx\\
&=-\frac{1}{28}\,\sqrt{7}\log\left(\frac{|4\,x-2\,\sqrt{7}+2|}{|4\,x+2\,\sqrt{7}+2|}\right)+\frac{1}{28}\,\sqrt{7}\log\left(\frac{\left|-2\,\sqrt{7}+\frac{6\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+4\right|}{\left|2\,\sqrt{7}+\frac{6\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+4\right|}\right)\\
&\quad-\frac{1}{28}\,\sqrt{7}\log\left(\frac{\left|-2\,\sqrt{7}+\frac{2\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}-4\right|}{\left|2\,\sqrt{7}+\frac{2\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}-4\right|}\right)\\
&\quad+\frac{1}{2}\arcsin\left(\frac{1}{2}\,x+\frac{1}{2}\right)+\frac{1}{4}\log\left(|2\,x^2+2\,x-3|\right)\\
&\quad+\frac{1}{4}\log\left(\left|\frac{4\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+\frac{3\left(\sqrt{-x^2-2\,x+3}-2\right)^2}{(x+1)^2}-1\right|\right)\\
&\quad-\frac{1}{4}\log\left(\left|-\frac{4\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+\frac{\left(\sqrt{-x^2-2\,x+3}-2\right)^2}{(x+1)^2}-3\right|\right)
\end{aligned}$$

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/28*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 1/28
*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/abs
(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 1/28*sqrt(7)*log(
abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7) +
2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) + 1/2*arcsin(1/2*x + 1/2) + 1/4*
log(abs(2*x^2 + 2*x - 3)) + 1/4*log(abs(4*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1
) + 3*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 1)) - 1/4*log(abs(-4*(sqrt(-
x^2 - 2*x + 3) - 2)/(x + 1) + (sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx = \int \frac{1}{x+\sqrt{-x^2-2\,x+3}}\,dx$$

```
[In] int(1/(x + (3 - x^2 - 2*x)^(1/2)),x)
```

```
[Out] int(1/(x + (3 - x^2 - 2*x)^(1/2)), x)
```

3.754 $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 172

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx = \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} + \frac{8\operatorname{arctanh}\left(\frac{3-x-\sqrt{3}x-\sqrt{3}\sqrt{3-2x-x^2}}{\sqrt{7}x}\right)}{7\sqrt{7}}$$

```
[Out] 8/49*arctanh(1/7*(3-x-x*3^(1/2)-3^(1/2)*(-x^2-2*x+3)^(1/2))/x*7^(1/2))*7^(1
/2)+2/7*(4-3^(1/2)+3*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x)/(2-3^(1/2)-2*(1+3^(1/2
))*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x+3^(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/2))^2/x^
2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1674, 12, 632, 212}

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx = \frac{8\operatorname{arctanh}\left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-\sqrt{3}x-x+3}{\sqrt{7}x}\right)}{7\sqrt{7}} + \frac{2\left(\frac{3\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+4\right)}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}$$

```
[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-2),x]
```

```
[Out] (2*(4 - Sqrt[3] + (3*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))/(7*(2 - Sqrt[3] -
 (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] -
Sqrt[3 - 2*x - x^2])^2)/x^2)) + (8*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sqr
t[3 - 2*x - x^2])/(Sqrt[7]*x)])/(7*Sqrt[7])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
```

```
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{-\sqrt{3}+2x+\sqrt{3}x^2}{\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}x^2\right)^2}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} - \frac{1}{14}\text{Subst}\left(\int -\frac{16}{2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}x^2}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} + \frac{8}{7}\text{Subst}\left(\int \frac{1}{2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}x^2}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} - \frac{16}{7}\text{Subst}\left(\int \frac{1}{28-x^2}\,dx, x, \frac{2\left(-3+x+\sqrt{3}x+\sqrt{3}\sqrt{3-2x-x^2}\right)}{x}\right)$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} + \frac{8\tanh^{-1}\left(\frac{3-x-\sqrt{3}x-\sqrt{3}\sqrt{3-2x-x^2}}{\sqrt{7}x}\right)}{7\sqrt{7}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.55

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx=\frac{3+6\sqrt{3-2x-x^2}-2x\left(4+\sqrt{3-2x-x^2}\right)}{14\left(-3+2x+2x^2\right)}+\frac{8\text{arctanh}\left(\frac{2-2x+\sqrt{3-2x-x^2}}{\sqrt{7}(-1+x)}\right)}{7\sqrt{7}}$$

```
[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-2),x]
```

```
[Out] (3 + 6*Sqrt[3 - 2*x - x^2] - 2*x*(4 + Sqrt[3 - 2*x - x^2]))/(14*(-3 + 2*x +
 2*x^2)) + (8*ArcTanh[(2 - 2*x + Sqrt[3 - 2*x - x^2])/(Sqrt[7]*(-1 + x))])/
(7*Sqrt[7])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.60

method	result
trager	$\frac{(-3+x)x}{14x^2+14x-21}-\frac{(-3+x)\sqrt{-x^2-2x+3}}{7(2x^2+2x-3)}+\frac{4\,\text{RootOf}\left(\_Z^2-7\right)\ln\left(-\frac{\text{RootOf}\left(\_Z^2-7\right)x-3\,\text{RootOf}\left(\_Z^2-7\right)+7\sqrt{-x^2-2x+3}}{\text{RootOf}\left(\_Z^2-7\right)x+x-3}\right)}{49}$
default	Expression too large to display

```
[In] int(1/(x+(-x^2-2*x+3)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(-3+x)*x/(2*x^2+2*x-3)-1/7*(-3+x)/(2*x^2+2*x-3)*(-x^2-2*x+3)^(1/2)+4/49
*RootOf(_Z^2-7)*ln(-(RootOf(_Z^2-7)*x-3*RootOf(_Z^2-7)+7*(-x^2-2*x+3)^(1/2)
)/(RootOf(_Z^2-7)*x+x-3))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx$$

$$=\frac{2\sqrt{7}(2\,x^2+2\,x-3)\log\left(\frac{x^4+44\,x^3-\sqrt{7}(3\,x^3+x^2-45\,x+45)\sqrt{-x^2-2\,x+3}+26\,x^2-276\,x+207}{4\,x^4+8\,x^3-8\,x^2-12\,x+9}\right)+4\sqrt{7}(2\,x^2+2\,x-3)\log\left(\right.}{98\left(2\,x^2+2\,x-3\right)}$$

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/98*(2*sqrt(7)*(2*x^2 + 2*x - 3)*log((x^4 + 44*x^3 - sqrt(7)*(3*x^3 + x^2
- 45*x + 45)*sqrt(-x^2 - 2*x + 3) + 26*x^2 - 276*x + 207)/(4*x^4 + 8*x^3 -
8*x^2 - 12*x + 9)) + 4*sqrt(7)*(2*x^2 + 2*x - 3)*log((2*x^2 + sqrt(7)*(2*x
+ 1) + 2*x + 4)/(2*x^2 + 2*x - 3)) - 14*sqrt(-x^2 - 2*x + 3)*(x - 3) - 56*x
 + 21)/(2*x^2 + 2*x - 3)
```

Sympy [F]

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx=\int \frac{1}{\left(x+\sqrt{-x^2-2x+3}\right)^2}\,dx$$

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2))**2,x)
```

```
[Out] Integral((x + sqrt(-x**2 - 2*x + 3))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx=\int \frac{1}{\left(x+\sqrt{-x^2-2\,x+3}\right)^2}\,dx$$

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 2*x + 3))^(-2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(132) = 264.

Time = 0.37 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.03

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx$$
$$= -\frac{2}{49}\sqrt{7}\log\left(\frac{|4x-2\sqrt{7}+2|}{|4x+2\sqrt{7}+2|}\right)+\frac{2}{49}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}\right)$$
$$-\frac{2}{49}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}{\left|2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}\right)-\frac{8x-3}{14(2x^2+2x-3)}$$
$$-\frac{8\left(\frac{5\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{26\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}+\frac{11\left(\sqrt{-x^2-2x+3}-2\right)^3}{(x+1)^3}-6\right)}{21\left(\frac{8\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{26\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}+\frac{8\left(\sqrt{-x^2-2x+3}-2\right)^3}{(x+1)^3}-\frac{3\left(\sqrt{-x^2-2x+3}-2\right)^4}{(x+1)^4}-3\right)}$$

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="giac")
[Out] -2/49*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 2/49
*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/abs
(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 2/49*sqrt(7)*log(
abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7) +
2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) - 1/14*(8*x - 3)/(2*x^2 + 2*x -
3) - 8/21*(5*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3)
- 2)^2/(x + 1)^2 + 11*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 6)/(8*(sqrt(
-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 +
8*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 3*(sqrt(-x^2 - 2*x + 3) - 2)^4/(
x + 1)^4 - 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx=\int \frac{1}{\left(x+\sqrt{-x^2-2x+3}\right)^2}\,dx$$

```
[In] int(1/(x + (3 - x^2 - 2*x)^(1/2))^2,x)
[Out] int(1/(x + (3 - x^2 - 2*x)^(1/2))^2, x)
```

3.755 $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 307

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx = -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} + \frac{2\left(18-43\sqrt{3}-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{147\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} + \frac{12\operatorname{arctanh}\left(\frac{3-x-\sqrt{3}x-\sqrt{3}\sqrt{3-2x-x^2}}{\sqrt{7}x}\right)}{49\sqrt{7}}$$

```
[Out] 12/343*arctanh(1/7*(3-x-x*3^(1/2)-3^(1/2)*(-x^2-2*x+3)^(1/2))/x*7^(1/2))*7^
(1/2)-4/21*(9-5*3^(1/2)+(21+5*3^(1/2))*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x)/(2-3
^(1/2)-2*(1+3^(1/2))*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x+3^(1/2)*(3^(1/2)-(-x^2-
2*x+3)^(1/2))^2/x^2)^2+2/147*(18-43*3^(1/2)-(18+49*3^(1/2))*(3^(1/2)-(-x^2-
2*x+3)^(1/2))/x)/(2-3^(1/2)-2*(1+3^(1/2))*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x+3^
(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/2))^2/x^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1674, 12, 632, 212}

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx$$
$$=\frac{12\text{arctanh}\left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-\sqrt{3}x-x+3}{\sqrt{7}x}\right)}{49\sqrt{7}}-\frac{4\left(\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-5\sqrt{3}+9\right)}{21\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)^2}+\frac{2\left(-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-43\sqrt{3}+18\right)}{147\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}$$

```
[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-3),x]
```

```
[Out] (-4*(9 - 5*Sqrt[3] + ((21 + 5*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))
/(21*(2 - Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (
Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2])^2)/x^2)^2) + (2*(18 - 43*Sqrt[3] -
((18 + 49*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))/(147*(2 - Sqrt[3] -
 (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] -
Sqrt[3 - 2*x - x^2])^2)/x^2)) + (12*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sq
rt[3 - 2*x - x^2])/(Sqrt[7]*x)])/(49*Sqrt[7])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{\sqrt{3}-2x-2x^3-\sqrt{3}x^4}{\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}x^2\right)^3}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} - \frac{1}{28}\text{Subst}\left(\int \frac{-\frac{8}{3}\left(21+16\sqrt{3}\right)-112x+56x^2}{\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}x^2\right)^2}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} + \frac{2\left(18-43\sqrt{3}-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{147\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} + \frac{1}{784}\text{Subst}\left(\int \frac{192}{2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}x^2}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$=-\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2}$$
$$+\frac{2\left(18-43\sqrt{3}-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{147\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)}$$
$$+\frac{12}{49}\text{Subst}\left(\int\frac{1}{2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}x^2}\,dx,x,\frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)$$

$$=-\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2}$$
$$+\frac{2\left(18-43\sqrt{3}-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{147\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)}$$
$$-\frac{24}{49}\text{Subst}\left(\int\frac{1}{28-x^2}\,dx,x,\frac{2\left(-3+x+\sqrt{3}x+\sqrt{3}\sqrt{3-2x-x^2}\right)}{x}\right)$$

$$=-\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2}$$
$$+\frac{2\left(18-43\sqrt{3}-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{147\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)}$$
$$+\frac{12\tanh^{-1}\left(\frac{3-x-\sqrt{3}x-\sqrt{3}\sqrt{3-2x-x^2}}{\sqrt{7}x}\right)}{49\sqrt{7}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.37

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx = \frac{\frac{7(-279+300x+26x^2-48x^3)}{(-3+2x+2x^2)^2}+\frac{14\sqrt{3-2x-x^2}(15+83x-58x^2-34x^3)}{(-3+2x+2x^2)^2}+48\sqrt{7}\operatorname{arctanh}\left(\frac{2-2x+\sqrt{3-2x-x^2}}{\sqrt{7}(-1+x)}\right)}{1372}$$

```
[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-3),x]
```

```
[Out] ((7*(-279 + 300*x + 26*x^2 - 48*x^3))/(-3 + 2*x + 2*x^2)^2 + (14*Sqrt[3 - 2
*x - x^2]*(15 + 83*x - 58*x^2 - 34*x^3))/(-3 + 2*x + 2*x^2)^2 + 48*Sqrt[7]*
ArcTanh[(2 - 2*x + Sqrt[3 - 2*x - x^2])/(Sqrt[7]*(-1 + x))])/1372
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.42

method	result
trager	$\frac{(62x^3+100x^2-111x-36)x}{98(2x^2+2x-3)^2}-\frac{(34x^3+58x^2-83x-15)\sqrt{-x^2-2x+3}}{98(2x^2+2x-3)^2}-\frac{6\operatorname{RootOf}\left(\_Z^2-7\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-7\right)x-3\operatorname{RootOf}\left(\_\right.}{\operatorname{RootOf}\left(\_Z^2-7\right)}\right.}{343}$
default	Expression too large to display

```
[In] int(1/(x+(-x^2-2*x+3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/98*(62*x^3+100*x^2-111*x-36)*x/(2*x^2+2*x-3)^2-1/98*(34*x^3+58*x^2-83*x-1
5)/(2*x^2+2*x-3)^2*(-x^2-2*x+3)^(1/2)-6/343*RootOf(_Z^2-7)*ln((RootOf(_Z^2-
7)*x-3*RootOf(_Z^2-7)-7*(-x^2-2*x+3)^(1/2))/(RootOf(_Z^2-7)*x-x+3))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx = -\frac{336\,x^3-6\,\sqrt{7}(4\,x^4+8\,x^3-8\,x^2-12\,x+9)\log\left(\frac{x^4+44\,x^3-\sqrt{7}(3\,x^3+x^2-45\,x+45)\sqrt{-x^2-2\,x+3}+26\,x^2-276\,x+207}{4\,x^4+8\,x^3-8\,x^2-12\,x+9}\right)-}{}$$

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -1/1372*(336*x^3 - 6*sqrt(7)*(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)*log((x^4 +
44*x^3 - sqrt(7)*(3*x^3 + x^2 - 45*x + 45)*sqrt(-x^2 - 2*x + 3) + 26*x^2 -
276*x + 207)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)) - 12*sqrt(7)*(4*x^4 + 8*x^
3 - 8*x^2 - 12*x + 9)*log((2*x^2 + sqrt(7)*(2*x + 1) + 2*x + 4)/(2*x^2 + 2*
x - 3)) - 182*x^2 + 14*(34*x^3 + 58*x^2 - 83*x - 15)*sqrt(-x^2 - 2*x + 3) -
 2100*x + 1953)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx = \int \frac{1}{\left(x+\sqrt{-x^2-2\,x+3}\right)^3}\,dx$$

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 2*x + 3))^(-3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx$$
$$= -\frac{3}{343}\sqrt{7}\log\left(\frac{\left|4x-2\sqrt{7}+2\right|}{\left|4x+2\sqrt{7}+2\right|}\right)+\frac{3}{343}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}\right)$$
$$-\frac{3}{343}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}{\left|2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}\right)-\frac{48x^3-26x^2-300x+279}{196\left(2x^2+2x-3\right)^2}$$
$$+\frac{4\left(\frac{231\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{3286\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}-\frac{4441\left(\sqrt{-x^2-2x+3}-2\right)^3}{(x+1)^3}-\frac{18906\left(\sqrt{-x^2-2x+3}-2\right)^4}{(x+1)^4}-\frac{12487\left(\sqrt{-x^2\cdots}\right.}{(x+\cdots}\right.}{441\left(\frac{8\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{26\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}+\frac{8\left(\sqrt{-x^2-2x+3}-2\right)^3}{(x+1)^3}-\cdots\right.}$$

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -3/343*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 3/3
43*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/a
bs(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 3/343*sqrt(7)*l
og(abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7)
 + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) - 1/196*(48*x^3 - 26*x^2 - 30
0*x + 279)/(2*x^2 + 2*x - 3)^2 + 4/441*(231*(sqrt(-x^2 - 2*x + 3) - 2)/(x +
 1) + 3286*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 4441*(sqrt(-x^2 - 2*x +
 3) - 2)^3/(x + 1)^3 - 18906*(sqrt(-x^2 - 2*x + 3) - 2)^4/(x + 1)^4 - 12487
*(sqrt(-x^2 - 2*x + 3) - 2)^5/(x + 1)^5 + 946*(sqrt(-x^2 - 2*x + 3) - 2)^6/
(x + 1)^6 + 1977*(sqrt(-x^2 - 2*x + 3) - 2)^7/(x + 1)^7 - 414)/(8*(sqrt(-x^
2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 + 8*(
sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 3*(sqrt(-x^2 - 2*x + 3) - 2)^4/(x +
 1)^4 - 3)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx=\int \frac{1}{\left(x+\sqrt{-x^2-2x+3}\right)^3}\,dx$$

```
[In] int(1/(x + (3 - x^2 - 2*x)^(1/2))^3,x)
```

```
[Out] int(1/(x + (3 - x^2 - 2*x)^(1/2))^3, x)
```

3.756 $\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx = -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+2\log\left(1-x-\sqrt{-3-2x+x^2}\right)-\frac{3}{2}\log\left(x+\sqrt{-3-2x+x^2}\right)$$

```
[Out] 2*ln(1-x-(x^2-2*x-3)^(1/2))-3/2*ln(x+(x^2-2*x-3)^(1/2))-2/(1-x-(x^2-2*x-3)^
(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2141, 907}

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx = -\frac{2}{-\sqrt{x^2-2x-3}-x+1}+2\log\left(-\sqrt{x^2-2x-3}-x+1\right)-\frac{3}{2}\log\left(\sqrt{x^2-2x-3}+x\right)$$

```
[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-1),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 2*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - (
3*Log[x + Sqrt[-3 - 2*x + x^2]])/2
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
```

```
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{-3-2x+x^2}{x(-2+2x)^2}\,dx, x, x+\sqrt{-3-2x+x^2}\right)\\ &= 2\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}+\frac{1}{-1+x}-\frac{3}{4x}\right)dx, x, x+\sqrt{-3-2x+x^2}\right)\\ &= -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+2\log\left(1-x-\sqrt{-3-2x+x^2}\right)-\frac{3}{2}\log\left(x+\sqrt{-3-2x+x^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx = \frac{1}{2}\Big(x-\sqrt{-3-2x+x^2}-\log\left(-1-x+\sqrt{-3-2x+x^2}\right)+4\log\left(1+x+\sqrt{-3-2x+x^2}\right)-3\log\left(3+3x+\sqrt{-3-2x+x^2}\right)\Big)$$

```
[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-1),x]
[Out] (x - Sqrt[-3 - 2*x + x^2] - Log[-1 - x + Sqrt[-3 - 2*x + x^2]] + 4*Log[1 +
x + Sqrt[-3 - 2*x + x^2]] - 3*Log[3 + 3*x + Sqrt[-3 - 2*x + x^2]])/2
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

method	result	size
default	$-\frac{\sqrt{4\left(x+\frac{3}{2}\right)^{2}-20x-21}}{4}+\frac{5\ln\left(-1+x+\sqrt{\left(x+\frac{3}{2}\right)^{2}-5x-\frac{21}{4}}\right)}{4}+\frac{3\,\mathrm{arctanh}\left(\frac{-2-\frac{10x}{3}}{\sqrt{4\left(x+\frac{3}{2}\right)^{2}-20x-21}}\right)}{4}+\frac{x}{2}-\frac{3\ln\left(2x+3\right)}{4}$	71
trager	$\frac{x}{2}-\frac{\sqrt{x^{2}-2x-3}}{2}-\frac{\ln\left(\sqrt{x^{2}-2x-3}\,x^{3}-x^{4}+3\sqrt{x^{2}-2x-3}\,x^{2}-2x^{3}+\sqrt{x^{2}-2x-3}\,x+4x^{2}-3\sqrt{x^{2}-2x-3}+12x+9\right)}{2}$	93

```
[In] int(1/(x+(x^2-2*x-3)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/4*(4*(x+3/2)^2-20*x-21)^(1/2)+5/4*ln(-1+x+((x+3/2)^2-5*x-21/4)^(1/2))+3/
4*arctanh(2/3*(-3-5*x)/(4*(x+3/2)^2-20*x-21)^(1/2))+1/2*x-3/4*ln(2*x+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx = \frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^{2}-2\,x-3}-\frac{3}{4}\,\log\left(2\,x+3\right) -\frac{5}{4}\,\log\left(-x+\sqrt{x^{2}-2\,x-3}+1\right) +\frac{3}{4}\,\log\left(-x+\sqrt{x^{2}-2\,x-3}\right) -\frac{3}{4}\,\log\left(-x+\sqrt{x^{2}-2\,x-3}-3\right)$$

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="fricas")
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*log(2*x + 3) - 5/4*log(-x + sqrt(x^2
- 2*x - 3) + 1) + 3/4*log(-x + sqrt(x^2 - 2*x - 3)) - 3/4*log(-x + sqrt(x^2
 - 2*x - 3) - 3)
```

Sympy [F]

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx = \int \frac{1}{x+\sqrt{x^{2}-2x-3}}\,dx$$

```
[In] integrate(1/(x+(x**2-2*x-3)**(1/2)),x)
[Out] Integral(1/(x + sqrt(x**2 - 2*x - 3)), x)
```

Maxima [F]

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx = \int \frac{1}{x+\sqrt{x^2-2\,x-3}}\,dx$$

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(x^2 - 2*x - 3)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx = \frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x^2-2\,x-3} - \frac{3}{4}\,\log\left(|2\,x+3|\right) - \frac{5}{4}\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}+1\right|\right) + \frac{3}{4}\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}\right|\right) - \frac{3}{4}\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}-3\right|\right)$$

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*log(abs(2*x + 3)) - 5/4*log(abs(-x +
sqrt(x^2 - 2*x - 3) + 1)) + 3/4*log(abs(-x + sqrt(x^2 - 2*x - 3))) - 3/4*lo
g(abs(-x + sqrt(x^2 - 2*x - 3) - 3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx = \frac{x}{2} - \frac{3\,\ln\left(x+\frac{3}{2}\right)}{4} - \int \frac{\sqrt{x^2-2\,x-3}}{2\,x+3}\,dx$$

```
[In] int(1/(x + (x^2 - 2*x - 3)^(1/2)),x)
```

```
[Out] x/2 - (3*log(x + 3/2))/4 - int((x^2 - 2*x - 3)^(1/2)/(2*x + 3), x)
```

3.757 $\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 83

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx = -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{2\left(x+\sqrt{-3-2x+x^2}\right)} + 4\log\left(1-x-\sqrt{-3-2x+x^2}\right) - 4\log\left(x+\sqrt{-3-2x+x^2}\right)$$

```
[Out] 4*ln(1-x-(x^2-2*x-3)^(1/2))-4*ln(x+(x^2-2*x-3)^(1/2))-2/(1-x-(x^2-2*x-3)^(1
/2))+3/2/(x+(x^2-2*x-3)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2141, 907}

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx = -\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{3}{2\left(\sqrt{x^2-2x-3}+x\right)} +4\log\left(-\sqrt{x^2-2x-3}-x+1\right)-4\log\left(\sqrt{x^2-2x-3}+x\right)$$

```
[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-2),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(2*(x + Sqrt[-3 - 2*x + x^2])) + 4*Lo
g[1 - x - Sqrt[-3 - 2*x + x^2]] - 4*Log[x + Sqrt[-3 - 2*x + x^2]]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{-3-2x+x^2}{x^2(-2+2x)^2}\,dx, x, x+\sqrt{-3-2x+x^2}\right)\\
&= 2\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}+\frac{2}{-1+x}-\frac{3}{4x^2}-\frac{2}{x}\right)\,dx, x, x+\sqrt{-3-2x+x^2}\right)\\
&= -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{2\left(x+\sqrt{-3-2x+x^2}\right)}\\
&\quad+4\log\left(1-x-\sqrt{-3-2x+x^2}\right)-4\log\left(x+\sqrt{-3-2x+x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx = \frac{-9+6x+4x^2-4(3+x)\sqrt{-3-2x+x^2}-32(3+2x)\text{arctanh}\left(\frac{1+x}{2+2x+\sqrt{-3-2x+x^2}}\right)}{4(3+2x)}$$

```
[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-2),x]
[Out] (-9 + 6*x + 4*x^2 - 4*(3 + x)*Sqrt[-3 - 2*x + x^2] - 32*(3 + 2*x)*ArcTanh[(
1 + x)/(2 + 2*x + Sqrt[-3 - 2*x + x^2])])/(4*(3 + 2*x))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

method	result
trager	$\frac{(3+x)x}{2x+3}-\frac{(3+x)\sqrt{x^2-2x-3}}{2x+3}+4\ln\left(-\frac{\sqrt{x^2-2x-3}+3+x}{2x+3}\right)$
default	$-2\ln\left(2x+3\right)+\frac{x}{2}-\frac{9}{4(2x+3)}-\frac{2\sqrt{4\left(x+\frac{3}{2}\right)^2-20x-21}}{3}+2\ln\left(-1+x+\sqrt{\left(x+\frac{3}{2}\right)^2-5x-\frac{21}{4}}\right)+2\ \mathrm{ar}$

```
[In] int(1/(x+(x^2-2*x-3)^(1/2))^2,x,method=_RETURNVERBOSE)
[Out] (3+x)*x/(2*x+3)-(3+x)/(2*x+3)*(x^2-2*x-3)^(1/2)+4*ln(-((x^2-2*x-3)^(1/2)+3+
x)/(2*x+3))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx$$
$$=\frac{4\,x^2-8\,(2\,x+3)\log\left(x^2-\sqrt{x^2-2\,x-3}(x+1)-3\right)-8\,(2\,x+3)\log\left(2\,x+3\right)+8\,(2\,x+3)\log\left(-x+\sqrt{\right.}}{4\,(2\,x+3)}$$

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
[Out] 1/4*(4*x^2 - 8*(2*x + 3)*log(x^2 - sqrt(x^2 - 2*x - 3)*(x + 1) - 3) - 8*(2*
x + 3)*log(2*x + 3) + 8*(2*x + 3)*log(-x + sqrt(x^2 - 2*x - 3)) - 4*sqrt(x^
2 - 2*x - 3)*(x + 3) + 2*x - 15)/(2*x + 3)
```

Sympy [F]

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx=\int \frac{1}{\left(x+\sqrt{x^2-2x-3}\right)^2}\,dx$$

```
[In] integrate(1/(x+(x**2-2*x-3)**(1/2))**2,x)
[Out] Integral((x + sqrt(x**2 - 2*x - 3))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx = \int \frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^2}\,dx$$

[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 - 2*x - 3))^(-2), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(69) = 138.

Time = 0.34 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.72

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx = \frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x^2-2\,x-3} - \frac{3\,\left(5\,x-5\,\sqrt{x^2-2\,x-3}+3\right)}{4\,\left(\left(x-\sqrt{x^2-2\,x-3}\right)^2+3\,x-3\,\sqrt{x^2-2\,x-3}\right)} - \frac{9}{4\,(2\,x+3)} - 2\,\log\left(|2\,x+3|\right) - 2\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}+1\right|\right) + 2\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}\right|\right) - 2\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}-3\right|\right)$$

[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="giac")

[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*(5*x - 5*sqrt(x^2 - 2*x - 3) + 3)/((x - sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 - 2*x - 3)) - 9/4/(2*x + 3) - 2*log(abs(2*x + 3)) - 2*log(abs(-x + sqrt(x^2 - 2*x - 3) + 1)) + 2*log(abs(-x + sqrt(x^2 - 2*x - 3))) - 2*log(abs(-x + sqrt(x^2 - 2*x - 3) - 3))

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx = \int \frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^2}\,dx$$

```
[In] int(1/(x + (x^2 - 2*x - 3)^(1/2))^2,x)
[Out] int(1/(x + (x^2 - 2*x - 3)^(1/2))^2, x)
```

3.758 $\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx = -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{4\left(x+\sqrt{-3-2x+x^2}\right)^2}+\frac{4}{x+\sqrt{-3-2x+x^2}}+6\log\left(1-x-\sqrt{-3-2x+x^2}\right)-6\log\left(x+\sqrt{-3-2x+x^2}\right)$$

```
[Out] 6*ln(1-x-(x^2-2*x-3)^(1/2))-6*ln(x+(x^2-2*x-3)^(1/2))-2/(1-x-(x^2-2*x-3)^(1
/2))+3/4/(x+(x^2-2*x-3)^(1/2))^2+4/(x+(x^2-2*x-3)^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2141, 907}

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx = -\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{4}{\sqrt{x^2-2x-3}+x}+\frac{3}{4\left(\sqrt{x^2-2x-3}+x\right)^2}+6\log\left(-\sqrt{x^2-2x-3}-x+1\right)-6\log\left(\sqrt{x^2-2x-3}+x\right)$$

```
[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-3),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(4*(x + Sqrt[-3 - 2*x + x^2])^2) + 4/
(x + Sqrt[-3 - 2*x + x^2]) + 6*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 6*Log[x
+ Sqrt[-3 - 2*x + x^2]]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{-3-2x+x^2}{x^3(-2+2x)^2}\,dx, x, x+\sqrt{-3-2x+x^2}\right)\\ &= 2\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}+\frac{3}{-1+x}-\frac{3}{4x^3}-\frac{2}{x^2}-\frac{3}{x}\right)\,dx, x, x+\sqrt{-3-2x+x^2}\right)\\ &= -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{4\left(x+\sqrt{-3-2x+x^2}\right)^2}+\frac{4}{x+\sqrt{-3-2x+x^2}}\\ &\quad+6\log\left(1-x-\sqrt{-3-2x+x^2}\right)-6\log\left(x+\sqrt{-3-2x+x^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx = -\frac{189+108x-48x^2-16x^3+4\sqrt{-3-2x+x^2}(33+31x+4x^2)+96(3+2x)^2\operatorname{arctanh}\left(\frac{1+x}{2+2x+\sqrt{-3-2x+x^2}}\right)}{8(3+2x)^2}$$

```
[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-3),x]
```

```
[Out] -1/8*(189 + 108*x - 48*x^2 - 16*x^3 + 4*Sqrt[-3 - 2*x + x^2]*(33 + 31*x + 4
*x^2) + 96*(3 + 2*x)^2*ArcTanh[(1 + x)/(2 + 2*x + Sqrt[-3 - 2*x + x^2])])/(
3 + 2*x)^2
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

method	result
trager	$\frac{\left(4x^{2}+33x+36\right)x}{2\left(2x+3\right)^{2}}-\frac{\left(4x^{2}+31x+33\right)\sqrt{x^{2}-2x-3}}{2\left(2x+3\right)^{2}}-6\ln\left(x+3-\sqrt{x^{2}-2x-3}\right)$
default	$-\frac{9}{2x+3}-3\ln\left(2x+3\right)+\frac{x}{2}+\frac{27}{8\left(2x+3\right)^{2}}-\frac{\left(\left(x+\frac{3}{2}\right)^{2}-5x-\frac{21}{4}\right)^{\frac{3}{2}}}{2\left(x+\frac{3}{2}\right)}-\sqrt{4\left(x+\frac{3}{2}\right)^{2}-20x-21}+3\ \mathrm{arctanh}$

```
[In] int(1/(x+(x^2-2*x-3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(4*x^2+33*x+36)*x/(2*x+3)^2-1/2*(4*x^2+31*x+33)/(2*x+3)^2*(x^2-2*x-3)^(
1/2)-6*ln(x+3-(x^2-2*x-3)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx$$
$$=\frac{8\,x^{3}-10\,x^{2}-12\,\left(4\,x^{2}+12\,x+9\right)\log\left(x^{2}-\sqrt{x^{2}-2\,x-3}(x+1)-3\right)-12\,\left(4\,x^{2}+12\,x+9\right)\log\left(2\,x+3\right.}{4\,(4\,x^{2}+1}$$

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*x^3 - 10*x^2 - 12*(4*x^2 + 12*x + 9)*log(x^2 - sqrt(x^2 - 2*x - 3)*(
x + 1) - 3) - 12*(4*x^2 + 12*x + 9)*log(2*x + 3) + 12*(4*x^2 + 12*x + 9)*lo
g(-x + sqrt(x^2 - 2*x - 3)) - 2*(4*x^2 + 31*x + 33)*sqrt(x^2 - 2*x - 3) - 1
56*x - 171)/(4*x^2 + 12*x + 9)
```

Sympy [F]

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx=\int \frac{1}{\left(x+\sqrt{x^2-2x-3}\right)^3}\,dx$$

```
[In] integrate(1/(x+(x**2-2*x-3)**(1/2))**3,x)
```

```
[Out] Integral((x + sqrt(x**2 - 2*x - 3))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx=\int \frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^3}\,dx$$

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 - 2*x - 3))^(-3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(85) = 170$.

Time = 0.34 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.82

$$\begin{aligned}\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx&=\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^2-2\,x-3}\\&\quad-\frac{104\,\left(x-\sqrt{x^2-2\,x-3}\right)^3+315\,\left(x-\sqrt{x^2-2\,x-3}\right)^2+162\,x-162\,\sqrt{x^2-2\,x-3}+27}{8\,\left(\left(x-\sqrt{x^2-2\,x-3}\right)^2+3\,x-3\,\sqrt{x^2-2\,x-3}\right)^2}\\&\quad-\frac{9\,(16\,x+21)}{8\,(2\,x+3)^2}-3\,\log\left(|2\,x+3|\right)-3\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}+1\right|\right)\\&\quad+3\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}\right|\right)-3\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}-3\right|\right)\end{aligned}$$

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 1/8*(104*(x - sqrt(x^2 - 2*x - 3))^3 + 31
5*(x - sqrt(x^2 - 2*x - 3))^2 + 162*x - 162*sqrt(x^2 - 2*x - 3) + 27)/((x -
 sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 - 2*x - 3))^2 - 9/8*(16*x + 21)/
(2*x + 3)^2 - 3*log(abs(2*x + 3)) - 3*log(abs(-x + sqrt(x^2 - 2*x - 3) + 1)
) + 3*log(abs(-x + sqrt(x^2 - 2*x - 3))) - 3*log(abs(-x + sqrt(x^2 - 2*x -
3) - 3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx=\int \frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^3}\,dx$$

```
[In] int(1/(x + (x^2 - 2*x - 3)^(1/2))^3,x)
[Out] int(1/(x + (x^2 - 2*x - 3)^(1/2))^3, x)
```

3.759 $\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 108

$$\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx = -\arctan\left(\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)-\sqrt{2}\arctan\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)+\frac{1}{2}\log(3+x)+\frac{1}{2}\log\left(\frac{3\sqrt{-1-x}+\sqrt{-1-x}x+x\sqrt{3+x}}{(3+x)^{3/2}}\right)$$

```
[Out] -arctan((-1-x)^(1/2)/(3+x)^(1/2))+1/2*ln(3+x)+1/2*ln((3*(-1-x)^(1/2)+x*(-1-
x)^(1/2)+x*(3+x)^(1/2))/(3+x)^(3/2))-arctan(1/2*(1-3*(-1-x)^(1/2)/(3+x)^(1/
2))*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {12, 1037, 648, 632, 210, 642, 649, 209, 266}

$$\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx = -\arctan\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)-\sqrt{2}\arctan\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)+\frac{1}{2}\log(x+3)+\frac{1}{2}\log\left(\frac{\sqrt{-x-1}x+\sqrt{x+3}x+3\sqrt{-x-1}}{(x+3)^{3/2}}\right)$$

```
[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-1),x]
```

```
[Out] -ArcTan[Sqrt[-1 - x]/Sqrt[3 + x]] - Sqrt[2]*ArcTan[(1 - (3*Sqrt[-1 - x])/Sq
rt[3 + x])/Sqrt[2]] + Log[3 + x]/2 + Log[(3*Sqrt[-1 - x] + Sqrt[-1 - x]*x +
 x*Sqrt[3 + x])/(3 + x)^(3/2)]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1037

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*
(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a
^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c
*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[b*
h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2), x]
, x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{2x}{(1+x^2)(1-2x+3x^2)}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) \\
&= 4\text{Subst}\left(\int \frac{x}{(1+x^2)(1-2x+3x^2)}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-2-2x}{1+x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{2+6x}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-2+6x}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)+2\text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) \\
&\quad - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) - \text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) \\
&= -\tan^{-1}\left(\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)+\frac{1}{2}\log(3+x)+\frac{1}{2}\log\left(\frac{3\sqrt{-1-x}+\sqrt{-1-x}x+x\sqrt{3+x}}{(3+x)^{3/2}}\right) \\
&\quad - 4\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, -2+\frac{6\sqrt{-1-x}}{\sqrt{3+x}}\right) \\
&= -\tan^{-1}\left(\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)-\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right) \\
&\quad +\frac{1}{2}\log(3+x)+\frac{1}{2}\log\left(\frac{3\sqrt{-1-x}+\sqrt{-1-x}x+x\sqrt{3+x}}{(3+x)^{3/2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.75

$$\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx=\frac{1}{2}\left(-2\arctan\left(\frac{\sqrt{-3-4x-x^2}}{3+x}\right)-2\sqrt{2}\arctan\left(\frac{\sqrt{2}(1+x)}{1+x+\sqrt{-3-4x-x^2}}\right)+\log\left(x+\sqrt{-3-4x-x^2}\right)\right)$$

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-1),x]
[Out] (-2*ArcTan[Sqrt[-3 - 4*x - x^2]/(3 + x)] - 2*Sqrt[2]*ArcTan[(Sqrt[2]*(1 + x
))/(1 + x + Sqrt[-3 - 4*x - x^2])] + Log[x + Sqrt[-3 - 4*x - x^2]])/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(85) = 170.

Time = 1.04 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.43

method	result
default	$\frac{\arcsin(x+2)}{2}-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)-\operatorname{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}}\right)\right)}{12\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(\frac{x}{-\frac{3}{2}-x}+1\right)^2}}\left(\frac{x}{-\frac{3}{2}-x}+1\right)}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{(\cdots}}{}$
trager	$\operatorname{RootOf}\left(4\_Z^2-4\_Z+3\right)\ln\left(4\operatorname{RootOf}\left(2\_Z^2+2\_Z+1\right)^2\operatorname{RootOf}\left(4\_Z^2-4\_Z+3\right)^2x+4\,\mathrm{R}\right.$

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsin(x+2)-1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*a
rctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2/
(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-3/2
-x)+1)+1/3*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(
-3/2-x)+1)*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2
-12)^(1/2)*2^(1/2))-1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2
)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3*x
^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-
3/2-x)+1)+1/4*ln(2*x^2+4*x+3)-1/2*2^(1/2)*arctan(1/4*(4+4*x)*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(85) = 170$.

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.73

$$\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right) + \frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) + \frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-4x-3}(x+2)}{x^2+4x+3}\right) + \frac{1}{4}\log\left(2x^2+4x+3\right) - \frac{1}{8}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right) + \frac{1}{8}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x +
3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/4*sqrt(2)*arctan(-1/2*(sqrt(
2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 1/2*arctan(sqrt(-x^2 -
4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/4*log(2*x^2 + 4*x + 3) - 1/8*log(-(2*
sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/8*log((2*sqrt(-x^2 - 4*x - 3)*x
- 4*x - 3)/x^2)
```

Sympy [F]

$$\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx = \int \frac{1}{x+\sqrt{-x^2-4x-3}}\,dx$$

```
[In] integrate(1/(x+(-x**2-4*x-3)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(-x**2 - 4*x - 3)), x)
```

Maxima [F]

$$\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx = \int \frac{1}{x+\sqrt{-x^2-4\,x-3}}\,dx$$

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(x + sqrt(-x^2 - 4*x - 3)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(85) = 170.

Time = 0.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.82

$$\begin{aligned}\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx = &-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)\\ &+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right)\\ &+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right)\\ &+\frac{1}{2}\arcsin\left(x+2\right)+\frac{1}{4}\log\left(2\,x^2+4\,x+3\right)\\ &+\frac{1}{4}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}\right.\\ &\left.+1\right)-\frac{1}{4}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}\right.\\ &\left.+\frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+3\right)\end{aligned}$$

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="giac")
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*arcsin(x + 2) + 1/4*log(2*x^2
+ 4*x + 3) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 -
4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x +
2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx=\int \frac{1}{x+\sqrt{-x^2-4\,x-3}}\,dx$$

```
[In] int(1/(x + (- 4*x - x^2 - 3)^(1/2)),x)
[Out] int(1/(x + (- 4*x - x^2 - 3)^(1/2)), x)
```

3.760 $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 87

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2} dx = \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}}+\frac{\arctan\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(1/2*(1-3*(-1-x)^(1/2)/(3+x)^(1/2))*2^(1/2))*2^(1/2)+(1-(-1-x)^(1
/2)/(3+x)^(1/2))/(1-3*(1+x)/(3+x)-2*(-1-x)^(1/2)/(3+x)^(1/2))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {12, 652, 632, 210}

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2} dx = \frac{\arctan\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}$$

```
[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-2),x]
```

```
[Out] (1 - Sqrt[-1 - x]/Sqrt[3 + x])/(1 - (3*(1 + x))/(3 + x) - (2*Sqrt[-1 - x])/
Sqrt[3 + x]) + ArcTan[(1 - (3*Sqrt[-1 - x])/Sqrt[3 + x])/Sqrt[2]]/Sqrt[2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int -\frac{2x}{(1-2x+3x^2)^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= -\left(4\text{Subst}\left(\int \frac{x}{(1-2x+3x^2)^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\right)\\
&= \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}} - \text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}} + 2\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, -2+\frac{6\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}} + \frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx = \frac{3+x+(3+2x)\sqrt{-3-4x-x^2}+\sqrt{2}(3+4x+2x^2)\arctan\left(\frac{\sqrt{2}(1+x)}{1+x+\sqrt{-3-4x-x^2}}\right)}{2\left(3+4x+2x^2\right)}$$

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-2),x]
[Out] (3 + x + (3 + 2*x)*Sqrt[-3 - 4*x - x^2] + Sqrt[2]*(3 + 4*x + 2*x^2)*ArcTan[
(Sqrt[2]*(1 + x))/(1 + x + Sqrt[-3 - 4*x - x^2])])/(2*(3 + 4*x + 2*x^2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

method	result
trager	$-\frac{(2x+3)x}{2(2x^2+4x+3)}+\frac{(2x+3)\sqrt{-x^2-4x-3}}{4x^2+8x+6}+\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{-2\operatorname{RootOf}\left(\_Z^2+2\right)x+2\sqrt{-x^2-4x-3}-3\operatorname{RootOf}\left(\_Z^2+2\right)}{\operatorname{RootOf}\left(\_Z^2+2\right)x+2x+3}\right)}{4}$
default	Expression too large to display

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2))^2,x,method=_RETURNVERBOSE)
[Out] -1/2*(2*x+3)*x/(2*x^2+4*x+3)+1/2*(2*x+3)/(2*x^2+4*x+3)*(-x^2-4*x-3)^(1/2)+1
/4*RootOf(_Z^2+2)*ln((-2*RootOf(_Z^2+2)*x+2*(-x^2-4*x-3)^(1/2)-3*RootOf(_Z^
2+2))/(RootOf(_Z^2+2)*x+2*x+3))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.39

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx = \frac{2\sqrt{2}(2\,x^2+4\,x+3)\arctan\left(\sqrt{2}(x+1)\right)-\sqrt{2}(2\,x^2+4\,x+3)\arctan\left(\frac{\sqrt{2}(6\,x^2+20\,x+15)\sqrt{-x^2-4\,x-3}}{4\,(2\,x^3+11\,x^2+18\,x+9)}\right)+4\,\sqrt{-x}}{8\,(2\,x^2+4\,x+3)}$$

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(2)*(2*x^2 + 4*x + 3)*arctan(sqrt(2)*(x + 1)) - sqrt(2)*(2*x^2 +
 4*x + 3)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*x^
3 + 11*x^2 + 18*x + 9)) + 4*sqrt(-x^2 - 4*x - 3)*(2*x + 3) + 4*x + 12)/(2*x
^2 + 4*x + 3)
```

Sympy [F]

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx=\int \frac{1}{\left(x+\sqrt{-x^2-4x-3}\right)^2}\,dx$$

```
[In] integrate(1/(x+(-x**2-4*x-3)**(1/2))**2,x)
```

```
[Out] Integral((x + sqrt(-x**2 - 4*x - 3))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx=\int \frac{1}{\left(x+\sqrt{-x^2-4\,x-3}\right)^2}\,dx$$

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 4*x - 3))^(-2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(72) = 144$.

Time = 0.34 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.02

$$\begin{aligned}&\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx\\&=\frac{1}{4}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right)\\&\quad-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right)+\frac{x+3}{2\left(2\,x^2+4\,x+3\right)}\\&\quad-\frac{\frac{10\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{7\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}-\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)^3}{(x+2)^3}+3}{3\left(\frac{8\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{14\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+\frac{8\left(\sqrt{-x^2-4\,x-3}-1\right)^3}{(x+2)^3}+\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^4}{(x+2)^4}+3\right)}\end{aligned}$$

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*(x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sq
rt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((sq
rt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*(x + 3)/(2*x^2 + 4*x + 3) - 1/3
*(10*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 7*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x
 + 2)^2 - 2*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3)/(8*(sqrt(-x^2 - 4*x
 - 3) - 1)/(x + 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 8*(sqrt(-x
^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 +
 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx=\int \frac{1}{{\left(x+\sqrt{-x^2-4\,x-3}\right)}^2}\,dx$$

```
[In] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^2,x)
```

```
[Out] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^2, x)
```

3.761 $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 149

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3} dx = -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)} - \frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} - \frac{3\arctan\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] -3/4*arctan(1/2*(1-3*(-1-x)^(1/2)/(3+x)^(1/2))*2^(1/2))*2^(1/2)+1/18*(-13+2
7*(-1-x)^(1/2)/(3+x)^(1/2))/(1-3*(1+x)/(3+x)-2*(-1-x)^(1/2)/(3+x)^(1/2))-2/
9*(2-(-1-x)^(1/2)/(3+x)^(1/2))/(1-3*(1+x)/(3+x)-2*(-1-x)^(1/2)/(3+x)^(1/2))
^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {12, 1674, 652, 632, 210}

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3} dx = -\frac{3\arctan\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{13-\frac{27\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)} - \frac{2\left(2-\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)}{9\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}$$

```
[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-3),x]
```

```
[Out] -1/18*(13 - (27*Sqrt[-1 - x])/Sqrt[3 + x])/(1 - (3*(1 + x))/(3 + x) - (2*Sq
rt[-1 - x])/Sqrt[3 + x]) - (2*(2 - Sqrt[-1 - x]/Sqrt[3 + x]))/(9*(1 - (3*(1
 + x))/(3 + x) - (2*Sqrt[-1 - x])/Sqrt[3 + x])^2) - (3*ArcTan[(1 - (3*Sqrt[
-1 - x])/Sqrt[3 + x])/Sqrt[2]])/(2*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{2x(1+x^2)}{(1-2x+3x^2)^3}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= 4\text{Subst}\left(\int \frac{x(1+x^2)}{(1-2x+3x^2)^3}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= -\frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} + \frac{1}{4}\text{Subst}\left(\int \frac{\frac{56}{9}+\frac{16x}{3}}{(1-2x+3x^2)^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)} - \frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)} - \frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} - 3\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, -2+\frac{6\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)} - \frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} - \frac{3\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx = -\frac{9+15x+16x^2+6x^3+\sqrt{-3-4x-x^2}(15+26x+22x^2+8x^3)+3\sqrt{2}(3+4x+2x^2)^2\arctan\left(\frac{\sqrt{2}(1+x)}{1+x+\sqrt{-3-4x-x^2}}\right)}{4\left(3+4x+2x^2\right)^2}$$

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-3),x]
```

```
[Out] -1/4*(9 + 15*x + 16*x^2 + 6*x^3 + Sqrt[-3 - 4*x - x^2]*(15 + 26*x + 22*x^2
+ 8*x^3) + 3*Sqrt[2]*(3 + 4*x + 2*x^2)^2*ArcTan[(Sqrt[2]*(1 + x))/(1 + x +
Sqrt[-3 - 4*x - x^2])])/(3 + 4*x + 2*x^2)^2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87

method	result
trager	$\frac{(4x^3+10x^2+12x+9)x}{4(2x^2+4x+3)^2}-\frac{(8x^3+22x^2+26x+15)\sqrt{-x^2-4x-3}}{4(2x^2+4x+3)^2}-\frac{3\,\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{-2\,\mathrm{RootOf}\left(\_Z^2+2\right)x+2\sqrt{-x^2-4x-3}-3\,\mathrm{R}}{\mathrm{RootOf}\left(\_Z^2+2\right)x+2x+3}\right)}{8}$
default	Expression too large to display

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(4*x^3+10*x^2+12*x+9)*x/(2*x^2+4*x+3)^2-1/4*(8*x^3+22*x^2+26*x+15)/(2*x
^2+4*x+3)^2*(-x^2-4*x-3)^(1/2)-3/8*RootOf(_Z^2+2)*ln((-2*RootOf(_Z^2+2)*x+2
*(-x^2-4*x-3)^(1/2)-3*RootOf(_Z^2+2))/(RootOf(_Z^2+2)*x+2*x+3))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx =$$

$$-\frac{24\,x^3+6\,\sqrt{2}(4\,x^4+16\,x^3+28\,x^2+24\,x+9)\arctan\left(\sqrt{2}(x+1)\right)-3\,\sqrt{2}(4\,x^4+16\,x^3+28\,x^2+24\,x+}{16\,(4\,x^4+16\,x^3+}$$

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -1/16*(24*x^3 + 6*sqrt(2)*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(sqrt(
2)*(x + 1)) - 3*sqrt(2)*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(1/4*sqr
t(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*x^3 + 11*x^2 + 18*x + 9))
+ 64*x^2 + 4*(8*x^3 + 22*x^2 + 26*x + 15)*sqrt(-x^2 - 4*x - 3) + 60*x + 36)
/(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)
```

Sympy [F]

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx = \int \frac{1}{\left(x+\sqrt{-x^2-4x-3}\right)^3}\,dx$$

```
[In] integrate(1/(x+(-x**2-4*x-3)**(1/2))**3,x)
[Out] Integral((x + sqrt(-x**2 - 4*x - 3))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx = \int \frac{1}{\left(x+\sqrt{-x^2-4\,x-3}\right)^3}\,dx$$

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="maxima")
[Out] integrate((x + sqrt(-x^2 - 4*x - 3))^(-3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(119) = 238.

Time = 0.33 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.46

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx$$
$$= -\frac{3}{8}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right) + \frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right) + \frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right) - \frac{6\,x^3+16\,x^2+15\,x+9}{4\,(2\,x^2+4\,x+3)^2} + \frac{\frac{618\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2} + \frac{1547\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2} + \frac{2362\left(\sqrt{-x^2-4\,x-3}-1\right)^3}{(x+2)^3} + \frac{2223\left(\sqrt{-x^2-4\,x-3}-1\right)^4}{(x+2)^4} + \frac{1174\left(\sqrt{-x^2-4\,x-3}\right.}{(x+2)^5}}{18\left(\frac{8\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2} + \frac{14\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2} + \frac{8\left(\sqrt{-x^2-4\,x-3}-1\right)^3}{(x+2)^3} + \frac{3\left(\sqrt{-\right.}}{}\right.}$$

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="giac")
[Out] -3/8*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*(s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*((s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*(6*x^3 + 16*x^2 + 15*x + 9)/(2
```

```
*x^2 + 4*x + 3)^2 + 1/18*(618*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1547*(sq
rt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 2362*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x
+ 2)^3 + 2223*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 1174*(sqrt(-x^2 - 4*
x - 3) - 1)^5/(x + 2)^5 + 377*(sqrt(-x^2 - 4*x - 3) - 1)^6/(x + 2)^6 + 6*(s
qrt(-x^2 - 4*x - 3) - 1)^7/(x + 2)^7 + 117)/(8*(sqrt(-x^2 - 4*x - 3) - 1)/(
x + 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 8*(sqrt(-x^2 - 4*x - 3
) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 3)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx=\int \frac{1}{\left(x+\sqrt{-x^2-4\,x-3}\right)^3}\,dx$$

```
[In] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^3,x)
[Out] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^3, x)
```

3.762 $\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 42

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx = -\frac{1}{15}\left(1-x^2-2x^3-x^4\right)^{3/2}\left(2+3x^2+6x^3+3x^4\right)$$

```
[Out] -1/15*(-x^4-2*x^3-x^2+1)^(3/2)*(3*x^4+6*x^3+3*x^2+2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1694, 12, 1261, 706, 643}

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx = -\frac{1}{5}x^2\left(-x^4-2x^3-x^2+1\right)^{3/2}(x+1)^2 - \frac{2}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}$$

```
[In] Int[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
```

```
[Out] (-2*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15 - (x^2*(1 + x)^2*(1 - x^2 - 2*x^3 - x
^4)^(3/2))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
```

```
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 706

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[2*d*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(b*(m + 2*p
 + 1))), x] + Dist[d^2*(m - 1)*((b^2 - 4*a*c)/(b^2*(m + 2*p + 1))), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{128}x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)\\
&= \frac{1}{128}\text{Subst}\left(\int x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)\\
&= \frac{1}{256}\text{Subst}\left(\int(-1+4x)^3\sqrt{15+8x-16x^2}\,dx, x, \left(\frac{1}{2}+x\right)^2\right)\\
&= -\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2}\\
&\quad+\frac{1}{40}\text{Subst}\left(\int(-1+4x)\sqrt{15+8x-16x^2}\,dx, x, \left(\frac{1}{2}+x\right)^2\right)\\
&= -\frac{2}{15}\left(1-x^2-2x^3-x^4\right)^{3/2}-\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx = \frac{1}{15}\left(-2-3x^2-6x^3-3x^4\right)\left(1-x^2-2x^3-x^4\right)^{3/2}$$

[In] Integrate[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]

[Out] ((-2 - 3*x^2 - 6*x^3 - 3*x^4)*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15

Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{\left(-x^4-2x^3-x^2+1\right)^{\frac{3}{2}}\left(x^4+2x^3+x^2+\frac{2}{3}\right)}{5}$
gosper	$\frac{\left(x^2+x+1\right)\left(x^2+x-1\right)\left(3x^4+6x^3+3x^2+2\right)\sqrt{-x^4-2x^3-x^2+1}}{15}$
trager	$\left(\frac{1}{5}x^8+\frac{4}{5}x^7+\frac{6}{5}x^6+\frac{4}{5}x^5+\frac{2}{15}x^4-\frac{2}{15}x^3-\frac{1}{15}x^2-\frac{2}{15}\right)\sqrt{-x^4-2x^3-x^2+1}$
risch	$-\frac{\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)\left(x^4+2x^3+x^2-1\right)}{15\sqrt{-x^4-2x^3-x^2+1}}$
default	$\frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}+\frac{x^8\sqrt{-x^4-2x^3-x}}{5}$
elliptic	$\frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}+\frac{x^8\sqrt{-x^4-2x^3-x}}{5}$

[In] int(x^3*(x+1)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/5*(-x^4-2*x^3-x^2+1)^(3/2)*(x^4+2*x^3+x^2+2/3)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx = \frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{-x^4-2\,x^3-x^2+1}$$

[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="fricas
")

[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(-x^4
 - 2*x^3 - x^2 + 1)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(36) = 72$.

Time = 0.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.33

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx = \frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{4x^7\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{6x^6\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{4x^5\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}$$

```
[In] integrate(x**3*(1+x)**3*(1+2*x)*(-x**4-2*x**3-x**2+1)**(1/2),x)
```

```
[Out] x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x**3 - x**2
+ 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**5*sqrt(-x**4 - 2*x
**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*x**3*sqrt
(-x**4 - 2*x**3 - x**2 + 1)/15 - x**2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 -
2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx = \frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{x^2+x+1}\sqrt{-x^2-x+1}$$

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(x^2
+ x + 1)*sqrt(-x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx = \frac{1}{5}\left(x^4+2\,x^3+x^2-1\right)^2\sqrt{-x^4-2\,x^3-x^2+1} - \frac{1}{3}\left(-x^4-2\,x^3-x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/5*(x^4 + 2*x^3 + x^2 - 1)^2*sqrt(-x^4 - 2*x^3 - x^2 + 1) - 1/3*(-x^4 - 2*
x^3 - x^2 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx = -\frac{\left(3\,x^4+6\,x^3+3\,x^2+2\right)\left(-x^4-2\,x^3-x^2+1\right)^{3/2}}{15}$$

```
[In] int(x^3*(2*x + 1)*(x + 1)^3*(1 - 2*x^3 - x^4 - x^2)^(1/2),x)
[Out] -((3*x^2 + 6*x^3 + 3*x^4 + 2)*(1 - 2*x^3 - x^4 - x^2)^(3/2))/15
```

3.763 $\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 42

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx=-\frac{1}{15}\left(1-x^2-2x^3-x^4\right)^{3/2}\left(2+3x^2+6x^3+3x^4\right)$$

```
[Out] -1/15*(-x^4-2*x^3-x^2+1)^(3/2)*(3*x^4+6*x^3+3*x^2+2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1607, 1694, 12, 1261, 706, 643}

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx=-\frac{1}{5}x^2\left(-x^4-2x^3-x^2+1\right)^{3/2}(x+1)^2-\frac{2}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}$$

```
[In] Int[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

```
[Out] (-2*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15 - (x^2*(1 + x)^2*(1 - x^2 - 2*x^3 - x
^4)^(3/2))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 706

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[2*d*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(b*(m + 2*p
 + 1))), x] + Dist[d^2*(m - 1)*((b^2 - 4*a*c)/(b^2*(m + 2*p + 1))), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^3(1+x)^3(1+2x)\sqrt{1-\left(x+x^2\right)^2}\,dx\\
&= \text{Subst}\left(\int \frac{1}{128}x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)\\
&= \frac{1}{128}\text{Subst}\left(\int x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x\right)
\end{aligned}
$$

$$= \frac{1}{256}\text{Subst}\left(\int (-1+4x)^3\sqrt{15+8x-16x^2}\,dx, x, \left(\frac{1}{2}+x\right)^2\right)$$

$$= -\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2} + \frac{1}{40}\text{Subst}\left(\int (-1+4x)\sqrt{15+8x-16x^2}\,dx, x, \left(\frac{1}{2}+x\right)^2\right)$$

$$= -\frac{2}{15}\left(1-x^2-2x^3-x^4\right)^{3/2} - \frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-(x+x^2)^2}\,dx = \frac{1}{15}\left(-2-3x^2-6x^3-3x^4\right)\left(1-x^2-2x^3-x^4\right)^{3/2}$$

```
[In] Integrate[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
[Out] ((-2 - 3*x^2 - 6*x^3 - 3*x^4)*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{\left(-x^4-2x^3-x^2+1\right)^{\frac{3}{2}}\left(x^4+2x^3+x^2+\frac{2}{3}\right)}{5}$
gosper	$\frac{\left(x^2+x+1\right)\left(x^2+x-1\right)\left(3x^4+6x^3+3x^2+2\right)\sqrt{-x^4-2x^3-x^2+1}}{15}$
trager	$\left(\frac{1}{5}x^8+\frac{4}{5}x^7+\frac{6}{5}x^6+\frac{4}{5}x^5+\frac{2}{15}x^4-\frac{2}{15}x^3-\frac{1}{15}x^2-\frac{2}{15}\right)\sqrt{-x^4-2x^3-x^2+1}$
risch	$-\frac{\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)\left(x^4+2x^3+x^2-1\right)}{15\sqrt{-x^4-2x^3-x^2+1}}$
default	$\frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}+\frac{x^8\sqrt{-x^4-2x^3-x^2}}{5}$
elliptic	$\frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}+\frac{x^8\sqrt{-x^4-2x^3-x^2}}{5}$

```
[In] int((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/5*(-x^4-2*x^3-x^2+1)^(3/2)*(x^4+2*x^3+x^2+2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx = \frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{-x^4-2\,x^3-x^2+1}$$

```
[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(-x^4
 - 2*x^3 - x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(36) = 72.

Time = 0.77 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.33

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx = \frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{4x^7\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{6x^6\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{4x^5\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}$$

```
[In] integrate((1+2*x)*(x**2+x)**3*(1-(x**2+x)**2)**(1/2),x)
[Out] x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x**3 - x**2
+ 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**5*sqrt(-x**4 - 2*x
**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*x**3*sqrt
(-x**4 - 2*x**3 - x**2 + 1)/15 - x**2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 -
2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx = \frac{1}{15}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)\sqrt{x^2+x+1}\sqrt{-x^2-x+1}$$

```
[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(x^2
+ x + 1)*sqrt(-x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx = \frac{1}{5}\left(x^4+2x^3+x^2-1\right)^2\sqrt{-x^4-2x^3-x^2+1} - \frac{1}{3}\left(-x^4-2x^3-x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="giac")
[Out] 1/5*(x^4 + 2*x^3 + x^2 - 1)^2*sqrt(-x^4 - 2*x^3 - x^2 + 1) - 1/3*(-x^4 - 2*
x^3 - x^2 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 18.47 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx = \sqrt{1-\left(x^2+x\right)^2}\left(\frac{x^8}{5}+\frac{4x^7}{5}+\frac{6x^6}{5}+\frac{4x^5}{5}+\frac{2x^4}{15} - \frac{2x^3}{15}-\frac{x^2}{15}-\frac{2}{15}\right)$$

```
[In] int((2*x + 1)*(1 - (x + x^2)^2)^(1/2)*(x + x^2)^3,x)
[Out] (1 - (x + x^2)^2)^(1/2)*((2*x^4)/15 - (2*x^3)/15 - x^2/15 + (4*x^5)/5 + (6*
x^6)/5 + (4*x^7)/5 + x^8/5 - 2/15)
```

3.764 $\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\frac{2}{35}\left(13-3(-1+x)^2\right)\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+\frac{16}{5}\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-\frac{176}{35}\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)$$

```
[Out] 1/7*(3-2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)-16/5*EllipticE(-1+x,1/3*I*3^(1/2))
*3^(1/2)+176/35*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+2/35*(13-3*(-1+x)^2)*
(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1120, 1105, 1190, 1194, 538, 435, 430}

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=-\frac{176}{35}\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)+\frac{16}{5}\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}\left(13-3(x-1)^2\right)(x-1)\sqrt{-(x-1)^4-2(x-1)^2+3}$$

```
[In] Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (2*(13 - 3*(-1 + x)^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + (
(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16*Sqrt[3]*EllipticE[A
rcSin[1 - x], -1/3])/5 - (176*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3])/35
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
```

```
b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \left(3-2x^2-x^4\right)^{3/2}\,dx, x, -1+x\right)\\
&= \frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)\\
&\quad + \frac{3}{7}\text{Subst}\left(\int \left(6-2x^2\right)\sqrt{3-2x^2-x^4}\,dx, x, -1+x\right)\\
&= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3-2(-1+x)^2\right.\\
&\quad \left.-(-1+x)^4\right)^{3/2}(-1+x)-\frac{1}{35}\text{Subst}\left(\int \frac{-192+112x^2}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)\\
&= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3-2(-1+x)^2\right.\\
&\quad \left.-(-1+x)^4\right)^{3/2}(-1+x)-\frac{2}{35}\text{Subst}\left(\int \frac{-192+112x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)\\
&= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)\\
&\quad +\frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)-\frac{16}{5}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x, -1+x\right)\\
&\quad + \frac{1056}{35}\text{Subst}\left(\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)\\
&= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3-2(-1+x)^2\right.\\
&\quad \left.-(-1+x)^4\right)^{3/2}(-1+x)\\
&\quad +\frac{16}{5}\sqrt{3}E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)-\frac{176}{35}\sqrt{3}F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.48 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.73

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\frac{896-1056x+352x^2+848x^3-1420x^4+1152x^5-602x^6+206x^7-45x^8+5x^9+\frac{112i\sqrt{2}(-2+\cdots}{\cdots}}{35\sqrt{-\cdots}}$$

```
[In] Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (896 - 1056*x + 352*x^2 + 848*x^3 - 1420*x^4 + 1152*x^5 - 602*x^6 + 206*x^7
 - 45*x^8 + 5*x^9 + ((112*I)*Sqrt[2]*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2]*E
llipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])
/(-I + Sqrt[3])])/Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)] - (304*I)*Sqrt[2
]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*Sqrt[(4 - 2*x + x^2)/x^2]*El
lipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/
(-I + Sqrt[3])])/(35*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 957 vs. 2(86) = 172.

Time = 2.64 (sec) , antiderivative size = 958, normalized size of antiderivative = 9.39

method	result	size
risch	Expression too large to display	958
default	Expression too large to display	1050
elliptic	Expression too large to display	1050

```
[In] int((-x^4+4*x^3-8*x^2+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*(5*x^5-25*x^4+66*x^3-98*x^2+32*x+20)*x*(x^3-4*x^2+8*x-8)/(-x*(x^3-4*x^
2+8*x-8))^(1/2)+32/7*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^
(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/
(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*
3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+
I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+64/5*(-1-I*3
^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2
))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(
-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF
```

```
(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))
/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3
^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1
/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-16/5*(x*(x-1+I*3^(1/2))*(x-1-I*3
^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)
^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2
))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2
))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2)
)/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I
*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^
(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/
(x-2))^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1
+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2
)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx = -\frac{112\,(-i\,x+i)E\left(\arcsin\left(\frac{1}{x-1}\right)\,|-3\right)+80\,(-i\,x+i)F\left(\arcsin\left(\frac{1}{x-1}\right)\,|-3\right)+\left(5\,x^6-30\,x^5+91\,x^4-164\,x^3\right.}{35\,(x-1)}$$

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/35*(112*(-I*x + I)*elliptic_e(arcsin(1/(x - 1)), -3) + 80*(-I*x + I)*ell
iptic_f(arcsin(1/(x - 1)), -3) + (5*x^6 - 30*x^5 + 91*x^4 - 164*x^3 + 130*x
^2 - 12*x - 132)*sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x))/(x - 1)
```

Sympy [F]

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx = \int \left(-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
```

```
[Out] Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Maxima [F]

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

Giac [F]

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{3/2}\,dx$$

```
[In] int((8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int((8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.765 $\int \sqrt{8x-8x^2+4x^3-x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx = \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{2E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)}{\sqrt{3}} - \frac{4\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] -2/3*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+4/3*EllipticF(-1+x,1/3*I*3^(1/2)
)*3^(1/2)+1/3*(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1120, 1105, 1194, 538, 435, 430}

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx = -\frac{4\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}} + \frac{2E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)}{\sqrt{3}} + \frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

```
[In] Int[Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*EllipticE[ArcSin[1 -
x], -1/3])/Sqrt[3] - (4*EllipticF[ArcSin[1 - x], -1/3])/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \sqrt{3-2x^2-x^4}\,dx, x, -1+x\right)$$

$$= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{1}{3}\text{Subst}\left(\int \frac{6-2x^2}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)$$

$$=\frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{2}{3}\text{Subst}\left(\int\frac{6-2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx,x,-1+x\right)$$

$$=\frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)-\frac{2}{3}\text{Subst}\left(\int\frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx,x,-1+x\right)+8\text{Subst}\left(\int\frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx,x,-1+x\right)$$

$$=\frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{2E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{4F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.42 (sec) , antiderivative size = 256, normalized size of antiderivative = 4.13

$$\int\sqrt{8x-8x^2+4x^3-x^4}\,dx=$$

$$-\frac{-16+24x-24x^2+14x^3-5x^4+x^5-\frac{2i\sqrt{2}(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}}E\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}}+8i\sqrt{2}\sqrt{-\frac{i(-2\cdots}{\left(-i+\cdots\right.}}}{3\sqrt{-x\left(-8+8x-4x^2+x^3\right)}}$$

```
[In] Integrate[Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] -1/3*(-16 + 24*x - 24*x^2 + 14*x^3 - 5*x^4 + x^5 - ((2*I)*Sqrt[2]*(-2 + x)*
x*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(S
qrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[((-I)*(-2 + x))/((-I +
Sqrt[3])*x)] + (8*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*S
qrt[(4 - 2*x + x^2)/x^2]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt
[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^
3))]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 935 vs. 2(54) = 108.

Time = 2.00 (sec) , antiderivative size = 936, normalized size of antiderivative = 15.10

method	result
risch	$-\frac{(x-1)x(x^3-4x^2+8x-8)}{3\sqrt{-x(x^3-4x^2+8x-8)}}+\frac{8(-1-i\sqrt{3})\sqrt{\frac{(-1+i\sqrt{3})x}{(1+i\sqrt{3})(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{(1-i\sqrt{3})(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{(1+i\sqrt{3})(x-2)}}\,F\left(\sqrt{\frac{(-1+i\sqrt{3})x}{(1+i\sqrt{3})(x-2)}},\sqrt{\frac{(1+i\sqrt{3}}{(-1+i}}\right.}{3(-1+i\sqrt{3})\sqrt{-x(x-2)(x-1+i\sqrt{3})(x-1-i\sqrt{3})}}$
default	$\frac{x\sqrt{-x^4+4x^3-8x^2+8x}}{3}-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{3}+\frac{8(-1-i\sqrt{3})\sqrt{\frac{(-1+i\sqrt{3})x}{(1+i\sqrt{3})(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{(1-i\sqrt{3})(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{(1+i\sqrt{3})(x-2)}}\,F\left(\sqrt{\frac{(}{(1}}\right.}{3(-1+i\sqrt{3})\sqrt{-x(x-2)(x-1+i\sqrt{3})(x-1-i\sqrt{}}}$
elliptic	$\frac{x\sqrt{-x^4+4x^3-8x^2+8x}}{3}-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{3}+\frac{8(-1-i\sqrt{3})\sqrt{\frac{(-1+i\sqrt{3})x}{(1+i\sqrt{3})(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{(1-i\sqrt{3})(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{(1+i\sqrt{3})(x-2)}}\,F\left(\sqrt{\frac{(}{(1}}\right.}{3(-1+i\sqrt{3})\sqrt{-x(x-2)(x-1+i\sqrt{3})(x-1-i\sqrt{}}}$

```
[In] int((-x^4+4*x^3-8*x^2+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(x-1)*x*(x^3-4*x^2+8*x-8)/(-x*(x^3-4*x^2+8*x-8))^(1/2)+8/3*(-1-I*3^(1/
2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(
1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I
*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I
*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*
3^(1/2))/(1-I*3^(1/2)))^(1/2))-2/3*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-1
-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^
(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/
2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(
1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2
)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-
2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)
)-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),
(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(
1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)+8/3
*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+
I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))
^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*E
llipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*
3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2))*
x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*(-
1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx = -\frac{2\left(-i\,x+i\right)E\left(\arcsin\left(\frac{1}{x-1}\right)\,|\,-3\right)+4\left(-i\,x+i\right)F\left(\arcsin\left(\frac{1}{x-1}\right)\,|\,-3\right)-\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}\left(x^2-\right.}{3\left(x-1\right)}$$

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="fricas")
[Out] -1/3*(2*(-I*x + I)*elliptic_e(arcsin(1/(x - 1)), -3) + 4*(-I*x + I)*ellipti
c_f(arcsin(1/(x - 1)), -3) - sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)*(x^2 - 2*x +
3))/(x - 1)
```

Sympy [F]

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx = \int \sqrt{-x^4+4x^3-8x^2+8x}\,dx$$

```
[In] integrate((-x**4+4*x**3-8*x**2+8*x)**(1/2),x)
[Out] Integral(sqrt(-x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Maxima [F]

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx = \int \sqrt{-x^4+4\,x^3-8\,x^2+8\,x}\,dx$$

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Giac [F]

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx = \int \sqrt{-x^4+4\,x^3-8\,x^2+8\,x}\,dx$$

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx = \int \sqrt{-x^4+4\,x^3-8\,x^2+8\,x}\,dx$$

```
[In] int((8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int((8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.766 $\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 17

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx=-\frac{\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1120, 1109, 430}

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx=-\frac{\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[In] Int[1/Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] -(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
```

```
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)\\ &= 2\text{Subst}\left(\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)\\ &= -\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.18

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx = \frac{\sqrt{-i+\sqrt{3}+\frac{4i}{x}}\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}x\left(-4+x-i\sqrt{3}x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{2}\sqrt{i+\sqrt{3}-\frac{4i}{x}}\sqrt{-x\left(-8+8x-4x^2+x^3\right)}}$$

```
[In] Integrate[1/Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (Sqrt[-I + Sqrt[3] + (4*I)/x]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x*(-
4 + x - I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/(Sqrt[2]*Sqrt[I + Sqrt[3] - (4*I)/x
]*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(15) = 30.

Time = 1.36 (sec) , antiderivative size = 200, normalized size of antiderivative = 11.76

method	result	size
default	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^{2}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-1-i\sqrt{3}\right)}{\left(-1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}}\right)}{\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$	200
elliptic	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^{2}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-1-i\sqrt{3}\right)}{\left(-1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}}\right)}{\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$	200

```
[In] int(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1
+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2)
)^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*Ell
ipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^
(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx = -\frac{1}{2}\,\sqrt{2}\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3\,x}\right)$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+4x^3-8x^2+8x}}\,dx$$

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{8x - 8x^2 + 4x^3 - x^4}} \, dx = \int \frac{1}{\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + 8\,x}} \, dx$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{8x - 8x^2 + 4x^3 - x^4}} \, dx = \int \frac{1}{\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + 8\,x}} \, dx$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{8x - 8x^2 + 4x^3 - x^4}} \, dx = \int \frac{1}{\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + 8\,x}} \, dx$$

```
[In] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.767 $\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{E\left(\arcsin(1-x)\,\middle|\,-\frac{1}{3}\right)}{8\sqrt{3}} - \frac{\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{4\sqrt{3}}$$

```
[Out] -1/24*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/12*EllipticF(-1+x,1/3*I*3^(1/
2))*3^(1/2)+1/24*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1120, 1106, 1194, 538, 435, 430}

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = -\frac{\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{4\sqrt{3}} + \frac{E\left(\arcsin(1-x)\,\middle|\,-\frac{1}{3}\right)}{8\sqrt{3}} + \frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

```
[In] Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + Elli
pticE[ArcSin[1 - x], -1/3]/(8*Sqrt[3]) - EllipticF[ArcSin[1 - x], -1/3]/(4*
Sqrt[3])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)\\
&= \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{48}\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)\\
&= \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{24}\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)\\
&= \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{24}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x, -1+x\right)\\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)\\
&= \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}} - \frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.67 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.58

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \frac{\sqrt{-x(-8+8x-4x^2+x^3)}\left(\frac{\sqrt{2}\left(-i+\sqrt{3}\right)\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}E\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{\frac{4-2x+x^2}{x^2}}}\right.}{24(-2+$$

```
[In] Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] (Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*((Sqrt[2]*(-I + Sqrt[3])*Sqrt[((-I)*(-
2 + x))/((-I + Sqrt[3])*x)]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(S
qrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[(4 - 2*x + x^2)/x^2] -
(2 + x^2 - (4*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*Sqrt[
(4 - 2*x + x^2)/x^2]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/(4 - 2*x + x^2)))/(24*(-2 + x)*x)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 931 vs. 2(61) = 122.

Time = 2.03 (sec) , antiderivative size = 932, normalized size of antiderivative = 12.77

method	result
risch	$\frac{x^3-3x^2+8x-6}{24\sqrt{-x(x^3-4x^2+8x-8)}}+\frac{\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-\right.}{\left(-1+i\sqrt{3}\right)}}\right.}{6\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$
default	Expression too large to display
elliptic	Expression too large to display

```
[In] int(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(x^3-3*x^2+8*x-6)/(-x*(x^3-4*x^2+8*x-8))^(1/2)+1/6*(-1-I*3^(1/2))*((-1
+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1
/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2)
)/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2)
)*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))
/(1-I*3^(1/2)))^(1/2))-1/24*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1
/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/
(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2
*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(
x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/
2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/
2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1
+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(-1-I*3
^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(
1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)+1/6*(-1-I*
3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/
2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/
(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*Elliptic
F(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2)
)/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2))*x/(1+I*
3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(
1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.63

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = -\frac{5\sqrt{2}(x^4-4x^3+8x^2-8x)\mathrm{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3x}\right)-6\sqrt{2}(x^4-4x^3+8x^2-8x)\mathrm{weierstras}}{72(x^4-4x^3+8x^2-}$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="fricas")
[Out] -1/72*(5*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassPInverse(-2/3, 7/54
, -1/3*(x - 3)/x) - 6*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassZeta(-
2/3, 7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 3*sqrt(-x^4 +
 4*x^3 - 8*x^2 + 8*x)*(x^2 + 2))/(x^4 - 4*x^3 + 8*x^2 - 8*x)
```

Sympy [F]

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4x^3-8x^2+8x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
[Out] Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4x^3-8x^2+8x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x)^{3/2}}\,dx$$

```
[In] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.768 $\int \frac{1}{\left(8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{1}{\left(8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx=\frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}}+\frac{\left(26+7(-1+x)^2\right)(-1+x)}{432\sqrt{3-2(-1+x)^2-(-1+x)^4}}+\frac{7E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)}{144\sqrt{3}}-\frac{11\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{144\sqrt{3}}$$

```
[Out] 1/72*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(3/2)-7/432*EllipticE(-1+x
,1/3*I*3^(1/2))*3^(1/2)+11/432*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/432*
(26+7*(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1120, 1106, 1192, 1194, 538, 435, 430}

$$\int \frac{1}{\left(8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx=-\frac{11\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{7E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)}{144\sqrt{3}}+\frac{\left(7(x-1)^2+26\right)(x-1)}{432\sqrt{-(x-1)^4-2(x-1)^2+3}}+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

```
[In] Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + ((
26 + 7*(-1 + x)^2)*(-1 + x))/(432*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (7
*EllipticE[ArcSin[1 - x], -1/3])/(144*Sqrt[3]) - (11*EllipticF[ArcSin[1 - x
], -1/3])/(144*Sqrt[3])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
```

```
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(3-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right)\\
&= \frac{(5+(-1+x)^2)(-1+x)}{72\,(3-2(-1+x)^2-(-1+x)^4)^{3/2}} - \frac{1}{144}\text{Subst}\left(\int \frac{-38-6x^2}{(3-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)\\
&= -\frac{(26+7(1-x)^2)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{(5+(-1+x)^2)(-1+x)}{72\,(3-2(-1+x)^2-(-1+x)^4)^{3/2}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{192-112x^2}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)}{6912}\\
&= -\frac{(26+7(1-x)^2)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{(5+(-1+x)^2)(-1+x)}{72\,(3-2(-1+x)^2-(-1+x)^4)^{3/2}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{192-112x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)}{3456}\\
&= -\frac{(26+7(1-x)^2)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{(5+(-1+x)^2)(-1+x)}{72\,(3-2(-1+x)^2-(-1+x)^4)^{3/2}}\\
&\quad - \frac{7}{432}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x, -1+x\right)\\
&\quad + \frac{11}{72}\text{Subst}\left(\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)\\
&= -\frac{(26+7(1-x)^2)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{(5+(-1+x)^2)(-1+x)}{72\,(3-2(-1+x)^2-(-1+x)^4)^{3/2}}\\
&\quad + \frac{7E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{144\sqrt{3}} - \frac{11F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{144\sqrt{3}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.85 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.73

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \frac{\frac{7i\sqrt{2}(-2+x)x^2\sqrt{\frac{4-2x+x^2}{x^2}}E\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}}}+\frac{36-232x+274x^2-226x^3+115x^4-3}{}}{432x\sqrt{-x(-8}}$$

```
[In] Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] (((7*I)*Sqrt[2]*(-2 + x)*x^2*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticE[ArcSin[Sqr
t[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/S
qrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)] + (36 - 232*x + 274*x^2 - 226*x^3 +
 115*x^4 - 37*x^5 + 7*x^6 - (19*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt
[3])*x)]*x^3*Sqrt[(4 - 2*x + x^2)/x^2]*(-8 + 8*x - 4*x^2 + x^3)*EllipticF[A
rcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqr
t[3])])/(-8 + 8*x - 4*x^2 + x^3))/(432*x*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))
])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 971 vs. 2(93) = 186.

Time = 2.07 (sec) , antiderivative size = 972, normalized size of antiderivative = 8.92

method	result	size
risch	Expression too large to display	972
default	Expression too large to display	1039
elliptic	Expression too large to display	1039

```
[In] int(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/432*(7*x^7-49*x^6+187*x^5-445*x^4+670*x^3-622*x^2+216*x+36)/(-x*(x^3-4*x^
2+8*x-8))^(1/2)/x/(x^3-4*x^2+8*x-8)+5/216*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/
(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1
/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x
-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/
2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2))
)^(1/2))+7/108*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*
(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3
^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2
)))^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3
```

```
^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((
-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1
+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-7/432*(x*(
x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1
/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1
-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*
EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I
*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE
(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))
/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^
(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1
/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I
*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.79

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx = -\frac{43\sqrt{2}(x^8-8x^7+32x^6-80x^5+128x^4-128x^3+64x^2)\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3x}\right)-84\sqrt{2}(x}{}$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2592*(43*sqrt(2)*(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64
*x^2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x) - 84*sqrt(2)*(x^8 - 8
*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)*weierstrassZeta(-2/3,
7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 6*(7*x^6 - 37*x^5
+ 115*x^4 - 226*x^3 + 274*x^2 - 232*x + 36)*sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x
))/(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)
```

Sympy [F]

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4x^3-8x^2+8x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(5/2),x)
```

```
[Out] Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(8x - 8x^2 + 4x^3 - x^4)^{5/2}}\, dx = \int \frac{1}{(-x^4 + 4\,x^3 - 8\,x^2 + 8\,x)^{\frac{5}{2}}}\, dx$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(8x - 8x^2 + 4x^3 - x^4)^{5/2}}\, dx = \int \frac{1}{(-x^4 + 4\,x^3 - 8\,x^2 + 8\,x)^{\frac{5}{2}}}\, dx$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="giac")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8x - 8x^2 + 4x^3 - x^4)^{5/2}}\, dx = \int \frac{1}{(-x^4 + 4\,x^3 - 8\,x^2 + 8\,x)^{5/2}}\, dx$$

```
[In] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
[Out] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

3.769 $\int ((2-x)x(4-2x+x^2))^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int ((2-x)x(4-2x+x^2))^{3/2}\,dx = \frac{2}{35}(13 - 3(-1+x)^2)\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) + \frac{16}{5}\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right) - \frac{176}{35}\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)$$

```
[Out] 1/7*(3-2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)-16/5*EllipticE(-1+x,1/3*I*3^(1/2))
*3^(1/2)+176/35*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+2/35*(13-3*(-1+x)^2)*
(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1120, 1105, 1190, 1194, 538, 435, 430}

$$\int ((2-x)x(4-2x+x^2))^{3/2}\,dx = -\frac{176}{35}\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right) + \frac{16}{5}\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right) + \frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2} + \frac{2}{35}\left(13-3(x-1)^2\right)(x-1)\sqrt{-(x-1)^4-2(x-1)^2+3}$$

```
[In] Int[((2 - x)*x*(4 - 2*x + x^2))^(3/2),x]
```

```
[Out] (2*(13 - 3*(-1 + x)^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + (
(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16*Sqrt[3]*EllipticE[A
rcSin[1 - x], -1/3])/5 - (176*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3])/35
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
```

```
b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \left(3-2x^2-x^4\right)^{3/2}\,dx, x, -1+x\right)\\
&= \frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)\\
&\quad + \frac{3}{7}\text{Subst}\left(\int \left(6-2x^2\right)\sqrt{3-2x^2-x^4}\,dx, x, -1+x\right)\\
&= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3-2(-1+x)^2\right.\\
&\quad \left.-(-1+x)^4\right)^{3/2}(-1+x)-\frac{1}{35}\text{Subst}\left(\int \frac{-192+112x^2}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)\\
&= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3-2(-1+x)^2\right.\\
&\quad \left.-(-1+x)^4\right)^{3/2}(-1+x)-\frac{2}{35}\text{Subst}\left(\int \frac{-192+112x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)\\
&= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)\\
&\quad +\frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)-\frac{16}{5}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x, -1+x\right)\\
&\quad +\frac{1056}{35}\text{Subst}\left(\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)\\
&= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)+\frac{1}{7}\left(3-2(-1+x)^2\right.\\
&\quad \left.-(-1+x)^4\right)^{3/2}(-1+x)\\
&\quad +\frac{16}{5}\sqrt{3}E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)-\frac{176}{35}\sqrt{3}F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 24.42 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.73

$$\int ((2-x)x(4-2x+x^2))^{3/2}\,dx = \frac{\sqrt{-x(-8+8x-4x^2+x^3)}\left(\sqrt{\frac{4-2x+x^2}{x^2}}(-224+152x+44x^2-228x^3+230x^4-116x^5+35x$$

```
[In] Integrate[((2 - x)*x*(4 - 2*x + x^2))^(3/2),x]
```

```
[Out] (Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*(Sqrt[(4 - 2*x + x^2)/x^2]*(-224 + 152
*x + 44*x^2 - 228*x^3 + 230*x^4 - 116*x^5 + 35*x^6 - 5*x^7) + 112*Sqrt[2]*(
-I + Sqrt[3])*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*EllipticE[ArcSin[Sqr
t[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])] +
(304*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*EllipticF[ArcSin[S
qrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])
)/(35*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 953 vs. 2(86) = 172.

Time = 2.22 (sec) , antiderivative size = 954, normalized size of antiderivative = 9.35

method	result	size
risch	Expression too large to display	954
default	Expression too large to display	1050
elliptic	Expression too large to display	1050

```
[In] int(((2-x)*x*(x^2-2*x+4))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*(5*x^5-25*x^4+66*x^3-98*x^2+32*x+20)*x*(x-2)*(x^2-2*x+4)/(-x*(x-2)*(x^
2-2*x+4))^(1/2)+32/7*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^
(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/
(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*
3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+
I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+64/5*(-1-I*3
^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2
))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(
-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF
```

```
(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))
/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3
^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1
/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-16/5*(x*(x-1+I*3^(1/2))*(x-1-I*3
^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)
^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2
))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2
))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2)
)/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I
*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^
(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/
(x-2))^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1
+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2
)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx = -\frac{112\,(-i\,x+i)E\left(\arcsin\left(\frac{1}{x-1}\right)\,|-3\right)+80\,(-i\,x+i)F\left(\arcsin\left(\frac{1}{x-1}\right)\,|-3\right)+(5\,x^6-30\,x^5+91\,x^4-164\,x^3}{35\,(x-1)}$$

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/35*(112*(-I*x + I)*elliptic_e(arcsin(1/(x - 1)), -3) + 80*(-I*x + I)*ell
iptic_f(arcsin(1/(x - 1)), -3) + (5*x^6 - 30*x^5 + 91*x^4 - 164*x^3 + 130*x
^2 - 12*x - 132)*sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x))/(x - 1)
```

Sympy [F]

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx = \int \left(x(2-x)\left(x^2-2x+4\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(((2-x)*x*(x**2-2*x+4))**(3/2),x)
```

```
[Out] Integral((x*(2 - x)*(x**2 - 2*x + 4))**(3/2), x)
```

Maxima [F]

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx=\int \left(-\left(x^2-2\,x+4\right)(x-2)x\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="maxima")
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2), x)
```

Giac [F]

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx=\int \left(-\left(x^2-2\,x+4\right)(x-2)x\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="giac")
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx=\int \left(-x\,(x-2)\,\left(x^2-2\,x+4\right)\right)^{3/2}\,dx$$

```
[In] int((-x*(x - 2)*(x^2 - 2*x + 4))^(3/2),x)
[Out] int((-x*(x - 2)*(x^2 - 2*x + 4))^(3/2), x)
```

3.770 $\int \sqrt{(2-x)x(4-2x+x^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 62

$$\int \sqrt{(2-x)x(4-2x+x^2)}\,dx = \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{2E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)}{\sqrt{3}} - \frac{4\operatorname{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] -2/3*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+4/3*EllipticF(-1+x,1/3*I*3^(1/2)
)*3^(1/2)+1/3*(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1120, 1105, 1194, 538, 435, 430}

$$\int \sqrt{(2-x)x(4-2x+x^2)}\,dx = -\frac{4\operatorname{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}} + \frac{2E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)}{\sqrt{3}} + \frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

```
[In] Int[Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] (Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*EllipticE[ArcSin[1 -
x], -1/3])/Sqrt[3] - (4*EllipticF[ArcSin[1 - x], -1/3])/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \sqrt{3-2x^2-x^4}\,dx, x, -1+x\right)$$

$$=\frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{1}{3}\text{Subst}\left(\int\frac{6-2x^2}{\sqrt{3-2x^2-x^4}}\,dx,x,-1+x\right)$$

$$=\frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{2}{3}\text{Subst}\left(\int\frac{6-2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx,x,-1+x\right)$$

$$=\frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)-\frac{2}{3}\text{Subst}\left(\int\frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx,x,-1+x\right)+8\text{Subst}\left(\int\frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx,x,-1+x\right)$$

$$=\frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{2E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{4F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 18.91 (sec) , antiderivative size = 256, normalized size of antiderivative = 4.13

$$\int\sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx$$

$$=\frac{\sqrt{-x\left(-8+8x-4x^2+x^3\right)}\left(\sqrt{\frac{4-2x+x^2}{x^2}}\left(-4+4x-3x^2+x^3\right)+2\sqrt{2}\left(-i+\sqrt{3}\right)\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}E\left(\arcsin\left(\right.\right.\right.}{3(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}}}$$

```
[In] Integrate[Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] (Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*(Sqrt[(4 - 2*x + x^2)/x^2]*(-4 + 4*x -
 3*x^2 + x^3) + 2*Sqrt[2]*(-I + Sqrt[3])*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3
])*x)]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*
Sqrt[3])/(-I + Sqrt[3])] + (8*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3
])*x)]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*
Sqrt[3])/(-I + Sqrt[3])]))/(3*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 931 vs. 2(54) = 108.

Time = 1.95 (sec) , antiderivative size = 932, normalized size of antiderivative = 15.03

method	result
risch	$-\frac{(x-1)x(x-2)(x^2-2x+4)}{3\sqrt{-x(x-2)(x^2-2x+4)}}+\frac{8\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\sqrt{\frac{(1+i\sqrt{}}{(-1+i}}\right.}{3\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$
default	$\frac{x\sqrt{-x^4+4x^3-8x^2+8x}}{3}-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{3}+\frac{8\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\frac{(}{(1}}\right.}{3\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{}\right.}}$
elliptic	$\frac{x\sqrt{-x^4+4x^3-8x^2+8x}}{3}-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{3}+\frac{8\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\frac{(}{(1}}\right.}{3\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{}\right.}}$

```
[In] int(((2-x)*x*(x^2-2*x+4))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(x-1)*x*(x-2)*(x^2-2*x+4)/(-x*(x-2)*(x^2-2*x+4))^(1/2)+8/3*(-1-I*3^(1/
2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(
1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I
*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I
*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*
3^(1/2))/(1-I*3^(1/2)))^(1/2))-2/3*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-1
-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^
(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/
2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(
1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2
)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-
2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)
)-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),
(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(
1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)+8/3
*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+
I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))
^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*E
llipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*
```

```
3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2))*
x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*(-
1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx = -\frac{2\left(-i\,x+i\right)E(\arcsin\left(\frac{1}{x-1}\right)\,|-3) + 4\left(-i\,x+i\right)F(\arcsin\left(\frac{1}{x-1}\right)\,|-3) - \sqrt{-x^4+4\,x^3-8\,x^2+8\,x}(x^2-}{3\left(x-1\right)}$$

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(-I*x + I)*elliptic_e(arcsin(1/(x - 1)), -3) + 4*(-I*x + I)*ellipti
c_f(arcsin(1/(x - 1)), -3) - sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)*(x^2 - 2*x +
3))/(x - 1)
```

Sympy [F]

$$\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx = \int \sqrt{x\left(2-x\right)\left(x^2-2x+4\right)}\,dx$$

```
[In] integrate(((2-x)*x*(x**2-2*x+4))**(1/2),x)
```

```
[Out] Integral(sqrt(x*(2 - x)*(x**2 - 2*x + 4)), x)
```

Maxima [F]

$$\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx = \int \sqrt{-(x^2-2\,x+4)(x-2)x}\,dx$$

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Giac [F]

$$\int \sqrt{(2-x)x\,(4-2x+x^2)}\,dx = \int \sqrt{-(x^2-2\,x+4)(x-2)x}\,dx$$

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{(2-x)x\,(4-2x+x^2)}\,dx = \int \sqrt{-x\,(x-2)\,(x^2-2\,x+4)}\,dx$$

```
[In] int((-x*(x - 2)*(x^2 - 2*x + 4))^(1/2),x)
[Out] int((-x*(x - 2)*(x^2 - 2*x + 4))^(1/2), x)
```

3.771 $\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{1}{\sqrt{(2-x)x\left(4-2x+x^2\right)}}\,dx=-\frac{\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1120, 1109, 430}

$$\int \frac{1}{\sqrt{(2-x)x\left(4-2x+x^2\right)}}\,dx=-\frac{\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[In] Int[1/Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
[Out] -(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)\\ &= 2\text{Subst}\left(\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)\\ &= -\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.86 (sec) , antiderivative size = 100, normalized size of antiderivative = 5.88

$$\int \frac{1}{\sqrt{(2-x)x\left(4-2x+x^2\right)}}\,dx = -\frac{\sqrt[3]{-1}(-2+x)^2\sqrt{\frac{x\left(-1+i\sqrt{3}+x\right)}{(-2+x)^2}}\sqrt{\frac{-2+x-\sqrt[3]{-1}x}{-2+x}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{(-1)^{2/3}x}{-2+x}}\right),(-1)^{2/3}\right)}{\sqrt{-x\left(-8+8x-4x^2+x^3\right)}}$$

```
[In] Integrate[1/Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
[Out] -((((-1)^(1/3)*(-2 + x)^2*Sqrt[(x*(-1 + I*Sqrt[3] + x))/(-2 + x)^2]*Sqrt[(-2
 + x - (-1)^(1/3)*x)/(-2 + x)]*EllipticF[ArcSin[Sqrt[-(((-1)^(2/3)*x)/(-2 +
 x))]], (-1)^(2/3)])/Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(15) = 30.

Time = 1.42 (sec) , antiderivative size = 200, normalized size of antiderivative = 11.76

method	result	size
default	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^{2}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-1-i\sqrt{3}\right)}{\left(-1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}}\right)}{\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$	200
elliptic	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^{2}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-1-i\sqrt{3}\right)}{\left(-1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}}\right)}{\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$	200

```
[In] int(1/((2-x)*x*(x^2-2*x+4))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1
+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2)
)^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*Ell
ipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^
(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{(2-x)x\,(4-2x+x^2)}}\,dx = -\frac{1}{2}\,\sqrt{2}\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3\,x}\right)$$

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{(2-x)x\,(4-2x+x^2)}}\,dx = \int \frac{1}{\sqrt{x\,(2-x)\,(x^2-2x+4)}}\,dx$$

```
[In] integrate(1/((2-x)*x*(x**2-2*x+4))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(2 - x)*(x**2 - 2*x + 4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}}\,dx = \int \frac{1}{\sqrt{-(x^2-2\,x+4)(x-2)x}}\,dx$$

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}}\,dx = \int \frac{1}{\sqrt{-(x^2-2\,x+4)(x-2)x}}\,dx$$

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}}\,dx = \int \frac{1}{\sqrt{-x\,(x-2)\,(x^2-2\,x+4)}}\,dx$$

```
[In] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(1/2),x)
[Out] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(1/2), x)
```

3.772 $\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}}\,dx = \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{E\left(\arcsin(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}} - \frac{\operatorname{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{4\sqrt{3}}$$

```
[Out] -1/24*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/12*EllipticF(-1+x,1/3*I*3^(1/
2))*3^(1/2)+1/24*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1120, 1106, 1194, 538, 435, 430}

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}}\,dx = -\frac{\operatorname{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{4\sqrt{3}} + \frac{E\left(\arcsin(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}} + \frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

```
[In] Int[((2 - x)*x*(4 - 2*x + x^2))^(-3/2),x]
```

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + Elli
pticE[ArcSin[1 - x], -1/3]/(8*Sqrt[3]) - EllipticF[ArcSin[1 - x], -1/3]/(4*
Sqrt[3])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)$$

$$= \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{48}\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)$$

$$= \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{24}\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)$$

$$= \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{24}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x, -1+x\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)$$

$$= \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}} - \frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{4\sqrt{3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 19.72 (sec) , antiderivative size = 298, normalized size of antiderivative = 4.08

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx = \frac{(-2+x)^2x(4-2x+x^2)\left(2(-1+x)x-3(4-2x+x^2)-\frac{3x(4-2x+x^2)}{-2+x}\right.}{}$$

```
[In] Integrate[((2 - x)*x*(4 - 2*x + x^2))^(-3/2),x]
```

```
[Out] ((-2 + x)^2*x*(4 - 2*x + x^2)*(2*(-1 + x)*x - 3*(4 - 2*x + x^2) - (3*x*(4 -
 2*x + x^2))/(-2 + x) - 4*(2 - x)*Sqrt[(4 - 2*x + x^2)/(-2 + x)^2]*(x*Sqrt[
(4 - 2*x + x^2)/(-2 + x)^2] - Sqrt[2]*(I + Sqrt[3])*Sqrt[(I*x)/((I + Sqrt[3
])*(-2 + x))]*EllipticE[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]
*3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])] + (4*I)*Sqrt[2]*Sqrt[(I*x)/((I + Sqr
t[3])*(-2 + x))]*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt
[2]*3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])])))/(96*(-(x*(-8 + 8*x - 4*x^2 + x
^3)))^(3/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 962 vs. $2(61) = 122$.

Time = 1.45 (sec) , antiderivative size = 963, normalized size of antiderivative = 13.19

method	result	size
default	Expression too large to display	963
elliptic	Expression too large to display	963

```
[In] int(1/((2-x)*x*(x^2-2*x+4))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(-x^3+4*x^2-8*x+8)/(x*(-x^3+4*x^2-8*x+8))^(1/2)+2*x*(1/24+1/192*x^2)/
(-x*(x^3-4*x^2+8*x-8))^(1/2)+1/6*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1
/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1
-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1
/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2)
)^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+
1/6*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x
-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-
2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(
2*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1
-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2
))*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))
*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-1/24*(x*(x-1+I*3^(1/2
))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^
(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/
(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((
-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-
1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1
/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/
2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+
I*3^(1/2))/(x-2))^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3
^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x
-1-I*3^(1/2)))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.63

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx = -\frac{5\sqrt{2}(x^4-4x^3+8x^2-8x)\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3x}\right)-6\sqrt{2}(x^4-4x^3+8x^2-8x)\text{weierstrass}}{72(x^4-4x^3+8x^2-}$$

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/72*(5*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassPInverse(-2/3, 7/54
, -1/3*(x - 3)/x) - 6*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassZeta(-
2/3, 7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 3*sqrt(-x^4 +
 4*x^3 - 8*x^2 + 8*x)*(x^2 + 2))/(x^4 - 4*x^3 + 8*x^2 - 8*x)
```

Sympy [F]

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx = \int \frac{1}{(x(2-x)(x^2-2x+4))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((2-x)*x*(x**2-2*x+4))**(3/2),x)
```

```
[Out] Integral((x*(2 - x)*(x**2 - 2*x + 4))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx = \int \frac{1}{(-(x^2-2x+4)(x-2)x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx = \int \frac{1}{(-(x^2-2\,x+4)(x-2)x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="giac")
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx = \int \frac{1}{(-x\,(x-2)\,(x^2-2\,x+4))^{3/2}}\,dx$$

```
[In] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(3/2),x)
[Out] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(3/2), x)
```

3.773 $\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx = \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{\left(26+7(-1+x)^2\right)(-1+x)}{432\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{7E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)}{144\sqrt{3}} - \frac{11\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{144\sqrt{3}}$$

```
[Out] 1/72*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(3/2)-7/432*EllipticE(-1+x
,1/3*I*3^(1/2))*3^(1/2)+11/432*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/432*
(26+7*(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1120, 1106, 1192, 1194, 538, 435, 430}

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx = -\frac{11\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{144\sqrt{3}} + \frac{7E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)}{144\sqrt{3}} + \frac{\left(7(x-1)^2+26\right)(x-1)}{432\sqrt{-(x-1)^4-2(x-1)^2+3}} + \frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

```
[In] Int[((2 - x)*x*(4 - 2*x + x^2))^(-5/2),x]
```

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + ((
26 + 7*(-1 + x)^2)*(-1 + x))/(432*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (7
*EllipticE[ArcSin[1 - x], -1/3])/(144*Sqrt[3]) - (11*EllipticF[ArcSin[1 - x
], -1/3])/(144*Sqrt[3])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
```

```
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(3-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right)\\
&= \frac{(5+(-1+x)^2)(-1+x)}{72\,(3-2(-1+x)^2-(-1+x)^4)^{3/2}} - \frac{1}{144}\text{Subst}\left(\int \frac{-38-6x^2}{(3-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)\\
&= -\frac{(26+7(1-x)^2)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{(5+(-1+x)^2)(-1+x)}{72\,(3-2(-1+x)^2-(-1+x)^4)^{3/2}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{192-112x^2}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)}{6912}\\
&= -\frac{(26+7(1-x)^2)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{(5+(-1+x)^2)(-1+x)}{72\,(3-2(-1+x)^2-(-1+x)^4)^{3/2}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{192-112x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)}{3456}\\
&= -\frac{(26+7(1-x)^2)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{(5+(-1+x)^2)(-1+x)}{72\,(3-2(-1+x)^2-(-1+x)^4)^{3/2}}\\
&\quad - \frac{7}{432}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x, -1+x\right)\\
&\quad + \frac{11}{72}\text{Subst}\left(\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)\\
&= -\frac{(26+7(1-x)^2)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{(5+(-1+x)^2)(-1+x)}{72\,(3-2(-1+x)^2-(-1+x)^4)^{3/2}}\\
&\quad + \frac{7E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{144\sqrt{3}} - \frac{11F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{144\sqrt{3}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.03 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.00

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{5/2}}\,dx = \frac{(-2+x)^3x^2(4-2x+x^2)^2\left(-\frac{7x(4-2x+x^2)}{-2+x}+\frac{36+216x-622x^2+670x^3-445x^4+187}{(-2+x)^2x(4-2x+x^2)}\right.}{}$$

[In] Integrate[((2 - x)*x*(4 - 2*x + x^2))^(-5/2),x]

[Out] ((-2 + x)^3*x^2*(4 - 2*x + x^2)^2*((-7*x*(4 - 2*x + x^2))/(-2 + x) + (36 + 216*x - 622*x^2 + 670*x^3 - 445*x^4 + 187*x^5 - 49*x^6 + 7*x^7)/((-2 + x)^2 *x*(4 - 2*x + x^2)) + ((7*I)*Sqrt[2]*x*Sqrt[(4 - 2*x + x^2)/(-2 + x)^2]*EllipticE[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])])/Sqrt[(I*x)/((I + Sqrt[3])*(-2 + x))] - (19*I)*Sqrt[2]*(-2 + x)*Sqrt[(I*x)/((I + Sqrt[3])*(-2 + x))]*Sqrt[(4 - 2*x + x^2)/(-2 + x)^2]*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])]))/(432*(-(x*(-8 + 8*x - 4*x^2 + x^3)))^(5/2))

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1038 vs. 2(93) = 186.

Time = 1.49 (sec) , antiderivative size = 1039, normalized size of antiderivative = 9.53

method	result	size
default	Expression too large to display	1039
elliptic	Expression too large to display	1039

[In] int(1/((2-x)*x*(x^2-2*x+4))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/768*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/x^2-1/96*(-x^3+4*x^2-8*x+8)/(x*(-x^3+4*x^2-8*x+8))^(1/2)+(1/36+1/288*x^2-1/96*x)*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/(x^3-4*x^2+8*x-8)^2+2*x*(53/3456+5/1728*x^2-19/4608*x)/(-x*(x^3-4*x^2+8*x-8))^(1/2)+5/216*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+7/108*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3

```
^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1
/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF(((-1+I*3
^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^
(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(
x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+
I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-7/432*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+
2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1
+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2)
)^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2))*x/(1+
I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3
^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I*3^(1/2)
)/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^
(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(
1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/
2))/(1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2
)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.79

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{5/2}}\,dx = -\frac{43\sqrt{2}(x^8-8x^7+32x^6-80x^5+128x^4-128x^3+64x^2)\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3x}\right)-84\sqrt{2}(x}{}$$

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2592*(43*sqrt(2)*(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64
*x^2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x) - 84*sqrt(2)*(x^8 - 8
*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)*weierstrassZeta(-2/3,
7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 6*(7*x^6 - 37*x^5
+ 115*x^4 - 226*x^3 + 274*x^2 - 232*x + 36)*sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x
))/(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)
```

Sympy [F]

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx = \int \frac{1}{\left(x\left(2-x\right)\left(x^2-2x+4\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/((2-x)*x*(x**2-2*x+4))**(5/2),x)
[Out] Integral((x*(2 - x)*(x**2 - 2*x + 4))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx = \int \frac{1}{\left(-(x^2-2\,x+4)(x-2)x\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="maxima")
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx = \int \frac{1}{\left(-(x^2-2\,x+4)(x-2)x\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="giac")
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx = \int \frac{1}{\left(-x\,\left(x-2\right)\,\left(x^2-2\,x+4\right)\right)^{5/2}}\,dx$$

```
[In] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(5/2),x)
[Out] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(5/2), x)
```

3.774 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 730

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx=\frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}$$
$$+\frac{2c\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\left(7c^3+20ad^2-3cd^2\left(\frac{c}{d}+x\right)^2\right)}{35d^2}$$
$$-\frac{16c^3(c^3+8ad^2)\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{35d^2\sqrt{c^3+4ad^2}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)}$$
$$+\frac{16c^{13/4}(c^3+4ad^2)^{3/4}(c^3+8ad^2)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)E\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right.\right.}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$
$$+\frac{8c^{7/4}(c^3+4ad^2)^{3/4}\left(\sqrt{c^3+4ad^2}(c^3+5ad^2)-c^{3/2}(c^3+8ad^2)\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

```
[Out] 1/7*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2)+2/35*c*(c/d+x)*(7*c^3
+20*a*d^2-3*c*d^2*(c/d+x)^2)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/d^2-
16/35*c^3*(8*a*d^2+c^3)*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/d
^2/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))/(4*a*d^2+c^3)^(1/2)+16/35*c^
(13/4)*(4*a*d^2+c^3)^(3/4)*(8*a*d^2+c^3)*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a
*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)
))*EllipticE(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^
(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2
))*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d
```

```
+x)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)/d^5/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)
^(1/2)+8/35*c^(7/4)*(4*a*d^2+c^3)^(3/4)*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a*
d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))
)*EllipticF(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(
3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2)
)*(-c^(3/2)*(8*a*d^2+c^3)+(5*a*d^2+c^3)*(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+
4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^
3)^(1/2))^2)^(1/2)/d^5/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1120, 1105, 1190, 1211, 1117, 1209}

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}dx = \frac{8c^{7/4}\left(4ad^2+c^3\right)^{3/4}\left(\sqrt{4ad^2+c^3}\left(5ad^2+c^3\right)-c^{3/2}\left(8ad^2+c^3\right)\right)\sqrt{\frac{d^2\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}{\left(4ad^2+c^3\right)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2(\cdots}{\sqrt{4a\cdots}}\right.}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3\cdots}}$$
$$+\frac{16c^{13/4}\left(4ad^2+c^3\right)^{3/4}\left(8ad^2+c^3\right)\sqrt{\frac{d^2\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}{\left(4ad^2+c^3\right)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)E\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)\cdots\right.}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$
$$+\frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}-\frac{16c^3\left(8ad^2+c^3\right)\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{35d^2\sqrt{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}+\frac{2c\left(\frac{c}{d}+x\cdots\right.}{\cdots}$$

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2),x]

[Out] ((c/d + x)*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2))/7 + (2*c*(c/d +
 x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]*(7*c^3 + 20*a*d^2 - 3*c*d
^2*(c/d + x)^2))/(35*d^2) - (16*c^3*(c^3 + 8*a*d^2)*(c/d + x)*Sqrt[4*a*c +
4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(35*d^2*Sqrt[c^3 + 4*a*d^2]*(Sqrt[c] + (d
^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])) + (16*c^(13/4)*(c^3 + 4*a*d^2)^(3/4)*
(c^3 + 8*a*d^2)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3
+ 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] +
 (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticE[2*ArcTan[(c + d*x)/(c^(1/
4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(35*d^5*S
qrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) + (8*c^(7/4)*(c^3 + 4*a*d^2)^
(3/4)*(Sqrt[c^3 + 4*a*d^2]*(c^3 + 5*a*d^2) - c^(3/2)*(c^3 + 8*a*d^2))*Sqrt[
(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] +
```

```
 (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sq
rt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1
/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(35*d^5*Sqrt[4*a*c + 4*c^2*x^2
 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^{3/2}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2} + \frac{3}{7}\text{Subst}\left(\int \left(2c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2} + \frac{2c(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\left(7c^3+20ad^2-3c(c+dx)^2\right)}{35d^3} + \frac{\text{Subst}\left(\int \frac{\frac{16c^2\left(c^3+4ad^2\right)\left(c^3+5ad^2\right)}{d^2}-16c^3\left(c^3+8ad^2\right)x^2}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}}\,dx, x, \frac{c}{d}+x\right)}{35d^2}$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2} + \frac{2c(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\left(7c^3+20ad^2-3c(c+dx)^2\right)}{35d^3} + \frac{\left(16c^{7/2}\sqrt{c^3+4ad^2}\left(c^3+8ad^2\right)\right)\text{Subst}\left(\int \frac{1-\frac{d^2x^2}{\sqrt{c}\sqrt{c^3+4ad^2}}}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}}\,dx, x, \frac{c}{d}+x\right)}{35d^4} + \frac{\left(16c^2\sqrt{c^3+4ad^2}\left(\sqrt{c^3+4ad^2}\left(c^3+5ad^2\right)-c^{3/2}\left(c^3+8ad^2\right)\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}}\,dx, x, \ldots\right)}{35d^4}$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}$$
$$+ \frac{2c(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}(7c^3+20ad^2-3c(c+dx)^2)}{35d^3}$$
$$- \frac{16c^3(c^3+8ad^2)(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{35d^3\sqrt{c^3+4ad^2}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)}$$
$$+ \frac{16c^{13/4}(c^3+4ad^2)^{3/4}(c^3+8ad^2)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)E\left(2\tan^{-1}\left(\frac{c}{\sqrt[4]{c}\sqrt[4]{c}}\right.\right.}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$
$$+ \frac{8c^{7/4}(c^3+4ad^2)^{3/4}\left(\sqrt{c^3+4ad^2}(c^3+5ad^2)-c^{3/2}(c^3+8ad^2)\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\right.}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.14 (sec) , antiderivative size = 10468, normalized size of antiderivative = 14.34

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2} dx = \text{Result too large to show}$$

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2),x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 5228 vs. 2(772) = 1544.

Time = 6.08 (sec) , antiderivative size = 5229, normalized size of antiderivative = 7.16

method	result	size
default	Expression too large to display	5229
elliptic	Expression too large to display	5229
risch	Expression too large to display	6018

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx=\int \left(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="fricas")
[Out] integral((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

Sympy [F]

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx=\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
[Out] Integral((4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)**(3/2), x)
```

Maxima [F]

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx=\int \left(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="maxima")
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

Giac [F]

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx=\int \left(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="giac")
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx=\int \left(4\,c^2\,x^2+4\,c\,d\,x^3+4\,a\,c+d^2\,x^4\right)^{3/2}\,dx$$

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2),x)
[Out] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2), x)
```

3.775 $\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 622

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$$

$$= \frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4} - \frac{2c^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{3\sqrt{c^3+4ad^2}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)}$$

$$+\frac{2c^{9/4}(c^3+4ad^2)^{3/4}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)E\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(1+\frac{}{\sqrt{c}}\right.\right.}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

$$+\frac{c^{3/4}\sqrt[4]{c^3+4ad^2}\left(c^3+4ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)\text{EllipticF}\left(2\,\text{ar}\right.}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

```
[Out] 1/3*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)-2/3*c^2*(c/d+x)*(d^2*
x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(
1/2))/(4*a*d^2+c^3)^(1/2)+2/3*c^(9/4)*(4*a*d^2+c^3)^(3/4)*(cos(2*arctan((d*
x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4
*a*d^2+c^3)^(1/4)))*EllipticE(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1
/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(
4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/
(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)/d^3/(d^2*x^4+4*c*d*x^3
+4*c^2*x^2+4*a*c)^(1/2)+1/3*c^(3/4)*(4*a*d^2+c^3)^(1/4)*(cos(2*arctan((d*x+
c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a
*d^2+c^3)^(1/4)))*EllipticF(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4
))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*
```

```
a*d^2+c^3)^(1/2))*(c^3+4*a*d^2-c^(3/2)*(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+4
*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3
)^(1/2))^2)^(1/2)/d^3/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1120, 1105, 1211, 1117, 1209}

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$$

$$= \frac{c^{3/4}\sqrt[4]{4ad^2+c^3}\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+c^3\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)\text{EllipticF}\left(2\,\text{a}\right.}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

$$+\frac{2c^{9/4}(4ad^2+c^3)^{3/4}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)E\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/}}{\sqrt{c^3+}}\right.\right.}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

$$+\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}-\frac{2c^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{3\sqrt{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}$$

```
[In] Int[Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] ((c/d + x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/3 - (2*c^2*(c/d +
 x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(3*Sqrt[c^3 + 4*a*d^2]*(
Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])) + (2*c^(9/4)*(c^3 + 4*a*d
^2)^(3/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*
d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*
(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticE[2*ArcTan[(c + d*x)/(c^(1/4)*(c^
3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(3*d^3*Sqrt[4*a
*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) + (c^(3/4)*(c^3 + 4*a*d^2)^(1/4)*(c^
3 + 4*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4
*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3
 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Elliptic
F[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^
3 + 4*a*d^2])/2])/(3*d^3*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
```

```
0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}\,dx, x, \frac{c}{d}+x\right)\\
&= \frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\\
&\quad+\frac{1}{3}\text{Subst}\left(\int \frac{2c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}}\,dx, x, \frac{c}{d}+x\right)
\end{aligned}$$

$$= \frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4} + \frac{\left(2c^{5/2}\sqrt{c^3+4ad^2}\right)\text{Subst}\left(\int \frac{1-\frac{d^2x^2}{\sqrt{c}\sqrt{c^3+4ad^2}}}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}}\,dx, x, \frac{c}{d}+x\right)}{3d^2} + \frac{\left(2c\left(c^3+4ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}}\,dx, x, \frac{c}{d}+x\right)}{3d^2}$$

$$= \frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4} - \frac{2c^2(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{3d\sqrt{c^3+4ad^2}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)} + \frac{2c^{9/4}(c^3+4ad^2)^{3/4}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)E\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\right.}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}} + \frac{c^{3/4}\sqrt[4]{c^3+4ad^2}\left(c^3+4ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)F\left(2\tan\right.}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.07 (sec) , antiderivative size = 5218, normalized size of antiderivative = 8.39

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4864 vs. 2(668) = 1336.

Time = 5.16 (sec) , antiderivative size = 4865, normalized size of antiderivative = 7.82

method	result	size
risch	Expression too large to display	4865
default	Expression too large to display	4890
elliptic	Expression too large to display	4890

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*(d*x+c)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/d+2/3*c/d*(8*a*d*((c+
(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-(c
+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^
(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)^2*((-(c+(2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*((-(c+(2*d*(-a*c
)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)/(-(c+
(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(d^2*(x
-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d
)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d))^(1/2)*EllipticF(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a
*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(
-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(
-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+
(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))-c*d*((x-(-c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1
/2)+c^2)^(1/2))/d)+((c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(-c+(2*d*(-a*c)^(1/
2)+c^2)^(1/2))/d)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/
2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)
^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)
^(1/2)+c^2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)^2*((-(c
+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-
c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(
-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))
^(1/2)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d))^(1/2)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d^2*(-c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d^2*(-c+(2*d*(-a*c)^(1/2
)+c^2)^(1/2))+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d^2*(c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))^2/d^2)/(-(c+(-2*d*(-a*c)^(1/2)+c
^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*EllipticF(((-(c+(-2*d*(-a*c
)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c
```

)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c
)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-(c+
(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((c+(
-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(-c+
(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((c+(
-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))
+(-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d
)*EllipticE(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)
+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)
+c^2)^(1/2))/d))^(1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d)*((c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(-c+(2*d*(-a*c
)^(1/2)+c^2)^(1/2))/d)/(-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(-c+(2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d)/((c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(c+(2*d*(-a*c)
^(1/2)+c^2)^(1/2))/d))^(1/2))/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d)+4*c/d/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c
+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*EllipticPi(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^
(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^
(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^
(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((c+(-2*d*(-a*c)^(1
/2)+c^2)^(1/2))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((c+(-2*d*(-a*c)^(1/
2)+c^2)^(1/2))/d-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d),((-(c+(2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((c+(-2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(-c+(-2*d*(-a*c)^(1/
2)+c^2)^(1/2))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((c+(-2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))))/(d^2*(x-(-c+(
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(
-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d
))^(1/2)-4*c^2*((c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(-c+(2*d*(-a*c)^(1/2)+c
^2)^(1/2))/d)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c
^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/
2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/
2)+c^2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)^2*((-(c+(2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/
2)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/
d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d))^(1/2)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d)/(d^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^
(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(
-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d*Elli
pticF(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2

```
))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^
(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^
(1/2))/d))^(1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/
2)+c^2)^(1/2))/d)*((c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(-c+(2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d)/(-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(-c+(2*d*(-a*c)^(1/
2)+c^2)^(1/2))/d)/((c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(c+(2*d*(-a*c)^(1/2)
+c^2)^(1/2))/d))^(1/2))+((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)
^(1/2)+c^2)^(1/2))/d)*EllipticPi(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+
(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-
(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x
+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((c+(-2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((c+(-2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))))
```

Fricas [F]

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \int \sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}\,dx$$

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Sympy [F]

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$$

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)
[Out] Integral(sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4), x)
```

Maxima [F]

$$\int \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}\, dx = \int \sqrt{d^2x^4 + 4\, cdx^3 + 4\, c^2x^2 + 4\, ac}\, dx$$

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Giac [F]

$$\int \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}\, dx = \int \sqrt{d^2x^4 + 4\, cdx^3 + 4\, c^2x^2 + 4\, ac}\, dx$$

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}\, dx = \int \sqrt{4\, c^2\, x^2 + 4\, c\, d\, x^3 + 4\, a\, c + d^2\, x^4}\, dx$$

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2),x)
[Out] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2), x)
```

3.776 $\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 227

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx = \frac{\sqrt[4]{c^3+4ad^2}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)\text{EllipticF}\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right),\frac{1}{2}\left(1+\frac{c^3}{\sqrt{c^3+}}\right.\right.}{2\sqrt[4]{c}d\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

```
[Out] 1/2*(4*a*d^2+c^3)^(1/4)*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))
^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))*EllipticF(sin(
2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^
3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+
4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^
3)^(1/2))^2)^(1/2)/c^(1/4)/d/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1120, 1117}

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx = \frac{\sqrt[4]{4ad^2+c^3}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right),\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}\right.\right.}{2\sqrt[4]{c}d\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

```
[In] Int[1/Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] ((c^3 + 4*a*d^2)^(1/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))
/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sq
rt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)
/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(2
*c^(1/4)*d*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{\sqrt[4]{c^3+4ad^2}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)F\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(1+\frac{c}{\sqrt{c^3 \ldots}}\right.\right.}{2\sqrt[4]{c}d\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.49 (sec) , antiderivative size = 822, normalized size of antiderivative = 3.62

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx$$

$$= \frac{2\left(-c+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}-dx\right)\left(c+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+dx\right)\sqrt{-\frac{\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\left(c-\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}+dx\right)\ldots}{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}\right)\left(-c+\sqrt{c^2-2i\sqrt{\ldots}}\right.}}}{d\sqrt{c^2-2i\sqrt{a}\sqrt{\ldots}}}$$

```
[In] Integrate[1/Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] (2*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x)*(c + Sqrt[c^2 - (2*I)*S
qrt[a]*Sqrt[c]*d] + d*x)*Sqrt[-((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*(c - S
qrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[
c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]
*Sqrt[c]*d] - d*x)))]*Sqrt[-((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*(c + Sqrt
[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*
d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sq
rt[c]*d] - d*x)))]*EllipticF[ArcSin[Sqrt[((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]
*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sq
rt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2 + (2*I)*S
qrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x))]], (Sq
rt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])^2/
(Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])
^2])/(d*Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*Sqrt[((Sqrt[c^2 - (2*I)*Sqrt[a]
*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(c + Sqrt[c^2 - (2*I)*Sq
rt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2 +
 (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x)
)]*Sqrt[4*a*c + x^2*(2*c + d*x)^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1055 vs. $2(259) = 518$.

Time = 1.04 (sec) , antiderivative size = 1056, normalized size of antiderivative = 4.65

method	result	size
default	Expression too large to display	1056
elliptic	Expression too large to display	1056

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)
*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)
*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)^2*((-(c+(2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1
/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1
/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*((-(c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)
/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/
(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(
```

```
d^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c
^2)^(1/2))/d))^(1/2)*EllipticF(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2
*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c
+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx = \int \frac{1}{\sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}}\,dx$$

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="fricas"
)
[Out] integral(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx = \int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx$$

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)
[Out] Integral(1/sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx = \int \frac{1}{\sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}}\,dx$$

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}\,dx = \int \frac{1}{\sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}}\,dx$$

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}\,dx = \int \frac{1}{\sqrt{4\,c^2\,x^2 + 4\,c\,d\,x^3 + 4\,a\,c + d^2\,x^4}}\,dx$$

```
[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2),x)
[Out] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2), x)
```

3.777 $\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 674

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx =$$

$$-\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\,(c^3+4ad^2)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}-\frac{d^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{8a\,(c^3+4ad^2)^{3/2}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)}$$

$$+\frac{\sqrt[4]{c}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)E\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(1+\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}\right)\right)}{8ad\sqrt[4]{c^3+4ad^2}\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

$$+\frac{\left(c^3+4ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)\text{EllipticF}\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+\cdots}}\right.\right.}{16ac^{5/4}d\,(c^3+4ad^2)^{3/4}\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

```
[Out] -1/8*(c/d+x)*(c^3-4*a*d^2-c*d^2*(c/d+x)^2)/a/c/(4*a*d^2+c^3)/(d^2*x^4+4*c*d
*x^3+4*c^2*x^2+4*a*c)^(1/2)-1/8*d^2*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*
a*c)^(1/2)/a/(4*a*d^2+c^3)^(3/2)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2)
)+1/8*c^(1/4)*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/
cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))*EllipticE(sin(2*arctan((
d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^
(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+4*c*d*x^3+
4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))^
2)^(1/2)/a/d/(4*a*d^2+c^3)^(1/4)/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)+
1/16*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arc
tan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))*EllipticF(sin(2*arctan((d*x+c)/c^
```

```
(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c
^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))*(c^3+4*a*d^2-c^(3/2)*(4*a*d^2+c^3
)^(1/2))*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^
2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)/a/c^(5/4)/d/(4*a*d^2+c^3)^(3/4)/(
d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1120, 1106, 1211, 1117, 1209}

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx = \frac{\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+c^3\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}\right.}{16ac^{5/4}d\left(4ad^2+c^3\right)^{3/4}\sqrt{4\ldots}}$$
$$+\frac{\sqrt[4]{c}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)E\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{8ad\sqrt[4]{4ad^2+c^3}\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$
$$-\frac{d^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{8a\left(4ad^2+c^3\right)^{3/2}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-3/2),x]
```

```
[Out] -1/8*((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(a*c*(c^3 + 4*a*d^2)*S
qrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) - (d^2*(c/d + x)*Sqrt[4*a*c +
 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(8*a*(c^3 + 4*a*d^2)^(3/2)*(Sqrt[c] + (d
^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])) + (c^(1/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x
^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sq
rt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*El
lipticE[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/S
qrt[c^3 + 4*a*d^2])/2])/(8*a*d*(c^3 + 4*a*d^2)^(1/4)*Sqrt[4*a*c + 4*c^2*x^2
 + 4*c*d*x^3 + d^2*x^4]) + ((c^3 + 4*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*S
qrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[
c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2
)/Sqrt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2
)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(16*a*c^(5/4)*d*(c^3 + 4*a
*d^2)^(3/4)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
```

```
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^{3/2}}\,dx, x, \frac{c}{d}+x\right)$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(c^3+4ad^2\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}} + \frac{\text{Subst}\left(\int \frac{2c\left(4a+\frac{c^3}{d^2}\right)d^2-2c^2d^2x^2}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}}dx,x,\frac{c}{d}+x\right)}{16ac^2\left(c^3+4ad^2\right)}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(c^3+4ad^2\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}} + \frac{\sqrt{c}\text{Subst}\left(\int \frac{1-\frac{d^2x^2}{\sqrt{c}\sqrt{c^3+4ad^2}}}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}}dx,x,\frac{c}{d}+x\right)}{8a\sqrt{c^3+4ad^2}} + \frac{\left(c^3+4ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}}dx,x,\frac{c}{d}+x\right)}{8ac\left(c^3+4ad^2\right)}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(c^3+4ad^2\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}} - \frac{d(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{8a\left(c^3+4ad^2\right)^{3/2}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)} + \frac{\sqrt[4]{c}\sqrt{\frac{d^2\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}{\left(c^3+4ad^2\right)\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)E\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(1+\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}\right)\right)}{8ad\sqrt[4]{c^3+4ad^2}\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}} + \frac{\left(c^3+4ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\sqrt{\frac{d^2\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}{\left(c^3+4ad^2\right)\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)F\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+}}\right.\right.}{16ac^{5/4}d\left(c^3+4ad^2\right)^{3/4}\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.12 (sec) , antiderivative size = 5276, normalized size of antiderivative = 7.83

$$\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-3/2),x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 5023 vs. 2(720) = 1440.

Time = 1.03 (sec) , antiderivative size = 5024, normalized size of antiderivative = 7.45

method	result	size
default	Expression too large to display	5024
elliptic	Expression too large to display	5024

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx = \int \frac{1}{(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="fricas"
)
[Out] integral(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)/(d^4*x^8 + 8*c*d^3*x
^7 + 24*c^2*d^2*x^6 + 32*c^3*d*x^5 + 32*a*c^2*d*x^3 + 32*a*c^3*x^2 + 8*(2*c
^4 + a*c*d^2)*x^4 + 16*a^2*c^2), x)
```

Sympy [F]

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx = \int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
[Out] Integral((4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx=\int \frac{1}{(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx=\int \frac{1}{(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx=\int \frac{1}{(4\,c^2\,x^2+4\,c\,d\,x^3+4\,a\,c+d^2\,x^4)^{3/2}}\,dx$$

```
[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2),x)
```

```
[Out] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2), x)
```

3.778 $\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 663

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$$
$$=\frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}-\frac{2d^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{\sqrt{5d^4+256ae^3}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)}$$
$$+\frac{d^2(5d^4+256ae^3)^{3/4}\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(5d^4+256ae^3)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)E\left(2\arctan\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\right.}{8\sqrt{2}e^2\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$
$$+\frac{\sqrt[4]{5d^4+256ae^3}\left(5d^4+256ae^3-3d^2\sqrt{5d^4+256ae^3}\right)\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(5d^4+256ae^3)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)\mathrm{El}}{48\sqrt{2}e^2\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

```
[Out] 1/3*(1/4*d/e+x)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)-2*d^2*(1/4*d/e+
x)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)/(1+16*e^2*(1/4*d/e+x)^2/(256
*a*e^3+5*d^4)^(1/2))/(256*a*e^3+5*d^4)^(1/2)+1/16*d^2*(256*a*e^3+5*d^4)^(3/
4)*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))^2)^(1/2)/cos(2*arctan(
(4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticE(sin(2*arctan((4*e*x+d)/(256*a
*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(1/2))^(1/2))*(1+16*e^2*
(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*
e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))^2)^
(1/2)/e^2*2^(1/2)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)+1/96*(256*a*e
^3+5*d^4)^(1/4)*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))^2)^(1/2)/
cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticF(sin(2*arctan((4*
```

```
e*x+d)/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(1/2))^(1/2
))*(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(5*d^4+256*a*e^3-3*d^2*
(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)/(256*a*e^
3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))^2)^(1/2)/e^2*2^(1
/2)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1120, 1105, 1211, 1117, 1209}

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$$

$$=\frac{\sqrt[4]{256ae^3+5d^4}\left(-3d^2\sqrt{256ae^3+5d^4}+256ae^3+5d^4\right)\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(256ae^3+5d^4)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)\text{Ellip}}{48\sqrt{2}e^2\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

$$+\frac{d^2(256ae^3+5d^4)^{3/4}\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(256ae^3+5d^4)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)E\left(2\arctan\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}(\right.}{8\sqrt{2}e^2\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

$$+\frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}$$

$$-\frac{2d^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{\sqrt{256ae^3+5d^4}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)}$$

```
[In] Int[Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] ((d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])/3 - (2*d^2*
(d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])/(Sqrt[5*d^4
+ 256*a*e^3]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])) + (d^2
*(5*d^4 + 256*a*e^3)^(3/4)*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*
e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticE[
2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 +
256*a*e^3])/2])/(8*Sqrt[2]*e^2*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4]) + ((5*d^4 + 256*a*e^3)^(1/4)*(5*d^4 + 256*a*e^3 - 3*d^2*Sqrt[5*d^4 + 2
56*a*e^3])*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 2
56*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (
16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticF[2*ArcTan[(d + 4*
```

```
e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/
(48*Sqrt[2]*e^2*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \sqrt{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}\,dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4} + \frac{1}{3}\text{Subst}\left(\int \frac{\frac{1}{16}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2}{\sqrt{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}}dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4} + \frac{\left(d^2\sqrt{5d^4+256ae^3}\right)\text{Subst}\left(\int \frac{1-\frac{16e^2x^2}{\sqrt{5d^4+256ae^3}}}{\sqrt{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}}dx, x, \frac{d}{4e}+x\right)}{16e} + \frac{\left(5d^4+256ae^3-3d^2\sqrt{5d^4+256ae^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}}dx, x, \frac{d}{4e}+x\right)}{48e}$$

$$= \frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4} - \frac{d^2(d+4ex)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{2e\sqrt{5d^4+256ae^3}\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)} + \frac{d^2\left(5d^4+256ae^3\right)^{3/4}\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(5d^4+256ae^3)\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)E\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right.\right.}{8\sqrt{2}e^2\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}} + \frac{\sqrt[4]{5d^4+256ae^3}\left(5d^4+256ae^3-3d^2\sqrt{5d^4+256ae^3}\right)\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(5d^4+256ae^3)\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae}\right.}{48\sqrt{2}e^2\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7543 vs. 2(663) = 1326.

Time = 13.92 (sec) , antiderivative size = 7543, normalized size of antiderivative = 11.38

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 7886 vs. 2(715) = 1430.

Time = 7.22 (sec) , antiderivative size = 7887, normalized size of antiderivative = 11.90

method	result	size
default	Expression too large to display	7887
elliptic	Expression too large to display	7887
risch	Expression too large to display	9561

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\, dx = \int \sqrt{8\, e^3x^4 + 8\, de^2x^3 - d^3x + 8\, ae^2}\, dx$$

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="fricas"
)
[Out] integral(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Sympy [F]

$$\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\, dx = \int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\, dx$$

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
[Out] Integral(sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4), x)
```

Maxima [F]

$$\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\, dx = \int \sqrt{8\, e^3x^4 + 8\, de^2x^3 - d^3x + 8\, ae^2}\, dx$$

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Giac [F]

$$\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx = \int \sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}\,dx$$

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx = \int \sqrt{-d^3\,x + 8\,d\,e^2\,x^3 + 8\,e^3\,x^4 + 8\,a\,e^2}\,dx$$

```
[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2),x)
[Out] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2), x)
```

3.779 $\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 235

$$\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx = \frac{\sqrt[4]{5d^4+256ae^3}\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(5d^4+256ae^3)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)\text{EllipticF}\left(2\arctan\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right),\frac{1}{2}\cdots\right.}{\sqrt{2}e\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

```
[Out] 1/2*(256*a*e^3+5*d^4)^(1/4)*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)
))^2)^(1/2)/cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticF(sin(
2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)
^(1/2))^(1/2))*(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x
^4+8*d*e^2*x^3-d^3*x+8*a*e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(25
6*a*e^3+5*d^4)^(1/2))^2)^(1/2)/e*2^(1/2)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1120, 1117}

$$\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx = \frac{\sqrt[4]{256ae^3+5d^4}\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(256ae^3+5d^4)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right),\frac{1}{2}\cdots\right.}{\sqrt{2}e\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

```
[In] Int[1/Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] ((5*d^4 + 256*a*e^3)^(1/4)*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*
e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticF[
2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 +
256*a*e^3])/2])/(Sqrt[2]*e*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\sqrt{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}}\,dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{\sqrt[4]{5d^4+256ae^3}\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(5d^4+256ae^3)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)F\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(1\right.\right.}{\sqrt{2}e\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1065 vs. 2(235) = 470.

Time = 11.41 (sec) , antiderivative size = 1065, normalized size of antiderivative = 4.53

$$\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx =$$

$$-\frac{\left(-d+\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-4ex\right)\left(d-\sqrt{3d^2+2\sqrt{d^4-64ae^3}}+4ex\right)\sqrt{-\frac{\sqrt{3d^2-2\sqrt{d^4-64ae}}}{\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-\sqrt{3d^2+2}\right.}}}{}$$

```
[In] Integrate[1/Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] -1/2*((-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x)*(d - Sqrt[3*d^2 +
 2*Sqrt[d^4 - 64*a*e^3]] + 4*e*x)*Sqrt[-((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^
3]]*(d + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]] + 4*e*x))/((Sqrt[3*d^2 - 2*Sq
rt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^
2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x)))]*Sqrt[(3*d^2 - 2*Sqrt[d^4 - 64*a*e^3
] - Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]*Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3
]] + d*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64
*a*e^3]]) + 4*e*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt
[d^4 - 64*a*e^3]])*x)/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 +
 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*
x))]*EllipticF[ArcSin[Sqrt[((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*
d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] +
4*e*x))/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 + 2*Sqrt[d^4 -
64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x))]], (Sqrt[3
*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])^2/(S
qrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])
^2])/(e*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 6
4*a*e^3]])*Sqrt[(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]*(-d + Sqrt[3*d^2 + 2*
Sqrt[d^4 - 64*a*e^3]] - 4*e*x))/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sq
rt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^
3]] - 4*e*x))]*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1703 vs. 2(273) = 546.

Time = 1.23 (sec) , antiderivative size = 1704, normalized size of antiderivative = 7.25

method	result	size
default	Expression too large to display	1704
elliptic	Expression too large to display	1704

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+
(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*((-1/4*(d*e+(3*d^2*e^2-
2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^
4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e
^2)^(1/2))/e^2)/(-1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e
^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(x+1/4*(d*
e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*(x+1/4*(d*e+(3
*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)^2*((-1/4*(d*e+(3*d^2*e^2+
2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d
^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*
```

```
e^2)^(1/2))/e^2)/(1/4*(-d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/
e^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(x+1/4*(d
*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*((-1/4*(d*e+(
3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2+2*(-
64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x+1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^
4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2
)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/
(x+1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)/(-1/
4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*d^2*e
^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*d^2*e^2+2*(-64*a*
e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)
*e^2)^(1/2))/e^2)*2^(1/2)/(e^3*(x-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1
/2)*e^2)^(1/2))/e^2)*(x+1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1
/2))/e^2)*(x-1/4*(-d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*
(x+1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*Elli
pticF(((-1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d
*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*d^2*
e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*d^2*e^2-2*(-64*a
*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2
)*e^2)^(1/2))/e^2)/(x+1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2
))/e^2))^(1/2),((-1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e
^2-1/4*(-d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(1/4*(-d*e
+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*d^2*e^2-2*(
-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^
4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^
(1/2))/e^2)/(-1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1
/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx = \int \frac{1}{\sqrt{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}}\,dx$$

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="frica
s")

[Out] integral(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx = \int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx$$

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
[Out] Integral(1/sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx = \int \frac{1}{\sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}}\,dx$$

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="maxim
a")
[Out] integrate(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx = \int \frac{1}{\sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}}\,dx$$

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="giac"
)
[Out] integrate(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx = \int \frac{1}{\sqrt{-d^3\,x + 8\,d\,e^2\,x^3 + 8\,e^3\,x^4 + 8\,a\,e^2}}\,dx$$

```
[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2),x)
[Out] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2), x)
```

3.780 $\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 748

$$\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}}\,dx = \frac{4e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$
$$+\frac{384d^2e^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{\left(d^4-64ae^3\right)\left(5d^4+256ae^3\right)^{3/2}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)}$$
$$-\frac{12\sqrt{2}d^2\sqrt{\frac{e\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}{\left(5d^4+256ae^3\right)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)E\left(2\arctan\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(1+\frac{3d^2}{\sqrt{5d^4+256ae^3}}\right.\right.}{\left(d^4-64ae^3\right)\sqrt[4]{5d^4+256ae^3}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$
$$-\frac{2\sqrt{2}\left(5d^4+256ae^3-3d^2\sqrt{5d^4+256ae^3}\right)\sqrt{\frac{e\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}{\left(5d^4+256ae^3\right)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)\text{EllipticF}\left(2\,\text{arct}\right.}{\left(d^4-64ae^3\right)\left(5d^4+256ae^3\right)^{3/4}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

```
[Out] 4*e*(1/4*d/e+x)*(13*d^4-256*a*e^3-48*d^2*e^2*(1/4*d/e+x)^2)/(-16384*a^2*e^6
-64*a*d^4*e^3+5*d^8)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)+384*d^2*e^
2*(1/4*d/e+x)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)/(-64*a*e^3+d^4)/(
256*a*e^3+5*d^4)^(3/2)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))-12*
d^2*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))^2)^(1/2)/cos(2*arctan
((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticE(sin(2*arctan((4*e*x+d)/(256*
a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(1/2))^(1/2))*2^(1/2)*(
1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d*e^2*x^3-d
^3*x+8*a*e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(
```

```
1/2))^2)^(1/2)/(-64*a*e^3+d^4)/(256*a*e^3+5*d^4)^(1/4)/(8*e^3*x^4+8*d*e^2*x
^3-d^3*x+8*a*e^2)^(1/2)-2*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))
^2)^(1/2)/cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticF(sin(2*
arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(
1/2))^(1/2))*2^(1/2)*(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(5*d^
4+256*a*e^3-3*d^2*(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+
8*a*e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))
^2)^(1/2)/(-64*a*e^3+d^4)/(256*a*e^3+5*d^4)^(3/4)/(8*e^3*x^4+8*d*e^2*x^3-d^
3*x+8*a*e^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1120, 1106, 1211, 1117, 1209}

$$\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^{3/2}}\,dx = \frac{4e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

$$-\frac{2\sqrt{2}\left(-3d^2\sqrt{256ae^3+5d^4}+256ae^3+5d^4\right)\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(256ae^3+5d^4)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)\text{EllipticF}\left(2\,a\right.}{(d^4-64ae^3)\,(256ae^3+5d^4)^{3/4}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3}}$$

$$-\frac{12\sqrt{2}d^2\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(256ae^3+5d^4)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)E\left(2\arctan\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+\right.\right.}{(d^4-64ae^3)\sqrt[4]{256ae^3+5d^4}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

$$+\frac{384d^2e^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{(d^4-64ae^3)\,(256ae^3+5d^4)^{3/2}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)}$$

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-3/2),x]
```

```
[Out] (4*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)^2))/((5*d
^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e
^3*x^4]) + (384*d^2*e^2*(d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 +
8*e^3*x^4])/((d^4 - 64*a*e^3)*(5*d^4 + 256*a*e^3)^(3/2)*(1 + (16*e^2*(d/(4*
e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])) - (12*Sqrt[2]*d^2*Sqrt[(e*(8*a*e^2 - d
^3*x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e)
 + x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*
d^4 + 256*a*e^3])*EllipticE[2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)]
, (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/((d^4 - 64*a*e^3)*(5*d^4 + 256*
a*e^3)^(1/4)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4]) - (2*Sqrt[2]*
```

```
(5*d^4 + 256*a*e^3 - 3*d^2*Sqrt[5*d^4 + 256*a*e^3])*Sqrt[(e*(8*a*e^2 - d^3*
x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) +
x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4
 + 256*a*e^3])*EllipticF[2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (
1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/((d^4 - 64*a*e^3)*(5*d^4 + 256*a*e
^3)^(3/4)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4\right)^{3/2}}\,dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{4e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(5d^8-64ad^4e^3-16384a^2e^6)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}} - \frac{8\text{Subst}\left(\int \frac{\frac{1}{2}e^3\left(\frac{5d^4}{e}+256ae^2\right)-24d^2e^4x^2}{\sqrt{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}}\,dx, x, \frac{d}{4e}+x\right)}{e\,(5d^8-64ad^4e^3-16384a^2e^6)}$$

$$= \frac{4e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(5d^8-64ad^4e^3-16384a^2e^6)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}} - \frac{(12d^2e)\,\text{Subst}\left(\int \frac{1-\frac{16e^2x^2}{\sqrt{5d^4+256ae^3}}}{\sqrt{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}}\,dx, x, \frac{d}{4e}+x\right)}{(d^4-64ae^3)\sqrt{5d^4+256ae^3}} - \frac{\left(4e\left(5d^4+256ae^3-3d^2\sqrt{5d^4+256ae^3}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}}\,dx, x, \frac{d}{4e}+x\right)}{5d^8-64ad^4e^3-16384a^2e^6}$$

$$= \frac{4e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(5d^8-64ad^4e^3-16384a^2e^6)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}} + \frac{96d^2e(d+4ex)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{(d^4-64ae^3)(5d^4+256ae^3)^{3/2}\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)} - \frac{12\sqrt{2}d^2\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(5d^4+256ae^3)\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)E\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(1+\frac{}{\sqrt{}}\right.\right.}{(d^4-64ae^3)\sqrt[4]{5d^4+256ae^3}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}} - \frac{2\sqrt{2}\left(5d^4+256ae^3-3d^2\sqrt{5d^4+256ae^3}\right)\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(5d^4+256ae^3)\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)F\left(2\,\text{ta}\right.}{(d^4-64ae^3)(5d^4+256ae^3)^{3/4}\sqrt{8ae^2-d^3x+8de^2x^3+}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7629 vs. 2(748) = 1496.

Time = 16.14 (sec) , antiderivative size = 7629, normalized size of antiderivative = 10.20

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^{3/2}} dx = \text{Result too large to show}$$

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-3/2),x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 8102 vs. 2(804) = 1608.

Time = 1.37 (sec) , antiderivative size = 8103, normalized size of antiderivative = 10.83

method	result	size
default	Expression too large to display	8103
elliptic	Expression too large to display	8103

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^{3/2}} dx = \int \frac{1}{(8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2)^{\frac{3}{2}}} dx$$

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="frica
s")
[Out] integral(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)/(64*e^6*x^8 + 128*
d*e^5*x^7 + 64*d^2*e^4*x^6 - 16*d^3*e^3*x^5 + 128*a*d*e^4*x^3 + d^6*x^2 - 1
6*a*d^3*e^2*x + 64*a^2*e^4 - 16*(d^4*e^2 - 8*a*e^5)*x^4), x)
```

Sympy [F]

$$\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^{3/2}}\,dx = \int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(3/2),x)
[Out] Integral((8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^{3/2}}\,dx = \int \frac{1}{(8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="maxim
a")
[Out] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^{3/2}}\,dx = \int \frac{1}{(8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="giac"
)
[Out] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^{3/2}}\,dx = \int \frac{1}{(-d^3\,x+8\,d\,e^2\,x^3+8\,e^3\,x^4+8\,a\,e^2)^{3/2}}\,dx$$

```
[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(3/2),x)
[Out] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(3/2), x)
```

3.781 $\int (a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 452

$$\int (a+8x-8x^2+4x^3-x^4)^{3/2}\,dx = -\frac{16(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{35\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{2}{35}\left(13+5a-3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) + \frac{16(7+2a)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{35\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{4(3+a)(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{35\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

```
[Out] 1/7*(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)-16/35*(7+2*a)*(-1+x)*(1+(-1+x)^2
/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+2/35*(13+
5*a-3*(-1+x)^2)*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+4/35*(3+a)*(16+5*a)*
(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*E
llipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-
2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)
^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))
/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+16/35*(7+2*a)*(1/(1+(-1+x)^2/(1+(4+a)^
(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)
^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(
1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^
```

```
(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+
x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1120, 1105, 1190, 1216, 545, 429, 506, 422}

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} dx = \frac{4(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{16(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2} + \frac{2}{35}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x-1)^2+3} - \frac{16(2a+7)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}\right.}{35\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (-16*(7 + 2*a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x
))/(35*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(13 + 5*a - 3*(-1 + x)
^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + ((3 + a - 2*(-1
+ x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16*(7 + 2*a)*(1 - Sqrt[4 + a])*Sq
rt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1
 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt
[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqr
t[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (4*(3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt
[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[
1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1 + (-1 +
 x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2
*(-1 + x)^2 - (-1 + x)^4])
```

<u>Rule 422</u>

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \left(3+a-2x^2-x^4\right)^{3/2}\,dx, x, -1+x\right)$$

$$= \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) + \frac{3}{7}\text{Subst}\left(\int \left(2(3+a)-2x^2\right)\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$

$$= -\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x) + \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) - \frac{1}{35}\text{Subst}\left(\int \frac{-4(3+a)(16+5a)+16(7+2a)x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)$$

$$= -\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x) + \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) - \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{-4(3+a)(16+5a)+16(7+2a)x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{35\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= -\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x) + \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) - \frac{\left(16(7+2a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{35\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\left(4(3+a)(16+5a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -\right.}{35\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{16(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{35\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$-\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x)$$
$$+\frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)$$
$$-\frac{4(3+a)(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{35\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$+\frac{\left(16(7+2a)\left(1-\sqrt{4+a}\right)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}{\left(1-\frac{2x^2}{-2-2\sqrt{4+a}}\right)^{3/2}}dx,x,-1\right.}{35\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{16(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{35\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$-\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x)$$
$$+\frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)$$
$$-\frac{16(7+2a)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{35\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$-\frac{4(3+a)(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{35\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6287 vs. $2(452) = 904$.

Time = 16.10 (sec) , antiderivative size = 6287, normalized size of antiderivative = 13.91

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} dx = \text{Result too large to show}$$

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2654 vs. 2(506) = 1012.

Time = 5.24 (sec) , antiderivative size = 2655, normalized size of antiderivative = 5.87

method	result	size
default	Expression too large to display	2655
elliptic	Expression too large to display	2655
risch	Expression too large to display	3593

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/7*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+5/7*x^4*(-x^4+4*x^3-8*x^2+a+8*x)^(1
/2)-66/35*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+14/5*x^2*(-x^4+4*x^3-8*x^2+a+8
*x)^(1/2)+(3/7*a-32/35)*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-3/7*a-4/7)*(-x^4
+4*x^3-8*x^2+a+8*x)^(1/2)-(a^2-(3/7*a-32/35)*a+12/7*a+16/7)*((-1-(a+4)^(1/2
))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/
2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))
^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1
+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(
-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2
))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1
/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+
(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^
(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2
))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)
-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(64/35*a+32/5)*((-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1
-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(
x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(
a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+
(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2)
)^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/
2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*
(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1
/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1
/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(
-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(
a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a
```

+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*E
llipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^
(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+
4)^(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-
(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-
1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+
4)^(1/2))^(1/2)))^(1/2)))+(-32/35*a-16/5)*((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-
1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(a+4)^(1/2))^(1
/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2
))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(
1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-
2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/
2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4
)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a
+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(
1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(
a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1
-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
)/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2
))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(
1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-
(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(
-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1
/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(
a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-
1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2
)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^
(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1
+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+
(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/
2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/
2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/
(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-
1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+
4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))
^(1/2)

Fricas [F]

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
[Out] integral((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Sympy [F]

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
[Out] Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Maxima [F]

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Giac [F]

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{3/2}\,dx$$

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.782 $\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 397

$$\begin{aligned}
&\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx \\
&= -\frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) \\
&\quad+\frac{2\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&\quad+\frac{2(3+a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}
\end{aligned}$$

```
[Out] -2/3*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-
1+x)^4)^(1/2)+1/3*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+2/3*(3+a)*(1/(1+(-
1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF
((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^
(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^
(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+
x)^2/(1+(4+a)^(1/2))))^(1/2)+2/3*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+
(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(
-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(
-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x
)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2
))))^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1120, 1105, 1216, 545, 429, 506, 422}

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx$$
$$= \frac{2(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$+\frac{2\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{2\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[In] Int[Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (-2*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)
^4]*(-1 + x))/3 + (2*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^
2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*
Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))
/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4
]) + (2*(3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*El
lipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[
4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + S
qrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \sqrt{3+a-2x^2-x^4}\,dx,x,-1+x\right)\\
&= \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x)\\
&\quad+\frac{1}{3}\text{Subst}\left(\int \frac{2(3+a)-2x^2}{\sqrt{3+a-2x^2-x^4}}\,dx,x,-1+x\right)
\end{aligned}$$

$$=\frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x)$$
$$+\frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{2(3+a)-2x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}dx,x,-1+x\right)}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$=\frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x)$$
$$-\frac{\left(2\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}dx,x,-1+x\right)}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{\left(2(3+a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}dx,x,-1+x\right)}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$=\frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}}+\frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1$$
$$+x)-\frac{2(3+a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$+\frac{\left(2\left(1-\sqrt{4+a}\right)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}{\left(1-\frac{2x^2}{-2-2\sqrt{4+a}}\right)^{3/2}}dx,x,-1+x\right)}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$=\frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}}+\frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1$$
$$+x)$$
$$-\frac{2\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$-\frac{2(3+a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3470 vs. 2(397) = 794.

Time = 16.06 (sec) , antiderivative size = 3470, normalized size of antiderivative = 8.74

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
[Out] (-1/3 + x/3)*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4] + (2*((4*(-Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[
((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4
+ a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-
1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4
 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[
-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt
[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], (
(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]]
+ Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]
])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4
+ a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x
^2 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt
[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 +
 a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sq
rt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(S
qrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Ellipt
icF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + S
qrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[
-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sq
rt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (4*(-Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 -
 Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 +
 Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 +
 Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1
```

```
- Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1
+ Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
 - Sqrt[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[A
rcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1
 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1
 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt
[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-
1 - Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x
))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4
 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3
 - x^4]) - ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]]
+ x)*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-
1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] -
 x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
+ x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
 + x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSi
n[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + S
qrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
 a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 -
Sqrt[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - S
qrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]]
 - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-
1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*
(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^
2])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x
^2 + 4*x^3 - x^4]))/3
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2518 vs. $2(455) = 910$.

Time = 4.20 (sec) , antiderivative size = 2519, normalized size of antiderivative = 6.35

method	result	size
default	Expression too large to display	2519
elliptic	Expression too large to display	2519
risch	Expression too large to display	3022

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/
3*a+4/3)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1
/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1
+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))
^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2
))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2))
)^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(
1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(
a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-
(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1
/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))
/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-4/3*((-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2)
)^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2)
)^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-
1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-
(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(
1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1
+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(
-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)
))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(
1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x
-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
```

```
/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/
2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1
/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^
(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)))-2/3*((x-1-(-1+(a+4)^(1/2))^(1/2))*(x
-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(a+4)^(1/2))^(
1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/
2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^
(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(
-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+
4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^
(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+
(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(
1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2
))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/
2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^
(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)
-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-
(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+
(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+
(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(
-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1
/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))
^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^
(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-
1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1
+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1
/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))
/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(
-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a
+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))
)^(1/2)
```

Fricas [F]

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,dx$$

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Sympy [F]

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
[Out] Integral(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Maxima [F]

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,dx$$

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Giac [F]

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,dx$$

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}\,dx$$

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.783 $\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 144

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

```
[Out] (1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*E
llipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-
2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)
^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))
/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1120, 1118, 429}

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[In] Int[1/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[
(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt
[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqr
t[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1118

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2
*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)$$

$$= \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= -\frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 540 vs. $2(144) = 288$.

Time = 11.03 (sec) , antiderivative size = 540, normalized size of antiderivative = 3.75

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$$
$$= \frac{2\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(1+\sqrt{-1+\sqrt{4+a}}-x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\sqrt{\frac{\cdots}{(-\cdots}}}{\sqrt{-1-\sqrt{4+a}}\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\cdots}{\left(\sqrt{-1-\sqrt{4+a}}+\cdots}}}$$

```
[In] Integrate[1/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (2*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(1 + Sqrt[
-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*Sqrt[(
Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((-Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Ellip
ticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + S
qrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(Sq
rt[-1 - Sqrt[4 + a]]*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]
)*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + S
qrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[a - x*(-8 + 8*x - 4*x^
2 + x^3)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(182) = 364$.

Time = 0.88 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.68

method	result
default	$-\frac{\left(\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\left(x-1-\sqrt{-1+\sqrt{a+4}}\right)}{\left(-\sqrt{-1-\sqrt{a+4}}-\sqrt{-1+\sqrt{a+4}}\right)\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)}}\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)^2\sqrt{-\frac{2\sqrt{-1+\sqrt{a+4}}\cdots}{\left(\sqrt{-1-\sqrt{a+4}}-\sqrt{-1\cdots}}}}{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{-\cdots}}$
elliptic	$-\frac{\left(\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\left(x-1-\sqrt{-1+\sqrt{a+4}}\right)}{\left(-\sqrt{-1-\sqrt{a+4}}-\sqrt{-1+\sqrt{a+4}}\right)\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)}}\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)^2\sqrt{-\frac{2\sqrt{-1+\sqrt{a+4}}\cdots}{\left(\sqrt{-1-\sqrt{a+4}}-\sqrt{-1\cdots}}}}{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{-\cdots}}$

[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] -((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))

Fricas [F]

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)

Sympy [F]

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{1}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)

[Out] Integral(1/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)

Maxima [F]

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)

Giac [F]

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}}\,dx$$

[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)

[Out] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)

3.784 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 437

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \frac{(5+a+(-1+x)^2)(-1+x)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{2(3+a)(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(3+a)(4+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(4+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

```
[Out] 1/2*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/2*
(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(a^2+7*a+12)/(3+a-2*(-1
+x)^2-(-1+x)^4)^(1/2)+1/2*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^
2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2
/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2
/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/
2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+1/2*(1
/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*Ell
ipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*
(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(
1/2))*(1+(4+a)^(1/2))^(1/2)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((
1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1120, 1106, 1216, 545, 429, 506, 422}

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\operatorname{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+3)(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}} - \frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)
^2 - (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(
-1 + x))/(2*(3 + a)*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1
- Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ell
ipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4
 + a])])/(2*(3 + a)*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-
1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sq
rt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1
 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(2*(4 +
a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]
))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\text{Subst}\left(\int \frac{-2(3+a)+2x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)}{4(12+7a+a^2)}$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{-2(3+a)+2x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{4(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\left((3+a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2(12+7a+a^2)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(5+a+(-1+x)^2)(-1+x)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(4+a)\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(\left(1-\sqrt{4+a}\right)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}{\left(1-\frac{2x^2}{-2-2\sqrt{4+a}}\right)^{3/2}}\,dx, x, -1+x\right)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{(1-\sqrt{4+a})\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2\,(12+7a+a^2)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(5+a+(-1+x)^2)\,(-1+x)}{2\,(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{(1-\sqrt{4+a})\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2\,(12+7a+a^2)\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(4+a)\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3526 vs. $2(437) = 874$.

Time = 16.07 (sec) , antiderivative size = 3526, normalized size of antiderivative = 8.07

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] ((6 + a - 8*x - a*x + 3*x^2 - x^3)*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4])/(2*
(3 + a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)) + ((4*(-Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((
-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 +
a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1
- Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 +
 a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1
 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4
+ a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-
1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-
Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])
*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 +
a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2
 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*
(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-
1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a
]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt
```

[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (4*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) - ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]

```
] - Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] -
 x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4
 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-
1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]
)/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x^2
 + 4*x^3 - x^4])/(2*(3 + a)*(4 + a))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2600 vs. $2(495) = 990$.

Time = 0.93 (sec) , antiderivative size = 2601, normalized size of antiderivative = 5.95

method	result	size
default	Expression too large to display	2601
elliptic	Expression too large to display	2601

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/4/(a^2+7*a+12)*x^3-3/4/(a^2+7*a+12)*x^2+1/4*(a+8)/(a^2+7*a+12)*x-1/4*(
6+a)/(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-((a+5)/(a^2+7*a+12)-1/2*(
a+8)/(a^2+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(
a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^
(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a
+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+
(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))
^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/
2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4
)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*
(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(
((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/
2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^
(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^
(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/
2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/(a^2+7
*a+12)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^
```

(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(
-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(
1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))
^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^
(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/
2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+
4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(
1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+
(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(
-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1
/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))
^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(
a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-
1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(
1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2
))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)))-1/2/(a^2+7*a+12
)*((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)
^(1/2))^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)
^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^
(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4
)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/
2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^
(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-
(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(
1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^
(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(
-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)
))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4
)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1
/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/
2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/

```
2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^
(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4
)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1
/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2)
)^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(
-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(
1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)
```

Fricas [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^8 - 8*x^7 + 32*x^6 - 2*(a
- 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x)
```

Sympy [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
[Out] Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx=\int \frac{1}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx=\int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{3/2}}\,dx$$

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.785 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 517

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \frac{(5+a+(-1+x)^2)(-1+x)}{6(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}}$$
$$+\frac{(104+47a+5a^2+4(7+2a)(-1+x)^2)(-1+x)}{12(3+a)^2(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$-\frac{(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{3(3+a)^2(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{(7+2a)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3(3+a)^2(4+a)^2\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{12(3+a)(4+a)^2\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

```
[Out] 1/6*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/12
*(104+47*a+5*a^2+4*(7+2*a)*(-1+x)^2)*(-1+x)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-
(-1+x)^4)^(1/2)-1/3*(7+2*a)*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1
/2))/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/12*(16+5*a)*(1/(1+(-1
+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF(
(-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(
1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(
1/2)/(3+a)/(4+a)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1
/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+1/3*(7+2*a)*(1/(1+(-1+x)^2/(1+(4+
```

```
a)^(1/2)))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4
+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a
)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2
))^(1/2)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+
a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1120, 1106, 1192, 1216, 545, 429, 506, 422}

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx=\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12(a+3)(a+4)^2\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3(a+3)^2(a+4)^2\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{(2a+7)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)^(3/2)) + ((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(-1 + x)^2)*(-1 +
x))/(12*(3 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((7
+ 2*a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*(3
 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 2*a)*(1
- Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ell
ipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4
 + a])])/(3*(3 + a)^2*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1
+ (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) +
 ((16 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellip
ticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 +
 a])])/(12*(3 + a)*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (
-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right)\\
&= \frac{(5+a+(-1+x)^2)\,(-1+x)}{6\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{4+2(3+a)-3(4+4(3+a))-6x^2}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)}{12\,(12+7a+a^2)}\\
&= -\frac{(104+47a+5a^2+4(7+2a)(1-x)^2)\,(1-x)}{12\,(12+7a+a^2)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}}\\
&\quad + \frac{(5+a+(-1+x)^2)\,(-1+x)}{6\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{4(3+a)(16+5a)-16(7+2a)x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)}{48\,(12+7a+a^2)^2}\\
&= -\frac{(104+47a+5a^2+4(7+2a)(1-x)^2)\,(1-x)}{12\,(12+7a+a^2)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}}\\
&\quad + \frac{(5+a+(-1+x)^2)\,(-1+x)}{6\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}}\\
&\quad + \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{4(3+a)(16+5a)-16(7+2a)x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{48\,(12+7a+a^2)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}
\end{aligned}$$

$$= -\frac{(104+47a+5a^2+4(7+2a)(1-x)^2)(1-x)}{12(12+7a+a^2)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(5+a+(-1+x)^2)(-1+x)}{6(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}} - \frac{\left((7+2a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}dx,x,-1+x\right)}{3(12+7a+a^2)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\left((3+a)(16+5a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}dx,x,-1\right.}{12(12+7a+a^2)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= -\frac{(104+47a+5a^2+4(7+2a)(1-x)^2)(1-x)}{12(12+7a+a^2)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3(12+7a+a^2)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(5+a+(-1+x)^2)(-1+x)}{6(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}} - \frac{(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{12(3+a)(4+a)^2\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left((7+2a)\left(1-\sqrt{4+a}\right)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}{\left(1-\frac{2x^2}{-2-2\sqrt{4+a}}\right)^{3/2}}dx,x,-1+x\right.}{3(12+7a+a^2)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= -\frac{(104+47a+5a^2+4(7+2a)(1-x)^2)\,(1-x)}{12\,(12+7a+a^2)^2\,\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(7+2a)\,(1-\sqrt{4+a})\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3\,(12+7a+a^2)^2\,\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(5+a+(-1+x)^2)\,(-1+x)}{6\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}} - \frac{(7+2a)\,(1-\sqrt{4+a})\,\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\,(12+7a+a^2)^2\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{12(3+a)(4+a)^2\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6386 vs. 2(517) = 1034.

Time = 16.14 (sec) , antiderivative size = 6386, normalized size of antiderivative = 12.35

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2756 vs. 2(571) = 1142.

Time = 0.96 (sec) , antiderivative size = 2757, normalized size of antiderivative = 5.33

method	result	size
default	Expression too large to display	2757
elliptic	Expression too large to display	2757

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x,method=_RETURNVERBOSE)
[Out] (1/6/(a^2+7*a+12)*x^3-1/2/(a^2+7*a+12)*x^2+1/6*(a+8)/(a^2+7*a+12)*x-1/6*(6+
a)/(a^2+7*a+12))*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2
*(1/6*(7+2*a)/(a^2+7*a+12)^2*x^3-1/2*(7+2*a)/(a^2+7*a+12)^2*x^2+1/24*(5*a^2
```

```
+71*a+188)/(a^2+7*a+12)^2*x-1/24*(5*a^2+55*a+132)/(a^2+7*a+12)^2)/(-x^4+4*x
^3-8*x^2+a+8*x)^(1/2)-(1/6*(5*a^2+47*a+104)/(a^2+7*a+12)^2-1/12*(5*a^2+71*a
+188)/(a^2+7*a+12)^2)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(
-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)
))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1
-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(
-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/
2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^
(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(
a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2
))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*Ellipti
cF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^
(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2
))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^
(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-2/3*(
7+2*a)/(a^2+7*a+12)^2*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(
-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)
))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1
-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(
-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/
2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^
(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(
a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2
))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1
+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2)
)^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+
(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(
a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)
^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^
(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),(-
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)))-
1/3*(7+2*a)/(a^2+7*a+12)^2*((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/
2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(
1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+
4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+
(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/
2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
```

```
/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*
(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2
))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1
+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/
2))+(1-(-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2
))^(1/2))^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/
2))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-
(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x
-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2)
)^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))
^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(
((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/
2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^
(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^
(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/
2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))/(-1+(a+4
)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticP
i(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(
1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2)
)^(1/2)))^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^
(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^
(1/2)))^(1/2))))/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2)
)*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)
```

Fricas [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^12 - 12*x^11 + 72*x^10 -
3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5*a -
 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^3 - 2
4*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(a-x^4+4x^3-8x^2+8x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
[Out] Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{5/2}}\,dx$$

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
[Out] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

3.786 $\int x(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 558

$$\int x(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx = \frac{3}{16}(4+a)\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} + \frac{1}{8}\left(1+(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2} - \frac{16(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{35\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{2}{35}\left(13+5a-3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) + \frac{3}{16}(4+a)^2\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right) + \frac{1}{\cdots}$$

```
[Out] 1/8*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/7*(3+a-2*(-1+x)^2-(-1+x)
^4)^(3/2)*(-1+x)+3/16*(4+a)^2*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)
^(1/2))-16/35*(7+2*a)*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(
3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+3/16*(4+a)*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+
x)^4)^(1/2)+2/35*(13+5*a-3*(-1+x)^2)*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)
+4/35*(3+a)*(16+5*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+
(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(
4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(
4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+
x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+16/35*(7+2*a)*(1/
(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*Elli
```

```
pticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(
4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1
/2))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-
(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1694, 1687, 1105, 1190, 1216, 545, 429, 506, 422, 1121, 626, 635, 210}

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\frac{3}{16}(a+4)^2\arctan\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)$$
$$+\frac{4(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$+\frac{16(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$+\frac{3}{16}(a+4)\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$
$$+\frac{1}{8}\left((x-1)^2+1\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$
$$+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{16(2}{}$$

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]

[Out] (3*(4 + a)*(1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/16 + (
(1 + (-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2))/8 - (16*(7 + 2*
a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(35*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(13 + 5*a - 3*(-1 + x)^2)*Sqrt[3 +
 a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + ((3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)^(3/2)*(-1 + x))/7 + (3*(4 + a)^2*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 +
a - 2*(-1 + x)^2 - (-1 + x)^4]])/16 + (16*(7 + 2*a)*(1 - Sqrt[4 + a])*Sqrt[
1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 +
x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1
 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3
 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (4*(3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt[4
+ a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 +
 Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1 + (-1 + x)
```

```
^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-
1 + x)^2 - (-1 + x)^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
```

```
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (1+x)\left(3+a-2x^2-x^4\right)^{3/2}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \left(3+a-2x^2-x^4\right)^{3/2}\,dx, x, -1+x\right) + \text{Subst}\left(\int x\left(3+a-2x^2-x^4\right)^{3/2}\,dx, x, -1+x\right)$$

$$= \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) + \frac{3}{7}\text{Subst}\left(\int \left(2(3+a)-2x^2\right)\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \frac{1}{2}\text{Subst}\left(\int \left(3+a-2x-x^2\right)^{3/2}\,dx, x, (-1+x)^2\right)$$

$$= \frac{1}{8}\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}\left(1+(-1+x)^2\right) - \frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x) + \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) - \frac{1}{35}\text{Subst}\left(\int \frac{-4(3+a)(16+5a)+16(7+2a)x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right) + \frac{1}{8}(3(4+a))\text{Subst}\left(\int \sqrt{3+a-2x-x^2}\,dx, x, (-1+x)^2\right)$$

$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{8}\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}\left(1+(-1+x)^2\right) - \frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x) + \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) + \frac{1}{16}\left(3(4+a)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x-x^2}}\,dx, x, (-1+x)^2\right) - \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{-4(3+a)(16+5a)+16(7+2a)x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{35\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{8}\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}\left(1+(-1+x)^2\right) - \frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x) + \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) + \frac{1}{8}\left(3(4+a)^2\right)\text{Subst}\left(\int\frac{1}{-4-x^2}\,dx, x, -\frac{2(1+(-1+x)^2)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right) - \frac{\left(16(7+2a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{35\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\left(4(3+a)(16+5a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -\right.}{35\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{8}\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}\left(1+(-1+x)^2\right) + \frac{16(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{35\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x) + \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) + \frac{3}{16}(4+a)^2\tan^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(1-x)^2-(1-x)^4}}\right) - \frac{4(3+a)(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{35\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(16(7+2a)\left(1-\sqrt{4+a}\right)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}{\left(1-\frac{2x^2}{-2-2\sqrt{4+a}}\right)^{3/2}}\,dx, x, -1\right.}{35\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\,(1+(-1+x)^2)+\frac{1}{8}\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}(1+(-1+x)^2)+\frac{16(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{35\sqrt{3+a-2(1-x)^2-(1-x)^4}}-\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+\frac{3}{16}(4+a)^2\tan^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(1-x)^2-(1-x)^4}}\right)-\frac{16(7+2a)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{35\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}-\frac{4(3+a)(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{35\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7235 vs. 2(558) = 1116.

Time = 17.10 (sec) , antiderivative size = 7235, normalized size of antiderivative = 12.97

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2693 vs. 2(600) = 1200.

Time = 4.90 (sec) , antiderivative size = 2694, normalized size of antiderivative = 4.83

method	result	size
default	Expression too large to display	2694
elliptic	Expression too large to display	2694
risch	Expression too large to display	3609

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x^6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+17/28*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^
(1/2)-43/28*x^4*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+74/35*x^3*(-x^4+4*x^3-8*x^2+
a+8*x)^(1/2)+(5/16*a-9/20)*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-11/56*a-29/
70)*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(11/56*a+13/14)*(-x^4+4*x^3-8*x^2+a+8*
x)^(1/2)-(-(-11/56*a-29/70)*a-11/14*a-26/7)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+
4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1
+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+
(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)
^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4
)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1
/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^
(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+
4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1
+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))
^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*
(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2
))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^
(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(
1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2)
)^(1/2)))^(1/2))-(a^2-2*(5/16*a-9/20)*a+55/14*a+62/5)*((-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
)*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1
/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2
*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2
)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)
^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+
4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(
1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a
+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)
^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(
1/2)*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+
(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(
-1+(a+4)^(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))
/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(
-1+(a+4)^(1/2))^(1/2)))^(1/2)))+(-32/35*a-16/5)*((x-1-(-1+(a+4)^(1/2))^(1/2
))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2)
)^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
```

```
/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2)
)^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-
1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-
(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(
a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1
+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/
2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^
(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2
))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)
-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/
2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1
-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(
x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2
))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1
/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1
/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x
-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2
))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(
-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(
1/2)))^(1/2)
```

Fricas [F]

$$\int x(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx=\int\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
[Out] integral(-(x^5 - 4*x^4 + 8*x^3 - a*x - 8*x^2)*sqrt(-x^4 + 4*x^3 - 8*x^2 + a
 + 8*x), x)
```

Sympy [F]

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int x\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
[Out] Integral(x*(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Maxima [F]

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x, x)
```

Giac [F]

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int x\,\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{3/2}\,dx$$

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.787 $\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 466

$$\begin{aligned}\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx &= \frac{1}{4}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} \\ &\quad -\frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) \\ &\quad +\frac{1}{4}(4+a)\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right) \\ &\quad +\frac{2\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\ &\quad +\frac{2(3+a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\end{aligned}$$

```
[Out] 1/4*(4+a)*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))-2/3*(-1+x)*(
1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)
+1/4*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/3*(-1+x)*(3+a-2*(-1+x)^
2-(-1+x)^4)^(1/2)+2/3*(3+a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x
)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)
^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)
^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/(
(1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+2/3*(1/(1+
```

```
(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*Ellipti
cE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a
)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2)
)*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+
a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1694, 1687, 1105, 1216, 545, 429, 506, 422, 1121, 626, 635, 210}

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$$
$$=\frac{1}{4}(a+4)\arctan\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)$$
$$+\frac{2(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\operatorname{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$+\frac{2\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$+\frac{1}{4}\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$
$$+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{2\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[In] Int[x*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ((1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/4 - (2*(1 - Sqrt
[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*Sqrt[3 + a - 2*(-1
 + x)^2 - (-1 + x)^4]) + (Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))
/3 + ((4 + a)*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4]])/4 + (2*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sq
rt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 +
a])/(1 - Sqrt[4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1
 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(
3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])
/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a
]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (1+x)\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int x\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$

$$= \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{1}{3}\text{Subst}\left(\int \frac{2(3+a)-2x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right) + \frac{1}{2}\text{Subst}\left(\int \sqrt{3+a-2x-x^2}\,dx, x, (-1+x)^2\right)$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{1}{4}(4+a)\text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x-x^2}}\,dx, x, (-1+x)^2\right) + \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{2(3+a)-2x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{1}{2}(4+a)\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, -\frac{2\left(1+(-1+x)^2\right)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right) - \frac{\left(2\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\left(2(3+a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{1}{4}(4+a)\tan^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(1-x)^2-(1-x)^4}}\right) - \frac{2(3+a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(2\left(1-\sqrt{4+a}\right)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}{\left(1-\frac{2x^2}{-2-2\sqrt{4+a}}\right)^{3/2}}\,dx,x,-1+x\right)}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{1}{4}(4+a)\tan^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(1-x)^2-(1-x)^4}}\right) - \frac{2\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{2(3+a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4389 vs. 2(466) = 932.

Time = 14.89 (sec) , antiderivative size = 4389, normalized size of antiderivative = 9.42

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Result too large to show}$$

```
[In] Integrate[x*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (1/6 - x/6 + x^2/4)*Sqrt[a - x*(-8 + 8*x - 4*x^2 + x^3)] + (Sqrt[a - x*(-8
+ 8*x - 4*x^2 + x^3)]*((-8*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]
])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(S
qrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(
Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Ellip
ticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 +
Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt
[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]]
+ Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (2*a*(-Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
+ x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[
-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt
[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])
*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqr
t[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]
] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - S
qrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
 Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[
-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a
 + 8*x - 8*x^2 + 4*x^3 - x^4]) + (40*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + S
qrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]
*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
```

- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))
]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)
)]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)
)]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + S
qrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + S
qrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (6*a*(-Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sq
rt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[
-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt
[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt
[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqr
t[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[
Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqr
t[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sq
rt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]]
 - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-1
- Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4
]) - (4*((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x
)*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
 Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)
)]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] +
x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[S
qrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt

```
[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt
[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqr
t[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]]
 - Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt
[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] -
x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 -
 Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt
[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])
/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))))/Sqrt[a + 8*x - 8*x^2
 + 4*x^3 - x^4]))/(6*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2550 vs. $2(516) = 1032$.

Time = 4.76 (sec) , antiderivative size = 2551, normalized size of antiderivative = 5.47

method	result	size
default	Expression too large to display	2551
elliptic	Expression too large to display	2551
risch	Expression too large to display	3034

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/6*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)
+1/6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(1/6*a-2/3)*((-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1
-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(
x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(
a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+
(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2)
)^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/
2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*
(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1
/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/
2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^
```

(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(1/2*a+10/3)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)))-2/3*((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/

```
2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2
)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/
((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1
/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1
-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(
1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2
)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/
2))))/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-
(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)
```

Fricas [F]

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x\,dx$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

Sympy [F]

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int x\sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
[Out] Integral(x*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Maxima [F]

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x\,dx$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

Giac [F]

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x\,dx$$

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int x\,\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}\,dx$$

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.788 $\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 179

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \frac{1}{2}\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right) + \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

```
[Out] 1/2*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))+(1/(1+(-1+x)^2/(1+
(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1
+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(
4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a
-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4
+a)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used

$= \{1694, 1687, 1118, 429, 1121, 635, 210\}$

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$$
$$= \frac{1}{2}\arctan\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)$$
$$+ \frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[In] Int[x/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]]/2 + (Sqrt[
1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 +
x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 +
(-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 +
a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1118

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2
*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1+x}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x-x^2}}\,dx, x, (-1+x)^2\right)\\
&\quad + \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\\
&= -\frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}\\
&\quad + \text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, -\frac{2(1+(-1+x)^2)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right)
\end{aligned}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right) - \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 813 vs. 2(179) = 358.

Time = 12.96 (sec) , antiderivative size = 813, normalized size of antiderivative = 4.54

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \frac{2\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(1+\sqrt{-1+\sqrt{4+a}}-x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\sqrt{\frac{}{(-\sqrt{}}}}{}$$

```
[In] Integrate[x/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (2*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(1 + Sqrt[
-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*Sqrt[(
Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((-Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*((1 +
 Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - S
qrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - S
qrt[-1 + Sqrt[4 + a]])^2] - 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
 Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a
]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
 Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]]
 + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a
]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[a - x*(-8
+ 8*x - 4*x^2 + x^3)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(213) = 426$.

Time = 1.04 (sec) , antiderivative size = 788, normalized size of antiderivative = 4.40

method	result
default	$-\frac{\left(\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\left(x-1-\sqrt{-1+\sqrt{a+4}}\right)}{\left(-\sqrt{-1-\sqrt{a+4}}-\sqrt{-1+\sqrt{a+4}}\right)\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)}}\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)^{2}\sqrt{-\frac{2\sqrt{-1+\sqrt{a+4}}}{\left(\sqrt{-1-\sqrt{a+4}}-\sqrt{-1}\right.}}}{}$
elliptic	$-\frac{\left(\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\left(x-1-\sqrt{-1+\sqrt{a+4}}\right)}{\left(-\sqrt{-1-\sqrt{a+4}}-\sqrt{-1+\sqrt{a+4}}\right)\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)}}\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)^{2}\sqrt{-\frac{2\sqrt{-1+\sqrt{a+4}}}{\left(\sqrt{-1-\sqrt{a+4}}-\sqrt{-1}\right.}}}{}$

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+
(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/
2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+
4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/
((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)
))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4
)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/
(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(
x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/
2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*
EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^
(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+
4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*(
(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)
)+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(
1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4
)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2
)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),
((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)))
```

Fricas [F]

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^4 - 4*x^3 + 8*x^2 - a -
 8*x), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
[Out] Integral(x/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx=\int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}}\,dx$$

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.789 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 474

$$
\begin{aligned}
\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx &= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&+ \frac{(5+a+(-1+x)^2)(-1+x)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&- \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{2(3+a)(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&+ \frac{\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(3+a)(4+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&+ \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(4+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}
\end{aligned}
$$

```
[Out] 1/2*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/2*(5+a+(-1+x)^2)*(
-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/2*(-1+x)*(1+(-1+x)^2/(
1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2
)+1/2*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(
1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1
/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1
+(4+a)^(1/2))^(1/2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(
4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+1/2*(1/(1+(-1+x)^2/(1+(4+a
)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+
```

```
a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)
^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2)
)^(1/2)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^
(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1694, 1687, 1106, 1216, 545, 429, 506, 422, 1121, 627}

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+3)(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{(x-1)^2+1}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}} - \frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]

[Out] (1 + (-1 + x)^2)/(2*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((5
+ a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 +
 x))/(2*(3 + a)*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1 - Sq
rt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellipti
cE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a
])])/(2*(3 + a)*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 +
x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[1
 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x
)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(2*(4 + a)*S
qrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*
Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
```

```
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1+x}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)\\
&\quad+\text{Subst}\left(\int \frac{x}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)\\
&= \frac{(5+a+(-1+x)^2)(-1+x)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\\
&\quad+\frac{1}{2}\text{Subst}\left(\int \frac{1}{(3+a-2x-x^2)^{3/2}}\,dx, x, (-1+x)^2\right)\\
&\quad-\frac{\text{Subst}\left(\int \frac{-2(3+a)+2x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)}{4(12+7a+a^2)}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\
&\quad + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&\quad - \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{-2(3+a)+2x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx,x,-1+x\right)}{4\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\
&\quad + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&\quad - \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx,x,-1+x\right)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&\quad + \frac{\left((3+a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx,x,-1+x\right)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\
&\quad + \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\
&\quad + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&\quad - \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(4+a)\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\
&\quad + \frac{\left(\left(1-\sqrt{4+a}\right)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}{\left(1-\frac{2x^2}{-2-2\sqrt{4+a}}\right)^{3/2}}\,dx,x,-1+x\right)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\
&+ \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\
&+ \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&- \frac{\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2\left(12+7a+a^2\right)\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\
&- \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(4+a)\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3593 vs. 2(474) = 948.

Time = 14.36 (sec) , antiderivative size = 3593, normalized size of antiderivative = 7.58

$$
\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] ((-a - 2*x + a*x - a*x^2 - x^3)*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2)/(2*(3 +
a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)*(a - x*(-8 + 8*x - 4*x^2 + x^3)
)^(3/2)) + ((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2)*((4*(-Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]
] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 -
Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a
]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 -
 Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 +
a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1
- Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sq
rt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(
-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]
]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 +
```

4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (4*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) - ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[

```
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[
-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt
[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]]
- Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x
))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4 +
 a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
+ Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[
-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/
(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x^2 +
 4*x^3 - x^4]))/(2*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 + x^3))^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2615 vs. $2(528) = 1056$.

Time = 1.12 (sec) , antiderivative size = 2616, normalized size of antiderivative = 5.52

method	result	size
default	Expression too large to display	2616
elliptic	Expression too large to display	2616

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/4/(a^2+7*a+12)*x^3+1/4/(a^2+7*a+12)*a*x^2-1/4*(a-2)/(a^2+7*a+12)*x+1/4
/(a^2+7*a+12)*a)/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/(a^2+7*a+12)+1/2*(a-2)/(
a^2+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(
1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4
)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*
(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1
/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^
(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2)
)/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1
/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2
))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(
-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2))
```

)^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))
^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1
/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-((1+a)/(a^2+7*
a+12)-1/(a^2+7*a+12)*a)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-
(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))
/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/
2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(
-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1
+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(
1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4
)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1
+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1
/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(
-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/
2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-
1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+
4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(
1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4
)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),
(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((
-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+
(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))
)-1/2/(a^2+7*a+12)*((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2
))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1
/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2)
)^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1
/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2
)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1
-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+
(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+
4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(
-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2)
)^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2
)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4
)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(
a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/
2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(
a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^

```
(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(
1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2
))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))
^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2))
)^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(
1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2
)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^
(1/2))))/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(
-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)
```

Fricas [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^8 - 8*x^7 + 32*x^6 - 2*(
a - 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x)
```

Sympy [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x}{(a-x^4+4x^3-8x^2+8x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
[Out] Integral(x/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{3/2}}\,dx$$

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.790 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 591

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}} dx = \frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}}$$
$$+\frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{(5+a+(-1+x)^2)(-1+x)}{6(12+7a+a^2)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}}$$
$$+\frac{(104+47a+5a^2+4(7+2a)(-1+x)^2)(-1+x)}{12(3+a)^2(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$-\frac{(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{3(3+a)^2(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{(7+2a)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3(3+a)^2(4+a)^2\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{12(3+a)(4+a)^2\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

```
[Out] 1/6*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/6*(5+a+(-1+x)^2)*(
-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/3*(1+(-1+x)^2)/(4+a)^2
/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/12*(104+47*a+5*a^2+4*(7+2*a)*(-1+x)^2)*(
-1+x)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/3*(7+2*a)*(-1+x)*(1+
(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-
```

```
1+x)^4)^(1/2)+1/12*(16+5*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x
)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)
^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)
^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a)/(4+a)^2/(3+a-2*(-1+x)^2-(-1
+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1
/2)+1/3*(7+2*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)
^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^
(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^
(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^
2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))
))^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1694, 1687, 1106, 1192, 1216, 545, 429, 506, 422, 1121, 628, 627}

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}} + \frac{(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\operatorname{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12(a+3)(a+4)^2\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3(a+3)^2(a+4)^2\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{(x-1)^2+1}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{(x-1)^2+1}{6(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}} - \frac{(2a+7)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

```
[Out] (1 + (-1 + x)^2)/(6*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + (1
 + (-1 + x)^2)/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((5
+ a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)^(3/2)) + ((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/
(12*(3 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((7 + 2*
```

```
a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*(3 + a
)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 2*a)*(1 - Sq
rt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellipti
cE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a
])])/(3*(3 + a)^2*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-
1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1
6 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF
[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])
])/(12*(3 + a)*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 +
 x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1+x}{(3+a-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right)\\
&\quad + \text{Subst}\left(\int \frac{x}{(3+a-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right)\\
&= \frac{(5+a+(-1+x)^2)(-1+x)}{6(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}}\\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(3+a-2x-x^2)^{5/2}}\,dx, x, (-1+x)^2\right)\\
&\quad - \frac{\text{Subst}\left(\int \frac{4+2(3+a)-3(4+4(3+a))-6x^2}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)}{12(12+7a+a^2)}\\
&= \frac{1+(-1+x)^2}{6(4+a)(3+a-2(1-x)^2-(1-x)^4)^{3/2}}\\
&\quad - \frac{(104+47a+5a^2+4(7+2a)(1-x)^2)(1-x)}{12(12+7a+a^2)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}}\\
&\quad + \frac{(5+a+(-1+x)^2)(-1+x)}{6(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{(3+a-2x-x^2)^{3/2}}\,dx, x, (-1+x)^2\right)}{3(4+a)}\\
&\quad + \frac{\text{Subst}\left(\int \frac{4(3+a)(16+5a)-16(7+2a)x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)}{48(12+7a+a^2)^2}
\end{aligned}$$

$$=\frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}}+\frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}}-\frac{\left(104+47a+5a^2+4(7+2a)(1-x)^2\right)(1-x)}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}}+\frac{\left(5+a+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}}+\frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{4(3+a)(16+5a)-16(7+2a)x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx,x,-1+x\right)}{48\left(12+7a+a^2\right)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$=\frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}}+\frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}}-\frac{\left(104+47a+5a^2+4(7+2a)(1-x)^2\right)(1-x)}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}}+\frac{\left(5+a+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}}-\frac{\left((7+2a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx,x,-1+x\right)}{3\left(12+7a+a^2\right)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{\left((3+a)(16+5a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx,x,-1\right.}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}} + \frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{\left(104+47a+5a^2+4(7+2a)(1-x)^2\right)(1-x)}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{12(3+a)(4+a)^2\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left((7+2a)\left(1-\sqrt{4+a}\right)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}{\left(1-\frac{2x^2}{-2-2\sqrt{4+a}}\right)^{3/2}}dx,x,-1+\right.}{3\left(12+7a+a^2\right)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}} + \frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{\left(104+47a+5a^2+4(7+2a)(1-x)^2\right)(1-x)}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3\left(12+7a+a^2\right)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{(7+2a)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\left(12+7a+a^2\right)^2\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{12(3+a)(4+a)^2\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6452 vs. 2(591) = 1182.

Time = 17.03 (sec) , antiderivative size = 6452, normalized size of antiderivative = 10.92

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2776 vs. 2(637) = 1274.

Time = 1.01 (sec) , antiderivative size = 2777, normalized size of antiderivative = 4.70

method	result	size
default	Expression too large to display	2777
elliptic	Expression too large to display	2777

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x,method=_RETURNVERBOSE)
[Out] (1/6/(a^2+7*a+12)*x^3+1/6/(a^2+7*a+12)*a*x^2-1/6*(a-2)/(a^2+7*a+12)*x+1/6/(
a^2+7*a+12)*a)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2*(
1/6*(7+2*a)/(a^2+7*a+12)^2*x^3+1/6*(a^2-12)/(a^2+7*a+12)^2*x^2-1/24*(3*a^2-
23*a-116)/(a^2+7*a+12)^2*x+1/24*(3*a^2-7*a-60)/(a^2+7*a+12)^2)/(-x^4+4*x^3-
8*x^2+a+8*x)^(1/2)-(1/6*(a^2+23*a+68)/(a^2+7*a+12)^2+1/12*(3*a^2-23*a-116)/
(a^2+7*a+12)^2)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4
)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(
a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/
2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)
^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+
4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1
/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))
^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(
1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-
1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-
(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))
/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/
2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(2/3*(a^2+2
*a-5)/(a^2+7*a+12)^2-2/3*(a^2-12)/(a^2+7*a+12)^2)*((-1-(a+4)^(1/2))^(1/2)+(
```

-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x
-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1
+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-
1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/
2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(
1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2)
)*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^
(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^
(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((
-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-
(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)
EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))(x-1-(-1+(a+4
)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(
a+4)^(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/
(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-
(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(
-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2)))^(1/2)))-1/3*(7+2*a)/(a^2+7*a+12)^2*((x-1-(-1+(a+4)^(1/2
))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2)
)^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+
4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)
(-2(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))
^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*((
1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(1
/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1/
2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)
^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2
)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^
(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2
))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2
))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(
1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+
4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4
)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(

```
a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1
/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))
^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a+
4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+
4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(
1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1
/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)))/(-(x-1-(-1+(a+4)^(1/2))^(1/2))
*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(
1/2))^(1/2)))^(1/2)
```

Fricas [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx=\int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^12 - 12*x^11 + 72*x^10
- 3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5*a
 - 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^3 -
 24*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

Sympy [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx=\int \frac{x}{(a-x^4+4x^3-8x^2+8x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

```
[Out] Integral(x/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx=\int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Giac [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{5/2}}\,dx$$

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
[Out] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

3.791 $\int x^2(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 585

$$\int x^2(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx = \frac{3}{8}(4+a)\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}$$
$$+\frac{1}{4}\left(1+(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}$$
$$+\frac{4(140+111a+21a^2)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{315\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{2}{315}\left(2(80+27a)+3(20+7a)(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x)$$
$$+\frac{1}{63}\left(15+7(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+\frac{3}{8}(4+a)^2\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-}}\right.$$

```
[Out] 1/4*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/63*(15+7*(-1+x)^2)*(3+a-
2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)+3/8*(4+a)^2*arctan((1+(-1+x)^2)/(3+a-2*(-
1+x)^2-(-1+x)^4)^(1/2))+4/315*(21*a^2+111*a+140)*(-1+x)*(1+(-1+x)^2/(1-(4+a
)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+3/8*(4+a)*(1+(-1+
x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+2/315*(160+54*a+3*(20+7*a)*(-1+x)^2)*
(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+4/315*(3+a)*(100+33*a)*(1/(1+(-1+x)^
2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+
x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)
/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)
/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/
(1+(4+a)^(1/2))))^(1/2)-4/315*(21*a^2+111*a+140)*(1/(1+(-1+x)^2/(1+(4+a)^(1
```

```
/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(
1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/
2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1
/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)
^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {1694, 1687, 1190, 1216, 545, 429, 506, 422, 12, 1121, 626, 635, 210}

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx =$$

$$-\frac{4(21a^2+111a+140)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{315\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$+\frac{4(21a^2+111a+140)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{315\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$+\frac{3}{8}(a+4)^2\arctan\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)$$

$$+\frac{4(a+3)(33a+100)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{315\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$+\frac{3}{8}(a+4)\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$+\frac{1}{4}\left((x-1)^2+1\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$+\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{315}(x-1)\left(3(7a+20)(x-1)^2+2(27a+80)\right)\sqrt{a-}$$

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (3*(4 + a)*(1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/8 + ((
1 + (-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2))/4 + (4*(140 + 11
1*a + 21*a^2)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x)
)/(315*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(2*(80 + 27*a) + 3*(20
 + 7*a)*(-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/315 +
 ((15 + 7*(-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/6
3 + (3*(4 + a)^2*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 +
x)^4]])/8 - (4*(140 + 111*a + 21*a^2)*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a
```

```
]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sq
rt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(315*Sqrt[(1 + (-1 + x)^2
/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1
+ x)^2 - (-1 + x)^4]) + (4*(3 + a)*(100 + 33*a)*Sqrt[1 + Sqrt[4 + a]]*(1 +
(-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a
]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(315*Sqrt[(1 + (-1 + x)^2/(1 - Sq
rt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int (1+x)^2\left(3+a-2x^2-x^4\right)^{3/2}\,dx,x,-1+x\right)\\
&= \text{Subst}\left(\int 2x\left(3+a-2x^2-x^4\right)^{3/2}\,dx,x,-1+x\right)\\
&\quad+\text{Subst}\left(\int \left(1+x^2\right)\left(3+a-2x^2-x^4\right)^{3/2}\,dx,x,-1+x\right)\\
&= \frac{1}{63}\left(15+7(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)\\
&\quad-\frac{1}{21}\text{Subst}\left(\int \left(-16(3+a)-2(20+7a)x^2\right)\sqrt{3+a-2x^2-x^4}\,dx,x,-1+x\right)\\
&\quad+2\text{Subst}\left(\int x\left(3+a-2x^2-x^4\right)^{3/2}\,dx,x,-1+x\right)\\
&= -\frac{2}{315}\left(2(80+27a)+3(20+7a)(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x)\\
&\quad+\frac{1}{63}\left(15+7(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)\\
&\quad+\frac{1}{315}\text{Subst}\left(\int \frac{4(3+a)(100+33a)+4\left(140+111a+21a^2\right)x^2}{\sqrt{3+a-2x^2-x^4}}\,dx,x,-1+x\right)\\
&\quad+\text{Subst}\left(\int \left(3+a-2x-x^2\right)^{3/2}\,dx,x,(-1+x)^2\right)\\
&= \frac{1}{4}\left(1+(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}\\
&\quad-\frac{2}{315}\left(2(80+27a)+3(20+7a)(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x)\\
&\quad+\frac{1}{63}\left(15+7(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)\\
&\quad+\frac{1}{4}(3(4+a))\text{Subst}\left(\int \sqrt{3+a-2x-x^2}\,dx,x,(-1+x)^2\right)\\
&\quad+\frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{4(3+a)(100+33a)+4\left(140+111a+21a^2\right)x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx,x,-1+x\right)}{315\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}
\end{aligned}$$

$$= \frac{3}{8}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right)$$
$$+\frac{1}{4}\left(1+(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}$$
$$-\frac{2}{315}\left(2(80+27a)+3(20+7a)(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x)$$
$$+\frac{1}{63}\left(15+7(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)$$
$$+\frac{1}{8}\left(3(4+a)^2\right)\text{Subst}\left(\int\frac{1}{\sqrt{3+a-2x-x^2}}\,dx,x,(-1+x)^2\right)$$
$$+\frac{\left(4(3+a)(100+33a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx,x\right.}{315\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{\left(4(140+111a+21a^2)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx\right.}{315\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{3}{8}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right)$$
$$+\frac{1}{4}\left(1+(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}$$
$$-\frac{4(140+111a+21a^2)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{315\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$-\frac{2}{315}\left(2(80+27a)+3(20+7a)(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x)$$
$$+\frac{1}{63}\left(15+7(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)$$
$$-\frac{4(3+a)(100+33a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{315\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

$$+\frac{1}{4}\left(3(4+a)^2\right)\text{Subst}\left(\int\frac{1}{-4-x^2}\,dx,x,-\frac{2\left(1+(-1+x)^2\right)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right)-\frac{\left(4(140+111a\right.}{}$$

$$
\begin{aligned}
&= \frac{3}{8}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) \\
&\quad + \frac{1}{4}\left(1+(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2} \\
&\quad - \frac{4(140+111a+21a^2)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{315\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\
&\quad - \frac{2}{315}\left(2(80+27a)+3(20+7a)(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x) \\
&\quad + \frac{1}{63}\left(15+7(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) \\
&\quad + \frac{3}{8}(4+a)^2\tan^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(1-x)^2-(1-x)^4}}\right) \\
&\quad + \frac{4(140+111a+21a^2)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{315\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\
&\quad - \frac{4(3+a)(100+33a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{315\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 8500 vs. $2(585) = 1170$.

Time = 17.05 (sec) , antiderivative size = 8500, normalized size of antiderivative = 14.53

$$
\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2732 vs. $2(624) = 1248$.

Time = 4.15 (sec) , antiderivative size = 2733, normalized size of antiderivative = 4.67

method	result	size
default	Expression too large to display	2733
elliptic	Expression too large to display	2733
risch	Expression too large to display	3625

[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/9*x^7*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+19/36*x^6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-163/126*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+71/42*x^4*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(11/45*a-16/63)*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-13/120*a-5/18)*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(9/140*a+23/63)*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(107/252*a+101/63)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(-(9/140*a+23/63)*a-107/63*a-404/63)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(-2*(-13/120*a-5/18)*a+827/315*a+76/9)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))^

(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2
))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)
^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^
(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2
))^(1/2)))^(1/2)))+(a^2-3*(11/45*a-16/63)*a+68/105*a+16/9)*((x-1-(-1+(a+4)^
(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+
(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(
-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(
1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1
-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(
1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1
/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/
2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2)
)^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)
^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(
a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^
(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^
(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(
1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-
1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^
(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2
))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1
+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-
(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4
)^(1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2)
)^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1
-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+
4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4
)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(a+4)^(1/2))^(1
/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+
4)^(1/2))^(1/2)))^(1/2)

Fricas [F]

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x^2\,dx$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
[Out] integral(-(x^6 - 4*x^5 + 8*x^4 - a*x^2 - 8*x^3)*sqrt(-x^4 + 4*x^3 - 8*x^2 +
 a + 8*x), x)
```

Sympy [F]

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int x^2\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
[Out] Integral(x**2*(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Maxima [F]

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x^2\,dx$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x^2\,dx$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx=\int x^2\,\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{3/2}\,dx$$

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.792 $\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 485

$$\begin{aligned}
&\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx \\
&= \frac{1}{2}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} \\
&\quad+\frac{2(8+3a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{15\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&\quad+\frac{1}{15}\left(7+3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) \\
&\quad+\frac{1}{2}(4+a)\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right) \\
&\quad-\frac{2(8+3a)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{15\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&\quad+\frac{8(3+a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{15\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}
\end{aligned}$$

```
[Out] 1/2*(4+a)*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))+2/15*(8+3*a)
*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)
^4)^(1/2)+1/2*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/15*(7+3*(-1+x)
^2)*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+8/15*(3+a)*(1/(1+(-1+x)^2/(1+(4+
a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4
+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a
```

```
)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*
(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)
^(1/2))))^(1/2)-2/15*(8+3*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+
x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x
)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x
)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-
(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))
^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {1694, 1687, 1190, 1216, 545, 429, 506, 422, 12, 1121, 626, 635, 210}

$$
\begin{aligned}
&\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx \\
&= \frac{1}{2}(a+4)\arctan\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right) \\
&\quad+\frac{8(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{15\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}} \\
&\quad-\frac{2(3a+8)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{15\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}} \\
&\quad+\frac{1}{2}\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3} \\
&\quad+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3} \\
&\quad+\frac{2(3a+8)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}
\end{aligned}
$$

```
[In] Int[x^2*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ((1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/2 + (2*(8 + 3*a)
*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(15*Sqrt[3
+ a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 3*(-1 + x)^2)*Sqrt[3 + a - 2*(-1
+ x)^2 - (-1 + x)^4]*(-1 + x))/15 + ((4 + a)*ArcTan[(1 + (-1 + x)^2)/Sqrt[3
 + a - 2*(-1 + x)^2 - (-1 + x)^4]])/2 - (2*(8 + 3*a)*(1 - Sqrt[4 + a])*Sqrt
[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 +
 x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(15*Sqrt[(
```

```
1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (8*(3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 +
 (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 +
a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(15*Sqrt[(1 + (-1 + x)^2/(1 - Sq
rt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
```

```
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (1+x)^2\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int 2x\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int \left(1+x^2\right)\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$

$$= \frac{1}{15}\left(7+3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) - \frac{1}{15}\text{Subst}\left(\int \frac{-8(3+a)-2(8+3a)x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right) + 2\text{Subst}\left(\int x\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$

$$= \frac{1}{15}\left(7+3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) - \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{-8(3+a)-2(8+3a)x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{15\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \text{Subst}\left(\int \sqrt{3+a-2x-x^2}\,dx, x, (-1+x)^2\right)$$

$$= \frac{1}{2}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} + \frac{1}{15}\left(7+3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{1}{2}(4+a)\text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x-x^2}}\,dx, x, (-1+x)^2\right) + \frac{\left(8(3+a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{15\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\left(2(8+3a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{15\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$
\begin{aligned}
&= \frac{1}{2}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} \\
&\quad - \frac{2(8+3a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{15\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\
&\quad + \frac{1}{15}\left(7+3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) \\
&\quad - \frac{8(3+a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{15\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\
&\quad + (4+a)\text{Subst}\left(\int\frac{1}{-4-x^2}\,dx, x, -\frac{2\left(1+(-1+x)^2\right)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right) \\
&\quad - \frac{\left(2(8+3a)\left(1-\sqrt{4+a}\right)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}{\left(1-\frac{2x^2}{-2-2\sqrt{4+a}}\right)^{3/2}}\,dx, x, -1+\right.}{15\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} \\
&\quad - \frac{2(8+3a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{15\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\
&\quad + \frac{1}{15}\left(7+3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) \\
&\quad + \frac{1}{2}(4+a)\tan^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(1-x)^2-(1-x)^4}}\right) \\
&\quad + \frac{2(8+3a)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{15\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\
&\quad - \frac{8(3+a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{15\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5647 vs. 2(485) = 970.

Time = 13.84 (sec) , antiderivative size = 5647, normalized size of antiderivative = 11.64

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Result too large to show}$$

```
[In] Integrate[x^2*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2581 vs. 2(535) = 1070.

Time = 4.11 (sec) , antiderivative size = 2582, normalized size of antiderivative = 5.32

method	result	size
default	Expression too large to display	2582
elliptic	Expression too large to display	2582
risch	Expression too large to display	3044

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/5*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/10*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1
/2)+1/15*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+1/3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2
)-(-1/15*a-4/3)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4
)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(
a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/
2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)
^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+
4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1
/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))
^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(
1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-
1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-
(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))
/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/
2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(1/5*a+28/1
5)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2
)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(
1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(
```

a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2)
)/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/
2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(
-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(
1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2)
)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4
)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(
a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/
2))+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)
^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a
+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)))+(2/5*a+16/15)*((x-1
-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))
^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))
^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/
2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+
(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^
(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2)
)^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(
-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))
^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-
(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/2))*(
1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1/2))^
(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-(a+4)
^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2
))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1
+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1
/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/
2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+
(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(
-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^(1/2)

```
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1
/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1
/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
)/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(a+4
)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(
x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)
```

Fricas [F]

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x^2\,dx$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

Sympy [F]

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int x^2\sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
[Out] Integral(x**2*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Maxima [F]

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x^2\,dx$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x^2\,dx$$

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int x^2\,\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}\,dx$$

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.793 $\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}} dx$

Optimal result

Integrand size = 28, antiderivative size = 388

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx &= \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right)\\ &\quad-\frac{\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\\ &\quad+\frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\end{aligned}$$

```
[Out] arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))+(-1+x)*(1+(-1+x)^2/(1-
(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+(1/(1+(-1+x)^
2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+
x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)
/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)
/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/
(1+(4+a)^(1/2))))^(1/2)-(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/
(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(
1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(
1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x
)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2
)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1694, 1687, 1216, 545, 429, 506, 422, 12, 1121, 635, 210}

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$$
$$=\arctan\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[In] Int[x^2/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/Sqrt[3 + a
- 2*(-1 + x)^2 - (-1 + x)^4] + ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 +
 x)^2 - (-1 + x)^4]] - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 +
x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (
-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a])
)/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4]) + (Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])
/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a])
)]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1+x)^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \frac{2x}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1+x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)\\
&= 2\text{Subst}\left(\int \frac{x}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)\\
&\quad+ \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{1+x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\\
&= \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\\
&\quad+ \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\\
&\quad+ \text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x-x^2}}\,dx, x, (-1+x)^2\right)
\end{aligned}$$

$$= -\frac{(1-\sqrt{4+a})\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} + 2\text{Subst}\left(\int\frac{1}{-4-x^2}dx, x, -\frac{2(1+(-1+x)^2)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right) - \frac{\left((1-\sqrt{4+a})\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}{\left(1-\frac{2x^2}{-2-2\sqrt{4+a}}\right)^{3/2}}dx, x, -1+x\right)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= -\frac{(1-\sqrt{4+a})\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \tan^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(1-x)^2-(1-x)^4}}\right) + \frac{(1-\sqrt{4+a})\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1145 vs. $2(388) = 776$.

Time = 15.19 (sec) , antiderivative size = 1145, normalized size of antiderivative = 2.95

$$\int\frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$$

$$= \frac{\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)+\frac{2\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{}\right.}{}}{}$$

```
[In] Integrate[x^2/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 +
Sqrt[-1 + Sqrt[4 + a]] + x) + (2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)^2*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(1
```

```
+ Sqrt[-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - S
qrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[-
((Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)))]*((1
 + Sqrt[-1 - Sqrt[4 + a]]*Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[Sqrt[((S
qrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]
] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - S
qrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(S
qrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] - (1 + 2*Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 - Sqrt[4 + a]]*Sqrt[-1 + Sqrt[4 + a]])*EllipticF[ArcSin[S
qrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt
[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt
[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 4*Sqrt[-1 - Sqr
t[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sq
rt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sq
rt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]]
- x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(1 + Sqrt[4 + a] + Sqrt[-1 - Sqrt[
4 + a]]*Sqrt[-1 + Sqrt[4 + a]]))/Sqrt[a - x*(-8 + 8*x - 4*x^2 + x^3)]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. $2(454) = 908$.

Time = 1.44 (sec) , antiderivative size = 1147, normalized size of antiderivative = 2.96

method	result	size
default	Expression too large to display	1147
elliptic	Expression too large to display	1147

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(
1/2))^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(
1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4
)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*
(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1
/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^
(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2)
)/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1
/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1
```

```
-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/
2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1
/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-
(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(
-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))
^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^
(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/
2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)
^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2
)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^
(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2
))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4
)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^
(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2
))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2
))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^
(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1
+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/
2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^4 - 4*x^3 + 8*x^2 - a
 - 8*x), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x^2}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}}\,dx$$

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.794 $\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 311

$$\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx = \frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{2(3+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(3+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

```
[Out] (1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/2*(4+a)*(2+(-1+x)^2)*(
-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/2*(-1+x)*(1+(-1+x)^2/(
1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/2*(
1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*El
lipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2
*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^
(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+
x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1694, 1687, 1192, 12, 1154, 506, 422, 1121, 627}

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+3)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{(x-1)^2+1}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}} - \frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (1 + (-1 + x)^2)/((4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((4 +
a)*(2 + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2
 - (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1
 + x))/(2*(3 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1 - Sqrt[4 +
 a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[Arc
Tan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(
2*(3 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt
[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1154

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(
b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[x^2/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqr
t[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && NegQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(1+x)^2}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{2x}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1+x^2}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)$$

$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + 2\text{Subst}\left(\int \frac{x}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right) - \frac{\text{Subst}\left(\int \frac{2(4+a)x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)}{4\left(12+7a+a^2\right)}$$

$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)}{2(3+a)} + \text{Subst}\left(\int \frac{1}{(3+a-2x-x^2)^{3/2}}\,dx, x, (-1+x)^2\right)$$

$$= \frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{2(3+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$=\frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2(3+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}}+\frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{\left(\left(1-\sqrt{4+a}\right)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}{\left(1-\frac{2x^2}{-2-2\sqrt{4+a}}\right)^{3/2}}dx,x,-1+x\right)}{2(3+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$=\frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2(3+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}}+\frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}-\frac{\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(3+a)\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2941 vs. $2(311) = 622$.

Time = 14.51 (sec) , antiderivative size = 2941, normalized size of antiderivative = 9.46

$$\int\frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx=\text{Result too large to show}$$

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] ((-a - 8*x - a*x + 6*x^2 + a*x^2 - 4*x^3 - a*x^3)*(a + 8*x - 8*x^2 + 4*x^3
- x^4)^2)/(2*(3 + a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)*(a - x*(-8 +
8*x - 4*x^2 + x^3))^(3/2)) - ((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2)*((2*(-S
qrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]
] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqr
t[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sq
rt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + S
```

qrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a
]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/(
(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 +
a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
+ Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqr
t[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt
[a + 8*x - 8*x^2 + 4*x^3 - x^4]) - (4*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))
]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)
)]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x
))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(
Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-
1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + ((-1 + Sqrt[-1 - S
qrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 + Sqrt[-1 + Sqrt[4 +
a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[
-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 +
a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4
+ a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4
+ a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-
1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*(((Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a
]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]]
, (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a
]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]) + ((-((-1 - Sqr
t[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])
) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[
4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4

```
 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[
-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 +
 a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4
 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
 a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]]
+ x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqr
t[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(-Sqrt[-1 - Sqrt[4 + a]]
+ Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]))/(2*(3 + a
)*(a - x*(-8 + 8*x - 4*x^2 + x^3))^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2606 vs. 2(331) = 662.

Time = 1.28 (sec) , antiderivative size = 2607, normalized size of antiderivative = 8.38

method	result	size
default	Expression too large to display	2607
elliptic	Expression too large to display	2607

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/4/(3+a)*x^3-1/4*(6+a)/(a^2+7*a+12)*x^2+1/4*(a+8)/(a^2+7*a+12)*x+1/4/(a
^2+7*a+12)*a)/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/(a^2+7*a+12)-1/2*(a+8)/(a^2
+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2
))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(
1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-
1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2)
)^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/
2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(
-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)
))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^
(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-
(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1
-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(
1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1
/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
)/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(-2/(a^2+7*a+12)+
(6+a)/(a^2+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-
(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(
-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))
```

```
^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(
a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1
+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2)
)^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1
/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+
4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))
*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(
a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/
2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^
(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a
+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(
1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1
/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(
a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(
a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)))-1/
2/(3+a)*((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1
-(a+4)^(1/2))^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2))
)^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-
(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-
1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2
))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(
1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(
1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1
/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-
(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF
(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1
/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))
^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)
^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1
/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(
-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))
^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/
2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+
4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-
(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2
))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(
1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-
```

```
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a
+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)
^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-
(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1
/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)
```

Fricas [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^8 - 8*x^7 + 32*x^6 - 2
*(a - 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x
)
```

Sympy [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x^2}{(a-x^4+4x^3-8x^2+8x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
[Out] Integral(x**2/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{3/2}}\,dx$$

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.795 $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 582

$$\begin{aligned}
\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx &= \frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} \\
&+ \frac{2(1+(-1+x)^2)}{3(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&+ \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} \\
&+ \frac{\left(29+7a+(13+3a)(-1+x)^2\right)(-1+x)}{12(3+a)^2(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&- \frac{(13+3a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{12(3+a)^2(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&+ \frac{(13+3a)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{12(3+a)^2(4+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&+ \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{12\left(12+7a+a^2\right)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}
\end{aligned}$$

```
[Out] 1/3*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/6*(4+a)*(2+(-1+x)^
2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+2/3*(1+(-1+x)^2)/(4+
a)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/12*(29+7*a+(13+3*a)*(-1+x)^2)*(-1+x)
/(3+a)^2/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/12*(13+3*a)*(-1+x)*(1+(-1+
```

```
x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a)^2/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^
4)^(1/2)+1/12*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(
1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1
/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1
/2)))*(1+(4+a)^(1/2))^(1/2)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((
1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+1/12*(13+3*
a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2
)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)
,(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4
+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a)^2/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1
/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1694, 1687, 1192, 1216, 545, 429, 506, 422, 12, 1121, 628, 627}

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx=\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12\left(a^2+7a+12\right)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{(3a+13)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12(a+3)^2(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{2\left((x-1)^2+1\right)}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{(x-1)\left((3a+13)(x-1)^2+7a+29\right)}{12(a+3)^2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{(3a+13)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{12(a+3)^2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

```
[Out] (1 + (-1 + x)^2)/(3*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + (2
*(1 + (-1 + x)^2))/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) +
((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)
^2 - (-1 + x)^4)^(3/2)) + ((29 + 7*a + (13 + 3*a)*(-1 + x)^2)*(-1 + x))/(12
*(3 + a)^2*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((13 + 3*a)*(
```

```
1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(12*(3 + a)^2
*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((13 + 3*a)*(1 - Sqrt[4
 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])
/(12*(3 + a)^2*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x
)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[1
+ Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)
/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(12*(12 + 7*a
 + a^2)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4
 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
```

```
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
```

```
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(1+x)^2}{(3+a-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{2x}{(3+a-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1+x^2}{(3+a-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right)$$

$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + 2\text{Subst}\left(\int \frac{x}{(3+a-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right) - \frac{\text{Subst}\left(\int \frac{-8(4+a)-6(4+a)x^2}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)}{12\left(12+7a+a^2\right)}$$

$$= -\frac{\left(29+7a+(13+3a)(1-x)^2\right)(1-x)}{12(3+a)^2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{4(3+a)(4+a)-4(4+a)(13+3a)x^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)}{48\left(12+7a+a^2\right)^2} + \text{Subst}\left(\int \frac{1}{(3+a-2x-x^2)^{5/2}}\,dx, x, (-1+x)^2\right)$$

$$= \frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{\left(29+7a+(13+3a)(1-x)^2\right)(1-x)}{12(3+a)^2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{(3+a-2x-x^2)^{3/2}}\,dx, x, (-1+x)^2\right)}{3(4+a)} + \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{4(3+a)(4+a)-4(4+a)(13+3a)x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+x\right)}{48\left(12+7a+a^2\right)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{2\left(1+(-1+x)^2\right)}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{\left(29+7a+(13+3a)(1-x)^2\right)(1-x)}{12(3+a)^2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{\left((3+a)(4+a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1+\right.}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\left((4+a)(13+3a)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{2x^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}\,dx, x, -1\right.}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{2(1+(-1+x)^2)}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{(29+7a+(13+3a)(1-x)^2)(1-x)}{12(3+a)^2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(13+3a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{12(3+a)^2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{12\left(12+7a+a^2\right)\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left((4+a)(13+3a)\left(1-\sqrt{4+a}\right)\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2x^2}{-2+2\sqrt{4+a}}}}{\left(1-\frac{2x^2}{-2-2\sqrt{4+a}}\right)^{3/2}}dx,\right.}{12\left(12+7a+a^2\right)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

$$= \frac{2(1+(-1+x)^2)}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{(29+7a+(13+3a)(1-x)^2)(1-x)}{12(3+a)^2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(13+3a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{12(3+a)^2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{(13+3a)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{12(3+a)^2(4+a)\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{12\left(12+7a+a^2\right)\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5812 vs. 2(582) = 1164.

Time = 17.14 (sec) , antiderivative size = 5812, normalized size of antiderivative = 9.99

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}} \, dx = \text{Result too large to show}$$

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2779 vs. 2(628) = 1256.

Time = 1.51 (sec) , antiderivative size = 2780, normalized size of antiderivative = 4.78

method	result	size
default	Expression too large to display	2780
elliptic	Expression too large to display	2780

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x,method=_RETURNVERBOSE)
[Out] (1/6/(3+a)*x^3-1/6*(6+a)/(a^2+7*a+12)*x^2+1/6*(a+8)/(a^2+7*a+12)*x+1/6/(a^2
+7*a+12)*a)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2*(1/2
4*(13+3*a)/(3+a)/(a^2+7*a+12)*x^3-1/24*(a^2+27*a+84)/(a^2+7*a+12)^2*x^2+1/6
*(9*a+32)/(a^2+7*a+12)^2*x+1/12*(3*a^2+7*a-12)/(a^2+7*a+12)^2)/(-x^4+4*x^3-
8*x^2+a+8*x)^(1/2)-(-1/6*(a^2-9*a-44)/(a^2+7*a+12)^2-1/3*(9*a+32)/(a^2+7*a+
12)^2)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^
(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(
-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(
1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))
^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^
(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/
2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+
4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)
^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(
(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(1/3*(a^2-a-16)/(a^2
+7*a+12)^2+1/6*(a^2+27*a+84)/(a^2+7*a+12)^2)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a
```

+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-
1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1
+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4
)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+
4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(
1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))
^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-
1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2)
)^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(
1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-
(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)
^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*Elli
pticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/
2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^
(1/2))^(1/2)))^(1/2),(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^
(1/2))^(1/2)))^(1/2)))-1/12*(13+3*a)/(3+a)/(a^2+7*a+12)*((x-1-(-1+(a+4)^(1/
2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))+((-1-(
a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+
4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+
(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2
))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-1/2*(
(1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))-(1-(-1-(a+4)^(1/2))^(
1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^(1/2))^(1/2))*(1-(-1+(a+4)^(1
/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+
(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/
2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)
^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/
2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(
a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/
2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^
(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a
+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(
1/2))^(1/2)))^(1/2))/(-1+(a+4)^(1/2))^(1/2)-4/(-(-1-(a+4)^(1/2))^(1/2)+(-1+

```
(a+4)^(1/2))^(1/2))*EllipticPi(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2)
)^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)),((-(-1-(a
+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^
(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(
1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(a+4)^(1/2))^(1/2)
)*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^
(1/2))^(1/2)))^(1/2)
```

Fricas [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^12 - 12*x^11 + 72*x^1
0 - 3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5
*a - 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^3
 - 24*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

Sympy [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x^2}{(a-x^4+4x^3-8x^2+8x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
[Out] Integral(x**2/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{5/2}}\,dx$$

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
[Out] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

3.796 $\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 129

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = -\frac{x^2\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)^2}}\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{4+x}{\sqrt{3}\sqrt[4]{29}x}\right),\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{8\sqrt{3}\sqrt[4]{29}\sqrt{8+8x-x^3+8x^4}}$$

```
[Out] -1/696*x^2*(cos(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))^2)^(1/2)/cos(2*arc
tan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))*EllipticF(sin(2*arctan(1/87*(4+x)*29^(3
/4)/x*3^(1/2))),1/58*(1682+58*29^(1/2))^(1/2))*(87+(4+x)^2*29^(1/2)/x^2)*((
261-6*(1+4/x)^2+(1+4/x)^4)/(87+(4+x)^2*29^(1/2)/x^2)^2)^(1/2)*29^(3/4)*3^(1
/2)/(8*x^4-x^3+8*x+8)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2094, 12, 6851, 1117}

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = -\frac{x^2\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)\text{EllipticF}\left(2\arctan\left(\frac{x+4}{\sqrt{3}\sqrt[4]{29}x}\right),\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{8\sqrt{3}\sqrt[4]{29}\sqrt{8x^4-x^3+8x+8}}$$

```
[In] Int[1/Sqrt[8 + 8*x - x^3 + 8*x^4],x]
```

```
[Out] -1/8*(x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4 + x)^
2)/x^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*EllipticF[2*ArcTan[(4 + x)/(Sqrt
[3]*29^(1/4)*x)], (29 + Sqrt[29])/58])/(Sqrt[3]*29^(1/4)*Sqrt[8 + 8*x - x^3
 + 8*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\text{integral} = -\left(1024\text{Subst}\left(\int \frac{1}{2\sqrt{2}(8-32x)^2\sqrt{\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}}}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(\left(256\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(8-32x)^2\sqrt{\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}}}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1069056-393216x^2+1048576x^4}}\,dx,\right.}{\sqrt{8+8x-x^3+8x^4}}$$

$$= -\frac{x^2\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)^2}}\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)F\left(2\tan^{-1}\left(\frac{4+x}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{8\sqrt{3}\sqrt[4]{29}\sqrt{8+8x-x^3+8x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 10.70 (sec) , antiderivative size = 927, normalized size of antiderivative = 7.19

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = -\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(x-\text{Root}[8\#1^4-\#1^3+8\#1+8\&,1])(\text{Root}[8\#1^4-\#1^3+8\#1+8\&,2]-\text{Root}[8\#1^4-\#1^3+8\#}{(x-\text{Root}[8\#1^4-\#1^3+8\#1+8\&,2])(\text{Root}[8\#1^4-\#1^3+8\#1+8\&,1]-\text{Root}[8\#1^4-\#1^3+8\#}}\right.\right.}{}$$

```
[In] Integrate[1/Sqrt[8 + 8*x - x^3 + 8*x^4],x]
```

```
[Out] (-2*EllipticF[ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], ((Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0
])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4
 & , 4, 0]))/((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2,
 0])^2*Sqrt[((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x -
Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))]*(Root[8 + 8*#1 - #1^3 +
 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])*Sqrt[((x - Roo
t[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1,
 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 4, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1
- #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^
2*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4
& , 4, 0])^2)]])/(Sqrt[8 + 8*x - x^3 + 8*x^4]*(-Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.13 (sec) , antiderivative size = 965, normalized size of antiderivative = 7.48

method	result	size
default	Expression too large to display	965
elliptic	Expression too large to display	965

```
[In] int(1/(8*x^4-x^3+8*x+8)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))
*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(
x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-R
ootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^
(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,i
ndex=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*
_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_
Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z^4-_Z^
3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+8
*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^
4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*
_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2)*El
lipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=3))*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,in
dex=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index
=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)
))^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}}\,dx$$

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8x^4-x^3+8x+8}}\,dx$$

```
[In] integrate(1/(8*x**4-x**3+8*x+8)**(1/2),x)
[Out] Integral(1/sqrt(8*x**4 - x**3 + 8*x + 8), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}}\,dx$$

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}}\,dx$$

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}}\,dx$$

```
[In] int(1/(8*x - x^3 + 8*x^4 + 8)^(1/2),x)
[Out] int(1/(8*x - x^3 + 8*x^4 + 8)^(1/2), x)
```

3.797 $\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 431

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = -\frac{\left(66-\left(1+\frac{4}{x}\right)^2\right)x^2}{1008\sqrt{8+8x-x^3+8x^4}} + \frac{\left(216-7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}} + \frac{7\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)\left(1+\frac{4}{x}\right)x^2}{432\sqrt{29}\sqrt{8+8x-x^3+8x^4}\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)} - \frac{7x^2\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)^2}}\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)E\left(2\arctan\left(\frac{4+x}{\sqrt{3}\sqrt[4]{29}x}\right)\middle|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{144\sqrt{3}29^{3/4}\sqrt{8+8x-x^3+8x^4}} + \frac{\left(14-5\sqrt{29}\right)x^2\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)^2}}\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{4+x}{\sqrt{3}\sqrt[4]{29}x}\right),\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{576\sqrt{3}29^{3/4}\sqrt{8+8x-x^3+8x^4}}$$

```
[Out] -1/1008*(66-(1+4/x)^2)*x^2/(8*x^4-x^3+8*x+8)^(1/2)+1/12528*(216-7*(1+4/x)^2
)*(1+4/x)*x^2/(8*x^4-x^3+8*x+8)^(1/2)+7/12528*(261-6*(1+4/x)^2+(1+4/x)^4)*(
1+4/x)*x^2*29^(1/2)/(87+(4+x)^2*29^(1/2)/x^2)/(8*x^4-x^3+8*x+8)^(1/2)-7/125
28*x^2*(cos(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))^2)^(1/2)/cos(2*arctan(
1/87*(4+x)*29^(3/4)/x*3^(1/2)))*EllipticE(sin(2*arctan(1/87*(4+x)*29^(3/4)/
x*3^(1/2))),1/58*(1682+58*29^(1/2))^(1/2))*(87+(4+x)^2*29^(1/2)/x^2)*((261-
6*(1+4/x)^2+(1+4/x)^4)/(87+(4+x)^2*29^(1/2)/x^2)^2)^(1/2)*29^(1/4)*3^(1/2)/
(8*x^4-x^3+8*x+8)^(1/2)+1/50112*x^2*(cos(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(
1/2)))^2)^(1/2)/cos(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))*EllipticF(sin(
2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2))),1/58*(1682+58*29^(1/2))^(1/2))*(14
-5*29^(1/2))*(87+(4+x)^2*29^(1/2)/x^2)*((261-6*(1+4/x)^2+(1+4/x)^4)/(87+(4+
x)^2*29^(1/2)/x^2)^2)^(1/2)*29^(1/4)*3^(1/2)/(8*x^4-x^3+8*x+8)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {2094, 12, 6851, 1687, 1692, 1211, 1117, 1209, 1261, 650}

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \frac{\left(14-5\sqrt{29}\right)\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)x^2\,\text{EllipticF}\left(2\arctan\left(\frac{x+4}{\sqrt{3}\sqrt[4]{29}x}\right),\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{576\sqrt{3}29^{3/4}\sqrt{8x^4-x^3+8x+8}} - \frac{7\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)x^2E\left(2\arctan\left(\frac{x+4}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{144\sqrt{3}29^{3/4}\sqrt{8x^4-x^3+8x+8}} - \frac{\left(66-\left(\frac{4}{x}+1\right)^2\right)x^2}{1008\sqrt{8x^4-x^3+8x+8}} + \frac{\left(216-7\left(\frac{4}{x}+1\right)^2\right)\left(\frac{4}{x}+1\right)x^2}{12528\sqrt{8x^4-x^3+8x+8}} + \frac{7\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)\left(\frac{4}{x}+1\right)x^2}{432\sqrt{29}\sqrt{8x^4-x^3+8x+8}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)}$$

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] -1/1008*((66 - (1 + 4/x)^2)*x^2)/Sqrt[8 + 8*x - x^3 + 8*x^4] + ((216 - 7*(1
 + 4/x)^2)*(1 + 4/x)*x^2)/(12528*Sqrt[8 + 8*x - x^3 + 8*x^4]) + (7*(261 - 6
*(1 + 4/x)^2 + (1 + 4/x)^4)*(1 + 4/x)*x^2)/(432*Sqrt[29]*Sqrt[8 + 8*x - x^3
 + 8*x^4]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)) - (7*x^2*Sqrt[(261 - 6*(1 + 4/x)
^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4 + x)^2)/x^2)^2]*(87 + (Sqrt[29]*(4 + x
)^2)/x^2)*EllipticE[2*ArcTan[(4 + x)/(Sqrt[3]*29^(1/4)*x)], (29 + Sqrt[29])
/58])/(144*Sqrt[3]*29^(3/4)*Sqrt[8 + 8*x - x^3 + 8*x^4]) + ((14 - 5*Sqrt[29
])*x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4 + x)^2)/
x^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*EllipticF[2*ArcTan[(4 + x)/(Sqrt[3]
*29^(1/4)*x)], (29 + Sqrt[29])/58])/(576*Sqrt[3]*29^(3/4)*Sqrt[8 + 8*x - x^
3 + 8*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
```

```
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\text{integral} = -\left(1024\text{Subst}\left(\int \frac{1}{16\sqrt{2}(8-32x)^2\left(\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}\right)^{3/2}}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(\left(32\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(8-32x)^2\left(\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}\right)^{3/2}}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{(8-32x)^4}{(1069056-393216x^2+1048576x^4)^{3/2}}\,dx\right.}{8\sqrt{8+8x-x^3+8x^4}}$$

$$= -\frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{x(-65536-1048576x^2)}{(1069056-393216x^2+1048576x^4)^{3/2}}\,dx\right.}{8\sqrt{8+8x-x^3+8x^4}}$$

$$-\frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{4096+393216x^2+1048576x^4}{(1069056-393216x^2+1048576x^4)^{3/2}}\,dx\right.}{8\sqrt{8+8x-x^3+8x^4}}$$

$$= \frac{\left(216 - 7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}} - \frac{\left(\sqrt{1069056 - 393216\left(\frac{1}{4}+\frac{1}{x}\right)^2 + 1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{2571273912251842560-1324058290446}{\sqrt{1069056-393216x^2+1048576}}\right.}{37026344336426532864\sqrt{8+8x-x^3+8x^4}} - \frac{\left(\sqrt{1069056 - 393216\left(\frac{1}{4}+\frac{1}{x}\right)^2 + 1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{-65536-1048576x}{(1069056-393216x+1048576x^2)^{3/2}}dx\right.}{16\sqrt{8+8x-x^3+8x^4}}$$

$$= -\frac{\left(66-\left(1+\frac{4}{x}\right)^2\right)x^2}{1008\sqrt{8+8x-x^3+8x^4}} + \frac{\left(216 - 7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}} - \frac{\left(7\sqrt{1069056 - 393216\left(\frac{1}{4}+\frac{1}{x}\right)^2 + 1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{1-\frac{16x^2}{3\sqrt{29}}}{\sqrt{1069056-393216x^2+1048576x^4}}dx\right.}{36\sqrt{29}\sqrt{8+8x-x^3+8x^4}} - \frac{\left(\left(145-14\sqrt{29}\right)\sqrt{1069056 - 393216\left(\frac{1}{4}+\frac{1}{x}\right)^2 + 1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1069056-39321}}\right.}{2088\sqrt{8+8x-x^3+8x^4}}$$

$$= -\frac{\left(66-\left(1+\frac{4}{x}\right)^2\right)x^2}{1008\sqrt{8+8x-x^3+8x^4}} + \frac{\left(216 - 7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}} + \frac{7\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)\left(1+\frac{4}{x}\right)x^2}{432\sqrt{29}\sqrt{8+8x-x^3+8x^4}\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)} - \frac{7x^2\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)^2}}\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)E\left(2\tan^{-1}\left(\frac{4+x}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{144\sqrt{3}29^{3/4}\sqrt{8+8x-x^3+8x^4}} + \frac{\left(14-5\sqrt{29}\right)x^2\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)^2}}\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)F\left(2\tan^{-1}\left(\frac{4+x}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{576\sqrt{3}29^{3/4}\sqrt{8+8x-x^3+8x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 16.15 (sec) , antiderivative size = 4865, normalized size of antiderivative = 11.29

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] (544 + 1539*x - 1146*x^2 + 784*x^3)/(21924*Sqrt[8 + 8*x - x^3 + 8*x^4]) + (
(28*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*(-(EllipticF[ArcSin[Sqr
t[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((-Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]
)*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4
& , 4, 0])))]*Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]) + EllipticPi[(-Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0
])/(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 4, 0]), ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Roo
t[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3,
 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 4, 0]))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 -
 #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])))]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1
, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]))*Sqrt[((-Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 &
, 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 +
 8*#1^4 & , 3, 0]))]*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 4, 0])*Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*
Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0
] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))])/(Sqrt[8 + 8*x - x^3 + 8*x^4
]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4
 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3
 + 8*#1^4 & , 4, 0])) + (842*EllipticF[ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 1, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 +
 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2
, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 4, 0]))]], ((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Ro
ot[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8
```

#1^4 & , 2, 0] - Root[8 + 8#1 - #1^3 + 8*#1^4 & , 4, 0]))]*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))]*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])*Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))])/(Sqrt[8 + 8*x - x^3 + 8*x^4]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])) - (224*((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]) + (x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))]*Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])*((EllipticE[ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])))]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]) + (EllipticF[ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 -

```
#1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] -
Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((-Root[8 + 8*#1 - #1^3
+ 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#
1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])))]*(
Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]) - Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 1, 0]*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 -
 #1^3 + 8*#1^4 & , 4, 0])))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Ro
ot[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])) - (EllipticPi[(-Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])/(
-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 &
, 4, 0]), ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0
]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8
+ 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])
*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 &
 , 4, 0]))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 4, 0])))]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0]
 - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4
& , 3, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/(-Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))))/Sqrt[8 +
 8*x - x^3 + 8*x^4])/6264
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.70 (sec) , antiderivative size = 4426, normalized size of antiderivative = 10.27

method	result	size
default	Expression too large to display	4426
risch	Expression too large to display	4426
elliptic	Expression too large to display	4426

```
[In] int(1/(8*x^4-x^3+8*x+8)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16*(-17/10962-57/12992*x+191/58464*x^2-7/3132*x^3)/(8*x^4-x^3+8*x+8)^(1/2)
+421/12528*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,in
dex=4))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
```

x=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))+7/6264*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2)*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)*EllipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))+(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*EllipticPi(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1

/2),(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))
/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)),((
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(Roo
tOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf
(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*
_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2)))-7/78
3*((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))+(RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index
=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=
1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))
/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+
8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8
_Z+8,index=1))(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+
8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8
_Z+8,index=2)))^(1/2)((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)*RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)*RootOf(8*_Z^4-_Z^3+8
*_Z+8,index=1)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)*RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)^2)/(RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*EllipticF(((RootOf(8*_Z^4-_Z^3+8*
_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_
Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(RootOf(8*_Z^4-_Z^3+
8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_
Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))+(RootOf(8*_Z^4-_Z^3+8
*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*EllipticE(((RootOf(8*_Z^
4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(
8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(RootOf(8*_
Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))/(RootOf(8*_Z
^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))-1/8/(RootOf(8*_
Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*EllipticPi(((R
ootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-Ro
otOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2
),(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(

```
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)),((Ro
otOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8
*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z
^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))))*2^(1/
2)/((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=4)))^(1/2)
```

Fricas [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-x^3+8\,x+8)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(8*x^4 - x^3 + 8*x + 8)/(64*x^8 - 16*x^7 + x^6 + 128*x^5 + 112
*x^4 - 16*x^3 + 64*x^2 + 128*x + 64), x)
```

Sympy [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8x^4-x^3+8x+8)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(8*x**4-x**3+8*x+8)**(3/2),x)
[Out] Integral((8*x**4 - x**3 + 8*x + 8)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-x^3+8\,x+8)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="maxima")
[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-x^3+8\,x+8)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="giac")
[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-x^3+8\,x+8)^{3/2}}\,dx$$

```
[In] int(1/(8*x - x^3 + 8*x^4 + 8)^(3/2),x)
[Out] int(1/(8*x - x^3 + 8*x^4 + 8)^(3/2), x)
```

3.798 $\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = -\frac{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}x^2\,\text{EllipticF}\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right),\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{1+4x+4x^2+4x^4}}$$

```
[Out] -1/10*x^2*(cos(2*arctan(1/5*(1+1/x)*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*(1+
1/x)*5^(3/4)))*EllipticF(sin(2*arctan(1/5*(1+1/x)*5^(3/4))),1/10*(50+10*5^(
1/2))^(1/2))*((1+1/x)^2+5^(1/2))*((5-2*(1+1/x)^2+(1+1/x)^4)/((1+1/x)^2+5^(1
/2))^2)^(1/2)*5^(3/4)/(4*x^4+4*x^2+4*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2094, 6851, 1117}

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = -\frac{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}x^2\,\text{EllipticF}\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right),\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{4x^4+4x^2+4x+1}}$$

```
[In] Int[1/Sqrt[1 + 4*x + 4*x^2 + 4*x^4],x]
```

```
[Out] -1/2*((Sqrt[5] + (1 + x^(-1))^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^
4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*x^2*EllipticF[2*ArcTan[(1 + x^(-1))/5^(1/4
)], (5 + Sqrt[5])/10])/(5^(1/4)*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\text{integral} = -\left(16\text{Subst}\left(\int \frac{1}{(4-4x)^2\sqrt{\frac{1280-512x^2+256x^4}{(4-4x)^4}}}\,dx, x, 1+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1280-512x^2+256x^4}}\,dx, x, 1+\frac{1}{x}\right)}{\sqrt{1+4x+4x^2+4x^4}}$$

$$= -\frac{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{1+4x+4x^2+4x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.38 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.31

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = \frac{(2-i)\sqrt{-\frac{1}{10}+\frac{i}{5}}\sqrt{\frac{\left(2i+\sqrt{-1-2i}-\sqrt{-1+2i}\right)\left(-i+\sqrt{-1-2i}-2x\right)}{\left(-2i+\sqrt{-1-2i}+\sqrt{-1+2i}\right)\left(i+\sqrt{-1-2i}+2x\right)}}\left(1+2x+2ix^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(2i+\sqrt{-1-2i}+\sqrt{-1+\ldots}}{\sqrt{-1+2i}(i+\ldots}}}{\sqrt{\ldots}}\right.\right.}{\sqrt{\frac{(1+2i)\left((-1+i)+\sqrt{-1-2i}\right)\left(1+2x+2ix^2\right)}{\left(i+\sqrt{-1-2i}+2x\right)^2}}\sqrt{1+4x+4x^2+4x^4}}$$

```
[In] Integrate[1/Sqrt[1 + 4*x + 4*x^2 + 4*x^4],x]
```

```
[Out] ((2 - I)*Sqrt[-1/10 + I/5]*Sqrt[((2*I + Sqrt[-1 - 2*I] - Sqrt[-1 + 2*I])*(-
I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(I + S
qrt[-1 - 2*I] + 2*x))]*(1 + 2*x + (2*I)*x^2)*EllipticF[ArcSin[Sqrt[((2*I +
Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[-1 + 2*I] + 2*x))/(Sqrt[-1 + 2*
I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5 - Sqrt[5])/2])/(Sqrt[((1 + 2*I
)*((-1 + I) + Sqrt[-1 - 2*I])*(1 + 2*x + (2*I)*x^2))/(I + Sqrt[-1 - 2*I] +
2*x)^2]*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.83 (sec) , antiderivative size = 961, normalized size of antiderivative = 8.90

method	result	size
default	Expression too large to display	961
elliptic	Expression too large to display	961

```
[In] int(1/(4*x^4+4*x^2+4*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))
*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2
))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,in
dex=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1
,index=2)))^(1/2)*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z^
4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*
_Z^4+4*_Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-RootOf(4
*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/
2)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index
=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,
```

```
index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z
+1,index=2)))^(1/2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_
Z^2+4*_Z+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=1))/((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4
*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(x-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)*EllipticF(((RootOf(4*_Z^4+4*_Z^
2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((Ro
otOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))/
(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))
)^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4x^4+4x^2+4x+1}}\,dx$$

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1)**(1/2),x)
[Out] Integral(1/sqrt(4*x**4 + 4*x**2 + 4*x + 1), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

```
[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(1/2),x)
[Out] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(1/2), x)
```

3.799 $\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 367

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}} \, dx = -\frac{\left(3-\left(1+\frac{1}{x}\right)^2\right)x^2}{\sqrt{1+4x+4x^2+4x^4}} + \frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}} + \frac{9\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)\left(1+\frac{1}{x}\right)x^2}{10\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{1+4x+4x^2+4x^4}} - \frac{9\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}x^2 E\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\,5^{3/4}\sqrt{1+4x+4x^2+4x^4}} + \frac{3\left(3-\sqrt{5}\right)\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}x^2\,\text{EllipticF}\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right),\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{4\,5^{3/4}\sqrt{1+4x+4x^2+4x^4}}$$

```
[Out] -(3-(1+1/x)^2)*x^2/(4*x^4+4*x^2+4*x+1)^(1/2)+1/10*(13-9*(1+1/x)^2)*(1+1/x)*
x^2/(4*x^4+4*x^2+4*x+1)^(1/2)+9/10*(5-2*(1+1/x)^2+(1+1/x)^4)*(1+1/x)*x^2/((
1+1/x)^2+5^(1/2))/(4*x^4+4*x^2+4*x+1)^(1/2)-9/10*x^2*(cos(2*arctan(1/5*(1+1
/x)*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*(1+1/x)*5^(3/4)))*EllipticE(sin(2*a
rctan(1/5*(1+1/x)*5^(3/4))),1/10*(50+10*5^(1/2))^(1/2))*((1+1/x)^2+5^(1/2))
*((5-2*(1+1/x)^2+(1+1/x)^4)/((1+1/x)^2+5^(1/2))^2)^(1/2)*5^(1/4)/(4*x^4+4*x
^2+4*x+1)^(1/2)+3/20*x^2*(cos(2*arctan(1/5*(1+1/x)*5^(3/4)))^2)^(1/2)/cos(2
*arctan(1/5*(1+1/x)*5^(3/4)))*EllipticF(sin(2*arctan(1/5*(1+1/x)*5^(3/4))),
1/10*(50+10*5^(1/2))^(1/2))*(3-5^(1/2))*((1+1/x)^2+5^(1/2))*((5-2*(1+1/x)^2
+(1+1/x)^4)/((1+1/x)^2+5^(1/2))^2)^(1/2)*5^(1/4)/(4*x^4+4*x^2+4*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {2094, 6851, 1687, 1692, 1211, 1117, 1209, 1261, 650}

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \frac{3(3-\sqrt{5})\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}x^2\,\text{EllipticF}\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right),\ldots\right)}{4\ 5^{3/4}\sqrt{4x^4+4x^2+4x+1}}$$
$$-\frac{9\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}x^2E\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}(5+\sqrt{5})\right)}{2\ 5^{3/4}\sqrt{4x^4+4x^2+4x+1}}$$
$$-\frac{\left(3-\left(\frac{1}{x}+1\right)^2\right)x^2}{\sqrt{4x^4+4x^2+4x+1}}+\frac{\left(13-9\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)x^2}{10\sqrt{4x^4+4x^2+4x+1}}$$
$$+\frac{9\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)\left(\frac{1}{x}+1\right)x^2}{10\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{4x^4+4x^2+4x+1}}$$

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-3/2),x]
```

```
[Out] -(((3 - (1 + x^(-1))^2)*x^2)/Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) + ((13 - 9*(1 +
 x^(-1))^2)*(1 + x^(-1))*x^2)/(10*Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) + (9*(5 -
2*(1 + x^(-1))^2 + (1 + x^(-1))^4)*(1 + x^(-1))*x^2)/(10*(Sqrt[5] + (1 + x^
(-1))^2)*Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) - (9*(Sqrt[5] + (1 + x^(-1))^2)*Sqr
t[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*x^2
*EllipticE[2*ArcTan[(1 + x^(-1))/5^(1/4)], (5 + Sqrt[5])/10])/(2*5^(3/4)*Sq
rt[1 + 4*x + 4*x^2 + 4*x^4]) + (3*(3 - Sqrt[5])*(Sqrt[5] + (1 + x^(-1))^2)*
Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*
x^2*EllipticF[2*ArcTan[(1 + x^(-1))/5^(1/4)], (5 + Sqrt[5])/10])/(4*5^(3/4)
*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\text{integral} = -\left(16\text{Subst}\left(\int \frac{1}{(4-4x)^2\left(\frac{1280-512x^2+256x^4}{(4-4x)^4}\right)^{3/2}}\,dx, x, 1+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{(4-4x)^4}{(1280-512x^2+256x^4)^{3/2}}\,dx, x, 1+\frac{1}{x}\right)}{\sqrt{1+4x+4x^2+4x^4}}$$

$$= -\frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{x(-1024-1024x^2)}{(1280-512x^2+256x^4)^{3/2}}\,dx, x, 1+\frac{1}{x}\right)}{\sqrt{1+4x+4x^2+4x^4}} - \frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{256+1536x^2+256x^4}{(1280-512x^2+256x^4)^{3/2}}\,dx, x, 1+\frac{1}{x}\right)}{\sqrt{1+4x+4x^2+4x^4}}$$

$$= \frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}} - \frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{2013265920-1207959552x^2}{\sqrt{1280-512x^2+256x^4}}\,dx, x, 1+\frac{1}{x}\right)}{1342177280\sqrt{1+4x+4x^2+4x^4}} - \frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{-1024-1024x}{(1280-512x+256x^2)^{3/2}}\,dx, x, \left(1+\frac{1}{x}\right)^2\right)}{2\sqrt{1+4x+4x^2+4x^4}}$$

$$= -\frac{\left(3-\left(1+\frac{1}{x}\right)^2\right)x^2}{\sqrt{1+4x+4x^2+4x^4}} + \frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}} - \frac{\left(9\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{1280-512x^2+256x^4}}\,dx, x, 1+\frac{1}{x}\right)}{2\sqrt{5}\sqrt{1+4x+4x^2+4x^4}} - \frac{\left(3\left(5-3\sqrt{5}\right)\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1280-512x^2+256x^4}}\,dx, x, 1+\frac{1}{x}\right)}{10\sqrt{1+4x+4x^2+4x^4}}$$

$$= -\frac{\left(3-\left(1+\frac{1}{x}\right)^2\right)x^2}{\sqrt{1+4x+4x^2+4x^4}} + \frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}} + \frac{9\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)\left(1+\frac{1}{x}\right)x^2}{10\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{1+4x+4x^2+4x^4}} - \frac{9\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}x^2E\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\,5^{3/4}\sqrt{1+4x+4x^2+4x^4}} + \frac{3\left(3-\sqrt{5}\right)\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{4\,5^{3/4}\sqrt{1+4x+4x^2+4x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.70 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.64

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \frac{19+42x-16x^2+36x^3+\frac{9}{2}\left(-i+\sqrt{-1-2i}-2x\right)\left(-i-\sqrt{-1+2i}+2x\right)}{}$$

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-3/2),x]
```

```
[Out] (19 + 42*x - 16*x^2 + 36*x^3 + (9*(-I + Sqrt[-1 - 2*I] - 2*x)*(-I - Sqrt[-1
 + 2*I] + 2*x)*(-I + Sqrt[-1 + 2*I] + 2*x))/2 - ((9*I)*Sqrt[-2/5 + (4*I)/5]
*(-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(2*I + Sqrt[-1 - 2*I] + Sqrt[-1 +
 2*I])*((I + Sqrt[-1 - 2*I])/2 + x)^2*Sqrt[((2*I + Sqrt[-1 - 2*I] - Sqrt[-1
 + 2*I])*(-I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2
*I])*(I + Sqrt[-1 - 2*I] + 2*x))]*Sqrt[((1 + 2*I)*((-1 + I) + Sqrt[-1 - 2*I
```

```
])*(1 + 2*x + (2*I)*x^2))/(I + Sqrt[-1 - 2*I] + 2*x)^2]*EllipticE[ArcSin[Sq
rt[((2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[-1 + 2*I] + 2*x))/(S
qrt[-1 + 2*I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5 - Sqrt[5])/2])/((-1
 + I) + Sqrt[-1 - 2*I]) + ((6 - 3*I)*Sqrt[-2/5 + (4*I)/5]*Sqrt[((2*I + Sqrt
[-1 - 2*I] - Sqrt[-1 + 2*I])*(-I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1
- 2*I] + Sqrt[-1 + 2*I])*(I + Sqrt[-1 - 2*I] + 2*x))]*(1 + 2*x + (2*I)*x^2)
*EllipticF[ArcSin[Sqrt[((2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[
-1 + 2*I] + 2*x))/(Sqrt[-1 + 2*I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5
 - Sqrt[5])/2])/Sqrt[((1 + 2*I)*((-1 + I) + Sqrt[-1 - 2*I])*(1 + 2*x + (2*I
)*x^2))/(I + Sqrt[-1 - 2*I] + 2*x)^2])/(10*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.81 (sec) , antiderivative size = 2564, normalized size of antiderivative = 6.99

method	result	size
default	Expression too large to display	2564
risch	Expression too large to display	2564
elliptic	Expression too large to display	2564

```
[In] int(1/(4*x^4+4*x^2+4*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8*(-9/20*x^3+1/5*x^2-21/40*x-19/80)/(4*x^4+4*x^2+4*x+1)^(1/2)+3/5*(RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))*((RootO
f(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-Ro
otOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2
)))^(1/2)*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z^4+4*_Z^2
+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_
Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*((Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)
-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index
=2)))^(1/2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z
+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_
Z+1,index=1))/((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(x-RootOf(4*
_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)*EllipticF(((RootOf(4*_Z^4+4*_Z^2+4*_Z+1
,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_
Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*
_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(RootOf(4*
```

_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4
*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))/(RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2))
-9/5*((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z
+1,index=3))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))+(RootOf(4*_Z^4+4*_Z^2
+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))*((RootOf(4*_Z^4+4*_Z
^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+
4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*(x-
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index
=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,in
dex=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,i
ndex=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*((RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4
+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^
4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*(
(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)*RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)*RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)+R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)*RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)+Ro
otOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)^2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)
-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2
)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*EllipticF(((RootOf(4*_Z^4+4*_Z^2+4*
_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2
+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^
2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((RootOf
(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(RootO
f(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(Root
Of(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))/(Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1
/2))+(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,inde
x=3))*EllipticE(((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2
+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^
4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-Ro
otOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,
index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))))/((x-RootOf(4*_Z^4+4*_Z^2+4
_Z+1,index=1))(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=3))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)

Fricas [F]

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \int \frac{1}{(4\,x^4+4\,x^2+4\,x+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(4*x^4 + 4*x^2 + 4*x + 1)/(16*x^8 + 32*x^6 + 32*x^5 + 24*x^4 +
 32*x^3 + 24*x^2 + 8*x + 1), x)
```

Sympy [F]

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \int \frac{1}{(4x^4+4x^2+4x+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1)**(3/2),x)
[Out] Integral((4*x**4 + 4*x**2 + 4*x + 1)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \int \frac{1}{(4\,x^4+4\,x^2+4\,x+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="maxima")
[Out] integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \int \frac{1}{(4\,x^4+4\,x^2+4\,x+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="giac")
[Out] integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \int \frac{1}{(4\,x^4+4\,x^2+4\,x+1)^{3/2}}\,dx$$

```
[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(3/2),x)
[Out] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(3/2), x)
```

3.800 $\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = -\frac{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}x^2\,\mathrm{EllipticF}\left(2\arctan\left(\frac{4+3x}{\sqrt[4]{517}x}\right),\frac{517+19\sqrt{517}}{1034}\right)}{8\sqrt[4]{517}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

```
[Out] -1/4136*x^2*(cos(2*arctan(1/517*(4+3*x)*517^(3/4)/x))^2)^(1/2)/cos(2*arctan
(1/517*(4+3*x)*517^(3/4)/x))*EllipticF(sin(2*arctan(1/517*(4+3*x)*517^(3/4)
/x)),1/1034*(534578+19646*517^(1/2))^(1/2))*((3+4/x)^2+517^(1/2))*((517-38*
(3+4/x)^2+(3+4/x)^4)/((3+4/x)^2+517^(1/2))^2)^(1/2)*517^(3/4)/(8*x^4-15*x^3
+8*x^2+24*x+8)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2094, 12, 6851, 1117}

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = -\frac{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2\,\mathrm{EllipticF}\left(2\arctan\left(\frac{3x+4}{\sqrt[4]{517}x}\right),\frac{517+19\sqrt{517}}{1034}\right)}{8\sqrt[4]{517}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

```
[In] Int[1/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4],x]
```

```
[Out] -1/8*((Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(
Sqrt[517] + (3 + 4/x)^2)^2]*x^2*EllipticF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)]
, (517 + 19*Sqrt[517])/1034])/(517^(1/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8
*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\text{integral} = -\left(1024\text{Subst}\left(\int\frac{1}{2\sqrt{2}(24-32x)^2\sqrt{\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}}}\,dx,x,\frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(\left(256\sqrt{2}\right)\text{Subst}\left(\int\frac{1}{(24-32x)^2\sqrt{\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}}}\,dx,x,\frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int\frac{1}{\sqrt{2117632-2490368x^2+1048576x^4}}\,dx\right.}{\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$= -\frac{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{4+3x}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{8\sqrt[4]{517}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 10.62 (sec) , antiderivative size = 1148, normalized size of antiderivative = 9.11

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx =$$

$$-\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(x-\text{Root}[8\#1^4-15\#1^3+8\#1^2+24\#1+8\&,1])(\text{Root}[8\#1^4-15\#1^3+8\#1^2+24\#1+8\&,2]-}{(x-\text{Root}[8\#1^4-15\#1^3+8\#1^2+24\#1+8\&,2])(\text{Root}[8\#1^4-15\#1^3+8\#1^2+24\#1+8\&,1]-}}\right.\right.}{}$$

```
[In] Integrate[1/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4],x]
```

```
[Out] (-2*EllipticF[ArcSin[Sqrt[((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
& , 1, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 +
 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 24*#1 + 8*#1^
2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))]], ((Ro
ot[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#1^
2 - 15*#1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))/((Root[
8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 -
 15*#1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 &
, 2, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))]*(x - Root
[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((Root[8 + 24*#1 +
 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 +
8*#1^4 & , 2, 0])*(x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 3, 0]
))/((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 24
*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1
^3 + 8*#1^4 & , 3, 0]))]*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1,
 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0])*Sqrt[((x - Root
[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 24*#1 + 8*#1^2
- 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 &
 , 2, 0])*(x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0])*(Root[
8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#1^2 -
 15*#1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#
1^4 & , 2, 0])^2*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Ro
ot[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0])^2)])/(Sqrt[8 + 24*x + 8
*x^2 - 15*x^3 + 8*x^4]*(-Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1,
```

```
0] + Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 24*#1
+ 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 +
 8*#1^4 & , 4, 0]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.16 (sec) , antiderivative size = 1180, normalized size of antiderivative = 9.37

method	result	size
default	Expression too large to display	1180
elliptic	Expression too large to display	1180

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1)-RootOf(8*_Z^4-15*_Z^3+8*
_Z^2+24*_Z+8,index=4))*((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-Root
Of(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+
24*_Z+8,index=1))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_
Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+
8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))^2*((Ro
otOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24
*_Z+8,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))/(RootOf(8
*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,
index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)))^(1/2)*((RootOf
(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+
8,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4))/(RootOf(8*_Z^
4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,inde
x=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z
^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,ind
ex=2))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3
+8*_Z^2+24*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,
index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_Z^
4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,
index=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-
RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_Z^4-15*_Z^3+8*_
Z^2+24*_Z+8,index=1))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf
(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24
*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-Root
Of(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))*(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24
*_Z+8,index=1)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4))/(RootOf(8*_Z^
4-15*_Z^3+8*_Z^2+24*_Z+8,index=1)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,inde
x=3))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+
8*_Z^2+24*_Z+8,index=4)))^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8x^4-15x^3+8x^2+24x+8}}\,dx$$

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(1/2),x)
[Out] Integral(1/sqrt(8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(1/2),x)
[Out] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(1/2), x)
```

3.801 $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 434

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = -\frac{\left(172-7\left(3+\frac{4}{x}\right)^2\right)x^2}{208\sqrt{8+24x+8x^2-15x^3+8x^4}}+\frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{322608\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$+\frac{2455\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\left(3+\frac{4}{x}\right)x^2}{322608\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$-\frac{2455\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}x^2E\left(2\arctan\left(\frac{4+3x}{\sqrt[4]{517}x}\right)\middle|\frac{517+19\sqrt{517}}{1034}\right)}{624\ 517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$+\frac{\left(4910-203\sqrt{517}\right)\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}x^2\operatorname{EllipticF}\left(2\arctan\left(\frac{4+3x}{\sqrt[4]{517}x}\right),\frac{517+19\sqrt{517}}{1034}\right)}{2496\ 517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

```
[Out] -1/208*(172-7*(3+4/x)^2)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/322608*(50
896-2455*(3+4/x)^2)*(3+4/x)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+2455/3226
08*(517-38*(3+4/x)^2+(3+4/x)^4)*(3+4/x)*x^2/((3+4/x)^2+517^(1/2))/(8*x^4-15
*x^3+8*x^2+24*x+8)^(1/2)-2455/322608*x^2*(cos(2*arctan(1/517*(4+3*x)*517^(3
/4)/x))^2)^(1/2)/cos(2*arctan(1/517*(4+3*x)*517^(3/4)/x))*EllipticE(sin(2*a
rctan(1/517*(4+3*x)*517^(3/4)/x)),1/1034*(534578+19646*517^(1/2))^(1/2))*((
3+4/x)^2+517^(1/2))*((517-38*(3+4/x)^2+(3+4/x)^4)/((3+4/x)^2+517^(1/2))^2)^
(1/2)*517^(1/4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/1290432*x^2*(cos(2*arct
an(1/517*(4+3*x)*517^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/517*(4+3*x)*517^(3/4
)/x))*EllipticF(sin(2*arctan(1/517*(4+3*x)*517^(3/4)/x)),1/1034*(534578+196
```

```
46*517^(1/2))^(1/2))*(4910-203*517^(1/2))*((3+4/x)^2+517^(1/2))*((517-38*(3
+4/x)^2+(3+4/x)^4)/((3+4/x)^2+517^(1/2))^2)^(1/2)*517^(1/4)/(8*x^4-15*x^3+8
*x^2+24*x+8)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2094, 12, 6851, 1687, 1692, 1211, 1117, 1209, 1261, 650}

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \frac{\left(4910-203\sqrt{517}\right)\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2\,\mathrm{Ell}}{2496\;517^{3/4}\sqrt{8x^4-15x^3+8x^2+}}$$
$$-\frac{2455\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2E\left(2\arctan\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{624\;517^{3/4}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$-\frac{\left(172-7\left(\frac{4}{x}+3\right)^2\right)x^2}{208\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(50896-2455\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{322608\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$+\frac{2455\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\left(\frac{4}{x}+3\right)x^2}{322608\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] -1/208*((172 - 7*(3 + 4/x)^2)*x^2)/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]
+ ((50896 - 2455*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(322608*Sqrt[8 + 24*x + 8*x^2
- 15*x^3 + 8*x^4]) + (2455*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)*(3 + 4/x)*x
^2)/(322608*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^
4]) - (2455*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x
)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]*x^2*EllipticE[2*ArcTan[(4 + 3*x)/(517^(1/
4)*x)], (517 + 19*Sqrt[517])/1034])/(624*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 -
15*x^3 + 8*x^4]) + ((4910 - 203*Sqrt[517])*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(
517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]*x^2*Ellipt
icF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (517 + 19*Sqrt[517])/1034])/(2496*51
7^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
```

```
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\text{integral} = -\left(1024\text{Subst}\left(\int \frac{1}{16\sqrt{2}(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right)^{3/2}}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(\left(32\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right)^{3/2}}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{(24-32x)^4}{(2117632-2490368x^2+1048576x^4)^{3/2}}\,d\right.}{8\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{x(-1769472-3145728x^2)}{(2117632-2490368x^2+1048576x^4)^{3/2}}\right.}{8\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$-\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{331776+3538944x^2+1048576x^4}{(2117632-2490368x^2+1048576x^4)^{3/2}}\right.}{8\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$= \frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{322608\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$-\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{29541080280760057856-22112674170}{\sqrt{2117632-2490368x^2+10485}}\right.}{45403039643335655424\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$-\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{-1769472-3145728x}{(2117632-2490368x+1048576x^2)^{3/2}}\right.}{16\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$= -\frac{\left(172-7\left(3+\frac{4}{x}\right)^2\right)x^2}{208\sqrt{8+24x+8x^2-15x^3+8x^4}}+\frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{322608\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$-\frac{\left(2455\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{1-\frac{16x^2}{\sqrt{517}}}{\sqrt{2117632-2490368x^2+104857}}\right.}{156\sqrt{517}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$-\frac{\left(\left(104951-4910\sqrt{517}\right)\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{}{\sqrt{2117}}\right.}{161304\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$= -\frac{\left(172-7\left(3+\frac{4}{x}\right)^2\right)x^2}{208\sqrt{8+24x+8x^2-15x^3+8x^4}}+\frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{322608\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$+\frac{2455\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\left(3+\frac{4}{x}\right)x^2}{322608\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$-\frac{2455\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}x^2E\left(2\tan^{-1}\left(\frac{4+3x}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{624\ 517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$+\frac{\left(4910-203\sqrt{517}\right)\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{4+3x}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{5}}{1034}\right.}{2496\ 517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 16.10 (sec) , antiderivative size = 6019, normalized size of antiderivative = 13.87

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-3/2),x]
[Out] Result too large to show
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.99 (sec) , antiderivative size = 5421, normalized size of antiderivative = 12.49

method	result	size
default	Expression too large to display	5421
risch	Expression too large to display	5421
elliptic	Expression too large to display	5421

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)/(64*x^8 - 240*x^7 + 353*x^
6 + 144*x^5 - 528*x^4 + 144*x^3 + 704*x^2 + 384*x + 64), x)
```

Sympy [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8x^4-15x^3+8x^2+24x+8)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(3/2),x)
[Out] Integral((8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="maxima")
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="giac")
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{3/2}}\,dx$$

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(3/2),x)
[Out] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(3/2), x)
```

3.802 $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 577

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = -\frac{\left(124415-6308\left(3+\frac{4}{x}\right)^2\right)x^2}{97344\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$-\frac{\left(64489-1399\left(3+\frac{4}{x}\right)^2\right)x^2}{624\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$+\frac{\left(18932921731-1086525994\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{78056941248\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$+\frac{\left(11921698-359497\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{483912\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$+\frac{543262997\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\left(3+\frac{4}{x}\right)x^2}{39028470624\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$-\frac{543262997\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}x^2E\left(2\arctan\left(\frac{4+3x}{\sqrt[4]{517}x}\right)\middle|\frac{517+19\sqrt{517}}{1034}\right)}{75490272\ 517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$+\frac{\left(4346103976-175318963\sqrt{517}\right)\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}x^2\,\text{EllipticF}\left(2\arctan\left(\frac{4+3x}{\sqrt[4]{517}x}\right.\right.}{1207844352\ 517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

```
[Out] -1/97344*(124415-6308*(3+4/x)^2)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)-1/62
4*(64489-1399*(3+4/x)^2)*x^2/(517-38*(3+4/x)^2+(3+4/x)^4)/(8*x^4-15*x^3+8*x
```

```
^2+24*x+8)^(1/2)+1/78056941248*(18932921731-1086525994*(3+4/x)^2)*(3+4/x)*x
^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/483912*(11921698-359497*(3+4/x)^2)*(
3+4/x)*x^2/(517-38*(3+4/x)^2+(3+4/x)^4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+5
43262997/39028470624*(517-38*(3+4/x)^2+(3+4/x)^4)*(3+4/x)*x^2/((3+4/x)^2+51
7^(1/2))/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)-543262997/39028470624*x^2*(cos(2
*arctan(1/517*(4+3*x)*517^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/517*(4+3*x)*517
^(3/4)/x))*EllipticE(sin(2*arctan(1/517*(4+3*x)*517^(3/4)/x)),1/1034*(53457
8+19646*517^(1/2))^(1/2))*((3+4/x)^2+517^(1/2))*((517-38*(3+4/x)^2+(3+4/x)^
4)/((3+4/x)^2+517^(1/2))^2)^(1/2)*517^(1/4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/
2)+1/624455529984*x^2*(cos(2*arctan(1/517*(4+3*x)*517^(3/4)/x))^2)^(1/2)/co
s(2*arctan(1/517*(4+3*x)*517^(3/4)/x))*EllipticF(sin(2*arctan(1/517*(4+3*x)
*517^(3/4)/x)),1/1034*(534578+19646*517^(1/2))^(1/2))*(4346103976-175318963
*517^(1/2))*((3+4/x)^2+517^(1/2))*((517-38*(3+4/x)^2+(3+4/x)^4)/((3+4/x)^2+
517^(1/2))^2)^(1/2)*517^(1/4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {2094, 12, 6851, 1687, 1692, 1211, 1117, 1209, 1677, 1674, 650}

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \frac{\left(4346103976-175318963\sqrt{517}\right)\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\cdots}{\left(\left(\frac{4}{x}+3\right)^2\cdots}}\cdots}{1207844352\ 517^{3/4}\sqrt{8x^4-15\cdots}}$$

$$-\frac{543262997\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}\,x^2 E\left(2\arctan\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{75490272\ 517^{3/4}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

$$-\frac{\left(124415-6308\left(\frac{4}{x}+3\right)^2\right)x^2}{97344\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(18932921731-1086525994\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{78056941248\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

$$+\frac{543262997\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\left(\frac{4}{x}+3\right)x^2}{39028470624\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

$$+\frac{\left(11921698-359497\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{483912\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

$$-\frac{\left(64489-1399\left(\frac{4}{x}+3\right)^2\right)x^2}{624\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-5/2),x]
```

```
[Out] -1/97344*((124415 - 6308*(3 + 4/x)^2)*x^2)/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 +
 8*x^4] - ((64489 - 1399*(3 + 4/x)^2)*x^2)/(624*(517 - 38*(3 + 4/x)^2 + (3
+ 4/x)^4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) + ((18932921731 - 108652
5994*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(78056941248*Sqrt[8 + 24*x + 8*x^2 - 15*x^
3 + 8*x^4]) + ((11921698 - 359497*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(483912*(517
- 38*(3 + 4/x)^2 + (3 + 4/x)^4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) +
(543262997*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)*(3 + 4/x)*x^2)/(39028470624
*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) - (5432
62997*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(
Sqrt[517] + (3 + 4/x)^2)^2]*x^2*EllipticE[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)]
, (517 + 19*Sqrt[517])/1034])/(75490272*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 1
5*x^3 + 8*x^4]) + ((4346103976 - 175318963*Sqrt[517])*(Sqrt[517] + (3 + 4/x
)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]
*x^2*EllipticF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (517 + 19*Sqrt[517])/1034
])/(1207844352*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
```

```
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

integral

$$= -\left(1024\text{Subst}\left(\int \frac{1}{128\sqrt{2}(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right)^{5/2}}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(\left(4\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right)^{5/2}}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{(24-32x)^8}{(2117632-2490368x^2+1048576x^4)^{5/2}}\,dx, x, \frac{3}{4}\right.}{64\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{x(-1174136684544-14611478740992x^2-2597596}{(2117632-2490368x^2+10485}\right.}{64\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{110075314176+5479304527872x^2+2435246456832}{(2117632-2490368x^2-}\right.}{64\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$= \frac{\left(11921698-359497\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{483912\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{440718049065141914354843648+960217998469209}{(2117632-2490368}\right.}{1089672951440055730176\sqrt{8+24x+8x^2-15x^3+8x}} - \frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{-1174136684544-14611478740992x-2597596220}{(2117632-2490368x+104857}\right.}{128\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$= -\frac{\left(64489 - 1399\left(3+\frac{4}{x}\right)^2\right)x^2}{624\left(517 - 38\left(3+\frac{4}{x}\right)^2 + \left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}} + \frac{\left(18932921731 - 1086525994\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{78056941248\sqrt{8+24x+8x^2-15x^3+8x^4}} + \frac{\left(11921698 - 359497\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{483912\left(517 - 38\left(3+\frac{4}{x}\right)^2 + \left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{\left(\sqrt{2117632 - 2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2 + 1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{718123303416888522576231512866863815065 6-}{\sqrt{2117632-2490}}\right.}{61843080265629273614808978354829818593 28\sqrt{8+24x+8x^2-}} - \frac{\left(\sqrt{2117632 - 2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2 + 1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{-3108699714785032273 92-252922155073127055}{(2117632-2490368x+1048576x^2)^{3/2}}\right.}{514571441799168\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$= -\frac{\left(124415 - 6308\left(3+\frac{4}{x}\right)^2\right)x^2}{97344\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{\left(64489 - 1399\left(3+\frac{4}{x}\right)^2\right)x^2}{624\left(517 - 38\left(3+\frac{4}{x}\right)^2 + \left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}} + \frac{\left(18932921731 - 1086525994\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{78056941248\sqrt{8+24x+8x^2-15x^3+8x^4}} + \frac{\left(11921698 - 359497\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{483912\left(517 - 38\left(3+\frac{4}{x}\right)^2 + \left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{\left(543262997\sqrt{2117632 - 2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2 + 1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{1-\frac{16x^2}{\sqrt{517}}}{\sqrt{2117632-2490368x^2+1048576x}}\right.}{18872568\sqrt{517}\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{\left(\left(90639903871 - 4346103976\sqrt{517}\right)\sqrt{2117632 - 2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2 + 1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int\right.}{78056941248\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

$$
\begin{aligned}
&= -\frac{\left(124415-6308\left(3+\frac{4}{x}\right)^2\right)x^2}{97344\sqrt{8+24x+8x^2-15x^3+8x^4}} \\
&- \frac{\left(64489-1399\left(3+\frac{4}{x}\right)^2\right)x^2}{624\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}} \\
&+ \frac{\left(18932921731-1086525994\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{78056941248\sqrt{8+24x+8x^2-15x^3+8x^4}} \\
&+ \frac{\left(11921698-359497\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{483912\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}} \\
&+ \frac{543262997\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\left(3+\frac{4}{x}\right)x^2}{39028470624\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{8+24x+8x^2-15x^3+8x^4}} \\
&- \frac{543262997\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}\,x^2E\left(2\tan^{-1}\left(\frac{4+3x}{\sqrt[4]{517}x}\right)\middle|\frac{517+19\sqrt{517}}{1034}\right)}{75490272\ 517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}} \\
&+ \frac{\left(4346103976-175318963\sqrt{517}\right)\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}\,x^2F\left(2\tan^{-1}\left(\frac{4+3x}{\sqrt[4]{517}x}\right)\middle|\frac{517\cdots}{}\right.}{1207844352\ 517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 16.10 (sec) , antiderivative size = 6084, normalized size of antiderivative = 10.54

$$
\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-5/2),x]
[Out] Result too large to show
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.84 (sec) , antiderivative size = 5441, normalized size of antiderivative = 9.43

method	result	size
risch	Expression too large to display	5441
default	Expression too large to display	5477
elliptic	Expression too large to display	5477

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)/(512*x^12 - 2880*x^11 + 69
36*x^10 - 4527*x^9 - 8808*x^8 + 16776*x^7 + 5528*x^6 - 17856*x^5 - 384*x^4
+ 20160*x^3 + 15360*x^2 + 4608*x + 512), x)
```

Sympy [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{1}{(8x^4-15x^3+8x^2+24x+8)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(5/2),x)
[Out] Integral((8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="maxima")
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="giac")
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{5/2}}\,dx$$

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(5/2),x)
[Out] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(5/2), x)
```

3.803 $\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = -\frac{\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)^2}}\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)x^2\,\text{EllipticF}\left(2\arctan\left(\frac{6-x}{\sqrt[4]{613}x}\right),\frac{613+91\sqrt{613}}{1226}\right)}{12\sqrt[4]{613}\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

```
[Out] -1/7356*x^2*(cos(2*arctan(1/613*(6-x)*613^(3/4)/x))^2)^(1/2)/cos(2*arctan(1
/613*(6-x)*613^(3/4)/x))*EllipticF(sin(2*arctan(1/613*(6-x)*613^(3/4)/x)),1
/1226*(751538+111566*613^(1/2))^(1/2))*((6-x)^2/x^2+613^(1/2))*((613-182*(1
-6/x)^2+(-1+6/x)^4)/((6-x)^2/x^2+613^(1/2))^2)^(1/2)*613^(3/4)/(3*x^4+15*x^
3-44*x^2-6*x+9)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2094, 12, 6851, 1110}

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = -\frac{\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2\,\text{EllipticF}\left(2\arctan\left(\frac{6-x}{\sqrt[4]{613}x}\right),\frac{613+91\sqrt{613}}{1226}\right)}{12\sqrt[4]{613}\sqrt{3x^4+15x^3-44x^2-6x+9}}$$

```
[In] Int[1/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4],x]
```

```
[Out] -1/12*(Sqrt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x)^2/x
^2)^2]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticF[2*ArcTan[(6 - x)/(613^(1/4
)*x)], (613 + 91*Sqrt[613])/1226])/(613^(1/4)*Sqrt[9 - 6*x - 44*x^2 + 15*x^
3 + 3*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1110

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\text{integral} = -\left(1296\text{Subst}\left(\int \frac{1}{3(-6-36x)^2\sqrt{\frac{794448-8491392x^2+1679616x^4}{(-6-36x)^4}}}\,dx, x, -\frac{1}{6}+\frac{1}{x}\right)\right)$$

$$= -\left(432\text{Subst}\left(\int \frac{1}{(-6-36x)^2\sqrt{\frac{794448-8491392x^2+1679616x^4}{(-6-36x)^4}}}\,dx, x, -\frac{1}{6}+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{794448-8491392x^2+1679616x^4}}\,d\right.}{\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$= -\frac{\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)^2}}\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)x^2F\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}x}\right)\middle|\frac{613+91\sqrt{613}}{1226}\right)}{12\sqrt[4]{613}\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 826, normalized size of antiderivative = 6.35

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx =$$

$$-\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(x-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1])(\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2]-}{(x-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2])(\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1]-}}\right.\right.}{}$$

```
[In] Integrate[1/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4],x]
```

```
[Out] (-2*EllipticF[ArcSin[Sqrt[((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
& , 1, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 -
 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((x - Root[9 - 6*#1 - 44*#1^
2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]], ((Ro
ot[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^
2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((Root[
9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 +
 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 &
, 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]*Sqrt[(x -
 Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])/(x - Root[9 - 6*#1 -
 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]*(x - Root[9 - 6*#1 - 44*#1^2 + 15*#
1^3 + 3*#1^4 & , 2, 0])^2*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#
1^4 & , 3, 0])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]*
Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0])/(x - Root[9
 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])])/Sqrt[(9 - 6*x - 44*x^2 +
15*x^3 + 3*x^4)*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Roo
t[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2
 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
& , 4, 0])]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.99 (sec) , antiderivative size = 1180, normalized size of antiderivative = 9.08

method	result	size
default	Expression too large to display	1180
elliptic	Expression too large to display	1180

```
[In] int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1)-RootOf(3*_Z^4+15*_Z^3-44
*_Z^2-6*_Z+9,index=4))*((RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-Root
Of(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2
-6*_Z+9,index=1))/(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-RootOf(3*_
Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+
9,index=2)))^(1/2)*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))^2*((Ro
otOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6
*_Z+9,index=1))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))/(RootOf(3
*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,
index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)))^(1/2)*((RootOf
(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+
9,index=1))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4))/(RootOf(3*_Z^
4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,inde
x=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)))^(1/2)/(RootOf(3*_Z
^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,ind
ex=2))/(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3
-44*_Z^2-6*_Z+9,index=1))*3^(1/2)/((x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,
index=1))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(x-RootOf(3*_Z^
4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,
index=4)))^(1/2)*EllipticF(((RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-
RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(x-RootOf(3*_Z^4+15*_Z^3-44*
_Z^2-6*_Z+9,index=1))/(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-RootOf
(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6
*_Z+9,index=2)))^(1/2),((RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-Root
Of(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))*(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6
*_Z+9,index=1)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4))/(RootOf(3*_Z^
4+15*_Z^3-44*_Z^2-6*_Z+9,index=1)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,inde
x=3))/(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-
44*_Z^2-6*_Z+9,index=4)))^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+15x^3-44x^2-6x+9}}\,dx$$

```
[In] integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(1/2),x)
[Out] Integral(1/sqrt(3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

```
[In] int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(1/2),x)
[Out] int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(1/2), x)
```

3.804 $\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 444

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = -\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{9-6x-44x^2+15x^3+3x^4}} + \frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{9-6x-44x^2+15x^3+3x^4}} + \frac{3722\left(613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4\right)\left(1-\frac{6}{x}\right)x^2}{31728267\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)\sqrt{9-6x-44x^2+15x^3+3x^4}} + \frac{3722\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)^2}}\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)x^2E\left(2\arctan\left(\frac{6-x}{\sqrt[4]{613}x}\right)\middle|\frac{613+91\sqrt{613}}{1226}\right)}{51759\ 613^{3/4}\sqrt{9-6x-44x^2+15x^3+3x^4}} - \frac{\left(7444-145\sqrt{613}\right)\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)^2}}\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)x^2\,\text{EllipticF}\left(2\arctan\left(\frac{6-x}{\sqrt[4]{613}x}\right),\frac{613+91\sqrt{613}}{1226}\right)}{207036\ 613^{3/4}\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

```
[Out] -1/51759*(176-23*(1-6/x)^2)*x^2/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)+1/3172826
7*(45401-3722*(1-6/x)^2)*(1-6/x)*x^2/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)+3722
/31728267*(613-182*(1-6/x)^2+(-1+6/x)^4)*(1-6/x)*x^2/((6-x)^2/x^2+613^(1/2)
)/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)+3722/31728267*x^2*(cos(2*arctan(1/613*(
6-x)*613^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/613*(6-x)*613^(3/4)/x))*Elliptic
E(sin(2*arctan(1/613*(6-x)*613^(3/4)/x)),1/1226*(751538+111566*613^(1/2))^(
1/2))*((6-x)^2/x^2+613^(1/2))*((613-182*(1-6/x)^2+(-1+6/x)^4)/((6-x)^2/x^2+
613^(1/2))^2)^(1/2)*613^(1/4)/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)-1/126913068
*x^2*(cos(2*arctan(1/613*(6-x)*613^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/613*(6
```

```
-x)*613^(3/4)/x))*EllipticF(sin(2*arctan(1/613*(6-x)*613^(3/4)/x)),1/1226*(
751538+111566*613^(1/2))^(1/2))*(7444-145*613^(1/2))*((6-x)^2/x^2+613^(1/2)
)*((613-182*(1-6/x)^2+(-1+6/x)^4)/((6-x)^2/x^2+613^(1/2))^2)^(1/2)*613^(1/4
)/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2094, 12, 6851, 1687, 1692, 1197, 1110, 1196, 1261, 650}

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = -\frac{\left(7444-145\sqrt{613}\right)\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2\operatorname{EllipticF}\left(2\arctan\left(\frac{6-x}{\sqrt[4]{613}x}\right),\frac{613+91\sqrt{613}}{1226}\right)}{207036\ 613^{3/4}\sqrt{3x^4+15x^3-44x^2-6x+9}} + \frac{3722\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2E\left(2\arctan\left(\frac{6-x}{\sqrt[4]{613}x}\right)|\frac{613+91\sqrt{613}}{1226}\right)}{51759\ 613^{3/4}\sqrt{3x^4+15x^3-44x^2-6x+9}} - \frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{3x^4+15x^3-44x^2-6x+9}} + \frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{3x^4+15x^3-44x^2-6x+9}} + \frac{3722\left(\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613\right)\left(1-\frac{6}{x}\right)x^2}{31728267\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)\sqrt{3x^4+15x^3-44x^2-6x+9}}$$

```
[In] Int[(9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4)^(-3/2),x]
```

```
[Out] -1/51759*((176 - 23*(1 - 6/x)^2)*x^2)/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^
4] + ((45401 - 3722*(1 - 6/x)^2)*(1 - 6/x)*x^2)/(31728267*Sqrt[9 - 6*x - 44
*x^2 + 15*x^3 + 3*x^4]) + (3722*(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)*(1 -
 6/x)*x^2)/(31728267*(Sqrt[613] + (6 - x)^2/x^2)*Sqrt[9 - 6*x - 44*x^2 + 15
*x^3 + 3*x^4]) + (3722*Sqrt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[61
3] + (6 - x)^2/x^2)^2]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticE[2*ArcTan[(
6 - x)/(613^(1/4)*x)], (613 + 91*Sqrt[613])/1226])/(51759*613^(3/4)*Sqrt[9
- 6*x - 44*x^2 + 15*x^3 + 3*x^4]) - ((7444 - 145*Sqrt[613])*Sqrt[(613 - 182
*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x)^2/x^2)^2]*(Sqrt[613] + (6
 - x)^2/x^2)*x^2*EllipticF[2*ArcTan[(6 - x)/(613^(1/4)*x)], (613 + 91*Sqrt[
613])/1226])/(207036*613^(3/4)*Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1110

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2
- 4*a*c, 0] && GtQ[c/a, 0] && LtQ[b/a, 0]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c
/a, 0] && LtQ[b/a, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\text{integral} = -\left(1296\text{Subst}\left(\int \frac{1}{27(-6-36x)^2\left(\frac{794448-8491392x^2+1679616x^4}{(-6-36x)^4}\right)^{3/2}}\,dx, x, -\frac{1}{6}+\frac{1}{x}\right)\right)$$

$$= -\left(48\text{Subst}\left(\int \frac{1}{(-6-36x)^2\left(\frac{794448-8491392x^2+1679616x^4}{(-6-36x)^4}\right)^{3/2}}\,dx, x, -\frac{1}{6}+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{(-6-36x)^4}{(794448-8491392x^2+1679616x^4)^{3/}}\right.}{9\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$= -\frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{x(31104+1119744x^2)}{(794448-8491392x^2+1679616x^4)^{3}}\right.}{9\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$-\frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{1296+279936x^2+1679616x^4}{(794448-8491392x^2+1679616x^4)^{3}}\right.}{9\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$= \frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$+\frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{4012069665987624960-120961970}{\sqrt{794448-8491392x^2+167}}\right.}{477380951360582713344\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$-\frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{31104+1119744x}{(794448-8491392x+1679616x^2)^{3/}}\right.}{18\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$= -\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{9-6x-44x^2+15x^3+3x^4}}+\frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$+\frac{\left(7444\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{1-\frac{36x^2}{\sqrt{613}}}{\sqrt{794448-8491392x^2+1679}}\right.}{17253\sqrt{613}\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$+\frac{\left(\left(88885-7444\sqrt{613}\right)\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}x^2\right)\text{Subst}\left(\int \frac{}{\sqrt{79}}\right.}{10576089\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$= -\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{9-6x-44x^2+15x^3+3x^4}}+\frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$+\frac{3722\left(613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4\right)\left(1-\frac{6}{x}\right)x^2}{31728267\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$+\frac{3722\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)^2}}\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)x^2E\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}x}\right)|\frac{613+91\sqrt{613}}{1226}\right)}{51759\ 613^{3/4}\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

$$-\frac{\left(7444-145\sqrt{613}\right)\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)^2}}\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)x^2F\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}x}\right)|\frac{613+91\sqrt{61}}{1226}\right.}{207036\ 613^{3/4}\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 15.99 (sec) , antiderivative size = 4974, normalized size of antiderivative = 11.20

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4)^(-3/2),x]
```

```
[Out] (2*(106926 + 592639*x - 232005*x^2 - 44664*x^3 + 81441*EllipticF[ArcSin[Sqr
t[((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])*(Root[9 - 6*#
1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^
3 + 3*#1^4 & , 4, 0]))/((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & ,
 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*
#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]], ((Root[9 - 6*#1 - 44*#1^2 +
15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & ,
 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*
#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((Root[9 - 6*#1 - 44*#1^2 + 15*
#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3,
 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1
- 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2
 + 15*#1^3 + 3*#1^4 & , 1, 0])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#
1^4 & , 2, 0])]*(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])^
2*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])/(x - Root
[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]*Sqrt[(x - Root[9 - 6*#1
- 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0])/((x - Root[9 - 6*#1 - 44*#1^2 + 15*
#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1,
 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1
- 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3
+ 3*#1^4 & , 4, 0]))] - (223320*(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*
#1^4 & , 2, 0])^2*(EllipticPi[(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 &
 , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0])/(Root[9 -
6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*
#1^3 + 3*#1^4 & , 4, 0]), ArcSin[Sqrt[((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1
^3 + 3*#1^4 & , 1, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0
] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((x - Root[9 - 6
*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#
1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4,
0]))]], -(((Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 -
 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15
*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4
, 0]))/((-Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] + Root[9 - 6
*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#
1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4,
```

0])))]*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#
1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0]) + EllipticF[ArcSin[Sqrt[((x - Roo
t[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])*(Root[9 - 6*#1 - 44*#1^2
 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
& , 4, 0]))/((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Ro
ot[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^
2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]], -(((Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 +
 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(
Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#
1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((-Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3
#1^4 & , 1, 0] + Root[9 - 6#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Ro
ot[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^
2 + 15*#1^3 + 3*#1^4 & , 4, 0])))]*Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1
^4 & , 2, 0])*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0
])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]*Sqrt[(x - Ro
ot[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0])/((x - Root[9 - 6*#1 - 4
4*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3
#1^4 & , 1, 0] - Root[9 - 6#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Ro
ot[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^
2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]*Sqrt[((x - Root[9 - 6*#1 - 44*#1^2 + 15*#
1^3 + 3*#1^4 & , 1, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2,
0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((x - Root[9 -
6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*
#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4,
 0]))]*(-Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] + Root[9 - 6*
#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/(-Root[9 - 6*#1 - 44*#1^2 + 15*
#1^3 + 3*#1^4 & , 2, 0] + Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4,
 0]) + 44664*((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])*(x
 - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(x - Root[9 - 6*#1
 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]) - ((x - Root[9 - 6*#1 - 44*#1^2 +
15*#1^3 + 3*#1^4 & , 2, 0])^2*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 +
 3*#1^4 & , 3, 0])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0
])]*Sqrt[((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0])*(Root[
9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 +
 15*#1^3 + 3*#1^4 & , 4, 0]))/((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#
1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root
[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0]))]*Sqrt[((x - Root[9 - 6*#
1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^
3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]
))/((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*
#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1
^3 + 3*#1^4 & , 4, 0]))]*(5*EllipticPi[(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 +
 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0])/(
Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#
1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]), ArcSin[Sqrt[((x - Root[9 - 6*#1 - 44*#1^

```
2 + 15*#1^3 + 3*#1^4 & , 1, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((x - R
oot[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1
^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^
4 & , 4, 0]))]], -(((Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] -
 Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*
#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#
1^4 & , 4, 0]))/((-Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] + R
oot[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1
^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^
4 & , 4, 0])))]*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Roo
t[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0]) + EllipticE[ArcSin[Sqrt[
((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])*(Root[9 - 6*#1
- 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3
+ 3*#1^4 & , 4, 0]))/((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2
, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1
 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]], -(((Root[9 - 6*#1 - 44*#1^2 +
15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & ,
 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*
#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((-Root[9 - 6*#1 - 44*#1^2 + 15
*#1^3 + 3*#1^4 & , 1, 0] + Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3
, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1
 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0])))]*(Root[9 - 6*#1 - 44*#1^2 + 15*#
1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3,
0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 -
 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]) + EllipticF[ArcSin[Sqrt[((x - Root[9
 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])*(Root[9 - 6*#1 - 44*#1^2 +
15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & ,
 4, 0]))/((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[
9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 +
 15*#1^3 + 3*#1^4 & , 4, 0]))]], -(((Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*
#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Roo
t[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2
 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((-Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1
^4 & , 1, 0] + Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[
9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 +
 15*#1^3 + 3*#1^4 & , 4, 0])))]*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
 & , 1, 0]*(-Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] + Root[9
- 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]) - Root[9 - 6*#1 - 44*#1^2 +
15*#1^3 + 3*#1^4 & , 2, 0]*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & ,
2, 0] + Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))))/(Root[9 -
6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*
#1^3 + 3*#1^4 & , 4, 0]))))/(10576089*Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^
4])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.34 (sec) , antiderivative size = 5421, normalized size of antiderivative = 12.21

method	result	size
default	Expression too large to display	5421
risch	Expression too large to display	5421
elliptic	Expression too large to display	5421

```
[In] int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+15\,x^3-44\,x^2-6\,x+9)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)/(9*x^8 + 90*x^7 - 39*x^6 -
 1356*x^5 + 1810*x^4 + 798*x^3 - 756*x^2 - 108*x + 81), x)
```

Sympy [F]

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+15x^3-44x^2-6x+9)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(3/2),x)
[Out] Integral((3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+15\,x^3-44\,x^2-6\,x+9)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="maxima")
[Out] integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+15\,x^3-44\,x^2-6\,x+9)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="giac")
[Out] integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+15\,x^3-44\,x^2-6\,x+9)^{3/2}}\,dx$$

```
[In] int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(3/2),x)
[Out] int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(3/2), x)
```

3.805 $\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 56

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx = -4x+12\arcsin\left(\frac{1-x}{2}\right)-24\sqrt{3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt{1+x}}{\sqrt{3-x}}\right)+21\log(x)-9\log(1+x)$$

```
[Out] -4*x-12*arcsin(-1/2+1/2*x)+21*ln(x)-9*ln(1+x)-24*arctanh(3^(1/2)*(1+x)^(1/2
)/(3-x)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6874, 36, 29, 31, 132, 55, 633, 222, 12, 95, 213}

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx = 12\arcsin\left(\frac{1-x}{2}\right)-24\sqrt{3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt{x+1}}{\sqrt{3-x}}\right)-4x+21\log(x)-9\log(x+1)$$

```
[In] Int[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]
```

```
[Out] -4*x + 12*ArcSin[(1 - x)/2] - 24*Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[1 + x])/Sqrt
[3 - x]] + 21*Log[x] - 9*Log[1 + x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-4+\frac{12}{x}+\frac{9}{x(1+x)}+\frac{12\sqrt{3-x}}{x\sqrt{1+x}}\right)\,dx \\
&= -4x+12\log(x)+9\int \frac{1}{x(1+x)}\,dx+12\int \frac{\sqrt{3-x}}{x\sqrt{1+x}}\,dx \\
&= -4x+12\log(x)+9\int \frac{1}{x}\,dx-9\int \frac{1}{1+x}\,dx \\
&\quad -12\int \frac{1}{\sqrt{3-x}\sqrt{1+x}}\,dx+12\int \frac{3}{\sqrt{3-x}x\sqrt{1+x}}\,dx \\
&= -4x+21\log(x)-9\log(1+x)-12\int \frac{1}{\sqrt{3+2x-x^2}}\,dx+36\int \frac{1}{\sqrt{3-x}x\sqrt{1+x}}\,dx \\
&= -4x+21\log(x)-9\log(1+x)+3\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx,x,2-2x\right) \\
&\quad +72\text{Subst}\left(\int \frac{1}{-1+3x^2}\,dx,x,\frac{\sqrt{1+x}}{\sqrt{3-x}}\right) \\
&= -4x+12\sin^{-1}\left(\frac{1-x}{2}\right)-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{1+x}}{\sqrt{3-x}}\right)+21\log(x)-9\log(1+x)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 404, normalized size of antiderivative = 7.21

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx = 12-24i\pi-4x-48\arctan\left(\frac{\sqrt{1+x}}{2+\sqrt{3-x}}\right)-42\log\left(-2+\sqrt{3-x}\right)-9\log(-1-x)+21\log\left(4+\sqrt{3}\left(-2+\sqrt{3-x}\right)-2\sqrt{3-x}-\sqrt{1+x}\right)-12\sqrt{3}\log\left(4+\sqrt{3}\left(-2+\sqrt{3-x}\right)-2\sqrt{3-x}-\sqrt{1+x}\right)+21\log\left(-4+\sqrt{3}\left(-2+\sqrt{3-x}\right)+2\sqrt{3-x}-\sqrt{1+x}\right)-12\sqrt{3}\log\left(-4+\sqrt{3}\left(-2+\sqrt{3-x}\right)+2\sqrt{3-x}-\sqrt{1+x}\right)+21\log\left(4+\sqrt{3}\left(-2+\sqrt{3-x}\right)-2\sqrt{3-x}+\sqrt{1+x}\right)+12\sqrt{3}\log\left(4+\sqrt{3}\left(-2+\sqrt{3-x}\right)-2\sqrt{3-x}+\sqrt{1+x}\right)+21\log\left(-4+\sqrt{3}\left(-2+\sqrt{3-x}\right)+2\sqrt{3-x}+\sqrt{1+x}\right)+12\sqrt{3}\log\left(-4+\sqrt{3}\left(-2+\sqrt{3-x}\right)+2\sqrt{3-x}+\sqrt{1+x}\right)$$

```
[In] Integrate[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]
```

```
[Out] 12 - (24*I)*Pi - 4*x - 48*ArcTan[Sqrt[1 + x]/(2 + Sqrt[3 - x])] - 42*Log[-2
 + Sqrt[3 - x]] - 9*Log[-1 - x] + 21*Log[4 + Sqrt[3]*(-2 + Sqrt[3 - x]) - 2
*Sqrt[3 - x] - Sqrt[1 + x]] - 12*Sqrt[3]*Log[4 + Sqrt[3]*(-2 + Sqrt[3 - x])
 - 2*Sqrt[3 - x] - Sqrt[1 + x]] + 21*Log[-4 + Sqrt[3]*(-2 + Sqrt[3 - x]) +
2*Sqrt[3 - x] - Sqrt[1 + x]] - 12*Sqrt[3]*Log[-4 + Sqrt[3]*(-2 + Sqrt[3 - x
]) + 2*Sqrt[3 - x] - Sqrt[1 + x]] + 21*Log[4 + Sqrt[3]*(-2 + Sqrt[3 - x]) -
 2*Sqrt[3 - x] + Sqrt[1 + x]] + 12*Sqrt[3]*Log[4 + Sqrt[3]*(-2 + Sqrt[3 - x
]) - 2*Sqrt[3 - x] + Sqrt[1 + x]] + 21*Log[-4 + Sqrt[3]*(-2 + Sqrt[3 - x])
+ 2*Sqrt[3 - x] + Sqrt[1 + x]] + 12*Sqrt[3]*Log[-4 + Sqrt[3]*(-2 + Sqrt[3 -
 x]) + 2*Sqrt[3 - x] + Sqrt[1 + x]]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

method	result	size
default	$-4x+21\ln\left(x\right)+\frac{12\sqrt{x+1}\,\sqrt{3-x}\,\left(-\arcsin\left(-\frac{1}{2}+\frac{x}{2}\right)-\sqrt{3}\,\operatorname{arctanh}\left(\frac{(3+x)\sqrt{3}}{3\sqrt{-x^{2}+2x+3}}\right)\right)}{\sqrt{-x^{2}+2x+3}}-9\ln\left(x+1\right)$	76

```
[In] int((2*(3-x)^(1/2)+3/(x+1)^(1/2))^2/x,x,method=_RETURNVERBOSE)
[Out] -4*x+21*ln(x)+12*(x+1)^(1/2)*(3-x)^(1/2)/(-x^2+2*x+3)^(1/2)*(-arcsin(-1/2+1
/2*x)-3^(1/2)*arctanh(1/3*(3+x)*3^(1/2)/(-x^2+2*x+3)^(1/2)))-9*ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx = 6\sqrt{3}\log\left(-\frac{\sqrt{3}(x+3)\sqrt{x+1}\sqrt{-x+3}+x^2-6x-9}{x^2}\right) - 4x + 12\arctan\left(\frac{\sqrt{x+1}(x-1)\sqrt{-x+3}}{x^2-2x-3}\right) - 9\log\left(x+1\right) + 21\log\left(x\right)$$

```
[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="fricas")
[Out] 6*sqrt(3)*log(-(sqrt(3)*(x + 3)*sqrt(x + 1)*sqrt(-x + 3) + x^2 - 6*x - 9)/x
^2) - 4*x + 12*arctan(sqrt(x + 1)*(x - 1)*sqrt(-x + 3)/(x^2 - 2*x - 3)) - 9
*log(x + 1) + 21*log(x)
```

Sympy [F]

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx = \int \frac{\left(2\sqrt{3-x}\sqrt{x+1}+3\right)^2}{x\,(x+1)}\,dx$$

```
[In] integrate((2*(3-x)**(1/2)+3/(1+x)**(1/2))**2/x,x)
[Out] Integral((2*sqrt(3 - x)*sqrt(x + 1) + 3)**2/(x*(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx = -12\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{-x^2+2x+3}}{|x|}+\frac{6}{|x|}+2\right)-4x+12\arcsin\left(-\frac{1}{2}x+\frac{1}{2}\right)-9\log(x+1)+21\log(x)$$

```
[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="maxima")
```

```
[Out] -12*sqrt(3)*log(2*sqrt(3)*sqrt(-x^2 + 2*x + 3)/abs(x) + 6/abs(x) + 2) - 4*x
 + 12*arcsin(-1/2*x + 1/2) - 9*log(x + 1) + 21*log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(44) = 88.

Time = 0.49 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.70

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx = 12\pi+12\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+\frac{6\left(\sqrt{x+1}-2\right)}{\sqrt{-x+3}}-\frac{6\sqrt{-x+3}}{\sqrt{x+1}-2}\right|}{\left|4\sqrt{3}+\frac{6\left(\sqrt{x+1}-2\right)}{\sqrt{-x+3}}-\frac{6\sqrt{-x+3}}{\sqrt{x+1}-2}\right|}\right)-4x+24\arctan\left(\frac{\sqrt{-x+3}\left(\frac{\left(\sqrt{x+1}-2\right)^2}{x-3}+1\right)}{2\left(\sqrt{x+1}-2\right)}\right)+21\log(|x|)-9\log(|-x-1|)+12$$

```
[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] 12*pi + 12*sqrt(3)*log(abs(-4*sqrt(3) + 6*(sqrt(x + 1) - 2)/sqrt(-x + 3) -
6*sqrt(-x + 3)/(sqrt(x + 1) - 2))/abs(4*sqrt(3) + 6*(sqrt(x + 1) - 2)/sqrt(
-x + 3) - 6*sqrt(-x + 3)/(sqrt(x + 1) - 2))) - 4*x + 24*arctan(1/2*sqrt(-x
+ 3)*((sqrt(x + 1) - 2)^2/(x - 3) + 1)/(sqrt(x + 1) - 2)) + 21*log(abs(x))
- 9*log(abs(-x - 1)) + 12
```

Mupad [B] (verification not implemented)

Time = 25.15 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.82

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx$$
$$= 48\,\mathrm{atan}\left(\frac{\sqrt{3-x}-4\sqrt{3}+3\sqrt{3}\sqrt{x+1}}{\sqrt{x+1}-3\sqrt{3}\sqrt{3-x}+8}\right)-9\ln\left(x+1\right)-4x+21\ln\left(x\right)$$
$$+12\sqrt{3}\ln\left(\frac{6x-12\sqrt{x+1}+4\sqrt{3}\sqrt{3-x}+2\sqrt{3}\sqrt{x+1}\sqrt{3-x}-6}{3x+6\sqrt{3}\sqrt{3-x}-18}\right)$$
$$-12\sqrt{3}\ln\left(\frac{\sqrt{x+1}-1}{\sqrt{3}-\sqrt{3-x}}\right)$$

```
[In] int((3/(x + 1)^(1/2) + 2*(3 - x)^(1/2))^2/x,x)
```

```
[Out] 48*atan(((3 - x)^(1/2) - 4*3^(1/2) + 3*3^(1/2)*(x + 1)^(1/2))/((x + 1)^(1/2
) - 3*3^(1/2)*(3 - x)^(1/2) + 8)) - 9*log(x + 1) - 4*x + 21*log(x) + 12*3^(
1/2)*log((6*x - 12*(x + 1)^(1/2) + 4*3^(1/2)*(3 - x)^(1/2) + 2*3^(1/2)*(x +
 1)^(1/2)*(3 - x)^(1/2) - 6)/(3*x + 6*3^(1/2)*(3 - x)^(1/2) - 18)) - 12*3^(
1/2)*log(((x + 1)^(1/2) - 1)/(3^(1/2) - (3 - x)^(1/2)))
```

3.806 $\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}} \, dx = -\frac{1}{x} - x + \sqrt{1+x^2} + \frac{\sqrt{1+x^2}}{x} + \frac{1}{2}x\sqrt{1+x^2} - \frac{\operatorname{arcsinh}(x)}{2} - \log\left(1+\sqrt{1+x^2}\right)$$

```
[Out] -1/x-x-1/2*arcsinh(x)-ln(1+(x^2+1)^(1/2))+(x^2+1)^(1/2)+(x^2+1)^(1/2)/x+1/2
*x*(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6874, 283, 221, 1605, 196, 45, 201}

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}} \, dx = -\frac{\operatorname{arcsinh}(x)}{2} + \frac{1}{2}\sqrt{x^2+1}x + \sqrt{x^2+1} + \frac{\sqrt{x^2+1}}{x} - \log\left(\sqrt{x^2+1}+1\right) - x - \frac{1}{x}$$

```
[In] Int[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) - x + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x + (x*Sqrt[1 + x^2])/2 - ArcSi
nh[x]/2 - Log[1 + Sqrt[1 + x^2]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{1+\sqrt{1+x^2}}+\frac{x}{1+\sqrt{1+x^2}}+\frac{x^2}{1+\sqrt{1+x^2}}\right) dx \\
&= -\int \frac{1}{1+\sqrt{1+x^2}}\,dx+\int \frac{x}{1+\sqrt{1+x^2}}\,dx+\int \frac{x^2}{1+\sqrt{1+x^2}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{x}}\,dx,x,1+x^2\right)+\int \left(-1+\sqrt{1+x^2}\right) dx-\int \left(-\frac{1}{x^2}+\frac{\sqrt{1+x^2}}{x^2}\right) dx \\
&= -\frac{1}{x}-x+\int \sqrt{1+x^2}\,dx-\int \frac{\sqrt{1+x^2}}{x^2}\,dx+\text{Subst}\left(\int \frac{x}{1+x}\,dx,x,\sqrt{1+x^2}\right) \\
&= -\frac{1}{x}-x+\frac{\sqrt{1+x^2}}{x}+\frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx \\
&\quad -\int \frac{1}{\sqrt{1+x^2}}\,dx+\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right) dx,x,\sqrt{1+x^2}\right) \\
&= -\frac{1}{x}-x+\sqrt{1+x^2}+\frac{\sqrt{1+x^2}}{x}+\frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\sinh^{-1}(x)-\log\left(1+\sqrt{1+x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$
\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx = \frac{-2(1+x^2)+\sqrt{1+x^2}(2+2x+x^2)+3x\log\left(-x+\sqrt{1+x^2}\right)-4x\log\left(1-x+\sqrt{1+x^2}\right)}{2x}
$$

```
[In] Integrate[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
[Out] (-2*(1 + x^2) + Sqrt[1 + x^2]*(2 + 2*x + x^2) + 3*x*Log[-x + Sqrt[1 + x^2]]
 - 4*x*Log[1 - x + Sqrt[1 + x^2]])/(2*x)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result	size
default	$-x-\frac{1}{x}-\frac{\operatorname{arcsinh}(x)}{2}-\frac{x\sqrt{x^2+1}}{2}+\sqrt{x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\ln(x)+\frac{(x^2+1)^{\frac{3}{2}}}{x}$	56
meijerg	$-\frac{x\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},2;-x^2\right)}{2}+\frac{x^3\,{}_3F_2\left(\frac{1}{2},1,\frac{3}{2};2,\frac{5}{2};-x^2\right)}{6}+\frac{-4\sqrt{\pi}+4\sqrt{\pi}\,\sqrt{x^2+1}-4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^2+1}}{2}\right)}{4\sqrt{\pi}}$	76
trager	$-\frac{(x-1)^2}{x}+\frac{(x^2+2x+2)\sqrt{x^2+1}}{2x}+\frac{\ln\left(\frac{\sqrt{x^2+1}\,x^2-x^3+2x\sqrt{x^2+1}-2x^2+2\sqrt{x^2+1}-2x-2}{x^4}\right)}{2}$	84

```
[In] int((x^2+x-1)/(1+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -x-1/x-1/2*arcsinh(x)-1/2*x*(x^2+1)^(1/2)+(x^2+1)^(1/2)-arctanh(1/(x^2+1)^(
1/2))-ln(x)+1/x*(x^2+1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx = -\frac{2\,x^2+2\,x\log(x)+2\,x\log\left(-x+\sqrt{x^2+1}+1\right)-x\log\left(-x+\sqrt{x^2+1}\right)-2\,x\log\left(-x+\sqrt{x^2+1}-1\right)}{2\,x}$$

```
[In] integrate((x^2+x-1)/(1+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^2 + 2*x*log(x) + 2*x*log(-x + sqrt(x^2 + 1) + 1) - x*log(-x + sqr
t(x^2 + 1)) - 2*x*log(-x + sqrt(x^2 + 1) - 1) - (x^2 + 2*x + 2)*sqrt(x^2 +
1) - 2*x + 2)/x
```

Sympy [A] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx = \frac{x\sqrt{x^2+1}}{2}-x+\frac{x}{\sqrt{x^2+1}}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{\operatorname{asinh}(x)}{2}-\frac{1}{x}+\frac{1}{x\sqrt{x^2+1}}$$

```
[In] integrate((x**2+x-1)/(1+(x**2+1)**(1/2)),x)
```

```
[Out] x*sqrt(x**2 + 1)/2 - x + x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(sqrt(x**2
+ 1) + 1) - asinh(x)/2 - 1/x + 1/(x*sqrt(x**2 + 1))
```

Maxima [F]

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx=\int \frac{x^2+x-1}{\sqrt{x^2+1}+1}\,dx$$

```
[In] integrate((x^2+x-1)/(1+(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*x - 5*arctan(1/2*x) + integrate((x^6 + x^5 - x^4)/(3*x^4 + 16*x^2 + (x^4
+ 8*x^2 + 16)*sqrt(x^2 + 1) + 16), x) + log(x^2 + 4)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.37

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx=\frac{1}{2}\sqrt{x^2+1}(x+2)-x-\frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1}-\frac{1}{x}+\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)-\log\left(|x|\right)-\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right)+\log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

```
[In] integrate((x^2+x-1)/(1+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 1)*(x + 2) - x - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + 1/2*l
og(-x + sqrt(x^2 + 1)) - log(abs(x)) - log(abs(-x + sqrt(x^2 + 1) + 1)) + l
og(abs(-x + sqrt(x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx=\left(\frac{x}{2}+1\right)\sqrt{x^2+1}-\frac{\mathrm{asinh}(x)}{2}-\ln\left(x\right)-x+\frac{\sqrt{x^2+1}}{x}-\frac{1}{x}+\mathrm{atan}\left(\sqrt{x^2+1}\,1\mathrm{i}\right)\,1\mathrm{i}$$

```
[In] int((x + x^2 - 1)/((x^2 + 1)^(1/2) + 1),x)
```

```
[Out] atan((x^2 + 1)^(1/2)*1i)*1i - x - asinh(x)/2 - log(x) + (x/2 + 1)*(x^2 + 1)
^(1/2) + (x^2 + 1)^(1/2)/x - 1/x
```

3.807 $\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx = \frac{1}{12}\left(6x^2+2x^3+\left(4-3x-2x^2\right)\sqrt{1+x^2}-3\text{arcsinh}(x)-6\log\left(1+\sqrt{1+x^2}\right)\right)$$

```
[Out] 1/2*x^2+1/6*x^3-1/4*arcsinh(x)-1/2*ln(1+(x^2+1)^(1/2))+1/12*(-2*x^2-3*x+4)*
(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.91, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6874, 2142, 907, 201, 221, 267}

$$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx = -\frac{\text{arcsinh}(x)}{4}+\frac{x^3}{6}+\frac{x^2}{2}-\frac{1}{4}\sqrt{x^2+1}x-\frac{1}{6}\left(x^2+1\right)^{3/2}+\frac{1}{2\left(\sqrt{x^2+1}+x\right)}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-\log\left(\sqrt{x^2+1}+x+1\right)+\frac{x}{2}$$

```
[In] Int[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]),x]
```

```
[Out] x/2 + x^2/2 + x^3/6 - (x*Sqrt[1 + x^2])/4 - (1 + x^2)^(3/2)/6 + 1/(2*(x + S
qrt[1 + x^2])) - ArcSinh[x]/4 + Log[x + Sqrt[1 + x^2]]/2 - Log[1 + x + Sqrt
[1 + x^2]]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{1}{1+x+\sqrt{1+x^2}}+\frac{x}{1+x+\sqrt{1+x^2}}+\frac{x^2}{1+x+\sqrt{1+x^2}}\right) dx \\ &= -\int \frac{1}{1+x+\sqrt{1+x^2}}\, dx+\int \frac{x}{1+x+\sqrt{1+x^2}}\, dx+\int \frac{x^2}{1+x+\sqrt{1+x^2}}\, dx \end{aligned}$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{2-2x+x^2}{(1-x)^2 x}\,dx, x, 1+x+\sqrt{1+x^2}\right)\right) + \int \left(\frac{1}{2}+\frac{x}{2}-\frac{\sqrt{1+x^2}}{2}\right)dx + \int \left(\frac{x}{2}+\frac{x^2}{2}-\frac{1}{2}x\sqrt{1+x^2}\right)dx$$

$$= \frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-\frac{1}{2}\int \sqrt{1+x^2}\,dx - \frac{1}{2}\int x\sqrt{1+x^2}\,dx - \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{(-1+x)^2}+\frac{2}{x}\right)dx, x, 1+x+\sqrt{1+x^2}\right)$$

$$= \frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-\frac{1}{4}x\sqrt{1+x^2}-\frac{1}{6}\left(1+x^2\right)^{3/2}+\frac{1}{2\left(x+\sqrt{1+x^2}\right)} + \frac{1}{2}\log\left(x+\sqrt{1+x^2}\right)-\log\left(1+x+\sqrt{1+x^2}\right)-\frac{1}{4}\int \frac{1}{\sqrt{1+x^2}}\,dx$$

$$= \frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-\frac{1}{4}x\sqrt{1+x^2}-\frac{1}{6}\left(1+x^2\right)^{3/2}+\frac{1}{2\left(x+\sqrt{1+x^2}\right)} - \frac{1}{4}\sinh^{-1}(x)+\frac{1}{2}\log\left(x+\sqrt{1+x^2}\right)-\log\left(1+x+\sqrt{1+x^2}\right)$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx = \frac{1}{12}\left(2x^2(3+x)+\left(4-3x-2x^2\right)\sqrt{1+x^2}+9\log\left(-x+\sqrt{1+x^2}\right) - 12\log\left(1-x+\sqrt{1+x^2}\right)\right)$$

```
[In] Integrate[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]),x]
[Out] (2*x^2*(3 + x) + (4 - 3*x - 2*x^2)*Sqrt[1 + x^2] + 9*Log[-x + Sqrt[1 + x^2]
] - 12*Log[1 - x + Sqrt[1 + x^2]])/12
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{x^2}{2}-\frac{\ln(x)}{2}+\frac{x^3}{6}-\frac{x\sqrt{x^2+1}}{4}-\frac{\text{arcsinh}(x)}{4}-\frac{(x^2+1)^{\frac{3}{2}}}{6}+\frac{\sqrt{x^2+1}}{2}-\frac{\text{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{2}$	58
trager	$\frac{(x^2+4x+4)(x-1)}{6}+\frac{\left(-\frac{1}{3}x^2-\frac{1}{2}x+\frac{2}{3}\right)\sqrt{x^2+1}}{2}+\frac{\ln\left(\frac{\sqrt{x^2+1}\,x^2-x^3+2x\sqrt{x^2+1}-2x^2+2\sqrt{x^2+1}-2x-2}{x^4}\right)}{4}$	86

```
[In] int((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-1/2*ln(x)+1/6*x^3-1/4*x*(x^2+1)^(1/2)-1/4*arcsinh(x)-1/6*(x^2+1)^(3
/2)+1/2*(x^2+1)^(1/2)-1/2*arctanh(1/(x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx = \frac{1}{6}\,x^3+\frac{1}{2}\,x^2-\frac{1}{12}\,(2\,x^2+3\,x-4)\sqrt{x^2+1} -\frac{1}{2}\,\log\left(x\right)-\frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}+1\right) +\frac{1}{4}\,\log\left(-x+\sqrt{x^2+1}\right)+\frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}-1\right)$$

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="fricas")
[Out] 1/6*x^3 + 1/2*x^2 - 1/12*(2*x^2 + 3*x - 4)*sqrt(x^2 + 1) - 1/2*log(x) - 1/2
*log(-x + sqrt(x^2 + 1) + 1) + 1/4*log(-x + sqrt(x^2 + 1)) + 1/2*log(-x + s
qrt(x^2 + 1) - 1)
```

Sympy [F]

$$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx = \int \frac{x^2+x-1}{x+\sqrt{x^2+1}+1}\,dx$$

```
[In] integrate((x**2+x-1)/(1+x+(x**2+1)**(1/2)),x)
[Out] Integral((x**2 + x - 1)/(x + sqrt(x**2 + 1) + 1), x)
```

Maxima [F]

$$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx = \int \frac{x^2+x-1}{x+\sqrt{x^2+1}+1}\,dx$$

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="maxima")
[Out] 1/4*x^2 - 3/56*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 3)) + 1/4*x + integrate((x
^4 + x^3 - x^2)/(4*x^5 + 12*x^4 + 19*x^3 + 19*x^2 + (4*x^4 + 12*x^3 + 17*x^
2 + 12*x + 4)*sqrt(x^2 + 1) + 12*x + 4), x) - 7/16*log(2*x^2 + 3*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx = \frac{1}{6}\,x^3+\frac{1}{2}\,x^2-\frac{1}{12}\,((2\,x+3)x-4)\sqrt{x^2+1} + \frac{1}{4}\,\log\left(-x+\sqrt{x^2+1}\right)-\frac{1}{2}\,\log\left(|x|\right) - \frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right)+\frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/2*x^2 - 1/12*((2*x + 3)*x - 4)*sqrt(x^2 + 1) + 1/4*log(-x + sqr
t(x^2 + 1)) - 1/2*log(abs(x)) - 1/2*log(abs(-x + sqrt(x^2 + 1) + 1)) + 1/2*
log(abs(-x + sqrt(x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx = \frac{x^2}{2}-\frac{\ln\left(x\right)}{2}-\sqrt{x^2+1}\left(\frac{x^2}{6}+\frac{x}{4}-\frac{1}{3}\right) - \frac{\mathrm{asinh}(x)}{4}+\frac{x^3}{6}+\frac{\mathrm{atan}\left(\sqrt{x^2+1}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

```
[In] int((x + x^2 - 1)/(x + (x^2 + 1)^(1/2) + 1),x)
```

```
[Out] (atan((x^2 + 1)^(1/2)*1i)*1i)/2 - asinh(x)/4 - log(x)/2 - (x^2 + 1)^(1/2)*(
x/4 + x^2/6 - 1/3) + x^2/2 + x^3/6
```

3.808 $\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 14

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x} \, dx = 2\sqrt{-1+x} + 2\log(x)$$

```
[Out] 2*ln(x)+2*(-1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6820}

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x} \, dx = 2\sqrt{x-1} + 2\log(x)$$

```
[In] Int[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x),x]
[Out] 2*Sqrt[-1 + x] + 2*Log[x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{\sqrt{-1+x}} + \frac{2}{x}\right) \, dx \\ &= 2\sqrt{-1+x} + 2\log(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x}\,dx = 2\sqrt{-1+x}+2\log(x)$$

```
[In] Integrate[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x),x]
[Out] 2*Sqrt[-1 + x] + 2*Log[x]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$2\ln\left(x\right)+2\sqrt{x-1}$	13
default	$2\ln\left(x\right)+2\sqrt{x-1}$	13
trager	$2\sqrt{x-1}-2\ln\left(\frac{1}{x}\right)$	15

```
[In] int((x+2*(x-1)^(1/2))/x/(x-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*ln(x)+2*(x-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x}\,dx = 2\sqrt{x-1}+2\log\left(x\right)$$

```
[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x - 1) + 2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x}\,dx = 2\sqrt{x-1}+2\log{(x)}$$

```
[In] integrate((x+2*(-1+x)**(1/2))/x/(-1+x)**(1/2),x)
[Out] 2*sqrt(x - 1) + 2*log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x}\,dx = 2\sqrt{x-1}+2\log{(x)}$$

```
[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x - 1) + 2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x}\,dx = 2\sqrt{x-1}+2\log{(x)}$$

```
[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x - 1) + 2*log(x)
```

Mupad [B] (verification not implemented)

Time = 20.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x}\,dx = 2\ln{(x)}+2\sqrt{x-1}$$

```
[In] int((x + 2*(x - 1)^(1/2))/(x*(x - 1)^(1/2)),x)
[Out] 2*log(x) + 2*(x - 1)^(1/2)
```

3.809 $\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2 dx$

Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2 dx = a^2x+\frac{4}{3}acx^{3/2}+\frac{6}{5}abx^{5/3}+\frac{c^2x^2}{2}+\frac{12}{13}bcx^{13/6}+\frac{3}{7}b^2x^{7/3}$$

```
[Out] a^2*x+4/3*a*c*x^(3/2)+6/5*a*b*x^(5/3)+1/2*c^2*x^2+12/13*b*c*x^(13/6)+3/7*b^
2*x^(7/3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6873, 6874}

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2 dx = a^2x+\frac{6}{5}abx^{5/3}+\frac{4}{3}acx^{3/2}+\frac{3}{7}b^2x^{7/3}+\frac{12}{13}bcx^{13/6}+\frac{c^2x^2}{2}$$

```
[In] Int[(a + c*Sqrt[x] + b*x^(2/3))^2,x]
```

```
[Out] a^2*x + (4*a*c*x^(3/2))/3 + (6*a*b*x^(5/3))/5 + (c^2*x^2)/2 + (12*b*c*x^(13
/6))/13 + (3*b^2*x^(7/3))/7
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = 6\text{Subst}\left(\int x^5\left(a+x^3(c+bx)\right)^2\,dx,x,\sqrt[6]{x}\right)$$
$$= 6\text{Subst}\left(\int x^5\left(a+cx^3+bx^4\right)^2\,dx,x,\sqrt[6]{x}\right)$$
$$= 6\text{Subst}\left(\int \left(a^2x^5+2acx^8+2abx^9+c^2x^{11}+2bcx^{12}+b^2x^{13}\right)\,dx,x,\sqrt[6]{x}\right)$$
$$= a^2x+\frac{4}{3}acx^{3/2}+\frac{6}{5}abx^{5/3}+\frac{c^2x^2}{2}+\frac{12}{13}bcx^{13/6}+\frac{3}{7}b^2x^{7/3}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2\,dx = \frac{2730a^2x+3640acx^{3/2}+3276abx^{5/3}+1365c^2x^2+2520bcx^{13/6}+1170b^2x^{7/3}}{2730}$$

```
[In] Integrate[(a + c*Sqrt[x] + b*x^(2/3))^2,x]
[Out] (2730*a^2*x + 3640*a*c*x^(3/2) + 3276*a*b*x^(5/3) + 1365*c^2*x^2 + 2520*b*c
*x^(13/6) + 1170*b^2*x^(7/3))/2730
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$a^2x+\frac{4acx^{\frac{3}{2}}}{3}+\frac{6abx^{\frac{5}{3}}}{5}+\frac{c^2x^2}{2}+\frac{12bcx^{\frac{13}{6}}}{13}+\frac{3b^2x^{\frac{7}{3}}}{7}$	44
default	$\frac{c^2x^2}{2}+2c\left(\frac{6bx^{\frac{13}{6}}}{13}+\frac{2ax^{\frac{3}{2}}}{3}\right)+a^2x+\frac{3b^2x^{\frac{7}{3}}}{7}+\frac{6abx^{\frac{5}{3}}}{5}$	46

```
[In] int((a+b*x^(2/3)+c*x^(1/2))^2,x,method=_RETURNVERBOSE)
[Out] a^2*x+4/3*a*c*x^(3/2)+6/5*a*b*x^(5/3)+1/2*c^2*x^2+12/13*b*c*x^(13/6)+3/7*b^
2*x^(7/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2\,dx=\frac{3}{7}\,b^2x^{\frac{7}{3}}+\frac{12}{13}\,bcx^{\frac{13}{6}}+\frac{1}{2}\,c^2x^2+\frac{6}{5}\,abx^{\frac{5}{3}}+\frac{4}{3}\,acx^{\frac{3}{2}}+a^2x$$

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="fricas")
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5/3) + 4/3*
a*c*x^(3/2) + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2\,dx=a^2x+\frac{6abx^{\frac{5}{3}}}{5}+\frac{4acx^{\frac{3}{2}}}{3}+\frac{3b^2x^{\frac{7}{3}}}{7}+\frac{12bcx^{\frac{13}{6}}}{13}+\frac{c^2x^2}{2}$$

```
[In] integrate((a+b*x**(2/3)+c*x**(1/2))**2,x)
[Out] a**2*x + 6*a*b*x**(5/3)/5 + 4*a*c*x**(3/2)/3 + 3*b**2*x**(7/3)/7 + 12*b*c*x
**(13/6)/13 + c**2*x**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2\,dx=\frac{3}{7}\,b^2x^{\frac{7}{3}}+\frac{12}{13}\,bcx^{\frac{13}{6}}+\frac{1}{2}\,c^2x^2+a^2x+\frac{2}{15}\left(9\,bx^{\frac{5}{3}}+10\,cx^{\frac{3}{2}}\right)a$$

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="maxima")
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + a^2*x + 2/15*(9*b*x^(5
/3) + 10*c*x^(3/2))*a
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2 \, dx = \frac{3}{7}\,b^2x^{\frac{7}{3}} + \frac{12}{13}\,bcx^{\frac{13}{6}} + \frac{1}{2}\,c^2x^2 + \frac{6}{5}\,abx^{\frac{5}{3}} + \frac{4}{3}\,acx^{\frac{3}{2}} + a^2x$$

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5/3) + 4/3*
a*c*x^(3/2) + a^2*x
```

Mupad [B] (verification not implemented)

Time = 21.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2 \, dx = a^2\,x + \frac{3\,b^2\,x^{7/3}}{7} + \frac{c^2\,x^2}{2} + \frac{6\,a\,b\,x^{5/3}}{5} + \frac{4\,a\,c\,x^{3/2}}{3} + \frac{12\,b\,c\,x^{13/6}}{13}$$

```
[In] int((a + b*x^(2/3) + c*x^(1/2))^2,x)
```

```
[Out] a^2*x + (3*b^2*x^(7/3))/7 + (c^2*x^2)/2 + (6*a*b*x^(5/3))/5 + (4*a*c*x^(3/2
))/3 + (12*b*c*x^(13/6))/13
```

3.810 $\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 114

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx=a^3x+2a^2cx^{3/2}+\frac{9}{5}a^2bx^{5/3}+\frac{3}{2}ac^2x^2+\frac{36}{13}abcx^{13/6}+\frac{9}{7}ab^2x^{7/3}+\frac{2}{5}c^3x^{5/2}+\frac{9}{8}bc^2x^{8/3}+\frac{18}{17}b^2cx^{17/6}+\frac{b^3x^3}{3}$$

```
[Out] a^3*x+2*a^2*c*x^(3/2)+9/5*a^2*b*x^(5/3)+3/2*a*c^2*x^2+36/13*a*b*c*x^(13/6)+
9/7*a*b^2*x^(7/3)+2/5*c^3*x^(5/2)+9/8*b*c^2*x^(8/3)+18/17*b^2*c*x^(17/6)+1/
3*b^3*x^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6873, 6874}

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx=a^3x+\frac{9}{5}a^2bx^{5/3}+2a^2cx^{3/2}+\frac{9}{7}ab^2x^{7/3}+\frac{36}{13}abcx^{13/6}+\frac{3}{2}ac^2x^2+\frac{b^3x^3}{3}+\frac{18}{17}b^2cx^{17/6}+\frac{9}{8}bc^2x^{8/3}+\frac{2}{5}c^3x^{5/2}$$

```
[In] Int[(a + c*Sqrt[x] + b*x^(2/3))^3,x]
```

```
[Out] a^3*x + 2*a^2*c*x^(3/2) + (9*a^2*b*x^(5/3))/5 + (3*a*c^2*x^2)/2 + (36*a*b*c
*x^(13/6))/13 + (9*a*b^2*x^(7/3))/7 + (2*c^3*x^(5/2))/5 + (9*b*c^2*x^(8/3))
/8 + (18*b^2*c*x^(17/6))/17 + (b^3*x^3)/3
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 6\text{Subst}\left(\int x^5\left(a+x^3(c+bx)\right)^3\,dx,x,\sqrt[6]{x}\right)\\
&= 6\text{Subst}\left(\int x^5\left(a+cx^3+bx^4\right)^3\,dx,x,\sqrt[6]{x}\right)\\
&= 6\text{Subst}\left(\int \left(a^3x^5+3a^2cx^8+3a^2bx^9+3ac^2x^{11}+6abcx^{12}+3ab^2x^{13}+c^3x^{14}\right.\right.\\
&\qquad \left.\left.+3bc^2x^{15}+3b^2cx^{16}+b^3x^{17}\right)\,dx,x,\sqrt[6]{x}\right)\\
&= a^3x+2a^2cx^{3/2}+\frac{9}{5}a^2bx^{5/3}+\frac{3}{2}ac^2x^2+\frac{36}{13}abcx^{13/6}\\
&\quad +\frac{9}{7}ab^2x^{7/3}+\frac{2}{5}c^3x^{5/2}+\frac{9}{8}bc^2x^{8/3}+\frac{18}{17}b^2cx^{17/6}+\frac{b^3x^3}{3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx=\frac{185640a^3x+371280a^2cx^{3/2}+334152a^2bx^{5/3}+278460ac^2x^2+514080abcx^{13/6}+238680ab^2x^{7\ldots}}{185640}$$

```
[In] Integrate[(a + c*Sqrt[x] + b*x^(2/3))^3,x]
```

```
[Out] (185640*a^3*x + 371280*a^2*c*x^(3/2) + 334152*a^2*b*x^(5/3) + 278460*a*c^2*
x^2 + 514080*a*b*c*x^(13/6) + 238680*a*b^2*x^(7/3) + 74256*c^3*x^(5/2) + 20
8845*b*c^2*x^(8/3) + 196560*b^2*c*x^(17/6) + 61880*b^3*x^3)/185640
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

method	result
derivativedivides	$a^{3}x+2a^{2}c\,x^{\frac{3}{2}}+\frac{9a^{2}b\,x^{\frac{5}{3}}}{5}+\frac{3a\,c^{2}x^{2}}{2}+\frac{36abc\,x^{\frac{13}{6}}}{13}+\frac{9a\,b^{2}x^{\frac{7}{3}}}{7}+\frac{2c^{3}x^{\frac{5}{2}}}{5}+\frac{9b\,c^{2}x^{\frac{8}{3}}}{8}+\frac{18b^{2}c\,x^{\frac{17}{6}}}{17}+\frac{b^{3}x^{3}}{3}$
default	$\frac{2c^{3}x^{\frac{5}{2}}}{5}+3c^{2}\left(\frac{3b\,x^{\frac{8}{3}}}{8}+\frac{a\,x^{2}}{2}\right)+3c\left(\frac{6b^{2}x^{\frac{17}{6}}}{17}+\frac{12ab\,x^{\frac{13}{6}}}{13}+\frac{2a^{2}x^{\frac{3}{2}}}{3}\right)+a^{3}x+\frac{b^{3}x^{3}}{3}+\frac{9a^{2}b\,x^{\frac{5}{3}}}{5}+\frac{9a\,b^{2}}{7}$

```
[In] int((a+b*x^(2/3)+c*x^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x+2*a^2*c*x^(3/2)+9/5*a^2*b*x^(5/3)+3/2*a*c^2*x^2+36/13*a*b*c*x^(13/6)+
9/7*a*b^2*x^(7/3)+2/5*c^3*x^(5/2)+9/8*b*c^2*x^(8/3)+18/17*b^2*c*x^(17/6)+1/
3*b^3*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.80

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx=\frac{1}{3}\,b^{3}x^{3}+\frac{18}{17}\,b^{2}cx^{\frac{17}{6}}+\frac{9}{7}\,ab^{2}x^{\frac{7}{3}}+\frac{36}{13}\,abcx^{\frac{13}{6}}+\frac{3}{2}\,ac^{2}x^{2}+a^{3}x+\frac{9}{40}\,\left(5\,bc^{2}x^{2}+8\,a^{2}bx\right)x^{\frac{2}{3}}+\frac{2}{5}\,\left(c^{3}x^{2}+5\,a^{2}cx\right)\sqrt{x}$$

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/7*a*b^2*x^(7/3) + 36/13*a*b*c*x^(13/
6) + 3/2*a*c^2*x^2 + a^3*x + 9/40*(5*b*c^2*x^2 + 8*a^2*b*x)*x^(2/3) + 2/5*(
c^3*x^2 + 5*a^2*c*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx=a^{3}x+\frac{9a^{2}bx^{\frac{5}{3}}}{5}+2a^{2}cx^{\frac{3}{2}}+\frac{9ab^{2}x^{\frac{7}{3}}}{7}+\frac{36abcx^{\frac{13}{6}}}{13}+\frac{3ac^{2}x^{2}}{2}+\frac{b^{3}x^{3}}{3}+\frac{18b^{2}cx^{\frac{17}{6}}}{17}+\frac{9bc^{2}x^{\frac{8}{3}}}{8}+\frac{2c^{3}x^{\frac{5}{2}}}{5}$$

```
[In] integrate((a+b*x**(2/3)+c*x**(1/2))**3,x)
```

```
[Out] a**3*x + 9*a**2*b*x**(5/3)/5 + 2*a**2*c*x**(3/2) + 9*a*b**2*x**(7/3)/7 + 36
*a*b*c*x**(13/6)/13 + 3*a*c**2*x**2/2 + b**3*x**3/3 + 18*b**2*c*x**(17/6)/1
7 + 9*b*c**2*x**(8/3)/8 + 2*c**3*x**(5/2)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3 \, dx = \frac{1}{3}\,b^3x^3+\frac{18}{17}\,b^2cx^{\frac{17}{6}}+\frac{9}{8}\,bc^2x^{\frac{8}{3}}+\frac{2}{5}\,c^3x^{\frac{5}{2}} + a^3x+\frac{1}{5}\left(9\,bx^{\frac{5}{3}}+10\,cx^{\frac{3}{2}}\right)a^2+\frac{3}{182}\left(78\,b^2x^{\frac{7}{3}}+168\,bcx^{\frac{13}{6}}+91\,c^2x^2\right)a$$

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*x^(5/2) +
a^3*x + 1/5*(9*b*x^(5/3) + 10*c*x^(3/2))*a^2 + 3/182*(78*b^2*x^(7/3) + 168*
b*c*x^(13/6) + 91*c^2*x^2)*a
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3 \, dx = \frac{1}{3}\,b^3x^3+\frac{18}{17}\,b^2cx^{\frac{17}{6}}+\frac{9}{8}\,bc^2x^{\frac{8}{3}}+\frac{2}{5}\,c^3x^{\frac{5}{2}} + \frac{9}{7}\,ab^2x^{\frac{7}{3}}+\frac{36}{13}\,abcx^{\frac{13}{6}}+\frac{3}{2}\,ac^2x^2+\frac{9}{5}\,a^2bx^{\frac{5}{3}}+2\,a^2cx^{\frac{3}{2}}+a^3x$$

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*x^(5/2) +
9/7*a*b^2*x^(7/3) + 36/13*a*b*c*x^(13/6) + 3/2*a*c^2*x^2 + 9/5*a^2*b*x^(5/3
) + 2*a^2*c*x^(3/2) + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3 \, dx = a^3\,x+\frac{b^3\,x^3}{3}+\frac{2\,c^3\,x^{5/2}}{5}+\frac{9\,a^2\,b\,x^{5/3}}{5}+\frac{9\,a\,b^2\,x^{7/3}}{7} + \frac{3\,a\,c^2\,x^2}{2}+2\,a^2\,c\,x^{3/2}+\frac{9\,b\,c^2\,x^{8/3}}{8}+\frac{18\,b^2\,c\,x^{17/6}}{17}+\frac{36\,a\,b\,c\,x^{13/6}}{13}$$

```
[In] int((a + b*x^(2/3) + c*x^(1/2))^3,x)
```

```
[Out] a^3*x + (b^3*x^3)/3 + (2*c^3*x^(5/2))/5 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x^
(7/3))/7 + (3*a*c^2*x^2)/2 + 2*a^2*c*x^(3/2) + (9*b*c^2*x^(8/3))/8 + (18*b^
2*c*x^(17/6))/17 + (36*a*b*c*x^(13/6))/13
```

3.811 $\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 58

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3} \, dx=\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] arctanh((a-b*(1-1/x^2))^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)+(a-b*(1-1/x^2))^(1/2
)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {528, 457, 81, 65, 214}

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3} \, dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}$$

```
[In] Int[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3),x]
```

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqrt[a - b]]/
Sqrt[a - b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1-\frac{1}{x^2}}{\sqrt{a-b+\frac{b}{x^2}}x}\,dx \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1-x}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right)
\end{aligned}$$

$$= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a-b}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\right)}{b}$$

$$= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.91

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx$$

$$= \frac{\sqrt{-a+b}(b+ax^2-bx^2) - 2bx\sqrt{b+ax^2-bx^2}\arctan\left(\frac{\sqrt{-a+b}x}{\sqrt{b}-\sqrt{b+(a-b)x^2}}\right)}{b\sqrt{-a+b}\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^2}$$

```
[In] Integrate[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3),x]
[Out] (Sqrt[-a + b]*(b + a*x^2 - b*x^2) - 2*b*x*Sqrt[b + a*x^2 - b*x^2]*ArcTan[(S
qrt[-a + b]*x)/(Sqrt[b] - Sqrt[b + (a - b)*x^2])])/(b*Sqrt[-a + b]*Sqrt[a +
 b*(-1 + x^(-2))]*x^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(50) = 100.

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.76

method	result	size
default	$\frac{\sqrt{a\,x^2-b\,x^2+b}\left(\ln\left(x\sqrt{a-b}+\sqrt{a\,x^2-b\,x^2+b}\right)bx+\sqrt{a\,x^2-b\,x^2+b}\,\sqrt{a-b}\right)}{\sqrt{\frac{a\,x^2-b\,x^2+b}{x^2}}\,x^2\sqrt{a-b}\,b}$	102
risch	$\frac{a\,x^2-b\,x^2+b}{b\,x^2\sqrt{\frac{a\,x^2-b\,x^2+b}{x^2}}} + \frac{\ln\left(x\sqrt{a-b}+\sqrt{x^2(a-b)+b}\right)\sqrt{a\,x^2-b\,x^2+b}}{\sqrt{a-b}\,\sqrt{\frac{a\,x^2-b\,x^2+b}{x^2}}\,x}$	110

```
[In] int((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (a*x^2-b*x^2+b)^(1/2)*(ln(x*(a-b)^(1/2)+(a*x^2-b*x^2+b)^(1/2))*b*x+(a*x^2-b
*x^2+b)^(1/2)*(a-b)^(1/2))/((a*x^2-b*x^2+b)/x^2)^(1/2)/x^2/(a-b)^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.10

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx = \left[\frac{\sqrt{a-b}b\log\left(-2(a-b)x^2-2\sqrt{a-b}x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}-b\right)+2(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{2(ab-b^2)}, \frac{\sqrt{-a+b}b\arctan\left(-\frac{\sqrt{\cdot}}{}\right.}{}\right.$$

```
[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*b*log(-2*(a - b)*x^2 - 2*sqrt(a - b)*x^2*sqrt(((a - b)*x^
2 + b)/x^2) - b) + 2*(a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2), (sqr
t(-a + b)*b*arctan(-sqrt(-a + b)*x^2*sqrt(((a - b)*x^2 + b)/x^2)/((a - b)*x
^2 + b)) + (a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx = -\frac{\begin{cases} -\frac{1}{\sqrt{a}x^2} & \text{for}\: b=0 \\ -\frac{2\sqrt{a-b+\frac{b}{x^2}}}{b} & \text{otherwise} \end{cases}}{2} - \frac{\begin{cases} \frac{2\operatorname{atan}\left(\frac{\sqrt{a-b+\frac{b}{x^2}}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} & \text{for}\: b\neq 0 \\ -\frac{\log(x^2)}{\sqrt{a}} & \text{otherwise} \end{cases}}{2}$$

```
[In] integrate((x**2-1)/x**3/(a-b+b/x**2)**(1/2),x)
```

```
[Out] -Piecewise((-1/(sqrt(a)*x**2), Eq(b, 0)), (-2*sqrt(a - b + b/x**2)/b, True)
)/2 - Piecewise((2*atan(sqrt(a - b + b/x**2)/sqrt(-a + b))/sqrt(-a + b), Ne
(b, 0)), (-log(x**2)/sqrt(a), True))/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}\,x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(44) = 88$.

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.59

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}\,x^3}\,dx = -\frac{\log\left(\left(\sqrt{a-b}x-\sqrt{ax^2-bx^2+b}\right)^2\right)}{2\sqrt{a-b}\operatorname{sgn}(x)} - \frac{2\sqrt{a-b}}{\left(\left(\sqrt{a-b}x-\sqrt{ax^2-bx^2+b}\right)^2-b\right)\operatorname{sgn}(x)}$$

```
[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log((sqrt(a - b)*x - sqrt(a*x^2 - b*x^2 + b))^2)/(sqrt(a - b)*sgn(x))
- 2*sqrt(a - b)/(((sqrt(a - b)*x - sqrt(a*x^2 - b*x^2 + b))^2 - b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 21.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}\,x^3}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a-b+\frac{b}{x^2}}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a-b+\frac{b}{x^2}}}{b}$$

```
[In] int((x^2 - 1)/(x^3*(a - b + b/x^2)^(1/2)),x)
```

```
[Out] atanh((a - b + b/x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) + (a - b + b/x^2)^
(1/2)/b
```

3.812 $\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3} dx = \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] arctanh((a-b*(1-1/x^2))^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)+(a-b*(1-1/x^2))^(1/2
)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2003, 528, 457, 81, 65, 214}

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}$$

```
[In] Int[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))]*x^3),x]
```

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqrt[a - b]]/
Sqrt[a - b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2003

```
Int[(Pq_)*(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*Pq*Expand
ToSum[u, x]^p, x] /; FreeQ[{c, m, p}, x] && PolyQ[Pq, x] && BinomialQ[u, x]
 &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx \\
&= \int \frac{1-\frac{1}{x^2}}{\sqrt{a-b+\frac{b}{x^2}}x}\,dx
\end{aligned}
$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1-x}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right)$$

$$= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a-b}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\right)}{b}$$

$$= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.91

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx$$

$$= \frac{\sqrt{-a+b}(b+ax^2-bx^2)-2bx\sqrt{b+ax^2-bx^2}\arctan\left(\frac{\sqrt{-a+b}x}{\sqrt{b}-\sqrt{b+(a-b)x^2}}\right)}{b\sqrt{-a+b}\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^2}$$

```
[In] Integrate[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))]*x^3),x]
[Out] (Sqrt[-a + b]*(b + a*x^2 - b*x^2) - 2*b*x*Sqrt[b + a*x^2 - b*x^2]*ArcTan[(S
qrt[-a + b]*x)/(Sqrt[b] - Sqrt[b + (a - b)*x^2])])/(b*Sqrt[-a + b]*Sqrt[a +
 b*(-1 + x^(-2))]*x^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(50) = 100.

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.76

method	result	size
default	$\frac{\sqrt{a\,x^2-b\,x^2+b}\,\left(\ln\left(x\sqrt{a-b}+\sqrt{a\,x^2-b\,x^2+b}\right)bx+\sqrt{a\,x^2-b\,x^2+b}\,\sqrt{a-b}\right)}{\sqrt{\frac{a\,x^2-b\,x^2+b}{x^2}}\,x^2\sqrt{a-b}\,b}$	102
risch	$\frac{a\,x^2-b\,x^2+b}{b\,x^2\sqrt{\frac{a\,x^2-b\,x^2+b}{x^2}}}+\frac{\ln\left(x\sqrt{a-b}+\sqrt{x^2(a-b)+b}\right)\sqrt{a\,x^2-b\,x^2+b}}{\sqrt{a-b}\,\sqrt{\frac{a\,x^2-b\,x^2+b}{x^2}}\,x}$	110

```
[In] int((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x^2-b*x^2+b)^(1/2)*(ln(x*(a-b)^(1/2)+(a*x^2-b*x^2+b)^(1/2))*b*x+(a*x^2-b
*x^2+b)^(1/2)*(a-b)^(1/2))/((a*x^2-b*x^2+b)/x^2)^(1/2)/x^2/(a-b)^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.10

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx = \left[\frac{\sqrt{a-b}b\log\left(-2\,(a-b)x^2-2\sqrt{a-b}x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}-b\right)+2\,(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{2\,(ab-b^2)}, \frac{\sqrt{-a+b}b\arctan\left(-\cdots\right.}{}\right.$$

```
[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*b*log(-2*(a - b)*x^2 - 2*sqrt(a - b)*x^2*sqrt(((a - b)*x^
2 + b)/x^2) - b) + 2*(a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2), (sqr
t(-a + b)*b*arctan(-sqrt(-a + b)*x^2*sqrt(((a - b)*x^2 + b)/x^2)/((a - b)*x
^2 + b)) + (a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2)]
```

Sympy [A] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx = -\frac{\begin{cases} -\frac{1}{\sqrt{a}x^2} & \text{for}\: b=0 \\ -\frac{2\sqrt{a-b+\frac{b}{x^2}}}{b} & \text{otherwise} \end{cases}}{2} - \frac{\begin{cases} \frac{2\operatorname{atan}\left(\frac{\sqrt{a-b+\frac{b}{x^2}}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} & \text{for}\: b\neq 0 \\ -\frac{\log\left(x^2\right)}{\sqrt{a}} & \text{otherwise} \end{cases}}{2}$$

```
[In] integrate((x**2-1)/x**3/(a+b*(-1+1/x**2))**(1/2),x)
```

```
[Out] -Piecewise((-1/(sqrt(a)*x**2), Eq(b, 0)), (-2*sqrt(a - b + b/x**2)/b, True)
)/2 - Piecewise((2*atan(sqrt(a - b + b/x**2)/sqrt(-a + b))/sqrt(-a + b), Ne
(b, 0)), (-log(x**2)/sqrt(a), True))/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(44) = 88$.

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.59

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx = -\frac{\log\left(\left(\sqrt{a-b}x-\sqrt{ax^2-bx^2+b}\right)^2\right)}{2\sqrt{a-b}\operatorname{sgn}(x)} - \frac{2\sqrt{a-b}}{\left(\left(\sqrt{a-b}x-\sqrt{ax^2-bx^2+b}\right)^2-b\right)\operatorname{sgn}(x)}$$

```
[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="giac")
[Out] -1/2*log((sqrt(a - b)*x - sqrt(a*x^2 - b*x^2 + b))^2)/(sqrt(a - b)*sgn(x))
- 2*sqrt(a - b)/(((sqrt(a - b)*x - sqrt(a*x^2 - b*x^2 + b))^2 - b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 21.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx = \frac{\sqrt{a+b\left(\frac{1}{x^2}-1\right)}}{b} + \frac{\ln\left(x^2\left(2a-2b+2\sqrt{a-b}\sqrt{a+b\left(\frac{1}{x^2}-1\right)}+\frac{b}{x^2}\right)\right)}{2\sqrt{a-b}}$$

```
[In] int((x^2 - 1)/(x^3*(a + b*(1/x^2 - 1))^(1/2)),x)
[Out] (a + b*(1/x^2 - 1))^(1/2)/b + log(x^2*(2*a - 2*b + 2*(a - b)^(1/2)*(a + b*(
1/x^2 - 1))^(1/2) + b/x^2))/(2*(a - b)^(1/2))
```

3.813 $\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{5}x}{2\sqrt{9+x^2}}\right)}{2\sqrt{5}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{9+x^2}}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] 1/10*arctan(1/2*x*5^(1/2)/(x^2+9)^(1/2))*5^(1/2)-1/5*arctanh(1/5*(x^2+9)^(1
/2)*5^(1/2))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1024, 385, 209, 455, 65, 213}

$$\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{5}x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+9}}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[In] Int[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]),x]
```

```
[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[9 + x^2])]/(2*Sqrt[5]) - ArcTanh[Sqrt[9 + x^2]/S
qrt[5]]/Sqrt[5]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(4+x^2)\sqrt{9+x^2}}\,dx + \int \frac{x}{(4+x^2)\sqrt{9+x^2}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(4+x)\sqrt{9+x}}\,dx, x, x^2\right) + \text{Subst}\left(\int \frac{1}{4+5x^2}\,dx, x, \frac{x}{\sqrt{9+x^2}}\right) \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{9+x^2}}\right)}{2\sqrt{5}} + \text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{9+x^2}\right) \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{9+x^2}}\right)}{2\sqrt{5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{9+x^2}}{\sqrt{5}}\right)}{\sqrt{5}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx = -\frac{\arctan\left(\frac{4+x^2-x\sqrt{9+x^2}}{2\sqrt{5}}\right)+2\text{arctanh}\left(\frac{\sqrt{9+x^2}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[In] Integrate[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]),x]
[Out] -1/2*(ArcTan[(4 + x^2 - x*Sqrt[9 + x^2])/(2*Sqrt[5])] + 2*ArcTanh[Sqrt[9 +
x^2]/Sqrt[5]])/Sqrt[5]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

method	result
default	$\frac{\arctan\left(\frac{x\sqrt{5}}{2\sqrt{x^2+9}}\right)\sqrt{5}}{10}-\frac{\text{arctanh}\left(\frac{\sqrt{x^2+9}\,\sqrt{5}}{5}\right)\sqrt{5}}{5}$
trager	$\text{RootOf}\left(1280\_Z^4-96\_Z^2+5\right)\ln\left(-\frac{-6400\,\text{RootOf}\left(1280\_Z^4-96\_Z^2+5\right)^5 x+1120\,\text{RootOf}\left(1280\_Z^4-96\_Z\right.}{}\right.$

```
[In] int((x+1)/(x^2+4)/(x^2+9)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/10*arctan(1/2*x*5^(1/2)/(x^2+9)^(1/2))*5^(1/2)-1/5*arctanh(1/5*(x^2+9)^(1
/2)*5^(1/2))*5^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.36

$$\begin{aligned}\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx = &-\frac{1}{20}\sqrt{5}\sqrt{4i+3}\log\left(-(i-2)\sqrt{5}\sqrt{4i+3}-5x+5\sqrt{x^2+9}-10i\right)\\ &+\frac{1}{20}\sqrt{5}\sqrt{4i+3}\log\left((i-2)\sqrt{5}\sqrt{4i+3}-5x+5\sqrt{x^2+9}-10i\right)\\ &-\frac{1}{20}\sqrt{5}\sqrt{-4i+3}\log\left((i+2)\sqrt{5}\sqrt{-4i+3}-5x+5\sqrt{x^2+9}\right.\\ &\left.+10i\right)+\frac{1}{20}\sqrt{5}\sqrt{-4i+3}\log\left(-(i+2)\sqrt{5}\sqrt{-4i+3}-5x\right.\\ &\left.+5\sqrt{x^2+9}+10i\right)\end{aligned}$$

```
[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(5)*sqrt(4*I + 3)*log(-(I - 2)*sqrt(5)*sqrt(4*I + 3) - 5*x + 5*sq
rt(x^2 + 9) - 10*I) + 1/20*sqrt(5)*sqrt(4*I + 3)*log((I - 2)*sqrt(5)*sqrt(4
*I + 3) - 5*x + 5*sqrt(x^2 + 9) - 10*I) - 1/20*sqrt(5)*sqrt(-4*I + 3)*log((
I + 2)*sqrt(5)*sqrt(-4*I + 3) - 5*x + 5*sqrt(x^2 + 9) + 10*I) + 1/20*sqrt(5
)*sqrt(-4*I + 3)*log(-(I + 2)*sqrt(5)*sqrt(-4*I + 3) - 5*x + 5*sqrt(x^2 + 9
) + 10*I)
```

Sympy [F]

$$\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx = \int \frac{x+1}{(x^2+4)\sqrt{x^2+9}}\,dx$$

```
[In] integrate((1+x)/(x**2+4)/(x**2+9)**(1/2),x)
```

```
[Out] Integral((x + 1)/((x**2 + 4)*sqrt(x**2 + 9)), x)
```

Maxima [F]

$$\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx = \int \frac{x+1}{\sqrt{x^2+9}(x^2+4)}\,dx$$

```
[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^2 + 9)*(x^2 + 4)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(38) = 76$.

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.32

$$\begin{aligned}\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx = &-\frac{1}{10}\sqrt{5}\arctan\left(\frac{1}{2}x-\frac{1}{2}\sqrt{5}-\frac{1}{2}\sqrt{x^2+9}\right)\\ &-\frac{1}{10}\sqrt{5}\arctan\left(-\frac{1}{2}x-\frac{1}{2}\sqrt{5}+\frac{1}{2}\sqrt{x^2+9}\right)\\ &+\frac{1}{10}\sqrt{5}\log\left(\left(x-\sqrt{x^2+9}\right)^2+2\sqrt{5}\left(x-\sqrt{x^2+9}\right)+9\right)\\ &-\frac{1}{10}\sqrt{5}\log\left(\left(x-\sqrt{x^2+9}\right)^2-2\sqrt{5}\left(x-\sqrt{x^2+9}\right)+9\right)\end{aligned}$$

```
[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*arctan(1/2*x - 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) - 1/10*sqrt(5
)*arctan(-1/2*x - 1/2*sqrt(5) + 1/2*sqrt(x^2 + 9)) + 1/10*sqrt(5)*log((x -
sqrt(x^2 + 9))^2 + 2*sqrt(5)*(x - sqrt(x^2 + 9)) + 9) - 1/10*sqrt(5)*log((x
 - sqrt(x^2 + 9))^2 - 2*sqrt(5)*(x - sqrt(x^2 + 9)) + 9)
```

Mupad [B] (verification not implemented)

Time = 20.64 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx = \sqrt{5}\left(\ln(x-2\mathrm{i})-\ln\left(\sqrt{5}\sqrt{x^2+9}+9+x\,2\mathrm{i}\right)\right)\left(\frac{1}{10}-\frac{1}{20}\mathrm{i}\right) + \sqrt{5}\left(\ln(x+2\mathrm{i})-\ln\left(\sqrt{5}\sqrt{x^2+9}+9-x\,2\mathrm{i}\right)\right)\left(\frac{1}{10}+\frac{1}{20}\mathrm{i}\right)$$

```
[In] int((x + 1)/((x^2 + 4)*(x^2 + 9)^(1/2)),x)
```

```
[Out] 5^(1/2)*(log(x - 2i) - log(x*2i + 5^(1/2)*(x^2 + 9)^(1/2) + 9))*(1/10 - 1i/
20) + 5^(1/2)*(log(x + 2i) - log(5^(1/2)*(x^2 + 9)^(1/2) - x*2i + 9))*(1/10
 + 1i/20)
```

3.814 $\int x\left(1+\sqrt{1-x^2}\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] 1/2*x^2-1/3*(-x^2+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {14, 267}

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

```
[In] Int[x*(1 + Sqrt[1 - x^2]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(x + x\sqrt{1-x^2}\right) dx \\ &= \frac{x^2}{2} + \int x\sqrt{1-x^2}\, dx \\ &= \frac{x^2}{2} - \frac{1}{3}\left(1-x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x\left(1+\sqrt{1-x^2}\right) dx = -\frac{1}{3}\left(1-x^2\right)^{3/2} + \frac{1}{2}\left(-1+x^2\right)$$

```
[In] Integrate[x*(1 + Sqrt[1 - x^2]),x]
[Out] -1/3*(1 - x^2)^(3/2) + (-1 + x^2)/2
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{x^2}{2} - \frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3}$	18
derivativedivides	$\frac{x^2}{2} - \frac{1}{2} - \frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3}$	19
trager	$\frac{x^2}{2} + \left(\frac{x^2}{3} - \frac{1}{3}\right)\sqrt{-x^2+1}$	24

```
[In] int(x*(1+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-1/3*(-x^2+1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{1}{2}\,x^2+\frac{1}{3}\,(x^2-1)\sqrt{-x^2+1}$$

```
[In] integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="fricas")
[Out] 1/2*x^2 + 1/3*(x^2 - 1)*sqrt(-x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{x^2\sqrt{1-x^2}}{3}+\frac{x^2}{2}-\frac{\sqrt{1-x^2}}{3}$$

```
[In] integrate(x*(1+(-x**2+1)**(1/2)),x)
[Out] x**2*sqrt(1 - x**2)/3 + x**2/2 - sqrt(1 - x**2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{1}{2}\,x^2-\frac{1}{3}\,(-x^2+1)^{\frac{3}{2}}$$

```
[In] integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{1}{2}\,x^2-\frac{1}{3}\,(-x^2+1)^{\frac{3}{2}}-\frac{1}{2}$$

```
[In] integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="giac")
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2) - 1/2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{x^2}{2}+\sqrt{1-x^2}\,\left(\frac{x^2}{3}-\frac{1}{3}\right)$$

```
[In] int(x*((1 - x^2)^(1/2) + 1),x)
[Out] x^2/2 + (1 - x^2)^(1/2)*(x^2/3 - 1/3)
```

3.815 $\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx=\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] 1/2*x^2-1/3*(-x^2+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {14, 267}

$$\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx=\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

```
[In] Int[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(x+x\sqrt{1-x^2}\right) dx\\ &= \frac{x^2}{2}+\int x\sqrt{1-x^2}\,dx\\ &= \frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right) dx = -\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}\left(-1+x^2\right)$$

```
[In] Integrate[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]
[Out] -1/3*(1 - x^2)^(3/2) + (-1 + x^2)/2
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{\sqrt{1-x}\,\sqrt{x+1}\,\left(x^2-1\right)}{3}+\frac{x^2}{2}$	26

```
[In] int(x*(1+(1-x)^(1/2)*(x+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/3*(1-x)^(1/2)*(x+1)^(1/2)*(x^2-1)+1/2*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right) dx = \frac{1}{2}\,x^2+\frac{1}{3}\,\left(x^2-1\right)\sqrt{x+1}\sqrt{-x+1}$$

```
[In] integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="fricas")
[Out] 1/2*x^2 + 1/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [F]

$$\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx=\int x\left(\sqrt{1-x}\sqrt{x+1}+1\right)\,dx$$

```
[In] integrate(x*(1+(1-x)**(1/2)*(1+x)**(1/2)),x)
[Out] Integral(x*(sqrt(1 - x)*sqrt(x + 1) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx=\frac{1}{2}\,x^2-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(17) = 34.

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.35

$$\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx=\frac{1}{2}\,(x+1)^2+\frac{1}{6}\,((2\,x-5)(x+1)+9)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\,\sqrt{x+1}(x-2)\sqrt{-x+1}-x-1$$

```
[In] integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="giac")
[Out] 1/2*(x + 1)^2 + 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*
sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - x - 1
```

Mupad [B] (verification not implemented)

Time = 21.38 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx=\frac{x^2}{2}-\frac{\sqrt{1-x}\,\left(-\frac{x^3}{3}-\frac{x^2}{3}+\frac{x}{3}+\frac{1}{3}\right)}{\sqrt{x+1}}$$

```
[In] int(x*((1 - x)^(1/2)*(x + 1)^(1/2) + 1),x)
[Out] x^2/2 - ((1 - x)^(1/2)*(x/3 - x^2/3 - x^3/3 + 1/3))/(x + 1)^(1/2)
```

3.816 $\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 33

$$\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx=\frac{x^2}{2}+\sqrt{2+x}\sqrt{3+x}-5\text{arcsinh}\left(\sqrt{2+x}\right)$$

```
[Out] 1/2*x^2-5*arcsinh((2+x)^(1/2))+(2+x)^(1/2)*(3+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {14, 81, 56, 221}

$$\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx=-5\text{arcsinh}\left(\sqrt{x+2}\right)+\frac{x^2}{2}+\sqrt{x+2}\sqrt{x+3}$$

```
[In] Int[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]
```

```
[Out] x^2/2 + Sqrt[2 + x]*Sqrt[3 + x] - 5*ArcSinh[Sqrt[2 + x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(x+\frac{x}{\sqrt{2+x}\sqrt{3+x}}\right) dx\\
&= \frac{x^2}{2}+\int \frac{x}{\sqrt{2+x}\sqrt{3+x}}\,dx\\
&= \frac{x^2}{2}+\sqrt{2+x}\sqrt{3+x}-\frac{5}{2}\int \frac{1}{\sqrt{2+x}\sqrt{3+x}}\,dx\\
&= \frac{x^2}{2}+\sqrt{2+x}\sqrt{3+x}-5\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{2+x}\right)\\
&= \frac{x^2}{2}+\sqrt{2+x}\sqrt{3+x}-5\sinh^{-1}\left(\sqrt{2+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

$$\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right) dx=\frac{x^2}{2}+\frac{\sqrt{3+x}}{\sqrt{2+x}\left(-1+\frac{3+x}{2+x}\right)}-5\text{arctanh}\left(\frac{\sqrt{3+x}}{\sqrt{2+x}}\right)$$

```
[In] Integrate[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]
[Out] x^2/2 + Sqrt[3 + x]/(Sqrt[2 + x]*(-1 + (3 + x)/(2 + x))) - 5*ArcTanh[Sqrt[3
 + x]/Sqrt[2 + x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.

Time = 1.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

method	result	size
default	$-\frac{\sqrt{x+2}\,\sqrt{3+x}\,\left(-2\sqrt{x^2+5x+6}+5\ln\left(\frac{5}{2}+x+\sqrt{x^2+5x+6}\right)\right)}{2\sqrt{x^2+5x+6}}+\frac{x^2}{2}$	58

[In] int(x*(1+1/(x+2)^(1/2)/(3+x)^(1/2)),x,method=_RETURNVERBOSE)

[Out] -1/2*(x+2)^(1/2)*(3+x)^(1/2)*(-2*(x^2+5*x+6)^(1/2)+5*ln(5/2+x+(x^2+5*x+6)^(1/2)))/(x^2+5*x+6)^(1/2)+1/2*x^2

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx=\frac{1}{2}\,x^2+\sqrt{x+3}\sqrt{x+2}+\frac{5}{2}\,\log\left(2\,\sqrt{x+3}\sqrt{x+2}-2\,x-5\right)$$

[In] integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="fricas")

[Out] 1/2*x^2 + sqrt(x + 3)*sqrt(x + 2) + 5/2*log(2*sqrt(x + 3)*sqrt(x + 2) - 2*x - 5)

Sympy [F]

$$\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx=\int\frac{x\left(\sqrt{x+2}\sqrt{x+3}+1\right)}{\sqrt{x+2}\sqrt{x+3}}\,dx$$

[In] integrate(x*(1+1/(2+x)**(1/2)/(3+x)**(1/2)),x)

[Out] Integral(x*(sqrt(x + 2)*sqrt(x + 3) + 1)/(sqrt(x + 2)*sqrt(x + 3)), x)

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx = \frac{1}{2}\,x^2+\sqrt{x^2+5\,x+6}-\frac{5}{2}\,\log\left(2\,x+2\,\sqrt{x^2+5\,x+6}+5\right)$$

```
[In] integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="maxima")
[Out] 1/2*x^2 + sqrt(x^2 + 5*x + 6) - 5/2*log(2*x + 2*sqrt(x^2 + 5*x + 6) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx = \frac{1}{2}\,(x+3)^2+\sqrt{x+3}\sqrt{x+2}-3\,x+5\,\log\left(\sqrt{x+3}-\sqrt{x+2}\right)-9$$

```
[In] integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="giac")
[Out] 1/2*(x + 3)^2 + sqrt(x + 3)*sqrt(x + 2) - 3*x + 5*log(sqrt(x + 3) - sqrt(x
+ 2)) - 9
```

Mupad [B] (verification not implemented)

Time = 25.92 (sec) , antiderivative size = 180, normalized size of antiderivative = 5.45

$$\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx = \frac{\frac{10\left(\sqrt{x+2}-\sqrt{2}\right)}{\sqrt{x+3}-\sqrt{3}}+\frac{10\left(\sqrt{x+2}-\sqrt{2}\right)^3}{\left(\sqrt{x+3}-\sqrt{3}\right)^3}-\frac{8\sqrt{6}\left(\sqrt{x+2}-\sqrt{2}\right)^2}{\left(\sqrt{x+3}-\sqrt{3}\right)^2}}{\frac{\left(\sqrt{x+2}-\sqrt{2}\right)^4}{\left(\sqrt{x+3}-\sqrt{3}\right)^4}-\frac{2\left(\sqrt{x+2}-\sqrt{2}\right)^2}{\left(\sqrt{x+3}-\sqrt{3}\right)^2}+1}-10\,\mathrm{atanh}\left(\frac{\sqrt{x+2}-\sqrt{2}}{\sqrt{x+3}-\sqrt{3}}\right)+\frac{x^2}{2}$$

```
[In] int(x*(1/((x + 2)^(1/2)*(x + 3)^(1/2)) + 1),x)
[Out] ((10*((x + 2)^(1/2) - 2^(1/2)))/((x + 3)^(1/2) - 3^(1/2)) + (10*((x + 2)^(1
/2) - 2^(1/2))^3)/((x + 3)^(1/2) - 3^(1/2))^3 - (8*6^(1/2)*((x + 2)^(1/2) -
 2^(1/2))^2)/((x + 3)^(1/2) - 3^(1/2))^2)/(((x + 2)^(1/2) - 2^(1/2))^4/((x
+ 3)^(1/2) - 3^(1/2))^4 - (2*((x + 2)^(1/2) - 2^(1/2))^2)/((x + 3)^(1/2) -
3^(1/2))^2 + 1) - 10*atanh(((x + 2)^(1/2) - 2^(1/2))/((x + 3)^(1/2) - 3^(1/
2))) + x^2/2
```

3.817 $\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 45

$$\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx=\frac{\arctan(x)}{2}+\frac{\sqrt{x^6}\arctan(x)}{2x^3}+\frac{\operatorname{arctanh}(x)}{2}-\frac{\sqrt{x^6}\operatorname{arctanh}(x)}{2x^3}$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*(
x^6)^(1/2)/x^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6857, 218, 212, 209, 15, 304}

$$\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx=\frac{\sqrt{x^6}\arctan(x)}{2x^3}+\frac{\arctan(x)}{2}-\frac{\sqrt{x^6}\operatorname{arctanh}(x)}{2x^3}+\frac{\operatorname{arctanh}(x)}{2}$$

```
[In] Int[(x - Sqrt[x^6])/(x*(1 - x^4)),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right) dx \\
&= \int \frac{1}{1-x^4}\, dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\, dx \\
&= \frac{1}{2}\int \frac{1}{1-x^2}\, dx+\frac{1}{2}\int \frac{1}{1+x^2}\, dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\, dx}{x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\, dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\, dx}{2x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx = \frac{\arctan(x)}{2}-\frac{1}{2}\arctan\left(\frac{\sqrt{x^6}}{x^4}\right) -\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)$$

```
[In] Integrate[(x - Sqrt[x^6])/(x*(1 - x^4)),x]
[Out] ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 -
x]/4 + Log[1 + x]/4
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\sqrt{x^6}\,(\ln(x-1)-\ln(x+1)+2\arctan(x))}{4x^3}+\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}$	35
meijerg	$-\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}+\frac{\sqrt{x^6}\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}$	80

```
[In] int((x-(x^6)^(1/2))/x/(-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*(x^6)^(1/2)*(ln(x-1)-ln(x+1)+2*arctan(x))/x^3+1/2*arctan(x)+1/2*arctanh
(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx = \arctan(x)$$

```
[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="fricas")
[Out] arctan(x)
```

Sympy [F]

$$\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx = -\int \frac{x}{x^5-x}\,dx - \int \left(-\frac{\sqrt{x^6}}{x^5-x}\right)dx$$

```
[In] integrate((x-(x**6)**(1/2))/x/(-x**4+1),x)
[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(x**6)/(x**5 - x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx = \arctan(x)$$

```
[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="maxima")
[Out] arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx = \frac{1}{2}(\mathrm{sgn}(x)+1)\arctan(x) - \frac{1}{4}(\mathrm{sgn}(x)-1)\log(|x+1|) + \frac{1}{4}(\mathrm{sgn}(x)-1)\log(|x-1|)$$

```
[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="giac")
[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x)
 - 1)*log(abs(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x-\sqrt{x^6}}{x\,(1-x^4)}\,dx=\int -\frac{x-\sqrt{x^6}}{x\,\left(x^4-1\right)}\,dx$$

```
[In] int(-(x - (x^6)^(1/2))/(x*(x^4 - 1)),x)
[Out] int(-(x - (x^6)^(1/2))/(x*(x^4 - 1)), x)
```

3.818 $\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 45

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx=\frac{\arctan(x)}{2}+\frac{\sqrt{x^6}\arctan(x)}{2x^3}+\frac{\operatorname{arctanh}(x)}{2}-\frac{\sqrt{x^6}\operatorname{arctanh}(x)}{2x^3}$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*(
x^6)^(1/2)/x^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6857, 218, 212, 209, 15, 304}

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx=\frac{\sqrt{x^6}\arctan(x)}{2x^3}+\frac{\arctan(x)}{2}-\frac{\sqrt{x^6}\operatorname{arctanh}(x)}{2x^3}+\frac{\operatorname{arctanh}(x)}{2}$$

```
[In] Int[(1 - Sqrt[x^6]/x)/(1 - x^4),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right) dx \\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx \\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx=\frac{\arctan(x)}{2}-\frac{1}{2}\arctan\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)$$

[In] Integrate[(1 - Sqrt[x^6]/x)/(1 - x^4),x]

[Out] ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 - x]/4 + Log[1 + x]/4

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\sqrt{x^6}\,(\ln(x-1)-\ln(x+1)+2\arctan(x))}{4x^3}+\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}$	35
meijerg	$-\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}+\frac{\sqrt{x^6}\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}$	80

[In] int((1-(x^6)^(1/2)/x)/(-x^4+1),x,method=_RETURNVERBOSE)

[Out] 1/4*(x^6)^(1/2)*(ln(x-1)-ln(x+1)+2*arctan(x))/x^3+1/2*arctan(x)+1/2*arctanh(x)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx=\arctan(x)$$

[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="fricas")

[Out] arctan(x)

Sympy [F]

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx = -\int \frac{x}{x^5-x}\,dx - \int \left(-\frac{\sqrt{x^6}}{x^5-x}\right)dx$$

```
[In] integrate((1-(x**6)**(1/2)/x)/(-x**4+1),x)
[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(x**6)/(x**5 - x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx = \arctan(x)$$

```
[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="maxima")
[Out] arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx = \frac{1}{2}(\operatorname{sgn}(x)+1)\arctan(x) - \frac{1}{4}(\operatorname{sgn}(x)-1)\log(|x+1|) + \frac{1}{4}(\operatorname{sgn}(x)-1)\log(|x-1|)$$

```
[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="giac")
[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x)
 - 1)*log(abs(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx=\int \frac{\frac{\sqrt{x^6}}{x}-1}{x^4-1}\,dx$$

```
[In] int(((x^6)^(1/2)/x - 1)/(x^4 - 1),x)
[Out] int(((x^6)^(1/2)/x - 1)/(x^4 - 1), x)
```

3.819 $\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx=\frac{\arctan(x)}{2}+\frac{\sqrt{x^6}\arctan(x)}{2x^3}+\frac{\operatorname{arctanh}(x)}{2}-\frac{\sqrt{x^6}\operatorname{arctanh}(x)}{2x^3}$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*(
x^6)^(1/2)/x^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1607, 6857, 218, 212, 209, 15, 304}

$$\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx=\frac{\sqrt{x^6}\arctan(x)}{2x^3}+\frac{\arctan(x)}{2}-\frac{\sqrt{x^6}\operatorname{arctanh}(x)}{2x^3}+\frac{\operatorname{arctanh}(x)}{2}$$

```
[In] Int[(x - Sqrt[x^6])/(x - x^5),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x-\sqrt{x^6}}{x\,(1-x^4)}\,dx\\ &= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\,(-1+x^4)}\right)dx\\ &= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\,(-1+x^4)}\,dx\end{aligned}$$

$$= \frac{1}{2}\int \frac{1}{1-x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx + \frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3}$$

$$= \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x) - \frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3} + \frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3}$$

$$= \frac{1}{2}\tan^{-1}(x) + \frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3} + \frac{1}{2}\tanh^{-1}(x) - \frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx = \frac{\arctan(x)}{2} - \frac{1}{2}\arctan\left(\frac{\sqrt{x^6}}{x^4}\right) - \frac{1}{2}\text{arctanh}\left(\frac{\sqrt{x^6}}{x^4}\right) - \frac{1}{4}\log(1-x) + \frac{1}{4}\log(1+x)$$

```
[In] Integrate[(x - Sqrt[x^6])/(x - x^5),x]
```

```
[Out] ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 -
x]/4 + Log[1 + x]/4
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\sqrt{x^6}\,(-\ln(x-1)+\ln(x+1)-2\arctan(x))}{4x^3}+\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}$	35
meijerg	$-\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}+\frac{\sqrt{x^6}\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}$	80

```
[In] int((x-(x^6)^(1/2))/(-x^5+x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(x^6)^(1/2)*(-ln(x-1)+ln(x+1)-2*arctan(x))/x^3+1/2*arctan(x)+1/2*arcta
nh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx = \arctan(x)$$

```
[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="fricas")
[Out] arctan(x)
```

Sympy [F]

$$\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx = -\int \frac{x}{x^5-x}\,dx - \int \left(-\frac{\sqrt{x^6}}{x^5-x}\right)\,dx$$

```
[In] integrate((x-(x**6)**(1/2))/(-x**5+x),x)
[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(x**6)/(x**5 - x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx = \arctan(x)$$

```
[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="maxima")
[Out] arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx = \frac{1}{2}\left(\mathrm{sgn}(x)+1\right)\arctan(x) - \frac{1}{4}\left(\mathrm{sgn}(x)-1\right)\log\left(|x+1|\right) + \frac{1}{4}\left(\mathrm{sgn}(x)-1\right)\log\left(|x-1|\right)$$

```
[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="giac")
[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x)
 - 1)*log(abs(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x - \sqrt{x^6}}{x - x^5} \, dx = \int \frac{x - \sqrt{x^6}}{x - x^5} \, dx$$

```
[In] int((x - (x^6)^(1/2))/(x - x^5),x)
[Out] int((x - (x^6)^(1/2))/(x - x^5), x)
```

3.820 $\int \frac{x}{x+\sqrt{x^6}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{x}{x+\sqrt{x^6}}\,dx=\frac{\arctan(x)}{2}+\frac{\sqrt{x^6}\arctan(x)}{2x^3}+\frac{\operatorname{arctanh}(x)}{2}-\frac{\sqrt{x^6}\operatorname{arctanh}(x)}{2x^3}$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*(
x^6)^(1/2)/x^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {6861, 1598, 6857, 218, 212, 209, 15, 304}

$$\int \frac{x}{x+\sqrt{x^6}}\,dx=\frac{\sqrt{x^6}\arctan(x)}{2x^3}+\frac{\arctan(x)}{2}-\frac{\sqrt{x^6}\operatorname{arctanh}(x)}{2x^3}+\frac{\operatorname{arctanh}(x)}{2}$$

```
[In] Int[x/(x + Sqrt[x^6]),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6861

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[u*((a*x^m - b*Sqrt[c*x^n])/(a^2*x^(2*m) - b^2*c*x^n)), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x\left(x - \sqrt{x^6}\right)}{x^2 - x^6}\, dx$$

$$
\begin{aligned}
&= \int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx \\
&= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x(-1+x^4)}\right)dx \\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x(-1+x^4)}\,dx \\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.79 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$
\int \frac{x}{x+\sqrt{x^6}}\,dx = \frac{\arctan(x)}{2}+\frac{1}{2}\arctan\left(\frac{\sqrt{x^6}}{x^2}\right) - \frac{1}{2}\text{arctanh}\left(\frac{\sqrt{x^6}}{x^2}\right)-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)
$$

```
[In] Integrate[x/(x + Sqrt[x^6]),x]
[Out] ArcTan[x]/2 + ArcTan[Sqrt[x^6]/x^2]/2 - ArcTanh[Sqrt[x^6]/x^2]/2 - Log[1 -
x]/4 + Log[1 + x]/4
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.44

method	result	size
meijerg	$\frac{x^{\frac{3}{2}}\arctan\left(\frac{(x^6)^{\frac{1}{4}}}{\sqrt{x}}\right)}{(x^6)^{\frac{1}{4}}}$	20
default	$\frac{\arctan\left(\sqrt{\frac{\sqrt{x^6}}{x^3}}\,x\right)}{\sqrt{\frac{\sqrt{x^6}}{x^3}}}$	27

```
[In] int(x/(x+(x^6)^(1/2)),x,method=_RETURNVERBOSE)
[Out] x^(3/2)/(x^6)^(1/4)*arctan((x^6)^(1/4)/x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{x}{x+\sqrt{x^6}}\,dx=\arctan\left(x\right)$$

```
[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="fricas")
[Out] arctan(x)
```

Sympy [F]

$$\int \frac{x}{x+\sqrt{x^6}}\,dx=\int \frac{x}{x+\sqrt{x^6}}\,dx$$

```
[In] integrate(x/(x+(x**6)**(1/2)),x)
[Out] Integral(x/(x + sqrt(x**6)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{x}{x+\sqrt{x^6}}\,dx=\arctan\left(x\right)$$

```
[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="maxima")
[Out] arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.27

$$\int \frac{x}{x+\sqrt{x^6}}\,dx=\frac{\arctan\left(x\sqrt{\operatorname{sgn}\left(x\right)}\right)}{\sqrt{\operatorname{sgn}\left(x\right)}}$$

```
[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="giac")
[Out] arctan(x*sqrt(sgn(x)))/sqrt(sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{x+\sqrt{x^6}}\,dx=\int \frac{x}{x+\sqrt{x^6}}\,dx$$

```
[In] int(x/(x + (x^6)^(1/2)),x)
[Out] int(x/(x + (x^6)^(1/2)), x)
```

3.821 $\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 52

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3} \, dx = \arctan \left(\sqrt{x}\right)+\frac{\sqrt{x^3} \arctan \left(\sqrt{x}\right)}{x^{3/2}}+\text{arctanh}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\text{arctanh}\left(\sqrt{x}\right)}{x^{3/2}}$$

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))+arctan(x^(1/2))*(x^3)^(1/2)/x^(3/2)-arctan
h(x^(1/2))*(x^3)^(1/2)/x^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1607, 6857, 335, 218, 212, 209, 15, 304}

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3} \, dx = \frac{\sqrt{x^3} \arctan \left(\sqrt{x}\right)}{x^{3/2}}+\arctan \left(\sqrt{x}\right)-\frac{\sqrt{x^3}\text{arctanh}\left(\sqrt{x}\right)}{x^{3/2}}+\text{arctanh}\left(\sqrt{x}\right)$$

```
[In] Int[(Sqrt[x] - Sqrt[x^3])/(x - x^3),x]
```

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] -
(Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{x}-\sqrt{x^3}}{x\left(1-x^2\right)}\,dx\\
&= \int \left(-\frac{1}{\sqrt{x}\left(-1+x^2\right)}+\frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\right)dx\\
&= -\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx+\int \frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\,dx\\
&= -\left(2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt{x}\right)\right)+\frac{\sqrt{x^3}\int \frac{\sqrt{x}}{-1+x^2}\,dx}{x^{3/2}}\\
&= \frac{\left(2\sqrt{x^3}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt{x}\right)}{x^{3/2}}\\
&\quad+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= \tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}+\frac{\sqrt{x^3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}\\
&= \tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$
\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx=\arctan\left(\sqrt{x}\right)+\arctan\left(\frac{\sqrt{x^3}}{x}\right)+\text{arctanh}\left(\sqrt{x}\right)-\text{arctanh}\left(\frac{\sqrt{x^3}}{x}\right)
$$

```
[In] Integrate[(Sqrt[x] - Sqrt[x^3])/(x - x^3),x]
[Out] ArcTan[Sqrt[x]] + ArcTan[Sqrt[x^3]/x] + ArcTanh[Sqrt[x]] - ArcTanh[Sqrt[x^3
]/x]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

method	result	si
default	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)+\frac{\sqrt{x^{3}}\,\left(\ln\left(-1+\sqrt{x}\right)-\ln\left(1+\sqrt{x}\right)+2\arctan\left(\sqrt{x}\right)\right)}{2x^{\frac{3}{2}}}$	41
meijerg	$-\frac{\sqrt{x}\,\left(\ln\left(1-\left(x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(x^{2}\right)^{\frac{1}{4}}\right)\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{x^{3}}\,\left(\ln\left(1-\left(x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{2}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(x^{2}\right)^{\frac{1}{4}}\right)\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}$	82

```
[In] int((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x,method=_RETURNVERBOSE)
[Out] arctan(x^(1/2))+arctanh(x^(1/2))+1/2*(x^3)^(1/2)*(ln(-1+x^(1/2))-ln(1+x^(1/
2))+2*arctan(x^(1/2)))/x^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{x}-\sqrt{x^{3}}}{x-x^{3}}\,dx=2\arctan\left(\sqrt{x}\right)$$

```
[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="fricas")
[Out] 2*arctan(sqrt(x))
```

Sympy [F]

$$\int \frac{\sqrt{x}-\sqrt{x^{3}}}{x-x^{3}}\,dx=-\int \frac{\sqrt{x}}{x^{3}-x}\,dx-\int \left(-\frac{\sqrt{x^{3}}}{x^{3}-x}\right)\,dx$$

```
[In] integrate((x**(1/2)-(x**3)**(1/2))/(-x**3+x),x)
[Out] -Integral(sqrt(x)/(x**3 - x), x) - Integral(-sqrt(x**3)/(x**3 - x), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx=\int \frac{\sqrt{x^3}-\sqrt{x}}{x^3-x}\,dx$$

```
[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x)) - integrate(1/2*sqrt(x)/(x + 1), x) + integrate(1/4/(sqrt(x
) + 1), x) + integrate(1/4/(sqrt(x) - 1), x) + 1/2*log(sqrt(x) + 1) - 1/2*l
og(sqrt(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx=2\arctan\left(\sqrt{x}\right)$$

```
[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx=-\int \frac{\sqrt{x^3}-\sqrt{x}}{x-x^3}\,dx$$

```
[In] int(-((x^3)^(1/2) - x^(1/2))/(x - x^3),x)
```

```
[Out] -int(((x^3)^(1/2) - x^(1/2))/(x - x^3), x)
```

3.822 $\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx=\arctan\left(\sqrt{x}\right)+\frac{\sqrt{x^3}\arctan\left(\sqrt{x}\right)}{x^{3/2}}+\operatorname{arctanh}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\operatorname{arctanh}\left(\sqrt{x}\right)}{x^{3/2}}$$

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))+arctan(x^(1/2))*(x^3)^(1/2)/x^(3/2)-arctan
h(x^(1/2))*(x^3)^(1/2)/x^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6861, 1607, 6857, 335, 218, 212, 209, 15, 304}

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx=\frac{\sqrt{x^3}\arctan\left(\sqrt{x}\right)}{x^{3/2}}+\arctan\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\operatorname{arctanh}\left(\sqrt{x}\right)}{x^{3/2}}+\operatorname{arctanh}\left(\sqrt{x}\right)$$

```
[In] Int[(Sqrt[x] + Sqrt[x^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] -
(Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6861

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[u*((a*x^m - b*Sqrt[c*x^n])/(a^2*x^(2*m) - b^2*c*x^n)), x] /; FreeQ[{a, b,
```

```
c, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx$$
$$= \int \frac{\sqrt{x}-\sqrt{x^3}}{x\left(1-x^2\right)}\,dx$$
$$= \int \left(-\frac{1}{\sqrt{x}\left(-1+x^2\right)}+\frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\right)dx$$
$$= -\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx+\int \frac{\sqrt{x^3}}{x\left(-1+x^2\right)}\,dx$$
$$= -\left(2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt{x}\right)\right)+\frac{\sqrt{x^3}\int \frac{\sqrt{x}}{-1+x^2}\,dx}{x^{3/2}}$$
$$= \frac{\left(2\sqrt{x^3}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt{x}\right)}{x^{3/2}}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)$$
$$= \tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}+\frac{\sqrt{x^3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}$$
$$= \tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx = \arctan\left(\sqrt{x}\right)+\arctan\left(\frac{\sqrt{x^3}}{x}\right)+\text{arctanh}\left(\sqrt{x}\right)-\text{arctanh}\left(\frac{\sqrt{x^3}}{x}\right)$$

```
[In] Integrate[(Sqrt[x] + Sqrt[x^3])^(-1),x]
[Out] ArcTan[Sqrt[x]] + ArcTan[Sqrt[x^3]/x] + ArcTanh[Sqrt[x]] - ArcTanh[Sqrt[x^3
]/x]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.40

method	result	size
meijerg	$\frac{2x^{\frac{3}{4}} \arctan\left(\frac{(x^3)^{\frac{1}{4}}}{x^{\frac{1}{4}}}\right)}{(x^3)^{\frac{1}{4}}}$	21
default	$\frac{2 \arctan\left(\sqrt{\frac{\sqrt{x^3}}{x^{\frac{3}{2}}}}\sqrt{x}\right)}{\sqrt{\frac{\sqrt{x^3}}{x^{\frac{3}{2}}}}}$	30

```
[In] int(1/(x^(1/2)+(x^3)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/(x^3)^(1/4)*x^(3/4)*arctan(1/x^(1/4)*(x^3)^(1/4))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.12

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}} \, dx = 2 \arctan\left(\sqrt{x}\right)$$

```
[In] integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="fricas")
[Out] 2*arctan(sqrt(x))
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}} \, dx = \int \frac{1}{\sqrt{x}+\sqrt{x^3}} \, dx$$

```
[In] integrate(1/(x**(1/2)+(x**3)**(1/2)),x)
[Out] Integral(1/(sqrt(x) + sqrt(x**3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.12

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx = 2\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="maxima")
[Out] 2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.12

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx = 2\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="giac")
[Out] 2*arctan(sqrt(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx = \int \frac{1}{\sqrt{x^3}+\sqrt{x}}\,dx$$

```
[In] int(1/((x^3)^(1/2) + x^(1/2)),x)
[Out] int(1/((x^3)^(1/2) + x^(1/2)), x)
```

3.823 $\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = \arctan\left(\sqrt{-1+x}\right)+\frac{\sqrt{(-1+x)^3}\arctan\left(\sqrt{-1+x}\right)}{(-1+x)^{3/2}} + \operatorname{arctanh}\left(\sqrt{-1+x}\right)-\frac{\sqrt{(-1+x)^3}\operatorname{arctanh}\left(\sqrt{-1+x}\right)}{(-1+x)^{3/2}}$$

```
[Out] arctan((-1+x)^(1/2))+arctanh((-1+x)^(1/2))+arctan((-1+x)^(1/2))*((-1+x)^3)^
(1/2)/(-1+x)^(3/2)-arctanh((-1+x)^(1/2))*((-1+x)^3)^(1/2)/(-1+x)^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {6861, 1607, 6857, 335, 218, 212, 209, 15, 304}

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = \frac{\sqrt{(x-1)^3}\arctan\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\arctan\left(\sqrt{x-1}\right) -\frac{\sqrt{(x-1)^3}\operatorname{arctanh}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\operatorname{arctanh}\left(\sqrt{x-1}\right)$$

```
[In] Int[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[-1 + x]] + (Sqrt[(-1 + x)^3]*ArcTan[Sqrt[-1 + x]])/(-1 + x)^(3/
2) + ArcTanh[Sqrt[-1 + x]] - (Sqrt[(-1 + x)^3]*ArcTanh[Sqrt[-1 + x]])/(-1 +
 x)^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rule 6861

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[u*((a*x^m - b*Sqrt[c*x^n])/(a^2*x^(2*m) - b^2*c*x^n)), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \frac{\sqrt{x}-\sqrt{x^3}}{x(1-x^2)}\,dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{x}(-1+x^2)}+\frac{\sqrt{x^3}}{x(-1+x^2)}\right)\,dx, x, -1+x\right)\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, -1+x\right)+\text{Subst}\left(\int \frac{\sqrt{x^3}}{x(-1+x^2)}\,dx, x, -1+x\right)\\
&= -\left(2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{-1+x}\right)\right)+\frac{\sqrt{(-1+x)^3}\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, -1+x\right)}{(-1+x)^{3/2}}\\
&= \frac{\left(2\sqrt{(-1+x)^3}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{-1+x}\right)}{(-1+x)^{3/2}}\\
&\quad+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{-1+x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\right)\\
&= \tan^{-1}\left(\sqrt{-1+x}\right)+\tanh^{-1}\left(\sqrt{-1+x}\right)\\
&\quad-\frac{\sqrt{(-1+x)^3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{-1+x}\right)}{(-1+x)^{3/2}}\\
&\quad+\frac{\sqrt{(-1+x)^3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\right)}{(-1+x)^{3/2}}\\
&= \tan^{-1}\left(\sqrt{-1+x}\right)+\frac{\sqrt{(-1+x)^3}\tan^{-1}\left(\sqrt{-1+x}\right)}{(-1+x)^{3/2}}\\
&\quad+\tanh^{-1}\left(\sqrt{-1+x}\right)-\frac{\sqrt{(-1+x)^3}\tanh^{-1}\left(\sqrt{-1+x}\right)}{(-1+x)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = \arctan\left(\sqrt{-1+x}\right)+\arctan\left(\frac{\sqrt{-1+3x-3x^2+x^3}}{-1+x}\right)+\operatorname{arctanh}\left(\sqrt{-1+x}\right)-\operatorname{arctanh}\left(\frac{\sqrt{-1+3x-3x^2+x^3}}{-1+x}\right)$$

```
[In] Integrate[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1),x]
[Out] ArcTan[Sqrt[-1 + x]] + ArcTan[Sqrt[-1 + 3*x - 3*x^2 + x^3]/(-1 + x)] + ArcT
anh[Sqrt[-1 + x]] - ArcTanh[Sqrt[-1 + 3*x - 3*x^2 + x^3]/(-1 + x)]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{2\arctan\left(\sqrt{\frac{\sqrt{(x-1)^3}}{(x-1)^{\frac{3}{2}}}}\,\sqrt{x-1}\right)}{\sqrt{\frac{\sqrt{(x-1)^3}}{(x-1)^{\frac{3}{2}}}}}$	40

```
[In] int(1/((x-1)^(1/2)+((x-1)^3)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/(((x-1)^3)^(1/2)/(x-1)^(3/2))^(1/2)*arctan((((x-1)^3)^(1/2)/(x-1)^(3/2))^
(1/2)*(x-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.12

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = 2\arctan\left(\sqrt{x-1}\right)$$

```
[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="fricas")
[Out] 2*arctan(sqrt(x - 1))
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = \int \frac{1}{\sqrt{x-1}+\sqrt{(x-1)^3}}\,dx$$

```
[In] integrate(1/((-1+x)**(1/2)+((-1+x)**3)**(1/2)),x)
[Out] Integral(1/(sqrt(x - 1) + sqrt((x - 1)**3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = \int \frac{1}{\sqrt{(x-1)^3}+\sqrt{x-1}}\,dx$$

```
[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x - 1) - integrate(sqrt(x - 1)/x, x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.12

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = 2\arctan\left(\sqrt{x-1}\right)$$

```
[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="giac")
[Out] 2*arctan(sqrt(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = \int -\frac{\sqrt{x-1}-\sqrt{(x-1)^3}}{(x-1)^3-x+1}\,dx$$

```
[In] int(1/((x - 1)^(1/2) + ((x - 1)^3)^(1/2)),x)
[Out] int(-((x - 1)^(1/2) - ((x - 1)^3)^(1/2))/((x - 1)^3 - x + 1), x)
```

3.824 $\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right) dx$

Optimal result

Integrand size = 35, antiderivative size = 31

$$\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right) dx = \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

```
[Out] 3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {817}

$$\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right) dx = \frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[In] Int[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 817

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*
g, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3}{5(4+5x)} - \int \frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\,dx \\
&= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$
\int \left(-\frac{3}{(4+5x)^2} - \frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right) dx = \frac{3+\frac{5}{\sqrt{1-x^2}} - \frac{5x^2}{\sqrt{1-x^2}}}{20+25x}
$$

```
[In] Integrate[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
[Out] (3 + 5/Sqrt[1 - x^2] - (5*x^2)/Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
trager	$-\frac{3x}{4(4+5x)} + \frac{\sqrt{-x^2+1}}{4+5x}$	29
default	$\frac{\sqrt{-\left(\frac{4}{5}+x\right)^2+\frac{8x}{5}+\frac{41}{25}}}{4+5x} + \frac{3}{5(4+5x)}$	32
risch	$\frac{3}{25\left(\frac{4}{5}+x\right)} - \frac{x^2-1}{(4+5x)\sqrt{-x^2+1}}$	32

```
[In] int(-3/(4+5*x)^2+(-4*x-5)/(4+5*x)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -3/4*x/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right) dx = \frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

```
[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="fric
as")
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

Sympy [F]

$$\begin{aligned}\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right) dx \\ = -\int \frac{4x}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx \\ -\int \frac{3\sqrt{1-x^2}}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx \\ -\int \frac{5}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx\end{aligned}$$

```
[In] integrate(-3/(4+5*x)**2+(-5-4*x)/(4+5*x)**2/(-x**2+1)**(1/2),x)
[Out] -Integral(4*x/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x
**2)), x) - Integral(3*sqrt(1 - x**2)/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1
 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(5/(25*x**2*sqrt(1 - x**2) + 40
*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right) dx = \frac{\sqrt{-x^2+1}}{5\,x+4}+\frac{3}{5\,(5\,x+4)}$$

```
[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="maxi
ma")
[Out] sqrt(-x^2 + 1)/(5*x + 4) + 3/5/(5*x + 4)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right)\,dx=\frac{\sqrt{\frac{8}{5\,x+4}+\frac{9}{(5\,x+4)^2}-1}}{5\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)}+\frac{3}{5\,(5\,x+4)}-\frac{1}{5}i\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)$$

```
[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] 1/5*sqrt(8/(5*x + 4) + 9/(5*x + 4)^2 - 1)/sgn(1/(5*x + 4)) + 3/5/(5*x + 4)
- 1/5*I*sgn(1/(5*x + 4))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right)\,dx=\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

```
[In] int(- 3/(5*x + 4)^2 - (4*x + 5)/((5*x + 4)^2*(1 - x^2)^(1/2)),x)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

3.825 $\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 31

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx=\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

```
[Out] 3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {6874, 745, 739, 212, 821}

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx=\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[In] Int[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{(4+5x)^2}-\frac{5}{(4+5x)^2\sqrt{1-x^2}}-\frac{4x}{(4+5x)^2\sqrt{1-x^2}}\right)dx\\
&= \frac{3}{5(4+5x)}-4\int \frac{x}{(4+5x)^2\sqrt{1-x^2}}\,dx-5\int \frac{1}{(4+5x)^2\sqrt{1-x^2}}\,dx\\
&= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx=\frac{3+5\sqrt{1-x^2}}{20+25x}$$

```
[In] Integrate[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
trager	$-\frac{3x}{4(4+5x)}+\frac{\sqrt{-x^{2}+1}}{4+5x}$	29
default	$\frac{\sqrt{-\left(\frac{4}{5}+x\right)^{2}+\frac{8x}{5}+\frac{41}{25}}}{4+5x}+\frac{3}{5(4+5x)}$	32

```
[In] int((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -3/4*x/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx=\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

```
[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

Sympy [F]

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx=-\int \frac{4x}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int \frac{3\sqrt{1-x^2}}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int \frac{5}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx$$

```
[In] integrate((-5-4*x-3*(-x**2+1)**(1/2))/(4+5*x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -Integral(4*x/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x
**2)), x) - Integral(3*sqrt(1 - x**2)/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1
 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(5/(25*x**2*sqrt(1 - x**2) + 40
*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx = \frac{5\,\sqrt{x+1}\sqrt{-x+1}+3}{5\,(5\,x+4)}$$

```
[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/5*(5*sqrt(x + 1)*sqrt(-x + 1) + 3)/(5*x + 4)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx = \frac{\sqrt{\frac{8}{5\,x+4}+\frac{9}{(5\,x+4)^2}-1}}{5\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)}+\frac{3}{5\,(5\,x+4)}-\frac{1}{5}i\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)$$

```
[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="
giac")
```

```
[Out] 1/5*sqrt(8/(5*x + 4) + 9/(5*x + 4)^2 - 1)/sgn(1/(5*x + 4)) + 3/5/(5*x + 4)
- 1/5*I*sgn(1/(5*x + 4))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx = \frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

```
[In] int(-(4*x + 3*(1 - x^2)^(1/2) + 5)/((5*x + 4)^2*(1 - x^2)^(1/2)),x)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

3.826 $\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 31

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx = \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

```
[Out] 3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {6874, 679, 222, 747, 858, 739, 212, 749}

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx = \frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[In] Int[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{(4+5x)^2}+\frac{\sqrt{1-x^2}}{18(-1+x)}-\frac{\sqrt{1-x^2}}{2(1+x)}-\frac{5\sqrt{1-x^2}}{(4+5x)^2}+\frac{20\sqrt{1-x^2}}{9(4+5x)}\right)\,dx\\
&= \frac{3}{5(4+5x)}+\frac{1}{18}\int \frac{\sqrt{1-x^2}}{-1+x}\,dx-\frac{1}{2}\int \frac{\sqrt{1-x^2}}{1+x}\,dx+\frac{20}{9}\int \frac{\sqrt{1-x^2}}{4+5x}\,dx-5\int \frac{\sqrt{1-x^2}}{(4+5x)^2}\,dx\\
&= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{1}{18}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{4}{9}\int \frac{5+4x}{(4+5x)\sqrt{1-x^2}}\,dx\\
&\quad -\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx+\int \frac{x}{(4+5x)\sqrt{1-x^2}}\,dx\\
&= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{5}{9}\sin^{-1}(x)+\frac{1}{5}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{16}{45}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx=\frac{3+5\sqrt{1-x^2}}{20+25x}$$

```
[In] Integrate[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1),x]
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
trager	$-\frac{3x}{4(4+5x)}+\frac{\sqrt{-x^2+1}}{4+5x}$	29
default	$\frac{3}{5(4+5x)}-\frac{\sqrt{-(x+1)^2+2x+2}}{2}+\frac{5\left(-\left(\frac{4}{5}+x\right)^2+\frac{8x}{5}+\frac{41}{25}\right)^{\frac{3}{2}}}{9\left(\frac{4}{5}+x\right)}+\frac{5x\sqrt{-\left(\frac{4}{5}+x\right)^2+\frac{8x}{5}+\frac{41}{25}}}{9}+\frac{\sqrt{-(x-1)^2-2x+2}}{18}$	81

```
[In] int(1/(-3*x^2+3+(-4*x-5)*(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -3/4*x/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx = \frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

```
[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="fricas")
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

Sympy [F]

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx = -\int \frac{1}{3x^2+4x\sqrt{1-x^2}+5\sqrt{1-x^2}-3}\,dx$$

```
[In] integrate(1/(-3*x**2+3+(-5-4*x)*(-x**2+1)**(1/2)),x)
[Out] -Integral(1/(3*x**2 + 4*x*sqrt(1 - x**2) + 5*sqrt(1 - x**2) - 3), x)
```

Maxima [F]

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx = \int -\frac{1}{3\,x^2+\sqrt{-x^2+1}(4\,x+5)-3}\,dx$$

```
[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="maxima")
[Out] -integrate(1/(3*x^2 + sqrt(-x^2 + 1)*(4*x + 5) - 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(27) = 54.

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.19

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx = \frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-2\right)}+\frac{3}{5\,(5\,x+4)}$$

```
[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="giac")
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2
 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3\left(1-x^2\right)}\,dx=\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

```
[In] int(-1/((4*x + 5)*(1 - x^2)^(1/2) + 3*x^2 - 3),x)
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

3.827 $\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 31

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx=\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

```
[Out] 3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6874, 679, 222, 747, 858, 739, 212, 749}

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx=\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[In] Int[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1),x]
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{3}{(4+5x)^2}+\frac{\sqrt{1-x^2}}{18(-1+x)}-\frac{\sqrt{1-x^2}}{2(1+x)}-\frac{5\sqrt{1-x^2}}{(4+5x)^2}+\frac{20\sqrt{1-x^2}}{9(4+5x)}\right)dx$$

$$=\frac{3}{5(4+5x)}+\frac{1}{18}\int\frac{\sqrt{1-x^2}}{-1+x}\,dx-\frac{1}{2}\int\frac{\sqrt{1-x^2}}{1+x}\,dx+\frac{20}{9}\int\frac{\sqrt{1-x^2}}{4+5x}\,dx-5\int\frac{\sqrt{1-x^2}}{(4+5x)^2}\,dx$$

$$=\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{1}{18}\int\frac{1}{\sqrt{1-x^2}}\,dx+\frac{4}{9}\int\frac{5+4x}{(4+5x)\sqrt{1-x^2}}\,dx$$
$$-\frac{1}{2}\int\frac{1}{\sqrt{1-x^2}}\,dx+\int\frac{x}{(4+5x)\sqrt{1-x^2}}\,dx$$

$$=\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{5}{9}\sin^{-1}(x)+\frac{1}{5}\int\frac{1}{\sqrt{1-x^2}}\,dx+\frac{16}{45}\int\frac{1}{\sqrt{1-x^2}}\,dx$$

$$=\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int\frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx=\frac{3+5\sqrt{1-x^2}}{20+25x}$$

```
[In] Integrate[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1),x]
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
trager	$-\frac{3x}{4(4+5x)}+\frac{\sqrt{-x^2+1}}{4+5x}$	29
default	$\frac{3}{5(4+5x)}-\frac{\sqrt{-(x+1)^2+2x+2}}{2}+\frac{5\left(-\left(\frac{4}{5}+x\right)^2+\frac{8x}{5}+\frac{41}{25}\right)^{\frac{3}{2}}}{9\left(\frac{4}{5}+x\right)}+\frac{5x\sqrt{-\left(\frac{4}{5}+x\right)^2+\frac{8x}{5}+\frac{41}{25}}}{9}+\frac{\sqrt{-(x-1)^2-2x+2}}{18}$	81

```
[In] int(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE
)
[Out] -3/4*x/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx = \frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

```
[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="fri
cas")
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

Sympy [F]

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx = -\int \frac{1}{3x^2+4x\sqrt{1-x^2}+5\sqrt{1-x^2}-3}\,dx$$

```
[In] integrate(1/(3-3*x**2-5*(-x**2+1)**(1/2)-4*x*(-x**2+1)**(1/2)),x)
[Out] -Integral(1/(3*x**2 + 4*x*sqrt(1 - x**2) + 5*sqrt(1 - x**2) - 3), x)
```

Maxima [F]

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx = \int -\frac{1}{3\,x^2+4\,\sqrt{-x^2+1}x+5\,\sqrt{-x^2+1}-3}\,dx$$

```
[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="max
ima")
[Out] -integrate(1/(3*x^2 + 4*sqrt(-x^2 + 1)*x + 5*sqrt(-x^2 + 1) - 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(27) = 54$.

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.19

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx = \frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-2\right)}+\frac{3}{5\,(5\,x+4)}$$

```
[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="gia
c")
```

```
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2
 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx=\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

```
[In] int(-1/(4*x*(1 - x^2)^(1/2) + 3*x^2 + 5*(1 - x^2)^(1/2) - 3),x)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

3.828 $\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 31

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx=\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

```
[Out] 3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.302, Rules used = {6874, 283, 222, 272, 52, 65, 212, 747, 858, 739, 749, 270, 745}

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx=\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[In] Int[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2),x]
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{\left(-2-x+2\sqrt{1-x^2}\right)^2}-\frac{1}{\sqrt{1-x^2}\left(-2-x+2\sqrt{1-x^2}\right)^2}\right)dx\\ &= \int \frac{1}{\left(-2-x+2\sqrt{1-x^2}\right)^2}\,dx-\int \frac{1}{\sqrt{1-x^2}\left(-2-x+2\sqrt{1-x^2}\right)^2}\,dx\end{aligned}$$

$$= -\int \left(\frac{1}{2x^2} - \frac{1}{x} + \frac{15}{2(4+5x)^2} + \frac{5}{4+5x} + \frac{1}{2x^2\sqrt{1-x^2}} - \frac{1}{x\sqrt{1-x^2}} + \frac{9}{2(4+5x)^2\sqrt{1-x^2}} + \frac{5}{(4+5x)\sqrt{1-x^2}}\right) dx + \int \left(\frac{1}{2x^2} - \frac{1}{x} + \frac{9}{2(4+5x)^2} + \frac{5}{4+5x} + \frac{\sqrt{1-x^2}}{2x^2} - \frac{\sqrt{1-x^2}}{x} + \frac{15\sqrt{1-x^2}}{2(4+5x)^2} + \frac{5\sqrt{1-x^2}}{4+5x}\right) dx$$

$$= \frac{3}{5(4+5x)} - \frac{1}{2}\int \frac{1}{x^2\sqrt{1-x^2}}\,dx + \frac{1}{2}\int \frac{\sqrt{1-x^2}}{x^2}\,dx - \frac{9}{2}\int \frac{1}{(4+5x)^2\sqrt{1-x^2}}\,dx - 5\int \frac{1}{(4+5x)\sqrt{1-x^2}}\,dx + 5\int \frac{\sqrt{1-x^2}}{4+5x}\,dx + \frac{15}{2}\int \frac{\sqrt{1-x^2}}{(4+5x)^2}\,dx + \int \frac{1}{x\sqrt{1-x^2}}\,dx - \int \frac{\sqrt{1-x^2}}{x}\,dx$$

$$= \frac{3}{5(4+5x)} + \sqrt{1-x^2} + \frac{\sqrt{1-x^2}}{4+5x} - \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx, x, x^2\right) - \frac{3}{2}\int \frac{x}{(4+5x)\sqrt{1-x^2}}\,dx + 2\int \frac{1}{(4+5x)\sqrt{1-x^2}}\,dx + 5\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{5+4x}{\sqrt{1-x^2}}\right) + \int \frac{5+4x}{(4+5x)\sqrt{1-x^2}}\,dx$$

$$= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} - \frac{1}{2}\sin^{-1}(x) + \frac{5}{3}\tanh^{-1}\left(\frac{5+4x}{3\sqrt{1-x^2}}\right) - \frac{3}{10}\int \frac{1}{\sqrt{1-x^2}}\,dx - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, x^2\right) + \frac{4}{5}\int \frac{1}{\sqrt{1-x^2}}\,dx + \frac{6}{5}\int \frac{1}{(4+5x)\sqrt{1-x^2}}\,dx + \frac{9}{5}\int \frac{1}{(4+5x)\sqrt{1-x^2}}\,dx - 2\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{5+4x}{\sqrt{1-x^2}}\right) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right)$$

$$= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} + \tanh^{-1}\left(\frac{5+4x}{3\sqrt{1-x^2}}\right) - \tanh^{-1}\left(\sqrt{1-x^2}\right) - \frac{6}{5}\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{5+4x}{\sqrt{1-x^2}}\right) - \frac{9}{5}\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{5+4x}{\sqrt{1-x^2}}\right) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right)$$

$$= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx=\frac{3+5\sqrt{1-x^2}}{20+25x}$$

```
[In] Integrate[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2),
x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
trager	$-\frac{3x}{4(4+5x)}+\frac{\sqrt{-x^2+1}}{4+5x}$	29
default	$\frac{\sqrt{-\left(\frac{4}{5}+x\right)^2+\frac{8x}{5}+\frac{41}{25}}}{4+5x}+\frac{3}{5(4+5x)}$	32

```
[In] int(((-x^2+1)^(1/2)-1)/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -3/4*x/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx=\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

```
[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, al
gorithm="fricas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

Sympy [F]

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx=\int \frac{\sqrt{1-x^2}-1}{\sqrt{-(x-1)(x+1)}\left(x-2\sqrt{1-x^2}+2\right)^2}\,dx$$

```
[In] integrate((-1+(-x**2+1)**(1/2))/(2+x-2*(-x**2+1)**(1/2))**2/(-x**2+1)**(1/2
),x)
[Out] Integral((sqrt(1 - x**2) - 1)/(sqrt(-(x - 1)*(x + 1))*(x - 2*sqrt(1 - x**2)
 + 2)**2), x)
```

Maxima [F]

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx=\int \frac{\sqrt{-x^2+1}-1}{\sqrt{-x^2+1}\left(x-2\sqrt{-x^2+1}+2\right)^2}\,dx$$

```
[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, al
gorithm="maxima")
[Out] -1/56*sqrt(7)*log((3*x - 2*sqrt(7) - 2)/(3*x + 2*sqrt(7) - 2)) - integrate(
-1/8*(100*x^7 + 285*x^6 + 264*x^5 + 80*x^4)/(21*x^9 + 278*x^8 + 283*x^7 - 2
022*x^6 - 3632*x^5 + 2256*x^4 + 7424*x^3 + 1536*x^2 - 8*(9*x^8 + 12*x^7 - 1
01*x^6 - 172*x^5 + 284*x^4 + 672*x^3 + 64*x^2 - 512*x - 256)*sqrt(x + 1)*sq
rt(-x + 1) - 4096*x - 2048), x) - 1/24*log(x + 2) + 1/16*log(x + 1) - 1/48*
log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(27) = 54$.

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.19

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx=\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-2\right)}+\frac{3}{5\,(5\,x+4)}$$

```
[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, al
gorithm="giac")
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2
 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

Mupad [B] (verification not implemented)

Time = 21.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx = \frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

```
[In] int(((1 - x^2)^(1/2) - 1)/((1 - x^2)^(1/2)*(x - 2*(1 - x^2)^(1/2) + 2)^2),x
)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

3.829 $\int \frac{a+bx^{-1+n}}{cx+dx^n} dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{a+bx^{-1+n}}{cx+dx^n} dx = \frac{b\log(x)}{d} - \frac{(bc-ad)\log(d+cx^{1-n})}{cd(1-n)}$$

```
[Out] b*ln(x)/d-(-a*d+b*c)*ln(d+c*x^(1-n))/c/d/(1-n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1607, 528, 457, 78}

$$\int \frac{a+bx^{-1+n}}{cx+dx^n} dx = \frac{b\log(x)}{d} - \frac{(bc-ad)\log(cx^{1-n}+d)}{cd(1-n)}$$

```
[In] Int[(a + b*x^(-1 + n))/(c*x + d*x^n),x]
```

```
[Out] (b*Log[x])/d - ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*d*(1 - n))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^{-n}(a+bx^{-1+n})}{d+cx^{1-n}}\,dx\\
&= \int \frac{b+ax^{1-n}}{x\,(d+cx^{1-n})}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{b+ax}{x(d+cx)}\,dx, x, x^{1-n}\right)}{1-n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{b}{dx}+\frac{-bc+ad}{d(d+cx)}\right)\,dx, x, x^{1-n}\right)}{1-n}\\
&= \frac{b\log(x)}{d}-\frac{(bc-ad)\log\left(d+cx^{1-n}\right)}{cd(1-n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$
\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx = \frac{b\log(x)+\frac{(bc-ad)\log\left(d+cx^{1-n}\right)}{c(-1+n)}}{d}
$$

```
[In] Integrate[(a + b*x^(-1 + n))/(c*x + d*x^n),x]
[Out] (b*Log[x] + ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*(-1 + n)))/d
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

method	result	size
norman	$\frac{(adn-bc)\ln(x)}{cd(-1+n)}-\frac{(ad-bc)\ln\left(cx+d\,\mathrm{e}^{n\ln(x)}\right)}{cd(-1+n)}$	58
risch	$\frac{b\ln(x)}{d}+\frac{n\ln(x)a}{c(-1+n)}-\frac{n\ln(x)b}{d(-1+n)}-\frac{\ln\left(x^n+\frac{xc}{d}\right)a}{c(-1+n)}+\frac{\ln\left(x^n+\frac{xc}{d}\right)b}{d(-1+n)}$	79

```
[In] int((a+b*x^(-1+n))/(c*x+d*x^n),x,method=_RETURNVERBOSE)
[Out] (a*d*n-b*c)/c/d/(-1+n)*ln(x)-(a*d-b*c)/c/d/(-1+n)*ln(c*x+d*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx=\frac{(bc-ad)\log\left(cx+dx^n\right)+(adn-bc)\log\left(x\right)}{cdn-cd}$$

```
[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="fricas")
[Out] ((b*c - a*d)*log(c*x + d*x^n) + (a*d*n - b*c)*log(x))/(c*d*n - c*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(29) = 58.

Time = 1.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 4.00

$$\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx =\begin{cases}\tilde{\infty}(a+b)\log\left(x\right) & \text{for } c=0\wedge d=0\wedge n=1\\ \frac{-\frac{ax}{nx^n-x^n}+\frac{bnx^n\log\left(x\right)}{nx^n-x^n}-\frac{bx^n\log\left(x\right)}{nx^n-x^n}}{d} & \text{for } c=0\\ \frac{\frac{an\log\left(x\right)}{n-1}-\frac{a\log\left(x\right)}{n-1}+\frac{bx^{n-1}}{n-1}}{c} & \text{for } d=0\\ \frac{(a+b)\log\left(x\right)}{c+d} & \text{for } n=1\\ \frac{adn\log\left(x\right)}{cdn-cd}-\frac{ad\log\left(x+\frac{dx^n}{c}\right)}{cdn-cd}-\frac{bc\log\left(x\right)}{cdn-cd}+\frac{bc\log\left(x+\frac{dx^n}{c}\right)}{cdn-cd} & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b*x**(-1+n))/(c*x+d*x**n),x)
```

```
[Out] Piecewise((zoo*(a + b)*log(x), Eq(c, 0) & Eq(d, 0) & Eq(n, 1)), ((-a*x/(n*x
**n - x**n) + b*n*x**n*log(x)/(n*x**n - x**n) - b*x**n*log(x)/(n*x**n - x**
n))/d, Eq(c, 0)), ((a*n*log(x)/(n - 1) - a*log(x)/(n - 1) + b*x**(n - 1)/(n
 - 1))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq(n, 1)), (a*d*n*log(x)/(c*d
*n - c*d) - a*d*log(x + d*x**n/c)/(c*d*n - c*d) - b*c*log(x)/(c*d*n - c*d)
+ b*c*log(x + d*x**n/c)/(c*d*n - c*d), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(40) = 80$.

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx = b\left(\frac{\log(x)}{d} - \frac{n\log(x)}{d(n-1)} + \frac{\log\left(\frac{cx+dx^n}{d}\right)}{d(n-1)}\right) + a\left(\frac{n\log(x)}{c(n-1)} - \frac{\log\left(\frac{cx+dx^n}{d}\right)}{c(n-1)}\right)$$

```
[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="maxima")
```

```
[Out] b*(log(x)/d - n*log(x)/(d*(n - 1)) + log((c*x + d*x^n)/d)/(d*(n - 1))) + a*
(n*log(x)/(c*(n - 1)) - log((c*x + d*x^n)/d)/(c*(n - 1)))
```

Giac [F]

$$\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx = \int \frac{bx^{n-1}+a}{cx+dx^n}\,dx$$

```
[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^(n - 1) + a)/(c*x + d*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx = \int \frac{a+b\,x^{n-1}}{d\,x^n+c\,x}\,dx$$

```
[In] int((a + b*x^(n - 1))/(d*x^n + c*x),x)
```

```
[Out] int((a + b*x^(n - 1))/(d*x^n + c*x), x)
```

3.830 $\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 42

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx=-\frac{1}{2x}+x+\frac{\sqrt{1+2x^2}}{2x}-\frac{\operatorname{arcsinh}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] -1/2/x+x-1/2*arcsinh(x*2^(1/2))*2^(1/2)+1/2*(2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6872, 6874, 283, 221}

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx=-\frac{\operatorname{arcsinh}\left(\sqrt{2}x\right)}{\sqrt{2}}+\frac{\sqrt{2x^2+1}}{2x}+x-\frac{1}{2x}$$

```
[In] Int[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
```

```
[Out] -1/2*1/x + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(1+\frac{1}{-1-\sqrt{1+2x^2}}\right)\,dx \\
&= x+\int \frac{1}{-1-\sqrt{1+2x^2}}\,dx \\
&= x+\int \left(\frac{1}{2x^2}-\frac{\sqrt{1+2x^2}}{2x^2}\right)\,dx \\
&= -\frac{1}{2x}+x-\frac{1}{2}\int \frac{\sqrt{1+2x^2}}{x^2}\,dx \\
&= -\frac{1}{2x}+x+\frac{\sqrt{1+2x^2}}{2x}-\int \frac{1}{\sqrt{1+2x^2}}\,dx \\
&= -\frac{1}{2x}+x+\frac{\sqrt{1+2x^2}}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx=\frac{-1+2x^2+\sqrt{1+2x^2}+\sqrt{2}x\log\left(-\sqrt{2}x+\sqrt{1+2x^2}\right)}{2x}$$

```
[In] Integrate[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
[Out] (-1 + 2*x^2 + Sqrt[1 + 2*x^2] + Sqrt[2]*x*Log[-(Sqrt[2]*x) + Sqrt[1 + 2*x^2
]])/(2*x)
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

method	result	size
default	$x-\frac{1}{2x}+\frac{\left(2x^{2}+1\right)^{\frac{3}{2}}}{2x}-x\sqrt{2x^{2}+1}-\frac{\operatorname{arcsinh}\left(x\sqrt{2}\right)\sqrt{2}}{2}$	45
trager	$\frac{(x-1)(1+2x)}{2x}+\frac{\sqrt{2x^{2}+1}}{2x}-\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}-2\right)x+\sqrt{2x^{2}+1}\right)}{2}$	56

```
[In] int((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] x-1/2/x+1/2/x*(2*x^2+1)^(3/2)-x*(2*x^2+1)^(1/2)-1/2*arcsinh(x*2^(1/2))*2^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx=\frac{\sqrt{2}x\log\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)+2\,x^2+\sqrt{2\,x^2+1}-1}{2\,x}$$

```
[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="fricas")
[Out] 1/2*(sqrt(2)*x*log(sqrt(2)*x - sqrt(2*x^2 + 1)) + 2*x^2 + sqrt(2*x^2 + 1) -
 1)/x
```

Sympy [F]

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx=\int \frac{\sqrt{2x^2+1}}{\sqrt{2x^2+1}+1}\,dx$$

```
[In] integrate((2*x**2+1)**(1/2)/(1+(2*x**2+1)**(1/2)),x)
[Out] Integral(sqrt(2*x**2 + 1)/(sqrt(2*x**2 + 1) + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{2\,x^2+1}+1}\,dx$$

```
[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="maxima")
[Out] x - integrate(1/(sqrt(2*x^2 + 1) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{2\,x^2+1}\right)+x-\frac{\sqrt{2}}{\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)^2-1}-\frac{1}{2\,x}$$

```
[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="giac")
[Out] 1/2*sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + x - sqrt(2)/((sqrt(2)*x - s
qrt(2*x^2 + 1))^2 - 1) - 1/2/x
```

Mupad [B] (verification not implemented)

Time = 20.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx = x-\frac{\sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\,x\right)}{2}+\frac{\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}-\frac{1}{2}}{x}$$

```
[In] int((2*x^2 + 1)^(1/2)/((2*x^2 + 1)^(1/2) + 1),x)
[Out] x - (2^(1/2)*asinh(2^(1/2)*x))/2 + ((2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1/2)/x
```

3.831 $\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 65

$$\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx=\frac{4x}{3}-\frac{1}{3}\sqrt{-1+4x^2}-\frac{\operatorname{arctanh}\left(\sqrt{3}x\right)}{3\sqrt{3}}+\frac{\operatorname{arctanh}\left(\sqrt{3}\sqrt{-1+4x^2}\right)}{3\sqrt{3}}$$

```
[Out] 4/3*x-1/9*arctanh(x*3^(1/2))*3^(1/2)+1/9*arctanh(3^(1/2)*(4*x^2-1)^(1/2))*3
^(1/2)-1/3*(4*x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6874, 455, 52, 65, 213, 396}

$$\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx=\frac{\operatorname{arctanh}\left(\sqrt{3}\sqrt{4x^2-1}\right)}{3\sqrt{3}}-\frac{\operatorname{arctanh}\left(\sqrt{3}x\right)}{3\sqrt{3}}-\frac{1}{3}\sqrt{4x^2-1}+\frac{4x}{3}$$

```
[In] Int[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]),x]
```

```
[Out] (4*x)/3 - Sqrt[-1 + 4*x^2]/3 - ArcTanh[Sqrt[3]*x]/(3*Sqrt[3]) + ArcTanh[Sqr
t[3]*Sqrt[-1 + 4*x^2]]/(3*Sqrt[3])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{x\sqrt{-1+4x^2}}{-1+3x^2}+\frac{-1+4x^2}{-1+3x^2}\right)\,dx \\
&= -\int \frac{x\sqrt{-1+4x^2}}{-1+3x^2}\,dx+\int \frac{-1+4x^2}{-1+3x^2}\,dx \\
&= \frac{4x}{3}+\frac{1}{3}\int \frac{1}{-1+3x^2}\,dx-\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-1+4x}}{-1+3x}\,dx,x,x^2\right)
\end{aligned}
$$

$$= \frac{4x}{3} - \frac{1}{3}\sqrt{-1+4x^2} - \frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{(-1+3x)\sqrt{-1+4x}}\,dx, x, x^2\right)$$

$$= \frac{4x}{3} - \frac{1}{3}\sqrt{-1+4x^2} - \frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}} - \frac{1}{12}\text{Subst}\left(\int \frac{1}{-\frac{1}{4}+\frac{3x^2}{4}}\,dx, x, \sqrt{-1+4x^2}\right)$$

$$= \frac{4x}{3} - \frac{1}{3}\sqrt{-1+4x^2} - \frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}} + \frac{\tanh^{-1}\left(\sqrt{3}\sqrt{-1+4x^2}\right)}{3\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx = \frac{1}{9}\left(12x - 3\sqrt{-1+4x^2} + 2\sqrt{3}\text{arctanh}\left(\frac{-2x+\sqrt{-1+4x^2}}{\sqrt{3}}\right)\right)$$

```
[In] Integrate[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]),x]
```

```
[Out] (12*x - 3*Sqrt[-1 + 4*x^2] + 2*Sqrt[3]*ArcTanh[(-2*x + Sqrt[-1 + 4*x^2])/Sq
rt[3]])/9
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

method	result
trager	$\frac{4x}{3} - \frac{\sqrt{4x^2-1}}{3} + \frac{\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{\text{RootOf}\left(\_Z^2-3\right)+3\sqrt{4x^2-1}}{\text{RootOf}\left(\_Z^2-3\right)x+1}\right)}{9}$
default	$\frac{4x}{3} - \frac{\text{arctanh}\left(x\sqrt{3}\right)\sqrt{3}}{9} - \frac{\sqrt{36\left(x+\frac{\sqrt{3}}{3}\right)^2-24\sqrt{3}\left(x+\frac{\sqrt{3}}{3}\right)+3}}{18} + \frac{\sqrt{3}\ln\left(\sqrt{4}\,x+\sqrt{4\left(x+\frac{\sqrt{3}}{3}\right)^2-\frac{8\sqrt{3}\left(x+\frac{\sqrt{3}}{3}\right)}{3}+\frac{1}{3}}\right)\sqrt{4}}{18} + \frac{\sqrt{3}\ \text{arc}}{}$

```
[In] int((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*x-1/3*(4*x^2-1)^(1/2)+1/9*RootOf(_Z^2-3)*ln(-(RootOf(_Z^2-3)+3*(4*x^2-1
)^(1/2))/(RootOf(_Z^2-3)*x+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx = \frac{1}{18}\,\sqrt{3}\log\left(\frac{6\,x^2+\sqrt{3}\sqrt{4\,x^2-1}-1}{3\,x^2-1}\right) + \frac{1}{18}\,\sqrt{3}\log\left(\frac{3\,x^2-2\,\sqrt{3}x+1}{3\,x^2-1}\right) + \frac{4}{3}\,x - \frac{1}{3}\,\sqrt{4\,x^2-1}$$

```
[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="fricas")
[Out] 1/18*sqrt(3)*log((6*x^2 + sqrt(3)*sqrt(4*x^2 - 1) - 1)/(3*x^2 - 1)) + 1/18*
sqrt(3)*log((3*x^2 - 2*sqrt(3)*x + 1)/(3*x^2 - 1)) + 4/3*x - 1/3*sqrt(4*x^2
 - 1)
```

Sympy [F]

$$\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx = \int \frac{\sqrt{(2x-1)\,(2x+1)}}{x+\sqrt{4x^2-1}}\,dx$$

```
[In] integrate((4*x**2-1)**(1/2)/(x+(4*x**2-1)**(1/2)),x)
[Out] Integral(sqrt((2*x - 1)*(2*x + 1))/(x + sqrt(4*x**2 - 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx = \int \frac{\sqrt{4\,x^2-1}}{x+\sqrt{4\,x^2-1}}\,dx$$

```
[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="maxima")
[Out] x - integrate(x/(sqrt(2*x + 1)*sqrt(2*x - 1) + x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(45) = 90$.

Time = 0.33 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx = \frac{1}{18}\sqrt{3}\log\left(\frac{|6\,x-2\,\sqrt{3}|}{|6\,x+2\,\sqrt{3}|}\right) - \frac{1}{18}\sqrt{3}\log\left(-\frac{\left|-12\,x-4\,\sqrt{3}+6\,\sqrt{4\,x^2-1}+\frac{6}{2\,x-\sqrt{4\,x^2-1}}\right|}{2\left(6\,x-2\,\sqrt{3}-3\,\sqrt{4\,x^2-1}-\frac{3}{2\,x-\sqrt{4\,x^2-1}}\right)}\right) + \frac{4}{3}\,x - \frac{1}{3}\,\sqrt{4\,x^2-1}$$

```
[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/18*sqrt(3)*log(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18*sqrt(3)*
log(-1/2*abs(-12*x - 4*sqrt(3) + 6*sqrt(4*x^2 - 1) + 6/(2*x - sqrt(4*x^2 -
1)))/(6*x - 2*sqrt(3) - 3*sqrt(4*x^2 - 1) - 3/(2*x - sqrt(4*x^2 - 1)))) + 4
/3*x - 1/3*sqrt(4*x^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx = \frac{4\,x}{3} + \frac{\sqrt{3}\,\ln\left(x-\frac{\sqrt{3}}{3}\right)}{18} - \frac{\sqrt{3}\,\ln\left(x+\frac{\sqrt{3}}{3}\right)}{18} + \frac{\sqrt{3}\,\mathrm{atanh}\left(\sqrt{3}\,\sqrt{4\,x^2-1}\right)}{9} - \frac{\sqrt{4\,x^2-1}}{3}$$

```
[In] int((4*x^2 - 1)^(1/2)/(x + (4*x^2 - 1)^(1/2)),x)
```

```
[Out] (4*x)/3 + (3^(1/2)*log(x - 3^(1/2)/3))/18 - (3^(1/2)*log(x + 3^(1/2)/3))/18
 + (3^(1/2)*atanh(3^(1/2)*(4*x^2 - 1)^(1/2)))/9 - (4*x^2 - 1)^(1/2)/3
```

3.832 $\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 195

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = -\frac{(cd^2-bde+ae^2)\sqrt{-1+x^2}}{2e(d^2-e^2)(d+ex)^2} + \frac{(c(d^3-4de^2)-e(3ade-b(d^2+2e^2)))\sqrt{-1+x^2}}{2e(d^2-e^2)^2(d+ex)} - \frac{(3bde-a(2d^2+e^2)-c(d^2+2e^2))\,\mathrm{arctanh}\left(\frac{e+dx}{\sqrt{d^2-e^2}\sqrt{-1+x^2}}\right)}{2(d^2-e^2)^{5/2}}$$

```
[Out] -1/2*(3*b*d*e-a*(2*d^2+e^2)-c*(d^2+2*e^2))*arctanh((d*x+e)/(d^2-e^2)^(1/2)/
(x^2-1)^(1/2))/(d^2-e^2)^(5/2)-1/2*(a*e^2-b*d*e+c*d^2)*(x^2-1)^(1/2)/e/(d^2
-e^2)/(e*x+d)^2+1/2*(c*(d^3-4*d*e^2)-e*(3*a*d*e-b*(d^2+2*e^2)))*(x^2-1)^(1/
2)/e/(d^2-e^2)^2/(e*x+d)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1665, 821, 739, 212}

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = -\frac{\mathrm{arctanh}\left(\frac{dx+e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right)(-a(2d^2+e^2)+3bde-c(d^2+2e^2))}{2(d^2-e^2)^{5/2}} - \frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2} + \frac{\sqrt{x^2-1}(c(d^3-4de^2)-e(3ade-b(d^2+2e^2)))}{2e(d^2-e^2)^2(d+ex)}$$

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]),x]
```

```
[Out] -1/2*((c*d^2 - b*d*e + a*e^2)*Sqrt[-1 + x^2])/(e*(d^2 - e^2)*(d + e*x)^2) +
 ((c*(d^3 - 4*d*e^2) - e*(3*a*d*e - b*(d^2 + 2*e^2)))*Sqrt[-1 + x^2])/(2*e*
(d^2 - e^2)^2*(d + e*x)) - ((3*b*d*e - a*(2*d^2 + e^2) - c*(d^2 + 2*e^2))*A
rcTanh[(e + d*x)/(Sqrt[d^2 - e^2]*Sqrt[-1 + x^2])])/(2*(d^2 - e^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(cd^2-bde+ae^2)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)(d+ex)^2}-\frac{\int \frac{-2(ad+cd-be)-\left(bd+\frac{cd^2}{e}-ae-2ce\right)x}{(d+ex)^2\sqrt{-1+x^2}}\,dx}{2\left(d^2-e^2\right)}\\
&= -\frac{(cd^2-bde+ae^2)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)(d+ex)^2}+\frac{\left(c(d^3-4de^2)-e(3ade-b(d^2+2e^2))\right)\sqrt{-1+x^2}}{2e\left(d^2-e^2\right)^2(d+ex)}\\
&\quad -\frac{\left(3bde-a(2d^2+e^2)-c(d^2+2e^2)\right)\int \frac{1}{(d+ex)\sqrt{-1+x^2}}\,dx}{2\left(d^2-e^2\right)^2}\end{aligned}$$

$$= -\frac{(cd^2 - bde + ae^2)\sqrt{-1+x^2}}{2e(d^2-e^2)(d+ex)^2} + \frac{(c(d^3-4de^2) - e(3ade - b(d^2+2e^2)))\sqrt{-1+x^2}}{2e(d^2-e^2)^2(d+ex)} + \frac{(3bde - a(2d^2+e^2) - c(d^2+2e^2))\,\text{Subst}\left(\int \frac{1}{d^2-e^2-x^2}\,dx, x, \frac{-e-dx}{\sqrt{-1+x^2}}\right)}{2(d^2-e^2)^2}$$

$$= -\frac{(cd^2 - bde + ae^2)\sqrt{-1+x^2}}{2e(d^2-e^2)(d+ex)^2} + \frac{(c(d^3-4de^2) - e(3ade - b(d^2+2e^2)))\sqrt{-1+x^2}}{2e(d^2-e^2)^2(d+ex)} - \frac{(3bde - a(2d^2+e^2) - c(d^2+2e^2))\tanh^{-1}\left(\frac{e+dx}{\sqrt{d^2-e^2}\sqrt{-1+x^2}}\right)}{2(d^2-e^2)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.92

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = \frac{\frac{(d-e)(d+e)\sqrt{-1+x^2}(ae(-4d^2+e^2-3dex)+cd(-3de+d^2x-4e^2x)+b(2d^3+de^2+d^2ex+2e^3x))}{(d+ex)^2} + 2\sqrt{-d^2+e^2}(-3bde+a(2d^2+e^2}{2(d-e)^3(d+e)^3}$$

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]),x]
```

```
[Out] (((d - e)*(d + e)*Sqrt[-1 + x^2]*(a*e*(-4*d^2 + e^2 - 3*d*e*x) + c*d*(-3*d*
e + d^2*x - 4*e^2*x) + b*(2*d^3 + d*e^2 + d^2*e*x + 2*e^3*x)))/(d + e*x)^2
+ 2*Sqrt[-d^2 + e^2]*(-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2))*ArcTan[
(d + e*(x - Sqrt[-1 + x^2]))/Sqrt[-d^2 + e^2]])/(2*(d - e)^3*(d + e)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(179) = 358$.

Time = 1.39 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.74

method	result
default	$-\frac{c\ln\left(\frac{\frac{2d^2-2e^2}{e^2}-\frac{2d\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{d^2-e^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2-\frac{2d\left(x+\frac{d}{e}\right)}{e}+\frac{d^2-e^2}{e^2}}}{x+\frac{d}{e}}\right)}{e^3\sqrt{\frac{d^2-e^2}{e^2}}}+\frac{(be-2cd)\left(-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right)^2-\frac{2d\left(x+\frac{d}{e}\right)}{e}+\frac{d^2-e^2}{e^2}}}{\left(d^2-e^2\right)\left(x+\frac{d}{e}\right)}-\frac{ed\ln\left(\frac{2d^2-}{e^2}\right.}{}\right.}{e}$

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -c/e^3/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2/e*d*(x+1/e*d)+2*((d^2-e^
2)/e^2)^(1/2)*((x+1/e*d)^2-2/e*d*(x+1/e*d)+(d^2-e^2)/e^2)^(1/2))/(x+1/e*d))
+(b*e-2*c*d)/e^4*(-1/(d^2-e^2)*e^2/(x+1/e*d)*((x+1/e*d)^2-2/e*d*(x+1/e*d)+(
d^2-e^2)/e^2)^(1/2)-e*d/(d^2-e^2)/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2
-2/e*d*(x+1/e*d)+2*((d^2-e^2)/e^2)^(1/2)*((x+1/e*d)^2-2/e*d*(x+1/e*d)+(d^2-
e^2)/e^2)^(1/2))/(x+1/e*d)))+(a*e^2-b*d*e+c*d^2)/e^5*(-1/2/(d^2-e^2)*e^2/(x
+1/e*d)^2*((x+1/e*d)^2-2/e*d*(x+1/e*d)+(d^2-e^2)/e^2)^(1/2)+3/2*e*d/(d^2-e^
2)*(-1/(d^2-e^2)*e^2/(x+1/e*d)*((x+1/e*d)^2-2/e*d*(x+1/e*d)+(d^2-e^2)/e^2)^
(1/2)-e*d/(d^2-e^2)/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2/e*d*(x+1/e*
d)+2*((d^2-e^2)/e^2)^(1/2)*((x+1/e*d)^2-2/e*d*(x+1/e*d)+(d^2-e^2)/e^2)^(1/2
))/(x+1/e*d)))+1/2/(d^2-e^2)*e^2/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-
2/e*d*(x+1/e*d)+2*((d^2-e^2)/e^2)^(1/2)*((x+1/e*d)^2-2/e*d*(x+1/e*d)+(d^2-e
^2)/e^2)^(1/2))/(x+1/e*d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(179) = 358$.

Time = 0.30 (sec) , antiderivative size = 1174, normalized size of antiderivative = 6.02

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c)*d^3*e
^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5
 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 + ((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 + (a
 + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^2 + 2*(
(2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*sqrt(d^2 - e^2)*log((
d^2*x + d*e + sqrt(d^2 - e^2)*(d*x + e) + (d^2 - e^2 + sqrt(d^2 - e^2)*d)*s
qrt(x^2 - 1))/(e*x + d)) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 + b
*d^3*e^4 + (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x + (2*b*d^5*e^2 - (4*a + 3*c)*
d^4*e^3 - b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 +
b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*
x)*sqrt(x^2 - 1))/(d^8*e^2 - 3*d^6*e^4 + 3*d^4*e^6 - d^2*e^8 + (d^6*e^4 - 3
*d^4*e^6 + 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e^3 - 3*d^5*e^5 + 3*d^3*e^7 - d*e
^9)*x), 1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c
)*d^3*e^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*
d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 - 2*((2*a + c)*d^4*e^2 - 3*b*d^3
*e^3 + (a + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*
x^2 + 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*sqrt(-d^2 +
e^2)*arctan(-(sqrt(-d^2 + e^2)*sqrt(x^2 - 1)*e - sqrt(-d^2 + e^2)*(e*x + d)
)/(d^2 - e^2)) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 + b*d^3*e^4 +
 (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x + (2*b*d^5*e^2 - (4*a + 3*c)*d^4*e^3 -
b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 + b*d^4*e^3
- (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x)*sqrt(x^
2 - 1))/(d^8*e^2 - 3*d^6*e^4 + 3*d^4*e^6 - d^2*e^8 + (d^6*e^4 - 3*d^4*e^6 +
 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e^3 - 3*d^5*e^5 + 3*d^3*e^7 - d*e^9)*x)]
```

Sympy [F]

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = \int \frac{a+bx+cx^2}{\sqrt{(x-1)(x+1)}\,(d+ex)^3}\,dx$$

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(x**2-1)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/(sqrt((x - 1)*(x + 1))*(d + e*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((e-d)*(e+d)>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(179) = 358$.

Time = 0.34 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.86

$$\int \frac{a + bx + cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = \frac{(2\,ad^2 + cd^2 - 3\,bde + ae^2 + 2\,ce^2)\arctan\left(-\frac{e\left(x-\sqrt{x^2-1}\right)+d}{\sqrt{-d^2+e^2}}\right)}{(d^4 - 2\,d^2e^2 + e^4)\sqrt{-d^2+e^2}} + \frac{2\,cd^4e\left(x-\sqrt{x^2-1}\right)^3 - 2\,ad^2e^3\left(x-\sqrt{x^2-1}\right)^3 - 5\,cd^2e^3\left(x-\sqrt{x^2-1}\right)^3 + 3\,bde^4\left(x-\sqrt{x^2-1}\right)^3 - ae^5}{}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] (2*a*d^2 + c*d^2 - 3*b*d*e + a*e^2 + 2*c*e^2)*arctan(-(e*(x - sqrt(x^2 - 1)
) + d)/sqrt(-d^2 + e^2))/((d^4 - 2*d^2*e^2 + e^4)*sqrt(-d^2 + e^2)) + (2*c*
d^4*e*(x - sqrt(x^2 - 1))^3 - 2*a*d^2*e^3*(x - sqrt(x^2 - 1))^3 - 5*c*d^2*e
^3*(x - sqrt(x^2 - 1))^3 + 3*b*d*e^4*(x - sqrt(x^2 - 1))^3 - a*e^5*(x - sqr
t(x^2 - 1))^3 + 2*c*d^5*(x - sqrt(x^2 - 1))^2 + 2*b*d^4*e*(x - sqrt(x^2 - 1
))^2 - 6*a*d^3*e^2*(x - sqrt(x^2 - 1))^2 - 7*c*d^3*e^2*(x - sqrt(x^2 - 1))^
2 + 5*b*d^2*e^3*(x - sqrt(x^2 - 1))^2 - 3*a*d*e^4*(x - sqrt(x^2 - 1))^2 - 4
*c*d*e^4*(x - sqrt(x^2 - 1))^2 + 2*b*e^5*(x - sqrt(x^2 - 1))^2 + 2*c*d^4*e*
(x - sqrt(x^2 - 1)) + 4*b*d^3*e^2*(x - sqrt(x^2 - 1)) - 10*a*d^2*e^3*(x - s
qrt(x^2 - 1)) - 11*c*d^2*e^3*(x - sqrt(x^2 - 1)) + 5*b*d*e^4*(x - sqrt(x^2
- 1)) + a*e^5*(x - sqrt(x^2 - 1)) + c*d^3*e^2 + b*d^2*e^3 - 3*a*d*e^4 - 4*c
*d*e^4 + 2*b*e^5)/((d^4*e^2 - 2*d^2*e^4 + e^6)*(e*(x - sqrt(x^2 - 1))^2 + 2
*d*(x - sqrt(x^2 - 1)) + e)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + bx + cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = \int \frac{c\,x^2 + b\,x + a}{\sqrt{x^2-1}\,{\left(d+e\,x\right)}^3}\,dx$$

```
[In] int((a + b*x + c*x^2)/((x^2 - 1)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int((a + b*x + c*x^2)/((x^2 - 1)^(1/2)*(d + e*x)^3), x)
```

3.833 $\int \frac{1+2x^8}{x(1+x^8)^{3/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 28

$$\int \frac{1+2x^8}{x(1+x^8)^{3/2}} \, dx = -\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4}\text{arctanh}\left(\sqrt{1+x^8}\right)$$

```
[Out] -1/4*arctanh((x^8+1)^(1/2))-1/4/(x^8+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {457, 79, 65, 213}

$$\int \frac{1+2x^8}{x(1+x^8)^{3/2}} \, dx = -\frac{1}{4}\text{arctanh}\left(\sqrt{x^8+1}\right) - \frac{1}{4\sqrt{x^8+1}}$$

```
[In] Int[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]
```

```
[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{x(1+x)^{3/2}}\,dx, x, x^8\right) \\
&= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{8}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^8\right) \\
&= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^8}\right) \\
&= -\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx = -\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4}\text{arctanh}\left(\sqrt{1+x^8}\right)$$

```
[In] Integrate[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]
[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$-\frac{1}{4\sqrt{x^8+1}}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^8+1}}\right)}{4}$	21
trager	$-\frac{1}{4\sqrt{x^8+1}}-\frac{\ln\left(\frac{\sqrt{x^8+1}+1}{x^4}\right)}{4}$	27
risch	$-\frac{1}{4\sqrt{x^8+1}}+\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^8+1}}{2}\right)+(-2\ln(2)+8\ln(x))\sqrt{\pi}}{8\sqrt{\pi}}$	47
meijerg	$\frac{-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{x^8+1}}-\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^8+1}}{2}\right)+\frac{(2-2\ln(2)+8\ln(x))\sqrt{\pi}}{2}}{4\sqrt{\pi}}+\frac{\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{x^8+1}}}{2\sqrt{\pi}}$	77

```
[In] int((2*x^8+1)/x/(x^8+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/4/(x^8+1)^(1/2)-1/4*arctanh(1/(x^8+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(20) = 40.

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx=-\frac{(x^8+1)\log\left(\sqrt{x^8+1}+1\right)-(x^8+1)\log\left(\sqrt{x^8+1}-1\right)+2\sqrt{x^8+1}}{8\left(x^8+1\right)}$$

```
[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="fricas")
[Out] -1/8*((x^8 + 1)*log(sqrt(x^8 + 1) + 1) - (x^8 + 1)*log(sqrt(x^8 + 1) - 1) +
 2*sqrt(x^8 + 1))/(x^8 + 1)
```

Sympy [A] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx=\frac{\log\left(\sqrt{x^8+1}-1\right)}{8}-\frac{\log\left(\sqrt{x^8+1}+1\right)}{8}-\frac{1}{4\sqrt{x^8+1}}$$

```
[In] integrate((2*x**8+1)/x/(x**8+1)**(3/2),x)
[Out] log(sqrt(x**8 + 1) - 1)/8 - log(sqrt(x**8 + 1) + 1)/8 - 1/(4*sqrt(x**8 + 1)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx = -\frac{1}{4\sqrt{x^8+1}} - \frac{1}{8}\log\left(\sqrt{x^8+1}+1\right) + \frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

```
[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="maxima")
[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1
)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx = -\frac{1}{4\sqrt{x^8+1}} - \frac{1}{8}\log\left(\sqrt{x^8+1}+1\right) + \frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

```
[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="giac")
[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1
)
```

Mupad [B] (verification not implemented)

Time = 21.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx = -\frac{\operatorname{atanh}\left(\sqrt{x^8+1}\right)}{4} - \frac{1}{4\sqrt{x^8+1}}$$

```
[In] int((2*x^8 + 1)/(x*(x^8 + 1)^(3/2)),x)
[Out] - atanh((x^8 + 1)^(1/2))/4 - 1/(4*(x^8 + 1)^(1/2))
```

3.834 $\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}} dx$

Optimal result

Integrand size = 29, antiderivative size = 28

$$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}} dx = -\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4}\operatorname{arctanh}\left(\sqrt{1+x^8}\right)$$

```
[Out] -1/4*arctanh((x^8+1)^(1/2))-1/4/(x^8+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1600, 1607, 457, 79, 65, 213}

$$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}} dx = -\frac{1}{4}\operatorname{arctanh}\left(\sqrt{x^8+1}\right) - \frac{1}{4\sqrt{x^8+1}}$$

```
[In] Int[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17),x]
```

```
[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1+2x^8}{\sqrt{1+x^8}\,(x+x^9)}\,dx\\
&= \int \frac{1+2x^8}{x\,(1+x^8)^{3/2}}\,dx\\
&= \frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{x(1+x)^{3/2}}\,dx,x,x^8\right)\\
&= -\frac{1}{4\sqrt{1+x^8}}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^8\right)
\end{aligned}$$

$$= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^8}\right)$$

$$= -\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}}\,dx = -\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4}\text{arctanh}\left(\sqrt{1+x^8}\right)$$

```
[In] Integrate[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17),x]
[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$-\frac{1}{4\sqrt{x^8+1}}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^8+1}}\right)}{4}$	21
trager	$-\frac{1}{4\sqrt{x^8+1}}+\frac{\ln\left(\frac{\sqrt{x^8+1}-1}{x^4}\right)}{4}$	27
risch	$-\frac{1}{4\sqrt{x^8+1}}+\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^8+1}}{2}\right)+(-2\ln(2)+8\ln(x))\sqrt{\pi}}{8\sqrt{\pi}}$	47

```
[In] int((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x,method=_RETURNVERBOSE)
[Out] -1/4/(x^8+1)^(1/2)-1/4*arctanh(1/(x^8+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(20) = 40.

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}}\,dx = -\frac{(x^8+1)\log\left(\sqrt{x^8+1}+1\right)-(x^8+1)\log\left(\sqrt{x^8+1}-1\right)+2\sqrt{x^8+1}}{8\,(x^8+1)}$$

```
[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="fricas")
```

```
[Out] -1/8*((x^8 + 1)*log(sqrt(x^8 + 1) + 1) - (x^8 + 1)*log(sqrt(x^8 + 1) - 1) +
 2*sqrt(x^8 + 1))/(x^8 + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}}\,dx = \int \frac{2x^8+1}{x\left(x^8+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((2*x**8+1)*(x**8+1)**(1/2)/(x**17+2*x**9+x),x)
```

```
[Out] Integral((2*x**8 + 1)/(x*(x**8 + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}}\,dx = \int \frac{(2\,x^8+1)\sqrt{x^8+1}}{x^{17}+2\,x^9+x}\,dx$$

```
[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="maxima")
```

```
[Out] integrate((2*x^8 + 1)*sqrt(x^8 + 1)/(x^17 + 2*x^9 + x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}}\,dx = -\frac{1}{4\sqrt{x^8+1}} - \frac{1}{8}\log\left(\sqrt{x^8+1}+1\right) + \frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

```
[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="giac")
```

```
[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}}\,dx = \int \frac{\sqrt{x^8+1}\,(2\,x^8+1)}{x^{17}+2\,x^9+x}\,dx$$

```
[In] int(((x^8 + 1)^(1/2)*(2*x^8 + 1))/(x + 2*x^9 + x^17),x)
[Out] int(((x^8 + 1)^(1/2)*(2*x^8 + 1))/(x + 2*x^9 + x^17), x)
```

3.835 $\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 22

$$\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx=x-3x^3-\frac{1}{9}\sqrt{1-9x^2}$$

```
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {267}

$$\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx=-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

```
[In] Int[1 - 9*x^2 + x/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral}&=x-3x^3+\int \frac{x}{\sqrt{1-9x^2}}\,dx\\ &=x-3x^3-\frac{1}{9}\sqrt{1-9x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx=x-3x^3-\frac{1}{9}\sqrt{1-9x^2}$$

```
[In] Integrate[1 - 9*x^2 + x/Sqrt[1 - 9*x^2],x]
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$x-3x^3-\frac{\sqrt{-9x^2+1}}{9}$	19
trager	$-\left(3x^2-1\right)x-\frac{\sqrt{-9x^2+1}}{9}$	23
risch	$-3x^3+x+\frac{9x^2-1}{9\sqrt{-9x^2+1}}$	26

```
[In] int(1-9*x^2+x/(-9*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx=-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

```
[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx=-3x^3+x-\frac{\sqrt{1-9x^2}}{9}$$

```
[In] integrate(1-9*x**2+x/(-9*x**2+1)**(1/2),x)
[Out] -3*x**3 + x - sqrt(1 - 9*x**2)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx=-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

```
[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx=-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

```
[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="giac")
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 21.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx=x-3\,x^3-\frac{\sqrt{\frac{1}{9}-x^2}}{3}$$

```
[In] int(x/(1 - 9*x^2)^(1/2) - 9*x^2 + 1,x)
[Out] x - 3*x^3 - (1/9 - x^2)^(1/2)/3
```

3.836 $\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 22

$$\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx = x-3x^3-\frac{1}{9}\sqrt{1-9x^2}$$

```
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6874, 267}

$$\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx = -3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

```
[In] Int[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2],x]
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)dx\\ &= x-3x^3+\int \frac{x}{\sqrt{1-9x^2}}\,dx\\ &= x-3x^3-\frac{1}{9}\sqrt{1-9x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx = x-3x^3-\frac{1}{9}\sqrt{1-9x^2}$$

```
[In] Integrate[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2],x]
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$x-3x^3-\frac{\sqrt{-9x^2+1}}{9}$	19
trager	$-(3x^2-1)\,x-\frac{\sqrt{-9x^2+1}}{9}$	23

```
[In] int((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx = -3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

```
[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx=-3x^3+x-\frac{\sqrt{1-9x^2}}{9}$$

```
[In] integrate((x+(-9*x**2+1)**(3/2))/(-9*x**2+1)**(1/2),x)
[Out] -3*x**3 + x - sqrt(1 - 9*x**2)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx=-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

```
[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx=-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

```
[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="giac")
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx=x-3\,x^3-\frac{\sqrt{\frac{1}{9}-x^2}}{3}$$

```
[In] int((x + (1 - 9*x^2)^(3/2))/(1 - 9*x^2)^(1/2),x)
[Out] x - 3*x^3 - (1/9 - x^2)^(1/2)/3
```

3.837 $\int \frac{(-3+2\sqrt{x})(-3\sqrt{x}+x)^{2/3}}{\sqrt{x}} dx$

Optimal result

Integrand size = 28, antiderivative size = 17

$$\int \frac{(-3+2\sqrt{x})\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}} dx = \frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}$$

```
[Out] 6/5*(x-3*x^(1/2))^(5/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2059, 643}

$$\int \frac{(-3+2\sqrt{x})\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}} dx = \frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

```
[In] Int[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x],x]
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
```

```
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx, x, \sqrt{x}\right)$$
$$= \frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx = \frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}$$

```
[In] Integrate[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x],x]
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{6\left(x-3\sqrt{x}\right)^{\frac{5}{3}}}{5}$	12
default	$\frac{6\left(x-3\sqrt{x}\right)^{\frac{5}{3}}}{5}$	12
meijerg	$-\frac{18\,3^{\frac{2}{3}}\,\mathrm{signum}\left(-1+\frac{\sqrt{x}}{3}\right)^{\frac{2}{3}} x^{\frac{5}{6}}\,{}_2F_1\left(-\frac{2}{3},\frac{5}{3};\frac{8}{3};\frac{\sqrt{x}}{3}\right)}{5\left(-\mathrm{signum}\left(-1+\frac{\sqrt{x}}{3}\right)\right)^{\frac{2}{3}}}+\frac{3\,3^{\frac{2}{3}}\,\mathrm{signum}\left(-1+\frac{\sqrt{x}}{3}\right)^{\frac{2}{3}} x^{\frac{4}{3}}\,{}_2F_1\left(-\frac{2}{3},\frac{8}{3};\frac{11}{3};\frac{\sqrt{x}}{3}\right)}{2\left(-\mathrm{signum}\left(-1+\frac{\sqrt{x}}{3}\right)\right)^{\frac{2}{3}}}$	84

```
[In] int((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 6/5*(x-3*x^(1/2))^(5/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx = \frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

```
[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(14) = 28.

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx = -\frac{18\sqrt{x}\left(-3\sqrt{x}+x\right)^{\frac{2}{3}}}{5}+\frac{6x\left(-3\sqrt{x}+x\right)^{\frac{2}{3}}}{5}$$

```
[In] integrate((x-3*x**(1/2))**(2/3)*(-3+2*x**(1/2))/x**(1/2),x)
[Out] -18*sqrt(x)*(-3*sqrt(x) + x)**(2/3)/5 + 6*x*(-3*sqrt(x) + x)**(2/3)/5
```

Maxima [F]

$$\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx = \int \frac{\left(x-3\sqrt{x}\right)^{\frac{2}{3}}\left(2\sqrt{x}-3\right)}{\sqrt{x}}\,dx$$

```
[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] integrate((x - 3*sqrt(x))^(2/3)*(2*sqrt(x) - 3)/sqrt(x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx = \frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

```
[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

Mupad [B] (verification not implemented)

Time = 22.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx=\frac{6\left(x-3\sqrt{x}\right)^{5/3}}{5}$$

```
[In] int(((x - 3*x^(1/2))^(2/3)*(2*x^(1/2) - 3))/x^(1/2),x)
[Out] (6*(x - 3*x^(1/2))^(5/3))/5
```

3.838 $\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 17

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx=\frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}$$

```
[Out] 6/5*(x-3*x^(1/2))^(5/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2068, 1645, 643}

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx=\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

```
[In] Int[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3),x]
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1645

```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2
```

```
)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Polyn
omialRemainder[Pq, b + c*x, x], 0]
```

Rule 2068

```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{d =
 Denominator[n]}, Dist[d, Subst[Int[x^(d - 1)*(SubstFor[x^n, Pq, x] /. x ->
 x^(d*n))*(a*x^(d*j) + b*x^(d*n))^p, x], x, x^(1/d)], x]] /; FreeQ[{a, b, j
, n, p}, x] && PolyQ[Pq, x^n] &&  !IntegerQ[p] && NeQ[n, j] && RationalQ[j,
 n] && IntegerQ[j/n] && LtQ[-1, n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx, x, \sqrt{x}\right)\\ &= \frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx = \frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}$$

```
[In] Integrate[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3),x]
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 7.35

method	result
meijerg	$\frac{18\,3^{\frac{2}{3}}\left(-\operatorname{signum}\left(-1+\frac{\sqrt{x}}{3}\right)\right)^{\frac{1}{3}} x^{\frac{5}{6}}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{8}{3};\frac{\sqrt{x}}{3}\right)}{5\operatorname{signum}\left(-1+\frac{\sqrt{x}}{3}\right)^{\frac{1}{3}}}+\frac{4\,3^{\frac{2}{3}}\left(-\operatorname{signum}\left(-1+\frac{\sqrt{x}}{3}\right)\right)^{\frac{1}{3}} x^{\frac{11}{6}}\,{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{14}{3};\frac{\sqrt{x}}{3}\right)}{11\operatorname{signum}\left(-1+\frac{\sqrt{x}}{3}\right)^{\frac{1}{3}}}-\frac{9\,3^{\frac{2}{3}}\left(-\operatorname{signum}\left(-\right.\right.}{4\,\mathrm{si}}$

```
[In] int((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x,method=_RETURNVERBOSE)
```

[Out] 18/5*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x^(5/6)*hypergeom([1/3,5/3],[8/3],1/3*x^(1/2))+4/11*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x^(11/6)*hypergeom([1/3,11/3],[14/3],1/3*x^(1/2))-9/4*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x^(4/3)*hypergeom([1/3,8/3],[11/3],1/3*x^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx=\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="fricas")

[Out] 6/5*(x - 3*sqrt(x))^(5/3)

Sympy [F]

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx=\int \frac{-9\sqrt{x}+2x+9}{\sqrt[3]{-3\sqrt{x}+x}}\,dx$$

[In] integrate((9+2*x-9*x**(1/2))/(x-3*x**(1/2))**(1/3),x)

[Out] Integral((-9*sqrt(x) + 2*x + 9)/(-3*sqrt(x) + x)**(1/3), x)

Maxima [F]

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx=\int \frac{2\,x-9\,\sqrt{x}+9}{\left(x-3\,\sqrt{x}\right)^{\frac{1}{3}}}\,dx$$

[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="maxima")

[Out] integrate((2*x - 9*sqrt(x) + 9)/(x - 3*sqrt(x))^(1/3), x)

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx = \frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

```
[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="giac")
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx = \int \frac{2\,x-9\,\sqrt{x}+9}{\left(x-3\,\sqrt{x}\right)^{1/3}}\,dx$$

```
[In] int((2*x - 9*x^(1/2) + 9)/(x - 3*x^(1/2))^(1/3),x)
[Out] int((2*x - 9*x^(1/2) + 9)/(x - 3*x^(1/2))^(1/3), x)
```

3.839 $\int \frac{1}{\sqrt{4-9x^2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{1}{\sqrt{4-9x^2}} \, dx = \frac{1}{3} \arcsin \left(\frac{3x}{2}\right)$$

```
[Out] 1/3*arcsin(3/2*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {222}

$$\int \frac{1}{\sqrt{4-9x^2}} \, dx = \frac{1}{3} \arcsin \left(\frac{3x}{2}\right)$$

```
[In] Int[1/Sqrt[4 - 9*x^2],x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\text{integral} = \frac{1}{3} \sin^{-1}\left(\frac{3x}{2}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{2}{3}\arctan\left(\frac{3x}{-2+\sqrt{4-9x^2}}\right)$$

```
[In] Integrate[1/Sqrt[4 - 9*x^2],x]
[Out] (2*ArcTan[(3*x)/(-2 + Sqrt[4 - 9*x^2])])/3
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$	7
meijerg	$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$	7
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-9x^2+4}}{3x}\right)}{3}$	18
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-9x^2+4}+3x\right)}{3}$	31

```
[In] int(1/(-9*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*arcsin(3/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(6) = 12$.

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx = -\frac{2}{3}\arctan\left(\frac{\sqrt{-9x^2+4}-2}{3x}\right)$$

```
[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="fricas")
[Out] -2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

```
[In] integrate(1/(-9*x**2+4)**(1/2),x)
[Out] asin(3*x/2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

```
[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="maxima")
[Out] 1/3*arcsin(3/2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{1}{2}\sqrt{-9\,x^2+4}x + \frac{2}{3}\arcsin\left(\frac{3}{2}x\right)$$

```
[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-9*x^2 + 4)*x + 2/3*arcsin(3/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{\operatorname{asin}\left(\frac{3\,x}{2}\right)}{3}$$

```
[In] int(1/(4 - 9*x^2)^(1/2),x)
[Out] asin((3*x)/2)/3
```

3.840 $\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 10

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx = \frac{1}{3}\arcsin\left(\frac{3x}{2}\right)$$

```
[Out] 1/3*arcsin(3/2*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {41, 222}

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx = \frac{1}{3}\arcsin\left(\frac{3x}{2}\right)$$

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{4-9x^2}}\,dx$$
$$= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx = \frac{2}{3}\arctan\left(\frac{3x}{-2+\sqrt{4-9x^2}}\right)$$

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]
[Out] (2*ArcTan[(3*x)/(-2 + Sqrt[4 - 9*x^2])])/3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(6) = 12.

Time = 1.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.40

method	result	size
default	$\frac{\sqrt{(2-3x)(3x+2)}\,\arcsin\left(\frac{3x}{2}\right)}{3\sqrt{2-3x}\,\sqrt{3x+2}}$	34

```
[In] int(1/(2-3*x)^(1/2)/(3*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*((2-3*x)*(3*x+2))^(1/2)/(2-3*x)^(1/2)/(3*x+2)^(1/2)*arcsin(3/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(6) = 12.

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx = -\frac{2}{3}\arctan\left(\frac{\sqrt{3x+2}\sqrt{-3x+2}-2}{3x}\right)$$

```
[In] integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
[Out] -2/3*arctan(1/3*(sqrt(3*x + 2)*sqrt(-3*x + 2) - 2)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 49, normalized size of antiderivative = 4.90

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx = \begin{cases} -\frac{2i\operatorname{acosh}\left(\frac{\sqrt{3}\sqrt{x+\frac{2}{3}}}{2}\right)}{3} & \text{for } \left|x+\frac{2}{3}\right| > \frac{4}{3} \\ \frac{2\operatorname{asin}\left(\frac{\sqrt{3}\sqrt{x+\frac{2}{3}}}{2}\right)}{3} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(2-3*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(3)*sqrt(x + 2/3)/2)/3, Abs(x + 2/3) > 4/3), (2*a
sin(sqrt(3)*sqrt(x + 2/3)/2)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx = \frac{1}{3}\arcsin\left(\frac{3}{2}\,x\right)$$

```
[In] integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(3/2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx = \frac{2}{3}\arcsin\left(\frac{1}{2}\sqrt{3\,x+2}\right)$$

```
[In] integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*arcsin(1/2*sqrt(3*x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 3.20

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx = -\frac{4\,\mathrm{atan}\left(\frac{\sqrt{2}-\sqrt{2-3\,x}}{\sqrt{2}-\sqrt{3\,x+2}}\right)}{3}$$

```
[In] int(1/((2 - 3*x)^(1/2)*(3*x + 2)^(1/2)),x)
[Out] -(4*atan((2^(1/2) - (2 - 3*x)^(1/2))/(2^(1/2) - (3*x + 2)^(1/2))))/3
```

3.841 $\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 10

$$\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx = \frac{1}{3}\arcsin\left(\frac{3x}{2}\right)$$

```
[Out] 1/3*arcsin(3/2*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1976, 222}

$$\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx = \frac{1}{3}\arcsin\left(\frac{3x}{2}\right)$$

```
[In] Int[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{4-9x^2}}\,dx$$
$$= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx = \frac{2}{3}\arctan\left(\frac{3x}{-2+\sqrt{4-9x^2}}\right)$$

```
[In] Integrate[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]
[Out] (2*ArcTan[(3*x)/(-2 + Sqrt[4 - 9*x^2])])/3
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$	7
meijerg	$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$	7
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-9x^2+4}}{3x}\right)}{3}$	18
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-9x^2+4}+3x\right)}{3}$	30

```
[In] int(1/((2-3*x)*(3*x+2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*arcsin(3/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(6) = 12$.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx = -\frac{2}{3}\,\arctan\left(\frac{\sqrt{-9\,x^2+4}-2}{3\,x}\right)$$

```
[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="fricas")
[Out] -2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx = \frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

```
[In] integrate(1/((2-3*x)*(2+3*x))**(1/2),x)
[Out] asin(3*x/2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx = \frac{1}{3}\,\arcsin\left(\frac{3}{2}\,x\right)$$

```
[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="maxima")
[Out] 1/3*arcsin(3/2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(6) = 12$.

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx = \frac{1}{2}\,\sqrt{-9\,x^2+4}x + \frac{2}{3}\,\arcsin\left(\frac{3}{2}\,x\right)$$

```
[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-9*x^2 + 4)*x + 2/3*arcsin(3/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx = \frac{\operatorname{asin}\left(\frac{3\,x}{2}\right)}{3}$$

```
[In] int(1/(-(3*x - 2)*(3*x + 2))^(1/2),x)
[Out] asin((3*x)/2)/3
```

3.842 $\int \frac{1}{\sqrt{15-2x-x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx = -\arcsin\left(\frac{1}{4}(-1-x)\right)$$

```
[Out] arcsin(1/4+1/4*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 222}

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx = -\arcsin\left(\frac{1}{4}(-x-1)\right)$$

```
[In] Int[1/Sqrt[15 - 2*x - x^2],x]
```

```
[Out] -ArcSin[(-1 - x)/4]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -2-2x\right)\right)\\ &= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{15-2x-x^2}}{5+x}\right)$$

```
[In] Integrate[1/Sqrt[15 - 2*x - x^2],x]
[Out] -2*ArcTan[Sqrt[15 - 2*x - x^2]/(5 + x)]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

method	result	size
default	$\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)$	7
trager	$\text{RootOf}\left(\_Z^2+1\right)\ln\left(-\text{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2-2x+15}-\text{RootOf}\left(\_Z^2+1\right)\right)$	39

```
[In] int(1/(-x^2-2*x+15)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(1/4+1/4*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(6) = 12.

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx = -\arctan\left(\frac{\sqrt{-x^2-2\,x+15}(x+1)}{x^2+2\,x-15}\right)$$

```
[In] integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^2 - 2*x + 15)*(x + 1)/(x^2 + 2*x - 15))
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx = \operatorname{asin}\left(\frac{x}{4}+\frac{1}{4}\right)$$

```
[In] integrate(1/(-x**2-2*x+15)**(1/2),x)
[Out] asin(x/4 + 1/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx = -\arcsin\left(-\frac{1}{4}\,x-\frac{1}{4}\right)$$

```
[In] integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/4*x - 1/4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(6) = 12.

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx = \frac{1}{2}\,\sqrt{-x^2-2\,x+15}(x+1)+8\,\arcsin\left(\frac{1}{4}\,x+\frac{1}{4}\right)$$

```
[In] integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 - 2*x + 15)*(x + 1) + 8*arcsin(1/4*x + 1/4)
```

Mupad [B] (verification not implemented)

Time = 21.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx = \operatorname{asin}\left(\frac{x}{4}+\frac{1}{4}\right)$$

```
[In] int(1/(15 - x^2 - 2*x)^(1/2),x)
[Out] asin(x/4 + 1/4)
```

3.843 $\int \frac{1}{\sqrt{3-x}\sqrt{5+x}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}} \, dx = -\arcsin\left(\frac{1}{4}(-1-x)\right)$$

```
[Out] arcsin(1/4+1/4*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {55, 633, 222}

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}} \, dx = -\arcsin\left(\frac{1}{4}(-x-1)\right)$$

```
[In] Int[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]
```

```
[Out] -ArcSin[(-1 - x)/4]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{15-2x-x^2}}\,dx\\ &= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -2-2x\right)\right)\\ &= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 44 vs. 2(12) = 24.

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 3.67

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx = \frac{2\sqrt{-3+x}\sqrt{5+x}\,\text{arctanh}\left(\frac{\sqrt{5+x}}{\sqrt{-3+x}}\right)}{\sqrt{-((-3+x)(5+x))}}$$

```
[In] Integrate[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]
[Out] (2*Sqrt[-3 + x]*Sqrt[5 + x]*ArcTanh[Sqrt[5 + x]/Sqrt[-3 + x]])/Sqrt[-((-3 +
 x)*(5 + x))]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(6) = 12.

Time = 1.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.58

method	result	size
default	$\frac{\sqrt{(3-x)(5+x)}\,\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)}{\sqrt{3-x}\,\sqrt{5+x}}$	31

```
[In] int(1/(3-x)^(1/2)/(5+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((3-x)*(5+x))^(1/2)/(3-x)^(1/2)/(5+x)^(1/2)*arcsin(1/4+1/4*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(6) = 12$.

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx = -\arctan\left(\frac{\sqrt{x+5}(x+1)\sqrt{-x+3}}{x^2+2\,x-15}\right)$$

```
[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(x + 5)*(x + 1)*sqrt(-x + 3)/(x^2 + 2*x - 15))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.25

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx = \begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+5}}{4}\right) & \text{for } |x+5| > 8 \\ 2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+5}}{4}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(3-x)**(1/2)/(5+x)**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/4), Abs(x + 5) > 8), (2*asin(sqrt
(2)*sqrt(x + 5)/4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx = -\arcsin\left(-\frac{1}{4}\,x - \frac{1}{4}\right)$$

```
[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/4*x - 1/4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(6) = 12$.

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx = 2\arcsin\left(\frac{1}{4}\sqrt{2}\sqrt{x+5}\right)$$

```
[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="giac")
[Out] 2*arcsin(1/4*sqrt(2)*sqrt(x + 5))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx = 4\,\mathrm{atan}\left(\frac{\sqrt{3}-\sqrt{3-x}}{\sqrt{x+5}-\sqrt{5}}\right)$$

```
[In] int(1/((3 - x)^(1/2)*(x + 5)^(1/2)),x)
[Out] 4*atan((3^(1/2) - (3 - x)^(1/2))/((x + 5)^(1/2) - 5^(1/2)))
```

3.844 $\int \frac{1}{\sqrt{(3-x)(5+x)}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{1}{\sqrt{(3-x)(5+x)}} \, dx = -\arcsin\left(\frac{1}{4}(-1-x)\right)$$

```
[Out] arcsin(1/4+1/4*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1976, 633, 222}

$$\int \frac{1}{\sqrt{(3-x)(5+x)}} \, dx = -\arcsin\left(\frac{1}{4}(-x-1)\right)$$

```
[In] Int[1/Sqrt[(3 - x)*(5 + x)],x]
```

```
[Out] -ArcSin[(-1 - x)/4]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{15-2x-x^2}}\,dx\\ &= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -2-2x\right)\right)\\ &= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 44 vs. 2(12) = 24.

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 3.67

$$\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx = \frac{2\sqrt{-3+x}\sqrt{5+x}\,\text{arctanh}\left(\frac{\sqrt{5+x}}{\sqrt{-3+x}}\right)}{\sqrt{-((-3+x)(5+x))}}$$

```
[In] Integrate[1/Sqrt[(3 - x)*(5 + x)],x]
```

```
[Out] (2*Sqrt[-3 + x]*Sqrt[5 + x]*ArcTanh[Sqrt[5 + x]/Sqrt[-3 + x]])/Sqrt[-((-3 +
 x)*(5 + x))]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

method	result	size
default	$\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)$	7
trager	$\text{RootOf}\left(\_Z^2+1\right)\ln\left(-\text{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2-2x+15}-\text{RootOf}\left(\_Z^2+1\right)\right)$	39

```
[In] int(1/((3-x)*(5+x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsin(1/4+1/4*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(6) = 12$.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx = -\arctan\left(\frac{\sqrt{-x^2-2\,x+15}(x+1)}{x^2+2\,x-15}\right)$$

```
[In] integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^2 - 2*x + 15)*(x + 1)/(x^2 + 2*x - 15))
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx = \operatorname{asin}\left(\frac{x}{4}+\frac{1}{4}\right)$$

```
[In] integrate(1/((3-x)*(5+x))**(1/2),x)
[Out] asin(x/4 + 1/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx = -\arcsin\left(-\frac{1}{4}\,x-\frac{1}{4}\right)$$

```
[In] integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/4*x - 1/4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(6) = 12$.

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx = \frac{1}{2}\,\sqrt{-x^2-2\,x+15}(x+1)+8\,\arcsin\left(\frac{1}{4}\,x+\frac{1}{4}\right)$$

```
[In] integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 - 2*x + 15)*(x + 1) + 8*arcsin(1/4*x + 1/4)
```

Mupad [B] (verification not implemented)

Time = 21.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx = \operatorname{asin}\left(\frac{x}{4}+\frac{1}{4}\right)$$

```
[In] int(1/(-(x - 3)*(x + 5))^(1/2),x)
[Out] asin(x/4 + 1/4)
```

3.845 $\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 4

$$\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx = \arcsin(4+x)$$

```
[Out] arcsin(4+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 222}

$$\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx = \arcsin(x+4)$$

```
[In] Int[1/Sqrt[-15 - 8*x - x^2],x]
```

```
[Out] ArcSin[4 + x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -8-2x\right)\right)$$
$$= \sin^{-1}(4+x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(4) = 8.

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 5.75

$$\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{-15-8x-x^2}}{5+x}\right)$$

```
[In] Integrate[1/Sqrt[-15 - 8*x - x^2],x]
[Out] -2*ArcTan[Sqrt[-15 - 8*x - x^2]/(5 + x)]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$\arcsin(x+4)$	5
trager	$\operatorname{RootOf}(\_Z^2+1)\ln\left(-\operatorname{RootOf}(\_Z^2+1)\,x+\sqrt{-x^2-8x-15}-4\operatorname{RootOf}(\_Z^2+1)\right)$	39

```
[In] int(1/(-x^2-8*x-15)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(x+4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(4) = 8.

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

$$\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx = -\arctan\left(\frac{\sqrt{-x^2-8\,x-15}(x+4)}{x^2+8\,x+15}\right)$$

```
[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^2 - 8*x - 15)*(x + 4)/(x^2 + 8*x + 15))
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx = \operatorname{asin}(x+4)$$

```
[In] integrate(1/(-x**2-8*x-15)**(1/2),x)
[Out] asin(x + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx = -\arcsin(-x-4)$$

```
[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-x - 4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(4) = 8$.

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 6.00

$$\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2-8\,x-15}(x+4) + \frac{1}{2}\arcsin(x+4)$$

```
[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 - 8*x - 15)*(x + 4) + 1/2*arcsin(x + 4)
```

Mupad [B] (verification not implemented)

Time = 21.15 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx = \operatorname{asin}(x+4)$$

```
[In] int(1/(- 8*x - x^2 - 15)^(1/2),x)
[Out] asin(x + 4)
```

3.846 $\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 4

$$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx = \arcsin(4+x)$$

```
[Out] arcsin(4+x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {55, 633, 222}

$$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx = \arcsin(x+4)$$

```
[In] Int[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]
```

```
[Out] ArcSin[4 + x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{-15-8x-x^2}}\,dx\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -8-2x\right)\right)\\ &= \sin^{-1}(4+x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 44 vs. $2(4) = 8$.

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 11.00

$$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx = \frac{2\sqrt{3+x}\sqrt{5+x}\,\text{arctanh}\left(\frac{\sqrt{5+x}}{\sqrt{3+x}}\right)}{\sqrt{-((3+x)(5+x))}}$$

```
[In] Integrate[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]
[Out] (2*Sqrt[3 + x]*Sqrt[5 + x]*ArcTanh[Sqrt[5 + x]/Sqrt[3 + x]])/Sqrt[-((3 + x)
*(5 + x))]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(4) = 8$.

Time = 1.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

method	result	size
default	$\frac{\sqrt{(-3-x)(5+x)}\,\arcsin(x+4)}{\sqrt{-3-x}\,\sqrt{5+x}}$	29

```
[In] int(1/(-3-x)^(1/2)/(5+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((-3-x)*(5+x))^(1/2)/(-3-x)^(1/2)/(5+x)^(1/2)*arcsin(x+4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(4) = 8$.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

$$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx = -\arctan\left(\frac{\sqrt{x+5}(x+4)\sqrt{-x-3}}{x^2+8\,x+15}\right)$$

```
[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(x + 5)*(x + 4)*sqrt(-x - 3)/(x^2 + 8*x + 15))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 39, normalized size of antiderivative = 9.75

$$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx = \begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+5}}{2}\right) & \text{for } |x+5| > 2 \\ 2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+5}}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(-3-x)**(1/2)/(5+x)**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/2), Abs(x + 5) > 2), (2*asin(sqrt
(2)*sqrt(x + 5)/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx = -\arcsin\left(-x-4\right)$$

```
[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-x - 4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(4) = 8$.

Time = 0.43 (sec) , antiderivative size = 13, normalized size of antiderivative = 3.25

$$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx = 2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+5}\right)$$

```
[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="giac")
[Out] 2*arcsin(1/2*sqrt(2)*sqrt(x + 5))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 8.25

$$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx = 4\,\mathrm{atan}\left(\frac{-\sqrt{-x-3}+\sqrt{3}\,1\mathrm{i}}{\sqrt{x+5}-\sqrt{5}}\right)$$

```
[In] int(1/((- x - 3)^(1/2)*(x + 5)^(1/2)),x)
[Out] 4*atan((3^(1/2)*1i - (- x - 3)^(1/2))/((x + 5)^(1/2) - 5^(1/2)))
```

3.847 $\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 4

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx = \arcsin(4+x)$$

```
[Out] arcsin(4+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1976, 633, 222}

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx = \arcsin(x+4)$$

```
[In] Int[1/Sqrt[(-3 - x)*(5 + x)],x]
```

```
[Out] ArcSin[4 + x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{-15-8x-x^2}}\,dx \\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -8-2x\right)\right) \\ &= \sin^{-1}(4+x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 44 vs. $2(4) = 8$.

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 11.00

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx = \frac{2\sqrt{3+x}\sqrt{5+x}\,\text{arctanh}\left(\frac{\sqrt{5+x}}{\sqrt{3+x}}\right)}{\sqrt{-((3+x)(5+x))}}$$

```
[In] Integrate[1/Sqrt[(-3 - x)*(5 + x)],x]
[Out] (2*Sqrt[3 + x]*Sqrt[5 + x]*ArcTanh[Sqrt[5 + x]/Sqrt[3 + x]])/Sqrt[-((3 + x)
*(5 + x))]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$\arcsin(x+4)$	5
trager	$\text{RootOf}\left(\_Z^2+1\right)\ln\left(-\text{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2-8x-15}-4\,\text{RootOf}\left(\_Z^2+1\right)\right)$	39

```
[In] int(1/((-3-x)*(5+x))^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(x+4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(4) = 8$.

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx = -\arctan\left(\frac{\sqrt{-x^2-8\,x-15}(x+4)}{x^2+8\,x+15}\right)$$

```
[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^2 - 8*x - 15)*(x + 4)/(x^2 + 8*x + 15))
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx = \operatorname{asin}(x+4)$$

```
[In] integrate(1/((-3-x)*(5+x))**(1/2),x)
[Out] asin(x + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx = -\arcsin(-x-4)$$

```
[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="maxima")
[Out] -arcsin(-x - 4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(4) = 8$.

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 6.00

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx = \frac{1}{2}\sqrt{-x^2-8\,x-15}(x+4) + \frac{1}{2}\arcsin(x+4)$$

```
[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 - 8*x - 15)*(x + 4) + 1/2*arcsin(x + 4)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx = \operatorname{asin}(x+4)$$

```
[In] int(1/(-(x + 3)*(x + 5))^(1/2),x)
[Out] asin(x + 4)
```

3.848 $\int \left(1-\sqrt{x}\right) dx$

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \left(1-\sqrt{x}\right) dx = x-\frac{2x^{3/2}}{3}$$

```
[Out] x-2/3*x^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(1-\sqrt{x}\right) dx = x-\frac{2x^{3/2}}{3}$$

```
[In] Int[1 - Sqrt[x],x]
```

```
[Out] x - (2*x^(3/2))/3
```

Rubi steps

$$\text{integral} = x-\frac{2x^{3/2}}{3}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \left(1-\sqrt{x}\right) \, dx = x-\frac{2x^{3/2}}{3}$$

```
[In] Integrate[1 - Sqrt[x],x]
[Out] x - (2*x^(3/2))/3
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
default	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
risch	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
parts	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
trager	$x-1-\frac{2x^{\frac{3}{2}}}{3}$	9

```
[In] int(1-x^(1/2),x,method=_RETURNVERBOSE)
[Out] x-2/3*x^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right) \, dx = -\frac{2}{3}\,x^{\frac{3}{2}}+x$$

```
[In] integrate(1-x^(1/2),x, algorithm="fricas")
[Out] -2/3*x^(3/2) + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \left(1-\sqrt{x}\right)\,dx = -\frac{2x^{\frac{3}{2}}}{3}+x$$

```
[In] integrate(1-x**(1/2),x)
[Out] -2*x**(3/2)/3 + x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right)\,dx = -\frac{2}{3}\,x^{\frac{3}{2}}+x$$

```
[In] integrate(1-x^(1/2),x, algorithm="maxima")
[Out] -2/3*x^(3/2) + x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right)\,dx = -\frac{2}{3}\,x^{\frac{3}{2}}+x$$

```
[In] integrate(1-x^(1/2),x, algorithm="giac")
[Out] -2/3*x^(3/2) + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right)\,dx = x-\frac{2\,x^{3/2}}{3}$$

```
[In] int(1 - x^(1/2),x)
[Out] x - (2*x^(3/2))/3
```

3.849 $\int \frac{1-x}{1+\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{1-x}{1+\sqrt{x}}\,dx = x-\frac{2x^{3/2}}{3}$$

```
[Out] x-2/3*x^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1412, 26, 45}

$$\int \frac{1-x}{1+\sqrt{x}}\,dx = x-\frac{2x^{3/2}}{3}$$

```
[In] Int[(1 - x)/(1 + Sqrt[x]),x]
```

```
[Out] x - (2*x^(3/2))/3
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= 2\text{Subst}\left(\int \frac{x(1-x^2)}{1+x}\,dx, x, \sqrt{x}\right) \\ &= 2\text{Subst}\left(\int (1-x)x\,dx, x, \sqrt{x}\right) \\ &= 2\text{Subst}\left(\int \left(x-x^2\right)\,dx, x, \sqrt{x}\right) \\ &= x-\frac{2x^{3/2}}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1-x}{1+\sqrt{x}}\,dx = x-\frac{2x^{3/2}}{3}$$

```
[In] Integrate[(1 - x)/(1 + Sqrt[x]),x]
[Out] x - (2*x^(3/2))/3
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
default	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
trager	$x-1-\frac{2x^{\frac{3}{2}}}{3}$	9
meijerg	$2\sqrt{x}-\frac{\sqrt{x}\,(4x-6\sqrt{x}+12)}{6}$	22

```
[In] int((1-x)/(1+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] x-2/3*x^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-x}{1+\sqrt{x}}\,dx=-\frac{2}{3}\,x^{\frac{3}{2}}+x$$

```
[In] integrate((1-x)/(1+x^(1/2)),x, algorithm="fricas")
[Out] -2/3*x^(3/2) + x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1-x}{1+\sqrt{x}}\,dx=-\frac{2x^{\frac{3}{2}}}{3}+x$$

```
[In] integrate((1-x)/(1+x**(1/2)),x)
[Out] -2*x**(3/2)/3 + x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-x}{1+\sqrt{x}}\,dx = -\frac{2}{3}\,x^{\frac{3}{2}} + x$$

```
[In] integrate((1-x)/(1+x^(1/2)),x, algorithm="maxima")
[Out] -2/3*x^(3/2) + x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-x}{1+\sqrt{x}}\,dx = -\frac{2}{3}\,x^{\frac{3}{2}} + x$$

```
[In] integrate((1-x)/(1+x^(1/2)),x, algorithm="giac")
[Out] -2/3*x^(3/2) + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-x}{1+\sqrt{x}}\,dx = x - \frac{2\,x^{3/2}}{3}$$

```
[In] int(-(x - 1)/(x^(1/2) + 1),x)
[Out] x - (2*x^(3/2))/3
```

3.850 $\int \sqrt{\frac{1}{1-x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \sqrt{\frac{1}{1-x^2}}\,dx = \sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\arcsin(x)$$

```
[Out] arcsin(x)*(1/(-x^2+1))^(1/2)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1972, 222}

$$\int \sqrt{\frac{1}{1-x^2}}\,dx = \sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\arcsin(x)$$

```
[In] Int[Sqrt[(1 - x^2)^(-1)],x]
```

```
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \left(\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \sqrt{\frac{1}{1-x^2}}\,dx = -2\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

```
[In] Integrate[Sqrt[(1 - x^2)^(-1)],x]
[Out] -2*Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	si
meijerg	$\arcsin(x)\sqrt{\frac{1}{-x^2+1}}\,\sqrt{-x^2+1}$	2
default	$\sqrt{-\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$	3
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{1}{x^2-1}}\,x^2+\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{1}{x^2-1}}+x\right)$	5

```
[In] int((1/(-x^2+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(x)*(1/(-x^2+1))^(1/2)*(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sqrt{\frac{1}{1-x^2}}\,dx = 2 \arctan\left(\frac{(x^2-1)\sqrt{-\frac{1}{x^2-1}}+1}{x}\right)$$

```
[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="fricas")
[Out] 2*arctan(((x^2 - 1)*sqrt(-1/(x^2 - 1)) + 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.26

$$\int \sqrt{\frac{1}{1-x^2}}\,dx = \begin{cases} \operatorname{asin}{\left(x \right)} & \text{for}\: x > -1 \wedge x < 1 \end{cases}$$

```
[In] integrate((1/(-x**2+1))**(1/2),x)
[Out] Piecewise((asin(x), (x > -1) & (x < 1)))
```

Maxima [F]

$$\int \sqrt{\frac{1}{1-x^2}}\,dx = \int \sqrt{-\frac{1}{x^2-1}}\,dx$$

```
[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-1/(x^2 - 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.37

$$\int \sqrt{\frac{1}{1-x^2}}\,dx = -\arcsin\left(x\right)\operatorname{sgn}\left(x^2-1\right)$$

```
[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="giac")
[Out] -arcsin(x)*sgn(x^2 - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{1}{1-x^2}}\,dx = \int \sqrt{-\frac{1}{x^2-1}}\,dx$$

```
[In] int((-1/(x^2 - 1))^(1/2),x)
[Out] int((-1/(x^2 - 1))^(1/2), x)
```

3.851 $\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 27

$$\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx = \sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\arcsin(x)$$

```
[Out] arcsin(x)*(1/(-x^2+1))^(1/2)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6820, 1972, 222}

$$\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx = \sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\arcsin(x)$$

```
[In] Int[Sqrt[(1 + x^2)/(1 - x^4)],x]
```

```
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt{\frac{1}{1-x^2}}\,dx\\ &= \left(\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\,\sin^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx = -2\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

```
[In] Integrate[Sqrt[(1 + x^2)/(1 - x^4)],x]
[Out] -2*Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

method	result	si
default	$\sqrt{-\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$	3
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{1}{x^2-1}}\,x^2+\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{1}{x^2-1}}+x\right)$	5

```
[In] int(((x^2+1)/(-x^4+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] (-1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx = 2\arctan\left(\frac{(x^2-1)\sqrt{-\frac{1}{x^2-1}}+1}{x}\right)$$

```
[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="fricas")
[Out] 2*arctan(((x^2 - 1)*sqrt(-1/(x^2 - 1)) + 1)/x)
```

Sympy [F]

$$\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx = \int \sqrt{\frac{x^2+1}{1-x^4}}\,dx$$

```
[In] integrate(((x**2+1)/(-x**4+1))**(1/2),x)
[Out] Integral(sqrt((x**2 + 1)/(1 - x**4)), x)
```

Maxima [F]

$$\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx = \int \sqrt{-\frac{x^2+1}{x^4-1}}\,dx$$

```
[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-(x^2 + 1)/(x^4 - 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.37

$$\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx = -\arcsin(x)\,\mathrm{sgn}\left(x^2-1\right)$$

```
[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="giac")
[Out] -arcsin(x)*sgn(x^2 - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx = \int \sqrt{-\frac{x^2+1}{x^4-1}}\,dx$$

```
[In] int((-(x^2 + 1)/(x^4 - 1))^(1/2),x)
[Out] int((-(x^2 + 1)/(x^4 - 1))^(1/2), x)
```

3.852 $\int \sqrt{\frac{1}{-1+x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \sqrt{\frac{1}{-1+x^2}}\,dx = \sqrt{1-x^2}\sqrt{\frac{1}{-1+x^2}}\arcsin(x)$$

```
[Out] arcsin(x)*(-x^2+1)^(1/2)*(1/(x^2-1))^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1973, 222}

$$\int \sqrt{\frac{1}{-1+x^2}}\,dx = \sqrt{1-x^2}\sqrt{\frac{1}{x^2-1}}\arcsin(x)$$

```
[In] Int[Sqrt[(-1 + x^2)^(-1)],x]
```

```
[Out] Sqrt[1 - x^2]*Sqrt[(-1 + x^2)^(-1)]*ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] && !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \left(\sqrt{1-x^2}\sqrt{\frac{1}{-1+x^2}}\right)\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \sqrt{1-x^2}\sqrt{\frac{1}{-1+x^2}}\sin^{-1}(x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

$$\int \sqrt{\frac{1}{-1+x^2}}\,dx = \frac{1}{2}\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\left(-\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)\right)$$

```
[In] Integrate[Sqrt[(-1 + x^2)^(-1)],x]
```

```
[Out] (Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 +
 x/Sqrt[-1 + x^2]]))/2
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

method	result	size
default	$\sqrt{\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$	28
trager	$\ln\left(\sqrt{\frac{1}{x^2-1}}\,x^2-\sqrt{\frac{1}{x^2-1}}+x\right)$	28
meijerg	$\frac{\sqrt{\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\sqrt{-\operatorname{signum}(x^2-1)}\,\arcsin(x)}{\sqrt{\operatorname{signum}(x^2-1)}}$	38

```
[In] int((1/(x^2-1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sqrt{\frac{1}{-1+x^2}}\,dx = -\log\left(-x+\sqrt{x^2-1}\right)$$

```
[In] integrate((1/(x^2-1))^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 - 1))
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int \sqrt{\frac{1}{-1+x^2}}\,dx = \begin{cases} \log\left(x+\sqrt{x^2-1}\right) & \text{for}\ x > -1 \wedge x < 1 \end{cases}$$

```
[In] integrate((1/(x**2-1))**(1/2),x)
[Out] Piecewise((log(x + sqrt(x**2 - 1)), (x > -1) & (x < 1)))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sqrt{\frac{1}{-1+x^2}}\,dx = \log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

```
[In] integrate((1/(x^2-1))^(1/2),x, algorithm="maxima")
[Out] log(2*x + 2*sqrt(x^2 - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \sqrt{\frac{1}{-1+x^2}}\,dx = \frac{1}{2}\,\sqrt{x^2-1}x + \frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

```
[In] integrate((1/(x^2-1))^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 - 1)*x + 1/2*log(abs(-x + sqrt(x^2 - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{1}{-1+x^2}}\,dx = \int \sqrt{\frac{1}{x^2-1}}\,dx$$

```
[In] int((1/(x^2 - 1))^(1/2),x)
[Out] int((1/(x^2 - 1))^(1/2), x)
```

3.853 $\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx = \sqrt{1-x^2}\sqrt{\frac{1}{-1+x^2}}\arcsin(x)$$

```
[Out] arcsin(x)*(-x^2+1)^(1/2)*(1/(x^2-1))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6820, 1973, 222}

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx = \sqrt{1-x^2}\sqrt{\frac{1}{x^2-1}}\arcsin(x)$$

```
[In] Int[Sqrt[(1 + x^2)/(-1 + x^4)],x]
```

```
[Out] Sqrt[1 - x^2]*Sqrt[(-1 + x^2)^(-1)]*ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] && !GeQ[a, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt{\frac{1}{-1+x^2}}\,dx \\ &= \left(\sqrt{1-x^2}\sqrt{\frac{1}{-1+x^2}}\right)\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \sqrt{1-x^2}\sqrt{\frac{1}{-1+x^2}}\sin^{-1}(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 56 vs. $2(25) = 50$.

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx = \frac{1}{2}\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\left(-\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)\right)$$

```
[In] Integrate[Sqrt[(1 + x^2)/(-1 + x^4)],x]
[Out] (Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 +
 x/Sqrt[-1 + x^2]]))/2
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

method	result	size
default	$\sqrt{\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$	28
trager	$\ln\left(\sqrt{\frac{1}{x^2-1}}\,x^2-\sqrt{\frac{1}{x^2-1}}+x\right)$	28

```
[In] int(((x^2+1)/(x^4-1))^(1/2),x,method=_RETURNVERBOSE)
[Out] (1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx = -\log\left(-x+\sqrt{x^2-1}\right)$$

```
[In] integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 - 1))
```

Sympy [F]

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx = \int \sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

```
[In] integrate(((x**2+1)/(x**4-1))**(1/2),x)
[Out] Integral(sqrt((x**2 + 1)/(x**4 - 1)), x)
```

Maxima [F]

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx = \int \sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

```
[In] integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((x^2 + 1)/(x^4 - 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx = -\log\left(\left|-x+\sqrt{x^2-1}\right|\right)\operatorname{sgn}\left(x^2-1\right)$$

```
[In] integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="giac")
[Out] -log(abs(-x + sqrt(x^2 - 1)))*sgn(x^2 - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx = \int \sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

```
[In] int(((x^2 + 1)/(x^4 - 1))^(1/2),x)
[Out] int(((x^2 + 1)/(x^4 - 1))^(1/2), x)
```

3.854 $\int \frac{1}{\sqrt{1-x}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\sqrt{1-x}$$

```
[Out] -2*(1-x)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\sqrt{1-x}$$

```
[In] Int[1/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 - x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -2\sqrt{1-x}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\sqrt{1-x}$$

```
[In] Integrate[1/Sqrt[1 - x],x]
[Out] -2*Sqrt[1 - x]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gosper	$-2\sqrt{1-x}$	10
derivativedivides	$-2\sqrt{1-x}$	10
default	$-2\sqrt{1-x}$	10
trager	$-2\sqrt{1-x}$	10
pseudoelliptic	$-2\sqrt{1-x}$	10
risch	$\frac{2x-2}{\sqrt{1-x}}$	13
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1-x}}{\sqrt{\pi}}$	24

```
[In] int(1/(1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(1-x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\,\sqrt{-x+1}$$

```
[In] integrate(1/(1-x)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\sqrt{1-x}$$

```
[In] integrate(1/(1-x)**(1/2),x)
[Out] -2*sqrt(1 - x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\,\sqrt{-x+1}$$

```
[In] integrate(1/(1-x)^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\,\sqrt{-x+1}$$

```
[In] integrate(1/(1-x)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(-x + 1)
```

Mupad [B] (verification not implemented)

Time = 21.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\,\sqrt{1-x}$$

```
[In] int(1/(1 - x)^(1/2),x)
[Out] -2*(1 - x)^(1/2)
```

3.855 $\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 11

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx = -2\sqrt{1-x}$$

```
[Out] -2*(1-x)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {26, 32}

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx = -2\sqrt{1-x}$$

```
[In] Int[Sqrt[1 + x]/Sqrt[1 - x^2],x]
```

```
[Out] -2*Sqrt[1 - x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{1-x}}\,dx\\ &= -2\sqrt{1-x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx = -\frac{2\sqrt{1-x^2}}{\sqrt{1+x}}$$

```
[In] Integrate[Sqrt[1 + x]/Sqrt[1 - x^2],x]
[Out] (-2*Sqrt[1 - x^2])/Sqrt[1 + x]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

method	result	size
default	$-\frac{2\sqrt{-x^2+1}}{\sqrt{x+1}}$	17
gosper	$\frac{2(x-1)\sqrt{x+1}}{\sqrt{-x^2+1}}$	20
risch	$\frac{2\sqrt{\frac{-x^2+1}{x+1}}\,\sqrt{x+1}\,(x-1)}{\sqrt{-x^2+1}\,\sqrt{1-x}}$	42

```
[In] int((x+1)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(-x^2+1)^(1/2)/(x+1)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx = -\frac{2\sqrt{-x^2+1}}{\sqrt{x+1}}$$

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-x^2 + 1)/sqrt(x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{x+1}}{\sqrt{-(x-1)(x+1)}}\,dx$$

```
[In] integrate((1+x)**(1/2)/(-x**2+1)**(1/2),x)
[Out] Integral(sqrt(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx = \frac{2(x-1)}{\sqrt{-x+1}}$$

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 2*(x - 1)/sqrt(-x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx = 2\sqrt{2} - 2\sqrt{-x+1}$$

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(2) - 2*sqrt(-x + 1)
```

Mupad [B] (verification not implemented)

Time = 21.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx = -\frac{2\sqrt{1-x^2}}{\sqrt{x+1}}$$

```
[In] int((x + 1)^(1/2)/(1 - x^2)^(1/2),x)
[Out] -(2*(1 - x^2)^(1/2))/(x + 1)^(1/2)
```

3.856 $\int \frac{1}{\sqrt{1+x}} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 9

$$\int \frac{1}{\sqrt{1+x}} \, dx = 2\sqrt{1+x}$$

```
[Out] 2*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\int \frac{1}{\sqrt{1+x}} \, dx = 2\sqrt{x+1}$$

```
[In] Int[1/Sqrt[1 + x],x]
```

```
[Out] 2*Sqrt[1 + x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = 2\sqrt{1+x}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x}}\,dx = 2\sqrt{1+x}$$

```
[In] Integrate[1/Sqrt[1 + x],x]
[Out] 2*Sqrt[1 + x]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
gosper	$2\sqrt{x+1}$	8
derivativedivides	$2\sqrt{x+1}$	8
default	$2\sqrt{x+1}$	8
trager	$2\sqrt{x+1}$	8
risch	$2\sqrt{x+1}$	8
meijerg	$\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x+1}}{\sqrt{\pi}}$	21

```
[In] int(1/(x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1+x}}\,dx = 2\,\sqrt{x+1}$$

```
[In] integrate(1/(1+x)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1+x}}\,dx = 2\sqrt{x+1}$$

```
[In] integrate(1/(1+x)**(1/2),x)
[Out] 2*sqrt(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1+x}}\,dx = 2\sqrt{x+1}$$

```
[In] integrate(1/(1+x)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1+x}}\,dx = 2\sqrt{x+1}$$

```
[In] integrate(1/(1+x)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x + 1)
```

Mupad [B] (verification not implemented)

Time = 21.41 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1+x}}\,dx = 2\sqrt{x+1}$$

```
[In] int(1/(x + 1)^(1/2),x)
[Out] 2*(x + 1)^(1/2)
```

3.857 $\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 9

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx = 2\sqrt{1+x}$$

```
[Out] 2*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {26, 32}

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx = 2\sqrt{x+1}$$

```
[In] Int[Sqrt[1 - x]/Sqrt[1 - x^2],x]
```

```
[Out] 2*Sqrt[1 + x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{1+x}}\,dx\\ &= 2\sqrt{1+x}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx = \frac{2\sqrt{1-x^2}}{\sqrt{1-x}}$$

```
[In] Integrate[Sqrt[1 - x]/Sqrt[1 - x^2],x]
[Out] (2*Sqrt[1 - x^2])/Sqrt[1 - x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(7) = 14.

Time = 1.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.44

method	result	size
gosper	$\frac{2(x+1)\sqrt{1-x}}{\sqrt{-x^2+1}}$	22
default	$-\frac{2\sqrt{1-x}\,\sqrt{-x^2+1}}{x-1}$	24
risch	$-\frac{2\sqrt{\frac{(1-x)\left(-x^2+1\right)}{(x-1)^2}}\,(x-1)\sqrt{x+1}}{\sqrt{1-x}\,\sqrt{-x^2+1}}$	47

```
[In] int((1-x)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(x+1)*(1-x)^(1/2)/(-x^2+1)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx = -\frac{2\sqrt{-x^2+1}\sqrt{-x+1}}{x-1}$$

```
[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-x^2 + 1)*sqrt(-x + 1)/(x - 1)
```

Sympy [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{1-x}}{\sqrt{-(x-1)(x+1)}}\,dx$$

```
[In] integrate((1-x)**(1/2)/(-x**2+1)**(1/2),x)
[Out] Integral(sqrt(1 - x)/sqrt(-(x - 1)*(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx = 2\sqrt{x+1}$$

```
[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx = -2\sqrt{2}+2\sqrt{x+1}$$

```
[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(2) + 2*sqrt(x + 1)
```

Mupad [B] (verification not implemented)

Time = 21.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\, dx = \frac{2\sqrt{1-x^2}}{\sqrt{1-x}}$$

```
[In] int((1 - x)^(1/2)/(1 - x^2)^(1/2),x)
[Out] (2*(1 - x^2)^(1/2))/(1 - x)^(1/2)
```

3.858 $\int \sqrt{1-x}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{1-x}\,dx = -\frac{2}{3}(1-x)^{3/2}$$

```
[Out] -2/3*(1-x)^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$\int \sqrt{1-x}\,dx = -\frac{2}{3}(1-x)^{3/2}$$

```
[In] Int[Sqrt[1 - x],x]
```

```
[Out] (-2*(1 - x)^(3/2))/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2}{3}(1-x)^{3/2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{1-x}\,dx = -\frac{2}{3}(1-x)^{3/2}$$

```
[In] Integrate[Sqrt[1 - x],x]
[Out] (-2*(1 - x)^(3/2))/3
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2(1-x)^{\frac{3}{2}}}{3}$	10
derivativedivides	$-\frac{2(1-x)^{\frac{3}{2}}}{3}$	10
default	$-\frac{2(1-x)^{\frac{3}{2}}}{3}$	10
pseudoelliptic	$-\frac{2(1-x)^{\frac{3}{2}}}{3}$	10
trager	$\left(\frac{2x}{3}-\frac{2}{3}\right)\sqrt{1-x}$	14
risch	$-\frac{2(x-1)^{2}}{3\sqrt{1-x}}$	15
meijerg	$\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(-2x+2)\sqrt{1-x}}{3}}{2\sqrt{\pi}}$	29

```
[In] int((1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(1-x)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \sqrt{1-x}\,dx = \frac{2}{3}\,(x-1)\sqrt{-x+1}$$

```
[In] integrate((1-x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(x - 1)*sqrt(-x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \sqrt{1-x}\,dx = -\frac{2(1-x)^{\frac{3}{2}}}{3}$$

```
[In] integrate((1-x)**(1/2),x)
[Out] -2*(1 - x)**(3/2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{1-x}\,dx = -\frac{2}{3}\,(-x+1)^{\frac{3}{2}}$$

```
[In] integrate((1-x)^(1/2),x, algorithm="maxima")
[Out] -2/3*(-x + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{1-x}\,dx = -\frac{2}{3}\,(-x+1)^{\frac{3}{2}}$$

```
[In] integrate((1-x)^(1/2),x, algorithm="giac")
[Out] -2/3*(-x + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{1-x}\,dx = -\frac{2\,(1-x)^{3/2}}{3}$$

```
[In] int((1 - x)^(1/2),x)
[Out] -(2*(1 - x)^(3/2))/3
```

3.859 $\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx = -\frac{2}{3}(1-x)^{3/2}$$

```
[Out] -2/3*(1-x)^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {26, 32}

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx = -\frac{2}{3}(1-x)^{3/2}$$

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 + x],x]
```

```
[Out] (-2*(1 - x)^(3/2))/3
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \int \sqrt{1-x}\,dx$$
$$= -\frac{2}{3}(1-x)^{3/2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx = -\frac{2(1-x^2)^{3/2}}{3(1+x)^{3/2}}$$

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 + x],x]
[Out] (-2*(1 - x^2)^(3/2))/(3*(1 + x)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 1.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

method	result	size
gosper	$\frac{2(x-1)\sqrt{-x^2+1}}{3\sqrt{x+1}}$	20
default	$\frac{2(x-1)\sqrt{-x^2+1}}{3\sqrt{x+1}}$	20
risch	$-\frac{2\sqrt{\frac{-x^2+1}{x+1}}\,\sqrt{x+1}\,(x-1)^2}{3\sqrt{-x^2+1}\,\sqrt{1-x}}$	44

```
[In] int((-x^2+1)^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(x-1)*(-x^2+1)^(1/2)/(x+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx = \frac{2\,\sqrt{-x^2+1}(x-1)}{3\,\sqrt{x+1}}$$

```
[In] integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(-x^2 + 1)*(x - 1)/sqrt(x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{x+1}}\,dx$$

```
[In] integrate((-x**2+1)**(1/2)/(1+x)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(x + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx = \frac{2}{3}\,(x-1)\sqrt{-x+1}$$

```
[In] integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x - 1)*sqrt(-x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx = -\frac{2}{3}\,(-x+1)^{\frac{3}{2}} + \frac{4}{3}\,\sqrt{2}$$

```
[In] integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] -2/3*(-x + 1)^(3/2) + 4/3*sqrt(2)
```

Mupad [B] (verification not implemented)

Time = 21.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx = \frac{\left(\frac{2x}{3}-\frac{2}{3}\right)\sqrt{1-x^2}}{\sqrt{x+1}}$$

```
[In] int((1 - x^2)^(1/2)/(x + 1)^(1/2),x)
[Out] (((2*x)/3 - 2/3)*(1 - x^2)^(1/2))/(x + 1)^(1/2)
```

3.860 $\int \sqrt{1+x}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2}$$

```
[Out] 2/3*(1+x)^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\int \sqrt{1+x}\,dx = \frac{2}{3}(x+1)^{3/2}$$

```
[In] Int[Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2))/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2}{3}(1+x)^{3/2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2}$$

```
[In] Integrate[Sqrt[1 + x],x]
[Out] (2*(1 + x)^(3/2))/3
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{2(x+1)^{\frac{3}{2}}}{3}$	8
derivativedivides	$\frac{2(x+1)^{\frac{3}{2}}}{3}$	8
default	$\frac{2(x+1)^{\frac{3}{2}}}{3}$	8
risch	$\frac{2(x+1)^{\frac{3}{2}}}{3}$	8
trager	$\left(\frac{2}{3}+\frac{2x}{3}\right)\sqrt{x+1}$	12
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(2x+2)\sqrt{x+1}}{3}}{2\sqrt{\pi}}$	27

```
[In] int((x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(x+1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \sqrt{1+x}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

```
[In] integrate((1+x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(x + 1)^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \sqrt{1+x}\,dx = \frac{2(x+1)^{\frac{3}{2}}}{3}$$

```
[In] integrate((1+x)**(1/2),x)
[Out] 2*(x + 1)**(3/2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \sqrt{1+x}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

```
[In] integrate((1+x)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \sqrt{1+x}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

```
[In] integrate((1+x)^(1/2),x, algorithm="giac")
[Out] 2/3*(x + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 21.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \sqrt{1+x}\,dx = \frac{2\,{(x+1)}^{3/2}}{3}$$

```
[In] int((x + 1)^(1/2),x)
[Out] (2*(x + 1)^(3/2))/3
```

3.861 $\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}} \, dx = \frac{2}{3}(1+x)^{3/2}$$

```
[Out] 2/3*(1+x)^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {26, 32}

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}} \, dx = \frac{2}{3}(x+1)^{3/2}$$

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 - x],x]
```

```
[Out] (2*(1 + x)^(3/2))/3
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \int \sqrt{1+x}\,dx$$
$$= \frac{2}{3}(1+x)^{3/2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 24 vs. $2(11) = 22$.

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx = \frac{2(1-x^2)^{3/2}}{3(1-x)^{3/2}}$$

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 - x],x]
[Out] (2*(1 - x^2)^(3/2))/(3*(1 - x)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.

Time = 1.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

method	result	size
gosper	$\frac{2(x+1)\sqrt{-x^2+1}}{3\sqrt{1-x}}$	22
default	$-\frac{2\sqrt{-x^2+1}\,\sqrt{1-x}\,(x+1)}{3(x-1)}$	27
risch	$-\frac{2\sqrt{\frac{(1-x)\left(-x^2+1\right)}{(x-1)^2}}\,(x-1)(x+1)^{\frac{3}{2}}}{3\sqrt{1-x}\,\sqrt{-x^2+1}}$	47

```
[In] int((-x^2+1)^(1/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(x+1)*(-x^2+1)^(1/2)/(1-x)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(7) = 14.

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx = -\frac{2\sqrt{-x^2+1}(x+1)\sqrt{-x+1}}{3(x-1)}$$

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(-x^2 + 1)*(x + 1)*sqrt(-x + 1)/(x - 1)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{1-x}}\,dx$$

```
[In] integrate((-x**2+1)**(1/2)/(1-x)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(1 - x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx = \frac{2}{3}(x+1)^{\frac{3}{2}}$$

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x + 1)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx = \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{4}{3}\sqrt{2}$$

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="giac")
[Out] 2/3*(x + 1)^(3/2) - 4/3*sqrt(2)
```

Mupad [B] (verification not implemented)

Time = 21.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx = \frac{\left(\frac{2\,x}{3}+\frac{2}{3}\right)\,\sqrt{1-x^2}}{\sqrt{1-x}}$$

```
[In] int((1 - x^2)^(1/2)/(1 - x)^(1/2),x)
[Out] (((2*x)/3 + 2/3)*(1 - x^2)^(1/2))/(1 - x)^(1/2)
```

3.862 $\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 35

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx = \sqrt{1+x}\sqrt{2+3x} - \frac{\operatorname{arcsinh}\left(\sqrt{2+3x}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arcsinh((2+3*x)^(1/2))*3^(1/2)+(1+x)^(1/2)*(2+3*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {52, 56, 221}

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{3x+2} - \frac{\operatorname{arcsinh}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

```
[In] Int[Sqrt[2 + 3*x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \sqrt{1+x}\sqrt{2+3x} - \frac{1}{2}\int \frac{1}{\sqrt{1+x}\sqrt{2+3x}}\,dx \\ &= \sqrt{1+x}\sqrt{2+3x} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2+3x}\right)}{\sqrt{3}} \\ &= \sqrt{1+x}\sqrt{2+3x} - \frac{\sinh^{-1}\left(\sqrt{2+3x}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx = \sqrt{1+x}\sqrt{2+3x} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2+3x}}{\sqrt{3}\sqrt{1+x}}\right)}{\sqrt{3}}$$

```
[In] Integrate[Sqrt[2 + 3*x]/Sqrt[1 + x],x]
[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcTanh[Sqrt[2 + 3*x]/(Sqrt[3]*Sqrt[1 + x])]/Sq
rt[3]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.

Time = 1.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.91

method	result	size
default	$\sqrt{x+1}\,\sqrt{3x+2}-\frac{\sqrt{(x+1)(3x+2)}\,\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^{2}+5x+2}\right)\sqrt{3}}{6\sqrt{3x+2}\,\sqrt{x+1}}$	67
risch	$\sqrt{x+1}\,\sqrt{3x+2}-\frac{\sqrt{(x+1)(3x+2)}\,\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^{2}+5x+2}\right)\sqrt{3}}{6\sqrt{3x+2}\,\sqrt{x+1}}$	67

```
[In] int((3*x+2)^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x+1)^(1/2)*(3*x+2)^(1/2)-1/6*((x+1)*(3*x+2))^(1/2)/(3*x+2)^(1/2)/(x+1)^(1/
2)*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx = \frac{1}{12}\,\sqrt{3}\log\left(-4\,\sqrt{3}(6\,x+5)\sqrt{3\,x+2}\sqrt{x+1}+72\,x^2+120\,x+49\right) + \sqrt{3\,x+2}\sqrt{x+1}$$

```
[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-4*sqrt(3)*(6*x + 5)*sqrt(3*x + 2)*sqrt(x + 1) + 72*x^2 +
120*x + 49) + sqrt(3*x + 2)*sqrt(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx = \begin{cases} \sqrt{x+1}\sqrt{3x+2} - \frac{\sqrt{3}\operatorname{acosh}\left(\sqrt{3}\sqrt{x+1}\right)}{3} & \text{for } |x+1| > \frac{1}{3} \\ \frac{\sqrt{3}i\operatorname{asin}\left(\sqrt{3}\sqrt{x+1}\right)}{3} - \frac{3i(x+1)^{\frac{3}{2}}}{\sqrt{-3x-2}} + \frac{i\sqrt{x+1}}{\sqrt{-3x-2}} & \text{otherwise} \end{cases}$$

```
[In] integrate((2+3*x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((sqrt(x + 1)*sqrt(3*x + 2) - sqrt(3)*acosh(sqrt(3)*sqrt(x + 1))/3
, Abs(x + 1) > 1/3), (sqrt(3)*I*asin(sqrt(3)*sqrt(x + 1))/3 - 3*I*(x + 1)**
(3/2)/sqrt(-3*x - 2) + I*sqrt(x + 1)/sqrt(-3*x - 2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx = -\frac{1}{6}\,\sqrt{3}\log\left(2\,\sqrt{3}\sqrt{3\,x^2+5\,x+2}+6\,x+5\right) + \sqrt{3\,x^2+5\,x+2}$$

```
[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5) + sqrt(3*x^2 +
5*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx = \frac{1}{3}\sqrt{3}\Big(\sqrt{3\,x+3}\sqrt{3\,x+2}+\log\left(\sqrt{3\,x+3}-\sqrt{3\,x+2}\right)\Big)$$

```
[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3*x + 3)*sqrt(3*x + 2) + log(sqrt(3*x + 3) - sqrt(3*x + 2
)))
```

Mupad [B] (verification not implemented)

Time = 24.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 4.91

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx = \frac{2\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\left(\sqrt{2}-\sqrt{3\,x+2}\right)}{3\,\left(\sqrt{x+1}-1\right)}\right)}{3} - \frac{\frac{30\,\left(\sqrt{2}-\sqrt{3\,x+2}\right)}{\sqrt{x+1}-1}+\frac{10\,\left(\sqrt{2}-\sqrt{3\,x+2}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{24\,\sqrt{2}\,\left(\sqrt{2}-\sqrt{3\,x+2}\right)^2}{\left(\sqrt{x+1}-1\right)^2}}{\frac{\left(\sqrt{2}-\sqrt{3\,x+2}\right)^4}{\left(\sqrt{x+1}-1\right)^4}-\frac{6\,\left(\sqrt{2}-\sqrt{3\,x+2}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+9}$$

```
[In] int((3*x + 2)^(1/2)/(x + 1)^(1/2),x)
```

```
[Out] (2*3^(1/2)*atanh((3^(1/2)*(2^(1/2) - (3*x + 2)^(1/2)))/(3*((x + 1)^(1/2) -
1))))/3 - ((30*(2^(1/2) - (3*x + 2)^(1/2)))/((x + 1)^(1/2) - 1) + (10*(2^(1
/2) - (3*x + 2)^(1/2))^3)/((x + 1)^(1/2) - 1)^3 + (24*2^(1/2)*(2^(1/2) - (3
*x + 2)^(1/2))^2)/((x + 1)^(1/2) - 1)^2)/((2^(1/2) - (3*x + 2)^(1/2))^4/((x
 + 1)^(1/2) - 1)^4 - (6*(2^(1/2) - (3*x + 2)^(1/2))^2)/((x + 1)^(1/2) - 1)^
2 + 9)
```

3.863 $\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 35

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = \sqrt{1+x}\sqrt{2+3x}-\frac{\operatorname{arcsinh}\left(\sqrt{2+3x}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arcsinh((2+3*x)^(1/2))*3^(1/2)+(1+x)^(1/2)*(2+3*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {26, 52, 56, 221}

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = \sqrt{x+1}\sqrt{3x+2}-\frac{\operatorname{arcsinh}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

```
[In] Int[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]
```

```
[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx\\ &= \sqrt{1+x}\sqrt{2+3x}-\frac{1}{2}\int \frac{1}{\sqrt{1+x}\sqrt{2+3x}}\,dx\\ &= \sqrt{1+x}\sqrt{2+3x}-\frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{2+3x}\right)}{\sqrt{3}}\\ &= \sqrt{1+x}\sqrt{2+3x}-\frac{\sinh^{-1}\left(\sqrt{2+3x}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx=\frac{3\sqrt{1+x}(2+3x)-\sqrt{6+9x}\,\text{arcsinh}\left(\sqrt{2+3x}\right)}{3\sqrt{2+3x}}$$

```
[In] Integrate[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]
[Out] (3*Sqrt[1 + x]*(2 + 3*x) - Sqrt[6 + 9*x]*ArcSinh[Sqrt[2 + 3*x]])/(3*Sqrt[2
+ 3*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(27) = 54$.

Time = 1.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.46

method	result	size
default	$\frac{\sqrt{1-x}\,\sqrt{3x+2}\,\sqrt{-x^{2}+1}\,\left(\ln\left(\frac{5\sqrt{3}}{6}+x\sqrt{3}+\sqrt{3x^{2}+5x+2}\right)\sqrt{3}-6\sqrt{3x^{2}+5x+2}\right)}{6(x-1)\sqrt{3x^{2}+5x+2}}$	86
risch	$-\frac{(x+1)\sqrt{3x+2}\,\sqrt{\frac{(1-x)(3x+2)\left(-x^{2}+1\right)}{(x-1)^{2}}}\,(x-1)}{\sqrt{(x+1)(3x+2)}\,\sqrt{1-x}\,\sqrt{-x^{2}+1}}+\frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^{2}+5x+2}\right)\sqrt{3}\,\sqrt{\frac{(1-x)(3x+2)\left(-x^{2}+1\right)}{(x-1)^{2}}}\,(x-1)}{6\sqrt{1-x}\,\sqrt{3x+2}\,\sqrt{-x^{2}+1}}$	149

```
[In] int((1-x)^(1/2)*(3*x+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1-x)^(1/2)*(3*x+2)^(1/2)*(-x^2+1)^(1/2)*(ln(5/6*3^(1/2)+x*3^(1/2)+(3*x
^2+5*x+2)^(1/2))*3^(1/2)-6*(3*x^2+5*x+2)^(1/2))/(x-1)/(3*x^2+5*x+2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(27) = 54$.

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.74

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = \frac{\sqrt{3}(x-1)\log\left(-\frac{72\,x^{3}+4\,\sqrt{3}\sqrt{-x^{2}+1}(6\,x+5)\sqrt{3\,x+2}\sqrt{-x+1}+48\,x^{2}-71\,x-49}{x-1}\right)-12\,\sqrt{-x^{2}+1}\sqrt{3\,x+2}\sqrt{-x+1}}{12\,(x-1)}$$

```
[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(3)*(x - 1)*log(-(72*x^3 + 4*sqrt(3)*sqrt(-x^2 + 1)*(6*x + 5)*sqr
t(3*x + 2)*sqrt(-x + 1) + 48*x^2 - 71*x - 49)/(x - 1)) - 12*sqrt(-x^2 + 1)*
sqrt(3*x + 2)*sqrt(-x + 1))/(x - 1)
```

Sympy [F]

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{1-x}\sqrt{3x+2}}{\sqrt{-(x-1)(x+1)}}\,dx$$

```
[In] integrate((1-x)**(1/2)*(2+3*x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - x)*sqrt(3*x + 2)/sqrt(-(x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{3\,x+2}\sqrt{-x+1}}{\sqrt{-x^2+1}}\,dx$$

```
[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(27) = 54$.

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = -\frac{1}{3}\sqrt{3}\left(\sqrt{5}\sqrt{3}\sqrt{2} + \log\left(\sqrt{3}\sqrt{2} - \sqrt{5}\right)\right) + \frac{1}{3}\sqrt{3}\log\left(\sqrt{3}\sqrt{x+1} - \sqrt{3\,x+2}\right) + \sqrt{3\,x+2}\sqrt{x+1}$$

```
[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/3*sqrt(3)*(sqrt(5)*sqrt(3)*sqrt(2) + log(sqrt(3)*sqrt(2) - sqrt(5))) + 1
/3*sqrt(3)*log(sqrt(3)*sqrt(x + 1) - sqrt(3*x + 2)) + sqrt(3*x + 2)*sqrt(x
+ 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{3\,x+2}\,\sqrt{1-x}}{\sqrt{1-x^2}}\,dx$$

```
[In] int(((3*x + 2)^(1/2)*(1 - x)^(1/2))/(1 - x^2)^(1/2),x)
[Out] int(((3*x + 2)^(1/2)*(1 - x)^(1/2))/(1 - x^2)^(1/2), x)
```

3.864 $\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx=\frac{4\sqrt{1+x}}{\sqrt{1-x}}-\arcsin(x)-\operatorname{arctanh}\left(\sqrt{1-x}\sqrt{1+x}\right)$$

```
[Out] -arcsin(x)-arctanh((1-x)^(1/2)*(1+x)^(1/2))+4*(1+x)^(1/2)/(1-x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {100, 21, 132, 41, 222, 94, 212}

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx=-\arcsin(x)-\operatorname{arctanh}\left(\sqrt{1-x}\sqrt{x+1}\right)+\frac{4\sqrt{x+1}}{\sqrt{1-x}}$$

```
[In] Int[(1 + x)^(3/2)/((1 - x)^(3/2)*x),x]
```

```
[Out] (4*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x] - ArcTanh[Sqrt[1 - x]*Sqrt[1 + x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
```

```
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - 2\int \frac{-\frac{1}{2}+\frac{x}{2}}{\sqrt{1-x}x\sqrt{1+x}}\,dx\\ &= \frac{4\sqrt{1+x}}{\sqrt{1-x}} + \int \frac{\sqrt{1-x}}{x\sqrt{1+x}}\,dx\end{aligned}$$

$$= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx + \int \frac{1}{\sqrt{1-x}x\sqrt{1+x}}\,dx$$
$$= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x^2}}\,dx - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x}\sqrt{1+x}\right)$$
$$= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x}\sqrt{1+x}\right)$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx = -\frac{4\sqrt{1-x^2}}{-1+x} + 4\arctan\left(\frac{\sqrt{1+x}}{\sqrt{2}-\sqrt{1-x}}\right) + 2\text{arctanh}\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

```
[In] Integrate[(1 + x)^(3/2)/((1 - x)^(3/2)*x),x]
[Out] (-4*Sqrt[1 - x^2])/(-1 + x) + 4*ArcTan[Sqrt[1 + x]/(Sqrt[2] - Sqrt[1 - x])]
 + 2*ArcTanh[Sqrt[1 - x^2]/(-1 + x)]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

method	result	size
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)x-\arcsin(x)x+\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)+\arcsin(x)-4\sqrt{-x^2+1}\right)\sqrt{1-x}\,\sqrt{x+1}}{(x-1)\sqrt{-x^2+1}}$	70
risch	$\frac{4\sqrt{x+1}\,\sqrt{(1-x)(x+1)}}{\sqrt{-(x-1)(x+1)}\,\sqrt{1-x}} - \frac{\left(\arcsin(x)+\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)\right)\sqrt{(1-x)(x+1)}}{\sqrt{1-x}\,\sqrt{x+1}}$	75

```
[In] int((x+1)^(3/2)/(1-x)^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] (-arctanh(1/(-x^2+1)^(1/2))*x-arcsin(x)*x+arctanh(1/(-x^2+1)^(1/2))+arcsin(
x)-4*(-x^2+1)^(1/2))*(1-x)^(1/2)*(x+1)^(1/2)/(x-1)/(-x^2+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(35) = 70$.

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx = \frac{2\,(x-1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + (x-1)\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + 4\,x - 4\,\sqrt{x+1}\sqrt{-x+}}{x-1}$$

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (2*(x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + (x - 1)*log((sqrt(x +
 1)*sqrt(-x + 1) - 1)/x) + 4*x - 4*sqrt(x + 1)*sqrt(-x + 1) - 4)/(x - 1)
```

Sympy [F]

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx = \int \frac{(x+1)^{\frac{3}{2}}}{x\,(1-x)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+x)**(3/2)/(1-x)**(3/2)/x,x)
```

```
[Out] Integral((x + 1)**(3/2)/(x*(1 - x)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx = \frac{4\,x}{\sqrt{-x^2+1}} + \frac{4}{\sqrt{-x^2+1}} - \arcsin\left(x\right) - \log\left(\frac{2\,\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x^2 + 1)/ab
s(x) + 2/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(35) = 70.

Time = 0.38 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.79

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx = -\pi - \frac{4\sqrt{x+1}\sqrt{-x+1}}{x-1} - 2\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right) - \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}+2\right|\right) + \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-2\right|\right)$$

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -pi - 4*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 2*arctan(1/2*sqrt(x + 1)*((sqrt(
2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1))) - log(abs(-(sqr
t(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) + 2
)) + log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) -
 sqrt(-x + 1)) - 2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx = \int \frac{(x+1)^{3/2}}{x\,(1-x)^{3/2}}\,dx$$

```
[In] int((x + 1)^(3/2)/(x*(1 - x)^(3/2)),x)
```

```
[Out] int((x + 1)^(3/2)/(x*(1 - x)^(3/2)), x)
```

3.865 $\int \frac{(1+x)^3}{x(1-x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{(1+x)^3}{x(1-x^2)^{3/2}} \, dx = \frac{4(1+x)}{\sqrt{1-x^2}} - \arcsin(x) - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

```
[Out] -arcsin(x)-arctanh((-x^2+1)^(1/2))+4*(1+x)/(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1819, 858, 222, 272, 65, 212}

$$\int \frac{(1+x)^3}{x(1-x^2)^{3/2}} \, dx = -\arcsin(x) - \operatorname{arctanh}\left(\sqrt{1-x^2}\right) + \frac{4(x+1)}{\sqrt{1-x^2}}$$

```
[In] Int[(1 + x)^3/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] (4*(1 + x))/Sqrt[1 - x^2] - ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4(1+x)}{\sqrt{1-x^2}} - \int \frac{-1+x}{x\sqrt{1-x^2}}\,dx \\
&= \frac{4(1+x)}{\sqrt{1-x^2}} - \int \frac{1}{\sqrt{1-x^2}}\,dx + \int \frac{1}{x\sqrt{1-x^2}}\,dx \\
&= \frac{4(1+x)}{\sqrt{1-x^2}} - \sin^{-1}(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, x^2\right) \\
&= \frac{4(1+x)}{\sqrt{1-x^2}} - \sin^{-1}(x) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right) \\
&= \frac{4(1+x)}{\sqrt{1-x^2}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{(1+x)^3}{x\left(1-x^2\right)^{3/2}}\,dx=-\frac{4\sqrt{1-x^2}}{-1+x}+2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)-2\text{arctanh}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

```
[In] Integrate[(1 + x)^3/(x*(1 - x^2)^(3/2)),x]
[Out] (-4*Sqrt[1 - x^2])/(-1 + x) + 2*ArcTan[Sqrt[1 - x^2]/(1 + x)] - 2*ArcTanh[S
qrt[1 - x^2]/(1 + x)]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result
risch	$\frac{4+4x}{\sqrt{-x^2+1}}-\arcsin\left(x\right)-\text{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$
default	$\frac{4x}{\sqrt{-x^2+1}}-\arcsin\left(x\right)+\frac{4}{\sqrt{-x^2+1}}-\text{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$
trager	$-\frac{4\sqrt{-x^2+1}}{x-1}+\text{RootOf}\left(\_Z^2+1\right)\ln\left(\text{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2+1}\right)+\ln\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$
meijerg	$\frac{-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{-x^2+1}}-\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)+\frac{(2-2\ln(2)+2\ln(x)+i\pi)\sqrt{\pi}}{2}}{\sqrt{\pi}}+\frac{i\left(-\frac{i\sqrt{\pi}\,x}{\sqrt{-x^2+1}}+i\sqrt{\pi}\,\arcsin(x)\right)}{\sqrt{\pi}}-\frac{3\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{-x^2+1}}\right)}{\sqrt{\pi}}+\frac{}{\sqrt{-}}$

```
[In] int((x+1)^3/x/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] 4*(x+1)/(-x^2+1)^(1/2)-arcsin(x)-arctanh(1/(-x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(31) = 62.

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{(1+x)^3}{x\left(1-x^2\right)^{3/2}}\,dx=\frac{2\,(x-1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+(x-1)\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+4\,x-4\,\sqrt{-x^2+1}-4}{x-1}$$

```
[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="fricas")
[Out] (2*(x - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) + (x - 1)*log((sqrt(-x^2 + 1) - 1
)/x) + 4*x - 4*sqrt(-x^2 + 1) - 4)/(x - 1)
```

Sympy [F]

$$\int \frac{(1+x)^3}{x(1-x^2)^{3/2}}\,dx = \int \frac{(x+1)^3}{x\left(-(x-1)(x+1)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((1+x)**3/x/(-x**2+1)**(3/2),x)
[Out] Integral((x + 1)**3/(x*(-(x - 1)*(x + 1))**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{(1+x)^3}{x(1-x^2)^{3/2}}\,dx = \frac{4\,x}{\sqrt{-x^2+1}} + \frac{4}{\sqrt{-x^2+1}} - \arcsin(x) - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

```
[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="maxima")
[Out] 4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x^2 + 1)/ab
s(x) + 2/abs(x))
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{(1+x)^3}{x(1-x^2)^{3/2}}\,dx = \frac{8}{\frac{\sqrt{-x^2+1}-1}{x}+1} - \arcsin(x) + \log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

```
[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="giac")
[Out] 8/((sqrt(-x^2 + 1) - 1)/x + 1) - arcsin(x) + log(-(sqrt(-x^2 + 1) - 1)/abs(
x))
```

Mupad [B] (verification not implemented)

Time = 25.78 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(1+x)^3}{x\left(1-x^2\right)^{3/2}}\,dx = \ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)-\mathrm{asin}(x)-\frac{4\sqrt{1-x^2}}{x-1}$$

```
[In] int((x + 1)^3/(x*(1 - x^2)^(3/2)),x)
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - asin(x) - (4*(1 - x^2)^(1/2))/(x -
 1)
```

3.866 $\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}} \, dx = \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - \arcsin(ax) - \operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)$$

```
[Out] -arcsin(a*x)-arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))+4*(a*x+1)^(1/2)/(-a*x+1)
^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {100, 21, 132, 41, 222, 94, 214}

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}} \, dx = -\arcsin(ax) - \operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right) + \frac{4\sqrt{ax+1}}{\sqrt{1-ax}}$$

```
[In] Int[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
```

```
[Out] (4*Sqrt[1 + a*x])/Sqrt[1 - a*x] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\text{integral} = \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{2\int \frac{-\frac{a}{2}+\frac{a^2x}{2}}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a}$$

$$= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} + \int \frac{\sqrt{1-ax}}{x\sqrt{1+ax}}\,dx$$

$$= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - a\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx + \int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx$$

$$= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx - a\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)$$

$$= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx = 4\left(\frac{\sqrt{1-a^2x^2}}{1-ax} + \arctan\left(\frac{\sqrt{1+ax}}{\sqrt{2-\sqrt{1-ax}}}\right)\right) + 2\text{arctanh}\left(\frac{\sqrt{1-a^2x^2}}{-1+ax}\right)$$

```
[In] Integrate[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
[Out] 4*(Sqrt[1 - a^2*x^2]/(1 - a*x) + ArcTan[Sqrt[1 + a*x]/(Sqrt[2] - Sqrt[1 - a
*x])]) + 2*ArcTanh[Sqrt[1 - a^2*x^2]/(-1 + a*x)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.63

method	result
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\operatorname{csgn}(a)ax-\arctan\left(\frac{\operatorname{csgn}(a)ax}{\sqrt{-(ax+1)(ax-1)}}\right)ax+\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\operatorname{csgn}(a)-4\sqrt{-a^2x^2+1}\operatorname{csgn}(a)+\arctan\right.}{(ax-1)\sqrt{-a^2x^2+1}}$

```
[In] int((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] (-arctanh(1/(-a^2*x^2+1)^(1/2))*csgn(a)*a*x-arctan(csgn(a)*a*x/(-(a*x+1)*(a
*x-1))^(1/2))*a*x+arctanh(1/(-a^2*x^2+1)^(1/2))*csgn(a)-4*(-a^2*x^2+1)^(1/2
)*csgn(a)+arctan(csgn(a)*a*x/(-(a*x+1)*(a*x-1))^(1/2)))*csgn(a)*(-a*x+1)^(1
/2)*(a*x+1)^(1/2)/(a*x-1)/(-a^2*x^2+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(43) = 86$.

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.82

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx = \frac{4\,ax + 2\,(ax-1)\arctan\left(\frac{\sqrt{ax+1}\sqrt{-ax+1}-1}{ax}\right) + (ax-1)\log\left(\frac{\sqrt{ax+1}\sqrt{-ax+1}-1}{x}\right) - 4\sqrt{ax+}}{ax-1}$$

[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="fricas")

[Out] (4*a*x + 2*(a*x - 1)*arctan((sqrt(a*x + 1)*sqrt(-a*x + 1) - 1)/(a*x)) + (a*x - 1)*log((sqrt(a*x + 1)*sqrt(-a*x + 1) - 1)/x) - 4*sqrt(a*x + 1)*sqrt(-a*x + 1) - 4)/(a*x - 1)

Sympy [F]

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx = \int \frac{(ax+1)^{\frac{3}{2}}}{x\,(-ax+1)^{\frac{3}{2}}}\,dx$$

[In] integrate((a*x+1)**(3/2)/x/(-a*x+1)**(3/2),x)

[Out] Integral((a*x + 1)**(3/2)/(x*(-a*x + 1)**(3/2)), x)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx = \frac{4\,ax}{\sqrt{-a^2x^2+1}} + \frac{4}{\sqrt{-a^2x^2+1}} - \arcsin\left(ax\right) - \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="maxima")

[Out] 4*a*x/sqrt(-a^2*x^2 + 1) + 4/sqrt(-a^2*x^2 + 1) - arcsin(a*x) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(43) = 86$.

Time = 0.43 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.84

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx = -\frac{\left(\pi + 2\arctan\left(\frac{\sqrt{ax+1}\left(\frac{\left(\sqrt{2}-\sqrt{-ax+1}\right)^2}{ax+1}-1\right)}{2\left(\sqrt{2}-\sqrt{-ax+1}\right)}\right)\right)a + a\log\left(\left|-\frac{\sqrt{2}-\sqrt{-ax+1}}{\sqrt{ax+1}} + \frac{\sqrt{ax+1}}{\sqrt{2}-\sqrt{-ax+1}} + 2\right|\right) - a\log\left(\left|-\frac{\sqrt{2}-\cdots}{\sqrt{\cdots}}\right.\right.}{a}$$

```
[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] -((pi + 2*arctan(1/2*sqrt(a*x + 1)*((sqrt(2) - sqrt(-a*x + 1))^2/(a*x + 1)
- 1)/(sqrt(2) - sqrt(-a*x + 1))))*a + a*log(abs(-(sqrt(2) - sqrt(-a*x + 1))
/sqrt(a*x + 1) + sqrt(a*x + 1)/(sqrt(2) - sqrt(-a*x + 1)) + 2)) - a*log(abs
(-(sqrt(2) - sqrt(-a*x + 1))/sqrt(a*x + 1) + sqrt(a*x + 1)/(sqrt(2) - sqrt(
-a*x + 1)) - 2)) + 4*sqrt(a*x + 1)*sqrt(-a*x + 1)*a/(a*x - 1))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx = \int \frac{(a\,x+1)^{3/2}}{x\,(1-a\,x)^{3/2}}\,dx$$

```
[In] int((a*x + 1)^(3/2)/(x*(1 - a*x)^(3/2)),x)
```

```
[Out] int((a*x + 1)^(3/2)/(x*(1 - a*x)^(3/2)), x)
```

3.867 $\int \frac{(1+ax)^3}{x(1-a^2x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 45

$$\int \frac{(1+ax)^3}{x\left(1-a^2x^2\right)^{3/2}} \, dx = \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - \arcsin(ax) - \operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -arcsin(a*x)-arctanh((-a^2*x^2+1)^(1/2))+4*(a*x+1)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1819, 858, 222, 272, 65, 214}

$$\int \frac{(1+ax)^3}{x\left(1-a^2x^2\right)^{3/2}} \, dx = -\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right) + \frac{4(ax+1)}{\sqrt{1-a^2x^2}} - \arcsin(ax)$$

```
[In] Int[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (4*(1 + a*x))/Sqrt[1 - a^2*x^2] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - \int \frac{-1+ax}{x\sqrt{1-a^2x^2}}\,dx \\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx + \int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - \sin^{-1}(ax) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - \sin^{-1}(ax) - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2} \\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

$$\int \frac{(1+ax)^3}{x(1-a^2x^2)^{3/2}}\,dx = -\frac{4\sqrt{1-a^2x^2}}{-1+ax} + 2\arctan\left(\frac{-1+\sqrt{1-a^2x^2}}{ax}\right) - \log(x) + \log\left(-1+\sqrt{1-a^2x^2}\right)$$

```
[In] Integrate[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-4*Sqrt[1 - a^2*x^2])/(-1 + a*x) + 2*ArcTan[(-1 + Sqrt[1 - a^2*x^2])/(a*x)
] - Log[x] + Log[-1 + Sqrt[1 - a^2*x^2]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(41) = 82.

Time = 1.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.20

method	result
default	$\frac{3ax}{\sqrt{-a^2x^2+1}} + \frac{4}{\sqrt{-a^2x^2+1}} - \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) + a^3\left(\frac{x}{a^2\sqrt{-a^2x^2+1}} - \frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{a^2\sqrt{a^2}}\right)$
meijerg	$\frac{-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{-a^2x^2+1}}-\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-a^2x^2+1}}{2}\right)+\frac{\left(2-2\ln(2)+2\ln(x)+\ln\left(-a^2\right)\right)\sqrt{\pi}}{2}}{\sqrt{\pi}} - \frac{a\left(\frac{\sqrt{\pi}\,x\left(-a^2\right)^{\frac{3}{2}}}{a^2\sqrt{-a^2x^2+1}} - \frac{\sqrt{\pi}\left(-a^2\right)^{\frac{3}{2}}\arcsin(ax)}{a^3}\right)}{\sqrt{\pi}\,\sqrt{-a^2}} - \frac{3(\sqrt{}}{}$

```
[In] int((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3*a*x/(-a^2*x^2+1)^(1/2)+4/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))
+a^3*(x/a^2/(-a^2*x^2+1)^(1/2)-1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2
*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.82

$$\int \frac{(1+ax)^3}{x(1-a^2x^2)^{3/2}}\,dx = \frac{4\,ax + 2\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + (ax-1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - 4\sqrt{-a^2x^2+1}}{ax-1}$$

```
[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (4*a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a*x - 1)*log
((sqrt(-a^2*x^2 + 1) - 1)/x) - 4*sqrt(-a^2*x^2 + 1) - 4)/(a*x - 1)
```

Sympy [F]

$$\int \frac{(1+ax)^3}{x(1-a^2x^2)^{3/2}}\,dx = \int \frac{(ax+1)^3}{x(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*x+1)**3/x/(-a**2*x**2+1)**(3/2),x)
[Out] Integral((a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{(1+ax)^3}{x(1-a^2x^2)^{3/2}}\,dx = \frac{4\,ax}{\sqrt{-a^2x^2+1}} + \frac{4}{\sqrt{-a^2x^2+1}} - \arcsin(ax) - \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

```
[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
[Out] 4*a*x/sqrt(-a^2*x^2 + 1) + 4/sqrt(-a^2*x^2 + 1) - arcsin(a*x) - log(2*sqrt(
-a^2*x^2 + 1)/abs(x) + 2/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(41) = 82.

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.93

$$\int \frac{(1+ax)^3}{x(1-a^2x^2)^{3/2}}\,dx = -\frac{a\arcsin(ax)\,\mathrm{sgn}(a)}{|a|} - \frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|} + \frac{8\,a}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)|a|}$$

```
[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
[Out] -a*arcsin(a*x)*sgn(a)/abs(a) - a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
 2*a)/(a^2*abs(x)))/abs(a) + 8*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x)
- 1)*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.82

$$\int \frac{(1+ax)^3}{x\,(1-a^2x^2)^{3/2}}\,dx = \frac{4\,a\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{a\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)$$

```
[In] int((a*x + 1)^3/(x*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (4*a*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- (a*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - atanh((1 - a^2*x^2)^(1/2))
```

3.868 $\int \frac{1}{\sqrt{1-x^2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 2

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin(x)$$

```
[Out] arcsin(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {222}

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin(x)$$

```
[In] Int[1/Sqrt[1 - x^2],x]
```

```
[Out] ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\text{integral} = \sin^{-1}(x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 20 vs. $2(2) = 4$.

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 10.00

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

```
[In] Integrate[1/Sqrt[1 - x^2],x]
[Out] -2*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\arcsin(x)$	3
meijerg	$\arcsin(x)$	3
pseudoelliptic	$-\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)$	17
trager	$\operatorname{RootOf}(\_Z^2+1)\ln\left(\operatorname{RootOf}(\_Z^2+1)\sqrt{-x^2+1}+x\right)$	27

```
[In] int(1/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(2) = 4$.

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 9.00

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \operatorname{asin}(x)$$

```
[In] integrate(1/(-x**2+1)**(1/2),x)
[Out] asin(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x)$$

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(2) = 4$.

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 8.50

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\,\sqrt{-x^2+1}x + \frac{1}{2}\,\arcsin(x)$$

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \operatorname{asin}(x)$$

```
[In] int(1/(1 - x^2)^(1/2),x)
[Out] asin(x)
```

3.869 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 2

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} \, dx = \arcsin(x)$$

```
[Out] arcsin(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {26, 222}

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} \, dx = \arcsin(x)$$

```
[In] Int[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]
```

```
[Out] ArcSin[x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sin^{-1}(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 32 vs. 2(2) = 4.

Time = 0.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 16.00

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = -\arctan\left(\frac{x\sqrt{1+x^2}\sqrt{1-x^4}}{-1+x^4}\right)$$

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]
[Out] -ArcTan[(x*Sqrt[1 + x^2]*Sqrt[1 - x^4])/(-1 + x^4)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(2) = 4.

Time = 1.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 14.50

method	result	size
default	$\frac{\sqrt{-x^4+1}\,\arcsin(x)}{\sqrt{x^2+1}\,\sqrt{-x^2+1}}$	29

```
[In] int((x^2+1)^(1/2)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(x^2+1)^(1/2)*(-x^4+1)^(1/2)/(-x^2+1)^(1/2)*arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(2) = 4.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 13.50

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = -\arctan\left(\frac{\sqrt{-x^4+1}\sqrt{x^2+1}}{x^3+x}\right)$$

```
[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
[Out] -arctan(sqrt(-x^4 + 1)*sqrt(x^2 + 1)/(x^3 + x))
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-(x-1)(x+1)(x^2+1)}}\,dx$$

```
[In] integrate((x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
[Out] Integral(sqrt(x**2 + 1)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-x^4+1}}\,dx$$

```
[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-x^4+1}}\,dx$$

```
[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{1-x^4}}\,dx$$

```
[In] int((x^2 + 1)^(1/2)/(1 - x^4)^(1/2),x)
[Out] int((x^2 + 1)^(1/2)/(1 - x^4)^(1/2), x)
```

3.870 $\int \frac{1}{\sqrt{1+x^2}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 2

$$\int \frac{1}{\sqrt{1+x^2}} \, dx = \text{arcsinh}(x)$$

```
[Out] arcsinh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {221}

$$\int \frac{1}{\sqrt{1+x^2}} \, dx = \text{arcsinh}(x)$$

```
[In] Int[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\text{integral} = \sinh^{-1}(x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs. $2(2) = 4$.

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 8.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = -\log\left(-x+\sqrt{1+x^2}\right)$$

```
[In] Integrate[1/Sqrt[1 + x^2],x]
[Out] -Log[-x + Sqrt[1 + x^2]]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\operatorname{arcsinh}(x)$	3
meijerg	$\operatorname{arcsinh}(x)$	3
trager	$\ln\left(x+\sqrt{x^2+1}\right)$	11
pseudoelliptic	$\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}}{x}\right)$	13

```
[In] int(1/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(2) = 4$.

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 7.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = -\log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{asinh}(x)$$

```
[In] integrate(1/(x**2+1)**(1/2),x)
[Out] asinh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{arsinh}(x)$$

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsinh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(2) = 4.

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 12.50

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \frac{1}{2}\sqrt{x^2+1}x - \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{asinh}(x)$$

```
[In] int(1/(x^2 + 1)^(1/2),x)
[Out] asinh(x)
```

3.871 $\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 2

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}} \, dx = \text{arcsinh}(x)$$

```
[Out] arcsinh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {26, 221}

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}} \, dx = \text{arcsinh}(x)$$

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
```

```
[Out] ArcSinh[x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{1+x^2}}\,dx$$
$$= \sinh^{-1}(x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(2) = 4.

Time = 0.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 21.00

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \log\left(1-x^2\right) - \log\left(-x+x^3+\sqrt{1-x^2}\sqrt{1-x^4}\right)$$

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
[Out] Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 - x^2]*Sqrt[1 - x^4]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(2) = 4.

Time = 1.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 14.50

method	result	size
default	$\frac{\sqrt{-x^4+1}\,\operatorname{arcsinh}(x)}{\sqrt{-x^2+1}\,\sqrt{x^2+1}}$	29

```
[In] int((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(-x^2+1)^(1/2)/(x^2+1)^(1/2)*(-x^4+1)^(1/2)*arcsinh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(2) = 4.

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 40.50

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = -\frac{1}{2}\log\left(\frac{x^3+\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right) + \frac{1}{2}\log\left(-\frac{x^3-\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)$$

```
[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log((x^3 + sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) + 1/2*log(-(x
^3 - sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x))
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{-(x-1)(x+1)(x^2+1)}}\,dx$$

```
[In] integrate((-x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{-x^4+1}}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{-x^4+1}}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx$$

```
[In] int((1 - x^2)^(1/2)/(1 - x^4)^(1/2),x)
[Out] int((1 - x^2)^(1/2)/(1 - x^4)^(1/2), x)
```

3.872 $\int \sqrt{1-x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2}+\frac{\arcsin(x)}{2}$$

```
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {201, 222}

$$\int \sqrt{1-x^2}\,dx = \frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x$$

```
[In] Int[Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

<u>Rule 201</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

<u>Rule 222</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2} - \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

```
[In] Integrate[Sqrt[1 - x^2],x]
[Out] (x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{x\sqrt{-x^2+1}}{2}+\frac{\arcsin(x)}{2}$	18
risch	$-\frac{x(x^2-1)}{2\sqrt{-x^2+1}}+\frac{\arcsin(x)}{2}$	23
pseudoelliptic	$\frac{x\sqrt{-x^2+1}}{2}-\frac{\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)}{2}$	30
meijerg	$\frac{i\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\,\arcsin(x)\right)}{4\sqrt{\pi}}$	32
trager	$\frac{x\sqrt{-x^2+1}}{2}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	41

```
[In] int((-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(-x^2+1)^(1/2)+1/2*arcsin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\,\sqrt{-x^2+1}x - \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*x - arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \sqrt{1-x^2}\,dx = \frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2}$$

```
[In] integrate((-x**2+1)**(1/2),x)
[Out] x*sqrt(1 - x**2)/2 + asin(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\,\sqrt{-x^2+1}x + \frac{1}{2}\,\arcsin(x)$$

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\,\sqrt{-x^2+1}x + \frac{1}{2}\,\arcsin(x)$$

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{\mathrm{asin}(x)}{2} + \frac{x\,\sqrt{1-x^2}}{2}$$

```
[In] int((1 - x^2)^(1/2),x)
[Out] asin(x)/2 + (x*(1 - x^2)^(1/2))/2
```

3.873 $\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}} dx = \frac{1}{2}x\sqrt{1-x^2} + \frac{\arcsin(x)}{2}$$

```
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {26, 201, 222}

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}} dx = \frac{\arcsin(x)}{2} + \frac{1}{2}\sqrt{1-x^2}x$$

```
[In] Int[Sqrt[1 - x^4]/Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt{1-x^2}\,dx\\ &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 50 vs. $2(23) = 46$.

Time = 0.61 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx = \frac{1}{2}\left(\frac{x\sqrt{1-x^4}}{\sqrt{1+x^2}}+\arctan\left(\frac{x\sqrt{1+x^2}}{\sqrt{1-x^4}}\right)\right)$$

```
[In] Integrate[Sqrt[1 - x^4]/Sqrt[1 + x^2],x]
[Out] ((x*Sqrt[1 - x^4])/Sqrt[1 + x^2] + ArcTan[(x*Sqrt[1 + x^2])/Sqrt[1 - x^4]])
/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.

Time = 1.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

method	result	size
default	$\frac{\sqrt{-x^4+1}\left(x\sqrt{-x^2+1}+\arcsin(x)\right)}{2\sqrt{x^2+1}\,\sqrt{-x^2+1}}$	42
risch	$-\frac{x(x^2-1)\sqrt{\frac{-x^4+1}{x^2+1}}\,\sqrt{x^2+1}}{2\sqrt{-x^2+1}\,\sqrt{-x^4+1}}+\frac{\arcsin(x)\sqrt{\frac{-x^4+1}{x^2+1}}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}$	89

```
[In] int((-x^4+1)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-x^4+1)^(1/2)/(x^2+1)^(1/2)*(x*(-x^2+1)^(1/2)+arcsin(x))/(-x^2+1)^(1/2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.61

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{-x^4+1}\sqrt{x^2+1}x - (x^2+1)\arctan\left(\frac{\sqrt{-x^4+1}\sqrt{x^2+1}}{x^3+x}\right)}{2\,(x^2+1)}$$

```
[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-x^4 + 1)*sqrt(x^2 + 1)*x - (x^2 + 1)*arctan(sqrt(-x^4 + 1)*sqrt(
x^2 + 1)/(x^3 + x)))/(x^2 + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((-x**4+1)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-x^4+1}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-x^4+1}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{1-x^4}}{\sqrt{x^2+1}}\,dx$$

```
[In] int((1 - x^4)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((1 - x^4)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.874 $\int \sqrt{1+x^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 21

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}x\sqrt{1+x^2} + \frac{\text{arcsinh}(x)}{2}$$

```
[Out] 1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {201, 221}

$$\int \sqrt{1+x^2}\,dx = \frac{\text{arcsinh}(x)}{2} + \frac{1}{2}\sqrt{x^2+1}x$$

```
[In] Int[Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx$$
$$= \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\sinh^{-1}(x)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}x\sqrt{1+x^2} - \frac{1}{2}\log\left(-x+\sqrt{1+x^2}\right)$$

```
[In] Integrate[Sqrt[1 + x^2],x]
[Out] (x*Sqrt[1 + x^2])/2 - Log[-x + Sqrt[1 + x^2]]/2
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\operatorname{arcsinh}(x)}{2}+\frac{x\sqrt{x^2+1}}{2}$	16
risch	$\frac{\operatorname{arcsinh}(x)}{2}+\frac{x\sqrt{x^2+1}}{2}$	16
trager	$\frac{x\sqrt{x^2+1}}{2}+\frac{\ln\left(x+\sqrt{x^2+1}\right)}{2}$	24
meijerg	$-\frac{-2\sqrt{\pi}\,x\sqrt{x^2+1}-2\sqrt{\pi}\,\operatorname{arcsinh}(x)}{4\sqrt{\pi}}$	27
pseudoelliptic	$\frac{x\sqrt{x^2+1}}{2}+\frac{\ln\left(\frac{x+\sqrt{x^2+1}}{x}\right)}{4}-\frac{\ln\left(\frac{\sqrt{x^2+1}-x}{x}\right)}{4}$	46

```
[In] int((x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}\,\sqrt{x^2+1}x - \frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate((x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{1+x^2}\,dx = \frac{x\sqrt{x^2+1}}{2} + \frac{\operatorname{asinh}(x)}{2}$$

```
[In] integrate((x**2+1)**(1/2),x)
[Out] x*sqrt(x**2 + 1)/2 + asinh(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}\,\sqrt{x^2+1}x + \frac{1}{2}\,\operatorname{arsinh}(x)$$

```
[In] integrate((x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^2 + 1)*x + 1/2*arcsinh(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}\,\sqrt{x^2+1}x - \frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate((x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{1+x^2}\,dx = \frac{\operatorname{asinh}(x)}{2} + \frac{x\,\sqrt{x^2+1}}{2}$$

```
[In] int((x^2 + 1)^(1/2),x)
[Out] asinh(x)/2 + (x*(x^2 + 1)^(1/2))/2
```

3.875 $\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} \, dx = \frac{1}{2}x\sqrt{1+x^2} + \frac{\operatorname{arcsinh}(x)}{2}$$

```
[Out] 1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {26, 201, 221}

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} \, dx = \frac{\operatorname{arcsinh}(x)}{2} + \frac{1}{2}\sqrt{x^2+1}x$$

```
[In] Int[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt{1+x^2}\,dx \\ &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx \\ &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\sinh^{-1}(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 70 vs. $2(21) = 42$.

Time = 0.62 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.33

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\left(\frac{x\sqrt{1-x^4}}{\sqrt{1-x^2}}+\log\left(1-x^2\right)-\log\left(-x+x^3+\sqrt{1-x^2}\sqrt{1-x^4}\right)\right)$$

```
[In] Integrate[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]
[Out] ((x*Sqrt[1 - x^4])/Sqrt[1 - x^2] + Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 - x
^2]*Sqrt[1 - x^4]])/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.

Time = 1.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.24

method	result	size
default	$-\frac{\sqrt{-x^4+1}\,\sqrt{-x^2+1}\,\left(x\sqrt{x^2+1}+\operatorname{arcsinh}(x)\right)}{2(x^2-1)\sqrt{x^2+1}}$	47
risch	$-\frac{x\sqrt{x^2+1}\,\sqrt{\frac{\left(-x^2+1\right)\left(-x^4+1\right)}{\left(x^2-1\right)^2}}\,\left(x^2-1\right)}{2\sqrt{-x^4+1}\,\sqrt{-x^2+1}}-\frac{\operatorname{arcsinh}(x)\sqrt{\frac{\left(-x^2+1\right)\left(-x^4+1\right)}{\left(x^2-1\right)^2}}\,\left(x^2-1\right)}{2\sqrt{-x^4+1}\,\sqrt{-x^2+1}}$	110

```
[In] int((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-x^4+1)^(1/2)*(-x^2+1)^(1/2)*(x*(x^2+1)^(1/2)+arcsinh(x))/(x^2-1)/(x^
2+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 5.71

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = -\frac{2\sqrt{-x^4+1}\sqrt{-x^2+1}x + (x^2-1)\log\left(\frac{x^3+\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right) - (x^2-1)\log\left(-\frac{x^3-\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)}{4(x^2-1)}$$

```
[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(-x^4 + 1)*sqrt(-x^2 + 1)*x + (x^2 - 1)*log((x^3 + sqrt(-x^4 +
1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) - (x^2 - 1)*log(-(x^3 - sqrt(-x^4 + 1)*sq
rt(-x^2 + 1) - x)/(x^3 - x)))/(x^2 - 1)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{\sqrt{-(x-1)(x+1)}}\,dx$$

```
[In] integrate((-x**4+1)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(-(x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{-x^4+1}}{\sqrt{-x^2+1}}\,dx$$

```
[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{-x^4+1}}{\sqrt{-x^2+1}}\,dx$$

```
[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx$$

```
[In] int((1 - x^4)^(1/2)/(1 - x^2)^(1/2),x)
[Out] int((1 - x^4)^(1/2)/(1 - x^2)^(1/2), x)
```

3.876 $\int \left(\frac{a+b+cx^2}{d}\right)^m dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx = \frac{dx\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^{1+m}\text{Hypergeometric2F1}\left(1,\frac{3}{2}+m,\frac{3}{2},-\frac{cx^2}{a+b}\right)}{a+b}$$

```
[Out] d*x*((a+b)/d+x^2*c/d)^(1+m)*hypergeom([1, 3/2+m],[3/2],-c*x^2/(a+b))/(a+b)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1997, 252, 251}

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx = x\left(\frac{cx^2}{a+b}+1\right)^{-m}\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},-\frac{cx^2}{a+b}\right)$$

```
[In] Int[((a + b + c*x^2)/d)^m,x]
```

```
[Out] (x*((a + b)/d + (c*x^2)/d)^m*Hypergeometric2F1[1/2, -m, 3/2, -((c*x^2)/(a +
 b))])/(1 + (c*x^2)/(a + b))^m
```

<u>Rule 251</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
```

```
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] && !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m dx \\ &= \left(\left(1+\frac{cx^2}{a+b}\right)^{-m}\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m\right)\int \left(1+\frac{cx^2}{a+b}\right)^m dx \\ &= x\left(1+\frac{cx^2}{a+b}\right)^{-m}\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};-\frac{cx^2}{a+b}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx = x\left(\frac{a+b+cx^2}{d}\right)^m \left(1+\frac{cx^2}{a+b}\right)^{-m} \text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},-\frac{cx^2}{a+b}\right)$$

```
[In] Integrate[((a + b + c*x^2)/d)^m,x]
[Out] (x*((a + b + c*x^2)/d)^m*Hypergeometric2F1[1/2, -m, 3/2, -((c*x^2)/(a + b))
])/(1 + (c*x^2)/(a + b))^m
```

Maple [F]

$$\int \left(\frac{c\,x^{2}+a+b}{d}\right)^{m}dx$$

```
[In] int(((c*x^2+a+b)/d)^m,x)
[Out] int(((c*x^2+a+b)/d)^m,x)
```

Fricas [F]

$$\int \left(\frac{a+b+cx^{2}}{d}\right)^{m}\,dx=\int \left(\frac{cx^{2}+a+b}{d}\right)^{m}\,dx$$

```
[In] integrate(((c*x^2+a+b)/d)^m,x, algorithm="fricas")
[Out] integral(((c*x^2 + a + b)/d)^m, x)
```

Sympy [F]

$$\int \left(\frac{a+b+cx^{2}}{d}\right)^{m}\,dx=\int \left(\frac{a+b+cx^{2}}{d}\right)^{m}\,dx$$

```
[In] integrate(((c*x**2+a+b)/d)**m,x)
[Out] Integral(((a + b + c*x**2)/d)**m, x)
```

Maxima [F]

$$\int \left(\frac{a+b+cx^{2}}{d}\right)^{m}\,dx=\int \left(\frac{cx^{2}+a+b}{d}\right)^{m}\,dx$$

```
[In] integrate(((c*x^2+a+b)/d)^m,x, algorithm="maxima")
[Out] integrate(((c*x^2 + a + b)/d)^m, x)
```

Giac [F]

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx = \int \left(\frac{cx^2+a+b}{d}\right)^m dx$$

```
[In] integrate(((c*x^2+a+b)/d)^m,x, algorithm="giac")
```

```
[Out] integrate(((c*x^2 + a + b)/d)^m, x)
```

Mupad [B] (verification not implemented)

Time = 23.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx = \frac{x\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m {}_2F_1\left(\frac{1}{2},-m;\ \frac{3}{2};\ -\frac{cx^2}{a+b}\right)}{\left(\frac{cx^2}{a+b}+1\right)^m}$$

```
[In] int(((a + b + c*x^2)/d)^m,x)
```

```
[Out] (x*((a + b)/d + (c*x^2)/d)^m*hypergeom([1/2, -m], 3/2, -(c*x^2)/(a + b)))/(
(c*x^2)/(a + b) + 1)^m
```

3.877 $\int \frac{1}{x-\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx = -\frac{x^2}{2}-\frac{1}{2}x\sqrt{1+x^2}-\frac{\operatorname{arcsinh}(x)}{2}$$

```
[Out] -1/2*x^2-1/2*arcsinh(x)-1/2*x*(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2131, 30, 201, 221}

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx = -\frac{\operatorname{arcsinh}(x)}{2}-\frac{x^2}{2}-\frac{1}{2}\sqrt{x^2+1}x$$

```
[In] Int[(x - Sqrt[1 + x^2])^(-1),x]
```

```
[Out] -1/2*x^2 - (x*Sqrt[1 + x^2])/2 - ArcSinh[x]/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2131

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Dist[-b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int x\,dx - \int \sqrt{1+x^2}\,dx \\
&= -\frac{x^2}{2} - \frac{1}{2}x\sqrt{1+x^2} - \frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx \\
&= -\frac{x^2}{2} - \frac{1}{2}x\sqrt{1+x^2} - \frac{1}{2}\sinh^{-1}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx = \frac{1}{2}\left(-x\left(x+\sqrt{1+x^2}\right) + \log\left(-x+\sqrt{1+x^2}\right)\right)$$

```
[In] Integrate[(x - Sqrt[1 + x^2])^(-1),x]
[Out] (-(x*(x + Sqrt[1 + x^2])) + Log[-x + Sqrt[1 + x^2]])/2
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{x^2}{2} - \frac{\operatorname{arcsinh}(x)}{2} - \frac{x\sqrt{x^2+1}}{2}$	21
trager	$-\frac{x^2}{2} - \frac{x\sqrt{x^2+1}}{2} + \frac{\ln\left(x-\sqrt{x^2+1}\right)}{2}$	31

```
[In] int(1/(x-(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2-1/2*arcsinh(x)-1/2*x*(x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx = -\frac{1}{2}\,x^2-\frac{1}{2}\,\sqrt{x^2+1}x+\frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="fricas")
[Out] -1/2*x^2 - 1/2*sqrt(x^2 + 1)*x + 1/2*log(-x + sqrt(x^2 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(22) = 44.

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.32

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx = -\frac{x\,\mathrm{asinh}\,(x)}{2x-2\sqrt{x^2+1}}+\frac{\sqrt{x^2+1}\,\mathrm{asinh}\,(x)}{2x-2\sqrt{x^2+1}}+\frac{\sqrt{x^2+1}}{2x-2\sqrt{x^2+1}}$$

```
[In] integrate(1/(x-(x**2+1)**(1/2)),x)
[Out] -x*asinh(x)/(2*x - 2*sqrt(x**2 + 1)) + sqrt(x**2 + 1)*asinh(x)/(2*x - 2*sqr
t(x**2 + 1)) + sqrt(x**2 + 1)/(2*x - 2*sqrt(x**2 + 1))
```

Maxima [F]

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx = \int \frac{1}{x-\sqrt{x^2+1}}\,dx$$

```
[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(x - sqrt(x^2 + 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx = -\frac{1}{2}\,x^2 - \frac{1}{2}\,\sqrt{x^2+1}x + \frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*x^2 - 1/2*sqrt(x^2 + 1)*x + 1/2*log(-x + sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx = -\frac{\operatorname{asinh}(x)}{2} - \frac{x\,\sqrt{x^2+1}}{2} - \frac{x^2}{2}$$

```
[In] int(1/(x - (x^2 + 1)^(1/2)),x)
```

```
[Out] - asinh(x)/2 - (x*(x^2 + 1)^(1/2))/2 - x^2/2
```

3.878 $\int \frac{1}{x-\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx=-\frac{\arcsin(x)}{2}-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{1-x^2}}\right)+\frac{1}{4}\log\left(1-2x^2\right)$$

```
[Out] -1/2*arcsin(x)-1/2*arctanh(x/(-x^2+1)^(1/2))+1/4*ln(-2*x^2+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6874, 266, 399, 222, 385, 213}

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx=-\frac{\arcsin(x)}{2}-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{1-x^2}}\right)+\frac{1}{4}\log\left(1-2x^2\right)$$

```
[In] Int[(x - Sqrt[1 - x^2])^(-1),x]
```

```
[Out] -1/2*ArcSin[x] - ArcTanh[x/Sqrt[1 - x^2]]/2 + Log[1 - 2*x^2]/4
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{x}{-1+2x^2}+\frac{\sqrt{1-x^2}}{-1+2x^2}\right)\,dx\\
&= \int \frac{x}{-1+2x^2}\,dx+\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx\\
&= \frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}\,(-1+2x^2)}\,dx\\
&= -\frac{1}{2}\sin^{-1}(x)+\frac{1}{4}\log\left(1-2x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)\\
&= -\frac{1}{2}\sin^{-1}(x)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)+\frac{1}{4}\log\left(1-2x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx = \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right) + \frac{1}{2}\log\left(-x+\sqrt{1-x^2}\right)$$

```
[In] Integrate[(x - Sqrt[1 - x^2])^(-1),x]
[Out] ArcTan[Sqrt[1 - x^2]/(1 + x)] + Log[-x + Sqrt[1 - x^2]]/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(29) = 58.

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.32

method	result
default	$\frac{\ln(2x^2-1)}{4} + \frac{\sqrt{2}\left(\frac{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{4} - \frac{\sqrt{2}\,\arcsin(x)}{4} - \frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(1-\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}\right)\sqrt{2}}{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{4}\right)}{2} - \frac{\sqrt{2}\left(\sqrt{-4\ldots}\right.}{}$
trager	$\operatorname{RootOf}\left(2\_Z^2+2\_Z+1\right)\ln\left(-\frac{\sqrt{-x^2+1}+x}{2x^2-1}\right) - \ln\left(\frac{-2\operatorname{RootOf}\left(2\_Z^2+2\_Z+1\right)^2 x^2-4\operatorname{RootOf}\left(2\_Z^2+2\_Z\ldots\right.}{2x^2-\ldots}\right.$

```
[In] int(1/(x-(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(2*x^2-1)+1/2*2^(1/2)*(1/4*(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^
(1/2)+2)^(1/2)-1/4*2^(1/2)*arcsin(x)-1/4*2^(1/2)*arctanh((1-(x-1/2*2^(1/2))
*2^(1/2))*2^(1/2)/(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/2))
)-1/2*2^(1/2)*(1/4*(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2)
+1/4*2^(1/2)*arcsin(x)-1/4*2^(1/2)*arctanh(((x+1/2*2^(1/2))*2^(1/2)+1)*2^(1
/2)/(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(29) = 58.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.27

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx = \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + \frac{1}{4}\log\left(2\,x^2-1\right) + \frac{1}{4}\log\left(-\frac{x^2+\sqrt{-x^2+1}(x+1)-x-1}{x^2}\right) - \frac{1}{4}\log\left(-\frac{x^2-\sqrt{-x^2+1}(x-1)+x-1}{x^2}\right)$$

```
[In] integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="fricas")
[Out] arctan((sqrt(-x^2 + 1) - 1)/x) + 1/4*log(2*x^2 - 1) + 1/4*log(-(x^2 + sqrt(
-x^2 + 1)*(x + 1) - x - 1)/x^2) - 1/4*log(-(x^2 - sqrt(-x^2 + 1)*(x - 1) +
x - 1)/x^2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.46

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx = \frac{\log\left(-x+\sqrt{1-x^2}\right)}{2} - \frac{\operatorname{asin}(x)}{2}$$

```
[In] integrate(1/(x-(-x**2+1)**(1/2)),x)
[Out] log(-x + sqrt(1 - x**2))/2 - asin(x)/2
```

Maxima [F]

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx = \int \frac{1}{x-\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(x - sqrt(-x^2 + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(29) = 58$.

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.78

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx = -\frac{1}{4}\,\pi\mathrm{sgn}(x) - \frac{1}{2}\,\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right) + \frac{1}{4}\,\log\left(\left|x+\frac{1}{2}\,\sqrt{2}\right|\right) + \frac{1}{4}\,\log\left(\left|x-\frac{1}{2}\,\sqrt{2}\right|\right) - \frac{1}{4}\,\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}+2\right|\right) + \frac{1}{4}\,\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}-2\right|\right)$$

```
[In] integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*pi*sgn(x) - 1/2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-
x^2 + 1) - 1)) + 1/4*log(abs(x + 1/2*sqrt(2))) + 1/4*log(abs(x - 1/2*sqrt(2
))) - 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x + 2)) +
1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.84

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx = \frac{\ln\left(x-\frac{\sqrt{2}}{2}\right)}{4} + \frac{\ln\left(x+\frac{\sqrt{2}}{2}\right)}{4} - \frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}-1\right)\,1\mathrm{i}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{2}}{2}}\right)}{4} + \frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}+1\right)\,1\mathrm{i}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{2}}{2}}\right)}{4} - \frac{\mathrm{asin}(x)}{2}$$

```
[In] int(1/(x - (1 - x^2)^(1/2)),x)
```

```
[Out] log(x - 2^(1/2)/2)/4 + log(x + 2^(1/2)/2)/4 - log((2^(1/2)*((2^(1/2)*x)/2 -
 1)*1i - (1 - x^2)^(1/2)*1i)/(x - 2^(1/2)/2))/4 + log((2^(1/2)*((2^(1/2)*x)
/2 + 1)*1i + (1 - x^2)^(1/2)*1i)/(x + 2^(1/2)/2))/4 - asin(x)/2
```

3.879 $\int \frac{1}{x-\sqrt{1+2x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{1}{x-\sqrt{1+2x^2}}\,dx = -\sqrt{2}\text{arcsinh}\left(\sqrt{2}x\right)+\text{arctanh}\left(\frac{x}{\sqrt{1+2x^2}}\right)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] arctanh(x/(2*x^2+1)^(1/2))-1/2*ln(x^2+1)-arcsinh(x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6874, 266, 399, 221, 385, 212}

$$\int \frac{1}{x-\sqrt{1+2x^2}}\,dx = -\sqrt{2}\text{arcsinh}\left(\sqrt{2}x\right)+\text{arctanh}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\frac{1}{2}\log\left(x^2+1\right)$$

```
[In] Int[(x - Sqrt[1 + 2*x^2])^(-1),x]
```

```
[Out] -(Sqrt[2]*ArcSinh[Sqrt[2]*x]) + ArcTanh[x/Sqrt[1 + 2*x^2]] - Log[1 + x^2]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{x}{1+x^2}-\frac{\sqrt{1+2x^2}}{1+x^2}\right)\,dx \\
&= -\int \frac{x}{1+x^2}\,dx-\int \frac{\sqrt{1+2x^2}}{1+x^2}\,dx \\
&= -\frac{1}{2}\log\left(1+x^2\right)-2\int \frac{1}{\sqrt{1+2x^2}}\,dx+\int \frac{1}{\left(1+x^2\right)\sqrt{1+2x^2}}\,dx \\
&= -\sqrt{2}\sinh^{-1}\left(\sqrt{2}x\right)-\frac{1}{2}\log\left(1+x^2\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{1+2x^2}}\right) \\
&= -\sqrt{2}\sinh^{-1}\left(\sqrt{2}x\right)+\tanh^{-1}\left(\frac{x}{\sqrt{1+2x^2}}\right)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.70

$$\int \frac{1}{x-\sqrt{1+2x^2}}\,dx = \left(1+\sqrt{2}\right)\log\left(2\left(-1+\sqrt{2}\right)x+\left(-2+\sqrt{2}\right)\sqrt{1+2x^2}\right) - \log\left(-2+\sqrt{2}-2x^2+x\sqrt{2+4x^2}\right)$$

```
[In] Integrate[(x - Sqrt[1 + 2*x^2])^(-1),x]
```

```
[Out] (1 + Sqrt[2])*Log[2*(-1 + Sqrt[2])*x + (-2 + Sqrt[2])*Sqrt[1 + 2*x^2]] - Lo
g[-2 + Sqrt[2] - 2*x^2 + x*Sqrt[2 + 4*x^2]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result
default	$\operatorname{arctanh}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\frac{\ln(x^2+1)}{2}-\operatorname{arcsinh}\left(x\sqrt{2}\right)\sqrt{2}$
trager	$\operatorname{RootOf}\left(\_Z^2-2\_Z-1\right)\ln\left(\frac{x+\sqrt{2x^2+1}}{x^2+1}\right)-\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-2\_Z-1\right)^2x^2+3\operatorname{RootOf}\left(\_Z^2-2\_Z-1\right)\sqrt{2x^2+}\right.}{}\right.$

```
[In] int(1/(x-(2*x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(x/(2*x^2+1)^(1/2))-1/2*ln(x^2+1)-arcsinh(x*2^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(32) = 64$.

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int \frac{1}{x-\sqrt{1+2x^2}}\,dx = \sqrt{2}\log\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)-\frac{1}{2}\log\left(x^2+1\right) - \frac{1}{2}\log\left(\frac{2\,x^2-\sqrt{2\,x^2+1}(x+1)+x+1}{x^2}\right) + \frac{1}{2}\log\left(\frac{2\,x^2+\sqrt{2\,x^2+1}(x-1)-x+1}{x^2}\right)$$

```
[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] sqrt(2)*log(sqrt(2)*x - sqrt(2*x^2 + 1)) - 1/2*log(x^2 + 1) - 1/2*log((2*x^
2 - sqrt(2*x^2 + 1)*(x + 1) + x + 1)/x^2) + 1/2*log((2*x^2 + sqrt(2*x^2 + 1
)*(x - 1) - x + 1)/x^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

$$\int \frac{1}{x-\sqrt{1+2x^2}}\,dx = -\log\left(-x+\sqrt{2x^2+1}\right)-\sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}x\right)$$

```
[In] integrate(1/(x-(2*x**2+1)**(1/2)),x)
[Out] -log(-x + sqrt(2*x**2 + 1)) - sqrt(2)*asinh(sqrt(2)*x)
```

Maxima [F]

$$\int \frac{1}{x-\sqrt{1+2x^2}}\,dx = \int \frac{1}{x-\sqrt{2\,x^2+1}}\,dx$$

```
[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(x - sqrt(2*x^2 + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(32) = 64.

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.20

$$\begin{aligned}\int \frac{1}{x-\sqrt{1+2x^2}}\,dx &= \sqrt{2}\log\left(-\sqrt{2}x+\sqrt{2\,x^2+1}\right)\\ &\quad+\frac{1}{2}\log\left(\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)^2+2\sqrt{2}+3\right)\\ &\quad-\frac{1}{2}\log\left(\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)^2-2\sqrt{2}+3\right)-\frac{1}{2}\log\left(x^2+1\right)\end{aligned}$$

```
[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="giac")
[Out] sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + 1/2*log((sqrt(2)*x - sqrt(2*x^2
 + 1))^2 + 2*sqrt(2) + 3) - 1/2*log((sqrt(2)*x - sqrt(2*x^2 + 1))^2 - 2*sqr
t(2) + 3) - 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

$$\int \frac{1}{x-\sqrt{1+2x^2}}\,dx = -\ln(x-\mathrm{i}) - \frac{\ln\left(x-\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}+\frac{1}{2}\mathrm{i}\right)}{2} + \frac{\ln\left(x+\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}-\frac{1}{2}\mathrm{i}\right)}{2} - \sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\,x\right)$$

```
[In] int(1/(x - (2*x^2 + 1)^(1/2)),x)
```

```
[Out] log(x + (2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1i/2)/2 - log(x - (2^(1/2)*(x^2 + 1
/2)^(1/2))/2 + 1i/2)/2 - log(x - 1i) - 2^(1/2)*asinh(2^(1/2)*x)
```

3.880 $\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 54

$$\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx=-\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log\left(1-x^2\right)$$

```
[Out] -1/4*x^2-1/2*arctanh(x/(-x^2+2)^(1/2))+1/4*ln(-x^2+1)+1/4*x*(-x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6857, 266, 272, 45, 489, 12, 385, 213}

$$\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx=-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{2-x^2}}\right)-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}x+\frac{1}{4}\log\left(1-x^2\right)$$

```
[In] Int[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2),x]
```

```
[Out] -1/4*x^2 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x^2]/
4
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 45</u>

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{x}{-1+x^2} - \frac{x^3}{2\left(-1+x^2\right)} + \frac{x^2\sqrt{2-x^2}}{2\left(-1+x^2\right)}\right) dx$$

$$
\begin{aligned}
&= -\left(\frac{1}{2}\int \frac{x^3}{-1+x^2}\,dx\right) + \frac{1}{2}\int \frac{x^2\sqrt{2-x^2}}{-1+x^2}\,dx + \int \frac{x}{-1+x^2}\,dx \\
&= \frac{1}{4}x\sqrt{2-x^2} + \frac{1}{2}\log\left(1-x^2\right) - \frac{1}{4}\int -\frac{2}{\sqrt{2-x^2}\left(-1+x^2\right)}\,dx - \frac{1}{4}\text{Subst}\left(\int \frac{x}{-1+x}\,dx, x, x^2\right) \\
&= \frac{1}{4}x\sqrt{2-x^2} + \frac{1}{2}\log\left(1-x^2\right) - \frac{1}{4}\text{Subst}\left(\int \left(1+\frac{1}{-1+x}\right)\,dx, x, x^2\right) \\
&\quad + \frac{1}{2}\int \frac{1}{\sqrt{2-x^2}\left(-1+x^2\right)}\,dx \\
&= -\frac{x^2}{4} + \frac{1}{4}x\sqrt{2-x^2} + \frac{1}{4}\log\left(1-x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \frac{x}{\sqrt{2-x^2}}\right) \\
&= -\frac{x^2}{4} + \frac{1}{4}x\sqrt{2-x^2} - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right) + \frac{1}{4}\log\left(1-x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.46

$$
\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx = \frac{1}{2}\left(\frac{1}{2}x\left(-x+\sqrt{2-x^2}\right) - \log\left(-2+\sqrt{4-2x^2}\right) + \log\left(-2+x^2+\sqrt{4-2x^2}+x\left(-\sqrt{2}+\sqrt{2-x^2}\right)\right)\right)
$$

```
[In] Integrate[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2),x]
```

```
[Out] ((x*(-x + Sqrt[2 - x^2]))/2 - Log[-2 + Sqrt[4 - 2*x^2]] + Log[-2 + x^2 + Sq
rt[4 - 2*x^2] + x*(-Sqrt[2] + Sqrt[2 - x^2])])/2
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result
trager	$-\frac{x^2}{4}+\frac{x\sqrt{-x^2+2}}{4}-\frac{\ln\left(-\frac{\sqrt{-x^2+2}+x}{(x-1)(x+1)}\right)}{2}$
default	$\frac{x\sqrt{-x^2+2}}{4}-\frac{\sqrt{-(x+1)^2+2x+3}}{4}+\frac{\operatorname{arctanh}\left(\frac{4+2x}{2\sqrt{-(x+1)^2+2x+3}}\right)}{4}+\frac{\sqrt{-(x-1)^2-2x+3}}{4}-\frac{\operatorname{arctanh}\left(\frac{4-2x}{2\sqrt{-(x-1)^2-2x+3}}\right)}{4}-$

```
[In] int((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^2+1/4*x*(-x^2+2)^(1/2)-1/2*ln(-((-x^2+2)^(1/2)+x)/(x-1)/(x+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int \frac{2x - x^3 + x^2\sqrt{2 - x^2}}{-2 + 2x^2} \, dx = -\frac{1}{4}x^2 + \frac{1}{4}\sqrt{-x^2 + 2}x + \frac{1}{4}\log(x^2 - 1) - \frac{1}{8}\log\left(-\frac{\sqrt{-x^2 + 2}x + 1}{x^2}\right) + \frac{1}{8}\log\left(\frac{\sqrt{-x^2 + 2}x - 1}{x^2}\right)$$

```
[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="fricas")
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) - 1/8*log(-(sqrt(-x^2 +
2)*x + 1)/x^2) + 1/8*log((sqrt(-x^2 + 2)*x - 1)/x^2)
```

Sympy [F]

$$\int \frac{2x - x^3 + x^2\sqrt{2 - x^2}}{-2 + 2x^2} \, dx = -\frac{\int \left(-\frac{2x}{x^2-1}\right) dx + \int \frac{x^3}{x^2-1} \, dx + \int \left(-\frac{x^2\sqrt{2-x^2}}{x^2-1}\right) dx}{2}$$

```
[In] integrate((2*x-x**3+x**2*(-x**2+2)**(1/2))/(2*x**2-2),x)
[Out] -(Integral(-2*x/(x**2 - 1), x) + Integral(x**3/(x**2 - 1), x) + Integral(-x
**2*sqrt(2 - x**2)/(x**2 - 1), x))/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(42) = 84.

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.74

$$\int \frac{2x - x^3 + x^2\sqrt{2 - x^2}}{-2 + 2x^2} \, dx = -\frac{1}{4}x^2 + \frac{1}{4}\sqrt{-x^2 + 2}x + \frac{1}{4}\log(x^2 - 1) + \frac{1}{4}\log\left(\frac{2\sqrt{-x^2 + 2}}{|2x + 2|} + \frac{2}{|2x + 2|} + 1\right) - \frac{1}{4}\log\left(\frac{2\sqrt{-x^2 + 2}}{|2x - 2|} + \frac{2}{|2x - 2|} - 1\right)$$

```
[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="maxima")
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) + 1/4*log(2*sqrt(-x^2 +
2)/abs(2*x + 2) + 2/abs(2*x + 2) + 1) - 1/4*log(2*sqrt(-x^2 + 2)/abs(2*x -
2) + 2/abs(2*x - 2) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(42) = 84$.

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.17

$$\int \frac{2x - x^3 + x^2\sqrt{2 - x^2}}{-2 + 2x^2}\,dx = -\frac{1}{4}\,x^2 + \frac{1}{4}\,\sqrt{-x^2 + 2}x + \frac{1}{4}\,\log\left(\left|x^2 - 1\right|\right) - \frac{1}{4}\,\log\left(\left|\frac{x}{\sqrt{2} - \sqrt{-x^2 + 2}} - \frac{\sqrt{2} - \sqrt{-x^2 + 2}}{x} + 2\right|\right) + \frac{1}{4}\,\log\left(\left|\frac{x}{\sqrt{2} - \sqrt{-x^2 + 2}} - \frac{\sqrt{2} - \sqrt{-x^2 + 2}}{x} - 2\right|\right)$$

```
[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="giac")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(abs(x^2 - 1)) - 1/4*log(abs(x/(sq
rt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x + 2)) + 1/4*log(abs(
x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x - 2))
```

Mupad [B] (verification not implemented)

Time = 21.96 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.59

$$\int \frac{2x - x^3 + x^2\sqrt{2 - x^2}}{-2 + 2x^2}\,dx = \frac{\ln\left(x - 1\right)}{4} + \frac{\ln\left(x + 1\right)}{4} - \frac{\ln\left(\frac{-x\,1\mathrm{i} + \sqrt{2 - x^2}\,1\mathrm{i} + 2\mathrm{i}}{x - 1}\right)}{4} + \frac{\ln\left(\frac{x\,1\mathrm{i} + \sqrt{2 - x^2}\,1\mathrm{i} + 2\mathrm{i}}{x + 1}\right)}{4} + \frac{x\,\sqrt{2 - x^2}}{4} - \frac{x^2}{4}$$

```
[In] int((2*x + x^2*(2 - x^2)^(1/2) - x^3)/(2*x^2 - 2),x)
```

```
[Out] log(x - 1)/4 + log(x + 1)/4 - log(((2 - x^2)^(1/2)*1i - x*1i + 2i)/(x - 1))
/4 + log((x*1i + (2 - x^2)^(1/2)*1i + 2i)/(x + 1))/4 + (x*(2 - x^2)^(1/2))/
4 - x^2/4
```

3.881 $\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 60

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx=-\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)$$

```
[Out] -1/4*x^2-1/2*arctanh(x/(-x^2+2)^(1/2))+1/4*ln(1-x)+1/4*ln(1+x)+1/4*x*(-x^2+
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6874, 201, 222, 711, 399, 385, 213}

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx=-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{2-x^2}}\right)-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}x+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(x+1)$$

```
[In] Int[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]
```

```
[Out] -1/4*x^2 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x]/4
+ Log[1 + x]/4
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\sqrt{2-x^2}}{2}+\frac{2-x^2}{4(-1+x)}+\frac{2-x^2}{4(1+x)}+\frac{\sqrt{2-x^2}}{2\left(-1+x^2\right)}\right) dx \\
&= \frac{1}{4}\int \frac{2-x^2}{-1+x}\,dx+\frac{1}{4}\int \frac{2-x^2}{1+x}\,dx+\frac{1}{2}\int \sqrt{2-x^2}\,dx+\frac{1}{2}\int \frac{\sqrt{2-x^2}}{-1+x^2}\,dx \\
&= \frac{1}{4}x\sqrt{2-x^2}+\frac{1}{4}\int \left(-1+\frac{1}{-1+x}-x\right) dx \\
&\quad +\frac{1}{4}\int \left(1-x+\frac{1}{1+x}\right) dx+\frac{1}{2}\int \frac{1}{\sqrt{2-x^2}\left(-1+x^2\right)}\,dx
\end{aligned}
$$

$$= -\frac{x^2}{4} + \frac{1}{4}x\sqrt{2-x^2} + \frac{1}{4}\log(1-x) + \frac{1}{4}\log(1+x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \frac{x}{\sqrt{2-x^2}}\right)$$
$$= -\frac{x^2}{4} + \frac{1}{4}x\sqrt{2-x^2} - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right) + \frac{1}{4}\log(1-x) + \frac{1}{4}\log(1+x)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx = \frac{1}{4}\left(x\left(-x+\sqrt{2-x^2}\right) - 2\log\left(-2+\sqrt{4-2x^2}\right) + 2\log\left(-2+x^2+\sqrt{4-2x^2}+x\left(-\sqrt{2}+\sqrt{2-x^2}\right)\right)\right)$$

```
[In] Integrate[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]
[Out] (x*(-x + Sqrt[2 - x^2]) - 2*Log[-2 + Sqrt[4 - 2*x^2]] + 2*Log[-2 + x^2 + Sq
rt[4 - 2*x^2] + x*(-Sqrt[2] + Sqrt[2 - x^2])])/4
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

method	result
trager	$-\frac{x^2}{4} + \frac{x\sqrt{-x^2+2}}{4} - \frac{\ln\left(-\frac{\sqrt{-x^2+2}+x}{(x-1)(x+1)}\right)}{2}$
default	$\frac{x\sqrt{-x^2+2}}{4} - \frac{\sqrt{-(x+1)^2+2x+3}}{4} + \frac{\operatorname{arctanh}\left(\frac{4+2x}{2\sqrt{-(x+1)^2+2x+3}}\right)}{4} + \frac{\sqrt{-(x-1)^2-2x+3}}{4} - \frac{\operatorname{arctanh}\left(\frac{4-2x}{2\sqrt{-(x-1)^2-2x+3}}\right)}{4} - \frac{x^2}{4}$

```
[In] int(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2+1/4*x*(-x^2+2)^(1/2)-1/2*ln(-((-x^2+2)^(1/2)+x)/(x-1)/(x+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx = -\frac{1}{4}x^2 + \frac{1}{4}\sqrt{-x^2+2}x + \frac{1}{4}\log\left(x^2-1\right) - \frac{1}{8}\log\left(-\frac{\sqrt{-x^2+2}x+1}{x^2}\right) + \frac{1}{8}\log\left(\frac{\sqrt{-x^2+2}x-1}{x^2}\right)$$

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="fricas")
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) - 1/8*log(-(sqrt(-x^2 +
2)*x + 1)/x^2) + 1/8*log((sqrt(-x^2 + 2)*x - 1)/x^2)
```

Sympy [F]

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx = \int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx$$

```
[In] integrate(x*(-x**2+2)**(1/2)/(x-(-x**2+2)**(1/2)),x)
[Out] Integral(x*sqrt(2 - x**2)/(x - sqrt(2 - x**2)), x)
```

Maxima [F]

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx = \int \frac{\sqrt{-x^2+2}x}{x-\sqrt{-x^2+2}}\,dx$$

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="maxima")
[Out] -1/2*x^2 - integrate(-x^2/(x - sqrt(-x^2 + 2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(46) = 92$.

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.95

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx = -\frac{1}{4}x^2 + \frac{1}{4}\sqrt{-x^2+2}x + \frac{1}{4}\log\left(\left|x^2-1\right|\right) - \frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}} - \frac{\sqrt{2}-\sqrt{-x^2+2}}{x} + 2\right|\right) + \frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}} - \frac{\sqrt{2}-\sqrt{-x^2+2}}{x} - 2\right|\right)$$

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="giac")
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(abs(x^2 - 1)) - 1/4*log(abs(x/(sq
rt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x + 2)) + 1/4*log(abs(
x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x - 2))
```

Mupad [B] (verification not implemented)

Time = 21.71 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx = \frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4} - \frac{\ln\left(\frac{-x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x-1}\right)}{4} + \frac{\ln\left(\frac{x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x+1}\right)}{4} + \frac{x\sqrt{2-x^2}}{4} - \frac{x^2}{4}$$

```
[In] int((x*(2 - x^2)^(1/2))/(x - (2 - x^2)^(1/2)),x)
```

```
[Out] log(x - 1)/4 + log(x + 1)/4 - log(((2 - x^2)^(1/2)*1i - x*1i + 2i)/(x - 1))
/4 + log((x*1i + (2 - x^2)^(1/2)*1i + 2i)/(x + 1))/4 + (x*(2 - x^2)^(1/2))/
4 - x^2/4
```

3.882 $\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx=-\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\text{arctanh}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)$$

```
[Out] -1/2*x+1/2*arctanh((-x^2+2*x)^(1/2))-1/2*ln(1-x)-1/2*(-x^2+2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6874, 699, 702, 213}

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx=\frac{1}{2}\text{arctanh}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\sqrt{2x-x^2}-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

```
[In] Int[x/(-x + Sqrt[2*x - x^2]),x]
[Out] -1/2*x - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 699

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[d*p*((b^2 - 4*a*c)/(b*e*(m + 2*p + 1))), Int[(d + e*x)^m*(a + b*x
```

```
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 702

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}-\frac{1}{2(-1+x)}+\frac{\sqrt{2x-x^2}}{2(1-x)}\right)dx\\
&= -\frac{x}{2}-\frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}\,dx\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}\,dx\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)-2\text{Subst}\left(\int \frac{1}{-4+4x^2}\,dx,x,\sqrt{2x-x^2}\right)\\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx = \frac{1}{2}\Big(i\pi - x - \sqrt{-((-2+x)x)} + \log(-2+x) - 2\log\left(-2+x+\sqrt{-((-2+x)x)}\right)\Big)$$

```
[In] Integrate[x/(-x + Sqrt[2*x - x^2]),x]
[Out] (I*Pi - x - Sqrt[-((-2 + x)*x)] + Log[-2 + x] - 2*Log[-2 + x + Sqrt[-((-2 +
 x)*x)]])/2
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

method	result	size
trager	$\frac{3}{2}-\frac{x}{2}-\frac{\sqrt{-x^{2}+2x}}{2}-\frac{\ln\left(\sqrt{-x^{2}+2x}-1\right)}{2}$	35
default	$-\frac{x}{2}-\frac{\ln(x-1)}{2}-\frac{\sqrt{-(x-1)^{2}+1}}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-(x-1)^{2}+1}}\right)}{2}$	38

```
[In] int(x/(-x+(-x^2+2*x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 3/2-1/2*x-1/2*(-x^2+2*x)^(1/2)-1/2*ln((-x^2+2*x)^(1/2)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx = -\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^{2}+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{x+\sqrt{-x^{2}+2\,x}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{-x^{2}+2\,x}}{x}\right)$$

```
[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="fricas")
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqrt(-x^2 + 2
*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

Sympy [F]

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx = -\int \frac{x}{x-\sqrt{-x^{2}+2x}}\,dx$$

```
[In] integrate(x/(-x+(-x**2+2*x)**(1/2)),x)
[Out] -Integral(x/(x - sqrt(-x**2 + 2*x)), x)
```

Maxima [F]

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx = \int -\frac{x}{x-\sqrt{-x^2+2\,x}}\,dx$$

```
[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="maxima")
[Out] -integrate(x/(x - sqrt(-x^2 + 2*x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx = -\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{-x^2+2\,x} - \frac{1}{2}\,\log\left(-\frac{2\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right) - \frac{1}{2}\,\log\left(|x-1|\right)$$

```
[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="giac")
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x
+ 2)) - 1/2*log(abs(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx = \int -\frac{x}{x-\sqrt{2\,x-x^2}}\,dx$$

```
[In] int(-x/(x - (2*x - x^2)^(1/2)),x)
[Out] int(-x/(x - (2*x - x^2)^(1/2)), x)
```

3.883 $\int \frac{x+\sqrt{2x-x^2}}{2-2x} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{x+\sqrt{2x-x^2}}{2-2x} \, dx = -\frac{x}{2} - \frac{1}{2}\sqrt{2x-x^2} + \frac{1}{2}\text{arctanh}\left(\sqrt{2x-x^2}\right) - \frac{1}{2}\log(1-x)$$

```
[Out] -1/2*x+1/2*arctanh((-x^2+2*x)^(1/2))-1/2*ln(1-x)-1/2*(-x^2+2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6874, 45, 699, 702, 213}

$$\int \frac{x+\sqrt{2x-x^2}}{2-2x} \, dx = \frac{1}{2}\text{arctanh}\left(\sqrt{2x-x^2}\right) - \frac{1}{2}\sqrt{2x-x^2} - \frac{x}{2} - \frac{1}{2}\log(1-x)$$

```
[In] Int[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]
```

```
[Out] -1/2*x - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 699

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[d*p*((b^2 - 4*a*c)/(b*e*(m + 2*p + 1))), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 702

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{x}{2(-1+x)}+\frac{\sqrt{2x-x^2}}{2(1-x)}\right)\,dx \\
&= -\left(\frac{1}{2}\int \frac{x}{-1+x}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}\,dx \\
&= -\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\int \left(1+\frac{1}{-1+x}\right)\,dx+\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}\,dx \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)-2\text{Subst}\left(\int \frac{1}{-4+4x^2}\,dx,x,\sqrt{2x-x^2}\right) \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx = \frac{1}{2}\Big(i\pi - x - \sqrt{-((-2+x)x)} + \log(-2+x) - 2\log\left(-2+x+\sqrt{-((-2+x)x)}\right)\Big)$$

```
[In] Integrate[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]
[Out] (I*Pi - x - Sqrt[-((-2 + x)*x)] + Log[-2 + x] - 2*Log[-2 + x + Sqrt[-((-2 +
 x)*x)]])/2
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

method	result	size
trager	$\frac{3}{2}-\frac{x}{2}-\frac{\sqrt{-x^2+2x}}{2}-\frac{\ln\left(\sqrt{-x^2+2x}-1\right)}{2}$	35
default	$-\frac{x}{2}-\frac{\ln(x-1)}{2}-\frac{\sqrt{-(x-1)^2+1}}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-(x-1)^2+1}}\right)}{2}$	38

```
[In] int((x+(-x^2+2*x)^(1/2))/(-2*x+2),x,method=_RETURNVERBOSE)
[Out] 3/2-1/2*x-1/2*(-x^2+2*x)^(1/2)-1/2*ln((-x^2+2*x)^(1/2)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx = -\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{-x^2+2\,x} - \frac{1}{2}\,\log\left(x-1\right) + \frac{1}{2}\,\log\left(\frac{x+\sqrt{-x^2+2\,x}}{x}\right) - \frac{1}{2}\,\log\left(-\frac{x-\sqrt{-x^2+2\,x}}{x}\right)$$

```
[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="fricas")
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqrt(-x^2 + 2
*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

Sympy [F]

$$\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx = -\frac{\int \frac{x}{x-1}\,dx + \int \frac{\sqrt{-x^2+2x}}{x-1}\,dx}{2}$$

```
[In] integrate((x+(-x**2+2*x)**(1/2))/(2-2*x),x)
[Out] -(Integral(x/(x - 1), x) + Integral(sqrt(-x**2 + 2*x)/(x - 1), x))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx = -\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{-x^2+2\,x} - \frac{1}{2}\,\log\left(x-1\right) + \frac{1}{2}\,\log\left(\frac{2\,\sqrt{-x^2+2\,x}}{|x-1|} + \frac{2}{|x-1|}\right)$$

```
[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="maxima")
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log(2*sqrt(-x^2 + 2*x)
/abs(x - 1) + 2/abs(x - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx = -\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{-x^2+2\,x} - \frac{1}{2}\,\log\left(-\frac{2\,\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right) - \frac{1}{2}\,\log\left(|x-1|\right)$$

```
[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="giac")
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x
+ 2)) - 1/2*log(abs(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx=\int -\frac{x+\sqrt{2\,x-x^2}}{2\,x-2}\,dx$$

```
[In] int(-(x + (2*x - x^2)^(1/2))/(2*x - 2),x)
[Out] int(-(x + (2*x - x^2)^(1/2))/(2*x - 2), x)
```

3.884 $\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x} dx$

Optimal result

Integrand size = 25, antiderivative size = 51

$$\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x} dx = -\frac{x}{2} - \frac{1}{2}\sqrt{2x-x^2} + \frac{1}{2}\text{arctanh}\left(\sqrt{2x-x^2}\right) - \frac{1}{2}\log(1-x)$$

```
[Out] -1/2*x+1/2*arctanh((-x^2+2*x)^(1/2))-1/2*ln(1-x)-1/2*(-x^2+2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6820, 2140, 6874, 45, 699, 702, 213}

$$\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x} dx = \frac{1}{2}\text{arctanh}\left(\sqrt{2x-x^2}\right) - \frac{1}{2}\sqrt{2x-x^2} - \frac{x}{2} - \frac{1}{2}\log(1-x)$$

```
[In] Int[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]
```

```
[Out] -1/2*x - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 699

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[d*p*((b^2 - 4*a*c)/(b*e*(m + 2*p + 1))), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 702

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 2140

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*((g_.) + (h_.)*(x_))^(m_.
), x_Symbol] :> Int[(g + h*x)^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandT
oSum[v, x]])^n, x] /; FreeQ[{f, g, h, j, k, m, n}, x] && LinearQ[u, x] && Q
uadraticQ[v, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[v, x] && (EqQ[j
, 0] || EqQ[f, 1])) && EqQ[(Coefficient[u, x, 1]*g - h*(Coefficient[u, x, 0
] + f*j))^2 - f^2*k^2*(Coefficient[v, x, 2]*g^2 - Coefficient[v, x, 1]*g*h
+ Coefficient[v, x, 0]*h^2), 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x+\sqrt{-((-2+x)x)}}{2-2x}\,dx \\
&= \int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx
\end{aligned}$$

$$= \int \left(-\frac{x}{2(-1+x)} + \frac{\sqrt{2x-x^2}}{2(1-x)}\right) dx$$

$$= -\left(\frac{1}{2}\int \frac{x}{-1+x}\, dx\right) + \frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}\, dx$$

$$= -\frac{1}{2}\sqrt{2x-x^2} - \frac{1}{2}\int \left(1+\frac{1}{-1+x}\right) dx + \frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}\, dx$$

$$= -\frac{x}{2} - \frac{1}{2}\sqrt{2x-x^2} - \frac{1}{2}\log(1-x) - 2\text{Subst}\left(\int \frac{1}{-4+4x^2}\, dx, x, \sqrt{2x-x^2}\right)$$

$$= -\frac{x}{2} - \frac{1}{2}\sqrt{2x-x^2} + \frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right) - \frac{1}{2}\log(1-x)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\, dx = \frac{1}{2}\Big(i\pi - x - \sqrt{-((-2+x)x)} + \log(-2+x) - 2\log\left(-2+x+\sqrt{-((-2+x)x)}\right)\Big)$$

```
[In] Integrate[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]
[Out] (I*Pi - x - Sqrt[-((-2 + x)*x)] + Log[-2 + x] - 2*Log[-2 + x + Sqrt[-((-2 +
 x)*x)]])/2
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\sqrt{2-x}\,\sqrt{x}\,\left(\sqrt{-x(x-2)}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x(x-2)}}\right)\right)}{2\sqrt{-x(x-2)}}-\frac{x}{2}-\frac{\ln(x-1)}{2}$	51

```
[In] int((x+(2-x)^(1/2)*x^(1/2))/(-2*x+2),x,method=_RETURNVERBOSE)
[Out] -1/2*(2-x)^(1/2)*x^(1/2)/(-x*(x-2))^(1/2)*((-x*(x-2))^(1/2)-arctanh(1/(-x*(
x-2))^(1/2)))-1/2*x-1/2*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx = -\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x}\sqrt{-x+2} - \frac{1}{2}\,\log\left(x-1\right) + \frac{1}{2}\,\log\left(\frac{x+\sqrt{x}\sqrt{-x+2}}{x}\right) - \frac{1}{2}\,\log\left(-\frac{x-\sqrt{x}\sqrt{-x+2}}{x}\right)$$

```
[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="fricas")
[Out] -1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(x - 1) + 1/2*log((x + sqrt(x)*s
qrt(-x + 2))/x) - 1/2*log(-(x - sqrt(x)*sqrt(-x + 2))/x)
```

Sympy [F]

$$\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx = -\frac{\int \frac{x}{x-1}\,dx + \int \frac{\sqrt{x}\sqrt{2-x}}{x-1}\,dx}{2}$$

```
[In] integrate((x+(2-x)**(1/2)*x**(1/2))/(2-2*x),x)
[Out] -(Integral(x/(x - 1), x) + Integral(sqrt(x)*sqrt(2 - x)/(x - 1), x))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx = -\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{-x^2+2\,x} - \frac{1}{2}\,\log\left(x-1\right) + \frac{1}{2}\,\log\left(\frac{2\,\sqrt{-x^2+2\,x}}{|x-1|} + \frac{2}{|x-1|}\right)$$

```
[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="maxima")
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log(2*sqrt(-x^2 + 2*x)
/abs(x - 1) + 2/abs(x - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(39) = 78.

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.14

$$\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx = -\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x}\sqrt{-x+2} - \frac{1}{2}\,\log\left(|x-1|\right) + \frac{1}{2}\,\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+2}}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{2}-\sqrt{-x+2}} + 2\right|\right) - \frac{1}{2}\,\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+2}}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{2}-\sqrt{-x+2}} - 2\right|\right)$$

```
[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="giac")
```

```
[Out] -1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(abs(x - 1)) + 1/2*log(abs(-(sqr
t(2) - sqrt(-x + 2))/sqrt(x) + sqrt(x)/(sqrt(2) - sqrt(-x + 2)) + 2)) - 1/2
*log(abs(-(sqrt(2) - sqrt(-x + 2))/sqrt(x) + sqrt(x)/(sqrt(2) - sqrt(-x + 2
)) - 2))
```

Mupad [B] (verification not implemented)

Time = 24.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx = \operatorname{atanh}\left(\frac{\sqrt{x}\,\left(\sqrt{2}-\sqrt{2-x}\right)}{x+\sqrt{2}\,\sqrt{2-x}-2}\right) - \frac{\ln\left(x-1\right)}{2} - \frac{x}{2} - \frac{\sqrt{x}\,\sqrt{2-x}}{2}$$

```
[In] int(-(x + x^(1/2)*(2 - x)^(1/2))/(2*x - 2),x)
```

```
[Out] atanh((x^(1/2)*(2^(1/2) - (2 - x)^(1/2)))/(x + 2^(1/2)*(2 - x)^(1/2) - 2))
- log(x - 1)/2 - x/2 - (x^(1/2)*(2 - x)^(1/2))/2
```

3.885 $\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}} dx$

Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}} dx = -\frac{1}{2}\sqrt{2-x}\sqrt{x} - \frac{x}{2} + \frac{1}{2}\text{arctanh}\left(\sqrt{2-x}\sqrt{x}\right) - \frac{1}{2}\log(1-x)$$

```
[Out] -1/2*x+1/2*arctanh((2-x)^(1/2)*x^(1/2))-1/2*ln(1-x)-1/2*(2-x)^(1/2)*x^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2130, 103, 12, 94, 212, 45}

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}} dx = \frac{1}{2}\text{arctanh}\left(\sqrt{2-x}\sqrt{x}\right) - \frac{x}{2} - \frac{1}{2}\sqrt{2-x}\sqrt{x} - \frac{1}{2}\log(1-x)$$

```
[In] Int[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]),x]
```

```
[Out] -1/2*(Sqrt[2 - x]*Sqrt[x]) - x/2 + ArcTanh[Sqrt[2 - x]*Sqrt[x]]/2 - Log[1 -
 x]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2130

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[e, Int[(u*Sqrt[a + b*x])/(a*e^2 - c*f^2 + (b*e^2 - d*f^2
)*x), x], x] - Dist[f, Int[(u*Sqrt[c + d*x])/(a*e^2 - c*f^2 + (b*e^2 - d*f^
2)*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a*e^2 - c*f^2, 0] && N
eQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{2-x}\sqrt{x}}{2-2x}\,dx + \int \frac{x}{2-2x}\,dx \\
&= -\frac{1}{2}\sqrt{2-x}\sqrt{x} + \frac{1}{2}\int \frac{2}{(2-2x)\sqrt{2-x}\sqrt{x}}\,dx + \int \left(-\frac{1}{2} - \frac{1}{2(-1+x)}\right)dx \\
&= -\frac{1}{2}\sqrt{2-x}\sqrt{x} - \frac{x}{2} - \frac{1}{2}\log(1-x) + \int \frac{1}{(2-2x)\sqrt{2-x}\sqrt{x}}\,dx \\
&= -\frac{1}{2}\sqrt{2-x}\sqrt{x} - \frac{x}{2} - \frac{1}{2}\log(1-x) + 2\text{Subst}\left(\int \frac{1}{4-4x^2}\,dx, x, \sqrt{2-x}\sqrt{x}\right)
\end{aligned}$$

$$= -\frac{1}{2}\sqrt{2-x}\sqrt{x} - \frac{x}{2} + \frac{1}{2}\tanh^{-1}\left(\sqrt{2-x}\sqrt{x}\right) - \frac{1}{2}\log(1-x)$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx = -\frac{x}{2} - \frac{1}{2}\sqrt{-((-2+x)x)} + \log\left(-2+\sqrt{4-2x}\right) - \log\left(-2+\sqrt{4-2x}-\sqrt{2}\sqrt{x}+x+\sqrt{-((-2+x)x)}\right)$$

```
[In] Integrate[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]),x]
```

```
[Out] -1/2*x - Sqrt[-((-2 + x)*x)]/2 + Log[-2 + Sqrt[4 - 2*x]] - Log[-2 + Sqrt[4
- 2*x] - Sqrt[2]*Sqrt[x] + x + Sqrt[-((-2 + x)*x)]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{\sqrt{2-x}\,\sqrt{x}\left(\sqrt{-x(x-2)}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x(x-2)}}\right)\right)}{2\sqrt{-x(x-2)}}-\frac{x}{2}-\frac{\ln(x-1)}{2}$	51

```
[In] int(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2-x)^(1/2)*x^(1/2)/(-x*(x-2))^(1/2)*((-x*(x-2))^(1/2)-arctanh(1/(-x*(
x-2))^(1/2)))-1/2*x-1/2*ln(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx = -\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x}\sqrt{-x+2} - \frac{1}{2}\,\log\left(x-1\right) + \frac{1}{2}\,\log\left(\frac{x+\sqrt{x}\sqrt{-x+2}}{x}\right) - \frac{1}{2}\,\log\left(-\frac{x-\sqrt{x}\sqrt{-x+2}}{x}\right)$$

```
[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(x - 1) + 1/2*log((x + sqrt(x)*s
qrt(-x + 2))/x) - 1/2*log(-(x - sqrt(x)*sqrt(-x + 2))/x)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx = \int \frac{\sqrt{x}}{-\sqrt{x}+\sqrt{2-x}}\,dx$$

```
[In] integrate(x**(1/2)/((2-x)**(1/2)-x**(1/2)),x)
[Out] Integral(sqrt(x)/(-sqrt(x) + sqrt(2 - x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx = \int -\frac{\sqrt{x}}{\sqrt{x}-\sqrt{-x+2}}\,dx$$

```
[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="maxima")
[Out] -integrate(sqrt(x)/(sqrt(x) - sqrt(-x + 2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(38) = 76$.

Time = 0.38 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx = -\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x}\sqrt{-x+2} - \frac{1}{2}\,\log\left(|x-1|\right) + \frac{1}{2}\,\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+2}}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{2}-\sqrt{-x+2}} + 2\right|\right) - \frac{1}{2}\,\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+2}}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{2}-\sqrt{-x+2}} - 2\right|\right)$$

```
[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="giac")
[Out] -1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(abs(x - 1)) + 1/2*log(abs(-(sqr
t(2) - sqrt(-x + 2))/sqrt(x) + sqrt(x)/(sqrt(2) - sqrt(-x + 2)) + 2)) - 1/2
*log(abs(-(sqrt(2) - sqrt(-x + 2))/sqrt(x) + sqrt(x)/(sqrt(2) - sqrt(-x + 2
)) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx = \operatorname{atanh}\left(\frac{\sqrt{x}\,\left(\sqrt{2}-\sqrt{2-x}\right)}{x+\sqrt{2}\,\sqrt{2-x}-2}\right)-\frac{\ln\left(x-1\right)}{2}-\frac{x}{2}-\frac{\sqrt{x}\,\sqrt{2-x}}{2}$$

```
[In] int(x^(1/2)/((2 - x)^(1/2) - x^(1/2)),x)
[Out] atanh((x^(1/2)*(2^(1/2) - (2 - x)^(1/2)))/(x + 2^(1/2)*(2 - x)^(1/2) - 2))
- log(x - 1)/2 - x/2 - (x^(1/2)*(2 - x)^(1/2))/2
```

3.886 $\int \frac{1}{((1+x)(-1+x^2))^{2/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{1}{((1+x)(-1+x^2))^{2/3}}\,dx = -\frac{3(1-x^2)}{2(-((1+x)(1-x^2)))^{2/3}}$$

```
[Out] -3/2*(-x^2+1)/(-(1+x)*(-x^2+1))^(2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2092, 2089, 37}

$$\int \frac{1}{((1+x)(-1+x^2))^{2/3}}\,dx = -\frac{3(1-x)(x+1)}{2(x^3+x^2-x-1)^{2/3}}$$

```
[In] Int[((1 + x)*(-1 + x^2))^(-2/3),x]
```

```
[Out] (-3*(1 - x)*(1 + x))/(2*(-1 - x + x^2 + x^3)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2089

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
```

```
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\left(-\frac{16}{27}-\frac{4x}{3}+x^3\right)^{2/3}}\,dx, x, \frac{1}{3}+x\right)$$

$$= \frac{\left(32\sqrt[3]{2}(-1-x)^{4/3}(-1+x)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{16}{9}-\frac{8x}{3}\right)^{4/3}\left(-\frac{16}{9}+\frac{4x}{3}\right)^{2/3}}\,dx, x, \frac{1}{3}+x\right)}{9\left(-1-x+x^2+x^3\right)^{2/3}}$$

$$= -\frac{3(1-x)(1+x)}{2\left(-1-x+x^2+x^3\right)^{2/3}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}}\,dx = \frac{3(-1+x)(1+x)}{2\left((-1+x)(1+x)^2\right)^{2/3}}$$

```
[In] Integrate[((1 + x)*(-1 + x^2))^(-2/3),x]
[Out] (3*(-1 + x)*(1 + x))/(2*((-1 + x)*(1 + x)^2)^(2/3))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{3(x-1)(x+1)}{2((x+1)(x^2-1))^{\frac{2}{3}}}$	20
risch	$\frac{3(x+1)(x-1)}{2\left((x+1)^2(x-1)\right)^{\frac{2}{3}}}$	20
trager	$\frac{3(x^3+x^2-x-1)^{\frac{1}{3}}}{2(x+1)}$	21

```
[In] int(1/((x+1)*(x^2-1))^(2/3),x,method=_RETURNVERBOSE)
[Out] 3/2*(x-1)*(x+1)/((x+1)*(x^2-1))^(2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{((1+x)(-1+x^2))^{2/3}}\,dx = \frac{3(x^3+x^2-x-1)^{\frac{1}{3}}}{2(x+1)}$$

```
[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="fricas")
[Out] 3/2*(x^3 + x^2 - x - 1)^(1/3)/(x + 1)
```

Sympy [F]

$$\int \frac{1}{((1+x)(-1+x^2))^{2/3}}\,dx = \int \frac{1}{((x+1)(x^2-1))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/((1+x)*(x**2-1))**(2/3),x)
[Out] Integral(((x + 1)*(x**2 - 1))**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{((1+x)(-1+x^2))^{2/3}}\,dx = \int \frac{1}{((x^2-1)(x+1))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="maxima")
[Out] integrate(((x^2 - 1)*(x + 1))^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}}\,dx = \int \frac{1}{\left((x^2-1)(x+1)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="giac")
[Out] integrate(((x^2 - 1)*(x + 1))^(-2/3), x)
```

Mupad [B] (verification not implemented)

Time = 23.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}}\,dx = \frac{3\left((x^2-1)\,(x+1)\right)^{1/3}}{2\,(x+1)}$$

```
[In] int(1/((x^2 - 1)*(x + 1))^(2/3),x)
[Out] (3*((x^2 - 1)*(x + 1))^(1/3))/(2*(x + 1))
```

3.887 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 14

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx = -\frac{2x}{\sqrt{x(1+x^2)}}$$

```
[Out] -2*x/(x*(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6851, 460}

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx = -\frac{2x}{\sqrt{x(x^2+1)}}$$

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]),x]
```

```
[Out] (-2*x)/Sqrt[x*(1 + x^2)]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
```

```
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}}\,dx}{\sqrt{x\left(1+x^2\right)}}\\ &= -\frac{2x}{\sqrt{x\left(1+x^2\right)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt{x\left(1+x^2\right)}}\,dx = -\frac{2x}{\sqrt{x+x^3}}$$

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]),x]
[Out] (-2*x)/Sqrt[x + x^3]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{2x}{\sqrt{x(x^2+1)}}$	13
default	$-\frac{2x}{\sqrt{x(x^2+1)}}$	13
risch	$-\frac{2x}{\sqrt{x(x^2+1)}}$	13
elliptic	$-\frac{2x}{\sqrt{x(x^2+1)}}$	13
pseudoelliptic	$-\frac{2x}{\sqrt{x(x^2+1)}}$	13
trager	$-\frac{2\sqrt{x^3+x}}{x^2+1}$	17
meijerg	$\frac{2x^{\frac{5}{2}}\,{}_2F_1\left(\frac{5}{4},\frac{3}{2};\frac{9}{4};-x^2\right)}{5}-2\sqrt{x}\,{}_2F_1\left(\frac{1}{4},\frac{3}{2};\frac{5}{4};-x^2\right)$	34

```
[In] int((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*x/(x*(x^2+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx = -\frac{2\sqrt{x^3+x}}{x^2+1}$$

```
[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(x^3 + x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx = \int \frac{(x-1)(x+1)}{\sqrt{x(x^2+1)}(x^2+1)}\,dx$$

```
[In] integrate((x**2-1)/(x**2+1)/(x*(x**2+1))**(1/2),x)
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx = \int \frac{x^2-1}{\sqrt{(x^2+1)x}(x^2+1)}\,dx$$

```
[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx = \int \frac{x^2-1}{\sqrt{(x^2+1)x}(x^2+1)}\,dx$$

```
[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="giac")
[Out] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 22.46 (sec) , antiderivative size = 138, normalized size of antiderivative = 9.86

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx = -\frac{2x}{\sqrt{x^3+x}} - \frac{\sqrt{1-x\,1\mathrm{i}}\,\sqrt{\frac{1}{2}+\frac{x\,1\mathrm{i}}{2}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{1-x\,1\mathrm{i}}\right)\middle|\frac{1}{2}\right)\sqrt{x\,1\mathrm{i}}\,2\mathrm{i}}{\sqrt{x^3+x}} + \frac{\sqrt{1-x\,1\mathrm{i}}\,\sqrt{\frac{1}{2}+\frac{x\,1\mathrm{i}}{2}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{1-x\,1\mathrm{i}}\right)\middle|\frac{1}{2}\right)\sqrt{x\,1\mathrm{i}}\,2\mathrm{i}}{\sqrt{x^3+x}} - \frac{\sqrt{1-x\,1\mathrm{i}}\,\sqrt{1+x\,1\mathrm{i}}\,\sqrt{-x\,1\mathrm{i}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-x\,1\mathrm{i}}\right)\middle|-1\right)\,1\mathrm{i}}{\sqrt{x^3+x}}$$

```
[In] int((x^2 - 1)/((x*(x^2 + 1))^(1/2)*(x^2 + 1)),x)
```

```
[Out] ((1 - x*1i)^(1/2)*((x*1i)/2 + 1/2)^(1/2)*ellipticF(asin((1 - x*1i)^(1/2)),
1/2)*(x*1i)^(1/2)*2i)/(x + x^3)^(1/2) - ((1 - x*1i)^(1/2)*((x*1i)/2 + 1/2)^
(1/2)*ellipticE(asin((1 - x*1i)^(1/2)), 1/2)*(x*1i)^(1/2)*2i)/(x + x^3)^(1/
2) - (2*x)/(x + x^3)^(1/2) - ((1 - x*1i)^(1/2)*(x*1i + 1)^(1/2)*(-x*1i)^(1/
2)*ellipticE(asin((-x*1i)^(1/2)), -1)*1i)/(x + x^3)^(1/2)
```

3.888 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 12

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx = -\frac{2x}{\sqrt{x+x^3}}$$

```
[Out] -2*x/(x^3+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2081, 460}

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx = -\frac{2x}{\sqrt{x^3+x}}$$

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
```

```
[Out] (-2*x)/Sqrt[x + x^3]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
```

```
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}}\,dx}{\sqrt{x+x^3}}$$
$$= -\frac{2x}{\sqrt{x+x^3}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx = -\frac{2x}{\sqrt{x+x^3}}$$

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
[Out] (-2*x)/Sqrt[x + x^3]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{2x}{\sqrt{x^3+x}}$	11
default	$-\frac{2x}{\sqrt{x(x^2+1)}}$	13
risch	$-\frac{2x}{\sqrt{x(x^2+1)}}$	13
elliptic	$-\frac{2x}{\sqrt{x(x^2+1)}}$	13
pseudoelliptic	$-\frac{2x}{\sqrt{x(x^2+1)}}$	13
trager	$-\frac{2\sqrt{x^3+x}}{x^2+1}$	17
meijerg	$\frac{2x^{\frac{5}{2}}\,{}_2F_1\left(\frac{5}{4},\frac{3}{2};\frac{9}{4};-x^2\right)}{5}-2\sqrt{x}\,{}_2F_1\left(\frac{1}{4},\frac{3}{2};\frac{5}{4};-x^2\right)$	34

```
[In] int((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*x/(x^3+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx = -\frac{2\sqrt{x^3+x}}{x^2+1}$$

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(x^3 + x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx = \int \frac{(x-1)(x+1)}{\sqrt{x(x^2+1)}\,(x^2+1)}\,dx$$

```
[In] integrate((x**2-1)/(x**2+1)/(x**3+x)**(1/2),x)
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx = \int \frac{x^2-1}{\sqrt{x^3+x}(x^2+1)}\,dx$$

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx = \int \frac{x^2-1}{\sqrt{x^3+x}(x^2+1)}\,dx$$

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx = -\frac{2\,x}{\sqrt{x^3+x}}$$

```
[In] int((x^2 - 1)/((x^2 + 1)*(x + x^3)^(1/2)),x)
[Out] -(2*x)/(x + x^3)^(1/2)
```

3.889 $\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2} dx = \frac{2x\sqrt{\frac{(1-x^2)^2}{x(1+x^2)}}}{1-x^2}$$

```
[Out] 2*x*((-x^2+1)^2/x/(x^2+1))^(1/2)/(-x^2+1)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {6850, 460}

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2} dx = \frac{2x\sqrt{\frac{(1-x^2)^2}{x(x^2+1)}}}{1-x^2}$$

```
[In] Int[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2),x]
[Out] (2*x*Sqrt[(1 - x^2)^2/(x*(1 + x^2))])/(1 - x^2)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 6850

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
a^IntPart[p]*((a*v^m*w^n*z^q)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[
p])*z^(q*FracPart[p]))), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{x}\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}\sqrt{1+x^2}\right)\int\frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}}\,dx}{-1+x^2}$$
$$= \frac{2x\sqrt{\frac{(1-x^2)^2}{x(1+x^2)}}}{1-x^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int\frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx = -\frac{2x\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{-1+x^2}$$

```
[In] Integrate[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2),x]
[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{2\sqrt{\frac{\left(x^2-1\right)^2}{x\left(x^2+1\right)}}\,x}{x^2-1}$	31
risch	$-\frac{2\sqrt{\frac{\left(x^2-1\right)^2}{x\left(x^2+1\right)}}\,x}{x^2-1}$	31
gosper	$-\frac{2x\sqrt{\frac{\left(x^2-1\right)^2}{x\left(x^2+1\right)}}}{(x-1)(x+1)}$	34
trager	$-\frac{2x\sqrt{-\frac{-x^4+2x^2-1}{x^3+x}}}{x^2-1}$	34

```
[In] int(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -2*((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2-1)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx = -\frac{2\,x\sqrt{\frac{x^4-2\,x^2+1}{x^3+x}}}{x^2-1}$$

```
[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] -2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{(x-1)^2(x+1)^2}{x^3+x}}}{x^2+1}\,dx$$

```
[In] integrate(((x**2-1)**2/x/(x**2+1))**(1/2)/(x**2+1),x)
[Out] Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{(x^2-1)^2}{(x^2+1)x}}}{x^2+1}\,dx$$

```
[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{(x^2-1)^2}{(x^2+1)x}}}{x^2+1}\,dx$$

```
[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="giac")
[Out] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 22.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx = -\frac{(2\,x^3+2\,x)\,\sqrt{\frac{1}{x^2+1}}\,\sqrt{(x^2-1)^2}\,\sqrt{\frac{1}{x}}}{(x^2-1)\,(x^2+1)}$$

```
[In] int(((x^2 - 1)^2/(x*(x^2 + 1)))^(1/2)/(x^2 + 1),x)
[Out] -((2*x + 2*x^3)*(1/(x^2 + 1))^(1/2)*((x^2 - 1)^2)^(1/2)*(1/x)^(1/2))/((x^2
- 1)*(x^2 + 1))
```

3.890 $\int \frac{\sqrt{\frac{\left(-1+x^2\right)^2}{x+x^3}}}{1+x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 33

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx=\frac{2x\sqrt{\frac{(1-x^2)^2}{x+x^3}}}{1-x^2}$$

```
[Out] 2*x*((-x^2+1)^2/(x^3+x))^(1/2)/(-x^2+1)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6851, 2081, 460}

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx=\frac{2x\sqrt{\frac{(1-x^2)^2}{x^3+x}}}{1-x^2}$$

```
[In] Int[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2),x]
[Out] (2*x*Sqrt[(1 - x^2)^2/(x + x^3)])/(1 - x^2)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{\frac{(-1+x^2)^2}{x+x^3}}\sqrt{x+x^3}\right)\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx}{-1+x^2}\\ &= \frac{\left(\sqrt{x}\sqrt{1+x^2}\sqrt{\frac{(-1+x^2)^2}{x+x^3}}\right)\int \frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}}\,dx}{-1+x^2}\\ &= \frac{2x\sqrt{\frac{(1-x^2)^2}{x+x^3}}}{1-x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx = -\frac{2x\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{-1+x^2}$$

```
[In] Integrate[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2),x]
[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{2\sqrt{\frac{\left(x^2-1\right)^2}{x\left(x^2+1\right)}}\, x}{x^2-1}$	31
risch	$-\frac{2\sqrt{\frac{\left(x^2-1\right)^2}{x\left(x^2+1\right)}}\, x}{x^2-1}$	31
gosper	$-\frac{2x\sqrt{\frac{\left(x^2-1\right)^2}{x\left(x^2+1\right)}}}{(x-1)(x+1)}$	34
trager	$-\frac{2x\sqrt{-\frac{-x^4+2x^2-1}{x^3+x}}}{x^2-1}$	34

```
[In] int(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -2*((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2-1)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx = -\frac{2\,x\sqrt{\frac{x^4-2\,x^2+1}{x^3+x}}}{x^2-1}$$

```
[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] -2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{(x-1)^2(x+1)^2}{x^3+x}}}{x^2+1}\,dx$$

```
[In] integrate(((x**2-1)**2/(x**3+x))**(1/2)/(x**2+1),x)
[Out] Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2+1}\,dx$$

```
[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2+1}\,dx$$

```
[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="giac")
[Out] integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx = -\frac{\sqrt{\frac{1}{x^3+x}}\,(2\,x^3+2\,x)\,\sqrt{(x^2-1)^2}}{(x^2-1)\,(x^2+1)}$$

```
[In] int(((x^2 - 1)^2/(x + x^3))^(1/2)/(x^2 + 1),x)
[Out] -((1/(x + x^3))^(1/2)*(2*x + 2*x^3)*((x^2 - 1)^2)^(1/2))/((x^2 - 1)*(x^2 +
1))
```

3.891 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{b+ax^2}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{b+ax^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,x}$$

```
[Out] arctanh(d^(1/2)*(a*x^2+b)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(a*x^2+b)^(1/2)/x/
a^(1/2)/d^(1/2)/(a+b/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {446, 455, 65, 223, 212}

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{ax^2+b}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{ax^2+b}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}x\sqrt{a+\frac{b}{x^2}}}$$

```
[In] Int[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b + a*x^2]*ArcTanh[(Sqrt[d]*Sqrt[b + a*x^2])/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(Sqrt[a]*Sqrt[d]*Sqrt[a + b/x^2]*x)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart[
q]), Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{b+ax^2}\int \frac{x}{\sqrt{b+ax^2}\sqrt{c+dx^2}}\,dx}{\sqrt{a+\frac{b}{x^2}}x}$$

$$= \frac{\sqrt{b+ax^2}\text{Subst}\left(\int \frac{1}{\sqrt{b+ax}\sqrt{c+dx}}\,dx, x, x^2\right)}{2\sqrt{a+\frac{b}{x^2}}x}$$

$$= \frac{\sqrt{b+ax^2}\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{bd}{a}+\frac{dx^2}{a}}}\,dx, x, \sqrt{b+ax^2}\right)}{a\sqrt{a+\frac{b}{x^2}}x}$$

$$= \frac{\sqrt{b+ax^2}\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{a}}\,dx, x, \frac{\sqrt{b+ax^2}}{\sqrt{c+dx^2}}\right)}{a\sqrt{a+\frac{b}{x^2}}x}$$

$$= \frac{\sqrt{b + ax^2} \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{b+ax^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\sqrt{a + \frac{b}{x^2}}x}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a + \frac{b}{x^2}}\sqrt{c + dx^2}}\,dx = \frac{\sqrt{b + ax^2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{b+ax^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\sqrt{a + \frac{b}{x^2}}x}$$

```
[In] Integrate[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b + a*x^2]*ArcTanh[(Sqrt[d]*Sqrt[b + a*x^2])/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(Sqrt[a]*Sqrt[d]*Sqrt[a + b/x^2]*x)
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.47

method	result	size
default	$\frac{\left(a\,x^{2}+b\right)\ln\left(\frac{2 a d\,x^{2}+2\sqrt{\left(a\,x^{2}+b\right)\left(d\,x^{2}+c\right)}\,\sqrt{a d}+a c+b d}{2\sqrt{a d}}\right)\sqrt{d\,x^{2}+c}}{2\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x\sqrt{a d}\,\sqrt{\left(a\,x^{2}+b\right)\left(d\,x^{2}+c\right)}}$	103

```
[In] int(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*x^2+b)/x^2)^(1/2)/x*(a*x^2+b)*ln(1/2*(2*a*d*x^2+2*((a*x^2+b)*(d*x^2
+c))^(1/2)*(a*d)^(1/2)+a*c+b*d)/(a*d)^(1/2))*(d*x^2+c)^(1/2)/(a*d)^(1/2)/((
a*x^2+b)*(d*x^2+c))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.97

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx = \left[\frac{\sqrt{ad}\log\left(8\,a^2d^2x^4+a^2c^2+6\,abcd+b^2d^2+8\,(a^2cd+abd^2)x^2+4\,(2\,adx^3+(ac+bd)x)\sqrt{dx^2+c}\sqrt{ad}\sqrt{}\right.}{4\,ad}\right.$$
$$\left. -\frac{\sqrt{-ad}\arctan\left(\frac{(2\,adx^3+(ac+bd)x)\sqrt{dx^2+c}\sqrt{-ad}\sqrt{\frac{ax^2+b}{x^2}}}{2\,(a^2d^2x^4+abcd+(a^2cd+abd^2)x^2)}\right)}{2\,ad}\right]$$

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a*d)*log(8*a^2*d^2*x^4 + a^2*c^2 + 6*a*b*c*d + b^2*d^2 + 8*(a^2*c
*d + a*b*d^2)*x^2 + 4*(2*a*d*x^3 + (a*c + b*d)*x)*sqrt(d*x^2 + c)*sqrt(a*d)
*sqrt((a*x^2 + b)/x^2))/(a*d), -1/2*sqrt(-a*d)*arctan(1/2*(2*a*d*x^3 + (a*c
 + b*d)*x)*sqrt(d*x^2 + c)*sqrt(-a*d)*sqrt((a*x^2 + b)/x^2)/(a^2*d^2*x^4 +
a*b*c*d + (a^2*c*d + a*b*d^2)*x^2))/(a*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx$$

```
[In] integrate(1/(a+b/x**2)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b/x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{dx^2+c}\sqrt{a+\frac{b}{x^2}}}\,dx$$

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(a + b/x^2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx = \frac{a\log\left(\left|-\sqrt{ad}\sqrt{b}+\sqrt{a^2c}\right|\right)\operatorname{sgn}(x)}{\sqrt{ad}|a|} - \frac{a\log\left(\left|-\sqrt{ax^2+b}\sqrt{ad}+\sqrt{a^2c+(ax^2+b)ad-abd}\right|\right)}{\sqrt{ad}|a|\operatorname{sgn}(x)}$$

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] a*log(abs(-sqrt(a*d)*sqrt(b) + sqrt(a^2*c)))*sgn(x)/(sqrt(a*d)*abs(a)) - a*
log(abs(-sqrt(a*x^2 + b)*sqrt(a*d) + sqrt(a^2*c + (a*x^2 + b)*a*d - a*b*d))
)/(sqrt(a*d)*abs(a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,\sqrt{d\,x^2+c}}\,dx$$

```
[In] int(1/((a + b/x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b/x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.892 $\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 83

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = \frac{2\sqrt{-2x^2+x^4}\arctan\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} - \frac{\sqrt{-2x^2+x^4}\arctan\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

```
[Out] 2/3*arctan(1/2*(x^2-2)^(1/2))*(x^4-2*x^2)^(1/2)/x/(x^2-2)^(1/2)-1/3*arctan(
(x^2-2)^(1/2))*(x^4-2*x^2)^(1/2)/x/(x^2-2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2081, 585, 85, 65, 209}

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = \frac{2\sqrt{x^4-2x^2}\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}} - \frac{\sqrt{x^4-2x^2}\arctan\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}$$

```
[In] Int[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)),x]
```

```
[Out] (2*Sqrt[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]/2])/(3*x*Sqrt[-2 + x^2]) - (Sqr
t[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{-2x^2+x^4}\int \frac{x\sqrt{-2+x^2}}{(-1+x^2)(2+x^2)}\,dx}{x\sqrt{-2+x^2}}\\
&= \frac{\sqrt{-2x^2+x^4}\text{Subst}\left(\int \frac{\sqrt{-2+x}}{(-1+x)(2+x)}\,dx,x,x^2\right)}{2x\sqrt{-2+x^2}}\\
&= -\frac{\sqrt{-2x^2+x^4}\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}(-1+x)}\,dx,x,x^2\right)}{6x\sqrt{-2+x^2}}\\
&\quad+\frac{\left(2\sqrt{-2x^2+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}(2+x)}\,dx,x,x^2\right)}{3x\sqrt{-2+x^2}}
\end{aligned}$$

$$= -\frac{\sqrt{-2x^2+x^4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{\left(4\sqrt{-2x^2+x^4}\right)\text{Subst}\left(\int \frac{1}{4+x^2}\,dx, x, \sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

$$= \frac{2\sqrt{-2x^2+x^4}\tan^{-1}\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} - \frac{\sqrt{-2x^2+x^4}\tan^{-1}\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = \frac{x\sqrt{-2+x^2}\left(2\arctan\left(\frac{1}{2}\sqrt{-2+x^2}\right) - \arctan\left(\sqrt{-2+x^2}\right)\right)}{3\sqrt{x^2(-2+x^2)}}$$

```
[In] Integrate[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)),x]
[Out] (x*Sqrt[-2 + x^2]*(2*ArcTan[Sqrt[-2 + x^2]/2] - ArcTan[Sqrt[-2 + x^2]]))/(3
*Sqrt[x^2*(-2 + x^2)])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{i\left(-i\arctan\left(\frac{1}{\sqrt{x^2-2}}\right)+\operatorname{arctanh}\left(\frac{(i\sqrt{2}-x)\sqrt{2}}{2\sqrt{x^2-2}}\right)+\operatorname{arctanh}\left(\frac{(x+i\sqrt{2})\sqrt{2}}{2\sqrt{x^2-2}}\right)\right)}{3}$
default	$\frac{\sqrt{x^4-2x^2}\left(\arctan\left(\frac{x+2}{\sqrt{x^2-2}}\right)-\arctan\left(\frac{x-2}{\sqrt{x^2-2}}\right)+4\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{6x\sqrt{x^2-2}}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^7-15\operatorname{RootOf}\left(\_Z^2+1\right)x^5+6\sqrt{x^4-2x^2}\,x^4+24\operatorname{RootOf}\left(\_Z^2+1\right)x^3-16\sqrt{x^4-2x^2}\,x}{\left(x^2+2\right)^2(x-1)(x+1)x}\right.}{6}$

```
[In] int((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x,method=_RETURNVERBOSE)
[Out] 1/3*I*(-I*arctan(1/(x^2-2)^(1/2))+arctanh(1/2*(I*2^(1/2)-x)*2^(1/2)/(x^2-2)
^(1/2))+arctanh(1/2*(x+I*2^(1/2))*2^(1/2)/(x^2-2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = -\frac{1}{3}\arctan\left(\frac{\sqrt{x^4-2\,x^2}}{x}\right)+\frac{2}{3}\arctan\left(\frac{\sqrt{x^4-2\,x^2}}{2\,x}\right)$$

```
[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="fricas")
[Out] -1/3*arctan(sqrt(x^4 - 2*x^2)/x) + 2/3*arctan(1/2*sqrt(x^4 - 2*x^2)/x)
```

Sympy [F]

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = \int \frac{\sqrt{x^2(x^2-2)}}{(x-1)(x+1)(x^2+2)}\,dx$$

```
[In] integrate((x**4-2*x**2)**(1/2)/(x**2-1)/(x**2+2),x)
[Out] Integral(sqrt(x**2*(x**2 - 2))/((x - 1)*(x + 1)*(x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = \int \frac{\sqrt{x^4-2\,x^2}}{(x^2+2)(x^2-1)}\,dx$$

```
[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 - 2*x^2)/((x^2 + 2)*(x^2 - 1)), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = \frac{1}{3}\left(\arctan\left(i\sqrt{2}\right)-2\arctan\left(\frac{1}{2}i\sqrt{2}\right)\right)\operatorname{sgn}(x)+\frac{2}{3}\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)\operatorname{sgn}(x)-\frac{1}{3}\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}(x)$$

```
[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="giac")
[Out] 1/3*(arctan(I*sqrt(2)) - 2*arctan(1/2*I*sqrt(2)))*sgn(x) + 2/3*arctan(1/2*s
qrt(x^2 - 2))*sgn(x) - 1/3*arctan(sqrt(x^2 - 2))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = \int \frac{\sqrt{x^4-2\,x^2}}{(x^2-1)\,(x^2+2)}\,dx$$

```
[In] int((x^4 - 2*x^2)^(1/2)/((x^2 - 1)*(x^2 + 2)),x)
[Out] int((x^4 - 2*x^2)^(1/2)/((x^2 - 1)*(x^2 + 2)), x)
```

3.893 $\int \frac{\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}}{2-x^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 47

$$\int \frac{\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}}{2-x^2} \, dx=\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\arctan\left(\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}}$$

```
[Out] (-x^2+1)*arctan((x^2-2)^(1/2))*(1-1/(-x^2+1)^2)^(1/2)/x/(x^2-2)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6854, 6857, 2015, 1160, 21, 267, 455, 52, 65, 209}

$$\int \frac{\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}}{2-x^2} \, dx=\frac{(1-x^2)\sqrt{x^4-2x^2}\sqrt{1-\frac{1}{(1-x^2)^2}}\arctan\left(\sqrt{x^2-2}\right)}{x\sqrt{x^2-2}\sqrt{(x^2-1)^2-1}}$$

```
[In] Int[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
```

```
[Out] ((1 - x^2)*Sqrt[-2*x^2 + x^4]*Sqrt[1 - (1 - x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2
]])/(x*Sqrt[-2 + x^2]*Sqrt[-1 + (-1 + x^2)^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1160

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 2015

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 6854

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\left((-1+x^2)\sqrt{1-\frac{1}{(-1+x^2)^2}}\right)\int \frac{\sqrt{-1+(-1+x^2)^2}}{(2-x^2)(-1+x^2)}\,dx}{\sqrt{-1+(-1+x^2)^2}}$$

$$= \frac{\left((-1+x^2)\sqrt{1-\frac{1}{(-1+x^2)^2}}\right)\int \left(\frac{\sqrt{-1+(-1+x^2)^2}}{2-x^2}+\frac{\sqrt{-1+(-1+x^2)^2}}{-1+x^2}\right)dx}{\sqrt{-1+(-1+x^2)^2}}$$

$$= \frac{\left((-1+x^2)\sqrt{1-\frac{1}{(-1+x^2)^2}}\right)\int \frac{\sqrt{-1+(-1+x^2)^2}}{2-x^2}\,dx}{\sqrt{-1+(-1+x^2)^2}} + \frac{\left((-1+x^2)\sqrt{1-\frac{1}{(-1+x^2)^2}}\right)\int \frac{\sqrt{-1+(-1+x^2)^2}}{-1+x^2}\,dx}{\sqrt{-1+(-1+x^2)^2}}$$

$$= \frac{\left((-1+x^2)\sqrt{1-\frac{1}{(-1+x^2)^2}}\right)\int \frac{\sqrt{-2x^2+x^4}}{2-x^2}\,dx}{\sqrt{-1+(-1+x^2)^2}} + \frac{\left((-1+x^2)\sqrt{1-\frac{1}{(-1+x^2)^2}}\right)\int \frac{\sqrt{-2x^2+x^4}}{-1+x^2}\,dx}{\sqrt{-1+(-1+x^2)^2}}$$

$$= \frac{\left((-1+x^2)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{(-1+x^2)^2}}\right)\int \frac{x\sqrt{-2+x^2}}{2-x^2}\,dx}{x\sqrt{-2+x^2}\sqrt{-1+(-1+x^2)^2}} + \frac{\left((-1+x^2)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{(-1+x^2)^2}}\right)\int \frac{x\sqrt{-2+x^2}}{-1+x^2}\,dx}{x\sqrt{-2+x^2}\sqrt{-1+(-1+x^2)^2}}$$

$$= \frac{\left((-1+x^2)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{(-1+x^2)^2}}\right)\text{Subst}\left(\int\frac{\sqrt{-2+x}}{-1+x}\,dx,x,x^2\right)}{2x\sqrt{-2+x^2}\sqrt{-1+(-1+x^2)^2}} - \frac{\left((-1+x^2)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{(-1+x^2)^2}}\right)\int\frac{x}{\sqrt{-2+x^2}}\,dx}{x\sqrt{-2+x^2}\sqrt{-1+(-1+x^2)^2}}$$

$$= -\frac{\left((-1+x^2)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{(-1+x^2)^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-2+x}(-1+x)}\,dx,x,x^2\right)}{2x\sqrt{-2+x^2}\sqrt{-1+(-1+x^2)^2}}$$

$$= -\frac{\left((-1+x^2)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{(-1+x^2)^2}}\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}\sqrt{-1+(-1+x^2)^2}}$$

$$= \frac{(1-x^2)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{(1-x^2)^2}}\tan^{-1}\left(\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}\sqrt{-1+(-1+x^2)^2}}$$

Mathematica [A] (verified)

Time = 5.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int\frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx = -\frac{(-1+x^2)\sqrt{1-\frac{1}{(-1+x^2)^2}}\arctan\left(\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}}$$

```
[In] Integrate[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
[Out] -(((-1 + x^2)*Sqrt[1 - (-1 + x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2]])/(x*Sqrt[-2
+ x^2]))
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

method	result	size
default	$\frac{\sqrt{\frac{(x^2-2)x^2}{(x^2-1)^2}}\,(x^2-1)\left(\arctan\left(\frac{x+2}{\sqrt{x^2-2}}\right)-\arctan\left(\frac{x-2}{\sqrt{x^2-2}}\right)\right)}{2x\sqrt{x^2-2}}$	63
trager	$-\frac{\text{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\text{RootOf}\left(\_Z^2+1\right)x^3+2x^2\sqrt{-\frac{-x^4+2x^2}{x^4-2x^2+1}}+3\,\text{RootOf}\left(\_Z^2+1\right)x-2\sqrt{-\frac{-x^4+2x^2}{x^4-2x^2+1}}}{(x+1)(x-1)x}\right)}{2}$	106

```
[In] int((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((x^2-2)*x^2/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((x+2)/(x^2-2)^(1/2))-arct
an((x-2)/(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx = -\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}}{x}\right)$$

```
[In] integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="fricas")
```

```
[Out] -arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)
```

Sympy [F]

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx = -\int \frac{\sqrt{\frac{x^4}{x^4-2x^2+1}-\frac{2x^2}{x^4-2x^2+1}}}{x^2-2}\,dx$$

```
[In] integrate((1-1/(x**2-1)**2)**(1/2)/(-x**2+2),x)
```

```
[Out] -Integral(sqrt(x**4/(x**4 - 2*x**2 + 1) - 2*x**2/(x**4 - 2*x**2 + 1))/(x**2
 - 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx = \int -\frac{\sqrt{-\frac{1}{(x^2-1)^2}+1}}{x^2-2}\,dx$$

```
[In] integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-1/(x^2 - 1)^2 + 1)/(x^2 - 2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx = -\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)$$

```
[In] integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="giac")
[Out] -arctan(sqrt(x^2 - 2))*sgn(x^3 - x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx = \int -\frac{\sqrt{1-\frac{1}{(x^2-1)^2}}}{x^2-2}\,dx$$

```
[In] int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2),x)
[Out] int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2), x)
```

3.894 $\int \frac{\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}}{2+x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 123

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = -\frac{2(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\arctan\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\arctan\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

```
[Out] -2/3*(-x^2+1)*arctan(1/2*(x^2-2)^(1/2))*((x^4-2*x^2)/(-x^2+1)^2)^(1/2)/x/(x
^2-2)^(1/2)+1/3*(-x^2+1)*arctan((x^2-2)^(1/2))*((x^4-2*x^2)/(-x^2+1)^2)^(1/
2)/x/(x^2-2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6851, 2081, 585, 85, 65, 209}

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = \frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\arctan\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}} - \frac{2(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}$$

```
[In] Int[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2),x]
```

```
[Out] (-2*(1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan[Sqrt[-2 + x^2]/2])/
(3*x*Sqrt[-2 + x^2]) + ((1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan
[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\text{integral} = \frac{\left((-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right)\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx}{\sqrt{-2x^2+x^4}}$$

$$= \frac{\left((-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right)\int \frac{x\sqrt{-2+x^2}}{(-1+x^2)(2+x^2)}\,dx}{x\sqrt{-2+x^2}}$$

$$= \frac{\left((-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right)\text{Subst}\left(\int \frac{\sqrt{-2+x}}{(-1+x)(2+x)}\,dx, x, x^2\right)}{2x\sqrt{-2+x^2}}$$

$$= -\frac{\left((-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}(-1+x)}\,dx, x, x^2\right)}{6x\sqrt{-2+x^2}} + \frac{\left(2(-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}(2+x)}\,dx, x, x^2\right)}{3x\sqrt{-2+x^2}}$$

$$= -\frac{\left((-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{\left(4(-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right)\text{Subst}\left(\int \frac{1}{4+x^2}\,dx, x, \sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

$$= -\frac{2(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = \frac{\sqrt{\frac{x^2(-2+x^2)}{(-1+x^2)^2}}(-1+x^2)\left(2\arctan\left(\frac{1}{2}\sqrt{-2+x^2}\right)-\arctan\left(\sqrt{-2+x^2}\right)\right)}{3x\sqrt{-2+x^2}}$$

```
[In] Integrate[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2),x]
```

```
[Out] (Sqrt[(x^2*(-2 + x^2))/(-1 + x^2)^2]*(-1 + x^2)*(2*ArcTan[Sqrt[-2 + x^2]/2]
 - ArcTan[Sqrt[-2 + x^2]]))/(3*x*Sqrt[-2 + x^2])
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

method	result
default	$\frac{\sqrt{\frac{\left(x^{2}-2\right)x^{2}}{\left(x^{2}-1\right)^{2}}}\,(x^{2}-1)\left(\arctan\left(\frac{x+2}{\sqrt{x^{2}-2}}\right)-\arctan\left(\frac{x-2}{\sqrt{x^{2}-2}}\right)+4\arctan\left(\frac{\sqrt{x^{2}-2}}{2}\right)\right)}{6x\sqrt{x^{2}-2}}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)x^{7}-6\sqrt{-\frac{-x^{4}+2x^{2}}{x^{4}-2x^{2}+1}}\,x^{6}-15\operatorname{RootOf}\left(\_Z^{2}+1\right)x^{5}+22x^{4}\sqrt{-\frac{-x^{4}+2x^{2}}{x^{4}-2x^{2}+1}}+24\operatorname{RootOf}\left(\_Z^{2}+1\right)x}{x\left(x^{2}+2\right)^{2}(x-1)(x+1)}\right.}{6}$

[In] int(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x,method=_RETURNVERBOSE)

[Out] 1/6*((x^2-2)*x^2/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((x+2)/(x^2-2)^(1/2))-arctan((x-2)/(x^2-2)^(1/2))+4*arctan(1/2*(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = -\frac{1}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}}{x}\right)+\frac{2}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}}{2\,x}\right)$$

[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="fricas")

[Out] -1/3*arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x) + 2/3*arctan(1/2*(x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)

Sympy [F]

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = \int \frac{\sqrt{\frac{x^2(x^2-2)}{x^4-2x^2+1}}}{x^2+2}\,dx$$

[In] integrate(((x**4-2*x**2)/(x**2-1)**2)**(1/2)/(x**2+2),x)

[Out] Integral(sqrt(x**2*(x**2 - 2)/(x**4 - 2*x**2 + 1))/(x**2 + 2), x)

Maxima [F]

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = \int \frac{\sqrt{\frac{x^4-2\,x^2}{(x^2-1)^2}}}{x^2+2}\,dx$$

```
[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="maxima")
[Out] integrate(sqrt((x^4 - 2*x^2)/(x^2 - 1)^2)/(x^2 + 2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = \frac{2}{3}\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right) - \frac{1}{3}\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)$$

```
[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="giac")
[Out] 2/3*arctan(1/2*sqrt(x^2 - 2))*sgn(x^3 - x) - 1/3*arctan(sqrt(x^2 - 2))*sgn(
x^3 - x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = \int \frac{\sqrt{-\frac{2\,x^2-x^4}{(x^2-1)^2}}}{x^2+2}\,dx$$

```
[In] int((-(2*x^2 - x^4)/(x^2 - 1)^2)^(1/2)/(x^2 + 2),x)
[Out] int((-(2*x^2 - x^4)/(x^2 - 1)^2)^(1/2)/(x^2 + 2), x)
```

3.895 $\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx = -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}-\frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3(1+x^2)} -\frac{(4+3x)(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{5\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\operatorname{arcsinh}(x)}{1+x}$$

```
[Out] -4/3*(1-2*x)*(1+x)*(1+2*x/(x^2+1))^(1/2)-1/3*(1-x)*(1+x)^3*(1+2*x/(x^2+1))^
(1/2)/(x^2+1)-(4+3*x)*(x^2+1)*(1+2*x/(x^2+1))^(1/2)/(1+x)+5*arcsinh(x)*(x^2
+1)^(1/2)*(1+2*x/(x^2+1))^(1/2)/(1+x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6855, 984, 753, 833, 794, 221}

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx = \frac{5\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\operatorname{arcsinh}(x)}{x+1}-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)^3}{3(x^2+1)} -\frac{4}{3}(1-2x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)-\frac{(3x+4)(x^2+1)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

```
[In] Int[(1 + (2*x)/(1 + x^2))^(5/2),x]
```

```
[Out] (-4*(1 - 2*x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)])/3 - ((1 - x)*(1 + x)^3*Sqr
t[1 + (2*x)/(1 + x^2)])/(3*(1 + x^2)) - ((4 + 3*x)*(1 + x^2)*Sqrt[1 + (2*x)
```

```
/(1 + x^2)])/(1 + x) + (5*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*ArcSinh[x
])/(1 + x)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 984

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6855

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
```

```
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int\frac{(1+2x+x^2)^{5/2}}{(1+x^2)^{5/2}}\,dx}{\sqrt{1+2x+x^2}}\\
&= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int\frac{(2+2x)^5}{(1+x^2)^{5/2}}\,dx}{16(2+2x)}\\
&= -\frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\,(1+x^2)}+\frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int\frac{(24-8x)(2+2x)^3}{(1+x^2)^{3/2}}\,dx}{48(2+2x)}\\
&= -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}-\frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\,(1+x^2)}\\
&\quad+\frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int\frac{(96-288x)(2+2x)}{\sqrt{1+x^2}}\,dx}{48(2+2x)}\\
&= -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}-\frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\,(1+x^2)}\\
&\quad-\frac{(4+3x)\,(1+x^2)\,\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{\left(10\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int\frac{1}{\sqrt{1+x^2}}\,dx}{2+2x}\\
&= -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}-\frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\,(1+x^2)}\\
&\quad-\frac{(4+3x)\,(1+x^2)\,\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{5\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\sinh^{-1}(x)}{1+x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.57

$$\int\left(1+\frac{2x}{1+x^2}\right)^{5/2}dx=\frac{(1+x)\left(-17-12x-18x^2-8x^3+3x^4-15(1+x^2)^{3/2}\log\left(-x+\sqrt{1+x^2}\right)\right)}{3\sqrt{\frac{(1+x)^2}{1+x^2}}\,(1+x^2)^2}$$

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(5/2),x]
[Out] ((1 + x)*(-17 - 12*x - 18*x^2 - 8*x^3 + 3*x^4 - 15*(1 + x^2)^(3/2)*Log[-x +
 Sqrt[1 + x^2]]))/(3*Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2)^2)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{\left(\frac{x^{2}+2x+1}{x^{2}+1}\right)^{\frac{5}{2}}\left(x^{2}+1\right)\left(15\,\mathrm{arcsinh}\left(x\right)\left(x^{2}+1\right)^{\frac{3}{2}}+3x^{4}-8x^{3}-18x^{2}-12x-17\right)}{3\left(x+1\right)^{5}}$	62
risch	$\frac{\left(3x^{4}-8x^{3}-18x^{2}-12x-17\right)\sqrt{\frac{\left(x+1\right)^{2}}{x^{2}+1}}}{3\left(x^{2}+1\right)\left(x+1\right)}+\frac{5\,\mathrm{arcsinh}\left(x\right)\sqrt{x^{2}+1}\,\sqrt{\frac{\left(x+1\right)^{2}}{x^{2}+1}}}{x+1}$	82
trager	$\frac{\left(3x^{4}-8x^{3}-18x^{2}-12x-17\right)\sqrt{-\frac{-x^{2}-2x-1}{x^{2}+1}}}{3\left(x^{2}+1\right)\left(x+1\right)}+5\ln\left(\frac{\sqrt{-\frac{-x^{2}-2x-1}{x^{2}+1}}\,x^{2}+x^{2}+\sqrt{-\frac{-x^{2}-2x-1}{x^{2}+1}}+x}{x+1}\right)$	117

[In] int((1+2*x/(x^2+1))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/3*((x^2+2*x+1)/(x^2+1))^(5/2)/(x+1)^5*(x^2+1)*(15*arcsinh(x)*(x^2+1)^(3/2)+3*x^4-8*x^3-18*x^2-12*x-17)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.88

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2}\,dx =$$

$$-\frac{8\,x^{3}+8\,x^{2}+15\,\left(x^{3}+x^{2}+x+1\right)\log\left(-\frac{x^{2}-\left(x^{2}+1\right)\sqrt{\frac{x^{2}+2\,x+1}{x^{2}+1}}+x}{x+1}\right)-\left(3\,x^{4}-8\,x^{3}-18\,x^{2}-12\,x-17\right)\sqrt{\frac{x^{2}+2}{x^{2}+}}}{3\,\left(x^{3}+x^{2}+x+1\right)}$$

[In] integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="fricas")

[Out] -1/3*(8*x^3 + 8*x^2 + 15*(x^3 + x^2 + x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) - (3*x^4 - 8*x^3 - 18*x^2 - 12*x - 17)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 8*x + 8)/(x^3 + x^2 + x + 1)

Sympy [F]

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx = \int \left(\frac{2x}{x^2+1}+1\right)^{\frac{5}{2}} dx$$

```
[In] integrate((1+2*x/(x**2+1))**(5/2),x)
[Out] Integral((2*x/(x**2 + 1) + 1)**(5/2), x)
```

Maxima [F]

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx = \int \left(\frac{2\,x}{x^2+1}+1\right)^{\frac{5}{2}} dx$$

```
[In] integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="maxima")
[Out] integrate((2*x/(x^2 + 1) + 1)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.65

$$\begin{aligned}\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx &= \left(\sqrt{2}+5\,\log\left(\sqrt{2}+1\right)\right)\operatorname{sgn}(x+1) \\ &- 5\,\log\left(-x+\sqrt{x^2+1}\right)\operatorname{sgn}(x+1) \\ &+ \frac{(((3\,x\operatorname{sgn}(x+1)-8\operatorname{sgn}(x+1))x-18\operatorname{sgn}(x+1))x-12\operatorname{sgn}(x+1))x-17\operatorname{sgn}(x+1)}{3\,(x^2+1)^{\frac{3}{2}}}\end{aligned}$$

```
[In] integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="giac")
[Out] (sqrt(2) + 5*log(sqrt(2) + 1))*sgn(x + 1) - 5*log(-x + sqrt(x^2 + 1))*sgn(x
 + 1) + 1/3*((((3*x*sgn(x + 1) - 8*sgn(x + 1))*x - 18*sgn(x + 1))*x - 12*sg
n(x + 1))*x - 17*sgn(x + 1))/(x^2 + 1)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx = \int \left(\frac{2\,x}{x^2+1}+1\right)^{5/2} dx$$

```
[In] int(((2*x)/(x^2 + 1) + 1)^(5/2),x)
[Out] int(((2*x)/(x^2 + 1) + 1)^(5/2), x)
```

3.896 $\int \left(1+\frac{2x}{1+x^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2}\,dx=-\left((1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}\right)-\frac{x(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{3\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\operatorname{arcsinh}(x)}{1+x}$$

```
[Out] -(1-x)*(1+x)*(1+2*x/(x^2+1))^(1/2)-x*(x^2+1)*(1+2*x/(x^2+1))^(1/2)/(1+x)+3*
arcsinh(x)*(x^2+1)^(1/2)*(1+2*x/(x^2+1))^(1/2)/(1+x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6855, 984, 753, 531, 396, 221}

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2}\,dx=\frac{3\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\operatorname{arcsinh}(x)}{x+1}-\left((1-x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)\right)-\frac{x(x^2+1)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

```
[In] Int[(1 + (2*x)/(1 + x^2))^(3/2),x]
```

```
[Out] -((1 - x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)]) - (x*(1 + x^2)*Sqrt[1 + (2*x)/
(1 + x^2)])/(1 + x) + (3*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*ArcSinh[x]
)/(1 + x)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 984

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6855

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int\frac{(1+2x+x^2)^{3/2}}{(1+x^2)^{3/2}}\,dx}{\sqrt{1+2x+x^2}}$$

$$= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int\frac{(2+2x)^3}{(1+x^2)^{3/2}}\,dx}{4(2+2x)}$$

$$= -\left((1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}\right) + \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int\frac{(8-8x)(2+2x)}{\sqrt{1+x^2}}\,dx}{4(2+2x)}$$

$$= -\left((1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}\right) + \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int\frac{16-16x^2}{\sqrt{1+x^2}}\,dx}{4(2+2x)}$$

$$= -\left((1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}\right) - \frac{x(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x} + \frac{\left(6\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int\frac{1}{\sqrt{1+x^2}}\,dx}{2+2x}$$

$$= -\left((1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}\right) - \frac{x(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x} + \frac{3\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\sinh^{-1}(x)}{1+x}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int\left(1+\frac{2x}{1+x^2}\right)^{3/2}dx = \frac{\sqrt{\frac{(1+x)^2}{1+x^2}}\left(-1-2x+x^2-3\sqrt{1+x^2}\log\left(-x+\sqrt{1+x^2}\right)\right)}{1+x}$$

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(3/2),x]
[Out] (Sqrt[(1 + x)^2/(1 + x^2)]*(-1 - 2*x + x^2 - 3*Sqrt[1 + x^2]*Log[-x + Sqrt[
1 + x^2]]))/(1 + x)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{\left(\frac{x^{2}+2x+1}{x^{2}+1}\right)^{\frac{3}{2}} \left(x^{2}+1\right) \left(3 \operatorname{arcsinh}\left(x\right) \sqrt{x^{2}+1}+x^{2}-2x-1\right)}{\left(x+1\right)^{3}}$	49
risch	$\frac{\left(x^{2}-2x-1\right) \sqrt{\frac{\left(x+1\right)^{2}}{x^{2}+1}}}{x+1}+\frac{3 \operatorname{arcsinh}\left(x\right) \sqrt{x^{2}+1}\, \sqrt{\frac{\left(x+1\right)^{2}}{x^{2}+1}}}{x+1}$	62
trager	$\frac{\left(x^{2}-2x-1\right) \sqrt{-\frac{-x^{2}-2x-1}{x^{2}+1}}}{x+1}+3 \ln\left(\frac{\sqrt{-\frac{-x^{2}-2x-1}{x^{2}+1}}\, x^{2}+x^{2}+\sqrt{-\frac{-x^{2}-2x-1}{x^{2}+1}}+x}{x+1}\right)$	97

```
[In] int((1+2*x/(x^2+1))^(3/2),x,method=_RETURNVERBOSE)
[Out] ((x^2+2*x+1)/(x^2+1))^(3/2)/(x+1)^3*(x^2+1)*(3*arcsinh(x)*(x^2+1)^(1/2)+x^2
-2*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx = -\frac{3\,(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)-(x^2-2\,x-1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+2\,x+2}{x+1}$$

```
[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="fricas")
[Out] -(3*(x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x +
 1)) - (x^2 - 2*x - 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 2*x + 2)/(x + 1)
```

Sympy [F]

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx = \int \left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}} dx$$

```
[In] integrate((1+2*x/(x**2+1))**(3/2),x)
[Out] Integral((2*x/(x**2 + 1) + 1)**(3/2), x)
```

Maxima [F]

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx = \int \left(\frac{2\,x}{x^2+1}+1\right)^{\frac{3}{2}} dx$$

```
[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="maxima")
[Out] integrate((2*x/(x^2 + 1) + 1)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx = -\left(\sqrt{2}-3\log\left(\sqrt{2}+1\right)\right)\mathrm{sgn}(x+1) - 3\log\left(-x+\sqrt{x^2+1}\right)\mathrm{sgn}(x+1) + \frac{(x\mathrm{sgn}(x+1)-2\,\mathrm{sgn}(x+1))x-\mathrm{sgn}(x+1)}{\sqrt{x^2+1}}$$

```
[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="giac")
[Out] -(sqrt(2) - 3*log(sqrt(2) + 1))*sgn(x + 1) - 3*log(-x + sqrt(x^2 + 1))*sgn(
x + 1) + ((x*sgn(x + 1) - 2*sgn(x + 1))*x - sgn(x + 1))/sqrt(x^2 + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx = \int \left(\frac{2\,x}{x^2+1}+1\right)^{3/2} dx$$

```
[In] int(((2*x)/(x^2 + 1) + 1)^(3/2),x)
[Out] int(((2*x)/(x^2 + 1) + 1)^(3/2), x)
```

3.897 $\int \sqrt{1+\frac{2x}{1+x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 61

$$\int \sqrt{1+\frac{2x}{1+x^2}}\,dx=\frac{(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\operatorname{arcsinh}(x)}{1+x}$$

```
[Out] (x^2+1)*(1+2*x/(x^2+1))^(1/2)/(1+x)+arcsinh(x)*(x^2+1)^(1/2)*(1+2*x/(x^2+1)
)^(1/2)/(1+x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6855, 984, 655, 221}

$$\int \sqrt{1+\frac{2x}{1+x^2}}\,dx=\frac{\sqrt{\frac{2x}{x^2+1}+1}\sqrt{x^2+1}\operatorname{arcsinh}(x)}{x+1}+\frac{\sqrt{\frac{2x}{x^2+1}+1}(x^2+1)}{x+1}$$

```
[In] Int[Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] ((1 + x^2)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x) + (Sqrt[1 + x^2]*Sqrt[1 + (2*
x)/(1 + x^2)]*ArcSinh[x])/(1 + x)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 984

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6855

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx}{\sqrt{1+2x+x^2}}\\
&= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{2+2x}{\sqrt{1+x^2}}\,dx}{2+2x}\\
&= \frac{(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{\left(2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{1}{\sqrt{1+x^2}}\,dx}{2+2x}\\
&= \frac{(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\sinh^{-1}(x)}{1+x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \sqrt{1+\frac{2x}{1+x^2}}\,dx = \frac{\sqrt{\frac{(1+x)^2}{1+x^2}}\left(1+x^2-\sqrt{1+x^2}\log\left(-x+\sqrt{1+x^2}\right)\right)}{1+x}$$

```
[In] Integrate[Sqrt[1 + (2*x)/(1 + x^2)],x]
[Out] (Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2 - Sqrt[1 + x^2]*Log[-x + Sqrt[1 + x^2]]
))/(1 + x)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\sqrt{\frac{x^2+2x+1}{x^2+1}}\,\sqrt{x^2+1}\left(\sqrt{x^2+1}+\operatorname{arcsinh}(x)\right)}{x+1}$	42
risch	$\frac{(x^2+1)\sqrt{\frac{(x+1)^2}{x^2+1}}}{x+1}+\frac{\operatorname{arcsinh}(x)\sqrt{x^2+1}\,\sqrt{\frac{(x+1)^2}{x^2+1}}}{x+1}$	58
trager	$\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,(x^2+1)}{x+1}+\ln\left(\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,x^2+x^2+\sqrt{-\frac{-x^2-2x-1}{x^2+1}}+x}{x+1}\right)$	92

```
[In] int((1+2*x/(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] ((x^2+2*x+1)/(x^2+1))^(1/2)/(x+1)*(x^2+1)^(1/2)*((x^2+1)^(1/2)+arcsinh(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int\sqrt{1+\frac{2x}{1+x^2}}\,dx=-\frac{(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)-(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}}{x+1}$$

```
[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="fricas")
[Out] -((x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1
)) - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)))/(x + 1)
```

Sympy [F]

$$\int\sqrt{1+\frac{2x}{1+x^2}}\,dx=\int\sqrt{\frac{2x}{x^2+1}+1}\,dx$$

```
[In] integrate((1+2*x/(x**2+1))**(1/2),x)
[Out] Integral(sqrt(2*x/(x**2 + 1) + 1), x)
```

Maxima [F]

$$\int \sqrt{1+\frac{2x}{1+x^2}}\,dx=\int \sqrt{\frac{2\,x}{x^2+1}+1}\,dx$$

```
[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(2*x/(x^2 + 1) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \sqrt{1+\frac{2x}{1+x^2}}\,dx=-\left(\sqrt{2}-\log\left(\sqrt{2}+1\right)\right)\operatorname{sgn}(x+1)-\log\left(-x+\sqrt{x^2+1}\right)\operatorname{sgn}(x+1)+\sqrt{x^2+1}\operatorname{sgn}(x+1)$$

```
[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="giac")
[Out] -(sqrt(2) - log(sqrt(2) + 1))*sgn(x + 1) - log(-x + sqrt(x^2 + 1))*sgn(x +
1) + sqrt(x^2 + 1)*sgn(x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\frac{2x}{1+x^2}}\,dx=\int \sqrt{\frac{2\,x}{x^2+1}+1}\,dx$$

```
[In] int(((2*x)/(x^2 + 1) + 1)^(1/2),x)
[Out] int(((2*x)/(x^2 + 1) + 1)^(1/2), x)
```

3.898 $\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 109

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx = \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(1+x)\text{arcsinh}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{\sqrt{2}(1+x)\text{arctanh}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

```
[Out] (1+x)/(1+2*x/(x^2+1))^(1/2)-(1+x)*arcsinh(x)/(x^2+1)^(1/2)/(1+2*x/(x^2+1))^
(1/2)-(1+x)*arctanh(1/2*(1-x)*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)/(x^2+1)^(1/2)/
(1+2*x/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6855, 984, 749, 858, 221, 739, 212}

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx = -\frac{(x+1)\text{arcsinh}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}} - \frac{\sqrt{2}(x+1)\text{arctanh}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}} + \frac{x+1}{\sqrt{\frac{2x}{x^2+1}+1}}$$

```
[In] Int[1/Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] (1 + x)/Sqrt[1 + (2*x)/(1 + x^2)] - ((1 + x)*ArcSinh[x])/(Sqrt[1 + x^2]*Sqr
t[1 + (2*x)/(1 + x^2)]) - (Sqrt[2]*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1
+ x^2])])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 984

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6855

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+2x+x^2}\int \frac{\sqrt{1+x^2}}{\sqrt{1+2x+x^2}}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{(2+2x)\int \frac{\sqrt{1+x^2}}{2+2x}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(2+2x)\int \frac{2-2x}{(2+2x)\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(2+2x)\int \frac{1}{\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(2(2+2x))\int \frac{1}{(2+2x)\sqrt{1+x^2}}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(2(2+2x))\text{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, \frac{2-2x}{\sqrt{1+x^2}}\right)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{\sqrt{2}(1+x)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx = \frac{(1+x)\left(\sqrt{1+x^2}+2\sqrt{2}\text{arctanh}\left(\frac{1+x-\sqrt{1+x^2}}{\sqrt{2}}\right)+\log\left(-x+\sqrt{1+x^2}\right)\right)}{\sqrt{\frac{(1+x)^2}{1+x^2}}\sqrt{1+x^2}}$$

```
[In] Integrate[1/Sqrt[1 + (2*x)/(1 + x^2)],x]
[Out] ((1 + x)*(Sqrt[1 + x^2] + 2*Sqrt[2]*ArcTanh[(1 + x - Sqrt[1 + x^2])/Sqrt[2]
] + Log[-x + Sqrt[1 + x^2]]))/(Sqrt[(1 + x)^2/(1 + x^2)]*Sqrt[1 + x^2])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x+1}{\sqrt{\frac{(x+1)^2}{x^2+1}}}+\frac{\left(-\operatorname{arcsinh}(x)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{(-2x+2)\sqrt{2}}{4\sqrt{(x+1)^2-2x}}\right)\right)(x+1)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\,\sqrt{x^2+1}}$
trager	$\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,(x^2+1)}{x+1}-\ln\left(\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,x^2+x^2+\sqrt{-\frac{-x^2-2x-1}{x^2+1}}+x}{x+1}\right)+\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{2\sqrt{-\frac{-x^2-2x-}{x^2+1}}}{}\right.$

```
[In] int(1/(1+2*x/(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/((x+1)^2/(x^2+1))^(1/2)*(x+1)+(-arcsinh(x)-2^(1/2)*arctanh(1/4*(-2*x+2)*2
^(1/2)/((x+1)^2-2*x)^(1/2)))/((x+1)^2/(x^2+1))^(1/2)/(x^2+1)^(1/2)*(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx = \frac{\sqrt{2}(x+1)\log\left(-\frac{x^2+\sqrt{2}(x^2-1)+\left(2x^2+\sqrt{2}(x^2+1)+2\right)\sqrt{\frac{x^2+2x+1}{x^2+1}}-1}{x^2+2x+1}\right)+(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}}+x}{x+1}\right)+(x}{x+1}$$

```
[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(x + 1)*log(-(x^2 + sqrt(2)*(x^2 - 1) + (2*x^2 + sqrt(2)*(x^2 + 1)
 + 2)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - 1)/(x^2 + 2*x + 1)) + (x + 1)*log(-
(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) + (x^2 + 1)*
sqrt((x^2 + 2*x + 1)/(x^2 + 1)))/(x + 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{2x}{x^2+1}+1}}\,dx$$

```
[In] integrate(1/(1+2*x/(x**2+1))**(1/2),x)
[Out] Integral(1/sqrt(2*x/(x**2 + 1) + 1), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{2x}{x^2+1}+1}}\,dx$$

```
[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x/(x^2 + 1) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx = \frac{\sqrt{2}\log\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+1}-2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+1}-2\right|}\right)}{\operatorname{sgn}(x+1)} + \frac{\log\left(-x+\sqrt{x^2+1}\right)}{\operatorname{sgn}(x+1)} + \frac{\sqrt{x^2+1}}{\operatorname{sgn}(x+1)}$$

```
[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="giac")
[Out] sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1) - 2)/abs(-2*x + 2*sqrt(2
) + 2*sqrt(x^2 + 1) - 2))/sgn(x + 1) + log(-x + sqrt(x^2 + 1))/sgn(x + 1) +
 sqrt(x^2 + 1)/sgn(x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx=\int \frac{1}{\sqrt{\frac{2\,x}{x^2+1}+1}}\,dx$$

```
[In] int(1/((2*x)/(x^2 + 1) + 1)^(1/2),x)
[Out] int(1/((2*x)/(x^2 + 1) + 1)^(1/2), x)
```

3.899 $\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 144

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}}-\frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}}-\frac{3(1+x)\operatorname{arcsinh}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}-\frac{9(1+x)\operatorname{arctanh}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{2\sqrt{2}\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

```
[Out] 3/2*(2+x)/(1+2*x/(x^2+1))^(1/2)+1/2*(-x^2-1)/(1+x)/(1+2*x/(x^2+1))^(1/2)-3*
(1+x)*arcsinh(x)/(x^2+1)^(1/2)/(1+2*x/(x^2+1))^(1/2)-9/4*(1+x)*arctanh(1/2*
(1-x)*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)/(x^2+1)^(1/2)/(1+2*x/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6855, 984, 747, 827, 858, 221, 739, 212}

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = -\frac{3(x+1)\operatorname{arcsinh}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{9(x+1)\operatorname{arctanh}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{2\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}+\frac{3(x+2)}{2\sqrt{\frac{2x}{x^2+1}+1}}-\frac{x^2+1}{2(x+1)\sqrt{\frac{2x}{x^2+1}+1}}$$

```
[In] Int[(1 + (2*x)/(1 + x^2))^(-3/2),x]
```

```
[Out] (3*(2 + x))/(2*Sqrt[1 + (2*x)/(1 + x^2)]) - (1 + x^2)/(2*(1 + x)*Sqrt[1 + (
2*x)/(1 + x^2)]) - (3*(1 + x)*ArcSinh[x])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1
+ x^2)]) - (9*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1 + x^2])])/(2*Sqrt[2]*
Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 984

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6855

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+2x+x^2}\int \frac{(1+x^2)^{3/2}}{(1+2x+x^2)^{3/2}}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{(4(2+2x))\int \frac{(1+x^2)^{3/2}}{(2+2x)^3}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= -\frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(3(2+2x))\int \frac{x\sqrt{1+x^2}}{(2+2x)^2}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(3(2+2x))\int \frac{-4+8x}{(2+2x)\sqrt{1+x^2}}\,dx}{8\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(3(2+2x))\int \frac{1}{\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(9(2+2x))\int \frac{1}{(2+2x)\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{3(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(9(2+2x))\text{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, \frac{2-2x}{\sqrt{1+x^2}}\right)}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{3(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{9(1+x)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{2\sqrt{2}\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \frac{(1+x)\left(\sqrt{1+x^2}(5+9x+2x^2)+9\sqrt{2}(1+x)^2\text{arctanh}\left(\frac{1+x-\sqrt{1+x^2}}{\sqrt{2}}\right)+6(1+x)^2\log\right.}{2\left(\frac{(1+x)^2}{1+x^2}\right)^{3/2}(1+x^2)^{3/2}}$$

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(-3/2),x]
```

```
[Out] ((1 + x)*(Sqrt[1 + x^2]*(5 + 9*x + 2*x^2) + 9*Sqrt[2]*(1 + x)^2*ArcTanh[(1
+ x - Sqrt[1 + x^2])/Sqrt[2]] + 6*(1 + x)^2*Log[-x + Sqrt[1 + x^2]]))/(2*((
1 + x)^2/(1 + x^2))^(3/2)*(1 + x^2)^(3/2))
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

method	result
risch	$\frac{2x^4+9x^3+7x^2+9x+5}{2(x+1)(x^2+1)\sqrt{\frac{(x+1)^2}{x^2+1}}}+\frac{\left(-3\,\text{arcsinh}(x)-\frac{9\sqrt{2}\,\text{arctanh}\left(\frac{(-2x+2)\sqrt{2}}{4\sqrt{(x+1)^2-2x}}\right)}{4}\right)(x+1)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\sqrt{x^2+1}}$
trager	$\frac{(x^2+1)(2x^2+9x+5)\sqrt{-\frac{-x^2-2x-1}{x^2+1}}}{2(x+1)^3}+3\ln\left(-\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,x^2-x^2+\sqrt{-\frac{-x^2-2x-1}{x^2+1}}-x}{x+1}\right)-\frac{9\,\text{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{2\sqrt{-}\right.}{}$
default	$-\frac{(x+1)\left(-(x^2+1)^{\frac{5}{2}}x+(x^2+1)^{\frac{3}{2}}x^3+(x^2+1)^{\frac{5}{2}}-(x^2+1)^{\frac{3}{2}}x^2-18\sqrt{2}\,\text{arctanh}\left(\frac{(x-1)\sqrt{2}}{2\sqrt{x^2+1}}\right)x^2-5x(x^2+1)^{\frac{3}{2}}+6\sqrt{x^2+1}\,x^3+24\,\text{arcsinh}\right.}{8(}$

```
[In] int(1/(1+2*x/(x^2+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*x^4+9*x^3+7*x^2+9*x+5)/(x+1)/(x^2+1)/((x+1)^2/(x^2+1))^(1/2)+(-3*arc
sinh(x)-9/4*2^(1/2)*arctanh(1/4*(-2*x+2)*2^(1/2)/((x+1)^2-2*x)^(1/2)))/((x+
1)^2/(x^2+1))^(1/2)/(x^2+1)^(1/2)*(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \frac{10\,x^3+9\sqrt{2}(x^3+3\,x^2+3\,x+1)\log\left(-\frac{x^2+\sqrt{2}(x^2-1)+\left(2\,x^2+\sqrt{2}(x^2+1)+2\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}-1}{x^2+2\,x+1}\right)+}{}$$

```
[In] integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="fricas")
[Out] 1/4*(10*x^3 + 9*sqrt(2)*(x^3 + 3*x^2 + 3*x + 1)*log(-(x^2 + sqrt(2)*(x^2 -
1) + (2*x^2 + sqrt(2)*(x^2 + 1) + 2)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - 1)/(
x^2 + 2*x + 1)) + 30*x^2 + 12*(x^3 + 3*x^2 + 3*x + 1)*log(-(x^2 - (x^2 + 1)
*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) + 2*(2*x^4 + 9*x^3 + 7*x^2 +
 9*x + 5)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 30*x + 10)/(x^3 + 3*x^2 + 3*x +
 1)
```

Sympy [F]

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(1+2*x/(x**2+1))**(3/2),x)
[Out] Integral((2*x/(x**2 + 1) + 1)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{2\,x}{x^2+1}+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="maxima")
[Out] integrate((2*x/(x^2 + 1) + 1)^(-3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \frac{9\sqrt{2}\log\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+1}-2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+1}-2\right|}\right)}{4\,\mathrm{sgn}\left(x+1\right)} + \frac{3\log\left(-x+\sqrt{x^2+1}\right)}{\mathrm{sgn}\left(x+1\right)} + \frac{\sqrt{x^2+1}}{\mathrm{sgn}\left(x+1\right)} + \frac{7\left(x-\sqrt{x^2+1}\right)^3+5\left(x-\sqrt{x^2+1}\right)^2-13x+13\sqrt{x^2+1}+5}{\left(\left(x-\sqrt{x^2+1}\right)^2+2x-2\sqrt{x^2+1}-1\right)^2\mathrm{sgn}\left(x+1\right)}$$

```
[In] integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] 9/4*sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1) - 2)/abs(-2*x + 2*sq
rt(2) + 2*sqrt(x^2 + 1) - 2))/sgn(x + 1) + 3*log(-x + sqrt(x^2 + 1))/sgn(x
+ 1) + sqrt(x^2 + 1)/sgn(x + 1) + (7*(x - sqrt(x^2 + 1))^3 + 5*(x - sqrt(x^
2 + 1))^2 - 13*x + 13*sqrt(x^2 + 1) + 5)/(((x - sqrt(x^2 + 1))^2 + 2*x - 2*
sqrt(x^2 + 1) - 1)^2*sgn(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{2x}{x^2+1}+1\right)^{3/2}}\,dx$$

```
[In] int(1/((2*x)/(x^2 + 1) + 1)^(3/2),x)
```

```
[Out] int(1/((2*x)/(x^2 + 1) + 1)^(3/2), x)
```

3.900 $\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 28

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2} \, dx = -\frac{(1-x)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}$$

```
[Out] -(1-x)*(1+2*x/(x^2+1))^(1/2)/(1+x)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6855, 984, 651}

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2} \, dx = -\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

```
[In] Int[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2),x]
[Out] -(((1 - x)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 984

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
```

```
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6855

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\sqrt{1+2x+x^2}}{(1+x^2)^{3/2}}\,dx}{\sqrt{1+2x+x^2}}$$
$$= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{2+2x}{(1+x^2)^{3/2}}\,dx}{2+2x}$$
$$= -\frac{(1-x)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx = \frac{(-1+x)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}$$

```
[In] Integrate[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2),x]
[Out] ((-1 + x)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x)
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{\sqrt{\frac{(x+1)^{2}}{x^{2}+1}}\,(x-1)}{x+1}$	25
gosper	$\frac{(x-1)\sqrt{\frac{x^{2}+2x+1}{x^{2}+1}}}{x+1}$	28
default	$\frac{(x-1)\sqrt{\frac{x^{2}+2x+1}{x^{2}+1}}}{x+1}$	28
trager	$\frac{(x-1)\sqrt{-\frac{-x^{2}-2x-1}{x^{2}+1}}}{x+1}$	31

```
[In] int((1+2*x/(x^2+1))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/(x+1)*((x+1)^2/(x^2+1))^(1/2)*(x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx = \frac{(x-1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x+1}{x+1}$$

```
[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] ((x - 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x + 1)/(x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{(x+1)^2}{x^2+1}}}{x^2+1}\,dx$$

```
[In] integrate((1+2*x/(x**2+1))**(1/2)/(x**2+1),x)
[Out] Integral(sqrt((x + 1)**2/(x**2 + 1))/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{2\,x}{x^2+1}+1}}{x^2+1}\,dx$$

```
[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt(2*x/(x^2 + 1) + 1)/(x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx = \sqrt{2}\mathrm{sgn}(x+1) + \frac{x\mathrm{sgn}(x+1) - \mathrm{sgn}(x+1)}{\sqrt{x^2+1}}$$

```
[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="giac")
[Out] sqrt(2)*sgn(x + 1) + (x*sgn(x + 1) - sgn(x + 1))/sqrt(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx = \frac{\sqrt{\frac{2\,x}{x^2+1}+1}\,(x-1)}{x+1}$$

```
[In] int((((2*x)/(x^2 + 1) + 1)^(1/2)/(x^2 + 1),x)
[Out] (((2*x)/(x^2 + 1) + 1)^(1/2)*(x - 1))/(x + 1)
```

3.901 $\int \sqrt{x-x^2} F(x) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \sqrt{x-x^2} F(x) \, dx = \operatorname{Int}\left(\sqrt{x-x^2} F(x), x\right)$$

```
[Out] CannotIntegrate(F(x)*(-x^2+x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{x-x^2} F(x) \, dx = \int \sqrt{x-x^2} F(x) \, dx$$

```
[In] Int[Sqrt[x - x^2]*F[x],x]
[Out] Defer[Int][Sqrt[x - x^2]*F[x], x]
```

Rubi steps

$$\text{integral} = \int \sqrt{x-x^2} F(x) \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \sqrt{x-x^2}F(x)\,dx=\int \sqrt{x-x^2}F(x)\,dx$$

```
[In] Integrate[Sqrt[x - x^2]*F[x],x]
[Out] Integrate[Sqrt[x - x^2]*F[x], x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int F(x)\sqrt{-x^2+x}dx$$

```
[In] int(F(x)*(-x^2+x)^(1/2),x)
[Out] int(F(x)*(-x^2+x)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{x-x^2}F(x)\,dx=\int \sqrt{-x^2+x}F(x)\,dx$$

```
[In] integrate(F(x)*(-x^2+x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^2 + x)*F(x), x)
```

Sympy [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{x-x^2}F(x)\,dx=\int \sqrt{-x\,(x-1)}F(x)\,dx$$

```
[In] integrate(F(x)*(-x**2+x)**(1/2),x)
[Out] Integral(sqrt(-x*(x - 1))*F(x), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{x-x^2}F(x)\,dx=\int \sqrt{-x^2+x}F(x)\,dx$$

```
[In] integrate(F(x)*(-x^2+x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + x)*F(x), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{x-x^2}F(x)\,dx=\int \sqrt{-x^2+x}F(x)\,dx$$

```
[In] integrate(F(x)*(-x^2+x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + x)*F(x), x)
```

Mupad [N/A]

Not integrable

Time = 20.75 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{x-x^2}F(x)\,dx=\int F(x)\,\sqrt{x-x^2}\,dx$$

```
[In] int(F(x)*(x - x^2)^(1/2),x)
[Out] int(F(x)*(x - x^2)^(1/2), x)
```

3.902 $\int \frac{F(x)}{\sqrt{x-x^2}} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{F(x)}{\sqrt{x-x^2}} dx = \operatorname{Int}\left(\frac{F(x)}{\sqrt{x-x^2}}, x\right)$$

```
[Out] CannotIntegrate(F(x)/(-x^2+x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{F(x)}{\sqrt{x-x^2}} dx = \int \frac{F(x)}{\sqrt{x-x^2}} dx$$

```
[In] Int[F[x]/Sqrt[x - x^2],x]
[Out] Defer[Int][F[x]/Sqrt[x - x^2], x]
```

Rubi steps

$$\text{integral} = \int \frac{F(x)}{\sqrt{x-x^2}} dx$$

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx = \int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

```
[In] Integrate[F[x]/Sqrt[x - x^2],x]
[Out] Integrate[F[x]/Sqrt[x - x^2], x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{F(x)}{\sqrt{-x^2+x}}dx$$

```
[In] int(F(x)/(-x^2+x)^(1/2),x)
[Out] int(F(x)/(-x^2+x)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx = \int \frac{F(x)}{\sqrt{-x^2+x}}\,dx$$

```
[In] integrate(F(x)/(-x^2+x)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^2 + x)*F(x)/(x^2 - x), x)
```

Sympy [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx = \int \frac{F(x)}{\sqrt{-x\,(x-1)}}\,dx$$

```
[In] integrate(F(x)/(-x**2+x)**(1/2),x)
[Out] Integral(F(x)/sqrt(-x*(x - 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx=\int \frac{F(x)}{\sqrt{-x^2+x}}\,dx$$

```
[In] integrate(F(x)/(-x^2+x)^(1/2),x, algorithm="maxima")
[Out] integrate(F(x)/sqrt(-x^2 + x), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx=\int \frac{F(x)}{\sqrt{-x^2+x}}\,dx$$

```
[In] integrate(F(x)/(-x^2+x)^(1/2),x, algorithm="giac")
[Out] integrate(F(x)/sqrt(-x^2 + x), x)
```

Mupad [N/A]

Not integrable

Time = 20.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx=\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

```
[In] int(F(x)/(x - x^2)^(1/2),x)
[Out] int(F(x)/(x - x^2)^(1/2), x)
```

3.903 $\int \sqrt{1-x}\sqrt{x}F(x)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \text{Int}\left(\sqrt{x-x^2}F(x),x\right)$$

```
[Out] CannotIntegrate(F(x)*(-x^2+x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \int \sqrt{1-x}\sqrt{x}F(x)\,dx$$

```
[In] Int[Sqrt[1 - x]*Sqrt[x]*F[x],x]
[Out] Defer[Int][Sqrt[x - x^2]*F[x], x]
```

Rubi steps

$$\text{integral} = \int \sqrt{x-x^2}F(x)\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \int \sqrt{1-x}\sqrt{x}F(x)\,dx$$

```
[In] Integrate[Sqrt[1 - x]*Sqrt[x]*F[x],x]
[Out] Integrate[Sqrt[1 - x]*Sqrt[x]*F[x], x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int F(x)\sqrt{1-x}\,\sqrt{x}dx$$

```
[In] int(F(x)*(1-x)^(1/2)*x^(1/2),x)
[Out] int(F(x)*(1-x)^(1/2)*x^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \int \sqrt{x}\sqrt{-x+1}F(x)\,dx$$

```
[In] integrate(F(x)*(1-x)^(1/2)*x^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

Sympy [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \int \sqrt{x}\sqrt{1-x}F(x)\,dx$$

```
[In] integrate(F(x)*(1-x)**(1/2)*x**(1/2),x)
[Out] Integral(sqrt(x)*sqrt(1 - x)*F(x), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \int \sqrt{x}\sqrt{-x+1}F(x)\,dx$$

```
[In] integrate(F(x)*(1-x)^(1/2)*x^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \int \sqrt{x}\sqrt{-x+1}F(x)\,dx$$

```
[In] integrate(F(x)*(1-x)^(1/2)*x^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

Mupad [N/A]

Not integrable

Time = 20.41 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \int \sqrt{x}\,F(x)\,\sqrt{1-x}\,dx$$

```
[In] int(x^(1/2)*F(x)*(1 - x)^(1/2),x)
[Out] int(x^(1/2)*F(x)*(1 - x)^(1/2), x)
```

3.904 $\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx = \text{Int}\left(\frac{F(x)}{\sqrt{x-x^2}},x\right)$$

```
[Out] CannotIntegrate(F(x)/(-x^2+x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx = \int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx$$

```
[In] Int[F[x]/(Sqrt[1 - x]*Sqrt[x]),x]
[Out] Defer[Int][F[x]/Sqrt[x - x^2], x]
```

Rubi steps

$$\text{integral} = \int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx = \int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx$$

```
[In] Integrate[F[x]/(Sqrt[1 - x]*Sqrt[x]),x]
[Out] Integrate[F[x]/(Sqrt[1 - x]*Sqrt[x]), x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{F(x)}{\sqrt{1-x}\,\sqrt{x}}dx$$

```
[In] int(F(x)/(1-x)^(1/2)/x^(1/2),x)
[Out] int(F(x)/(1-x)^(1/2)/x^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx = \int \frac{F(x)}{\sqrt{x}\sqrt{-x+1}}\,dx$$

```
[In] integrate(F(x)/(1-x)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(x)*sqrt(-x + 1)*F(x)/(x^2 - x), x)
```

Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx = \int \frac{F(x)}{\sqrt{x}\sqrt{1-x}}\,dx$$

```
[In] integrate(F(x)/(1-x)**(1/2)/x**(1/2),x)
[Out] Integral(F(x)/(sqrt(x)*sqrt(1 - x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx = \int \frac{F(x)}{\sqrt{x}\sqrt{-x+1}}\,dx$$

```
[In] integrate(F(x)/(1-x)^(1/2)/x^(1/2),x, algorithm="maxima")
[Out] integrate(F(x)/(sqrt(x)*sqrt(-x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx = \int \frac{F(x)}{\sqrt{x}\sqrt{-x+1}}\,dx$$

```
[In] integrate(F(x)/(1-x)^(1/2)/x^(1/2),x, algorithm="giac")
[Out] integrate(F(x)/(sqrt(x)*sqrt(-x + 1)), x)
```

Mupad [N/A]

Not integrable

Time = 20.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx = \int \frac{F(x)}{\sqrt{x}\,\sqrt{1-x}}\,dx$$

```
[In] int(F(x)/(x^(1/2)*(1 - x)^(1/2)),x)
[Out] int(F(x)/(x^(1/2)*(1 - x)^(1/2)), x)
```

3.905 $\int F\left(\frac{a+bx}{x}\right) dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int F\left(\frac{a+bx}{x}\right) dx = \text{Int}\left(F\left(b+\frac{a}{x}\right),x\right)$$

```
[Out] CannotIntegrate(F(b+a/x),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F\left(\frac{a+bx}{x}\right) dx = \int F\left(\frac{a+bx}{x}\right) dx$$

```
[In] Int[F[(a + b*x)/x],x]
```

```
[Out] Defer[Int][F[b + a/x], x]
```

Rubi steps

$$\text{integral} = \int F\left(b+\frac{a}{x}\right) dx$$

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{a+bx}{x}\right) dx = \int F\left(\frac{a+bx}{x}\right) dx$$

```
[In] Integrate[F[(a + b*x)/x],x]
[Out] Integrate[F[(a + b*x)/x], x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int F\left(\frac{bx+a}{x}\right) dx$$

```
[In] int(F((b*x+a)/x),x)
[Out] int(F((b*x+a)/x),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{a+bx}{x}\right) dx = \int F\left(\frac{bx+a}{x}\right) dx$$

```
[In] integrate(F((b*x+a)/x),x, algorithm="fricas")
[Out] integral(F((b*x + a)/x), x)
```

Sympy [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int F\left(\frac{a+bx}{x}\right) dx = \int F\left(\frac{a+bx}{x}\right) dx$$

```
[In] integrate(F((b*x+a)/x),x)
[Out] Integral(F((a + b*x)/x), x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{a+bx}{x}\right)\,dx=\int F\left(\frac{bx+a}{x}\right)\,dx$$

```
[In] integrate(F((b*x+a)/x),x, algorithm="maxima")
[Out] integrate(F((b*x + a)/x), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{a+bx}{x}\right)\,dx=\int F\left(\frac{bx+a}{x}\right)\,dx$$

```
[In] integrate(F((b*x+a)/x),x, algorithm="giac")
[Out] integrate(F((b*x + a)/x), x)
```

Mupad [N/A]

Not integrable

Time = 20.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{a+bx}{x}\right)\,dx=\int F\left(\frac{a+b\,x}{x}\right)\,dx$$

```
[In] int(F((a + b*x)/x),x)
[Out] int(F((a + b*x)/x), x)
```

3.906 $\int F\left(\frac{a+bx^2}{x^2}\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int F\left(\frac{a+bx^2}{x^2}\right) dx = \text{Int}\left(F\left(b+\frac{a}{x^2}\right),x\right)$$

```
[Out] CannotIntegrate(F(b+a/x^2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F\left(\frac{a+bx^2}{x^2}\right) dx = \int F\left(\frac{a+bx^2}{x^2}\right) dx$$

```
[In] Int[F[(a + b*x^2)/x^2],x]
[Out] Defer[Int][F[b + a/x^2], x]
```

Rubi steps

$$\text{integral} = \int F\left(b+\frac{a}{x^2}\right) dx$$

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{a+bx^2}{x^2}\right)\,dx=\int F\left(\frac{a+bx^2}{x^2}\right)\,dx$$

```
[In] Integrate[F[(a + b*x^2)/x^2],x]
[Out] Integrate[F[(a + b*x^2)/x^2], x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int F\left(\frac{b\,x^2+a}{x^2}\right)dx$$

```
[In] int(F((b*x^2+a)/x^2),x)
[Out] int(F((b*x^2+a)/x^2),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{a+bx^2}{x^2}\right)\,dx=\int F\left(\frac{bx^2+a}{x^2}\right)\,dx$$

```
[In] integrate(F((b*x^2+a)/x^2),x, algorithm="fricas")
[Out] integral(F((b*x^2 + a)/x^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int F\left(\frac{a+bx^2}{x^2}\right)\,dx=\int F\left(\frac{a+bx^2}{x^2}\right)dx$$

```
[In] integrate(F((b*x**2+a)/x**2),x)
[Out] Integral(F((a + b*x**2)/x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{a+bx^2}{x^2}\right)\,dx=\int F\left(\frac{bx^2+a}{x^2}\right)\,dx$$

```
[In] integrate(F((b*x^2+a)/x^2),x, algorithm="maxima")
[Out] integrate(F((b*x^2 + a)/x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{a+bx^2}{x^2}\right)\,dx=\int F\left(\frac{bx^2+a}{x^2}\right)\,dx$$

```
[In] integrate(F((b*x^2+a)/x^2),x, algorithm="giac")
[Out] integrate(F((b*x^2 + a)/x^2), x)
```

Mupad [N/A]

Not integrable

Time = 20.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{a+bx^2}{x^2}\right)\,dx=\int F\left(\frac{b\,x^2+a}{x^2}\right)\,dx$$

```
[In] int(F((a + b*x^2)/x^2),x)
[Out] int(F((a + b*x^2)/x^2), x)
```

3.907 $\int F\left(\frac{x}{a+bx}\right)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int F\left(\frac{x}{a+bx}\right)\,dx=\operatorname{Int}\left(F\left(\frac{x}{a+bx}\right),x\right)$$

```
[Out] CannotIntegrate(F(x/(b*x+a)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F\left(\frac{x}{a+bx}\right)\,dx=\int F\left(\frac{x}{a+bx}\right)\,dx$$

```
[In] Int[F[x/(a + b*x)],x]
```

```
[Out] Defer[Int][F[x/(a + b*x)], x]
```

Rubi steps

$$\text{integral}=\int F\left(\frac{x}{a+bx}\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{x}{a+bx}\right)\,dx=\int F\left(\frac{x}{a+bx}\right)\,dx$$

```
[In] Integrate[F[x/(a + b*x)],x]
[Out] Integrate[F[x/(a + b*x)], x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int F\left(\frac{x}{bx+a}\right)dx$$

```
[In] int(F(x/(b*x+a)),x)
[Out] int(F(x/(b*x+a)),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{x}{a+bx}\right)\,dx=\int F\left(\frac{x}{bx+a}\right)\,dx$$

```
[In] integrate(F(x/(b*x+a)),x, algorithm="fricas")
[Out] integral(F(x/(b*x + a)), x)
```

Sympy [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int F\left(\frac{x}{a+bx}\right)\,dx=\int F\left(\frac{x}{a+bx}\right)dx$$

```
[In] integrate(F(x/(b*x+a)),x)
[Out] Integral(F(x/(a + b*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{x}{a+bx}\right) dx = \int F\left(\frac{x}{bx+a}\right) dx$$

```
[In] integrate(F(x/(b*x+a)),x, algorithm="maxima")
[Out] integrate(F(x/(b*x + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{x}{a+bx}\right) dx = \int F\left(\frac{x}{bx+a}\right) dx$$

```
[In] integrate(F(x/(b*x+a)),x, algorithm="giac")
[Out] integrate(F(x/(b*x + a)), x)
```

Mupad [N/A]

Not integrable

Time = 20.48 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{x}{a+bx}\right) dx = \int F\left(\frac{x}{a+b\,x}\right) dx$$

```
[In] int(F(x/(a + b*x)),x)
[Out] int(F(x/(a + b*x)), x)
```

3.908 $\int F\left(\frac{x^2}{a+bx^2}\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int F\left(\frac{x^2}{a+bx^2}\right)\,dx=\operatorname{Int}\left(F\left(\frac{x^2}{a+bx^2}\right),x\right)$$

```
[Out] CannotIntegrate(F(x^2/(b*x^2+a)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F\left(\frac{x^2}{a+bx^2}\right)\,dx=\int F\left(\frac{x^2}{a+bx^2}\right)\,dx$$

```
[In] Int[F[x^2/(a + b*x^2)],x]
```

```
[Out] Defer[Int][F[x^2/(a + b*x^2)], x]
```

Rubi steps

$$\text{integral}=\int F\left(\frac{x^2}{a+bx^2}\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^2}{a+bx^2}\right) dx = \int F\left(\frac{x^2}{a+bx^2}\right) dx$$

```
[In] Integrate[F[x^2/(a + b*x^2)],x]
[Out] Integrate[F[x^2/(a + b*x^2)], x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F\left(\frac{x^2}{b\,x^2+a}\right) dx$$

```
[In] int(F(x^2/(b*x^2+a)),x)
[Out] int(F(x^2/(b*x^2+a)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^2}{a+bx^2}\right) dx = \int F\left(\frac{x^2}{bx^2+a}\right) dx$$

```
[In] integrate(F(x^2/(b*x^2+a)),x, algorithm="fricas")
[Out] integral(F(x^2/(b*x^2 + a)), x)
```

Sympy [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int F\left(\frac{x^2}{a+bx^2}\right) dx = \int F\left(\frac{x^2}{a+bx^2}\right) dx$$

```
[In] integrate(F(x**2/(b*x**2+a)),x)
[Out] Integral(F(x**2/(a + b*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^2}{a+bx^2}\right)\,dx=\int F\left(\frac{x^2}{bx^2+a}\right)\,dx$$

```
[In] integrate(F(x^2/(b*x^2+a)),x, algorithm="maxima")
[Out] integrate(F(x^2/(b*x^2 + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^2}{a+bx^2}\right)\,dx=\int F\left(\frac{x^2}{bx^2+a}\right)\,dx$$

```
[In] integrate(F(x^2/(b*x^2+a)),x, algorithm="giac")
[Out] integrate(F(x^2/(b*x^2 + a)), x)
```

Mupad [N/A]

Not integrable

Time = 21.68 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^2}{a+bx^2}\right)\,dx=\int F\left(\frac{x^2}{b\,x^2+a}\right)\,dx$$

```
[In] int(F(x^2/(a + b*x^2)),x)
[Out] int(F(x^2/(a + b*x^2)), x)
```

3.909 $\int F\left(\frac{x^2}{(a+bx)^2}\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx = \text{Int}\left(F\left(\frac{x^2}{(a+bx)^2}\right), x\right)$$

```
[Out] CannotIntegrate(F(x^2/(b*x+a)^2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx = \int F\left(\frac{x^2}{(a+bx)^2}\right) dx$$

```
[In] Int[F[x^2/(a + b*x)^2],x]
[Out] Defer[Int][F[x^2/(a + b*x)^2], x]
```

Rubi steps

$$\text{integral} = \int F\left(\frac{x^2}{(a+bx)^2}\right) dx$$

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{x^2}{(a+bx)^2}\right)\,dx=\int F\left(\frac{x^2}{(a+bx)^2}\right)\,dx$$

```
[In] Integrate[F[x^2/(a + b*x)^2],x]
[Out] Integrate[F[x^2/(a + b*x)^2], x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int F\left(\frac{x^2}{\left(bx+a\right)^2}\right)dx$$

```
[In] int(F(x^2/(b*x+a)^2),x)
[Out] int(F(x^2/(b*x+a)^2),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int F\left(\frac{x^2}{(a+bx)^2}\right)\,dx=\int F\left(\frac{x^2}{\left(bx+a\right)^2}\right)\,dx$$

```
[In] integrate(F(x^2/(b*x+a)^2),x, algorithm="fricas")
[Out] integral(F(x^2/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

Sympy [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int F\left(\frac{x^2}{(a+bx)^2}\right)\,dx=\int F\left(\frac{x^2}{\left(a+bx\right)^2}\right)dx$$

```
[In] integrate(F(x**2/(b*x+a)**2),x)
[Out] Integral(F(x**2/(a + b*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx = \int F\left(\frac{x^2}{(bx+a)^2}\right) dx$$

```
[In] integrate(F(x^2/(b*x+a)^2),x, algorithm="maxima")
[Out] integrate(F(x^2/(b*x + a)^2), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx = \int F\left(\frac{x^2}{(bx+a)^2}\right) dx$$

```
[In] integrate(F(x^2/(b*x+a)^2),x, algorithm="giac")
[Out] integrate(F(x^2/(b*x + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 21.46 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx = \int F\left(\frac{x^2}{(a+b\,x)^2}\right) dx$$

```
[In] int(F(x^2/(a + b*x)^2),x)
[Out] int(F(x^2/(a + b*x)^2), x)
```

3.910 $\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx = \text{Int}\left(F\left(\frac{x^4}{(a+bx^2)^2}\right), x\right)$$

```
[Out] CannotIntegrate(F(x^4/(b*x^2+a)^2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx = \int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$

```
[In] Int[F[x^4/(a + b*x^2)^2],x]
```

```
[Out] Defer[Int][F[x^4/(a + b*x^2)^2], x]
```

Rubi steps

$$\text{integral} = \int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right)\,dx=\int F\left(\frac{x^4}{(a+bx^2)^2}\right)\,dx$$

```
[In] Integrate[F[x^4/(a + b*x^2)^2],x]
[Out] Integrate[F[x^4/(a + b*x^2)^2], x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F\left(\frac{x^4}{(b\,x^2+a)^2}\right)dx$$

```
[In] int(F(x^4/(b*x^2+a)^2),x)
[Out] int(F(x^4/(b*x^2+a)^2),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right)\,dx=\int F\left(\frac{x^4}{(bx^2+a)^2}\right)\,dx$$

```
[In] integrate(F(x^4/(b*x^2+a)^2),x, algorithm="fricas")
[Out] integral(F(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)), x)
```

Sympy [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right)\,dx=\int F\left(\frac{x^4}{(a+bx^2)^2}\right)dx$$

```
[In] integrate(F(x**4/(b*x**2+a)**2),x)
[Out] Integral(F(x**4/(a + b*x**2)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right)\,dx=\int F\left(\frac{x^4}{(bx^2+a)^2}\right)\,dx$$

```
[In] integrate(F(x^4/(b*x^2+a)^2),x, algorithm="maxima")
[Out] integrate(F(x^4/(b*x^2 + a)^2), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right)\,dx=\int F\left(\frac{x^4}{(bx^2+a)^2}\right)\,dx$$

```
[In] integrate(F(x^4/(b*x^2+a)^2),x, algorithm="giac")
[Out] integrate(F(x^4/(b*x^2 + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 19.76 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right)\,dx=\int F\left(\frac{x^4}{(b\,x^2+a)^2}\right)\,dx$$

```
[In] int(F(x^4/(a + b*x^2)^2),x)
[Out] int(F(x^4/(a + b*x^2)^2), x)
```

3.911 $\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} \, dx$

Optimal result

Integrand size = 37, antiderivative size = 47

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)*b^(1/2)/(b*x^2+(b^2*x^4+a)^(1/2))^(1/2))*2^(1/2)/b^(1
/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2157, 212}

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

```
[In] Int[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[
b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2157

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{1-2bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+\sqrt{a+b^2x^4}}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \frac{\log\left(i\sqrt{b}\left(bx^2+\sqrt{a+b^2x^4}+\sqrt{2}\sqrt{b}x\sqrt{bx^2+\sqrt{a+b^2x^4}}\right)\right)}{\sqrt{2}\sqrt{b}}$$

```
[In] Integrate[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
[Out] Log[I*Sqrt[b]*(b*x^2 + Sqrt[a + b^2*x^4] + Sqrt[2]*Sqrt[b]*x*Sqrt[b*x^2 + S
qrt[a + b^2*x^4]])]/(Sqrt[2]*Sqrt[b])
```

Maple [F]

$$\int \frac{\sqrt{b\,x^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}dx$$

```
[In] int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
[Out] int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 1.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.87

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \left[\frac{\sqrt{2}\log\left(4\,b^2x^4+4\,\sqrt{b^2x^4+a}bx^2+2\left(\sqrt{2}b^{\frac{3}{2}}x^3+\sqrt{2}\sqrt{b^2x^4+a}\sqrt{b}x\right)\sqrt{bx^2+\sqrt{b^2x^4+a}}+a\right)}{4\,\sqrt{b}},\ -\frac{1}{2}\,\sqrt{2}\sqrt{-\frac{1}{b}}\arctan\left(\frac{\sqrt{2}\sqrt{bx^2+\sqrt{b^2x^4+a}}\sqrt{-\frac{1}{b}}}{2\,x}\right)\right]$$

```
[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/4*sqrt(2)*log(4*b^2*x^4 + 4*sqrt(b^2*x^4 + a)*b*x^2 + 2*(sqrt(2)*b^(3/2)
*x^3 + sqrt(2)*sqrt(b^2*x^4 + a)*sqrt(b)*x)*sqrt(b*x^2 + sqrt(b^2*x^4 + a))
 + a)/sqrt(b), -1/2*sqrt(2)*sqrt(-1/b)*arctan(1/2*sqrt(2)*sqrt(b*x^2 + sqrt
(b^2*x^4 + a))*sqrt(-1/b)/x)]
```

Sympy [F]

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$$

```
[In] integrate((b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \int \frac{\sqrt{bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

```
[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \int \frac{\sqrt{bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

```
[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="g
iac")
[Out] integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \int \frac{\sqrt{\sqrt{b^2\,x^4+a}+b\,x^2}}{\sqrt{b^2\,x^4+a}}\,dx$$

```
[In] int(((a + b^2*x^4)^(1/2) + b*x^2)^(1/2)/(a + b^2*x^4)^(1/2),x)
[Out] int(((a + b^2*x^4)^(1/2) + b*x^2)^(1/2)/(a + b^2*x^4)^(1/2), x)
```

3.912 $\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 48

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}$$

```
[Out] 1/2*arctan(x*2^(1/2)*b^(1/2)/(-b*x^2+(b^2*x^4+a)^(1/2))^(1/2))*2^(1/2)/b^(1
/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2157, 209}

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

```
[In] Int[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqr
t[b])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2157

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{1+2bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \frac{i\log\left(-ib^{3/2}x^2+i\sqrt{b}\sqrt{a+b^2x^4}+\sqrt{2}bx\sqrt{-bx^2+\sqrt{a+b^2x^4}}\right)}{\sqrt{2}\sqrt{b}}$$

```
[In] Integrate[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
[Out] (I*Log[(-I)*b^(3/2)*x^2 + I*Sqrt[b]*Sqrt[a + b^2*x^4] + Sqrt[2]*b*x*Sqrt[-(
b*x^2) + Sqrt[a + b^2*x^4]]])/(Sqrt[2]*Sqrt[b])
```

Maple [F]

$$\int \frac{\sqrt{-b\,x^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}dx$$

```
[In] int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
[Out] int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 1.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.04

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \left[\frac{1}{4}\,\sqrt{2}\sqrt{-\frac{1}{b}}\log\left(4\,b^2x^4-4\,\sqrt{b^2x^4+a}bx^2 + 2\left(\sqrt{2}b^2x^3\sqrt{-\frac{1}{b}}-\sqrt{2}\sqrt{b^2x^4+a}bx\sqrt{-\frac{1}{b}}\right)\sqrt{-bx^2+\sqrt{b^2x^4+a}}+a\right), -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-bx^2+\sqrt{b^2x^4+a}}}{2\sqrt{b}x}\right)}{2\sqrt{b}}\right]$$

```
[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(-1/b)*log(4*b^2*x^4 - 4*sqrt(b^2*x^4 + a)*b*x^2 + 2*(sqrt
(2)*b^2*x^3*sqrt(-1/b) - sqrt(2)*sqrt(b^2*x^4 + a)*b*x*sqrt(-1/b))*sqrt(-b*
x^2 + sqrt(b^2*x^4 + a)) + a), -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-b*x^2
+ sqrt(b^2*x^4 + a))/(sqrt(b)*x))/sqrt(b)]
```

Sympy [F]

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$$

```
[In] integrate((-b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \int \frac{\sqrt{-bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

```
[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx=\int \frac{\sqrt{-bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

```
[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx=\int \frac{\sqrt{\sqrt{b^2\,x^4+a}-b\,x^2}}{\sqrt{b^2\,x^4+a}}\,dx$$

```
[In] int(((a + b^2*x^4)^(1/2) - b*x^2)^(1/2)/(a + b^2*x^4)^(1/2),x)
```

```
[Out] int(((a + b^2*x^4)^(1/2) - b*x^2)^(1/2)/(a + b^2*x^4)^(1/2), x)
```

3.913 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 169

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx = \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\arctan\left(\frac{\sqrt{3}d+2icx}{\sqrt{2ic^2-\sqrt{3}d^2}\sqrt{\sqrt{3}-2ix^2}}\right)}{\sqrt{2ic^2-\sqrt{3}d^2}} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{3}d-2icx}{\sqrt{2ic^2+\sqrt{3}d^2}\sqrt{\sqrt{3}+2ix^2}}\right)}{\sqrt{2ic^2+\sqrt{3}d^2}}$$

```
[Out] (1/2-1/2*I)*arctan((2*I*c*x+d*3^(1/2))/(-2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2-d^
2*3^(1/2))^(1/2))/(2*I*c^2-d^2*3^(1/2))^(1/2)-(1/2+1/2*I)*arctanh((-2*I*c*x
+d*3^(1/2))/(2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2+d^2*3^(1/2))^(1/2))/(2*I*c^2+d
^2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2158, 739, 210, 212}

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx = \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\arctan\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}d^2+2ic^2}} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{\sqrt{3}d^2+2ic^2}}$$

```
[In] Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]),x]
```

```
[Out] ((1/2 - I/2)*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - Sqrt[3]*d^2]*
Sqrt[Sqrt[3] - (2*I)*x^2])])/Sqrt[(2*I)*c^2 - Sqrt[3]*d^2] - ((1/2 + I/2)*A
rcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*Sqrt[Sqrt[3]
+ (2*I)*x^2])])/Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2158

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}-2ix^2}}\,dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}+2ix^2}}\,dx\\
&= \left(-\frac{1}{2}-\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{2ic^2+\sqrt{3}d^2-x^2}\,dx,x,\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}}\right)\\
&\quad+\left(-\frac{1}{2}+\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{-2ic^2+\sqrt{3}d^2-x^2}\,dx,x,\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}}\right)\\
&= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{2ic^2-\sqrt{3}d^2}\sqrt{\sqrt{3}-2ix^2}}\right)}{\sqrt{2ic^2-\sqrt{3}d^2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{\sqrt{3}d-2icx}{\sqrt{2ic^2+\sqrt{3}d^2}\sqrt{\sqrt{3}+2ix^2}}\right)}{\sqrt{2ic^2+\sqrt{3}d^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.89 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.15

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx = -\frac{\sqrt{-2c^2-\sqrt{4c^4+3d^4}}\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}\right)}{\sqrt{4c^4+3d^4}} + \frac{\sqrt{-2c^2+\sqrt{4c^4+3d^4}}\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2+\sqrt{4c^4+3d^4}}}\right)}{\sqrt{4c^4+3d^4}} + c\text{RootSum}\Bigg[9d^2-24c^2\#1-6d^2\#1^2-8c^2\#1^3 + d^2\#1^4\&, \frac{-3\log\left(2x^2+\sqrt{3+4x^4}+2x\sqrt{2x^2+\sqrt{3+4x^4}}-\#1\right)-\log\left(2x^2+\sqrt{3+4x^4}+2x\sqrt{2x^2+\sqrt{3+4x^4}}-\#1\right)\#1^2}{-6c^2-3d^2\#1-6c^2\#1^2+d^2\#1^3}\&\Bigg]$$

```
[In] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]),x]
```

```
[Out] -((Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4
]])/Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]])/Sqrt[4*c^4 + 3*d^4]) + (Sqrt[-2*c^
2 + Sqrt[4*c^4 + 3*d^4]]*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[-2*c
^2 + Sqrt[4*c^4 + 3*d^4]]])/Sqrt[4*c^4 + 3*d^4] + c*RootSum[9*d^2 - 24*c^2*
#1 - 6*d^2*#1^2 - 8*c^2*#1^3 + d^2*#1^4 & , (-3*Log[2*x^2 + Sqrt[3 + 4*x^4]
 + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1] - Log[2*x^2 + Sqrt[3 + 4*x^4] +
2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1]*#1^2)/(-6*c^2 - 3*d^2*#1 - 6*c^2*#1
^2 + d^2*#1^3) & ]
```

Maple [F]

$$\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{(dx+c)\sqrt{4x^4+3}}dx$$

```
[In] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)
```

```
[Out] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx=\text{Timed out}$$

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx=\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{(c+dx)\sqrt{4x^4+3}}\,dx$$

```
[In] integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)/(4*x**4+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/((c + d*x)*sqrt(4*x**4 + 3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx=\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}(dx+c)}\,dx$$

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx=\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}(dx+c)}\,dx$$

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx = \int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,\left(c+d\,x\right)}\,dx$$

```
[In] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)),x)
[Out] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)), x)
```

3.914 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 268

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx = \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}d^2\right)(c+dx)} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}d^2\right)(c+dx)} + \frac{(1+i)c\arctan\left(\frac{\sqrt{3}d+2icx}{\sqrt{2ic^2-\sqrt{3}d^2}\sqrt{\sqrt{3}-2ix^2}}\right)}{\left(2ic^2-\sqrt{3}d^2\right)^{3/2}} + \frac{(1-i)c\,\text{arctanh}\left(\frac{\sqrt{3}d-2icx}{\sqrt{2ic^2+\sqrt{3}d^2}\sqrt{\sqrt{3}+2ix^2}}\right)}{\left(2ic^2+\sqrt{3}d^2\right)^{3/2}}$$

```
[Out] (1+I)*c*arctan((2*I*c*x+d*3^(1/2))/(-2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2-d^2*3^
(1/2))^(1/2))/(2*I*c^2-d^2*3^(1/2))^(3/2)+(1-I)*c*arctanh((-2*I*c*x+d*3^(1/
2))/(2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2+d^2*3^(1/2))^(1/2))/(2*I*c^2+d^2*3^(1/
2))^(3/2)+(1/2-1/2*I)*d*(-2*I*x^2+3^(1/2))^(1/2)/(d*x+c)/(2*I*c^2-d^2*3^(1/
2))-(1/2+1/2*I)*d*(2*I*x^2+3^(1/2))^(1/2)/(d*x+c)/(2*I*c^2+d^2*3^(1/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{2158, 745, 739, 210, 212\}$

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx = \frac{(1+i)c\arctan\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\left(-\sqrt{3}d^2+2ic^2\right)^{3/2}} + \frac{(1-i)c\,\text{arctanh}\left(\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}d^2+2ic^2}}\right)}{\left(\sqrt{3}d^2+2ic^2\right)^{3/2}} + \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(-\sqrt{3}d^2+2ic^2\right)(c+dx)} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(\sqrt{3}d^2+2ic^2\right)(c+dx)}$$

```
[In] Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]),x]
```

```
[Out] ((1/2 - I/2)*d*Sqrt[Sqrt[3] - (2*I)*x^2])/(((2*I)*c^2 - Sqrt[3]*d^2)*(c + d
*x)) - ((1/2 + I/2)*d*Sqrt[Sqrt[3] + (2*I)*x^2])/(((2*I)*c^2 + Sqrt[3]*d^2)
*(c + d*x)) + ((1 + I)*c*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - S
qrt[3]*d^2]*Sqrt[Sqrt[3] - (2*I)*x^2])])/((2*I)*c^2 - Sqrt[3]*d^2)^(3/2) +
((1 - I)*c*ArcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*S
qrt[Sqrt[3] + (2*I)*x^2])])/((2*I)*c^2 + Sqrt[3]*d^2)^(3/2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2158

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(c+dx)^2\sqrt{\sqrt{3}-2ix^2}}\,dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(c+dx)^2\sqrt{\sqrt{3}+2ix^2}}\,dx \\
&= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}d^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}d^2\right)(c+dx)} \\
&\quad+\frac{((1+i)c)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}+2ix^2}}\,dx}{2c^2-i\sqrt{3}d^2}+\frac{((1-i)c)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}-2ix^2}}\,dx}{2c^2+i\sqrt{3}d^2} \\
&= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}d^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}d^2\right)(c+dx)} \\
&\quad+-\frac{((1+i)c)\text{Subst}\left(\int \frac{1}{2ic^2+\sqrt{3}d^2-x^2}\,dx,x,\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}}\right)}{2c^2-i\sqrt{3}d^2} \\
&\quad+-\frac{((1-i)c)\text{Subst}\left(\int \frac{1}{-2ic^2+\sqrt{3}d^2-x^2}\,dx,x,\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}}\right)}{2c^2+i\sqrt{3}d^2} \\
&= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}d^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}d^2\right)(c+dx)} \\
&\quad+\frac{(1+i)c\tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{2ic^2-\sqrt{3}d^2}\sqrt{\sqrt{3}-2ix^2}}\right)}{\left(2ic^2-\sqrt{3}d^2\right)^{3/2}}+\frac{(1-i)c\tanh^{-1}\left(\frac{\sqrt{3}d-2icx}{\sqrt{2ic^2+\sqrt{3}d^2}\sqrt{\sqrt{3}+2ix^2}}\right)}{\left(2ic^2+\sqrt{3}d^2\right)^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.42 (sec) , antiderivative size = 1464, normalized size of antiderivative = 5.46

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$$
$$= -\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}\left(3d^2\left(2x^2+\sqrt{3+4x^4}\right)+2c^2\left(3+8x^4+4x^2\sqrt{3+4x^4}\right)\right)}{\left(4c^4+3d^4\right)(c+dx)\left(3+8x^4+4x^2\sqrt{3+4x^4}\right)}$$
$$+\frac{8c^5\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}\right)}{\left(4c^4+3d^4\right)^{3/2}\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}-\frac{6cd^4\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}\right)}{\left(4c^4+3d^4\right)^{3/2}\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}$$
$$+\frac{4c^3\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}\right)}{\left(4c^4+3d^4\right)\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}-\frac{8c^5\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2+\sqrt{4c^4+3d^4}}}\right)}{\left(4c^4+3d^4\right)^{3/2}\sqrt{-2c^2+\sqrt{4c^4+3d^4}}}$$
$$+\frac{6cd^4\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2+\sqrt{4c^4+3d^4}}}\right)}{\left(4c^4+3d^4\right)^{3/2}\sqrt{-2c^2+\sqrt{4c^4+3d^4}}}+\frac{4c^3\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2+\sqrt{4c^4+3d^4}}}\right)}{\left(4c^4+3d^4\right)\sqrt{-2c^2+\sqrt{4c^4+3d^4}}}$$
$$-\frac{\text{RootSum}\Big[9d^2-24c^2\#1-6d^2\#1^2-8c^2\#1^3+d^2\#1^4\&,\ \frac{128c^4\log\left(2x^2+\sqrt{3+4x^4}+2x\sqrt{2x^2+\sqrt{3+4x^4}}-\#1\right)+3d^4\log\left(2\right.}{}}{}$$
$$+\frac{\text{RootSum}\Big[9d^2-24c^2\#1-6d^2\#1^2-8c^2\#1^3+d^2\#1^4\&,\ \frac{512c^8\log\left(2x^2+\sqrt{3+4x^4}+2x\sqrt{2x^2+\sqrt{3+4x^4}}-\#1\right)+408c^4d^4\,\mathrm{l}}{}}{}$$

```
[In] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]),x]
```

```
[Out] -((d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]*(3*d^2*(2*x^2 + Sqrt[3 + 4*x^4]) + 2*c^2
*(3 + 8*x^4 + 4*x^2*Sqrt[3 + 4*x^4])))/((4*c^4 + 3*d^4)*(c + d*x)*(3 + 8*x^
4 + 4*x^2*Sqrt[3 + 4*x^4]))) + (8*c^5*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4
]])/Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]])/((4*c^4 + 3*d^4)^(3/2)*Sqrt[-2*c^2
 - Sqrt[4*c^4 + 3*d^4]]) - (6*c*d^4*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]
)/Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]])/((4*c^4 + 3*d^4)^(3/2)*Sqrt[-2*c^2 -
 Sqrt[4*c^4 + 3*d^4]]) + (4*c^3*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sq
rt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]])/((4*c^4 + 3*d^4)*Sqrt[-2*c^2 - Sqrt[4*c^
4 + 3*d^4]]) - (8*c^5*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[-2*c^2
+ Sqrt[4*c^4 + 3*d^4]]])/((4*c^4 + 3*d^4)^(3/2)*Sqrt[-2*c^2 + Sqrt[4*c^4 +
3*d^4]]) + (6*c*d^4*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[-2*c^2 +
Sqrt[4*c^4 + 3*d^4]]])/((4*c^4 + 3*d^4)^(3/2)*Sqrt[-2*c^2 + Sqrt[4*c^4 + 3*
d^4]]) + (4*c^3*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[-2*c^2 + Sqrt
[4*c^4 + 3*d^4]]])/((4*c^4 + 3*d^4)*Sqrt[-2*c^2 + Sqrt[4*c^4 + 3*d^4]]) - R
ootSum[9*d^2 - 24*c^2*#1 - 6*d^2*#1^2 - 8*c^2*#1^3 + d^2*#1^4 & , (128*c^4*
Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1] + 3*d
^4*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1] +
16*c^2*d^2*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]
- #1]*#1 + d^4*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^
```

```
4]] - #1]*#1^2)/(6*c^2 + 3*d^2*#1 + 6*c^2*#1^2 - d^2*#1^3) & ]/d^4 + RootSu
m[9*d^2 - 24*c^2*#1 - 6*d^2*#1^2 - 8*c^2*#1^3 + d^2*#1^4 & , (512*c^8*Log[2
*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1] + 408*c^4*
d^4*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1] +
 9*d^8*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1
] + 64*c^6*d^2*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^
4]] - #1]*#1 + 36*c^2*d^6*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sq
rt[3 + 4*x^4]] - #1]*#1 + 8*c^4*d^4*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[
2*x^2 + Sqrt[3 + 4*x^4]] - #1]*#1^2 + 3*d^8*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2
*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1]*#1^2)/(6*c^2 + 3*d^2*#1 + 6*c^2*#1^2
 - d^2*#1^3) & ]/(4*c^4*d^4 + 3*d^8)
```

Maple [F]

$$\int \frac{\sqrt{2x^{2}+\sqrt{4x^{4}+3}}}{\left(dx+c\right)^{2}\sqrt{4x^{4}+3}}dx$$

```
[In] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)
[Out] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx=\text{Timed out}$$

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algori
thm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx=\int \frac{\sqrt{2x^{2}+\sqrt{4x^{4}+3}}}{\left(c+dx\right)^{2}\sqrt{4x^{4}+3}}\,dx$$

```
[In] integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)**2/(4*x**4+3)**(1/2),x)
[Out] Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/((c + d*x)**2*sqrt(4*x**4 + 3)), x
)
```

Maxima [F]

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx=\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}(dx+c)^2}\,dx$$

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algori
thm="maxima")
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2), x)
```

Giac [F]

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx=\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}(dx+c)^2}\,dx$$

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algori
thm="giac")
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx=\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,(c+d\,x)^2}\,dx$$

```
[In] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)^2),x)
[Out] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)^2), x)
```

3.915 $\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+30\arctan\left(\sqrt[6]{x}\right)$$

```
[Out] -30*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+30*arctan(x^(1/6))+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1854, 1634, 52, 65, 209}

$$\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = 30\arctan\left(\sqrt[6]{x}\right)+\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-30\sqrt[6]{x}$$

```
[In] Int[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] -30*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 30*ArcTan[x^(1/6)
]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1854

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g =
Denominator[n]}, Dist[g, Subst[Int[x^(g*(m + 1) - 1)*(Pq /. x -> x^g)*(a +
b*x^(g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, b, m, p}, x] && PolyQ[Pq, x
] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{\sqrt{x}(-4+x^3)}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= 3\text{Subst}\left(\int \left(\sqrt{x}-x^{3/2}+x^{5/2}-\frac{5\sqrt{x}}{1+x}\right)\,dx, x, \sqrt[3]{x}\right)\\
&= 2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}-15\text{Subst}\left(\int \frac{\sqrt{x}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+15\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x)}\,dx, x, \sqrt[3]{x}\right)\\
&= -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+30\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+30\tan^{-1}\left(\sqrt[6]{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{2}{35}\left(-525\sqrt[6]{x}+35\sqrt{x}-21x^{5/6}+15x^{7/6}\right)+30\arctan\left(\sqrt[6]{x}\right)$$

```
[In] Integrate[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]
[Out] (2*(-525*x^(1/6) + 35*Sqrt[x] - 21*x^(5/6) + 15*x^(7/6)))/35 + 30*ArcTan[x^
(1/6)]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$-30x^{\frac{1}{6}}-\frac{6x^{\frac{5}{6}}}{5}+\frac{6x^{\frac{7}{6}}}{7}+30\arctan\left(x^{\frac{1}{6}}\right)+2\sqrt{x}$	28
default	$-30x^{\frac{1}{6}}-\frac{6x^{\frac{5}{6}}}{5}+\frac{6x^{\frac{7}{6}}}{7}+30\arctan\left(x^{\frac{1}{6}}\right)+2\sqrt{x}$	28
meijerg	$-\frac{2x^{\frac{1}{6}}\left(-45x+63x^{\frac{2}{3}}-105x^{\frac{1}{3}}+315\right)}{105}+30\arctan\left(x^{\frac{1}{6}}\right)-24x^{\frac{1}{6}}$	33

```
[In] int((x-4)/(1+x^(1/3))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -30*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+30*arctan(x^(1/6))+2*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{6}{7}\,(x-35)x^{\frac{1}{6}}-\frac{6}{5}\,x^{\frac{5}{6}}+2\,\sqrt{x}+30\,\arctan\left(x^{\frac{1}{6}}\right)$$

```
[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="fricas")
[Out] 6/7*(x - 35)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 30*arctan(x^(1/6))
```

Sympy [A] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} - 30\sqrt[6]{x} + 2\sqrt{x} + 30\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

```
[In] integrate((-4+x)/(1+x**(1/3))/x**(1/2),x)
[Out] 6*x**(7/6)/7 - 6*x**(5/6)/5 - 30*x**(1/6) + 2*sqrt(x) + 30*atan(x**(1/6))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{6}{7}\,x^{\frac{7}{6}} - \frac{6}{5}\,x^{\frac{5}{6}} + 2\,\sqrt{x} - 30\,x^{\frac{1}{6}} + 30\,\arctan\left(x^{\frac{1}{6}}\right)$$

```
[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="maxima")
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 30*x^(1/6) + 30*arctan(x^(1/6))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{6}{7}\,x^{\frac{7}{6}} - \frac{6}{5}\,x^{\frac{5}{6}} + 2\,\sqrt{x} - 30\,x^{\frac{1}{6}} + 30\,\arctan\left(x^{\frac{1}{6}}\right)$$

```
[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="giac")
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 30*x^(1/6) + 30*arctan(x^(1/6))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = 30\,\mathrm{atan}\left(x^{1/6}\right) + 2\,\sqrt{x} - 30\,x^{1/6} - \frac{6\,x^{5/6}}{5} + \frac{6\,x^{7/6}}{7}$$

```
[In] int((x - 4)/(x^(1/2)*(x^(1/3) + 1)),x)
[Out] 30*atan(x^(1/6)) + 2*x^(1/2) - 30*x^(1/6) - (6*x^(5/6))/5 + (6*x^(7/6))/7
```

3.916 $\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}} \, dx = 3\sqrt[3]{x}+6\arctan\left(\sqrt[6]{x}\right)-3\log\left(1+\sqrt[3]{x}\right)$$

```
[Out] 3*x^(1/3)+6*arctan(x^(1/6))-3*ln(1+x^(1/3))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1607, 1833, 1824, 649, 209, 266}

$$\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}} \, dx = 6\arctan\left(\sqrt[6]{x}\right)+3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)$$

```
[In] Int[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]
```

```
[Out] 3*x^(1/3) + 6*ArcTan[x^(1/6)] - 3*Log[1 + x^(1/3)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right) x^{5/6}}\,dx\\
&= 6\text{Subst}\left(\int \frac{1+x^3}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= 6\text{Subst}\left(\int \left(x+\frac{1-x}{1+x^2}\right)\,dx, x, \sqrt[6]{x}\right)\\
&= 3\sqrt[3]{x}+6\text{Subst}\left(\int \frac{1-x}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= 3\sqrt[3]{x}+6\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)-6\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= 3\sqrt[3]{x}+6\tan^{-1}\left(\sqrt[6]{x}\right)-3\log\left(1+\sqrt[3]{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx = 3\sqrt[3]{x}+6\arctan\left(\sqrt[6]{x}\right)-3\log\left(1+\sqrt[3]{x}\right)$$

```
[In] Integrate[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]
[Out] 3*x^(1/3) + 6*ArcTan[x^(1/6)] - 3*Log[1 + x^(1/3)]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$3x^{\frac{1}{3}}+6\arctan\left(x^{\frac{1}{6}}\right)-3\ln\left(1+x^{\frac{1}{3}}\right)$$

```
[In] int((1+x^(1/2))/(x^(5/6)+x^(7/6)),x)
[Out] 3*x^(1/3)+6*arctan(x^(1/6))-3*ln(1+x^(1/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx = 3\,x^{\frac{1}{3}}+6\,\arctan\left(x^{\frac{1}{6}}\right)-3\,\log\left(x^{\frac{1}{3}}+1\right)$$

```
[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="fricas")
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx = 3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)+6\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

```
[In] integrate((1+x**(1/2))/(x**(5/6)+x**(7/6)),x)
[Out] 3*x**(1/3) - 3*log(x**(1/3) + 1) + 6*atan(x**(1/6))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx = 3\,x^{\frac{1}{3}} + 6\,\arctan\left(x^{\frac{1}{6}}\right) - 3\,\log\left(x^{\frac{1}{3}} + 1\right)$$

```
[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="maxima")
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx = 3\,x^{\frac{1}{3}} + 6\,\arctan\left(x^{\frac{1}{6}}\right) - 3\,\log\left(x^{\frac{1}{3}} + 1\right)$$

```
[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="giac")
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx = 6\,\mathrm{atan}\left(x^{1/6}\right) - 3\,\ln\left(36\,x^{1/3} + 36\right) + 3\,x^{1/3}$$

```
[In] int((x^(1/2) + 1)/(x^(5/6) + x^(7/6)),x)
[Out] 6*atan(x^(1/6)) - 3*log(36*x^(1/3) + 36) + 3*x^(1/3)
```

3.917 $\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 42

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\arctan\left(\sqrt[6]{x}\right)+3\log\left(1+\sqrt[3]{x}\right)$$

```
[Out] 6*x^(1/6)-3*x^(1/3)+3/2*x^(2/3)-6*arctan(x^(1/6))+3*ln(1+x^(1/3))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6820, 1607, 1816, 649, 209, 266}

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = -6\arctan\left(\sqrt[6]{x}\right)+\frac{3x^{2/3}}{2}-3\sqrt[3]{x}+6\sqrt[6]{x}+3\log\left(\sqrt[3]{x}+1\right)$$

```
[In] Int[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 - 6*ArcTan[x^(1/6)] + 3*Log[1 + x^(1/
3)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1+\frac{1}{\sqrt{x}}}{1+\sqrt[3]{x}}\,dx \\
&= 6\text{Subst}\left(\int \frac{x^2+x^5}{1+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= 6\text{Subst}\left(\int \frac{x^2(1+x^3)}{1+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= 6\text{Subst}\left(\int \left(1-x+x^3-\frac{1-x}{1+x^2}\right)\,dx, x, \sqrt[6]{x}\right) \\
&= 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\text{Subst}\left(\int \frac{1-x}{1+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)+6\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\tan^{-1}\left(\sqrt[6]{x}\right)+3\log\left(1+\sqrt[3]{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{3}{2}\left(4-2\sqrt[6]{x}+\sqrt{x}\right)\sqrt[6]{x}-6\arctan\left(\sqrt[6]{x}\right)+3\log\left(1+\sqrt[3]{x}\right)$$

```
[In] Integrate[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] (3*(4 - 2*x^(1/6) + Sqrt[x])*x^(1/6))/2 - 6*ArcTan[x^(1/6)] + 3*Log[1 + x^(
1/3)]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

method	result	s
derivativedivides	$6x^{\frac{1}{6}}-3x^{\frac{1}{3}}+\frac{3x^{\frac{2}{3}}}{2}-6\arctan\left(x^{\frac{1}{6}}\right)+3\ln\left(1+x^{\frac{1}{3}}\right)$	3
meijerg	$6x^{\frac{1}{6}}-6\arctan\left(x^{\frac{1}{6}}\right)-\frac{x^{\frac{1}{3}}\left(-3x^{\frac{1}{3}}+6\right)}{2}+3\ln\left(1+x^{\frac{1}{3}}\right)$	3
default	$\ln\left(x+1\right)+\frac{3x^{\frac{2}{3}}}{2}-\ln\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right)+2\ln\left(1+x^{\frac{1}{3}}\right)-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\arctan\left(x^{\frac{1}{6}}\right)$	4

```
[In] int((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6*x^(1/6)-3*x^(1/3)+3/2*x^(2/3)-6*arctan(x^(1/6))+3*ln(1+x^(1/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}}-3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}-6\,\arctan\left(x^{\frac{1}{6}}\right)+3\,\log\left(x^{\frac{1}{3}}+1\right)$$

```
[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1
)
```

Sympy [A] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = 6\sqrt[6]{x} + \frac{3x^{\frac{2}{3}}}{2} - 3\sqrt[3]{x} + 3\log\left(\sqrt[3]{x}+1\right) - 6\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

```
[In] integrate((1+x**(1/2))/(1+x**(1/3))/x**(1/2),x)
[Out] 6*x**(1/6) + 3*x**(2/3)/2 - 3*x**(1/3) + 3*log(x**(1/3) + 1) - 6*atan(x**(1
/6))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\arctan\left(x^{\frac{1}{6}}\right) + 3\,\log\left(x^{\frac{1}{3}}+1\right)$$

```
[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="maxima")
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1
)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\arctan\left(x^{\frac{1}{6}}\right) + 3\,\log\left(x^{\frac{1}{3}}+1\right)$$

```
[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="giac")
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1
)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{3\,x^{2/3}}{2} + 3\,\ln\left(\left(-6+x^{1/6}\,6\mathrm{i}\right)\,\left(6+x^{1/6}\,6\mathrm{i}\right)\right) - 3\,x^{1/3} - 6\,\mathrm{atan}\left(x^{1/6}\right) + 6\,x^{1/6}$$

```
[In] int((x^(1/2) + 1)/(x^(1/2)*(x^(1/3) + 1)),x)
```

```
[Out] 3*log((x^(1/6)*6i - 6)*(x^(1/6)*6i + 6)) - 6*atan(x^(1/6)) - 3*x^(1/3) + (3
*x^(2/3))/2 + 6*x^(1/6)
```

3.918 $\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2} dx = -\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

```
[Out] -arccsch(x*2^(1/2)/b^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {25, 342, 221}

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2} dx = -\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

```
[In] Int[Sqrt[2 + b/x^2]/(b + 2*x^2),x]
```

```
[Out] -(ArcCsch[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{2+\frac{b}{x^2}}x^2}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\text{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx = -\frac{\sqrt{2+\frac{b}{x^2}}x\text{arctanh}\left(\frac{\sqrt{b+2x^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+2x^2}}$$

```
[In] Integrate[Sqrt[2 + b/x^2]/(b + 2*x^2),x]
[Out] -((Sqrt[2 + b/x^2]*x*ArcTanh[Sqrt[b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[b + 2*
x^2]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(14) = 28.

Time = 1.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.50

method	result	size
default	$-\frac{\sqrt{\frac{2x^2+b}{x^2}}\,x\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{2x^2+b}}{x}\right)}{\sqrt{2x^2+b}\,\sqrt{b}}$	50

```
[In] int((2+b/x^2)^(1/2)/(2*x^2+b),x,method=_RETURNVERBOSE)
```

```
[Out] -((2*x^2+b)/x^2)^(1/2)*x/(2*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(2*x^2+b)^(1
/2)+b)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(14) = 28.

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.75

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx = \left[\frac{\log\left(-\frac{x^2-\sqrt{b}x\sqrt{\frac{2x^2+b}{x^2}}+b}{x^2}\right)}{2\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{2x^2+b}{x^2}}}{2x^2+b}\right)}{b}\right]$$

```
[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(x^2 - sqrt(b)*x*sqrt((2*x^2 + b)/x^2) + b)/x^2)/sqrt(b), sqrt(-b
)*arctan(sqrt(-b)*x*sqrt((2*x^2 + b)/x^2)/(2*x^2 + b))/b]
```

Sympy [F]

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx = \int \frac{\sqrt{\frac{b}{x^2}+2}}{b+2x^2}\,dx$$

```
[In] integrate((2+b/x**2)**(1/2)/(2*x**2+b),x)
```

```
[Out] Integral(sqrt(b/x**2 + 2)/(b + 2*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx = \int \frac{\sqrt{\frac{b}{x^2}+2}}{2\,x^2+b}\,dx$$

```
[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b/x^2 + 2)/(2*x^2 + b), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(14) = 28$.

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx = \frac{\arctan\left(\frac{\sqrt{2\,x^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} - \frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}$$

```
[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="giac")
```

```
[Out] arctan(sqrt(2*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - arctan(sqrt(b)/sqrt(-b))
*sgn(x)/sqrt(-b)
```

Mupad [B] (verification not implemented)

Time = 19.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx = -\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}}{2\,x}\right)}{\sqrt{b}}$$

```
[In] int((b/x^2 + 2)^(1/2)/(b + 2*x^2),x)
```

```
[Out] -asinh((2^(1/2)*b^(1/2))/(2*x))/b^(1/2)
```

3.919 $\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 20

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2} \, dx = -\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

```
[Out] -arccsc(x*2^(1/2)/b^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {25, 342, 222}

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2} \, dx = -\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

```
[In] Int[Sqrt[2 - b/x^2]/(-b + 2*x^2),x]
```

```
[Out] -(ArcCsc[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{2-\frac{b}{x^2}}x^2}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 52 vs. 2(20) = 40.

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2}\,dx = \frac{\sqrt{2-\frac{b}{x^2}}x\arctan\left(\frac{\sqrt{-b+2x^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{-b+2x^2}}$$

```
[In] Integrate[Sqrt[2 - b/x^2]/(-b + 2*x^2),x]
[Out] (Sqrt[2 - b/x^2]*x*ArcTan[Sqrt[-b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[-b + 2*x
^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(14) = 28.

Time = 1.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.05

method	result	size
default	$-\frac{\sqrt{-\frac{-2x^2+b}{x^2}}\,x\ln\left(\frac{-2b+2\sqrt{-b}\,\sqrt{2x^2-b}}{x}\right)}{\sqrt{2x^2-b}\,\sqrt{-b}}$	61

```
[In] int((2-b/x^2)^(1/2)/(2*x^2-b),x,method=_RETURNVERBOSE)
[Out] -(-(-2*x^2+b)/x^2)^(1/2)*x/(2*x^2-b)^(1/2)/(-b)^(1/2)*ln(2*((-b)^(1/2)*(2*x
^2-b)^(1/2)-b)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 4.20

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2}\,dx = \left[-\frac{\sqrt{-b}\log\left(-\frac{x^2-\sqrt{-b}x\sqrt{\frac{2\,x^2-b}{x^2}}-b}{x^2}\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{b}x\sqrt{\frac{2\,x^2-b}{x^2}}}{2\,x^2-b}\right)}{\sqrt{b}}\right]$$

```
[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="fricas")
[Out] [-1/2*sqrt(-b)*log(-(x^2 - sqrt(-b)*x*sqrt((2*x^2 - b)/x^2) - b)/x^2)/b, -a
rctan(sqrt(b)*x*sqrt((2*x^2 - b)/x^2)/(2*x^2 - b))/sqrt(b)]
```

Sympy [F]

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2}\,dx = \int \frac{\sqrt{-\frac{b}{x^2}+2}}{-b+2x^2}\,dx$$

```
[In] integrate((2-b/x**2)**(1/2)/(2*x**2-b),x)
[Out] Integral(sqrt(-b/x**2 + 2)/(-b + 2*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2}\,dx = \int \frac{\sqrt{-\frac{b}{x^2}+2}}{2\,x^2-b}\,dx$$

```
[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="maxima")
[Out] integrate(sqrt(-b/x^2 + 2)/(2*x^2 - b), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(14) = 28$.

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2}\,dx = \frac{\arctan\left(\frac{\sqrt{2\,x^2-b}}{\sqrt{b}}\right)\operatorname{sgn}(x)}{\sqrt{b}} - \frac{\arctan\left(\frac{\sqrt{-b}}{\sqrt{b}}\right)\operatorname{sgn}(x)}{\sqrt{b}}$$

```
[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="giac")
```

```
[Out] arctan(sqrt(2*x^2 - b)/sqrt(b))*sgn(x)/sqrt(b) - arctan(sqrt(-b)/sqrt(b))*s
gn(x)/sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 19.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2}\,dx = -\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{-b}}{2\,x}\right)}{\sqrt{-b}}$$

```
[In] int(-(2 - b/x^2)^(1/2)/(b - 2*x^2),x)
```

```
[Out] -asinh((2^(1/2)*(-b)^(1/2))/(2*x))/(-b)^(1/2)
```

3.920 $\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex} dx$

Optimal result

Integrand size = 19, antiderivative size = 121

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex} dx = \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e} - \frac{\sqrt{ad^2+ce^2}\operatorname{arctanh}\left(\frac{ad-\frac{ce}{x}}{\sqrt{ad^2+ce^2}\sqrt{a+\frac{c}{x^2}}}\right)}{de} - \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}}{\sqrt{a+\frac{c}{x^2}}x}\right)}{d}$$

```
[Out] arctanh((a+c/x^2)^(1/2)/a^(1/2))*a^(1/2)/e-arctanh(c^(1/2)/x/(a+c/x^2)^(1/2
))*c^(1/2)/d-arctanh((a*d-c*e/x)/(a*d^2+c*e^2)^(1/2)/(a+c/x^2)^(1/2))*(a*d^
2+c*e^2)^(1/2)/d/e
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1458, 1489, 910, 272, 65, 214, 858, 223, 212, 739}

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex} dx = -\frac{\sqrt{ad^2+ce^2}\operatorname{arctanh}\left(\frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\sqrt{ad^2+ce^2}}\right)}{de} - \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}}{x\sqrt{a+\frac{c}{x^2}}}\right)}{d} + \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

```
[In] Int[Sqrt[a + c/x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + c/x^2]/Sqrt[a]])/e - (Sqrt[a*d^2 + c*e^2]*ArcTanh
[(a*d - (c*e)/x)/(Sqrt[a*d^2 + c*e^2]*Sqrt[a + c/x^2])])/(d*e) - (Sqrt[c]*A
rcTanh[Sqrt[c]/(Sqrt[a + c/x^2]*x)])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 910

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[Simp[c*d*f + a*e*g - c*(e*
f - d*g)*x, x]*((a + c*x^2)^(p - 1)/(f + g*x)), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rule 1458

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{a+\frac{c}{x^2}}}{\left(e+\frac{d}{x}\right)x}\,dx\\
&= -\text{Subst}\left(\int \frac{\sqrt{a+cx^2}}{x(e+dx)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\text{Subst}\left(\int \frac{ad-cex}{(e+dx)\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)}{e} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)}{e}\\
&= -\frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)}{d} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, \frac{1}{x^2}\right)}{2e}\\
&\quad + \left(\frac{ad}{e}+\frac{ce}{d}\right)\text{Subst}\left(\int \frac{1}{(e+dx)\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{c\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{c}{x^2}}\,x}\right)}{d} - \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+\frac{c}{x^2}}\right)}{ce}\\
&\quad + \left(-\frac{ad}{e}-\frac{ce}{d}\right)\text{Subst}\left(\int \frac{1}{ad^2+ce^2-x^2}\,dx, x, \frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}}\right)
\end{aligned}$$

$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e} - \frac{\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ad-\frac{ce}{x}}{\sqrt{ad^2+ce^2}\sqrt{a+\frac{c}{x^2}}}\right)}{de} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{a+\frac{c}{x^2}}x}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx = -\frac{\sqrt{a+\frac{c}{x^2}}x\left(2\sqrt{-ad^2-ce^2}\arctan\left(\frac{\sqrt{a}(d+ex)-e\sqrt{c+ax^2}}{\sqrt{-ad^2-ce^2}}\right)-2\sqrt{c}e\,\mathrm{arctanh}\left(\frac{\sqrt{a}x-\sqrt{c+ax^2}}{\sqrt{c}}\right)+\sqrt{a}d\log\left(-\sqrt{a}x\right.\right.}{de\sqrt{c+ax^2}}$$

```
[In] Integrate[Sqrt[a + c/x^2]/(d + e*x),x]
```

```
[Out] -((Sqrt[a + c/x^2]*x*(2*Sqrt[-(a*d^2) - c*e^2]*ArcTan[(Sqrt[a]*(d + e*x) -
e*Sqrt[c + a*x^2])/Sqrt[-(a*d^2) - c*e^2]] - 2*Sqrt[c]*e*ArcTanh[(Sqrt[a]*x
 - Sqrt[c + a*x^2])/Sqrt[c]] + Sqrt[a]*d*Log[-(Sqrt[a]*x) + Sqrt[c + a*x^2]
]))/(d*e*Sqrt[c + a*x^2]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(103) = 206.

Time = 1.10 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.04

method	result
default	$-\frac{\sqrt{\frac{a\,x^2+c}{x^2}}\,x\left(\sqrt{c}\sqrt{\frac{a\,d^2+c\,e^2}{e^2}}\ln\left(\frac{2c+2\sqrt{c}\sqrt{a\,x^2+c}}{x}\right)e^2-\sqrt{a}\,d\ln\left(\frac{\sqrt{a\,x^2+c}\sqrt{a}+ax}{\sqrt{a}}\right)e\sqrt{\frac{a\,d^2+c\,e^2}{e^2}}-\ln\left(\frac{2\sqrt{a\,x^2+c}\sqrt{\frac{a\,d^2+c\,e^2}{e^2}}\,e-2adx+\right.\right.}{ex+d}}{\sqrt{a\,x^2+c}\,d\,e^2\sqrt{\frac{a\,d^2+c\,e^2}{e^2}}}$

```
[In] int((a+c/x^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -((a*x^2+c)/x^2)^(1/2)*x*(c^(1/2)*((a*d^2+c*e^2)/e^2)^(1/2)*ln(2*(c^(1/2)*(
a*x^2+c)^(1/2)+c)/x)*e^2-a^(1/2)*d*ln(((a*x^2+c)^(1/2)*a^(1/2)+a*x)/a^(1/2)
)*e*((a*d^2+c*e^2)/e^2)^(1/2)-ln(2*((a*x^2+c)^(1/2)*((a*d^2+c*e^2)/e^2)^(1/
2)*e-a*d*x+c*e)/(e*x+d))*a*d^2-ln(2*((a*x^2+c)^(1/2)*((a*d^2+c*e^2)/e^2)^(1
/2)*e-a*d*x+c*e)/(e*x+d))*c*e^2)/(a*x^2+c)^(1/2)/d/e^2/((a*d^2+c*e^2)/e^2)^
(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.77 (sec) , antiderivative size = 1532, normalized size of antiderivative = 12.66

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx = \text{Too large to display}$$

```
[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + s
qrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) + sqrt
(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e
^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e
^2*x^2 + 2*d*e*x + d^2)))/(d*e), -1/2*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqr
t((a*x^2 + c)/x^2)/(a*x^2 + c)) - sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt(
(a*x^2 + c)/x^2) + 2*c)/x^2) - sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d
^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2
 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(sq
rt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + sqrt(c)*e
*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) + 2*sqrt(-a*d^
2 - c*e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x
^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*e), -1/2*(2*sqrt(-a)
*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) - sqrt(c)*e*log(-
(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) - 2*sqrt(-a*d^2 - c*
e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x^2)/(a
*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*e), 1/2*(2*sqrt(-c)*e*arct
an(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) + sqrt(a)*d*log(-2*a*x^2 -
 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + sqrt(a*d^2 + c*e^2)*log((2*a*c*
d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*
x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(
d*e), -1/2*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x^2)/(a*x^2 +
 c)) - 2*sqrt(-c)*e*arctan(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) -
sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a
*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2)
)/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(2*sqrt(-c)*e*arctan(sqrt(-c)*x*sq
rt((a*x^2 + c)/x^2)/(a*x^2 + c)) + sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*s
qrt((a*x^2 + c)/x^2) - c) + 2*sqrt(-a*d^2 - c*e^2)*arctan((a*d*x^2 - c*e*x)
*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 +
 a*c*e^2)*x^2)))/(d*e), -(sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x
^2)/(a*x^2 + c)) - sqrt(-c)*e*arctan(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^
2 + c)) - sqrt(-a*d^2 - c*e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2
)*sqrt((a*x^2 + c)/x^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*
e)]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx$$

```
[In] integrate((a+c/x**2)**(1/2)/(e*x+d),x)
[Out] Integral(sqrt(a + c/x**2)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{c}{x^2}}}{ex+d}\,dx$$

```
[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(a + c/x^2)/(e*x + d), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{c}{x^2}}}{d+e\,x}\,dx$$

```
[In] int((a + c/x^2)^(1/2)/(d + e*x),x)
[Out] int((a + c/x^2)^(1/2)/(d + e*x), x)
```

3.921 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 181

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx = \frac{\sqrt{a}\,\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e} - \frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{d} - \frac{\sqrt{ad^2-e(bd-ce)}\,\text{arctanh}\left(\frac{2ad-be+\frac{bd-2ce}{x}}{2\sqrt{ad^2-e(bd-ce)}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{de}$$

```
[Out] arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))*a^(1/2)/e-arctanh(1/2*(b
+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/2))*c^(1/2)/d-arctanh(1/2*(2*a*d-b*e+(b*d-
2*c*e)/x)/(a*d^2-e*(b*d-c*e))^(1/2)/(a+c/x^2+b/x)^(1/2))*(a*d^2-e*(b*d-c*e)
)^(1/2)/d/e
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1457, 1488, 909, 738, 212, 857, 635}

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx = -\frac{\sqrt{ad^2-e(bd-ce)}\,\text{arctanh}\left(\frac{2ad+\frac{bd-2ce}{x}-be}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{ad^2-e(bd-ce)}}\right)}{de} - \frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{d} + \frac{\sqrt{a}\,\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

```
[In] Int[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]
```

```
[Out] (Sqrt[a]*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/e - (Sqrt[
c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/d - (Sqrt[a*d^
2 - e*(b*d - c*e)]*ArcTanh[(2*a*d - b*e + (b*d - 2*c*e)/x)/(2*Sqrt[a*d^2 -
e*(b*d - c*e)]*Sqrt[a + c/x^2 + b/x])])/(d*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 909

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p -
1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 1457

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)
^(n2_.))^(p_.), x_Symbol] :> Int[((e + d*x^n)^q*(a + b*x^n + c*x^(2*n))^p)/
x^(n*q), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[n2, 2*n] && EqQ[mn, -
```

```
n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{\left(e+\frac{d}{x}\right)x}\,dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x(e+dx)}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\text{Subst}\left(\int \frac{ad-be-cex}{(e+dx)\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{e} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{e} \\
&= -\frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{d} + \frac{(2a)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e} \\
&\quad + \left(-b+\frac{ad}{e}+\frac{ce}{d}\right)\text{Subst}\left(\int \frac{1}{(e+dx)\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e} - \frac{(2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+\frac{2c}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{d} + \left(2\left(b-\frac{ad}{e}\right.\right. \\
&\quad \left.\left.-\frac{ce}{d}\right)\right)\text{Subst}\left(\int \frac{1}{4ad^2-4bde+4ce^2-x^2}\,dx, x, \frac{2ad-be-\frac{-bd+2ce}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right) \\
&= \frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{d} \\
&\quad - \frac{\sqrt{ad^2-e(bd-ce)}\tanh^{-1}\left(\frac{2ad-be+\frac{bd-2ce}{x}}{2\sqrt{ad^2-e(bd-ce)}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{de}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx = \frac{x\sqrt{a+\frac{c+bx}{x^2}}\left(-2\sqrt{-ad^2+bde-ce^2}\arctan\left(\frac{\sqrt{a}(d+ex)-e\sqrt{c+x(b+ax)}}{\sqrt{-ad^2+bde-ce^2}}\right)+2\sqrt{c}e\operatorname{arctanh}\left(\frac{\sqrt{a}x-\sqrt{c+x(b+ax)}}{\sqrt{c}}\right)-\sqrt{}\right.}{de\sqrt{c+x(b+ax)}}$$

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*(-2*Sqrt[-(a*d^2) + b*d*e - c*e^2]*ArcTan[(Sqrt[
a]*(d + e*x) - e*Sqrt[c + x*(b + a*x)])/Sqrt[-(a*d^2) + b*d*e - c*e^2]] + 2
*Sqrt[c]*e*ArcTanh[(Sqrt[a]*x - Sqrt[c + x*(b + a*x)])/Sqrt[c]] - Sqrt[a]*d
*Log[e*(b + 2*a*x - 2*Sqrt[a]*Sqrt[c + x*(b + a*x)])]))/(d*e*Sqrt[c + x*(b
+ a*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(157) = 314.

Time = 1.06 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.19

method	result
default	$-\frac{\sqrt{\frac{a\,x^2+bx+c}{x^2}}\,x\left(\sqrt{\frac{a\,d^2-bde+c\,e^2}{e^2}}\,\sqrt{a}\,\sqrt{c}\,\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a\,x^2+bx+c}}{x}\right)e^2-\sqrt{\frac{a\,d^2-bde+c\,e^2}{e^2}}\,\ln\left(\frac{2\sqrt{a\,x^2+bx+c}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)ade-a^{\frac{3}{2}}\right.}{}$

```
[In] int((a+c/x^2+b/x)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -((a*x^2+b*x+c)/x^2)^(1/2)*x*(((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*a^(1/2)*c^(1/
2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*e^2-((a*d^2-b*d*e+c*e^2)/e
^2)^(1/2)*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*d*e-a^(
3/2)*ln((2*(a*x^2+b*x+c)^(1/2)*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*e-2*a*d*x+b*
e*x-b*d+2*c*e)/(e*x+d))*d^2+a^(1/2)*ln((2*(a*x^2+b*x+c)^(1/2)*((a*d^2-b*d*e
+c*e^2)/e^2)^(1/2)*e-2*a*d*x+b*e*x-b*d+2*c*e)/(e*x+d))*b*d*e-a^(1/2)*ln((2*
(a*x^2+b*x+c)^(1/2)*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*e-2*a*d*x+b*e*x-b*d+2*c
*e)/(e*x+d))*c*e^2)/(a*x^2+b*x+c)^(1/2)/d/e^2/a^(1/2)/((a*d^2-b*d*e+c*e^2)/
e^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 43.57 (sec) , antiderivative size = 2411, normalized size of antiderivative = 13.32

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex} \, dx = \text{Too large to display}$$

```
[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*
sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + sqrt(c)*e*log(-(8*b*c*x + (b^2 + 4*a
*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^
2) + sqrt(a*d^2 - b*d*e + c*e^2)*log((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)
*d^2 - (8*a^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b
*c*e^2 - (3*b^2 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)*((2*a*d - b
*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^2 + 2*d*e*x
+ d^2)))/(d*e), -1/2*(2*sqrt(-a)*d*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt
((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - sqrt(c)*e*log(-(8*b*c*x
+ (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x +
 c)/x^2))/x^2) - sqrt(a*d^2 - b*d*e + c*e^2)*log((8*b*c*d*e - 8*c^2*e^2 - (
b^2 + 4*a*c)*d^2 - (8*a^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a
*b*d^2 + 4*b*c*e^2 - (3*b^2 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)
*((2*a*d - b*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^
2 + 2*d*e*x + d^2)))/(d*e), 1/2*(sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x - b^2 -
 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + sqrt(c)*e
*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt
((a*x^2 + b*x + c)/x^2))/x^2) - 2*sqrt(-a*d^2 + b*d*e - c*e^2)*arctan(-1/2*
sqrt(-a*d^2 + b*d*e - c*e^2)*((2*a*d - b*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*
x^2 + b*x + c)/x^2)/(a*c*d^2 - b*c*d*e + c^2*e^2 + (a^2*d^2 - a*b*d*e + a*c
*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b*c*e^2)*x)))/(d*e), -1/2*(2*sqrt(-a)*d*ar
ctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*
b*x + a*c)) - sqrt(c)*e*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^
2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) + 2*sqrt(-a*d^2 + b*d*
e - c*e^2)*arctan(-1/2*sqrt(-a*d^2 + b*d*e - c*e^2)*((2*a*d - b*e)*x^2 + (b
*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*d^2 - b*c*d*e + c^2*e^2 + (
a^2*d^2 - a*b*d*e + a*c*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b*c*e^2)*x)))/(d*e)
, 1/2*(2*sqrt(-c)*e*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x +
 c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x - b^
2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + sqrt(a
*d^2 - b*d*e + c*e^2)*log((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*a
^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b*c*e^2 - (3
*b^2 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)*((2*a*d - b*e)*x^2 + (
b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d
```

```
*e), -1/2*(2*sqrt(-a)*d*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b
*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - 2*sqrt(-c)*e*arctan(1/2*(b*x^2 + 2*
c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) - sqrt(a
*d^2 - b*d*e + c*e^2)*log((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*a
^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b*c*e^2 - (3
*b^2 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)*((2*a*d - b*e)*x^2 + (
b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d
*e), 1/2*(2*sqrt(-c)*e*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*
x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x -
 b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) - 2*s
qrt(-a*d^2 + b*d*e - c*e^2)*arctan(-1/2*sqrt(-a*d^2 + b*d*e - c*e^2)*((2*a*
d - b*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*d^2 - b*c*
d*e + c^2*e^2 + (a^2*d^2 - a*b*d*e + a*c*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b*
c*e^2)*x)))/(d*e), -(sqrt(-a)*d*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a
*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - sqrt(-c)*e*arctan(1/2*(b*x^
2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) +
sqrt(-a*d^2 + b*d*e - c*e^2)*arctan(-1/2*sqrt(-a*d^2 + b*d*e - c*e^2)*((2*a
*d - b*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*d^2 - b*c
*d*e + c^2*e^2 + (a^2*d^2 - a*b*d*e + a*c*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b
*c*e^2)*x)))/(d*e)]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx=\int \frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{d+ex}\,dx$$

```
[In] integrate((a+c/x**2+b/x)**(1/2)/(e*x+d),x)
[Out] Integral(sqrt(a + b/x + c/x**2)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx=\int \frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{ex+d}\,dx$$

```
[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(a + b/x + c/x^2)/(e*x + d), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{d+e\,x}\,dx$$

```
[In] int((a + b/x + c/x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + b/x + c/x^2)^(1/2)/(d + e*x), x)
```

3.922 $\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}} \, dx = \frac{3x^{2/3}}{2} + \frac{10}{11}\sqrt{x}\sqrt[5]{x^3}$$

```
[Out] 3/2*x^(2/3)+10/11*(x^3)^(1/5)*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {14, 15, 30}

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}} \, dx = \frac{3x^{2/3}}{2} + \frac{10}{11}\sqrt[5]{x^3}\sqrt{x}$$

```
[In] Int[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]
[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
```

```
&& !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{\sqrt[3]{x}}+\frac{\sqrt[5]{x^3}}{\sqrt{x}}\right) dx \\
&= \frac{3x^{2/3}}{2}+\int \frac{\sqrt[5]{x^3}}{\sqrt{x}}\, dx \\
&= \frac{3x^{2/3}}{2}+\frac{\sqrt[5]{x^3}\int \sqrt[10]{x}\, dx}{x^{3/5}} \\
&= \frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt{x}\sqrt[5]{x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 6.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\, dx=\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt{x}\sqrt[5]{x^3}$$

```
[In] Integrate[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]
[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{3x^{\frac{2}{3}}}{2}+\frac{10\left(x^{3}\right)^{\frac{1}{5}}\sqrt{x}}{11}$	17

```
[In] int((x^(1/6)+(x^3)^(1/5))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 3/2*x^(2/3)+10/11*(x^3)^(1/5)*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx=\frac{10}{11}\left(x^3\right)^{\frac{1}{5}}\sqrt{x}+\frac{3}{2}\,x^{\frac{2}{3}}$$

```
[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="fricas")
[Out] 10/11*(x^3)^(1/5)*sqrt(x) + 3/2*x^(2/3)
```

Sympy [A] (verification not implemented)

Time = 63.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx=\frac{3x^{\frac{2}{3}}}{2}+\frac{10\sqrt{x}\sqrt[5]{x^3}}{11}$$

```
[In] integrate((x**(1/6)+(x**3)**(1/5))/x**(1/2),x)
[Out] 3*x**(2/3)/2 + 10*sqrt(x)*(x**3)**(1/5)/11
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx=\frac{10}{11}\left(x^3\right)^{\frac{1}{5}}\sqrt{x}+\frac{3}{2}\,x^{\frac{2}{3}}$$

```
[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="maxima")
[Out] 10/11*(x^3)^(1/5)*sqrt(x) + 3/2*x^(2/3)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx=\frac{10}{11}\,x^{\frac{11}{10}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

```
[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="giac")
[Out] 10/11*x^(11/10) + 3/2*x^(2/3)
```

Mupad [B] (verification not implemented)

Time = 19.99 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx=\frac{10\,\sqrt{x}\,{\left(x^3\right)}^{1/5}}{11}+\frac{3\,x^{2/3}}{2}$$

```
[In] int(((x^3)^(1/5) + x^(1/6))/x^(1/2),x)
```

```
[Out] (10*x^(1/2)*(x^3)^(1/5))/11 + (3*x^(2/3))/2
```

3.923 $\int \frac{2+x}{\sqrt{4x-x^2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 26

$$\int \frac{2+x}{\sqrt{4x-x^2}} \, dx = -\sqrt{4x-x^2} - 4\arcsin\left(1-\frac{x}{2}\right)$$

```
[Out] 4*arcsin(-1+1/2*x)-(-x^2+4*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {654, 633, 222}

$$\int \frac{2+x}{\sqrt{4x-x^2}} \, dx = -4\arcsin\left(1-\frac{x}{2}\right) - \sqrt{4x-x^2}$$

```
[In] Int[(2 + x)/Sqrt[4*x - x^2],x]
```

```
[Out] -Sqrt[4*x - x^2] - 4*ArcSin[1 - x/2]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\sqrt{4x-x^2}+4\int \frac{1}{\sqrt{4x-x^2}}\,dx\\ &= -\sqrt{4x-x^2}-\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx,x,4-2x\right)\\ &= -\sqrt{4x-x^2}-4\sin^{-1}\left(1-\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int \frac{2+x}{\sqrt{4x-x^2}}\,dx=\frac{(-4+x)x-8\sqrt{-4+x}\sqrt{x}\log\left(\sqrt{-4+x}-\sqrt{x}\right)}{\sqrt{-((-4+x)x)}}$$

```
[In] Integrate[(2 + x)/Sqrt[4*x - x^2],x]
[Out] ((-4 + x)*x - 8*Sqrt[-4 + x]*Sqrt[x]*Log[Sqrt[-4 + x] - Sqrt[x]])/Sqrt[-((-
4 + x)*x)]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result
default	$4\arcsin\left(-1+\frac{x}{2}\right)-\sqrt{-x^2+4x}$
risch	$\frac{x(x-4)}{\sqrt{-x(x-4)}}+4\arcsin\left(-1+\frac{x}{2}\right)$
pseudoelliptic	$-\sqrt{-x\left(x-4\right)}-8\arctan\left(\frac{\sqrt{-x(x-4)}}{x}\right)$
meijerg	$4\arcsin\left(\frac{\sqrt{x}}{2}\right)+\frac{4i\left(\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{-\frac{x}{4}+1}}{2}-i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{x}}{2}\right)\right)}{\sqrt{\pi}}$
trager	$-\sqrt{-x^2+4x}-4\,\text{RootOf}\left(\_Z^2+1\right)\ln\left(\text{RootOf}\left(\_Z^2+1\right)x-2\,\text{RootOf}\left(\_Z^2+1\right)+\sqrt{-\right.$

```
[In] int((x+2)/(-x^2+4*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 4*arcsin(-1+1/2*x)-(-x^2+4*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{2+x}{\sqrt{4x-x^2}}\,dx = -\sqrt{-x^2+4\,x} - 8\,\arctan\left(\frac{\sqrt{-x^2+4\,x}}{x}\right)$$

```
[In] integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 4*x) - 8*arctan(sqrt(-x^2 + 4*x)/x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{2+x}{\sqrt{4x-x^2}}\,dx = -\sqrt{-x^2+4x} + 4\,\mathrm{asin}\left(\frac{x}{2}-1\right)$$

```
[In] integrate((2+x)/(-x**2+4*x)**(1/2),x)
[Out] -sqrt(-x**2 + 4*x) + 4*asin(x/2 - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{2+x}{\sqrt{4x-x^2}}\,dx = -\sqrt{-x^2+4\,x} - 4\,\arcsin\left(-\frac{1}{2}\,x+1\right)$$

```
[In] integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 4*x) - 4*arcsin(-1/2*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{2+x}{\sqrt{4x-x^2}}\,dx = -\sqrt{-x^2+4\,x} + 4\,\arcsin\left(\frac{1}{2}\,x-1\right)$$

```
[In] integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 4*x) + 4*arcsin(1/2*x - 1)
```

Mupad [B] (verification not implemented)

Time = 19.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{2+x}{\sqrt{4x-x^2}}\,dx = 4\,\mathrm{asin}\left(\frac{x}{2}-1\right) - \sqrt{4\,x-x^2}$$

```
[In] int((x + 2)/(4*x - x^2)^(1/2),x)
[Out] 4*asin(x/2 - 1) - (4*x - x^2)^(1/2)
```

3.924 $\int \frac{3+x}{\sqrt[3]{6x+x^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}} \, dx = \frac{3}{4}\left(6x+x^2\right)^{2/3}$$

```
[Out] 3/4*(x^2+6*x)^(2/3)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {643}

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}} \, dx = \frac{3}{4}\left(x^2+6x\right)^{2/3}$$

```
[In] Int[(3 + x)/(6*x + x^2)^(1/3),x]
```

```
[Out] (3*(6*x + x^2)^(2/3))/4
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{3}{4}\left(6x+x^2\right)^{2/3}$$

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx=\frac{3}{4}(x(6+x))^{2/3}$$

[In] Integrate[(3 + x)/(6*x + x^2)^(1/3),x]

[Out] (3*(x*(6 + x))^(2/3))/4

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{3(x(6+x))^{\frac{2}{3}}}{4}$	10
default	$\frac{3(x^2+6x)^{\frac{2}{3}}}{4}$	12
trager	$\frac{3(x^2+6x)^{\frac{2}{3}}}{4}$	12
risch	$\frac{3x(6+x)}{4(x(6+x))^{\frac{1}{3}}}$	14
gosper	$\frac{3x(6+x)}{4(x^2+6x)^{\frac{1}{3}}}$	16
meijerg	$\frac{3\,9^{\frac{1}{3}}2^{\frac{2}{3}}x^{\frac{2}{3}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{x}{6}\right)}{4}+\frac{9^{\frac{1}{3}}2^{\frac{2}{3}}x^{\frac{5}{3}}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{8}{3};-\frac{x}{6}\right)}{10}$	42

[In] int((3+x)/(x^2+6*x)^(1/3),x,method=_RETURNVERBOSE)

[Out] 3/4*(x*(6+x))^(2/3)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx=\frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

[In] integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="fricas")

[Out] 3/4*(x^2 + 6*x)^(2/3)

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx = \frac{3(x^2+6x)^{\frac{2}{3}}}{4}$$

```
[In] integrate((3+x)/(x**2+6*x)**(1/3),x)
[Out] 3*(x**2 + 6*x)**(2/3)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx = \frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

```
[In] integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="maxima")
[Out] 3/4*(x^2 + 6*x)^(2/3)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx = \frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

```
[In] integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="giac")
[Out] 3/4*(x^2 + 6*x)^(2/3)
```

Mupad [B] (verification not implemented)

Time = 19.90 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx = \frac{3\,\left(x\,\left(x+6\right)\right)^{2/3}}{4}$$

```
[In] int((x + 3)/(6*x + x^2)^(1/3),x)
[Out] (3*(x*(x + 6))^(2/3))/4
```

3.925 $\int \frac{4+x}{(6x-x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 22

$$\int \frac{4+x}{(6x-x^2)^{3/2}} \, dx = -\frac{12-7x}{9\sqrt{6x-x^2}}$$

```
[Out] 1/9*(-12+7*x)/(-x^2+6*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {650}

$$\int \frac{4+x}{(6x-x^2)^{3/2}} \, dx = -\frac{12-7x}{9\sqrt{6x-x^2}}$$

```
[In] Int[(4 + x)/(6*x - x^2)^(3/2),x]
[Out] -1/9*(12 - 7*x)/Sqrt[6*x - x^2]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = -\frac{12-7x}{9\sqrt{6x-x^2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx = \frac{-12+7x}{9\sqrt{-((-6+x)x)}}$$

```
[In] Integrate[(4 + x)/(6*x - x^2)^(3/2),x]
[Out] (-12 + 7*x)/(9*Sqrt[-((-6 + x)*x)])
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{-12+7x}{9\sqrt{-x(-6+x)}}$	16
pseudoelliptic	$\frac{-12+7x}{9\sqrt{-x(-6+x)}}$	16
gosper	$-\frac{x(-6+x)(-12+7x)}{9(-x^2+6x)^{\frac{3}{2}}}$	23
trager	$-\frac{(-12+7x)\sqrt{-x^2+6x}}{9x(-6+x)}$	27
default	$\frac{1}{\sqrt{-x^2+6x}}-\frac{7(-2x+6)}{18\sqrt{-x^2+6x}}$	31
meijerg	$-\frac{2\sqrt{3}\,\sqrt{2}\,(1-\frac{x}{3})}{9\sqrt{x}\,\sqrt{1-\frac{x}{6}}}+\frac{\sqrt{x}\,\sqrt{6}}{18\sqrt{1-\frac{x}{6}}}$	40

```
[In] int((x+4)/(-x^2+6*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/9*(-12+7*x)/(-x*(-6+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx = -\frac{\sqrt{-x^2+6\,x}(7\,x-12)}{9\,(x^2-6\,x)}$$

```
[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="fricas")
[Out] -1/9*sqrt(-x^2 + 6*x)*(7*x - 12)/(x^2 - 6*x)
```

Sympy [F]

$$\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx = \int \frac{x+4}{(-x\,(x-6))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((4+x)/(-x**2+6*x)**(3/2),x)
[Out] Integral((x + 4)/(-x*(x - 6))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx = \frac{7\,x}{9\,\sqrt{-x^2+6\,x}} - \frac{4}{3\,\sqrt{-x^2+6\,x}}$$

```
[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="maxima")
[Out] 7/9*x/sqrt(-x^2 + 6*x) - 4/3/sqrt(-x^2 + 6*x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx = -\frac{\sqrt{-x^2+6\,x}(7\,x-12)}{9\,(x^2-6\,x)}$$

```
[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="giac")
[Out] -1/9*sqrt(-x^2 + 6*x)*(7*x - 12)/(x^2 - 6*x)
```

Mupad [B] (verification not implemented)

Time = 19.85 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx = \frac{7\,x-12}{9\,\sqrt{6\,x-x^2}}$$

```
[In] int((x + 4)/(6*x - x^2)^(3/2),x)
[Out] (7*x - 12)/(9*(6*x - x^2)^(1/2))
```

3.926 $\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = \arctan\left(\sqrt{2x+x^2}\right)$$

```
[Out] arctan((x^2+2*x)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {702, 209}

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = \arctan\left(\sqrt{x^2+2x}\right)$$

```
[In] Int[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] ArcTan[Sqrt[2*x + x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 702

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
```

```
qQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= 4\text{Subst}\left(\int \frac{1}{4+4x^2}\,dx, x, \sqrt{2x+x^2}\right) \\ &= \tan^{-1}\left(\sqrt{2x+x^2}\right) \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 41 vs. $2(12) = 24$.

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.42

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = -\frac{2\sqrt{x}\sqrt{2+x}\arctan\left(1+x-\sqrt{x}\sqrt{2+x}\right)}{\sqrt{x(2+x)}}$$

```
[In] Integrate[1/((1 + x)*Sqrt[2*x + x^2]),x]
[Out] (-2*Sqrt[x]*Sqrt[2 + x]*ArcTan[1 + x - Sqrt[x]*Sqrt[2 + x]])/Sqrt[x*(2 + x)
]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$\arctan\left(\sqrt{x\,(x+2)}\right)$	9
default	$-\arctan\left(\frac{1}{\sqrt{(x+1)^2-1}}\right)$	13
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+1\right)+\sqrt{x^2+2x}}{x+1}\right)$	31

```
[In] int(1/(x+1)/(x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan((x*(x+2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = 2 \arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="fricas")
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Sympy [F]

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = \int \frac{1}{\sqrt{x\,(x+2)}\,(x+1)}\,dx$$

```
[In] integrate(1/(1+x)/(x**2+2*x)**(1/2),x)
[Out] Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = -\arcsin\left(\frac{1}{|x+1|}\right)$$

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(1/abs(x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = 2 \arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="giac")
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2\,x}\,\,(x+1)}\,dx$$

```
[In] int(1/((2*x + x^2)^(1/2)*(x + 1)),x)
[Out] int(1/((2*x + x^2)^(1/2)*(x + 1)), x)
```

3.927 $\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx = \arctan\left(2\sqrt{x+x^2}\right)$$

```
[Out] arctan(2*(x^2+x)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {702, 209}

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx = \arctan\left(2\sqrt{x^2+x}\right)$$

```
[In] Int[1/((1 + 2*x)*Sqrt[x + x^2]),x]
[Out] ArcTan[2*Sqrt[x + x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 702

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
```

```
qQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= 4\text{Subst}\left(\int \frac{1}{2+8x^2}\,dx, x, \sqrt{x+x^2}\right) \\ &= \tan^{-1}\left(2\sqrt{x+x^2}\right) \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. $2(12) = 24$.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.58

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx = -\frac{2\sqrt{x}\sqrt{1+x}\arctan\left(1+2x-2\sqrt{x}\sqrt{1+x}\right)}{\sqrt{x(1+x)}}$$

```
[In] Integrate[1/((1 + 2*x)*Sqrt[x + x^2]),x]
[Out] (-2*Sqrt[x]*Sqrt[1 + x]*ArcTan[1 + 2*x - 2*Sqrt[x]*Sqrt[1 + x]])/Sqrt[x*(1
+ x)]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
pseudoelliptic	$\arctan\left(2\sqrt{(x+1)\,x}\right)$	11
default	$-\arctan\left(\frac{1}{\sqrt{4\left(x+\frac{1}{2}\right)^2-1}}\right)$	15
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+1\right)+2\sqrt{x^2+x}}{1+2x}\right)$	33

```
[In] int(1/(1+2*x)/(x^2+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan(2*((x+1)*x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx = 2\,\arctan\left(-2\,x+2\,\sqrt{x^2+x}-1\right)$$

```
[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="fricas")
[Out] 2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)
```

Sympy [F]

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx = \int \frac{1}{\sqrt{x\,(x+1)}\,(2x+1)}\,dx$$

```
[In] integrate(1/(1+2*x)/(x**2+x)**(1/2),x)
[Out] Integral(1/(sqrt(x*(x + 1))*(2*x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx = -\arcsin\left(\frac{1}{|2\,x+1|}\right)$$

```
[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(1/abs(2*x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx = 2\,\arctan\left(-2\,x+2\,\sqrt{x^2+x}-1\right)$$

```
[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="giac")
[Out] 2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx = \int \frac{1}{(2\,x+1)\,\sqrt{x^2+x}}\,dx$$

```
[In] int(1/((2*x + 1)*(x + x^2)^(1/2)),x)
```

```
[Out] int(1/((2*x + 1)*(x + x^2)^(1/2)), x)
```

3.928 $\int \frac{-1+x}{\sqrt{2x-x^2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{2x-x^2}$$

```
[Out] -(-x^2+2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {643}

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{2x-x^2}$$

```
[In] Int[(-1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] -Sqrt[2*x - x^2]
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\sqrt{2x-x^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{-((-2+x)x)}$$

```
[In] Integrate[(-1 + x)/Sqrt[2*x - x^2],x]
[Out] -Sqrt[-((-2 + x)*x)]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

method	result	size
pseudoelliptic	$-\sqrt{-x\,(x-2)}$	11
default	$-\sqrt{-x^2+2x}$	14
trager	$-\sqrt{-x^2+2x}$	14
risch	$\frac{x(x-2)}{\sqrt{-x(x-2)}}$	14
gosper	$\frac{x(x-2)}{\sqrt{-x^2+2x}}$	17
meijerg	$-2\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)+\frac{2i\left(\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{1-\frac{x}{2}}}{2}-i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)\right)}{\sqrt{\pi}}$	54

```
[In] int((x-1)/(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-x*(x-2))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{-x^2+2\,x}$$

```
[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx=-\sqrt{-x^2+2x}$$

```
[In] integrate((-1+x)/(-x**2+2*x)**(1/2),x)
[Out] -sqrt(-x**2 + 2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx=-\sqrt{-x^2+2\,x}$$

```
[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx=-\sqrt{-x^2+2\,x}$$

```
[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 2*x)
```

Mupad [B] (verification not implemented)

Time = 20.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx=-\sqrt{-x\,\left(x-2\right)}$$

```
[In] int((x - 1)/(2*x - x^2)^(1/2),x)
[Out] -(-x*(x - 2))^(1/2)
```

3.929 $\int \frac{\sqrt{x-x^2}}{1+x} dx$

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \frac{\sqrt{x-x^2}}{1+x} dx = \sqrt{x-x^2} - \frac{3}{2}\arcsin(1-2x) + \sqrt{2}\arctan\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right)$$

```
[Out] 3/2*arcsin(-1+2*x)+arctan(1/4*(1-3*x)*2^(1/2)/(-x^2+x)^(1/2))*2^(1/2)+(-x^2
+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {748, 857, 633, 222, 738, 210}

$$\int \frac{\sqrt{x-x^2}}{1+x} dx = -\frac{3}{2}\arcsin(1-2x) + \sqrt{2}\arctan\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right) + \sqrt{x-x^2}$$

```
[In] Int[Sqrt[x - x^2]/(1 + x),x]
```

```
[Out] Sqrt[x - x^2] - (3*ArcSin[1 - 2*x])/2 + Sqrt[2]*ArcTan[(1 - 3*x)/(2*Sqrt[2]
*Sqrt[x - x^2])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \sqrt{x-x^2}-\frac{1}{2}\int\frac{1-3x}{(1+x)\sqrt{x-x^2}}\,dx\\
&= \sqrt{x-x^2}+\frac{3}{2}\int\frac{1}{\sqrt{x-x^2}}\,dx-2\int\frac{1}{(1+x)\sqrt{x-x^2}}\,dx\\
&= \sqrt{x-x^2}-\frac{3}{2}\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)+4\text{Subst}\left(\int\frac{1}{-8-x^2}\,dx,x,\frac{-1+3x}{\sqrt{x-x^2}}\right)\\
&= \sqrt{x-x^2}-\frac{3}{2}\sin^{-1}(1-2x)+\sqrt{2}\tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{x-x^2}}{1+x}\,dx = \frac{\sqrt{-((-1+x)x)}\left(\sqrt{-1+x}\sqrt{x}-6\text{arctanh}\left(\frac{\sqrt{-1+x}}{-1+\sqrt{x}}\right)+2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}}{\sqrt{\frac{-1+x}{x}}}\right)\right)}{\sqrt{-1+x}\sqrt{x}}$$

[In] Integrate[Sqrt[x - x^2]/(1 + x),x]

[Out] (Sqrt[-((-1 + x)*x)]*(Sqrt[-1 + x]*Sqrt[x] - 6*ArcTanh[Sqrt[-1 + x]/(-1 + Sqrt[x])] + 2*Sqrt[2]*ArcTanh[Sqrt[2]/Sqrt[(-1 + x)/x]]))/(Sqrt[-1 + x]*Sqrt[x])

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$2\sqrt{2}\,\arctan\left(\frac{\sqrt{-(x-1)x}\,\sqrt{2}}{2x}\right)+\sqrt{-\left(x-1\right)x}-3\arctan\left(\frac{\sqrt{-(x-1)x}}{x}\right)$
default	$\sqrt{-\left(x+1\right)^{2}+3x+1}+\frac{3\arcsin(2x-1)}{2}-\sqrt{2}\,\arctan\left(\frac{(3x-1)\sqrt{2}}{4\sqrt{-(x+1)^{2}+3x+1}}\right)$
risch	$-\frac{(x-1)x}{\sqrt{-(x-1)x}}+\frac{3\arcsin(2x-1)}{2}-\sqrt{2}\,\arctan\left(\frac{(3x-1)\sqrt{2}}{4\sqrt{-(x+1)^{2}+3x+1}}\right)$
trager	$\sqrt{-x^{2}+x}+\operatorname{RootOf}\left(\_Z^{2}+2\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^{2}+2\right)x+4\sqrt{-x^{2}+x}-\operatorname{RootOf}\left(\_Z^{2}+2\right)}{x+1}\right)+\frac{3\operatorname{RootOf}(\_$

[In] int((-x^2+x)^(1/2)/(x+1),x,method=_RETURNVERBOSE)

[Out] 2*2^(1/2)*arctan(1/2*(-(x-1)*x)^(1/2)/x*2^(1/2))+(-(x-1)*x)^(1/2)-3*arctan((-(x-1)*x)^(1/2)/x)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{x-x^2}}{1+x}\,dx = 2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+x}}{2\,x}\right)+\sqrt{-x^2+x}-3\,\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

```
[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="fricas")
[Out] 2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + x)/x) + sqrt(-x^2 + x) - 3*arctan(
sqrt(-x^2 + x)/x)
```

Sympy [F]

$$\int \frac{\sqrt{x-x^2}}{1+x}\,dx = \int \frac{\sqrt{-x\,(x-1)}}{x+1}\,dx$$

```
[In] integrate((-x**2+x)**(1/2)/(1+x),x)
[Out] Integral(sqrt(-x*(x - 1))/(x + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{x-x^2}}{1+x}\,dx = -\sqrt{2}\arcsin\left(\frac{3\,x}{|x+1|}-\frac{1}{|x+1|}\right)+\sqrt{-x^2+x}+\frac{3}{2}\,\arcsin\left(2\,x-1\right)$$

```
[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="maxima")
[Out] -sqrt(2)*arcsin(3*x/abs(x + 1) - 1/abs(x + 1)) + sqrt(-x^2 + x) + 3/2*arcsi
n(2*x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{x-x^2}}{1+x}\,dx = 2\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(\frac{3\left(2\sqrt{-x^2+x}-1\right)}{2x-1}-1\right)\right) + \sqrt{-x^2+x} + \frac{3}{2}\arcsin\left(2x-1\right)$$

```
[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="giac")
[Out] 2*sqrt(2)*arctan(1/4*sqrt(2)*(3*(2*sqrt(-x^2 + x) - 1)/(2*x - 1) - 1)) + sq
rt(-x^2 + x) + 3/2*arcsin(2*x - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x-x^2}}{1+x}\,dx = \int \frac{\sqrt{x-x^2}}{x+1}\,dx$$

```
[In] int((x - x^2)^(1/2)/(x + 1),x)
[Out] int((x - x^2)^(1/2)/(x + 1), x)
```

3.930 $\int \sqrt{\sqrt[4]{x}+x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 59

$$\int \sqrt{\sqrt[4]{x}+x}\,dx=\frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{3}\operatorname{arctanh}\left(\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right)$$

```
[Out] -1/3*arctanh(x^(1/2)/(x^(1/4)+x)^(1/2))+1/3*x^(1/4)*(x^(1/4)+x)^(1/2)+2/3*x
*(x^(1/4)+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {2029, 2043, 2049, 2054, 212}

$$\int \sqrt{\sqrt[4]{x}+x}\,dx=-\frac{1}{3}\operatorname{arctanh}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt[4]{x}}}\right)+\frac{2}{3}\sqrt{x+\sqrt[4]{x}}x+\frac{1}{3}\sqrt{x+\sqrt[4]{x}}\sqrt[4]{x}$$

```
[In] Int[Sqrt[x^(1/4) + x],x]
```

```
[Out] (x^(1/4)*Sqrt[x^(1/4) + x])/3 + (2*x*Sqrt[x^(1/4) + x])/3 - ArcTanh[Sqrt[x]
/Sqrt[x^(1/4) + x]]/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{3}x\sqrt{\sqrt[4]{x}+x}+\frac{1}{4}\int\frac{\sqrt[4]{x}}{\sqrt{\sqrt[4]{x}+x}}\,dx\\
&= \frac{2}{3}x\sqrt{\sqrt[4]{x}+x}+\text{Subst}\left(\int\frac{x^4}{\sqrt{x+x^4}}\,dx,x,\sqrt[4]{x}\right)\\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{2}\text{Subst}\left(\int\frac{x}{\sqrt{x+x^4}}\,dx,x,\sqrt[4]{x}\right)\\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{3}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right)\\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \sqrt{\sqrt[4]{x}+x}\,dx=\frac{1}{3}\sqrt{\sqrt[4]{x}+x}(\sqrt[4]{x}+2x)-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right)$$

```
[In] Integrate[Sqrt[x^(1/4) + x],x]
[Out] (Sqrt[x^(1/4) + x]*(x^(1/4) + 2*x))/3 - ArcTanh[Sqrt[x]/Sqrt[x^(1/4) + x]]/
3
```

Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

method	result
meijerg	$-\frac{2\left(-\frac{\sqrt{\pi}\,x^{\frac{3}{8}}\left(6x^{\frac{3}{4}}+3\right)\sqrt{1+x^{\frac{3}{4}}}}{6}+\frac{\sqrt{\pi}\,\text{arcsinh}\left(x^{\frac{3}{8}}\right)}{2}\right)}{3\sqrt{\pi}}$
derivativedivides	$\frac{1}{6\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}-1\right)^{2}}+\frac{1}{\frac{6\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}-6}+\frac{\ln\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}-1\right)}{6}-\frac{1}{6\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}+1\right)^{2}}+\frac{1}{\frac{6\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}+6}-\frac{\ln\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}+1\right)}{6}$
default	$\frac{1}{6\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}-1\right)^{2}}+\frac{1}{\frac{6\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}-6}+\frac{\ln\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}-1\right)}{6}-\frac{1}{6\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}+1\right)^{2}}+\frac{1}{\frac{6\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}+6}-\frac{\ln\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}+1\right)}{6}$

```
[In] int((x^(1/4)+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3/Pi^(1/2)*(-1/6*Pi^(1/2)*x^(3/8)*(6*x^(3/4)+3)*(1+x^(3/4))^(1/2)+1/2*Pi
^(1/2)*arcsinh(x^(3/8)))
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{\sqrt[4]{x}+x}\,dx=\text{Timed out}$$

```
[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \sqrt{\sqrt[4]{x}+x}\,dx=\int \sqrt{\sqrt[4]{x}+x}\,dx$$

```
[In] integrate((x**(1/4)+x)**(1/2),x)
[Out] Integral(sqrt(x**(1/4) + x), x)
```

Maxima [F]

$$\int \sqrt{\sqrt[4]{x}+x}\,dx=\int \sqrt{x+x^{\frac{1}{4}}}\,dx$$

```
[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x + x^(1/4)), x)
```

Giac [A] (verification not implemented)

none

Time = 1.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

$$\int \sqrt{\sqrt[4]{x}+x}\,dx=\frac{1}{3}\sqrt{x+x^{\frac{1}{4}}}x^{\frac{1}{4}}\left(2\,x^{\frac{3}{4}}+1\right)-\frac{1}{6}\log\left(\sqrt{\frac{1}{x^{\frac{3}{4}}}+1}+1\right)+\frac{1}{6}\log\left(\left|\sqrt{\frac{1}{x^{\frac{3}{4}}}+1}-1\right|\right)$$

```
[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(x + x^(1/4))*x^(1/4)*(2*x^(3/4) + 1) - 1/6*log(sqrt(1/x^(3/4) + 1)
 + 1) + 1/6*log(abs(sqrt(1/x^(3/4) + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 20.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.46

$$\int \sqrt{\sqrt[4]{x}+x}\,dx=\frac{8\,x\,\sqrt{x+x^{1/4}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{3}{2};\ \frac{5}{2};\ -x^{3/4}\right)}{9\,\sqrt{x^{3/4}+1}}$$

```
[In] int((x + x^(1/4))^(1/2),x)
[Out] (8*x*(x + x^(1/4))^(1/2)*hypergeom([-1/2, 3/2], 5/2, -x^(3/4)))/(9*(x^(3/4)
 + 1)^(1/2))
```

3.931 $\int \sqrt{x+x^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 59

$$\int \sqrt{x+x^{3/2}}\,dx=\frac{32\left(x+x^{3/2}\right)^{3/2}}{105x^{3/2}}-\frac{16\left(x+x^{3/2}\right)^{3/2}}{35x}+\frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}}$$

```
[Out] 32/105*(x+x^(3/2))^(3/2)/x^(3/2)-16/35*(x+x^(3/2))^(3/2)/x+4/7*(x+x^(3/2))^
(3/2)/x^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2027, 2041, 2039}

$$\int \sqrt{x+x^{3/2}}\,dx=\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{16\left(x^{3/2}+x\right)^{3/2}}{35x}+\frac{32\left(x^{3/2}+x\right)^{3/2}}{105x^{3/2}}$$

```
[In] Int[Sqrt[x + x^(3/2)],x]
```

```
[Out] (32*(x + x^(3/2))^(3/2))/(105*x^(3/2)) - (16*(x + x^(3/2))^(3/2))/(35*x) +
(4*(x + x^(3/2))^(3/2))/(7*Sqrt[x])
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}}-\frac{4}{7}\int\frac{\sqrt{x+x^{3/2}}}{\sqrt{x}}\,dx\\ &= -\frac{16\left(x+x^{3/2}\right)^{3/2}}{35x}+\frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}}+\frac{8}{35}\int\frac{\sqrt{x+x^{3/2}}}{x}\,dx\\ &= \frac{32\left(x+x^{3/2}\right)^{3/2}}{105x^{3/2}}-\frac{16\left(x+x^{3/2}\right)^{3/2}}{35x}+\frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int\sqrt{x+x^{3/2}}\,dx=\frac{4\sqrt{x+x^{3/2}}\left(8-4\sqrt{x}+3x+15x^{3/2}\right)}{105\sqrt{x}}$$

```
[In] Integrate[Sqrt[x + x^(3/2)],x]
[Out] (4*Sqrt[x + x^(3/2)]*(8 - 4*Sqrt[x] + 3*x + 15*x^(3/2)))/(105*Sqrt[x])
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

method	result	size
derivativedivides	$\frac{4\sqrt{x+x^{\frac{3}{2}}}\,(1+\sqrt{x})\,(15x-12\sqrt{x}+8)}{105\sqrt{x}}$	28
default	$\frac{4\sqrt{x+x^{\frac{3}{2}}}\,(1+\sqrt{x})\,(15x-12\sqrt{x}+8)}{105\sqrt{x}}$	28
meijerg	$-\frac{\frac{32\sqrt{\pi}}{105}-\frac{4\sqrt{\pi}\,(1+\sqrt{x})^{\frac{3}{2}}\,(15x-12\sqrt{x}+8)}{105}}{\sqrt{\pi}}$	34

```
[In] int((x+x^(3/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 4/105*(x+x^(3/2))^(1/2)*(1+x^(1/2))*(15*x-12*x^(1/2)+8)/x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.51

$$\int \sqrt{x+x^{3/2}}\,dx=\frac{4\left(15\,x^2+(3\,x+8)\sqrt{x}-4\,x\right)\sqrt{x^{\frac{3}{2}}+x}}{105\,x}$$

```
[In] integrate((x+x^(3/2))^(1/2),x, algorithm="fricas")
[Out] 4/105*(15*x^2 + (3*x + 8)*sqrt(x) - 4*x)*sqrt(x^(3/2) + x)/x
```

Sympy [F]

$$\int \sqrt{x+x^{3/2}}\,dx=\int \sqrt{x^{\frac{3}{2}}+x}\,dx$$

```
[In] integrate((x+x**(3/2))**(1/2),x)
[Out] Integral(sqrt(x**(3/2) + x), x)
```

Maxima [F]

$$\int \sqrt{x + x^{3/2}}\, dx = \int \sqrt{x^{\frac{3}{2}} + x}\, dx$$

```
[In] integrate((x+x^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^(3/2) + x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.56

$$\int \sqrt{x + x^{3/2}}\, dx = \frac{4}{105} \left(15 \left(\sqrt{x} + 1\right)^{\frac{7}{2}} - 42 \left(\sqrt{x} + 1\right)^{\frac{5}{2}} + 35 \left(\sqrt{x} + 1\right)^{\frac{3}{2}} - 8\right) \operatorname{sgn}(x)$$

```
[In] integrate((x+x^(3/2))^(1/2),x, algorithm="giac")
[Out] 4/105*(15*(sqrt(x) + 1)^(7/2) - 42*(sqrt(x) + 1)^(5/2) + 35*(sqrt(x) + 1)^(
3/2) - 8)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 19.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.46

$$\int \sqrt{x + x^{3/2}}\, dx = \frac{2\, x \sqrt{x + x^{3/2}}\; {}_2F_1\left(-\frac{1}{2}, 3;\ 4;\ -\sqrt{x}\right)}{3 \sqrt{\sqrt{x} + 1}}$$

```
[In] int((x + x^(3/2))^(1/2),x)
[Out] (2*x*(x + x^(3/2))^(1/2)*hypergeom([-1/2, 3], 4, -x^(1/2)))/(3*(x^(1/2) + 1
)^(1/2))
```

3.932 $\int x\sqrt{x+x^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int x\sqrt{x+x^{3/2}}\,dx=-\frac{32}{99}\left(x+x^{3/2}\right)^{3/2}+\frac{512\left(x+x^{3/2}\right)^{3/2}}{3465x^{3/2}}-\frac{256\left(x+x^{3/2}\right)^{3/2}}{1155x}+\frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}}+\frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}$$

```
[Out] -32/99*(x+x^(3/2))^(3/2)+512/3465*(x+x^(3/2))^(3/2)/x^(3/2)-256/1155*(x+x^(
3/2))^(3/2)/x+64/231*(x+x^(3/2))^(3/2)/x^(1/2)+4/11*(x+x^(3/2))^(3/2)*x^(1/
2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2041, 2027, 2039}

$$\int x\sqrt{x+x^{3/2}}\,dx=\frac{4}{11}\sqrt{x}\left(x^{3/2}+x\right)^{3/2}+\frac{64\left(x^{3/2}+x\right)^{3/2}}{231\sqrt{x}}-\frac{256\left(x^{3/2}+x\right)^{3/2}}{1155x}+\frac{512\left(x^{3/2}+x\right)^{3/2}}{3465x^{3/2}}-\frac{32}{99}\left(x^{3/2}+x\right)^{3/2}$$

```
[In] Int[x*Sqrt[x + x^(3/2)],x]
```

```
[Out] (-32*(x + x^(3/2))^(3/2))/99 + (512*(x + x^(3/2))^(3/2))/(3465*x^(3/2)) - (
256*(x + x^(3/2))^(3/2))/(1155*x) + (64*(x + x^(3/2))^(3/2))/(231*Sqrt[x])
+ (4*Sqrt[x]*(x + x^(3/2))^(3/2))/11
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}-\frac{8}{11}\int\sqrt{x}\sqrt{x+x^{3/2}}\,dx\\
&= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2}+\frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}+\frac{16}{33}\int\sqrt{x+x^{3/2}}\,dx\\
&= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2}+\frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}}+\frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}-\frac{64}{231}\int\frac{\sqrt{x+x^{3/2}}}{\sqrt{x}}\,dx\\
&= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2}-\frac{256\left(x+x^{3/2}\right)^{3/2}}{1155x}+\frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}}\\
&\quad+\frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}+\frac{128\int\frac{\sqrt{x+x^{3/2}}}{x}\,dx}{1155}\\
&= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2}+\frac{512\left(x+x^{3/2}\right)^{3/2}}{3465x^{3/2}}-\frac{256\left(x+x^{3/2}\right)^{3/2}}{1155x}\\
&\quad+\frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}}+\frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

$$\int x\sqrt{x+x^{3/2}}\,dx = \frac{4\sqrt{x+x^{3/2}}\left(128-64\sqrt{x}+48x-40x^{3/2}+35x^2+315x^{5/2}\right)}{3465\sqrt{x}}$$

```
[In] Integrate[x*Sqrt[x + x^(3/2)],x]
[Out] (4*Sqrt[x + x^(3/2)]*(128 - 64*Sqrt[x] + 48*x - 40*x^(3/2) + 35*x^2 + 315*x
^(5/2)))/(3465*Sqrt[x])
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.40

method	result	size
derivativedivides	$\frac{4\sqrt{x+x^{\frac{3}{2}}}\,(1+\sqrt{x})\left(315x^2-280x^{\frac{3}{2}}+240x-192\sqrt{x}+128\right)}{3465\sqrt{x}}$	38
default	$\frac{4\sqrt{x+x^{\frac{3}{2}}}\,(1+\sqrt{x})\left(315x^2-280x^{\frac{3}{2}}+240x-192\sqrt{x}+128\right)}{3465\sqrt{x}}$	38
meijerg	$-\frac{\frac{512\sqrt{\pi}}{3465}-\frac{4\sqrt{\pi}\,(1+\sqrt{x})^{\frac{3}{2}}\left(315x^2-280x^{\frac{3}{2}}+240x-192\sqrt{x}+128\right)}{3465}}{\sqrt{\pi}}$	44

```
[In] int(x*(x+x^(3/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 4/3465*(x+x^(3/2))^(1/2)*(1+x^(1/2))*(315*x^2-280*x^(3/2)+240*x-192*x^(1/2)
+128)/x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int x\sqrt{x+x^{3/2}}\,dx = \frac{4\left(315\,x^3-40\,x^2+(35\,x^2+48\,x+128)\sqrt{x}-64\,x\right)\sqrt{x^{\frac{3}{2}}+x}}{3465\,x}$$

```
[In] integrate(x*(x+x^(3/2))^(1/2),x, algorithm="fricas")
[Out] 4/3465*(315*x^3 - 40*x^2 + (35*x^2 + 48*x + 128)*sqrt(x) - 64*x)*sqrt(x^(3/
2) + x)/x
```

Sympy [F]

$$\int x\sqrt{x+x^{3/2}}\,dx=\int x\sqrt{x^{\frac{3}{2}}+x}\,dx$$

```
[In] integrate(x*(x+x**(3/2))**(1/2),x)
[Out] Integral(x*sqrt(x**(3/2) + x), x)
```

Maxima [F]

$$\int x\sqrt{x+x^{3/2}}\,dx=\int \sqrt{x^{\frac{3}{2}}+x}x\,dx$$

```
[In] integrate(x*(x+x^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^(3/2) + x)*x, x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

$$\int x\sqrt{x+x^{3/2}}\,dx=\frac{4}{3465}\left(315\left(\sqrt{x}+1\right)^{\frac{11}{2}}-1540\left(\sqrt{x}+1\right)^{\frac{9}{2}}+2970\left(\sqrt{x}+1\right)^{\frac{7}{2}}-2772\left(\sqrt{x}+1\right)^{\frac{5}{2}}+1155$$

```
[In] integrate(x*(x+x^(3/2))^(1/2),x, algorithm="giac")
[Out] 4/3465*(315*(sqrt(x) + 1)^(11/2) - 1540*(sqrt(x) + 1)^(9/2) + 2970*(sqrt(x)
 + 1)^(7/2) - 2772*(sqrt(x) + 1)^(5/2) + 1155*(sqrt(x) + 1)^(3/2) - 128)*sg
n(x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{x+x^{3/2}}\,dx=\int x\,\sqrt{x+x^{3/2}}\,dx$$

```
[In] int(x*(x + x^(3/2))^(1/2),x)
[Out] int(x*(x + x^(3/2))^(1/2), x)
```

3.933 $\int (1-x^2) \sqrt{\frac{1}{2-x^2}}\, dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int (1-x^2) \sqrt{\frac{1}{2-x^2}}\, dx = \frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

```
[Out] 1/2*x/(1/(-x^2+2))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1972, 391}

$$\int (1-x^2) \sqrt{\frac{1}{2-x^2}}\, dx = \frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

```
[In] Int[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]
```

```
[Out] x/(2*Sqrt[(2 - x^2)^(-1)])
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
```

```
/; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \left(\sqrt{\frac{1}{2-x^2}}\sqrt{2-x^2}\right)\int \frac{1-x^2}{\sqrt{2-x^2}}\,dx$$
$$= \frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (1-x^2)\sqrt{\frac{1}{2-x^2}}\,dx = \frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

```
[In] Integrate[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]
[Out] x/(2*Sqrt[(2 - x^2)^(-1)])
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

method	result	size
gosper	$-\frac{(x^2-2)x\sqrt{-\frac{1}{x^2-2}}}{2}$	20
default	$-\frac{(x^2-2)x\sqrt{-\frac{1}{x^2-2}}}{2}$	20
trager	$-\frac{(x^2-2)x\sqrt{-\frac{1}{x^2-2}}}{2}$	20
risch	$-\frac{(x^2-2)x\sqrt{-\frac{1}{x^2-2}}}{2}$	20
meijerg	$\sqrt{\frac{1}{-x^2+2}}\,\sqrt{-x^2+2}\,\arcsin\left(\frac{x\sqrt{2}}{2}\right)-\frac{i\sqrt{\frac{1}{-x^2+2}}\,\sqrt{-x^2+2}\left(\frac{i\sqrt{\pi}\,x\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}}{2}-i\sqrt{\pi}\,\arcsin\left(\frac{x\sqrt{2}}{2}\right)\right)}{\sqrt{\pi}}$	89

```
[In] int((-x^2+1)*(1/(-x^2+2))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(x^2-2)*x*(-1/(x^2-2))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (1-x^2)\sqrt{\frac{1}{2-x^2}}\,dx = -\frac{1}{2}\left(x^3-2\,x\right)\sqrt{-\frac{1}{x^2-2}}$$

```
[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="fricas")
[Out] -1/2*(x^3 - 2*x)*sqrt(-1/(x^2 - 2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int (1-x^2)\sqrt{\frac{1}{2-x^2}}\,dx = -\frac{x^3\sqrt{-\frac{1}{x^2-2}}}{2}+x\sqrt{-\frac{1}{x^2-2}}$$

```
[In] integrate((-x**2+1)*(1/(-x**2+2))**(1/2),x)
[Out] -x**3*sqrt(-1/(x**2 - 2))/2 + x*sqrt(-1/(x**2 - 2))
```

Maxima [F]

$$\int (1-x^2)\sqrt{\frac{1}{2-x^2}}\,dx = \int -\left(x^2-1\right)\sqrt{-\frac{1}{x^2-2}}\,dx$$

```
[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)*sqrt(-1/(x^2 - 2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (1-x^2)\sqrt{\frac{1}{2-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+2}x\mathrm{sgn}\left(x^2-2\right)$$

```
[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^2 + 2)*x*sgn(x^2 - 2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \left(1-x^2\right) \sqrt{\frac{1}{2-x^2}}\, dx = -\frac{x\,(x^2-2)\,\sqrt{-\frac{1}{x^2-2}}}{2}$$

```
[In] int(-(x^2 - 1)*(-1/(x^2 - 2))^(1/2),x)
[Out] -(x*(x^2 - 2)*(-1/(x^2 - 2))^(1/2))/2
```

3.934 $\int \sqrt{x^2+x^3-x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \sqrt{x^2+x^3-x^4}\,dx=-\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x}-\frac{(1+x-x^2)\sqrt{x^2+x^3-x^4}}{3x}-\frac{5\sqrt{x^2+x^3-x^4}\arcsin\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{1+x-x^2}}$$

```
[Out] -1/8*(1-2*x)*(-x^4+x^3+x^2)^(1/2)/x-1/3*(-x^2+x+1)*(-x^4+x^3+x^2)^(1/2)/x-5
/16*arcsin(1/5*(1-2*x)*5^(1/2))*(-x^4+x^3+x^2)^(1/2)/x/(-x^2+x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1917, 654, 626, 633, 222}

$$\int \sqrt{x^2+x^3-x^4}\,dx=-\frac{5\sqrt{-x^4+x^3+x^2}\arcsin\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{-x^2+x+1}}-\frac{\sqrt{-x^4+x^3+x^2}(1-2x)}{8x}-\frac{(-x^2+x+1)\sqrt{-x^4+x^3+x^2}}{3x}$$

```
[In] Int[Sqrt[x^2 + x^3 - x^4],x]
```

```
[Out] -1/8*((1 - 2*x)*Sqrt[x^2 + x^3 - x^4])/x - ((1 + x - x^2)*Sqrt[x^2 + x^3 -
x^4])/(3*x) - (5*Sqrt[x^2 + x^3 - x^4]*ArcSin[(1 - 2*x)/Sqrt[5]])/(16*x*Sqr
t[1 + x - x^2])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1917

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{x^2+x^3-x^4}\int x\sqrt{1+x-x^2}\,dx}{x\sqrt{1+x-x^2}}\\
&= -\frac{(1+x-x^2)\sqrt{x^2+x^3-x^4}}{3x}+\frac{\sqrt{x^2+x^3-x^4}\int\sqrt{1+x-x^2}\,dx}{2x\sqrt{1+x-x^2}}\\
&= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x}-\frac{(1+x-x^2)\sqrt{x^2+x^3-x^4}}{3x}+\frac{\left(5\sqrt{x^2+x^3-x^4}\right)\int\frac{1}{\sqrt{1+x-x^2}}\,dx}{16x\sqrt{1+x-x^2}}\\
&= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x}-\frac{(1+x-x^2)\sqrt{x^2+x^3-x^4}}{3x}\\
&\quad-\frac{\left(\sqrt{5}\sqrt{x^2+x^3-x^4}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{5}}}\,dx,x,1-2x\right)}{16x\sqrt{1+x-x^2}}
\end{aligned}$$

$$= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{(1+x-x^2)\sqrt{x^2+x^3-x^4}}{3x} - \frac{5\sqrt{x^2+x^3-x^4}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{1+x-x^2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \sqrt{x^2+x^3-x^4}\,dx$$
$$= \frac{\sqrt{x^2+x^3-x^4}\left(2\sqrt{-1-x+x^2}(-11-2x+8x^2)+15\log\left(1-2x+2\sqrt{-1-x+x^2}\right)\right)}{48x\sqrt{-1-x+x^2}}$$

```
[In] Integrate[Sqrt[x^2 + x^3 - x^4],x]
[Out] (Sqrt[x^2 + x^3 - x^4]*(2*Sqrt[-1 - x + x^2]*(-11 - 2*x + 8*x^2) + 15*Log[1
 - 2*x + 2*Sqrt[-1 - x + x^2]]))/(48*x*Sqrt[-1 - x + x^2])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

method	result
pseudoelliptic	$\frac{(16x^2-4x-22)\sqrt{-x^2(x^2-x-1)}+15\arcsin\left(\frac{\sqrt{5}\,(2x-1)}{5}\right)x}{48x}$
trager	$\frac{(8x^2-2x-11)\sqrt{-x^4+x^3+x^2}}{24x}+\frac{5\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-2\,\mathrm{RootOf}\left(\_Z^2+1\right)x^2+\mathrm{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^4+x^3+x^2}}{x}\right)}{16}$
default	$-\frac{\sqrt{-x^4+x^3+x^2}\left(16(-x^2+x+1)^{\frac{3}{2}}-12x\sqrt{-x^2+x+1}+6\sqrt{-x^2+x+1}-15\arcsin\left(\frac{\sqrt{5}\,(2x-1)}{5}\right)\right)}{48x\sqrt{-x^2+x+1}}$
risch	$\frac{(8x^2-2x-11)\sqrt{-x^2(x^2-x-1)}}{24x}-\frac{5\arcsin\left(\frac{2\sqrt{5}\,\left(x-\frac{1}{2}\right)}{5}\right)\sqrt{-x^2(x^2-x-1)}\,\sqrt{-x^2+x+1}}{16x(x^2-x-1)}$

```
[In] int((-x^4+x^3+x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/48*((16*x^2-4*x-22)*(-x^2*(x^2-x-1))^(1/2)+15*arcsin(1/5*5^(1/2)*(2*x-1))
*x)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \sqrt{x^2+x^3-x^4}\,dx = -\frac{15\,x\arctan\left(-\frac{x-\sqrt{-x^4+x^3+x^2}}{x^2}\right)-\sqrt{-x^4+x^3+x^2}(8\,x^2-2\,x-11)+11\,x}{24\,x}$$

```
[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="fricas")
[Out] -1/24*(15*x*arctan(-(x - sqrt(-x^4 + x^3 + x^2))/x^2) - sqrt(-x^4 + x^3 + x
^2)*(8*x^2 - 2*x - 11) + 11*x)/x
```

Sympy [F]

$$\int \sqrt{x^2+x^3-x^4}\,dx = \int \sqrt{-x^4+x^3+x^2}\,dx$$

```
[In] integrate((-x**4+x**3+x**2)**(1/2),x)
[Out] Integral(sqrt(-x**4 + x**3 + x**2), x)
```

Maxima [F]

$$\int \sqrt{x^2+x^3-x^4}\,dx = \int \sqrt{-x^4+x^3+x^2}\,dx$$

```
[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + x^3 + x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.56

$$\int \sqrt{x^2+x^3-x^4}\,dx = \frac{1}{48}\left(15\arcsin\left(\frac{1}{5}\sqrt{5}\right)+22\right)\mathrm{sgn}(x) + \frac{5}{16}\arcsin\left(\frac{1}{5}\sqrt{5}(2\,x-1)\right)\mathrm{sgn}(x) + \frac{1}{24}\left(2\left(4\,x\mathrm{sgn}(x)-\mathrm{sgn}(x)\right)x-11\,\mathrm{sgn}(x)\right)\sqrt{-x^2+x+1}$$

```
[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(15*arcsin(1/5*sqrt(5)) + 22)*sgn(x) + 5/16*arcsin(1/5*sqrt(5)*(2*x -
1))*sgn(x) + 1/24*(2*(4*x*sgn(x) - sgn(x))*x - 11*sgn(x))*sqrt(-x^2 + x + 1
)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x^2 + x^3 - x^4}\, dx = \int \sqrt{-x^4 + x^3 + x^2}\, dx$$

```
[In] int((x^2 + x^3 - x^4)^(1/2),x)
```

```
[Out] int((x^2 + x^3 - x^4)^(1/2), x)
```

3.935 $\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx = \frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$$

```
[Out] x*(a^2+x^2)/a^2/((a^2+x^2)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1973, 197}

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx = \frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$$

```
[In] Int[1/Sqrt[(a^2 + x^2)^3],x]
[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
```

```
x], x] /; FreeQ[{a, b, c, n, p, q}, x] && !GeQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(1+\frac{x^2}{a^2}\right)^{3/2}\int \frac{1}{\left(1+\frac{x^2}{a^2}\right)^{3/2}}\,dx}{\sqrt{(a^2+x^2)^3}}$$
$$= \frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx = \frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$$

```
[In] Integrate[1/Sqrt[(a^2 + x^2)^3],x]
[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])
```

Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$	24
default	$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$	24
risch	$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$	24
trager	$\frac{x\sqrt{a^6+3a^4x^2+3a^2x^4+x^6}}{a^2(a^2+x^2)^2}$	40

```
[In] int(1/((a^2+x^2)^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] x*(a^2+x^2)/a^2/((a^2+x^2)^3)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(23) = 46$.

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.56

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx = \frac{a^4+2\,a^2x^2+x^4+\sqrt{a^6+3\,a^4x^2+3\,a^2x^4+x^6}x}{a^6+2\,a^4x^2+a^2x^4}$$

```
[In] integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="fricas")
[Out] (a^4 + 2*a^2*x^2 + x^4 + sqrt(a^6 + 3*a^4*x^2 + 3*a^2*x^4 + x^6)*x)/(a^6 +
2*a^4*x^2 + a^2*x^4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx = \int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx$$

```
[In] integrate(1/((a**2+x**2)**3)**(1/2),x)
[Out] Integral(1/sqrt((a**2 + x**2)**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx = \frac{x}{\sqrt{a^2+x^2}a^2}$$

```
[In] integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="maxima")
[Out] x/(sqrt(a^2 + x^2)*a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx = \frac{x}{\sqrt{a^2+x^2}a^2}$$

```
[In] integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="giac")
[Out] x/(sqrt(a^2 + x^2)*a^2)
```

Mupad [B] (verification not implemented)

Time = 21.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx = \frac{x\,\sqrt{(a^2+x^2)^3}}{a^2\,(a^2+x^2)^2}$$

```
[In] int(1/((a^2 + x^2)^3)^(1/2),x)
[Out] (x*((a^2 + x^2)^3)^(1/2))/(a^2*(a^2 + x^2)^2)
```

3.936 $\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx = 2\sqrt{x}-\frac{2\arctan\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}-\log\left(1+\sqrt{x}+x\right)$$

```
[Out] -ln(1+x+x^(1/2))-2/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1371, 717, 648, 632, 210, 642}

$$\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx = -\frac{2\arctan\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}+2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)$$

```
[In] Int[Sqrt[x]/(1 + Sqrt[x] + x),x]
```

```
[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + Sqrt[x] +
 x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^2}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= 2\sqrt{x} + 2\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= 2\sqrt{x} - \text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt{x}\right) - \text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= 2\sqrt{x} - \log\left(1+\sqrt{x}+x\right) + 2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt{x}\right)\\
&= 2\sqrt{x} - \frac{2\tan^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1+\sqrt{x}+x\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx = 2\sqrt{x}-\frac{2\arctan\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}-\log\left(1+\sqrt{x}+x\right)$$

```
[In] Integrate[Sqrt[x]/(1 + Sqrt[x] + x),x]
[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + Sqrt[x] +
 x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

method	result
derivativedivides	$-\ln\left(1+x+\sqrt{x}\right)-\frac{2\arctan\left(\frac{(1+2\sqrt{x})\sqrt{3}}{3}\right)\sqrt{3}}{3}+2\sqrt{x}$
default	$-\ln\left(1+x+\sqrt{x}\right)-\frac{2\arctan\left(\frac{(1+2\sqrt{x})\sqrt{3}}{3}\right)\sqrt{3}}{3}+2\sqrt{x}$
trager	$2\sqrt{x}-2\ln\left(1+x+\sqrt{x}\right)\text{RootOf}\left(3\_Z^2+3\_Z+1\right)+2\ln\left(-3\,\text{RootOf}\left(3\_Z^2+3\_Z\right.\right.$

```
[In] int(x^(1/2)/(1+x+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -ln(1+x+x^(1/2))-2/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)+2*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x}+\frac{1}{3}\sqrt{3}\right)+2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)$$

```
[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="fricas")
[Out] -2/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x) + 1/3*sqrt(3)) + 2*sqrt(x) - log(x
+ sqrt(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx = 2\sqrt{x} - \log\left(4\sqrt{x}+4x+4\right) - \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate(x**(1/2)/(1+x+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) - log(4*sqrt(x) + 4*x + 4) - 2*sqrt(3)*atan(2*sqrt(3)*sqrt(x)/3 +
 sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\sqrt{x}+1)\right) + 2\sqrt{x} - \log\left(x+\sqrt{x}+1\right)$$

```
[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - log(x + sqrt
(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\sqrt{x}+1)\right) + 2\sqrt{x} - \log\left(x+\sqrt{x}+1\right)$$

```
[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - log(x + sqrt
(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx = 2\sqrt{x} - \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\sqrt{x}}{3}\right)}{3} - \ln\left(x+\sqrt{x}+1\right)$$

```
[In] int(x^(1/2)/(x + x^(1/2) + 1),x)
[Out] 2*x^(1/2) - (2*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^(1/2))/3))/3 - log(x +
 x^(1/2) + 1)
```

3.937 $\int \frac{x}{1+\sqrt{x}+x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{x}{1+\sqrt{x}+x} \, dx = -2\sqrt{x}+x+\frac{4\arctan\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] x+4/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)-2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1371, 715, 632, 210}

$$\int \frac{x}{1+\sqrt{x}+x} \, dx = \frac{4\arctan\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}+x-2\sqrt{x}$$

```
[In] Int[x/(1 + Sqrt[x] + x),x]
```

```
[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^3}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x+x^2}\right)\,dx, x, \sqrt{x}\right)\\
&= -2\sqrt{x}+x+2\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\
&= -2\sqrt{x}+x-4\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt{x}\right)\\
&= -2\sqrt{x}+x+\frac{4\tan^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+\sqrt{x}+x}\,dx = -2\sqrt{x}+x+\frac{4\arctan\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Integrate[x/(1 + Sqrt[x] + x),x]
[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$x+\frac{4\arctan\left(\frac{\left(1+2\sqrt{x}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}-2\sqrt{x}$	26
default	$x+\frac{4\arctan\left(\frac{\left(1+2\sqrt{x}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}-2\sqrt{x}$	26
trager	$x-1-2\sqrt{x}-\frac{2\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\sqrt{x}-x+1}{\mathrm{RootOf}\left(\_Z^2+3\right)x-\mathrm{RootOf}\left(\_Z^2+3\right)+3x+3}\right)}{3}$	58

```
[In] int(x/(1+x+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] x+4/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)-2*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{x}{1+\sqrt{x}+x}\,dx=\frac{4}{3}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\sqrt{x}+\frac{1}{3}\,\sqrt{3}\right)+x-2\,\sqrt{x}$$

```
[In] integrate(x/(1+x+x^(1/2)),x, algorithm="fricas")
[Out] 4/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x) + 1/3*sqrt(3)) + x - 2*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{x}{1+\sqrt{x}+x}\,dx=-2\sqrt{x}+x+\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate(x/(1+x+x**(1/2)),x)
[Out] -2*sqrt(x) + x + 4*sqrt(3)*atan(2*sqrt(3)*sqrt(x)/3 + sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x}{1+\sqrt{x}+x}\,dx = \frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+x-2\sqrt{x}$$

```
[In] integrate(x/(1+x+x^(1/2)),x, algorithm="maxima")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x}{1+\sqrt{x}+x}\,dx = \frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+x-2\sqrt{x}$$

```
[In] integrate(x/(1+x+x^(1/2)),x, algorithm="giac")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 21.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{x}{1+\sqrt{x}+x}\,dx = x+\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\sqrt{x}}{3}\right)}{3}-2\sqrt{x}$$

```
[In] int(x/(x + x^(1/2) + 1),x)
[Out] x + (4*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^(1/2))/3))/3 - 2*x^(1/2)
```

3.938 $\int \frac{1}{\sqrt{x}(1+\sqrt{x}+x)^{7/2}} dx$

Optimal result

Integrand size = 18, antiderivative size = 76

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}} dx = \frac{4(1+2\sqrt{x})}{15\left(1+\sqrt{x}+x\right)^{5/2}} + \frac{64(1+2\sqrt{x})}{135\left(1+\sqrt{x}+x\right)^{3/2}} + \frac{512(1+2\sqrt{x})}{405\sqrt{1+\sqrt{x}+x}}$$

```
[Out] 4/15*(1+2*x^(1/2))/(1+x+x^(1/2))^(5/2)+64/135*(1+2*x^(1/2))/(1+x+x^(1/2))^(
3/2)+512/405*(1+2*x^(1/2))/(1+x+x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1366, 628, 627}

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}} dx = \frac{512(2\sqrt{x}+1)}{405\sqrt{x+\sqrt{x}+1}} + \frac{64(2\sqrt{x}+1)}{135\left(x+\sqrt{x}+1\right)^{3/2}} + \frac{4(2\sqrt{x}+1)}{15\left(x+\sqrt{x}+1\right)^{5/2}}$$

```
[In] Int[1/(Sqrt[x]*(1 + Sqrt[x] + x)^(7/2)),x]
```

```
[Out] (4*(1 + 2*Sqrt[x]))/(15*(1 + Sqrt[x] + x)^(5/2)) + (64*(1 + 2*Sqrt[x]))/(13
5*(1 + Sqrt[x] + x)^(3/2)) + (512*(1 + 2*Sqrt[x]))/(405*Sqrt[1 + Sqrt[x] +
x])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{1}{(1+x+x^2)^{7/2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{4(1+2\sqrt{x})}{15\left(1+\sqrt{x}+x\right)^{5/2}}+\frac{32}{15}\text{Subst}\left(\int \frac{1}{(1+x+x^2)^{5/2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{4(1+2\sqrt{x})}{15\left(1+\sqrt{x}+x\right)^{5/2}}+\frac{64(1+2\sqrt{x})}{135\left(1+\sqrt{x}+x\right)^{3/2}}+\frac{256}{135}\text{Subst}\left(\int \frac{1}{(1+x+x^2)^{3/2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{4(1+2\sqrt{x})}{15\left(1+\sqrt{x}+x\right)^{5/2}}+\frac{64(1+2\sqrt{x})}{135\left(1+\sqrt{x}+x\right)^{3/2}}+\frac{512(1+2\sqrt{x})}{405\sqrt{1+\sqrt{x}+x}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx=\frac{4(1+2\sqrt{x})\left(203+304\sqrt{x}+432x+256x^{3/2}+128x^2\right)}{405\left(1+\sqrt{x}+x\right)^{5/2}}$$

```
[In] Integrate[1/(Sqrt[x]*(1 + Sqrt[x] + x)^(7/2)),x]
[Out] (4*(1 + 2*Sqrt[x])*(203 + 304*Sqrt[x] + 432*x + 256*x^(3/2) + 128*x^2))/(40
5*(1 + Sqrt[x] + x)^(5/2))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{\frac{4}{15}+\frac{8\sqrt{x}}{15}}{\left(1+x+\sqrt{x}\right)^{\frac{5}{2}}}+\frac{\frac{64}{135}+\frac{128\sqrt{x}}{135}}{\left(1+x+\sqrt{x}\right)^{\frac{3}{2}}}+\frac{\frac{512}{405}+\frac{1024\sqrt{x}}{405}}{\sqrt{1+x+\sqrt{x}}}$	53
default	$\frac{\frac{4}{15}+\frac{8\sqrt{x}}{15}}{\left(1+x+\sqrt{x}\right)^{\frac{5}{2}}}+\frac{\frac{64}{135}+\frac{128\sqrt{x}}{135}}{\left(1+x+\sqrt{x}\right)^{\frac{3}{2}}}+\frac{\frac{512}{405}+\frac{1024\sqrt{x}}{405}}{\sqrt{1+x+\sqrt{x}}}$	53

```
[In] int(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/15*(1+2*x^(1/2))/(1+x+x^(1/2))^(5/2)+64/135*(1+2*x^(1/2))/(1+x+x^(1/2))^(
3/2)+512/405*(1+2*x^(1/2))/(1+x+x^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx =$$
$$-\frac{4\left(128\,x^{5}+272\,x^{4}+455\,x^{3}+232\,x^{2}-\left(256\,x^{5}+736\,x^{4}+1366\,x^{3}+1427\,x^{2}+839\,x+101\right)\sqrt{x}-128\,x-2}{405\left(x^{6}+3\,x^{5}+6\,x^{4}+7\,x^{3}+6\,x^{2}+3\,x+1\right)}$$

```
[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="fricas")
```

```
[Out] -4/405*(128*x^5 + 272*x^4 + 455*x^3 + 232*x^2 - (256*x^5 + 736*x^4 + 1366*x
^3 + 1427*x^2 + 839*x + 101)*sqrt(x) - 128*x - 203)*sqrt(x + sqrt(x) + 1)/(
x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx = \int \frac{1}{\sqrt{x}\left(\sqrt{x}+x+1\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/x**(1/2)/(1+x+x**(1/2))**(7/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(sqrt(x) + x + 1)**(7/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx = \int \frac{1}{\left(x+\sqrt{x}+1\right)^{\frac{7}{2}}\sqrt{x}}\,dx$$

```
[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="maxima")
[Out] integrate(1/((x + sqrt(x) + 1)^(7/2)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx = \frac{4\left(2\left(8\left(2\left(4\sqrt{x}(2\sqrt{x}+5)+35\right)\sqrt{x}+65\right)\sqrt{x}+355\right)\sqrt{x}+203\right)}{405\left(x+\sqrt{x}+1\right)^{\frac{5}{2}}}$$

```
[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="giac")
[Out] 4/405*(2*(8*(2*(4*sqrt(x)*(2*sqrt(x) + 5) + 35)*sqrt(x) + 65)*sqrt(x) + 355
)*sqrt(x) + 203)/(x + sqrt(x) + 1)^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx = \int \frac{1}{\sqrt{x}\left(x+\sqrt{x}+1\right)^{7/2}}\,dx$$

```
[In] int(1/(x^(1/2)*(x + x^(1/2) + 1)^(7/2)),x)
[Out] int(1/(x^(1/2)*(x + x^(1/2) + 1)^(7/2)), x)
```

3.939 $\int \frac{-1+x}{1+\sqrt{1+x^2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \frac{-1+x}{1+\sqrt{1+x^2}} \, dx = -\frac{1}{x}+\sqrt{1+x^2}+\frac{\sqrt{1+x^2}}{x}-\operatorname{arcsinh}(x)-\log\left(1+\sqrt{1+x^2}\right)$$

```
[Out] -1/x-arcsinh(x)-ln(1+(x^2+1)^(1/2))+(x^2+1)^(1/2)+(x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6874, 283, 221, 1605, 196, 45}

$$\int \frac{-1+x}{1+\sqrt{1+x^2}} \, dx = -\operatorname{arcsinh}(x)+\frac{\sqrt{x^2+1}}{x}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}$$

```
[In] Int[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x - ArcSinh[x] - Log[1 + Sqrt[1 + x
^2]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{1+\sqrt{1+x^2}}+\frac{x}{1+\sqrt{1+x^2}}\right) dx \\
&= -\int \frac{1}{1+\sqrt{1+x^2}}\,dx+\int \frac{x}{1+\sqrt{1+x^2}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{x}}\,dx,x,1+x^2\right)-\int \left(-\frac{1}{x^2}+\frac{\sqrt{1+x^2}}{x^2}\right) dx \\
&= -\frac{1}{x}-\int \frac{\sqrt{1+x^2}}{x^2}\,dx+\text{Subst}\left(\int \frac{x}{1+x}\,dx,x,\sqrt{1+x^2}\right) \\
&= -\frac{1}{x}+\frac{\sqrt{1+x^2}}{x}-\int \frac{1}{\sqrt{1+x^2}}\,dx+\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right) dx,x,\sqrt{1+x^2}\right) \\
&= -\frac{1}{x}+\sqrt{1+x^2}+\frac{\sqrt{1+x^2}}{x}-\sinh^{-1}(x)-\log\left(1+\sqrt{1+x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx = -\frac{1}{x}+\frac{(1+x)\sqrt{1+x^2}}{x}-4\text{arctanh}\left(1-2x+2\sqrt{1+x^2}\right)$$

```
[In] Integrate[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
[Out] -x^(-1) + ((1 + x)*Sqrt[1 + x^2])/x - 4*ArcTanh[1 - 2*x + 2*Sqrt[1 + x^2]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

method	result	size
trager	$\frac{x-1}{x}+\frac{(x+1)\sqrt{x^2+1}}{x}+2\ln\left(-\frac{\sqrt{x^2+1}-1-x}{x}\right)$	43
default	$-\frac{1}{x}+\sqrt{x^2+1}-\text{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\ln(x)+\frac{(x^2+1)^{\frac{3}{2}}}{x}-x\sqrt{x^2+1}-\text{arcsinh}(x)$	53
meijerg	$-\frac{x\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},2;-x^2\right)}{2}+\frac{-4\sqrt{\pi}+4\sqrt{\pi}\,\sqrt{x^2+1}-4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^2+1}}{2}\right)}{4\sqrt{\pi}}$	58

```
[In] int((x-1)/(1+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] (x-1)/x+(x+1)/x*(x^2+1)^(1/2)+2*ln(-((x^2+1)^(1/2)-1-x)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx = \frac{x\log\left(2x^2-\sqrt{x^2+1}(2x+1)+x+1\right)-x\log(x)-x\log\left(-x+\sqrt{x^2+1}+1\right)+\sqrt{x^2+1}(x+1)+x-1}{x}$$

```
[In] integrate((-1+x)/(1+(x^2+1)^(1/2)),x, algorithm="fricas")
[Out] (x*log(2*x^2 - sqrt(x^2 + 1)*(2*x + 1) + x + 1) - x*log(x) - x*log(-x + sqr
t(x^2 + 1) + 1) + sqrt(x^2 + 1)*(x + 1) + x - 1)/x
```

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx = \frac{x}{\sqrt{x^2+1}} + \sqrt{x^2+1} - \log\left(\sqrt{x^2+1}+1\right) - \operatorname{asinh}(x) - \frac{1}{x} + \frac{1}{x\sqrt{x^2+1}}$$

```
[In] integrate((-1+x)/(1+(x**2+1)**(1/2)),x)
[Out] x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(sqrt(x**2 + 1) + 1) - asinh(x) - 1/
x + 1/(x*sqrt(x**2 + 1))
```

Maxima [F]

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx = \int \frac{x-1}{\sqrt{x^2+1}+1}\,dx$$

```
[In] integrate((-1+x)/(1+(x^2+1)^(1/2)),x, algorithm="maxima")
[Out] 1/4*x^2 - 1/2*x - integrate(1/2*(x^3 - x^2)/(x^2 + 2*sqrt(x^2 + 1) + 2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.72

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx = \sqrt{x^2+1} - \frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1} - \frac{1}{x} + \log\left(-x+\sqrt{x^2+1}\right) - \log\left(|x|\right) - \log\left(\left|-x+\sqrt{x^2+1}+1\right|\right) + \log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

```
[In] integrate((-1+x)/(1+(x^2+1)^(1/2)),x, algorithm="giac")
[Out] sqrt(x^2 + 1) - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + log(-x + sqrt(x^2 + 1
)) - log(abs(x)) - log(abs(-x + sqrt(x^2 + 1) + 1)) + log(abs(-x + sqrt(x^2
 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 20.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx = \sqrt{x^2+1} - \ln(x) - \operatorname{asinh}(x) + \frac{\sqrt{x^2+1}}{x} - \frac{1}{x} + \operatorname{atan}\left(\sqrt{x^2+1}\,1\mathrm{i}\right)\,1\mathrm{i}$$

```
[In] int((x - 1)/((x^2 + 1)^(1/2) + 1),x)
```

```
[Out] atan((x^2 + 1)^(1/2)*1i)*1i - asinh(x) - log(x) + (x^2 + 1)^(1/2) + (x^2 +
1)^(1/2)/x - 1/x
```

3.940 $\int \frac{1}{(1+x)^{2/3}(-1+x^2)^{2/3}} dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{1}{(1+x)^{2/3} (-1+x^2)^{2/3}} dx = \frac{3\sqrt[3]{-1+x^2}}{2(1+x)^{2/3}}$$

```
[Out] 3/2*(x^2-1)^(1/3)/(1+x)^(2/3)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {665}

$$\int \frac{1}{(1+x)^{2/3} (-1+x^2)^{2/3}} dx = \frac{3\sqrt[3]{x^2-1}}{2(x+1)^{2/3}}$$

```
[In] Int[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)),x]
[Out] (3*(-1 + x^2)^(1/3))/(2*(1 + x)^(2/3))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\text{integral} = \frac{3\sqrt[3]{-1+x^2}}{2(1+x)^{2/3}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)^{2/3}\left(-1+x^2\right)^{2/3}}\,dx=\frac{3\sqrt[3]{-1+x^2}}{2(1+x)^{2/3}}$$

```
[In] Integrate[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)),x]
[Out] (3*(-1 + x^2)^(1/3))/(2*(1 + x)^(2/3))
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
gosper	$\frac{3(x-1)(x+1)^{\frac{1}{3}}}{2(x^2-1)^{\frac{2}{3}}}$	18
risch	$\frac{3(x+1)^{\frac{1}{3}}\left(\frac{(x^2-1)^2}{x+1}\right)^{\frac{1}{3}}(x-1)}{2(x^2-1)^{\frac{2}{3}}\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}$	44

```
[In] int(1/(x+1)^(2/3)/(x^2-1)^(2/3),x,method=_RETURNVERBOSE)
[Out] 3/2*(x-1)*(x+1)^(1/3)/(x^2-1)^(2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{(1+x)^{2/3}\left(-1+x^2\right)^{2/3}}\,dx=\frac{3\left(x^2-1\right)^{\frac{1}{3}}}{2\,(x+1)^{\frac{2}{3}}}$$

```
[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="fricas")
[Out] 3/2*(x^2 - 1)^(1/3)/(x + 1)^(2/3)
```

Sympy [F]

$$\int \frac{1}{(1+x)^{2/3}(-1+x^2)^{2/3}}\,dx = \int \frac{1}{((x-1)(x+1))^{\frac{2}{3}}(x+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(1+x)**(2/3)/(x**2-1)**(2/3),x)
[Out] Integral(1/(((x - 1)*(x + 1))**(2/3)*(x + 1)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x)^{2/3}(-1+x^2)^{2/3}}\,dx = \int \frac{1}{(x^2-1)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+x)^{2/3}(-1+x^2)^{2/3}}\,dx = \frac{3}{2}\left(-\frac{2}{x+1}+1\right)^{\frac{1}{3}}$$

```
[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="giac")
[Out] 3/2*(-2/(x + 1) + 1)^(1/3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)^{2/3}(-1+x^2)^{2/3}}\,dx = \int \frac{1}{(x^2-1)^{2/3}(x+1)^{2/3}}\,dx$$

```
[In] int(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)),x)
[Out] int(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)
```

3.941 $\int \left((1-x^6)^{2/3}+\frac{(1-x^6)^{2/3}}{x^6}\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 35

$$\int \left((1-x^6)^{2/3}+\frac{(1-x^6)^{2/3}}{x^6}\right)\,dx=-\frac{(1-x^6)^{2/3}}{5x^5}+\frac{1}{5}x(1-x^6)^{2/3}$$

```
[Out] -1/5*(-x^6+1)^(2/3)/x^5+1/5*x*(-x^6+1)^(2/3)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2 in optimal.

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {251, 371}

$$\int \left((1-x^6)^{2/3}+\frac{(1-x^6)^{2/3}}{x^6}\right)\,dx=x\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{6},\frac{7}{6},x^6\right)-\frac{\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{2}{3},\frac{1}{6},x^6\right)}{5x^5}$$

```
[In] Int[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]
```

```
[Out] -1/5*Hypergeometric2F1[-5/6, -2/3, 1/6, x^6]/x^5 + x*Hypergeometric2F1[-2/3
, 1/6, 7/6, x^6]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(1-x^6\right)^{2/3}\,dx + \int \frac{\left(1-x^6\right)^{2/3}}{x^6}\,dx \\ &= -\frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5} + x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};x^6\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int \left(\left(1-x^6\right)^{2/3}+\frac{\left(1-x^6\right)^{2/3}}{x^6}\right)dx = -\frac{\left(1-x^6\right)^{5/3}}{5x^5}$$

```
[In] Integrate[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]
[Out] -1/5*(1 - x^6)^(5/3)/x^5
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

method	result	size
trager	$\frac{\left(x^{6}-1\right)\left(-x^{6}+1\right)^{\frac{2}{3}}}{5 x^{5}}$	20
risch	$-\frac{x^{12}-2 x^{6}+1}{5 x^{5}\left(-x^{6}+1\right)^{\frac{1}{3}}}$	25
meijerg	$x{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};x^{6}\right)-\frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^{6}\right)}{5 x^{5}}$	27
gosper	$\frac{\left(-x^{6}+1\right)^{\frac{2}{3}}\left(x^{2}-x+1\right)\left(x^{2}+x+1\right)\left(x+1\right)\left(x-1\right)}{5 x^{5}}$	35

```
[In] int((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(x^6-1)/x^5*(-x^6+1)^(2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.54

$$\int \left(\left(1-x^6\right)^{2/3} + \frac{\left(1-x^6\right)^{2/3}}{x^6} \right) dx = \frac{(x^6-1)(-x^6+1)^{\frac{2}{3}}}{5\,x^5}$$

```
[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="fricas")
[Out] 1/5*(x^6 - 1)*(-x^6 + 1)^(2/3)/x^5
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.94

$$\int \left(\left(1-x^6\right)^{2/3} + \frac{\left(1-x^6\right)^{2/3}}{x^6} \right) dx = \frac{x\Gamma\left(\frac{1}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{6} \\ \frac{7}{6} \end{matrix}\middle| x^6 e^{2i\pi}\right)}{6\Gamma\left(\frac{7}{6}\right)} + \frac{\Gamma\left(-\frac{5}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{6}, -\frac{2}{3} \\ \frac{1}{6} \end{matrix}\middle| x^6 e^{2i\pi}\right)}{6x^5\Gamma\left(\frac{1}{6}\right)}$$

```
[In] integrate((-x**6+1)**(2/3)+(-x**6+1)**(2/3)/x**6,x)
[Out] x*gamma(1/6)*hyper((-2/3, 1/6), (7/6,), x**6*exp_polar(2*I*pi))/(6*gamma(7/
6)) + gamma(-5/6)*hyper((-5/6, -2/3), (1/6,), x**6*exp_polar(2*I*pi))/(6*x*
*5*gamma(1/6))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \left(\left(1-x^6\right)^{2/3} + \frac{\left(1-x^6\right)^{2/3}}{x^6} \right) dx = \frac{(x^6-1)(x^2+x+1)^{\frac{2}{3}}(-x^2+x-1)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}(x-1)^{\frac{2}{3}}}{5\,x^5}$$

```
[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="maxima")
[Out] 1/5*(x^6 - 1)*(x^2 + x + 1)^(2/3)*(-x^2 + x - 1)^(2/3)*(x + 1)^(2/3)*(x - 1
)^(2/3)/x^5
```

Giac [F]

$$\int \left((1-x^6)^{2/3} + \frac{(1-x^6)^{2/3}}{x^6} \right) dx = \int (-x^6+1)^{\frac{2}{3}} + \frac{(-x^6+1)^{\frac{2}{3}}}{x^6} \, dx$$

```
[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="giac")
[Out] integrate((-x^6 + 1)^(2/3) + (-x^6 + 1)^(2/3)/x^6, x)
```

Mupad [B] (verification not implemented)

Time = 19.88 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.40

$$\int \left((1-x^6)^{2/3} + \frac{(1-x^6)^{2/3}}{x^6} \right) dx = -\frac{(1-x^6)^{5/3}}{5\,x^5}$$

```
[In] int((1 - x^6)^(2/3)/x^6 + (1 - x^6)^(2/3),x)
[Out] -(1 - x^6)^(5/3)/(5*x^5)
```

3.942 $\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 15

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2\left(a+bx^n\right)^{3/2}}\,dx=\frac{x^m}{\sqrt{a+bx^n}}$$

```
[Out] x^m/(a+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {12, 460}

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2\left(a+bx^n\right)^{3/2}}\,dx=\frac{x^m}{\sqrt{a+bx^n}}$$

```
[In] Int[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x^n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
```

```
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\int \frac{x^{-1+m}(2am + b(2m-n)x^n)}{(a+bx^n)^{3/2}}\,dx$$
$$= \frac{x^m}{\sqrt{a+bx^n}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 7.40

$$\int \frac{x^{-1+m}(2am + b(2m-n)x^n)}{2\,(a+bx^n)^{3/2}}\,dx = \frac{x^m\sqrt{1+\frac{bx^n}{a}}\left(2a(m+n)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m}{n},\frac{m+n}{n},-\frac{bx^n}{a}\right)+b(2\cdots\right.}{2a(m+n)\sqrt{a+b\cdots}}$$

```
[In] Integrate[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^n)/a]*(2*a*(m + n)*Hypergeometric2F1[3/2, m/n, (m + n)/n
, -((b*x^n)/a)] + b*(2*m - n)*x^n*Hypergeometric2F1[3/2, (m + n)/n, 2 + m/n
, -((b*x^n)/a)]))/(2*a*(m + n)*Sqrt[a + b*x^n])
```

Maple [F]

$$\int \frac{x^{-1+m}(2am + b(2m-n)\,x^n)}{2\,(a+b\,x^n)^{\frac{3}{2}}}dx$$

```
[In] int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x)
```

```
[Out] int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{x^{-1+m}(2am + b(2m-n)x^n)}{2\,(a+bx^n)^{3/2}}\,dx = \frac{xx^{m-1}}{\sqrt{bx^n+a}}$$

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
fricas")
```

```
[Out] x*x^(m - 1)/sqrt(b*x^n + a)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 8.13

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2\left(a+bx^n\right)^{3/2}}\,dx = \frac{aa^{\frac{m}{n}}a^{-\frac{m}{n}-\frac{3}{2}}mx^m\Gamma\left(\frac{m}{n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{n}\\ \frac{m}{n}+1\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1\right)} + \frac{a^{-\frac{m}{n}-\frac{5}{2}}a^{\frac{m}{n}+1}bx^{m+n}(2m-n)\,\Gamma\left(\frac{m}{n}+1\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{n}+1\\ \frac{m}{n}+2\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{2n\Gamma\left(\frac{m}{n}+2\right)}$$

```
[In] integrate(1/2*x**(-1+m)*(2*a*m+b*(2*m-n)*x**n)/(a+b*x**n)**(3/2),x)
```

```
[Out] a*a**(m/n)*a**(-m/n - 3/2)*m*x**m*gamma(m/n)*hyper((3/2, m/n), (m/n + 1,),
b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1)) + a**(-m/n - 5/2)*a**(m/n + 1)
*b*x**(m + n)*(2*m - n)*gamma(m/n + 1)*hyper((3/2, m/n + 1), (m/n + 2,), b*
x**n*exp_polar(I*pi)/a)/(2*n*gamma(m/n + 2))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2\left(a+bx^n\right)^{3/2}}\,dx = \frac{x^m}{\sqrt{bx^n+a}}$$

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
maxima")
```

```
[Out] x^m/sqrt(b*x^n + a)
```

Giac [F]

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2\left(a+bx^n\right)^{3/2}}\,dx = \int \frac{(b(2\,m-n)x^n+2\,am)x^{m-1}}{2\left(bx^n+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(1/2*(b*(2*m - n)*x^n + 2*a*m)*x^(m - 1)/(b*x^n + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 20.46 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2\,(a+bx^n)^{3/2}}\,dx=\frac{x^m}{\sqrt{a+b\,x^n}}$$

```
[In] int((x^(m - 1)*(2*a*m + b*x^n*(2*m - n)))/(2*(a + b*x^n)^(3/2)),x)
[Out] x^m/(a + b*x^n)^(1/2)
```

3.943 $\int \frac{x-2x^3}{\sqrt{2+3x}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{x-2x^3}{\sqrt{2+3x}} \, dx = -\frac{4}{81}\sqrt{2+3x}-\frac{10}{81}(2+3x)^{3/2}+\frac{8}{135}(2+3x)^{5/2}-\frac{4}{567}(2+3x)^{7/2}$$

```
[Out] -10/81*(2+3*x)^(3/2)+8/135*(2+3*x)^(5/2)-4/567*(2+3*x)^(7/2)-4/81*(2+3*x)^(
1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1607, 786}

$$\int \frac{x-2x^3}{\sqrt{2+3x}} \, dx = -\frac{4}{567}(3x+2)^{7/2}+\frac{8}{135}(3x+2)^{5/2}-\frac{10}{81}(3x+2)^{3/2}-\frac{4}{81}\sqrt{3x+2}$$

```
[In] Int[(x - 2*x^3)/Sqrt[2 + 3*x],x]
```

```
[Out] (-4*Sqrt[2 + 3*x])/81 - (10*(2 + 3*x)^(3/2))/81 + (8*(2 + 3*x)^(5/2))/135 -
 (4*(2 + 3*x)^(7/2))/567
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(1-2x^2)}{\sqrt{2+3x}}\,dx\\ &= \int \left(-\frac{2}{27\sqrt{2+3x}}-\frac{5}{9}\sqrt{2+3x}+\frac{4}{9}(2+3x)^{3/2}-\frac{2}{27}(2+3x)^{5/2}\right)dx\\ &= -\frac{4}{81}\sqrt{2+3x}-\frac{10}{81}(2+3x)^{3/2}+\frac{8}{135}(2+3x)^{5/2}-\frac{4}{567}(2+3x)^{7/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.53

$$\int \frac{x-2x^3}{\sqrt{2+3x}}\,dx = -\frac{2\sqrt{2+3x}(164-123x-216x^2+270x^3)}{2835}$$

```
[In] Integrate[(x - 2*x^3)/Sqrt[2 + 3*x],x]
[Out] (-2*Sqrt[2 + 3*x]*(164 - 123*x - 216*x^2 + 270*x^3))/2835
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.45

method	result	size
trager	$\left(-\frac{4}{21}x^3+\frac{16}{105}x^2+\frac{82}{945}x-\frac{328}{2835}\right)\sqrt{3x+2}$	24
gosper	$-\frac{2(270x^3-216x^2-123x+164)\sqrt{3x+2}}{2835}$	25
risch	$-\frac{2(270x^3-216x^2-123x+164)\sqrt{3x+2}}{2835}$	25
pseudoelliptic	$-\frac{2(270x^3-216x^2-123x+164)\sqrt{3x+2}}{2835}$	25
derivativedivides	$-\frac{10(3x+2)^{\frac{3}{2}}}{81}+\frac{8(3x+2)^{\frac{5}{2}}}{135}-\frac{4(3x+2)^{\frac{7}{2}}}{567}-\frac{4\sqrt{3x+2}}{81}$	38
default	$-\frac{10(3x+2)^{\frac{3}{2}}}{81}+\frac{8(3x+2)^{\frac{5}{2}}}{135}-\frac{4(3x+2)^{\frac{7}{2}}}{567}-\frac{4\sqrt{3x+2}}{81}$	38
meijerg	$-\frac{16\sqrt{2}\left(\frac{32\sqrt{\pi}}{35}-\frac{\sqrt{\pi}\left(-135x^3+108x^2-96x+128\right)\sqrt{1+\frac{3x}{2}}}{140}\right)}{81\sqrt{\pi}}+\frac{2\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-6x+8\right)\sqrt{1+\frac{3x}{2}}}{6}\right)}{9\sqrt{\pi}}$	74

```
[In] int((-2*x^3+x)/(3*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-4/21*x^3+16/105*x^2+82/945*x-328/2835)*(3*x+2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.45

$$\int \frac{x-2x^3}{\sqrt{2+3x}}\,dx=-\frac{2}{2835}\left(270\,x^3-216\,x^2-123\,x+164\right)\sqrt{3\,x+2}$$

```
[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="fricas")
[Out] -2/2835*(270*x^3 - 216*x^2 - 123*x + 164)*sqrt(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{x-2x^3}{\sqrt{2+3x}}\,dx=-\frac{4(3x+2)^{\frac{7}{2}}}{567}+\frac{8(3x+2)^{\frac{5}{2}}}{135}-\frac{10(3x+2)^{\frac{3}{2}}}{81}-\frac{4\sqrt{3x+2}}{81}$$

```
[In] integrate((-2*x**3+x)/(2+3*x)**(1/2),x)
[Out] -4*(3*x + 2)**(7/2)/567 + 8*(3*x + 2)**(5/2)/135 - 10*(3*x + 2)**(3/2)/81 -
 4*sqrt(3*x + 2)/81
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x-2x^3}{\sqrt{2+3x}}\,dx=-\frac{4}{567}\,(3\,x+2)^{\frac{7}{2}}+\frac{8}{135}\,(3\,x+2)^{\frac{5}{2}}-\frac{10}{81}\,(3\,x+2)^{\frac{3}{2}}-\frac{4}{81}\,\sqrt{3\,x+2}$$

```
[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="maxima")
[Out] -4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^(3/2) - 4/
81*sqrt(3*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x-2x^3}{\sqrt{2+3x}}\,dx=-\frac{4}{567}\,(3\,x+2)^{\frac{7}{2}}+\frac{8}{135}\,(3\,x+2)^{\frac{5}{2}}-\frac{10}{81}\,(3\,x+2)^{\frac{3}{2}}-\frac{4}{81}\,\sqrt{3\,x+2}$$

```
[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="giac")
[Out] -4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^(3/2) - 4/
81*sqrt(3*x + 2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x-2x^3}{\sqrt{2+3x}}\,dx=\frac{8\,(3\,x+2)^{5/2}}{135}-\frac{10\,(3\,x+2)^{3/2}}{81}-\frac{4\,\sqrt{3\,x+2}}{81}-\frac{4\,(3\,x+2)^{7/2}}{567}$$

```
[In] int((x - 2*x^3)/(3*x + 2)^(1/2),x)
[Out] (8*(3*x + 2)^(5/2))/135 - (10*(3*x + 2)^(3/2))/81 - (4*(3*x + 2)^(1/2))/81
- (4*(3*x + 2)^(7/2))/567
```

3.944 $\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = -4\sqrt[4]{1+x}+2\sqrt{1+x}+4\log\left(1+\sqrt[4]{1+x}\right)$$

```
[Out] -4*(1+x)^(1/4)+4*ln(1+(1+x)^(1/4))+2*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2037, 1607, 272, 45}

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = 2\sqrt{x+1}-4\sqrt[4]{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

```
[In] Int[((1 + x)^(1/4) + Sqrt[1 + x])^(-1),x]
```

```
[Out] -4*(1 + x)^(1/4) + 2*Sqrt[1 + x] + 4*Log[1 + (1 + x)^(1/4)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2037

```
Int[((a_.)*(u_)^(j_.) + (b_.)*(u_)^(n_.))^(p_), x_Symbol] :> Dist[1/Coeffic
ient[u, x, 1], Subst[Int[(a*x^j + b*x^n)^p, x], x, u], x] /; FreeQ[{a, b, j
, n, p}, x] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx, x, 1+x\right)\\
&= \text{Subst}\left(\int \frac{1}{\left(1+\sqrt[4]{x}\right)\sqrt[4]{x}}\,dx, x, 1+x\right)\\
&= 4\text{Subst}\left(\int \frac{x^2}{1+x}\,dx, x, \sqrt[4]{1+x}\right)\\
&= 4\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x}\right)\,dx, x, \sqrt[4]{1+x}\right)\\
&= -4\sqrt[4]{1+x}+2\sqrt{1+x}+4\log\left(1+\sqrt[4]{1+x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = 2\left(-2\sqrt[4]{1+x}+\sqrt{1+x}+2\log\left(1+\sqrt[4]{1+x}\right)\right)$$

```
[In] Integrate[((1 + x)^(1/4) + Sqrt[1 + x])^(-1),x]
[Out] 2*(-2*(1 + x)^(1/4) + Sqrt[1 + x] + 2*Log[1 + (1 + x)^(1/4)])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-4(x+1)^{\frac{1}{4}}+4\ln\left(1+(x+1)^{\frac{1}{4}}\right)+2\sqrt{x+1}$	26
default	$-4(x+1)^{\frac{1}{4}}+4\ln\left(1+(x+1)^{\frac{1}{4}}\right)+2\sqrt{x+1}$	26

```
[In] int(1/((x+1)^(1/4)+(x+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -4*(x+1)^(1/4)+4*ln(1+(x+1)^(1/4))+2*(x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = 2\,\sqrt{x+1}-4\,(x+1)^{\frac{1}{4}}+4\,\log\left((x+1)^{\frac{1}{4}}+1\right)$$

```
[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = -4\sqrt[4]{x+1}+2\sqrt{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

```
[In] integrate(1/((1+x)**(1/4)+(1+x)**(1/2)),x)
[Out] -4*(x + 1)**(1/4) + 2*sqrt(x + 1) + 4*log((x + 1)**(1/4) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 4\,(x+1)^{\frac{1}{4}} + 4\,\log\left((x+1)^{\frac{1}{4}} + 1\right)$$

```
[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 4\,(x+1)^{\frac{1}{4}} + 4\,\log\left((x+1)^{\frac{1}{4}} + 1\right)$$

```
[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = 4\,\ln\left((x+1)^{1/4} + 1\right) + 2\sqrt{x+1} - 4\,(x+1)^{1/4}$$

```
[In] int(1/((x + 1)^(1/2) + (x + 1)^(1/4)),x)
[Out] 4*log((x + 1)^(1/4) + 1) + 2*(x + 1)^(1/2) - 4*(x + 1)^(1/4)
```

3.945 $\int \frac{1+2x}{\sqrt{x+x^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{1+2x}{\sqrt{x+x^2}} \, dx = 2\sqrt{x+x^2}$$

```
[Out] 2*(x^2+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {643}

$$\int \frac{1+2x}{\sqrt{x+x^2}} \, dx = 2\sqrt{x^2+x}$$

```
[In] Int[(1 + 2*x)/Sqrt[x + x^2],x]
```

```
[Out] 2*Sqrt[x + x^2]
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = 2\sqrt{x+x^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1+2x}{\sqrt{x+x^2}}\,dx = 2\sqrt{x(1+x)}$$

```
[In] Integrate[(1 + 2*x)/Sqrt[x + x^2],x]
[Out] 2*Sqrt[x*(1 + x)]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$2\sqrt{x^2+x}$	10
default	$2\sqrt{x^2+x}$	10
trager	$2\sqrt{x^2+x}$	10
pseudoelliptic	$2\sqrt{(x+1)\,x}$	10
gosper	$\frac{2(x+1)x}{\sqrt{x^2+x}}$	14
risch	$\frac{2(x+1)x}{\sqrt{(x+1)x}}$	14
meijerg	$2\,\text{arcsinh}\left(\sqrt{x}\right)+\frac{2\sqrt{\pi}\,\sqrt{x}\,\sqrt{x+1}-2\sqrt{\pi}\,\text{arcsinh}\left(\sqrt{x}\right)}{\sqrt{\pi}}$	35

```
[In] int((1+2*x)/(x^2+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(x^2+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+2x}{\sqrt{x+x^2}}\,dx = 2\sqrt{x^2+x}$$

```
[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x^2 + x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1+2x}{\sqrt{x+x^2}}\,dx = 2\sqrt{x^2+x}$$

```
[In] integrate((1+2*x)/(x**2+x)**(1/2),x)
[Out] 2*sqrt(x**2 + x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+2x}{\sqrt{x+x^2}}\,dx = 2\,\sqrt{x^2+x}$$

```
[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x^2 + x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+2x}{\sqrt{x+x^2}}\,dx = 2\,\sqrt{x^2+x}$$

```
[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x^2 + x)
```

Mupad [B] (verification not implemented)

Time = 21.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+2x}{\sqrt{x+x^2}}\,dx = 2\,\sqrt{x\,(x+1)}$$

```
[In] int((2*x + 1)/(x + x^2)^(1/2),x)
[Out] 2*(x*(x + 1))^(1/2)
```

3.946 $\int \frac{1}{2\sqrt{x}(1+x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 6

$$\int \frac{1}{2\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {12, 65, 209}

$$\int \frac{1}{2\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)$$

```
[In] Int[1/(2*Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \frac{1}{\sqrt{x}(1+x)}\,dx\\ &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\ &= \tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{2\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)$$

```
[In] Integrate[1/(2*Sqrt[x]*(1 + x)),x]
[Out] ArcTan[Sqrt[x]]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)$	5
default	$\arctan\left(\sqrt{x}\right)$	5
meijerg	$\arctan\left(\sqrt{x}\right)$	5
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{x}+x-1}{x+1}\right)}{2}$	30

```
[In] int(1/2/(x+1)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{2\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)$$

```
[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="fricas")
[Out] arctan(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{1}{2\sqrt{x}(1+x)}\,dx = \operatorname{atan}\left(\sqrt{x}\right)$$

```
[In] integrate(1/2/(1+x)/x**(1/2),x)
[Out] atan(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{2\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)$$

```
[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="maxima")
[Out] arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{2\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)$$

```
[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 21.16 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{2\sqrt{x}(1+x)}\,dx = \operatorname{atan}\left(\sqrt{x}\right)$$

```
[In] int(1/(2*x^(1/2)*(x + 1)),x)
[Out] atan(x^(1/2))
```

3.947 $\int \frac{1}{x\sqrt{6x-x^2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{1}{x\sqrt{6x-x^2}} dx = -\frac{\sqrt{6x-x^2}}{3x}$$

```
[Out] -1/3*(-x^2+6*x)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {664}

$$\int \frac{1}{x\sqrt{6x-x^2}} dx = -\frac{\sqrt{6x-x^2}}{3x}$$

```
[In] Int[1/(x*Sqrt[6*x - x^2]),x]
```

```
[Out] -1/3*Sqrt[6*x - x^2]/x
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{6x-x^2}}{3x}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = \frac{-6+x}{3\sqrt{-((-6+x)x)}}$$

```
[In] Integrate[1/(x*Sqrt[6*x - x^2]),x]
[Out] (-6 + x)/(3*Sqrt[-((-6 + x)*x)])
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{-6+x}{3\sqrt{-x(-6+x)}}$	14
pseudoelliptic	$-\frac{\sqrt{-x(-6+x)}}{3x}$	14
gosper	$\frac{-6+x}{3\sqrt{-x^{2}+6x}}$	17
default	$-\frac{\sqrt{-x^{2}+6x}}{3x}$	17
trager	$-\frac{\sqrt{-x^{2}+6x}}{3x}$	17
meijerg	$-\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{1-\frac{x}{6}}}{3\sqrt{x}}$	19

```
[In] int(1/x/(-x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(-6+x)/(-x*(-6+x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = -\frac{\sqrt{-x^2+6\,x}}{3\,x}$$

```
[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(-x^2 + 6*x)/x
```

Sympy [F]

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = \int \frac{1}{x\sqrt{-x(x-6)}}\,dx$$

```
[In] integrate(1/x/(-x**2+6*x)**(1/2),x)
[Out] Integral(1/(x*sqrt(-x*(x - 6))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = -\frac{\sqrt{-x^2+6\,x}}{3\,x}$$

```
[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(-x^2 + 6*x)/x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = \frac{2}{3\left(\frac{\sqrt{-x^2+6\,x}-3}{x-3}-1\right)}$$

```
[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="giac")
[Out] 2/3/((sqrt(-x^2 + 6*x) - 3)/(x - 3) - 1)
```

Mupad [B] (verification not implemented)

Time = 21.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = -\frac{\sqrt{6\,x-x^2}}{3\,x}$$

```
[In] int(1/(x*(6*x - x^2)^(1/2)),x)
[Out] -(6*x - x^2)^(1/2)/(3*x)
```

3.948 $\int (1+\sqrt{x}) \sqrt{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int (1+\sqrt{x}) \sqrt{x} \, dx = \frac{2x^{3/2}}{3} + \frac{x^2}{2}$$

```
[Out] 2/3*x^(3/2)+1/2*x^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\int (1+\sqrt{x}) \sqrt{x} \, dx = \frac{2x^{3/2}}{3} + \frac{x^2}{2}$$

```
[In] Int[(1 + Sqrt[x])*Sqrt[x],x]
[Out] (2*x^(3/2))/3 + x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\sqrt{x}+x\right) \, dx \\ &= \frac{2x^{3/2}}{3} + \frac{x^2}{2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx=\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

```
[In] Integrate[(1 + Sqrt[x])*Sqrt[x],x]
[Out] (2*x^(3/2))/3 + x^2/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}+\frac{x^2}{2}$	12
default	$\frac{2x^{\frac{3}{2}}}{3}+\frac{x^2}{2}$	12
trager	$\frac{(x-1)(x+1)}{2}+\frac{2x^{\frac{3}{2}}}{3}$	15

```
[In] int(x^(1/2)*(1+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/3*x^(3/2)+1/2*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx=\frac{1}{2}\,x^2+\frac{2}{3}\,x^{\frac{3}{2}}$$

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="fricas")
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx = \frac{2x^{\frac{3}{2}}}{3} + \frac{x^2}{2}$$

```
[In] integrate(x**(1/2)*(1+x**(1/2)),x)
[Out] 2*x**(3/2)/3 + x**2/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx = \frac{1}{2}\left(\sqrt{x}+1\right)^4 - \frac{4}{3}\left(\sqrt{x}+1\right)^3 + \left(\sqrt{x}+1\right)^2$$

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="maxima")
[Out] 1/2*(sqrt(x) + 1)^4 - 4/3*(sqrt(x) + 1)^3 + (sqrt(x) + 1)^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx = \frac{1}{2}\,x^2 + \frac{2}{3}\,x^{\frac{3}{2}}$$

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="giac")
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx = \frac{x^2}{2} + \frac{2\,x^{3/2}}{3}$$

```
[In] int(x^(1/2)*(x^(1/2) + 1),x)
[Out] x^2/2 + (2*x^(3/2))/3
```

3.949 $\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\, dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\, dx = \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

```
[Out] 3/2*x^(2/3)-6/7*x^(7/6)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\, dx = \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

```
[In] Int[(1 - Sqrt[x])/x^(1/3),x]
```

```
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{\sqrt[3]{x}} - \sqrt[6]{x}\right) dx \\ &= \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx = \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

```
[In] Integrate[(1 - Sqrt[x])/x^(1/3),x]
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\frac{3x^{\frac{2}{3}}}{2}-\frac{6x^{\frac{7}{6}}}{7}$	12
default	$\frac{3x^{\frac{2}{3}}}{2}-\frac{6x^{\frac{7}{6}}}{7}$	12

```
[In] int((1-x^(1/2))/x^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/2*x^(2/3)-6/7*x^(7/6)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx = -\frac{6}{7}\,x^{\frac{7}{6}} + \frac{3}{2}\,x^{\frac{2}{3}}$$

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="fricas")
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx = -\frac{6x^{\frac{7}{6}}}{7} + \frac{3x^{\frac{2}{3}}}{2}$$

```
[In] integrate((1-x**(1/2))/x**(1/3),x)
[Out] -6*x**(7/6)/7 + 3*x**(2/3)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx = -\frac{6}{7}\,x^{\frac{7}{6}} + \frac{3}{2}\,x^{\frac{2}{3}}$$

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="maxima")
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx = -\frac{6}{7}\,x^{\frac{7}{6}} + \frac{3}{2}\,x^{\frac{2}{3}}$$

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="giac")
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx = -\frac{3\,x^{2/3}\,\left(4\,\sqrt{x}-7\right)}{14}$$

```
[In] int(-(x^(1/2) - 1)/x^(1/3),x)
[Out] -(3*x^(2/3)*(4*x^(1/2) - 7))/14
```

3.950 $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}} \, dx = -6\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+6\arctan\left(\sqrt[6]{x}\right)$$

```
[Out] -6*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+6*arctan(x^(1/6))+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {348, 52, 65, 209}

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}} \, dx = 6\arctan\left(\sqrt[6]{x}\right)+\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-6\sqrt[6]{x}$$

```
[In] Int[Sqrt[x]/(1 + x^(1/3)),x]
```

```
[Out] -6*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 6*ArcTan[x^(1/6)]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{x^{7/2}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{6x^{7/6}}{7} - 3\text{Subst}\left(\int \frac{x^{5/2}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 3\text{Subst}\left(\int \frac{x^{3/2}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} - 3\text{Subst}\left(\int \frac{\sqrt{x}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 3\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x)}\,dx, x, \sqrt[3]{x}\right)\\
&= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 6\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 6\tan^{-1}\left(\sqrt[6]{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = \frac{2}{35}\left(-105\sqrt[6]{x}+35\sqrt{x}-21x^{5/6}+15x^{7/6}\right)+6\arctan\left(\sqrt[6]{x}\right)$$

```
[In] Integrate[Sqrt[x]/(1 + x^(1/3)),x]
[Out] (2*(-105*x^(1/6) + 35*Sqrt[x] - 21*x^(5/6) + 15*x^(7/6)))/35 + 6*ArcTan[x^(
1/6)]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$-6x^{\frac{1}{6}}-\frac{6x^{\frac{5}{6}}}{5}+\frac{6x^{\frac{7}{6}}}{7}+6\arctan\left(x^{\frac{1}{6}}\right)+2\sqrt{x}$	28
default	$-6x^{\frac{1}{6}}-\frac{6x^{\frac{5}{6}}}{5}+\frac{6x^{\frac{7}{6}}}{7}+6\arctan\left(x^{\frac{1}{6}}\right)+2\sqrt{x}$	28
meijerg	$-\frac{2x^{\frac{1}{6}}\left(-45x+63x^{\frac{2}{3}}-105x^{\frac{1}{3}}+315\right)}{105}+6\arctan\left(x^{\frac{1}{6}}\right)$	28

```
[In] int(x^(1/2)/(1+x^(1/3)),x,method=_RETURNVERBOSE)
[Out] -6*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+6*arctan(x^(1/6))+2*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = \frac{6}{7}\left(x-7\right)x^{\frac{1}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}+6\arctan\left(x^{\frac{1}{6}}\right)$$

```
[In] integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="fricas")
[Out] 6/7*(x - 7)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 6*arctan(x^(1/6))
```

Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = \frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} - 6\sqrt[6]{x} + 2\sqrt{x} + 6\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

```
[In] integrate(x**(1/2)/(1+x**(1/3)),x)
[Out] 6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 2*sqrt(x) + 6*atan(x**(1/6))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = \frac{6}{7}\,x^{\frac{7}{6}} - \frac{6}{5}\,x^{\frac{5}{6}} + 2\,\sqrt{x} - 6\,x^{\frac{1}{6}} + 6\,\arctan\left(x^{\frac{1}{6}}\right)$$

```
[In] integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="maxima")
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 6*x^(1/6) + 6*arctan(x^(1/6))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = \frac{6}{7}\,x^{\frac{7}{6}} - \frac{6}{5}\,x^{\frac{5}{6}} + 2\,\sqrt{x} - 6\,x^{\frac{1}{6}} + 6\,\arctan\left(x^{\frac{1}{6}}\right)$$

```
[In] integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="giac")
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 6*x^(1/6) + 6*arctan(x^(1/6))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = 6\,\mathrm{atan}\left(x^{1/6}\right) + 2\,\sqrt{x} - 6\,x^{1/6} - \frac{6\,x^{5/6}}{5} + \frac{6\,x^{7/6}}{7}$$

```
[In] int(x^(1/2)/(x^(1/3) + 1),x)
[Out] 6*atan(x^(1/6)) + 2*x^(1/2) - 6*x^(1/6) - (6*x^(5/6))/5 + (6*x^(7/6))/7
```

3.951 $\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx = 6\sqrt[3]{1+\sqrt{x}} - 2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1+\sqrt{x}}}{\sqrt{3}}\right) + 3\log\left(1-\sqrt[3]{1+\sqrt{x}}\right) - \frac{\log(x)}{2}$$

```
[Out] -1/2*ln(x)+3*ln(1-(1+x^(1/2))^(1/3))-2*arctan(1/3*(1+2*(1+x^(1/2))^(1/3))*3
^(1/2))*3^(1/2)+6*(1+x^(1/2))^(1/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {272, 52, 59, 632, 210, 31}

$$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx = -2\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right) + 6\sqrt[3]{\sqrt{x}+1} + 3\log\left(1-\sqrt[3]{\sqrt{x}+1}\right) - \frac{\log(x)}{2}$$

```
[In] Int[(1 + Sqrt[x])^(1/3)/x,x]
```

```
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/3))/Sqrt[3
]] + 3*Log[1 - (1 + Sqrt[x])^(1/3)] - Log[x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{x}\,dx, x, \sqrt{x}\right)\\ &= 6\sqrt[3]{1+\sqrt{x}} + 2\text{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx, x, \sqrt{x}\right)\end{aligned}$$

$$= 6\sqrt[3]{1+\sqrt{x}} - \frac{\log(x)}{2} - 3\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+\sqrt{x}}\right) - 3\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1+\sqrt{x}}\right)$$

$$= 6\sqrt[3]{1+\sqrt{x}} + 3\log\left(1-\sqrt[3]{1+\sqrt{x}}\right) - \frac{\log(x)}{2} + 6\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+\sqrt{x}}\right)$$

$$= 6\sqrt[3]{1+\sqrt{x}} - 2\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1+\sqrt{x}}}{\sqrt{3}}\right) + 3\log\left(1-\sqrt[3]{1+\sqrt{x}}\right) - \frac{\log(x)}{2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx = 6\sqrt[3]{1+\sqrt{x}} - 2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1+\sqrt{x}}}{\sqrt{3}}\right) + 2\log\left(-1+\sqrt[3]{1+\sqrt{x}}\right) - \log\left(1+\sqrt[3]{1+\sqrt{x}}+\left(1+\sqrt{x}\right)^{2/3}\right)$$

```
[In] Integrate[(1 + Sqrt[x])^(1/3)/x,x]
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/3))/Sqrt[3
]] + 2*Log[-1 + (1 + Sqrt[x])^(1/3)] - Log[1 + (1 + Sqrt[x])^(1/3) + (1 + S
qrt[x])^(2/3)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

method	result
meijerg	$-\frac{2\left(-\Gamma\left(\frac{2}{3}\right)\sqrt{x}\,{}_3F_2\left(\frac{2}{3},1,1;2,2;-\sqrt{x}\right)-3\left(3+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+\frac{\ln(x)}{2}\right)\Gamma\left(\frac{2}{3}\right)\right)}{3\Gamma\left(\frac{2}{3}\right)}$
derivativedivides	$6\left(1+\sqrt{x}\right)^{\frac{1}{3}}-\ln\left(\left(1+\sqrt{x}\right)^{\frac{2}{3}}+\left(1+\sqrt{x}\right)^{\frac{1}{3}}+1\right)-2\arctan\left(\frac{\left(1+2\left(1+\sqrt{x}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}+2$
default	$6\left(1+\sqrt{x}\right)^{\frac{1}{3}}-\ln\left(\left(1+\sqrt{x}\right)^{\frac{2}{3}}+\left(1+\sqrt{x}\right)^{\frac{1}{3}}+1\right)-2\arctan\left(\frac{\left(1+2\left(1+\sqrt{x}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}+2$

```
[In] int((1+x^(1/2))^(1/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/GAMMA(2/3)*(-GAMMA(2/3)*x^(1/2)*hypergeom([2/3,1,1],[2,2],-x^(1/2))-3*
(3+1/6*Pi*3^(1/2)-3/2*ln(3)+1/2*ln(x))*GAMMA(2/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx = -2\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\sqrt{x}+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}} - \log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right)$$

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*arctan(2/3*sqrt(3)*(sqrt(x) + 1)^(1/3) + 1/3*sqrt(3)) + 6*(sqrt(
x) + 1)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log(
(sqrt(x) + 1)^(1/3) - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx = -\frac{2\sqrt[6]{x}\,\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{e^{i\pi}}{\sqrt{x}}\right)}{\Gamma\left(\frac{2}{3}\right)}$$

```
[In] integrate((1+x**(1/2))**(1/3)/x,x)
```

```
[Out] -2*x**(1/6)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), exp_polar(I*pi)/sqrt(x)
)/gamma(2/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx = -2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}} - \log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right)$$

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="maxima")
[Out] -2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(x) + 1)^(1/3) + 1)) + 6*(sqrt(x) + 1
)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log((sqrt(
x) + 1)^(1/3) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx = -2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}} - \log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left|\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="giac")
[Out] -2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(x) + 1)^(1/3) + 1)) + 6*(sqrt(x) + 1
)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log(abs((s
qrt(x) + 1)^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 21.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx = 2\ln\left(\left(\sqrt{x}+1\right)^{1/3}-1\right)+6\left(\sqrt{x}+1\right)^{1/3} +\ln\left(\left(\sqrt{x}+1\right)^{1/3}+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)-\ln\left(\left(\sqrt{x}+1\right)^{1/3}+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)$$

```
[In] int((x^(1/2) + 1)^(1/3)/x,x)
[Out] 2*log((x^(1/2) + 1)^(1/3) - 1) + 6*(x^(1/2) + 1)^(1/3) + log((x^(1/2) + 1)^
(1/3) - (3^(1/2)*1i)/2 + 1/2)*(3^(1/2)*1i - 1) - log((3^(1/2)*1i)/2 + (x^(1
/2) + 1)^(1/3) + 1/2)*(3^(1/2)*1i + 1)
```

3.952 $\int (1-\sqrt{x}) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int (1-\sqrt{x}) \, dx = x - \frac{2x^{3/2}}{3}$$

```
[Out] x-2/3*x^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (1-\sqrt{x}) \, dx = x - \frac{2x^{3/2}}{3}$$

```
[In] Int[1 - Sqrt[x],x]
[Out] x - (2*x^(3/2))/3
```

Rubi steps

$$\text{integral} = x - \frac{2x^{3/2}}{3}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \left(1-\sqrt{x}\right) \, dx=x-\frac{2x^{3/2}}{3}$$

```
[In] Integrate[1 - Sqrt[x],x]
[Out] x - (2*x^(3/2))/3
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
default	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
risch	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
parts	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
trager	$x-1-\frac{2x^{\frac{3}{2}}}{3}$	9

```
[In] int(1-x^(1/2),x,method=_RETURNVERBOSE)
[Out] x-2/3*x^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right) \, dx=-\frac{2}{3}\,x^{\frac{3}{2}}+x$$

```
[In] integrate(1-x^(1/2),x, algorithm="fricas")
[Out] -2/3*x^(3/2) + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \left(1-\sqrt{x}\right) \, dx = -\frac{2x^{\frac{3}{2}}}{3} + x$$

```
[In] integrate(1-x**(1/2),x)
[Out] -2*x**(3/2)/3 + x
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right) \, dx = -\frac{2}{3} \, x^{\frac{3}{2}} + x$$

```
[In] integrate(1-x^(1/2),x, algorithm="maxima")
[Out] -2/3*x^(3/2) + x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right) \, dx = -\frac{2}{3} \, x^{\frac{3}{2}} + x$$

```
[In] integrate(1-x^(1/2),x, algorithm="giac")
[Out] -2/3*x^(3/2) + x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right) \, dx = x - \frac{2\,x^{3/2}}{3}$$

```
[In] int(1 - x^(1/2),x)
[Out] x - (2*x^(3/2))/3
```

3.953 $\int \left(1-\sqrt[4]{x}\right) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \left(1-\sqrt[4]{x}\right) \, dx=x-\frac{4x^{5/4}}{5}$$

```
[Out] x-4/5*x^(5/4)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(1-\sqrt[4]{x}\right) \, dx=x-\frac{4x^{5/4}}{5}$$

```
[In] Int[1 - x^(1/4),x]
```

```
[Out] x - (4*x^(5/4))/5
```

Rubi steps

$$\text{integral}=x-\frac{4x^{5/4}}{5}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \left(1-\sqrt[4]{x}\right)\,dx=x-\frac{4x^{5/4}}{5}$$

```
[In] Integrate[1 - x^(1/4),x]
[Out] x - (4*x^(5/4))/5
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$x-\frac{4x^{\frac{5}{4}}}{5}$	8
default	$x-\frac{4x^{\frac{5}{4}}}{5}$	8
risch	$x-\frac{4x^{\frac{5}{4}}}{5}$	8
parts	$x-\frac{4x^{\frac{5}{4}}}{5}$	8
trager	$x-1-\frac{4x^{\frac{5}{4}}}{5}$	9

```
[In] int(1-x^(1/4),x,method=_RETURNVERBOSE)
[Out] x-4/5*x^(5/4)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt[4]{x}\right)\,dx=-\frac{4}{5}\,x^{\frac{5}{4}}+x$$

```
[In] integrate(1-x^(1/4),x, algorithm="fricas")
[Out] -4/5*x^(5/4) + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \left(1-\sqrt[4]{x}\right)\,dx=-\frac{4x^{\frac{5}{4}}}{5}+x$$

```
[In] integrate(1-x**(1/4),x)
[Out] -4*x**(5/4)/5 + x
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt[4]{x}\right)\,dx=-\frac{4}{5}\,x^{\frac{5}{4}}+x$$

```
[In] integrate(1-x^(1/4),x, algorithm="maxima")
[Out] -4/5*x^(5/4) + x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt[4]{x}\right)\,dx=-\frac{4}{5}\,x^{\frac{5}{4}}+x$$

```
[In] integrate(1-x^(1/4),x, algorithm="giac")
[Out] -4/5*x^(5/4) + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt[4]{x}\right)\,dx=x-\frac{4\,x^{5/4}}{5}$$

```
[In] int(1 - x^(1/4),x)
[Out] x - (4*x^(5/4))/5
```

3.954 $\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 11

$$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx=x-\frac{4x^{5/4}}{5}$$

```
[Out] x-4/5*x^(5/4)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {26}

$$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx=x-\frac{4x^{5/4}}{5}$$

```
[In] Int[(1 - Sqrt[x])/(1 + x^(1/4)),x]
[Out] x - (4*x^(5/4))/5
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(1-\sqrt[4]{x}\right)\,dx \\ &= x-\frac{4x^{5/4}}{5} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx = x-\frac{4x^{5/4}}{5}$$

[In] Integrate[(1 - Sqrt[x])/(1 + x^(1/4)),x]

[Out] x - (4*x^(5/4))/5

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result
derivativedivides	$x-\frac{4x^{\frac{5}{4}}}{5}$
meijerg	$\frac{x^{\frac{1}{4}}\left(4\sqrt{x}-6x^{\frac{1}{4}}+12\right)}{3}-\frac{x^{\frac{1}{4}}\left(12x-15x^{\frac{3}{4}}+20\sqrt{x}-30x^{\frac{1}{4}}+60\right)}{15}$
default	$-\frac{4x^{\frac{5}{4}}}{5}+x+2\ln\left(1+x^{\frac{1}{4}}\right)-\ln\left(1-x\right)-\ln\left(-1+\sqrt{x}\right)+\ln\left(1+\sqrt{x}\right)+2\ln\left(x^{\frac{1}{4}}-1\right)$

[In] int((1-x^(1/2))/(1+x^(1/4)),x,method=_RETURNVERBOSE)

[Out] x-4/5*x^(5/4)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx = -\frac{4}{5}\,x^{\frac{5}{4}}+x$$

[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="fricas")

[Out] -4/5*x^(5/4) + x

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx = -\frac{4x^{\frac{5}{4}}}{5} + x$$

```
[In] integrate((1-x**(1/2))/(1+x**(1/4)),x)
[Out] -4*x**(5/4)/5 + x
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx = -\frac{4}{5}\,x^{\frac{5}{4}} + x$$

```
[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="maxima")
[Out] -4/5*x^(5/4) + x
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx = -\frac{4}{5}\,x^{\frac{5}{4}} + x$$

```
[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="giac")
[Out] -4/5*x^(5/4) + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx = x - \frac{4\,x^{5/4}}{5}$$

```
[In] int(-(x^(1/2) - 1)/(x^(1/4) + 1),x)
[Out] x - (4*x^(5/4))/5
```

3.955 $\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+(bc+ad)x+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] arctanh(1/2*(2*b*d*x+a*d+b*c)/b^(1/2)/d^(1/2)/(a*c+(a*d+b*c)*x+b*d*x^2)^(1/
2))/b^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1976, 635, 212}

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(ad+bc)+ac+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[In] Int[1/Sqrt[(a + b*x)*(c + d*x)],x]
```

```
[Out] ArcTanh[(b*c + a*d + 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c + a*d)*x +
 b*d*x^2])]/(Sqrt[b]*Sqrt[d])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\sqrt{ac+(bc+ad)x+bdx^2}}\,dx\\
&= 2\text{Subst}\left(\int \frac{1}{4bd-x^2}\,dx, x, \frac{bc+ad+2bdx}{\sqrt{ac+(bc+ad)x+bdx^2}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+(bc+ad)x+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx = \frac{2\sqrt{a+bx}\sqrt{c+dx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}$$

```
[In] Integrate[1/Sqrt[(a + b*x)*(c + d*x)],x]
[Out] (2*Sqrt[a + b*x]*Sqrt[c + d*x]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqr
t[a + b*x])])/(Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{ac+(ad+bc)x+bd\,x^2}\right)}{\sqrt{bd}}$	49

```
[In] int(1/((b*x+a)*(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(a*c+(a*d+b*c)*x+b*d*x^2)^(1/2))/(b*
d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.15

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx = \left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,\sqrt{bdx^2+ac+(bc+ad)x}(2\,bdx+bc+ad)\sqrt{bd}+8\,(b^2cd+\right.}{2\,bd}\right.$$
$$\left.-\frac{\sqrt{-bd}\arctan\left(\frac{\sqrt{bdx^2+ac+(bc+ad)x}(2\,bdx+bc+ad)\sqrt{-bd}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)}{bd}\right]$$

```
[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*sqrt(b
*d*x^2 + a*c + (b*c + a*d)*x)*(2*b*d*x + b*c + a*d)*sqrt(b*d) + 8*(b^2*c*d
+ a*b*d^2)*x)/(b*d), -sqrt(-b*d)*arctan(1/2*sqrt(b*d*x^2 + a*c + (b*c + a*d
)*x)*(2*b*d*x + b*c + a*d)*sqrt(-b*d)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a
*b*d^2)*x))/(b*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(58) = 116.

Time = 1.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.66

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx = \begin{cases} \frac{\log\left(ad+bc+2bdx+2\sqrt{bd}\sqrt{ac+bdx^2+x(ad+bc)}\right)}{\sqrt{bd}} & \text{for}\: bd \neq 0 \wedge ac-\frac{(ad+bc)^2}{4bd} \neq 0 \\ \frac{\left(x-\frac{-ad-bc}{2bd}\right)\log\left(x-\frac{-ad-bc}{2bd}\right)}{\sqrt{bd\left(x-\frac{-ad-bc}{2bd}\right)^2}} & \text{for}\: bd \neq 0 \\ \frac{2\sqrt{ac+x(ad+bc)}}{ad+bc} & \text{for}\: ad+bc \neq 0 \\ \frac{x}{\sqrt{ac}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/((b*x+a)*(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((log(a*d + b*c + 2*b*d*x + 2*sqrt(b*d)*sqrt(a*c + b*d*x**2 + x*(a
*d + b*c)))/sqrt(b*d), Ne(b*d, 0) & Ne(a*c - (a*d + b*c)**2/(4*b*d), 0)), (
(x - (-a*d - b*c)/(2*b*d))*log(x - (-a*d - b*c)/(2*b*d))/sqrt(b*d*(x - (-a*
d - b*c)/(2*b*d))**2), Ne(b*d, 0)), (2*sqrt(a*c + x*(a*d + b*c))/(a*d + b*c
), Ne(a*d + b*c, 0)), (x/sqrt(a*c), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(49) = 98$.

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.02

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx = \frac{1}{4}\sqrt{bdx^2+bcx+adx+ac}\left(2\,x+\frac{bc+ad}{bd}\right) + \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(\left|-bc-ad-2\,\sqrt{bd}\left(\sqrt{bd}x-\sqrt{bdx^2+bcx+adx+ac}\right)\right|\right)}{8\,\sqrt{bd}bd}$$

```
[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*d*x^2 + b*c*x + a*d*x + a*c)*(2*x + (b*c + a*d)/(b*d)) + 1/8*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(-b*c - a*d - 2*sqrt(b*d)*(sqrt(b*d)*x
- sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))))/(sqrt(b*d)*b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx = \int \frac{1}{\sqrt{(a+b\,x)\,\left(c+d\,x\right)}}\,dx$$

```
[In] int(1/((a + b*x)*(c + d*x))^(1/2),x)
[Out] int(1/((a + b*x)*(c + d*x))^(1/2), x)
```

3.956 $\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = -\frac{\arctan\left(\frac{bc-ad-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+(bc-ad)x-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] -arctan(1/2*(-2*b*d*x-a*d+b*c)/b^(1/2)/d^(1/2)/(a*c+(-a*d+b*c)*x-b*d*x^2)^(
1/2))/b^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1976, 635, 210}

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = -\frac{\arctan\left(\frac{-ad+bc-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(bc-ad)+ac-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[In] Int[1/Sqrt[(a + b*x)*(c - d*x)],x]
```

```
[Out] -(ArcTan[(b*c - a*d - 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c - a*d)*x
- b*d*x^2])]/(Sqrt[b]*Sqrt[d]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{ac+(bc-ad)x-bdx^2}}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{-4bd-x^2}\,dx, x, \frac{bc-ad-2bdx}{\sqrt{ac+(bc-ad)x-bdx^2}}\right)\\ &= -\frac{\tan^{-1}\left(\frac{bc-ad-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+(bc-ad)x-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = -\frac{2\sqrt{a+bx}\sqrt{c-dx}\arctan\left(\frac{\sqrt{b}\sqrt{c-dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c-dx)}}$$

```
[In] Integrate[1/Sqrt[(a + b*x)*(c - d*x)],x]
[Out] (-2*Sqrt[a + b*x]*Sqrt[c - d*x]*ArcTan[(Sqrt[b]*Sqrt[c - d*x])/(Sqrt[d]*Sqr
t[a + b*x])])/(Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c - d*x)])
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{bd}\left(x-\frac{-ad+bc}{2bd}\right)}{\sqrt{ac+(-ad+bc)x-bd\,x^2}}\right)}{\sqrt{bd}}$	55

```
[In] int(1/((b*x+a)*(-d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*d)^(1/2)*arctan((b*d)^(1/2)*(x-1/2*(-a*d+b*c)/b/d)/(a*c+(-a*d+b*c)*x-b
*d*x^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 3.11

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = \left[-\frac{\sqrt{-bd}\log\left(8\,b^2d^2x^2+b^2c^2-6\,abcd+a^2d^2-4\,\sqrt{-bdx^2+ac+(bc-ad)x}(2\,bdx-bc+ad)\sqrt{-bd}-8\right.}{2\,bd}, -\frac{\sqrt{bd}\arctan\left(\frac{\sqrt{-bdx^2+ac+(bc-ad)x}(2\,bdx-bc+ad)\sqrt{bd}}{2\,(b^2d^2x^2-abcd-(b^2cd-abd^2)x)}\right)}{bd}\right]$$

```
[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 - 4*sqrt
(-b*d*x^2 + a*c + (b*c - a*d)*x)*(2*b*d*x - b*c + a*d)*sqrt(-b*d) - 8*(b^2*
c*d - a*b*d^2)*x)/(b*d), -sqrt(b*d)*arctan(1/2*sqrt(-b*d*x^2 + a*c + (b*c -
 a*d)*x)*(2*b*d*x - b*c + a*d)*sqrt(b*d)/(b^2*d^2*x^2 - a*b*c*d - (b^2*c*d
- a*b*d^2)*x))/(b*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(60) = 120.

Time = 1.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.49

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = \begin{cases} \frac{\log\left(-ad+bc-2bdx+2\sqrt{-bd}\sqrt{ac-bdx^2+x(-ad+bc)}\right)}{\sqrt{-bd}} & \text{for } bd\neq 0\wedge ac+\frac{(ad-bc)^2}{4bd}\neq 0 \\ \frac{\left(x+\frac{ad-bc}{2bd}\right)\log\left(x+\frac{ad-bc}{2bd}\right)}{\sqrt{-bd\left(x+\frac{ad-bc}{2bd}\right)^2}} & \text{for } bd\neq 0 \\ \frac{2\sqrt{ac+x(-ad+bc)}}{-ad+bc} & \text{for } ad-bc\neq 0 \\ \frac{x}{\sqrt{ac}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/((b*x+a)*(-d*x+c))**(1/2),x)
```

```
[Out] Piecewise((log(-a*d + b*c - 2*b*d*x + 2*sqrt(-b*d)*sqrt(a*c - b*d*x**2 + x*
(-a*d + b*c)))/sqrt(-b*d), Ne(b*d, 0) & Ne(a*c + (a*d - b*c)**2/(4*b*d), 0)
), ((x + (a*d - b*c)/(2*b*d))*log(x + (a*d - b*c)/(2*b*d))/sqrt(-b*d*(x + (
a*d - b*c)/(2*b*d))**2), Ne(b*d, 0)), (2*sqrt(a*c + x*(-a*d + b*c))/(-a*d +
 b*c), Ne(a*d - b*c, 0)), (x/sqrt(a*c), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(53) = 106$.

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.02

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = \frac{1}{4}\sqrt{-bdx^2+bcx-adx+ac}\left(2\,x-\frac{bc-ad}{bd}\right) - \frac{(b^2c^2+2\,abcd+a^2d^2)\log\left(\left|-bc+ad-2\,\sqrt{-bd}\left(\sqrt{-bd}x-\sqrt{-bdx^2+bcx-adx+ac}\right)\right|\right)}{8\,\sqrt{-bd}bd}$$

```
[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(-b*d*x^2 + b*c*x - a*d*x + a*c)*(2*x - (b*c - a*d)/(b*d)) - 1/8*(b
^2*c^2 + 2*a*b*c*d + a^2*d^2)*log(abs(-b*c + a*d - 2*sqrt(-b*d)*(sqrt(-b*d)
*x - sqrt(-b*d*x^2 + b*c*x - a*d*x + a*c))))/(sqrt(-b*d)*b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = \int \frac{1}{\sqrt{(a+b\,x)\,(c-d\,x)}}\,dx$$

```
[In] int(1/((a + b*x)*(c - d*x))^(1/2),x)
[Out] int(1/((a + b*x)*(c - d*x))^(1/2), x)
```

3.957 $\int \frac{1}{\sqrt{x}(1-x^2)} dx$

Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx = \arctan\left(\sqrt{x}\right) + \operatorname{arctanh}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {335, 218, 212, 209}

$$\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx = \arctan\left(\sqrt{x}\right) + \operatorname{arctanh}\left(\sqrt{x}\right)$$

```
[In] Int[1/(Sqrt[x]*(1 - x^2)),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right)\\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\
&= \tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx = \arctan\left(\sqrt{x}\right) + \text{arctanh}\left(\sqrt{x}\right)$$

```
[In] Integrate[1/(Sqrt[x]*(1 - x^2)),x]
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
default	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
meijerg	$-\frac{\sqrt{x}\left(\ln\left(1-(x^{2})^{\frac{1}{4}}\right)-\ln\left(1+(x^{2})^{\frac{1}{4}}\right)-2\arctan\left((x^{2})^{\frac{1}{4}}\right)\right)}{2(x^{2})^{\frac{1}{4}}}$	40
trager	$\frac{\ln\left(\frac{2\sqrt{x}+1+x}{x-1}\right)}{2}-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)x+2\sqrt{x}-\operatorname{RootOf}\left(\_Z^{2}+1\right)}{x+1}\right)}{2}$	56

```
[In] int(1/(-x^2+1)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx = \arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

```
[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="fricas")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx = -\frac{\log\left(\sqrt{x}-1\right)}{2}+\frac{\log\left(\sqrt{x}+1\right)}{2}+\operatorname{atan}\left(\sqrt{x}\right)$$

```
[In] integrate(1/(-x**2+1)/x**(1/2),x)
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx = \arctan\left(\sqrt{x}\right) + \frac{1}{2}\,\log\left(\sqrt{x}+1\right) - \frac{1}{2}\,\log\left(\sqrt{x}-1\right)$$

```
[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="maxima")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(9) = 18$.

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx = \arctan\left(\sqrt{x}\right) + \frac{1}{2}\,\log\left(\sqrt{x}+1\right) - \frac{1}{2}\,\log\left(\left|\sqrt{x}-1\right|\right)$$

```
[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx = \mathrm{atan}\left(\sqrt{x}\right) + \mathrm{atanh}\left(\sqrt{x}\right)$$

```
[In] int(-1/(x^(1/2)*(x^2 - 1)),x)
[Out] atan(x^(1/2)) + atanh(x^(1/2))
```

3.958 $\int \frac{\sqrt{x}}{x-x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{\sqrt{x}}{x-x^3} \, dx = \arctan\left(\sqrt{x}\right) + \text{arctanh}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1598, 335, 218, 212, 209}

$$\int \frac{\sqrt{x}}{x-x^3} \, dx = \arctan\left(\sqrt{x}\right) + \text{arctanh}\left(\sqrt{x}\right)$$

```
[In] Int[Sqrt[x]/(x - x^3),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\sqrt{x}\left(1-x^2\right)}\,dx\\
&= 2\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right)\\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\
&= \tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{x-x^3}\,dx=\arctan\left(\sqrt{x}\right)+\text{arctanh}\left(\sqrt{x}\right)$$

```
[In] Integrate[Sqrt[x]/(x - x^3),x]
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
default	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
meijerg	$-\frac{\sqrt{x}\left(\ln\left(1-(x^2)^{\frac{1}{4}}\right)-\ln\left(1+(x^2)^{\frac{1}{4}}\right)-2\arctan\left((x^2)^{\frac{1}{4}}\right)\right)}{2(x^2)^{\frac{1}{4}}}$	40
trager	$\frac{\ln\left(\frac{2\sqrt{x}+1+x}{x-1}\right)}{2}-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x+2\sqrt{x}-\operatorname{RootOf}\left(\_Z^2+1\right)}{x+1}\right)}{2}$	56

```
[In] int(x^(1/2)/(-x^3+x),x,method=_RETURNVERBOSE)
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{x}}{x-x^3}\,dx = \arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

```
[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="fricas")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{x}}{x-x^3}\,dx = -\frac{\log\left(\sqrt{x}-1\right)}{2}+\frac{\log\left(\sqrt{x}+1\right)}{2}+\operatorname{atan}\left(\sqrt{x}\right)$$

```
[In] integrate(x**(1/2)/(-x**3+x),x)
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{x}}{x - x^3} dx = \arctan\left(\sqrt{x}\right) + \frac{1}{2} \log\left(\sqrt{x} + 1\right) - \frac{1}{2} \log\left(\sqrt{x} - 1\right)$$

```
[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="maxima")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(9) = 18$.

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{x}}{x - x^3} dx = \arctan\left(\sqrt{x}\right) + \frac{1}{2} \log\left(\sqrt{x} + 1\right) - \frac{1}{2} \log\left(\left|\sqrt{x} - 1\right|\right)$$

```
[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="giac")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{x}}{x - x^3} dx = \mathrm{atan}\left(\sqrt{x}\right) + \mathrm{atanh}\left(\sqrt{x}\right)$$

```
[In] int(x^(1/2)/(x - x^3),x)
[Out] atan(x^(1/2)) + atanh(x^(1/2))
```

3.959 $\int \frac{x}{2-\sqrt{3}+(1+\sqrt{3})x+x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx=\sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{1+\sqrt{3}+2x}{\sqrt{2\left(-2+3\sqrt{3}\right)}}\right)+\frac{1}{2}\log\left(2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2\right)$$

```
[Out] 1/2*ln(2+x^2-3^(1/2)+x*(1+3^(1/2)))+1/23*arctanh((1+2*x+3^(1/2))/(-4+6*3^(1
/2))^(1/2))*(299+184*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {648, 632, 212, 642}

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx=\sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{2x+\sqrt{3}+1}{\sqrt{2\left(3\sqrt{3}-2\right)}}\right)+\frac{1}{2}\log\left(x^2+\left(1+\sqrt{3}\right)x-\sqrt{3}+2\right)$$

```
[In] Int[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2),x]
```

```
[Out] Sqrt[(13 + 8*Sqrt[3])/23]*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[2*(-2 + 3*Sqrt[3
])]] + Log[2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\int \frac{1+\sqrt{3}+2x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx + \frac{1}{2}\left(-1-\sqrt{3}\right)\int \frac{1}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx$$

$$= \frac{1}{2}\log\left(2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2\right) + \left(1+\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-2\left(2-3\sqrt{3}\right)-x^2}\,dx, x, 1+\sqrt{3}+2x\right)$$

$$= \sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\tanh^{-1}\left(\frac{1+\sqrt{3}+2x}{\sqrt{2\left(-2+3\sqrt{3}\right)}}\right) + \frac{1}{2}\log\left(2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx=\frac{\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{1+\sqrt{3}+2x}{\sqrt{-4+6\sqrt{3}}}\right)}{\sqrt{-4+6\sqrt{3}}}+\frac{1}{2}\log\left(2-\sqrt{3}+x+\sqrt{3}x+x^2\right)$$

```
[In] Integrate[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2),x]
```

```
[Out] ((1 + Sqrt[3])*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[-4 + 6*Sqrt[3]]])/Sqrt[-4 +
 6*Sqrt[3]] + Log[2 - Sqrt[3] + x + Sqrt[3]*x + x^2]/2
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\ln\left(x\sqrt{3}+x^2-\sqrt{3}+x+2\right)}{2}-\frac{2\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)\operatorname{arctanh}\left(\frac{1+2x+\sqrt{3}}{\sqrt{-4+6\sqrt{3}}}\right)}{\sqrt{-4+6\sqrt{3}}}$	58

```
[In] int(x/(2+x^2-3^(1/2)+(1+3^(1/2))*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x*3^(1/2)+x^2-3^(1/2)+x+2)-2*(-1/2-1/2*3^(1/2))/(-4+6*3^(1/2))^(1/2)
*arctanh((1+2*x+3^(1/2))/(-4+6*3^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(51) = 102.

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.82

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx=\frac{1}{46}\sqrt{23}\sqrt{8\sqrt{3}+13}\log\left(\frac{23x^4+46x^3+\sqrt{23}\left(11x^3+24x^2-\sqrt{3}(5x^3+13x^2-6x-4)-4x+5\right)\sqrt{8\sqrt{}}}{x^4+2x^3+2x^2+10x+1}\right.$$
$$+\frac{1}{2}\log\left(x^2+\sqrt{3}(x-1)+x+2\right)$$

```
[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="fricas")
```

```
[Out] 1/46*sqrt(23)*sqrt(8*sqrt(3) + 13)*log((23*x^4 + 46*x^3 + sqrt(23)*(11*x^3
+ 24*x^2 - sqrt(3)*(5*x^3 + 13*x^2 - 6*x - 4) - 4*x + 5)*sqrt(8*sqrt(3) + 1
3) + 23*sqrt(3)*(3*x^2 + 5*x + 4) - 23*x + 138)/(x^4 + 2*x^3 + 2*x^2 + 10*x
 + 1)) + 1/2*log(x^2 + sqrt(3)*(x - 1) + x + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(58) = 116.

Time = 0.70 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.81

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx = \left(\frac{\sqrt{5+4\sqrt{3}}}{2\cdot\left(2-3\sqrt{3}\right)}+\frac{1}{2}\right)\log\left(x-\frac{5\sqrt{3}}{5+4\sqrt{3}}+\left(\frac{\sqrt{5+4\sqrt{3}}}{2\cdot\left(2-3\sqrt{3}\right)}+\frac{1}{2}\right)\left(\frac{47}{22+13\sqrt{3}}+\frac{33\sqrt{3}}{22+13\sqrt{3}}\right)+\frac{11}{5+4\sqrt{3}}\right)+\left(\frac{1}{2}-\frac{\sqrt{5+4\sqrt{3}}}{2\cdot\left(2-3\sqrt{3}\right)}\right)\log\left(x-\frac{5\sqrt{3}}{5+4\sqrt{3}}+\frac{11}{5+4\sqrt{3}}+\left(\frac{1}{2}-\frac{\sqrt{5+4\sqrt{3}}}{2\cdot\left(2-3\sqrt{3}\right)}\right)\left(\frac{47}{22+13\sqrt{3}}+\frac{33\sqrt{3}}{22+13\sqrt{3}}\right)\right)$$

```
[In] integrate(x/(2+x**2-3**(1/2)+x*(1+3**(1/2))),x)
```

```
[Out] (sqrt(5 + 4*sqrt(3))/(2*(2 - 3*sqrt(3))) + 1/2)*log(x - 5*sqrt(3)/(5 + 4*sq
rt(3)) + (sqrt(5 + 4*sqrt(3))/(2*(2 - 3*sqrt(3))) + 1/2)*(47/(22 + 13*sqrt(
3)) + 33*sqrt(3)/(22 + 13*sqrt(3))) + 11/(5 + 4*sqrt(3))) + (1/2 - sqrt(5 +
 4*sqrt(3))/(2*(2 - 3*sqrt(3))))*log(x - 5*sqrt(3)/(5 + 4*sqrt(3)) + 11/(5
+ 4*sqrt(3)) + (1/2 - sqrt(5 + 4*sqrt(3))/(2*(2 - 3*sqrt(3))))*(47/(22 + 13
*sqrt(3)) + 33*sqrt(3)/(22 + 13*sqrt(3))))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx = -\frac{\left(\sqrt{3}+1\right)\log\left(\frac{2x+\sqrt{3}-\sqrt{6\sqrt{3}-4}+1}{2x+\sqrt{3}+\sqrt{6\sqrt{3}-4}+1}\right)}{2\sqrt{6\sqrt{3}-4}}+\frac{1}{2}\log\left(x^2+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right)$$

```
[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(3) + 1)*log((2*x + sqrt(3) - sqrt(6*sqrt(3) - 4) + 1)/(2*x + sqr
t(3) + sqrt(6*sqrt(3) - 4) + 1))/sqrt(6*sqrt(3) - 4) + 1/2*log(x^2 + x*(sqr
t(3) + 1) - sqrt(3) + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx = -\frac{\left(\sqrt{3}+1\right)\log\left(\frac{\left|2\,x+\sqrt{3}-\sqrt{6\,\sqrt{3}-4}+1\right|}{\left|2\,x+\sqrt{3}+\sqrt{6\,\sqrt{3}-4}+1\right|}\right)}{2\,\sqrt{6\,\sqrt{3}-4}} + \frac{1}{2}\,\log\left(\left|x^2+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right|\right)$$

```
[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(3) + 1)*log(abs(2*x + sqrt(3) - sqrt(6*sqrt(3) - 4) + 1)/abs(2*x
 + sqrt(3) + sqrt(6*sqrt(3) - 4) + 1))/sqrt(6*sqrt(3) - 4) + 1/2*log(abs(x^
2 + x*(sqrt(3) + 1) - sqrt(3) + 2))
```

Mupad [B] (verification not implemented)

Time = 20.60 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.24

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx = \ln\left(x-\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}+\frac{1}{2}\right)\left(2x+\sqrt{3}+1\right)\right)\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}+\frac{1}{2}\right)-\ln\left(x+\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}-\frac{1}{2}\right)\left(2x+\sqrt{3}+1\right)\right)\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}-\frac{1}{2}\right)$$

```
[In] int(x/(x*(3^(1/2) + 1) - 3^(1/2) + x^2 + 2),x)
```

```
[Out] log(x - ((((3^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(
3^(1/2) + 7))^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) + 1/2)*(2*x + 3^(1/2)
 + 1))*((((3^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(3
^(1/2) + 7))^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) + 1/2) - log(x + ((((3
^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(3^(1/2) + 7))
^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) - 1/2)*(2*x + 3^(1/2) + 1))*((((3^
(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(3^(1/2) + 7))^
(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) - 1/2)
```

3.960 $\int \sqrt{x^2+x^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \sqrt{x^2+x^3}\,dx=-\frac{4(x^2+x^3)^{3/2}}{15x^3}+\frac{2(x^2+x^3)^{3/2}}{5x^2}$$

```
[Out] -4/15*(x^3+x^2)^(3/2)/x^3+2/5*(x^3+x^2)^(3/2)/x^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 2039}

$$\int \sqrt{x^2+x^3}\,dx=\frac{2(x^3+x^2)^{3/2}}{5x^2}-\frac{4(x^3+x^2)^{3/2}}{15x^3}$$

```
[In] Int[Sqrt[x^2 + x^3],x]
```

```
[Out] (-4*(x^2 + x^3)^(3/2))/(15*x^3) + (2*(x^2 + x^3)^(3/2))/(5*x^2)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
```

```
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(x^2+x^3)^{3/2}}{5x^2}-\frac{2}{5}\int\frac{\sqrt{x^2+x^3}}{x}\,dx\\ &= -\frac{4(x^2+x^3)^{3/2}}{15x^3}+\frac{2(x^2+x^3)^{3/2}}{5x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int\sqrt{x^2+x^3}\,dx=\frac{2(x^2(1+x))^{3/2}\,(-2+3x)}{15x^3}$$

```
[In] Integrate[Sqrt[x^2 + x^3],x]
[Out] (2*(x^2*(1 + x))^(3/2)*(-2 + 3*x))/(15*x^3)
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{2(x+1)(3x-2)\sqrt{x^3+x^2}}{15x}$	23
default	$\frac{2(x+1)(3x-2)\sqrt{x^3+x^2}}{15x}$	23
trager	$\frac{2(3x^2+x-2)\sqrt{x^3+x^2}}{15x}$	23
risch	$\frac{2\sqrt{x^2(x+1)}\,(3x^2+x-2)}{15x}$	23
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(x+1)^{\frac{3}{2}}(2-3x)}{15}}{2\sqrt{\pi}}$	27

```
[In] int((x^3+x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(x+1)*(3*x-2)*(x^3+x^2)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \sqrt{x^2+x^3}\,dx = \frac{2\,\sqrt{x^3+x^2}(3\,x^2+x-2)}{15\,x}$$

```
[In] integrate((x^3+x^2)^(1/2),x, algorithm="fricas")
[Out] 2/15*sqrt(x^3 + x^2)*(3*x^2 + x - 2)/x
```

Sympy [F]

$$\int \sqrt{x^2+x^3}\,dx = \int \sqrt{x^3+x^2}\,dx$$

```
[In] integrate((x**3+x**2)**(1/2),x)
[Out] Integral(sqrt(x**3 + x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.41

$$\int \sqrt{x^2+x^3}\,dx = \frac{2}{15}\,\left(3\,x^2+x-2\right)\sqrt{x+1}$$

```
[In] integrate((x^3+x^2)^(1/2),x, algorithm="maxima")
[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \sqrt{x^2+x^3}\,dx = \frac{2}{15}\left(3\,(x+1)^{\frac{5}{2}}-10\,(x+1)^{\frac{3}{2}}+15\,\sqrt{x+1}\right)\mathrm{sgn}(x) + \frac{2}{3}\left((x+1)^{\frac{3}{2}}-3\,\sqrt{x+1}\right)\mathrm{sgn}(x)+\frac{4}{15}\,\mathrm{sgn}(x)$$

```
[In] integrate((x^3+x^2)^(1/2),x, algorithm="giac")
[Out] 2/15*(3*(x + 1)^(5/2) - 10*(x + 1)^(3/2) + 15*sqrt(x + 1))*sgn(x) + 2/3*((x
 + 1)^(3/2) - 3*sqrt(x + 1))*sgn(x) + 4/15*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 19.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \sqrt{x^2+x^3}\,dx=\frac{2\,\sqrt{x^3+x^2}\,\left(3\,x^2+x-2\right)}{15\,x}$$

```
[In] int((x^2 + x^3)^(1/2),x)
[Out] (2*(x^2 + x^3)^(1/2)*(x + 3*x^2 - 2))/(15*x)
```

3.961 $\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = \arctan\left(\sqrt{2x+x^2}\right)$$

```
[Out] arctan((x^2+2*x)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {702, 209}

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = \arctan\left(\sqrt{x^2+2x}\right)$$

```
[In] Int[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] ArcTan[Sqrt[2*x + x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 702

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
```

```
qQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 4\text{Subst}\left(\int \frac{1}{4+4x^2}\,dx, x, \sqrt{2x+x^2}\right)\\
&= \tan^{-1}\left(\sqrt{2x+x^2}\right)
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 41 vs. 2(12) = 24.

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.42

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = -\frac{2\sqrt{x}\sqrt{2+x}\arctan\left(1+x-\sqrt{x}\sqrt{2+x}\right)}{\sqrt{x(2+x)}}$$

```
[In] Integrate[1/((1 + x)*Sqrt[2*x + x^2]),x]
[Out] (-2*Sqrt[x]*Sqrt[2 + x]*ArcTan[1 + x - Sqrt[x]*Sqrt[2 + x]])/Sqrt[x*(2 + x)
]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$\arctan\left(\sqrt{x\left(x+2\right)}\right)$	9
default	$-\arctan\left(\frac{1}{\sqrt{\left(x+1\right)^{2}-1}}\right)$	13
trager	$\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)+\sqrt{x^{2}+2x}}{x+1}\right)$	31

```
[In] int(1/(x+1)/(x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan((x*(x+2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = 2\,\arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="fricas")
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Sympy [F]

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = \int \frac{1}{\sqrt{x\,(x+2)}\,(x+1)}\,dx$$

```
[In] integrate(1/(1+x)/(x**2+2*x)**(1/2),x)
[Out] Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = -\arcsin\left(\frac{1}{|x+1|}\right)$$

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="maxima")
[Out] -arcsin(1/abs(x + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = 2\,\arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="giac")
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2\,x}\,\left(x+1\right)}\,dx$$

```
[In] int(1/((2*x + x^2)^(1/2)*(x + 1)),x)
[Out] int(1/((2*x + x^2)^(1/2)*(x + 1)), x)
```

3.962 $\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx=\frac{9}{32}(1+2\sqrt{x})\sqrt{1-\sqrt{x}-x}+\frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}+\frac{45}{64}\arcsin\left(\frac{1+2\sqrt{x}}{\sqrt{5}}\right)$$

```
[Out] 45/64*arcsin(1/5*(1+2*x^(1/2))*5^(1/2))+5/12*(1-x-x^(1/2))^(3/2)-1/2*(1-x-x
^(1/2))^(3/2)*x^(1/2)+9/32*(1+2*x^(1/2))*(1-x-x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1371, 756, 654, 626, 633, 222}

$$\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx=\frac{45}{64}\arcsin\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)-\frac{1}{2}\sqrt{x}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{5}{12}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{9}{32}(2\sqrt{x}+1)\sqrt{-x-\sqrt{x}+1}$$

```
[In] Int[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]
```

```
[Out] (9*(1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/32 + (5*(1 - Sqrt[x] - x)^(3/2))/
12 - ((1 - Sqrt[x] - x)^(3/2)*Sqrt[x])/2 + (45*ArcSin[(1 + 2*Sqrt[x])/Sqrt[
5]])/64
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int x^2\sqrt{1-x-x^2}\,dx,x,\sqrt{x}\right)\\ &= -\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}-\frac{1}{2}\text{Subst}\left(\int\left(-1+\frac{5x}{2}\right)\sqrt{1-x-x^2}\,dx,x,\sqrt{x}\right)\end{aligned}$$

$$
\begin{aligned}
&= \frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}+\frac{9}{8}\text{Subst}\left(\int \sqrt{1-x-x^2}\,dx,x,\sqrt{x}\right)\\
&= \frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}+\frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}\\
&\quad -\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}+\frac{45}{64}\text{Subst}\left(\int \frac{1}{\sqrt{1-x-x^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}+\frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}\\
&\quad -\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}-\frac{1}{64}\left(9\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{5}}}\,dx,x,-1-2\sqrt{x}\right)\\
&= \frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}+\frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}\\
&\quad -\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}+\frac{45}{64}\sin^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{5}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.75

$$
\begin{aligned}
\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx &= \frac{1}{96}\sqrt{1-\sqrt{x}-x}\left(67-34\sqrt{x}+8x+48x^{3/2}\right)\\
&\quad +\frac{45}{32}\arctan\left(\frac{\sqrt{x}}{-1+\sqrt{1-\sqrt{x}-x}}\right)
\end{aligned}
$$

```
[In] Integrate[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]
[Out] (Sqrt[1 - Sqrt[x] - x]*(67 - 34*Sqrt[x] + 8*x + 48*x^(3/2)))/96 + (45*ArcTa
n[Sqrt[x]/(-1 + Sqrt[1 - Sqrt[x] - x])])/32
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{(1-x-\sqrt{x})^{\frac{3}{2}}\sqrt{x}}{2}+\frac{5(1-x-\sqrt{x})^{\frac{3}{2}}}{12}-\frac{9(-2\sqrt{x}-1)\sqrt{1-x-\sqrt{x}}}{32}+\frac{45\arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{64}$	67
default	$-\frac{(1-x-\sqrt{x})^{\frac{3}{2}}\sqrt{x}}{2}+\frac{5(1-x-\sqrt{x})^{\frac{3}{2}}}{12}-\frac{9(-2\sqrt{x}-1)\sqrt{1-x-\sqrt{x}}}{32}+\frac{45\arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{64}$	67

```
[In] int(x^(1/2)*(1-x-x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(1-x-x^(1/2))^(3/2)*x^(1/2)+5/12*(1-x-x^(1/2))^(3/2)-9/32*(-2*x^(1/2)-
1)*(1-x-x^(1/2))^(1/2)+45/64*arcsin(2/5*5^(1/2)*(x^(1/2)+1/2))
```

Fricas [A] (verification not implemented)

none

Time = 1.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx = \frac{1}{96}\left(2\left(24\,x-17\right)\sqrt{x}+8\,x+67\right)\sqrt{-x-\sqrt{x}+1} - \frac{45}{128}\arctan\left(-\frac{\left(8\,x^{2}-\left(16\,x^{2}-38\,x+11\right)\sqrt{x}-9\,x+3\right)\sqrt{-x-\sqrt{x}+1}}{4\left(4\,x^{3}-13\,x^{2}+7\,x-1\right)}\right)$$

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(2*(24*x - 17)*sqrt(x) + 8*x + 67)*sqrt(-x - sqrt(x) + 1) - 45/128*arc
tan(-1/4*(8*x^2 - (16*x^2 - 38*x + 11)*sqrt(x) - 9*x + 3)*sqrt(-x - sqrt(x)
 + 1)/(4*x^3 - 13*x^2 + 7*x - 1))
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.59

$$\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx = 2\sqrt{-\sqrt{x}-x+1}\left(\frac{x^{\frac{3}{2}}}{4}-\frac{17\sqrt{x}}{96}+\frac{x}{24}+\frac{67}{192}\right)+\frac{45\operatorname{asin}\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{64}$$

```
[In] integrate(x**(1/2)*(1-x-x**(1/2))**(1/2),x)
```

```
[Out] 2*sqrt(-sqrt(x) - x + 1)*(x**(3/2)/4 - 17*sqrt(x)/96 + x/24 + 67/192) + 45*
asin(2*sqrt(5)*(sqrt(x) + 1/2)/5)/64
```

Maxima [F]

$$\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx = \int \sqrt{x}\sqrt{-x-\sqrt{x}+1}\,dx$$

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)*sqrt(-x - sqrt(x) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

$$\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx = \frac{1}{96}\left(2\left(4\sqrt{x}(6\sqrt{x}+1)-17\right)\sqrt{x}+67\right)\sqrt{-x-\sqrt{x}+1} + \frac{45}{64}\arcsin\left(\frac{1}{5}\sqrt{5}(2\sqrt{x}+1)\right)$$

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="giac")
[Out] 1/96*(2*(4*sqrt(x)*(6*sqrt(x) + 1) - 17)*sqrt(x) + 67)*sqrt(-x - sqrt(x) +
1) + 45/64*arcsin(1/5*sqrt(5)*(2*sqrt(x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx = \int \sqrt{x}\,\sqrt{1-\sqrt{x}-x}\,dx$$

```
[In] int(x^(1/2)*(1 - x^(1/2) - x)^(1/2),x)
[Out] int(x^(1/2)*(1 - x^(1/2) - x)^(1/2), x)
```

3.963 $\int \sqrt[3]{1+\sqrt{-3+x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = -\frac{3}{2}\left(1+\sqrt{-3+x}\right)^{4/3}+\frac{6}{7}\left(1+\sqrt{-3+x}\right)^{7/3}$$

```
[Out] -3/2*(1+(-3+x)^(1/2))^(4/3)+6/7*(1+(-3+x)^(1/2))^(7/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {253, 196, 45}

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = \frac{6}{7}\left(\sqrt{x-3}+1\right)^{7/3}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{4/3}$$

```
[In] Int[(1 + Sqrt[-3 + x])^(1/3),x]
```

```
[Out] (-3*(1 + Sqrt[-3 + x])^(4/3))/2 + (6*(1 + Sqrt[-3 + x])^(7/3))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
 - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
```

```
IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \sqrt[3]{1+\sqrt{x}}\,dx, x, -3+x\right)\\
&= 2\text{Subst}\left(\int x\sqrt[3]{1+x}\,dx, x, \sqrt{-3+x}\right)\\
&= 2\text{Subst}\left(\int \left(-\sqrt[3]{1+x}+(1+x)^{4/3}\right)\,dx, x, \sqrt{-3+x}\right)\\
&= -\frac{3}{2}\left(1+\sqrt{-3+x}\right)^{4/3}+\frac{6}{7}\left(1+\sqrt{-3+x}\right)^{7/3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = \frac{3}{14}\left(1+\sqrt{-3+x}\right)^{4/3}\left(-3+4\sqrt{-3+x}\right)$$

```
[In] Integrate[(1 + Sqrt[-3 + x])^(1/3),x]
[Out] (3*(1 + Sqrt[-3 + x])^(4/3)*(-3 + 4*Sqrt[-3 + x]))/14
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{3\left(1+\sqrt{-3+x}\right)^{\frac{4}{3}}}{2}+\frac{6\left(1+\sqrt{-3+x}\right)^{\frac{7}{3}}}{7}$	24
default	$-\frac{3\left(1+\sqrt{-3+x}\right)^{\frac{4}{3}}}{2}+\frac{6\left(1+\sqrt{-3+x}\right)^{\frac{7}{3}}}{7}$	24

```
[In] int((1+(-3+x)^(1/2))^(1/3),x,method=_RETURNVERBOSE)
[Out] -3/2*(1+(-3+x)^(1/2))^(4/3)+6/7*(1+(-3+x)^(1/2))^(7/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = \frac{3}{14}\left(4\,x+\sqrt{x-3}-15\right)\left(\sqrt{x-3}+1\right)^{\frac{1}{3}}$$

```
[In] integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="fricas")
[Out] 3/14*(4*x + sqrt(x - 3) - 15)*(sqrt(x - 3) + 1)^(1/3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(29) = 58.

Time = 0.64 (sec) , antiderivative size = 184, normalized size of antiderivative = 5.26

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = \frac{12(x-3)^{\frac{7}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14\,(x-3)^{\frac{5}{2}}+14\,(x-3)^{2}} - \frac{6(x-3)^{\frac{5}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14\,(x-3)^{\frac{5}{2}}+14\,(x-3)^{2}} + \frac{9(x-3)^{\frac{5}{2}}}{14\,(x-3)^{\frac{5}{2}}+14\,(x-3)^{2}} + \frac{15(x-3)^{3}\sqrt[3]{\sqrt{x-3}+1}}{14\,(x-3)^{\frac{5}{2}}+14\,(x-3)^{2}} - \frac{9(x-3)^{2}\sqrt[3]{\sqrt{x-3}+1}}{14\,(x-3)^{\frac{5}{2}}+14\,(x-3)^{2}} + \frac{9(x-3)^{2}}{14\,(x-3)^{\frac{5}{2}}+14\,(x-3)^{2}}$$

```
[In] integrate((1+(-3+x)**(1/2))**(1/3),x)
[Out] 12*(x - 3)**(7/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)*
*2) - 6*(x - 3)**(5/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x
- 3)**2) + 9*(x - 3)**(5/2)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 15*(x - 3
)**3*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) - 9*(x -
3)**2*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 9*(x -
 3)**2/(14*(x - 3)**(5/2) + 14*(x - 3)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = \frac{6}{7}\left(\sqrt{x-3}+1\right)^{\frac{7}{3}} - \frac{3}{2}\left(\sqrt{x-3}+1\right)^{\frac{4}{3}}$$

```
[In] integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="maxima")
[Out] 6/7*(sqrt(x - 3) + 1)^(7/3) - 3/2*(sqrt(x - 3) + 1)^(4/3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = \frac{6}{7}\left(\sqrt{x-3}+1\right)^{\frac{7}{3}} - \frac{3}{2}\left(\sqrt{x-3}+1\right)^{\frac{4}{3}}$$

```
[In] integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="giac")
[Out] 6/7*(sqrt(x - 3) + 1)^(7/3) - 3/2*(sqrt(x - 3) + 1)^(4/3)
```

Mupad [B] (verification not implemented)

Time = 19.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.46

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = (x-3)\ {}_2F_1\left(-\frac{1}{3},2;\ 3;\ -\sqrt{x-3}\right)$$

```
[In] int(((x - 3)^(1/2) + 1)^(1/3),x)
[Out] (x - 3)*hypergeom([-1/3, 2], 3, -(x - 3)^(1/2))
```

3.964 $\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}} \, dx = -6\sqrt{3+\sqrt{-1+2x}}+\frac{2}{3}\left(3+\sqrt{-1+2x}\right)^{3/2}$$

```
[Out] 2/3*(3+(-1+2*x)^(1/2))^(3/2)-6*(3+(-1+2*x)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {253, 196, 45}

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}} \, dx = \frac{2}{3}\left(\sqrt{2x-1}+3\right)^{3/2}-6\sqrt{\sqrt{2x-1}+3}$$

```
[In] Int[1/Sqrt[3 + Sqrt[-1 + 2*x]],x]
```

```
[Out] -6*Sqrt[3 + Sqrt[-1 + 2*x]] + (2*(3 + Sqrt[-1 + 2*x])^(3/2))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
```

```
IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+\sqrt{x}}}\,dx, x, -1+2x\right)\\
&= \text{Subst}\left(\int \frac{x}{\sqrt{3+x}}\,dx, x, \sqrt{-1+2x}\right)\\
&= \text{Subst}\left(\int \left(-\frac{3}{\sqrt{3+x}}+\sqrt{3+x}\right)\,dx, x, \sqrt{-1+2x}\right)\\
&= -6\sqrt{3+\sqrt{-1+2x}}+\frac{2}{3}\left(3+\sqrt{-1+2x}\right)^{3/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx = \frac{2}{3}\left(-6+\sqrt{-1+2x}\right)\sqrt{3+\sqrt{-1+2x}}$$

```
[In] Integrate[1/Sqrt[3 + Sqrt[-1 + 2*x]],x]
[Out] (2*(-6 + Sqrt[-1 + 2*x])*Sqrt[3 + Sqrt[-1 + 2*x]])/3
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2\left(3+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-6\sqrt{3+\sqrt{2x-1}}$	28
default	$\frac{2\left(3+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-6\sqrt{3+\sqrt{2x-1}}$	28

```
[In] int(1/(3+(2*x-1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(3+(2*x-1)^(1/2))^(3/2)-6*(3+(2*x-1)^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx = \frac{2}{3}\sqrt{\sqrt{2\,x-1}+3}\left(\sqrt{2\,x-1}-6\right)$$

```
[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(sqrt(2*x - 1) + 3)*(sqrt(2*x - 1) - 6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(31) = 62.

Time = 0.62 (sec) , antiderivative size = 265, normalized size of antiderivative = 7.16

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx = -\frac{6\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}} + \frac{36\sqrt{2}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}} + \frac{4\sqrt{3}\left(x-\frac{1}{2}\right)^{3}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}} - \frac{36\sqrt{3}\left(x-\frac{1}{2}\right)^{2}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}} + \frac{108\left(x-\frac{1}{2}\right)^{2}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}$$

```
[In] integrate(1/(3+(-1+2*x)**(1/2))**(1/2),x)
[Out] -6*sqrt(6)*(x - 1/2)**(5/2)*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(6)*(x -
 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 36*sqrt(2)*(x - 1/2)**(5/2)/(3*sqr
t(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 4*sqrt(3)*(x - 1/2)**3*sq
rt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x -
1/2)**2) - 36*sqrt(3)*(x - 1/2)**2*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(
6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 108*(x - 1/2)**2/(3*sqrt(6)
*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx = \frac{2}{3}\left(\sqrt{2\,x-1}+3\right)^{\frac{3}{2}} - 6\,\sqrt{\sqrt{2\,x-1}+3}$$

```
[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] 2/3*(sqrt(2*x - 1) + 3)^(3/2) - 6*sqrt(sqrt(2*x - 1) + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx = \frac{2}{3}\left(\sqrt{2\,x-1}+3\right)^{\frac{3}{2}} - 6\,\sqrt{\sqrt{2\,x-1}+3}$$

```
[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
[Out] 2/3*(sqrt(2*x - 1) + 3)^(3/2) - 6*sqrt(sqrt(2*x - 1) + 3)
```

Mupad [B] (verification not implemented)

Time = 20.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx = \frac{\sqrt{3}\,(2\,x-1)\;{}_2\mathrm{F}_1\left(\frac{1}{2},2;\;3;\;-\frac{\sqrt{2\,x-1}}{3}\right)}{6}$$

```
[In] int(1/((2*x - 1)^(1/2) + 3)^(1/2),x)
[Out] (3^(1/2)*(2*x - 1)*hypergeom([1/2, 2], 3, -(2*x - 1)^(1/2)/3))/6
```

3.965 $\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 29

$$\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx = -\left(\left(2-\sqrt{x}\right)\sqrt{1-x}\right) - \arcsin\left(\sqrt{x}\right)$$

```
[Out] -arcsin(x^(1/2))-(1-x)^(1/2)*(2-x^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1412, 799, 794, 222}

$$\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx = -\arcsin\left(\sqrt{x}\right) - \sqrt{1-x}\left(2-\sqrt{x}\right)$$

```
[In] Int[Sqrt[1 - x]/(1 + Sqrt[x]),x]
[Out] -((2 - Sqrt[x])*Sqrt[1 - x]) - ArcSin[Sqrt[x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] && !ILtQ[p - 1/2, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{x\sqrt{1-x^2}}{1+x}\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\\ &= -\left(\left(2-\sqrt{x}\right)\sqrt{1-x}\right) - \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\\ &= -\left(\left(2-\sqrt{x}\right)\sqrt{1-x}\right) - \sin^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx = \left(-2+\sqrt{x}\right)\sqrt{1-x} + 2\arctan\left(\frac{\sqrt{1-x}}{1+\sqrt{x}}\right)$$

```
[In] Integrate[Sqrt[1 - x]/(1 + Sqrt[x]),x]
[Out] (-2 + Sqrt[x])*Sqrt[1 - x] + 2*ArcTan[Sqrt[1 - x]/(1 + Sqrt[x])]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(23) = 46.

Time = 1.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

method	result	size
default	$-\frac{\sqrt{1-x}\,\sqrt{x}\,\left(-2\sqrt{-(x-1)x}+\arcsin(2x-1)\right)}{2\sqrt{-(x-1)x}}-2\sqrt{1-x}$	48

```
[In] int((1-x)^(1/2)/(1+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/2*(1-x)^(1/2)*x^(1/2)*(-2*(-(x-1)*x)^(1/2)+arcsin(2*x-1))/(-(x-1)*x)^(1/
2)-2*(1-x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx = \sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} + \arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

```
[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="fricas")
[Out] sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arctan(sqrt(-x + 1)/sqrt(x))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx = i\sqrt{x}\sqrt{x-1} - 2i\sqrt{x-1} + i\operatorname{asinh}\left(\sqrt{x-1}\right)$$

```
[In] integrate((1-x)**(1/2)/(1+x**(1/2)),x)
[Out] I*sqrt(x)*sqrt(x - 1) - 2*I*sqrt(x - 1) + I*asinh(sqrt(x - 1))
```

Maxima [F]

$$\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx = \int \frac{\sqrt{-x+1}}{\sqrt{x}+1}\,dx$$

```
[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x + 1)/(sqrt(x) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx = \sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} + \arcsin\left(\sqrt{-x+1}\right)$$

```
[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arcsin(sqrt(-x + 1))
```

Mupad [B] (verification not implemented)

Time = 21.87 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{1-x} - 2\,\sqrt{1-x} - 2\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)$$

```
[In] int((1 - x)^(1/2)/(x^(1/2) + 1),x)
```

```
[Out] x^(1/2)*(1 - x)^(1/2) - 2*(1 - x)^(1/2) - 2*atan(x^(1/2)/((1 - x)^(1/2) - 1
))
```

3.966 $\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx=-\left(\left(2+\sqrt{x}\right)\sqrt{1-x}\right)+\arcsin\left(\sqrt{x}\right)$$

```
[Out] arcsin(x^(1/2))-(1-x)^(1/2)*(2+x^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1412, 799, 794, 222}

$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx=\arcsin\left(\sqrt{x}\right)-\left(\sqrt{x}+2\right)\sqrt{1-x}$$

```
[In] Int[Sqrt[1 - x]/(1 - Sqrt[x]),x]
```

```
[Out] -((2 + Sqrt[x])*Sqrt[1 - x]) + ArcSin[Sqrt[x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] && !ILtQ[p - 1/2, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x\sqrt{1-x^2}}{1-x}\,dx, x, \sqrt{x}\right)\\
&= -\left(2\text{Subst}\left(\int \frac{(-1-x)x}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\right)\\
&= -\left(\left(2+\sqrt{x}\right)\sqrt{1-x}\right) + \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\\
&= -\left(\left(2+\sqrt{x}\right)\sqrt{1-x}\right) + \sin^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$
\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx = \left(-2-\sqrt{x}\right)\sqrt{1-x} + 2\arctan\left(\frac{\sqrt{x}}{-1+\sqrt{1-x}}\right)
$$

```
[In] Integrate[Sqrt[1 - x]/(1 - Sqrt[x]),x]
[Out] (-2 - Sqrt[x])*Sqrt[1 - x] + 2*ArcTan[Sqrt[x]/(-1 + Sqrt[1 - x])]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(19) = 38$.

Time = 1.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

method	result	size
default	$-2\sqrt{1-x}+\frac{\sqrt{1-x}\,\sqrt{x}\,\left(-2\sqrt{-\left(x-1\right)x}+\arcsin\left(2x-1\right)\right)}{2\sqrt{-\left(x-1\right)x}}$	48

```
[In] int((1-x)^(1/2)/(1-x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -2*(1-x)^(1/2)+1/2*(1-x)^(1/2)*x^(1/2)*(-2*(-(x-1)*x)^(1/2)+arcsin(2*x-1))/
(-(x-1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx = -\sqrt{x}\sqrt{-x+1} - 2\,\sqrt{-x+1} - \arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

```
[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="fricas")
[Out] -sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arctan(sqrt(-x + 1)/sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx = -2\left(\begin{cases} \frac{\sqrt{x}\sqrt{1-x}}{2} + \sqrt{1-x} - \frac{\operatorname{asin}\left(\sqrt{x}\right)}{2} & \text{for}\: \sqrt{x} > -1 \wedge \sqrt{x} < 1 \end{cases}\right)$$

```
[In] integrate((1-x)**(1/2)/(1-x**(1/2)),x)
[Out] -2*Piecewise((sqrt(x)*sqrt(1 - x)/2 + sqrt(1 - x) - asin(sqrt(x))/2, (sqrt(
x) > -1) & (sqrt(x) < 1)))
```

Maxima [F]

$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx = \int -\frac{\sqrt{-x+1}}{\sqrt{x}-1}\,dx$$

[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="maxima")

[Out] -integrate(sqrt(-x + 1)/(sqrt(x) - 1), x)

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx = -\sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} - \arcsin\left(\sqrt{-x+1}\right)$$

[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="giac")

[Out] -sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arcsin(sqrt(-x + 1))

Mupad [B] (verification not implemented)

Time = 22.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx = 2\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right) - 2\sqrt{1-x} - \sqrt{x}\,\sqrt{1-x}$$

[In] int(-(1 - x)^(1/2)/(x^(1/2) - 1),x)

[Out] 2*atan(x^(1/2)/((1 - x)^(1/2) - 1)) - 2*(1 - x)^(1/2) - x^(1/2)*(1 - x)^(1/2)

3.967 $\int \frac{x}{x-\sqrt{1+x^2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{x}{x-\sqrt{1+x^2}} \, dx = -\frac{x^3}{3} - \frac{1}{3}\left(1+x^2\right)^{3/2}$$

```
[Out] -1/3*x^3-1/3*(x^2+1)^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2131, 30, 267}

$$\int \frac{x}{x-\sqrt{1+x^2}} \, dx = -\frac{x^3}{3} - \frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[In] Int[x/(x - Sqrt[1 + x^2]),x]
```

```
[Out] -1/3*x^3 - (1 + x^2)^(3/2)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2131

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Dist[-b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int x^2\,dx - \int x\sqrt{1+x^2}\,dx\\ &= -\frac{x^3}{3} - \frac{1}{3}\left(1+x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x}{x-\sqrt{1+x^2}}\,dx = -\frac{x^3}{3} - \frac{1}{3}\left(1+x^2\right)^{3/2}$$

```
[In] Integrate[x/(x - Sqrt[1 + x^2]),x]
[Out] -1/3*x^3 - (1 + x^2)^(3/2)/3
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{x^3}{3}-\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$	16
trager	$-\frac{x^3}{3}+\left(-\frac{x^2}{3}-\frac{1}{3}\right)\sqrt{x^2+1}$	22

```
[In] int(x/(x-(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/3*x^3-1/3*(x^2+1)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{x}{x-\sqrt{1+x^2}}\,dx = -\frac{1}{3}\,x^3 - \frac{1}{3}\,\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/3*x^3 - 1/3*(x^2 + 1)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(15) = 30.

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.67

$$\int \frac{x}{x-\sqrt{1+x^2}}\,dx = \frac{2x^2}{3x-3\sqrt{x^2+1}} - \frac{x\sqrt{x^2+1}}{3x-3\sqrt{x^2+1}} + \frac{1}{3x-3\sqrt{x^2+1}}$$

```
[In] integrate(x/(x-(x**2+1)**(1/2)),x)
```

```
[Out] 2*x**2/(3*x - 3*sqrt(x**2 + 1)) - x*sqrt(x**2 + 1)/(3*x - 3*sqrt(x**2 + 1))
 + 1/(3*x - 3*sqrt(x**2 + 1))
```

Maxima [F]

$$\int \frac{x}{x-\sqrt{1+x^2}}\,dx = \int \frac{x}{x-\sqrt{x^2+1}}\,dx$$

```
[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(x - sqrt(x^2 + 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{x}{x-\sqrt{1+x^2}}\,dx = -\frac{1}{3}\,x^3 - \frac{1}{3}\,\left(x^2+1\right)^{\frac{3}{2}}$$

```
[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/3*x^3 - 1/3*(x^2 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{x}{x-\sqrt{1+x^2}}\,dx = -\sqrt{x^2+1}\left(\frac{x^2}{3}+\frac{1}{3}\right)-\frac{x^3}{3}$$

```
[In] int(x/(x - (x^2 + 1)^(1/2)),x)
[Out] - (x^2 + 1)^(1/2)*(x^2/3 + 1/3) - x^3/3
```

3.968 $\int \frac{x}{x-\sqrt{1-x^2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \frac{x}{x-\sqrt{1-x^2}} \, dx = \frac{x}{2}+\frac{\sqrt{1-x^2}}{2}-\frac{\operatorname{arctanh}\left(\sqrt{2}x\right)}{2\sqrt{2}}-\frac{\operatorname{arctanh}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}}$$

```
[Out] 1/2*x-1/4*arctanh(x*2^(1/2))*2^(1/2)-1/4*arctanh(2^(1/2)*(-x^2+1)^(1/2))*2^
(1/2)+1/2*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2132, 327, 212, 455, 52, 65, 213}

$$\int \frac{x}{x-\sqrt{1-x^2}} \, dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}}-\frac{\operatorname{arctanh}\left(\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\sqrt{1-x^2}}{2}+\frac{x}{2}$$

```
[In] Int[x/(x - Sqrt[1 - x^2]),x]
```

```
[Out] x/2 + Sqrt[1 - x^2]/2 - ArcTanh[Sqrt[2]*x]/(2*Sqrt[2]) - ArcTanh[Sqrt[2]*Sq
rt[1 - x^2]]/(2*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2132

```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] :> Dist[-d, Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
+ Dist[c, Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x]
, x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\int \frac{x^2}{1-2x^2}\,dx - \int \frac{x\sqrt{1-x^2}}{1-2x^2}\,dx$$

$$= \frac{x}{2} - \frac{1}{2}\int \frac{1}{1-2x^2}\,dx - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{1-2x}\,dx, x, x^2\right)$$
$$= \frac{x}{2} + \frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{(1-2x)\sqrt{1-x}}\,dx, x, x^2\right)$$
$$= \frac{x}{2} + \frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sqrt{1-x^2}\right)$$
$$= \frac{x}{2} + \frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\tanh^{-1}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx = \frac{1}{2}\left(x + \sqrt{1-x^2} + \sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}x}{-1-x+\sqrt{1-x^2}}\right)\right)$$

```
[In] Integrate[x/(x - Sqrt[1 - x^2]),x]
[Out] (x + Sqrt[1 - x^2] + Sqrt[2]*ArcTanh[(Sqrt[2]*x)/(-1 - x + Sqrt[1 - x^2])])
/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

method	result
trager	$\frac{x}{2} + \frac{\sqrt{-x^2+1}}{2} - \frac{\text{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{2\sqrt{-x^2+1}+\text{RootOf}\left(\_Z^2-2\right)}{\text{RootOf}\left(\_Z^2-2\right)x-1}\right)}{4}$
default	$\frac{x}{2} - \frac{\text{arctanh}\left(x\sqrt{2}\right)\sqrt{2}}{4} + \frac{\sqrt{-4\left(x+\frac{\sqrt{2}}{2}\right)^2+4\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{8} - \frac{\sqrt{2}\,\text{arctanh}\left(\frac{\left(\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+1\right)\sqrt{2}}{\sqrt{-4\left(x+\frac{\sqrt{2}}{2}\right)^2+4\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{8} + \sqrt{-4\left(x-\right.}$

```
[In] int(x/(x-(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*(-x^2+1)^(1/2)-1/4*RootOf(_Z^2-2)*ln(-(2*(-x^2+1)^(1/2)+RootOf(_Z
^2-2))/(RootOf(_Z^2-2)*x-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{6x^2-2\sqrt{2}(2x^2-3)+2\sqrt{-x^2+1}(3\sqrt{2}-4)-9}{2x^2-1}\right) + \frac{1}{8}\sqrt{2}\log\left(\frac{2x^2-2\sqrt{2}x+1}{2x^2-1}\right) + \frac{1}{2}x + \frac{1}{2}\sqrt{-x^2+1}$$

```
[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="fricas")
[Out] 1/8*sqrt(2)*log((6*x^2 - 2*sqrt(2)*(2*x^2 - 3) + 2*sqrt(-x^2 + 1)*(3*sqrt(2
) - 4) - 9)/(2*x^2 - 1)) + 1/8*sqrt(2)*log((2*x^2 - 2*sqrt(2)*x + 1)/(2*x^2
 - 1)) + 1/2*x + 1/2*sqrt(-x^2 + 1)
```

Sympy [F]

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx = \int \frac{x}{x-\sqrt{1-x^2}}\,dx$$

```
[In] integrate(x/(x-(-x**2+1)**(1/2)),x)
[Out] Integral(x/(x - sqrt(1 - x**2)), x)
```

Maxima [F]

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx = \int \frac{x}{x-\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(x/(x - sqrt(-x^2 + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(45) = 90$.

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{\left|4x-2\sqrt{2}\right|}{\left|4x+2\sqrt{2}\right|}\right) - \frac{1}{8}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}{\left|4\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}\right) + \frac{1}{2}x + \frac{1}{2}\sqrt{-x^2+1}$$

```
[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(4*x - 2*sqrt(2))/abs(4*x + 2*sqrt(2))) - 1/8*sqrt(2)*lo
g(abs(-4*sqrt(2) + 2*(sqrt(-x^2 + 1) - 1)^2/x^2 - 6)/abs(4*sqrt(2) + 2*(sqr
t(-x^2 + 1) - 1)^2/x^2 - 6)) + 1/2*x + 1/2*sqrt(-x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 21.67 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.95

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx = \frac{x}{2} - \frac{\sqrt{2}\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}x}{2}-1\right)1\mathrm{i}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{2}}{2}}\right)}{8} - \frac{\sqrt{2}\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}x}{2}+1\right)1\mathrm{i}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{2}}{2}}\right)}{8} + \frac{\sqrt{2}\ln\left(x-\frac{\sqrt{2}}{2}\right)}{8} - \frac{\sqrt{2}\ln\left(x+\frac{\sqrt{2}}{2}\right)}{8} + \frac{\sqrt{1-x^2}}{2}$$

```
[In] int(x/(x - (1 - x^2)^(1/2)),x)
```

```
[Out] x/2 - (2^(1/2)*log((2^(1/2)*((2^(1/2)*x)/2 - 1)*1i - (1 - x^2)^(1/2)*1i)/(x
 - 2^(1/2)/2)))/8 - (2^(1/2)*log((2^(1/2)*((2^(1/2)*x)/2 + 1)*1i + (1 - x^2
)^(1/2)*1i)/(x + 2^(1/2)/2)))/8 + (2^(1/2)*log(x - 2^(1/2)/2))/8 - (2^(1/2)
*log(x + 2^(1/2)/2))/8 + (1 - x^2)^(1/2)/2
```

3.969 $\int \frac{x}{x-\sqrt{1+2x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \frac{x}{x-\sqrt{1+2x^2}}\,dx = -x-\sqrt{1+2x^2}+\arctan(x)+\arctan\left(\sqrt{1+2x^2}\right)$$

```
[Out] -x+arctan(x)+arctan((2*x^2+1)^(1/2))-(2*x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2132, 327, 209, 455, 52, 65}

$$\int \frac{x}{x-\sqrt{1+2x^2}}\,dx = \arctan\left(\sqrt{2x^2+1}\right)+\arctan(x)-\sqrt{2x^2+1}-x$$

```
[In] Int[x/(x - Sqrt[1 + 2*x^2]),x]
```

```
[Out] -x - Sqrt[1 + 2*x^2] + ArcTan[x] + ArcTan[Sqrt[1 + 2*x^2]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2132

```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] :> Dist[-d, Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
+ Dist[c, Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x]
, x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{x^2}{1+x^2}\,dx - \int \frac{x\sqrt{1+2x^2}}{1+x^2}\,dx \\
&= -x - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1+2x}}{1+x}\,dx, x, x^2\right) + \int \frac{1}{1+x^2}\,dx \\
&= -x - \sqrt{1+2x^2} + \tan^{-1}(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{1+2x}}\,dx, x, x^2\right) \\
&= -x - \sqrt{1+2x^2} + \tan^{-1}(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{x^2}{2}}\,dx, x, \sqrt{1+2x^2}\right)
\end{aligned}
$$

$$= -x - \sqrt{1+2x^2} + \tan^{-1}(x) + \tan^{-1}\left(\sqrt{1+2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{x}{x-\sqrt{1+2x^2}}\,dx = -x - \sqrt{1+2x^2} + 2\arctan\left(\left(2+\sqrt{2}\right)x - \left(1+\sqrt{2}\right)\sqrt{1+2x^2}\right)$$

```
[In] Integrate[x/(x - Sqrt[1 + 2*x^2]),x]
[Out] -x - Sqrt[1 + 2*x^2] + 2*ArcTan[(2 + Sqrt[2])*x - (1 + Sqrt[2])*Sqrt[1 + 2*
x^2]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
default	$-x+\arctan\left(x\right)+\arctan\left(\sqrt{2x^2+1}\right)-\sqrt{2x^2+1}$	28
trager	$-x-\sqrt{2x^2+1}+\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{2x^2+1}+\operatorname{RootOf}\left(\_Z^2+1\right)}{\operatorname{RootOf}\left(\_Z^2+1\right)x+1}\right)$	53

```
[In] int(x/(x-(2*x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -x+arctan(x)+arctan((2*x^2+1)^(1/2))-(2*x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{x}{x-\sqrt{1+2x^2}}\,dx = -x - \sqrt{2\,x^2+1} + \arctan\left(x\right) - \arctan\left(-\frac{x^2-\sqrt{2\,x^2+1}+1}{x^2}\right)$$

```
[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="fricas")
[Out] -x - sqrt(2*x^2 + 1) + arctan(x) - arctan(-(x^2 - sqrt(2*x^2 + 1) + 1)/x^2)
```

Sympy [F]

$$\int \frac{x}{x-\sqrt{1+2x^2}}\,dx=\int \frac{x}{x-\sqrt{2x^2+1}}\,dx$$

```
[In] integrate(x/(x-(2*x**2+1)**(1/2)),x)
[Out] Integral(x/(x - sqrt(2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{x}{x-\sqrt{1+2x^2}}\,dx=\int \frac{x}{x-\sqrt{2\,x^2+1}}\,dx$$

```
[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(x/(x - sqrt(2*x^2 + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(27) = 54$.

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

$$\int \frac{x}{x-\sqrt{1+2x^2}}\,dx=-\frac{1}{2}\,\pi-x-\sqrt{2\,x^2+1}+\arctan\left(x\right)+\arctan\left(-\frac{\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)^2+1}{2\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)}\right)$$

```
[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="giac")
[Out] -1/2*pi - x - sqrt(2*x^2 + 1) + arctan(x) + arctan(-1/2*((sqrt(2)*x - sqrt(
2*x^2 + 1))^2 + 1)/(sqrt(2)*x - sqrt(2*x^2 + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.06

$$\int \frac{x}{x-\sqrt{1+2x^2}}\,dx=-x-\sqrt{2}\sqrt{x^2+\frac{1}{2}}-\ln\left(x-\mathrm{i}\right)\,1\mathrm{i}+\frac{\ln\left(x-\frac{\sqrt{2}\sqrt{x^2+\frac{1}{2}}}{2}+\frac{1}{2}\mathrm{i}\right)\,1\mathrm{i}}{2}+\frac{\ln\left(x+\frac{\sqrt{2}\sqrt{x^2+\frac{1}{2}}}{2}-\frac{1}{2}\mathrm{i}\right)\,1\mathrm{i}}{2}$$

```
[In] int(x/(x - (2*x^2 + 1)^(1/2)),x)
```

```
[Out] (log(x - (2^(1/2)*(x^2 + 1/2)^(1/2))/2 + 1i/2)*1i)/2 - log(x - 1i)*1i - x +
 (log(x + (2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1i/2)*1i)/2 - 2^(1/2)*(x^2 + 1/2)
^(1/2)
```

3.970 $\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 82

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx = \frac{5}{32}(1+2\sqrt{x})\sqrt{\sqrt{x}+x}-\frac{5}{12}(\sqrt{x}+x)^{3/2}+\frac{1}{2}\sqrt{x}(\sqrt{x}+x)^{3/2}-\frac{5}{32}\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$

```
[Out] -5/32*arctanh(x^(1/2)/(x+x^(1/2))^(1/2))-5/12*(x+x^(1/2))^(3/2)+1/2*x^(1/2)
*(x+x^(1/2))^(3/2)+5/32*(1+2*x^(1/2))*(x+x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2043, 684, 654, 626, 634, 212}

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx = -\frac{5}{32}\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)+\frac{1}{2}\sqrt{x}(x+\sqrt{x})^{3/2}-\frac{5}{12}(x+\sqrt{x})^{3/2}+\frac{5}{32}(2\sqrt{x}+1)\sqrt{x+\sqrt{x}}$$

```
[In] Int[Sqrt[x]*Sqrt[Sqrt[x] + x],x]
```

```
[Out] (5*(1 + 2*Sqrt[x])*Sqrt[Sqrt[x] + x])/32 - (5*(Sqrt[x] + x)^(3/2))/12 + (Sq
rt[x]*(Sqrt[x] + x)^(3/2))/2 - (5*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]])/32
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^2\sqrt{x+x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} - \frac{5}{4}\text{Subst}\left(\int x\sqrt{x+x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{5}{12}\left(\sqrt{x}+x\right)^{3/2} + \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} + \frac{5}{8}\text{Subst}\left(\int \sqrt{x+x^2}\,dx, x, \sqrt{x}\right)
\end{aligned}
$$

$$= \frac{5}{32}(1+2\sqrt{x})\sqrt{\sqrt{x}+x} - \frac{5}{12}(\sqrt{x}+x)^{3/2} + \frac{1}{2}\sqrt{x}(\sqrt{x}+x)^{3/2} - \frac{5}{64}\text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,dx, x, \sqrt{x}\right)$$

$$= \frac{5}{32}(1+2\sqrt{x})\sqrt{\sqrt{x}+x} - \frac{5}{12}(\sqrt{x}+x)^{3/2} + \frac{1}{2}\sqrt{x}(\sqrt{x}+x)^{3/2} - \frac{5}{32}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$

$$= \frac{5}{32}(1+2\sqrt{x})\sqrt{\sqrt{x}+x} - \frac{5}{12}(\sqrt{x}+x)^{3/2} + \frac{1}{2}\sqrt{x}(\sqrt{x}+x)^{3/2} - \frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx = \frac{1}{96}\sqrt{\sqrt{x}+x}(15-10\sqrt{x}+8x+48x^{3/2}) - \frac{5}{32}\text{arctanh}\left(\frac{\sqrt{\sqrt{x}+x}}{\sqrt{x}}\right)$$

```
[In] Integrate[Sqrt[x]*Sqrt[Sqrt[x] + x],x]
[Out] (Sqrt[Sqrt[x] + x]*(15 - 10*Sqrt[x] + 8*x + 48*x^(3/2)))/96 - (5*ArcTanh[Sq
rt[Sqrt[x] + x]/Sqrt[x]])/32
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.56

method	result	size
meijerg	$-\frac{-\frac{\sqrt{\pi}\,x^{\frac{1}{4}}\left(336x^{\frac{3}{2}}+56x-70\sqrt{x}+105\right)\sqrt{1+\sqrt{x}}}{672}+\frac{5\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{1}{4}}\right)}{32}}{\sqrt{\pi}}$	46
derivativedivides	$\frac{\sqrt{x}\,(x+\sqrt{x})^{\frac{3}{2}}}{2}-\frac{5(x+\sqrt{x})^{\frac{3}{2}}}{12}+\frac{5(1+2\sqrt{x})\sqrt{x+\sqrt{x}}}{32}-\frac{5\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)}{64}$	54
default	$\frac{\sqrt{x}\,(x+\sqrt{x})^{\frac{3}{2}}}{2}-\frac{5(x+\sqrt{x})^{\frac{3}{2}}}{12}+\frac{5(1+2\sqrt{x})\sqrt{x+\sqrt{x}}}{32}-\frac{5\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)}{64}$	54

```
[In] int(x^(1/2)*(x+x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/Pi^(1/2)*(-1/672*Pi^(1/2)*x^(1/4)*(336*x^(3/2)+56*x-70*x^(1/2)+105)*(1+x
^(1/2))^(1/2)+5/32*Pi^(1/2)*arcsinh(x^(1/4)))
```

Fricas [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx = \frac{1}{96}\left(2\left(24\,x-5\right)\sqrt{x}+8\,x+15\right)\sqrt{x+\sqrt{x}} + \frac{5}{128}\,\log\left(4\,\sqrt{x+\sqrt{x}}(2\,\sqrt{x}+1)-8\,x-8\,\sqrt{x}-1\right)$$

```
[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/96*(2*(24*x - 5)*sqrt(x) + 8*x + 15)*sqrt(x + sqrt(x)) + 5/128*log(4*sqrt
(x + sqrt(x))*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.71

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx = 2\sqrt{\sqrt{x}+x}\left(\frac{x^{\frac{3}{2}}}{4}-\frac{5\sqrt{x}}{96}+\frac{x}{24}+\frac{5}{64}\right)-\frac{5\log\left(2\sqrt{x}+2\sqrt{\sqrt{x}+x}+1\right)}{64}$$

```
[In] integrate(x**(1/2)*(x+x**(1/2))**(1/2),x)
[Out] 2*sqrt(sqrt(x) + x)*(x**(3/2)/4 - 5*sqrt(x)/96 + x/24 + 5/64) - 5*log(2*sqr
t(x) + 2*sqrt(sqrt(x) + x) + 1)/64
```

Maxima [F]

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx = \int \sqrt{x+\sqrt{x}}\sqrt{x}\,dx$$

```
[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(x))*sqrt(x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.61

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx = \frac{1}{96}\left(2\left(4\sqrt{x}(6\sqrt{x}+1)-5\right)\sqrt{x}+15\right)\sqrt{x+\sqrt{x}} + \frac{5}{64}\log\left(-2\sqrt{x+\sqrt{x}}+2\sqrt{x}+1\right)$$

```
[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="giac")
[Out] 1/96*(2*(4*sqrt(x)*(6*sqrt(x) + 1) - 5)*sqrt(x) + 15)*sqrt(x + sqrt(x)) + 5
/64*log(-2*sqrt(x + sqrt(x)) + 2*sqrt(x) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx = \int \sqrt{x}\,\sqrt{x+\sqrt{x}}\,dx$$

```
[In] int(x^(1/2)*(x + x^(1/2))^(1/2),x)
[Out] int(x^(1/2)*(x + x^(1/2))^(1/2), x)
```

3.971 $\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}} \, dx = -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5} - 2\sqrt{3}\arctan\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right) - 4\log\left(1+\sqrt[6]{x}\right) - \log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)$$

```
[Out] -3*x^(1/3)+6/5*x^(5/6)-4*ln(1+x^(1/6))-ln(1-x^(1/6)+x^(1/3))-2*arctan(1/3*(
1-2*x^(1/6))*3^(1/2))*3^(1/2)+2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1607, 1901, 1888, 31, 648, 632, 210, 642}

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}} \, dx = -2\sqrt{3}\arctan\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)+\frac{6x^{5/6}}{5}+2\sqrt{x} - 3\sqrt[3]{x} - 4\log\left(\sqrt[6]{x}+1\right) - \log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)$$

```
[In] Int[(1 + x^(1/3))/(1 + Sqrt[x]),x]
```

```
[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/S
qrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1888

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[q*((A - B*q
+ C*q^2)/(3*a)), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 6\text{Subst}\left(\int \frac{x^5+x^7}{1+x^3}\,dx, x, \sqrt[6]{x}\right)\\
&= 6\text{Subst}\left(\int \frac{x^5(1+x^2)}{1+x^3}\,dx, x, \sqrt[6]{x}\right)\\
&= 6\text{Subst}\left(\int \left(-x+x^2+x^4+\frac{(1-x)x}{1+x^3}\right)\,dx, x, \sqrt[6]{x}\right)\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}+6\text{Subst}\left(\int \frac{(1-x)x}{1+x^3}\,dx, x, \sqrt[6]{x}\right)\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}+2\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, \sqrt[6]{x}\right)-4\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[6]{x}\right)\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-4\log\left(1+\sqrt[6]{x}\right)\\
&\quad+3\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[6]{x}\right)-\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}\\
&\quad-4\log\left(1+\sqrt[6]{x}\right)-\log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)-6\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[6]{x}\right)\\
&= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)-4\log\left(1+\sqrt[6]{x}\right)-\log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$
\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx = -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-2\sqrt{3}\arctan\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)-4\log\left(1+\sqrt[6]{x}\right)-\log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)
$$

```
[In] Integrate[(1 + x^(1/3))/(1 + Sqrt[x]),x]
[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/S
qrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{6x^{\frac{5}{6}}}{5}+2\sqrt{x}-3x^{\frac{1}{3}}-4\ln\left(1+x^{\frac{1}{6}}\right)-\ln\left(1-x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)+2\sqrt{3}\arctan\left(\frac{\left(2x^{\frac{1}{6}}-1\right)\sqrt{3}}{3}\right)$
default	$\frac{6x^{\frac{5}{6}}}{5}+2\sqrt{x}-3x^{\frac{1}{3}}-4\ln\left(1+x^{\frac{1}{6}}\right)-\ln\left(1-x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)+2\sqrt{3}\arctan\left(\frac{\left(2x^{\frac{1}{6}}-1\right)\sqrt{3}}{3}\right)$
meijerg	$2\sqrt{x}-2\ln\left(1+\sqrt{x}\right)-\frac{3x^{\frac{1}{3}}\left(-8\sqrt{x}+20\right)}{20}+2x^{\frac{1}{3}}\left(-\frac{\ln\left(1+x^{\frac{1}{6}}\right)}{x^{\frac{1}{3}}}+\frac{\ln\left(1-x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)}{2x^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\,x}{2-x}\right.}{x^{\frac{1}{3}}}\right.$

```
[In] int((1+x^(1/3))/(1+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 6/5*x^(5/6)+2*x^(1/2)-3*x^(1/3)-4*ln(1+x^(1/6))-ln(1-x^(1/6)+x^(1/3))+2*3^(
1/2)*arctan(1/3*(2*x^(1/6)-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx = 2\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x} - 3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

```
[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/6) - 1/3*sqrt(3)) + 6/5*x^(5/6) + 2*sqrt(
x) - 3*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.09

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx = \frac{16x^{\frac{5}{6}}\Gamma\left(\frac{8}{3}\right)}{5\Gamma\left(\frac{11}{3}\right)} - \frac{8\sqrt[3]{x}\Gamma\left(\frac{8}{3}\right)}{\Gamma\left(\frac{11}{3}\right)} + 2\sqrt{x} - 2\log\left(\sqrt{x}+1\right) - \frac{16e^{-\frac{2i\pi}{3}}\log\left(-\sqrt[6]{x}e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)} - \frac{16\log\left(-\sqrt[6]{x}e^{i\pi}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)} - \frac{16e^{\frac{2i\pi}{3}}\log\left(-\sqrt[6]{x}e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

```
[In] integrate((1+x**(1/3))/(1+x**(1/2)),x)
[Out] 16*x**(5/6)*gamma(8/3)/(5*gamma(11/3)) - 8*x**(1/3)*gamma(8/3)/gamma(11/3)
+ 2*sqrt(x) - 2*log(sqrt(x) + 1) - 16*exp(-2*I*pi/3)*log(-x**(1/6)*exp_pola
r(I*pi/3) + 1)*gamma(8/3)/(3*gamma(11/3)) - 16*log(-x**(1/6)*exp_polar(I*pi
) + 1)*gamma(8/3)/(3*gamma(11/3)) - 16*exp(2*I*pi/3)*log(-x**(1/6)*exp_pola
r(5*I*pi/3) + 1)*gamma(8/3)/(3*gamma(11/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx = 2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right) + \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} - 3x^{\frac{1}{3}} - \log\left(x^{\frac{1}{3}} - x^{\frac{1}{6}} + 1\right) - 4\log\left(x^{\frac{1}{6}}+1\right)$$

```
[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*sqrt(x) - 3
*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx = 2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right) + \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} - 3x^{\frac{1}{3}} - \log\left(x^{\frac{1}{3}} - x^{\frac{1}{6}} + 1\right) - 4\log\left(x^{\frac{1}{6}}+1\right)$$

```
[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*sqrt(x) - 3
*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

Mupad [B] (verification not implemented)

Time = 20.54 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx = 2\sqrt{x} + \ln\left(\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(27+\sqrt{3}\,9\mathrm{i}\right) + 36\,x^{1/6} + 36\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right) - \ln\left(\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(-27+\sqrt{3}\,9\mathrm{i}\right) + 36\,x^{1/6} + 36\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right) - 4\,\ln\left(36\,x^{1/6}+36\right) - 3\,x^{1/3} + \frac{6\,x^{5/6}}{5}$$

```
[In] int((x^(1/3) + 1)/(x^(1/2) + 1),x)
```

```
[Out] log((3^(1/2)*1i - 1)*(3^(1/2)*9i + 27) + 36*x^(1/6) + 36)*(3^(1/2)*1i - 1)
- 4*log(36*x^(1/6) + 36) - log((3^(1/2)*1i + 1)*(3^(1/2)*9i - 27) + 36*x^(1
/6) + 36)*(3^(1/2)*1i + 1) + 2*x^(1/2) - 3*x^(1/3) + (6*x^(5/6))/5
```

3.972 $\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 115

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx=12\sqrt[12]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}-2\sqrt{x}+\frac{12x^{7/12}}{7}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{13/12}}{13}+4\sqrt{3}\arctan\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)-8\log\left(1+\sqrt[12]{x}\right)-2\log\left(1-\sqrt[12]{x}+\sqrt[6]{x}\right)$$

```
[Out] 12*x^(1/12)+4*x^(1/4)-3*x^(1/3)+12/7*x^(7/12)+4/3*x^(3/4)-6/5*x^(5/6)+12/13
*x^(13/12)-8*ln(1+x^(1/12))-2*ln(1-x^(1/12)+x^(1/6))+4*arctan(1/3*(1-2*x^(1
/12))*3^(1/2))*3^(1/2)-2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1607, 1850, 1901, 1888, 31, 648, 632, 210, 642}

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx=4\sqrt{3}\arctan\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)+\frac{12x^{13/12}}{13}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}+\frac{12x^{7/12}}{7}-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}+12\sqrt[12]{x}-8\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[6]{x}-\sqrt[12]{x}+1\right)$$

```
[In] Int[(1 + x^(1/3))/(1 + x^(1/4)),x]
```

```
[Out] 12*x^(1/12) + 4*x^(1/4) - 3*x^(1/3) - 2*Sqrt[x] + (12*x^(7/12))/7 + (4*x^(3
/4))/3 - (6*x^(5/6))/5 + (12*x^(13/12))/13 + 4*Sqrt[3]*ArcTan[(1 - 2*x^(1/1
2))/Sqrt[3]] - 8*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12) + x^(1/6)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1888

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[q*((A - B*q
+ C*q^2)/(3*a)), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 12\text{Subst}\left(\int \frac{x^{11}+x^{15}}{1+x^3}\,dx, x, \sqrt[12]{x}\right)\\
&= 12\text{Subst}\left(\int \frac{x^{11}(1+x^4)}{1+x^3}\,dx, x, \sqrt[12]{x}\right)\\
&= \frac{12x^{13/12}}{13} + \frac{12}{13}\text{Subst}\left(\int \frac{(13-13x)x^{11}}{1+x^3}\,dx, x, \sqrt[12]{x}\right)\\
&= \frac{12x^{13/12}}{13}\\
&\quad + \frac{12}{13}\text{Subst}\left(\int \left(13+13x^2-13x^3-13x^5+13x^6+13x^8-13x^9-\frac{13(1+x^2)}{1+x^3}\right)\,dx, x, \sqrt[12]{x}\right)\\
&= 12\sqrt[12]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}-2\sqrt{x}+\frac{12x^{7/12}}{7}+\frac{4x^{3/4}}{3}\\
&\quad -\frac{6x^{5/6}}{5}+\frac{12x^{13/12}}{13}-12\text{Subst}\left(\int \frac{1+x^2}{1+x^3}\,dx, x, \sqrt[12]{x}\right)\\
&= 12\sqrt[12]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}-2\sqrt{x}+\frac{12x^{7/12}}{7}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{13/12}}{13}\\
&\quad -4\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \sqrt[12]{x}\right)-8\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[12]{x}\right)\\
&= 12\sqrt[12]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}-2\sqrt{x}+\frac{12x^{7/12}}{7}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{13/12}}{13}\\
&\quad -8\log\left(1+\sqrt[12]{x}\right)-2\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[12]{x}\right)-6\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[12]{x}\right)\\
&= 12\sqrt[12]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}-2\sqrt{x}+\frac{12x^{7/12}}{7}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{13/12}}{13}\\
&\quad -8\log\left(1+\sqrt[12]{x}\right)-2\log\left(1-\sqrt[12]{x}+\sqrt[6]{x}\right)+12\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[12]{x}\right)
\end{aligned}
$$

$$= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} + 4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right) - 8\log\left(1+\sqrt[12]{x}\right) - 2\log\left(1-\sqrt[12]{x}+\sqrt[6]{x}\right)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx = \frac{16380\sqrt[12]{x} + 5460\sqrt[4]{x} - 4095\sqrt[3]{x} - 2730\sqrt{x} + 2340x^{7/12} + 1820x^{3/4} - 1638x^{5/6} + 1260x^{13/12}}{1365} + 4\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[12]{x}}{\sqrt{3}}\right) - 8\log\left(1+\sqrt[12]{x}\right) - 2\log\left(1-\sqrt[12]{x}+\sqrt[6]{x}\right)$$

```
[In] Integrate[(1 + x^(1/3))/(1 + x^(1/4)),x]
[Out] (16380*x^(1/12) + 5460*x^(1/4) - 4095*x^(1/3) - 2730*Sqrt[x] + 2340*x^(7/12
) + 1820*x^(3/4) - 1638*x^(5/6) + 1260*x^(13/12))/1365 + 4*Sqrt[3]*ArcTan[1
/Sqrt[3] - (2*x^(1/12))/Sqrt[3]] - 8*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12)
 + x^(1/6)]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{12x^{\frac{13}{12}}}{13} - \frac{6x^{\frac{5}{6}}}{5} + \frac{4x^{\frac{3}{4}}}{3} + \frac{12x^{\frac{7}{12}}}{7} - 2\sqrt{x} - 3x^{\frac{1}{3}} + 4x^{\frac{1}{4}} + 12x^{\frac{1}{12}} - 8\ln\left(1+x^{\frac{1}{12}}\right) - 2\ln\left(1-x\right.$
default	$\frac{12x^{\frac{13}{12}}}{13} - \frac{6x^{\frac{5}{6}}}{5} + \frac{4x^{\frac{3}{4}}}{3} + \frac{12x^{\frac{7}{12}}}{7} - 2\sqrt{x} - 3x^{\frac{1}{3}} + 4x^{\frac{1}{4}} + 12x^{\frac{1}{12}} - 8\ln\left(1+x^{\frac{1}{12}}\right) - 2\ln\left(1-x\right.$
meijerg	$\frac{x^{\frac{1}{4}}\left(4\sqrt{x}-6x^{\frac{1}{4}}+12\right)}{3} - 4\ln\left(1+x^{\frac{1}{4}}\right) + \frac{3x^{\frac{1}{12}}\left(560x-728x^{\frac{3}{4}}+1040\sqrt{x}-1820x^{\frac{1}{4}}+7280\right)}{1820} - 4x^{\frac{1}{12}}\left(\frac{\ln\left(1+x^{1}\right.}{x^{\frac{1}{12}}}\right.$

```
[In] int((1+x^(1/3))/(1+x^(1/4)),x,method=_RETURNVERBOSE)
[Out] 12/13*x^(13/12)-6/5*x^(5/6)+4/3*x^(3/4)+12/7*x^(7/12)-2*x^(1/2)-3*x^(1/3)+4
*x^(1/4)+12*x^(1/12)-8*ln(1+x^(1/12))-2*ln(1-x^(1/12)+x^(1/6))-4*3^(1/2)*ar
ctan(1/3*(2*x^(1/12)-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx = -4\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{12}} - \frac{1}{3}\sqrt{3}\right) + \frac{12}{13}(x+13)x^{\frac{1}{12}} - \frac{6}{5}x^{\frac{5}{6}} + \frac{4}{3}x^{\frac{3}{4}} + \frac{12}{7}x^{\frac{7}{12}} - 2\sqrt{x} - 3x^{\frac{1}{3}} + 4x^{\frac{1}{4}} - 2\log\left(x^{\frac{1}{6}} - x^{\frac{1}{12}} + 1\right) - 8\log\left(x^{\frac{1}{12}} + 1\right)$$

```
[In] integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="fricas")
[Out] -4*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/12) - 1/3*sqrt(3)) + 12/13*(x + 13)*x^(1
/12) - 6/5*x^(5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) +
4*x^(1/4) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.91 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.92

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx = \frac{64x^{\frac{13}{12}}\Gamma\left(\frac{16}{3}\right)}{13\Gamma\left(\frac{19}{3}\right)} + \frac{64x^{\frac{7}{12}}\Gamma\left(\frac{16}{3}\right)}{7\Gamma\left(\frac{19}{3}\right)} + \frac{64\sqrt[12]{x}\Gamma\left(\frac{16}{3}\right)}{\Gamma\left(\frac{19}{3}\right)} - \frac{32x^{\frac{5}{6}}\Gamma\left(\frac{16}{3}\right)}{5\Gamma\left(\frac{19}{3}\right)} + \frac{4x^{\frac{3}{4}}}{3} + 4\sqrt[4]{x} - \frac{16\sqrt[3]{x}\Gamma\left(\frac{16}{3}\right)}{\Gamma\left(\frac{19}{3}\right)} - 2\sqrt{x} - 4\log\left(\sqrt[4]{x}+1\right) + \frac{64e^{-\frac{i\pi}{3}}\log\left(-\sqrt[12]{x}e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{16}{3}\right)}{3\Gamma\left(\frac{19}{3}\right)} - \frac{64\log\left(-\sqrt[12]{x}e^{i\pi}+1\right)\Gamma\left(\frac{16}{3}\right)}{3\Gamma\left(\frac{19}{3}\right)} + \frac{64e^{\frac{i\pi}{3}}\log\left(-\sqrt[12]{x}e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{16}{3}\right)}{3\Gamma\left(\frac{19}{3}\right)}$$

```
[In] integrate((1+x**(1/3))/(1+x**(1/4)),x)
[Out] 64*x**(13/12)*gamma(16/3)/(13*gamma(19/3)) + 64*x**(7/12)*gamma(16/3)/(7*ga
mma(19/3)) + 64*x**(1/12)*gamma(16/3)/gamma(19/3) - 32*x**(5/6)*gamma(16/3)
/(5*gamma(19/3)) + 4*x**(3/4)/3 + 4*x**(1/4) - 16*x**(1/3)*gamma(16/3)/gamm
a(19/3) - 2*sqrt(x) - 4*log(x**(1/4) + 1) + 64*exp(-I*pi/3)*log(-x**(1/12)*
exp_polar(I*pi/3) + 1)*gamma(16/3)/(3*gamma(19/3)) - 64*log(-x**(1/12)*exp_
polar(I*pi) + 1)*gamma(16/3)/(3*gamma(19/3)) + 64*exp(I*pi/3)*log(-x**(1/12
)*exp_polar(5*I*pi/3) + 1)*gamma(16/3)/(3*gamma(19/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx = -4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}} - 2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-8\log\left(x^{\frac{1}{12}}+1\right)$$

```
[In] integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="maxima")
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*x^(13/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) + 4*x^(1/4) + 12
*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx = -4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}} - 2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-8\log\left(x^{\frac{1}{12}}+1\right)$$

```
[In] integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="giac")
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*x^(13/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) + 4*x^(1/4) + 12
*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.13

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx = 4x^{1/4} + \ln\left(\left(-2+\sqrt{3}\,2i\right)\left(54-36x^{1/12}+\sqrt{3}\,18i\right)-144x^{1/12}+144\right)\left(-2+\sqrt{3}\,2i\right)-\ln\left(\left(2+\sqrt{3}\,2i\right)\left(36x^{1/12}-\right.\right.$$

```
[In] int((x^(1/3) + 1)/(x^(1/4) + 1),x)
[Out] log((3^(1/2)*2i - 2)*(3^(1/2)*18i - 36*x^(1/12) + 54) - 144*x^(1/12) + 144)
*(3^(1/2)*2i - 2) - 8*log(144*x^(1/12) + 144) - log((3^(1/2)*2i + 2)*(3^(1/
2)*18i + 36*x^(1/12) - 54) - 144*x^(1/12) + 144)*(3^(1/2)*2i + 2) - 2*x^(1/
2) - 3*x^(1/3) + 4*x^(1/4) + (4*x^(3/4))/3 - (6*x^(5/6))/5 + 12*x^(1/12) +
(12*x^(7/12))/7 + (12*x^(13/12))/13
```

3.973 $\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 4

$$\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx = x+\arcsin(x)$$

```
[Out] x+arcsin(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2187, 8, 222}

$$\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx = \arcsin(x)+x$$

```
[In] Int[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]
```

```
[Out] x + ArcSin[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int -1\,dx + \int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= x + \sin^{-1}(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 22 vs. 2(4) = 8.

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 5.50

$$\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx = x + 2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

```
[In] Integrate[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]
[Out] x + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

method	result	size
trager	$x+\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)$	29
default	$x+\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}+\operatorname{arctanh}(x)+\frac{\sqrt{-(x+1)^2+2x+2}}{2}+\arcsin(x)-\frac{\sqrt{-(x-1)^2-2x+2}}{2}$	51

```
[In] int(x^2/(-1+x^2+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] x+RootOf(_Z^2+1)*ln(RootOf(_Z^2+1)*(-x^2+1)^(1/2)+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(4) = 8$.

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 5.00

$$\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx = x - 2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="fricas")
[Out] x - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [F]

$$\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx = \int \frac{x^2}{x^2+\sqrt{1-x^2}-1}\,dx$$

```
[In] integrate(x**2/(-1+x**2+(-x**2+1)**(1/2)),x)
[Out] Integral(x**2/(x**2 + sqrt(1 - x**2) - 1), x)
```

Maxima [F]

$$\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx = \int \frac{x^2}{x^2+\sqrt{-x^2+1}-1}\,dx$$

```
[In] integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(x^2/(x^2 + sqrt(-x^2 + 1) - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx = x + \arcsin(x)$$

```
[In] integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="giac")
[Out] x + arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx = x + \operatorname{asin}(x)$$

```
[In] int(x^2/(x^2 + (1 - x^2)^(1/2) - 1),x)
[Out] x + asin(x)
```

3.974 $\int \sqrt{\frac{1+x}{x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \sqrt{\frac{1+x}{x}}\,dx = \sqrt{1+\frac{1}{x}}x + \operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\right)$$

```
[Out] arctanh((1+1/x)^(1/2))+x*(1+1/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1997, 248, 43, 65, 213}

$$\int \sqrt{\frac{1+x}{x}}\,dx = \operatorname{arctanh}\left(\sqrt{\frac{1}{x}+1}\right) + \sqrt{\frac{1}{x}+1}x$$

```
[In] Int[Sqrt[(1 + x)/x],x]
```

```
[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \sqrt{1+\frac{1}{x}}\,dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{1+\frac{1}{x}}x - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{1+\frac{1}{x}}x - \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\right) \\
&= \sqrt{1+\frac{1}{x}}x + \tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt{\frac{1+x}{x}}\,dx = \sqrt{1+\frac{1}{x}}x + \operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\right)$$

```
[In] Integrate[Sqrt[(1 + x)/x],x]
[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(18) = 36.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

method	result	size
trager	$x\sqrt{-\frac{-x-1}{x}}-\frac{\ln\left(2x\sqrt{-\frac{-x-1}{x}}-2x-1\right)}{2}$	39
default	$\frac{\sqrt{\frac{x+1}{x}}\,x\left(2\sqrt{x^{2}+x}+\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)\right)}{2\sqrt{(x+1)x}}$	41
risch	$x\sqrt{\frac{x+1}{x}}+\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)\sqrt{\frac{x+1}{x}}\,\sqrt{(x+1)x}}{2x+2}$	47

```
[In] int(((x+1)/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] x*(-(-x-1)/x)^(1/2)-1/2*ln(2*x*(-(-x-1)/x)^(1/2)-2*x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(18) = 36.

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \sqrt{\frac{1+x}{x}}\,dx = x\sqrt{\frac{x+1}{x}} + \frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

```
[In] integrate(((1+x)/x)^(1/2),x, algorithm="fricas")
[Out] x*sqrt((x + 1)/x) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sqrt((x + 1)/x)
- 1)
```

Sympy [F]

$$\int \sqrt{\frac{1+x}{x}}\,dx = \int \sqrt{\frac{x+1}{x}}\,dx$$

```
[In] integrate(((1+x)/x)**(1/2),x)
[Out] Integral(sqrt((x + 1)/x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \sqrt{\frac{1+x}{x}}\,dx = \frac{\sqrt{\frac{x+1}{x}}}{\frac{x+1}{x}-1} + \frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

```
[In] integrate(((1+x)/x)^(1/2),x, algorithm="maxima")
[Out] sqrt((x + 1)/x)/((x + 1)/x - 1) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sq
rt((x + 1)/x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \sqrt{\frac{1+x}{x}}\,dx = -\frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x) + \sqrt{x^2+x}\operatorname{sgn}(x)$$

```
[In] integrate(((1+x)/x)^(1/2),x, algorithm="giac")
[Out] -1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + sqrt(x^2 + x)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 21.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sqrt{\frac{1+x}{x}}\,dx = \operatorname{atanh}\left(\sqrt{\frac{1}{x}+1}\right) + x\sqrt{\frac{1}{x}+1}$$

```
[In] int(((x + 1)/x)^(1/2),x)
```

```
[Out] atanh((1/x + 1)^(1/2)) + x*(1/x + 1)^(1/2)
```

3.975 $\int \sqrt{\frac{1-x}{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \sqrt{\frac{1-x}{x}}\,dx = \sqrt{-1+\frac{1}{x}}x - \arctan\left(\sqrt{-1+\frac{1}{x}}\right)$$

```
[Out] -arctan((-1+1/x)^(1/2))+x*(-1+1/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1997, 248, 43, 65, 209}

$$\int \sqrt{\frac{1-x}{x}}\,dx = \sqrt{\frac{1}{x}-1}x - \arctan\left(\sqrt{\frac{1}{x}-1}\right)$$

```
[In] Int[Sqrt[(1 - x)/x],x]
```

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sqrt{-1+\frac{1}{x}}\,dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{-1+\frac{1}{x}}x - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{-1+\frac{1}{x}}x - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\frac{1}{x}}\right) \\
&= \sqrt{-1+\frac{1}{x}}x - \tan^{-1}\left(\sqrt{-1+\frac{1}{x}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sqrt{\frac{1-x}{x}}\,dx = \sqrt{-1+\frac{1}{x}}x - \arctan\left(\sqrt{-1+\frac{1}{x}}\right)$$

```
[In] Integrate[Sqrt[(1 - x)/x],x]
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

method	result	size
default	$\frac{\sqrt{-\frac{x-1}{x}}\,x\left(2\sqrt{-x^{2}+x}+\arcsin\left(2x-1\right)\right)}{2\sqrt{-\left(x-1\right)x}}$	40
risch	$\sqrt{-\frac{x-1}{x}}\,x-\frac{\arcsin\left(2x-1\right)\sqrt{-\frac{x-1}{x}}\,\sqrt{-\left(x-1\right)x}}{2\left(x-1\right)}$	45
trager	$\sqrt{-\frac{x-1}{x}}\,x+\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(2\sqrt{-\frac{x-1}{x}}\,x-2\operatorname{RootOf}\left(\_Z^{2}+1\right)x+\operatorname{RootOf}\left(\_Z^{2}+1\right)\right)}{2}$	52

```
[In] int(((1-x)/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(-(x-1)/x)^(1/2)*x*(2*(-x^2+x)^(1/2)+arcsin(2*x-1))/(-(x-1)*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \sqrt{\frac{1-x}{x}}\,dx = x\sqrt{-\frac{x-1}{x}} - \arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="fricas")
[Out] x*sqrt(-(x - 1)/x) - arctan(sqrt(-(x - 1)/x))
```

Sympy [F]

$$\int \sqrt{\frac{1-x}{x}}\,dx = \int \sqrt{\frac{1-x}{x}}\,dx$$

```
[In] integrate(((1-x)/x)**(1/2),x)
[Out] Integral(sqrt((1 - x)/x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \sqrt{\frac{1-x}{x}}\,dx = -\frac{\sqrt{-\frac{x-1}{x}}}{\frac{x-1}{x}-1} - \arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-(x - 1)/x)/((x - 1)/x - 1) - arctan(sqrt(-(x - 1)/x))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \sqrt{\frac{1-x}{x}}\,dx = \frac{1}{4}\,\pi \mathrm{sgn}(x) + \frac{1}{2}\,\arcsin\left(2\,x-1\right)\mathrm{sgn}(x) + \sqrt{-x^2+x}\mathrm{sgn}(x)$$

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="giac")
[Out] 1/4*pi*sgn(x) + 1/2*arcsin(2*x - 1)*sgn(x) + sqrt(-x^2 + x)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 20.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \sqrt{\frac{1-x}{x}}\,dx = x\sqrt{\frac{1}{x}-1} - \mathrm{atan}\left(\sqrt{\frac{1}{x}-1}\right)$$

```
[In] int((-(x - 1)/x)^(1/2),x)
[Out] x*(1/x - 1)^(1/2) - atan((1/x - 1)^(1/2))
```

3.976 $\int \sqrt{\frac{-1+x}{x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \sqrt{\frac{-1+x}{x}}\,dx = \sqrt{-1+x}\sqrt{x} - \operatorname{arcsinh}\left(\sqrt{-1+x}\right)$$

```
[Out] -arcsinh((-1+x)^(1/2))+(-1+x)^(1/2)*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1997, 248, 43, 65, 212}

$$\int \sqrt{\frac{-1+x}{x}}\,dx = \sqrt{\frac{x-1}{x}}x - \operatorname{arctanh}\left(\sqrt{\frac{x-1}{x}}\right)$$

```
[In] Int[Sqrt[(-1 + x)/x],x]
[Out] Sqrt[(-1 + x)/x]*x - ArcTanh[Sqrt[(-1 + x)/x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \sqrt{1-\frac{1}{x}}\,dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{\frac{-1+x}{x}}x + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{\frac{-1+x}{x}}x - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\frac{-1+x}{x}}\right) \\
&= \sqrt{\frac{-1+x}{x}}x - \tanh^{-1}\left(\sqrt{\frac{-1+x}{x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \sqrt{\frac{-1+x}{x}}\,dx = \sqrt{-1+x}\sqrt{x} - 2\text{arctanh}\left(\frac{\sqrt{-1+x}}{-1+\sqrt{x}}\right)$$

```
[In] Integrate[Sqrt[(-1 + x)/x],x]
[Out] Sqrt[-1 + x]*Sqrt[x] - 2*ArcTanh[Sqrt[-1 + x]/(-1 + Sqrt[x])]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(18) = 36.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

method	result	size
trager	$\sqrt{-\frac{1-x}{x}}\,x+\frac{\ln\left(2\sqrt{-\frac{1-x}{x}}\,x-2x+1\right)}{2}$	39
default	$-\frac{\sqrt{\frac{x-1}{x}}\,x\left(-2\sqrt{x^{2}-x}+\ln\left(x-\frac{1}{2}+\sqrt{x^{2}-x}\right)\right)}{2\sqrt{(x-1)x}}$	45
risch	$x\sqrt{\frac{x-1}{x}}-\frac{\ln\left(x-\frac{1}{2}+\sqrt{x^{2}-x}\right)\sqrt{\frac{x-1}{x}}\,\sqrt{(x-1)x}}{2(x-1)}$	49

```
[In] int(((x-1)/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-(1-x)/x)^(1/2)*x+1/2*ln(2*(-(1-x)/x)^(1/2)*x-2*x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(18) = 36.

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \sqrt{\frac{-1+x}{x}}\,dx = x\sqrt{\frac{x-1}{x}} - \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}-1\right)$$

```
[In] integrate(((-1+x)/x)^(1/2),x, algorithm="fricas")
[Out] x*sqrt((x - 1)/x) - 1/2*log(sqrt((x - 1)/x) + 1) + 1/2*log(sqrt((x - 1)/x)
- 1)
```

Sympy [F]

$$\int \sqrt{\frac{-1+x}{x}}\,dx = \int \sqrt{\frac{x-1}{x}}\,dx$$

```
[In] integrate(((-1+x)/x)**(1/2),x)
[Out] Integral(sqrt((x - 1)/x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(18) = 36$.

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \sqrt{\frac{-1+x}{x}}\,dx = -\frac{\sqrt{\frac{x-1}{x}}}{\frac{x-1}{x}-1} - \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}-1\right)$$

```
[In] integrate(((-1+x)/x)^(1/2),x, algorithm="maxima")
[Out] -sqrt((x - 1)/x)/((x - 1)/x - 1) - 1/2*log(sqrt((x - 1)/x) + 1) + 1/2*log(s
qrt((x - 1)/x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \sqrt{\frac{-1+x}{x}}\,dx = \frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right)\mathrm{sgn}(x) + \sqrt{x^2-x}\mathrm{sgn}(x)$$

```
[In] integrate(((-1+x)/x)^(1/2),x, algorithm="giac")
[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))*sgn(x) + sqrt(x^2 - x)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sqrt{\frac{-1+x}{x}}\,dx = x\sqrt{1-\frac{1}{x}} - \operatorname{atanh}\left(\sqrt{1-\frac{1}{x}}\right)$$

```
[In] int(((x - 1)/x)^(1/2),x)
[Out] x*(1 - 1/x)^(1/2) - atanh((1 - 1/x)^(1/2))
```

3.977 $\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx = -2\sqrt{1+\frac{1}{x}} + 2\operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\right)$$

```
[Out] 2*arctanh((1+1/x)^(1/2))-2*(1+1/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1998, 272, 52, 65, 213}

$$\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx = 2\operatorname{arctanh}\left(\sqrt{\frac{1}{x}+1}\right) - 2\sqrt{\frac{1}{x}+1}$$

```
[In] Int[Sqrt[(1 + x)/x]/x,x]
```

```
[Out] -2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1998

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ[u, x
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1+\frac{1}{x}}}{x}\,dx\\
&= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -2\sqrt{1+\frac{1}{x}} - \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)\\
&= -2\sqrt{1+\frac{1}{x}} - 2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\right)\\
&= -2\sqrt{1+\frac{1}{x}} + 2\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx = -2\sqrt{1+\frac{1}{x}} + 2\text{arctanh}\left(\sqrt{1+\frac{1}{x}}\right)$$

```
[In] Integrate[Sqrt[(1 + x)/x]/x,x]
[Out] -2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

method	result	size
trager	$-2\sqrt{-\frac{-x-1}{x}} - \ln\left(2x\sqrt{-\frac{-x-1}{x}} - 2x - 1\right)$	39
risch	$-2\sqrt{\frac{x+1}{x}} + \frac{\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\sqrt{\frac{x+1}{x}}\,\sqrt{(x+1)x}}{x+1}$	46
default	$-\frac{\sqrt{\frac{x+1}{x}}\left(2(x^2+x)^{\frac{3}{2}}-2\sqrt{x^2+x}\,x^2-\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)x^2\right)}{x\sqrt{(x+1)x}}$	60

```
[In] int(((x+1)/x)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] -2*(-(-x-1)/x)^(1/2)-ln(2*x*(-(-x-1)/x)^(1/2)-2*x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx = -2\sqrt{\frac{x+1}{x}} + \log\left(\sqrt{\frac{x+1}{x}}+1\right) - \log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

```
[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="fricas")
[Out] -2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/x) - 1)
```

Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx = -2\sqrt{1+\frac{1}{x}} - \log\left(\sqrt{1+\frac{1}{x}}-1\right) + \log\left(\sqrt{1+\frac{1}{x}}+1\right)$$

```
[In] integrate(((1+x)/x)**(1/2)/x,x)
[Out] -2*sqrt(1 + 1/x) - log(sqrt(1 + 1/x) - 1) + log(sqrt(1 + 1/x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx = -2\sqrt{\frac{x+1}{x}} + \log\left(\sqrt{\frac{x+1}{x}}+1\right) - \log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

```
[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="maxima")
[Out] -2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx = -\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x) + \frac{2\operatorname{sgn}(x)}{x-\sqrt{x^2+x}}$$

```
[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="giac")
[Out] -log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + 2*sgn(x)/(x - sqrt(x^2 + x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx = 2\,\text{atanh}\left(\sqrt{\frac{1}{x}+1}\right) - 2\,\sqrt{\frac{1}{x}+1}$$

```
[In] int(((x + 1)/x)^(1/2)/x,x)
[Out] 2*atanh((1/x + 1)^(1/2)) - 2*(1/x + 1)^(1/2)
```

3.978 $\int \sqrt{\frac{x}{1+x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \sqrt{\frac{x}{1+x}}\,dx = \sqrt{x}\sqrt{1+x} - \operatorname{arcsinh}\left(\sqrt{x}\right)$$

```
[Out] -arcsinh(x^(1/2))+x^(1/2)*(1+x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1978, 52, 56, 221}

$$\int \sqrt{\frac{x}{1+x}}\,dx = \sqrt{x}\sqrt{x+1} - \operatorname{arcsinh}\left(\sqrt{x}\right)$$

```
[In] Int[Sqrt[x/(1 + x)],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\ &= \sqrt{x}\sqrt{1+x} - \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\ &= \sqrt{x}\sqrt{1+x} - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)\\ &= \sqrt{x}\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(22) = 44.

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \sqrt{\frac{x}{1+x}}\,dx = \frac{\sqrt{\frac{x}{1+x}}\left(\sqrt{x}(1+x) + \sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)\right)}{\sqrt{x}}$$

```
[In] Integrate[Sqrt[x/(1 + x)],x]
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) + Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]]
))/Sqrt[x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(16) = 32$.

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

method	result	size
default	$\frac{\sqrt{\frac{x}{x+1}}\,(x+1)\left(2\sqrt{x^{2}+x}-\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)\right)}{2\sqrt{(x+1)x}}$	45
risch	$(x+1)\sqrt{\frac{x}{x+1}}-\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)\sqrt{\frac{x}{x+1}}\,\sqrt{(x+1)x}}{2x}$	47
trager	$2\left(\frac{x}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{x+1}}+\frac{\ln\left(2\sqrt{\frac{x}{x+1}}\,x+2\sqrt{\frac{x}{x+1}}-2x-1\right)}{2}$	49

```
[In] int((x/(x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(x/(x+1))^(1/2)*(x+1)*(2*(x^2+x)^(1/2)-ln(x+1/2+(x^2+x)^(1/2)))/((x+1)*
x)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \sqrt{\frac{x}{1+x}}\,dx = (x+1)\sqrt{\frac{x}{x+1}}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="fricas")
[Out] (x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x
+ 1)) - 1)
```

Sympy [F]

$$\int \sqrt{\frac{x}{1+x}}\,dx = \int \sqrt{\frac{x}{x+1}}\,dx$$

```
[In] integrate((x/(1+x))**(1/2),x)
[Out] Integral(sqrt(x/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(16) = 32$.

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \sqrt{\frac{x}{1+x}}\,dx = -\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1} - \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="maxima")
[Out] -sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(s
qrt(x/(x + 1)) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{x}{1+x}}\,dx = \frac{1}{2}\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1) + \sqrt{x^2+x}\operatorname{sgn}(x+1)$$

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="giac")
[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x +
 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{x}{1+x}}\,dx = -\operatorname{atanh}\left(\sqrt{\frac{x}{x+1}}\right) - \frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

```
[In] int((x/(x + 1))^(1/2),x)
[Out] - atanh((x/(x + 1))^(1/2)) - (x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

3.979 $\int \frac{1}{\sqrt{\frac{-1-x}{x}}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}} \, dx = -x\sqrt{-\frac{1+x}{x}} + \arctan\left(\sqrt{-\frac{1+x}{x}}\right)$$

```
[Out] arctan(((-1-x)/x)^(1/2))-x*((-1-x)/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1997, 248, 44, 65, 210}

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}} \, dx = \arctan\left(\sqrt{-\frac{x+1}{x}}\right) - x\sqrt{-\frac{x+1}{x}}$$

```
[In] Int[1/Sqrt[(-1 - x)/x],x]
```

```
[Out] -(x*Sqrt[-((1 + x)/x)]) + ArcTan[Sqrt[-((1 + x)/x)]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\sqrt{-1-\frac{1}{x}}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}x^2}\,dx, x, \frac{1}{x}\right)\\
&= -x\sqrt{-\frac{1+x}{x}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}x}\,dx, x, \frac{1}{x}\right)\\
&= -x\sqrt{-\frac{1+x}{x}} - \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{-\frac{1+x}{x}}\right)\\
&= -x\sqrt{-\frac{1+x}{x}} + \tan^{-1}\left(\sqrt{-\frac{1+x}{x}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.79

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx = \frac{\sqrt{x}(1+x)+\sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)}{\sqrt{x}\sqrt{-\frac{1+x}{x}}}$$

```
[In] Integrate[1/Sqrt[(-1 - x)/x],x]
[Out] (Sqrt[x]*(1 + x) + Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]])/(Sqrt[x]*Sqrt[-
((1 + x)/x)])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

method	result	size
default	$\frac{(x+1)\left(2\sqrt{-x^{2}-x}+\arcsin(1+2x)\right)}{2\sqrt{-\frac{x+1}{x}}\,\sqrt{-(x+1)x}}$	44
risch	$\frac{x+1}{\sqrt{-\frac{x+1}{x}}}-\frac{\arcsin(1+2x)\sqrt{-(x+1)x}}{2\sqrt{-\frac{x+1}{x}}\,x}$	45
trager	$-\sqrt{-\frac{x+1}{x}}\,x-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(2\sqrt{-\frac{x+1}{x}}\,x-2\operatorname{RootOf}\left(\_Z^{2}+1\right)x-\operatorname{RootOf}\left(\_Z^{2}+1\right)\right)}{2}$	55

```
[In] int(1/((-x-1)/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(x+1)*(2*(-x^2-x)^(1/2)+arcsin(1+2*x))/(-(x+1)/x)^(1/2)/(-(x+1)*x)^(1/2
)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx = -x\sqrt{-\frac{x+1}{x}} + \arctan\left(\sqrt{-\frac{x+1}{x}}\right)$$

```
[In] integrate(1/((-1-x)/x)^(1/2),x, algorithm="fricas")
[Out] -x*sqrt(-(x + 1)/x) + arctan(sqrt(-(x + 1)/x))
```

Sympy [F]

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{-x-1}{x}}}\,dx$$

```
[In] integrate(1/((-1-x)/x)**(1/2),x)
[Out] Integral(1/sqrt((-x - 1)/x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx = -\frac{\sqrt{-\frac{x+1}{x}}}{\frac{x+1}{x}-1} + \arctan\left(\sqrt{-\frac{x+1}{x}}\right)$$

```
[In] integrate(1/((-1-x)/x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-(x + 1)/x)/((x + 1)/x - 1) + arctan(sqrt(-(x + 1)/x))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx = \frac{1}{4}\,\pi \mathrm{sgn}(x) - \frac{\arcsin\left(2\,x+1\right)}{2\,\mathrm{sgn}\left(x\right)} - \frac{\sqrt{-x^2-x}}{\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/((-1-x)/x)^(1/2),x, algorithm="giac")
[Out] 1/4*pi*sgn(x) - 1/2*arcsin(2*x + 1)/sgn(x) - sqrt(-x^2 - x)/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx = \operatorname{atan}\left(\sqrt{-\frac{1}{x}-1}\right) - x\sqrt{-\frac{1}{x}-1}$$

```
[In] int(1/(-(x + 1)/x)^(1/2),x)
[Out] atan((- 1/x - 1)^(1/2)) - x*(- 1/x - 1)^(1/2)
```

3.980 $\int \sqrt{(4-x)x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \sqrt{(4-x)x}\,dx = -\frac{1}{2}(2-x)\sqrt{4x-x^2} - 2\arcsin\left(1-\frac{x}{2}\right)$$

```
[Out] 2*arcsin(-1+1/2*x)-1/2*(2-x)*(-x^2+4*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1976, 626, 633, 222}

$$\int \sqrt{(4-x)x}\,dx = -2\arcsin\left(1-\frac{x}{2}\right) - \frac{1}{2}\sqrt{4x-x^2}(2-x)$$

```
[In] Int[Sqrt[(4 - x)*x],x]
```

```
[Out] -1/2*((2 - x)*Sqrt[4*x - x^2]) - 2*ArcSin[1 - x/2]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sqrt{4x-x^2}\,dx \\
&= -\frac{1}{2}(2-x)\sqrt{4x-x^2}+2\int \frac{1}{\sqrt{4x-x^2}}\,dx \\
&= -\frac{1}{2}(2-x)\sqrt{4x-x^2}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx,x,4-2x\right) \\
&= -\frac{1}{2}(2-x)\sqrt{4x-x^2}-2\sin^{-1}\left(1-\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \sqrt{(4-x)x}\,dx = \frac{1}{2}\sqrt{-((-4+x)x)}\left(-2+x-\frac{16\text{arctanh}\left(\frac{\sqrt{-4+x}}{-2+\sqrt{x}}\right)}{\sqrt{-4+x}\sqrt{x}}\right)$$

```
[In] Integrate[Sqrt[(4 - x)*x],x]
[Out] (Sqrt[-((-4 + x)*x)]*(-2 + x - (16*ArcTanh[Sqrt[-4 + x]/(-2 + Sqrt[x])])/(S
qrt[-4 + x]*Sqrt[x])))/2
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{(x-2)x(x-4)}{2\sqrt{-x(x-4)}}+2\arcsin\left(-1+\frac{x}{2}\right)$
default	$-\frac{(4-2x)\sqrt{-x^{2}+4x}}{4}+2\arcsin\left(-1+\frac{x}{2}\right)$
pseudoelliptic	$-4\arctan\left(\frac{\sqrt{-x(x-4)}}{x}\right)+\frac{(x-2)\sqrt{-x(x-4)}}{2}$
meijerg	$-\frac{8i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\left(-\frac{3x}{2}+3\right)\sqrt{-\frac{x}{4}+1}}{12}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{x}}{2}\right)}{2}\right)}{\sqrt{\pi}}$
trager	$\left(-1+\frac{x}{2}\right)\sqrt{-x^{2}+4x}+2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{2}+1\right)x+\sqrt{-x^{2}+4x}+2\,\mathrm{I}\right.$

```
[In] int(((4-x)*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(x-2)*x*(x-4)/(-x*(x-4))^(1/2)+2*arcsin(-1+1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \sqrt{(4-x)x}\,dx=\frac{1}{2}\,\sqrt{-x^{2}+4\,x}(x-2)-4\,\arctan\left(\frac{\sqrt{-x^{2}+4\,x}}{x}\right)$$

```
[In] integrate(((4-x)*x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 4*x)*(x - 2) - 4*arctan(sqrt(-x^2 + 4*x)/x)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \sqrt{(4-x)x}\,dx=\left(\frac{x}{2}-1\right)\sqrt{-x^{2}+4x}+2\,\mathrm{asin}\left(\frac{x}{2}-1\right)$$

```
[In] integrate(((4-x)*x)**(1/2),x)
[Out] (x/2 - 1)*sqrt(-x**2 + 4*x) + 2*asin(x/2 - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \sqrt{(4-x)x}\,dx = \frac{1}{2}\sqrt{-x^2+4\,x}x - \sqrt{-x^2+4\,x} - 2\,\arcsin\left(-\frac{1}{2}\,x+1\right)$$

```
[In] integrate(((4-x)*x)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 4*x)*x - sqrt(-x^2 + 4*x) - 2*arcsin(-1/2*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \sqrt{(4-x)x}\,dx = \frac{1}{2}\sqrt{-x^2+4\,x}(x-2) + 2\,\arcsin\left(\frac{1}{2}\,x-1\right)$$

```
[In] integrate(((4-x)*x)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 4*x)*(x - 2) + 2*arcsin(1/2*x - 1)
```

Mupad [B] (verification not implemented)

Time = 21.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \sqrt{(4-x)x}\,dx = 2\,\mathrm{asin}\left(\frac{x}{2}-1\right) + \left(\frac{x}{2}-1\right)\,\sqrt{4\,x-x^2}$$

```
[In] int((-x*(x - 4))^(1/2),x)
[Out] 2*asin(x/2 - 1) + (x/2 - 1)*(4*x - x^2)^(1/2)
```

3.981 $\int \frac{1}{\sqrt{(1-x)x}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{1}{\sqrt{(1-x)x}} \, dx = -\arcsin(1-2x)$$

```
[Out] arcsin(-1+2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1976, 633, 222}

$$\int \frac{1}{\sqrt{(1-x)x}} \, dx = -\arcsin(1-2x)$$

```
[In] Int[1/Sqrt[(1 - x)*x],x]
```

```
[Out] -ArcSin[1 - 2*x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{x-x^2}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1-2x\right)\\ &= -\sin^{-1}(1-2x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(8) = 16.

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 5.00

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = -\frac{2\sqrt{-1+x}\sqrt{x}\log\left(\sqrt{-1+x}-\sqrt{x}\right)}{\sqrt{-((-1+x)x)}}$$

```
[In] Integrate[1/Sqrt[(1 - x)*x],x]
[Out] (-2*Sqrt[-1 + x]*Sqrt[x]*Log[Sqrt[-1 + x] - Sqrt[x]])/Sqrt[-((-1 + x)*x)]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\arcsin(2x-1)$	7
meijerg	$2\arcsin\left(\sqrt{x}\right)$	7
pseudoelliptic	$-2\arctan\left(\frac{\sqrt{-(x-1)x}}{x}\right)$	16
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-2\operatorname{RootOf}\left(\_Z^2+1\right)x+\operatorname{RootOf}\left(\_Z^2+1\right)+2\sqrt{-x^2+x}\right)$	36

```
[In] int(1/((1-x)*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(2*x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = -2 \arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

```
[In] integrate(1/((1-x)*x)^(1/2),x, algorithm="fricas")
[Out] -2*arctan(sqrt(-x^2 + x)/x)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = \operatorname{asin}(2x-1)$$

```
[In] integrate(1/((1-x)*x)**(1/2),x)
[Out] asin(2*x - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = \arcsin(2\,x-1)$$

```
[In] integrate(1/((1-x)*x)^(1/2),x, algorithm="maxima")
[Out] arcsin(2*x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(6) = 12$.

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 3.12

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = \frac{1}{4}\sqrt{-x^2+x}(2\,x-1) + \frac{1}{8}\arcsin(2\,x-1)$$

```
[In] integrate(1/((1-x)*x)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(-x^2 + x)*(2*x - 1) + 1/8*arcsin(2*x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = \operatorname{asin}(2\,x-1)$$

```
[In] int(1/(-x*(x - 1))^(1/2),x)
[Out] asin(2*x - 1)
```

3.982 $\int \frac{x}{(x(2+x))^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{x}{(x(2+x))^{3/2}}\,dx = \frac{x}{\sqrt{2x+x^2}}$$

```
[Out] x/(x^2+2*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1976, 650}

$$\int \frac{x}{(x(2+x))^{3/2}}\,dx = \frac{x}{\sqrt{x^2+2x}}$$

```
[In] Int[x/(x*(2 + x))^(3/2),x]
```

```
[Out] x/Sqrt[2*x + x^2]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
```

```
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{(2x+x^2)^{3/2}}\,dx$$
$$= \frac{x}{\sqrt{2x+x^2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{(x(2+x))^{3/2}}\,dx = \frac{x}{\sqrt{x(2+x)}}$$

```
[In] Integrate[x/(x*(2 + x))^(3/2),x]
[Out] x/Sqrt[x*(2 + x)]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{x}{\sqrt{x(x+2)}}$	10
pseudoelliptic	$\frac{x}{\sqrt{x(x+2)}}$	10
gosper	$\frac{x^2(x+2)}{(x(x+2))^{\frac{3}{2}}}$	15
trager	$\frac{\sqrt{x^2+2x}}{x+2}$	16
meijerg	$\frac{\sqrt{2}\,\sqrt{x}}{2\sqrt{1+\frac{x}{2}}}$	16
default	$-\frac{1}{\sqrt{x^2+2x}}+\frac{2x+2}{2\sqrt{x^2+2x}}$	29

```
[In] int(x/(x*(x+2))^(3/2),x,method=_RETURNVERBOSE)
[Out] x/(x*(x+2))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{x}{(x(2+x))^{3/2}}\,dx = \frac{x+\sqrt{x^2+2\,x}+2}{x+2}$$

```
[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="fricas")
[Out] (x + sqrt(x^2 + 2*x) + 2)/(x + 2)
```

Sympy [F]

$$\int \frac{x}{(x(2+x))^{3/2}}\,dx = \int \frac{x}{(x\,(x+2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(x*(2+x))**(3/2),x)
[Out] Integral(x/(x*(x + 2))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{(x(2+x))^{3/2}}\,dx = \frac{x}{\sqrt{x^2+2\,x}}$$

```
[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="maxima")
[Out] x/sqrt(x^2 + 2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{x}{(x(2+x))^{3/2}}\,dx = \frac{2}{x-\sqrt{x^2+2\,x}+2}$$

```
[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="giac")
[Out] 2/(x - sqrt(x^2 + 2*x) + 2)
```

Mupad [B] (verification not implemented)

Time = 22.39 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x}{(x(2+x))^{3/2}} \, dx = \frac{\sqrt{x\,(x+2)}}{x+2}$$

```
[In] int(x/(x*(x + 2))^(3/2),x)
[Out] (x*(x + 2))^(1/2)/(x + 2)
```

3.983 $\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2} dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{x}}}{\sqrt{2}}\right)$$

```
[Out] arctanh(1/2*(1+1/x)^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1460, 1483, 641, 65, 213}

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2} dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{x}+1}}{\sqrt{2}}\right)$$

```
[In] Int[Sqrt[1 + x^(-1)]/(1 - x^2),x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + x^(-1)]/Sqrt[2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 1460

```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_S
ymbol] :> Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x] /; FreeQ[{a,
c, d, e, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 1483

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1+\frac{1}{x}}}{\left(-1+\frac{1}{x^2}\right)x^2}\,dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{-1+x^2}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{1+x}}\,dx, x, \frac{1}{x}\right) \\
&= -\left(2\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\right)\right) \\
&= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{x}}}{\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\,dx=\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1+x}{x}}}{\sqrt{2}}\right)$$

```
[In] Integrate[Sqrt[1 + x^(-1)]/(1 - x^2),x]
[Out] Sqrt[2]*ArcTanh[Sqrt[(1 + x)/x]/Sqrt[2]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(17) = 34.

Time = 1.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

method	result	size
default	$\frac{\sqrt{\frac{x+1}{x}}\, x\sqrt{2}\, \operatorname{arctanh}\left(\frac{(1+3x)\sqrt{2}}{4\sqrt{x^2+x}}\right)}{2\sqrt{(x+1)x}}$	41
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{3\operatorname{RootOf}\left(\_Z^2-2\right)x-4x\sqrt{-\frac{-x-1}{x}}+\operatorname{RootOf}\left(\_Z^2-2\right)}{x-1}\right)}{2}$	48

```
[In] int((1+1/x)^(1/2)/(-x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*((x+1)/x)^(1/2)*x/((x+1)*x)^(1/2)*2^(1/2)*arctanh(1/4*(1+3*x)*2^(1/2)/(
x^2+x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\,dx=\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}x\sqrt{\frac{x+1}{x}}+3x+1}{x-1}\right)$$

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*log(-(2*sqrt(2)*x*sqrt((x + 1)/x) + 3*x + 1)/(x - 1))
```

Sympy [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\,dx = -\int \frac{\sqrt{1+\frac{1}{x}}}{x^2-1}\,dx$$

```
[In] integrate((1+1/x)**(1/2)/(-x**2+1),x)
[Out] -Integral(sqrt(1 + 1/x)/(x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\,dx = \int -\frac{\sqrt{\frac{1}{x}+1}}{x^2-1}\,dx$$

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="maxima")
[Out] -integrate(sqrt(1/x + 1)/(x^2 - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(17) = 34$.

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.32

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\,dx = \frac{1}{2}\sqrt{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\mathrm{sgn}(x) - \frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+x}+2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+x}+2\right|}\right)\mathrm{sgn}(x)$$

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="giac")
[Out] 1/2*sqrt(2)*log((sqrt(2) - 1)/(sqrt(2) + 1))*sgn(x) - 1/2*sqrt(2)*log(abs(-
2*x - 2*sqrt(2) + 2*sqrt(x^2 + x) + 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2 +
x) + 2))*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 22.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\,dx = \sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}+1}}{2}\right)$$

```
[In] int(-(1/x + 1)^(1/2)/(x^2 - 1),x)
[Out] 2^(1/2)*atanh((2^(1/2)*(1/x + 1)^(1/2))/2)
```

3.984 $\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 24

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx=\frac{1}{2}\arctan\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x\right)$$

```
[Out] 1/2*arctan(x*(1/2*5^(1/2)-1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6, 209}

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx=\frac{1}{2}\arctan\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x\right)$$

```
[In] Int[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[(3 - Sqrt[5])/2]*x]/2
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\text{integral} = \int \frac{1}{1+\sqrt{5}+\left(-1+\sqrt{5}\right)x^2}\,dx$$
$$= \frac{1}{2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx = \frac{1}{4}i\log\left(1+\sqrt{5}-2ix\right)-\frac{1}{4}i\log\left(1+\sqrt{5}+2ix\right)$$

```
[In] Integrate[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1),x]
[Out] (I/4)*Log[1 + Sqrt[5] - (2*I)*x] - (I/4)*Log[1 + Sqrt[5] + (2*I)*x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 1.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
risch	$\frac{\arctan\left(\frac{2x}{\sqrt{5}+1}\right)}{4}-\frac{\arctan\left(\frac{2x}{-\sqrt{5}-1}\right)}{4}$	30
parallelrisch	$-\frac{i\ln\left(-\frac{i\sqrt{5}}{2}-\frac{i}{2}+x\right)}{4}+\frac{i\ln\left(\frac{i\sqrt{5}}{2}+\frac{i}{2}+x\right)}{4}$	30
default	$\frac{4\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{\left(\sqrt{5}-1\right)\left(2\sqrt{5}+2\right)}$	32
meijerg	$\frac{\arctan\left(\frac{x\sqrt{\sqrt{5}-1}}{\sqrt{\sqrt{5}+1}}\right)}{\sqrt{\sqrt{5}+1}\sqrt{\sqrt{5}-1}}$	33

```
[In] int(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/4*arctan(2*x/(5^(1/2)+1))-1/4*arctan(2*x/(-5^(1/2)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx = \frac{1}{2}\,\arctan\left(\frac{1}{2}\,\sqrt{5}x-\frac{1}{2}\,x\right)$$

```
[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="fricas")
[Out] 1/2*arctan(1/2*sqrt(5)*x - 1/2*x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx = -\frac{\operatorname{atan}\left(x\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\right)}{2}$$

```
[In] integrate(1/(1-x**2+5**(1/2)+x**2*5**(1/2)),x)
[Out] -atan(x*(1/2 - sqrt(5)/2))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx = \frac{1}{2}\,\arctan\left(\frac{1}{2}\,x\left(\sqrt{5}-1\right)\right)$$

```
[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="maxima")
[Out] 1/2*arctan(1/2*x*(sqrt(5) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx = \frac{1}{2}\,\arctan\left(\frac{2\,x}{\sqrt{5}+1}\right)$$

```
[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="giac")
[Out] 1/2*arctan(2*x/(sqrt(5) + 1))
```

Mupad [B] (verification not implemented)

Time = 22.71 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx=\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\left(\sqrt{5}+1\right)}{4\,\left(\frac{\sqrt{5}}{4}+\frac{1}{4}\right)\,\sqrt{\sqrt{5}+3}}\right)\,\left(\sqrt{5}+1\right)}{4\,\sqrt{\sqrt{5}+3}}$$

```
[In] int(1/(5^(1/2) + 5^(1/2)*x^2 - x^2 + 1),x)
[Out] (2^(1/2)*atan((2^(1/2)*x*(5^(1/2) + 1))/(4*(5^(1/2)/4 + 1/4)*(5^(1/2) + 3)^
(1/2)))*(5^(1/2) + 1))/(4*(5^(1/2) + 3)^(1/2))
```

3.985 $\int \frac{1}{\sqrt{ax+bx^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {634, 212}

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[In] Int[1/Sqrt[a*x + b*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

```
[In] Integrate[1/Sqrt[a*x + b*x^2],x]
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{\sqrt{b}}$	23
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

```
[In] int(1/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

```
[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(26) = 52.

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ b\neq 0\wedge \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{for}\ b\neq 0 \\ \frac{2\sqrt{ax}}{a} & \text{for}\ a\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(b*x**2+a*x)**(1/2),x)
```

```
[Out] Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(b, 0)
& Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2),
 Ne(b, 0)), (2*sqrt(a*x)/a, Ne(a, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}$$

```
[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(22) = 44$.

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}}$$

```
[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*x
^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 21.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

```
[In] int(1/(a*x + b*x^2)^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.986 $\int \frac{1}{\sqrt{x(a+bx)}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1976, 634, 212}

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[In] Int[1/Sqrt[x*(a + b*x)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{ax+bx^2}}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

```
[In] Integrate[1/Sqrt[x*(a + b*x)],x]
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$\frac{2\ \operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{\sqrt{b}}$	23
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

```
[In] int(1/(x*(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = \left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

```
[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(26) = 52.

Time = 0.71 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = \begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } b\neq 0 \wedge \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{for } b\neq 0 \\ \frac{2\sqrt{ax}}{a} & \text{for } a\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(x*(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(b, 0)
& Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2),
 Ne(b, 0)), (2*sqrt(a*x)/a, Ne(a, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = \frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}$$

```
[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(22) = 44$.

Time = 0.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}}$$

```
[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*x
^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 21.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = \frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

```
[In] int(1/(x*(a + b*x))^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.987 $\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2004, 634, 212}

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[In] Int[1/Sqrt[(b + a/x)*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{ax+bx^2}}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

```
[In] Integrate[1/Sqrt[(b + a/x)*x^2],x]
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

```
[In] int(1/((b+a/x)*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx = \left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

```
[In] integrate(1/((b+a/x)*x^2)^(1/2),x, algorithm="fricas")
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(26) = 52.

Time = 0.66 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx = \begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } b\neq 0 \wedge \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{for } b\neq 0 \\ \frac{2\sqrt{ax}}{a} & \text{for } a\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

```
[In] integrate(1/((b+a/x)*x**2)**(1/2),x)
[Out] Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(b, 0)
& Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2),
 Ne(b, 0)), (2*sqrt(a*x)/a, Ne(a, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx = \frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}$$

```
[In] integrate(1/((b+a/x)*x^2)^(1/2),x, algorithm="maxima")
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(22) = 44$.

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}}$$

```
[In] integrate(1/((b+a/x)*x^2)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*x
^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 21.69 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx = \frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

```
[In] int(1/(x^2*(b + a/x))^(1/2),x)
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.988 $\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2004, 634, 212}

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[In] Int[1/Sqrt[(a/x^2 + b/x)*x^3],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{ax+bx^2}}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

```
[In] Integrate[1/Sqrt[(a/x^2 + b/x)*x^3],x]
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

```
[In] int(1/((a/x^2+b/x)*x^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx = \left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

```
[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="fricas")
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(26) = 52.

Time = 0.70 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx = \begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } b\neq 0\wedge\frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{for } b\neq 0 \\ \frac{2\sqrt{ax}}{a} & \text{for } a\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

```
[In] integrate(1/((a/x**2+b/x)*x**3)**(1/2),x)
[Out] Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(b, 0)
& Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2),
 Ne(b, 0)), (2*sqrt(a*x)/a, Ne(a, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx = \frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}$$

```
[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="maxima")
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(22) = 44$.

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(2x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8b^{\frac{3}{2}}}$$

```
[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*x
^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 22.67 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx = \frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

```
[In] int(1/(x^3*(a/x^2 + b/x))^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.989 $\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}} \, dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2004, 634, 212}

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}} \, dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[In] Int[1/Sqrt[(a*x^2 + b*x^3)/x],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{\sqrt{ax+bx^2}}\,dx \\ &= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right) \\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

```
[In] Integrate[1/Sqrt[(a*x^2 + b*x^3)/x],x]
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

```
[In] int(1/((b*x^3+a*x^2)/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx = \left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

```
[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="fricas")
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(26) = 52.

Time = 0.66 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx = \begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } b\neq 0\wedge\frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{for } b\neq 0 \\ \frac{2\sqrt{ax}}{a} & \text{for } a\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

```
[In] integrate(1/((b*x**3+a*x**2)/x)**(1/2),x)
[Out] Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(b, 0)
& Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2),
 Ne(b, 0)), (2*sqrt(a*x)/a, Ne(a, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx = \frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}$$

```
[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="maxima")
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(22) = 44$.

Time = 0.35 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}}$$

```
[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*x
^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 22.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx = \frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

```
[In] int(1/((a*x^2 + b*x^3)/x)^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.990 $\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2004, 634, 212}

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[In] Int[1/Sqrt[(a*x^3 + b*x^4)/x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{ax+bx^2}}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

```
[In] Integrate[1/Sqrt[(a*x^3 + b*x^4)/x^2],x]
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

```
[In] int(1/((b*x^4+a*x^3)/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = \left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

```
[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(26) = 52.

Time = 0.69 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = \begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } b\neq 0\wedge \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{for } b\neq 0 \\ \frac{2\sqrt{ax}}{a} & \text{for } a\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

```
[In] integrate(1/((b*x**4+a*x**3)/x**2)**(1/2),x)
```

```
[Out] Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(b, 0)
& Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2),
 Ne(b, 0)), (2*sqrt(a*x)/a, Ne(a, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = \frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}$$

```
[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(22) = 44$.

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = \frac{1}{4}\,\sqrt{bx^2+ax}\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}}$$

```
[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*x
^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 21.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = \frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

```
[In] int(1/((a*x^3 + b*x^4)/x^2)^(1/2),x)
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.991 $\int \frac{1}{\sqrt{acx+bcx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{1}{\sqrt{acx+bcx^2}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] 2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {634, 212}

$$\int \frac{1}{\sqrt{acx+bcx^2}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[In] Int[1/Sqrt[a*c*x + b*c*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{acx+bcx^2}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

```
[In] Integrate[1/Sqrt[a*c*x + b*c*x^2],x]
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[c*x*(a + b*x)])
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{cx(bx+a)}}{x\sqrt{bc}}\right)}{\sqrt{bc}}$	28
default	$\frac{\ln\left(\frac{\frac{1}{2}ac+bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$	37

```
[In] int(1/(b*c*x^2+a*c*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/(b*c)^(1/2)*arctanh((c*x*(b*x+a))^(1/2)/x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{acx+bcx^2}}\,dx = \left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\sqrt{bc}\right)}{bc}, -\frac{2\,\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

```
[In] integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="fricas")
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(39) = 78.

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{acx+bcx^2}}\,dx = \begin{cases} \frac{\log\left(ac+2bcx+2\sqrt{bc}\sqrt{acx+bcx^2}\right)}{\sqrt{bc}} & \text{for } bc\neq 0 \wedge \frac{a^2c}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{bc\left(\frac{a}{2b}+x\right)^2}} & \text{for } bc\neq 0 \\ \frac{2\sqrt{acx}}{ac} & \text{for } ac\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(b*c*x**2+a*c*x)**(1/2),x)
[Out] Piecewise((log(a*c + 2*b*c*x + 2*sqrt(b*c)*sqrt(a*c*x + b*c*x**2))/sqrt(b*c
), Ne(b*c, 0) & Ne(a**2*c/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*c*
(a/(2*b) + x)**2), Ne(b*c, 0)), (2*sqrt(a*c*x)/(a*c), Ne(a*c, 0)), (zoo*x,
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{acx + bcx^2}}\,dx = \frac{\log\left(2\,bcx + ac + 2\,\sqrt{bcx^2 + acx}\sqrt{bc}\right)}{\sqrt{bc}}$$

```
[In] integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="maxima")
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(30) = 60.

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{acx + bcx^2}}\,dx = \frac{a^2c\log\left(\left|-ac - 2\,\sqrt{bc}\left(\sqrt{bc}x - \sqrt{bcx^2 + acx}\right)\right|\right)}{8\,\sqrt{bc}b} + \frac{1}{4}\,\sqrt{bcx^2 + acx}\left(2\,x + \frac{a}{b}\right)$$

```
[In] integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="giac")
[Out] 1/8*a^2*c*log(abs(-a*c - 2*sqrt(b*c)*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x)))
)/(sqrt(b*c)*b) + 1/4*sqrt(b*c*x^2 + a*c*x)*(2*x + a/b)
```

Mupad [B] (verification not implemented)

Time = 21.40 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{acx + bcx^2}}\,dx = \frac{\ln\left(a\,c + 2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a + b\,x\right)} + 2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

```
[In] int(1/(a*c*x + b*c*x^2)^(1/2),x)
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

3.992 $\int \frac{1}{\sqrt{c(ax+bx^2)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{1}{\sqrt{c\,(ax+bx^2)}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] 2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2004, 634, 212}

$$\int \frac{1}{\sqrt{c\,(ax+bx^2)}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[In] Int[1/Sqrt[c*(a*x + b*x^2)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{acx+bcx^2}}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{c\,(ax+bx^2)}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

```
[In] Integrate[1/Sqrt[c*(a*x + b*x^2)],x]
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[c*x*(a + b*x)])
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{cx(bx+a)}}{x\sqrt{bc}}\right)}{\sqrt{bc}}$	28
default	$\frac{\ln\left(\frac{\frac{1}{2}ac+bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$	37

```
[In] int(1/(c*(b*x^2+a*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/(b*c)^(1/2)*arctanh((c*x*(b*x+a))^(1/2)/x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{c\,(ax+bx^2)}}\,dx = \left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\sqrt{bc}\right)}{bc},\ -\frac{2\,\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

```
[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(39) = 78$.

Time = 0.66 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{c\,(ax+bx^2)}}\,dx = \begin{cases} \frac{\log\left(ac+2bcx+2\sqrt{bc}\sqrt{acx+bcx^2}\right)}{\sqrt{bc}} & \text{for } bc \neq 0 \wedge \frac{a^2c}{b} \neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{bc\left(\frac{a}{2b}+x\right)^2}} & \text{for } bc \neq 0 \\ \frac{2\sqrt{acx}}{ac} & \text{for } ac \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(c*(b*x**2+a*x))**(1/2),x)
```

```
[Out] Piecewise((log(a*c + 2*b*c*x + 2*sqrt(b*c)*sqrt(a*c*x + b*c*x**2))/sqrt(b*c
), Ne(b*c, 0) & Ne(a**2*c/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*c*
(a/(2*b) + x)**2), Ne(b*c, 0)), (2*sqrt(a*c*x)/(a*c), Ne(a*c, 0)), (zoo*x,
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{c\,(ax+bx^2)}}\,dx = \frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\sqrt{bc}\right)}{\sqrt{bc}}$$

```
[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="maxima")
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(30) = 60.

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{c\,(ax+bx^2)}}\,dx = \frac{a^2c\log\left(\left|-ac-2\,\sqrt{bc}\left(\sqrt{bc}x-\sqrt{bcx^2+acx}\right)\right|\right)}{8\,\sqrt{bc}b} + \frac{1}{4}\,\sqrt{bcx^2+acx}\left(2\,x+\frac{a}{b}\right)$$

```
[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="giac")
[Out] 1/8*a^2*c*log(abs(-a*c - 2*sqrt(b*c)*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x)))
)/(sqrt(b*c)*b) + 1/4*sqrt(b*c*x^2 + a*c*x)*(2*x + a/b)
```

Mupad [B] (verification not implemented)

Time = 20.97 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{c\,(ax+bx^2)}}\,dx = \frac{\ln\left(a\,c+2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a+b\,x\right)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

```
[In] int(1/(c*(a*x + b*x^2))^(1/2),x)
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

3.993 $\int \frac{1}{\sqrt{cx(a+bx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] 2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1976, 634, 212}

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[In] Int[1/Sqrt[c*x*(a + b*x)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\sqrt{acx+bcx^2}}\,dx \\
&= 2\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right) \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

```
[In] Integrate[1/Sqrt[c*x*(a + b*x)],x]
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[c*x*(a + b*x)])
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$\frac{2\ \text{arctanh}\left(\frac{\sqrt{cx(bx+a)}}{x\sqrt{bc}}\right)}{\sqrt{bc}}$	28
default	$\frac{\ln\left(\frac{\frac{1}{2}ac+bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$	37

```
[In] int(1/(c*x*(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/(b*c)^(1/2)*arctanh((c*x*(b*x+a))^(1/2)/x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = \left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\sqrt{bc}\right)}{bc}, -\frac{2\,\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

```
[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(39) = 78.

Time = 0.71 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = \begin{cases} \frac{\log\left(ac+2bcx+2\sqrt{bc}\sqrt{acx+bcx^2}\right)}{\sqrt{bc}} & \text{for } bc\neq 0\wedge\frac{a^2c}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{bc\left(\frac{a}{2b}+x\right)^2}} & \text{for } bc\neq 0 \\ \frac{2\sqrt{acx}}{ac} & \text{for } ac\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(c*x*(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((log(a*c + 2*b*c*x + 2*sqrt(b*c)*sqrt(a*c*x + b*c*x**2))/sqrt(b*c
), Ne(b*c, 0) & Ne(a**2*c/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*c*
(a/(2*b) + x)**2), Ne(b*c, 0)), (2*sqrt(a*c*x)/(a*c), Ne(a*c, 0)), (zoo*x,
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = \frac{\log\left(2\,bcx + ac + 2\,\sqrt{bcx^2 + acx}\sqrt{bc}\right)}{\sqrt{bc}}$$

```
[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="maxima")
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(30) = 60.

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = \frac{a^2c\log\left(\left|-ac - 2\,\sqrt{bc}\left(\sqrt{bc}x - \sqrt{bcx^2 + acx}\right)\right|\right)}{8\,\sqrt{bc}b} + \frac{1}{4}\,\sqrt{bcx^2 + acx}\left(2\,x + \frac{a}{b}\right)$$

```
[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="giac")
[Out] 1/8*a^2*c*log(abs(-a*c - 2*sqrt(b*c)*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x)))
)/(sqrt(b*c)*b) + 1/4*sqrt(b*c*x^2 + a*c*x)*(2*x + a/b)
```

Mupad [B] (verification not implemented)

Time = 22.70 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = \frac{\ln\left(a\,c + 2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a + b\,x\right)} + 2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

```
[In] int(1/(c*x*(a + b*x))^(1/2),x)
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

3.994 $\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] 2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2004, 634, 212}

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[In] Int[1/Sqrt[c*(b + a/x)*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{\sqrt{acx+bcx^2}}\,dx \\ &= 2\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right) \\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

```
[In] Integrate[1/Sqrt[c*(b + a/x)*x^2],x]
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[c*x*(a + b*x)])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2}ac+bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$	37

```
[In] int(1/(c*(b+a/x)*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx = \left[\frac{\sqrt{bc}\log\left(2\,bcx + ac + 2\,\sqrt{bcx^2 + acx}\sqrt{bc}\right)}{bc},\; -\frac{2\,\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

```
[In] integrate(1/(c*(b+a/x)*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(39) = 78.

Time = 0.69 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx = \begin{cases} \frac{\log\left(ac+2bcx+2\sqrt{bc}\sqrt{acx+bcx^2}\right)}{\sqrt{bc}} & \text{for } bc \neq 0 \wedge \frac{a^2c}{b} \neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{bc\left(\frac{a}{2b}+x\right)^2}} & \text{for } bc \neq 0 \\ \frac{2\sqrt{acx}}{ac} & \text{for } ac \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(c*(b+a/x)*x**2)**(1/2),x)
```

```
[Out] Piecewise((log(a*c + 2*b*c*x + 2*sqrt(b*c)*sqrt(a*c*x + b*c*x**2))/sqrt(b*c
), Ne(b*c, 0) & Ne(a**2*c/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*c*
(a/(2*b) + x)**2), Ne(b*c, 0)), (2*sqrt(a*c*x)/(a*c), Ne(a*c, 0)), (zoo*x,
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx=\frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\sqrt{bc}\right)}{\sqrt{bc}}$$

```
[In] integrate(1/(c*(b+a/x)*x^2)^(1/2),x, algorithm="maxima")
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(30) = 60.

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx=\frac{a^2c\log\left(\left|-ac-2\,\sqrt{bc}\left(\sqrt{bc}x-\sqrt{bcx^2+acx}\right)\right|\right)}{8\,\sqrt{bc}b}+\frac{1}{4}\,\sqrt{bcx^2+acx}\left(2\,x+\frac{a}{b}\right)$$

```
[In] integrate(1/(c*(b+a/x)*x^2)^(1/2),x, algorithm="giac")
[Out] 1/8*a^2*c*log(abs(-a*c - 2*sqrt(b*c)*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x)))
)/(sqrt(b*c)*b) + 1/4*sqrt(b*c*x^2 + a*c*x)*(2*x + a/b)
```

Mupad [B] (verification not implemented)

Time = 23.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx=\frac{\ln\left(a\,c+2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a+b\,x\right)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

```
[In] int(1/(c*x^2*(b + a/x))^(1/2),x)
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

3.995 $\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx = \frac{1}{4}\left(3x+\sqrt{-1+x^2}\right)\sqrt{1-x^2+x\sqrt{-1+x^2}} + \frac{3\arcsin\left(x-\sqrt{-1+x^2}\right)}{4\sqrt{2}}$$

```
[Out] 3/8*arcsin(x-(x^2-1)^(1/2))*2^(1/2)+1/4*(3*x+(x^2-1)^(1/2))*(1-x^2+x*(x^2-1
)^(1/2))^(1/2)
```

Rubi [F]

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx = \int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

```
[In] Int[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
[Out] Defer[Int][Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x]
```

Rubi steps

$$\text{integral} = \int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.75

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx = \frac{1}{8}\left(\frac{2(-1+x^2)\left(3x+\sqrt{-1+x^2}\right)}{\sqrt{1-x^2+x\sqrt{-1+x^2}}\left(-1+x^2+x\sqrt{-1+x^2}\right)} - 3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-1+x^2}}{\sqrt{1-x^2+x\sqrt{-1+x^2}}}\right)\right)$$

```
[In] Integrate[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
```

```
[Out] ((2*(-1 + x^2)*(3*x + Sqrt[-1 + x^2]))/(Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]]*(-
1 + x^2 + x*Sqrt[-1 + x^2])) - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[-1 + x^2])/Sq
rt[1 - x^2 + x*Sqrt[-1 + x^2]]])/8
```

Maple [F]

$$\int \sqrt{1-x^2+x\sqrt{x^2-1}}dx$$

```
[In] int((1-x^2+x*(x^2-1)^(1/2))^(1/2),x)
```

```
[Out] int((1-x^2+x*(x^2-1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.64 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx = \frac{1}{4}\sqrt{-x^2+\sqrt{x^2-1}x+1}\left(3\,x+\sqrt{x^2-1}\right) + \frac{3}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+\sqrt{x^2-1}x+1}}{2\sqrt{x^2-1}}\right)$$

```
[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-x^2 + sqrt(x^2 - 1)*x + 1)*(3*x + sqrt(x^2 - 1)) + 3/8*sqrt(2)*ar
ctan(1/2*sqrt(2)*sqrt(-x^2 + sqrt(x^2 - 1)*x + 1)/sqrt(x^2 - 1))
```

Sympy [F]

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx=\int \sqrt{-x^2+x\sqrt{x^2-1}+1}\,dx$$

```
[In] integrate((1-x**2+x*(x**2-1)**(1/2))**(1/2),x)
[Out] Integral(sqrt(-x**2 + x*sqrt(x**2 - 1) + 1), x)
```

Maxima [F]

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx=\int \sqrt{-x^2+\sqrt{x^2-1}x+1}\,dx$$

```
[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)
```

Giac [F]

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx=\int \sqrt{-x^2+\sqrt{x^2-1}x+1}\,dx$$

```
[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx=\int \sqrt{x\,\sqrt{x^2-1}-x^2+1}\,dx$$

```
[In] int((x*(x^2 - 1)^(1/2) - x^2 + 1)^(1/2),x)
[Out] int((x*(x^2 - 1)^(1/2) - x^2 + 1)^(1/2), x)
```

3.996 $\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 66

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}} \, dx = \frac{1}{2}\left(\sqrt{x}+3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\sqrt{1+x}}-\frac{3\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)}{2\sqrt{2}}$$

```
[Out] -3/4*arcsin(x^(1/2)-(1+x)^(1/2))*2^(1/2)+1/2*(x^(1/2)+3*(1+x)^(1/2))*(-x+x^
(1/2)*(1+x)^(1/2))^(1/2)
```

Rubi [F]

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}} \, dx = \int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}} \, dx$$

```
[In] Int[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] 2*Defer[Subst][Defer[Int][Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x], x, Sqrt[1 +
 x]]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \sqrt{1-x^2+x\sqrt{-1+x^2}} \, dx, x, \sqrt{1+x}\right)$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx = \frac{1}{4}\left(2\left(\sqrt{x}+3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\sqrt{1+x}} - 3\sqrt{2}\arctan\left(\frac{\sqrt{-2x+2\sqrt{x}\sqrt{1+x}}}{-\sqrt{x}+\sqrt{1+x}}\right)\right)$$

```
[In] Integrate[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x],x]
[Out] (2*(Sqrt[x] + 3*Sqrt[1 + x])*Sqrt[-x + Sqrt[x]*Sqrt[1 + x]] - 3*Sqrt[2]*Arc
Tan[Sqrt[-2*x + 2*Sqrt[x]*Sqrt[1 + x]]/(-Sqrt[x] + Sqrt[1 + x])])/4
```

Maple [F]

$$\int \frac{\sqrt{-x+\sqrt{x}\,\sqrt{x+1}}}{\sqrt{x+1}}dx$$

```
[In] int((-x+x^(1/2)*(x+1)^(1/2))^(1/2)/(x+1)^(1/2),x)
[Out] int((-x+x^(1/2)*(x+1)^(1/2))^(1/2)/(x+1)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx = \text{Timed out}$$

```
[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx = \int \frac{\sqrt{\sqrt{x}\sqrt{x+1}-x}}{\sqrt{x+1}}\,dx$$

```
[In] integrate((-x+x**(1/2)*(1+x)**(1/2))**(1/2)/(1+x)**(1/2),x)
[Out] Integral(sqrt(sqrt(x)*sqrt(x + 1) - x)/sqrt(x + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx = \int \frac{\sqrt{\sqrt{x+1}\sqrt{x}-x}}{\sqrt{x+1}}\,dx$$

```
[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx = \int \frac{\sqrt{\sqrt{x+1}\sqrt{x}-x}}{\sqrt{x+1}}\,dx$$

```
[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx = \int \frac{\sqrt{\sqrt{x}\,\sqrt{x+1}-x}}{\sqrt{x+1}}\,dx$$

```
[In] int((x^(1/2)*(x + 1)^(1/2) - x)^(1/2)/(x + 1)^(1/2),x)
[Out] int((x^(1/2)*(x + 1)^(1/2) - x)^(1/2)/(x + 1)^(1/2), x)
```

3.997 $\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 78

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = -\sqrt{2\left(1+\sqrt{5}\right)}\arctan\left(\sqrt{-2+\sqrt{5}}\left(x+\sqrt{1+x^2}\right)\right) + \sqrt{2\left(-1+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{2+\sqrt{5}}\left(x+\sqrt{1+x^2}\right)\right)$$

```
[Out] arctanh((x+(x^2+1)^(1/2))*(2+5^(1/2))^(1/2))*(-2+2*5^(1/2))^(1/2)-arctan((x
+(x^2+1)^(1/2))*(-2+5^(1/2))^(1/2))*(2+2*5^(1/2))^(1/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 319 vs. 2(78) = 156.

Time = 0.56 (sec) , antiderivative size = 319, normalized size of antiderivative = 4.09, number of steps used = 25, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules

used = {6874, 267, 1144, 209, 213, 1265, 838, 721, 1107, 1261, 713, 1293}

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = -\sqrt{\frac{2}{5}\left(\sqrt{5}-1\right)}\arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right) - \sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right) - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) - 2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) + \sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x^2+1}\right) - \sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x^2+1}\right) + \sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right) - 2\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\operatorname{arctanh}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)$$

```
[In] Int[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]
```

```
[Out] -2*Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[(1 + Sq
rt[5])/10]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[2/(5*(-1 + Sqrt[5]))]*Arc
Tan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] - Sqrt[(2*(-1 + Sqrt[5]))/5]*ArcT
an[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] - 2*Sqrt[2/(5*(-1 + Sqrt[5]))]*Arc
Tanh[Sqrt[2/(-1 + Sqrt[5])]*x] + Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(-1
 + Sqrt[5])]*x] - Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*S
qrt[1 + x^2]] + Sqrt[(2*(1 + Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqr
t[1 + x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 713

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 721

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Sym
bol] :> Dist[2*e, Subst[Int[1/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 838

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[
(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \left(\frac{x}{x+x^3+\sqrt{1+x^2}}+\frac{2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\right)dx \\
&= -\left(2\int \frac{\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx\right)-\int \frac{x}{x+x^3+\sqrt{1+x^2}}\,dx \\
&= -\left(2\int \left(1+\frac{x\sqrt{1+x^2}}{-1+x^2+x^4}-\frac{x^2(1+x^2)}{-1+x^2+x^4}\right)dx\right) \\
&\quad -\int \left(\frac{x}{\sqrt{1+x^2}}+\frac{x^2}{-1+x^2+x^4}-\frac{x^3\sqrt{1+x^2}}{-1+x^2+x^4}\right)dx \\
&= -2x-2\int \frac{x\sqrt{1+x^2}}{-1+x^2+x^4}\,dx+2\int \frac{x^2(1+x^2)}{-1+x^2+x^4}\,dx \\
&\quad -\int \frac{x}{\sqrt{1+x^2}}\,dx-\int \frac{x^2}{-1+x^2+x^4}\,dx+\int \frac{x^3\sqrt{1+x^2}}{-1+x^2+x^4}\,dx
\end{aligned}$$

$$= -\sqrt{1+x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{1+x}}{-1+x+x^2}\,dx, x, x^2\right) + 2\int \frac{1}{-1+x^2+x^4}\,dx + \frac{1}{10}\left(-5+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx - \frac{1}{10}\left(5+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx - \text{Subst}\left(\int \frac{\sqrt{1+x}}{-1+x+x^2}\,dx, x, x^2\right)$$

$$= -\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) + \sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x}\,(-1+x+x^2)}\,dx, x, x^2\right) - 2\text{Subst}\left(\int \frac{x^2}{-1-x^2+x^4}\,dx, x, \sqrt{1+x^2}\right) + \frac{2\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx}{\sqrt{5}} - \frac{2\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx}{\sqrt{5}}$$

$$= -2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) - 2\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right) + \sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right) - \frac{1}{5}\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, \sqrt{1+x^2}\right) - \frac{1}{5}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, \sqrt{1+x^2}\right) + \text{Subst}\left(\int \frac{1}{-1-x^2+x^4}\,dx, x, \sqrt{1+x^2}\right)$$

$$= -2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)$$
$$- \sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{1+x^2}\right)$$
$$- 2\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)$$
$$+ \sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)$$
$$+ \sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{1+x^2}\right)$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, \sqrt{1+x^2}\right)}{\sqrt{5}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, \sqrt{1+x^2}\right)}{\sqrt{5}}$$

$$= -2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)$$
$$- \sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{1+x^2}\right)$$
$$- \sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{1+x^2}\right)$$
$$- 2\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)$$
$$+ \sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)$$
$$- \sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{1+x^2}\right)$$
$$+ \sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{1+x^2}\right)$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = -\sqrt{2\left(1+\sqrt{5}\right)}\arctan\left(\sqrt{2+\sqrt{5}}\left(x-\sqrt{1+x^2}\right)\right) - \sqrt{2\left(-1+\sqrt{5}\right)}\text{arctanh}\left(\sqrt{-2+\sqrt{5}}\left(x-\sqrt{1+x^2}\right)\right)$$

```
[In] Integrate[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]
[Out] -(Sqrt[2*(1 + Sqrt[5])]*ArcTan[Sqrt[2 + Sqrt[5]]*(x - Sqrt[1 + x^2])]) - Sq
rt[2*(-1 + Sqrt[5])]*ArcTanh[Sqrt[-2 + Sqrt[5]]*(x - Sqrt[1 + x^2])]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.33 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.78

method	result
trager	$-\text{RootOf}\left(\text{RootOf}\left(\_Z^4+\_Z^2-1\right)^2+\_Z^2+1\right)\ln\left(-\frac{\text{RootOf}\left(\text{RootOf}\left(\_Z^4+\_Z^2-1\right)^2+\_Z^2+1\right)\text{Ro}}{1+\text{RootO}}\right.$
default	$-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\text{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}+\frac{2\left(\sqrt{5}+1\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}-\frac{2\sqrt{5}\left(\sqrt{5}-1\right)\text{arct}}{5\sqrt{2\text{v}}}$

```
[In] int((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -RootOf(RootOf(_Z^4+_Z^2-1)^2+_Z^2+1)*ln(-(RootOf(RootOf(_Z^4+_Z^2-1)^2+_Z^
2+1)*RootOf(_Z^4+_Z^2-1)^2-(x^2+1)^(1/2)*RootOf(_Z^4+_Z^2-1)^2-RootOf(RootO
f(_Z^4+_Z^2-1)^2+_Z^2+1))/(1+RootOf(RootOf(_Z^4+_Z^2-1)^2+_Z^2+1)*RootOf(_Z
^4+_Z^2-1)^2*x))+RootOf(_Z^4+_Z^2-1)*ln(-((x^2+1)^(1/2)*RootOf(_Z^4+_Z^2-1)
^2+RootOf(_Z^4+_Z^2-1)^3+(x^2+1)^(1/2)+2*RootOf(_Z^4+_Z^2-1))/(RootOf(_Z^4+
_Z^2-1)^3*x+RootOf(_Z^4+_Z^2-1)*x+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(58) = 116$.

Time = 0.29 (sec) , antiderivative size = 434, normalized size of antiderivative = 5.56

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = \frac{1}{4}\sqrt{2}\sqrt{-\sqrt{5}-1}\log\left(4x^2 - \sqrt{x^2+1}\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-1}+4x\right) + \left(\sqrt{5}\sqrt{2}x-\sqrt{2}x\right)\sqrt{-\sqrt{5}-1}+4\right) - \frac{1}{4}\sqrt{2}\sqrt{-\sqrt{5}-1}\log\left(4x^2 + \sqrt{x^2+1}\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-1}-4x\right) - \left(\sqrt{5}\sqrt{2}x-\sqrt{2}x\right)\sqrt{-\sqrt{5}-1}+4\right) - \frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(4x^2 - 4\sqrt{x^2+1}x + \left(\sqrt{5}\sqrt{2}x - \sqrt{x^2+1}\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)+\sqrt{2}x\right)\sqrt{\sqrt{5}-1}+4\right) + \frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(4x^2 - 4\sqrt{x^2+1}x - \left(\sqrt{5}\sqrt{2}x - \sqrt{x^2+1}\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)+\sqrt{2}x\right)\sqrt{\sqrt{5}-1}+4\right) - \frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(2x+\sqrt{2}\sqrt{\sqrt{5}-1}\right) + \frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(2x-\sqrt{2}\sqrt{\sqrt{5}-1}\right) - \frac{1}{4}\sqrt{2}\sqrt{-\sqrt{5}-1}\log\left(2x+\sqrt{2}\sqrt{-\sqrt{5}-1}\right) + \frac{1}{4}\sqrt{2}\sqrt{-\sqrt{5}-1}\log\left(2x-\sqrt{2}\sqrt{-\sqrt{5}-1}\right)$$

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(-sqrt(5) - 1)*log(4*x^2 - sqrt(x^2 + 1)*((sqrt(5)*sqrt(2)
- sqrt(2))*sqrt(-sqrt(5) - 1) + 4*x) + (sqrt(5)*sqrt(2)*x - sqrt(2)*x)*sqrt
(-sqrt(5) - 1) + 4) - 1/4*sqrt(2)*sqrt(-sqrt(5) - 1)*log(4*x^2 + sqrt(x^2 +
 1)*((sqrt(5)*sqrt(2) - sqrt(2))*sqrt(-sqrt(5) - 1) - 4*x) - (sqrt(5)*sqrt(
2)*x - sqrt(2)*x)*sqrt(-sqrt(5) - 1) + 4) - 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*l
og(4*x^2 - 4*sqrt(x^2 + 1)*x + (sqrt(5)*sqrt(2)*x - sqrt(x^2 + 1)*(sqrt(5)*
```

```
sqrt(2) + sqrt(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4) + 1/4*sqrt(2)*sqrt(s
qrt(5) - 1)*log(4*x^2 - 4*sqrt(x^2 + 1)*x - (sqrt(5)*sqrt(2)*x - sqrt(x^2 +
 1)*(sqrt(5)*sqrt(2) + sqrt(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4) - 1/4*s
qrt(2)*sqrt(sqrt(5) - 1)*log(2*x + sqrt(2)*sqrt(sqrt(5) - 1)) + 1/4*sqrt(2)
*sqrt(sqrt(5) - 1)*log(2*x - sqrt(2)*sqrt(sqrt(5) - 1)) - 1/4*sqrt(2)*sqrt(
-sqrt(5) - 1)*log(2*x + sqrt(2)*sqrt(-sqrt(5) - 1)) + 1/4*sqrt(2)*sqrt(-sqr
t(5) - 1)*log(2*x - sqrt(2)*sqrt(-sqrt(5) - 1))
```

Sympy [F]

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = -\int \frac{x}{x^3+x+\sqrt{x^2+1}}\,dx - \int \frac{2\sqrt{x^2+1}}{x^3+x+\sqrt{x^2+1}}\,dx$$

```
[In] integrate((-x-2*(x**2+1)**(1/2))/(x+x**3+(x**2+1)**(1/2)),x)
[Out] -Integral(x/(x**3 + x + sqrt(x**2 + 1)), x) - Integral(2*sqrt(x**2 + 1)/(x*
*3 + x + sqrt(x**2 + 1)), x)
```

Maxima [F]

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = \int -\frac{x+2\sqrt{x^2+1}}{x^3+x+\sqrt{x^2+1}}\,dx$$

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="maxima")
[Out] -x - 1/2*arctan(x) + integrate(1/2*(2*x^6 + 3*x^4 - x^2 - 1)/(x^6 + 2*x^4 +
 2*x^2 + 2*(x^3 + x)*sqrt(x^2 + 1) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(58) = 116$.

Time = 0.41 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.79

$$
\begin{aligned}
&\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx \\
&= -\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(-\frac{x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}}{\sqrt{2\sqrt{5}-2}}\right) \\
&\quad -\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) \\
&\quad +\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(-x+\sqrt{x^2+1}+\sqrt{2\sqrt{5}+2}-\frac{1}{x-\sqrt{x^2+1}}\right) \\
&\quad -\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) \\
&\quad -\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\left|-x+\sqrt{x^2+1}-\sqrt{2\sqrt{5}+2}-\frac{1}{x-\sqrt{x^2+1}}\right|\right)
\end{aligned}
$$

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2*sqrt(5) + 2)*arctan(-(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 + 1))
)/sqrt(2*sqrt(5) - 2)) - 1/2*sqrt(2*sqrt(5) + 2)*arctan(x/sqrt(1/2*sqrt(5)
+ 1/2)) + 1/4*sqrt(2*sqrt(5) - 2)*log(-x + sqrt(x^2 + 1) + sqrt(2*sqrt(5) +
 2) - 1/(x - sqrt(x^2 + 1))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(x + sqrt(1/2
*sqrt(5) - 1/2))) + 1/4*sqrt(2*sqrt(5) - 2)*log(abs(x - sqrt(1/2*sqrt(5) -
1/2))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(-x + sqrt(x^2 + 1) - sqrt(2*sqrt(5
) + 2) - 1/(x - sqrt(x^2 + 1))))
```

Mupad [B] (verification not implemented)

Time = 20.65 (sec) , antiderivative size = 649, normalized size of antiderivative = 8.32

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = \frac{\ln\left(x+\frac{\sqrt{2}\sqrt{\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}-\frac{5}{2}\right)}{2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}} - \frac{\ln\left(x-\frac{\sqrt{2}\sqrt{\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}-\frac{5}{2}\right)}{2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}$$
$$+\frac{\ln\left(x-\frac{\sqrt{2}\sqrt{-\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{5}{2}\right)}{2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}} - \frac{\ln\left(x+\frac{\sqrt{2}\sqrt{-\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{5}{2}\right)}{2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}$$
$$-\frac{\left(\ln\left(x-\frac{\sqrt{2}\sqrt{\sqrt{5}-1}}{2}\right)-\ln\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{5}-1}}{2}+\frac{\sqrt{2}\sqrt{x^2+1}\sqrt{\sqrt{5}+1}}{2}+1\right)\right)\left(\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}$$
$$-\frac{\left(\ln\left(x+\frac{\sqrt{2}\sqrt{\sqrt{5}-1}}{2}\right)-\ln\left(\frac{\sqrt{2}\sqrt{x^2+1}\sqrt{\sqrt{5}+1}}{2}-\frac{\sqrt{2}\,x\sqrt{\sqrt{5}-1}}{2}+1\right)\right)\left(\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}$$
$$+\frac{\left(\ln\left(\frac{\sqrt{2}\sqrt{x^2+1}\sqrt{1-\sqrt{5}}}{2}-\frac{\sqrt{2}\,x\sqrt{-\sqrt{5}-1}}{2}+1\right)-\ln\left(x+\frac{\sqrt{2}\sqrt{-\sqrt{5}-1}}{2}\right)\right)\left(\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}$$
$$+\frac{\left(\ln\left(\frac{\sqrt{2}\,x\sqrt{-\sqrt{5}-1}}{2}+\frac{\sqrt{2}\sqrt{x^2+1}\sqrt{1-\sqrt{5}}}{2}+1\right)-\ln\left(x-\frac{\sqrt{2}\sqrt{-\sqrt{5}-1}}{2}\right)\right)\left(\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}$$

```
[In] int(-(x + 2*(x^2 + 1)^(1/2))/(x + (x^2 + 1)^(1/2) + x^3),x)
```

```
[Out] (log(x + (2^(1/2)*(5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 - 5/2))/(2*(5^(1/2)/2
- 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2)) - (log(x - (2^(1/2)*(5^(1/2) - 1)
^(1/2))/2)*(5^(1/2)/2 - 5/2))/(2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1
/2)^(3/2)) + (log(x - (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 + 5/2))
/(2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2)) - (log(x + (2^
(1/2)*(- 5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 + 5/2))/(2*(- 5^(1/2)/2 - 1/2)^(
1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2)) - ((log(x - (2^(1/2)*(5^(1/2) - 1)^(1/2
))/2) - log((2^(1/2)*x*(5^(1/2) - 1)^(1/2))/2 + (2^(1/2)*(x^2 + 1)^(1/2)*(5
^(1/2) + 1)^(1/2))/2 + 1))*((5^(1/2)/2 - 1/2)^(1/2) + 2*(5^(1/2)/2 - 1/2)^(
3/2)))/((2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2
+ 1/2)^(1/2)) - ((log(x + (2^(1/2)*(5^(1/2) - 1)^(1/2))/2) - log((2^(1/2)*(
x^2 + 1)^(1/2)*(5^(1/2) + 1)^(1/2))/2 - (2^(1/2)*x*(5^(1/2) - 1)^(1/2))/2 +
 1))*((5^(1/2)/2 - 1/2)^(1/2) + 2*(5^(1/2)/2 - 1/2)^(3/2)))/((2*(5^(1/2)/2
- 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 1/2)^(1/2)) + ((log(
```

```
(2^(1/2)*(x^2 + 1)^(1/2)*(1 - 5^(1/2))^(1/2))/2 - (2^(1/2)*x*(- 5^(1/2) - 1
)^(1/2))/2 + 1) - log(x + (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2))*((- 5^(1/2)/2
 - 1/2)^(1/2) + 2*(- 5^(1/2)/2 - 1/2)^(3/2)))/((2*(- 5^(1/2)/2 - 1/2)^(1/2)
 + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(1/2 - 5^(1/2)/2)^(1/2)) + ((log((2^(1/2)*x
*(- 5^(1/2) - 1)^(1/2))/2 + (2^(1/2)*(x^2 + 1)^(1/2)*(1 - 5^(1/2))^(1/2))/2
 + 1) - log(x - (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2))*((- 5^(1/2)/2 - 1/2)^(1
/2) + 2*(- 5^(1/2)/2 - 1/2)^(3/2)))/((2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^
(1/2)/2 - 1/2)^(3/2))*(1/2 - 5^(1/2)/2)^(1/2))
```

3.998 $\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx = -\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\arctan\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{2+2x+x^2}}\right) -\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{2\sqrt{5}+\left(5-\sqrt{5}\right)x}{\sqrt{10\left(-1+\sqrt{5}\right)}\sqrt{2+2x+x^2}}\right)$$

```
[Out] -1/2*arctanh((x*(5-5^(1/2))+2*5^(1/2))/(x^2+2*x+2)^(1/2)/(-10+10*5^(1/2))^(
1/2))*(-2+2*5^(1/2))^(1/2)-1/2*arctan((2*5^(1/2)-x*(5+5^(1/2)))/(x^2+2*x+2)
^(1/2)/(10+10*5^(1/2))^(1/2))*(2+2*5^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1050, 1044, 213, 209}

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx = -\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\arctan\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{x^2+2x+2}}\right) -\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\operatorname{arctanh}\left(\frac{\left(5-\sqrt{5}\right)x+2\sqrt{5}}{\sqrt{10\left(\sqrt{5}-1\right)}\sqrt{x^2+2x+2}}\right)$$

```
[In] Int[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]),x]
```

```
[Out] -(Sqrt[(1 + Sqrt[5])/2]*ArcTan[(2*Sqrt[5] - (5 + Sqrt[5])*x)/(Sqrt[10*(1 +
Sqrt[5])]*Sqrt[2 + 2*x + x^2])]) - Sqrt[(-1 + Sqrt[5])/2]*ArcTanh[(2*Sqrt[5
] + (5 - Sqrt[5])*x)/(Sqrt[10*(-1 + Sqrt[5])]*Sqrt[2 + 2*x + x^2])]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{-5-\sqrt{5}-2\sqrt{5}x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx}{2\sqrt{5}}+\frac{\int \frac{-5+\sqrt{5}+2\sqrt{5}x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx}{2\sqrt{5}}\\
&= \left(2\left(5-\sqrt{5}\right)\right)\operatorname{Subst}\left(\int \frac{1}{20\left(1-\sqrt{5}\right)+2x^2}\,dx,x,\frac{2\sqrt{5}+\left(5-\sqrt{5}\right)x}{\sqrt{2+2x+x^2}}\right)\\
&\quad+\left(2\left(5+\sqrt{5}\right)\right)\operatorname{Subst}\left(\int \frac{1}{20\left(1+\sqrt{5}\right)+2x^2}\,dx,x,\frac{-2\sqrt{5}+\left(5+\sqrt{5}\right)x}{\sqrt{2+2x+x^2}}\right)
\end{aligned}
$$

$$= -\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{2+2x+x^2}}\right) - \sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{2\sqrt{5}+\left(5-\sqrt{5}\right)x}{\sqrt{10\left(-1+\sqrt{5}\right)}\sqrt{2+2x+x^2}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.77

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx = \text{RootSum}\Big[8-8\#1 +\#1^4\&, \frac{-\log\left(-x+\sqrt{2+2x+x^2}-\#1\right)-\log\left(-x+\sqrt{2+2x+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{2+2x+x^2}\right.}{-2+\#1^3}$$

```
[In] Integrate[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]),x]
[Out] RootSum[8 - 8*#1 + #1^4 & , (-Log[-x + Sqrt[2 + 2*x + x^2] - #1] - Log[-x +
 Sqrt[2 + 2*x + x^2] - #1]*#1 + Log[-x + Sqrt[2 + 2*x + x^2] - #1]*#1^2)/(-
2 + #1^3) & ]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.67 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.41

method	result
trager	$-\frac{\text{RootOf}\left(\_Z^2+4\,\text{RootOf}\left(16\_Z^4+8\_Z^2+5\right)^2+2\right)\ln\left(\frac{32x\,\text{RootOf}\left(16\_Z^4+8\_Z^2+5\right)^4\text{RootOf}\left(\_Z^2+4\,\text{RootOf}\left(16\_Z^4+8\_\right.\right.}{}\right.}{}$
default	Expression too large to display

```
[In] int((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*RootOf(_Z^2+4*RootOf(16*_Z^4+8*_Z^2+5)^2+2)*ln((32*x*RootOf(16*_Z^4+8*
_Z^2+5)^4*RootOf(_Z^2+4*RootOf(16*_Z^4+8*_Z^2+5)^2+2)+52*RootOf(_Z^2+4*Root
Of(16*_Z^4+8*_Z^2+5)^2+2)*RootOf(16*_Z^4+8*_Z^2+5)^2*x+80*RootOf(16*_Z^4+8*
_Z^2+5)^2*RootOf(_Z^2+4*RootOf(16*_Z^4+8*_Z^2+5)^2+2)+96*(x^2+2*x+2)^(1/2)*
RootOf(16*_Z^4+8*_Z^2+5)^2-7*RootOf(_Z^2+4*RootOf(16*_Z^4+8*_Z^2+5)^2+2)*x-
```

```
10*RootOf(_Z^2+4*RootOf(16*_Z^4+8*_Z^2+5)^2+2)+38*(x^2+2*x+2)^(1/2))/(4*x*R
ootOf(16*_Z^4+8*_Z^2+5)^2+x+2))-RootOf(16*_Z^4+8*_Z^2+5)*ln(-(-32*RootOf(16
*_Z^4+8*_Z^2+5)^5*x+20*RootOf(16*_Z^4+8*_Z^2+5)^3*x+48*(x^2+2*x+2)^(1/2)*Ro
otOf(16*_Z^4+8*_Z^2+5)^2+80*RootOf(16*_Z^4+8*_Z^2+5)^3+25*RootOf(16*_Z^4+8*
_Z^2+5)*x+5*(x^2+2*x+2)^(1/2)+50*RootOf(16*_Z^4+8*_Z^2+5))/(4*x*RootOf(16*_
Z^4+8*_Z^2+5)^2+x-2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx = \frac{1}{2}\sqrt{2i-1}\log\left(-x+i\sqrt{2i-1}+\sqrt{x^2+2x+2}-i\right) - \frac{1}{2}\sqrt{2i-1}\log\left(-x-i\sqrt{2i-1}+\sqrt{x^2+2x+2}-i\right) - \frac{1}{2}\sqrt{-2i-1}\log\left(-x+i\sqrt{-2i-1}+\sqrt{x^2+2x+2}+i\right) + \frac{1}{2}\sqrt{-2i-1}\log\left(-x-i\sqrt{-2i-1}+\sqrt{x^2+2x+2}+i\right)$$

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2*I - 1)*log(-x + I*sqrt(2*I - 1) + sqrt(x^2 + 2*x + 2) - I) - 1/2
*sqrt(2*I - 1)*log(-x - I*sqrt(2*I - 1) + sqrt(x^2 + 2*x + 2) - I) - 1/2*sq
rt(-2*I - 1)*log(-x + I*sqrt(-2*I - 1) + sqrt(x^2 + 2*x + 2) + I) + 1/2*sqr
t(-2*I - 1)*log(-x - I*sqrt(-2*I - 1) + sqrt(x^2 + 2*x + 2) + I)
```

Sympy [F]

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx = \int \frac{2x+1}{(x^2+1)\sqrt{x^2+2x+2}}\,dx$$

```
[In] integrate((1+2*x)/(x**2+1)/(x**2+2*x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + 2*x + 2)), x)
```

Maxima [F]

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx = \int \frac{2\,x+1}{\sqrt{x^2+2\,x+2}(x^2+1)}\,dx$$

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x + 1)/(sqrt(x^2 + 2*x + 2)*(x^2 + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(93) = 186$.

Time = 0.38 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.52

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx$$
$$= \frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(256\left(\sqrt{5}\left(x-\sqrt{x^2+2\,x+2}\right)-2\,x+\sqrt{5}\sqrt{\sqrt{5}-2}+\sqrt{5}+2\sqrt{x^2+2\,x+2}-2\sqrt{\sqrt{5}\right.\right.$$
$$\left.+256\left(\sqrt{5}\left(x-\sqrt{x^2+2\,x+2}\right)-2\,x-\sqrt{5}+2\sqrt{x^2+2\,x+2}+\sqrt{\sqrt{5}-2}+2\right)^2\right)$$
$$-\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(256\left(\sqrt{5}\left(x-\sqrt{x^2+2\,x+2}\right)-2\,x-\sqrt{5}\sqrt{\sqrt{5}-2}+\sqrt{5}+2\sqrt{x^2+2\,x+2}+2\sqrt{\right.\right.$$
$$\left.+256\left(\sqrt{5}\left(x-\sqrt{x^2+2\,x+2}\right)-2\,x-\sqrt{5}+2\sqrt{x^2+2\,x+2}-\sqrt{\sqrt{5}-2}+2\right)^2\right)$$
$$+\frac{\left(\pi+4\arctan\left(\frac{1}{2}\left(x-\sqrt{x^2+2\,x+2}\right)\left(2\sqrt{5}\sqrt{\sqrt{5}-2}+\sqrt{5}+4\sqrt{\sqrt{5}-2}+3\right)+\frac{3}{2}\sqrt{5}\sqrt{\sqrt{5}-2}+\frac{1}{2}\right.\right.}{4\left(\sqrt{5}-1\right)}$$
$$-\frac{\left(\pi+4\arctan\left(-\frac{1}{2}\left(x-\sqrt{x^2+2\,x+2}\right)\left(2\sqrt{5}\sqrt{\sqrt{5}-2}-\sqrt{5}+4\sqrt{\sqrt{5}-2}-3\right)-\frac{3}{2}\sqrt{5}\sqrt{\sqrt{5}-2}+\right.\right.}{4\left(\sqrt{5}-1\right)}$$

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(2*sqrt(5) - 2)*log(256*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x +
sqrt(5)*sqrt(sqrt(5) - 2) + sqrt(5) + 2*sqrt(x^2 + 2*x + 2) - 2*sqrt(sqrt(5
) - 2) - 2)^2 + 256*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x - sqrt(5) + 2*
sqrt(x^2 + 2*x + 2) + sqrt(sqrt(5) - 2) + 2)^2) - 1/4*sqrt(2*sqrt(5) - 2)*l
og(256*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x - sqrt(5)*sqrt(sqrt(5) - 2)
 + sqrt(5) + 2*sqrt(x^2 + 2*x + 2) + 2*sqrt(sqrt(5) - 2) - 2)^2 + 256*(sqrt
(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x - sqrt(5) + 2*sqrt(x^2 + 2*x + 2) - sqr
t(sqrt(5) - 2) + 2)^2) + 1/4*(pi + 4*arctan(1/2*(x - sqrt(x^2 + 2*x + 2))*(
```

```
2*sqrt(5)*sqrt(sqrt(5) - 2) + sqrt(5) + 4*sqrt(sqrt(5) - 2) + 3) + 3/2*sqrt
(5)*sqrt(sqrt(5) - 2) + 1/2*sqrt(5) + 7/2*sqrt(sqrt(5) - 2) + 3/2))*sqrt(2*
sqrt(5) - 2)/(sqrt(5) - 1) - 1/4*(pi + 4*arctan(-1/2*(x - sqrt(x^2 + 2*x +
2))*(2*sqrt(5)*sqrt(sqrt(5) - 2) - sqrt(5) + 4*sqrt(sqrt(5) - 2) - 3) - 3/2
*sqrt(5)*sqrt(sqrt(5) - 2) + 1/2*sqrt(5) - 7/2*sqrt(sqrt(5) - 2) + 3/2))*sq
rt(2*sqrt(5) - 2)/(sqrt(5) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx=\int \frac{2\,x+1}{\left(x^2+1\right)\,\sqrt{x^2+2\,x+2}}\,dx$$

```
[In] int((2*x + 1)/((x^2 + 1)*(2*x + x^2 + 2)^(1/2)),x)
[Out] int((2*x + 1)/((x^2 + 1)*(2*x + x^2 + 2)^(1/2)), x)
```

3.999 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 22

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \arctan\left(\frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)$$

```
[Out] arctan(x/(-x^2+(x^4+1)^(1/2))^(1/2))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2153, 209}

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \arctan\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

```
[In] Int[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2153

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
```

```
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right) \\ &= \tan^{-1}\left(\frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right) \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.36

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = i\text{arctanh}\left(\sqrt{2}+\sqrt{2}x^4-ix^3\sqrt{-x^2+\sqrt{1+x^4}} + \frac{\sqrt{1+x^4}\left(-2x^2+i\sqrt{2}x\sqrt{-x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}\right)$$

```
[In] Integrate[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
[Out] I*ArcTanh[Sqrt[2] + Sqrt[2]*x^4 - I*x^3*Sqrt[-x^2 + Sqrt[1 + x^4]] + (Sqrt[
1 + x^4]*(-2*x^2 + I*Sqrt[2]*x*Sqrt[-x^2 + Sqrt[1 + x^4]]))/Sqrt[2]]
```

Maple [F]

$$\int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}dx$$

```
[In] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
[Out] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(18) = 36$.

Time = 0.63 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.82

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{4}\arctan\left(\frac{4\left(10\,x^7-6\,x^3+(7\,x^5-x)\sqrt{x^4+1}\right)\sqrt{-x^2+\sqrt{x^4+1}}}{17\,x^8-46\,x^4+1}\right)$$

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
[Out] -1/4*arctan(4*(10*x^7 - 6*x^3 + (7*x^5 - x)*sqrt(x^4 + 1))*sqrt(-x^2 + sqrt
(x^4 + 1))/(17*x^8 - 46*x^4 + 1))
```

Sympy [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{-x^2+\sqrt{x^4+1}}\,(x^4+1)}\,dx$$

```
[In] integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)
[Out] Integral(1/(sqrt(-x**2 + sqrt(x**4 + 1))*(x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

Giac [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{\sqrt{x^4+1}-x^2}\,(x^4+1)}\,dx$$

```
[In] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)),x)
[Out] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)), x)
```

3.1000 $\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 40

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}$$

```
[Out] arctanh(x*c^(1/2)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2))/a/c^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2153, 212}

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}+cx^2}}\right)}{a\sqrt{c}}$$

```
[In] Int[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2153

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
```

```
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx = \frac{\text{arctanh}\left(\frac{\sqrt{cx^2+d\sqrt{a+bx^4}}}{\sqrt{c}x}\right)}{a\sqrt{c}}$$

```
[In] Integrate[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]
[Out] ArcTanh[Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]/(Sqrt[c]*x)]/(a*Sqrt[c])
```

Maple [F]

$$\int \frac{1}{(b\,x^4+a)\sqrt{c\,x^2+d\sqrt{b\,x^4+a}}}dx$$

```
[In] int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
[Out] int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx$$

```
[In] integrate(1/(b*x**4+a)/(c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
[Out] Integral(1/((a + b*x**4)*sqrt(c*x**2 + d*sqrt(a + b*x**4))), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{cx^2+\sqrt{bx^4+a}d}}\,dx$$

```
[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{cx^2+\sqrt{bx^4+a}d}}\,dx$$

```
[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{(b\,x^4+a)\,\sqrt{d\,\sqrt{b\,x^4+a}+c\,x^2}}\,dx$$

```
[In] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) + c*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) + c*x^2)^(1/2)), x)
```

3.1001 $\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 41

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx = \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{-cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}$$

```
[Out] arctan(x*c^(1/2)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2))/a/c^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2153, 209}

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx = \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}-cx^2}}\right)}{a\sqrt{c}}$$

```
[In] Int[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2153

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
```

```
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx, x, \frac{x}{\sqrt{-cx^2+d\sqrt{a+bx^4}}}\right)}{a}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{-cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx = -\frac{\arctan\left(\frac{\sqrt{-cx^2+d\sqrt{a+bx^4}}}{\sqrt{c}x}\right)}{a\sqrt{c}}$$

```
[In] Integrate[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]
[Out] -(ArcTan[Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]/(Sqrt[c]*x)]/(a*Sqrt[c]))
```

Maple [F]

$$\int \frac{1}{(b\,x^4+a)\sqrt{-c\,x^2+d\sqrt{b\,x^4+a}}}dx$$

```
[In] int(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
[Out] int(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="fricas
")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx$$

```
[In] integrate(1/(b*x**4+a)/(-c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
[Out] Integral(1/((a + b*x**4)*sqrt(-c*x**2 + d*sqrt(a + b*x**4))), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{-cx^2+\sqrt{bx^4+a}\,d}}\,dx$$

```
[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="maxima
")
[Out] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{-cx^2+\sqrt{bx^4+a}\,d}}\,dx$$

```
[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{(b\,x^4+a)\,\sqrt{d\,\sqrt{b\,x^4+a}-c\,x^2}}\,dx$$

```
[In] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) - c*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) - c*x^2)^(1/2)), x)
```

3.1002 $\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$

Optimal result

Integrand size = 51, antiderivative size = 184

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}\right)}{2\sqrt{b}d^2} - \frac{c\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

```
[Out] 1/2*arctanh(d^2*(c/d+x)^2*b^(1/2)/(a+b*d^4*(c/d+x)^4)^(1/2))/d^2/b^(1/2)-1/
2*c*(cos(2*arctan(b^(1/4)*(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(
d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*(d*x+c)/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+d^2*(c/d+x)^2*b^(1/2))*((a+b*d^4*(c/d+x)^4)/(a^(1/2)+d^2*(c/d+
x)^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/d^2/(a+b*d^4*(c/d+x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used

$= \{1694, 1899, 226, 281, 223, 212\}$

$$\int \frac{x}{\sqrt{a + bc^4 + 4bc^3dx + 6bc^2d^2x^2 + 4bcd^3x^3 + bd^4x^4}}\,dx$$
$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}\right)}{2\sqrt{b}d^2} - \frac{c\left(\sqrt{a} + \sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a + bd^4\left(\frac{c}{d}+x\right)^4}}$$

```
[In] Int[x/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d^
4*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*d^2*(c/d + x)^2)/Sqrt[a + b*d^4*(c/d + x)^4]]/(2*Sqrt[b]*d
^2) - (c*(Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)*Sqrt[(a + b*d^4*(c/d + x)^4)/(
Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x)
)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*d^2*Sqrt[a + b*d^4*(c/d + x)^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
```

```
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{-\frac{c}{d}+x}{\sqrt{a+bd^4x^4}}\,dx, x, \frac{c}{d}+x\right)\\
&= \text{Subst}\left(\int \left(-\frac{c}{d\sqrt{a+bd^4x^4}}+\frac{x}{\sqrt{a+bd^4x^4}}\right)dx, x, \frac{c}{d}+x\right)\\
&= -\frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+bd^4x^4}}\,dx, x, \frac{c}{d}+x\right)}{d}+\text{Subst}\left(\int \frac{x}{\sqrt{a+bd^4x^4}}\,dx, x, \frac{c}{d}+x\right)\\
&= -\frac{c\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a+b(c+dx)^4}}\\
&\quad+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a+bd^4x^2}}\,dx, x, \left(\frac{c}{d}+x\right)^2\right)\\
&= -\frac{c\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a+b(c+dx)^4}}\\
&\quad+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-bd^4x^2}\,dx, x, \frac{\left(\frac{c}{d}+x\right)^2}{\sqrt{a+b(c+dx)^4}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a+b(c+dx)^4}}\right)}{2\sqrt{b}d^2}-\frac{c\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a+b(c+dx)^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.36 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.11

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$$
$$= \frac{\sqrt[4]{-1}\left(\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)\right)^2\sqrt{\frac{(1-i)\left((-1)^{3/4}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)}}\sqrt{-\frac{i\left(\sqrt[4]{-1}\sqrt[4]{a}+\sqrt[4]{b}(c+dx)\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)}}\sqrt{-\frac{(1+i)\left((-1)^{3/4}\sqrt[4]{a}\right.}{\sqrt[4]{-1}\sqrt[4]{a}-}}}{}$$

```
[In] Integrate[x/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3
+ b*d^4*x^4],x]
```

```
[Out] ((-1)^(1/4)*((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))^2*Sqrt[((1 - I)*((-1)^
(3/4)*a^(1/4) - b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x)
)]*Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4)
 - b^(1/4)*(c + d*x))]*Sqrt[((-1 - I)*((-1)^(3/4)*a^(1/4) + b^(1/4)*(c + d*
x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*(((-1)^(1/4)*a^(1/4) - b^(1/
4)*c)*EllipticF[ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))
/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1] - 2*(-1)^(1/4)*a^(1/4)*Ell
ipticPi[-I, ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-
1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1]))/(a^(1/4)*Sqrt[b]*d^2*Sqrt[a
+ b*(c + d*x)^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.99 (sec) , antiderivative size = 1528, normalized size of antiderivative = 8.30

method	result	size
default	Expression too large to display	1528
elliptic	Expression too large to display	1528

```
[In] int(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 2*(-(-I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)*(((-I/b*(-a*b^3)^(1
/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*
b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/
2)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*(((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^
(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)/((-1/b*(-a*b^3)^(1/4)-c)/d-(1/b*(
```

```
-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(((I/b*(-a*b^3)^(1/
4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*
b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/
2)/((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4
)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(b*d^4*(x-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(I/
b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4
)-c)/d))^(1/2)*((I/b*(-a*b^3)^(1/4)-c)/d*EllipticF((((-I/b*(-a*b^3)^(1/4)-c
)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^
(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2),((
(I/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)*(-(-I/b*(-a*b^3)^(1/4)-
c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/
4)-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d))^(1/2))+(-(I/
b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)*EllipticPi((((-I/b*(-a*b^3)
^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(
-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^
(1/2),((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^
(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d),(((I/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b
^3)^(1/4)-c)/d)*(-(-I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)/((1/b
*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(
-I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)))
```

Fricas [F]

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{x}{\sqrt{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}}\,dx$$

```
[In] integrate(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x +
 b*c^4 + a), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$$

```
[In] integrate(x/(b*d**4*x**4+4*b*c*d**3*x**3+6*b*c**2*d**2*x**2+4*b*c**3*d*x+b*
c**4+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3
*x**3 + b*d**4*x**4), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{x}{\sqrt{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}}\,dx$$

```
[In] integrate(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{x}{\sqrt{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}}\,dx$$

```
[In] integrate(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{x}{\sqrt{b\,c^4+4\,b\,c^3\,d\,x+6\,b\,c^2\,d^2\,x^2+4\,b\,c\,d^3\,x^3+b\,d^4\,x^4+a}}\,dx$$

```
[In] int(x/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x^
3)^(1/2),x)
```

```
[Out] int(x/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x^
3)^(1/2), x)
```

3.1003 $\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 131

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

```
[Out] 1/2*(cos(2*arctan(b^(1/4)*(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(
d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*(d*x+c)/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+d^2*(c/d+x)^2*b^(1/2))*((a+b*d^4*(c/d+x)^4)/(a^(1/2)+d^2*(c/d+
x)^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/d/(a+b*d^4*(c/d+x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.041, Rules used = {1120, 226}

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

```
[In] Int[1/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d^
4*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)*Sqrt[(a + b*d^4*(c/d + x)^4)/(Sqrt[a]
+ Sqrt[b]*d^2*(c/d + x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4
)], 1/2])/(2*a^(1/4)*b^(1/4)*d*Sqrt[a + b*d^4*(c/d + x)^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{\sqrt{a+bd^4x^4}}\,dx, x, \frac{c}{d}+x\right)$$
$$= \frac{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$$
$$= -\frac{i\sqrt{\frac{a+b(c+dx)^4}{a}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}(c+dx)\right),-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}d\sqrt{a+b(c+dx)^4}}$$

```
[In] Integrate[1/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3
+ b*d^4*x^4],x]
```

```
[Out] ((-I)*Sqrt[(a + b*(c + d*x)^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt
[a]]*(c + d*x)], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*d*Sqrt[a + b*(c + d*x)^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 1036, normalized size of antiderivative = 7.91

method	result	size
default	Expression too large to display	1036
elliptic	Expression too large to display	1036

```
[In] int(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 2*(-(-I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)*(((-I/b*(-a*b^3)^(1
/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*
b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/
2)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*(((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^
(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)/((-1/b*(-a*b^3)^(1/4)-c)/d-(1/b*(
-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(((I/b*(-a*b^3)^(1/
4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*
b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/
2)/((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4
)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(b*d^4*(x-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(I/
b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4
)-c)/d))^(1/2)*EllipticF((((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)
/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(
1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2),(((I/b*(-a*b^3)^(1/4)-c)/d-(
-1/b*(-a*b^3)^(1/4)-c)/d)*(-(-I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c
)/d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1
/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d))^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{1}{\sqrt{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}}\,dx$$

```
[In] integrate(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x +
 b*c^4 + a), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$$

```
[In] integrate(1/(b*d**4*x**4+4*b*c*d**3*x**3+6*b*c**2*d**2*x**2+4*b*c**3*d*x+b*
c**4+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3
*x**3 + b*d**4*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{1}{\sqrt{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}}\,dx$$

```
[In] integrate(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{1}{\sqrt{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}}\,dx$$

```
[In] integrate(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$$
$$=\int \frac{1}{\sqrt{b\,c^4+4\,b\,c^3\,d\,x+6\,b\,c^2\,d^2\,x^2+4\,b\,c\,d^3\,x^3+b\,d^4\,x^4+a}}\,dx$$

```
[In] int(1/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x^
3)^(1/2),x)
[Out] int(1/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x^
3)^(1/2), x)
```

3.1004 $\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}(ad+aex^2+cdx^4)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 54

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{bd-ae}\,x}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}$$

```
[Out] arctanh(x*(-a*e+b*d)^(1/2)/d^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^(1/2)/(-a*e+b*d
)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2137, 214}

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = \frac{\operatorname{arctanh}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}$$

```
[In] Int[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

```
[Out] ArcTanh[(Sqrt[b*d - a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2 + c*x^4])]/(Sqrt[d]*Sqr
t[b*d - a*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2137

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= a\text{Subst}\left(\int \frac{1}{ad-(abd-a^2e)\,x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2+cx^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{bd-ae}x}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 12.41 (sec) , antiderivative size = 419, normalized size of antiderivative = 7.76

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx$$
$$= \frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right), \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right) - \text{EllipticPi}\left(\frac{(\ldots}{ae-\ldots}\right.\right.}{\sqrt{2}\ldots}$$

```
[In] Integrate[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x
]
```

```
[Out] (I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
- EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(a*e - Sqrt[a]*Sqrt[-4*c*d^2 + a*e
^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/
(a*e + Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[
b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt
[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\arctan\left(\frac{d\sqrt{c\,x^{4}+b\,x^{2}+a}}{x\sqrt{(ae-bd)d}}\right)}{\sqrt{(ae-bd)d}}$	47
elliptic	$-\frac{\arctan\left(\frac{d\sqrt{c\,x^{4}+b\,x^{2}+a}}{x\sqrt{(ae-bd)d}}\right)}{\sqrt{(ae-bd)d}}$	47
pseudoelliptic	$-\frac{\arctan\left(\frac{d\sqrt{c\,x^{4}+b\,x^{2}+a}}{x\sqrt{(ae-bd)d}}\right)}{\sqrt{(ae-bd)d}}$	47

```
[In] int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/((a*e-b*d)*d)^(1/2)*arctan(d*(c*x^4+b*x^2+a)^(1/2)/x/((a*e-b*d)*d)^(1/2)
)
```

Fricas [A] (verification not implemented)

none

Time = 7.88 (sec) , antiderivative size = 305, normalized size of antiderivative = 5.65

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx$$
$$=\left[\frac{\log\left(-\frac{c^2d^2x^8+2\,(4\,bcd^2-3\,acde)x^6-(8\,abde-a^2e^2-2\,(4\,b^2+ac)d^2)x^4+a^2d^2+2\,(4\,abd^2-3\,a^2de)x^2+4\,(cdx^5+(2\,bd-ae)x^3+adx)\sqrt{cx^4+bx^2+}}{c^2d^2x^8+2\,acdex^6+2\,a^2dex^2+(2\,acd^2+a^2e^2)x^4+a^2d^2}\right.}{4\sqrt{bd^2-ade}}\right.$$
$$\left.-\frac{\sqrt{-bd^2+ade}\arctan\left(\frac{2\sqrt{cx^4+bx^2+a}\sqrt{-bd^2+ade}x}{cdx^4+(2\,bd-ae)x^2+ad}\right)}{2\,(bd^2-ade)}\right]$$

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*log(-(c^2*d^2*x^8 + 2*(4*b*c*d^2 - 3*a*c*d*e)*x^6 - (8*a*b*d*e - a^2*e
^2 - 2*(4*b^2 + a*c)*d^2)*x^4 + a^2*d^2 + 2*(4*a*b*d^2 - 3*a^2*d*e)*x^2 + 4
*(c*d*x^5 + (2*b*d - a*e)*x^3 + a*d*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(b*d^2 -
 a*d*e))/(c^2*d^2*x^8 + 2*a*c*d*e*x^6 + 2*a^2*d*e*x^2 + (2*a*c*d^2 + a^2*e^
2)*x^4 + a^2*d^2))/sqrt(b*d^2 - a*d*e), -1/2*sqrt(-b*d^2 + a*d*e)*arctan(2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-b*d^2 + a*d*e)*x/(c*d*x^4 + (2*b*d - a*e)*x^2
 + a*d))/(b*d^2 - a*d*e)]
```

Sympy [F]

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx$$
$$= -\int \left(-\frac{a}{ad\sqrt{a+bx^2+cx^4}+aex^2\sqrt{a+bx^2+cx^4}+cdx^4\sqrt{a+bx^2+cx^4}}\right)dx$$
$$-\int \frac{cx^4}{ad\sqrt{a+bx^2+cx^4}+aex^2\sqrt{a+bx^2+cx^4}+cdx^4\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*sqrt(a + b*x**2 + c*x**4) + a*e*x**2*sqrt(a + b*x**2 + c*
x**4) + c*d*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*sqrt
(a + b*x**2 + c*x**4) + a*e*x**2*sqrt(a + b*x**2 + c*x**4) + c*d*x**4*sqrt(
a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = \int -\frac{cx^4-a}{(cdx^4+aex^2+ad)\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a)),
 x)
```

Giac [F]

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = \int -\frac{cx^4-a}{(cdx^4+aex^2+ad)\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx=\int \frac{a-c\,x^4}{\left(c\,d\,x^4+a\,e\,x^2+a\,d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.1005 $\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}(ad+aex^2+cdx^4)}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 53

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx=\frac{\arctan\left(\frac{\sqrt{bd+ae}x}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd+ae}}$$

```
[Out] arctan(x*(a*e+b*d)^(1/2)/d^(1/2)/(c*x^4-b*x^2+a)^(1/2))/d^(1/2)/(a*e+b*d)^(
1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2137, 211}

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx=\frac{\arctan\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae+bd}}$$

```
[In] Int[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

```
[Out] ArcTan[(Sqrt[b*d + a*e]*x)/(Sqrt[d]*Sqrt[a - b*x^2 + c*x^4])]/(Sqrt[d]*Sqrt
[b*d + a*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2137

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a\text{Subst}\left(\int \frac{1}{ad-(-abd-a^2e)\,x^2}\,dx, x, \frac{x}{\sqrt{a-bx^2+cx^4}}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{bd+ae}\,x}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd+ae}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 11.91 (sec) , antiderivative size = 416, normalized size of antiderivative = 7.85

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = \frac{i\sqrt{2+\frac{4cx^2}{-b+\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{-b+\sqrt{b^2-4ac}}}x\right), \frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)-\text{EllipticPi}\left(\right.\right.}{2}$$

```
[In] Integrate[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x
]
```

```
[Out] ((I/2)*Sqrt[2 + (4*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4
*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])] - EllipticPi[(
(b - Sqrt[b^2 - 4*a*c])*d)/(-(a*e) + Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*Arc
Sinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(
b + Sqrt[b^2 - 4*a*c])] - EllipticPi[((-b + Sqrt[b^2 - 4*a*c])*d)/(a*e + Sq
rt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*
a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])]))/(Sqrt[c/(-b +
 Sqrt[b^2 - 4*a*c])]*d*Sqrt[a - b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\arctan\left(\frac{d\sqrt{c\,x^{4}-b\,x^{2}+a}}{x\sqrt{(ae+bd)d}}\right)}{\sqrt{(ae+bd)d}}$	46
elliptic	$-\frac{\arctan\left(\frac{d\sqrt{c\,x^{4}-b\,x^{2}+a}}{x\sqrt{(ae+bd)d}}\right)}{\sqrt{(ae+bd)d}}$	46
pseudoelliptic	$-\frac{\arctan\left(\frac{d\sqrt{c\,x^{4}-b\,x^{2}+a}}{x\sqrt{(ae+bd)d}}\right)}{\sqrt{(ae+bd)d}}$	46

```
[In] int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/((a*e+b*d)*d)^(1/2)*arctan(d*(c*x^4-b*x^2+a)^(1/2)/x/((a*e+b*d)*d)^(1/2)
)
```

Fricas [A] (verification not implemented)

none

Time = 7.66 (sec) , antiderivative size = 304, normalized size of antiderivative = 5.74

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx$$
$$= \left[-\frac{\sqrt{-bd^2-ade}\log\left(-\frac{c^2d^2x^8-2\,(4\,bcd^2+3\,acde)x^6+(8\,abde+a^2e^2+2\,(4\,b^2+ac)d^2)x^4+a^2d^2-2\,(4\,abd^2+3\,a^2de)x^2+4\,(cdx^5-(2\,bd+a}{c^2d^2x^8+2\,acdex^6+2\,a^2dex^2+(2\,acd^2+a^2e^2)x^4+a^2d^2}\right.}{4\,(bd^2+ade)}\right.$$

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/4*sqrt(-b*d^2 - a*d*e)*log(-(c^2*d^2*x^8 - 2*(4*b*c*d^2 + 3*a*c*d*e)*x^
6 + (8*a*b*d*e + a^2*e^2 + 2*(4*b^2 + a*c)*d^2)*x^4 + a^2*d^2 - 2*(4*a*b*d^
2 + 3*a^2*d*e)*x^2 + 4*(c*d*x^5 - (2*b*d + a*e)*x^3 + a*d*x)*sqrt(c*x^4 - b
*x^2 + a)*sqrt(-b*d^2 - a*d*e))/(c^2*d^2*x^8 + 2*a*c*d*e*x^6 + 2*a^2*d*e*x^
2 + (2*a*c*d^2 + a^2*e^2)*x^4 + a^2*d^2))/(b*d^2 + a*d*e), 1/2*arctan(2*sqr
t(c*x^4 - b*x^2 + a)*sqrt(b*d^2 + a*d*e)*x/(c*d*x^4 - (2*b*d + a*e)*x^2 + a
*d))/sqrt(b*d^2 + a*d*e)]
```

Sympy [F]

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = -\int \left(-\frac{a}{ad\sqrt{a-bx^2+cx^4}+aex^2\sqrt{a-bx^2+cx^4}+cdx^4\sqrt{a-bx^2+cx^4}}\right)dx - \int \frac{cx^4}{ad\sqrt{a-bx^2+cx^4}+aex^2\sqrt{a-bx^2+cx^4}+cdx^4\sqrt{a-bx^2+cx^4}}\,dx$$

```
[In] integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4-b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*sqrt(a - b*x**2 + c*x**4) + a*e*x**2*sqrt(a - b*x**2 + c*
x**4) + c*d*x**4*sqrt(a - b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*sqrt
(a - b*x**2 + c*x**4) + a*e*x**2*sqrt(a - b*x**2 + c*x**4) + c*d*x**4*sqrt(
a - b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = \int -\frac{cx^4-a}{(cdx^4+aex^2+ad)\sqrt{cx^4-bx^2+a}}\,dx$$

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*x^2 + a)),
 x)
```

Giac [F]

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = \int -\frac{cx^4-a}{(cdx^4+aex^2+ad)\sqrt{cx^4-bx^2+a}}\,dx$$

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*x^2 + a)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx=\int \frac{a-c\,x^4}{(c\,d\,x^4+a\,e\,x^2+a\,d)\,\sqrt{c\,x^4-b\,x^2+a}}\,dx$$

```
[In] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a - b*x^2 + c*x^4)^(1/2)),x)
[Out] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a - b*x^2 + c*x^4)^(1/2)), x)
```

3.1006 $\int \frac{1}{\sqrt{5-2x+x^2}(8+x^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx = -\frac{\arctan\left(\frac{1-x}{\sqrt{3}\sqrt{5-2x+x^2}}\right)}{4\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{7-3x}{\sqrt{13}\sqrt{5-2x+x^2}}\right)}{12\sqrt{13}} + \frac{1}{12}\operatorname{arctanh}\left(\sqrt{5-2x+x^2}\right)$$

```
[Out] 1/12*arctanh((x^2-2*x+5)^(1/2))-1/12*arctan(1/3*(1-x)*3^(1/2)/(x^2-2*x+5)^(
1/2))*3^(1/2)-1/156*arctanh(1/13*(7-3*x)*13^(1/2)/(x^2-2*x+5)^(1/2))*13^(1/
2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2099, 738, 212, 1039, 996, 209, 1038, 213}

$$\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx = -\frac{\arctan\left(\frac{1-x}{\sqrt{3}\sqrt{x^2-2x+5}}\right)}{4\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}} + \frac{1}{12}\operatorname{arctanh}\left(\sqrt{x^2-2x+5}\right)$$

```
[In] Int[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]
```

```
[Out] -1/4*ArcTan[(1 - x)/(Sqrt[3]*Sqrt[5 - 2*x + x^2])]/Sqrt[3] - ArcTanh[(7 - 3
*x)/(Sqrt[13]*Sqrt[5 - 2*x + x^2])]/(12*Sqrt[13]) + ArcTanh[Sqrt[5 - 2*x +
x^2]]/12
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1038

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1039

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
```

```
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{12(2+x)\sqrt{5-2x+x^2}}+\frac{4-x}{12\left(4-2x+x^2\right)\sqrt{5-2x+x^2}}\right)dx\\
&= \frac{1}{12}\int \frac{1}{(2+x)\sqrt{5-2x+x^2}}\,dx+\frac{1}{12}\int \frac{4-x}{\left(4-2x+x^2\right)\sqrt{5-2x+x^2}}\,dx\\
&= -\left(\frac{1}{24}\int \frac{-2+2x}{\left(4-2x+x^2\right)\sqrt{5-2x+x^2}}\,dx\right)\\
&\quad -\frac{1}{6}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{14-6x}{\sqrt{5-2x+x^2}}\right)\\
&\quad +\frac{1}{4}\int \frac{1}{\left(4-2x+x^2\right)\sqrt{5-2x+x^2}}\,dx\\
&= -\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{5-2x+x^2}}\right)}{12\sqrt{13}}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{-2+2x^2}\,dx,x,\sqrt{5-2x+x^2}\right)\\
&\quad +\text{Subst}\left(\int \frac{1}{24+2x^2}\,dx,x,\frac{-2+2x}{\sqrt{5-2x+x^2}}\right)\\
&= \frac{\tan^{-1}\left(\frac{-2+2x}{2\sqrt{3}\sqrt{5-2x+x^2}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{5-2x+x^2}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{5-2x+x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\begin{aligned}
&\int \frac{1}{\sqrt{5-2x+x^2}\left(8+x^3\right)}\,dx\\
&= \frac{1}{156}\left(-13\sqrt{3}\arctan\left(\frac{4-2x+x^2-(-1+x)\sqrt{5-2x+x^2}}{\sqrt{3}}\right)\right.\\
&\qquad \left.+13\text{arctanh}\left(\sqrt{5-2x+x^2}\right)+2\sqrt{13}\text{arctanh}\left(\frac{2+x-\sqrt{5-2x+x^2}}{\sqrt{13}}\right)\right)
\end{aligned}$$

```
[In] Integrate[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]
```

```
[Out] (-13*Sqrt[3]*ArcTan[(4 - 2*x + x^2 - (-1 + x)*Sqrt[5 - 2*x + x^2])/Sqrt[3]]
 + 13*ArcTanh[Sqrt[5 - 2*x + x^2]] + 2*Sqrt[13]*ArcTanh[(2 + x - Sqrt[5 - 2
*x + x^2])/Sqrt[13]])/156
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

method	result
default	$-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{(14-6x)\sqrt{13}}{26\sqrt{(x+2)^2-6x+1}}\right)}{156}+\frac{\operatorname{arctanh}\left(\sqrt{x^2-2x+5}\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2x-2)}{6\sqrt{x^2-2x+5}}\right)}{12}$
trager	$\operatorname{RootOf}\left(144\_Z^2+12\_Z+1\right)\ln\left(\frac{2880\operatorname{RootOf}\left(144\_Z^2+12\_Z+1\right)^2 x+126\operatorname{RootOf}\left(144\_Z^2+12\_Z+1\right)\sqrt{x^2-2}}{12\operatorname{RootOf}}\right.$

```
[In] int(1/(x^3+8)/(x^2-2*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/156*13^(1/2)*arctanh(1/26*(14-6*x)*13^(1/2)/((x+2)^2-6*x+1)^(1/2))+1/12*
arctanh((x^2-2*x+5)^(1/2))+1/12*3^(1/2)*arctan(1/6*3^(1/2)/(x^2-2*x+5)^(1/2
)*(2*x-2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(64) = 128.

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.83

$$\begin{aligned}
&\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx \\
&= \frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x-2)+\frac{1}{3}\sqrt{3}\sqrt{x^2-2x+5}\right) \\
&\quad -\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^2-2x+5}\right) \\
&\quad +\frac{1}{156}\sqrt{13}\log\left(-\frac{\sqrt{13}(3x-7)+\sqrt{x^2-2x+5}(3\sqrt{13}+13)+9x-21}{x+2}\right) \\
&\quad +\frac{1}{24}\log\left(x^2-\sqrt{x^2-2x+5}(x-2)-3x+6\right)-\frac{1}{24}\log\left(x^2-\sqrt{x^2-2x+5}x-x+4\right)
\end{aligned}$$

```
[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - 2) + 1/3*sqrt(3)*sqrt(x^2 - 2*x + 5))
 - 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 - 2*x + 5)) +
1/156*sqrt(13)*log(-(sqrt(13)*(3*x - 7) + sqrt(x^2 - 2*x + 5)*(3*sqrt(13) +
```

```
13) + 9*x - 21)/(x + 2)) + 1/24*log(x^2 - sqrt(x^2 - 2*x + 5)*(x - 2) - 3*
x + 6) - 1/24*log(x^2 - sqrt(x^2 - 2*x + 5)*x - x + 4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx = \int \frac{1}{(x+2)\,(x^2-2x+4)\,\sqrt{x^2-2x+5}}\,dx$$

```
[In] integrate(1/(x**3+8)/(x**2-2*x+5)**(1/2),x)
[Out] Integral(1/((x + 2)*(x**2 - 2*x + 4)*sqrt(x**2 - 2*x + 5)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx = \int \frac{1}{(x^3+8)\sqrt{x^2-2\,x+5}}\,dx$$

```
[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 8)*sqrt(x^2 - 2*x + 5)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(64) = 128.

Time = 0.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.95

$$\begin{aligned}\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx &= -\frac{1}{12}\,\sqrt{3}\arctan\left(-\frac{1}{3}\,\sqrt{3}\left(x-\sqrt{x^2-2\,x+5}\right)\right) \\ &\quad+\frac{1}{12}\,\sqrt{3}\arctan\left(-\frac{1}{3}\,\sqrt{3}\left(x-\sqrt{x^2-2\,x+5}-2\right)\right) \\ &\quad+\frac{1}{156}\,\sqrt{13}\log\left(\frac{\left|-2\,x-2\,\sqrt{13}+2\,\sqrt{x^2-2\,x+5}-4\right|}{\left|-2\,x+2\,\sqrt{13}+2\,\sqrt{x^2-2\,x+5}-4\right|}\right) \\ &\quad+\frac{1}{24}\,\log\left(\left(x-\sqrt{x^2-2\,x+5}\right)^2-4\,x+4\,\sqrt{x^2-2\,x+5}\right. \\ &\quad\left.+7\right)-\frac{1}{24}\,\log\left(\left(x-\sqrt{x^2-2\,x+5}\right)^2+3\right)\end{aligned}$$

```
[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="giac")
[Out] -1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 - 2*x + 5))) + 1/12*sqrt(3)
*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 - 2*x + 5) - 2)) + 1/156*sqrt(13)*log(ab
s(-2*x - 2*sqrt(13) + 2*sqrt(x^2 - 2*x + 5) - 4)/abs(-2*x + 2*sqrt(13) + 2*
sqrt(x^2 - 2*x + 5) - 4)) + 1/24*log((x - sqrt(x^2 - 2*x + 5))^2 - 4*x + 4*
sqrt(x^2 - 2*x + 5) + 7) - 1/24*log((x - sqrt(x^2 - 2*x + 5))^2 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx = \int \frac{1}{(x^3+8)\,\sqrt{x^2-2\,x+5}}\,dx$$

```
[In] int(1/((x^3 + 8)*(x^2 - 2*x + 5)^(1/2)),x)
[Out] int(1/((x^3 + 8)*(x^2 - 2*x + 5)^(1/2)), x)
```

3.1007 $\int \sqrt{\frac{x^2}{1+x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \frac{\sqrt{x^2}\sqrt{1+x^2}}{x}$$

```
[Out] (x^2)^(1/2)*(x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1978, 15, 267}

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \frac{\sqrt{x^2}\sqrt{x^2+1}}{x}$$

```
[In] Int[Sqrt[x^2/(1 + x^2)],x]
```

```
[Out] (Sqrt[x^2]*Sqrt[1 + x^2])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt{x^2}}{\sqrt{1+x^2}}\,dx\\ &= \frac{\sqrt{x^2}\int \frac{x}{\sqrt{1+x^2}}\,dx}{x}\\ &= \frac{\sqrt{x^2}\sqrt{1+x^2}}{x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \frac{x}{\sqrt{\frac{x^2}{1+x^2}}}$$

```
[In] Integrate[Sqrt[x^2/(1 + x^2)],x]
[Out] x/Sqrt[x^2/(1 + x^2)]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

method	result	size
gosper	$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23
default	$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23
trager	$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23
risch	$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23

```
[In] int((x^2/(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] (x^2+1)/x*(x^2/(x^2+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$$

```
[In] integrate((x^2/(x^2+1))^(1/2),x, algorithm="fricas")
[Out] (x^2 + 1)*sqrt(x^2/(x^2 + 1))/x
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = x\sqrt{\frac{x^2}{x^2+1}} + \frac{\sqrt{\frac{x^2}{x^2+1}}}{x}$$

```
[In] integrate((x**2/(x**2+1))**(1/2),x)
[Out] x*sqrt(x**2/(x**2 + 1)) + sqrt(x**2/(x**2 + 1))/x
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.35

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \sqrt{x^2+1}$$

```
[In] integrate((x^2/(x^2+1))^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \sqrt{x^2+1}\text{sgn}(x) - \text{sgn}(x)$$

```
[In] integrate((x^2/(x^2+1))^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 + 1)*sgn(x) - sgn(x)
```

Mupad [B] (verification not implemented)

Time = 21.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \frac{\sqrt{x^4+x^2}}{x}$$

```
[In] int((x^2/(x^2 + 1))^(1/2),x)
[Out] (x^2 + x^4)^(1/2)/x
```

3.1008 $\int \sqrt{\frac{x^n}{1+x^n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx=\frac{2x\sqrt{x^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-x^n\right)}{2+n}$$

```
[Out] 2*x*hypergeom([1/2, 1/2+1/n],[3/2+1/n],-x^n)*(x^n)^(1/2)/(2+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1978, 15, 371}

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx=\frac{2x\sqrt{x^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-x^n\right)}{n+2}$$

```
[In] Int[Sqrt[x^n/(1 + x^n)],x]
```

```
[Out] (2*x*Sqrt[x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3 + 2/n)/2, -x^n])/(2 +
 n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt{x^n}}{\sqrt{1+x^n}}\,dx\\ &= \left(x^{-n/2}\sqrt{x^n}\right)\int \frac{x^{n/2}}{\sqrt{1+x^n}}\,dx\\ &= \frac{2x\sqrt{x^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{2+n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \frac{2x\sqrt{x^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n},\frac{3}{2}+\frac{1}{n},-x^n\right)}{2+n}$$

```
[In] Integrate[Sqrt[x^n/(1 + x^n)],x]
[Out] (2*x*Sqrt[x^n]*Hypergeometric2F1[1/2, 1/2 + n^(-1), 3/2 + n^(-1), -x^n])/(2
 + n)
```

Maple [F]

$$\int \sqrt{\frac{x^n}{1+x^n}}dx$$

```
[In] int((x^n/(1+x^n))^(1/2),x)
[Out] int((x^n/(1+x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((x^n/(1+x^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

```
[In] integrate((x**n/(1+x**n))**(1/2),x)
[Out] Integral(sqrt(x**n/(x**n + 1)), x)
```

Maxima [F]

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

```
[In] integrate((x^n/(1+x^n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^n/(x^n + 1)), x)
```

Giac [F]

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

```
[In] integrate((x^n/(1+x^n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^n/(x^n + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

```
[In] int((x^n/(x^n + 1))^(1/2),x)
[Out] int((x^n/(x^n + 1))^(1/2), x)
```

3.1009 $\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}} \, dx$

Optimal result

Integrand size = 52, antiderivative size = 88

$$\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}} \, dx = \frac{ef \arctan\left(\frac{ab+(4a^2+b^2-2ac)x+abx^2}{2a\sqrt{2a-c}\sqrt{a+bx+cx^2+bx^3+ax^4}}\right)}{a\sqrt{2a-c}d}$$

```
[Out] e*f*arctan(1/2*(a*b+(4*a^2-2*a*c+b^2)*x+a*b*x^2)/a/(2*a-c)^(1/2)/(a*x^4+b*x
^3+c*x^2+b*x+a)^(1/2))/a/d/(2*a-c)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {2109}

$$\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}} \, dx = \frac{ef \arctan\left(\frac{x(4a^2-2ac+b^2)+abx^2+ab}{2a\sqrt{2a-c}\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a-c}}$$

```
[In] Int[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 +
 a*x^4]),x]
```

```
[Out] (e*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a - c]*Sq
rt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(a*Sqrt[2*a - c]*d)
```

Rule 2109

```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[a*(f/(d*Rt[a^2*(2*a - c), 2]))*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b
*x^2)/(2*Rt[a^2*(2*a - c), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g, 0] &&
PosQ[a^2*(2*a - c)]
```

Rubi steps

$$\text{integral} = \frac{ef\tan^{-1}\left(\frac{ab+(4a^2+b^2-2ac)x+abx^2}{2a\sqrt{2a-c}\sqrt{a+bx+cx^2+bx^3+ax^4}}\right)}{a\sqrt{2a-c}d}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 246 vs. $2(88) = 176$.

Time = 0.94 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.80

$$\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx = -\frac{ef\left(-2\sqrt{-2a+c}\arctan\left(\frac{b\left(-\sqrt{-2a+c}x+\sqrt{a+bx+cx^2+bx^3+ax^4}\right)}{2a\sqrt{2a-c}(1+x^2)}\right)+\sqrt{2a-c}\left(2\log\left(-\sqrt{-2a+c}x+\sqrt{a+bx\right.\right.\right.}$$

```
[In] Integrate[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b
*x^3 + a*x^4]),x]
[Out] -1/2*(e*f*(-2*Sqrt[-2*a + c]*ArcTan[(b*(-(Sqrt[-2*a + c]*x) + Sqrt[a + b*x
+ c*x^2 + b*x^3 + a*x^4]))/(2*a*Sqrt[2*a - c]*(1 + x^2))] + Sqrt[2*a - c]*(
2*Log[-(Sqrt[-2*a + c]*x) + Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]] - Log[a*
b^2*(-1 + x^2)^2 + 8*a^3*(1 + x^2)^2 - 4*a^2*c*(1 + x^2)^2 + b^2*x*(b + 2*c
*x + b*x^2 - 2*Sqrt[-2*a + c]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])))/(a
*Sqrt[-(-2*a + c)^2]*d)
```

Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{ef\ln\left(\frac{2\sqrt{-2a+c}\,\sqrt{a\,x^4+b\,x^3+c\,x^2+bx+a}\,a-4a^2x+\left(-b\,x^2+2cx-b\right)a-b^2x}{a\,x^2+bx+a}\right)}{d\sqrt{-2a+c}\,a}$	92
pseudoelliptic	$\frac{ef\ln\left(\frac{2\sqrt{-2a+c}\,\sqrt{a\,x^4+b\,x^3+c\,x^2+bx+a}\,a-4a^2x+\left(-b\,x^2+2cx-b\right)a-b^2x}{a\,x^2+bx+a}\right)}{d\sqrt{-2a+c}\,a}$	92
elliptic	Expression too large to display	254498

```
[In] int((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] e*f/d/(-2*a+c)^(1/2)*ln((2*(-2*a+c)^(1/2)*(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2)*a
-4*a^2*x+(-b*x^2+2*c*x-b)*a-b^2*x)/(a*x^2+b*x+a))/a
```

Fricas [A] (verification not implemented)

none

Time = 2.16 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.68

$$\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx = \left[-\frac{\sqrt{-2\,a+c}ef\log\left(\frac{2\,ab^3x^3+2\,ab^3x-(8\,a^4-a^2b^2-4\,a^3c)x^4-8\,a^4+a^2b^2+4\,a^3c+(16\,a^4+10\,a^2b^2+b^4+8\,a^2c^2-4\,(6\,a^3+ab^2)c)x^2-4\,(a^2bx^2+\cdots}{a^2x^4+2\,abx^3+2\,abx+(2\,a^2+b^2)x^2+a^2}\right)}{2\,(2\,a^2-ac)d} - \frac{\sqrt{2\,a-c}ef\arctan\left(\frac{2\,\sqrt{ax^4+bx^3+cx^2+bx+a}\sqrt{2\,a-c}a}{abx^2+ab+(4\,a^2+b^2-2\,ac)x}\right)}{(2\,a^2-ac)d}\right]$$

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2
),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*a + c)*e*f*log((2*a*b^3*x^3 + 2*a*b^3*x - (8*a^4 - a^2*b^2 -
4*a^3*c)*x^4 - 8*a^4 + a^2*b^2 + 4*a^3*c + (16*a^4 + 10*a^2*b^2 + b^4 + 8*a
^2*c^2 - 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b + (4*a^3 + a*b^2 -
 2*a^2*c)*x)*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(-2*a + c))/(a^2*x^4
 + 2*a*b*x^3 + 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/((2*a^2 - a*c)*d), -sqrt
(2*a - c)*e*f*arctan(2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(2*a - c)*
a/(a*b*x^2 + a*b + (4*a^2 + b^2 - 2*a*c)*x))/((2*a^2 - a*c)*d)]
```

Sympy [F]

$$\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx = -\frac{ef\left(\int \frac{x^2}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2}+a\sqrt{ax^4+a+bx^3+bx+cx^2}+bx\sqrt{ax^4+a+bx^3+bx+cx^2}}\,dx + \int \left(-\frac{\cdots}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2}+a\sqrt{ax^4+\cdots}}\right.\right.}{d}$$

```
[In] integrate((-e*f*x**2+e*f)/(a*d*x**2+b*d*x+a*d)/(a*x**4+b*x**3+c*x**2+b*x+a)
**(1/2),x)
```

```
[Out] -e*f*(Integral(x**2/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + a*sq
rt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a + b*x**3 + b*x
```

```
 + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x*
*2) + a*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a + b*
x**3 + b*x + c*x**2)), x))/d
```

Maxima [F]

$$\int \frac{ef - efx^2}{(ad + bdx + adx^2)\sqrt{a + bx + cx^2 + bx^3 + ax^4}}\,dx = \int -\frac{efx^2 - ef}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}(adx^2 + bdx + ad)}\,dx$$

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2
),x, algorithm="maxima")
```

```
[Out] -integrate((e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2
+ b*d*x + a*d)), x)
```

Giac [F]

$$\int \frac{ef - efx^2}{(ad + bdx + adx^2)\sqrt{a + bx + cx^2 + bx^3 + ax^4}}\,dx = \int -\frac{efx^2 - ef}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}(adx^2 + bdx + ad)}\,dx$$

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2
),x, algorithm="giac")
```

```
[Out] integrate(-(e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2
+ b*d*x + a*d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ef - efx^2}{(ad + bdx + adx^2)\sqrt{a + bx + cx^2 + bx^3 + ax^4}}\,dx = \int \frac{e\,f - e\,f\,x^2}{\left(a\,d\,x^2 + b\,d\,x + a\,d\right)\,\sqrt{a\,x^4 + b\,x^3 + c\,x^2 + b\,x + a}}\,dx$$

```
[In] int((e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*(a + b*x + a*x^4 + b*x^3 + c*x
^2)^(1/2)),x)
```

```
[Out] int((e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*(a + b*x + a*x^4 + b*x^3 + c*x
^2)^(1/2)), x)
```

3.1010 $\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx$

Optimal result

Integrand size = 57, antiderivative size = 88

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx = \frac{ef\operatorname{arctanh}\left(\frac{ab-(4a^2+b^2+2ac)x+abx^2}{2a\sqrt{2a+c}\sqrt{-a+bx+cx^2+bx^3-ax^4}}\right)}{a\sqrt{2a+c}d}$$

```
[Out] e*f*arctanh(1/2*(a*b-(4*a^2+2*a*c+b^2)*x+a*b*x^2)/a/(2*a+c)^(1/2)/(-a*x^4+b
*x^3+c*x^2+b*x-a)^(1/2))/a/d/(2*a+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {2110}

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx = \frac{ef\operatorname{arctanh}\left(\frac{-x(4a^2+2ac+b^2)+abx^2+ab}{2a\sqrt{2a+c}\sqrt{-ax^4-a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a+c}}$$

```
[In] Int[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2 + b*x
^3 - a*x^4]),x]
```

```
[Out] (e*f*ArcTanh[(a*b - (4*a^2 + b^2 + 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a + c]*S
qrt[-a + b*x + c*x^2 + b*x^3 - a*x^4])])/(a*Sqrt[2*a + c]*d)
```

Rule 2110

```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[(-a)*(f/(d*Rt[(-a^2)*(2*a - c), 2]))*ArcTanh[(a*b + (4*a^2 + b^2 - 2*a*c)*
x + a*b*x^2)/(2*Rt[(-a^2)*(2*a - c), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^
4])], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f +
 g, 0] && NegQ[a^2*(2*a - c)]
```

Rubi steps

$$\text{integral} = \frac{ef\tanh^{-1}\left(\frac{ab-(4a^2+b^2+2ac)x+abx^2}{2a\sqrt{2a+c}\sqrt{-a+bx+cx^2+bx^3-ax^4}}\right)}{a\sqrt{2a+c}d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 213 vs. $2(88) = 176$.

Time = 0.92 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.42

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx$$
$$= \frac{ef\left(2\operatorname{arctanh}\left(\frac{b\left(x-\frac{\sqrt{x(b+cx+bx^2)-a(1+x^4)}}{\sqrt{2a+c}}\right)}{2a(1+x^2)}\right)+2\log\left(-\sqrt{2a+c}x+\sqrt{x(b+cx+bx^2)-a(1+x^4)}\right)-\log\right.}{}$$

```
[In] Integrate[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2
 + b*x^3 - a*x^4]),x]
```

```
[Out] (e*f*(2*ArcTanh[(b*(x - Sqrt[x*(b + c*x + b*x^2) - a*(1 + x^4)]/Sqrt[2*a +
c]))/(2*a*(1 + x^2))] + 2*Log[-(Sqrt[2*a + c]*x) + Sqrt[x*(b + c*x + b*x^2)
 - a*(1 + x^4)]] - Log[a*b^2*(-1 + x^2)^2 + 8*a^3*(1 + x^2)^2 + 4*a^2*c*(1
+ x^2)^2 - b^2*x*(b + 2*c*x + b*x^2 - 2*Sqrt[2*a + c]*Sqrt[x*(b + c*x + b*x
^2) - a*(1 + x^4)])]))/(2*a*Sqrt[2*a + c]*d)
```

Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09

method	result	size
default	$-\frac{ef\ln\left(\frac{2\sqrt{2a+c}\,\sqrt{-a\,x^{4}+b\,x^{3}+c\,x^{2}+bx-a}\,a+4a^{2}x+\left(-b\,x^{2}+2cx-b\right)a+b^{2}x}{a\,x^{2}-bx+a}\right)}{d\sqrt{2a+c}\,a}$	96
pseudoelliptic	$-\frac{ef\ln\left(\frac{2\sqrt{2a+c}\,\sqrt{-a\,x^{4}+b\,x^{3}+c\,x^{2}+bx-a}\,a+4a^{2}x+\left(-b\,x^{2}+2cx-b\right)a+b^{2}x}{a\,x^{2}-bx+a}\right)}{d\sqrt{2a+c}\,a}$	96
elliptic	Expression too large to display	281960

```
[In] int((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1/2),x,
method=_RETURNVERBOSE)
```

```
[Out] -e*f/d/(2*a+c)^(1/2)*ln((2*(2*a+c)^(1/2)*(-a*x^4+b*x^3+c*x^2+b*x-a)^(1/2)*a
+4*a^2*x+(-b*x^2+2*c*x-b)*a+b^2*x)/(a*x^2-b*x+a))/a
```

Fricas [A] (verification not implemented)

none

Time = 2.10 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.76

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx$$

$$= \left[\frac{\sqrt{2\,a+c}ef\log\left(\frac{2\,ab^3x^3+2\,ab^3x+(8\,a^4-a^2b^2+4\,a^3c)x^4+8\,a^4-a^2b^2+4\,a^3c-(16\,a^4+10\,a^2b^2+b^4+8\,a^2c^2+4\,(6\,a^3+ab^2)c)x^2-4\,(a^2bx^2+a^2}{a^2x^4-2\,abx^3-2\,abx+(2\,a^2+b^2)x^2+a^2}\right.}{2\,(2\,a^2+ac)d} - \frac{\sqrt{-2\,a-c}ef\arctan\left(\frac{2\,\sqrt{-ax^4+bx^3+cx^2+bx-a}a\sqrt{-2\,a-c}}{abx^2+ab-(4\,a^2+b^2+2\,ac)x}\right)}{(2\,a^2+ac)d}\right]$$

```
[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2*a + c)*e*f*log((2*a*b^3*x^3 + 2*a*b^3*x + (8*a^4 - a^2*b^2 + 4*
a^3*c)*x^4 + 8*a^4 - a^2*b^2 + 4*a^3*c - (16*a^4 + 10*a^2*b^2 + b^4 + 8*a^2
*c^2 + 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b - (4*a^3 + a*b^2 + 2
*a^2*c)*x)*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*sqrt(2*a + c))/(a^2*x^4 -
 2*a*b*x^3 - 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/((2*a^2 + a*c)*d), -sqrt(-
2*a - c)*e*f*arctan(2*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*a*sqrt(-2*a -
c)/(a*b*x^2 + a*b - (4*a^2 + b^2 + 2*a*c)*x))/((2*a^2 + a*c)*d)]
```

Sympy [F]

$$\int \frac{ef - efx^2}{(-ad + bdx - adx^2)\sqrt{-a + bx + cx^2 + bx^3 - ax^4}}\,dx$$
$$= \frac{ef\left(\int \frac{x^2}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+a\sqrt{-ax^4-a+bx^3+bx+cx^2}-bx\sqrt{-ax^4-a+bx^3+bx+cx^2}}\,dx + \int\left(-\frac{}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+a\cdots}\right.\right.}{d}$$

```
[In] integrate((-e*f*x**2+e*f)/(-a*d*x**2+b*d*x-a*d)/(-a*x**4+b*x**3+c*x**2+b*x-
a)**(1/2),x)
```

```
[Out] e*f*(Integral(x**2/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) + a*sq
rt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4 - a + b*x**3 + b
*x + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x + c
*x**2) + a*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4 - a
 + b*x**3 + b*x + c*x**2)), x))/d
```

Maxima [F]

$$\int \frac{ef - efx^2}{(-ad + bdx - adx^2)\sqrt{-a + bx + cx^2 + bx^3 - ax^4}}\,dx$$
$$= \int \frac{efx^2 - ef}{\sqrt{-ax^4 + bx^3 + cx^2 + bx - a}(adx^2 - bdx + ad)}\,dx$$

```
[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2
- b*d*x + a*d)), x)
```

Giac [F]

$$\int \frac{ef - efx^2}{(-ad + bdx - adx^2)\sqrt{-a + bx + cx^2 + bx^3 - ax^4}}\,dx$$
$$= \int \frac{efx^2 - ef}{\sqrt{-ax^4 + bx^3 + cx^2 + bx - a}(adx^2 - bdx + ad)}\,dx$$

```
[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1
/2),x, algorithm="giac")
```

```
[Out] integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2
- b*d*x + a*d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ef - efx^2}{(-ad + bdx - adx^2)\sqrt{-a + bx + cx^2 + bx^3 - ax^4}}\,dx$$
$$= \int -\frac{e\,f - e\,f\,x^2}{(a\,d\,x^2 - b\,d\,x + a\,d)\,\sqrt{-a\,x^4 + b\,x^3 + c\,x^2 + b\,x - a}}\,dx$$

```
[In] int(-(e*f - e*f*x^2)/((a*d - b*d*x + a*d*x^2)*(b*x - a - a*x^4 + b*x^3 + c*
x^2)^(1/2)),x)
[Out] int(-(e*f - e*f*x^2)/((a*d - b*d*x + a*d*x^2)*(b*x - a - a*x^4 + b*x^3 + c*
x^2)^(1/2)), x)
```

3.1011 $\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal result

Integrand size = 59, antiderivative size = 46

$$\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\frac{\sqrt{2}b\text{arcsinh}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arcsinh((a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2155, 221}

$$\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\frac{\sqrt{2}b\text{arcsinh}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Int[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2) + (a^
2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[
a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2155

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[Sqrt[2]*(b/a), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{2}b\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$= \frac{\sqrt{2}b\sinh^{-1}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 107 vs. $2(46) = 92$.

Time = 5.00 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.33

$$\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx = -\frac{\sqrt{2}b\sqrt{x\left(-ax+b\sqrt{\frac{a(-1+ax^2)}{b^2}}\right)}\sqrt{x\left(ax+b\sqrt{\frac{a(-1+ax^2)}{b^2}}\right)}\arctan\left(\sqrt{2}\sqrt{x\left(-ax+b\sqrt{\frac{a(-1+ax^2)}{b^2}}\right)}\right)}{ax}$$

```
[In] Integrate[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2)
 + (a^2*x^2)/b^2]),x]

[Out] -((Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*Sqrt[x*(a*x +
b*Sqrt[(a*(-1 + a*x^2))/b^2])]*ArcTan[Sqrt[2]*Sqrt[x*(-(a*x) + b*Sqrt[(a*(-
1 + a*x^2))/b^2])]])/(a*x))
```

Maple [F]

$$\int \frac{\sqrt{a\,x^{2}+bx\sqrt{-\frac{a}{b^{2}}+\frac{a^{2}x^{2}}{b^{2}}}}}{x\sqrt{-\frac{a}{b^{2}}+\frac{a^{2}x^{2}}{b^{2}}}}dx$$

```
[In] int((a*x^2+b*x*(-a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

```
[Out] int((a*x^2+b*x*(-a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

Fricas [A] (verification not implemented)

none

Time = 5.58 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{ax^{2}+bx\sqrt{-\frac{a}{b^{2}}+\frac{a^{2}x^{2}}{b^{2}}}}}{x\sqrt{-\frac{a}{b^{2}}+\frac{a^{2}x^{2}}{b^{2}}}}\,dx = \left[\frac{\sqrt{2}b\log\left(-4\,ax^{2}-4\,bx\sqrt{\frac{a^{2}x^{2}-a}{b^{2}}}-2\,\sqrt{ax^{2}+bx\sqrt{\frac{a^{2}x^{2}-a}{b^{2}}}}\left(\sqrt{2}\sqrt{a}x+\frac{\sqrt{2}b\sqrt{\frac{a^{2}x^{2}-a}{b^{2}}}}{\sqrt{a}}\right)+1\right)}{2\,\sqrt{a}}, -\sqrt{2}b\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{ax^{2}+bx\sqrt{\frac{a^{2}x^{2}-a}{b^{2}}}}\sqrt{-\frac{1}{a}}}{2\,x}\right)\right]$$

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*log(-4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) - 2*sqrt(a*x^2
+ b*x*sqrt((a^2*x^2 - a)/b^2))*(sqrt(2)*sqrt(a)*x + sqrt(2)*b*sqrt((a^2*x^2
 - a)/b^2)/sqrt(a)) + 1)/sqrt(a), -sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(2)*
sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-1/a)/x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}\,dx = \int \frac{\sqrt{x\left(ax + b\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2-1)}{b^2}}}\,dx$$

```
[In] integrate((a*x**2+b*x*(-a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(-a/b**2+a*
*2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))/(x*sqrt(a*(a*x**2
- 1)/b**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}\,dx = \int \frac{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}bx}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}x}\,dx$$

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

Giac [F]

$$\int \frac{\sqrt{ax^2 + bx\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}\,dx = \int \frac{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}bx}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}x}\,dx$$

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{a\,x^2+b\,x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}}{x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}\,dx$$

```
[In] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)),x)
```

```
[Out] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)), x)
```

3.1012 $\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal result

Integrand size = 58, antiderivative size = 46

$$\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\frac{\sqrt{2}b\arcsin\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arcsin((a*x-b*(a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2155, 222}

$$\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\frac{\sqrt{2}b\arcsin\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Int[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 + (a^2*x
^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2155

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[Sqrt[2]*(b/a), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(\sqrt{2}b\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$= \frac{\sqrt{2}b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 114 vs. $2(46) = 92$.

Time = 4.44 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

$$= \frac{\sqrt{2}b\sqrt{x\left(-ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}\sqrt{ax\left(ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}\arctan\left(\frac{\sqrt{2}\sqrt{ax\left(ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}}{\sqrt{a}}\right)}{a^{3/2}x}$$

```
[In] Integrate[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 +
(a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*Sqrt[a*x*(a*x + b
*Sqrt[(a*(1 + a*x^2))/b^2])]*ArcTan[(Sqrt[2]*Sqrt[a*x*(a*x + b*Sqrt[(a*(1 +
 a*x^2))/b^2])])/Sqrt[a]])/(a^(3/2)*x)
```

Maple [F]

$$\int \frac{\sqrt{-a\,x^{2}+bx\sqrt{\frac{a}{b^{2}}+\frac{a^{2}x^{2}}{b^{2}}}}}{x\sqrt{\frac{a}{b^{2}}+\frac{a^{2}x^{2}}{b^{2}}}}dx$$

```
[In] int((-a*x^2+b*x*(a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/2
),x)
[Out] int((-a*x^2+b*x*(a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/2
),x)
```

Fricas [A] (verification not implemented)

none

Time = 5.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{-ax^{2}+bx\sqrt{\frac{a}{b^{2}}+\frac{a^{2}x^{2}}{b^{2}}}}}{x\sqrt{\frac{a}{b^{2}}+\frac{a^{2}x^{2}}{b^{2}}}}\,dx = \left[\frac{1}{2}\sqrt{2}b\sqrt{-\frac{1}{a}}\log\left(4\,ax^{2}-4\,bx\sqrt{\frac{a^{2}x^{2}+a}{b^{2}}} + 2\sqrt{-ax^{2}+bx\sqrt{\frac{a^{2}x^{2}+a}{b^{2}}}}\left(\sqrt{2}ax\sqrt{-\frac{1}{a}}-\sqrt{2}b\sqrt{-\frac{1}{a}}\sqrt{\frac{a^{2}x^{2}+a}{b^{2}}}\right)+1\right), -\frac{\sqrt{2}b\arctan\left(\frac{\sqrt{2}\sqrt{-ax^{2}+bx\sqrt{\frac{a^{2}x^{2}+a}{b^{2}}}}}{2\sqrt{a}x}\right)}{\sqrt{a}}\right]$$

```
[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="fricas")
[Out] [1/2*sqrt(2)*b*sqrt(-1/a)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 + a)/b^2) + 2*s
qrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))*(sqrt(2)*a*x*sqrt(-1/a) - sqrt(2)
*b*sqrt(-1/a)*sqrt((a^2*x^2 + a)/b^2)) + 1), -sqrt(2)*b*arctan(1/2*sqrt(2)*
sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))/(sqrt(a)*x))/sqrt(a)]
```

Sympy [F]

$$\int \frac{\sqrt{-ax^2 + bx\sqrt{\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}\,dx = \int \frac{\sqrt{-x\left(ax - b\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2+1)}{b^2}}}\,dx$$

```
[In] integrate((-a*x**2+b*x*(a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(a/b**2+a**
2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x*(a*x - b*sqrt(a**2*x**2/b**2 + a/b**2)))/(x*sqrt(a*(a*x**2
 + 1)/b**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{-ax^2 + bx\sqrt{\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}\,dx = \int \frac{\sqrt{-ax^2 + \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}bx}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}x}\,dx$$

```
[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

Giac [F]

$$\int \frac{\sqrt{-ax^2 + bx\sqrt{\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}\,dx = \int \frac{\sqrt{-ax^2 + \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}bx}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}x}\,dx$$

```
[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{b\,x\,\sqrt{\frac{a}{b^2}+\frac{a^2\,x^2}{b^2}}-a\,x^2}}{x\,\sqrt{\frac{a}{b^2}+\frac{a^2\,x^2}{b^2}}}\,dx$$

```
[In] int((b*x*(a/b^2 + (a^2*x^2)/b^2)^(1/2) - a*x^2)^(1/2)/(x*(a/b^2 + (a^2*x^2)
/b^2)^(1/2)),x)
[Out] int((b*x*(a/b^2 + (a^2*x^2)/b^2)^(1/2) - a*x^2)^(1/2)/(x*(a/b^2 + (a^2*x^2)
/b^2)^(1/2)), x)
```

3.1013 $\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal result

Integrand size = 58, antiderivative size = 46

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\frac{\sqrt{2}b\operatorname{arcsinh}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arcsinh((a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2156, 2155, 221}

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\frac{\sqrt{2}b\operatorname{arcsinh}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Int[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt[-(a/b^2) + (a^
2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[
a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2155

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[Sqrt[2]*(b/a), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rule 2156

```
Int[Sqrt[(e_.)*(x_)*((a_.)*(x_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2])]/((x_)*S
qrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Int[Sqrt[a*e*x^2 + b*e*x*Sqrt[c + d
*x^2]]/(x*Sqrt[c + d*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^
2*d, 0] && EqQ[b^2*c*e + a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx\\ &= \frac{\left(\sqrt{2}b\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}\\ &= \frac{\sqrt{2}b\sinh^{-1}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 107 vs. $2(46) = 92$.

Time = 0.01 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.33

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx = -\frac{\sqrt{2}b\sqrt{x\left(-ax+b\sqrt{\frac{a(-1+ax^2)}{b^2}}\right)}\sqrt{x\left(ax+b\sqrt{\frac{a(-1+ax^2)}{b^2}}\right)}\arctan\left(\sqrt{2}\sqrt{x\left(-ax+b\sqrt{\frac{a(-1+ax^2)}{b^2}}\right)}\right)}{ax}$$

```
[In] Integrate[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt[-(a/b^2)
 + (a^2*x^2)/b^2]),x]
```

```
[Out] -((Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*Sqrt[x*(a*x +
b*Sqrt[(a*(-1 + a*x^2))/b^2])]*ArcTan[Sqrt[2]*Sqrt[x*(-(a*x) + b*Sqrt[(a*(-
1 + a*x^2))/b^2])]])/(a*x))
```

Maple [F]

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}dx$$

```
[In] int((x*(a*x+b*(-a/b^2+a^2/b^2*x^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

```
[Out] int((x*(a*x+b*(-a/b^2+a^2/b^2*x^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

Fricas [A] (verification not implemented)

none

Time = 5.51 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx = \left[\frac{\sqrt{2}b\log\left(-4\,ax^2-4\,bx\sqrt{\frac{a^2x^2-a}{b^2}}-2\,\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\left(\sqrt{2}\sqrt{a}x+\frac{\sqrt{2}b\sqrt{\frac{a^2x^2-a}{b^2}}}{\sqrt{a}}\right)+1\right)}{2\,\sqrt{a}},\ -\sqrt{2}b\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\sqrt{-\frac{1}{a}}}{2\,x}\right)\right]$$

```
[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*log(-4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) - 2*sqrt(a*x^2
+ b*x*sqrt((a^2*x^2 - a)/b^2))*(sqrt(2)*sqrt(a)*x + sqrt(2)*b*sqrt((a^2*x^2
```

```
- a)/b^2)/sqrt(a)) + 1)/sqrt(a), -sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(2)*
sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-1/a)/x)]
```

Sympy [F]

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{x\left(ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2-1)}{b^2}}}\,dx$$

```
[In] integrate((x*(a*x+b*(-a/b**2+a**2*x**2/b**2)**(1/2)))**(1/2)/x/(-a/b**2+a**
2*x**2/b**2)**(1/2),x)
[Out] Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))/(x*sqrt(a*(a*x**2
- 1)/b**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{\left(ax+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}x}\,dx$$

```
[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

Giac [F]

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{\left(ax+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}x}\,dx$$

```
[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{x\,\left(a\,x+b\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}\right)}}{x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}\,dx$$

```
[In] int((x*(a*x + b*((a^2*x^2)/b^2 - a/b^2)^(1/2)))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)),x)
[Out] int((x*(a*x + b*((a^2*x^2)/b^2 - a/b^2)^(1/2)))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)), x)
```

3.1014 $\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal result

Integrand size = 57, antiderivative size = 46

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\frac{\sqrt{2}b\arcsin\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arcsin((a*x-b*(a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2156, 2155, 222}

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\frac{\sqrt{2}b\arcsin\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Int[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 + (a^2*x
^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2155

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[Sqrt[2]*(b/a), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rule 2156

```
Int[Sqrt[(e_.)*(x_)*((a_.)*(x_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2])]/((x_)*S
qrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Int[Sqrt[a*e*x^2 + b*e*x*Sqrt[c + d
*x^2]]/(x*Sqrt[c + d*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^
2*d, 0] && EqQ[b^2*c*e + a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx \\
&= -\frac{\left(\sqrt{2}b\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a} \\
&= \frac{\sqrt{2}b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 114 vs. $2(46) = 92$.

Time = 0.01 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

$$= \frac{\sqrt{2}b\sqrt{x\left(-ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}\sqrt{ax\left(ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}\arctan\left(\frac{\sqrt{2}\sqrt{ax\left(ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}}{\sqrt{a}}\right)}{a^{3/2}x}$$

```
[In] Integrate[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 +
(a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*Sqrt[a*x*(a*x + b
*Sqrt[(a*(1 + a*x^2))/b^2])]*ArcTan[(Sqrt[2]*Sqrt[a*x*(a*x + b*Sqrt[(a*(1 +
 a*x^2))/b^2])])/Sqrt[a]])/(a^(3/2)*x)
```

Maple [F]

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}dx$$

```
[In] int((x*(-a*x+b*(a/b^2+a^2/b^2*x^2)^(1/2)))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/2
),x)
```

```
[Out] int((x*(-a*x+b*(a/b^2+a^2/b^2*x^2)^(1/2)))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/2
),x)
```

Fricas [A] (verification not implemented)

none

Time = 5.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx = \left[\frac{1}{2}\sqrt{2}b\sqrt{-\frac{1}{a}}\log\left(4\,ax^2-4\,bx\sqrt{\frac{a^2x^2+a}{b^2}}+2\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}\left(\sqrt{2}ax\sqrt{-\frac{1}{a}}-\sqrt{2}b\sqrt{-\frac{1}{a}}\sqrt{\frac{a^2x^2+a}{b^2}}\right)+1\right),\ -\frac{\sqrt{2}b\arctan\left(\frac{\sqrt{2}\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}}{2\sqrt{a}x}\right)}{\sqrt{a}}\right]$$

```
[In] integrate((x*(-a*x+b*(a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*sqrt(-1/a)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 + a)/b^2) + 2*s
qrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))*(sqrt(2)*a*x*sqrt(-1/a) - sqrt(2)
*b*sqrt(-1/a)*sqrt((a^2*x^2 + a)/b^2)) + 1), -sqrt(2)*b*arctan(1/2*sqrt(2)*
sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))/(sqrt(a)*x))/sqrt(a)]
```

Sympy [F]

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{-x\left(ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2+1)}{b^2}}}\,dx$$

```
[In] integrate((x*(-a*x+b*(a/b**2+a**2*x**2/b**2)**(1/2)))**(1/2)/x/(a/b**2+a**2
*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x*(a*x - b*sqrt(a**2*x**2/b**2 + a/b**2)))/(x*sqrt(a*(a*x**2
 + 1)/b**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{-\left(ax-\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}x}\,dx$$

```
[In] integrate((x*(-a*x+b*(a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

Giac [F]

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{-\left(ax-\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}x}\,dx$$

```
[In] integrate((x*(-a*x+b*(a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{-x\left(a\,x-b\sqrt{\frac{a}{b^2}+\frac{a^2\,x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2\,x^2}{b^2}}}\,dx$$

```
[In] int((-x*(a*x - b*(a/b^2 + (a^2*x^2)/b^2)^(1/2)))^(1/2)/(x*(a/b^2 + (a^2*x^2
)/b^2)^(1/2)),x)
```

```
[Out] int((-x*(a*x - b*(a/b^2 + (a^2*x^2)/b^2)^(1/2)))^(1/2)/(x*(a/b^2 + (a^2*x^2
)/b^2)^(1/2)), x)
```

3.1015 $\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)\left(4-5x+x^2\right)}\,dx$

Optimal result

Integrand size = 66, antiderivative size = 19

$$\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)\left(4-5x+x^2\right)}\,dx=2\log\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)$$

```
[Out] 2*ln(1+(-4+x)^(1/2)+(-1+x)^(1/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6820, 1600, 6816}

$$\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)\left(4-5x+x^2\right)}\,dx=2\log\left(\sqrt{x-4}+\sqrt{x-1}+1\right)$$

```
[In] Int[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-1 + x]*x)/((1
+ Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]
```

```
[Out] 2*Log[1 + Sqrt[-4 + x] + Sqrt[-1 + x]]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6816

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt{-1+x}\left(-4+\sqrt{-4+x}\sqrt{-1+x}+x\right)}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)\left(4-5x+x^2\right)}\,dx\\ &= \int \frac{-4+\sqrt{-4+x}\sqrt{-1+x}+x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(-4+x)\sqrt{-1+x}}\,dx\\ &= 2\log\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)\left(4-5x+x^2\right)}\,dx = 4\text{arctanh}\left(1-\frac{2\sqrt{-4+x}}{3}+\frac{2\sqrt{-1+x}}{3}\right)$$

```
[In] Integrate[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-1 + x]*x
)/((1 + Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]
[Out] 4*ArcTanh[1 - (2*Sqrt[-4 + x])/3 + (2*Sqrt[-1 + x])/3]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(15) = 30.

Time = 1.38 (sec) , antiderivative size = 147, normalized size of antiderivative = 7.74

method	result
default	$\frac{\ln(-5+x)}{2}-\frac{\ln(1+\sqrt{x-4})}{2}+\frac{\ln(-1+\sqrt{x-4})}{2}+\frac{\ln(\sqrt{x-1}+2)}{2}-\frac{\ln(\sqrt{x-1}-2)}{2}+\frac{7\sqrt{x-4}\,\sqrt{x-1}\,\operatorname{arctanh}\left(\frac{-17+5x}{4\sqrt{x^2-5x+4}}\right)}{4\sqrt{x^2-5x+4}}+$

```
[In] int((-(x-4)^(1/2)+x*(x-4)^(1/2)-4*(x-1)^(1/2)+x*(x-1)^(1/2))/(x^2-5*x+4)/(1
+(x-4)^(1/2)+(x-1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(-5+x)-1/2*ln(1+(x-4)^(1/2))+1/2*ln(-1+(x-4)^(1/2))+1/2*ln((x-1)^(1/2
)+2)-1/2*ln((x-1)^(1/2)-2)+7/4*(x-4)^(1/2)*(x-1)^(1/2)/(x^2-5*x+4)^(1/2)*ar
ctanh(1/4*(-17+5*x)/(x^2-5*x+4)^(1/2))+1/4*(x-4)^(1/2)*(x-1)^(1/2)*(2*ln(-5
/2+x+(x^2-5*x+4)^(1/2))-5*arctanh(1/4*(-17+5*x)/(x^2-5*x+4)^(1/2)))/(x^2-5*
x+4)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(15) = 30$.

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 5.05

$$\begin{aligned}\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}\,dx &= -\frac{1}{2}\log\left(-(4\,x-11)\sqrt{x-1}\sqrt{x-4}+4\,x^2-21\,x+23\right) \\ &\quad+\frac{1}{2}\log\left(\sqrt{x-1}\sqrt{x-4}-x+7\right)+\frac{1}{2}\log\left(x-5\right)+\frac{1}{2}\log\left(\sqrt{x-1}+2\right) \\ &\quad-\frac{1}{2}\log\left(\sqrt{x-1}-2\right)-\frac{1}{2}\log\left(\sqrt{x-4}+1\right)+\frac{1}{2}\log\left(\sqrt{x-4}-1\right)\end{aligned}$$

```
[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2
-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*log(-(4*x - 11)*sqrt(x - 1)*sqrt(x - 4) + 4*x^2 - 21*x + 23) + 1/2*log
(sqrt(x - 1)*sqrt(x - 4) - x + 7) + 1/2*log(x - 5) + 1/2*log(sqrt(x - 1) +
2) - 1/2*log(sqrt(x - 1) - 2) - 1/2*log(sqrt(x - 4) + 1) + 1/2*log(sqrt(x -
 4) - 1)
```

Sympy [A] (verification not implemented)

Time = 8.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}\,dx = 2\log\left(\sqrt{x-4}+\sqrt{x-1}+1\right)$$

```
[In] integrate((-(-4+x)**(1/2)+x*(-4+x)**(1/2)-4*(-1+x)**(1/2)+x*(-1+x)**(1/2))/
(x**2-5*x+4)/(1+(-4+x)**(1/2)+(-1+x)**(1/2)),x)
```

```
[Out] 2*log(sqrt(x - 4) + sqrt(x - 1) + 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 4.95

$$\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}\,dx$$
$$=\frac{1}{2}\log(x-1)$$
$$+\frac{1}{2}\log\left(\frac{2x^2+2\left((x-1)\sqrt{x-4}+2x-6\right)\sqrt{x-1}+2(2x-3)\sqrt{x-4}-7x+3}{2\left((x-1)\sqrt{x-4}+2x-6\right)}\right)$$
$$+\frac{1}{2}\log\left(\frac{(x-1)\sqrt{x-4}+2x-6}{x-1}\right)$$

```
[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2
-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*log(x - 1) + 1/2*log(1/2*(2*x^2 + 2*((x - 1)*sqrt(x - 4) + 2*x - 6)*sqr
t(x - 1) + 2*(2*x - 3)*sqrt(x - 4) - 7*x + 3)/((x - 1)*sqrt(x - 4) + 2*x -
6)) + 1/2*log(((x - 1)*sqrt(x - 4) + 2*x - 6)/(x - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(15) = 30$.

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.05

$$\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}\,dx$$
$$=-\log\left(\sqrt{x-1}-\sqrt{x-4}+1\right)-\log\left(\sqrt{x-1}-\sqrt{x-4}\right)$$
$$+\log\left(\sqrt{x-1}+2\right)+\log\left(\left|-\sqrt{x-1}+\sqrt{x-4}-3\right|\right)$$

```
[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2
-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -log(sqrt(x - 1) - sqrt(x - 4) + 1) - log(sqrt(x - 1) - sqrt(x - 4)) + log(
sqrt(x - 1) + 2) + log(abs(-sqrt(x - 1) + sqrt(x - 4) - 3))
```

Mupad [B] (verification not implemented)

Time = 22.53 (sec) , antiderivative size = 132, normalized size of antiderivative = 6.95

$$\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)\left(4-5x+x^2\right)}\,dx$$
$$=\frac{\ln\left(x-5\right)}{2}+2\,\mathrm{atanh}\left(\frac{\sqrt{x-1}-\sqrt{3}}{\sqrt{x-4}}\right)+\frac{7\,\mathrm{atanh}\left(\frac{4\left(\sqrt{x-1}-\sqrt{3}\right)}{\left(\frac{\left(\sqrt{x-1}-\sqrt{3}\right)^2}{x-4}+1\right)\sqrt{x-4}}\right)}{2}$$
$$-\frac{5\,\mathrm{atanh}\left(\frac{194400\left(\sqrt{x-1}-\sqrt{3}\right)}{\left(\frac{48600\left(\sqrt{x-1}-\sqrt{3}\right)^2}{x-4}+48600\right)\sqrt{x-4}}\right)}{2}-\mathrm{atanh}\left(\sqrt{x-4}\right)+\mathrm{atanh}\left(\frac{\sqrt{x-1}}{2}\right)$$

```
[In] int((x*(x - 1)^(1/2) + x*(x - 4)^(1/2) - 4*(x - 1)^(1/2) - (x - 4)^(1/2))/(
(x^2 - 5*x + 4)*((x - 1)^(1/2) + (x - 4)^(1/2) + 1)),x)
```

```
[Out] log(x - 5)/2 + 2*atanh(((x - 1)^(1/2) - 3^(1/2))/(x - 4)^(1/2)) + (7*atanh(
(4*((x - 1)^(1/2) - 3^(1/2)))/((((x - 1)^(1/2) - 3^(1/2))^2/(x - 4) + 1)*(x
 - 4)^(1/2))))/2 - (5*atanh((194400*((x - 1)^(1/2) - 3^(1/2)))/(((48600*((x
 - 1)^(1/2) - 3^(1/2))^2)/(x - 4) + 48600)*(x - 4)^(1/2))))/2 - atanh((x -
4)^(1/2)) + atanh((x - 1)^(1/2)/2)
```

3.1016 $\int \frac{1}{x(3+3x+x^2)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 90

$$\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx=-\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}}-\frac{\log\left(1-(1+x)^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}(1+x)-\sqrt[3]{2+(1+x)^3}\right)}{2\sqrt[3]{3}}$$

```
[Out] -1/3*arctan(1/3*(1+2*3^(1/3)*(1+x)/(2+(1+x)^3)^(1/3))*3^(1/2))*3^(1/6)-1/18
*ln(1-(1+x)^3)*3^(2/3)+1/6*ln(3^(1/3)*(1+x)-(2+(1+x)^3)^(1/3))*3^(2/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {444, 442, 384}

$$\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = -\frac{\arctan\left(\frac{\frac{2\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}} - \frac{\log\left(1-(x+1)^3\right)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}(x+1)-\sqrt[3]{(x+1)^3+2}\right)}{2\sqrt[3]{3}}$$

```
[In] Int[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[(1 + (2*3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3))/Sqrt[3]]/3^(5/6))
- Log[1 - (1 + x)^3]/(6*3^(1/3)) + Log[3^(1/3)*(1 + x) - (2 + (1 + x)^3)^(1
/3)]/(2*3^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 442

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rule 444

```
Int[(u_)^(p_.)*(v_)^(q_.)*(x_)^(m_.), x_Symbol] :> Int[NormalizePseudoBinom
ial[x^(m/p)*u, x]^p*NormalizePseudoBinomial[v, x]^q, x] /; FreeQ[{p, q}, x]
 && IntegersQ[p, m/p] && PseudoBinomialPairQ[x^(m/p)*u, v, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\left(-1+(1+x)^3\right)\sqrt[3]{2+(1+x)^3}}\,dx$$

$$= \text{Subst}\left(\int \frac{1}{\left(-1+x^3\right)\sqrt[3]{2+x^3}}\,dx, x, 1+x\right)$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}} - \frac{\log\left(1-(1+x)^3\right)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}(1+x)-\sqrt[3]{2+(1+x)^3}\right)}{2\sqrt[3]{3}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.00

$$\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \frac{\arctan\left(\frac{\sqrt{3}\sqrt[3]{3+3x+3x^2+x^3}}{2\sqrt[3]{3}+2\sqrt[3]{3}x+\sqrt[3]{3+3x+3x^2+x^3}}\right)}{3^{5/6}} + \frac{2\log\left(\sqrt[3]{3}+\sqrt[3]{3}x-\sqrt[3]{3+3x+3x^2+x^3}\right)-\log\left(3^{2/3}+2\,3^{2/3}x+3^{2/3}x^2+\sqrt[3]{3}(1+x)\sqrt[3]{3+3x+3x^2+x^3}+\ldots\right)}{6\sqrt[3]{3}}$$

```
[In] Integrate[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*(3 + 3*x + 3*x^2 + x^3)^(1/3))/(2*3^(1/3) + 2*3^(1/3)*x + (
3 + 3*x + 3*x^2 + x^3)^(1/3))]/3^(5/6) + (2*Log[3^(1/3) + 3^(1/3)*x - (3 +
3*x + 3*x^2 + x^3)^(1/3)] - Log[3^(2/3) + 2*3^(2/3)*x + 3^(2/3)*x^2 + 3^(1/
3)*(1 + x)*(3 + 3*x + 3*x^2 + x^3)^(1/3) + (3 + 3*x + 3*x^2 + x^3)^(2/3)])/
(6*3^(1/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 17.66 (sec) , antiderivative size = 1673, normalized size of antiderivative = 18.59

method	result	size
trager	Expression too large to display	1673

```
[In] int(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*RootOf(_Z^3-9)*ln(-(34139998872*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^
3-9)+81*_Z^2)*x^3+102419996616*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+
81*_Z^2)*x^2-6984737648*RootOf(_Z^3-9)*x^3-20954212944*RootOf(_Z^3-9)*x^2-2
0954212944*RootOf(_Z^3-9)*x+102419996616*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootO
f(_Z^3-9)+81*_Z^2)*x-15322002984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(_Z^3-9)^2*R
ootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x+1828928511*RootOf(Roo
tOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3-374182374
*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3-
8232012228*RootOf(_Z^3-9)+2619276618*RootOf(_Z^3-9)^3*RootOf(RootOf(_Z^3-9)
```

^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+40236427242*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+20428536825*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)*x^2+40857073650*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x+5107334328*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2-8512490709*(x^3+3*x^2+3*x+3)^(2/3)*x-8512490709*(x^3+3*x^2+3*x+3)^(2/3)-15322002984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+5107334328*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x^2+10214668656*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2*x+20428536825*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)-12802499577*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2+5486785533*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2-1122547122*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^2+5486785533*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x-1122547122*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x)/x/(x^2+3*x+3))+RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*ln(-(-59494997466*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3-178484992398*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2+3590118929*RootOf(_Z^3-9)*x^3+10770356787*RootOf(_Z^3-9)*x^2+10770356787*RootOf(_Z^3-9)*x-178484992398*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x+15322002984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x+3367641366*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3-203214279*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3+5893214091*RootOf(_Z^3-9)+1422499953*RootOf(_Z^3-9)^3*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)-97661599614*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)-25537472127*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)*x^2-51074944254*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x-5107334328*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2+6809512275*(x^3+3*x^2+3*x+3)^(2/3)*x+6809512275*(x^3+3*x^2+3*x+3)^(2/3)+15322002984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)-5107334328*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x^2-10214668656*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2*x-25537472127*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)-23573489562*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2+10102924098*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2-609642837*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^2+10102924098*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x-609642837*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x)/x/(x^2+3*x+3))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(71) = 142$.

Time = 5.03 (sec) , antiderivative size = 458, normalized size of antiderivative = 5.09

$$\int \frac{1}{x(3+3x+x^2)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = -\frac{1}{54} \cdot 3^{\frac{2}{3}} \log \left(\frac{3 \cdot 3^{\frac{2}{3}}(7\,x^4+28\,x^3+42\,x^2+30\,x+9)(x^3+3\,x^2+3\,x+3)^{\frac{2}{3}}+3^{\frac{1}{3}}(31\,x^6+186\,x^5+465\,x^4+666\,x^3 \cdots}{x^6+6\,x^5+15\,x^4 \cdots} \right. + \frac{1}{27} \cdot 3^{\frac{2}{3}} \log \left(\frac{2 \cdot 3^{\frac{2}{3}}(x^3+3\,x^2+3\,x)-9 \cdot 3^{\frac{1}{3}}(x^3+3\,x^2+3\,x+3)^{\frac{1}{3}}(x^2+2\,x+1)+9\,(x^3+3\,x^2+3\,x+3)^{\frac{2}{3}}(x \cdots}{x^3+3\,x^2+3\,x} \right. - \frac{1}{9} \cdot 3^{\frac{1}{6}} \arctan \left(\frac{3^{\frac{1}{6}}\left(12 \cdot 3^{\frac{2}{3}}(7\,x^7+49\,x^6+147\,x^5+240\,x^4+225\,x^3+117\,x^2+27\,x)(x^3+3\,x^2+3\,x+3)^{\frac{2}{3}} - \cdots\right.}{\cdots} \right.$$

```
[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 28*x^3 + 42*x^2 + 30*x + 9)*(x^3 + 3*
x^2 + 3*x + 3)^(2/3) + 3^(1/3)*(31*x^6 + 186*x^5 + 465*x^4 + 666*x^3 + 603*
x^2 + 324*x + 81) + 9*(5*x^5 + 25*x^4 + 50*x^3 + 54*x^2 + 33*x + 9)*(x^3 +
3*x^2 + 3*x + 3)^(1/3))/(x^6 + 6*x^5 + 15*x^4 + 18*x^3 + 9*x^2)) + 1/27*3^(
2/3)*log((2*3^(2/3)*(x^3 + 3*x^2 + 3*x) - 9*3^(1/3)*(x^3 + 3*x^2 + 3*x + 3)
^(1/3)*(x^2 + 2*x + 1) + 9*(x^3 + 3*x^2 + 3*x + 3)^(2/3)*(x + 1))/(x^3 + 3*
x^2 + 3*x)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*x^7 + 49*x^6 +
147*x^5 + 240*x^4 + 225*x^3 + 117*x^2 + 27*x)*(x^3 + 3*x^2 + 3*x + 3)^(2/3)
 - 3^(1/3)*(127*x^9 + 1143*x^8 + 4572*x^7 + 11070*x^6 + 18414*x^5 + 22032*x
^4 + 18900*x^3 + 11178*x^2 + 4131*x + 729) - 18*(31*x^8 + 248*x^7 + 868*x^6
 + 1782*x^5 + 2400*x^4 + 2196*x^3 + 1332*x^2 + 486*x + 81)*(x^3 + 3*x^2 + 3
*x + 3)^(1/3))/(251*x^9 + 2259*x^8 + 9036*x^7 + 21546*x^6 + 34398*x^5 + 385
56*x^4 + 30348*x^3 + 16038*x^2 + 5103*x + 729))
```

Sympy [F]

$$\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx=\int \frac{1}{x\left(x^2+3x+3\right)\sqrt[3]{x^3+3x^2+3x+3}}\,dx$$

```
[In] integrate(1/x/(x**2+3*x+3)/(x**3+3*x**2+3*x+3)**(1/3),x)
[Out] Integral(1/(x*(x**2 + 3*x + 3)*(x**3 + 3*x**2 + 3*x + 3)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx=\int \frac{1}{\left(x^3+3\,x^2+3\,x+3\right)^{\frac{1}{3}}\left(x^2+3\,x+3\right)x}\,dx$$

```
[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx=\int \frac{1}{\left(x^3+3\,x^2+3\,x+3\right)^{\frac{1}{3}}\left(x^2+3\,x+3\right)x}\,dx$$

```
[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="giac")
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx=\int \frac{1}{x\,\left(x^2+3\,x+3\right)\,{\left(x^3+3\,x^2+3\,x+3\right)}^{1/3}}\,dx$$

```
[In] int(1/(x*(3*x + x^2 + 3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)),x)
[Out] int(1/(x*(3*x + x^2 + 3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)), x)
```

3.1017 $\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 103

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx=\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(1+2(1-x)^3-x^3\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}(1-x)+\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/4*ln(1+2*(1-x)^3-x^3)*2^(1/3)+3/4*ln(2^(1/3)*(1-x)+(-x^3+1)^(1/3))*2^(1/
3)+1/2*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(1/
3)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 425 vs. 2(103) = 206.

Time = 0.46 (sec) , antiderivative size = 425, normalized size of antiderivative = 4.13, number of steps used = 42, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.517, Rules used = {2183, 421, 251, 420, 493, 298, 31, 648, 631, 210, 642, 503, 455, 59, 494}

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx = \frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(x^3+1)}{3\ 2^{2/3}} + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right) - \frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\log\left(-\sqrt[3]{1-x^3}\right.}{2\ 2^{2/3}}$$

```
[In] Int[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

```
[Out] (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]
) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) +
 ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 +
x^3]/(3*2^(2/3)) + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log
[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3
)]/(3*2^(2/3)) + (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - L
og[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3
)]/(6*2^(2/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3)) - Log[-(2^(1/3)
*x) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
```

```
x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
```

```
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{(1-x^3)^{2/3}(1+x^3)} + \frac{x}{(1-x^3)^{2/3}(1+x^3)} - \frac{x^2}{(1-x^3)^{2/3}(1+x^3)} - \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\right) dx$$

$$= \int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx + \int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx - \int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx - \int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1+x^3)}{6\ 2^{2/3}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right) + \frac{1}{2}\int \frac{1}{(1-x^3)^{2/3}}\,dx + \frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx - \int \frac{1}{(1-x^3)^{2/3}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right) + \frac{\log(1+x^3)}{3\ 2^{2/3}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \frac{1}{2}\int \frac{1}{(1-x^3)^{2/3}}\,dx + \frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx - \frac{9}{2}\text{Subst}\left(\int \frac{x}{(4-x^3)(1+2x^3)}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{}{2^2}\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1+x^3)}{3\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{1}{2}\text{Subst}\left(\int \frac{x}{4-x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{9}{2}\text{Subst}\left(\int \frac{x}{(4-x^3)(1+2x^3)}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1+x^3)}{3\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{1}{2}\text{Subst}\left(\int \frac{x}{4-x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{2^{2/3}-x}{2\sqrt[3]{2}+2^{2/3}x+ \cdots}\right.}{6\ 2^{\cdots}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1+x^3)}{3\ 2^{2/3}} + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int \frac{2^{2/3}+2x}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}} - \cdots$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1+x^3)}{3\ 2^{2/3}} + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\log\left(\cdots\right.}{\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1+x^3)}{3\ 2^{2/3}} + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\log\left(-\right.}{}$$

$$= \frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1+x^3)}{3\ 2^{2/3}} + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\log\left(-\right.}{}$$

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.76

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(1-x^3)^{2/3}}{2^{2/3}+2^{2/3}x+2^{2/3}x^2-(1-x^3)^{2/3}}\right) - 2\log\left(2^{2/3}+2^{2/3}x+2^{2/3}x^2+2(1-x^3)^{2/3}\right) + \log\left(-\left((1+x+\right.\right.}{2\ 2^{2/3}}$$

```
[In] Integrate[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(2/3))/(2^(2/3) + 2^(2/3)*x + 2^(
2/3)*x^2 - (1 - x^3)^(2/3))] - 2*Log[2^(2/3) + 2^(2/3)*x + 2^(2/3)*x^2 + 2*
(1 - x^3)^(2/3)] + Log[-((1 + x + x^2)*(2^(1/3) + 2^(1/3)*x^2 - (2 - 2*x^3)
^(2/3) + 2*(1 - x^3)^(1/3) + x*(2^(1/3) - 2*(1 - x^3)^(1/3))))])/2^(2/3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 10.01 (sec) , antiderivative size = 692, normalized size of antiderivative = 6.72

method	result	size
trager	Expression too large to display	692

```
[In] int((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(-(-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^
3-2)^4*x+2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^
2)*RootOf(_Z^3-2)^2-RootOf(_Z^3-2)^2*x^2+4*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^
3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+RootOf(_Z^3-2)^2*x-4*(-x^3+1)^(1/3)*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-RootOf(_Z^3-2)^2)/(x^2-x+
1))*RootOf(_Z^3-2)-ln(-(-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z
^2)*RootOf(_Z^3-2)^4*x+2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf
(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2-RootOf(_Z^3-2)^2*x^2+4*(-x^3+1)^(1/3)*Roo
tOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+RootOf(_Z^3-2)^2*x-4*(-x
^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-RootOf(_Z^3
-2)^2)/(x^2-x+1))*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+RootO
f(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln((-2*RootOf(RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^4*x+2*(-x^3+1)^(2/3)*RootOf(R
ootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2+RootOf(_Z^3-2)
^2*x^2+2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)*x+4*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^
3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x-3*RootOf(_Z^3-2)^2*x+2*(-x^3+1)^(2/3)-
2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)-4*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*
_Z*RootOf(_Z^3-2)+4*_Z^2)+RootOf(_Z^3-2)^2)/(x^2-x+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(78) = 156.

Time = 2.92 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.81

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx = -\frac{1}{6}$$
$$\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\Big(2\cdot 4^{\frac{2}{3}}(x^5-x^4-3\,x^3+3\,x^2+x-1)(-x^3+1)^{\frac{1}{3}}+4\,(x^4-4\,x^3+5\,x^2-4\,x+1)(-x^3}{6\,(3\,x^6-9\,x^5+6\,x^4-x^3+6\,x^2-9\,x+}\right.$$
$$-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}(-x^3+1)^{\frac{2}{3}}(x^2-3\,x+1)-4^{\frac{2}{3}}(x^4-3\,x^2+1)-8\,(-x^3+1)^{\frac{1}{3}}(x^2-x)}{x^4-2\,x^3+3\,x^2-2\,x+1}\right)$$
$$+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-\frac{4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}(x-1)-4^{\frac{1}{3}}(x^2-x+1)-2\,(-x^3+1)^{\frac{2}{3}}}{x^2-x+1}\right)$$

```
[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*4^(2/3)*(x^5 - x^4 - 3*x
^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 4*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-
x^3 + 1)^(2/3) + 4^(1/3)*(x^6 - 7*x^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))
/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x^2 - 9*x + 3)) - 1/24*4^(2/3)*log((2*4^(
1/3)*(-x^3 + 1)^(2/3)*(x^2 - 3*x + 1) - 4^(2/3)*(x^4 - 3*x^2 + 1) - 8*(-x^3
 + 1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/12*4^(2/3)*log(
-(4^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) - 4^(1/3)*(x^2 - x + 1) - 2*(-x^3 + 1)^(
2/3))/(x^2 - x + 1))
```

Sympy [F]

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx = -\int \frac{x^2}{x^2\,(1-x^3)^{\frac{2}{3}}-x\,(1-x^3)^{\frac{2}{3}}+(1-x^3)^{\frac{2}{3}}}\,dx$$
$$-\int\left(-\frac{1}{x^2\,(1-x^3)^{\frac{2}{3}}-x\,(1-x^3)^{\frac{2}{3}}+(1-x^3)^{\frac{2}{3}}}\right)dx$$

```
[In] integrate((-x**2+1)/(x**2-x+1)/(-x**3+1)**(2/3),x)
```

```
[Out] -Integral(x**2/(x**2*(1 - x**3)**(2/3) - x*(1 - x**3)**(2/3) + (1 - x**3)**
(2/3)), x) - Integral(-1/(x**2*(1 - x**3)**(2/3) - x*(1 - x**3)**(2/3) + (1
 - x**3)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx=\int -\frac{x^2-1}{(-x^3+1)^{\frac{2}{3}}(x^2-x+1)}\,dx$$

```
[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)
```

Giac [F]

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx=\int -\frac{x^2-1}{(-x^3+1)^{\frac{2}{3}}(x^2-x+1)}\,dx$$

```
[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="giac")
[Out] integrate(-(x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx=-\int \frac{x^2-1}{(1-x^3)^{2/3}\,(x^2-x+1)}\,dx$$

```
[In] int(-(x^2 - 1)/((1 - x^3)^(2/3)*(x^2 - x + 1)),x)
[Out] -int((x^2 - 1)/((1 - x^3)^(2/3)*(x^2 - x + 1)), x)
```

3.1018 $\int \frac{x^2}{\sqrt{-1+x^4}(1+x^4)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx = -\frac{1}{4}\arctan\left(\frac{1+x^2}{x\sqrt{-1+x^4}}\right) - \frac{1}{4}\operatorname{arctanh}\left(\frac{1-x^2}{x\sqrt{-1+x^4}}\right)$$

```
[Out] -1/4*arctan((x^2+1)/x/(x^4-1)^(1/2))-1/4*arctanh((-x^2+1)/x/(x^4-1)^(1/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {504, 1225, 228, 1713, 212, 209}

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx = \left(\frac{1}{8}+\frac{i}{8}\right)\operatorname{arctanh}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right) - \left(\frac{1}{8}+\frac{i}{8}\right)\arctan\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)$$

```
[In] Int[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
```

```
[Out] (-1/8 - I/8)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] + (1/8 + I/8)*ArcTanh[((1 +
 I)*x)/Sqrt[-1 + x^4]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 228

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1225

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1713

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \frac{1}{(i-x^2)\sqrt{-1+x^4}}\,dx\right) + \frac{1}{2}\int \frac{1}{(i+x^2)\sqrt{-1+x^4}}\,dx \\
&= -\left(\frac{1}{4}i\int \frac{i-x^2}{(i+x^2)\sqrt{-1+x^4}}\,dx\right) + \frac{1}{4}i\int \frac{i+x^2}{(i-x^2)\sqrt{-1+x^4}}\,dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{i-2x^2}\,dx, x, \frac{x}{\sqrt{-1+x^4}}\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{i+2x^2}\,dx, x, \frac{x}{\sqrt{-1+x^4}}\right) \\
&= \left(-\frac{1}{8}-\frac{i}{8}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right) + \left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx = \left(-\frac{1}{8}-\frac{i}{8}\right)\arctan\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right) + \left(\frac{1}{8}-\frac{i}{8}\right)\arctan\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-1+x^4}}{x}\right)$$

```
[In] Integrate[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
[Out] (-1/8 - I/8)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] + (1/8 - I/8)*ArcTan[((1/2
 + I/2)*Sqrt[-1 + x^4])/x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\left(\frac{1}{8}-\frac{i}{8}\right)\left(\ln(2)+\ln\left(\frac{(1-i)\sqrt{x^4-1}-2ix}{x^2+i}\right)+\arctan\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^4-1}}{x}\right)\right)$
default	$\frac{\ln\left(\frac{1+\frac{x^4-1}{2x^2}+\frac{\sqrt{x^4-1}}{x}}{1+\frac{x^4-1}{2x^2}-\frac{\sqrt{x^4-1}}{x}}\right)}{16}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}+1\right)}{8}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}-1\right)}{8}$
elliptic	$\frac{\ln\left(\frac{1+\frac{x^4-1}{2x^2}+\frac{\sqrt{x^4-1}}{x}}{1+\frac{x^4-1}{2x^2}-\frac{\sqrt{x^4-1}}{x}}\right)}{16}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}+1\right)}{8}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}-1\right)}{8}$
trager	$\operatorname{RootOf}\left(32\_Z^2-8\_Z+1\right)\ln\left(-\frac{8\operatorname{RootOf}\left(32\_Z^2-8\_Z+1\right)x+\sqrt{x^4-1}}{8x^2\operatorname{RootOf}\left(32\_Z^2-8\_Z+1\right)-x^2+1}\right)+\frac{\ln\left(\frac{8\operatorname{RootOf}\left(32\_Z^2-8\_Z\ldots\right.}{8x^2\operatorname{RootOf}\left(32\_Z^2-8\ldots\right.}\right.}{4}$

```
[In] int(x^2/(x^4+1)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] (1/8-1/8*I)*(ln(2)+ln(((1-I)*(x^4-1)^(1/2)-2*I*x)/(x^2+I))+arctan((1/2+1/2*
I)*(x^4-1)^(1/2)/x))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx=\frac{1}{4}\arctan\left(\frac{\sqrt{x^4-1}x}{x^2+1}\right)+\frac{1}{8}\log\left(\frac{x^4+2\,x^2+2\,\sqrt{x^4-1}x-1}{x^4+1}\right)$$

```
[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="fricas")
[Out] 1/4*arctan(sqrt(x^4 - 1)*x/(x^2 + 1)) + 1/8*log((x^4 + 2*x^2 + 2*sqrt(x^4 -
 1)*x - 1)/(x^4 + 1))
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx=\int \frac{x^2}{\sqrt{(x-1)\,(x+1)\,(x^2+1)}\,(x^4+1)}\,dx$$

```
[In] integrate(x**2/(x**4+1)/(x**4-1)**(1/2),x)
[Out] Integral(x**2/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx=\int \frac{x^2}{(x^4+1)\sqrt{x^4-1}}\,dx$$

```
[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx=\int \frac{x^2}{(x^4+1)\sqrt{x^4-1}}\,dx$$

```
[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx = \int \frac{x^2}{\sqrt{x^4-1}\,(x^4+1)}\,dx$$

```
[In] int(x^2/((x^4 - 1)^(1/2)*(x^4 + 1)),x)
[Out] int(x^2/((x^4 - 1)^(1/2)*(x^4 + 1)), x)
```

3.1019 $\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 80

$$\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}}$$

```
[Out] arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d
^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2137, 211}

$$\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\frac{\arctan\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

```
[In] Int[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*
x^4])]/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2137

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\text{integral} = a\text{Subst}\left(\int \frac{1}{ade - (abde - a(cd^2 + ae^2))x^2}\,dx, x, \frac{x}{\sqrt{a + bx^2 + cx^4}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{cd^2 - bde + ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a + bx^2 + cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{cd^2 - bde + ae^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 10.57 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.79

$$\int \frac{a - cx^4}{(ae + cdx^2)(d + ex^2)\sqrt{a + bx^2 + cx^4}}\,dx$$
$$= \frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1 + \frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right), \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right) - \text{EllipticPi}\left(\frac{(b+\sqrt{}}{}\right.\right.}{\sqrt{2}\sqrt{\frac{}{b+\sqrt{b}}}}$$

```
[In] Integrate[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])
,x]
```

```
[Out] (I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
- EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(2*a*e), I*ArcSinh[Sqrt[2]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt
[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a
*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4
])
```

Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\arctan\left(\frac{de\sqrt{c\,x^4+b\,x^2+a}}{x\sqrt{\left(a\,e^2-bde+c\,d^2\right)de}}\right)}{\sqrt{(a\,e^2-bde+c\,d^2)de}}$	66
elliptic	$-\frac{\arctan\left(\frac{de\sqrt{c\,x^4+b\,x^2+a}}{x\sqrt{\left(a\,e^2-bde+c\,d^2\right)de}}\right)}{\sqrt{(a\,e^2-bde+c\,d^2)de}}$	66
pseudoelliptic	$-\frac{\arctan\left(\frac{de\sqrt{c\,x^4+b\,x^2+a}}{x\sqrt{\left(a\,e^2-bde+c\,d^2\right)de}}\right)}{\sqrt{(a\,e^2-bde+c\,d^2)de}}$	66

```
[In] int((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/((a*e^2-b*d*e+c*d^2)*d*e)^(1/2)*arctan(d*e*(c*x^4+b*x^2+a)^(1/2)/x/((a*e
^2-b*d*e+c*d^2)*d*e)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 28.97 (sec) , antiderivative size = 472, normalized size of antiderivative = 5.90

$$\int \frac{a-cx^4}{(ae+cdx^2)\,(d+ex^2)\,\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \left[-\frac{\sqrt{-cd^3e+bd^2e^2-ade^3}\log\left(-\frac{c^2d^2e^2x^8-2\,(3\,c^2d^3e-4\,bcd^2e^2+3\,acde^3)x^6+a^2d^2e^2+(c^2d^4-8\,bcd^3e-8\,abde^3+a^2e^4+4\,(2\,b^2+ac}{c^2d^2e^2x^8+2\,(c^2d^3e+acde^3)x^6+}\right.}{4\,(cd^3e-bd^2e^2+}\right.$$

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*sqrt(-c*d^3*e + b*d^2*e^2 - a*d*e^3)*log(-(c^2*d^2*e^2*x^8 - 2*(3*c^2
*d^3*e - 4*b*c*d^2*e^2 + 3*a*c*d*e^3)*x^6 + a^2*d^2*e^2 + (c^2*d^4 - 8*b*c*
d^3*e - 8*a*b*d*e^3 + a^2*e^4 + 4*(2*b^2 + a*c)*d^2*e^2)*x^4 - 2*(3*a*c*d^3
*e - 4*a*b*d^2*e^2 + 3*a^2*d*e^3)*x^2 + 4*(c*d*e*x^5 + a*d*e*x - (c*d^2 - 2
*b*d*e + a*e^2)*x^3)*sqrt(-c*d^3*e + b*d^2*e^2 - a*d*e^3)*sqrt(c*x^4 + b*x^
2 + a))/(c^2*d^2*e^2*x^8 + 2*(c^2*d^3*e + a*c*d*e^3)*x^6 + a^2*d^2*e^2 + (c
^2*d^4 + 4*a*c*d^2*e^2 + a^2*e^4)*x^4 + 2*(a*c*d^3*e + a^2*d*e^3)*x^2))/(c*
d^3*e - b*d^2*e^2 + a*d*e^3), 1/2*arctan(2*sqrt(c*d^3*e - b*d^2*e^2 + a*d*e
^3)*sqrt(c*x^4 + b*x^2 + a)*x/(c*d*e*x^4 + a*d*e - (c*d^2 - 2*b*d*e + a*e^2
)*x^2))/sqrt(c*d^3*e - b*d^2*e^2 + a*d*e^3)]
```

Sympy [F]

$$\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\int \left(-\frac{a}{ade\sqrt{a+bx^2+cx^4}+ae^2x^2\sqrt{a+bx^2+cx^4}+cd^2x^2\sqrt{a+bx^2+cx^4}+cdex^4\sqrt{a+bx^2+cx^4}}\right)dx - \int \frac{cx^4}{ade\sqrt{a+bx^2+cx^4}+ae^2x^2\sqrt{a+bx^2+cx^4}+cd^2x^2\sqrt{a+bx^2+cx^4}+cdex^4\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((-c*x**4+a)/(c*d*x**2+a*e)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*e*sqrt(a + b*x**2 + c*x**4) + a*e**2*x**2*sqrt(a + b*x**2
 + c*x**4) + c*d**2*x**2*sqrt(a + b*x**2 + c*x**4) + c*d*e*x**4*sqrt(a + b*
x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*e*sqrt(a + b*x**2 + c*x**4) + a*
e**2*x**2*sqrt(a + b*x**2 + c*x**4) + c*d**2*x**2*sqrt(a + b*x**2 + c*x**4)
 + c*d*e*x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int -\frac{cx^4-a}{\sqrt{cx^4+bx^2+a}(cdx^2+ae)(ex^2+d)}\,dx$$

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(e*x^2 + d)
), x)
```

Giac [F]

$$\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int -\frac{cx^4-a}{\sqrt{cx^4+bx^2+a}(cdx^2+ae)(ex^2+d)}\,dx$$

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(e*x^2 + d)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{a-c\,x^4}{(e\,x^2+d)\,\left(c\,d\,x^2+a\,e\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((a - c*x^4)/((d + e*x^2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((a - c*x^4)/((d + e*x^2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.1020 $\int \left(x+\frac{1-x^2}{1+x}\right) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 1

$$\int \left(x+\frac{1-x^2}{1+x}\right) \, dx=x$$

```
[Out] x
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(x+\frac{1-x^2}{1+x}\right) \, dx=x$$

```
[In] Int[x + (1 - x^2)/(1 + x),x]
```

```
[Out] x
```

Rubi steps

$$\text{integral}=x$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(x+\frac{1-x^2}{1+x}\right) dx = x$$

```
[In] Integrate[x + (1 - x^2)/(1 + x),x]
[Out] x
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
default	x	2
norman	x	2
risch	x	2

```
[In] int(x+(-x^2+1)/(x+1),x,method=_RETURNVERBOSE)
[Out] x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(x+\frac{1-x^2}{1+x}\right) dx = x$$

```
[In] integrate(x+(-x^2+1)/(1+x),x, algorithm="fricas")
[Out] x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 0, normalized size of antiderivative = 0.00

$$\int \left(x+\frac{1-x^2}{1+x}\right) dx = x$$

```
[In] integrate(x+(-x**2+1)/(1+x),x)
[Out] x
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(x+\frac{1-x^2}{1+x}\right) dx = x$$

```
[In] integrate(x+(-x^2+1)/(1+x),x, algorithm="maxima")
[Out] x
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(x+\frac{1-x^2}{1+x}\right) dx = x$$

```
[In] integrate(x+(-x^2+1)/(1+x),x, algorithm="giac")
[Out] x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(x+\frac{1-x^2}{1+x}\right) dx = x$$

```
[In] int(x - (x^2 - 1)/(x + 1),x)
[Out] x
```

3.1021 $\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx = \arcsin(x) - \frac{\arctan\left(\frac{1+4x\sqrt{1-x^2}}{\sqrt{3}(1-2x^2)}\right)}{\sqrt{3}}$$

```
[Out] arcsin(x)-1/3*arctan(1/3*(1+4*x*(-x^2+1)^(1/2))*3^(1/2)/(-2*x^2+1))*3^(1/2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.90, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6874, 1121, 632, 210, 1307, 222, 1188, 385, 211}

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx = \arcsin(x) - \frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}$$

```
[In] Int[(x^(-1) + Sqrt[1 - x^2])^(-1),x]
```

```
[Out] ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sqr
t[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/Sqrt[3] - ArcTan[(Sqrt[-((I - Sqrt[3]
)/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1307

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[e*(f^2/c), Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[a
*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d,
```

```
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{x}{1-x^2+x^4} - \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\right) dx$$

$$= \int \frac{x}{1-x^2+x^4}\,dx - \int \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^2\right) + \int \frac{1}{\sqrt{1-x^2}}\,dx - \int \frac{1}{\sqrt{1-x^2}\,(1-x^2+x^4)}\,dx$$

$$= \sin^{-1}(x) + \frac{(2i)\int \frac{1}{\sqrt{1-x^2}\left(-1-i\sqrt{3}+2x^2\right)}\,dx}{\sqrt{3}} - \frac{(2i)\int \frac{1}{\sqrt{1-x^2}\left(-1+i\sqrt{3}+2x^2\right)}\,dx}{\sqrt{3}} - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^2\right)$$

$$= \sin^{-1}(x) + \frac{\tan^{-1}\left(\frac{-1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{(2i)\text{Subst}\left(\int \frac{1}{-1+i\sqrt{3}-\left(-1-i\sqrt{3}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{3}} + \frac{(2i)\text{Subst}\left(\int \frac{1}{-1-i\sqrt{3}-\left(-1+i\sqrt{3}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}$$

$$= \sin^{-1}(x) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}x}{\sqrt{1-x^2}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{-1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.07

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right) + \text{RootSum}\Big[1+2\#1+2\#1^2-2\#1^3 +\#1^4\&, \frac{-\log(-1+x)+\log\left(\sqrt{1-x^2}+\#1-x\#1\right)-\log(-1+x)\#1^2+\log\left(\sqrt{1-x^2}+\#1-x\#1\right)\#}{1+2\#1-3\#1^2+2\#1^3}$$

```
[In] Integrate[(x^(-1) + Sqrt[1 - x^2])^(-1),x]
```

```
[Out] -2*ArcTan[Sqrt[1 - x^2]/(1 + x)] + RootSum[1 + 2*#1 + 2*#1^2 - 2*#1^3 + #1^
4 & , (-Log[-1 + x] + Log[Sqrt[1 - x^2] + #1 - x*#1] - Log[-1 + x]*#1^2 + L
og[Sqrt[1 - x^2] + #1 - x*#1]*#1^2)/(1 + 2*#1 - 3*#1^2 + 2*#1^3) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.12

method	result
trager	$\text{RootOf}\left(\_Z^2+1\right)\ln\left(\text{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)+\frac{\text{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{2\,\text{RootOf}\left(\_Z^2+3\right)x^2+3x\sqrt{}}{\text{RootOf}\left(\_Z^2\right.}\right.}{3}$
default	$\frac{i\sqrt{3}\,\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{\left(-1+i\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)}{6}-\frac{i\sqrt{3}\,\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{\left(-1-i\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)}{6}+\frac{i\sqrt{3}\,\ln\left(\frac{\left(\sqrt{-x^2\right.}{x}\right.}{}$

```
[In] int(1/(1/x+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] RootOf(_Z^2+1)*ln(RootOf(_Z^2+1)*(-x^2+1)^(1/2)+x)+1/3*RootOf(_Z^2+3)*ln((2
*RootOf(_Z^2+3)*x^2+3*x*(-x^2+1)^(1/2)-RootOf(_Z^2+3))/(RootOf(_Z^2+3)*x^2-
x^2+2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right) + \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,x^2-1)\sqrt{-x^2+1}}{3\,(x^3-x)}\right) - 2\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x^2 - 1)*sqrt(-x^2 + 1)/(x^3 - x)) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [F]

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx = \int \frac{x}{x\sqrt{1-x^2}+1}\,dx$$

```
[In] integrate(1/(1/x+(-x**2+1)**(1/2)),x)
[Out] Integral(x/(x*sqrt(1 - x**2) + 1), x)
```

Maxima [F]

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}+\frac{1}{x}}\,dx$$

```
[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 + 1) + 1/x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(37) = 74$.

Time = 0.37 (sec) , antiderivative size = 193, normalized size of antiderivative = 4.60

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx$$
$$= \frac{1}{2}\,\pi\mathrm{sgn}(x) - \frac{1}{6}\,\sqrt{3}\left(\pi\mathrm{sgn}(x) + 2\,\arctan\left(-\frac{\sqrt{3}x\left(\frac{\sqrt{-x^2+1}-1}{x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$
$$- \frac{1}{6}\,\sqrt{3}\left(\pi\mathrm{sgn}(x) + 2\,\arctan\left(\frac{\sqrt{3}x\left(\frac{\sqrt{-x^2+1}-1}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$
$$+ \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right) + \arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

```
[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x^
2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - 1/
6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x - (sq
rt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sq
rt(-x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 20.26 (sec) , antiderivative size = 549, normalized size of antiderivative = 13.07

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx = \operatorname{asin}(x) - \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{\frac{\sqrt{3}}{2}-x+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} + \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} - \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)+1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} - \frac{\ln\left(x-\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}} - \frac{\ln\left(x+\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}} + \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)+1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} + \frac{\ln\left(x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}} + \frac{\ln\left(x+\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}}$$

```
[In] int(1/(1/x + (1 - x^2)^(1/2)),x)
```

```
[Out] asin(x) - log((((x*(3^(1/2)/2 + 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(
1/2) - (1 - x^2)^(1/2)*1i)/(3^(1/2)/2 - x + 1i/2))/((1 - (3^(1/2)/2 + 1i/2)
^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + log((((x*(3^(1/2)/2 -
```

1i/2) - 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - 3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - log((((x*(3^(1/2)/2 - 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 - 1i/2))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - (log(x - 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i) - (log(x + 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i) + log((((x*(3^(1/2)/2 + 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2 + 1i/2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + (log(x - 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i) + (log(x + 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)

3.1022 $\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 42

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx=\arcsin(x)-\frac{\arctan\left(\frac{1+4x\sqrt{1-x^2}}{\sqrt{3}(1-2x^2)}\right)}{\sqrt{3}}$$

```
[Out] arcsin(x)-1/3*arctan(1/3*(1+4*x*(-x^2+1)^(1/2))*3^(1/2)/(-2*x^2+1))*3^(1/2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.55, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {6874, 1307, 222, 1188, 385, 211, 1265, 787, 632, 210}

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx=\arcsin(x)-\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{x^2}{2}+\frac{1}{4}(1-x)^2+\frac{1}{4}(x+1)^2$$

```
[In] Int[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]
```

```
[Out] (1 - x)^2/4 - x^2/2 + (1 + x)^2/4 + ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]
/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/S
```

```
qrt[3] - ArcTan[(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqr
t[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1307

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[e*(f^2/c), Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[a
*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}(-1+x)+\frac{1+x}{2}-\frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}+\frac{x^3(1-x^2)}{1-x^2+x^4}\right)\,dx\\
&= \frac{1}{4}(1-x)^2+\frac{1}{4}(1+x)^2-\int \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\,dx+\int \frac{x^3(1-x^2)}{1-x^2+x^4}\,dx\\
&= \frac{1}{4}(1-x)^2+\frac{1}{4}(1+x)^2+\frac{1}{2}\text{Subst}\left(\int \frac{(1-x)x}{1-x+x^2}\,dx,x,x^2\right)\\
&\quad+\int \frac{1}{\sqrt{1-x^2}}\,dx-\int \frac{1}{\sqrt{1-x^2}\,(1-x^2+x^4)}\,dx\\
&= \frac{1}{4}(1-x)^2-\frac{x^2}{2}+\frac{1}{4}(1+x)^2+\sin^{-1}(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)\\
&\quad+\frac{(2i)\int \frac{1}{\sqrt{1-x^2}\left(-1-i\sqrt{3}+2x^2\right)}\,dx}{\sqrt{3}}-\frac{(2i)\int \frac{1}{\sqrt{1-x^2}\left(-1+i\sqrt{3}+2x^2\right)}\,dx}{\sqrt{3}}\\
&= \frac{1}{4}(1-x)^2-\frac{x^2}{2}+\frac{1}{4}(1+x)^2+\sin^{-1}(x)-\frac{(2i)\text{Subst}\left(\int \frac{1}{-1+i\sqrt{3}-\left(-1-i\sqrt{3}\right)x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}\\
&\quad+\frac{(2i)\text{Subst}\left(\int \frac{1}{-1-i\sqrt{3}-\left(-1+i\sqrt{3}\right)x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)
\end{aligned}
$$

$$= \frac{1}{4}(1-x)^2 - \frac{x^2}{2} + \frac{1}{4}(1+x)^2 + \sin^{-1}(x) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}x}{\sqrt{1-x^2}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{-1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.07

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right) + \text{RootSum}\Big[1+2\#1+2\#1^2-2\#1^3 +\#1^4\&, \frac{-\log(-1+x)+\log\left(\sqrt{1-x^2}+\#1-x\#1\right)-\log(-1+x)\#1^2+\log\left(\sqrt{1-x^2}+\#1-x\#1\right)\#}{1+2\#1-3\#1^2+2\#1^3}$$

```
[In] Integrate[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]
[Out] -2*ArcTan[Sqrt[1 - x^2]/(1 + x)] + RootSum[1 + 2*#1 + 2*#1^2 - 2*#1^3 + #1^
4 & , (-Log[-1 + x] + Log[Sqrt[1 - x^2] + #1 - x*#1] - Log[-1 + x]*#1^2 + L
og[Sqrt[1 - x^2] + #1 - x*#1]*#1^2)/(1 + 2*#1 - 3*#1^2 + 2*#1^3) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.12

method	result
trager	$\text{RootOf}\left(\_Z^2+1\right)\ln\left(\text{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)+\frac{\text{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{2\,\text{RootOf}\left(\_Z^2+3\right)x^2+3x\sqrt{}}{\text{RootOf}\left(\_Z^2\right.}\right.}{3}$
default	$\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{3}+\left(\frac{1}{4}+\frac{i\sqrt{3}}{12}\right)\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{\left(-1+i\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)+\left(-\frac{i\sqrt{3}}{12}+\frac{1}{4}\right)\ln\left(\right.$

```
[In] int(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] RootOf(_Z^2+1)*ln(RootOf(_Z^2+1)*(-x^2+1)^(1/2)+x)+1/3*RootOf(_Z^2+3)*ln((2
*RootOf(_Z^2+3)*x^2+3*x*(-x^2+1)^(1/2)-RootOf(_Z^2+3))/(RootOf(_Z^2+3)*x^2-
x^2+2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right) + \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,x^2-1)\sqrt{-x^2+1}}{3\,(x^3-x)}\right) - 2\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x^2 - 1)*sqrt(-x^2 + 1)/(x^3 - x)) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [F]

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx = -\int \frac{x\sqrt{1-x^2}}{x^3-x-\sqrt{1-x^2}}\,dx$$

```
[In] integrate(x*(-x**2+1)**(1/2)/(x-x**3+(-x**2+1)**(1/2)),x)
```

```
[Out] -Integral(x*sqrt(1 - x**2)/(x**3 - x - sqrt(1 - x**2)), x)
```

Maxima [F]

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx = \int -\frac{\sqrt{-x^2+1}x}{x^3-x-\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + integrate(-(x^4 - x^2)/(x^3 - x - sqrt(x + 1)*sqrt(-x + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(37) = 74$.

Time = 0.35 (sec) , antiderivative size = 193, normalized size of antiderivative = 4.60

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx$$
$$= \frac{1}{2}\,\pi \mathrm{sgn}(x) - \frac{1}{6}\,\sqrt{3}\left(\pi \mathrm{sgn}(x) + 2\,\arctan\left(-\frac{\sqrt{3}x\left(\frac{\sqrt{-x^2+1}-1}{x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$
$$- \frac{1}{6}\,\sqrt{3}\left(\pi \mathrm{sgn}(x) + 2\,\arctan\left(\frac{\sqrt{3}x\left(\frac{\sqrt{-x^2+1}-1}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$
$$+ \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right) + \arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x^
2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - 1/
6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x - (sq
rt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sq
rt(-x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 549, normalized size of antiderivative = 13.07

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx = \operatorname{asin}(x) - \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{\frac{\sqrt{3}}{2}-x+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} + \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} - \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)+1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} - \frac{\ln\left(x-\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}} - \frac{\ln\left(x+\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}} + \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)+1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} + \frac{\ln\left(x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}} + \frac{\ln\left(x+\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}}$$

```
[In] int((x*(1 - x^2)^(1/2))/(x - x^3 + (1 - x^2)^(1/2)),x)
```

```
[Out] asin(x) - log((((x*(3^(1/2)/2 + 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(
1/2) - (1 - x^2)^(1/2)*1i)/(3^(1/2)/2 - x + 1i/2))/((1 - (3^(1/2)/2 + 1i/2)
^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + log((((x*(3^(1/2)/2 -
```

1i/2) - 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - 3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - log((((x*(3^(1/2)/2 - 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 - 1i/2))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - (log(x - 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i) - (log(x + 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i) + log((((x*(3^(1/2)/2 + 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2 + 1i/2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + (log(x - 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i) + (log(x + 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)

3.1023 $\int (1+x+x^2+x^3)^{-n} (1-x^4)^n \, dx$

Optimal result

Integrand size = 23, antiderivative size = 34

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx = -\frac{(1-x) \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n}{1+n}$$

```
[Out] -(1-x)*(-x^4+1)^n/(1+n)/((x^3+x^2+x+1)^n)
```

Rubi [F]

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx = \int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx$$

```
[In] Int[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]
[Out] Defer[Int][(1 - x^4)^n/(1 + x + x^2 + x^3)^n, x]
```

Rubi steps

$$\text{integral} = \int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx = \frac{(-1+x) \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n}{1+n}$$

```
[In] Integrate[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]
[Out] ((-1 + x)*(1 - x^4)^n)/((1 + n)*(1 + x + x^2 + x^3)^n)
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{(x-1)\left(-x^{4}+1\right)^{n}\left(x^{3}+x^{2}+x+1\right)^{-n}}{1+n}$
parallelrisch	$-\frac{\left(-x\left(-x^{4}+1\right)^{n} n+\left(-x^{4}+1\right)^{n} n\right)\left(x^{3}+x^{2}+x+1\right)^{-n}}{n(1+n)}$
norman	$\left(\frac{x\,\mathrm{e}^{n \ln\left(-x^{4}+1\right)}}{1+n}-\frac{\mathrm{e}^{n \ln\left(-x^{4}+1\right)}}{1+n}\right) \mathrm{e}^{-n \ln\left(x^{3}+x^{2}+x+1\right)}$
risch	$\frac{(x-1)(x-1)^{n}\mathrm{e}^{-\frac{in\pi\left(-\mathrm{csgn}\left(i(x-1)\left(x^{3}+x^{2}+x+1\right)\right)^{3}-\mathrm{csgn}\left(i(x-1)\left(x^{3}+x^{2}+x+1\right)\right)^{2}\mathrm{csgn}(i(x-1))-\mathrm{csgn}\left(i(x-1)\left(x^{3}+x^{2}+x+1\right)\right)^{2}\mathrm{csgn}\left(i(\right.\right.}{2}}}{1+n}$

```
[In] int((-x^4+1)^n/((x^3+x^2+x+1)^n),x,method=_RETURNVERBOSE)
[Out] (x-1)/(1+n)*(-x^4+1)^n/((x^3+x^2+x+1)^n)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx = \frac{(-x^4+1)^n(x-1)}{(x^3+x^2+x+1)^n(n+1)}$$

```
[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="fricas")
[Out] (-x^4 + 1)^n*(x - 1)/((x^3 + x^2 + x + 1)^n*(n + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(24) = 48.

Time = 17.96 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.15

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx = \begin{cases} \frac{x(1-x^4)^n}{n(x^3+x^2+x+1)^n+(x^3+x^2+x+1)^n} - \frac{(1-x^4)^n}{n(x^3+x^2+x+1)^n+(x^3+x^2+x+1)^n} & \text{for } n \neq -1 \\ -\log(x-1) & \text{otherwise} \end{cases}$$

```
[In] integrate((-x**4+1)**n/((x**3+x**2+x+1)**n),x)
[Out] Piecewise((x*(1 - x**4)**n/(n*(x**3 + x**2 + x + 1)**n + (x**3 + x**2 + x +
 1)**n) - (1 - x**4)**n/(n*(x**3 + x**2 + x + 1)**n + (x**3 + x**2 + x + 1)
**n), Ne(n, -1)), (-log(x - 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.47

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx = \frac{(x-1)(-x+1)^n}{n+1}$$

```
[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="maxima")
[Out] (x - 1)*(-x + 1)^n/(n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(31) = 62.

Time = 0.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.38

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx = \frac{\frac{xe^{\left(n \log\left(x^3+x^2+x+1\right)+n \log(-x+1)\right)}}{\left(x^3+x^2+x+1\right)^n} - \frac{e^{\left(n \log\left(x^3+x^2+x+1\right)+n \log(-x+1)\right)}}{\left(x^3+x^2+x+1\right)^n}}{n+1}$$

```
[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="giac")
[Out] (x*e^(n*log(x^3 + x^2 + x + 1) + n*log(-x + 1))/(x^3 + x^2 + x + 1)^n - e^(
n*log(x^3 + x^2 + x + 1) + n*log(-x + 1))/(x^3 + x^2 + x + 1)^n)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 19.81 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx = \frac{\left(1-x^4\right)^n (x-1)}{(n+1) \left(x^3+x^2+x+1\right)^n}$$

```
[In] int((1 - x^4)^n/(x + x^2 + x^3 + 1)^n,x)
[Out] ((1 - x^4)^n*(x - 1))/((n + 1)*(x + x^2 + x^3 + 1)^n)
```

3.1024 $\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 177

$$\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx \\ = \frac{\log\left(207380736000000000b^8c^4+5970056970240000000b^6c^6x^2+25831007059968000000b^5c^7x^3+95105071448\right.}{}$$

```
[Out] 1/18432*ln(207380736000000000*b^8*c^4+5970056970240000000*b^6*c^6*x^2+2583100
705996800000*b^5*c^7*x^3+95105071448064000*b^4*c^8*x^4+2164168736951500800
0*b^3*c^9*x^5+32462531054272512000*b^2*c^10*x^6+149587343098087735296*c^12*
x^8+5308416*(12230590464*c^10*x^6+1990656000*b^2*c^8*x^4+1105920000*b^3*c^7
*x^3+38880000*b^4*c^6*x^2+79200000*b^5*c^5*x+12203125*b^6*c^4)*(5308416*c^4
*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(1/2))/c^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2107}

$$\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx \\ = \frac{\log\left(207380736000000000b^8c^4+5970056970240000000b^6c^6x^2+25831007059968000000b^5c^7x^3+95105071448\right.}{}$$

```
[In] Int[x/Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416*c^4*x
^4],x]
```

```
[Out] Log[207380736000000000*b^8*c^4 + 5970056970240000000*b^6*c^6*x^2 + 2583100705
996800000*b^5*c^7*x^3 + 95105071448064000*b^4*c^8*x^4 + 216416873695150080
```

```
00*b^3*c^9*x^5 + 32462531054272512000*b^2*c^10*x^6 + 149587343098087735296*
c^12*x^8 + 5308416*Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 +
5308416*c^4*x^4]*(12203125*b^6*c^4 + 79200000*b^5*c^5*x + 38880000*b^4*c^6*
x^2 + 1105920000*b^3*c^7*x^3 + 1990656000*b^2*c^8*x^4 + 12230590464*c^10*x^
6)]/(18432*c^2)
```

Rule 2107

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (e_.)*(x_)^4], x_Symbol] :
> With[{Px = (1/320)*(33*b^2*c + 6*a*c^2 + 40*a^2*e) - (22/5)*a*c*e*x^2 + (
22/15)*b*c*e*x^3 + (1/4)*e*(5*c^2 + 4*a*e)*x^4 + (4/3)*b*e^2*x^5 + 2*c*e^2*
x^6 + e^3*x^8}, Simp[(1/(8*Rt[e, 2]))*Log[Px + Dist[1/(8*Rt[e, 2]*x), D[Px,
 x], x]*Sqrt[a + b*x + c*x^2 + e*x^4]], x]] /; FreeQ[{a, b, c, e}, x] && Eq
Q[71*c^2 + 100*a*e, 0] && EqQ[1152*c^3 - 125*b^2*e, 0]
```

Rubi steps

integral

$$= \frac{\log \left(207380736000000000 b^8 c^4 + 597005697024000000 b^6 c^6 x^2 + 2583100705996800000 b^5 c^7 x^3 + 9510507144\right.}{}$$

Mathematica [A] (verified)

Time = 5.60 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03

$$\int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}}\,dx =$$
$$-\frac{\log \left(-20386355871744000000000000000 b^{13} c^7 - 58688048040247296000000000000 b^{11} c^9 x^2 - 25392913\right.}{}$$

```
[In] Integrate[x/Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416
*c^4*x^4],x]
```

```
[Out] -1/18432*Log[-20386355871744000000000000000*b^13*c^7 - 586880480402472960000
000000000*b^11*c^9*x^2 - 253929131802309427200000000000*b^10*c^10*x^3 - 9
349208943630483456000000000000*b^9*c^11*x^4 - 212746443517280334643200000000
00*b^8*c^12*x^5 - 319119665275920501964800000000*b^7*c^13*x^6 - 147050341
759144167305379840000000*b^5*c^15*x^8 + Sqrt[-44375*b^4 + 576000*b^3*c*x +
576000*b^2*c^2*x^2 + 5308416*c^4*x^4]*(636806076825600000000000000*b^11*c^7
+ 4132961129594880000000000000*b^10*c^8*x + 20289081908920320000000000*b^9*
c^9*x^2 + 577111663187066880000000000*b^8*c^10*x^3 + 103880099373672038400
00000000*b^7*c^11*x^4 + 638239330551841003929600000000*b^5*c^13*x^6)]/c^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 1597, normalized size of antiderivative = 9.02

method	result	size
default	Expression too large to display	1597
elliptic	Expression too large to display	1597

```
[In] int(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/1152*(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*
_Z^2+96*_Z-71,index=4)*b/c)*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/
c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z
^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5
/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+
96*_Z-71,index=2)*b/c))^(1/2)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)
*b/c)^2*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4+1
0*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)
*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c-5/48*RootOf(_Z^4+10*_
Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/
c))^(1/2)*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4
+10*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=
4)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10
*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*
b/c))^(1/2)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^
4+10*_Z^2+96*_Z-71,index=2)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2
)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(c^4*(x-5/48*RootOf(_
Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,ind
ex=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)*(x-5/48*RootO
f(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2)*(5/48*RootOf(_Z^4+10*_Z^2+96*_
Z-71,index=2)*b/c*EllipticF(((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/
c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z
^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5
/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+
96*_Z-71,index=2)*b/c))^(1/2),((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*
b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)*(5/48*RootOf(_Z^4+10*_Z
^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c)/(5
/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_
Z-71,index=3)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*Roo
tOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2))+(5/48*RootOf(_Z^4+10*_Z^2+9
6*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*Ellipt
icPi(((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*_
Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/
```

```
c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*_Z^2
+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c))
^(1/2),(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*
_Z^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c
-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c),((5/48*RootOf(_Z^4+10*_Z^2
+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)*(5/4
8*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-
71,index=4)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootO
f(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,in
dex=2)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.93

$$\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx$$
$$= \frac{\log\left(28179280429056\,c^8x^8+6115295232000\,b^2c^6x^6+4076863488000\,b^3c^5x^5+179159040000\,b^4c^4x^4+486\right.}{}$$

```
[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(
1/2),x, algorithm="fricas")
```

```
[Out] 1/18432*log(28179280429056*c^8*x^8 + 6115295232000*b^2*c^6*x^6 + 4076863488
000*b^3*c^5*x^5 + 179159040000*b^4*c^4*x^4 + 486604800000*b^5*c^3*x^3 + 112
464000000*b^6*c^2*x^2 + 3906640625*b^8 + (12230590464*c^6*x^6 + 1990656000*
b^2*c^4*x^4 + 1105920000*b^3*c^3*x^3 + 38880000*b^4*c^2*x^2 + 79200000*b^5*
c*x + 12203125*b^6)*sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*
c*x - 44375*b^4))/c^2
```

Sympy [F]

$$\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx = \int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx$$

```
[In] integrate(x/(5308416*c**4*x**4+576000*b**2*c**2*x**2+576000*b**3*c*x-44375*
b**4)**(1/2),x)
```

```
[Out] Integral(x/sqrt(-44375*b**4 + 576000*b**3*c*x + 576000*b**2*c**2*x**2 + 530
8416*c**4*x**4), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}}\,dx = \int \frac{x}{\sqrt{5308416\,c^4x^4 + 576000\,b^2c^2x^2 + 576000\,b^3cx - 44375\,b^4}}\,dx$$

```
[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 44
375*b^4), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}}\,dx = \int \frac{x}{\sqrt{5308416\,c^4x^4 + 576000\,b^2c^2x^2 + 576000\,b^3cx - 44375\,b^4}}\,dx$$

```
[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 44
375*b^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}}\,dx = \int \frac{x}{\sqrt{-44375\,b^4 + 576000\,b^3\,c\,x + 576000\,b^2\,c^2\,x^2 + 5308416\,c^4\,x^4}}\,dx$$

```
[In] int(x/(5308416*c^4*x^4 - 44375*b^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x)^(
1/2),x)
```

```
[Out] int(x/(5308416*c^4*x^4 - 44375*b^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x)^(
1/2), x)
```

3.1025 $\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}} dx$

Optimal result

Integrand size = 30, antiderivative size = 100

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx = \frac{1}{16}\log\Big(921+2864x+9280x^2+13440x^3+17024x^4 + 19456x^5+12288x^6+8192x^7+4096x^8 + \sqrt{9+120x+64x^2+64x^3+64x^4}\,(179+444x + 744x^2+1280x^3+960x^4+768x^5+512x^6)\Big)$$

```
[Out] 1/16*ln(921+2864*x+9280*x^2+13440*x^3+17024*x^4+19456*x^5+12288*x^6+8192*x^
7+4096*x^8+(512*x^6+768*x^5+960*x^4+1280*x^3+744*x^2+444*x+179)*(64*x^4+64*
x^3+64*x^2+120*x+9)^(1/2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 243 vs. $2(100) = 200$.

Time = 0.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.43, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2108, 2107}

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx = \frac{1}{16}\log\Big(4096x^8+8192x^7+12288x^6+19456x^5 + 17024x^4+13440x^3+9280x^2 + 960\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^4 + 1280\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^3 + 744\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^2 + 444\sqrt{64x^4+64x^3+64x^2+120x+9}\,x + 179\sqrt{64x^4+64x^3+64x^2+120x+9} + 512\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^6 + 768\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^5 + 2864x+921\Big)$$

```
[In] Int[(1 + 4*x)/Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4],x]
```

```
[Out] Log[921 + 2864*x + 9280*x^2 + 13440*x^3 + 17024*x^4 + 19456*x^5 + 12288*x^6
 + 8192*x^7 + 4096*x^8 + 179*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 4
44*x*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 744*x^2*Sqrt[9 + 120*x +
64*x^2 + 64*x^3 + 64*x^4] + 1280*x^3*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*
x^4] + 960*x^4*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 768*x^5*Sqrt[9
+ 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 512*x^6*Sqrt[9 + 120*x + 64*x^2 + 64*
x^3 + 64*x^4]]/16
```

Rule 2107

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (e_.)*(x_)^4], x_Symbol] :
> With[{Px = (1/320)*(33*b^2*c + 6*a*c^2 + 40*a^2*e) - (22/5)*a*c*e*x^2 + (
22/15)*b*c*e*x^3 + (1/4)*e*(5*c^2 + 4*a*e)*x^4 + (4/3)*b*e^2*x^5 + 2*c*e^2*
x^6 + e^3*x^8}, Simp[(1/(8*Rt[e, 2]))*Log[Px + Dist[1/(8*Rt[e, 2]*x), D[Px,
 x], x]*Sqrt[a + b*x + c*x^2 + e*x^4]], x]] /; FreeQ[{a, b, c, e}, x] && Eq
Q[71*c^2 + 100*a*e, 0] && EqQ[1152*c^3 - 125*b^2*e, 0]
```

Rule 2108

```
Int[((A_) + (B_.)*(x_))/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^
3 + (e_.)*(x_)^4], x_Symbol] :> Dist[B, Subst[Int[x/Sqrt[(-3*d^4 + 16*c*d^2
*e - 64*b*d*e^2 + 256*a*e^3)/(256*e^3) + (d^3 - 4*c*d*e + 8*b*e^2)*(x/(8*e^
2)) - (3*d^2 - 8*c*e)*(x^2/(8*e)) + e*x^4], x], x, d/(4*e) + x], x] /; Free
Q[{a, b, c, d, e, A, B}, x] && EqQ[B*d - 4*A*e, 0] && EqQ[d*(141*d^3 - 752*
```

```
c*d*e - 400*b*e^2) + 16*e^2*(71*c^2 + 100*a*e), 0] && EqQ[144*(3*d^2 - 8*c*
e)^3 + 125*(d^3 - 4*c*d*e + 8*b*e^2)^2, 0]
```

Rubi steps

$$\text{integral} = 4\text{Subst}\left(\int \frac{x}{\sqrt{-\frac{71}{4}+96x+40x^2+64x^4}}\,dx, x, \frac{1}{4}+x\right)$$

$$= \frac{1}{16}\log\Big(921+2864x+9280x^2+13440x^3+17024x^4+19456x^5+12288x^6 + 8192x^7+4096x^8+179\sqrt{9+120x+64x^2+64x^3+64x^4} + 444x\sqrt{9+120x+64x^2+64x^3+64x^4}+744x^2\sqrt{9+120x+64x^2+64x^3+64x^4} + 1280x^3\sqrt{9+120x+64x^2+64x^3+64x^4} + 960x^4\sqrt{9+120x+64x^2+64x^3+64x^4} + 768x^5\sqrt{9+120x+64x^2+64x^3+64x^4} + 512x^6\sqrt{9+120x+64x^2+64x^3+64x^4}\Big)$$

Mathematica [A] (verified)

Time = 3.63 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx = -\frac{1}{16}\log\Big(-921-2864x-9280x^2-13440x^3 -17024x^4-19456x^5-12288x^6-8192x^7-4096x^8 +\sqrt{9+120x+64x^2+64x^3+64x^4}\big(179+444x +744x^2+1280x^3+960x^4+768x^5+512x^6\big)\Big)$$

```
[In] Integrate[(1 + 4*x)/Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4],x]
```

```
[Out] -1/16*Log[-921 - 2864*x - 9280*x^2 - 13440*x^3 - 17024*x^4 - 19456*x^5 - 12
288*x^6 - 8192*x^7 - 4096*x^8 + Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4]*
(179 + 444*x + 744*x^2 + 1280*x^3 + 960*x^4 + 768*x^5 + 512*x^6)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(96) = 192.

Time = 2.07 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.28

method	result
trager	$-\frac{\ln\left(4096x^8-512\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^6+8192x^7-768\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^5+12288x^6-960\sqrt{64x^4+64x^3+64x^2}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

```
[In] int((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*ln(4096*x^8-512*(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2)*x^6+8192*x^7-768
*(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2)*x^5+12288*x^6-960*(64*x^4+64*x^3+64*x
^2+120*x+9)^(1/2)*x^4+19456*x^5-1280*(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2)*x
^3+17024*x^4-744*(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2)*x^2+13440*x^3-444*x*(
64*x^4+64*x^3+64*x^2+120*x+9)^(1/2)+9280*x^2-179*(64*x^4+64*x^3+64*x^2+120*
x+9)^(1/2)+2864*x+921)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx = \frac{1}{16}\log\Big(-4096\,x^8-8192\,x^7-12288\,x^6-19456\,x^5-17024\,x^4-13440\,x^3-9280\,x^2 - \left(512\,x^6+768\,x^5+960\,x^4+1280\,x^3+744\,x^2+444\,x+179\right)\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9} - 2864\,x-921\Big)$$

```
[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/16*log(-4096*x^8 - 8192*x^7 - 12288*x^6 - 19456*x^5 - 17024*x^4 - 13440*x
^3 - 9280*x^2 - (512*x^6 + 768*x^5 + 960*x^4 + 1280*x^3 + 744*x^2 + 444*x +
 179)*sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9) - 2864*x - 921)
```

Sympy [F]

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx = \int \frac{4x+1}{\sqrt{64x^4+64x^3+64x^2+120x+9}}\,dx$$

```
[In] integrate((1+4*x)/(64*x**4+64*x**3+64*x**2+120*x+9)**(1/2),x)
[Out] Integral((4*x + 1)/sqrt(64*x**4 + 64*x**3 + 64*x**2 + 120*x + 9), x)
```

Maxima [F]

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx = \int \frac{4\,x+1}{\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9}}\,dx$$

```
[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="maxima
")
[Out] integrate((4*x + 1)/sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9), x)
```

Giac [F]

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx = \int \frac{4\,x+1}{\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9}}\,dx$$

```
[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="giac")
[Out] integrate((4*x + 1)/sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx = \int \frac{4\,x+1}{\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9}}\,dx$$

```
[In] int((4*x + 1)/(120*x + 64*x^2 + 64*x^3 + 64*x^4 + 9)^(1/2),x)
[Out] int((4*x + 1)/(120*x + 64*x^2 + 64*x^3 + 64*x^4 + 9)^(1/2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_coun
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation ="none"
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```